UTIAS Space Robotics Group

**High-Frequency Algorithmic Trading
with Momentum and Order Imbalance**

My goal is to establish and solve the stochastic optimal control problem that captures the momentum and order imbalance dynamics of the Limit Order Book (LOB). The solution will yield an optimal trading strategy that will permit statistical arbitrage of the underlying stock, which will then be backtested on historical data.

Progress Timeline

DATE	THESIS	STA4505
~~Dec 2014~~	~~Complete CTMC calibration~~	
~~Dec 2014~~	~~Backtest naive strategies based on CTMC~~	
~~Jan-May~~	~~Study stochastic controls: ECE1639, STA4505~~	
~~Jun 5~~	~~Establish models~~	~~Exam Study~~
~~Jun 12~~	~~Establish performance criteria~~	~~Exam Study~~
~~Jun 15~~	~~Derive DPP/DPE~~	~~EXAM~~
~~Jun 19~~	~~Derive DPP/DPE~~	
~~Jun 26~~	~~Derive continuous-time equations~~	
~~Jul 3~~	~~Derive discrete-time equations~~	
~~Jul 10~~	~~Set up MATLAB numerical integration~~	
Jul 17	Integrate functions and plot dynamics	~~Integrate and analyze~~
Jul 24	More dynamics, and calib/choose parameters	
Jul 31	Backtest on historical data	~~Simulate results~~
Aug 7	More backtesting, comparing with previous	
Aug 14	Dissertation writeup / buffer	Project writeup
Aug 21	Dissertation writeup / buffer	
Aug 28	Dissertation writeup	Presentation

Whiteboard Inspirational Quote of the Week

> *Solving a problem for which you know there's an answer is like climbing a mountain with a guide, along a trail someone else has laid. In mathematics, the truth is somewhere out there in a place no one knows, beyond all the beaten paths. And it's not always at the top of the mountain. It might be in a crack on the smoothest cliff or somewhere deep in the valley.*
> – Yoko Ogawa, *The Housekeeper and the Professor*

For Our Readers in the Middle East...

The Academic Week in Review

$$
\begin{aligned}
h_k(\boldsymbol{z}, q) = \max\Bigg\{ & q\mathbb{E}[\eta_{0,\boldsymbol{z}} T(\boldsymbol{z}, \omega)^{(2)}] + \frac{1}{\kappa}(p(\delta^{+*}) + p(\delta^{-*})) - 2\pi p(\delta^{+*})p(\delta^{-*})\mathbb{1}_{q=0} \\
& \quad + p(\delta^{+*})p(\delta^{-*}) \sum_{\mathbf{j}} \boldsymbol{P}_{\boldsymbol{z},\mathbf{j}} \left[h_{k+1}(\mathbf{j}, q-1) + h_{k+1}(\mathbf{j}, q+1) - 2h_{k+1}(\mathbf{j}, q)\right] \\
& \quad + \sum_{\mathbf{j}} \boldsymbol{P}_{\boldsymbol{z},\mathbf{j}} h_{k+1}(\mathbf{j}, q) \ ; \\
& (q+1)\mathbb{E}[\eta_{0,\boldsymbol{z}} T(\boldsymbol{z}, \omega)^{(2)}] + \frac{1}{\kappa}(p(\delta_2^{+*}) + p(\delta_2^{-*})) - 2\pi p(\delta_2^{+*})p(\delta_2^{-*})\mathbb{1}_{q=-1} \\
& \quad + p(\delta_2^{+*})p(\delta_2^{-*}) \sum_{\mathbf{j}} \boldsymbol{P}_{\boldsymbol{z},\mathbf{j}} \left[h_{k+1}(\mathbf{j}, q) + h_{k+1}(\mathbf{j}, q+2) - 2h_{k+1}(\mathbf{j}, q+1)\right] \\
& \quad + \sum_{\mathbf{j}} \boldsymbol{P}_{\boldsymbol{z},\mathbf{j}} h_{k+1}(\mathbf{j}, q+1) \ ; \\
& (q-1)\mathbb{E}[\eta_{0,\boldsymbol{z}} T(\boldsymbol{z}, \omega)^{(2)}] + \frac{1}{\kappa}(p(\delta_3^{+*}) + p(\delta_3^{-*})) - 2\pi p(\delta_3^{+*})p(\delta_3^{-*})\mathbb{1}_{q=1} \\
& \quad + p(\delta_3^{+*})p(\delta_3^{-*}) \sum_{\mathbf{j}} \boldsymbol{P}_{\boldsymbol{z},\mathbf{j}} \left[h_{k+1}(\mathbf{j}, q-2) + h_{k+1}(\mathbf{j}, q) - 2h_{k+1}(\mathbf{j}, q-1)\right] \\
& \quad + \sum_{\mathbf{j}} \boldsymbol{P}_{\boldsymbol{z},\mathbf{j}} h_{k+1}(\mathbf{j}, q-1) \Bigg\}
\end{aligned} \tag{1}
$$

We now have an explicit means of numerically solving for the optimal posting depths. Since we know the function h at the terminal timestep T, we can take one step back to $T-1$ and solve for each of the optimal posting depths in each of the cases. With these values we are then able to calculate the value function h_{T-1}, and in doing so determine whether to execute market orders in addition to posting limit orders (essentially taking the arg max instead of the max in equation 1). This process then repeats for each step backward.

Looking Ahead