CONFERENCE ON 4–MANIFOLDS AND KNOT CONCORDANCE

MPIM, OCTOBER 17-21, 2016

1. 50min talks

R. İnanç Baykur (University of Massachusetts Amherst): Small symplectic and exotic 4-manifolds via positive factorizations. We will discuss new ideas and techniques for producing positive Dehn twist factorizations of surface mapping classes (joint work with Mustafa Korkmaz), which yield novel constructions of interesting symplectic and smooth 4-manifolds, such as small symplectic Calabi-Yau surfaces and exotic rational surfaces, via Lefschetz fibrations and pencils.

Jae Choon Cha (POSTECH): 4-dimensional L^2-acyclic bordism and Whitney towers. Recently Sylvain Cappell, Jim Davis and Shmuel Weinberger studied L^2-acyclic bordism groups in high dimensions. In this talk we address 4-dimensional L^2-acyclic bordism between 3-manifolds. We introduce a Whitney tower approach to study the structure in this dimension, and show that Cheeger-Gromov invariants over amenable groups give obstructions. Also we answer some questions of Cappell, Davis and Weinberger which concern the relationship of L^2-acyclic bordism and knot concordance.

David Gabai (Princeton University): On Mazur's telescoping argument. In 1958 Mazur gave a remarkable telescoping argument to show that a smooth 3-sphere in 4-space bounds a topological 4-ball. We discuss some applications of his argument.

David Gay (University of Georgia): Functions on surfaces and constructions of 3-, 4- and 5-manifolds. I'll steal ideas from Lickorish's proof that 3-manifolds bound 4-manifolds and Hatcher and Thurston's proof that the mapping class group of a surface is finitely presented to give, among other things, a new proof that $\Omega_4 = \mathbb{Z}$ (using trisections where Lickorish uses Heegaard splittings). The key idea is that generic n-parameter families of functions on surfaces describe $(n+3)$-manifolds, at least for $n \leq 2$.

Robert Gompf (The University of Texas at Austin): Group actions on exotic smoothings of $\mathbb{R}^4$. It has been known for more than three decades that there are uncountably many diffeomorphism types of smoothings on $\mathbb{R}^4$, so in some sense there is a shortage of diffeomorphisms. However, nothing has been known about the self-diffeomorphisms of such a manifold up to smooth isotopy, except that there is only one (preserving orientation) for the standard smoothing. This talk will cover a recent breakthrough in the matter.

Shelly Harvey (Rice University): A rational valued metric on the knot concordance group coming from gropes. Most of the 50-year history of the study of the set of smooth knot concordance classes, $\mathcal{C}$, has focused on its structure as an abelian group. A few years ago, Tim Cochran and I took a different approach, namely we studied $\mathcal{C}$ as a metric space (with the slice genus metric or the homology metrics) admitting many natural geometric operators, especially satellite operators. The hope was to give evidence that the knot concordance is a fractal space. However, both of these metrics are integer valued metrics and so induce the discrete topology. Here (with Mark Powell) we define a family of metrics, called the q-grope metrics, which take values in the real numbers. We will show that there are sequences of knots whose q-norms get arbitrarily small for $q > 1$. We will also show that for any winding number 0 satellite operator, $R\colon \mathcal{C} \to \mathcal{C}$, is a contraction for q large enough. This is joint work with Tim Cochran and Mark Powell.

Matthew Hedden (Michigan State University): Knot theory and complex curves in subcritical Stein domains. Beautiful results of Rudolph and Boileau-Orevkov characterize those links in the 3-sphere which arise as transverse intersections with algebraic curves in $\mathbb{C}^2$. I'll discuss work in progress which provides a corresponding characterization of such links in connected sums of $S^1 \times S^2$, viewed as the boundary of a subcritical Stein domain (the ball union Stein 1-handles). Time permitting, I'll talk about further generalizations being pursued with Baykur, Etnyre, Kawamuro, and Van Horn-Morris.

Jennifer Hom (Georgia Tech): Knot concordance in homology spheres. The knot concordance group $\mathcal{C}$ consists of knots in S^3 modulo knots that bound smooth disks in B^4. We consider $\widehat{\mathcal{C}}_{\mathbb{Z}}$, the group of knots in homology spheres that bound homology balls modulo knots that bound smooth disks in a homology ball. Matsumoto asked if the natural map from $\mathcal{C}$ to $\widehat{\mathcal{C}}_{\mathbb{Z}}$ is an isomorphism. Adam Levine answered this question in the negative by showing the map is not surjective. We show that the image of $\mathcal{C}$ in $\widehat{\mathcal{C}}_{\mathbb{Z}}$ is of infinite index; more specifically, it contains a subgroup isomorphic to the integers. This is joint work with Adam Levine and Tye Lidman.

Adam Levine (Princeton University): Heegaard Floer invariants for homology $S^1 \times S^3$s. Using Heegaard Floer homology, we construct a numerical invariant for any smooth, oriented 4-manifold X with the homology of $S^1 \times S^3$. Specifically, we show that for any smoothly embedded 3-manifold Y representing a generator of $H_3(X)$, a suitable version of the Heegaard Floer d invariant of Y, defined using twisted coefficients, is a diffeomorphism invariant of X. We show how this invariant can be used to obstruct embeddings of certain types of 3-manifolds, including those obtained as a connected sum of a rational homology 3-sphere and any number of copies of $S^1 \times S^2$. We also give similar obstructions to embeddings in certain open 4-manifolds, including exotic $\mathbb{R}^4$s. This is joint work with Danny Ruberman.

Brendan Owens (University of Glasgow): Searching for slice alternating knots. Which alternating knots are slice? Which are ribbon? For which does the double branched cover bound a rational homology ball? Unfortunately I can't answer any of these questions. I will report on a computer search with Frank Swenton which found approximately 30,000 new examples of slice alternating knots, and discuss some related questions.

Daniel Ruberman (Brandeis University): Invariants of 4-manifolds with the homology of $S^1 \times S^3$. Gauge theoretic invariants of a closed 4-manifold X usually are defined only in the setting where the intersection form has $b_2^+ > 0$. This condition rules out reducible solutions. Many interesting questions remain, however, in the setting where there is no second homology at all. We relate several invariants that have been previously defined in this context. We relate the invariant $\lambda_{SW}(X)$ defined by Mrowka-Ruberman-Saveliev to two invariants introduced by Frøyshov for a rational homology sphere Y. One is the *correction term* $h(Y)$; if Y is embedded in X, then $h(Y)$ is actually an invariant of X alone. The other is the Lefschetz number of the map on reduced monopole homology induced by the cobordism W obtained from cutting X open along Y. We show that $\lambda_{SW}(X) + h(Y) = Lef(W_* : HM_{red}(Y) \to HM_{red}(Y))$. This is joint work with Jianfeng Lin and Nikolai Saveliev.

Rob Schneiderman (City University of New York): Whitney towers: What are they? And what are they good for? This talk will sketch an introduction to current theories of Whitney towers, highlight recent results and applications, and point out some open problems.

2. 25min talks

Tetsuya Abe (Osaka City University Advanced Mathematical Institute): The slice-ribbon conjecture and related topics. We survey the recent progress on the slice-ribbon conjecture, which is one of the biggest conjectures in knot concordance theory. In this talk, we will give some potential counterexamples of the slice-ribbon conjecture using annulus twist, which is a certain operation on knots. Also, we will discuss related topics.

Marco Golla (Uppsala University): Heegaard Floer homology and cobordisms of algebraic knots. In this talk I will discuss how correction terms in Heegaard Floer homology can be used to study smooth cobordisms of algebraic knots and complex deformations of cusp singularities of curves. The main tool will be the concordance invariant ν^+ and its subadditivity with respect to connected sums.

Marc Kegel (University of Cologne): The knot complement problem for Legendrian and transverse knots. The famous knot complement theorem by Gordon and Luecke states that two knots in the 3-sphere are equivalent if and only if their complements are homeomorphic. In this talk I want to discuss the same question for Legendrian and transverse knots and links in contact 3-manifolds. The main results are that Legendrian as well as transverse knots in the tight contact 3-sphere are equivalent if and only if their exteriors are contactomorphic.

Alexandra Kjuchukova (University of Wisconsin-Madison): Linking numbers of pseudo-branch curves. Let K be a knot in S^3 and $f\colon M \to S^3$ a cover branched along K. Under certain hypotheses, the linking numbers in M between the components of $f^{-1}(K)$ are an invariant of K. This invariant was crucial for expanding the knot table to include knots of more than 8 crossings, among other uses. Also important but less well-studied are linking numbers between "pseudo-branch curves", or lifts to M of simple closed curves in the complement of K. I describe a method for computing such linking numbers. I will also explain the motivation for this work, and how it can be used in the classification of branched covers between four-manifolds with singular branching sets. Joint with Patricia Cahn.

Kyle Larson (Michigan State University): Embedding of 3-manifolds in spin 4-manifolds. An invariant of orientable 3-manifolds is defined by taking the minimum n such that a given 3-manifold embeds in the connected sum of n copies of $S^2 \times S^2$, and we call this n the *embedding number* of the 3-manifold. We discuss some general properties of this invariant, and discuss some calculations for families of lens spaces and Brieskorn spheres. We can construct rational and integral homology spheres whose embedding numbers grow arbitrarily large, and which can be calculated by exactly if we assume the 11/8-Conjecture. This is joint work with Paolo Aceto and Marco Golla.

Ana Lecuona (Institut de Mathématiques de Marseille): Slopes, colored links and Kojima's η concordance invariant. In this talk we will introduce an invariant, the slope, for a colored link in a homology sphere together with a suitable multiplicative character defined on the link group. The slope takes values in the complex number union infinity and it is real for finite order characters. It is a generalization of Kojima η-invariants and can be expressed as a quotient of Conway polynomials. It is also related to the correction term in Wall's non-additivity formula for the signatures of 4-manifolds, and as such it appears naturally as a correction term in the expression of the signature formula for the splice of two colored links. This is work in progress with Alex Degtyarev and Vincent Florens.

Boris Lishak (Max Planck Institute for Mathematics): Geometric complexity of 4-manifolds. We find a lower bound on how geometrically complicated 4-manifolds might be. Results of this type are, probably, most difficult for the sphere. We find Riemannian metrics (or triangulations) of the sphere which make it look very different from the round S^4. More precisely, for each closed four-dimensional smooth manifold M and for each sufficiently small positive ϵ the set of isometry classes of Riemannian metrics with volume equal to 1 and injectivity radius greater than ϵ is disconnected. For each closed four-dimensional PL-manifold M and any m there exist arbitrarily large values of N such that some two triangulations of M with $< N$ simplices cannot be connected by any sequence of $< \exp_m(N)$ bistellar transformations, where $\exp_m(N) = \exp(\exp(\ldots\exp(N)))$ (m times).

We construct families of trivial 2-knots K_i in $\mathbb{R}^4$ such that the maximal complexity of 2-knots in any isotopy connecting K_i with the standard unknot grows faster than a tower of exponentials of any fixed height of the complexity of K_i. Here we can either construct K_i as smooth embeddings and measure their complexity as the ropelength (a.k.a the crumpledness) or constrcut PL-knots K_i, consider isotopies through PL-knots, and measure the complexity of a PL-knot as the minimal number of flat 2-simplices in its triangulation.

Taylor Martin (Sam Houston State University): Lower order quotients in the n-solvable filtration. We establish several results about two short exact sequences involving lower terms of the n-solvable filtration, $\{\mathcal{F}_n^m\}$ of the string link concordance group $\mathcal{C}^m$, which was introduced by Cochran, Orr, and Teichner in the late 90's. We show that the short exact sequence

$$0 \to \mathcal{F}_0^m/\mathcal{F}_{0.5}^m \to \mathcal{F}_{-0.5}^m/\mathcal{F}_{0.5}^m \to \mathcal{F}_{-0.5}^m/\mathcal{F}_0^m \to 0$$

does not split for links of two or more components, in contrast to the fact that it splits for knots. Considering lower terms of the filtration $\{\mathcal{F}_n^m\}$ in the short exact sequence

$$0 \to \mathcal{F}_{-0.5}^m/\mathcal{F}_0^m \to \mathcal{C}^m/\mathcal{F}_0^m \to \mathcal{C}^m/\mathcal{F}_{-0.5}^m \to 0,$$

we show that while the sequence does not split for $m \geq 3$, it does indeed split for $m = 2$. This allows us to determine that the quotient $\mathcal{C}^2/\mathcal{F}_0^2$ is an abelian group. This is joint work with Carolyn Otto.

Kyungbae Park (Korea Institute for Advanced Study): An infinite-rank summand of knots with trivial Alexander polynomial. The subgroup $\mathcal{T}$ of the smooth knot concordance group generated by topologically slice knots portrays the significant difference between the topological and smooth categories in 4–dimension. Last decade, some structural problems (such as splitting and divisibility) of $\mathcal{T}$ have been answered due to the development of homological invariants of knots. In this talk I will review some recent results related to this topic and show that there exists an infinite rank summand in the subgroup of $\mathcal{T}$ generated by knots with trivial Alexander polynomial. To this end we use the invariant Upsilon recently introduced by Ozsváth, Stipsicz and Szabó using knot Floer homology. This is joint work with Min Hoon Kim.

Juanita Pinzón-Caicedo (University of Georgia): Iterated Whitehead doubles are independent. In the 1980's Furuta and Fintushel-Stern applied the theory of instantons and Chern-Simons invariants to develop a criterion for a collection of Seifert fibered homology spheres to be independent in the homology cobordism group of oriented homology 3-spheres. These results, together with some 4-dimensional constructions, can be used to show that iterated Whitehead doubles of positive torus knots are independent in the smooth knot concordance group.

Andrew Ranicki (University of Edinburgh): The mod 8 signature of a surface bundle. The talk will report on the current status of a joint project with Dave Benson, Caterina Campagnolo and Carmen Rovi. Werner Meyer (Bonn thesis 1972) proved that the signature $\sigma(E) \in \mathbb{Z}$ of a surface bundle $\Sigma_g \to E \to \Sigma_h$ is divisible by 4, and can be computed from a cohomology class $\tau \in H^2(\mathrm{Sp}(2g, \mathbb{Z}); \mathbb{Z})$ and the monodromy $\pi_1(\Sigma_h) \to \mathrm{Sp}(2g, \mathbb{Z})$. In our project we prove that $\sigma(E)/4 \in \mathbb{Z}_2$ can be computed from a cohomology class $\tau_8 \in H^2(\mathrm{Sp}(2g, \mathbb{Z}_4); \mathbb{Z}_2)$ and the monodromy $\pi_1(\Sigma_h) \to \mathrm{Sp}(2g, \mathbb{Z}_4)$. The symmetric L-theory of $\mathbb{Z}_4$ is used to construct an explicit cocycle for τ_8.

Motoo Tange (University of Tsukuba): Cork twist and infinite exotic families. Cork twist is a 4-dimensional surgery by a contractible submanifold. This twist is a significant objet in terms of which any exotic pair is related by a cork wist each other. In this talk I will explain some corks (with finite order or infinite order) and show that some infinite exotic families are not related by any infinite order cork.

Biji Wong (Brandeis University): Equivariant corks and Heegaard Floer homology. A cork is a contractible smooth 4-manifold with an involution on its boundary that does not extend to a diffeomorphism of the entire manifold. Corks can be used to produce exotic structures; in fact any two smooth structures on a closed simply-connected 4-manifold are related by a cork twist. Recently, Auckly-Kim-Melvin-Ruberman showed that for any finite subgroup of SO(4) there exists a contractible 4-manifold with an effective G-action on its boundary so that the twists associated to the elements of G don't extend to diffeomorphisms of the entire manifold. We use a Heegaard Floer theory argument originating in work of Akbulut-Karakurt to reprove this fact.