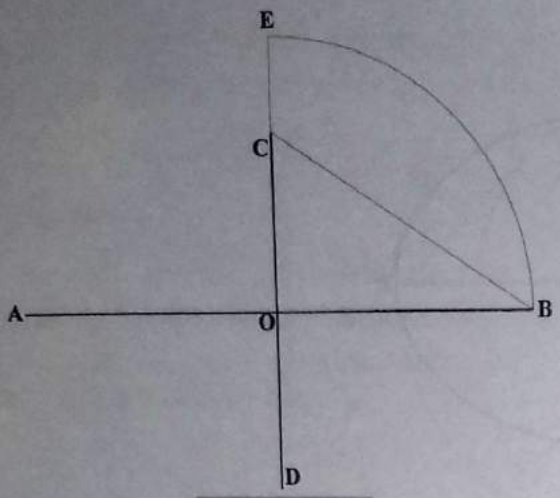


Figure 2.50(c)

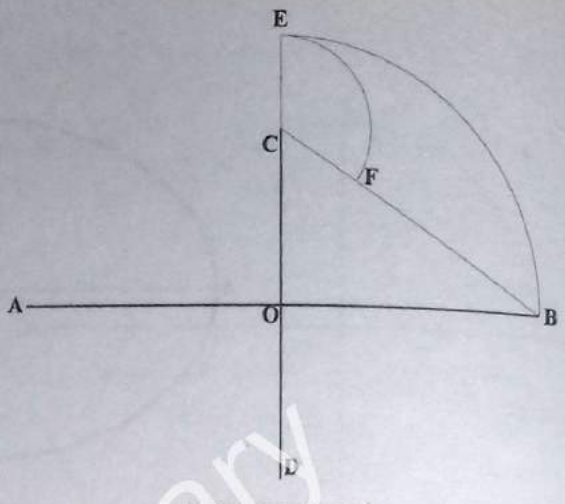
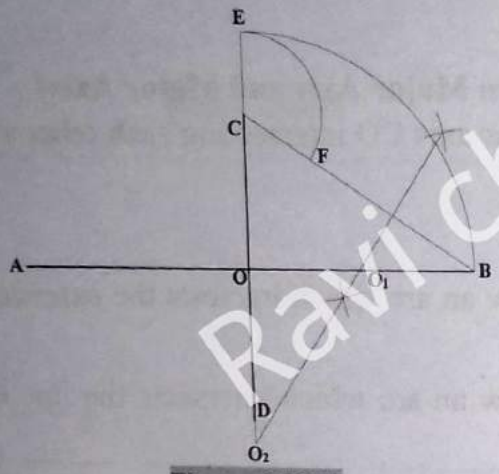


Figure 2.50(d)



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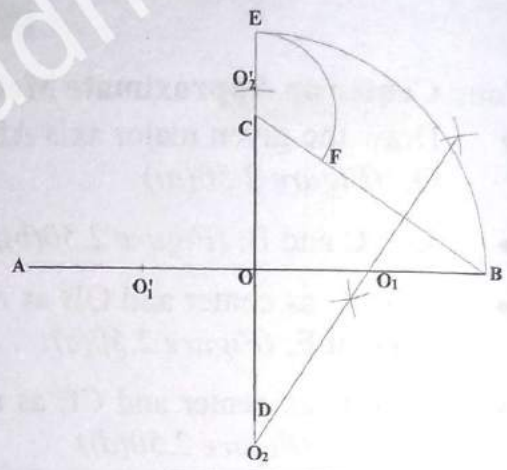


Figure 2.50(f)

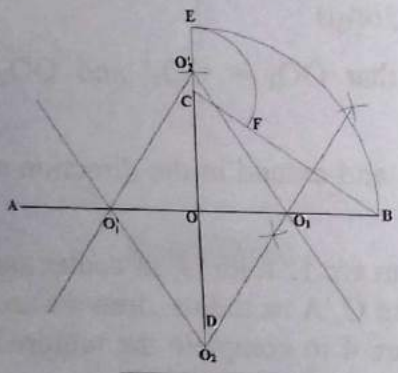


Figure 2.50(g)

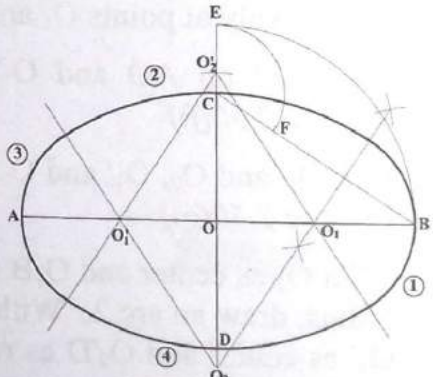


Figure 2.50(h)

2.4.4 Construction of a Hyperbola

(a) Definition Method (Given Eccentricity and Focus)

- Draw a straight line AB as the directrix of the hyperbola and draw another line OC perpendicular to AB and passing through the midpoint of the AB as an axis of the hyperbola. Locate the given focus F on the axis OC according to the given distance. *(Figure 2.51(a))*
- Assume that given eccentricity is 1.5. Divide OF in the proportion of 2:3 to locate vertex V of the hyperbola. *(Figure 2.51(b))*
- Mark any point M on the axis OC at any convenient distance, say 30. Draw a line perpendicular to the axis OC at point M. *(Figure 2.51(c))*

- With O as center and radius equal to 1.5 (eccentricity) times the OM (= 45), draw an arc intersecting the vertical line passing through M at point N. *(Figure 2.51(d))*
- Draw any number of lines parallel to the directrix AB and right to the vertex V which intersect the axis at points 1, 2, 3.... and the line ON at 1', 2', 3', ... respectively. *(Figure 2.51(e))*
- With F as a center and the O1' as radius, draw arcs on both sides of the axis intersecting the line passing through 1 at points P_1 and P_1'. *(Figure 2.51(f))*
- Again with F as a center and O2' as radius, draw arcs on both sides of the axis intersecting the line passing through 2 at points P_2 and P_2'. *(Figure 2.51(g))*
- In the similar way, determine points P_3, P_3', P_4, P_4', ... and so on. *(Figure 2.45(h))*
- Join all the points V, P_1, P_2, P_3, P_4, ... and P_1', P_2', P_3', P_4', ... by a smooth curve to get the required hyperbola. *(Figure 2.51(i))*

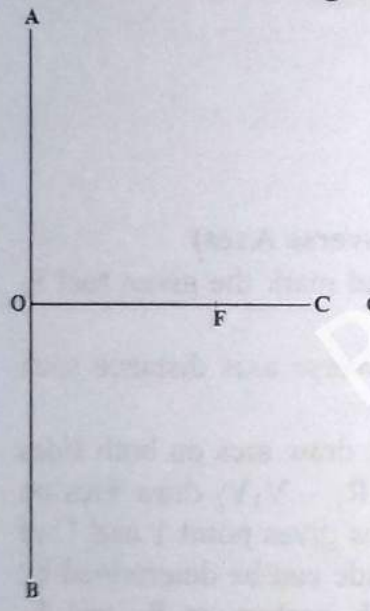


Figure 2.51(a)

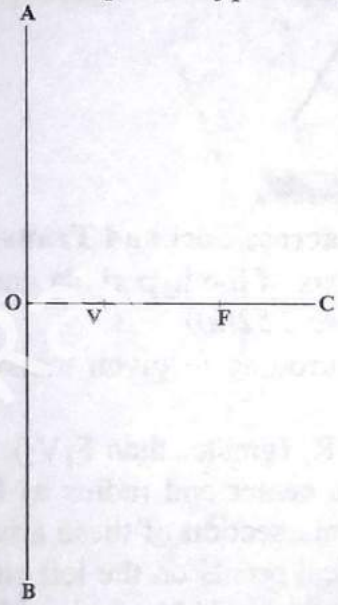


Figure 2.51(b)

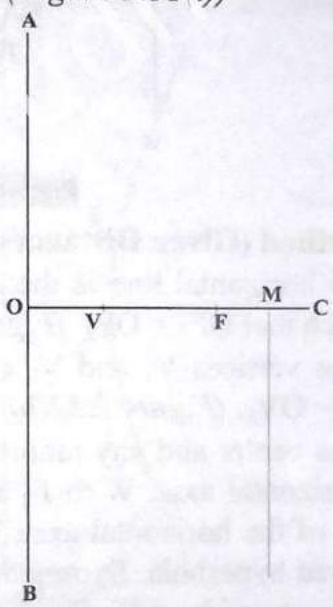
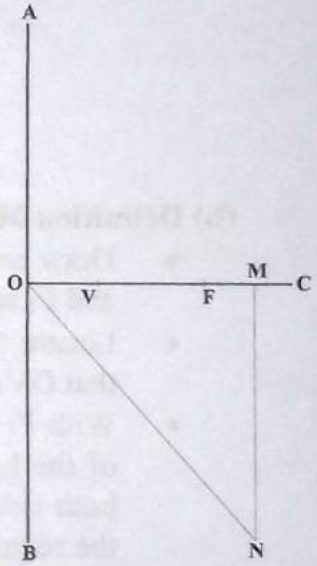


Figure 2.51(c)



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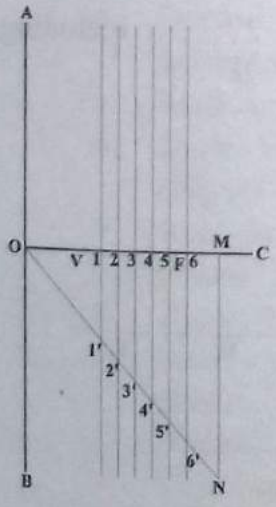

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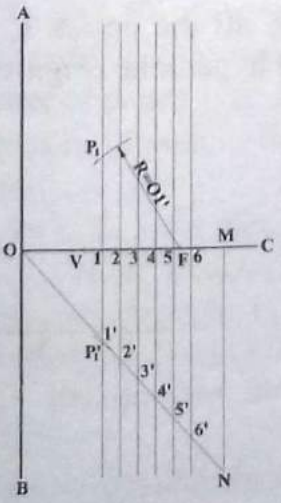


Figure 2.51(f)

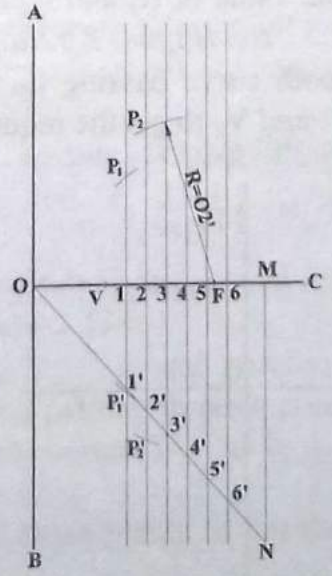


Figure 2.51(g)

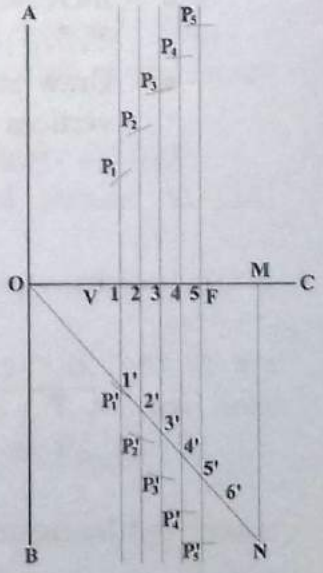


Figure 2.51(h)

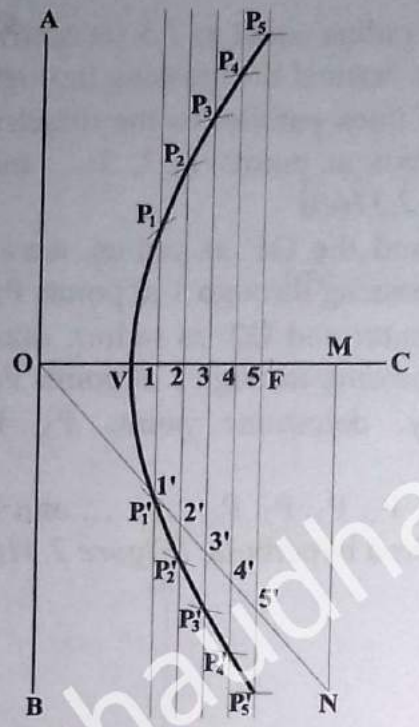


Figure 2.51(i)

(b) Definition Method (Given Distances across Foci and Transverse Axes)

- Draw any horizontal line as the axis of the hyperbola and mark the given foci F_1 and F_2 such that $OF_1 = OF_2$. *(Figure 2.52(a))*
- Locate the vertices V_1 and V_2 corroding to given transverse axis distance such that $OV_1 = OV_2$. *(Figure 2.52(b))*
- With F_1 as center and any radius R_1 (greater than F_1V_2), draw arcs on both sides of the horizontal axis. With F_2 as center and radius as $R_1 - V_1V_2$ draw arcs on both side of the horizontal axis. Intersection of these arcs gives point 1 and 1' of the required hyperbola. Symmetrical points on the left side can be determined by drawing arcs with radii R_1 and $R_1 - V_1V_2$ and with centers at F_2 and F_1 respectively. *(Figure 2.52(c))*
- Increase the value of R_1 and repeat the same procedure to get the other points 2, 2', 3, 3', ... etc. *(Figure 2.52(d))*
- Draw smooth curve passing through all the points 1, 1', 2, 2', ... including vertices V_1 and V_2 to get the required hyperbola. *(Figure 2.52(e))*

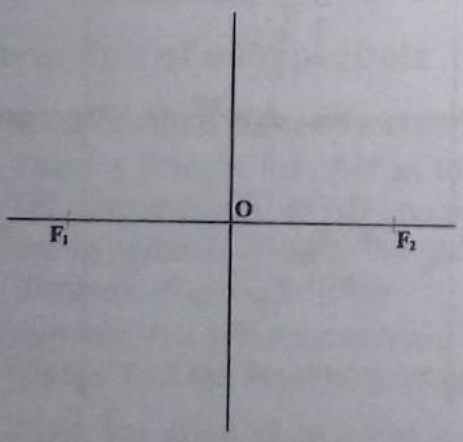


Figure 2.52(a)

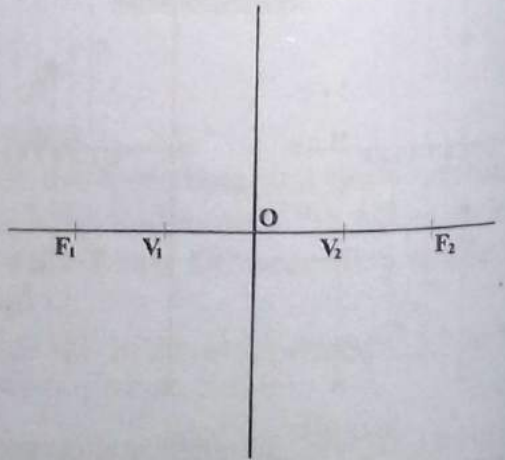
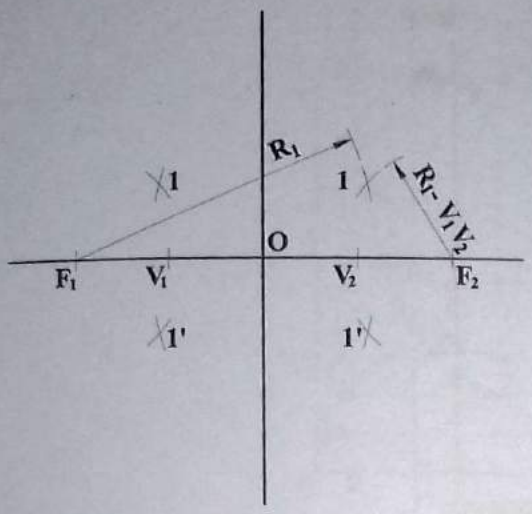


Figure 2.52(b)



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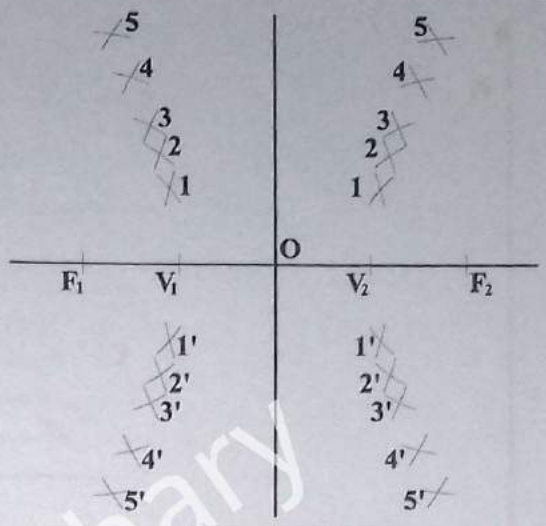


Figure 2.52(d)

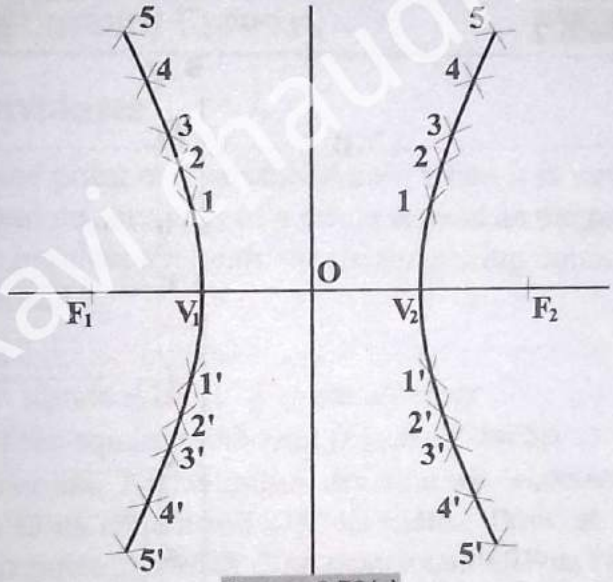


Figure 2.52(e)

(c) Rectangular Hyperbola (Given Asymptotes and Any Point of the Hyperbola)

- Draw given asymptotes OA and OB and mark given point P_0. *(Figure 2.53(a))*
- Draw lines CD and EF passing through point P_0 and parallel to the given asymptotes OA and OB. *(Figure 2.53(b))*
- Mark any number of points 1, 2, 3, … on line segment EP_0. *(Figure 2.53(c))*
- Draw lines passing through the points 1, 2, 3, …. and parallel to OA. *(Figure 2.53(d))*
- Joint each points 1, 2, 3, … with the point O and extend them to intersect CD at points 1', 2', 3', …. respectively. *(Figure 2.53(e))*
- Draw lines passing through 1', 2', 3', ….. and parallel to OB. Mark the intersection of lines parallel to OA and passing through points 1, 2, 3… and lines parallel to OB and passing through points 1', 2', 3', … as P_1, P_2, P_3, …. *(Figure 2.53(f))*
- Draw smooth curve passing through these points to get the required hyperbola. *(Figure 2.53(g))*

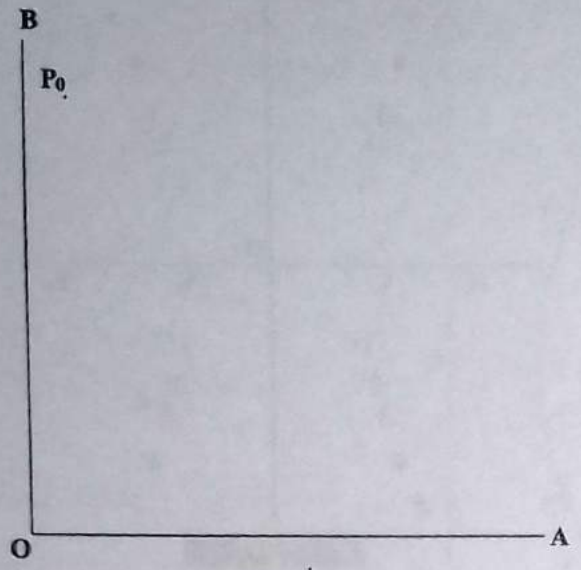
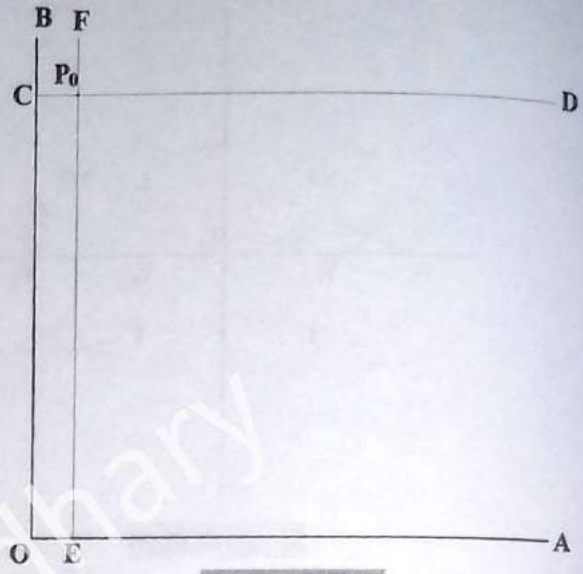


Figure 2.53(a)



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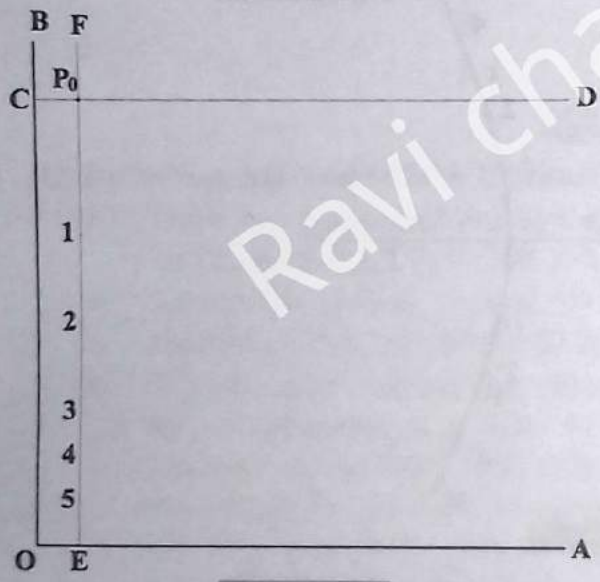

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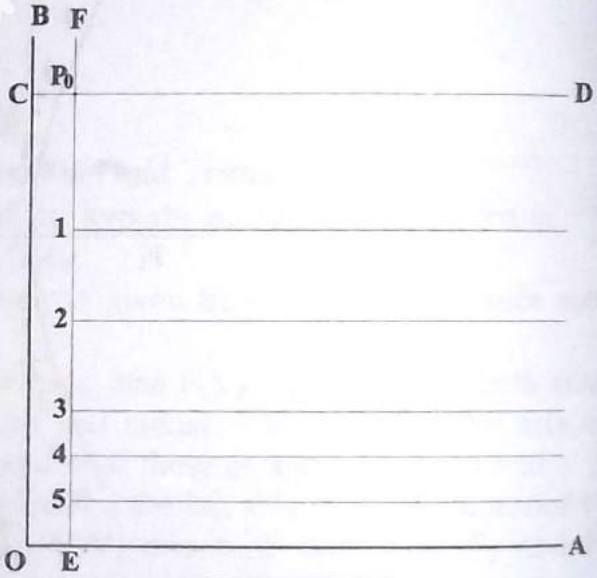

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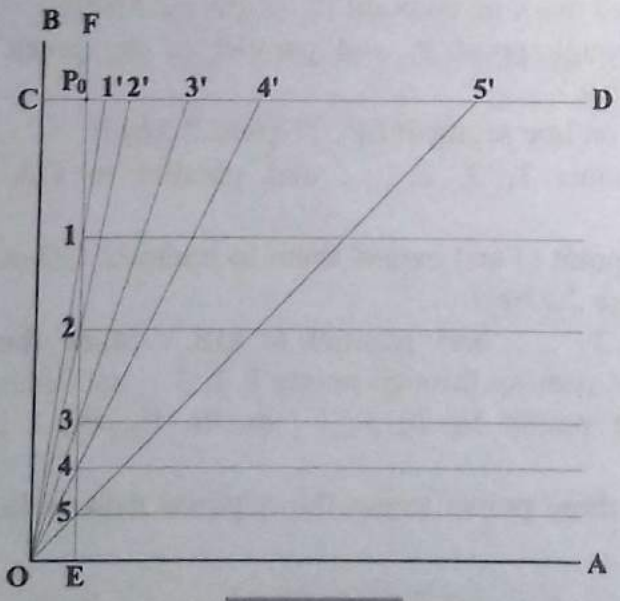

Figure 2.53(e)

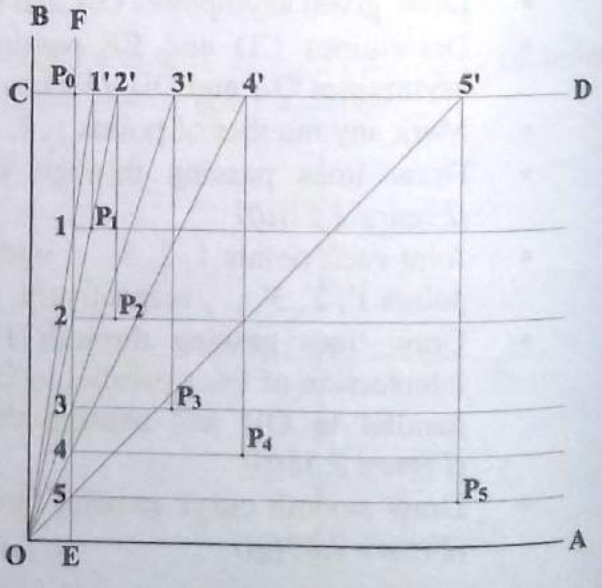

Figure 2.53(f)

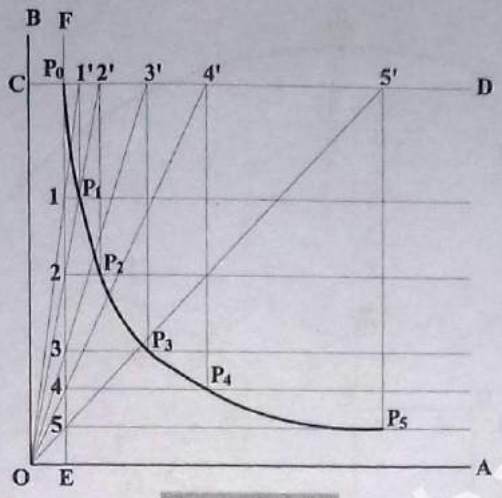

Figure 2.53(g)

2.5 Construction of Standard Curves

2.5.1 Construction of Involutes

The curve traced by a fixed point on a stretched cord when it is unwounded from a circle or a polygon is called an involute. Involute of a circle is used as the profile of gear teeth. Cams are often designed to the involute shape to ensure the rolling contact between the roller and the follower at constant speed.

(a) Involute of a Square

- Draw the given square ABCD. *(Figure 2.54(a))*
- Extend sides of the square as shown. *(Figure 2.54(b))*
- With A as center and AB as radius draw an arc intersecting the extended DA at point A'. With D as center and DA' as radius, draw an arc which intersects the extended CD at point D'. With C as center and CD' as radius, draw an arc which intersects the extended BC at point C'. With B as center and BC' as radius, draw an arc which intersects the extended AB at point B'. *(Figure 2.54(c))*

When one turn of the cord is taken out from the square, the length of the cord (BB') will be equal to perimeter of the square.

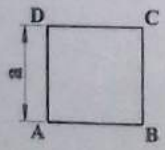

Figure 2.54(a)

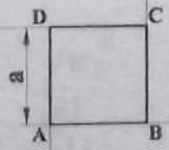

Figure 2.54(b)