CS3231
Tutorial 10

1. We say that a TM enumerates a language L, iff the TM (irrespective of the input) works as follows: On the output tape, it prints elements x_1, x_2, ..., such that $L = \{x_1, x_2, \ldots\}$. Note that elements may be printed in arbitrary order.

   Show that L is RE iff some TM enumerates L as above.

2. Consider the language $\{wcw^R \mid w \in \{a,b\}^*\}$. Here w^R denotes w in reverse (that is, if $w = w_1w_2\ldots w_{n-1}w_n$, with each $w_i \in \{a,b\}$, then $w^R = w_nw_{n-1}\ldots w_2w_1$).

   Give a description of a 1-tape Turing Machine which can accept the above language. What is the time complexity of your machine?

   Give a description of a 2-tape Turing Machine which can accept the above language in linear time.

   You would note that the complexity of 2-tape Turing Machine is better. It can be shown that no 1-tape Turing Machine can accept the above language in linear time.

   Thus in some cases one can improve the complexity of accepting languages using more tapes.

3. Show that any context free language can be accepted by some Turing Machine.

4. Show that the recursively enumerable languages are closed under union.

5. Suppose L_1 is recursive and L_2 is recursively enumerable. Then show that $L_2 - L_1$ must be recursively enumerable. Could you say anything about $L_1 - L_2$?

6. (a) Show that every finite language is recursive.

   (b) Show that every co-finite language is recursive.
   (Note: A language L is said to be co-finite if the complement of L, $\Sigma^* - L$, is finite.)

   (c) Suppose that L is a recursive language and D is a finite language. Then show that $L\boldsymbol{\Delta}D$ must be recursive. (Here $L\boldsymbol{\Delta}D$ denotes the symmetric difference, $(L - D) \cup (D - L)$, of L and D).

   (d) Suppose L is a recusively enumerable language which is not recursive. Suppose **M** is a Turing Machine which accepts L. Then show that there must be infinitely many inputs on which **M** does not halt.

7. (a) Suppose L is a recursive language. Then show that $\{x \mid xx \in L\}$ is also recursive.

   (b) Suppose L is a recursive language. Then show that $\{x \mid \text{some prefix of } x \text{ is in } L\}$ is also recursive.

8. Use the construction of Universal Turing Machine to show that the following language is recursive:

   $\{(a^i, a^j, a^t) : M_i \text{ accepts } w_j \text{ in } \leq t \text{ time steps }\}$

   Here M_i is the i-th Turing Machine, w_j is the j-th string and t is a natural number.

9. In the construction of Universal Turing Machine done in class, suppose we allow the coded Turing Machine to be non-deterministic. Then give a construction of a non-deterministic Universal Turing Machine which takes two inputs: p, a code for non-detereministic Turing Machine, and x; and then simulates the Turing Machine with code p on input x (for language acceptance).

   You need not describe the whole construction, but just give the main points about where the above Universal Turing Machine differs from the Universal Turing Machine for deterministic TMs.