**What is Set Theory? Definition, Types, Operations**

**Set Theory** is a branch of logical mathematics that studies the collection of objects and operations based on it. A set is simply a collection of objects or a group of objects. For example, a group of players in a football team is a set and the players in the team are its objects.

The words collection, aggregate, and class are synonymous with set. On the other hand elements, members, and objects are synonymous and stand for the members of the set of which the set is comprised.

In this article, we will learn about the set theory and cover sets in detail. Look at the content guide that shows all the topics, we will be covering in this article.

**Set Theory Definition**

Sets are defined as ”a well-defined collection of objects”. Let’s say we have a set of [Natural Numbers](https://www.geeksforgeeks.org/what-are-natural-numbers/) then it will have all the natural numbers as its members and the collection of the numbers is well defined that they are natural numbers.

**Note:**A set is always denoted by a capital letter.

A set of Natural Numbers is given by:

***N = {1, 2, 3, 4…..}***

The above example is a collection of natural numbers and is also well-defined. Well-defined means, that anyone should be able to tell whether the object belongs to the set or not.

***Note:****The term ‘well defined’ should be taken care of, as if we try to make a set of best players, then the term ‘best’ is not well defined. The concept of best, worst, beautiful, powerful, etc. varies according to the notions, assumptions, likes, dislikes, and biases of a person.*

This explains what is a set, now let’s look at some set terms:

**History of Set Theory**

The concept of Set Theory was propounded in the year **1874**by **Georg Cantor** in his paper name ‘**On a Property of Collection of All Real Algebraic Numbers**‘.

His concept of Set Theory was later used by other mathematicians in giving various other theories such as **Klein’s Encyclopedia**and **Russell Paradox**. Sets Theory is a foundation for a better understanding of **topology**, **abstract algebra**, and [**discrete mathematics**](https://www.geeksforgeeks.org/discrete-mathematics-tutorial/).

Understanding set theory will also help in understanding other mathematical concepts like [**relations**](https://www.geeksforgeeks.org/relation-in-maths/), [**functions**](https://www.geeksforgeeks.org/what-is-a-function/), [probability](https://www.geeksforgeeks.org/probability-in-maths/), etc.

**Examples of Sets**

Some common examples of sets are mentioned below:

* Set of [Natural Numbers](https://www.geeksforgeeks.org/what-are-natural-numbers/): N = {1, 2, 3, 4….}
* Set of [Even Numbers](https://www.geeksforgeeks.org/even-numbers/): E = {2, 4, 6, 8…}
* Set of [Prime Numbers](https://www.geeksforgeeks.org/prime-number/): P = {2, 3, 5, 7,….}
* Set of [Integers](https://www.geeksforgeeks.org/integers): Z = {…, -4, -3, -2, -1, 0, 1, 2,….}

**Some Common Sets**

Below are the symbols used to represent common sets in set theory.

| **Symbol** | **Set Name** | **Description** | **Example** |
| --- | --- | --- | --- |
| **N** | Set of Natural Numbers | The set of all positive integers. Starting from 1. | {1, 2, 3, 4, …} |
| **Z** | Set of Integers | The set of all positive, negative, and zero integers. | {…, -3, -2, -1, 0, 1, 2, 3, …} |
| **R** | Set of Real Numbers | The set of all real numbers, including positive and negative rational and irrational numbers. | {…, -3.14, -2, -1, 0, 1, 2, 3.14, …} |
| **C** | Set of Complex Numbers | The set of all numbers that can be expressed as a + bi, where a and b are real numbers and i is the imaginary unit (√(-1)). | {1 + 2i, 3 – 5i, √2, -πi, …} |
| **Q** | Set of Rational Numbers | The set of all numbers that can be expressed as a fraction p/q, where p and q are integers and q ≠ 0. | {1/2, -3/4, 5, 0.75, …} |

**Important Terms Related to Set Theory**

Some of the important terms related to sets are mentioned below. These terms will be used several times in this article, and knowing these terms will help you learn set theory.

**Elements of a Set**

The objects contained by a set are called the elements of the set.

They are represented using the ∈ symbol which means “belongs to”.

**For Example:**

*In the set of Natural Numbers, 1, 2, 3, etc. are the objects, hence they are the elements of the set of Natural Numbers.*

*We can also say that 1 belongs to set N and it is represented as****1 ∈ N.***

**Cardinal Number of a Set**

The number of elements present in a set is called the Cardinal Number of a Set.

**For Example:**

*Suppose P is a set of the first five*[*prime numbers*](https://www.geeksforgeeks.org/prime-number/)*given by P = {2, 3, 5, 7, 11}, then the Cardinal Number of set P is 5.*

*The Cardinal Number of Set P is represented by n(P) or |P| = 5.*

**Representation of Sets**

Sets are primarily represented in two forms:

* Roster Form
* Set Builder Form

**Roster Form**

In the [**Roster Form**](https://www.geeksforgeeks.org/roster-form/) of the set, the elements are placed inside braces {} and are separated by commas.

Let’s say we have a set of the first five prime numbers then it will be represented by P = {2, 3, 5, 7, 11}. Here the set P is an example of a finite set as the number of elements is finite, however, we can come across a set that has infinite elements then in that case the roster form is represented in the manner that some elements are placed followed by dots to represent infinity inside the braces.

Let’s say we have to represent a set of Natural Numbers in Roster Form then its Roster Form is given as N = {1, 2, 3, 4…..}.

In roster form representation, the set does not contain duplicate elements. **For Example,**If A represents a set that contains all letters of the word TREE, then the correct roster form representation will be:

A = {T,R,E} = {E,R,T}

A={T,R,E,E} is the wrong representation, therefore A≠ {T,R,E,E}

**Set Builder Form**

In [**Set Builder Form**](https://www.geeksforgeeks.org/set-builder-notation/), a rule or a statement describing the common characteristics of all the elements is written instead of writing the elements directly inside the braces.

**For Example,**A set of all the prime numbers less than or equal to 10 is given as P = {p : p is a prime number ≤ 10}. In another example, the set of Natural Numbers in set builder form is given as N = {n : n is a natural number}.

**Read More:** [**Representation of Sets**](https://www.geeksforgeeks.org/representation-of-a-set/)

**Types of Sets**

There are different types of sets categorized on various parameters. Some types of sets are mentioned below:

| **Set Name** | **Description** | **Example** |
| --- | --- | --- |
| [Empty Set](https://www.geeksforgeeks.org/empty-set/) | A set containing no elements whatsoever. | {} |
| [Singleton Set](https://www.geeksforgeeks.org/singleton-set/) | A set containing exactly one element. | {1} |
| [Finite Set](https://www.geeksforgeeks.org/finite-sets/) | A set with a limited, countable number of elements. | {apple, banana, orange} |
| [Infinite Set](https://www.geeksforgeeks.org/infinite-set/) | A set with an uncountable number of elements. | {natural numbers (1, 2, 3, …)} |
| **Equivalent Sets** | Sets that have the same number of elements and their elements can be paired one-to-one. | Set A = {1, 2, 3} and Set B = {a, b, c} (assuming a corresponds to 1, b to 2, and c to 3) |
| [Equal Sets](https://www.geeksforgeeks.org/equal-sets/) | Sets that contain exactly the same elements. | Set A = {1, 2} and Set B = {1, 2} |
| [Universal Set](https://www.geeksforgeeks.org/universal-set/) | A set containing all elements relevant to a specific discussion. | The set of all students in a school (when discussing student grades) |
| **Unequal Sets** | Sets that do not have all the same elements. | Set A = {1, 2, 3} and Set B = {a, b} |
| [Power Set](https://www.geeksforgeeks.org/power-set/) | The set contains all possible subsets of a given set. | Power Set of {a, b} = { {}, {a}, {b}, {a, b} } |
| **Overlapping Sets** | Sets that share at least one common element. | Set A = {1, 2, 3} and Set B = {2, 4, 5} |
| [Disjoint Sets](https://www.geeksforgeeks.org/disjoint-sets/) | Sets that have no elements in common. | Set A = {1, 2, 3} and Set B = {a, b, c} |
| [Subset](https://www.geeksforgeeks.org/subsets/) | A set where all elements are also members of another set. | {1, 2} is a subset of {1, 2, 3} |

***Read More on,***[***Types of Sets***](https://www.geeksforgeeks.org/types-of-sets/)

**Set Theory Symbols**

Various symbols are used in Sets Theory. The notations and their explanation are tabulated below:

| **Symbol** | **Explanation** |
| --- | --- |
| {} | Set |
| x ∈ A | x is an element of set A |
| x ∉ A | x is not an element of set A |
| ∃ or ∄ | There exist or there doesn’t exist |
| Φ | Empty Set |
| A = B | Equal Sets |
| n(A) | Cardinal Number of Set A |
| P(A) | Power Set |
| A ⊆ B | A is a subset of B |
| A ⊂ B | A is the Proper subset of B |
| A ⊈ B | A is not a subset of B |
| B ⊇ A | B is the superset of A |
| B ⊃ A | B is a proper superset of A |
| B ⊉ A | B is not a superset of A |
| A ∪ B | A union B |
| A ∩ B | A intersection B |
| A’ | Complement of Set A |

**Read More:**[**Set Theory Symbols**](https://www.geeksforgeeks.org/set-theory-symbols/)**.**

**Set Theory Operations**

Some commonly used set operations are:

| **Set Operation** | **Description** | **Example** |
| --- | --- | --- |
| [Union (U)](https://www.geeksforgeeks.org/union-of-sets/) | The combination of elements from two sets, including duplicates. | if Set A = {1, 3, 5} and Set B = {2, 3, 4} ,then  A U B = {1, 2, 3, 4, 5} |
| [Intersection (∩)](https://www.geeksforgeeks.org/intersection-of-sets/) | The collection of elements that are in both sets. | if Set A = {1, 3, 5} and Set B = {2, 3, 4}, then  A ∩ B = {3} |
| [Difference ( \ )](https://www.geeksforgeeks.org/difference-of-sets/) | The collection of elements in the first set that are not in the second set. | if Set A = {1, 3, 5} and Set B = {2, 3, 4} , then  A \ B = {1, 5} |
| [Complement (A’)](https://www.geeksforgeeks.org/complement-of-a-set/) | The collection of elements that are not in the first set, but are in the universal set (all elements under consideration). | if Universal Set = {1, 2, 3, 4, 5} and Set A = {2, 4} ,then  A’ = {1, 3, 5} |
| [Cartesian Product (A x B)](https://www.geeksforgeeks.org/cartesian-product-of-sets/) | The collection of ordered pairs where the first element comes from the first set and the second element comes from the second set. | if Set A = {1, 2} and Set B = {a, b}, then  A x B = {(1, a), (1, b), (2, a), (2, b)} |

***Learn More:***[***Operation on Sets***](https://www.geeksforgeeks.org/set-operations/)

**Properties of Set Operations**

The various properties followed by sets are tabulated below:

| **Property** | **Expression** |
| --- | --- |
| [Commutative Property](https://www.geeksforgeeks.org/commutative-property/) | A ∪ B = B ∪ A  A ∩ B = B ∩ A |
| [Associative Property](https://www.geeksforgeeks.org/associative-property/) | (A ∩ B) ∩ C = A ∩ (B ∩ C)  (A ∪ B) ∪ C = A ∪ (B ∪ C) |
| [Distributive Property](https://www.geeksforgeeks.org/distributive-property/) | A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)  A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) |
| [Identity Property](https://www.geeksforgeeks.org/identity-property/) | A ∪ Φ = A  A ∩ U = A |
| Complement Property | A ∪ A’ = U |
| Idempotent Property | A ∪ A = A ∩ A = A |

**Set Theory Formulas**

The [set theory formulas](https://www.geeksforgeeks.org/set-theory-formulas/) are given for two sets – overlapping and disjoint sets. Let’s learn them separately

**Overlapping Set Formulas**

Given that two sets A and B are overlapping, the formulas are as follows:

| n(A ∪ B) | n(A) + n(B) – n(A ∩ B) |
| --- | --- |
| n(A ∩ B) | n(A) + n(B) – n(A ∪ B) |
| n(A) | n(A ∪ B) + n(A ∩ B) – n(B) |
| n(B) | n(A ∪ B) + n(A ∩ B) – n(A) |
| n(A – B) | n(A ∪ B) – n(B) |
| n(A – B) | n(A) – n(A ∩ B) |
| n(A ∪ B ∪ C) | n(A) + n(B) + n(C) – n(A∩B) – n(B∩C) – n(C∩A) + n(A∩B∩C) |

**Disjoint Set Formula**

If two sets A and B are disjoint sets

| n(A ∪ B) | n(A) + n(B) |
| --- | --- |
| (A ∩ B) | Φ |
| n(A – B) | n(A) |

**De Morgan’s Laws**

[De Morgan’s Law](https://www.geeksforgeeks.org/de-morgans-law/) is applicable in relating the union and intersection of two sets via their complements. There are two laws under De Morgan’s Law. Let’s learn them briefly

**De Morgan’s Law of Union**

De Morgan’s Law of Union states that the complement of the union of two sets is equal to the intersection of the complement of individual sets. Mathematically it can be expressed as

***(A ∪ B)’ = A’ ∩ B’***

**De Morgan’s Law of Intersection**

De Morgan’s Law of Intersection states that the complement of the intersection of two sets is equal to the union of the complement of individual sets. Mathematically it can be expressed as

***(A ∩ B)’ = A’ ∪ B’***

**Visual Representation of Sets Using Venn Diagram**

[Venn Diagram](https://www.geeksforgeeks.org/venn-diagram/) is a technique for representing the relation between two sets with the help of circles, generally intersecting.

**For Example:**

Two circles intersecting with each other with the common area merged into them represent the union of sets, and two intersecting circles with a common area highlighted represent the intersection of sets while two circles separated from each other represent the two disjoint sets.

A rectangular box surrounding the circle represents the universal set. The Venn diagrams for various operations of sets are listed below:

**Conclusion – Set Theory**

We have covered all the concepts required to learn the set theory. We have covered the history, definition, examples, symbols, operations, and formulas of set theory.

Set theory is an important topic and many questions come from set theory in many competitive Exams. Students should focus on set theory and practice with some questions provided in this article.

**Solved Examples on Set Theory**

**Example 1:**If A and B are two sets such that n(A) = 17, n(B) = 23 and n(A ∪ B) = 38 then find n(A ∩ B).

**Solution:**

*We know that n(A ∪ B) = n(A) + n(B) – n(A ∩ B)*

*⇒ 38 = 17 + 23 – n(A ∩ B)*

*⇒ n(A ∩ B) = 40 – 38 = 2*

**Example 2:** If X = {1, 2, 3, 4, 5}, Y = {4, 5, 6, 7, 8}, and Z = {7, 8, 9, 10, 11}, find (X ∪ Y), (X ∪ Z), (Y ∪ Z), (X ∪ Y ∪ Z), and X ∩ (Y ∪ Z)

**Solution:**

*(X ∪ Y) = {1, 2, 3, 4, 5} ∪ {4, 5, 6, 7, 8} = {1, 2, 3, 4, 5, 6, 7, 8}*

*(X ∪ Z) = {1, 2, 3, 4, 5} ∪ {7, 8, 9, 10, 11} = {1, 2, 3, 4, 5, 7, 8, 9, 10, 11}*

*(Y ∪ Z) = {4, 5, 6, 7, 8} ∪ {7, 8, 9, 10, 11} = {4, 5, 6, 7, 8, 9, 10, 11}*

*(X ∪ Y ∪ Z) = {1, 2, 3, 4, 5} ∪ {4, 5, 6, 7, 8} ∪ {7, 8, 9, 10, 11} = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}*

*X ∩ (Y ∪ Z) = {1, 2, 3, 4, 5} ∩ {4, 5, 6, 7, 8, 9, 10, 11} = {4, 5}*

**Practice Problems on Set Theory**

**1. Given two sets *A*= {1,3,5,7} and *B*= {1,2,3,4}, find:**

* *A*∪*B* (Union)
* *A*∩*B* (Intersection)
* *A*−*B* (Difference)
* *B*−*A* (Difference)
* *A*×*B* (Cartesian product)

**2. Let the universal set 𝑈={1,2,3,4,5,6,7,8,9,10} and *C*={2,4,6,8,10}. Find the complement of C with respect to U.**

**3. Define a set 𝐷={x | x is a letter in the word “HELLO”}. List all the subsets of D and then find the power set of D.**

**4. In a survey of 100 students, it is found that:**

* 60 students like chocolate ice cream.
* 30 students like vanilla ice cream.
* 10 students like neither. Find how many students like both chocolate and vanilla ice cream, assuming these are the only flavors available.