Approximation of the inventory volume integral

Here is a quick overview of the approximation used to compute the volume integral :

The mass of tritium present at the site (in units of activity i.e. Ci) can be computed as follows :

$m_{tritium} = \int_D nCdV$

where n is the porosity and C is the activity concentration. The domain for this integral corresponds to the drainage area downgradient of the F-area seepage basins and the aquifers below. The domain is not a regular polyhedron, and we do not know the integrand at every point :

$m_{tritium} = \iiint_D n(x,y,z)C(x,y,z)dxdydz$

The first approximation is the use of and average porosity over the whole domain :

$m_{tritium} \approx \frac{1}{V_{tot}} \iiint_D n(x,y,z)dxdydz \cdot \iiint_D C(x,y,z)dxdydz$
$m_{tritium} \approx \bar{n} \int_D C(x,y,z)dxdydz$

The second approximation comes from the fact that we do not know the concentration at every x,y and z. We know for a fact that the concentration is not uniform in z, but the measurements collected provide the concentration average over the screen length $h(x,y)$, which in most cases we can replace by the aquifer thickness $h(x,y)$ in the case of fully penetrating wells i.e.:

$\overline{C_z(x,y)} = \frac{1}{h(x,y)} \int_h C(x,y,z)dz$

$m_{tritium} \approx \bar{n} \iint_{D_{xy}} \left(\int_h C(x,y,z)dz\right) dxdy$
$m_{tritium} \approx \bar{n} \iint_{D_{xy}} \overline{C_z(x,y)}h(x,y)dxdy$

There are several ways to deal with that final integral. One can look at the available samples for $\overline{C_z(x,y)}$ and $h(x,y)$, and use a bootstrap estimate to provide mean and standard errors. We approximate the intergral by a point by point computation using an interpolant on a regular grid :

$m_{tritium} \approx \bar{n} \sum_{i,j} \overline{C_z(x_i,y_j)}h(x_i,y_j)A_{ij}$
$m_{tritium} \approx \bar{n}A_{tot} \frac{1}{n_i n_j} \sum_{i,j} \overline{C_z(x_i,y_j)}h(x_i,y_j)$

As one can see this is equivalent to computing a mean of the population and multiply this by the total area. This estimate could be obtained by Monte-Carlo Integration