CS3231
Tutorial 1

1. Let $\Sigma = \{a, b, c\}$.

   Let $L_1 = \{a^n b^m \mid n, m \geq 0\}$, $L_2 = \{b^m c^n \mid n, m \geq 0\}$, $L_3 = \{a^m b^n c^r \mid n, m, r \geq 0\}$.

   (a) What is $L_1 \cap L_2$?

   (b) What is $L_1 \cup L_2$?

   (c) What is L_1^*?

   (d) Is $L_1 \cdot L_2 = L_3$?

   (e) Is $L_1 \cup L_2 = L_3$?

   (f) Is $L_1 \cap L_2 = L_3$?

2. Show by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

3. (a) For a particular alphabet set Σ, how many strings of length n are there in Σ^*? How many strings in Σ^* have length $\leq n$?

   (b) Suppose w and u are strings. Find an expression for the length of $(w \cdot u \cdot u)^{i^3 + i^2 * j^3} \cdot u^j \cdot w^i$, in terms of $i, j, |w|, |u|$.

   (c) Find all the substrings of the string "SINGAPORE".

4. Suppose A, B_i are languages over Σ.

   Show that $A \cdot (\bigcup_{i=1}^{\infty} B_i) = \bigcup_{i=1}^{\infty} (A \cdot B_i)$.

5. Show that $(A^*)^+ = (A^+)^* = A^*$.

6. Suppose $\Sigma = \{0, 1\}$.

   (a) Let $L = \{w \mid$ number of 0's in w is of form $5i + 3$, for some natural number $i\}$. Give a DFA for L.

   (b) Let $L = \{w \mid w$ has 01001 as a substring $\}$. Give a DFA for L.

7. What is the language accepted by the following DFA.