Module I

Signal:

A signal is defined as a function of one or more variables, which conveys information on the nature of a physical phenomenon.

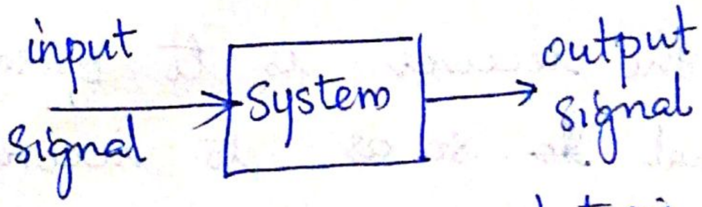
When the function depends on a single variable, the signal is called <u>one-dimentional</u>. eg: Speech signal.

When the function depends on two or more variable, the signal is called <u>multi-dimentional</u> eg: image signal.

A signal is defined as a single valued function of one or more independent variables which contains some information. $x(t) \rightarrow$ independent variable; x — dependent variable

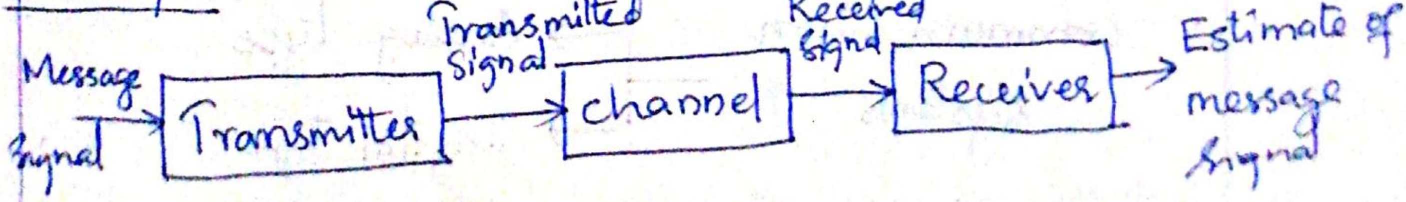
System:

A system is defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals.

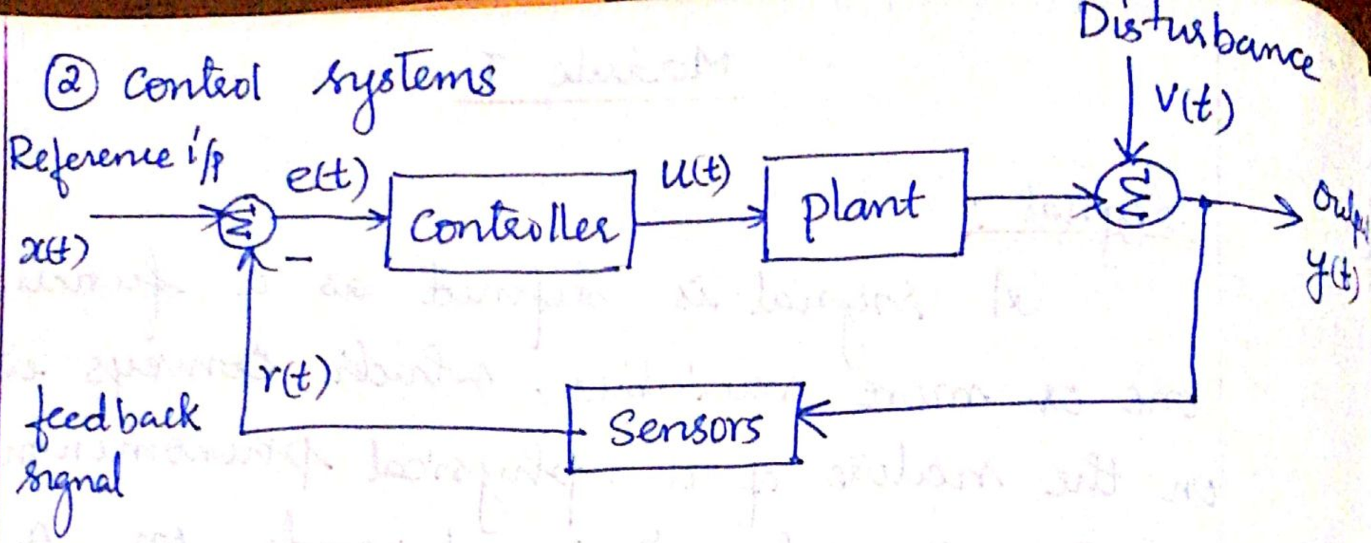


Block diagram representation of a system.

Example: ① Communication systems



② Control systems



Communication system:

The basic elements of every communication system are transmitter, channel and receiver. The transmitter converts the message signal produced by a source of information into a form suitable for transmission over the channel. The message signal could be a speech signal, video signal or computer data. As the transmitted signal propagates over the channel, it is distorted due to the physical characteristics of the channel. The channel may be an optical fiber, coaxial cable, satellite channel etc. The function of the receiver is to operate on the received signal ~~so~~ so as to reconstruct a recognizable form of the original message signal and deliver it to the user. destination.

Communication systems
→ Analog type
→ Digital type

Representation of signals:

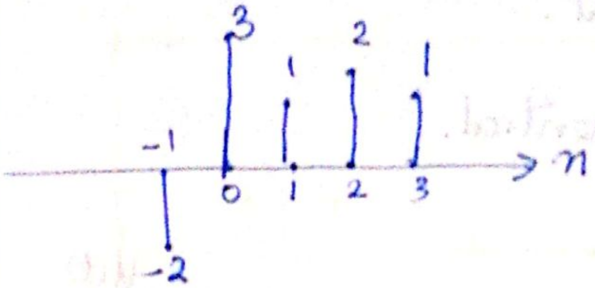
In general signals are classified as continuous time signals or discrete time signals. Continuous time signals are defined for all instants of time, Discrete time signals are defined only at discrete instants of time.

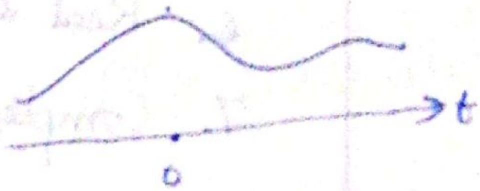
continuous time → $x(t)$

discrete time → $x[n]$ $n = 0, 1, 2, \cdot$

1. Graphical representation.

$x(n)$ $x(t)$





2. Functional representation.

$$x(n) = \begin{cases} 0 & n = -1 \\ 2 & n = 0 \\ 4 & n = +1 \end{cases}$$

$x(t) = \cos(t)$

$x(t) = 2t$

3. Tabular representation. ⇝ (in case of discrete)

n	-2	-1	0	1	2	3
$x(n)$	-3	2	0	3	1	2

4. Sequence representation.

$$x(n) = \{-2, 0, \underset{\uparrow}{1}, 4, 6, 7\}$$

↑ denotes the $n = 0$ term.

Elementary Signals.

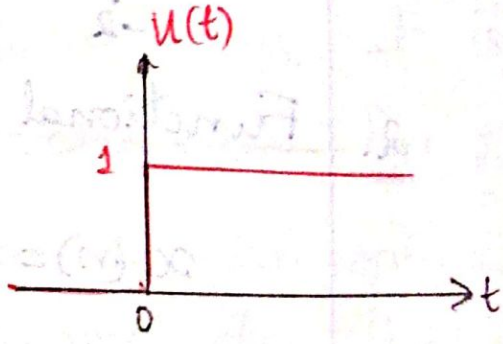
Basic building blocks for the construction of more complex signals.

Standard Signals are :

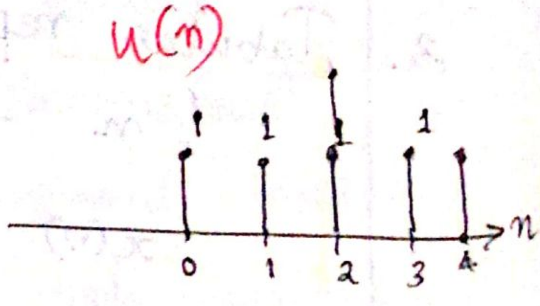
1. Unit step function
2. Unit ramp function
3. Parabolic
4. Impulse.
5. Sinusoidal
6. Real exponential.
7. Complex exponential.

1. Unit Step function

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



2. Unit Ramp function

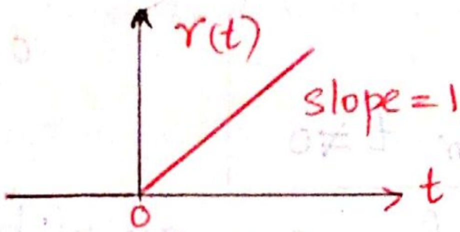
$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

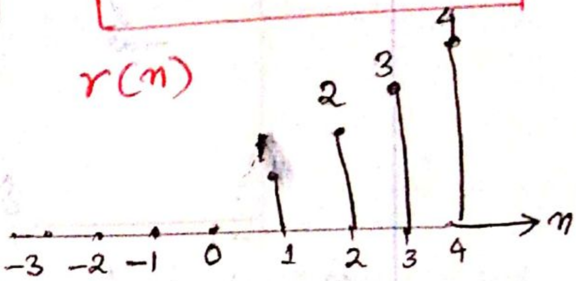
$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$r(t) = \int u(t)\, d(t)$$

$$u(t) = \frac{d}{dt} r(t)$$

$$r(n) = n\, u(n)$$





3. Unit Parabolic function

$$p(t) = \begin{cases} t^2/2 & \text{for } t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$p(t) = \iint u(t)\, d(t)$$

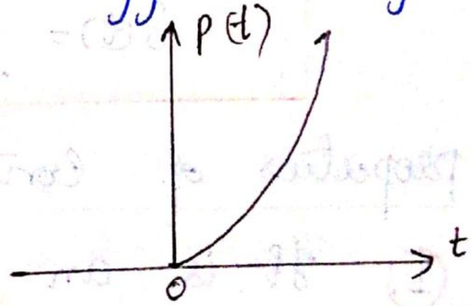
$$= \int r(t)\, dt$$

$$= \int t\, dt = t^2/2 \quad t \geq 0$$

$$r(t) = \frac{d}{dt} p(t)$$

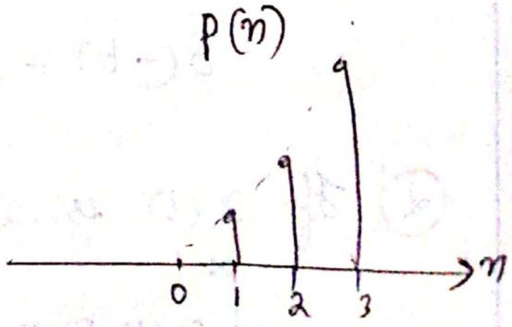
$$u(t) = \frac{d^2}{dt^2} p(t)$$

$$p(t) = \iint u(t)\, dt = \int r(t)\, dt$$



$$p(n) = \begin{cases} n^2/2 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$p(n) = n^2/2\; u(n)$$



4. Unit Impulse function.

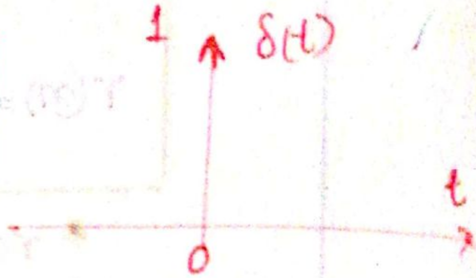
The continuous time unit impulse function $\delta(t)$ is called Dirac Delta function.

It is defined as

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

and

$$\delta(t) = 0 \quad \text{for } t \neq 0$$



The impulse function has zero amplitude everywhere except at $t=0$. At $t=0$, the amplitude is infinity so that the area under the curve is unity.

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\, dt$$

$$\delta(t) = \frac{d}{dt} u(t)$$

properties of continuous-time unit impulse function

(1) It is an even function of time t

$$\delta(-t) = \delta(t)$$

(2) If $x(t)$ is a continuous time signal, then

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\, dt = x(0); \quad \int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

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$\delta(n) = u(n) - u(n-1)$

$\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$

$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

$\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

③ Time scaling property,

$$\delta(at) = \frac{1}{|a|}\delta(t), \quad a>0$$

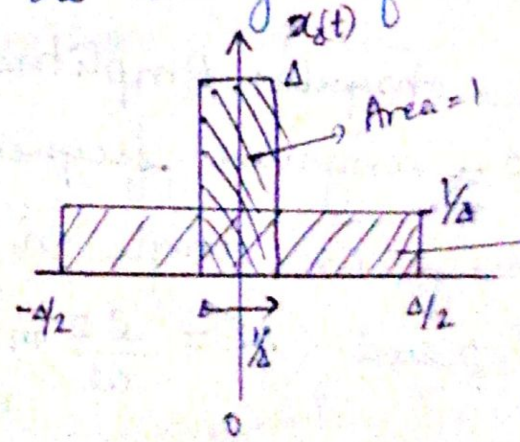
④. $$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) = x(t_0).$$

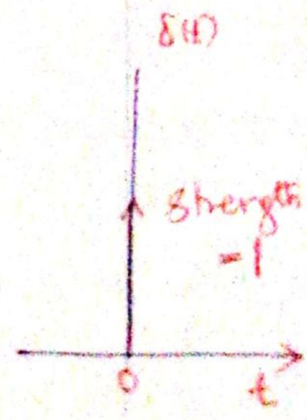
$$x(t)\,\delta(t) = x(0)\,\delta(t) = x(0)$$

⑤ $$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$\delta(t)$ can be obtained as the by limiting form of a rectangular pulse of unit area. The duration of the pulse is decreased, and its amplitude is increased, such that the area under the pulse is maintained constant at unity.

$$\boxed{\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)}$$, where $x_\Delta(t)$ is any pulse that is an even function of time t with duration Δ and unit area. The area under the pulse defines the strength of the pulse.



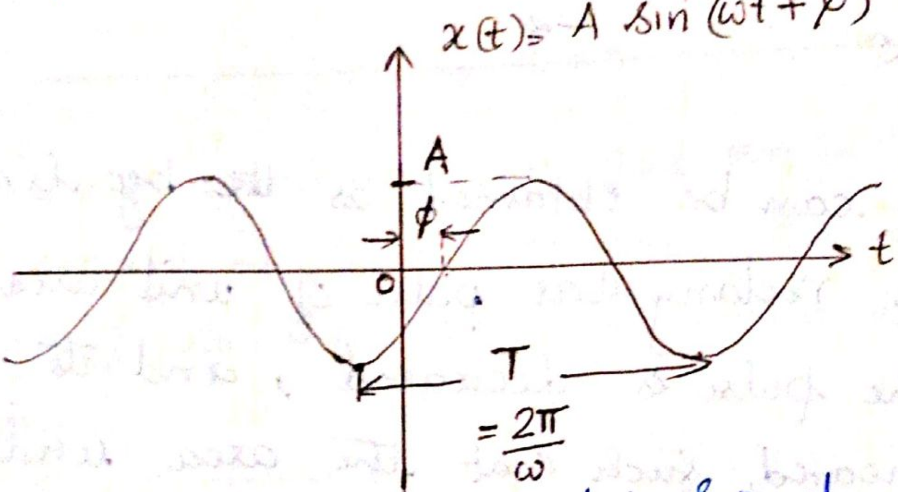


5. Sinusoidal Signal

continuous time. Sinusoidal signal is defined as

$$x(t) = A \sin(\omega t + \phi)$$

A = Amplitude
ω = Angular frequency in radians
ϕ = phase angle in radians.



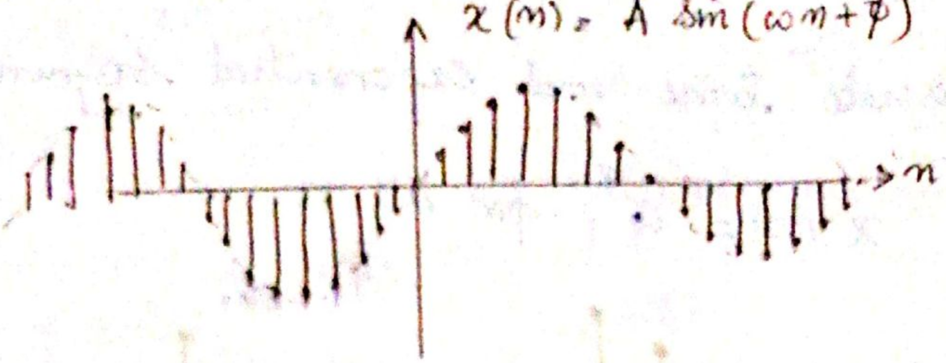
Sinusoidal signal is a periodic signal. The time period $T = \frac{2\pi}{\omega}$

Discrete-time sinusoidal sequence is given by

$$x(n) = A \sin(\omega n + \phi)$$

A = ~~Angular~~ Amplitude
ω = angular frequency in radians
ϕ = phase angle in radians

Time period $N = \frac{2\pi}{\omega} m$ where N & m are integers

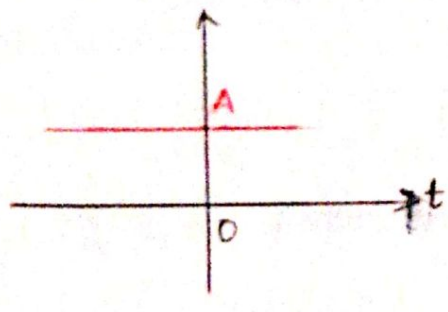


All continuous time sinusoidal signals are periodic but discrete-time sinusoidal sequences may or may not be periodic. To be periodic, the angular frequency ω must be a rational multiple of 2π.

6 Real Exponential Signal

Continuous time real exponential signal in general form $x(t) = A e^{\alpha t}$

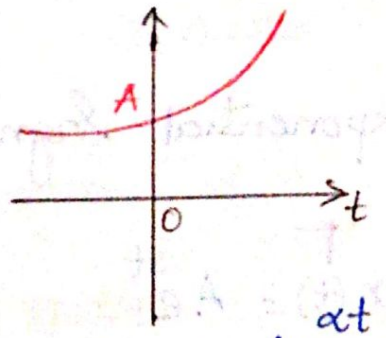
where A & α are real



$x(t) = A e^{\alpha t}$

for $\alpha = 0$

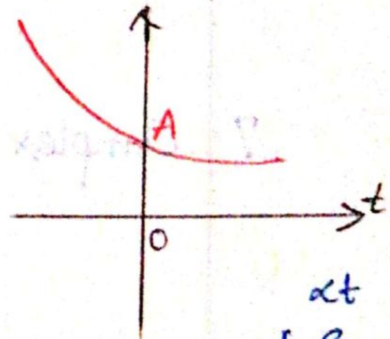
is constant amplitute for all times



$x(t) = A e^{\alpha t}$

$\alpha > 0$

$x(t)$ is a growing exponential signal



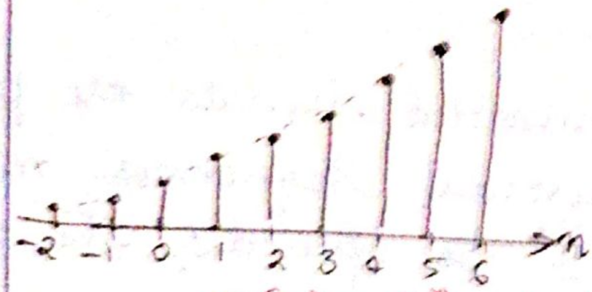
$x(t) = A e^{\alpha t}$

$\alpha < 0$

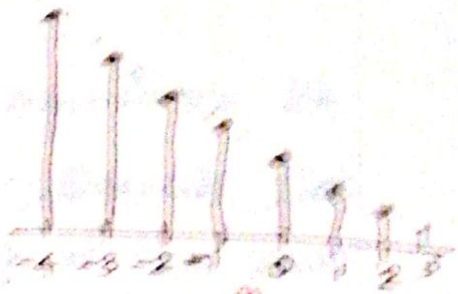
$x(t)$ is a decaying exponential signal.

Discrete time real exponential sequence.

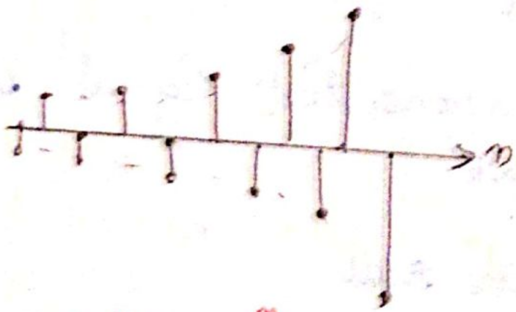
$$x(n) = a^n \text{ for all } n$$



$x(n) = a^n$; $a>1$



$x(n) = a^n$, $0<a<1$



$x(n) = a^n$; $a<-1$

$x(n) = a^n$, $-1<a<0$

7 Complex Exponential Signal.

$$x(t) = Ae^{st}$$

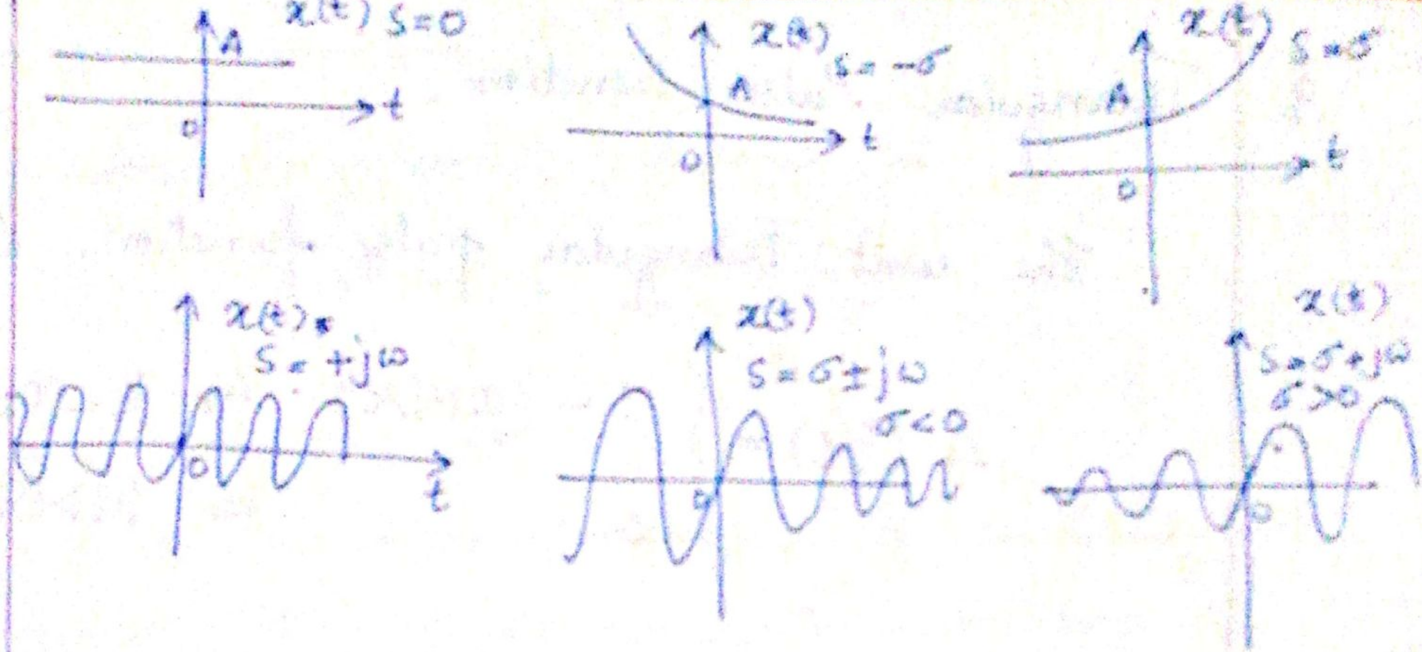
A = Amplitude.

S = Complex variable

$$S = \sigma + j\omega$$

$$\therefore x(t) = Ae^{(\sigma+j\omega)t}$$

$$= Ae^{\sigma t}[\cos \omega t + j \sin \omega t]$$



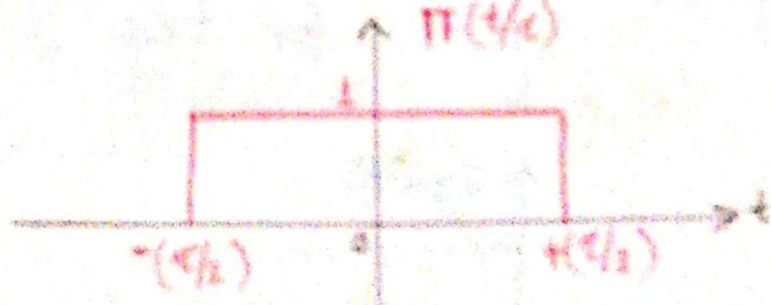
$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$$= a^n \cos(\omega_0 n + \phi) + j a^n \sin(\omega_0 n + \phi)$$

8. Rectangular Pulse function -

$$\Pi(t/\tau) = \begin{cases} 1 & \text{for } |t| \leq \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

is an even function of t



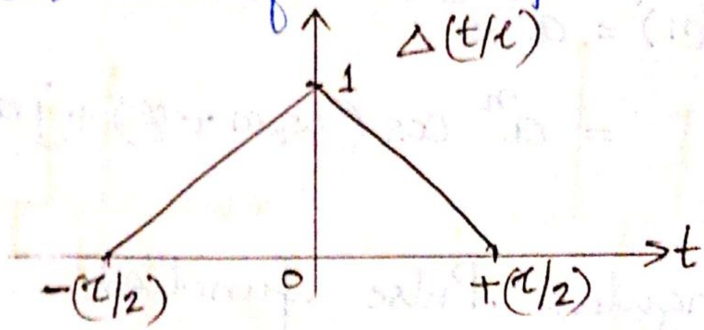
Rectangular pulse function

9. Triangular Pulse function.

The unit triangular pulse function. $\Delta(t/\tau)$

$$\Delta(t/\tau) = \begin{cases} 1 - (2|t|/\tau) & \text{for } |t| < (\tau/2) \\ 0 & \text{for } |t| > (\tau/2) \end{cases}$$

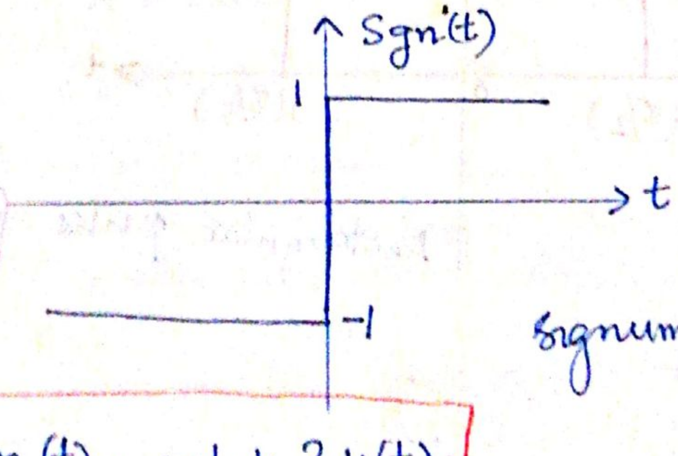
It is an even function of t.



10. Signum Function.

Is defined as

$$Sgn(t) = \begin{cases} 1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$$



signum function.

$$\boxed{Sgn(t) = -1 + 2u(t)}$$

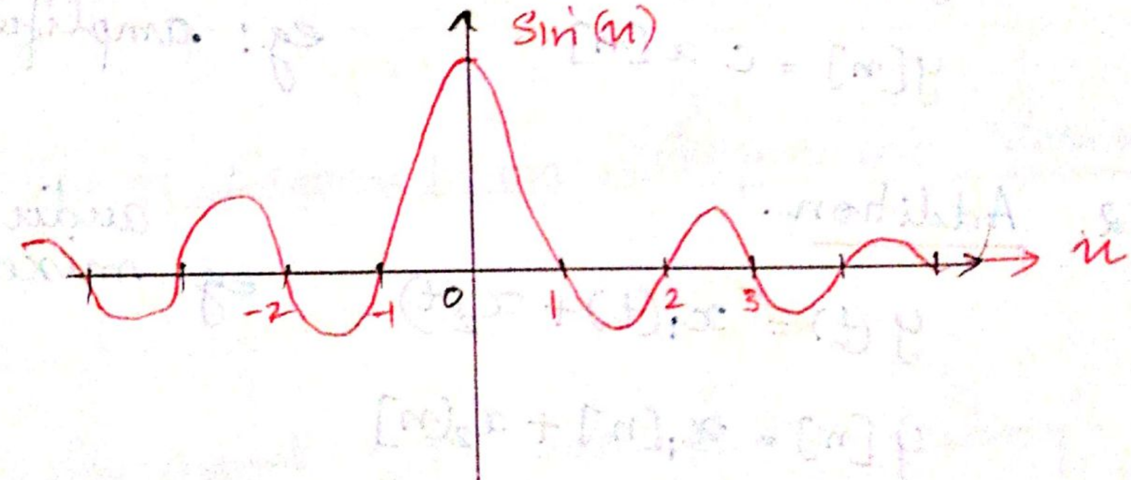
11 Sinc Function.

Sinc function is defined as

$$\text{Sinc}(u) = \frac{\sin(\pi u)}{(\pi u)}$$

Sinc (u) = 0 at $u = \pm 1, \pm 2, \ldots$

It is an even function. It oscillates with period 2π and decays with increasing u.



12 Gaussian function.

$$g_a(t) = e^{-at^2} \text{ for } -\alpha < t < \alpha.$$

$$g(t) = \left[\frac{1}{\sqrt{2\pi}}\right] e^{-t^2/2}$$