Location of the zeros of linear phase FIR filters:

The transfer function of a linear phase FIR filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}.$$

where $h(n) \longrightarrow$ impulse response of FIR filter.

If z_0 is a non zero finite 'zero' of $H(z)$, then

$$H(z)\Big|_{z=z_0} = H(z_0) = \sum_{n=0}^{N-1} h(n)\, z_0^{-n} = 0$$

$$H(z_0) = h(0) + h(1)\, z_0^{-1} + \ldots\, h(N-1)\, z_0^{-(N-1)} = 0.$$

For linear phase filter

$$h(n) = h(N-1-n).$$

$$\therefore H(z_0) = h(N-1) + h(N-2)\, z_0^{-1} + \ldots\, h(0)\, z_0^{-(N-1)} = 0.$$

$$= z_0^{-(N-1)} \left[h(N-1)\, z_0^{N-1} + h(N-2)\, z_0^{N-2} + \ldots\, h(1)\, z_0^{1} + h(0) \right] = 0.$$

$$H(z_0) = z_0^{-(N-1)} \sum_{n=0}^{N-1} h(n)\, z_0^{n}$$

$$H(z_0) = z_0^{-(N-1)} \sum_{n=0}^{N-1} h(n)\, (z_0^{-1})^{-n}.$$

$$H(z_0) = z_0^{-(N-1)} H(z_0^{-1}) = 0.$$

That indicates if $H(z_0) = 0$ then $H(z_0^{-1}) = 0$.

for non zero value of 'z_0'

ie $H(z_0) = H(z_0^{-1}) = 0$

From above equation it is clear that if 'z_0' is a 'zero' of $H(z)$, then z_0^{-1} is also a 'zero'.