CS3231
Tutorial 11–12

1. Suppse L_1, L_2 are recursively enumerable sets.

   (a) Is $L_1 \cap L_2$ recursively enumerable?

   (b) Is $L_1 - L_2$ recursively enumerable?

2. The state entry problem for Turing Machines can be described as follows. Suppose a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$, a state $q \in Q$, and input $w \in \Sigma^*$ is given. Does M ever enter the state q when input is string w?

   Show that above problem is undecidable using appropriate reduction from the Halting problem.

3. Recall that W_i denotes the language accepted by the Turing Machine with code i. That is, $W_i = L(M_i)$.

   (a) Show that $L_{inf} = \{1^i \mid W_i \text{ is infinite }\}$ is not RE.

   (b) Show that $L_{fin} = \{1^i \mid W_i \text{ is finite }\}$ is not RE.

   (c) Show that $\{w_i \mid a \in W_i \text{ or } b \notin W_i\}$ is not RE, where $a \neq b$.

   (d) Show that $L_a = \{w_i \mid a \notin W_i\}$ is not RE.

   Note that Rice's theorem cannot be used to show a language over machines is not RE. So you would need to use some of the techniques used in class.

4. Which of the following problems are decidable and why?

   (a) Given two DFA's M_1 and M_2, is $L(M_1) \cap L(M_2) = \emptyset$?

   (b) Given a CFG G, and a DFA M, is $L(G) \cap L(M) = \emptyset$?

   (c) Given a TM M', and a DFA M, is $L(M') \cap L(M) = \emptyset$?

5. Show that the following languages are undecidable, using Rice's theorem.

   (a) $\{M \mid \text{number of elements in } L(M) \leq 20\}$.

   (b) $\{M \mid a \in L(M)\}$.

   (c) $\{M \mid a \in L(M) \text{ and } b \notin L(M)\}$.

   (d) $\{M \mid a \in L(M) \text{ or } b \notin L(M)\}$.

6. Prove or disprove:

   If L^* is recursive. Then L is recursive.

7. Show that the following problems are undecidable:

   (a) Given two CFG, G_1 and G_2, is $L(G_1) = L(G_2)$?

   (b) Given two CFG, G_1 and G_2, is $L(G_1) \subseteq L(G_2)$?

8. Give unrestricted grammar for
   (a) $\{a^i b^j c^k \mid i \leq j \leq k\}$.
   (b) $\{w \mid w \in \{a, b, c\}^*$ and w contains equal number of a's, b's and c's$\}$.
   (c) $\{ww \mid w \in \{a, b\}^*\}$.
9. Show that every RE language can be generated by an unrestricted grammar.