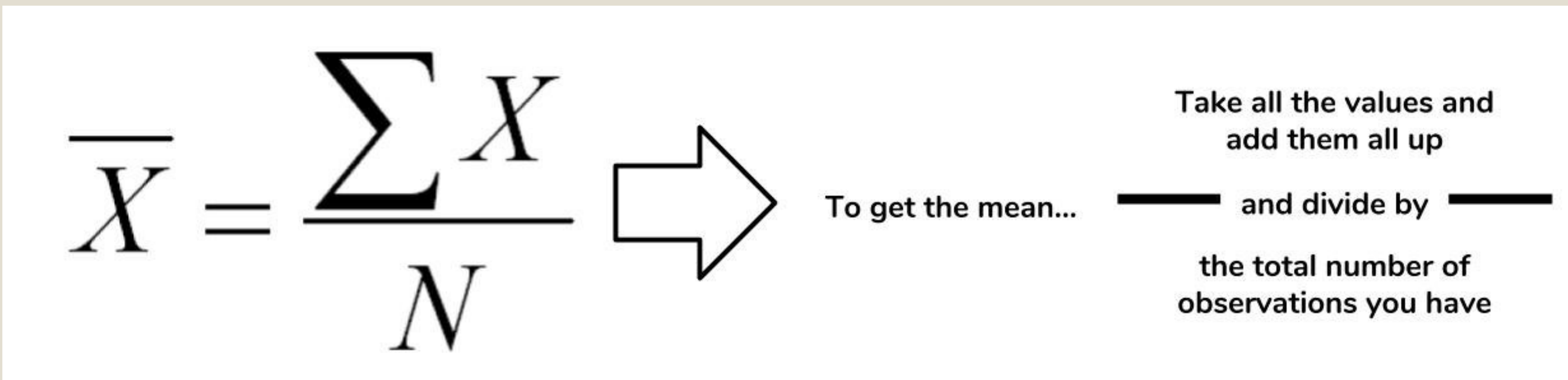
DESCRIPTIVE STATISTICS THROUGH EXAMPLES

Arun P R

Mean



MIN, MAX, RANGE, MODE

[Ref: Basic Statistics in Python: Descriptive Statistics - KDnuggets]

Median

Take your observations:	80, 87, 95, 83, 92
Rearrange your observations into ascending order:	80, 83, 87, 92, 95
The middle value is your median	80, 83, **87**, 92, 95
If there are an even amount of observations, average the middle two	**89.5** 80, 83, **87, 92**, 95, 98

IQR

interquartile range (IQR)

The interquartile range is a measure of the spread equal to the upper quartile (Q3) minus the lower quartile (Q1) of an ordered set of data values.

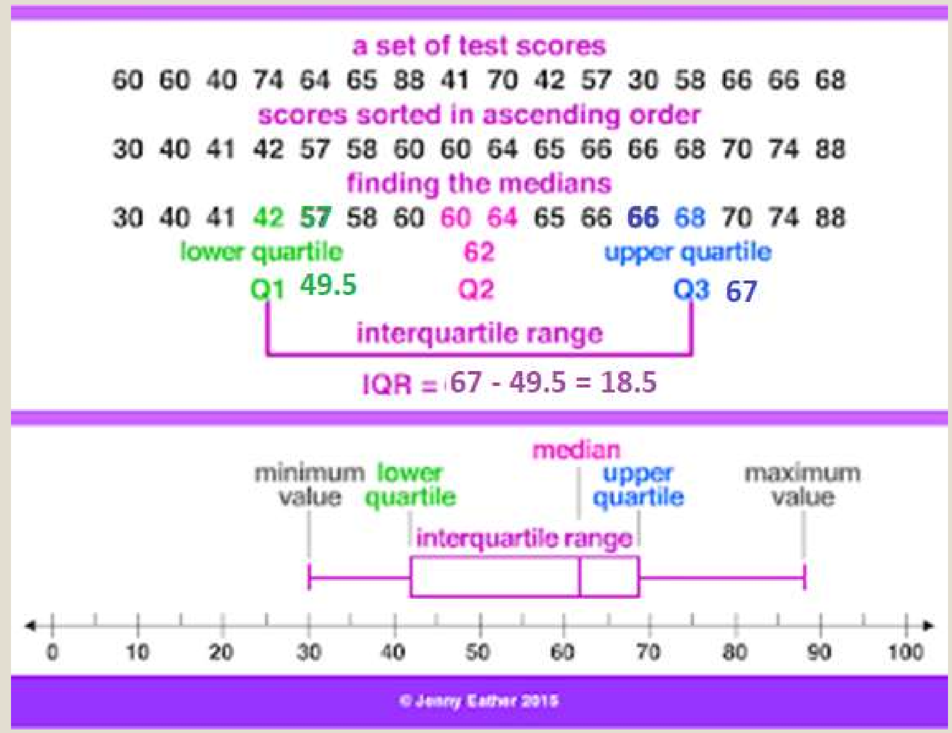
$$IQR = Q3 - Q1$$

Quartiles divide a sorted data set into quarters by finding the median of all the scores, then finding the median of the lower and upper halves of the scores.

If the number of scores is even, the median is the average of the two middle scores.

finding the quartiles

1. Arrange the data scores in ascending order.
2. Find the median of the data set (the number in the middle) … Q2.
3. Find the median of the lower half of the scores … Q1.
4. Find the median of the upper half of the scores … Q3.



Mean Absolute Deviation about Mean

Steps	Formula
1. Take the mean of the observations	$mean = \bar{x} = \dfrac{a + b + c \ldots}{n}$
2. Subtract the mean from each observation	$a - \bar{m}, b - \bar{m}\ etc.$
3. Take the absolute value of step 2	$\lvert a - \bar{m} \rvert, \lvert b - \bar{m} \rvert, etc.$
4. Add together step 3	$\lvert a - \bar{m} \rvert + \lvert b - \bar{m} \rvert + etc.$
5. Divide by the number of observations	$\dfrac{\lvert a - \bar{m} \rvert + \lvert b - \bar{m} \rvert + etc.}{n}$

Population and Sample Variance

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{N} \qquad s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Sample Standard Deviation

3. Sum up all the differences

1. For each observation, subtract each observation from the average

2. Square each difference

Greek letter sigma: used to represent population standard deviation

$$\mathbf{s} = \sqrt{\frac{\sum_{i=1}^{n}\underbrace{(x_i - \overline{x}\,)}_{}^2}{n-1}}$$

5. Square root the result

4. Divide sum by the total number of observations minus 1

PROBABILITY DISTRIBUTIONS

Arun P R

Contents

Random Variables

- Consider a random experiment with sample space S.
- A random variable X is a single-valued real function that assigns a real number to each sample point of S
- A random variable is called discrete if its range (the set of values that it can take) is finite or at most countably infinite.
  - The random variable that associates with a the numerical value sgn(a) = 1 if a > 0, 0 if a = 0, −1 if a < 0, is discrete.
  - A (discrete) random variable has an associated probability mass function (PMF), which gives the probability of each numerical value that the random variable can take.
  - If x is any possible value of X, the probability mass of x, denoted p(x), is the probability of the event {X = x} consisting of all outcomes that give rise to a value of X equal to x: p(x) = P({X = x})
  - For example, let the experiment consist of two independent tosses of a fair coin, and let X be the number of heads obtained, define p(x) for x = 0,1,2,….Ans:¼,1/2,1/4
- A random variable that can take an uncountably infinite number of values is not discrete.
  - For an example, consider the experiment of choosing a point a from the interval [−1, 1].

Expectation, Mean & Variance

- The PMF of a random variable X provides us with several numbers, the probabilities of all the possible values of X. It would be desirable to summarize this information in a single representative number. This is accomplished by the expectation of X, which is a weighted (in proportion to probabilities) average of the possible values of X.
- We define the expected value (also called the expectation or the mean) of a random variable X, with PMF p(x), by $E[X] = \Sigma\, xp(x)$
- The most important quantity associated with a random variable X, other than the mean, is its variance, which is denoted by var(X) and is defined as the expected value of the random variable $(X - E[X])^2$
  - $var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$
- The variance provides a measure of dispersion of X around its mean. Another measure of dispersion is the standard deviation of X, which is defined as the square root of the variance and is denoted by σ

Expectation, Mean & Variance

- Consider two independent coin tosses, each with a 3/4 probability of a head, and let X be the number of heads obtained. This is a binomial random variable with parameters n = 2 and p = 3/4. Find E[X]
  - Ans:0*1/16+1*3/8+2*9/16=3/2

- Find E[X] and var(X) where X is the event of throwing a die
  - Ans:3.5, 35/12

Expectation, Mean & Variance

- A player tosses 3 coins. He gains Rs.500 if 3 heads occur, Rs.300 if 2 heads occur and Rs.100 if one head occurs and he loses Rs.1500 if all three are tails. Find if the game is profitable to the player.
  - Ans:(1700-1500)/8=25

- A dealer plays a gambling game where probability that a player wins is 0.01. Each player has to give Rs.x to enter the game and if he wins dealer will give him Rs.y or else the player loses the money he paid to enter the game. Find the break even point for the dealer in terms of y/x
  - Ans:0.99*x-0.01*y=0

Expectation, Mean & Variance

- Consider a quiz game where a person is given two questions and must decide which question to answer first.
  - Question 1 will be answered correctly with probability 0.8, and the person will then receive as prize Rs.100, while question 2 will be answered correctly with probability 0.5, and the person will then receive as prize Rs. 200.
  - If the first question attempted is answered incorrectly, the quiz terminates, i.e., the person is not allowed to attempt the second question.
  - If the first question is answered correctly, the person is allowed to attempt the second question.
  - Which question should be answered first to maximize the expected value of the total prize money received?

  - Ans:
    - 0.2*0+0.8*0.5*100+0.8*0.5*300=160
    - 0.5*0+0.2*0.5*200+0.8*0.5*300=140

Expectation, Mean & Variance

- A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.
  - E(X) = (0) 1 /35 + (1) 12 /35 + (2) 18 /35 + (3) 4/ 35 = 12 /7 = 1.7.
- Find mean transit time
  - Messages transmitted by a computer in Boston through a data network are destined
    - for New York with probability 0.5,
    - for Chicago with probability 0.3, and
    - for San Francisco with probability 0.2.
  - The transit time X of a message is random. Its mean is
    - 0.05 secs if it is destined for New York,
    - 0.1 secs if it is destined for Chicago, and
    - 0.3 secs if it is destined for San Francisco.
  - Ans:0.115s

Play with Cards

- A card is drawn at random from a pack of cards. Find probability that
  - It is a face card=12/52
  - Neither a diamond nor a face card=1-(13+12-3)/52=30/52
  - Neither a 10 nor a King=1-(4+4)/52=44/52

- 4 cards are drawn at random from a pack of cards. Find probability that:
  - All 4 cards are from same suite=4*C(13,4)/C(52,4)

  - No two cards are from same suite=C(13,1)^4/C(52,4)

Conditional Probability

- Probability that a car will be ready for shipment on time is 0.85 and it is ready for shipment on time and delivered on time is 0.75. Find the probability that it will be delivered on time given it was ready for shipment on time.
  - 0.75/0.85 = 0.88

- For doing a job, 0.65 is probability that there will be a strike. The job will be completed on time with a probability 0.8 if there is no strike, and 0.32 if there is a strike. Find probability that the job will be completed on time.
  - 0.65*0.32 + 0.35*0.8

Baye’s Thorem Application

- 60% of the employees of the company are college graduates. Of these, 10% are in the sales dept. Of the employees who did not graduate from college, 80% are in sales dept. A person is selected at random. Find probability that
  - The person is in the sales dept
  - The person is neither in sales dept nor is a college graduate

  - 100->60,40
  - 60->6,54
  - 40->32,8
- The person is in the sales dept 38/100
- The person is neither in sales dept nor is a college graduate 8/100

Baye's Thorem Application

	P(A is true)	P(A is not true)	
P(Test for A is false)	P(A ∩ TA') – Type I Error (α)	P(A' ∩ TA')	P(TA')
P(Test for A is true)	P(A ∩ TA)	P(A' ∩ TA) – Type II Error (β)	P(TA)
	P(A)	P(A')	1

	Graduate (A)	Not Graduate(A')	
Not Sales Dept (TA')	54	8	= 62
Sales Dept (TA)	6	32	= 38
	= 60	= 40	100

Baye's Thorem Application

	Covid +ve (A)	Covid –ve (A')	
Tested Covid -ve (TA')	1	93	= 94
Tested Covid +ve (TA)	4	2	= 6
	= 5	= 95	100

Find P(A), P(A'), P(TA), P(TA'), P(A/TA),P(A/TA'),P(A'/TA),P(A'/TA'),P(TA/A),P(TA/A'),P(TA'/A),P(TA'/A')

	Covid +ve (A)	Covid –ve (A')	
Tested Covid -ve (TA')	$P(A \cap TA') = 0.01$	$P(A' \cap TA') = 0.93$	$P(TA') = 0.94$
Tested Covid +ve (TA)	$P(A \cap TA) = 0.04$	$P(A' \cap TA) = 0.02$	$P(TA) = 0.06$
	$P(A) = 0.05$	$P(A') = 0.95$	1.00

- $P(A \cap TA) = P(A \mid TA) * P(TA) = P(TA \mid A) * P(A)$
- $P(A \mid TA) = P(A \cap TA) / P(TA) = 0.04 / 0.06$
  - Thus, there is 66.66% probability that somebody who turns positive in Covid test is actually having Covid
- $P(TA \mid A) = P(A \cap TA) / P(A) = 0.04 / 0.05$
  - Thus, there is 80% probability that somebody who has Covid turns positive in Covid test as well

Baye's Thorem Application

- "A patient goes to see a doctor. The doctor performs a test with 99 percent reliability--that is, 99 percent of people who are sick test positive and 99 percent of the healthy people test negative. The doctor knows that only 1 percent of the people in the country are sick. Now the question is: if the patient tests positive, what are the chances the patient is sick?"

Refer the link for worked out examples :

1. 4.1 - The Motivation | STAT 414 (psu.edu)
2. Bayes's Theorem (bu.edu)

Play with balls

- There are two bags A,B. It contains White and Red balls as follows:
  - A ball is drawn at random from one of the bags and found to be red
  - Find the probability that it was from bag B
    - P(A)=P(B)=1/2
    - P(R/A)=3/5,P(R/B)=5/9
    - P(R∩B)=5/9 * ½ = 5/18

	W	R
A	2	3
B	4	5

- There are 3 bags A,B,C each with some number of Blue, Red, and Green balls as given below
  - A bag is drawn at random and two balls are taken from it. They are found to be one blue and one red
  - Find the probability that the selected balls are from bag C
    - P(A)=P(B)=P©=1/3
    - P(E)=P(E int A)+P(E int B)+P(E int C)=P(A)*P(E/A)+…=11/45
    - P(E/A)=C(1,1)*C(2,1)/C(6,2)=2/15

    P(E/B)=C(2,1)*C(3,1)/C(6,2)=6/15

    P(E/C)=C(3,1)*C(1,1)/C(6,2)=3/15
    - P(C/E)=1/3 * 3/15/ 11/45 = 3/11
    - P(B/E)=6/11
    - P(A/E)=2/11

	B	R	G
A	1	2	3
B	2	3	1
C	3	1	2

Probability Distribution

- Two dice are thrown. Find probability distribution for the sum of dots on the dice.

	1,1	1,2 2,1	1,3 2,2 3,1	1,4 2,3 3,2 4,1	1,5 2,4 3,3 4,2 5,1	1,6 2,5 3,4 4,3 5,2 6,1	2,6 3,5 4,4 5,3 6,2	3,6 4,5 5,4 6,2	4,6 5,5 6,4	5,6 6,5	6,6
X	2	3	4	5	6	7	8	9	10	11	12
f(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

- Find P (3 < X <= 4) = 3/36
- Find P (3 <= X <= 4)= 5/36

Discrete Probability Distributions

Discrete Uniform over $[a, b]$:

$$p_X(k) = \begin{cases} \dfrac{1}{b-a+1} & \text{if } k = a, a+1, \ldots, b, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathbf{E}[X] = \frac{a+b}{2}, \qquad \text{var}(X) = \frac{(b-a)(b-a+2)}{12}.$$

Bernoulli with Parameter p: (Describes the success or failure in a single trial.)

$$p_X(k) = \begin{cases} p & \text{if } k = 1, \\ 1-p & \text{if } k = 0, \end{cases}$$

$$\mathbf{E}[X] = p, \qquad \text{var}(X) = p(1-p).$$

Discrete Probability Distributions

Binomial with Parameters p and n: (Describes the number of successes in n independent Bernoulli trials.)

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \ldots, n,$$

$$\mathbf{E}[X] = np, \qquad \mathrm{var}(X) = np(1-p).$$

Geometric with Parameter p: (Describes the number of trials until the first success, in a sequence of independent Bernoulli trials.)

$$p_X(k) = (1-p)^{k-1} p, \qquad k = 1, 2, \ldots,$$

$$\mathbf{E}[X] = \frac{1}{p}, \qquad \mathrm{var}(X) = \frac{1-p}{p^2}.$$

Poisson with Parameter λ: (Approximates the binomial PMF when n is large, p is small, and $\lambda = np$.)

$$p_X(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \qquad k = 0, 1, \ldots,$$

$$\mathbf{E}[X] = \lambda, \qquad \mathrm{var}(X) = \lambda.$$

Discrete Probability Distributions

- Discrete Uniform over an interval [a,b]
  - Describes equally likely events
- Bernoulli Trial with parameter p
  - Describes the success or failure in a single trial.
- Binomial Distribution with parameters p and n
  - Describes the number of successes in n independent Bernoulli trials
- Geometric Distribution with parameter p
  - Describes the number of trials until the first success, in a sequence of independent Bernoulli trials
- Poisson Distribution with parameter λ
  - Approximates the binomial PMF when n is large, p is small, and $\lambda = np$

Discrete Probability Distributions

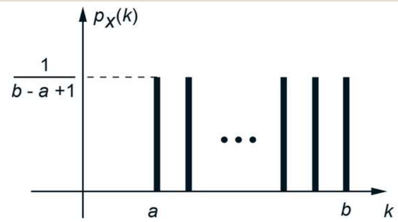


Figure 2.9: PMF of the discrete random variable that is uniformly distributed between two integers a and b. Its mean and variance are

$$\mathbf{E}[X] = \frac{a+b}{2}, \qquad \mathrm{var}(X) = \frac{(b-a)(b-a+2)}{12}.$$

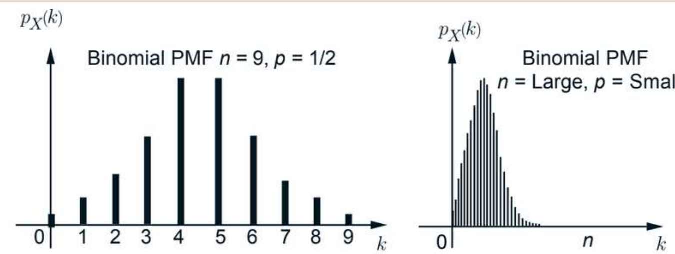


Figure 2.3: The PMF of a binomial random variable. If $p = 1/2$, the PMF is symmetric around $n/2$. Otherwise, the PMF is skewed towards 0 if $p < 1/2$, and towards n if $p > 1/2$.

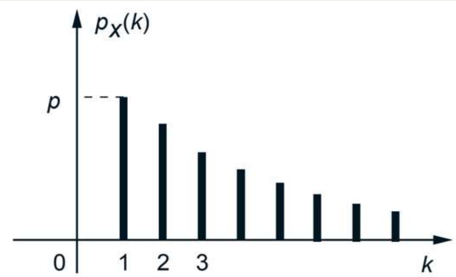


Figure 2.4: The PMF

$$p_X(k) = (1-p)^{k-1}p, \qquad k = 1, 2, \ldots,$$

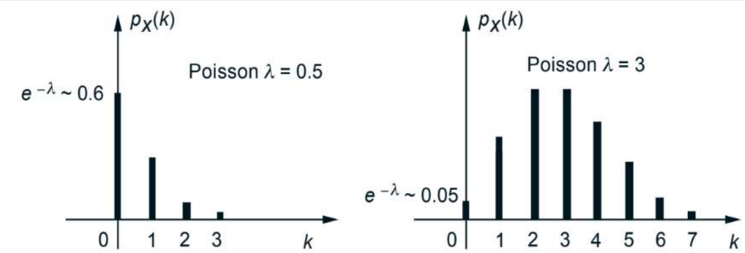


Figure 2.5: The PMF $e^{-\lambda}\frac{\lambda^k}{k!}$ of the Poisson random variable for different values of λ. Note that if $\lambda < 1$, then the PMF is monotonically decreasing, while if $\lambda > 1$, the PMF first increases and then decreases as the value of k increases (this is shown in the end-of-chapter problems).

Binomial Distribution

- There are a fixed number of trials. Think of trials as repetitions of an experiment. The letter n denotes the number of trials.
- There are only two possible outcomes, called "success" and "failure," for each trial. The letter p denotes the probability of a success on one trial, and q denotes the probability of a failure on one trial. $p + q = 1$
- The n trials are independent and are repeated using identical conditions. Because the n trials are independent, the outcome of one trial does not help in predicting the outcome of another trial.
  - Another way of saying this is that for each individual trial, the probability, p, of a success and probability, q, of a failure remain the same.
  - For example, randomly guessing at a true-false statistics question has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly.
  - Suppose Joe always guesses correctly on any statistics true-false question with probability $p = 0.6$. Then, $q = 0.4$. This means that for every true-false statistics question Joe answers, his probability of success ($p = 0.6$) and his probability of failure ($q = 0.4$) remain the same.

Toss Coins & Roll Dice

- 3 coins are tossed. Find probability of following events
  - Getting atmost one head->P(X<=1) = 1/8+3/8 = ½
  - Getting atleast one tail->P(X>=1) = 1 - 1/8 = 7/8
  - Atleast one head and one tail->6/8
  - Atleast one head and atmost one tail->1/2
- A coin is tossed 6 times. Find probability that no of heads is more than no of tails ->(C(6,6)+C(6,5)+C(6,4))/64=22/64
- A coin is tossed 4 times. Find probability of atleast 2 head and atleast 2 tails ->C(4,2)/16=6/16
- Two dice are rolled twice. Find the probability of following events
  - Getting a sum 7 twice->6/36*6/36 = 1/36
  - Getting a sum 7 exactly once->1/6*5/6 + 5/6*1/6 = 10/36
  - Not getting a sum 7 in both trials->5/6*5/6 = 25/36
  - Getting a sum 7 atleast once->1-5/6*5/6=11/36
  - Sum of two numbers is 4 given the numbers which come up where different->2/36 / 30/36
- A die is rolled 180 times. Find expected no of times that the face 4 will turn up. Find variance->180*1/6=30,variance=npq=25

Coin 1	Coin 2	Coin 3
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

Binomial Distribution Applications

- Find probability of getting a 9 exactly 2 out of 3 times with a pair of dice
  - P=4/36, q=32/36,C(3,2)*p^x*q*(n-x)=8/243
- 2 dice are rolled 120 times. Find the average no of times in which the number on the first die exceeds the no on the second die
  - Equal cases 6/36,others 30/36, half are first exceeds, other half second exceeds
  - P=15/36,n=120
  - E[X]=np=50
- There are 5% defective bulbs in a lot. Find prob that sample of 10 bulbs will include not more than 1 defective bulb
  - 19/20 ^10 + C(10,2)*1/20 * 19/20^9
- A man hits a target with probability 1/3. In 5 shots, what is probability that he has hit the target atleast twice → contrast with geometric distribution
  - P(x>=2)=1-p(x<2)=1-(p x=1 + p x =0)=1-[2/3 ^5 + C(5,1)* 1/3 * 2/3 ^4]=131/243

Poisson Distribution Applications

- For No of complaints received in an office per day λ=3.3. Find prob of
  - Exactly 2 complaints on any given day 3.3^2 e^-3.3 /2!
  - At most 2 complaints on any given day P 0 + p 1 + p 2

- A telephone switch receives 20 calls on an average in an hour. Find the probability for 5 minutes, for 5 minutes Lambda = 1.65 = 20/12
  - No call is received E^-1.65*1.65^0/0!
  - Exactly 3 calls are received E^-1.65*(1.65)^3/3!
  - At least 2 calls are received
    - 1-p(x<2)=1-(p x=0 + p x = 1)

PDF and CDF

- A random variable X is called continuous if its probability law can be described in terms of a nonnegative function f_X, called the probability density function of X, or PDF for short
  - The probability that the value of X falls within an interval is $P(a \leq X \leq b)$ and can be interpreted as the area under the graph of the PDF
  - The entire area under the graph of the PDF must be equal to 1.
  - For any single value a, we have $P(X = a) = 0$
- We have been dealing with discrete and continuous random variables in a somewhat different manner, using PMFs and PDFs, respectively.
- It would be desirable to describe all kinds of random variables with a single mathematical concept.
- This is accomplished by the cumulative distribution function, or CDF for short. The CDF of a random variable X is denoted by F_X and provides the probability $P(X \leq x)$

Continuous Probability Distributions

Continuous Uniform Over $[a, b]$:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathbf{E}[X] = \frac{a+b}{2}, \qquad \text{var}(X) = \frac{(b-a)^2}{12}.$$

Exponential with Parameter λ:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \ge 0, \\ 0 & \text{otherwise,} \end{cases} \qquad F_X(x) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \ge 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathbf{E}[X] = \frac{1}{\lambda}, \qquad \text{var}(X) = \frac{1}{\lambda^2}.$$

Normal with Parameters μ and σ^2:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2},$$

$$\mathbf{E}[X] = \mu, \qquad \text{var}(X) = \sigma^2.$$

Continuous Probability Distributions

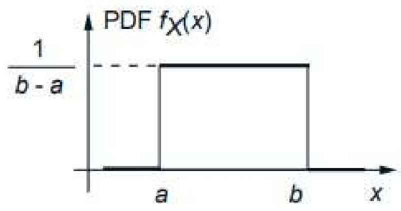


Figure 3.3: The PDF of a uniform random variable.

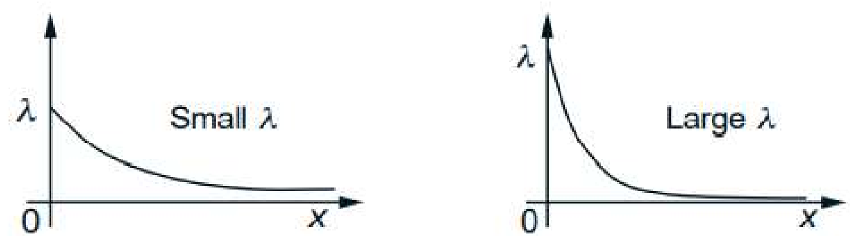


Figure 3.5: The PDF $\lambda e^{-\lambda x}$ of an exponential random variable.

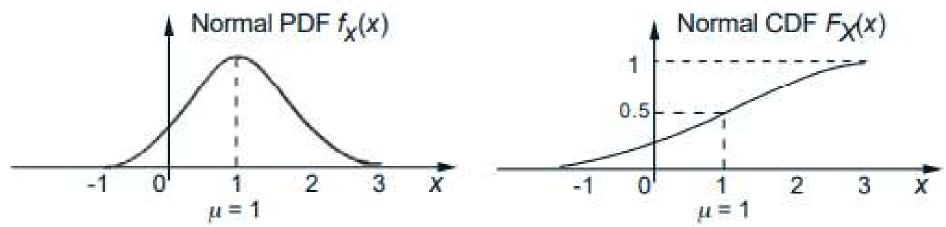


Figure 3.9: A normal PDF and CDF, with $\mu = 1$ and $\sigma^2 = 1$. We observe that the PDF is symmetric around its mean μ, and has a characteristic bell-shape. As x gets further from μ, the term $e^{-(x-\mu)^2/2\sigma^2}$ decreases very rapidly. In this figure, the PDF is very close to zero outside the interval $[-1, 3]$.

Standard Normal Distribution

- A normal random variable Y with zero mean and unit variance is said to be a standard normal.
- Let X be a normal random variable with mean μ and variance σ^2. We "standardize" X by defining a new random variable Z given by $Z = (X - \mu) / \sigma$.

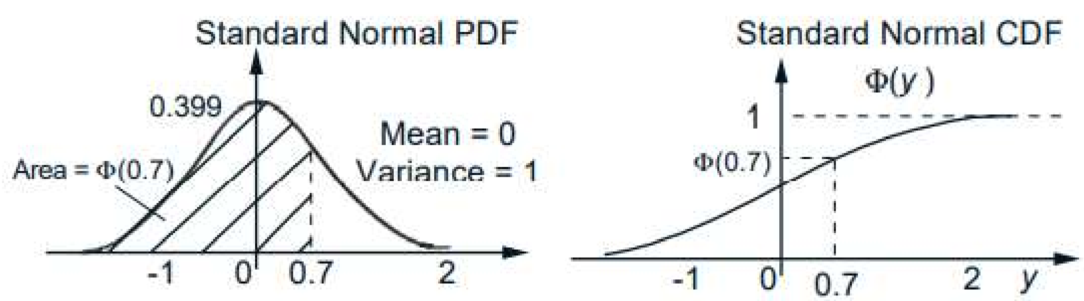


Figure 3.10: The PDF

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}$$

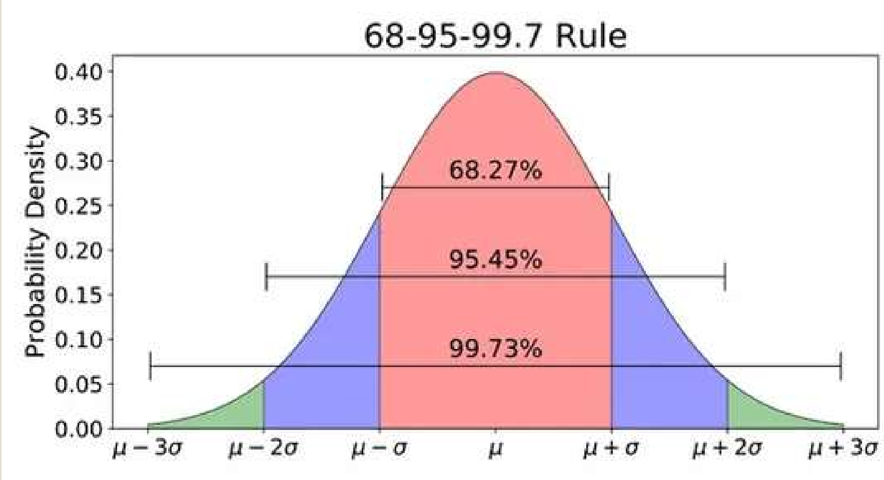


Table entry for z is the area under the standard normal curve to the left of z.

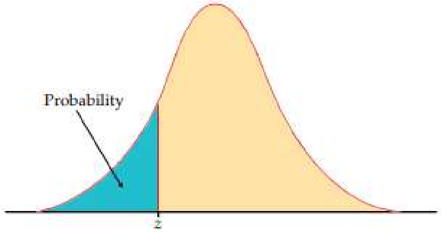


TABLE A

Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard normal curve to the left of z.

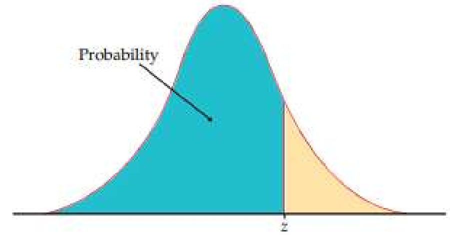


TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Standard Normal Distribution Examples

- Mean 20, Std dev 3.33,
  - find prob less than 21.11 and 26.66
  - find prob less than 26.66
  - find prob bw 21.11 and 26.66
  - find prob more than 26.66

- Mean 30, std dev 5, find prob bw $|x-30|>5$

Using the Normal Curve in Reverse

- Given a normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of x that has
  - 45% of the area to the left
  - 14% of the area to the right.

Standard Normal Distribution Examples

- A die is rolled 180 times. Find prob that the face 4 will turn up atleast 35 times

- The annual snowfall at a particular geographic location is modeled as a normal random variable with a mean of $\mu = 60$ inches, and a standard deviation of $\sigma = 20$. What is the probability that this year's snowfall will be at least 80 inches?

- An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Thank You !!!