Homomorphism Question from Tutorial 3

Suppose $(Q, \Sigma, \delta, q_0, F)$ is a DFA for L. Let h be the homomorphism from Σ to Γ^*.

Then, ϵ-NFA for $h(L)$ is $A_h = (Q \cup Q', \Gamma, \delta', q_0, F)$,

where

$Q' = \{p_{q,a,i} : q \in Q, a \in \Sigma, i \leq |h(a)|\}$.

For $q \in Q$, $\delta'(q, \epsilon) = \{p_{q,a,0} : a \in \Sigma\}$;

Suppose $q \in Q$, $a \in \Sigma$, and $h(a) = b_1, b_2, \ldots, b_n$. Then, $\delta'(p_{q,a,i}, b_{i+1}) = \{p_{q,a,i+1}\}$, for $i < n$ and, $\delta'(p_{q,a,n}, \epsilon) = \{\delta(q, a)\}$.

If $\delta(q, a) = q'$, then $q' \in \hat{\delta}'(q, h(a))$. Thus, if $\hat{\delta}(q_0, w) \in F$ then, $\hat{\delta}'(q_0, h(w)) \in F$. Thus, $h(L) \subseteq L(A_h)$.

Now suppose $w \in L(A_h)$. Suppose the states (from Q) in the accepting path for w by A_h are $q_0, q_1, q_2, \ldots, q_r$, where $q_r \in F$. Suppose, in going from q_i to q_{i+1}, the automata consumed part w_i of the input string w, where $w = w_0 w_1 \ldots w_{r-1}$. Then, for $i < r$, by construction, there exists an $a_i \in \Sigma$ such that $h(a_i) = w_i$ and $\delta(q_i, a_i) = q_{i+1}$, Thus, $\hat{\delta}(a_0 a_1 \ldots a_{r-1}) = q_r \in F$, and $h(a_0 a_1 \ldots a_{r-1}) = w$. Hence, $w \in h(L)$.