Q) A source generates three symbols with probabilities 0.25, 0.25 & 0.50 at a rate of 3000 symbols per sec. Assuming independent generation of symbols, calculate the average bit rate.

Ans) $R = ?$

X:

x_0 — $P(x_0) = 0.25$

x_1 — $P(x_1) = 0.25$

x_2 — $P(x_2) = 0.50$

$r =$ symbol rate $= 3000$

$R = r H(x)$

$\therefore R = 3000 \times 1.5$

$= 4500$ bits/sec

$$H(x) = 0.25 \log_2\left(1/0.25\right) + 0.25 \log_2\left(1/0.25\right) + 0.50 \log_2\left(1/0.50\right)$$

$= 1.5$ bits/symbol

(Q) A certain RV has the cdf given by

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ Kx^2 & 0 < x \le 10 \\ 100K & x > 10 \end{cases}$$

(1) Calculate the value of K.

(2) Find the values of $P(x \le 5)$ & $P(5 < x \le 7)$

(3) plot the corresponding pdf.

Ans) (1) $F_X(x) = 100K$

we know $F_X(x) = 1$ for $x > x_n$

$100K = 1$

$K = 1/100$

(2) $P(x \le 5) = F_X(5)$

$= Kx^2 \Big|_{x=5}$

$= 1/100 \times 5^2$

$= 0.25$

$P(5 < x \leq 7) = F_X(7) - F_X(5)$

$= Kx^2 \Big|_{x=7} - Kx^2 \Big|_{x=5}$

$= \frac{1}{100} \times 49 - \frac{1}{100} \times 25$

$= \underline{\underline{0.24}}$

(B) $pdf = \frac{d}{dx} F_X(x)$

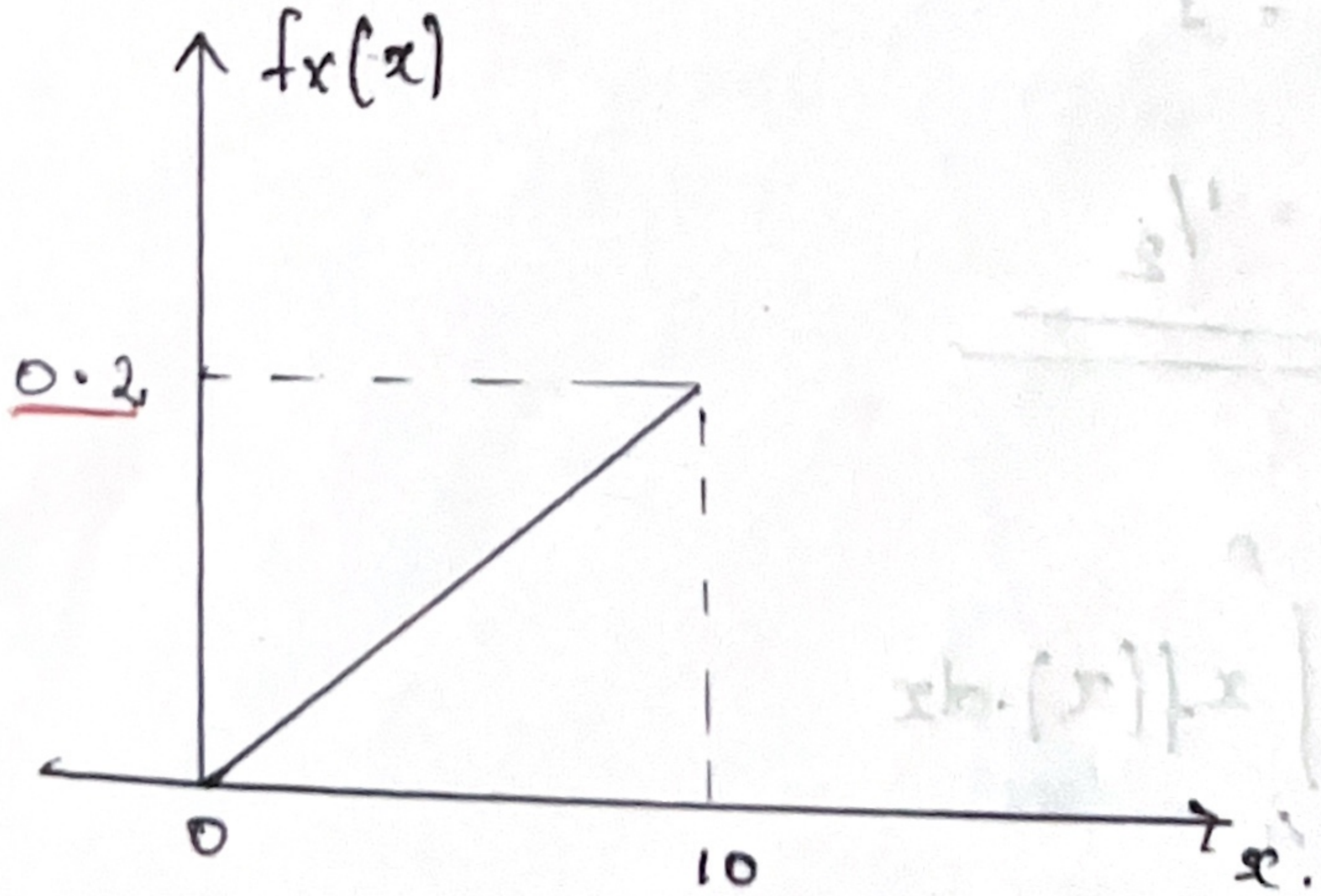
$= \frac{d}{dx} Kx^2 \qquad 0 < x \leq 10$

$= \frac{1}{100} \times 2x$

$= \underline{\underline{0.02x}}$

$y = mx + c$

slope $= 0.02$.



$\frac{?}{10} = 0.02$

$\underline{\underline{? = 0.2}}$

(Q) A continuous RV has a pdf of

$$f(x) = Kx^2 e^{-x} \quad 0 < x \leq 1$$

find the value of K, mean and variance.

Ans) We know $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^{\infty} x^{n-1} e^{-x}\,dx = \Gamma n \qquad \text{Gamma form}$$

$$= (n-1)!$$

$$\int_0^{\infty} Kx^2 e^{-x}.\,dx = 1 \qquad n-1=2$$

$$K \times 2! = 1$$

$$K = 1/2$$

mean: $\mu = \int_{-\infty}^{\infty} x f(x).dx$

$$= \int_0^{\infty} x \times 1/2\, x^2 e^{-x}.\,dx$$

$$= 1/2 \times 3! = 3$$

$$\sigma^2 = E[x^2) - [E[x]]^2$$

$$E[x^2] = \int x^2 \cdot \frac{1}{2} x^2 e^{-x} \cdot dx$$

$$= \frac{1}{2} \int x^4 e^{-x} \cdot dx$$

$$= \frac{1}{2} \times 4!$$

$$= \frac{1}{2} \times (4 \times 3 \times 2 \times 1) = \underline{12}$$

$$\therefore \sigma^2 = 12 - (3)^2$$

$$= 3 //$$