①

Arun. (.)
ECB - 54

Q1) find the inverse transform of X[z)

$$X[z] = \frac{3 - \frac{5}{6} z^{-1}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z^{-1})}$$

A)

$$X[z] = \frac{3 - \frac{5}{6} z^{-1}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z^{-1})} = \frac{A}{1 - \frac{1}{4} z^{-1}} + \frac{B}{1 - \frac{1}{3} z^{-1}}$$

$$= A - \frac{1}{3} A z^{-1} + B - \frac{1}{4} B z^{-1} = 3 - \frac{5}{6} z^{-1}$$

$$\therefore A + B = 3$$

$$-\frac{1}{3} A + -\frac{1}{4} B = -\frac{5}{6}$$

$$\frac{A}{3} + \frac{B}{4} = \frac{5}{6}$$

$$\frac{4A + 3B}{12} = \frac{10}{12}$$

$$4A + 3B = 10$$

$$\therefore A + B = 3$$

$$\therefore A = 1$$

$$B = 3 - A$$

$$= 2$$

(2) A

Anun. (.)

$$\therefore X(z) = \frac{1}{(1-\frac{1}{4}z^{-1})} + \frac{2}{1-\frac{1}{3}z^{-1}}$$

$$\therefore x(n) = \left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{3}\right)^n u[n]$$

2) State and prove the time reversal property of z-transform

A)

If $x[n] \xleftrightarrow{z} X[z]$ with ROC $= R$,

then $x[-n] \xleftrightarrow{z} X(\frac{1}{z})$ with ROC $= \frac{1}{R}$

$$x[n] \longleftrightarrow X[z]$$

$$X[z] = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}$$

take $n = -n$

$$x[-n] = \sum_{n=-\infty}^{\infty} x(-n) \cdot z^{-n}$$

put $-n = m$

$$\therefore x[-n] = \sum_{m=-\infty}^{\infty} x(m)\, z^{m}$$

② ~~A~~

③ ~~Ans~~

$$= \sum_{m=-\infty}^{\infty} x(m) \cdot (z^{-1})^{-m}$$

$$= X[z^{-1}]$$

Q3) State any 6 properties of ROC of Z-transform

1) If $x[n]$ is a left sided sequence and if the circle $|z| = r_0$ is inside the ROC then all values of z for which $0 < |z| < r_0$ will be inside the ROC

2) If $x[n]$ is a right sided sequence, and if the circle $|z| = r_0$ is in the ROC, then all finite values of z for which $|z| > r_0$ will be in the ROC

3) If $x(n)$ is a two sided sequence and $|z| = r_0$ is in the ROC, then the ROC will consist of a ring in the z plane that includes the circle $|z| = r_0$.

4) If $x[n]$ is of finite duration, then ROC is the entire z-plane except $z=0$, $z=\infty$

5) ROC will not contain any poles.

6) ROC of $X(z)$ is consist of a ring in the z-plane centered about the origin.

4) Compute the z transform and ROC of $x[n] = 7(1/3)^n u[n] - 6(1/2)^n u[n]$

A)

$$x(n) \xleftrightarrow{z} \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$x(n) = 7(1/3)^n u[n] - 6(1/2)^n u[n]$$

$$\therefore X[z] = \sum_{n=-\infty}^{\infty} \{7(1/3)^n u[n] - 6(1/2)^n u[n]\} z^{-n}$$

$$= 7\sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^n \cdot u[n] \cdot z^{-n} - 6\sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n u[n] z^{-n}$$

$$= 7\sum_{n=0}^{\infty}\left(\frac{1}{3}z^{-1}\right)^n - 6\sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^n$$

$$= \frac{7}{1-\frac{1}{3}z^{-1}} - \frac{6}{1-\frac{1}{2}z^{-1}}$$

5a) Using the properties of z-transform find the inverse z-transform of

$$X(z) = \log(1 + az^{-1}), \ |z| > |a|$$

cont

$X(z) = \log(1+az^{-1}), |z|>a$

then

$$n x[n] \xleftrightarrow{z} -z\frac{dX(z)}{dz} = \frac{az^{-1}}{1+a/z}$$

$$a(a)^n u[n] \xleftrightarrow{z} \frac{a}{1+a/z}$$

$$a(-a)^{n-1} u[n-1] \longleftrightarrow \frac{az^{-1}}{1+az^{-1}}$$

$\therefore$

$$x[n] = \frac{-(-a)^n}{n} u[n-1]$$