ROBOTIC PERCEPTION - ASSIGNMENT 4

$$\begin{bmatrix} x_K \\ y_K \\ \theta_K \end{bmatrix} = \begin{bmatrix} x_{K-1} \\ y_{K-1} \\ \theta_{K-1} \end{bmatrix} + T \begin{bmatrix} \cos\theta_{K-1} & 0 \\ \sin\theta_{K-1} & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} V_K \\ \omega_K \end{bmatrix} + W_K \right)$$

$$w_K \sim \mathcal{N}(0, Q)$$

$$\begin{bmatrix} r_K \\ \phi_K \end{bmatrix} = \begin{bmatrix} \sqrt{x_K^2 + y_K^2} \\ \operatorname{atan2}(-y_K, -x_K) - \theta_K \end{bmatrix} + n_K \qquad n_K \sim \mathcal{N}(0, R)$$

Given motion model is similar to Odometry motion model

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \underbrace{\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \delta_{trans} \cos(\theta + \delta_{rot1}) \\ \delta_{trans} \sin(\theta + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{bmatrix}}_{g(u_t, x_{t-1})} + \mathcal{N}(0, R_t)$$

Linearization:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t(x_{t-1}, \mu_{t-1}).$$

$$G_t = \frac{\delta g(u_t, \mu_{t-1})}{\delta x_{t-1}} = \begin{pmatrix} \frac{\delta x'}{\delta \mu_{t-1,x}} & \frac{\delta x'}{\delta \mu_{t-1,y}} & \frac{\delta x'}{\delta \mu_{t-1,\theta}} \\ \frac{\delta y'}{\delta \mu_{t-1,x}} & \frac{\delta y'}{\delta \mu_{t-1,y}} & \frac{\delta y'}{\delta \mu_{t-1,\theta}} \\ \frac{\delta \theta'}{\delta \mu_{t-1,x}} & \frac{\delta \theta'}{\delta \mu_{t-1,y}} & \frac{\delta \theta'}{\delta \mu_{t-1,\theta}} \end{pmatrix}$$

Comparing given equations with the odometry equations

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \underbrace{\begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + T \begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix}}_{g(u_t,\, x_{t-1})}$$

$$+ T \begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k$$

$$g\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} = \begin{bmatrix} x_{k-1} + T\cos\theta_{k-1} u_k \\ y_{k-1} + T\sin\theta_{k-1} u_k \\ \theta_{k-1} + T w_k \end{bmatrix}$$

$$G_t = \begin{bmatrix} \frac{\partial x_k}{\partial x_{k-1}} & \frac{\partial x_k}{\partial y_{k-1}} & \frac{\partial x_k}{\partial \theta_{k-1}} \\ \frac{\partial y_k}{\partial x_{k-1}} & \frac{\partial y_k}{\partial y_{k-1}} & \frac{\partial y_k}{\partial \theta_{k-1}} \\ \frac{\partial \theta_k}{\partial x_{k-1}} & \frac{\partial \theta_k}{\partial y_{k-1}} & \frac{\partial \theta_k}{\partial \theta_{k-1}} \end{bmatrix}$$

$$\frac{\partial(x_{k-1} + T\cos\theta_{k-1} u_k)}{\partial x_{k-1}} = \frac{\partial x_{k-1}}{\partial x_{k-1}} + \frac{T\cos\theta_{k-1} u_k}{\partial x_{k-1}}$$

$$= 1$$

$$\frac{\partial x_k}{\partial \theta_{k-1}} = \frac{\partial(x_{k-1} + T\cos\theta_{k-1} u_k)}{\partial \theta_{k-1}} = -T\sin\theta_{k-1} u_k$$

$$\frac{\partial y_k}{\partial y_{k-1}} = \frac{\partial(y_{k-1} + T\sin\theta_{k-1} u_k)}{\partial y_{k-1}} = 1$$

$$\frac{\partial y_k}{\partial \theta_{k-1}} = T\cos\theta_{k-1}\, u_k$$

$$\frac{\partial \theta_k}{\partial \theta_{k-1}} = \frac{\partial(\theta_{k-1} + T w_k)}{\partial \theta_{k-1}} = 1$$

$$F = \begin{bmatrix} 1 & 0 & -T\sin\theta_{k-1}\, u_k \\ 0 & 1 & T\cos\theta_{k-1}\, u_k \\ 0 & 0 & 1 \end{bmatrix}$$

Since mean of Gaussian error is 0

$$\mu_{w_t} = 0.$$

Modified covariance:

$$\bar{Q} = E\{(w_t - \mu_{w_t})(w_t - \mu_{w_t})^T\} \times \frac{1}{m}$$

$$= \frac{E\{w_t w_t^T\}}{m}$$

$$= E\left\{\frac{T a w_k w_k^T a^T T}{m}\right\}$$

Assuming T to be a scalar

$$= T^2 a\, E\left\{\frac{w_k w_k^T}{m}\right\} a^T$$

$$E\left\{\frac{w_k w_k^T}{m}\right\} = \text{covariance}$$

$$Q_t = E\left\{\frac{w_k w_k^T}{m}\right\}$$

$$Q_t = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix}$$

$$Q = T^2 a \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} a^T$$

$$Q = T^2 \begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} \begin{bmatrix} \cos\theta_{k-1} & \sin\theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= T^2 \begin{bmatrix} \sigma_a^2\cos\theta_{k-1} & \sigma_{ab}\cos\theta_{k-1} \\ \sigma_a^2\sin\theta_{k-1} & \sigma_{ab}\sin\theta_{k-1} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} \begin{bmatrix} \cos\theta_{k-1} & \sin\theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q = T^2 \begin{bmatrix} \sigma_a^2\cos^2\theta_{k-1} & \sigma_1^2\cos\theta_{k-1}\sin\theta_{k-1} & \sigma_{ab}\cos\theta_{k-1} \\ \sigma_a^2\sin\theta_{k-1}\cos\theta_{k-1} & \sigma_1^2\sin^2\theta_{k-1} & \sigma_{ab}\sin\theta_{k-1} \\ \sigma_{ba}\cos\theta_{k-1} & \sigma_{ba}\sin\theta_{k-1} & \sigma_b^2 \end{bmatrix}$$

MEASUREMENT:

$$\begin{bmatrix} r_k \\ \phi_k \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \operatorname{atan2}(y_k, -x_k) - \theta_k \end{bmatrix} + n_k \qquad n_k \sim \mathcal{N}(0, R).$$

Comparing with odometry model

$$z_t^i = \begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \operatorname{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \end{pmatrix}}_{h(x,t)} + \mathcal{N}(0, Q_t)$$

$$h(x_t, m) \approx h(\bar{\mu}_t) + H_t^i (x_t - \bar{\mu}_t)$$

$$H_t = \begin{bmatrix} \dfrac{\delta r_t}{\delta \mu_{t,x}} & \dfrac{\delta r_t}{\delta \mu_{t,y}} & \dfrac{\delta r_t}{\delta \mu_{t,\theta}} \\[2ex] \dfrac{\delta \phi_t}{\delta \mu_{t,x}} & \dfrac{\delta \phi_t}{\delta \mu_{t,y}} & \dfrac{\delta \phi_t}{\delta \mu_{t,\theta}} \end{bmatrix}$$

$$h(x) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\[2ex] \operatorname{atan2}(y_k/x_k) - \theta_k \end{bmatrix}$$

$$\frac{\delta r_t}{\partial x} = \frac{\delta\left(\sqrt{x_k^2+y_k^2}\right)}{\delta x_k} = \frac{2x_k}{2\sqrt{x_k^2+y_k^2}} = \frac{x_k}{\sqrt{x_k^2+y_k^2}}$$

$$\frac{\delta r_t}{\delta y} = \frac{\delta\left(\sqrt{x_k^2+y_k^2}\right)}{\delta y_k} = \frac{y_k}{\sqrt{x_k^2+y_k^2}}$$

$$\frac{\partial \phi_k}{\partial x_k} = \frac{\partial\left(\tan^{-1}(y_k/x_k) - \theta_k\right)}{\partial x_k} = \frac{1}{1+(y_k/x_k)^2}\left(\frac{-y_k}{x_k^2}\right)$$

$$= \frac{-y_k}{x_k^2+y_k^2}$$

$$\frac{\partial \phi_k}{\partial y_k} = \frac{+x_k}{x_k^2+y_k^2}$$

$$\frac{\partial \phi_k}{\partial \theta_k} = -1$$

$$H = \begin{bmatrix} \dfrac{x_k}{\sqrt{x_k^2+y_k^2}} & \dfrac{y_k}{\sqrt{x_k^2+y_k^2}} & 0 \\[2ex] \dfrac{-y_k}{x_k^2+y_k^2} & \dfrac{x_k}{\sqrt{x_k^2+y_k^2}} & -1 \end{bmatrix}$$

Modified covariance $\bar{R}$ is same as the noise covariance R.