CS3231, Mid Term 2

Answer all questions.

1. $S \rightarrow aSc|X|Y$

   $X \rightarrow aXb|\epsilon$

   $Y \rightarrow bYc|\epsilon$

2. (a) Consider derivation of aab. It has two derivations corresponding to two different trees.

   $S \rightarrow aS \rightarrow aaSb \rightarrow aab$

   and

   $S \rightarrow aSb \rightarrow aaSb \rightarrow aab$

   (b) An unambiguous grammar for the above language would be:

   $S \rightarrow aS|T$

   $T \rightarrow aTb|\epsilon$

3. Acceptance by empty stack.

   NPDA is ($Q = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}, \Gamma = \{a, b, Z\}, \delta, q_0, Z, F$) (where F is not relevant).

   Transition function δ is as given below. Intuitively, we do similar to accepting equal number of a's and b's except that when we get b in the input, we think of it as three b's.

   $\delta(q_0, Z) = \{(q_0, \epsilon)\}$

   $\delta(q_0, a, Z) = \{(q_0, aZ)\}$

   $\delta(q_0, a, a) = \{(q_0, aa)\}$

   $\delta(q_0, a, b) = \{(q_0, \epsilon)\}$

   $\delta(q_0, b, Z) = \{(q_0, bbbZ)\}$

   $\delta(q_0, b, b) = \{(q_0, bbbb)\}$

   $\delta(q_0, b, a) = \{(q_1, \epsilon)\}$

   $\delta(q_1, \epsilon, a) = \{(q_2, \epsilon)\}$

   $\delta(q_1, \epsilon, Z) = \{(q_0, bbZ)\}$

   $\delta(q_2, \epsilon, a) = \{(q_0, \epsilon)\}$

   $\delta(q_2, \epsilon, Z) = \{(q_0, bZ)\}$

4. (5 marks) Yes, it is context free.

   Suppose (V, Σ, S, P) is the grammar for L.

   The new grammar for $Suff(L)$ is $(V \cup \{A^s : A \in V\}, \Sigma, S^s, P')$, where P' is defined as follows.

   Intuitively, A^s generates suffixes of all strings which are generated by A.

For each production $A \rightarrow \alpha$ in P, we have the following productions in P':

(a) production $A \rightarrow \alpha$

(b) all productions of form $A^s \rightarrow \gamma$, where γ is a suffix of α.

(c) all productions of form $A^s \rightarrow B^s\gamma$, where $B\gamma$ is a suffix of α.

Second Method: Suppose PDA for accepting L is $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Below $s(q)$ are new states (which are used for guessing the missing prefix). The new NPDA for $Suff(L)$ is defined as follows:

$(Q \cup \{s(q) : q \in Q\}, \Sigma, \Gamma, \delta', s(q_0), Z_0, F)$ where,

$\delta'(q, a, X) = \delta(q, a, X)$, for $q \in Q, a \in \Sigma, X \in \Gamma$.

For $q \in Q$ and $X \in \Gamma$, $\delta'(s(q), \epsilon, X) = \{(s(p), \alpha) : (p, \alpha) \in \delta(q, a, X)$, for some $a \in \Sigma \cup \{\epsilon\}\} \cup \{(q, X)\}$.

Intuitively, initially, the PDA just guesses the inputs. The states $s(q)$ do this part. Then, nondeterministically PDA switches to real state q (from previous state $s(q)$), to follow the original NPDA to check if the 'suffix' would lead to acceptance.

5. Suppose by way of contradiction that $L = \{a^i b^j$: for some natural number $k \quad [i = j * k]\}$ is context free. Let n be as in the pumping lemma for context free languages. Let $p > n$.

   Then, $z = a^p b^{p^2} \in L$. Let $z = uvwxy$ be as in pumping lemma, and consider the following cases.

   Case 1: v or x contains both a, b. Then uv^2wx^2y is not in L, as uv^2wx^2y is not a member of a^*b^*.

   For following cases, we assume that v contains only one of a, b, and x contains only one of a, b.

   Case 2: $vx \in a^*$. Then $uv^{p^2+1}wx^{p^2+1}y$ is not in L, as number of a's is $>$ number of b's.

   Case 3: $vx \in b^*$. Then uv^2wx^2y is not in L, as number of b's is $p^2 + |vx|$ which is not divisible by p (as $1 \leq |vx| \leq n < p$).

   Case 4: $v = a^i$ and $x = b^j$, where $i > 0, j > 0$. Take $k = p^2 + 1$. Then, for $uv^kwx^ky = a^{p+p^2i}b^{p^2+p^2j}$ to be in L we must have that $p + p^2i$ divides $p^2 + p^2j$. Thus, $1 + pi$ divides $p(1 + j)$.

   Suppose $(1 + pi) * r = p(1 + j)$. Then, $r = p(1 + j - r * i)$. Thus, r must be divisible by p, but $r < 1 + j \leq 1 + n \leq p$, a contradiction.

   As above cases are exhaustive we have that L is not context free.