CSC236 / Introduction to the Theory of Computation

Tutorial Exercises

May 23, 2006

1. Consider the principle $(*)$ below.

> **Principle** $(*)$
>
> Let S be a set with the following properties.
>
> 1. $0 \in S$
> 2. $1 \in S$
> 3. For all $x \in \mathbb{N}$, if $x \in S$ then $x + 2 \in S$
>
> Then $\mathbb{N} \subseteq S$.

Prove that $(*)$ is equivalent to the principle of induction.

2. Textbook exercise 1.13

3. A full binary tree is a non-empty binary tree in which every internal node has exactly two children. Use induction to prove that every full binary tree has an odd number of nodes. Now, prove the same statement using the well-ordering principle.