5.14 Realization of Digital filters

A digital filter transfer function can be realized in a variety of ways. There are two types of realizations 1. Recursive, 2. Nonrecursive

1. For recursive realization the current output $y(n)$ is a function of past outputs, past and present inputs. This form corresponds to an Infinite Impulse Response (IIR) digital filter. In this section we discuss this type of realization.

2. For non-recursive realization current output sample $y(n)$ is a function of only past and present inputs. This form corresponds to a Finite Impulse Response (FIR) digital filter.

IIR filter can be realized in many forms. They are

1. Direct form - I realization,
2. Direct form - II realization,
3. Transposed direct form realization,
4. Cascade form realization,
5. Parallel form realization,
6. Lattice - ladder structure. x

5.14.1 *Direct Form I realization*

Let us consider an LTI recursive system described by the difference equation.

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \tag{5.102}$$

$$= -a_1 y(n-1) - a_2 y(n-2) \ldots - a_{N-1} y(n-N+1)$$
$$- a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) \tag{5.103}$$

Let

$$b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) = w(n). \tag{5.104}$$

then $y(n) = -a_1 y(n-1) - a_2 y(n-2) + \ldots - a_N y(n-N) + w(n)$ (5.105)

The Eq. (5.104) can be realized as shown in Fig. 5.28.

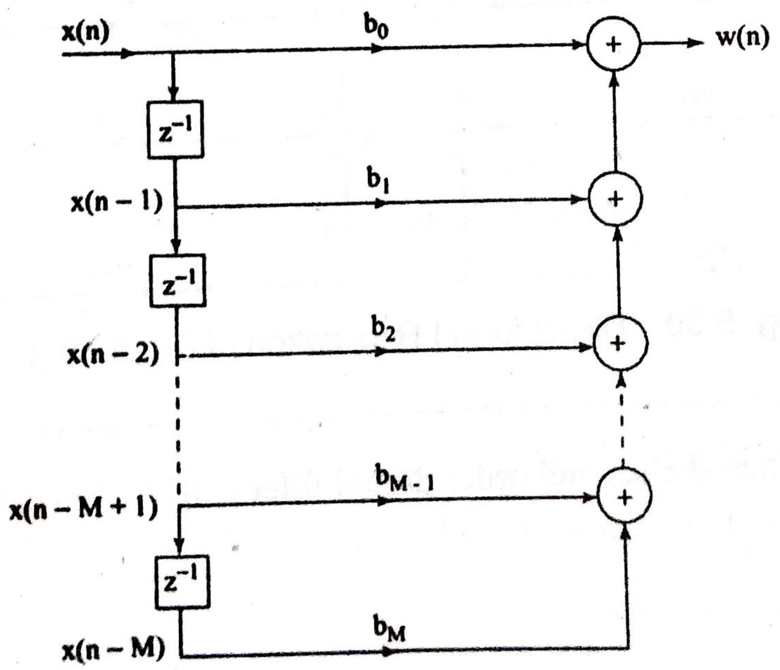

Fig. 5.28 Realization structure of Eq. (5.104)

Similarly the Eq.(5.105) can be realized as shown in Fig. 5.29.
To realize the difference Eq.(5.103) combine Fig. (5.28) and Fig. (5.29).

The structure shown in Fig. 5.30 is called direct form I, which used separate delays for both input and output. This realization requires $M + N + 1$ multiplications $M + N$ additions and $M + N + 1$ memory locations.

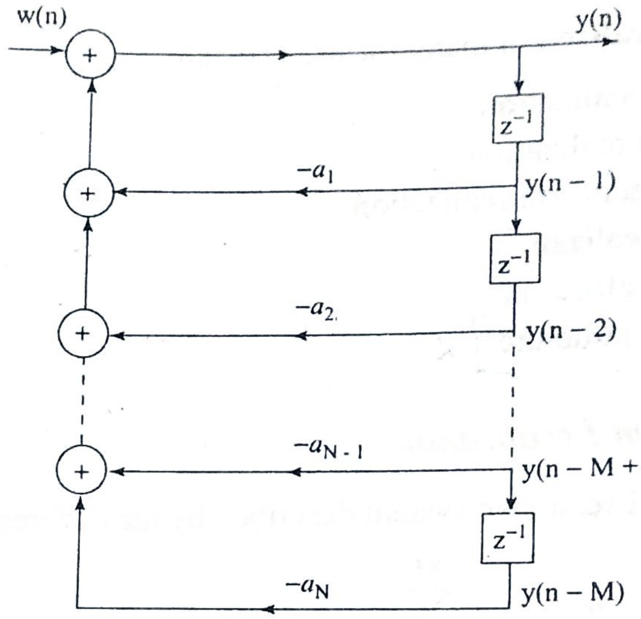

Fig. 5.29 Realization Structure of Eq. (5.105)

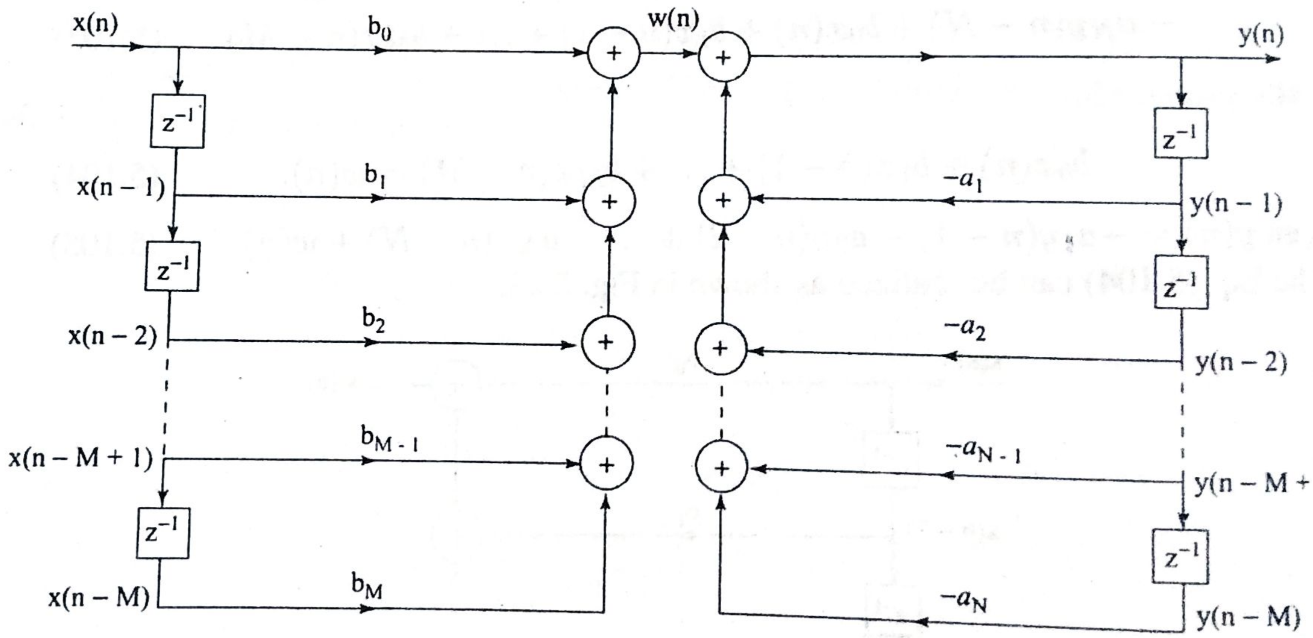

Fig. 5.30 Direct form I Realization of Eq.(5.103).

Example 5.20 Realize the second order digital filter $y(n) = 2r\cos(\omega_0)y(n-1)$ $r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$

Solution

Let

$$x(n) - r\cos(\omega_0)x(n-1) = w(n) \tag{5.106}$$

then

$$y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + w(n) \tag{5.107}$$

Realizing Eq.(5.106) we get

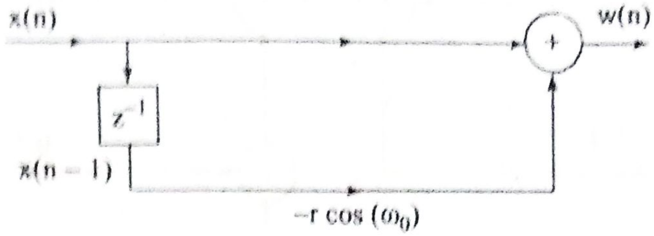

Fig. 5.31 Realization of Equation 5.106

Realizing Eq.(5.107) we obtain

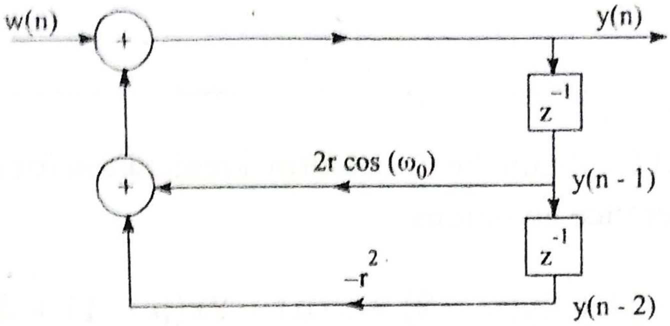

Fig. 5.32 Realization of Equation 5.107

If we combine both figures, we obtain the realization of the second order digital filter as shown in Fig. 5.33.

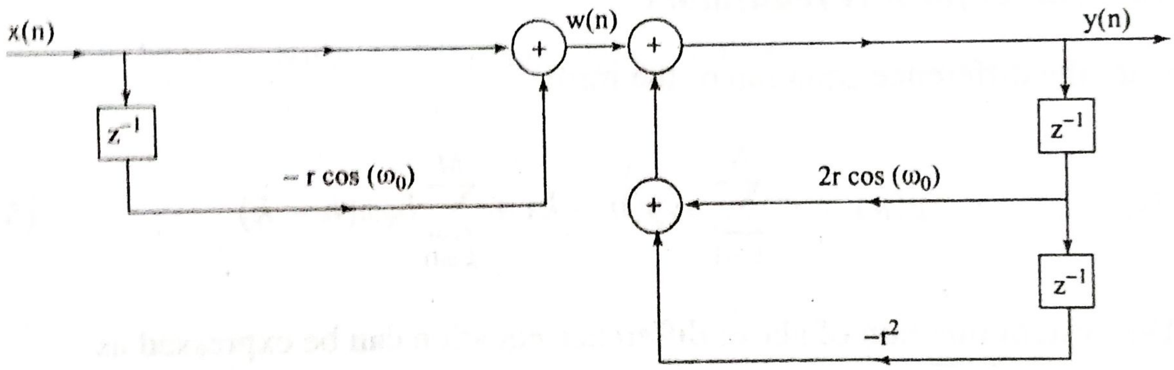

Fig. 5.33 Realization of Example 5.20

Example 5.21 Obtain the direct form-I realization for the system described by difference equation $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

Solution

Let

$$x(n) + 0.4x(n-1) = w(n) \tag{5.108}$$

then

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + w(n) \tag{5.109}$$

Realizing Eq. (5.108) and Eq. (5.109) and combining we get

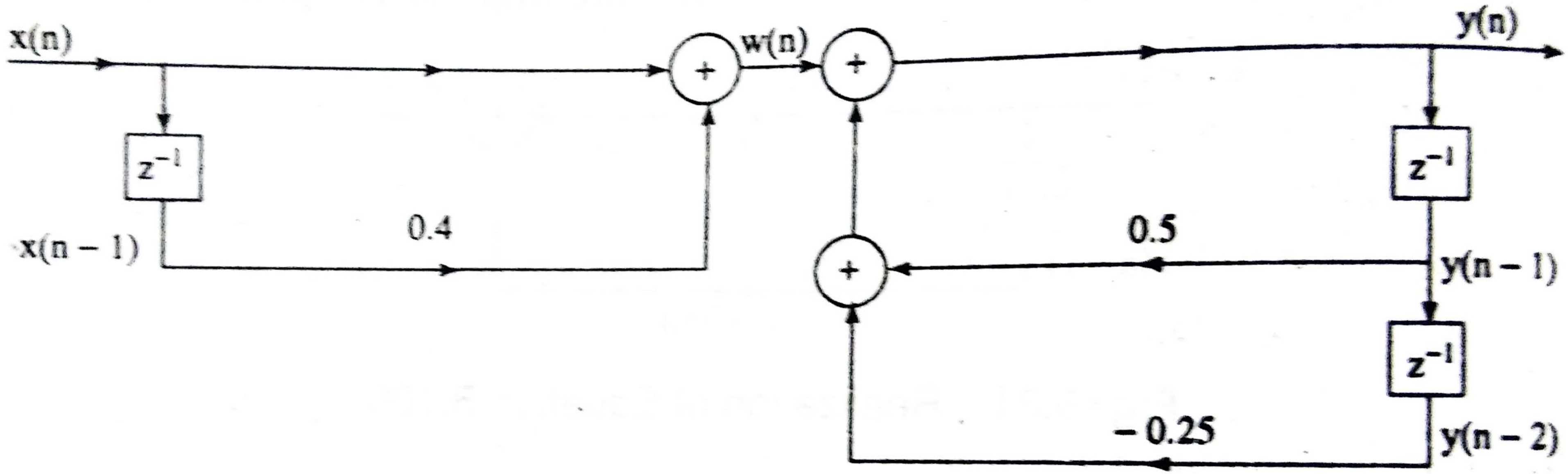

Fig. 5.34 Realization of Example 5.21

9.3.1 Direct-Form Structures

The rational system function as given by (9.1.2) that characterizes an IIR system can be viewed as two systems in cascade, that is,

$$H(z) = H_1(z)H_2(z) \tag{9.3.1}$$

where $H_1(z)$ consists of the zeros of $H(z)$, and $H_2(z)$ consists of the poles of $H(z)$.

$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \tag{9.3.2}$$

and

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{9.3.3}$$

In Section 2.5 we described two different direct-form realizations, characterized by whether $H_1(z)$ precedes $H_2(z)$, or vice versa. Since $H_1(z)$ is an FIR system, its direct-form realization was illustrated in Fig. 9.2.1. By attaching the all-pole system in cascade with $H_1(z)$, we obtain the direct form I realization depicted in Fig. 9.3.1. This realization requires $M+N+1$ multiplications, $M+N$ additions, and $M+N+1$ memory locations.

If the all-pole filter $H_2(z)$ is placed before the all-zero filter $H_1(z)$, a more compact structure is obtained as illustrated in Section 2.5. Recall that the difference

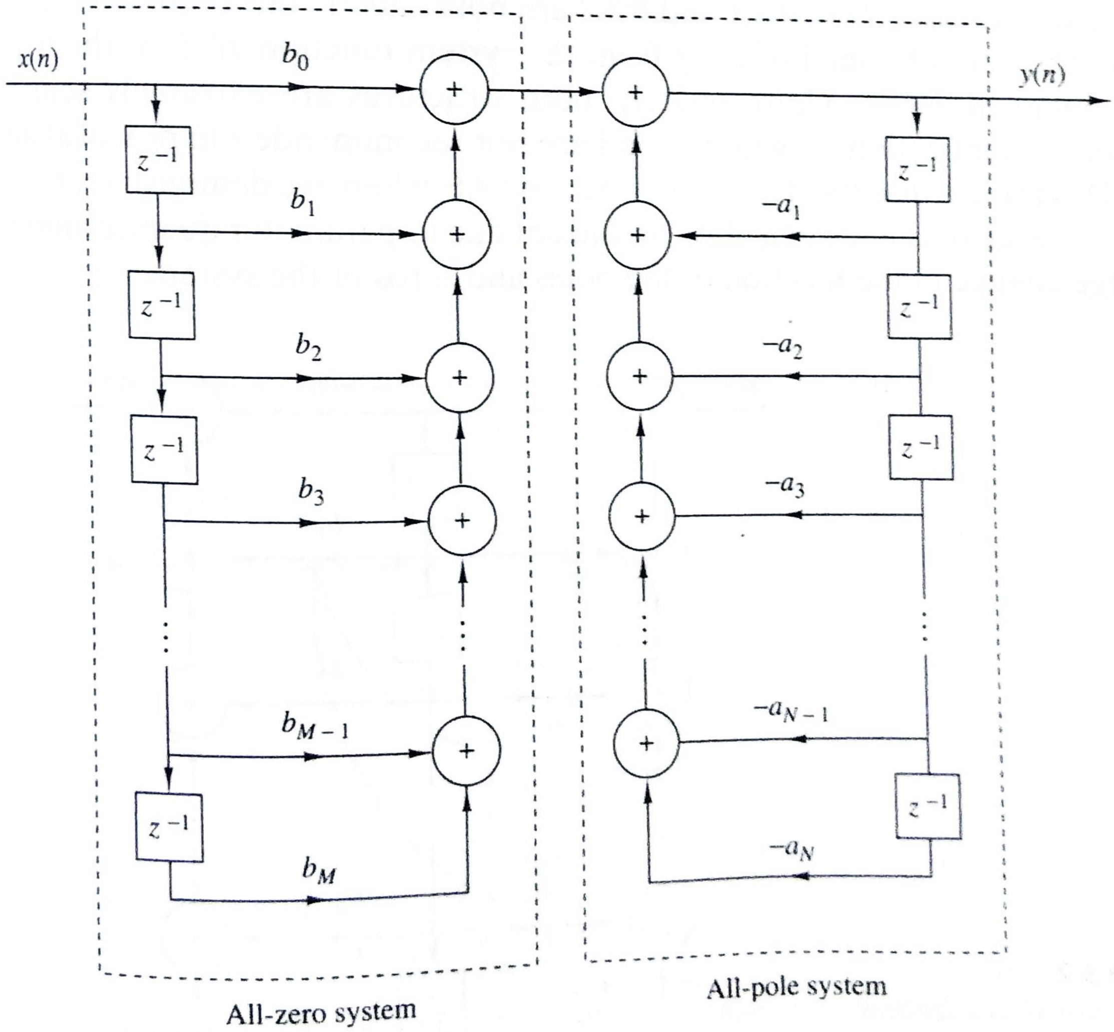

Figure 9.3.1 Direct form I realization.