CS3231, Mid Term 1

Answer all questions.

1. Let $\#_a(w)$ denote the number of a's in the string w. Let $\#_b(w)$ denote the number of b's in the string w.

   (a) (4 marks) Give a DFA which accepts the language:

   $L = \{w \in \{a,b\}^* : (\#_a(w) = 1 \text{ and } \#_b(w) \text{ is even}) \text{ or } (\#_a(w) = 2 \text{ and } \#_b(w) \text{ is odd})\}$.

   Answer: Let $A = (Q, \{a,b\}, \delta, q_0, \{q_2, q_5\})$, where

   $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$.

   $\delta(q_0, a) = q_2$, $\delta(q_0, b) = q_1$,

   $\delta(q_1, a) = q_3$, $\delta(q_1, b) = q_0$,

   $\delta(q_2, a) = q_4$, $\delta(q_2, b) = q_3$,

   $\delta(q_3, a) = q_5$, $\delta(q_3, b) = q_2$,

   $\delta(q_4, a) = q_6$, $\delta(q_4, b) = q_5$,

   $\delta(q_5, a) = q_6$, $\delta(q_5, b) = q_4$,

   $\delta(q_6, a) = q_6$, $\delta(q_6, b) = q_6$.

   (b) (4 marks) What is $\hat{\delta}(q_0, bbab)$ for the DFA constructed in part (a)? (where q_0 is the start state for your DFA). Please clearly give the names of your states for this question.

   Answer: $\hat{\delta}(q_0, bbab) = q_3$.

2. (4 marks) Give a deterministic finite state automata, with minimum number of states, which accepts the same language as accepted by the automata in figure 1. Please use the method given in class for minimizing automata, and show your work.

   Answer: Note that all the states are reachable.

   In the base case we will get, $(q_1, q_2), (q_4, q_2)$, $(q_1, q_3), (q_4, q_3)$, $(q_1, q_5), (q_4, q_5)$ as distinguishable pairs, as q_1, q_4 are non-accepting whereas q_3, q_4, q_5 are accepting states.

   In the first induction step, we will add (q_3, q_2) and (q_3, q_5) as distinguishable pairs, as $(\delta(q_3, b), \delta(q_2, b))$ are distinguishable, and $(\delta(q_3, b), \delta(q_5, b))$ are distinguishable.

   There will be no more additions, and we will have that q_1, q_4 are equialent, and q_2, q_5 are equivalent.

   Thus, the minimal automata will consist of three states, called q_{14}, q_3, and q_{25}, where q_{14} is the starting state and q_3 and q_{25} are accepting states.

   The transition function would be:

   $\delta(q_{14}, a) = q_{14}$; $\delta(q_{14}, b) = q_3$

   $\delta(q_{25}, a) = q_{25}$; $\delta(q_{25}, b) = q_{14}$

   $\delta(q_3, a) = q_{25}$; $\delta(q_3, b) = q_{25}$

3. Prove or Disprove:

(a) (4 marks) Suppose L and S are regular languages. Then, $\{xy : x \in L, y \notin S\}$ is also regular.

Answer: True. As S is regular, $\overline{S}$ is also regular. Thus, $L \cdot \overline{S}$ is also regular, which is same $\{xy : x \in L, y \notin S\}$.

(b) (4 marks) Suppose R and S are regular expressions. Then $L(S(R+S)^*S) = L((SR^*S)^+)$.

(Here $L(E)$ denotes the language associated with the regular expression E).

Answer: False. Let $R = \{a\}$ and $S = \{b\}$. Then, LHS contains the string bbb. However, RHS does not contain bbb (it contains only strings with non-zero even number of b's).

4. (5 marks) Show that $\{a^m : m = 3i^2 + 51$ for some natural number $i\}$ is not a regular langauge.

Hint: Note that the gap between squares ($1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $\ldots$) grows unbounded.

Proof: Let $L = \{a^m : m = 3i^2 + 51$ for some natural number $i\}$.

Suppose by way of contradiction that the above language is regular. Then, let n be as in the pumping lemma. Without loss of generality assume that $n \geq 1$.

Let $w = a^{3n^2+51}$. Let x, y, z be as in the pumping lemma. Let $|y| = j$.

Then, consider $xy^2z = a^{3n^2+51+j}$.

Note that $3n^2+51 < 3n^2+51+j \leq 3n^2+51+n < 3n^2+51+6n+3 = 3(n+1)^2+51$.

Thus, there exists no natural number i such that $3n^2 + 51 + j = 3i^2 + 51$.

Hence $xy^2z \notin L$, a contradiction to the pumping lemma.

QED.