COUNTEREXAMPLES IN 4-MANIFOLD TOPOLOGY

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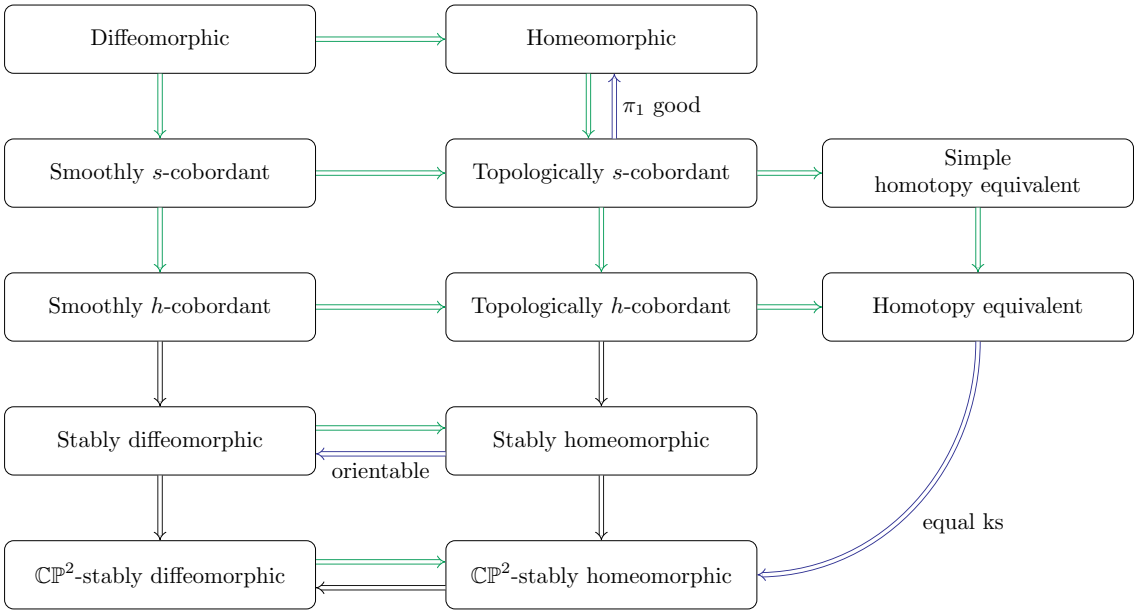

FIGURE 1. Equivalence relations on 4-manifolds. The implications shown in green are immediate. The blue implications hold when the corresponding condition is true, e.g. homotopy equivalent manifolds are $\mathbb{CP}^2$-stably homeomorphic if their Kirby-Siebenmann invariants coincide. Where necessary for an implication to make sense, we assume that the manifolds are smooth. For example, the black arrow in the bottom row means that closed, *smooth* $\mathbb{CP}^2$-stably homeomorphic 4-manifolds are $\mathbb{CP}^2$-stably diffeomorphic, since the latter notion is only defined for smooth manifolds.

This document can also be found at tinyurl.com/4dcounterexamples.

Examples	Properties			Equivalence relations												
	smooth	oriented	$\pi_1 = 1$	equal χ	$S^2 \times S^2$-stably homeo.	$\mathbb{CP}^2$-stably homeo.	$S^2 \times S^2$-stably diffeo.	$\mathbb{CP}^2$-stably diffeo.	homotopy equiv.	simple homotopy equiv.	top. h-cobordant	top. s-cobordant	smoothly h-cobordant	smoothly s-cobordant	homeomorphic	diffeomorphic
S^4 and $S^2 \times S^2$	✓	✓	✓	✗	✓	✓	✓	✓	✗	✗	✗	✗	✗	✗	✗	✗
$S^2 \times S^2$ and $S^2 \widetilde{\times} S^2$	✓	✓	✓	✓	✗	✓	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗
$\mathbb{CP}^2$ and $*\mathbb{CP}^2$	✗	✓	✓	✓	✗	✗	n/a	n/a	✓	✓	✗	✗	n/a	n/a	✗	n/a
$\mathbb{RP}^4 \# \mathbb{CP}^2$ and $\mathcal{R} \# *\mathbb{CP}^2$	✓	✗	✗	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗	✗
$K3 \# \mathbb{RP}^4$ and $\#^{11} S^2 \times S^2 \# \mathbb{RP}^4$	✓	✗	✗	✓	✓	✓	✗	✓	✓	✓	✓	✓	✗	✗	✓	✗
$\mathbb{RP}^4$ and R	✓	✗	✗	✓	✓	✓	✗	✓	✓	✓	✓	✓	✗	✗	✓	✗
$L_{p,q_1} \times S^1, \ldots, L_{p,q_k} \times S^1$, with $L_{p,q_1} \simeq L_{p,q_2}$ and $L_{p,q_1} \ncong L_{p,q_2}$	✓	✓	✗	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗	✗
$E(1)$ and $E(1)_{2,3}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗
$\#^3 E_8$ and Le	✗	✓	✓	✓	✓	✓	n/a	n/a	✗	✗	✗	✗	n/a	n/a	✗	n/a
Kreck-Schafer manifolds	✓	✓	✗	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗	✗	✗	✗
Teichner's $E\#E\# \#^k(S^2 \times S^2)$ and $*E\#*E\# \#^k(S^2 \times S^2)$	✓	✓	✗	✓	✗	✓	✗	✓	✓	✓	✗	✗	✗	✗	✗	✗
Akbulut's P and Q	✓	✗	✗	✓	✓	✓	✗	✓	✓	✓	✓	✓	✗	✗	✓	✗
$\mathcal{M}(L_{p,q} \times S^1)$, p odd, ∞ set	?	✓	✗	✓	✓	✓	n/a	n/a	✓	✓	✗	✗	n/a	n/a	✗	n/a
$\{M_r(\kappa)\}_{\kappa \in K}$	?	✓	✗	✓	✓	✓	n/a	n/a	✓	✓	✓	✗	n/a	n/a	✗	n/a

TABLE 1. Counterexamples in 4-manifold topology.