

ROBOTIC PERCEPTION - ASSIGNMENT 4

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + T \begin{bmatrix} \cos \theta_{k-1} & 0 \\ \sin \theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} v_k \\ w_k \end{bmatrix} + w_k \right)$$

$$w_k \sim N(0, Q)$$

$$\begin{bmatrix} r_k \\ \phi_k \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \text{atan2}(-y_k, -x_k) - \theta_k \end{bmatrix} + n_k \quad n_k \sim N(0, R)$$

Given motion model is similar to Odometry motion model

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \delta_{\text{trans}} \cos(\theta + \delta_{\text{rot}_1}) \\ \delta_{\text{trans}} \sin(\theta + \delta_{\text{rot}_1}) \\ \delta_{\text{rot}_1} + \delta_{\text{rot}_2} \end{bmatrix} + N(0, R_t)$$

Linearization:

$$g(u_t, \mu_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t(x_{t-1}, \mu_{t-1}).$$

$$G_t = \frac{\partial g}{\partial x_{t-1}}(u_t, \mu_{t-1}) + \begin{pmatrix} \frac{\partial g}{\partial u_t, x} \frac{\partial g}{\partial \mu_{t-1}, x} & \frac{\partial g}{\partial u_t, y} \frac{\partial g}{\partial \mu_{t-1}, y} & \frac{\partial g}{\partial u_t, \theta} \frac{\partial g}{\partial \mu_{t-1}, \theta} \\ \frac{\partial g}{\partial u_t, y} \frac{\partial g}{\partial \mu_{t-1}, x} & \frac{\partial g}{\partial u_t, y} \frac{\partial g}{\partial \mu_{t-1}, y} & \frac{\partial g}{\partial u_t, \theta} \frac{\partial g}{\partial \mu_{t-1}, \theta} \\ \frac{\partial g}{\partial u_t, \theta} \frac{\partial g}{\partial \mu_{t-1}, x} & \frac{\partial g}{\partial u_t, \theta} \frac{\partial g}{\partial \mu_{t-1}, y} & \frac{\partial g}{\partial u_t, \theta} \frac{\partial g}{\partial \mu_{t-1}, \theta} \end{pmatrix}$$

$$(u_t, \delta_{\text{trans}}, \delta_{\text{rot}}) \cdot \begin{pmatrix} \frac{\partial g}{\partial u_t, x} & \frac{\partial g}{\partial u_t, y} & \frac{\partial g}{\partial u_t, \theta} \\ \frac{\partial g}{\partial \delta_{\text{trans}}, x} & \frac{\partial g}{\partial \delta_{\text{trans}}, y} & \frac{\partial g}{\partial \delta_{\text{trans}}, \theta} \\ \frac{\partial g}{\partial \delta_{\text{rot}}, x} & \frac{\partial g}{\partial \delta_{\text{rot}}, y} & \frac{\partial g}{\partial \delta_{\text{rot}}, \theta} \end{pmatrix}$$

Comparing given equations with the odometry equations

$$g(x_t, z_{t-1})$$

$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} = \begin{pmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{pmatrix} + T \begin{bmatrix} \cos \theta_{k-1} & 0 \\ \sin \theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix}$$

$$(s.o) W + w + T \begin{bmatrix} \cos \theta_{k-1} & 0 \\ \sin \theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k$$

$$(s.o) W + w + \begin{bmatrix} x_{k-1} + T \cos \theta_{k-1} u_k \\ y_{k-1} + T \sin \theta_{k-1} u_k \\ \theta_{k-1} + T w_k \end{bmatrix}$$

$$g \begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} = \begin{bmatrix} x_{k-1} + T \cos \theta_{k-1} u_k \\ y_{k-1} + T \sin \theta_{k-1} u_k \\ \theta_{k-1} + T w_k \end{bmatrix}$$

$$G_t = \begin{bmatrix} \frac{\partial(x_{k-1} + T \cos \theta_{k-1} u_k)}{\partial x_{k-1}} & \frac{\partial(x_{k-1} + T \cos \theta_{k-1} u_k)}{\partial y_{k-1}} & \frac{\partial(x_{k-1} + T \cos \theta_{k-1} u_k)}{\partial \theta_{k-1}} \\ \frac{\partial(y_{k-1} + T \sin \theta_{k-1} u_k)}{\partial x_{k-1}} & \frac{\partial(y_{k-1} + T \sin \theta_{k-1} u_k)}{\partial y_{k-1}} & \frac{\partial(y_{k-1} + T \sin \theta_{k-1} u_k)}{\partial \theta_{k-1}} \\ \frac{\partial(\theta_{k-1} + T w_k)}{\partial x_{k-1}} & \frac{\partial(\theta_{k-1} + T w_k)}{\partial y_{k-1}} & \frac{\partial(\theta_{k-1} + T w_k)}{\partial \theta_{k-1}} \end{bmatrix}$$

$$\frac{\partial(x_{k-1} + T \cos \theta_{k-1} u_k)}{\partial x_{k-1}} = \frac{\partial x_{k-1}}{\partial x_{k-1}} + T \frac{\partial \cos \theta_{k-1}}{\partial x_{k-1}} u_k$$

$$= 1$$

$$\frac{\partial x_k}{\partial \theta_{k-1}} = \frac{\partial(x_{k-1} + T \cos \theta_{k-1} u_k)}{\partial \theta_{k-1}} = -T \sin \theta_{k-1} u_k$$

$$\frac{\partial y_k}{\partial \theta_{k-1}} = \frac{\partial(y_{k-1} + T \sin \theta_{k-1} u_k)}{\partial \theta_{k-1}} = 1$$

$$\frac{\partial y_k}{\partial \theta_{k-1}} = T \cos \theta_{k-1} u_k \begin{bmatrix} \sin \theta_{k-1} & \cos \theta_{k-1} \\ -\cos \theta_{k-1} & \sin \theta_{k-1} \end{bmatrix} \begin{bmatrix} T & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{c} \frac{\partial \theta_{k-1}}{\partial \theta_{k-1}} \\ \frac{\partial \theta_{k-1}}{\partial \theta_{k-1}} \end{array} \right] \left[\begin{array}{c} \frac{\partial (y_k + T u_k)}{\partial \theta_{k-1}} \\ \frac{\partial (y_k + T u_k)}{\partial \theta_{k-1}} \end{array} \right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T & 0 \\ 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & -T \sin \theta_{k-1} u_k \\ 0 & 1 & T \cos \theta_{k-1} u_k \\ 0 & 0 & 1 \end{bmatrix}$$

Since mean of Gaussian error is 0

$$\mu_{w_t} = 0$$

Modified covariance:

$$\bar{\Omega} = E \{ (w_t - \mu_{w_t})(w_t - \mu_{w_t})^T \} \times \frac{1}{m}$$

$$= E \{ w_t w_t^T \}$$

$$= E \{ T a \omega_k \omega_k^T a^T T^T \} \times \frac{1}{m}$$

Assuming T to be a scalar and a a unit vector

$$= T^2 a E \{ \omega_k \omega_k^T \} a^T$$

$$\frac{E \{ \omega_k \omega_k^T \}}{m} = \text{covariance}$$

$$\Omega_t = E \{ \omega_k \omega_k^T \} \times \frac{1}{m}$$

$$\Omega_t = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} \quad d \in (m, x)$$

$$Q = T^2 \alpha \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} \alpha^T$$

$$Q = T^2 \begin{bmatrix} \cos \theta_{k-1} & 0 \\ \sin \theta_{k-1} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} \begin{bmatrix} \cos \theta_{k-1} & \sin \theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q = T^2 \begin{bmatrix} \sigma_a^2 \cos \theta_{k-1} & \sigma_{ab} \cos \theta_{k-1} \\ \sigma_a^2 \sin \theta_{k-1} & \sigma_{ab} \sin \theta_{k-1} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} \begin{bmatrix} \cos \theta_{k-1} & \sin \theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q = T^2 \begin{bmatrix} \sigma_a^2 \cos^2 \theta_{k-1} & \sigma_a^2 \cos \theta_{k-1} \sin \theta_{k-1} & \sigma_{ab} \cos \theta_{k-1} \\ \sigma_a^2 \sin \theta_{k-1} \cos \theta_{k-1} & \sigma_a^2 \sin^2 \theta_{k-1} & \sigma_{ab} \sin \theta_{k-1} \\ \sigma_{ba} \cos \theta_{k-1} & \sigma_{ba} \sin \theta_{k-1} & \sigma_b^2 \end{bmatrix}$$

$$\Sigma = \{ (w_1 - \bar{w})(w_2 - \bar{w}) \} \dots$$

MEASUREMENT :

$$\begin{bmatrix} r_k \\ \phi_k \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \text{atan2}(y_k, -x_k) - \theta_{k-1} \end{bmatrix} + n_k \quad n_k \sim N(0, R).$$

Comparing with odometry model

$$z_t^i = \begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix} = \begin{bmatrix} \sqrt{(m_j.x - x)^2 + (m_j.y - y)^2} \\ \text{atan2}(m_j.y - y, m_j.x - x) - \theta \end{bmatrix} + N(0, Q_t)$$

$\underbrace{\qquad\qquad\qquad}_{h(x_t, m)}$

$$h(x_t, m) \approx h(\bar{x}_t) + H_t^i(x_t - \bar{x}_t)$$

$$H_t = \begin{bmatrix} \frac{\delta r_t}{\delta x_t, x} & \frac{\delta r_t}{\delta x_t, y} & \frac{\delta r_t}{\delta x_t, \theta} \\ \frac{\delta \phi_t}{\delta x_t, x} & \frac{\delta \phi_t}{\delta x_t, y} & \frac{\delta \phi_t}{\delta x_t, \theta} \end{bmatrix}$$

$$h(x) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \arctan 2(y_k/x_k) - \theta_k \end{bmatrix}$$

$$\frac{\delta r_t}{\delta x} = \frac{\delta(\sqrt{x_k^2 + y_k^2})}{\delta x_k} = \frac{2x_k}{2\sqrt{x_k^2 + y_k^2}} = \frac{x_k}{\sqrt{x_k^2 + y_k^2}}$$

$$\frac{\delta r_t}{\delta y} = \frac{\delta(\sqrt{x_k^2 + y_k^2})}{\delta y_k} = \frac{y_k}{\sqrt{x_k^2 + y_k^2}}$$

Writing Φ in term of x_k

$$\frac{\partial \Phi_k}{\partial x_k} = \frac{\partial \left(\tan^{-1}(y_k/x_k) - \theta_k \right)}{\partial x_k} \cdot \frac{1}{1 + (y_k/x_k)^2} \left(\frac{-y_k}{x_k^2} \right)$$

$$= \frac{-y_k}{x_k^2 + y_k^2}$$

$$\frac{\partial \Phi_k}{\partial y_k} = \frac{x_k}{x_k^2 + y_k^2}$$

$$\frac{\partial \Phi_k}{\partial \theta_k} = -1$$

$$H = \begin{bmatrix} \frac{x_k}{\sqrt{x_k^2 + y_k^2}} & \frac{y_k}{\sqrt{x_k^2 + y_k^2}} & 0 \\ \frac{-y_k}{x_k^2 + y_k^2} & \frac{x_k}{\sqrt{x_k^2 + y_k^2}} & -1 \end{bmatrix}$$

Modified covariance \bar{R} is same as the noise covariance R .

$$\begin{bmatrix} 0.12 & 0.08 & 0.08 \\ 0.08 & 0.12 & 0.08 \\ 0.08 & 0.08 & 0.12 \end{bmatrix}$$

$$\begin{bmatrix} 0.12 + 0.08 & 0 \\ 0 & 0.12 + 0.08 \end{bmatrix} = (r) I$$

$$0.2 - (0.2 \times 0.2) \leq \text{const}$$

$$\begin{bmatrix} 0.12 & 0.08 & 0.08 \\ 0.08 & 0.12 & 0.08 \\ 0.08 & 0.08 & 0.12 \end{bmatrix} = \begin{bmatrix} 0.12 + 0.08 & 0 \\ 0 & 0.12 + 0.08 \end{bmatrix} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$\begin{bmatrix} 0.12 & 0.08 & 0.08 \\ 0.08 & 0.12 & 0.08 \\ 0.08 & 0.08 & 0.12 \end{bmatrix} = \begin{bmatrix} 0.12 + 0.08 & 0 \\ 0 & 0.12 + 0.08 \end{bmatrix} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$

sub for const in the previous

$$\left(\frac{0.12}{0.2} \right) \frac{1}{c} \cdot \frac{(0.2 - (0.2 \times 0.2))^{\text{const}}}{0.12} = \frac{0.12}{0.12} = 1$$

$$\begin{bmatrix} 0.12 & 0.08 & 0.08 \\ 0.08 & 0.12 & 0.08 \\ 0.08 & 0.08 & 0.12 \end{bmatrix}$$

$$\begin{bmatrix} 0.12 & 0.08 & 0.08 \\ 0.08 & 0.12 & 0.08 \\ 0.08 & 0.08 & 0.12 \end{bmatrix}$$

$$1 - \frac{0.06}{0.12} = 1 - 0.5 = 0.5$$

$$\begin{bmatrix} 0 & 0.12 & 0.08 \\ 0.08 & 0.12 & 0.08 \\ 0.08 & 0.08 & 0.12 \end{bmatrix} = I$$