Parallel Simulation of 2-DFAs

Suppose $A = (Q, \Sigma, \delta, q_0, F)$ and $A' = (Q', \Sigma, \delta', q_0', F')$ are two DFAs.

Then, one can construct another DFA as:

$A'' = (Q \times Q', \Sigma, \delta'', (q_0, q_0'), F'')$,

where $\delta''((q, q'), a) = (\delta(q, a), \delta(q', a))$, and F'' depends on the need.

The above DFA A'' simulates the two DFAs A and A' in parallel. Note that alphabet Σ is same for all the three DFAs.

If we take $F'' = F \times F'$, then the new DFA A'' accepts the the intersection of the languages accepted by A and A'.

If we take $F'' = F \times Q' \cup Q \times F'$, then the new DFA A'' accepts the union of the languages accepted by A and A'.

By taking other appropriate F'', one can build the DFA for other boolean combination of languages accepted by A and A'.