Syndrome calculation –error detection and correction

- When a transmitted code vector C is passed through a noisy channel, the code vector R is received, R may not be the same as that of C
- R has 2^k code vectors similar to that of C.

Decoder

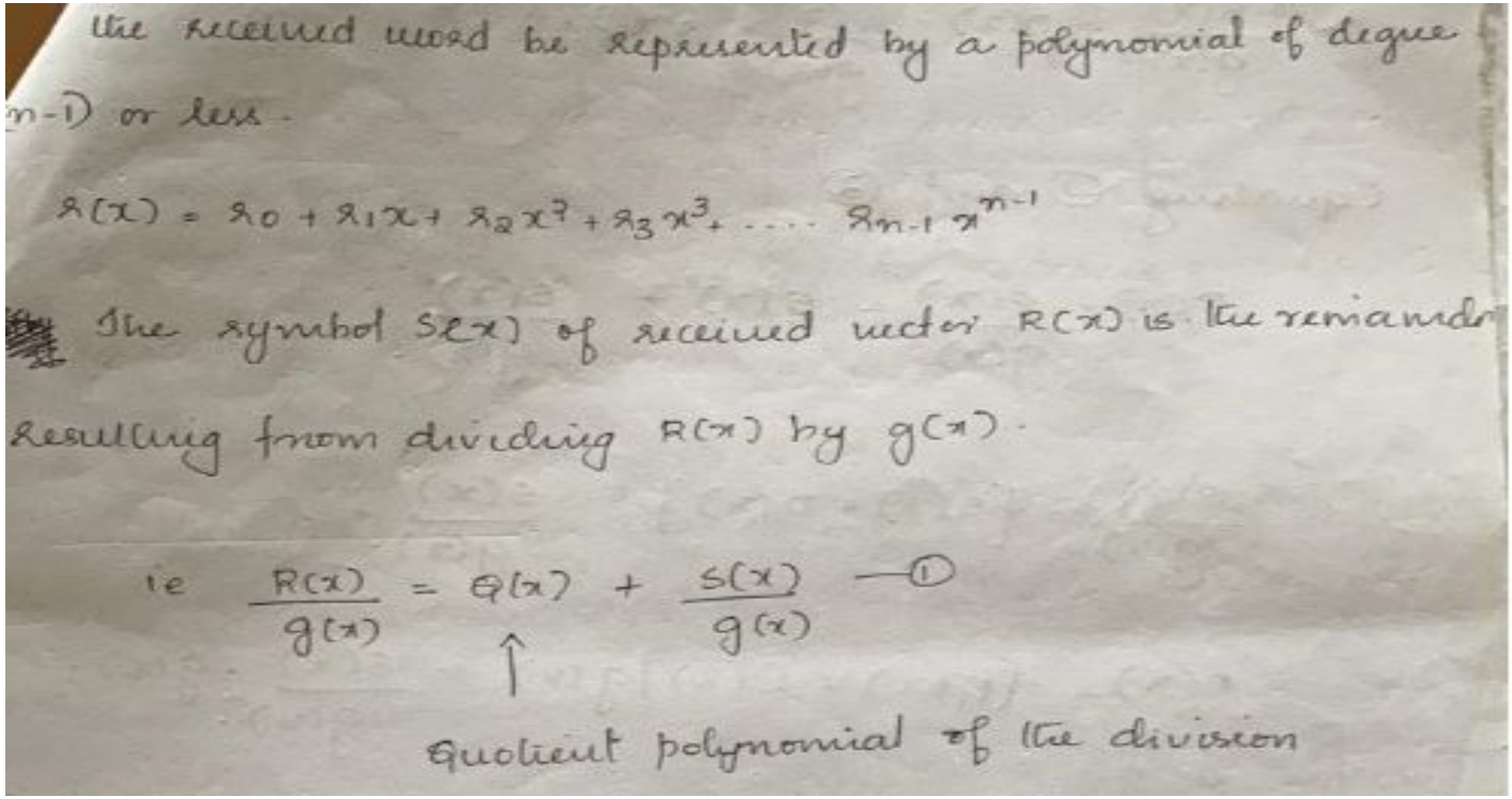
Function of decoder.

→ To determine the transmitted code vector C based on the received vector R.

→ The decoder first tests whether or not the received vector R is a valid code vector by calculating the syndrome of the received vector.

If the syndrome is zero, the received vector polynomial is divisible by the generator polynomial and received vector is a valid code vector. The decoder accepts the received vector R(x) as transmitted code vector.

- A non zero syndrome indicated error is present



Syndrom $s(x)$ is polynomial of degree $n-k-1$ or less.

If $e(x)$ is the error pattern caused by the channel,

then $R(x) = c(x) + e(x)$

$$\frac{R(x)}{g(x)} = \frac{c(x)}{g(x)} + \frac{e(x)}{g(x)}$$

we know that $c(x) = D(x) \cdot g(x)$.

$$\frac{R(x)}{g(x)} = D(x) + \frac{e(x)}{g(x)} \quad \text{—②}$$

Equating ① and ②

$$D(x) + \frac{e(x)}{g(x)} = q(x) + \frac{s(x)}{g(x)}$$

$$\frac{e(x)}{g(x)} = [q(x) + D(x)] + \frac{s(x)}{g(x)}$$

$$e(x) = [q(x) + D(x)]g(x) + \frac{s(x)}{g(x)} \cdot g(x)$$

$$= [q(x) + D(x)]\, g(x) + s(x)$$

Hence the syndrome of R(x) is equal to the Remainder resulting from ~~before~~ dividing the error pattern by generator polynomial

The syndrome contains information about the error pattern that can be used for error correction

Syndrome calculation circuit

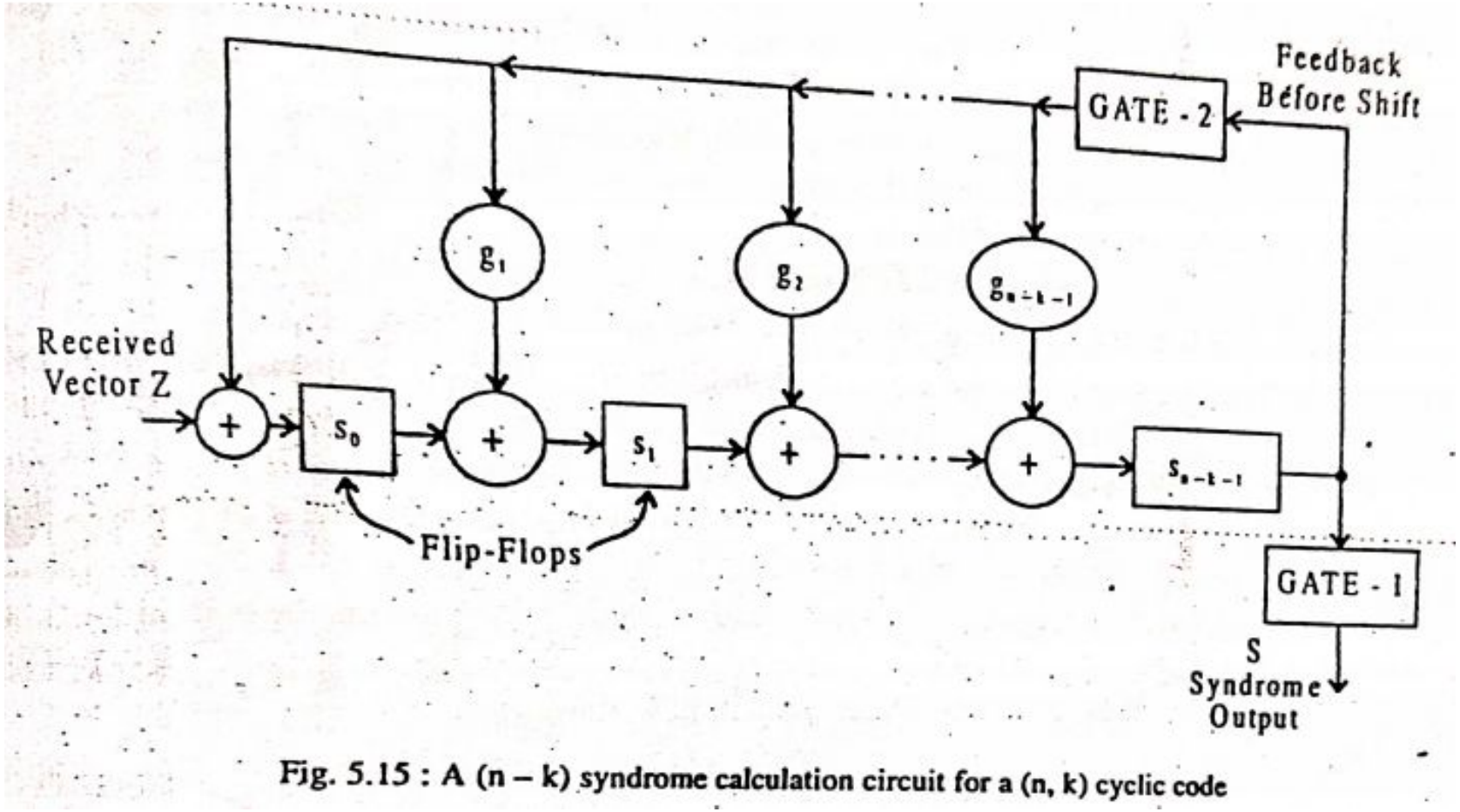


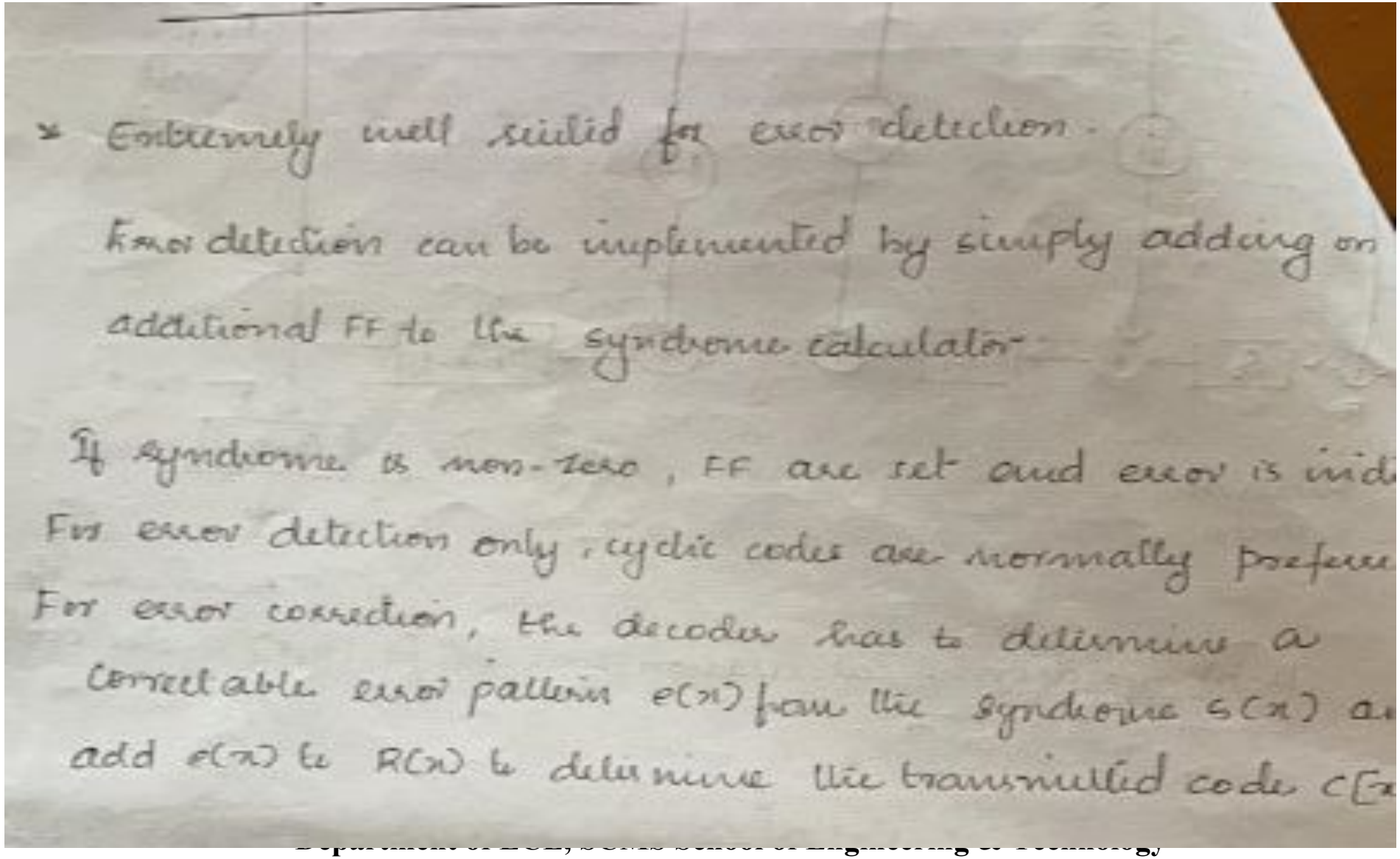
Fig. 5.15 : A (n – k) syndrome calculation circuit for a (n, k) cyclic code

Operation of syndrome decoder

The syndrome calculations are carried out as below :

1. The register is first initialized. The with gate-2 turned ON and gate-1 OFF, the received vector Z is entered into shift register.
2. After the entire received vector is shifted into the register, the contents of the register will be syndrome. Now gate-2 is turned OFF, gate-1 ON, and the syndrome vector is shifted out of the register. The circuit is ready for processing the next received vector.

Advantages of cyclic codes



- Extremely well suited for error detection.

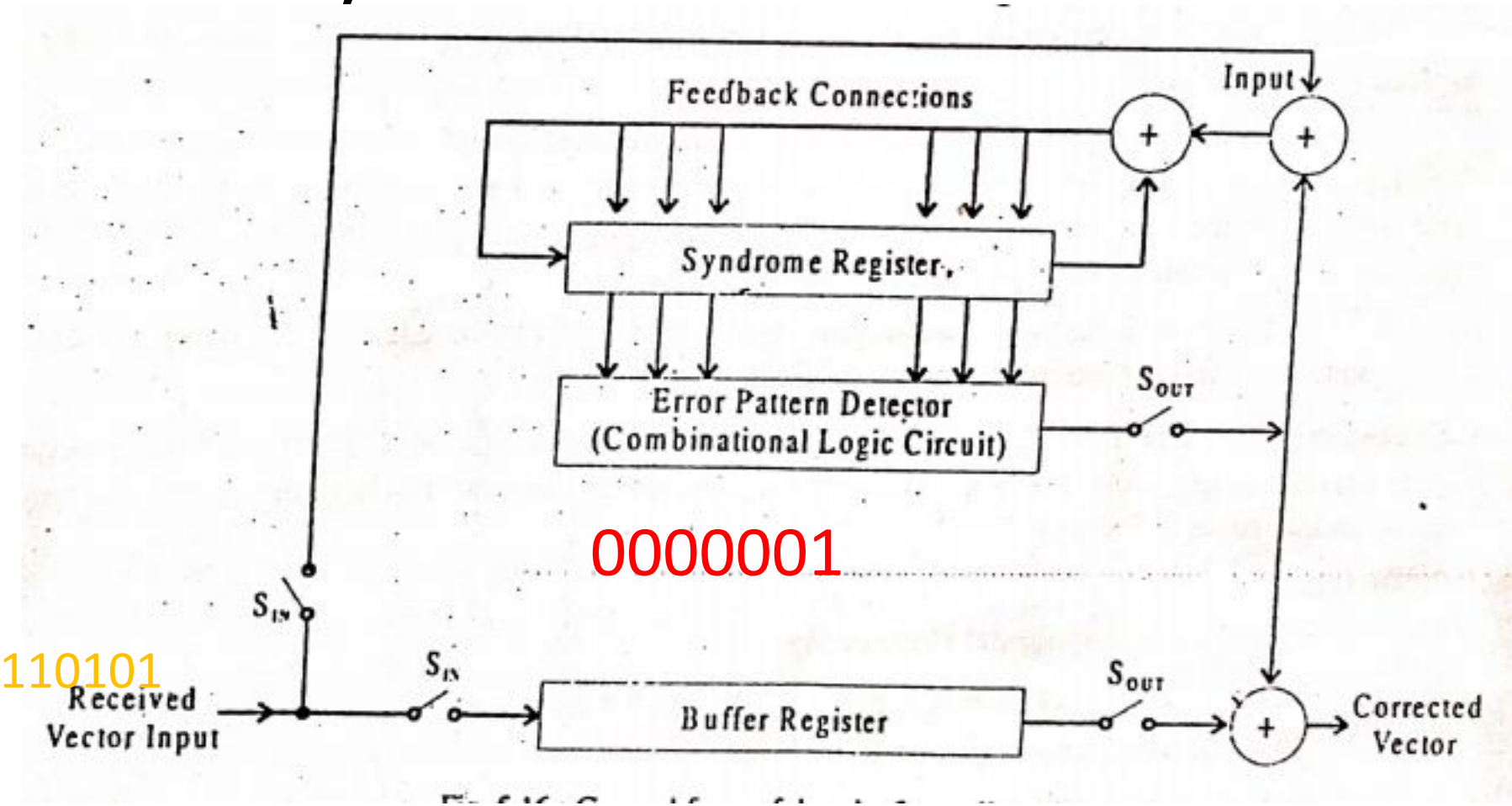
Error detection can be implemented by simply adding on additional FF to the syndrome calculator

If syndrome is non-zero, FF are set and error is indi

For error detection only, cyclic codes are normally prefer

For error correction, the decoder has to determine a correctable error pattern $e(x)$ from the syndrome $s(x)$ a add $e(x)$ to $R(x)$ to determine the transmitted code $c(x$

General form of decoder(with error correction)



Correspondin g to the syndrome S error pattern is generated E which is then added with the R to get the Corrected code vector C

E + R = C

Steps

1- The received vector is shifted into the buffer register & the syndrome register.

2. After the syndrome for received vector is calculated and placed in syndrome register, the contents of syndrome register is read into the detector.

Detector is combinational circuit designed to output a 1 iff the syndrome in ~~Reg.~~ syndrome ~~the~~ corresponds to a correctable error pattern with an error @ the highest order position x^{n-1}

detector o/p is 1, received digit at the right most stage of the buffer register is erroneous and hence is corrected

If detector o/p is zero, right most stage of buffer reg is assumed to be correct.

Thus the detector o/p is the estimated error value for digit coming out of the buffer Reg.

If the first received digit is in error, detector o/p is 1, which is used for corresponding the first received digit. The o/p of detector, ~~the~~ is also fed into the syndrome register to modify the syndrome.

This results in a new syndrome corresponding to the actual received vector shifted ~~to~~ ~~right~~ to right by one place.

④ The new syndrome is now used to check whether or not the 2nd received digit, now at the right most stage of the buffer is an erroneous digit.

If so, it is corrected, a new syndrome is calculated as in step ③ and procedure is repeated.

The decoder operates on the received vector digit by digit until entire received vector is shifted out of the buffer.

At the end of the decoding operations, the Syndrome register will contain all 0's.