Контрольная работа №1 Вариант 2.

1. $\frac{(1+i)^7}{(1-i)^5} = \frac{(1+i)^{12}}{(1+1)^5} = \frac{(1+i)^{12}}{2^5} \ominus$

$|z| = \sqrt{1+1} = \sqrt{2}$ $\varphi = arctg\frac{1}{1} = arctg\,1 = \frac{\pi}{4}$.

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\ominus \frac{\sqrt{2}^{12}\left(\cos\frac{12\pi}{4} + i\sin\frac{12\pi}{4}\right)}{2^5} =$

$= \frac{2^6(\cos 3\pi + i\sin 3\pi)}{2^5} = 2(-1 + i\cdot 0) =$

$= -2$.

2. $z = -5+5i$ $|z| = \sqrt{(-5)^2+5^2} = \sqrt{50} = 5\sqrt{2}$.

$x = -5 < 0$ $y = 5 > 0$ $\varphi = arctg\frac{5}{-5} + 2\pi = arctg\,{-1} + 2\pi =$

$= -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$.

$z = 5\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$.

3. $z^6 + 64 = 0$ $z^6 = -2^6$ $\tilde{z} = -2^6$

$|\tilde{z}| = \sqrt{((-2)^6)^2 + 0} = 2^6$

$\tilde{z} = 2^6(\cos\pi + i\sin\pi)$

$\sqrt[6]{z} = \sqrt[6]{\tilde{z}} = \sqrt[6]{2^6}\left(\cos\frac{\pi+2\pi k}{6} + i\sin\frac{\pi+2\pi k}{6}\right)$

$k=0$ $z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = \sqrt{3} + i$.

$k=1$ $z = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i$.

$k=2$ $z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -\sqrt{3} + i$.

$k=3$ $z = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2} - \frac{i}{2}\right) = -\sqrt{3} - i$.

$K=4 \quad z = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = 2i.$

$K=5 \quad z = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right) = \sqrt{3} - i.$

(4) $3|z| - \operatorname{Re} z = 12$

$z = x + iy$

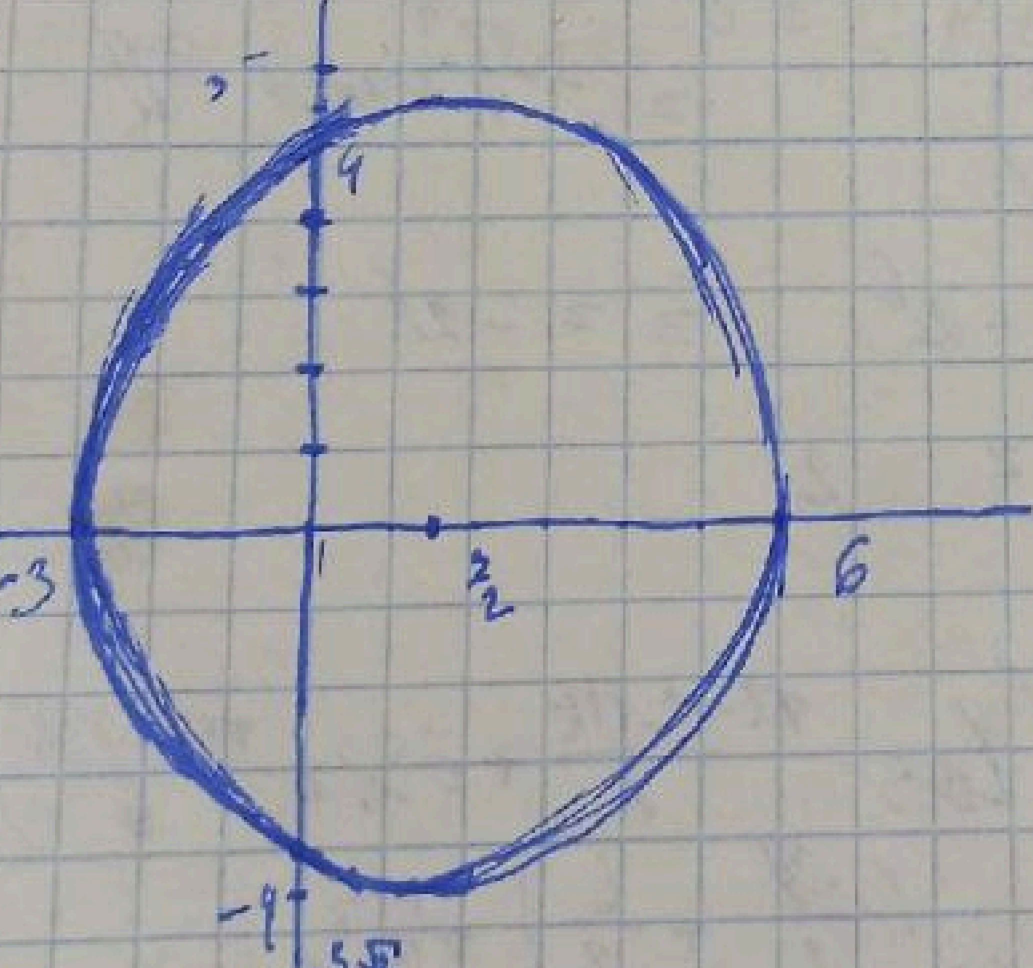
$$3\sqrt{x^2+y^2} - x = 12 \Leftrightarrow \begin{cases} 9(x^2+y^2) = (12+x)^2 \\ x \geq -12 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 9x^2+9y^2 = 144+24x+x^2 \\ x \geq -12 \end{cases} \Leftrightarrow \begin{cases} 8\left(x - \frac{3}{2}\right)^2 + 9y^2 = 162 \\ x \geq -12 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \dfrac{\left(x-\frac{3}{2}\right)^2}{\frac{9}{2}} + \dfrac{y^2}{(3\sqrt{2})^2} = 1 \\ x \geq -12 \end{cases} \Leftrightarrow \frac{\left(x-\frac{3}{2}\right)^2}{\left(\frac{9}{2}\right)^2} + \frac{y^2}{(3\sqrt{2})^2} = 1.$$

(наименьшее значение x будет $\frac{3}{2} - \frac{9}{2} = -3 \geq -2$)

это эллипс с центром в $\left(\frac{3}{2}; 0\right)$ и полуосями $\frac{9}{2}$ и $3\sqrt{2}$.



5) $v(x,y) = 1 - \frac{y}{x^2+y^2}$

$\frac{\partial v}{\partial x} = \frac{2xy}{(x^2+y^2)^2} = -\frac{\partial u}{\partial y}$

$u(x,y) = -\int \frac{2xy}{(x^2+y^2)^2}\,dy = -\int \frac{x\,d(x^2+y^2)}{(x^2+y^2)^2} =$

$= \frac{x}{(x^2+y^2)} + C(x)$

$\frac{\partial u}{\partial y} = -\frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

$\frac{\partial u}{\partial x} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} + C'(x) = \frac{y^2-x^2}{(x^2+y^2)^2} + C'(x)$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Leftrightarrow C'(x) = 0 \Leftrightarrow C(x) = C$

$f(x,y) = u(x,y) + i\,v(x,y) = \frac{x}{(x^2+y^2)} + C +$

$+ i\left(1 - \frac{y}{x^2+y^2}\right) = C + i + \frac{x-iy}{(x+iy)(x-iy)} =$

$= C + i + \frac{1}{x+iy} = C + i + \frac{1}{z}.$

$$\sin z = \frac{e^{iz}-e^{iz}}{2i} \quad \cos z = \frac{e^{iz}+e^{iz}}{2} \quad sh z = \frac{e^{z}-e^{-z}}{2} \quad ch z = \frac{e^{z}+e^{-z}}{2}$$

$$tg z = \frac{1}{i}\cdot\frac{e^{iz}-e^{-iz}}{e^{iz}+e^{-iz}} \quad ctg z = i\cdot\frac{e^{iz}+e^{-iz}}{e^{iz}-e^{-iz}}$$

$$th z = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}} \quad cth = \frac{e^{z}+e^{-z}}{e^{z}-e^{-z}} \qquad \sin iz = i\, sh z$$

$$e^{iz} = \cos z + i\sin z \quad e^{-iz} = \cos z - i\sin z. \qquad \cos iz = ch z$$

$$e^{z} = ch z + sh z \quad e^{-z} = ch z - sh z. \qquad sh iz = i\sin z$$

$$\sin^2 z + \cos^2 z = 1 \quad ch^2 z - sh^2 z = 1. \qquad ch iz = \cos z.$$

$$\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \cos z_1 \sin z_2$$

$$\cos(z_1 \pm z_2) = \cos z_1 \cos z_2 \mp \sin z_1 \sin z_2 \qquad e^{w} = z$$

$$\Downarrow$$

$$sh(z_1 \pm z_2) = sh z_1\, ch z_2 \pm ch z_1\, sh z_2 \qquad \ln z = w$$

$$ch(z_1 \pm z_2) = ch z_1\, ch z_2 \pm sh z_1\, sh z_2.$$

$$\mathrm{Arcsin}\, z = \frac{1}{i} Ln(iz + \sqrt{1-z^2}) \quad \mathrm{Arsh}\, z = Ln(z + \sqrt{1+z^2})$$

$$\mathrm{Arccos}\, z = \frac{1}{i} Ln(z + \sqrt{z^2-1}) \quad \mathrm{Arch}\, z = Ln(z + \sqrt{z^2-1})$$

$$\mathrm{Arctg}\, z = -\frac{i}{2} Ln \frac{1+iz}{1-iz} \quad \mathrm{Arth}\, z = \frac{1}{2} Ln \frac{1+z}{1-z}.$$

$$\mathrm{Arcctg}\, z = \frac{i}{2} Ln \frac{z-i}{z+i} \quad \mathrm{Arcth}\, z = \frac{1}{2} Ln \frac{z+1}{z-1}.$$