Контрольная работа №1. Вариант 1

1) $\frac{(1+i)^{10}}{(1-i)^6} = \frac{(1+i)^{16}}{(1-i^2)^6} = \frac{(1+i)^{16}}{2^6}$ ②

$|z| = \sqrt{1+1} = \sqrt{2}$ $\varphi = arctg\frac{1}{1} = arctg 1 = \frac{\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

② $\frac{\sqrt{2}^{16}\left(\cos\frac{16\pi}{4} + i\sin\frac{16\pi}{4}\right)}{2^6} = \frac{2^8(\cos 4\pi + i\cdot\sin 4\pi)}{2^6}$

$= 2^2(\cos 0 + i\sin 0) = 2^2(1 + i\cdot 0) = 2^2 = 4$

2) $z = 3+3i.$

$|z| = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}.$

$x = 3 > 0$ $y = 3 > 0$ $\varphi = arctg\frac{3}{3} + 2\pi = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}.$

$z = 3\sqrt{2}\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)$

3) $z^6 + 1 = 0$ $z^6 = -1^6$ $\tilde{z} = -1^6$

$|\tilde{z}| = \sqrt{(-1^6)^2 + 0} = \sqrt{1} = 1^6$

$\tilde{z} = 1^6(\cos\pi + i\sin\pi)$

$\sqrt[6]{z} = \sqrt[6]{\tilde{z}} = \sqrt[6]{1^6}\left(\cos\frac{\pi+2\pi k}{6} + i\sin\frac{\pi+2\pi k}{6}\right).$

$k=0$ $z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{i}{2}$

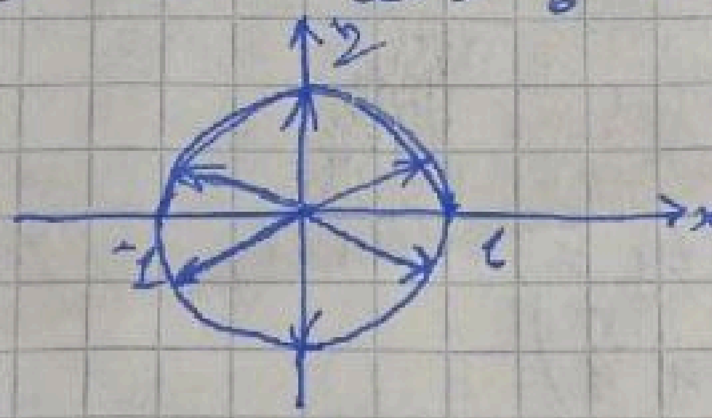
$k=1$ $z = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$k=2$ $z = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{i}{2}.$

$k=3$ $z = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{i}{2}$

$K=4 \quad z = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$

$K=5 \quad z = \cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6} = \frac{\sqrt{3}}{2} - \frac{i}{2}.$



(4) $|z| - 3\,\mathrm{Im}\,z = 6$

$z = x+iy. \quad \sqrt{x^2+y^2} - 3y = 6$

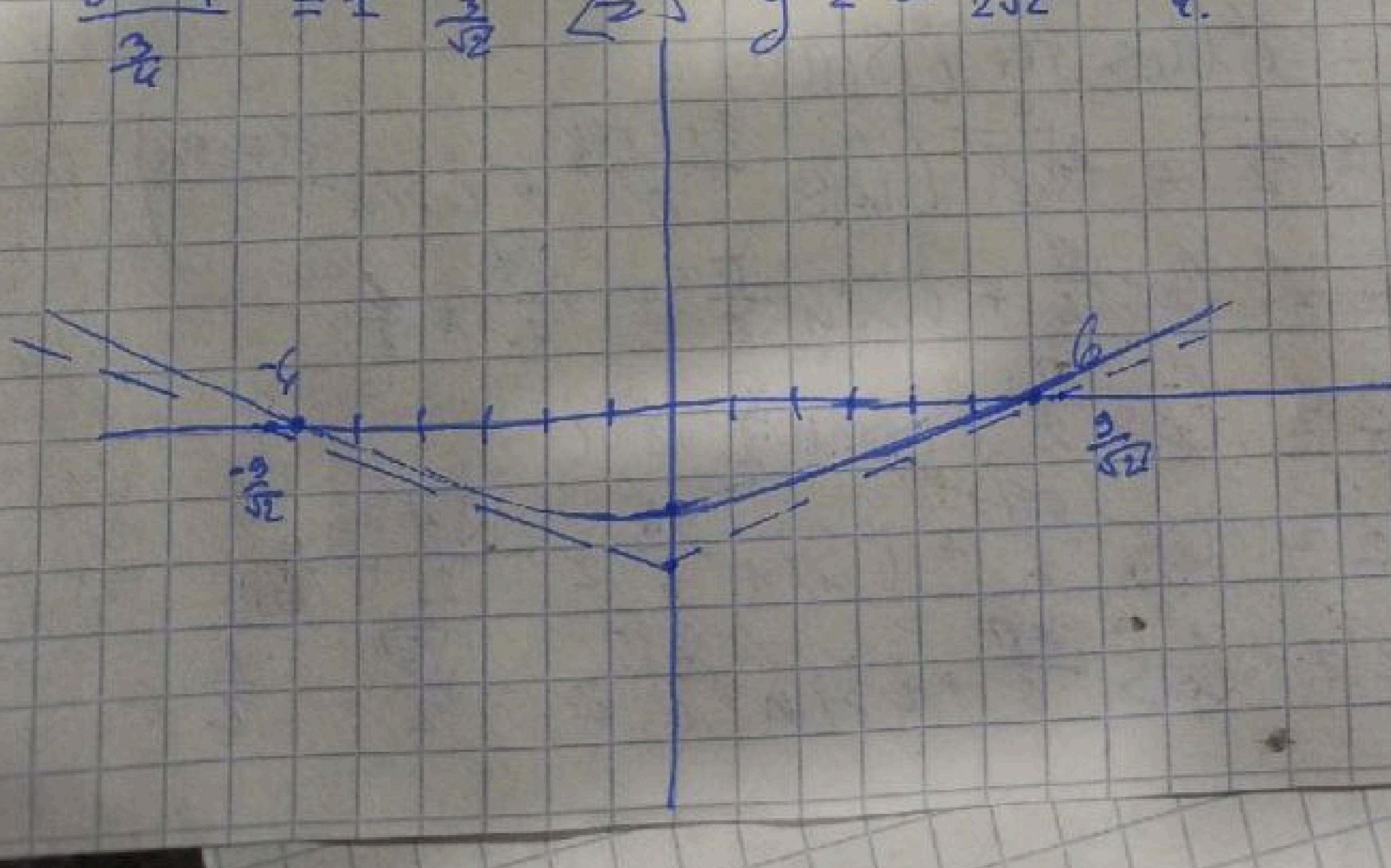
$\begin{cases} x^2+y^2 = (6+3y)^2 \\ 6+3y \geq 0 \end{cases} \Leftrightarrow \begin{cases} x^2+y^2 = 36+36y+9y^2 \\ y \geq -2 \end{cases}$

$\Leftrightarrow \begin{cases} 8y^2+36y+36-x^2=0 \\ y \geq -2 \end{cases} \Leftrightarrow \begin{cases} 8(y+\frac{9}{4})^2 - \frac{9}{2} - x^2 = 0 \\ y \geq -2 \end{cases}$

$\begin{cases} 8(y+\frac{9}{4})^2 - x^2 = \frac{9}{2} \\ y \geq -2 \end{cases} \Leftrightarrow \begin{cases} \frac{(y+\frac{9}{4})^2}{(\frac{3}{4})^2} - \frac{x^2}{(\frac{3}{\sqrt{2}})^2} = 1 \\ y \geq -2 \end{cases}$

Это верхняя половина гиперболы, её асимптоты

$\frac{y+\frac{9}{4}}{\frac{3}{4}} = \pm\frac{x}{\frac{3}{\sqrt{2}}} \Leftrightarrow y = \pm\frac{x}{2\sqrt{2}} - \frac{9}{4}.$



(5) $v(x,y) = y - \dfrac{y}{x^2+y^2}$

$$\frac{\partial v}{\partial x} = \frac{2xy}{(x^2+y^2)^2} = -\frac{\partial u}{\partial y}$$

$$u = -\int \frac{2xy}{(x^2+y^2)^2}\,dy = -\int \frac{x\,d(x^2+y^2)}{(x^2+y^2)^2} =$$

$$= \frac{x}{(x^2+y^2)} + C(x)$$

$$\frac{\partial v}{\partial y} = 1 - \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{1+y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^2)-2x^2}{(x^2+y^2)^2} + C'(x) = \frac{y^2-x^2}{(x^2+y^2)^2} + C'(x)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Leftrightarrow C'(x) = 1 \Leftrightarrow C(x) = x + C$$

$$f(x,y) = u(x,y) + i\,v(x,y) = \frac{x}{(x^2+y^2)} + x + C +$$

$$+ i\left(y - \frac{y}{x^2+y^2}\right) = x + iy + \frac{x-iy}{x^2+y^2} + C =$$

$$= z + \frac{x-iy}{(x+iy)(x-iy)} + C = z + \frac{1}{z} + C.$$