CS3231
Tutorial 3

1. Give a regular expression for the language accepted by the automata in Q8 of Tutorial 2.
2. (a) Show $(R+S)^* = (R^*S^*)^*$, for any regular expressions R and S.

   (b) Show $(RS+R)^*R = R(SR+R)^*$, for any regular expressions R and S.

   (c) Show $(R+S)^*S = (R^*S)^+$, for any regular expressions R and S.
3. Use the method discussed in class, to give a regular expression for the language accepted by the DFA $(\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_1\})$, where δ is defined as follows.

   $\delta(q_0, 1) = q_0$. $\delta(q_0, 0) = q_1$. $\delta(q_1, 1) = q_1$. $\delta(q_1, 0) = q_0$.
4. Give the minimal DFA which is equivalent to the DFA in Figure 1.
5. Prove or Disprove:

   (a) For all regular languages L_1 and L_2: $\overline{L_1 \cdot L_2} = (\overline{L_1}) \cdot (\overline{L_2})$.

   (b) If L is not regular, then $\overline{L}$ is not regular.

   (c) Suppose L is a regular language. Show that $L^R = \{x^R \mid x \in L\}$ is also a regular language. Here x^R denotes the reverse of a string x.

   (d) If L_1 is regular, and $L_2 \subseteq L_1$, then L_2 is regular.

   (e) For any language L, let $MIN(L) = \{x \mid \text{no proper prefix of } x \text{ is in } L\}$. If L is regular then so is $MIN(L)$.
6. Suppose Σ and Γ are two alphabets. Suppose h is a mapping from Σ to Γ^*. Extend h to strings as follows.

   $h(\epsilon) = \epsilon$.

   $h(aw) = h(a) \cdot h(w)$, for any $a \in \Sigma, w \in \Sigma^*$.

   Show: If L is regular then $h(L) = \{h(w) \mid w \in L\}$ is also regular.