MPO 762 – Problem Set 5

1 The energy conserving discrete equations

The nonlinear shallow water equations are to be discretized using finite differences and using a staggered configuration for the different variable according to the Arakawa C-grid shown in figure 1. The following finite difference scheme conserves energy discretely:

$$
\begin{aligned}
\frac{\partial u}{\partial t} - \overline{q\overline{V}^{x}}^{y} + \delta_x \Phi &= 0 && (1)\\
\frac{\partial v}{\partial t} + \overline{q\overline{U}^{y}}^{x} + \delta_y \Phi &= 0 && (2)\\
\frac{\partial \eta}{\partial t} + \delta_x U + \delta_y V &= 0 && (3)
\end{aligned}
$$

The new variables are defined on the Arakawa C-grid as follows:

$$
\begin{array}{rcll}
\text{x-Mass Fluxes} & U = \overline{h}^{x} u & u\text{-point} & (4)\\
\text{y-Mass Fluxes} & V = \overline{h}^{y} v & v\text{-point} & (5)\\
\text{Total Head} & \Phi = g\eta + \dfrac{\overline{u^2}^{x} + \overline{v^2}^{y}}{2} & \eta\text{-point} & (6)\\
\text{Potential vorticity} & q = \dfrac{\delta_x v - \delta_y u + f}{\overline{h}^{xy}} & \zeta\text{-point} & (7)
\end{array}
$$

Extend the code you have developed for the linearized SWE equation to include the non-linear terms in the momentum and continuity equation, and and the coriolis force.

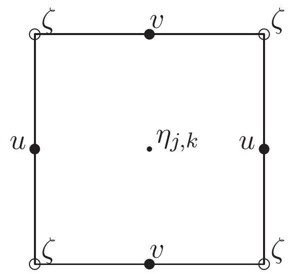

Figure 1: Arakawa C-grid configuration. The depth and pressure are defined at the center of cell (j, k). The x-component of the velocity is defined on vertical cell edges $(j+\frac{1}{2}, k)$ and the y-components at the centers of horizontal edges $(j, k+\frac{1}{2})$. The vorticity, coriolis force, and enstrophy are defined on cell corners $(j+\frac{1}{2}, k+\frac{1}{2})$.

2 Equatorial Rossby Soliton

Use the nonlinear code to simulate the propagation of an equatorial Rossby soliton. The Rossby soliton dynamics hinges on the interplay between the wave-steepening caused by the non-linear terms and the dispersion caused by the Coriolis term; the two are exactly balanced and the soliton propagates without change of shape for long distances. Boyd (1980) investigated the equatorial Rossby soliton dynamics and provided an asymptotic solution in dimensionless form.

The domain is an equatorial β-plane where $-24 \leq x \leq 24$, and $-8 \leq y \leq 8$. The Coriolis variation is given by $f = y$. The depth of the fluid is $H = 1$ and the gravitational acceleration is $g = 1$. The final integration time is $t = 40$. A stable time step for the coarsest resolution is $\Delta t = 0.02$ (stable time-step); please explain what sets it. The boundaries are closed: so that the boundary conditions are: $u(\pm 24, y) = 0$, $v(x, \pm 8) = 0$; use symmetry when updating variables associated with the vorticity. The initial conditions are:

$$u = \frac{6y^2 - 9}{4}\phi(x)e^{-\frac{y^2}{2}} \tag{8}$$

$$v = 2y\frac{\partial \phi}{\partial x}e^{-\frac{y^2}{2}} \tag{9}$$

$$\eta = \frac{6y^2 + 3}{4}\phi(x)e^{-\frac{y^2}{2}} \tag{10}$$

$$\phi(x) = 0.771B^2 \mathrm{sech}^2 Bx \tag{11}$$

$$\frac{\partial \phi}{\partial x} = -2B \tanh Bx\, \phi \tag{12}$$

$$B = 0.395 \tag{13}$$

You should monitor the evolution of the perturbations (current minus initial) in energy, mass and potential enstrophy during the course of the simulation. Provide contour lines of the initial, mid-, and final states. Try 3 spatial resolutions: $\Delta x = \Delta y = 0.5, 0.25, 0.125$.

A fortran code to initialize the Rossby soliton problem is in available.

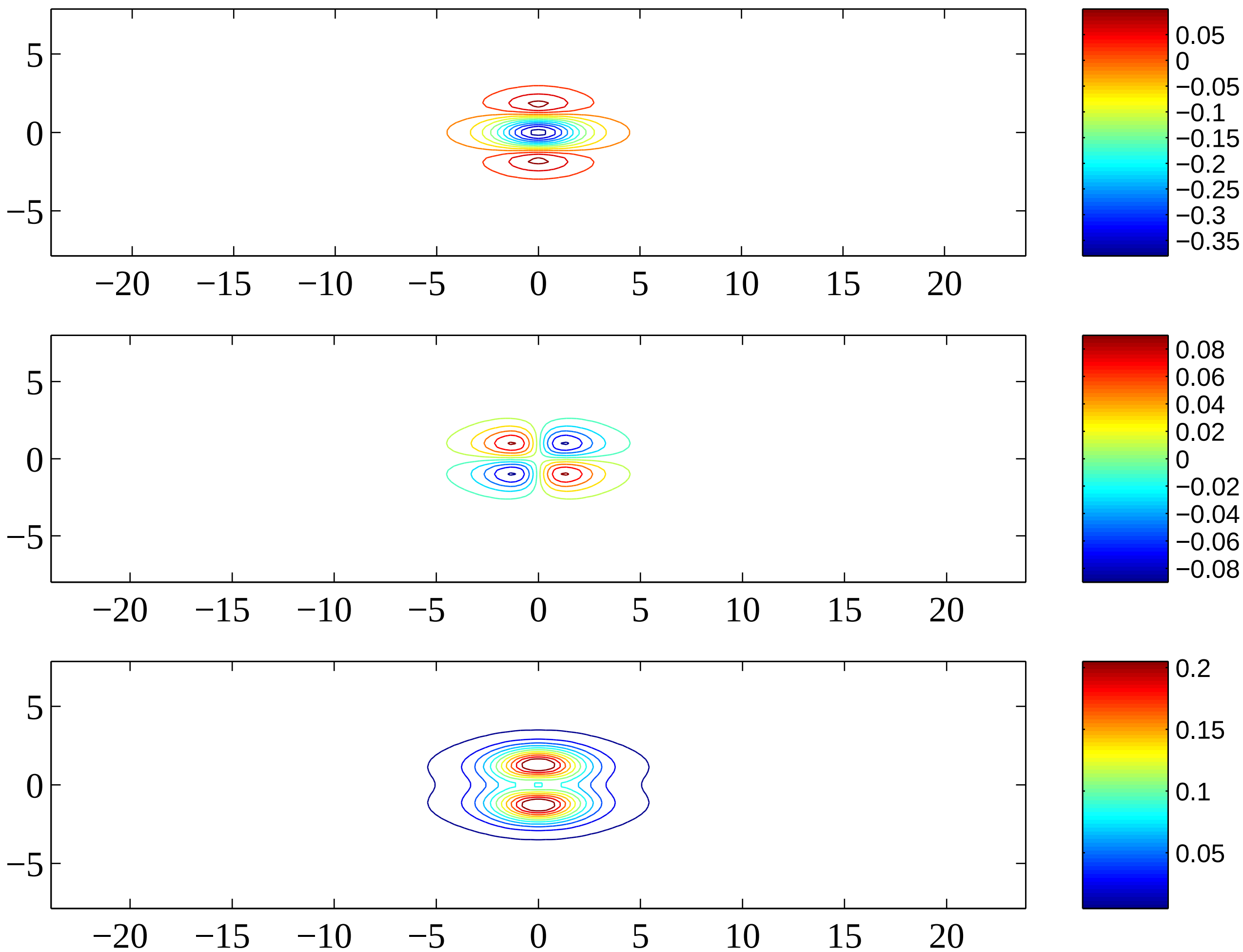

Figure 2: Equatorial Rossby Soliton initial conditions, from top to bottom:u, v and η, respectively.

References

Boyd, J. P., 1980. Equatorial solitary waves part i: Rossby solitons. Journal of Physical Oceanography 10 (11), 1699–1717.

SEE ME IF YOU HAVE ANY QUESTIONS