```
> library(tidyverse)
> n <- 100
> k <- 8
> Sigma <- 64  * matrix(c(1, .75, .5, .75, 1, .5, .5, .5, 1), 3, 3)
> m <- MASS::mvrnorm(n, rep(0, 3), Sigma)
> m <- m[order(rowMeans(m), decreasing = TRUE),]
> y <- m %x% matrix(rep(1, k), nrow = 1) + matrix(rnorm(matrix(n*k*3)), n, k*3)
> colnames(y) <- c(paste(rep("Math",k), 1:k, sep="_"),
+                  paste(rep("Science",k), 1:k, sep="_"),
+                  paste(rep("Arts",k), 1:k, sep="_"))
>
>
> dim(m)
[1] 100   3
> head(m , 1)
         [,1]     [,2]     [,3]
[1,] 14.68714 13.17245 9.587728
>
> dim(y)
[1] 100  24
> head(y, 1)
       Math_1   Math_2   Math_3   Math_4   Math_5   Math_6   Math_7   Math_8
[1,] 15.11119 14.80123 14.61508 16.49006 14.51317 13.91765 14.39053 14.08034
     Science_1 Science_2 Science_3 Science_4 Science_5 Science_6 Science_7
[1,]  12.94543   13.9061  11.55866  14.12669  12.72028  12.55856  13.29831
     Science_8   Arts_1   Arts_2   Arts_3   Arts_4   Arts_5  Arts_6   Arts_7
[1,]  14.59861 8.667345 9.353845 9.639527 10.51491 10.49026 7.67721 9.266137
       Arts_8
[1,] 9.451579
>
>
>
> my_image <- function(x, zlim = range(x), ...){
+ colors = rev(RColorBrewer::brewer.pal(9, "RdBu"))
+ cols <- 1:ncol(x)
+ rows <- 1:nrow(x)
+ image(cols, rows, t(x[rev(rows),,drop=FALSE]), xaxt = "n", yaxt = "n",
+ xlab="", ylab="",  col = colors, zlim = zlim, ...)
+ abline(h=rows + 0.5, v = cols + 0.5)
+ axis(side = 1, cols, colnames(x), las = 2)
+ }
>
> my_image(y)
>
>
>
> my_image(cor(y), zlim = c(-1,1))
> range(cor(y))
[1] 0.4959604 1.0000000
> axis(side = 2, 1:ncol(y), rev(colnames(y)), las = 2)
>
>
>
> s <- svd(y)
> names(s)
[1] "d" "u" "v"
> y_svd <- s$u %*% diag(s$d) %*% t(s$v)
> max(abs(y - y_svd))
[1] 5.329071e-14
>
> ss_y <- apply(y^2, 2, sum)
> sum(ss_y)
[1] 175434.6
>
> y %>% dim()
```

```
[1] 100  24
> y %>% head(1)
       Math_1   Math_2   Math_3   Math_4   Math_5   Math_6   Math_7   Math_8
[1,] 20.09892 18.75238 19.18081 18.57126 18.87608 18.35266 19.37919 19.19652
     Science_1 Science_2 Science_3 Science_4 Science_5 Science_6 Science_7
[1,]  17.57078  17.78818  15.64214  17.37406  20.50545  16.99999  17.65281
     Science_8   Arts_1   Arts_2  Arts_3  Arts_4   Arts_5   Arts_6   Arts_7
[1,]  15.78353 16.74699 16.38969 14.7141 16.7063 14.78854 15.86147 16.02914
      Arts_8
[1,] 17.8744
> s$v %>% dim()
[1] 24 24
> s$v %>% head(1)
           [,1]      [,2]      [,3]      [,4]      [,5]        [,6]      [,7]
[1,] -0.2087448 0.1384358 0.2398709 -0.227831 0.2003452 -0.06861614 0.1203056
            [,8]      [,9]        [,10]     [,11]       [,12]     [,13]
[1,] -0.06231442 0.2155731 -0.002597498 0.3305406 -0.04183518 0.2267724
         [,14]     [,15]       [,16]     [,17]      [,18]     [,19]      [,20]
[1,] -0.101486 0.3230971 -0.02305995 -0.141546 -0.3031618 0.1187632 -0.1672883
          [,21]      [,22]      [,23]      [,24]
[1,] 0.02796905 0.01289183 -0.5252116 0.09823785
> y %*% s$v %>% head(1)
          [,1]        [,2]     [,3]      [,4]       [,5]      [,6]      [,7]
[1,] -86.01066 -0.07442552 4.166728 0.9494567 0.06902383 0.9035352 0.3461261
         [,8]      [,9]     [,10]    [,11]      [,12]     [,13]      [,14]
[1,] -2.28862 -2.076847 -1.226179 2.375902 -0.1307859 0.6573099 -0.5468135
        [,15]      [,16]      [,17]      [,18]     [,19]      [,20]      [,21]
[1,] 1.792848 -0.3515728 -0.7816362 -0.2016614 0.2791805 0.05912085 -0.3337954
         [,22]     [,23]      [,24]
[1,] 0.1665105 0.0897204 -0.4400632
>
> ss_yv <- apply((y %*% s$v)^2, 2, sum)
> sum(ss_yv)
[1] 175434.6
>
>
>
> plot(1:ncol(y), ss_y)
> plot(1:ncol(y), ss_yv)
>
>
>
> plot(s$d,sqrt(ss_yv))
>
>
>
> sum(s$d[1:3]^2) / sum(s$d^2)
[1] 0.9877922
>
>
>
> identical(s$u %*% diag(s$d), sweep(s$u, 2, s$d, FUN = "*"))
[1] TRUE
>
>
>
> plot(-s$u[,1]*s$d[1], rowMeans(y))
>
>
>
> my_image(s$v)
>
```