3130CIT Theory of Computation

Tutorial problems: Regular languages

1. Prove informally and by mathematical induction that every finite language is regular.

2. Write regular expressions for the languages in Exercises 3.1.1 and 3.1.2 of IATLC (Hopcroft *et al.*).

3. Write a regular expression that captures the different forms of Australian phone numbers, e.g. +61 7 3875 5047, (07) 3875 5047, 07 3875 5047, 3875 5047.

4. State which of the statements about regular expressions in Exercise 3.4.2 of IATLC is true and which is false, and justify your answers.

5. Simplify the regular expression in Exercise 3.4.3 of IATLC.

6. Construct a finite-state automaton that recognises the language in $\{0, 1\}^*$ of (binary) numbers divisible by three. Show the corresponding regular expression.

7. For $N \geq 1$, let

   $$L_N = \{\, s \in \{0, 1\}^* \mid |s| \geq N \text{ and the } N\text{th symbol from the right is } 1 \,\}$$

   Construct a finite-state automaton that recognises L_3.

8. (a) Given a DFA that accepts a language L, construct a DFA that accepts the complement L' of L.

   (b) Given DFAs that accept languages L_1 and L_2, construct a DFA that accepts the symmetric difference $L_1 \Delta L_2$ of L_1 and L_2.

9. If L is a language and a a symbol, then define L/a to be the set of strings w such that $wa \in L$. For example, if L is the language of the regular expression $(01)^*$, then $L/1$ has the regular expression $(01)^*0$ and $L/0$ is empty. Prove that if L is regular, then so is L/a. (Hint: Use a construction based on a DFA for L.)

10. Convert the following regular expressions to Λ-NFAs:

    (a) 01^*

    (b) $(0 + 1)01$

    (c) $00(0 + 1)^*$

11. Convert each of the Λ-NFAs in Question 10 to NFAs (by eliminating the Λ-transitions).

12. Convert each of the NFAs in Exercises 2.3.1 to 2.3.3 of Hopcroft et al. to DFAs (and informally describe the language accepted in each case).

13. Consider the 3-state DFA for the set of strings in $\{0, 1\}^*$ representing binary numbers divisible by 3. Use the state-elimination construction to construct a regular expression equivalent to this language.

14. Construct a regular expression for the set of strings in $\{0, 1\}^*$ that, when intepreted in *reverse* as a binary number, is divisible by 3.

15. Consider the 4-state DFA for the set of strings in $\{0, 1\}^*$ that contain an even number of 0s and an even number of 1s. Use the state-elimination construction to construct a regular expression equivalent to this language.

16. Construct a regular expression for the DFAs in Exercises 3.2.1 to 3.2.3 of IATLC.

17. Use the pumping lemma for regular languages to prove that none of the following languages is regular:

    (a) $\{\, 0^n 1 0^n \mid n \geq 1 \,\}$

    (b) $\{\, 0^n 1^m \mid n \leq m \,\}$

    (c) $\{\, 0^n \mid n$ is a power of 2 $\}$

    (d) $\{\, ww \mid w \in \{a, b\}^* \,\}$

18. Describe what happens when you attempt to use the pumping lemma to show that some finite (and hence regular) language is not regular.

19. Describe a decision algorithms to answer each of the questions in Exercises 4.3.3 to 4.3.5 of IATLC.