RT 801 - Security in Computing

Module III Cryptography

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Module 3

Cryptography: Basic Encryption & Decryption – Transposition & substitution ciphers – Caesar substitution – Polyalphabetic substitutions – Crypt analysis

Symmetric key algorithms – Fiestel Networks – Confusion – Diffusion – DES Algorithm – Strength of DES – Comparison & important features of modern symmetric key algorithms

Public key cryptosystems – The RSA Algorithm – Diffie Hellman key exchange – comparison of RSA & DES – Message Authentication & Hash functions – Digital signature

Cryptography

- In Greek means "secret writing"
- An outsider (interceptor/intruder/adversary) can make following threats:
 - Block message (affecting availability)
 - Intercept message (affecting secrecy)
 - Modify message (affecting integrity)
- Cryptography is the fundamental technique to counter these threats because the outsider does not understand the meaning of messages

- Cryptography: Study of mathematical techniques related to certain aspects of information security, such as confidentiality, data integrity, entity authentication, and data origin authentication.
 - The basic component of cryptography is a cryptosystem
- Cryptanalyst: Person working for unauthorized interceptor
- Cryptology: Study of encryption and decryption, including cryptography and cryptanalysis.

Cryptanalysis

- Means "breaking the code".
- Cryptanalysis relies on a knowledge of the encryption algorithm and some knowledge of the possible structure of the plaintext (such as the structure of a typical inter-bank financial transaction) for a partial or full reconstruction of the plaintext from ciphertext.
- Additionally, the goal is to also infer the key for decryption of future messages.

Cryptography issues

- Confidentiality: only sender, intended receiver should "understand" message contents
 - -sender encrypts message
 - -receiver decrypts message
- End-Point Authentication: sender, receiver want to confirm identity of each other
- Message Integrity: sender, receiver want to ensure message not altered (in transit, or afterwards) without detection.

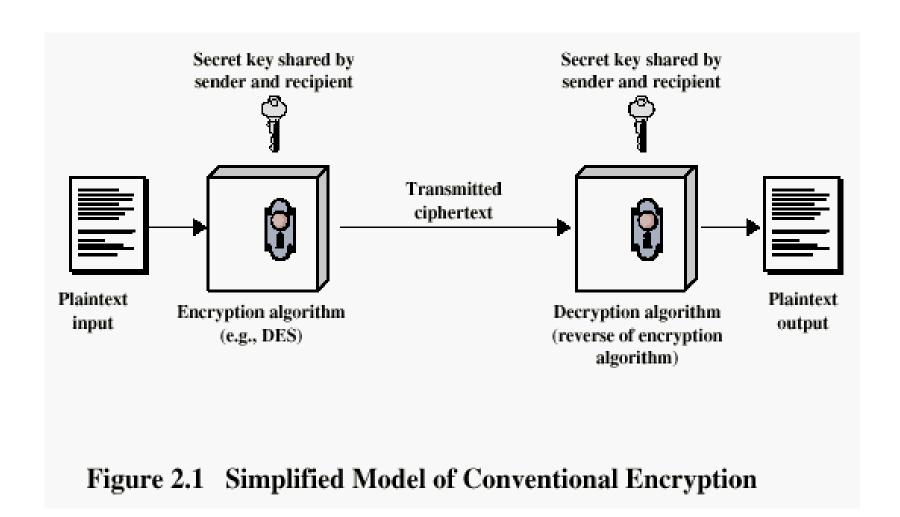
Cryptosystem

- A cryptosystem is a 5-tuple (E, D, M, K, C), where M is the set of plaintexts, K is the set of keys, C is the set of ciphertexts,
 - $-E: M \times K \rightarrow C$ is the set of encipher (**encryption**) functions, and
 - D: C × K→M is the set of deciphering (decryption) functions.
 - Plaintext M: set of messages in original form
 - Ciphertext C: set of messages in encrypted form
- Security depends on the secrecy of the key, not the secrecy of the algorithm

Cryptosystem....

- Encryption: Process of encoding (enciphering) a message so that its meaning is not obvious.
 - Provide confidentiality
 - Principle of Encryption
 - Very hard (impossible) to find out the message without knowing the key
 - Very easy (and fast) to find out the message knowing the key
- Decryption: Process of decoding (deciphering or transforming) an encrypted message to its original form.

Conventional Encryption Principles



Brute-force attack

- When encryption and decryption algorithms are publicly available
 - a brute-force attack means trying every possible key on a piece of ciphertext until an intelligible translation into plaintext is obtained.

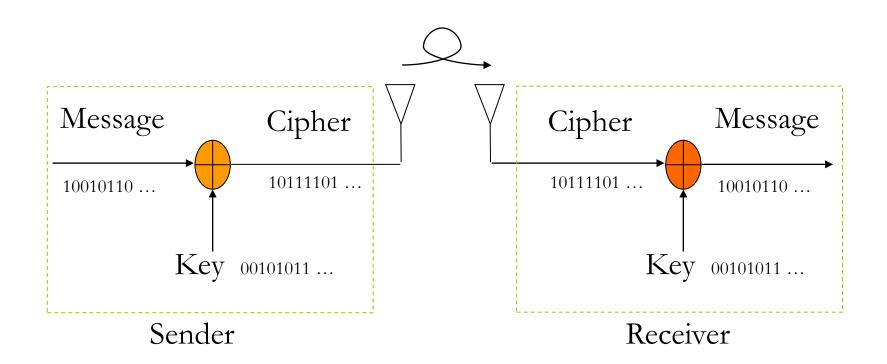
Key space

- The total number of all possible keys that can be used in a cryptographic system.
- For example, DES uses a 56-bit key.
 - So the key space is of size 2^{56} , which is approximately the same as 7.2×10^{16} .

One-Time Pad

- OTP takes a string of random numbers as long as the message
 - OTP uses a symmetric key, which should be statistically proven to be *random*
- Combines the random numbers with the message
 - Perform XOR arithmetic
- This produces ciphertext
- Because all random strings are equally likely, cryptanalysis is impossible

One-Time Pad

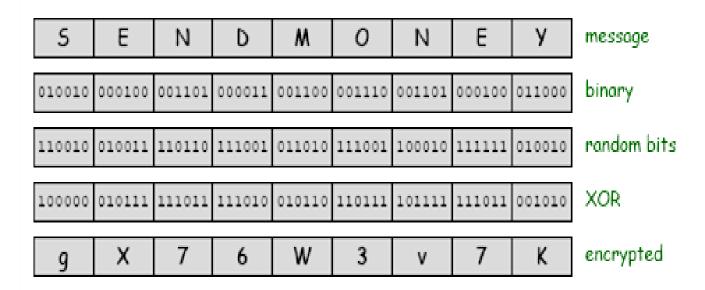


One Time Pad ...

- A and B wish to communicate privately using the one-time pad (it uses a symmetric key K)
- They have previously agreed upon secret key K which is a string of n randomly chosen bits
- If A wishes to send an n-bit message M to B, A sends to B the ciphertext C = M XOR K,
- The received ciphertext can be decrypted by B to obtain M, since M = C XOR K.
- When another message is to be sent, another key K must be used, hence the name "one-time pad"

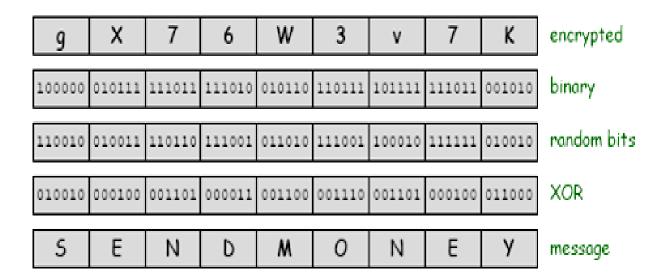
One-Time Pad Encryption (example)

- Convert text message to N bits.
- Generate N random bits (one-time pad).
- Take bitwise XOR of two strings.
- Convert binary back into text.



One-Time Pad Decryption

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two strings.
- Convert back into text.



Why Does It Work?

- Crucial property: (a ^ b) ^ b = a.
- Decrypted message = original message.
- Use properties of XOR.

$$(a \land b) \land b = a \land (b \land b) = a \land 0 = a$$

Notation	Meaning
а	original message
Ь	one-time pad
^	XOR operator
a^b	encrypted message
(a ^ b) ^ b	decrypted message

Difficulties of OTP

- To ensure the security of the system, key size should not be less than message size
- Generating a fully random key is practically very difficult
- Sending an unrepeated key with the same size of the message through a secure channel to the receiver is not practical

Classical Cryptography

- Basic techniques for classical ciphers
 - Substitution: One letter is exchanged for another
 - Transposition: The order of the letters is rearranged (also referred to as permutation)
- Classical ciphers
 - Mono-alphabetic: Letters of the plaintext alphabet are mapped into other unique letters
 - Poly-alphabetic: Letters of the plaintext alphabet are mapped into letters of the ciphertext space depending on their positions in the text

Substitution

- Substitute each letter in the plaintext for another one
 - Monoalphabetic cipher: substitute one letter for another
 - Ciphers in which the cipher alphabet remains unchanged throughout the message are called Monoalphabetic Substitution Ciphers.
- Goal: Confusion difficult to determine how a message and key were transformed into ciphertext.
- Example (Caesar Cipher)
 - abcdefghijklmnopqrstuvwxyz
 - qeryuiopasdfgwhjklzxcvbnmt
 under attack we need help
 - cwyul qxxqrd bu wuuy pufj

Caesar Cipher

- This is the earliest known example of a substitution cipher.
- Each character of a message is replaced by a character three position down in the alphabet.

plaintext: are you ready

ciphertext: DUH BRX UHDGB

Caesar Cipher.....

 If we represent each letter of the alphabet by an integer that corresponds to its position in the alphabet, the formula for replacing each character 'p' of the plaintext with a character 'C' of the ciphertext can be expressed as

$$C = E(3, p) = (p + 3) \mod 26$$

Caesar Cipher.....

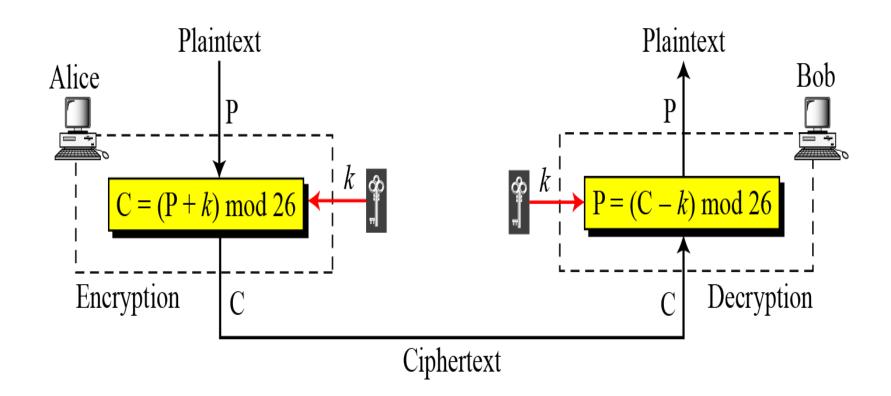
 A more general version of this cipher that allows for any degree of shift would be expressed by

$$C = E(k, p) = (p + k) \mod 26$$

The formula for decryption would be

$$p = D(k, C) = (C - k) \mod 26$$

Caesar Cipher.....



Pigpen Cipher

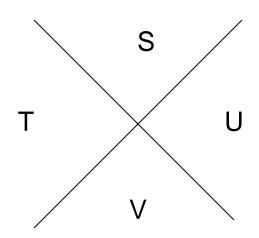
- Monoalphabetic cipher
- Pigpen cipher is a variation on letter substitution

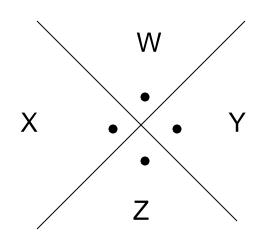
Alphabets are arranged as follows:

Α	В	С
D	Е	F
G	Н	I

J	K •	L
M •	Z •	• 0
P	• Q	• R

Pigpen Cipher diagram





Pigpen Cipher

- Alphabets will be represented by the corresponding diagram
- E.g., WAG would be



This is a weak cipher

In monoalphabetic substitution, the relationship between a symbol in the plaintext to a symbol in the ciphertext is always one-to-one.

Polyalphabetic Cipher

- In monoalphabetic cipher the problem was that each character was substituted by a single character.
- They are easy to break because they reflect the frequency data of the original alphabet.
- Polyalphabetic cipher's goal is to make this process difficult.

Polyalphabetic Cipher

- In polyalphabetic cipher, each plaintext character may be replaced by more than one character
- Since there are only 26 alphabets this process will require using a different representation than the alphabets
- Alphabets 'A' through 'Z' are replaced by 00, 01, 02, ..., 25

Polyalphabetic Cipher

- The most common method used is Vigenère cipher
- Vigenère cipher starts with a 26 x 26 matrix of alphabets in sequence. First row starts with 'A', second row starts with 'B', etc.
- This cipher requires a keyword that the sender and receiver know ahead of time.
- Each character of the message is combined with the characters of the keyword to find the ciphertext character.

Vigenère Cipher Table

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ BABCDEFGHIJKLMNOPQRSTUVWXY CDEFGHIJKLMNOPQRSTUVWXYZAB DEFGHIJKLMNOPQRSTUVWXYZABC EFGHIJKLMNOPQRSTUVWXYZABCD FGHIJKLMNOPQRSTUVWXYZABCDE GHIJKLMNOPQRSTUVWXYZABCDEF HIJKLMNOPQRSTUVWXYZABCDEFG IJKLMNOPQRSTUVWXYZABCDEFGH J K L M N O P Q R S T U V W X Y Z A B C D E F G H I K K L M N O P Q R S T U V W X Y Z A B C D E F G H I J L L M N O P Q R S T U V W X Y Z A B C D E F G H I J K MMNOPQRSTUVWXYZABCDEFGHIJKL

Vigenère Cipher Table

ABCDEFGHIJKLMNOPQRSTUVWXYZ

NOPQRSTUVWXYZABCDEFGHIJKLM OPQRSTUVWXYZABCDEFGHIJKLMN PQRSTUVWXYZABCDEFGHIJKLMNO QRSTUVWXYZABCDEFGHIJKLMNOP RSTUVWXYZABCDEFGHIJKLMNOPQ STUVWXYZABCDEFGHIJKLMNOPQR TUVWXYZABCDEFGHIJKLMNOPQRS UVWXYZABCDEFGHIJKLMNOPQRST V W X Y Z A B C D E F G H I J K L M N O P Q R S T U WXYZABCDEFGHIJKLMNOPQRSTUV XYZABCDEFGHIJKLMNOPQRSTUVW YZABCDEFGHIJKLMNOPQRSTUVWX Z¹ZABCDEFGHIJKLMNOPQRSTUVWXY

Vigenère Cipher Table (Full)

	a	b	с	d	e	f	g	h	i	j	k	1	m	n	О	p	q	r	s	t	v	v	W	X	У	Z
A	Α	В	C	D	E	F	G	Н	I	J	K	L	M	N	О	P	Q	R	S	T	U	V	W	X	Y	Z
В	В	\mathbf{C}	D	E	F	G	Н	I	J	K	L	M	N	O	P	Q	R	S	T	\mathbf{U}	V	W	X	Y	Z	Α
C	C	D	E	F	G	Н	I	J	K	L	\mathbf{M}	N	О	P	Q	R	S	T	U	V	W	X	Y	Z	Α	В
D	D	\mathbf{E}	F	G	Н	I	J	K	L	\mathbf{M}	N	O	P	Q	R	S	T	\mathbf{U}	V	W	X	Y	Z	Α	В	C
E	Е	F	\mathbf{G}	Н	I	J	K	L	\mathbf{M}	N	O	P	Q	R	S	T	\mathbf{U}	V	W	X	Y	Z	Α	В	\mathbf{C}	D
$oldsymbol{F}$	F	G	Н	I	J	K	L	\mathbf{M}	N	О	P	Q	R	S	T	\mathbf{U}	V	W	X	Y	Z	Α	В	C	D	Е
G	G	Н	I	J	K	L	\mathbf{M}	N	О	P	Q	R	S	T	\mathbf{U}	\mathbf{V}	W	X	Y	Z	A	В	\mathbf{C}	D	E	F
H	Н	I	J	K	L	\mathbf{M}	N	О	P	Q	R	S	T	\mathbf{U}	V	W	X	Y	Z	Α	В	C	D	E	F	G
I	I	J	K	L	M	N	О	P	Q	R	S	T	U	V	W	\mathbf{X}	Y	\mathbf{Z}	Α	В	C	D	E	F	G	Н
J	J	K	L	M	N	О	P	Q	R	S	T	U	V	W	X	Y	Z	A	В	C	D	E	F	G	Н	Ι
K	K	L	M	N	О	P	Q	R	S	T	U	V	W	X	Y	Z	A	В	C	D	E	F	G	Н	I	J
L	L	M	N	О	P	Q	R	S	T	U	V	W	X	Y	Z	Α	В	C	D	E	F	G	Н	Ι	J	K
M	M	N	О	P	Q	R	S	T	U	V	W	X	Y	Z	A	В	C	D	Е	F	G	Н	Ι	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	Α	В	C	D	E	F	G	Η	I	J	K	L	M
0	О	P	Q	R	S	T	U	V	W	X	Y	Z	Α	В	C	D	E	F	G	Н	Ι	J	K	L	\mathbf{M}	Ν
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	В	C	D	Е	F	G	Н	Ι	J	K	L	M	N	О
Q	Q	R	S	Т	U	V	W	X	Y	Z	Α	В	C	D	E	F	G	Н	Ι	J	K	L	M	N	О	P
R	R	S	Т	U	V	W	X	Y	Z	Α	В	C	D	E	F	G	Н	Ι	J	K	L	М	N	О	Р	Q
S	S	Т	U	V	W	X	Y	Z	A	В	C	D	Е	F	G	Н	Ι	J	K	L	М	N	О	Р	Q	R
T	Т	U	V	W	X	Y	Z	Α	В	C	D	Е	F	G	Н	I	J	K	L	M	N	О	Р	Q	R	S
U	U	V	W	X	Y	Z	Α	В	C	D	Е	F	G	Н	Ι	J	K	L	M	N	О	Р	Q	R	S	Т
V	V	W	X	Y	Z	Α	В	C	D	Е	F	G	Н	Ι	J	K	L	M	N	О	Р	Q	R	S	Т	U
W	W	X	Y	Z	A	В	C	D	Е	F	G	Н	Ι	J	K	L	M	N	О	P	Q	R	S	Т	U	V
X	X	Y	Z	Α	В	C	D	Е	F	G	Н	Ι	J	K	L	M	N	О	Р	Q	R	S	Т	U	V	W
Y	Y	Z	Α	В	С	D	Е	F	G	Н	Ι	J	K	L	M	N	О	P	Q	R	S	Т	U	V	W	X
\boldsymbol{z}	Z	A	В	С	D	Е	F	G	Н	Ι	J	K	L	M	N	О	P	Q	R	S	Т	U	V	W	X	Y

Vigenère cipher

- E.g., Message = SEE ME IN MALL
- Take keyword as INFOSEC
- Vigenère cipher works as follows:

SEEMEINMALL INFOSECINFO

ARJAWMPUNQZ

Vigenère cipher

- To decrypt, the receiver places the keyword characters below each ciphertext character
- Using the table, choose the row corresponding to the keyword character and look for the ciphertext character in that row
- Plaintext character is then at the top of that column

Vigenère cipher

Decryption of ciphertext:

ARJAWMPUNQZ

INFOSECINFO

SEEMEIN MALL

 Best feature is that same plaintext character is substituted by different ciphertext characters (i.e., polyalphabetic)

Polyalphbetic Ciphers Strength

 There are multiple ciphertext letters for each plaintext letter, one for each unique letter of the keyword. Thus, the letter frequency information is hidden.

TRANSPOSITION CIPHERS

A transposition cipher does not substitute one symbol for another, instead it changes the location of the symbols.

A transposition cipher reorders symbols.

Keyless Transposition Ciphers

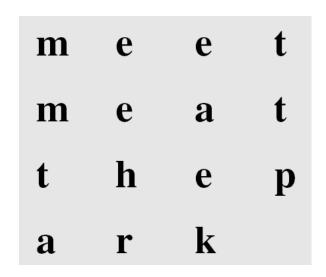
Simple transposition ciphers, which were used in the past, are keyless.

A good example of a keyless cipher using the first method is the rail fence cipher. The ciphertext is created reading the pattern row by row.

For example, to send the message "Meet me at the park" to Bob, Alice writes

She then creates the ciphertext "MEMATEAKETETHPR".

Alice and Bob can agree on the number of columns and use the second method. Alice writes the same plaintext, row by row, in a table of four columns.



She then creates the ciphertext "MMTAEEHREAEKTTP".

Transposition (using keys)

- Ciphertext: write out the columns in an order specified by a key.
- Goal: Diffusion spread the information from the message or the key across the ciphertext.



Encryption --- 3 (third column) becomes 1 (first column) and so on Decryption – 1 (first column) becomes 3 (third column) and so on

The cipher can be made more secure by performing multiple rounds of such permutations.

Alice needs to send the message "Enemy attacks tonight" to Bob...

enemy attac kston ightz

The permutation yields

E E M Y N T A A C T T K O N S H I T Z G

Enemy Attac kston ightz

e	n	e	m	y
a	t	t	a	c
k	s	t	0	n
i	g	h	t	Z

Encryption ↓

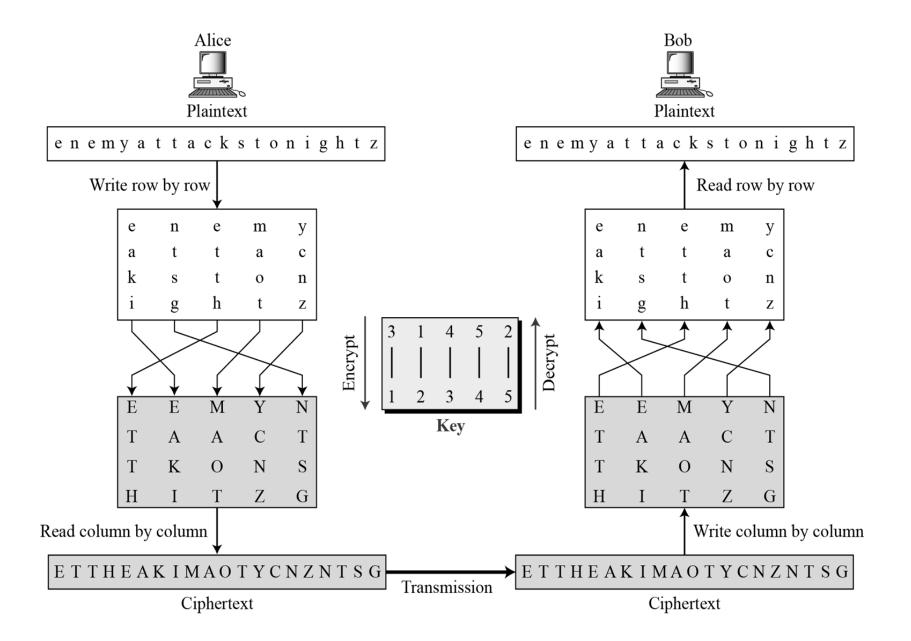
3	1	4	5	2
1	2	3	4	5

↑ Decryption

e	e	m	y	n
t	a	a	c	t
t	k	0	n	s
h	i	t	Z	g

Cipher Text

etth eaki maot ycnz ntsg



Confusion and Diffusion

Confusion

 A technique that seeks to make the relationship between the statistics of the ciphertext and the value of the encryption keys as complex as possible. Cipher uses key and plaintext.

Diffusion

 A technique that seeks to obscure the statistical structure of the plaintext by spreading out the influence of each individual plaintext digit over many ciphertext digits.

Types of Cryptosystems

 Symmetric cryptosystems (also called single-key cryptosystems) are classical cryptosystems:

$$M = D(K, E(K, M))$$

- The encryption key and decryption key are the same.
- Asymmetric cryptosystem:

$$M = D(K_d, E(K_e, M))$$

- $-K_{d}$ is the decryption key and K_{e} is the encryption key
- $-K_{\rm d} \neq K_{\rm e}$

Classical Cryptography: Secret-Key or Symmetric Cryptography

- Alice and Bob agree on an encryption method and a shared *key*.
- Alice uses the key and the encryption method to encrypt (or encipher) a message and sends it to Bob.
- Bob uses the same key and the related decryption method to decrypt (or decipher) the message.

Four Secure Key Distribution Strategies for Symmetric Cryptosystems

- A key K can be selected by A to be shared with B, and K needs to be physically delivered to B
- 2. A third party can select the same key K and physically deliver K to A and B
- 3. If A and B have previously used a key K', one party can transmit the new key K to the other, encrypted using the old key K'
- 4. If A and B each has an encrypted connection to a third party C, C can transmit the new key K on the encrypted links to both A and B

Public-Key Cryptography: Asymmetric Cryptography

- Alice generates a key value (usually a number or pair of related numbers) which she makes public.
- Alice uses her public key (and some additional information) to determine a second key (her *private key*).
- Alice keeps her private key (and the additional information she used to construct it) secret.

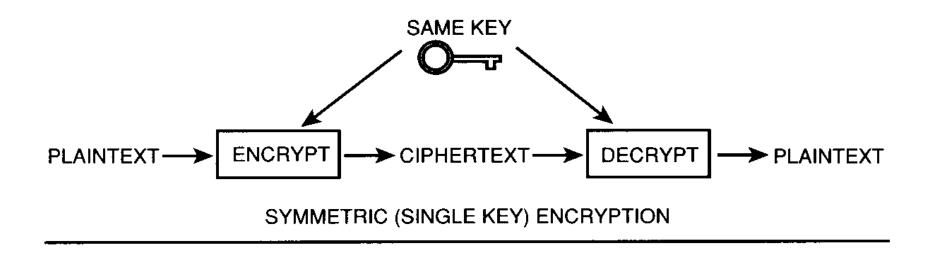
Public-Key Cryptography

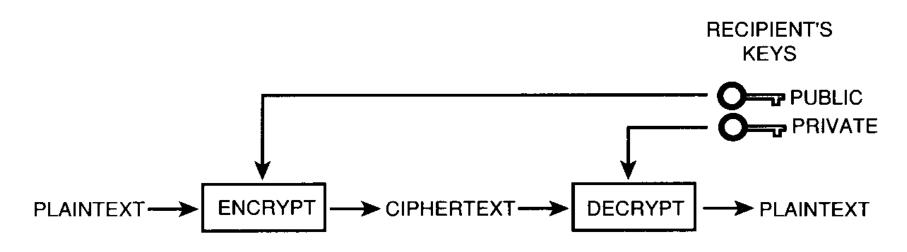
- Bob (or Carol, or anyone else) can use Alice's public key to encrypt a message for Alice.
- Alice can use her private key to decrypt this message.
- No- one without access to Alice's private key can easily decrypt the message.

An Example: Internet Commerce

- Bob wants to use his credit card to buy some items from Alice over the Internet.
- Alice sends her public key to Bob.
- Bob uses this key to encrypt his credit-card number and sends the encrypted number to Alice.
- Alice uses her private key to decrypt this message (and get Bob's credit-card number).

Symmetric and Asymmetric Encryption



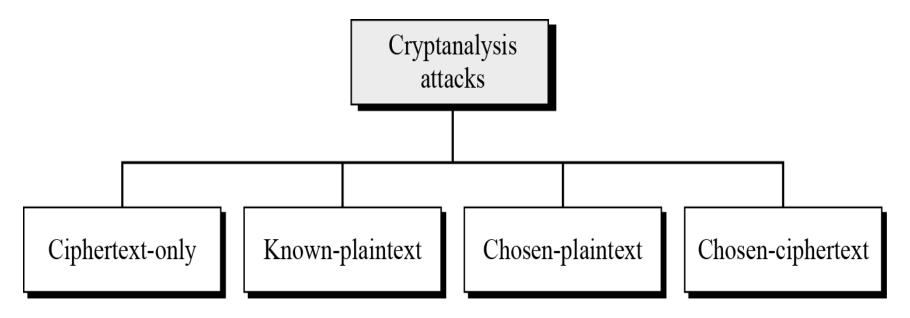


Kerckhoff's Principle

A cryptosystem should be secure even if everything about the system, except the key, is public knowledge.

Cryptanalysis

As cryptography is the science and art of creating secret codes, cryptanalysis is the science and art of breaking those codes.



May be classified by how much information needed by the attacker

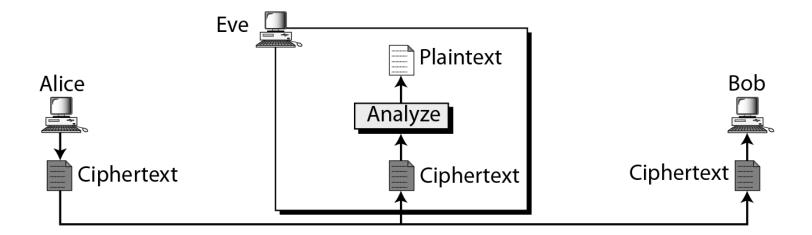
Ciphertext-Only Attack

- In ciphertext only attack, encryption algorithm and ciphertext are known to the cryptanalyst.
- With simple ciphers, such as the Caesar Cipher, frequency analysis can be used to break the cipher.
- Monoalphabetic substitution ciphers are vulnerable to ciphertext-only attacks.
 - Each character was substituted by a single character.
 - Easy to break because they reflect the frequency data of the original alphabet.

Ciphertext-Only Attack

Given: a ciphertext c

Q: what is the plaintext m?



Known plaintext attack

- In *known plaintext attack*, information known includes: encryption algorithm, ciphertext, and one or more plaintext-ciphertext pairs formed with the secret key.
- Seeks to discover a correlation between plaintext and the corresponding ciphertext.
 - The attacker knows or can guess the plaintext for some parts of the ciphertext.
 - The task is to decrypt the rest of the ciphertext blocks using this information.
 - This may be done by determining the key used to encrypt the data.

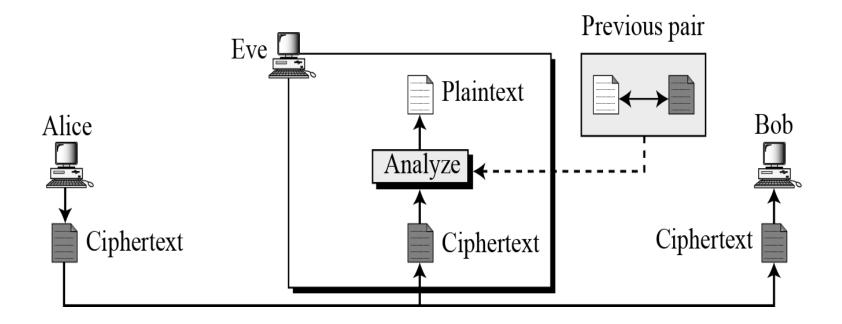
Known-Plaintext Attack

The attacker has samples of both the plaintext and its encrypted version (ciphertext).

Given: (m1,c1), (m2,c2), ..., (mk, ck) and a new ciphertext c.

Q: what is the plaintext of c?

Q: what is the secret key in use?



- Plain Text + Key = Cipher Text
- Cipher Text Plain Text = Key

Known-Plaintext Attack – example 1

- Alice sends Bob an email and attaches her favorite holiday snapshot. The email is encrypted.
- She sends the same holiday snapshot to her mother in plain text.
- Steve, who wishes to spy on Alice and Bob, was able to intercept her email to Mom and now has a copy of "myholiday.jpg".

Known-Plaintext Attack – example 1.....

- If the picture consisted of 200 Kilobytes of data and Alice included only a short personal message to Bob with the picture (say 50 letters)
 - then Steve already knows 99% of the message contents prior to encryption and now has greatly improved chances of breaking Alice's key if he comes into possession of the corresponding cipher text.

Known-Plaintext Attack – example 2

- An example with monoalphabetic substitution cipher.
- The following ciphertext is intercepted and is known to contain information about a person called "ANDERSON" and a place called "MISSISSIPPI".

JZKGXAHZDAVWGZGBWGJKHUAIDGADZEDAADAADIID

JZKGXAHZDAVWGZGBWGJKHUAIDGADZEDAADAADIID

- We could use our information of the plaintext to devise stronger attack.
- MISSISSIPPI we look for a sequence of 11 letters where the 3rd, 4th, 6th and 7th letters are the same and so are the 2nd, 5th, 8th and 11th.
 - The sequence 'EDAADAADIID' is the ciphertext for 'MISSISSIPPI'.

JZKGXAHZDAVWGZGBWGJKHUAIDGADZEDAADAADIID

For ANDERSON

- Look for a sequence of 8 letters where all characters are different except for the 2nd and the 8th.
- Eventually the attacker will find that 'JZKGXAHZ' represents 'ANDERSON'

ANDERSON DAVWGZGBWGJKHUAIDGADZ MISSISSIPPI

ANDERSON IS VWGZGBWGJKHUAIDGADZ MISSISSIPPI

JZKGXAHZDAVWGZGBWGJKHUAIDGADZEDAADADIID

 With the newly gained information so far, the ciphertext has been decrypted to

ANDERSON IS VWENEBWEADOU SPIES IN MISSISSIPPI

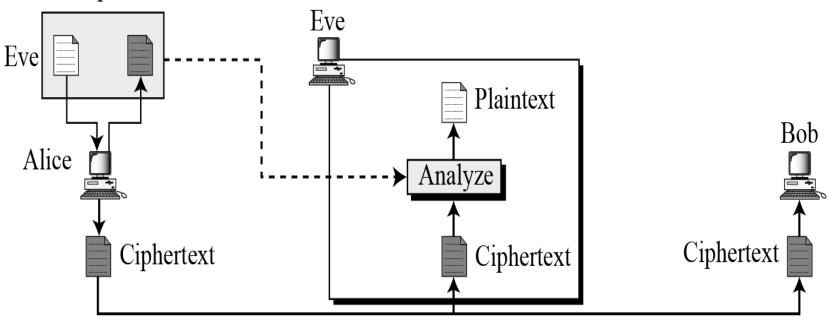
 Subsequent effort of cryptanalysis may eventually reveal the secret

ANDERSON IS THE NEW HEAD OF SPIES IN MISSISSIPPI

Chosen-Plaintext Attack

- In *chosen plaintext attack*, information known includes: encryption algorithm, ciphertext, and chosen plaintext and its corresponding ciphertext generated with the secret key.
- Given: (m1,c1), (m2,c2), ..., (mk,ck), where m1, m2, ..., mk are chosen by the adversary; and a new ciphertext c.
 - Q: what is the plaintext of c, or what is the secret key?
- For a chosen plaintext attack, an attacker chooses plaintext and submits it to be encrypted.
- Attackers can then analyze the ciphertext that corresponds to the chosen plaintext, identify subtle differences and patterns, and quickly break the encryption.

Pair created from chosen plaintext



Given: P_1 , $C_1 = E_k(P_1)$, P_2 , $C_2 = E_k(P_2)$, ... P_i , $C_i = E_k(P_i)$, where the cryptanalyst gets to choose P_1 , P_2 , ... P_i

Chosen-Plaintext Attack

- For example, suppose we want to attack communication from Alice to Bob which is encrypted by monoalphabetic substitution cipher.
- Assume the intercepted messages so far could not be solved using frequency analysis.
- If we can get Alice to send an encrypted message to Bob which contains the word 'MISSISSIPPI'.
 - Here, we can send an email to Alice, "Please tell Bob that saying Mississippi will take exactly one second".
 - Assume Alice simply forwards our written email.
 - We can intercept the message and obtain some information about the mapping of plaintext to ciphertext used in encryption of communication from Alice to Bob.

Example: chosen-plaintext attack

- In 1942, US Navy cryptanalysts discovered that Japan was planning an attack on "AF".
- They believed that "AF" means Midway island.
- Pentagon didn't think so.
- US forces in Midway sent a plain message that their fresh water supplies were low.
- Shortly, US intercepted a Japanese ciphertext saying that "AF" was low on water.
- This proved that "AF" is Midway.

Chosen-Ciphertext Attack

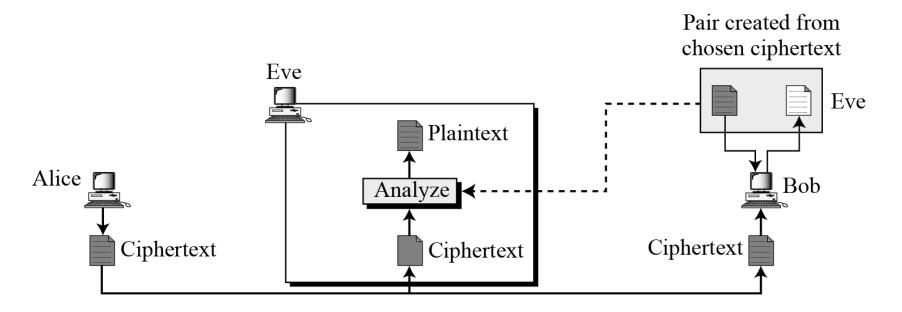
Given: (m1,c1), (m2,c2), ..., (mk,ck), where c1, c2, ..., ck are chosen by the adversary; and a new ciphertext c.

Q: what is the plaintext of c, or what is the secret key of sender?

A chosen ciphertext attack is an attack where a cryptanalyst chooses a ciphertext and attempts to find a matching plaintext.

An attacker gains access to an unattended decryption machine.

Decrypted with an unknown key



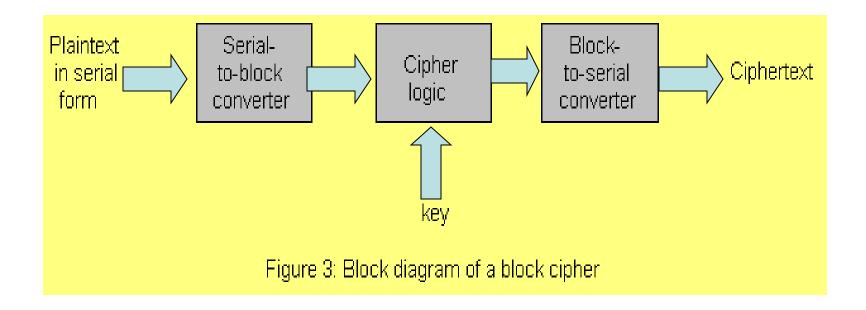
- In chosen ciphertext attack, information known includes: encryption algorithm, ciphertext, and chosen ciphertext and its corresponding decrypted plaintext with the secret key.
- it has also been called a "lunch-time" or "midnight" attack.

Cryptographic Systems - Classification

- The type of operations used for transforming plaintext to ciphertext
 - Substitution
 - Transposition
 - Product systems multiple stages of substitutions and transpositions
- The number of keys used
 - Symmetric
 - Asymmetric
- The way in which the plaintext is processed
 - Block cipher
 - Stream cipher

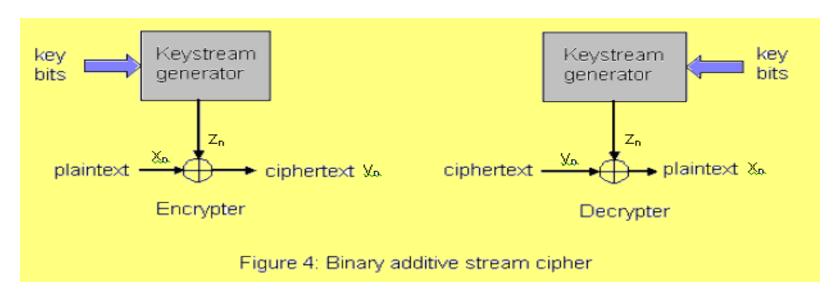
Block Ciphers

- Encrypt data one block at a time
- Operate on a plaintext block of n bits
- Produce a ciphertext block of n bits
- Typical block size 64 128 bits



Stream ciphers

 Whereas block ciphers operate on large data on a block-byblock basis, stream ciphers operate on individual bits.



Let x_n -> Plaintext bit; y ->ciphertext bit; z ->keystream bit at n^{th} instant

For encryption: $y_n = x_n \oplus z_n$, n=1, 2, ..., N

For decryption: $x_n = y_n \oplus z_n$, n=1, 2, ..., N

Steganography

Secure Communication

Two parties, Alice and Bob, can exchange information over an insecure medium in such a way that even if an intruder (Willie) is able to intercept, read and perform computation on the intercepted information, Willie will not be able to decipher the content of the exchanged information.

Encryption may not be enough

- Prisoners Problem:
 - Alice and Bob are in jail and wish to hatch an escape plan. All their communications pass through the warden, Willie, and if Willie detects any encrypted messages, he can simply stop the communication.
- So they must find some way of hiding their secret message in an innocuous looking text.

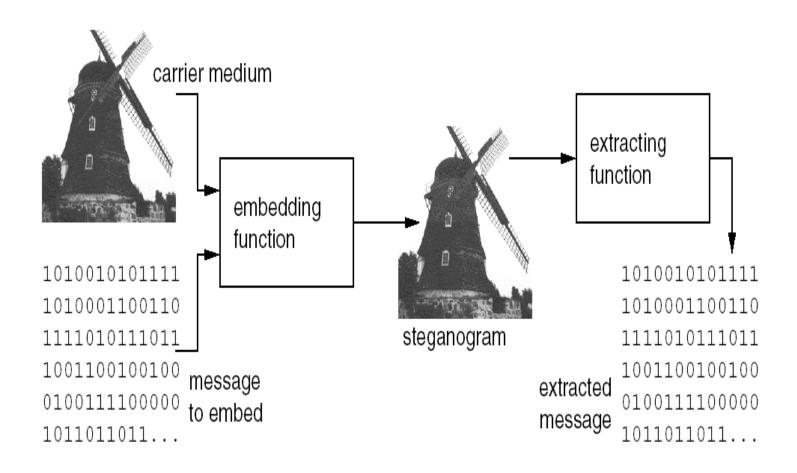
Steganography

- The art of hiding information in ways that prevent detection of hidden messages.
- In Greek means "covered writing"
- Steganography and cryptography are cousins in the spy craft family
- While the goal of the cryptography system is to conceal the content of the messages, the goal of information hiding or steganography is to conceal their existence

Steganography

- What to hide
 - Texts
 - Images
 - Sound
- How to hide
 - embed text in text/images/audio/video files
 - embed image in text/images/audio/video files
 - embed sound in text/images/audio/video files

Steganographic System

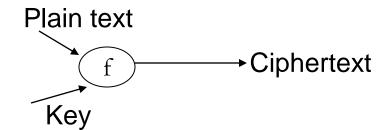


Comparison

Cryptography

$$C = E_k(P)$$

$$P = D_k(C)$$



Steganography

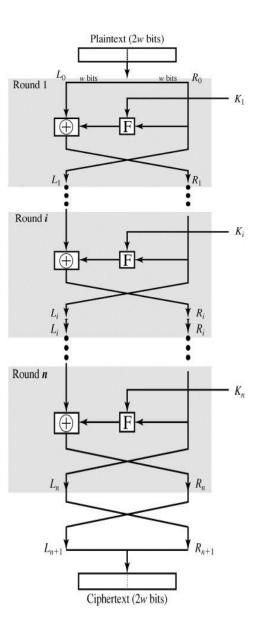
A Real Example

- During WW2 the following cipher message was actually sent by a German spy
 - "Apparently neutral's protest is thoroughly discounted and ignored. Isman hard hit. Blockade issue affects pretext for embargo on by-products, ejecting suets and vegetable oils"
- Hidden Message
 - "Pershing sails from NY June 1"
 - Can be obtained by extracting the second letter in each word of the message sent

- A Feistel cipher is a symmetric structure used in the construction of block ciphers
- Named after the German-born physicist and cryptographer Horst Feistel who did pioneering research while working for IBM (USA).
- It is also commonly known as a Feistel network.
- A large proportion of block ciphers use the scheme, including the Data Encryption Standard (DES).

(from Wiki)

- The Fiestel structure has the advantage that encryption and decryption operations are very similar, even identical in some cases, requiring only a reversal of the key schedule.
- The ciphertext is calculated from the plaintext by repeated application of same transformation or round function.



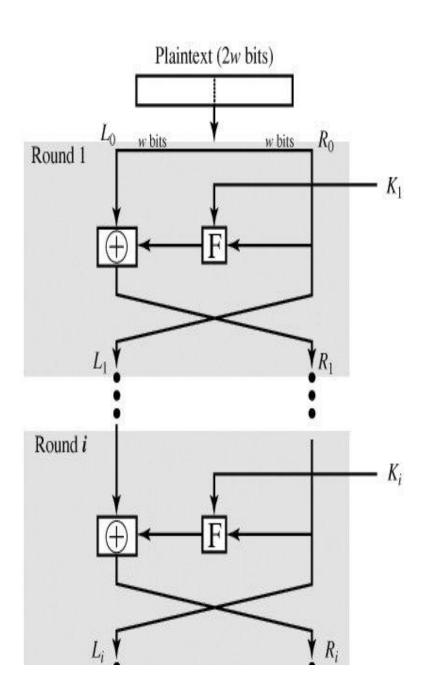
- Substitution performed to left half
 - apply round function F to right half
 - take XOR of output with left half
 - F is parameterized by round subkey Ki
- Permutation of left and right halves
 - interchange left and right halves (swapping)

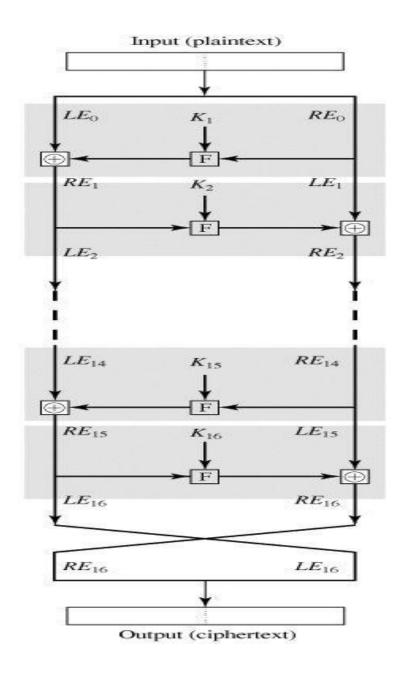
How Feistel Cipher Scheme Works

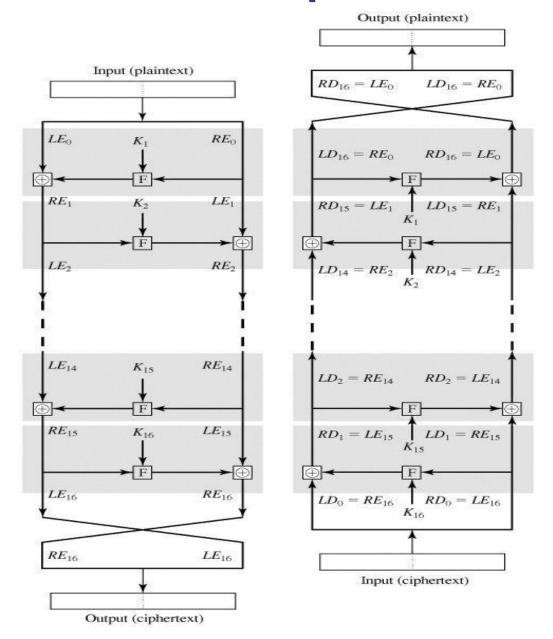
- The input to the encryption algorithm are a plaintext block of length 2w bits and a key k
- The plain text block is divided into two halves: Li and Ri
- The two halves pass through n rounds of processing and then combine to produce the ciphertext block.
- All rounds have the same structure which involves substitution and transposition using a round function F and subkey Ki.

Design Parameters

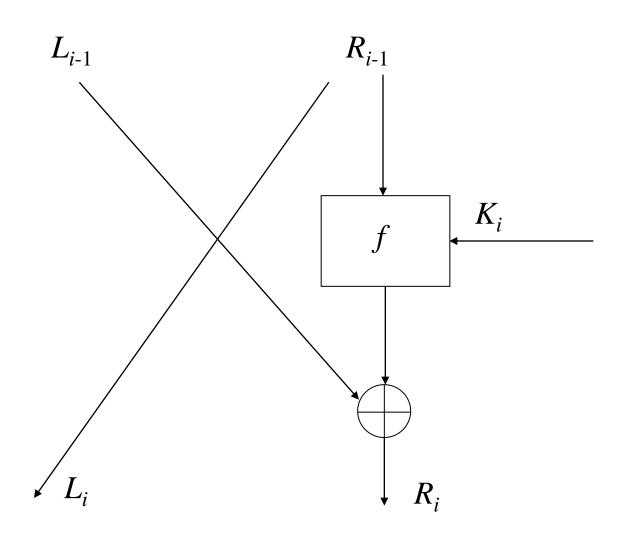
- Block size
 - larger: greater security (diffusion)
 - smaller: faster encryption, decryption
 - typical: 64 bit, 128 bit AES
- Key size
 - larger: greater security (brute-force resist)
 - smaller: faster encryption, decryption
 - typical: 128 bit
- Number of rounds
 - multiple rounds increase security
 - typical: 16
- Round function
 - complexity makes cryptanalysis difficult



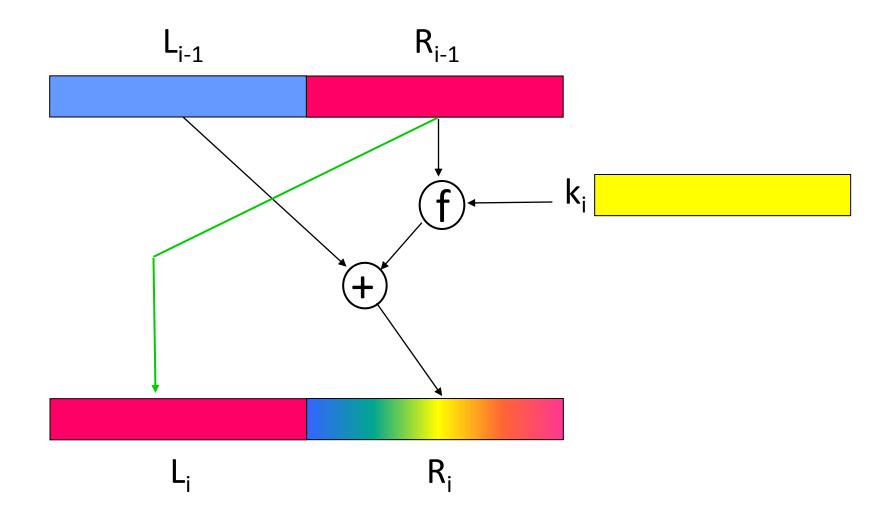




One round of a Feistel system



Round i



Fiestel Cipher Decryption Algorithm

- Ciphertext is used as input
- Use subkeys Ki in reverse order
- Same algorithm is used
- Notation
 - LEi: left half in encryption algorithm
 - REi: right half in encryption algorithm
 - LDi : left half in decryption algorithm
 - RDi : right half in decryption algorithm
- Output of ith encryption round input to (16-i)th decryption round swapped

Fiestel Cipher Decryption Proof

Encryption side

```
LE16 = RE15
RE16 = LE15 \oplus F(RE15, K16)
```

Decryption side

```
LD1 = RD0 = LE16 = RE15

LD0 = RE16

RD1 = LD0\oplus F(RD0, K16)

= RE16\oplus F(RE15, K16)

= [LE15\oplusF(RE15, K16)] \oplusF(RE15, K16)
```

Data Encryption Standard (DES)

- In 1974, IBM submitted an algorithm called LUCIFER for the National Bureau of Standards (NBS).
- The NBS forwarded it to the National Security Agency (NSA), which reviewed it, and returned a version called the Data Encryption Standard (DES) algorithm.
- In 1977, NBS made it the official data encryption standard for use on all unclassified government communications.
- This was probably the result of a misunderstanding between NSA and NBS.
 - The NSA thought DES was hardware-only.
- But NBS published enough details so that people could write DES software.

- DES is a block cipher; it encrypts data in 64-bit blocks.
- A 64-bit block of plaintext goes in one end of the algorithm and a 64bit block of Ciphertext comes out the other end.
- DES is a symmetric algorithm.
- The actual mechanics of how Encryption/Decryption is done is often called a Feistel system.
- DES uses 56-bit key.
- The overall procedure can be given as

$$P^{-1}\{F[P(X)]\}$$

where, X->plaintext

P->certain permutation

F->certain transposition & substitution

F is obtained by cascading a certain function f, with each stage of cascade referred as around.

There are 16 rounds of operations.

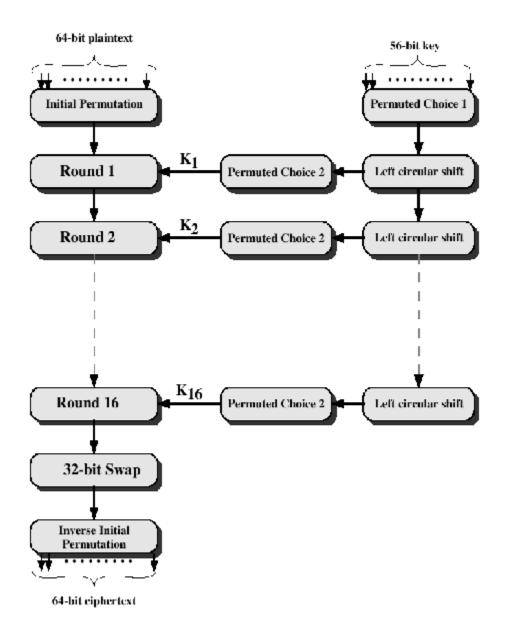
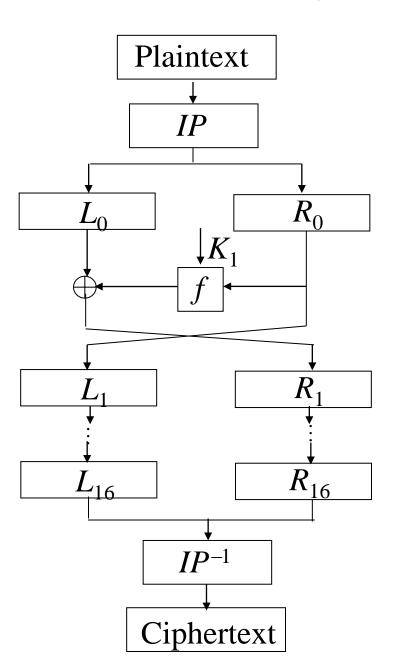


Figure 2.3 General Depiction of DES Encryption Algorithm

Description of DES Algorithm



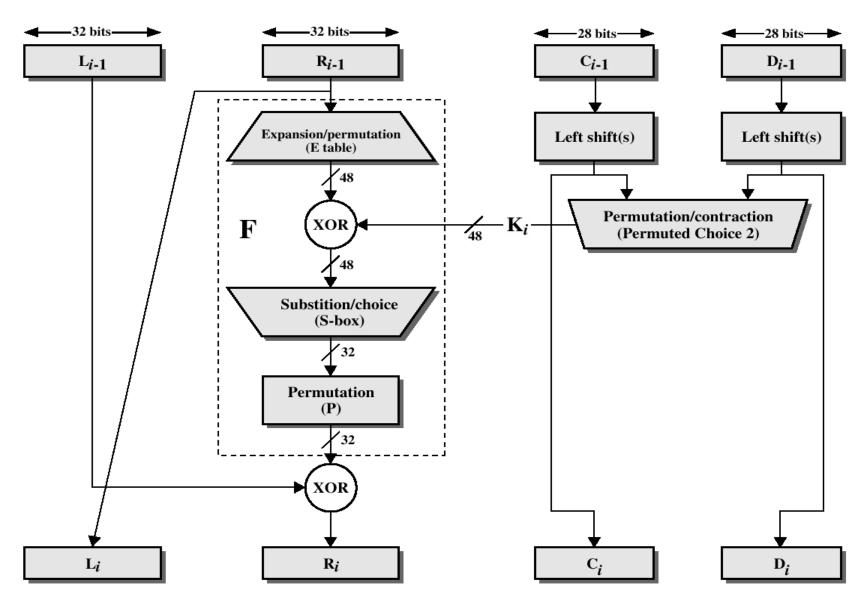


Figure 2.4 Single Round of DES Algorithm

How DES works?

 DES operates on 64-bit of data. Each block of 64 bits is divided into two blocks of 32 bits each, a left half block L and a right half R.

- **M** = 0123456789ABCDEF
- **M** = 0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1011 1010 1011 1100 1101 1110 1111
- L = 0000 0001 0010 0011 0100 0101 0110 0111
- R = 1000 1001 1010 1011 1100 1101 1110 1111

Encoding Data - Initial Permutation

 There is an *initial permutation*, IP of the 64 bits of the message data, M. This rearranges the bits according to the following table.

50	42	34	26	18	10	2
52	44	36	28	20	12	4
54	46	38	30	22	14	6
56	48	40	32	24	16	8
49	41	33	25	17	9	1
51	43	35	27	19	11	3
53	45	37	29	21	13	5
55	47	39	31	23	15	7
	52 54 56 49 51 53	 52 44 54 46 56 48 49 41 51 43 53 45 	52 44 36 54 46 38 56 48 40 49 41 33 51 43 35 53 45 37	52 44 36 28 54 46 38 30 56 48 40 32 49 41 33 25 51 43 35 27 53 45 37 29	52 44 36 28 20 54 46 38 30 22 56 48 40 32 24 49 41 33 25 17 51 43 35 27 19 53 45 37 29 21	52 44 36 28 20 12 54 46 38 30 22 14 56 48 40 32 24 16 49 41 33 25 17 9 51 43 35 27 19 11 53 45 37 29 21 13

M = 0000 0001 0010 0011 0100 0101 0110 0111
1000 1001 1010 1011 1100 1101 1110 1111
IP = 1100 1100 0000 0000 1100 1100 1111 1111
1111 0000 1010 1010 1111 0000 1010 1010

Next divide the permuted block **IP** into a left half L_0 of 32 bits, and a right half R_0 of 32bits.

 $L_0 = 1100 \ 1100 \ 0000 \ 0000 \ 1100 \ 1100 \ 1111 \ 1111$

 $R_0 = 1111\ 0000\ 1010\ 1010\ 1111\ 0000\ 1010\ 1010$

Key Generation (K1....K16)

- 64-bit key is used as input to the algorithm
- Every eight bit is ignored
- The key is first subjected to a permutation Permuted Choice One table
- Resulting 56-bit key treated as two 28-bit quantities labeled C0 and D0

Key Computation

 The 64-bit key is permuted according to the following table & 56-bit key is calculated from it.

Permuted Choice One table

57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	25	27
19	11	3	60	52	44	36
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

LET

K = 00010011 00110100 01010111 01111001 10011011 10111100 11011111 11110001

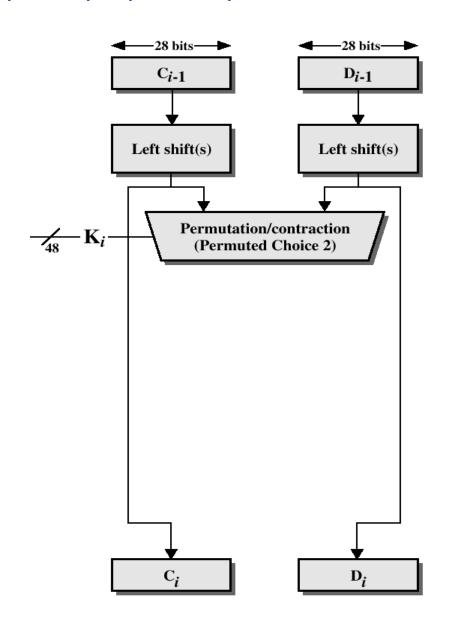
The 56-bit permutation:

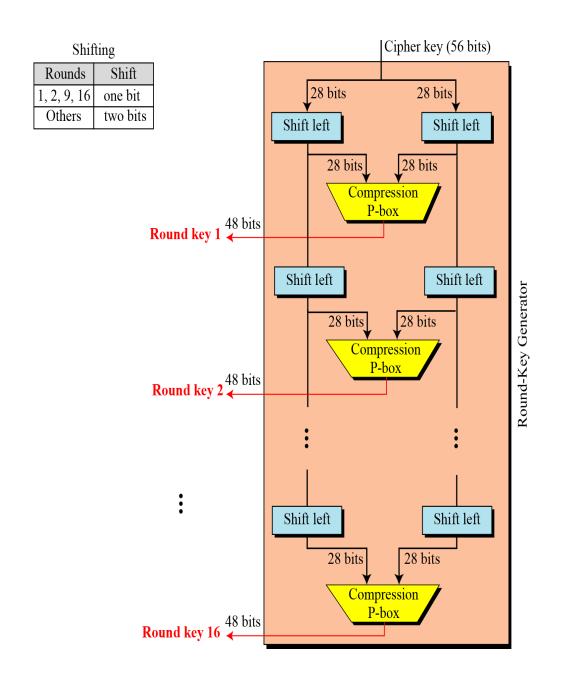
K+ = 1111000 0110011 0010101 0101111 0101010 1011001 1001111 0001111

From the permuted key K+, we get

 $C_0 = 1111000 0110011 0010101 0101111$ $D_0 = 0101010 1011001 1001111 0001111$

(C1, D1)....(C16,D16) - for each round





Key generation

Key Computation ...

With C₀ and D₀ defined, we now create sixteen blocks C_n and D_n,
 1<=i<=16. Each pair of blocks C_i and D_i is formed from the previous pair C_{i-1} and D_{i-1}, respectively, for i = 1, 2, ..., 16, using the following schedule of "left shifts" of the previous block.

Iteration Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Number of Left Shifts	1	1	2	2	2	2	2	2	1	2	2	2	2	2	2	1

C0 = 111100001100110010101011111

D0 = 0101010101100110011110001111

C1 = 1110000110011001010101011111

D1 = 1010101011001100111100011110

C2 = 1100001100110010101010111111

D2 = 0101010110011001111000111101

and so on upto C_{16} & D_{16} .

Key Computation...

• We now form the keys Ki, for 1 <= i <= 16, by applying the following permutation table to each of the concatenated pairs $C_n D_n$.

14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

Thus the 16, 48-bit subkeys are obtained.

 C_1D_1 = 1110000 1100110 0101010 1011111
1010101 0110011 0011110 0011110 K_1 = 000110 110000 001011 101111
111111 000111 000001 110010
Similarly, K_2 = 011110 011010 111011 011001
110110 111100 100111 100101

 K_3 = 010101 011111 110010 001010 010000 101100 111110 011001 and so on upto K_{16} .

Encoding Data ...

We now proceed through 16 iterations, for 1<=i<=16, using a function, F which operates on two blocks - a data block of 32 bits and a key K_i of 48 bits - to produce a block of 32 bits.

$$L_{i} = R_{i-1}$$
 $R_{n} = L_{i-1}$ $F(R_{i-1}, K_{i})$

 $L_0 = 1100 \ 1100 \ 0000 \ 0000 \ 1100 \ 1100 \ 1111 \ 1111$

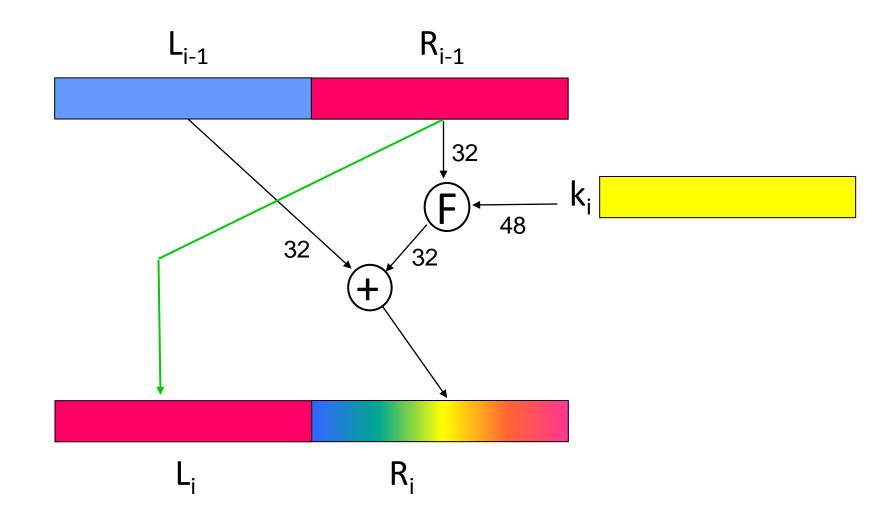
 $R_0 = 1111\ 0000\ 1010\ 1010\ 1111\ 0000\ 1010\ 1010$

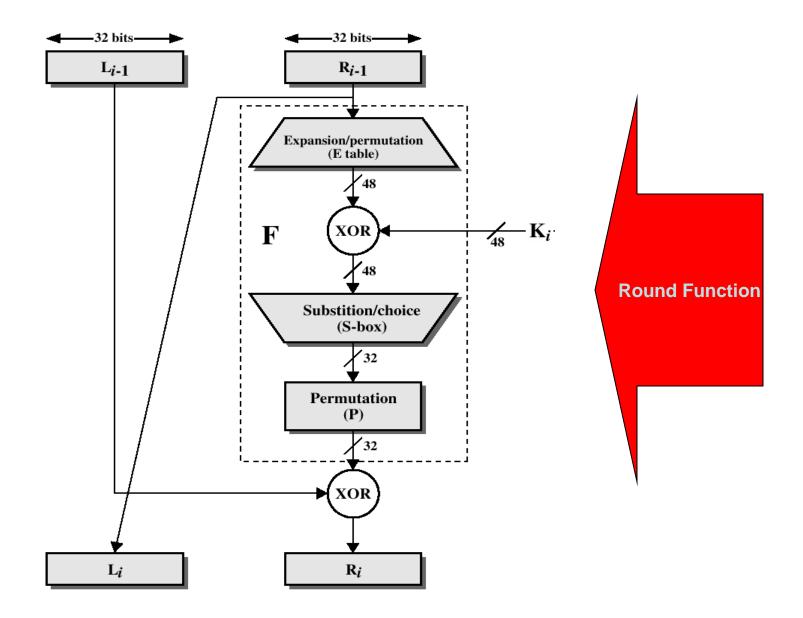
For n = 1, we have

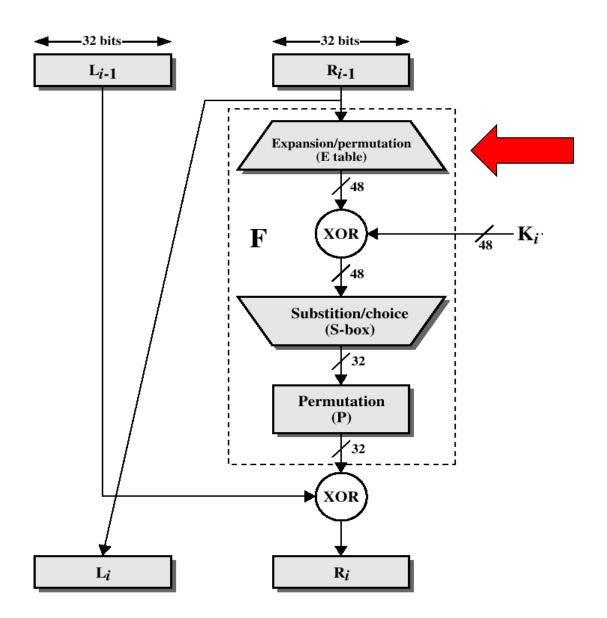
 $K_1 = 000110 \ 110000 \ 001011 \ 101111 \ 111111 \ 000111 \ 000001 \ 110010$ $L_1 = R_0 = 1111 \ 0000 \ 1010 \ 1010 \ 1111 \ 0000 \ 1010 \ 1010$ $R_1 = L_0 + f(R_0, K_1)$

➤It remains to explain how the function **f** works.

Round i of DES







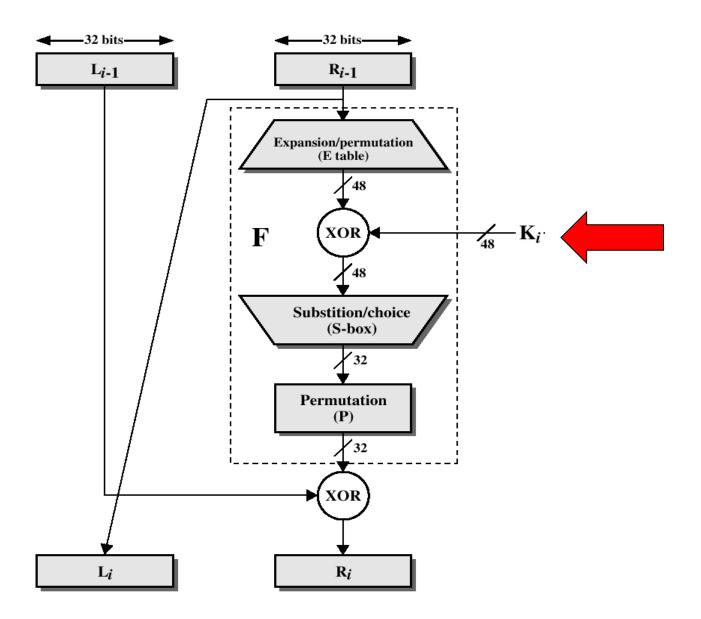
Encoding Data ...

- To calculate F, we first expand each block R_{i-1} from 32 bits to 48 bits.
- This is done by using a selection table called E-table that repeats some of the bits in R_{i-1} .

E-table

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

We calculate $\mathbf{E}(\mathbf{R}_o)$ from \mathbf{R}_o as follows:

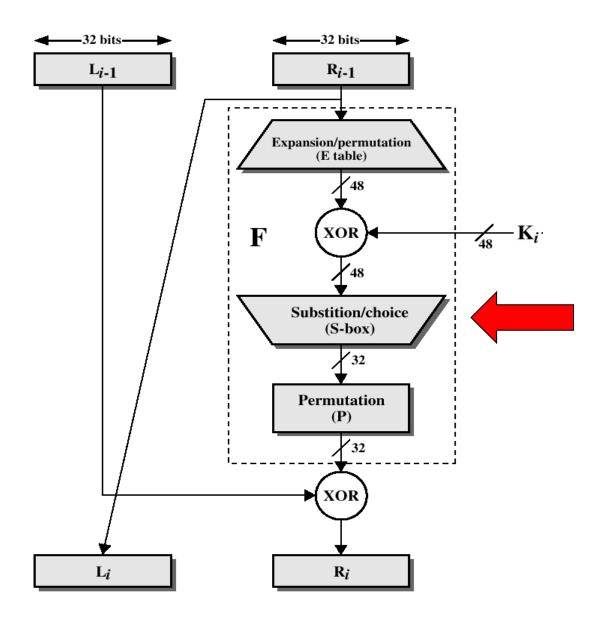


Encoding Data...

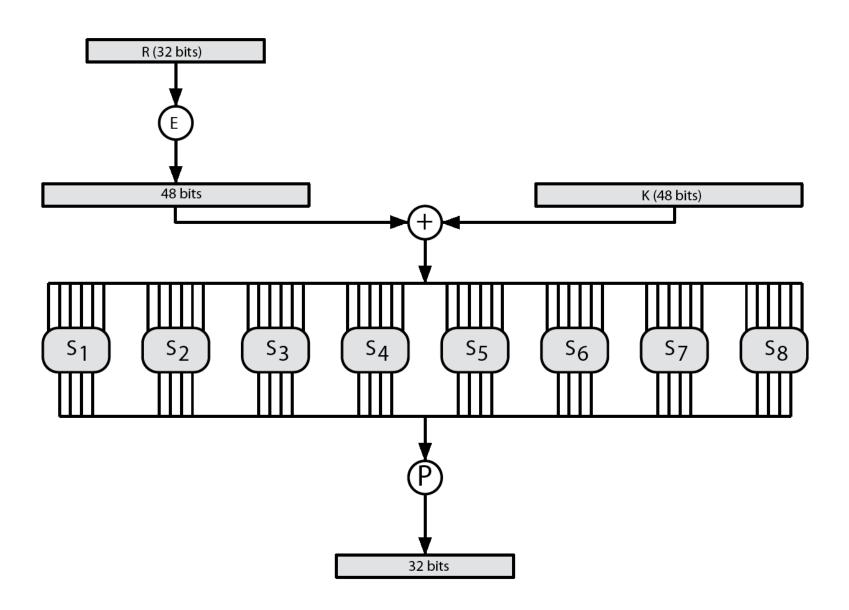
• Next in the f calculation, we XOR the output $E(R_{i-1})$ with the key K_i :

- For
$$K_1$$
, $E(R_0)$, we have $K_i \oplus E(R_{i-1})$

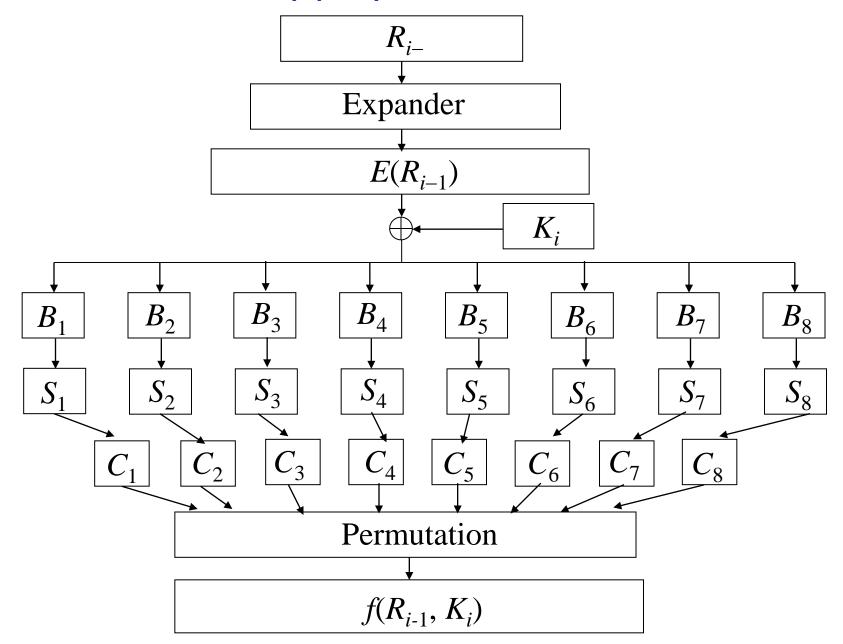
 $K_1 = 000110 \ 110000 \ 001011 \ 101111 \ 111111 \ 000111 \ 000001 \ 110010$ $E(R_0) = 011110 \ 100001 \ 010101 \ 010101 \ 011110 \ 100001 \ 100110 \ 010100$ $K_1 \oplus E(R_0) = 011000 \ 010001 \ 011110 \ 111010 \ 100001 \ 100110 \ 010100$



The F function of DES



The Function $f(R_{i-1}, K_i)$

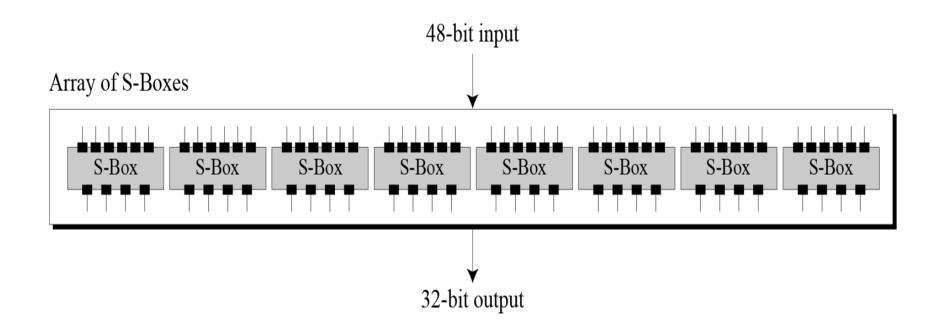


The S-Boxes

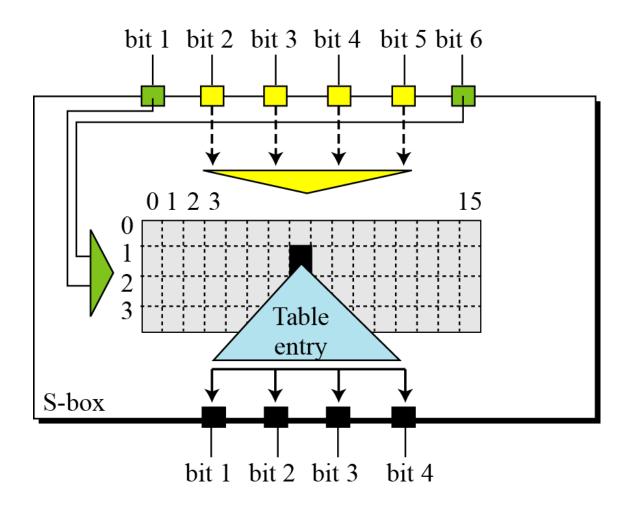
- There are eight S-boxes.
- Each S-box is specified as a 4 x 16 table.
 - each row is a permutation of 0-15
 - outer bits 1 & 6 of input are used to select one of the four rows
 - inner 4 bits of input are used to select a column
- All the eight boxes are different.
- The net result is that the eight groups of 6 bits are transformed into eight groups of 4 bits (the 4-bit outputs from the S boxes) for 32 bits total.

S-Boxes

The S-boxes do the real mixing (confusion). 8 S-boxes, each with a 6-bit input and a 4-bit output.



S-box rule



Encoding Data...

 $K1 \oplus E(R0) = 011000 \ 010001 \ 011110 \ 111010 \ 100001 \ 100110 \ 010100 \ 100111$ S1 S2 S3 S4 S5 S6 S7 S8

S₁ Box

R

Column number

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	14	4	13	1	3	15	11	8	3	10	6	12	5	9	0	7
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

Here $S_1(011011) = 0101$

Similarly, there exists S_1 , S_2 ,..., S_8

For the first round, we obtain as the output of the eight S boxes:

S = 0101 1100 1000 0010 1011 0101 1001 0111

S-Boxes S - box 18 3 10 2 15 11 6 12 S-box 27 2

S-Boxes (Continued) S-box 3

S-box 4 1 2 8 5

S-Boxes (Continued)

S-box 5

2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3

S - box 6

12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13

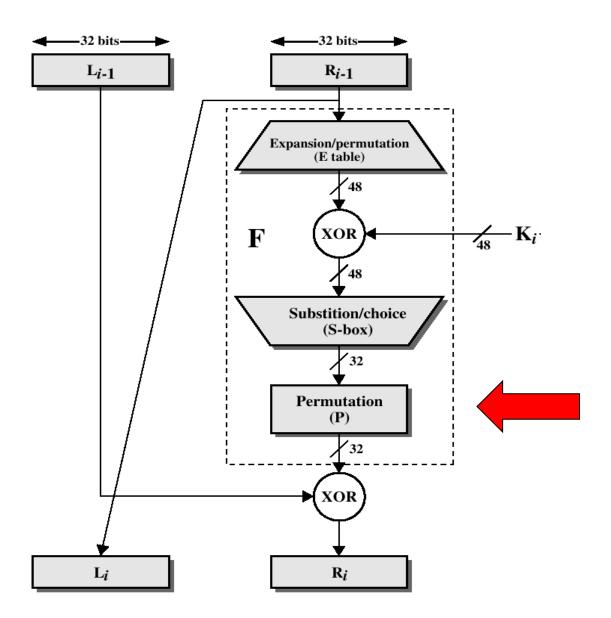
S-Boxes (Continued)

S-box 7

3 12

S-box 8

() $\mathbf{0}$



Encoding Data ...

 The final stage in the calculation of F is to do a permutation P of the S-box output to obtain the final value of f:

$$f = P(S)$$

The permutation **P** is defined in the following table. **P** yields a 32-bit output from a 32-bit input by permuting the bits of the input block.

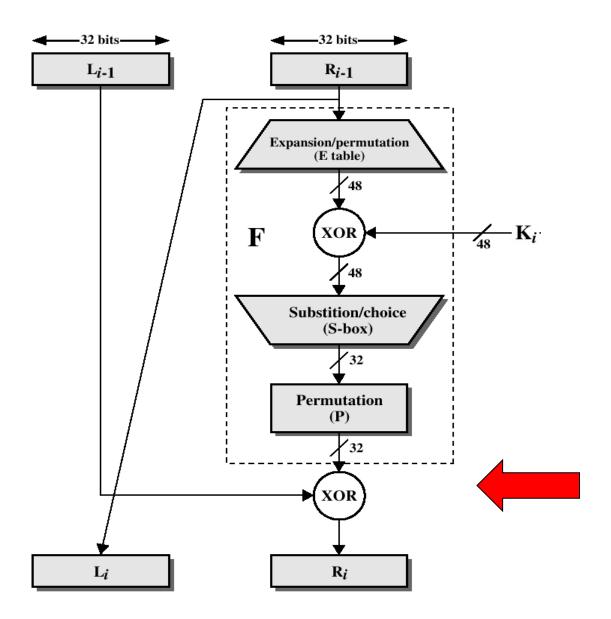
P

16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

From

S = 0101 1100 1000 0010 1011 0101 1001 0111

f = 0010 0011 0100 1010 1010 1001 1011 1011

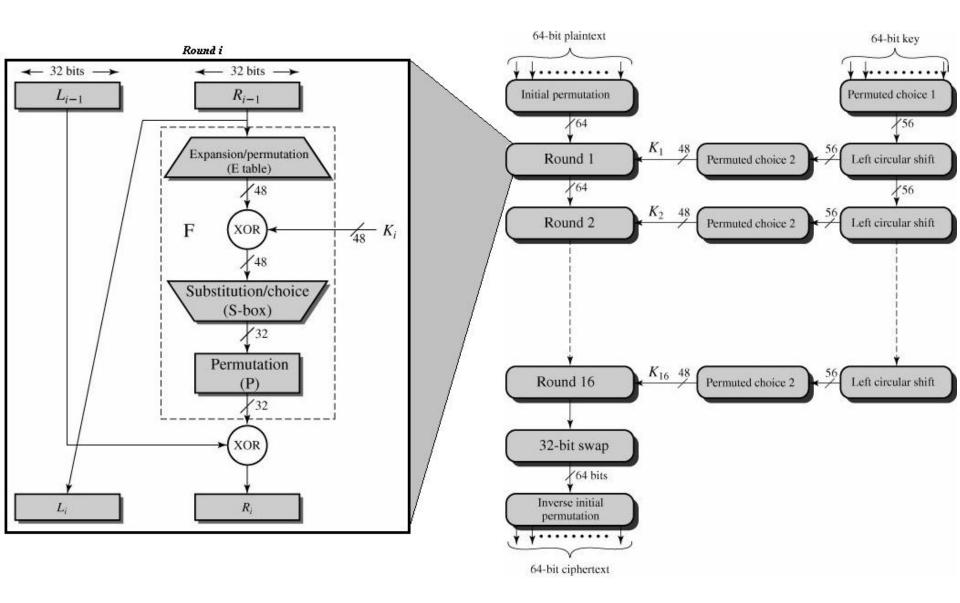


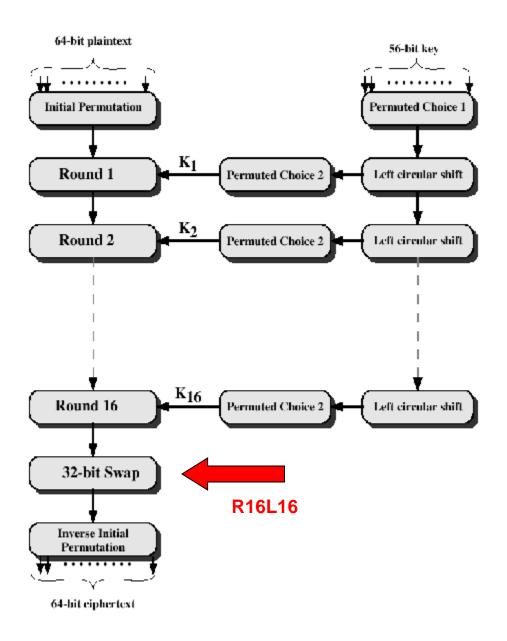
Encoding Data ...

• $R_1 = L_0 \oplus f(R_0, K_1)$

- = 1100 1100 0000 0000 1100 1100 1111 1111
- = 1110 1111 0100 1010 0110 0101 0100 0100

- Proceeding like this we obtain L₁R₁, L₂R₂,..., L₁₆R₁₆.
- At the end of the sixteenth round we have the blocks L16 and R16. We then
 reverse the order of the two blocks into the 64-bit block R₁₆L₁₆ and apply a
 permutation IP-1.





•At the end of the sixteenth round we have the blocks L16 and R16. We then reverse the order of the two blocks into the 64-bit block R16L16 and apply a permutation IP-1.

Figure 2.3 General Depiction of DES Encryption Algorithm

Encoding Data ... IP-1

IP-1

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

LET

 $R_{16}L_{16} = 00001010 \ 01001100$ 11011001 10010101 01000011
01000010 00110010 00110100

IP-1 = 1000 0101 1110 1000 0001 0011 0101 0100 0000 1111 0000 1010 1011 0100 0000 0101 which in hexadecimal format is 85E813540F0AB405.

Thus the encrypted form of M = 0123456789ABCDEF:

namely, **C** = 85E813540F0AB405

Decryption

As with any Feistel cipher, decryption uses the same algorithm as encryption, except that the application of the subkeys is reversed.

Avalanche Effect

Avalanche effect:

- A small change in the plaintext or key results in a significant change in the ciphertext.
- This indicates a high degree of diffusion and confusion
- Is a desirable property of any encryption algorithm
- DES exhibits a strong avalanche effect
 - Changing 1 bit in the plaintext affects 34 bits in the ciphertext on average.
 - 1-bit change in the key affects 35 bits in the ciphertext on average.

To check the avalanche effect in DES, let us encrypt two plaintext blocks (with the same key) that differ only in one bit and observe the differences in the number of bits in each round.

Plaintext: 0000000000000000 Key: 22234512987ABB23

Ciphertext: 4789FD476E82A5F1

Ciphertext: 0A4ED5C15A63FEA3

Key: 22234512987ABB23

Although the two plaintext blocks differ only in the rightmost bit, the ciphertext blocks differ in 29 bits. This means that changing approximately 1.5 percent of the plaintext creates a change of approximately 45 percent in the ciphertext.

Number of bit differences

Rounds	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Bit differences	1	6	20	29	30	33	32	29	32	39	33	28	30	31	30	29

The Strength of DES

1. The Use of 56-Bit Keys

With a key length of 56 bits, there are $\mathbf{2}^{56}$ possible keys, which is approximately 7.2×10^{16}

Thus a brute-force attack appears impractical. Assuming that, on average, half the key space has to be searched, a single machine performing one DES encryption per microsecond would take more than a thousand years to break the cipher.

DES is now considered to be insecure for many applications. This is chiefly due to the 56-bit key size being too small

In July 1998, when the Electronic Frontier Foundation (EFF) announced that it had broken a DES encryption using a special-purpose "DES cracker" machine that was built for less than \$250,000. The attack took less than three days.

The Strength of DES

2. The Nature of the DES Algorithm

The focus of concern has been on the eight substitution tables, or S-boxes, that are used in each iteration.

No one has so far succeeded in discovering the fatal weaknesses in the Sboxes

The Strength of Des

3. Timing Attacks

A timing attack is one in which information about the key or the plaintext is obtained by observing how long it takes a given implementation to perform decryptions on various ciphertexts.

A timing attack exploits the fact that an encryption or decryption algorithm often takes slightly different amounts of time on different inputs.

DES appears to be fairly resistant to a successful timing attack.

Triple DES

- Triple DES (3DES[1]) is the common name for the Triple Data Encryption Algorithm (TDEA or Triple DEA) block cipher, which applies the Data Encryption Standard (DES) cipher algorithm three times to each data block.
- Because of the availability of increasing computational power, the key size of the original DES cipher was becoming subject to brute force attacks
- Triple DES was designed to provide a relatively simple method of increasing the key size of DES to protect against such attacks, without designing a completely new block cipher algorithm.

3DES

- Triple DES uses a "key bundle" which comprises three DES keys, K₁, K₂ and K₃, each of 56 bits (excluding parity bits).
- The encryption algorithm is:
 - ciphertext = $E_{K3}(D_{K2}(E_{K1}(plaintext)))$ i.e., DES encrypt with K_1 , DES decrypt with K_2 , then DES encrypt with K_3 .
- Decryption is the reverse:
 - plaintext = $D_{K1}(E_{K2}(D_{K3}(ciphertext)))$ I.e., decrypt with K_3 , encrypt with K_2 , then decrypt with K_1 .
- Each triple encryption encrypts one block of 64 bits of data.

3DES Keying options

- The standards define three keying options:
 - Keying option 1: All three keys are independent.
 - Keying option 2: K_1 and K_2 are independent, and $K_3 = K_1$.
 - Keying option 3: All three keys are identical, i.e. $K_1 = K_2 = K_3$.
- Keying option 1 is the strongest, with $3 \times 56 = 168$ independent key bits.
- Keying option 2 provides less security, with 2 x 56 = 112 key bits.
- Keying option 3 is equivalent to DES, with only 56 key bits.

Private-Key Cryptography

- Traditional private/secret/single key cryptography uses one key
- shared by both sender and receiver
- if this key is disclosed communications are compromised
- also is symmetric, parties are equal
- hence does not protect sender from receiver forging a message & claiming is sent by sender

Public-Key Cryptography

- Public-key/two-key/asymmetric cryptography involves the use of two keys:
 - a public-key, which may be known by anybody, and can be used to encrypt messages, and verify signatures
 - a private-key, known only to the recipient, used to decrypt messages, and sign (create) signatures
- is asymmetric because
 - those who encrypt messages cannot decrypt messages

Public-Key Characteristics

- Public-Key algorithms rely on two keys with the characteristics that it is:
 - computationally infeasible to find decryption key knowing only algorithm & encryption key
 - computationally easy to en/decrypt messages when the relevant (en/decrypt) key is know

Requirements for public key cryptography

- It is computationally easy for a party B to generate a public key and private key pair.
- It is computationally easy for a sender A, knowing the public key and the message to be encrypted M to generate the corresponding cipher text
- It is computationally easy for the receiver B to decrypt the resulting ciphertext using the private key to recover the original message.
- It is computationally infeasible for adversary knowing the public key to determine the private key.
- It is computationally infeasible for adversary knowing the public key and a ciphertext to recover the original message.
- The two keys can be applied in either order.

Public-Key Applications

- Can classify uses into 3 categories:
 - encryption/decryption (provide secrecy)
 - digital signatures (provide authentication)
 - key exchange (of session keys)

Rivest-Shamir-Adleman (RSA)

- Widely used public key encryption algorithm.
- by Rivest, Shamir & Adleman of MIT in 1977
- RSA is believed to be secure if its keys have a length of al least 1024-bits

RSA

It is a block cipher based upon the fact that finding a random prime number of large size (e.g., 100 digit) is computationally easy, but factoring the product of two such numbers is considered computationally infeasible.

- The security of the RSA cryptosystem is based on the widely believed difficulty of factoring large numbers.
- In 1999, a 512-bit number was factored in 4 months using the following computers:
 - 160 175-400 MHz SGI and Sun
 - 8 250 MHz SGI Origin
 - 120 300-450 MHz Pentium II
 - 4 500 MHz Digital/Compaq
- Estimated resources needed to factor a number within one year

Bits	PCs	Memory
430	1	128MB
760	215,000	4GB
1,020	342×10 ⁶	170GB
1,620	1.6×10 ¹⁵	120TB

- A factor is a number which divides INTO another number without leaving a remainder.
 - For example, 6 is a factor of 12, but 6 is not a factor of 13.
- A prime factor is a number such as 2, 3, 5, 7, 11, 13, or 17 (to name a few) --- a number that can only be divided by itself and 1.

RSA Key Generation Algorithm

- 1. Generate two large prime numbers, p and q
- 2. Let $n = p^*q$
- 3. Let $\emptyset(n) = (p-1)^*(q-1)$
- 4. Choose an integer e, $1 < e < \emptyset(n)$ such that: gcd $(e, \emptyset(n))=1$

[gcd – greatest common denominator]

5. Find d, $1 < d < \emptyset(n)$ such that $ed \equiv 1 \pmod{\emptyset(n)}$ i.e. compute $d = e^{-1} \pmod{\phi(n)}$ Extended Euclidean algorithm

To be secure, very large numbers must be used for p and q - 100 decimal digits at the very least.

- The public key is (e, n) and the private key (d,n)
- The values p,q and ø(n) are private
- To encrypt a plaintext message block M, compute C=Me mod n
- To decrypt the block,
 - compute M=C^d mod n

- e public exponent (give this to others)
- d private exponent (keep this secret)

Key Generation

Select p, q p and q both prime, $p \neq q$

Calculate $n = p \times q$

Calculate $\phi(n) = (p-1)(q-1)$

Select integer e $\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$

Calculate $d \equiv e^{-1} \pmod{\phi(n)}$

Public key $PU = \{e, n\}$

Private key $PR = \{d, n\}$

Encryption

Plaintext: M < n

Ciphertext: $C = M^{\epsilon} \mod n$

Decryption

Ciphertext: C

Plaintext: $M = C^d \mod n$

- Knowing C and e, it is "infeasible" to calculate m without knowing d and n
- Knowing d and n, it is easy to find the original plaintext message m = C^d mod n

RSA Factoring Challenge



http://www.rsa.com

- The RSA Factoring Challenge was a challenge put forward by RSA Laboratories on March 18, 1991 to encourage research into computational number theory and the practical difficulty of factoring large integers and cracking RSA keys used in cryptography.
- Let n be an RSA Number. RSA Laboratories states there are prime numbers p and q such that n = p × q, the problem is to find these two primes, given only n.

RSA Number	Decimal digits	Binary digits	Cash prize offered	Factored on	Factored by
RSA-100	100	330	\$1,000 USD	April 1, 1991	Arjen K. Lenstra
RSA-110	110	364	\$4,429 USD	April 14, 1992	Arjen K. Lenstra and M.S. Manasse
RSA-120	120	397	\$5,898	June 9, 1993	T. Denny et al.
RSA-129	129	426	\$100 USD ^[4]	April 26, 1994	Arjen K. Lenstra et al.
RSA-130	130	430	\$14,527 USD	April 10, 1996	Arjen K. Lenstra et al.
RSA-140	140	463	\$17,226 USD	February 2, 1999	Herman te Riele et al.
RSA-150 ^[5]	150	496		April 16, 2004	Kazumaro Aoki et al.
RSA-155	155	512	\$9,383	August 22, 1999	Herman te Riele et al.
RSA-160	160	530		April 1, 2003	Jens Franke et al., University of Bonn
RSA-170	170	563		December 29, 2009	D. Bonenberger and M. Krone
RSA-576	174	576	\$10,000 USD	December 3, 2003	Jens Franke et al., University of Bonn
RSA-180	180	596		May 8, 2010	S. A. Danilov and I. A. Popovyan, Moscow State University
RSA-190	190	629		November 8, 2010	A. Timofeev and I. A. Popovyan
RSA-640	193	640	\$20,000 USD	November 2, 2005	Jens Franke et al., University of Bonn
RSA-200	200	663		May 9, 2005	Jens Franke et al., University of Bonn

RSA-768 is factored!

- A six-institution research team led by T. Kleinjung has successfully factored the RSA-768 (Decimal digits:232 Binary digits:768) challenge number.
- The effort took almost 2000 2.2GHz-Opteron-CPU years
- The factors are:

334780716989568987860441698482126908177047 949837137685689124313889828837938780022876 14711652531743087737814467999489

and

3674604366679959042824463379962795263227915 8164343087642676032283815739666511279233373 417143396810270092798736308917

Cash prize offered: \$30,000 USD

Example: Bob chooses his public key

- He randomly chooses two primes, 59 and 67, respectively (p = 59, q = 67)
- N = pxq = 3953
- $\phi(n) = (58)(66) = 3828$
- Pick a random e, less than 3828 but > 1
 try 2669, gcd(2669, 3828) = 1
- $d = e^{-1} \mod \phi(n)$
 - d = 1625
- Bob's private key (d, n) is (1625,3953), so now Bob publishes his public key (e, n) as (2669, 3953)

Alice wants to send Bob a message, m...

- Alice has plaintext 3128 to send. She will send E(m):
 - Alice encrypts with public key (e,n) or (2669,3953)
 - $-C=E(m) = 3128^{2669} \mod 3953 = 3541$
- Bob receives the ciphertext 3541:
 - Bob decrypts with private key (d,n) or (1625,3953)
 - $-M = 3541^{1625} \mod 3953 = 3128$
- •C=Me mod n
 •M=Cd mod n

Euclid's Algorithm

- To find the greatest common divisor of two numbers
- If you want to find the gcd(x,y), you take the larger of the two, so if x>y, we would write,
 - $-x=m_1^*y+r_1$, where m1 is our first multiplier and r_1 is our first remainder
 - If $r_1 = 1$ then we have found that gcd(x,y)=1
 - if $r_1 = 0$ then gcd(x,y) = y

- If r₁>1 then we must continue our process
- y=m₂*r₁+r₂, where m₂ and r₂ are our next multiplier and remainder
- Now if we have r2=1 then gcd(x,y)=1
- If $r_2=0$, then gcd(x,y)=r1

 If r2>1, then we must do another step, and continue this process until we reach a remainder of either 0 or 1.

```
x=m1*y+r1
y=m2*r1+r2
r1=m3*r2+r3
r2=m4*r3+r4
.....
ri=mi+2 * ri+1 + ri+2
```

Euclid's Algorithm Example

p=7, q=11, n=pq=77
$$\emptyset(n)=(7-1)$$
 (11-1)=60, choose k=13, find gcd(60,13)

find gcd(60,13)

$$60=(4)(13)+8$$

$$13=(1)(8)+5$$

$$8=(1)(5)+3$$

$$5=(1)(3)+2$$

$$3=(1)(2)+1$$

$$2=(2)(1)+0$$

$$gcd(60,13)=1$$

•If r2=0, then gcd(x,y)=r1

ed
$$\equiv 1 \pmod{\emptyset(n)}$$

d= $e^{-1} \pmod{\emptyset(n)}$

Inverses mod n

Extended Euclidean algorithm

Inverses mod n

We will number the steps of the Euclidean algorithm starting with step 0. The quotient obtained at step i will be denoted by q. As we carry out each step of the Euclidean algorithm, we will also calculate an auxillary number, p_i. For the first two steps, the value of this number is given: $p_0 = 0$ and $p_1 = 1$. For the remainder of the steps, we recursively calculate $p_i = p_{i,2} - p_{i,1} q_{i,2} \pmod{n}$. Continue this calculation for one step beyond the last step of the Euclidean algorithm.

$$p_i = p_{i-2} - p_{i-1} q_{i-2} \pmod{n}$$

Example

Find the inverse of 15 mod 26.

Step 0:
$$26 = 1(15) + 11$$
 $p_0 = 0$
Step 1: $15 = 1(11) + 4$ $p_1 = 1$
Step 2: $11 = 2(4) + 3$ $p_2 = 0 - 1(1) \mod 26 = 25$
Step 3: $4 = 1(3) + 1$ $p_3 = 1 - 25(1) \mod 26 = -24 \mod 26 = 2$
Step 4: $3 = 3(1) + 0$ $p_4 = 25 - 2(2) \mod 26 = 21$
 $p_5 = 2 - 21(1) \mod 26 = -19 \mod 26 = 7$

$decrypt(855) = (855^{2753}) \mod 3233$

= 50432888958416068734422899127394466631453878360035509315554967564501 05562861208255997874424542811005438349865428933638493024645144150785 17209179665478263530709963803538732650089668607477182974582295034295 04079035818459409563779385865989368838083602840132509768620766977396 67533250542826093475735137988063256482639334453092594385562429233017 51977190016924916912809150596019178760171349725439279215696701789902 54965202919790505781532871558392070303159585937493663283548602090830 63550704455658896319318011934122017826923344101330116480696334024075 $04695258866987658669006224024102088466507530263953870526631933584734\\ \ 81094876156227126037327597360375237388364148088948438096157757045380$ $08107946980066734877795883758289985132793070353355127509043994817897\\ 9054899338121732945853544741326805698108726334828546381688504882434658897839333466254454006619645218766694795528023088412465948239275105\\ 77049113329025684306505229256142730389832089007051511055250618994171$ 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69907690169025946468104141214204472402661658275680524166861473393322$ 13992163738356814149047932037426337301987825405699619163520193896982 54478631309773749154478427634532593998741700138163198116645377208944 00285485000269685982644562183794116702151847721909339232185087775790 95933267631141312961939849592613898790166971088102766386231676940572 $95932538078643444100512138025081797622723797210352196773268441946486\\ 16402961059899027710532570457016332613431076417700043237152474626393$ 99011899727845362949303636914900881060531231630009010150839331880116 68215163893104666659513782749892374556051100401647771682271626727078 37012242465512648784549235041852167426383189733332434674449039780017 84689726405462148024124125833843501704885320601475687862318094090012 $\frac{63241969092252022679880113408073012216264404133887392600523096072386}{99227975522771848648475326124302804177943090938992370938053652046462}\\ 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\frac{15855496515800103474611979213076722454380367188325370860671331132581}{59227975527773274119265709116613580084145421487687310394441054$ 79639308530896880365608504772144592172500126500717068969428154627563 70458838904219177398190648731908014828739058159462227867277418610111 02763247972904122211994117388204526335701759090678628159281519982214 57652796853892517218720090070389138562840007332258507590485348046564 84202455109029882398517953684125891446352791897307683834073696131409 74522985638668272691043357517677128894527881368623965066654089894394 95161912002160777898876864736481837825324846699168307281220310791935 64666840159148582699993374427677252275403853322196852298590851548110 $\frac{40229657916338257385513314823459591633281445819843614596306024993617}{46279764077057074816406450769779869955106180046471937808223250148934} 07851137833251073753823403466269553292608813843895784099804170410417$ 77608463062862610614059615207066695243018438575031762939543026312673 77406936404705896083462601885911184367532529845888040849710922999195 65539701911191919188327308603766775339607722455632113506572191067587 51186812786344197572392195263333856538388240057190102564949233944519 65959203992392217400247234147190970964562108299547746193228981181286 05556588093851898811812905614274085809168765711911224763288658712755 38928438126611991937924624112632990739867854558756652453056197509891 14578114735771283607554001774268660965093305172102723066635739462334 13638045914237759965220309418558880039496755829711258361621890140359 54234930424749053693992776114261796407100127643280428706083531594582

Assignments

- 1. Perform encryption and decryption using RSA algorithm, for the following:
 - ① p = 3; q = 11, e = 7; M = 5
 - ② p = 5; q = 11, e = 3; M = 9
- 2. In a public-key system using RSA, you intercept the ciphertext C = 10 sent to a user whose public key is e = 5, $\emptyset(n) = 35$. What is the plaintext M?

RSA for digital signing and verification

Digital Signing

- In order to sign a message the sender does the following:
 - produces a hash value of the message
 - uses his/her private key (n,d) to compute the signature
 - S=Md mod n
 - Sends the signature S to the recipient

Signature Verification

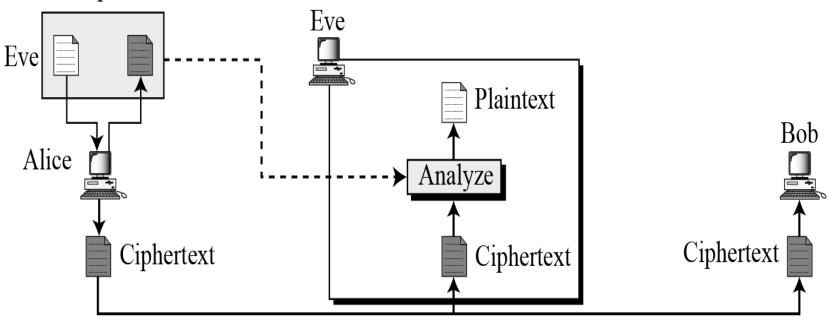
- The recipient does the following in order to verify the message:
 - Uses the sender's public key (n,e) to compute the hash value $V = S^e \mod n$
 - Extracts the hash value from the message
 - If both hash values are identical then signature is valid.

• Anyone else who wants to compute d, must first know $\varphi(n)$, but to know $\varphi(n)$, one must know p and q. In other words, they must factor n.

DES vs. RSA

- An RSA operation is can be performed by a series of modular multiplications.
 - DES several rounds Feistel Structure
- By comparison, DES is much faster than RSA.
 - In software, DES is generally at least 100 times as fast as RSA.
 - Generation of numbers used in RSA can take time
 - In hardware, DES is between 1,000 and 10,000 times as fast, depending on the implementation.
- RSA can be used both for encryption and for digitally signing.
- RSA is generally considered to be secure when sufficiently long keys are used (512 bits is insecure, 768 bits is moderately secure, and 1024 bits is good, for now).
- The security of RSA relies on the difficulty of factoring large integers.
 Dramatic advances in factoring large integers would make RSA vulnerable.
- RSA is very vulnerable to chosen plaintext_attacks.

Pair created from chosen plaintext



Given: P_1 , $C_1 = E_k(P_1)$, P_2 , $C_2 = E_k(P_2)$, ... P_i , $C_i = E_k(P_i)$, where the cryptanalyst gets to choose P_1 , P_2 , ... P_i

Advanced Encryption Standard

- AES was published by NIST (National Institute of Standards and Technology) in 2001.
- Symmetric block cipher
- Block length 128 bits
- Key length 128, 192 and 256 bits
- AES is not a Fiestel structure
- AES process the entire data block in parallel during each round using substitutions and permutations.
- Rounds: 10,12 or 14 (depending on key size)

Key Management

- Public key encryption schemes are secure only if the authenticity of the public key is assured.
- Key management deals with the secure generation, distribution, and storage of keys.
 Secure methods of key management are extremely important.
- Once a key is randomly generated, it must remain secret to avoid unfortunate mishaps (such as impersonation).
- In practice, most attacks on public-key systems will probably be aimed at the key management level, rather than at the cryptographic algorithm itself.

- Users must be able to securely obtain a key pair suited to their efficiency and security needs.
- There must be a way to look up other people's public keys and to publicize one's own public key.
- Users must be able to legitimately obtain others' public keys; otherwise, an intruder can either change public keys listed in a directory, or impersonate another user.

Distribution of Public Keys

- Public announcement of public key
 - Public key made public
 - RSA any participant can send his public key to any other participant or broadcast the key to the community at large
 - Disadv.
 - Any one can forge such a public announcement
 - i.e. some user could pretend to be user A and send a public key to another participant or broadcast such a public key

Public Key Authority

- Stronger control over the distribution of public keys from a directory
- A central authority maintains a dynamic directory of public keys of all participants
- Each participant reliably know a public key of the authority, with only the authority knows the corresponding private key.

Public Key Authority – steps

- A send a time stamped message to the public key authority containing a request for the current public key of B
- The authority responds with a message that is encrypted using authorities private key PRauth. Thus A is able to decrypt the message using the authority's public key
 - The message includes
 - B's public key, PUb which A can use to encrypt messages destined for B
 - The original time stamp, so A can determine that this is not an old message from authority.
 - The original request, sent by A to authority to ensure that the request was not altered.
- A stores B's public key and also uses it to encrypt a message to B containing an identifier of A(IDA) and a transaction id (N1)
- B retrieves A's public key from authority in the same manner as A retrieved B's public key

Public-key Certificates

- Use certificates that can be used by participants to exchange keys without contacting a public key authority.
- A certificate consists of a public key plus an identifier of the key owner with the whole block signed by a trusted third party (certificate authority a Govt. agency, financial institution)
- A user can present his/her public key to the authority in a secure manner and obtain a certificate. The user can then publish the certificate.
- Anyone needed this users public key can obtain the certificate and verify that it is a valid by way of the attached trusted signature.



Diffie-Hellman Key Exchange Protocol

- Diffie-Hellman key exchange protocol allows two parties that have no prior knowledge of each other to jointly establish a shared secret key over an insecure communications channel.
- This key can then be used to encrypt subsequent communications using a symmetric key cipher.
- Developed by Diffie and Hellman in 1976 and published in the ground-breaking paper "New Directions in Cryptography.

- The protocol has two system parameters p and g.
- They are both public and may be used by all the users in a system.
- Parameter p is a prime number
- and parameter g (usually called a generator) is an integer less than p, with the following property:
 - for every number n between 1 and p-1 inclusive, there is a power k of g such that $n = gk \mod p$.

- Suppose Alice and Bob want to agree on a shared secret key using the Diffie-Hellman key agreement protocol.
- First, Alice generates a random private value a and Bob generates a random private value b.
- Both a and b are drawn from the set of integers.
- Then they derive their public values using parameters p and g and their private values.
- Alice's public value is g^a mod p and Bob's public value is g^b mod p.
- They then exchange their public values.

- Finally, Alice computes $g^{ab} = (g^b)^a \mod p$, i.e. $(g^b \mod p)^a \mod p = (g^b)^a \mod p$
- Bob computes $g^{ba} = (g^a)^b \mod p$ i.e. $(g^a \mod p)^b \mod p = (g^b)^a \mod p$
- Since $g^{ab} = g^{ba} = k$, Alice and Bob now have a shared secret key k.

It assumes that it is computationally infeasible to calculate the shared secret key k = g^{ab} mod p given the two public values g^a mod p and g^b mod p when the prime p is sufficiently large.

- 1. Alice and Bob agree to use a prime number p=23 and base g=5.
- 2. Alice chooses a secret integer a=6, then sends Bob A = g^a mod p
 - $A = 5^6 \mod 23$
 - $A = 15,625 \mod 23$
 - A = 8
- 3. Bob chooses a secret integer b=15, then sends Alice B = $g^b \mod p$
 - $B = 5^{15} \mod 23$
 - B = 30,517,578,125 mod 23
 - B = 19
- 4. Alice computes $\mathbf{s} = B^a \mod p$
 - $\mathbf{s} = 19^6 \mod 23$
 - $\mathbf{s} = 47,045,881 \mod 23$
 - s = 2
- 5. Bob computes $\mathbf{s} = A^b \mod p$
 - $\mathbf{s} = 8^{15} \mod 23$
 - $\mathbf{s} = 35,184,372,088,832 \mod 23$
 - s = 2

Man-in-the-middle attack

- The Diffie-Hellman key exchange is vulnerable to a man-inthe-middle attack.
- An opponent Carol intercepts Alice's public value and sends her own public value to Bob.
- When Bob transmits his public value, Carol substitutes it with her own and sends it to Alice.
- Carol and Alice thus agree on one shared key and Carol and Bob agree on another shared key.
- After this exchange, Carol simply decrypts any messages sent out by Alice or Bob, and then reads and possibly modifies them before re-encrypting with the appropriate key and transmitting them to the other party.
- This vulnerability is present because Diffie-Hellman key exchange does not authenticate the participants.
- Possible solutions include the use of digital signatures.

Message Authentication

What is Authentication?

 A procedure to verify that received messages come from the alleged source and have not been altered.

- Message authentication is a mechanism or service used to verify the integrity of a message.
- Message authentication assures that data received are exactly as sent by (i.e., contain no modification, insertion, deletion, or replay) and that the purported identity of the sender is valid.

- Message authentication involves two aspects:
 - Source authentication, which verifies the identity of the source, prevents the acceptance of messages from a fraudulent source.
 - Data integrity, which protects the data from modification.

Authentication Requirements

Disclosure

 Release of message contents to any person or process not possessing the appropriate cryptographic key

Traffic analysis

 process of intercepting and examining messages in order to deduce information from patterns in communication

Masquerade

 Insertion of messages into the network from a fraudulent source.

Authentication Requirements

Content modification

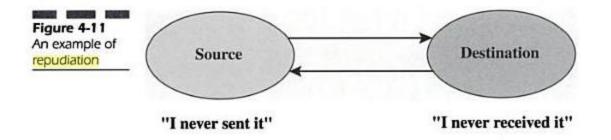
 Changes to the contents of a message, including insertion, deletion, transposition and modification

Sequence modification

 Any modification to a sequence of messages between parties, including insertion, deletion and reordering.

Authentication Requirements

- Timing modification
 - Delay or replay of messages
- Source repudiation
 - Denial of transmission of message by source.
- Destination repudiation
 - Denial of receipt of message by destination



Nonrepudiation?

- Nonrepudiation means to ensure that a transferred message has been sent and received by the parties claiming to have sent and received the message.
- Nonrepudiation is a way to guarantee that the sender of a message cannot later deny having sent the message and that the recipient cannot deny having received the message.
- Nonrepudiation can be obtained through the use of digital signatures.

Measures to deal with the attacks

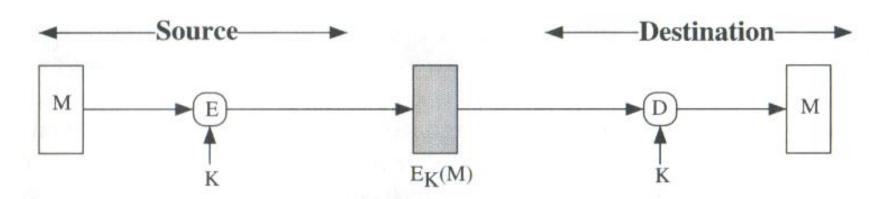
- Message confidentiality
 - Disclosure, Traffic analysis
- Message Authentication
 - Masquerade, Content modification, Sequence modification, Timing modification
- Digital Signature
 - Source and Destination repudiation

Authentication Functions

- 3 Types of cryptographic operations related to authentication:
 - Message encryption: The ciphertext of the entire message serves as its authenticator.
 - Message authentication code (MAC): A
 function of the message and a secret key that
 produces a fixed-length value that serves as the
 authenticator.
 - Hash function: A function that maps a message of any length into a fixed-length hash value, which serves as the authenticator.

Message Encryption

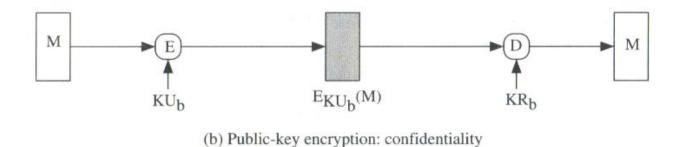
- if symmetric encryption is used then:
 - receiver know sender must have created it
 - since only sender and receiver Know key used



(a) Conventional encryption: confidentiality and authentication

Message Encryption

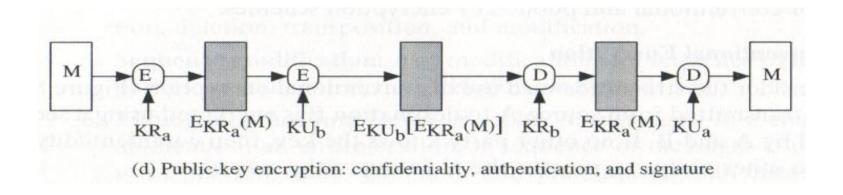
Public-key
 Encryption



Provides confidentiality but not authentication

No authentication – because any opponent could also use recipient's (B) public key to encrypt a message claiming to be the sender A

- if public-key encryption is used:
 - encryption provides no confidence of sender
 - since anyone potentially knows public-key
 - however if
 - sender signs message using their private-key (signature)
 - then encrypts with recipients public key (confidentiality)
 - have both secrecy and authentication
 - Disadv: public key algorithm exercised four times rather than two in each communication



Message Authentication Code

 An alternative authentication technique involves the use of a secret key to generate a small fixed-size block of data, known as a cryptographic checksum or MAC that is appended to the message.

Message Authentication Code

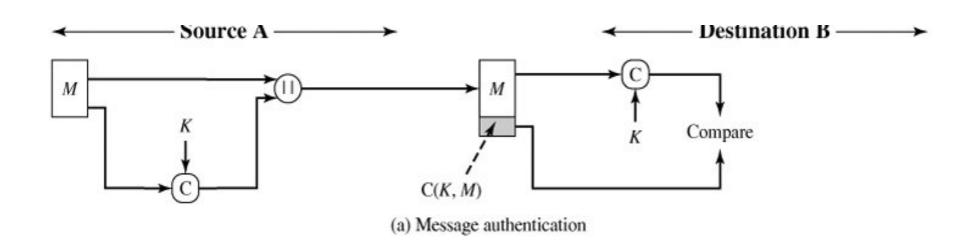
- generated by an algorithm that creates a small fixed-sized block
 - depending on both message and some key
 - like encryption though need not be reversible
- appended to message as a signature
- receiver performs same computation on message and checks it matches the MAC
- provides assurance that message is unaltered and comes from sender

MAC

 This technique assumes that two communicating parties, say A and B, share a common secret key K. When A has a message to send to B, it calculates the MAC as a function of the message and the key:

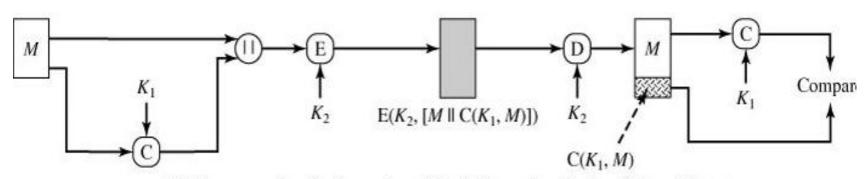
Basic Uses of MAC

a) The receiver is assured that the message has not been altered. If an attacker alters the message but does not alter the MAC, then the receiver's calculation of the MAC will differ from the received MAC. Because the attacker is assumed not to know the secret key, the attacker cannot alter the MAC to correspond to the alterations in the message.



Basic Uses of MAC

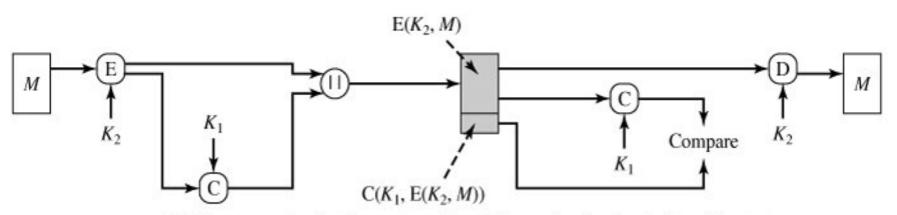
b) The receiver is assured that the message is from the alleged sender. Because no one else knows the secret key, no one else could prepare a message with a proper MAC.



(b) Message authentication and confidentiality; authentication tied to plaintext

Basic Uses of MAC

c) authentication tied to ciphertext.



(c) Message authentication and confidentiality; authentication tied to ciphertext

(a) Message authentication

- A →B: M||C(K, M)
 - Provides authenticationOnly A and B share K
 - (b) Message authentication and confidentiality: authentication tied to plaintext
- A → B:E(K₂, [M||C(K1, M)])
 - Provides authentication
 Only A and B share K₁
 - Provides confidentiality
 Only A and B share K₂
 - (c) Message authentication and confidentiality: authentication tied to ciphertext
- $A \rightarrow B:E(K_2, M)||C(K_1, E(K_2, M))|$
 - Provides authentication
 Using K₁
 - Provides confidentiality
 Using K₂

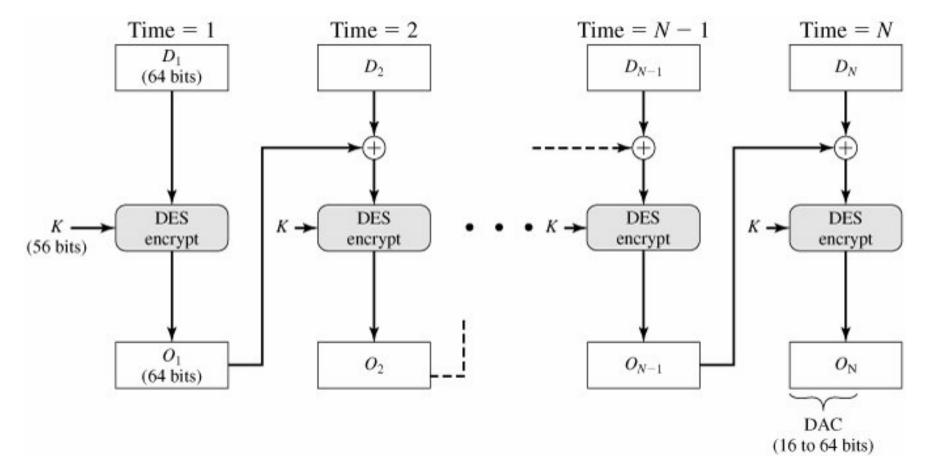
MAC Requirements

- The MAC function should satisfy the following requirements:
- 1. If an opponent observes M and C(K, M), it should be computationally infeasible for the opponent to construct a message M' such that C(K, M') = C(K, M).
 - (it should infeasible that opponent is able to construct a new message to match a given MAC)
- 1. C(K, M) should be uniformly distributed in the sense that for randomly chosen messages, M and M', the probability that C(K, M) = C(K, M') is 2^{-n} , where n is the number of bits in the MAC.
- 2. Let M' be equal to some known transformation on M. That is, M' = f(M). For example, f may involve inverting one or more specific bits. In that case, $Pr[C(K, M) = C(K, M')] = 2^{-n}$.

MAC based on DES

- The Data Authentication Algorithm, based on DES, has been one of the most widely used MACs for a number of years.
- The algorithm can be defined as using the cipher block chaining (CBC) mode of operation of DES with an initialization vector of zero.
- The message is encrypted with some block cipher algorithm in CBC mode to create a chain of blocks such that each block depends on the proper encryption of the previous block.

- The data (e.g., message, record, file, or program) to be authenticated are grouped into contiguous 64-bit blocks: D₁, D₂,..., D_N.
- If necessary, the final block is padded on the right with zeroes to form a full 64-bit block.
- Using the DES encryption algorithm, E, and a secret key, K, a data authentication code (DAC) is calculated.



$$O_1 = E(K, D_1)$$
 $O_2 = E(K, [D_2 \oplus O_1)$
 $O_3 = E(K, [D_3 \oplus O_2)$

 $ON=E(K, [D_N \oplus O_{N-1}))$

Hash Functions

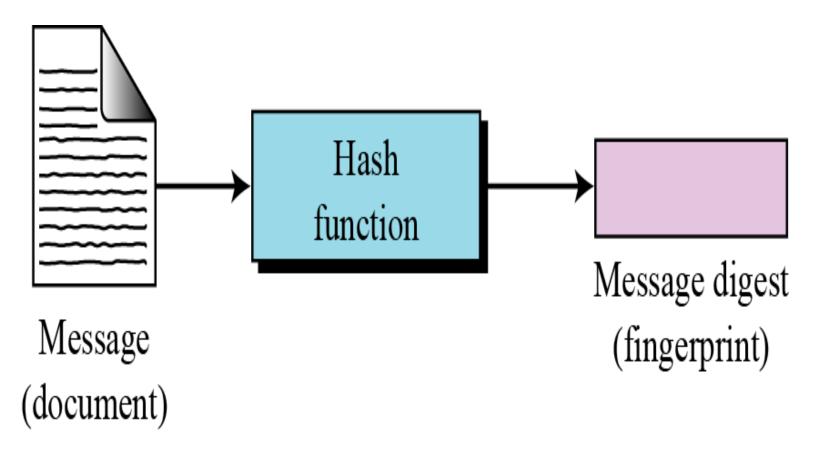
- Hash functions take a message as input and produce an output referred to as a hash code, hash value or simply hash.
 - More precisely, a hash function h maps bit strings of arbitrary finite length to strings of fixed length.

Hash Functions

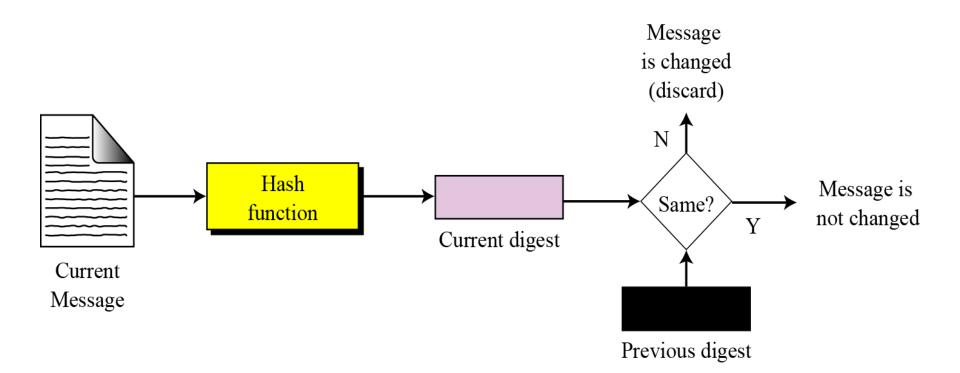
- A (one-way) hash function accepts a variablesize message M as input and produces a fixedsize hash code H(M) as output (called Message Digest)
- Hash code provides error detection -> a change in one bit of message results in a change to the hash code.

- The purpose of a hash function is to produce a "fingerprint" of a file, message or other block of data.
- The hash value is appended to the message at the source.
- The receiver authenticates that message by recomputing the hash value.

Message and digest



Checking integrity



Requirements for a hash function

- H can be applied to a block of data of any size
- H produces a fixed length output
- It is easy to compute the hash value for any given message.
- It is infeasible to find a message that has a given hash. This
 is referred to as "one way property" (preimage resistance
 property)
- For any given block x, it is infeasible to find y≠x such that H(y)=H(x), this is known as "weak collision resistance"
- It is infeasible to modify a message without changing its hash.

- Weak collision resistance
 - Guarantees that an alternative message hashing to the same value as a given message cannot be found.
- Strong Collision resistance
 - Refers to how resistant the hash function is to a type of attack known as birthday attack.

Birthday attack

 If an encrypted hash code c is transmitted with the corresponding unencrypted message m the opponent can find m' such that H(m')=H(m) and substitute another message and fool the receiver..i.e the opponent tries about 263 messages to find one that matches the hash code of the intercepted message.

Simple Hash Functions

- based on XOR of message blocks
 - -divide the message into equal size blocks
 - -perform XOR operation block by block
 - -final output is the hash
- not very secure

Simple Hash Function

Bit-by-bit exclusive-OR (XOR)

$$C_i = b_{i1} \oplus b_{i2} \oplus \ldots \oplus b_{im}$$

where C_i = ith bit of the hash code, $1 \le i \le n$

m = no. of n-bit blocks in the input

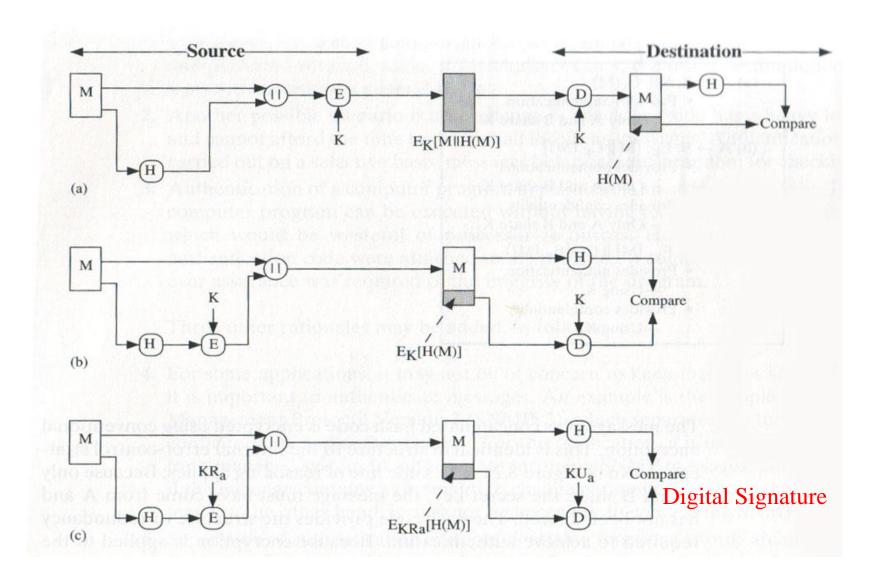
 b_{ii} = ith bit in jth block

 \oplus = XOR operation

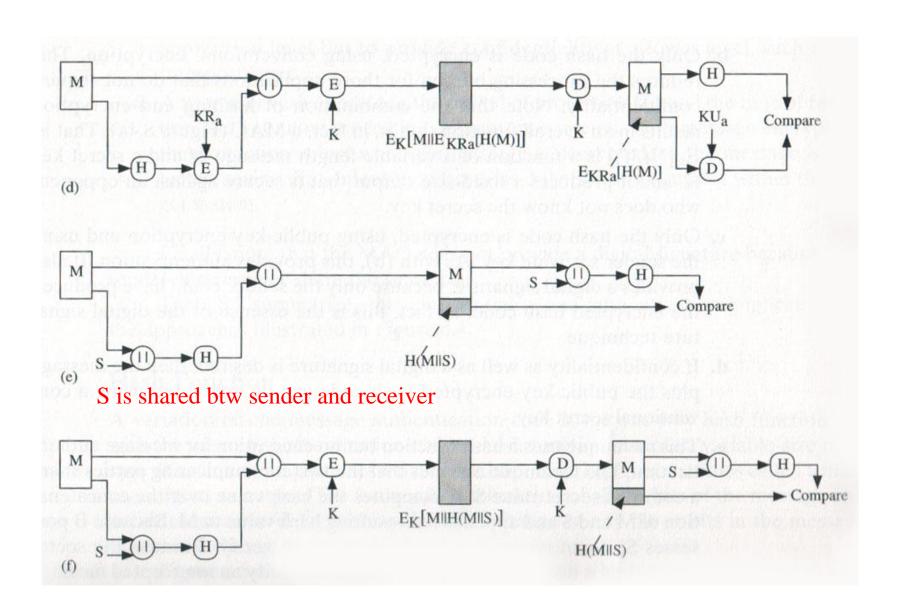
unencrypte anae	Bit 1	Bit 2	CERTAIN . I I COLUE	Bit n
Block 1	b ₁₁	b ₂₁	To make and	b_{n1}
Block 2	b ₁₂	b ₂₂	vppends.	b_{n2}
Haveley	•	Kalada is poss	ale, pas • lens the	battado• p
CARDANCES PAR	•	May lo own 5 x	•	
a. media	•	thing. a me	and Consords	die at
Block m	b_{1m}	b_{2m}	八十二十八十十二十二十二十二十二十二十二十二十二十二十二十二十二十二十二十二十	b _{nm}
Hash code	C_1	C_2		C_n

Figure 8.7 Simple Hash Function Using Bitwise XOR.

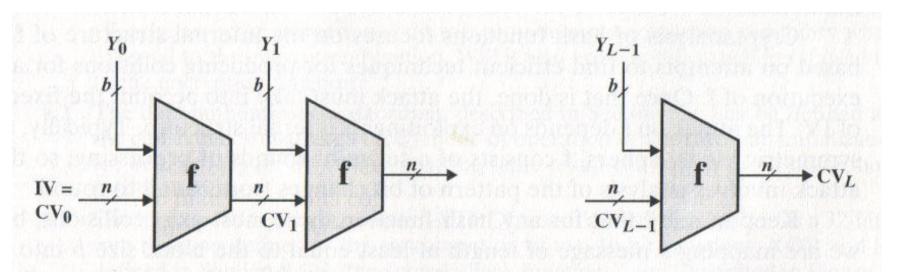
Basic Uses of Hash Functions



Basic Uses of Hash Functions



General Structure of Hash Function



IV = initial value

CV = chaining variable

 $Y_i = i$ th input block

f = compression algorithm

L = number of input blocks

n =length of hash code

b = length of input block

f: compression function taking two inputs and producing n-bit output
 CV₀ = IV = initial n-bit value
 CV_i = f(CV_{i-1}, Y_{i-1}), 1 ≤ i ≤ L

 $H(M) = CV_L$

Figure 8.10 General Structure of Secure Hash Code.

DIGITAL SIGNATURE

- A sends a message to B with an authentication code, B modifies the message and claims that it has come from A. The modified message has an authentication code generated using the shared key of both A and B. There is no way to prove that A did in fact send the message.
- Hence, in situations where there is not complete trust between sender and receiver, something more than authentication is needed
 - The most attractive solution is DIGITAL SIGNATURE

Digital Signatures

- A signature is a technique for non-repudiation based on the public key cryptography.
- The creator of a message can attach a code, the signature, which guarantees the source and integrity of the message.

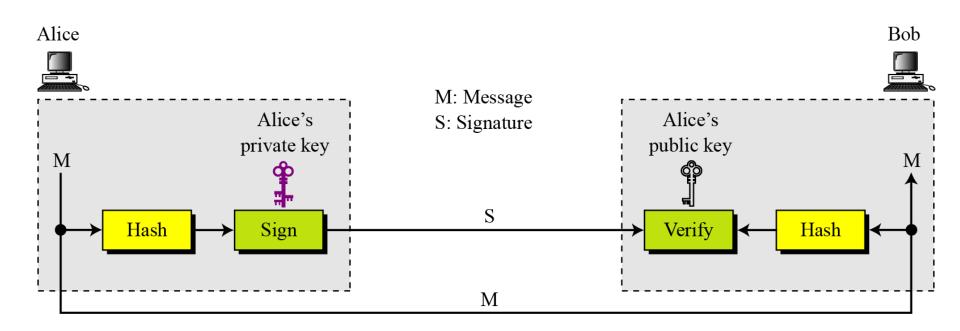
Properties of Signatures

- Similar to handwritten signatures, digital signatures must fulfill the following:
 - ✓ Must not be forgeable
 - ✓ Recipients must be able to verify them
 - ✓ Signers must not be able to repudiate them later
- In addition, digital signatures cannot be constant and must be a function of the entire document it signs

- The sender uses a signing algorithm to sign the message. The message and the signature are sent to the recipient.
- The recipient receives the message and the signature and applies the verifying algorithm to the combination. If the result is true, the message is accepted, otherwise it is rejected.

Signing the digest

Asymmetric-key cryptosystems are very inefficient when dealing with long messages. In a digital signature system, the messages are normally long, but we have to use asymmetric-key schemes. The solution is to sign a digest of the message, which is much shorter than the message itself.



Two categories of DS

- Direct DS
- Arbitrated DS

Direct DS

- involves only the communication parties
- The destination knows the public key of the source
- A DS may be formed by encrypting the encrypting the entire message with the sender's private key or by encrypting a hash code of the message with the sender's private key.
- In case of dispute, some third party must view the message and its signature

Arbitrated DS

- Every signed message from a sender X to a receiver Y goes first to an arbiter A, who subjects the message and its signature to a number of tests to check the origin and content
- The message is then dated and sent to Y with an indication that it has been verified to the satisfaction of the arbiter.

The Digital Signature Standard

- Proposed by NIST National Institute of Standards and Technology
- The DSA includes signature generation and verification.
- Generation makes use of a private key to generate a digital signature.
- Verification of the signature makes use of a public key.
- Each user possess a private and public key pair.
- Public keys are assumed to be known to all members of a group.
- Signature generation can be performed only by the possessor of the user's private key.

Digital Signature Algorithm (DSA)

- A hash function is used in the signature generation process to obtain a condensed version of data called message digest.
- The message digest is then signed.
- The digital signature is sent to the intended recipient along with signed data.
- The recipient of the message and signature verifies the signature by using the sender's public key.
- The same hash function must be used in the verification process.

Digital Signature Algorithm (DSA)

- creates a 320 bit signature, with 512-1024 bit security
- smaller and faster than RSA
- DSA authenticates the integrity of the signed data and the identity of the signer.
- The DSA may also be used in proving to a third party that data was actually signed by the generator of the signature.
- The DSA is intended for use in electronic mail, software distribution, electronic funds transfer, data storage etc...

DSA Key Generation

- have shared global public key values (p,q,g):
 - choose q, a 160 bit
 - choose a large prime $p = 2^{L}$
 - where L= 512 to 1024 bits and is a multiple of 64
 - and q is a prime factor of (p-1)
 - choose $q = h^{(p-1)/q}$
 - where h < p-1, $h^{(p-1)/q} \pmod{p} > 1$
- users choose private & compute public key:
 - choose x<q</p>
 - compute $y = g^x \pmod{p}$

DSA Signature Creation

- to sign a message M the sender:
 - generates a random signature key k, k<q
 - k must be random, destroyed after use, and never be reused
- then computes signature pair:

```
r = (g^{k} (mod p)) (mod q)

s = (k^{-1}.H(M) + x.r) (mod q)
```

• sends signature (r,s) with message M

DSA Signature Verification

- having received M & signature (r,s)
- to verify a signature, recipient computes:

```
w = s^{-1} \pmod{q}

u1 = (H(M).w) \pmod{q}

u2 = (r.w) \pmod{q}

v = (g^{u1}.y^{u2} \pmod{p}) \pmod{q}
```

if v=r then signature is verified

