Motion Models

$$\underset{\text{Yaw rate}}{\dot{\theta}} = 0$$

$$\underset{\text{Final x position}}{x_f} = \underset{\text{Initial x position}}{x_0} + \underset{\text{Velocity}}{v}\underset{\text{Time elapsed}}{(dt)}\underset{\text{X-component of velocity}}{(cos(\theta_0))}$$

$$\underset{\text{Final y position}}{y_f} = \underset{\text{Initial y position}}{y_0} + \underset{\text{Velocity}}{v}\underset{\text{Time elapsed}}{(dt)}\underset{\text{Y-component of velocity}}{(sin(\theta_0))}$$

$$\underset{\text{Final yaw}}{\theta_f} = \underset{\text{Initial yaw}}{\theta_0}$$

$$\underset{\text{Yaw rate}}{\dot{\theta}} \neq 0$$

$$\underset{\text{Final x position}}{x_f} = \underset{\text{Initial x position}}{x_0} + \frac{\overset{\text{Velocity}}{v}}{\underset{\text{Yaw rate}}{\dot{\theta}}}[sin(\underset{\text{Initial yaw}}{\theta_0} + \underset{\text{Yaw rate}}{\dot{\theta}}\underset{\text{Time elapsed}}{(dt)}) - sin(\underset{\text{Initial yaw}}{\theta_0})]$$

$$\underset{\text{Final y position}}{y_f} = \underset{\text{Initial y position}}{y_0} + \frac{\overset{\text{Velocity}}{v}}{\underset{\text{Yaw rate}}{\dot{\theta}}}[cos(\underset{\text{Initial yaw}}{\theta_0}) - cos(\underset{\text{Initial yaw}}{\theta_0} + \underset{\text{Yaw rate}}{\dot{\theta}}\underset{\text{Time elapsed}}{(dt)})]$$

$$\underset{\text{Final yaw}}{\theta_f} = \underset{\text{Initial yaw}}{\theta_0} + \underset{\text{Yaw rate}}{\dot{\theta}}\underset{\text{Time elapsed}}{(dt)}$$

Calculating the Particle's Final Weight

Now we that we have done the measurement transformations and associations, we have all the pieces we need to calculate the particle's final weight. The particles final weight will be calculated as the product of each measurement's Multivariate-Gaussian probability density.

The Multivariate-Gaussian probability density has two dimensions, x and y. The mean of the Multivariate-Gaussian is the measurement's associated landmark position and the Multivariate-Gaussian's standard deviation is described by our initial uncertainty in the x and y ranges. The Multivariate-Gaussian is evaluated at the point of the transformed measurement's position. The formula for the Multivariate-Gaussian can be seen below.

$$P(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{(x-\mu_x)^2}{2\sigma_x^2} + \frac{(y-\mu_y)^2}{2\sigma_y^2}\right)}$$

Homogenous Transformation

$$\begin{bmatrix} \mathrm{x}_m \\ \mathrm{y}_m \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \mathrm{x}_p \\ \sin\theta & \cos\theta & \mathrm{y}_p \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \mathrm{x}_c \\ \mathrm{y}_c \\ 1 \end{bmatrix}$$

Matrix multiplication results in:

$$\mathrm{x}_m = \mathrm{x}_p + (\cos\theta \times \mathrm{x}_c) - (\sin\theta \times \mathrm{y}_c)$$

$$\mathrm{y}_m = \mathrm{y}_p + (\sin\theta \times \mathrm{x}_c) + (\cos\theta \times \mathrm{y}_c)$$