1/Juli 24

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Latihan Bab 3

1. Tentukan solusi SPL berikut :

$$2a - 8b = 12$$
$$3a - 6b = 9$$
$$-a + 2b = -4$$

2. Tentukan solusi SPL :

$$2p - 2q - r + 3s = 4$$
$$p - q + 2s = 1$$
$$-2p + 2q - 4s = -2$$

3. Tentukan solusi SPL homogen berikut :

$$p - 5q - 4r - 7t = 0$$
$$2p + 10q - 7r + s - 7t = 0$$
$$r + s + 7t = 0$$
$$-2p - 10q + 8r + s + 18t = 0$$

4. Diketahui SPL $AX = B$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ dan } B = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Tentukan solusi SPL diatas dengan menggunakan :

- Operasi baris elementer (OBE)
- Invers matriks
- Aturan Cramer

5. Diketahui

$$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} X - X \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 5 & 4 \end{bmatrix}$$

Tentukan $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ yang memenuhi.

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6. SPL homogen (dengan perubahan p, q dan r)

$p + 2q + r = 0$

$q + 2r = 0$

$k^2 p + (k+1) q + r = 0$

Tentukan nilai k ~~sekarang~~ sehingga SPL punya solusi tunggal

7. Misalkan

$B = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$

Tentukan vektor tak nol $\bar{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ sehingga $B\bar{u} = 6\bar{u}$

Jawaban

1. $2a - 8b = 12$

$3a - 6b = 9$

$-a + 2b = -4$

Ubah ke matriks

$\left[\begin{array}{ccc|c} 2 & -8 & 0 & 12 \\ 3 & -6 & 0 & 9 \\ -1 & 2 & 0 & -4 \end{array}\right] B_1 \Rightarrow \frac{1}{2} B_1 \left[\begin{array}{ccc|c} 1 & -4 & 0 & 6 \\ 3 & -6 & 0 & 9 \\ -1 & 2 & 0 & -4 \end{array}\right] B_2 \Rightarrow B_2 - 3B_1 \left[\begin{array}{ccc|c} 1 & -4 & 0 & 6 \\ 0 & 6 & 0 & -9 \\ -1 & 2 & 0 & -4 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & -4 & 0 & 6 \\ 0 & 6 & 0 & -9 \\ -1 & 2 & 0 & -4 \end{array}\right] B_3 \Rightarrow B_3 + B_1 \left[\begin{array}{ccc|c} 1 & -4 & 0 & 6 \\ 0 & 6 & 0 & -9 \\ 0 & -2 & 0 & 2 \end{array}\right] B_2 \rightarrow \frac{1}{6} B_2 \left[\begin{array}{ccc|c} 1 & -4 & 0 & 6 \\ 0 & 1 & 0 & -1{,}5 \\ 0 & -2 & 0 & 2 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & -4 & 0 & 6 \\ 0 & 1 & 0 & -1{,}5 \\ 0 & -2 & 0 & 2 \end{array}\right] B_3 \rightarrow B_3 + 2B_2 \left[\begin{array}{ccc|c} 1 & -4 & 0 & 6 \\ 0 & 1 & 0 & -1{,}5 \\ 0 & 0 & 0 & -1 \end{array}\right] B_1 \rightarrow B_1 + 4B_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1{,}5 \\ 0 & 0 & 0 & -1 \end{array}\right]$

karna C, 0 = -1 jadi sistem persamaan linear ini tidak memiliki solusi.

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2. $2p - 2q - r + 3s = 4$

$p - q + 2s = 1$

$-2p + 2q - 4s = -2$

ubah ke matriks

$$\left[\begin{array}{cccc|c} 2 & -2 & -1 & 3 & 4 \\ 1 & -1 & 0 & 2 & 1 \\ -2 & 2 & 0 & -4 & -2 \end{array}\right] \xrightarrow{B_2 \to B_2 - \frac{1}{2}B_1} \left[\begin{array}{cccc|c} 2 & -2 & -1 & 3 & 4 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & -1 \\ -2 & 2 & 0 & -4 & -2 \end{array}\right] \xrightarrow{B_3 \to B_3 + B_1} \left[\begin{array}{cccc|c} 2 & -2 & -1 & 3 & 4 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 0 & -1 & -1 & 2 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 2 & -2 & -1 & 3 & 4 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 0 & -1 & -1 & 2 \end{array}\right] \xrightarrow{B_3 \to B_3 + 2B_2} \left[\begin{array}{cccc|c} 2 & -2 & -1 & 3 & 4 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{B_2 \to 2B_2} \left[\begin{array}{cccc|c} 2 & -2 & -1 & 3 & 4 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{B_1 \to B_1 - B_2}$$

$$\left[\begin{array}{cccc|c} 2 & -2 & -2 & 2 & 6 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{B_1 \to \frac{B_1}{2}} \left[\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 3 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Persamaan yang tersisa adalah

$p - q - r + s = 3$

$r + s = -2$

dari persamaan kedua dipindah ruaskan untuk mendapatkan nilai r :

$r + s = -2 \Rightarrow r = -2 - s$

substitusi nilai r ke persamaan pertama

jadi, $p - q - (-2 - s) + s = 3$

$= p - q + 2 - s + s = 3$

$= p - q + 2 = 3$

$= p - q = 3 - 2$

$p - q = 1$

$p = q + 1$

jadi solusi umum untuk sistem persamaan ini adalah

$p = q + 1$

$r = -2 - s$

$q = q$

$s = s$

Dimana q dan s adalah parameter bebas.

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3. $p - 5q - 4r - 7t = 0$

$2p + 10q - 7r + s - 7t = 0$

$r + s + 7t = 0$

$-2p - 10q + 8r + s + 18t = 0$

Dengan eliminasi Gauss Jordan

$$\begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 2 & 10 & -7 & 1 & -7 \\ 0 & 0 & 1 & 1 & 7 \\ -2 & -10 & 8 & 1 & 18 \end{bmatrix} (-2B_1 + B_2) \begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 20 & 1 & 1 & 7 \\ 0 & 0 & 1 & 1 & 7 \\ -2 & -10 & 8 & 1 & 18 \end{bmatrix} (2B_1 + B_4) \begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 20 & 1 & 1 & 7 \\ 0 & 0 & 1 & 1 & 7 \\ 0 & -20 & 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 20 & 1 & 1 & 7 \\ 0 & 0 & 1 & 1 & 7 \\ 0 & -20 & 0 & 1 & 4 \end{bmatrix} (B_2 + B_4) \begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 20 & 1 & 1 & 7 \\ 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 2 & 11 \end{bmatrix} (\tfrac{1}{20} B_2) \begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 1 & \frac{1}{20} & \frac{1}{20} & \frac{7}{20} \\ 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 2 & 11 \end{bmatrix} (-B_3 + B_4)$$

$$\begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 1 & \frac{1}{20} & \frac{1}{20} & \frac{7}{20} \\ 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} (-B_4 + B_3) \begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 1 & \frac{1}{20} & \frac{1}{20} & \frac{7}{20} \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} (-\tfrac{1}{20} B_3 + B_2) \begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 & -4 & 0 & -7 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} (5B_2 + B_1) \begin{bmatrix} 1 & 0 & -4 & 0 & -7 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} (4B_3 + B_1) \begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Diperoleh :

$p + 5t = 0$

$q = 0$

$r + 3t = 0$

$s + 4t = 0$

Dengan :

misalkan t dalam paramenter i,

maka solusinya

$\{(-5i, 0, -3i, -4i, i)\}$

4. $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ dan $B = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

✱ Menggunakan OBE

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 2 & 1 & 1 \end{array}\right) B_3 + B_2 \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right) -B_1 + B_2 \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 \end{array}\right) -2B_2 + B_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right) -B_3 + B_1 \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \quad \text{jadi} \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$$

✱ Menggunakan invers Matriks

$$A = \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \end{array}\right) B_3 + B_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right) -B_1 + B_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right) -2B_2 + B_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array}\right) -B_3 + B_1 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array}\right)$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1}B = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 1 & 1 \\ 2 & -2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 1 + 2 \cdot -1 + 1 \cdot 1 \\ -1 \cdot 1 + 1 \cdot -1 + 1 \cdot 1 \\ 2 \cdot 1 + 2 \cdot 1 + -1 \cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix}$$

✱ Menggunakan aturan cramer

$$DET(A) = \begin{vmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{vmatrix}\begin{matrix} 0 & 1 \\ -1 & 0 \\ 2 & 1 \end{matrix} = |-1+0+2| - |0+0+0| = 1$$

$$\rightarrow x_1 = \frac{\det(A_1)}{\det(A)} = \begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & 0 \\ 1 & 2 & 1 \end{vmatrix}\begin{matrix} 1 & 0 \\ -1 & -1 \\ 1 & 2 \end{matrix} = (-1+0+(-2)) - (0+0+(-1)) = -2$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix}\begin{matrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{matrix} = (-1+0+1) - (1+0+0) = -1$$

$$x_3 = \frac{\det(A_3)}{\det(A)} = \begin{vmatrix} 1 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix}\begin{matrix} 1 & 0 \\ 1 & -1 \\ 0 & 2 \end{matrix} = (-1+0+2) - (0+(-2)+0) = 3$$

5. Misalkan

$$A = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} \qquad B = \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix}$$

* Hitung AX :

$$AX = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix}\begin{vmatrix} x_1 & x_2 \\ x_3 & x_4 \end{vmatrix} = \begin{vmatrix} 3x_1 + x_3 & 3x_2 + x_4 \\ -x_1 + 2x_3 & -x_2 + 2x_4 \end{vmatrix}$$

* Hitung XB :

$$XB = \begin{vmatrix} x_1 & x_2 \\ x_3 & x_4 \end{vmatrix}\begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} = \begin{vmatrix} x_1 + 2x_2 & 4x_1 \\ x_3 + 2x_4 & 4x_3 \end{vmatrix}$$

* Persamaan AX - XB :

$$AX - XB = \begin{vmatrix} 2 & -2 \\ 5 & 4 \end{vmatrix} : \begin{vmatrix} 3x_1 + x_3 & 3x_2 + x_4 \\ -x_1 + 2x_3 & -x_2 + 2x_4 \end{vmatrix} - \begin{vmatrix} x_1 + 2x_2 & 4x_1 \\ x_3 + 2x_4 & 4x_3 \end{vmatrix} = \begin{vmatrix} 2 & -2 \\ 5 & 4 \end{vmatrix}$$

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* Persamaan matriks menjadi persamaan skalar

→ Elemen pertama

$(3x_1 + x_3) - (x_1 + 2x_2) = 2$

$3x_1 + x_3 - x_1 - 2x_2 = 2$

$2x_1 + x_3 - 2x_2 = 2$

→ Elemen kedua

$(3x_2 + x_4) - 4x_1 = -2$

$3x_2 + x_4 - 4x_1 = -2$

→ Elemen ketiga

$(-x_1 + 2x_3) - (x_3 + 2x_4) = 5$

$-x_1 + 2x_3 - x_3 - 2x_4 = 5$

$-x_1 + x_3 - 2x_4 = 5$

→ Elemen keempat

$(-x_2 + 2x_4) - 4x_3 = 4$

$-x_2 + 2x_4 - 4x_3 = 4$

Sehingga :

1. $2x_1 + x_3 - 2x_2 = 2$
2. $3x_2 + x_4 - 4x_1 = -2$
3. $-x_1 + x_3 - 2x_4 = 5$
4. $-x_2 + 2x_4 - 4x_3 = 4$

* Penyelesaian sistem persamaan linear

$x_1 = -\frac{113}{37}$ $\quad x_2 = -\frac{160}{37}$

$x_3 = -\frac{20}{37}$ $\quad x_4 = -\frac{46}{37}$

Jadi matriks x yang memenuhi persamaan tsb adalah :

$$X = \begin{bmatrix} -\frac{113}{37} & -\frac{160}{37} \\ -\frac{20}{37} & -\frac{46}{37} \end{bmatrix}$$

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6. $p + 2q + r = 0$
$q + 2r = 0$
$k^2 p + (k+1) q + r = 0$

$$A = \left|\begin{array}{ccc|cc} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ k^2 & (k+1) & 1 & (k+1) & 1 \end{array}\right| = (1 + 4k^2 + 0) - (0 + 2k + k^2)$$

$$\text{Det}(A) = (1 + 4k^2 + 0) - (k^2 + 2k + 0)$$
$$= 1 + 4k^2 - k^2 - 2k$$
$$= 3k^2 - 2k + 1$$

Dit: $3k^2 - 2k + 1 \neq 0$

Rumus $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$k = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3}$$
$$= \frac{2 \pm \sqrt{4 - 4 \cdot 3 \cdot 1}}{6}$$
$$= \frac{2 \pm \sqrt{-8}}{6}$$
$$= \frac{2 \pm 2i\sqrt{2}}{6}$$
$$= \frac{1 \pm i\sqrt{2}}{3}$$

Hasilnya
$k = \dfrac{1 + i\sqrt{2}}{3}$ atau $k = \dfrac{1 - i\sqrt{2}}{3}$

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7. $B = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} \quad \bar{u} = \begin{bmatrix} x \\ y \end{bmatrix}$

$B\bar{u} = 6\bar{u}$

$\begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 6 \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} x + 3y \\ 5x + 3y \end{bmatrix} = \begin{bmatrix} 6x \\ 6y \end{bmatrix}$

$x + 3y = 6x$
$3y = 6x - x$
$3y = 5x$
$y = \frac{5}{3} x$

$x = 3$
$y = 5$

$5x + 3y = 6y$
$5x = 6y - 3y$
$5x = 3y$
$x = \frac{3}{5} y$

$x = 3$
$y = 5$

pembuktian

$\begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 + 15 \\ 15 + 15 \end{bmatrix}$

$= \begin{bmatrix} 18 \\ 30 \end{bmatrix}$

$= 6 \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

'Nama kelompok'

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