A Glimpse of the Large-Scale Equatorial Dynamics:

The Equatorial β-Plane Approximation

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Geophysical Fluid Dynamics I
Class project

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Outline

1. **Introduction**

2. **Equatorial β-plane approximation: Characteristics and a few solutions**
   - The equatorial β-plane
   - Length and time scales of motions influenced by rotation
   - A note on geostrophy near the Equator
   - Free waves solutions and the equatorial waveguide

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3. A lot of important phenomena going on in the GFD context

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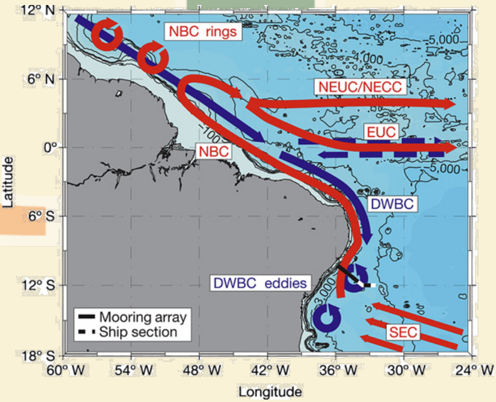
Mass and energy transfer between hemispheres and basins



Source: Dengler et al. (2004)

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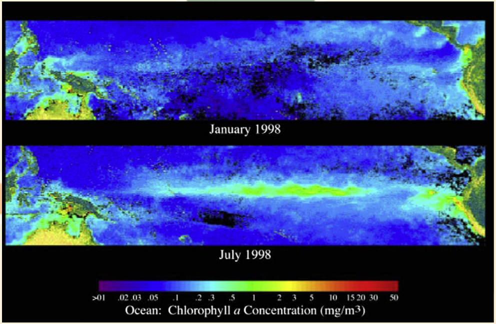
Climate, biological productivity, human activities ...



Source: Talley et al. (2011)

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I am going to talk about ocean !

Outline

The equatorial β-plane

Basically, it is the representation of the equations of motion in the Cartesian coordinate system which keeps the effect of the Earth's curvature around the Equator.

$$\frac{D\vec{v}}{Dt} + f\vec{k} \times \vec{v} = -\frac{\nabla p}{\rho_0} - g'\vec{k} + \nabla \cdot \left(\frac{\mu}{\rho_0} \nabla \vec{v}\right)$$

$$f = 2\Omega sin(\theta_0) + \frac{2\Omega}{a} cos(\theta_0) y, \; f \approx \beta y$$

$$\beta \approx 2.3 \times 10^{-11} m^{-1} s^{-1}$$

(Cushman-Roisin and Beckers, 1994)

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Length and time scales of motions influenced by rotation

Rossby Deformation Radius and Inertial Period

Equatorial

$$R_{di} = \sqrt{\frac{\sqrt{g'H}}{\beta}} \approx 200 \text{ km}$$
$$T \approx 2 \text{ days}$$

Mid-latitudes

$$R_{di} = \frac{\sqrt{g'H}}{f} \approx 10 \text{ km - } 50 \text{ km}$$
$$T \approx 15 \text{ hours - } 1 \text{ day}$$

Equatorial fluid motions must have larger length scales and longer time scales !

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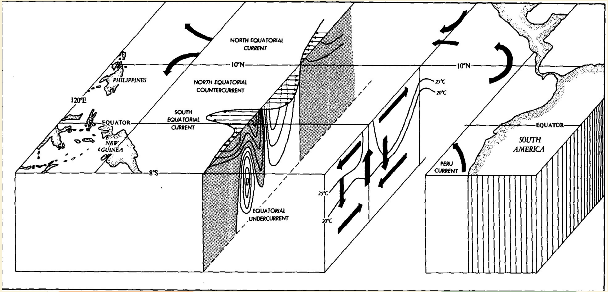
A Few Solutions of the Equations of Motion

A steady solution: geostrophy near the Equator

Source: Philander (1990)



EUC and SEC are indeed in geostrophic balance !
(e.g., Lukas and Firing, 1984)

Equatorial

$$\beta u_{eq} = -\frac{1}{\rho_0}\frac{\partial^2 p}{\partial y^2}$$

Mid-latitudes

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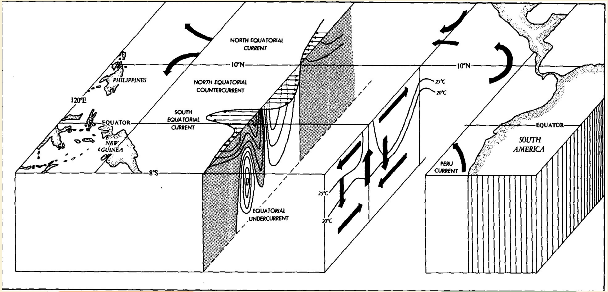

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Linear free waves solutions

Reduced gravity model (Philander, 1990)

x-momentum:

$$\frac{\partial u}{\partial t} - \beta y v = -g' \frac{\partial \eta}{\partial x}$$

y-momentum:

$$\frac{\partial v}{\partial t} + \beta y u = -g' \frac{\partial \eta}{\partial y}$$

continuity:

$$g' \frac{\partial \eta}{\partial t} + c^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

where $c^2 = g'H$ and at the Equator $y = 0$

Combining the equations and assuming velocity wave-like perturbations:

e.g., $v(x, y, t) = \hat{V}(y) exp[i(kx - \omega t)]$

$$\frac{\partial^2 \hat{V}}{\partial y^2} + \frac{1}{R_{eqi}^4}(Y^2 - y^2)\hat{V} = 0$$

where

$$Y^2 = \left[\left(\frac{\omega}{c}\right)^2 - k^2 - \frac{\beta k}{\omega}\right] R_{eqi}^4$$

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$$\frac{\partial^2 \hat{V}}{\partial y^2} + \frac{1}{R_{eqi}^4}(Y^2 - y^2)\hat{V} = 0$$

$\hat{V}(y)$ is sinusoidal function that decays towards $\pm Y$

where

$$Y^2 = [(\frac{\omega}{c})^2 - k^2 - \frac{\beta k}{\omega}]R_{eqi}^4$$

Linear free waves solutions

Using the same reasoning and assuming no cross-equatorial water transport (i.e. $v = 0$):

$$\eta = e^{-\frac{y^2}{2R_{eqi}^2}} e^{i(kx-\omega t)}$$

$$u = \left(\frac{g'}{c}\right) e^{-\frac{y^2}{2R_{eqi}^2}} e^{i(kx-\omega t)}$$

$$v = 0$$

$c = \frac{R_{eqi}^2}{\beta} = \sqrt{g'H}$

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(Kelvin Waves)

Linear free waves solutions: meridional propagation?

What about $v(x, y, t) = \hat{V} exp[i(kx + ly - \omega t)]$ **?**

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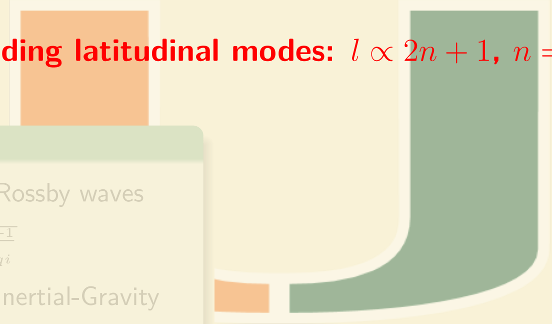

Standing latitudinal modes: $l \propto 2n+1$, $n = 1, 2, 3...$

3 wave classes

1. Equatorial Rossby waves
   $\omega \approx \frac{-\beta k}{k^2 + \frac{2n+1}{R^2_{eqi}}}$
2. Equatorial Inertial-Gravity waves
   $\omega^2 \approx (2n+1)\beta c + k^2 c^2$
3. Rossby-Gravity (Yanai) waves
   $\frac{c}{\beta}[(\frac{\omega}{c})^2 - k^2 - \frac{\beta k}{\omega}] = 1$

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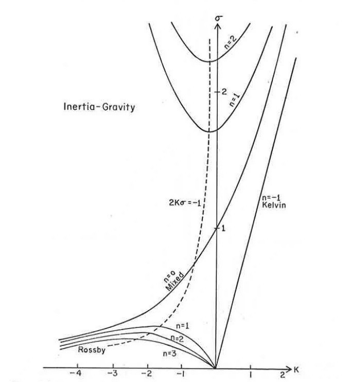
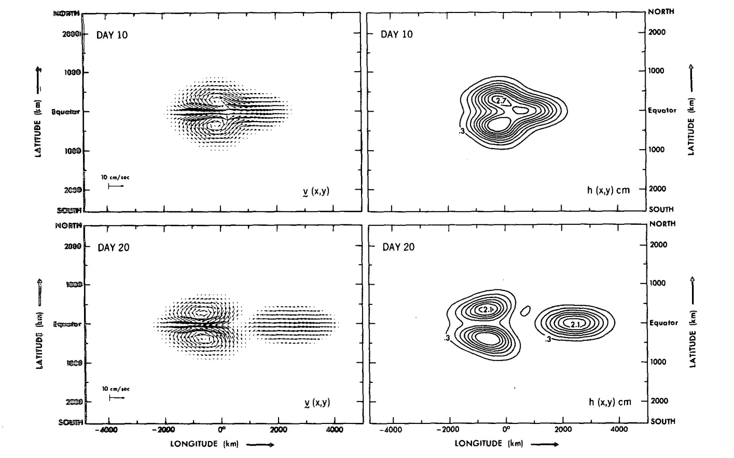


Source: Cane and Sarachik (1976)

Linear free waves solutions: praticle example

Simulations from Philander et al. (1984)



General behavior of the thermocline displacements during an El Niño event

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THANK YOU

Questions ?