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(a) $T = \begin{pmatrix} R(\theta) & t \\ 0 & 1 \end{pmatrix}$, $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, $t = \begin{pmatrix} x \\ y \end{pmatrix}$

So, $T = \begin{pmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{pmatrix}$

Now, $x_1 = (x_1, y_1, \theta_1)^T$, $l_{x_1} = (l_x, l_y)$ wrt local frame.

So, $l_g = T_{x_1 g}\, l_{x_1}$, where T_{ab} = homogeneous form of pose a wrt pose b.

$$\Rightarrow l_g = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & x_1 \\ \sin\theta_1 & \cos\theta_1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_x \\ l_y \\ 1 \end{bmatrix} = \begin{bmatrix} l_x\cos\theta_1 - l_y\sin\theta_1 + x_1 \\ l_x\sin\theta_1 + l_y\cos\theta_1 + y_1 \\ 1 \end{bmatrix}$$

So, $l_g = (l_x\cos\theta_1 - l_y\sin\theta_1 + x_1,\ l_x\sin\theta_1 + l_y\cos\theta_1 + y_1)$

(b) Now, $l_g = T_{x_1 g}\, l_{x_1}$

$\Rightarrow l_{x_1} = (T_{x_1 g}^{-1})(l_g)$

(c) $l_g = T_{x_2 g}\, l_{x_2}$

$l_g = T_{x_1 g}\, l_{x_1}$

and $l_{x_1} = T_{12}\, l_{x_2} \Rightarrow (T_{x_1 g}^{-1})\, l_g = T_{12} \cdot (T_{x_2 g}^{-1})\, l_g$

$\Rightarrow T_{x_1 g}^{-1} = T_{12}\, T_{x_2 g}^{-1} \Rightarrow T_{12} = (T_{x_1 g}^{-1})(T_{x_2 g})$

(d) $l_{x_1} = T_{12}\, l_{x_2}$

$\Rightarrow l_{x_2} = (T_{12}^{-1})\, l_{x_1}$

$\Rightarrow l_{x_2} = (T_{x_2 g}^{-1})(T_{x_1 g})\, l_{x_1}$