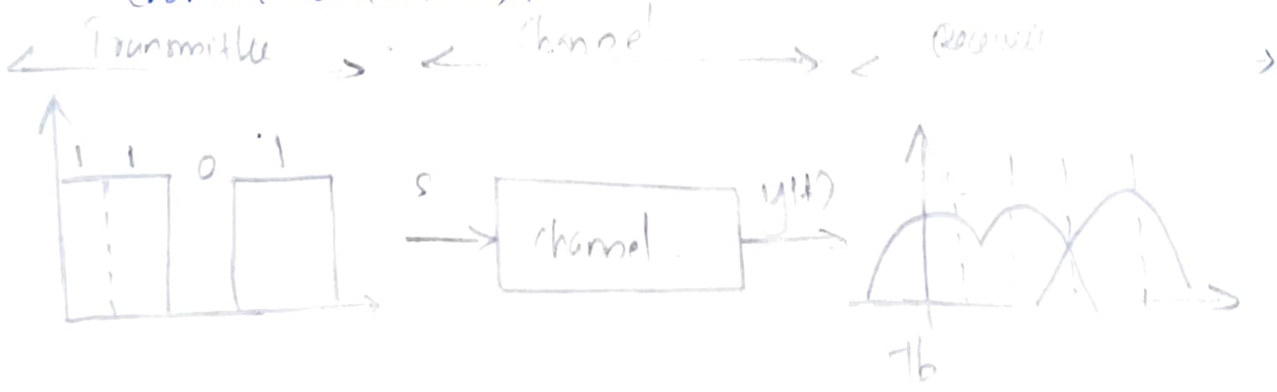
Mod- 4 (Blue)

(Additive)

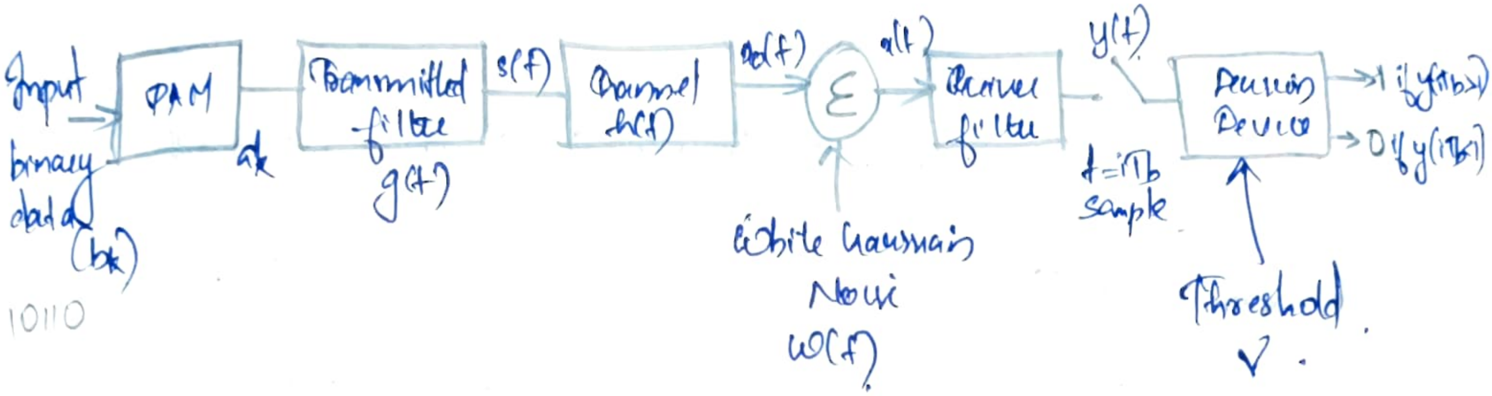
Base Band Transmission through AWGN

Inter Symbol Interference (ISI)

When a pulse of short duration is transmitted through a bandlimited communication channel, then the frequency components contained in the i/p pulse are differentially activated as well as delayed by the channel. As the result, the pulses will ~~overlap each~~ spill over to the adjacent time slots. The spreading of pulse beyond the time period is called inter symbol interference (ISI). ISI is caused by non-ideal channels that are not distortionless over the entire sig bandwidth, since the sig bandwidth is more than the available channel bandwidth.



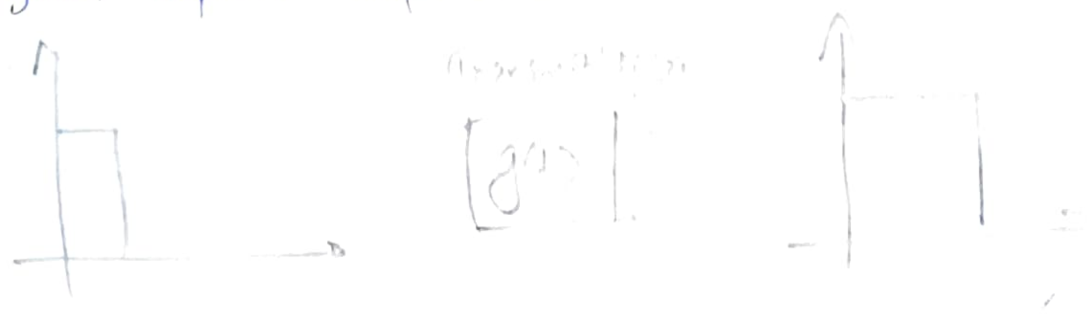
To illustrate the concept of ISI, consider a PAM s/m shown below.



The incoming binary sequence, b_k consists of symbols 1 and 0 each of duration T_b. The PAM modifies this binary sequence into a new sequence of narrow pulse whose amplitude is represented in polar form i.e

$$a_k = \begin{cases} +1 & \text{if } b_k = 1 \\ -1 & \text{if } b_k = 0 \end{cases}$$

These pulses are ~~so narrow that~~ then transmitted through the transmit filter with impulse response $g(t)$ to get a weighted impulse response.



The transmitted slg at the o/p of transmit filter is represented as

$$s(t) = \sum_k a_k g(t - kT_b)$$

The slg s(t) is passed through a channel of impulse response h(t) and adds random noise w(t) at the channel. The resulting noise corrupted slg x(t) is then passed through a received filter with impulse response c(t). The o/p of the received filter y(t) is then sampled at $t = iT_b$.

The sequence of samples thus obtained is used to reconstruct the origi. data sequence by means of a decision device. In decision device, the amplitude of each sample is compared with the threshold value V. ~~If threshold value V exceeded~~ ~~the~~ the sampled slg exceed the threshold value, then decision is made in favour of bit '1', otherwise in favour of bit '0'.

The receiver filter o/p is written as,

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) + n(t)$$

where μ is the scaling factor and $n(t)$ is the noise produced at the o/p of the received filter and $p(t)$ is the pulse shaping waveform. which is obtained by the double convol^n i.e,

$$\mu p(t) = g(t) * h(t) * c(t)$$

with the assumption that the pulse $p(t)$ is normalised by setting $p(0) = 1$.

In frequency domain, $\mu p(f) = g(f)$ — $G(F) \cdot H(F) \cdot C(F)$.

The received filter o/p is then sampled at $t = iT_b$.

$$y(iT_b) = \mu \sum_{k=-\infty}^{\infty} a_k p(iT_b - kT_b) + n(iT_b)$$

when $k = i$,

$$\boxed{y(iT_b) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(iT_b - kT_b) + n(iT_b)}$$

In the above eqn, first term represents the contribution of i^{th} transmitted bit and the 2nd term represents the residual effect of all other transmitted bits.

This residual effect due to the occurence of pulse before and after the sampling instant is called ISI.

In the absence of both ISI and channel noise,

$$\boxed{y(iT_b) = \mu a_i.}$$

ie the i^{th} transmitted bit can be decoded correctly in the absence of ISI and channel noise.

→ Nyquist criteria for Distortionless Transmission

[Nyquist criteria for 0 ISI]

We know that, the sampled slg,

$$y(iT_b) = \mu a_i + \sum_{k=-\infty}^{\infty} a_k P(iT_b - kT_b) + n(iT_b)$$

Inorder to avoid ISI in the received slg, select a suitable pulse shape $p(t)$. i.e during the decoding of ith transmitted pulse, the contribution of $p(iTb - kTb)$ for $k = i$ should be free from ISI i.e $p(iTb - kTb) = \begin{cases} 1 & \text{for } k = i \\ 0 & \text{o.w.} \end{cases}$

where $p(0)$ is $= 1$. By normalisation, if $p(t)$ satisfies the condn, the o/p $y(iTb)$ becomes μa_i for all i.

For 0 ISI, the nyquist criteria states that the pulse shaping function $p(t)$ with fourier transform $P(f)$ satisfies

$$\sum_{n=-\infty}^{\infty} P(f - nRb) = Tb$$. has 0 ISI.

Ideal Nyquist channel / Ideal soln

we know that ISI can be minimized by controlling p(t) in time domain or P(f) in frequency domain

i.e for zero ISI

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b$$

i.e The sum of shifted version of F.T of p(t) should be constant.

> Inorder to avoid ISI, select any pulse shaping function, p(t) such that the sum of shifted version of FT of p(t) should be constant.

> The simplest way of satisfying the Nyquist criteria is to specify frequency function P(f) to be in the form of rectangular function as

$$P(f) = \begin{cases} \frac{1}{2w} & -w \le f \le w \\ 0 & |f| > w \end{cases}$$

$$= \frac{1}{2w} \text{rect}(f/2w)$$

where rect(f) represent a rectangular function of unit amplitude and is centered around $f = 0$

> The overall s/m B.W 'w' is defined as

$$w = \frac{R_b}{2} = \frac{1}{2T_b}$$

$$\boxed{R_b = \frac{1}{T_b}}$$

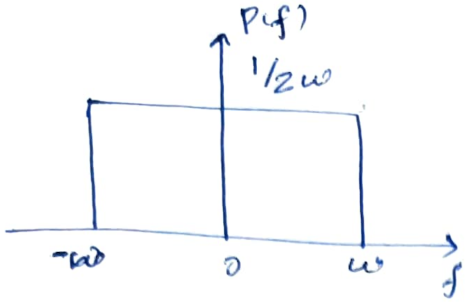
R_b = Bit rate.

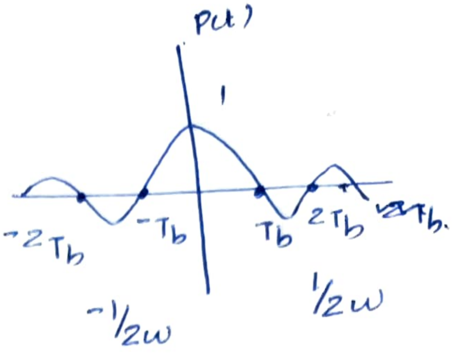
> $R_b = 2w \rightarrow$ Nyquistic B.W

> P(f) represented in time domain as

$$p(t) = \frac{\sin 2\pi wt}{2\pi wt}$$

i.e [p(t) = sinc(2wt)]



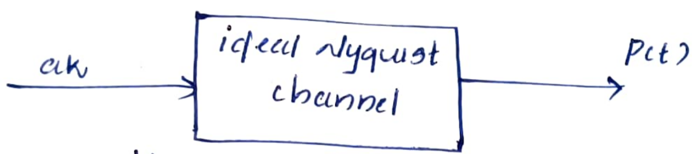


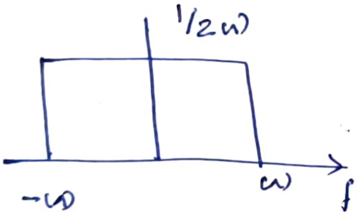
$\frac{1}{2w}$ rect$(f/2w)$

Ideal frequency response

Ideal Nyquist channel.

ideal pulse response.





Gains

Limitations of ideal Nyquist channel.

(1) The magnitude response of P(f) should be flat from $-w$ to $+w$ and zero elsewhere. This is practically un-realizable because the abrupt transaction at the band edges $\pm w$

(2) ISI can be avoided in Nyquist channel only if the o/p sinc pulses from receive filter is sampled exactly at $t = 0, T_b, 2T_b, \ldots$ and even a slight sampling filter will introduce the ISI

(3) ~~Thrown~~ The rate of decrease of sinc pulses $p(t)$ is only $\frac{1}{|t|}$ even at large value of 't'. So the side lobe amplitude do not decrease for t. This results in considerable ISI even for small errors in sampling time.

Practical sol^n - The raised cosine spectrum

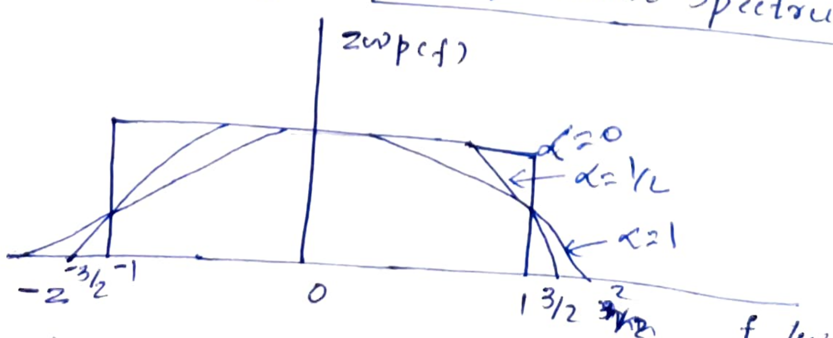
> Due to the practical limitations in the ideal Nyquist channel, we need to reshape the received pulse other than the sinc pulse.

> Here we extend the band width from a minimum value $w = R_b/2$ to an adjustable value b/w w and $2w$

> Correspondingly, the frequency response $P(f)$ of the modified pulse shaping function $p(t)$ has a flat portion and a roll off portion that has a sinusoidal form as follows.

$$P(f) = \begin{cases} \frac{1}{2w} & 0 \le |f| \le f \\ \frac{1}{4w}\left[1 - \sin\left(\frac{\pi(|f| - w)}{2w - 2f_1}\right)\right] & f_1 \le |f| < 2w - f_1 \\ 0 & |f| \ge 2w - f_1 \end{cases}$$

> The spectrum of the modified pulse shaping function is called Raised cosine spectrum



~~Raised~~ Raised cosine spectrum.

> The frequency parameters f_1 and bandwidth are related by

$$\alpha = 1 - f_1/W$$

where α is called roll of factor it indicates the excess bandwidth over the ideal soln W

Now the transmission B.W

$$B_T = 2W - f_1$$

$$\alpha = 1 - \frac{f_1}{W}$$

$$\alpha W = W - f_1$$

$$f_1 = W - \alpha W$$

$$= 2W - (W - W\alpha)$$

$$= W + W\alpha$$

$$B_T = W(1+\alpha)$$

Correspondingley p(t) is given by

$$p(t) = \text{sinc}(2Wt)\left(\frac{\cos(2\pi\alpha W t)}{1 - 16\alpha^2 W^2 t^2}\right)$$

The raised cosine pulse p(t) consists of two factors

1. The factor sinc(2Wt) characterizing the ideal Nyquist channel.

2. The factor $\left(\frac{\cos(2\pi\alpha W t)}{1-16\alpha^2 W^2 t^2}\right)$ that decreases as $\frac{1}{t^2}$ for a large 't'.

(Qn) A base band digital s/m uses 4-level PAM along with the received cosine pulse The s/m has a frequency response of 3.2 KHz. If the binary data is transmitted at 9600 bps data rate, then what would be the symbol rate and roll off factor of the transmitted pulse shape for zero ISI of $L = 4$ $R_b = 9600$ bps

Symbol rate, $R_s = \dfrac{R_b}{\log_2 L}$

$= \dfrac{9600}{\log_2 4} = 4800 \text{bps}$

minimum s/m bandwidth, B.W is $W = \dfrac{R_s}{2}$

$= \dfrac{4800}{2} = \underline{\underline{2400 \text{ Hz}}}$

Given frequency response of s/m is 3.2KHz

$B_T = 3.2 \text{KHz}, \ W = 2.4 \text{ KHz}$

$B_T = W(1+\alpha)$

$\alpha = \underline{\underline{0.33}}$

<u>Correlative ~~Correlatic~~ level coding</u>

(partial response ~~signal~~ signalling)

> We knows that ISI produces decoding errors and theore by degrades the s/m perfomance

> By adding ISI to the transmitted s/g in a controlled manner, it is possibel to achieve a signalling rate equal to Nyquist rate of 2W symbols per second in a channel of BW W Hz. Such scheme ~~ms~~ are called correlative level coding as partial ~~signalsstic~~ response signalling.

> The basic principle behind this is that if ISI is introduced into the txd signal is known, its effect can be interpreted at the receiver in a deterministic manner.

> So this is regarded as the practical method for achieving the theoretical max signalling rate of 2W symbols per second in a B.W of W Hz

> Correlative level coding can be of
1) Duo ~~bi~~ binary encoding
2) Duo ~~bi~~ binary encoder with precoding
3) modified duo binary signalling

1) Duo binary signalling

> This corelative level of coding is also called class I ~~a~~ partial response Because the response to an input pulse is spread over ~~the~~ two signalling interval.

> Here the message signal consist of binary ~~sigl~~ symbols 1 and 0's each having duration T_b.

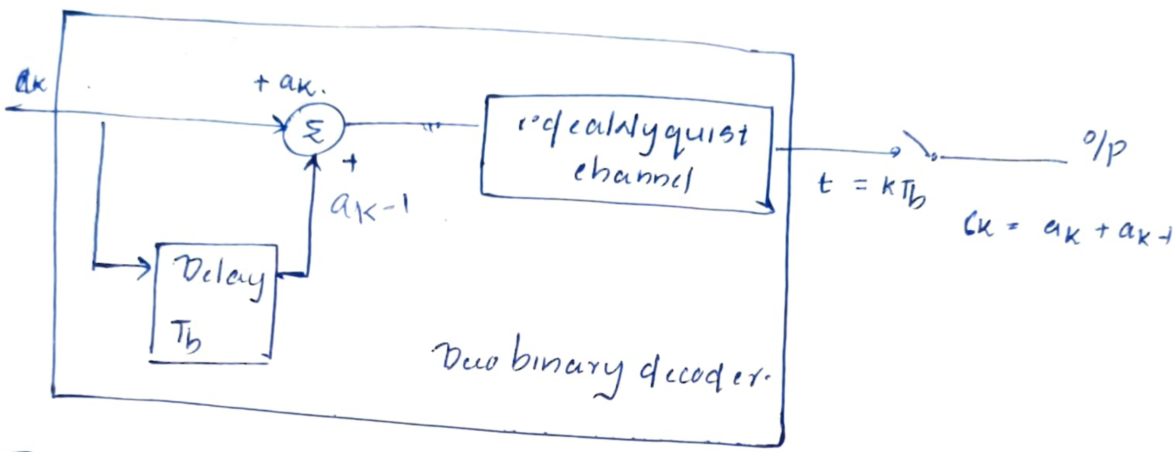
> This sequence is applied to a PAM which produces two level sequence of short duration ~~wav~~ i'e,

$$a_k = \begin{cases} +1 & \text{of} \quad b_k = 1 \\ -1 & \text{of} \quad b_k = 0 \end{cases}$$

> when this (+1, −1) is applied to a duo binary encoder, it is converted into a ~~theoretical~~ ~~three level (−2, 0, +2)~~ three level sequence (−2, 0, +2)

↳ duo binary encoder o/p is expressed as

$$c_k = a_k + a_{k-1}$$



> The above eqn shows that the encoder converts the i/p sequence (a_k) of uncorrelated two level pulses to an o/p sequence $\{c_k\}$ of correlated three ~~three~~ level pulse.

> This correlation b/w adjacent pulses may be viewed as introducing ISI into the transmitted sig in an artificial manner.

> Frequency response of duo binary encoder. (impulse response).

→ The duo binary encoder is the cascade of delay line filter and ideal Nyquist channel.

→ The delay line ~~level~~ element produces a delay on T_b - seconds and FR of $e^{-j2\pi f T_b}$

So The frequency response of delay line ~~level filter is $1+e^{-j2\pi T_b f}$~~ so the frequency response filter is $1+e^{-j2\pi f T_b}$

Hence over all frequency response

$$H_1(f) = H_{nyquist}(f) \cdot (1 + e^{-j2\pi f T_b})$$

$$= H_{nyquist}(f)\, e^{-j2\pi f T_b/2} \left[e^{+j2\pi f \frac{T_b}{2}} + e^{-j2\pi f T_b/2} \right]$$

$$= H_{nyquist}(f)\, e^{-j\pi f T_b} \left[e^{j\pi f T_b} + e^{-j\pi f T_b} \right]$$

$$H_1(f) = H_{nyquist}(f)\, e^{-j\pi f T_b} \left[2 \cos \pi f T_b \right]$$

$$\left[\because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \right]$$

$H_1(f)$ indicates the class-I partial response.

→ For an ideal nyquist channel of bandwidth ~~2W~~

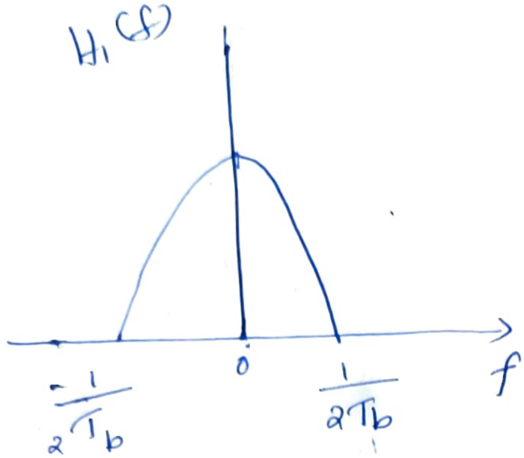
$W = \frac{1}{2T_b}$, we have

$$H_{nyquist}(f) = \begin{cases} T_b & |f| \le \frac{1}{2T_b} \\ 0 & \text{otherwise} \end{cases}$$

→ Thus the overall frequency response of the duobinary encoder has the form of half cycle cosine function

ie

$$H_1(f) = \begin{cases} 2T_b \cos(\pi f T_b)\, e^{-j\pi f T_b} & |f| \le \frac{1}{2T_b} \\ 0 & \text{otherwise} \end{cases}$$

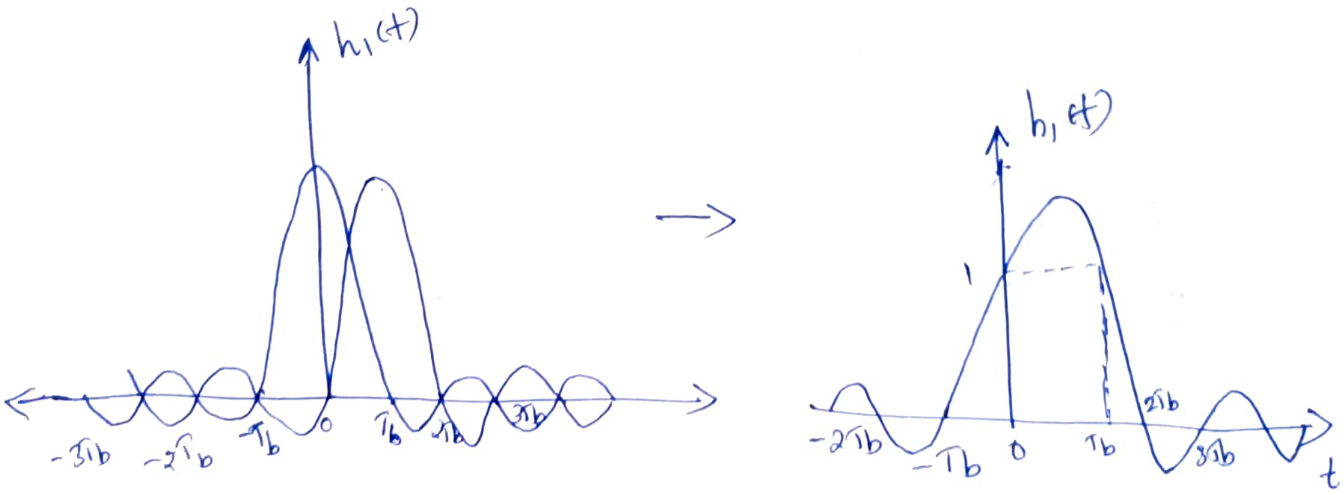


Frequency response of duobinary encoded.

→ Impulse response of duobinary encoded

$$h_1(f) = \int_{-\infty}^{\infty} H_1(f)\, e^{j2\pi f t}\, df$$

$$h_1(t) = \text{sinc}\left(\frac{t}{2T_b}\right) + \text{sinc}\left(\frac{t-T_b}{T_b}\right)$$



$\text{sinc}\,\frac{t}{T_b} + \text{sinc}\left(\frac{t-T_b}{T_b}\right)$

$h_1(t)$

Impulse response of duobinary encoded.

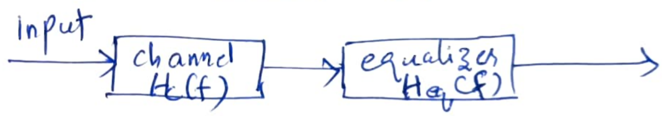
Equalizers

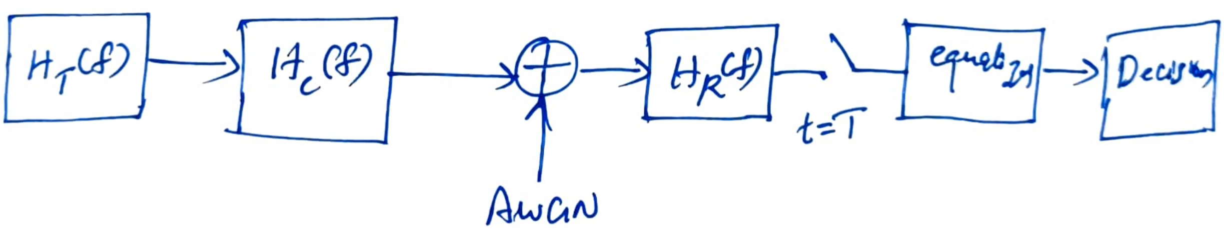
→ In digital communication its purpose is to reduce intersymbol interference to allow recovery of the transmit symbols.

→ we choose a filter which take samples at intervals T and put a digital filter called equalizer at the output to eliminate ISI. as shown

This approach to remove ISI is usually known as equalization.

equaliser Block diagram





→ The Types of equalizers commonly used are

(i) linear equalizer
(ii) Decision feed back equalizer
(iii) Blind equalizer
(iv) Adaptive equalizer
(v) Zero forcing equalizer.

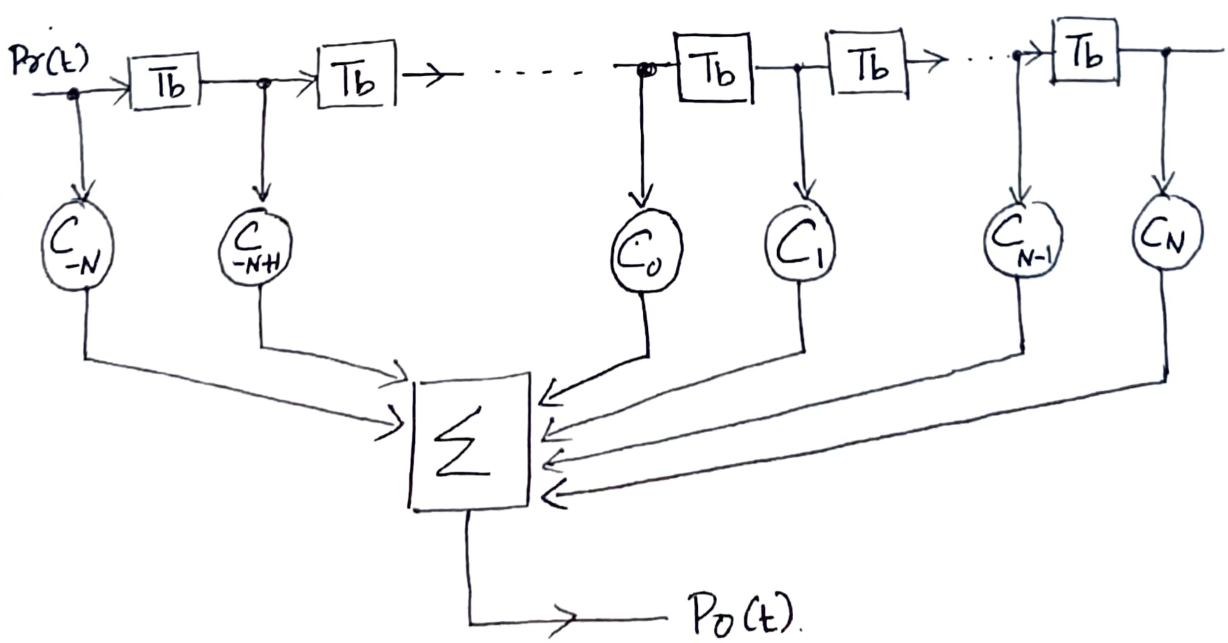
Zero forcing Equalization.

→ When we transmit nyquist criterion pulse for zero ISI through communication channel, then because of channel non-linearities, distortions in the pulse occurs.

→ In other words, pulse which is present at the receiver input is a distorted pulse with non zero values at sampling instants.

→ In order to compensate these distortions at the receiver, to get back the nyquist criterion pulse, we use equalizer.

→ One such equalizer is a <u>Zero-forcing Equalizer</u> as shown as,



Fig(1) ⇒ Zero forcing Equalizer

→ where $P_r(t)$ = input pulse

$P_o(t)$ = output pulse

T_b = Delay Elements

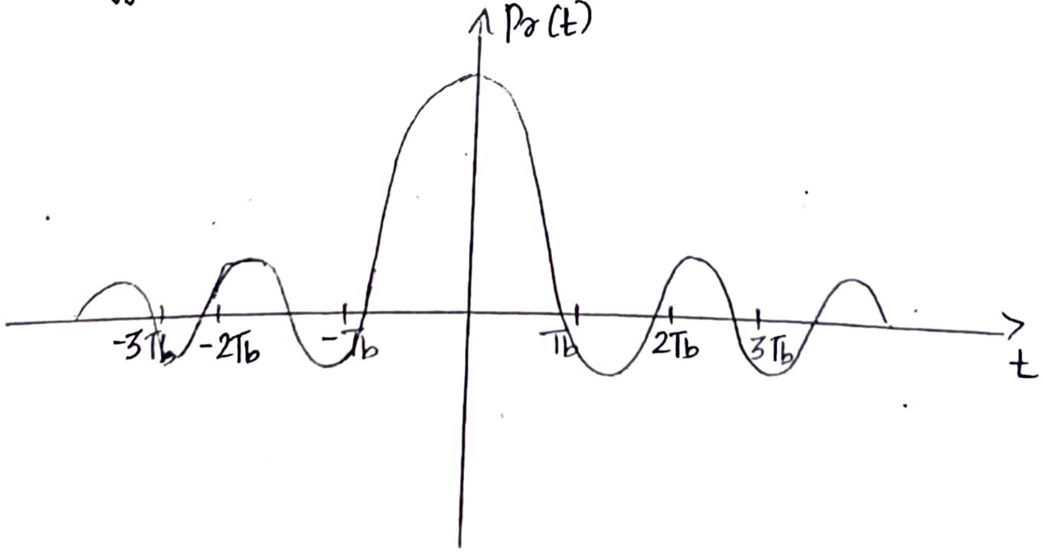
$C_{-N}, C_{-N+1} \cdots C_N$ = Coefficients

→ For zero ISI pulse, we need to select these coefficients in such a way that, the output pulse must satisfy nyquist criteria.

→ The structure shown above is like a tap-delay line filter or FIR filter. It has $-N$ to $+N$ coefficients or taps i.e $2N+1$ taps ⇒ $2N+1$ tap equalizer.

→ Here the number of delay elements are $2N$ while the coefficients are $2N+1$.

→



Fig(2) ⇒ Received Pulse

→ Consider a pulse $P_r(t)$ shown above, we can ~~easily~~ seen that at the sampling instant T_b is not zero, at $2T_b$ is not zero, at $3T_b$ it is not zero & so on... ||ly in negative side also.

→ This implies that, $P_r(t)$ is not a nyquist criterion pulse at the receiver input.

→ So we need to design the equalizer in such a way that, at these sampling instants, the values must be zero. So the effect of ISI can be removed.

→ From the figure (1), the o/p pulse $P_o(t)$ can be written as,

$$P_o(t) = \sum_{n=-N}^{+N} C_n P_r(t - nT_b) \quad \text{———} \quad (1)$$

→ At sampling instant, $t = kT_b$, where k is an integer, where $k = 0, \pm 1, \pm 2 \ldots\ldots$

$$P_o(kT_b) = \sum_{n=-N}^{+N} C_n P_r(kT_b - nT_b) \quad \text{———} \quad (2)$$

→ If $kT_b = k$ & $nT_b = n$, we can rewrite eqn (2) as,

$$P_o(k) = \sum_{n=-N}^{+N} C_n P_r(k - n) \; ; \; k = 0, \pm 1, \pm 2 \ldots \quad \text{———} \quad (3)$$

where n = Tap values [$-N$ to N]

k = Sampling instants

→ The aim is to choose the value of coefficient C_n in such a way that the o/p pulse $P_o(k)$ satisfy the nyquist criteria.

→ The nyquist criteria is defined by,

$$\left.\begin{aligned} P_o(k) &= 1 \quad ; k = 0 \\ &= 0 \quad ; k \neq 0 \end{aligned}\right\} \text{—— (4)}$$

→ Expanding the eqn. (3) for different values of k from $-N$ to $+N$.,

$$P_o(-N) = C_{-N} P_r(0) + C_{-N+1} P_r(-1) + \cdots\cdots + C_N P_r(-2N)$$

$$P_o(-N+1) = C_{-N} P_r(1) + C_{-N+1} P_r(0) + \cdots\cdots + C_N P_r(-2N+1)$$

—— (5)

→ The above eqns.(5), can be written in matrix form as,

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} P_r(0) & P_r(-1) & \cdots & P_r(-2N) \\ P_r(1) & P_r(0) & \cdots & P_r(-2N+1) \\ \vdots & & & \\ \vdots & & & \\ P_r(2N) & \cdots\cdots & \cdots & P_r(0) \end{bmatrix} \begin{bmatrix} C_{-N} \\ C_{-N+1} \\ \vdots \\ C_0 \\ \vdots \\ C_N \end{bmatrix}$$

—— (6)

⇒ Eqn (6) shows P_r matrices, Coefficients matrices & P_o matrices.

⇒ The condition P_o for the given values of P_r, we can easily find out the values of coefficients from C_{-N} to C_N. & then condition of Nyquist criterion will satisfied.

<u>Example</u>

a) Design a three tap zero forcing equalizer if following is given:

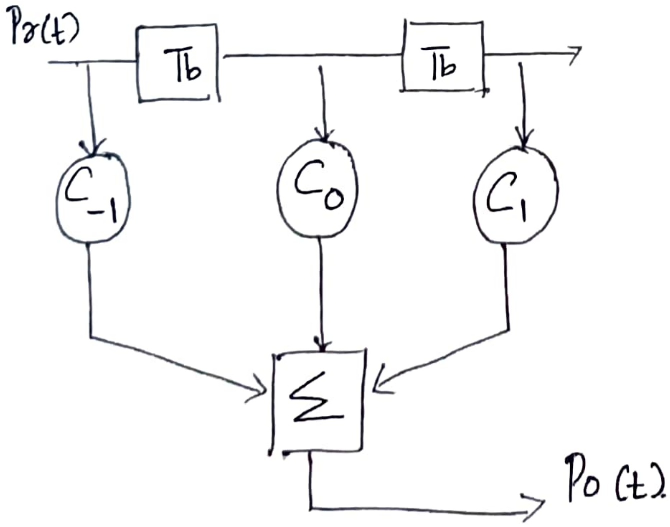
$P_r[0] = 1$ $P_r[2] = 0.1$

$P_r[-1] = -0.2$ $P_r[-2] = 0.05$

$P_r[1] = -0.3$

∴ Here $2N+1 = 3 \Rightarrow N = 1 \Rightarrow 2N \Rightarrow 2$ Delay Elements.

<u>First step</u>: <u>Structure of a 3 tap Equalizer</u>



Step 2 : Put the values or parameters into the matrices of the form,

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} P_r(0) & P_r(-1) & P_r(-2) \\ P_r(1) & P_r(0) & P_r(-1) \\ P_r(2) & P_r(1) & P_r(0) \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix}$$

Putting the values,

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -0.2 & 0.05 \\ -0.3 & 1 & -0.2 \\ 0.1 & -0.3 & .1 \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix}$$

→ Solving these equations :

$$0 = 1 \cdot C_{-1} + -0.2 C_0 + 0.05 C_1$$

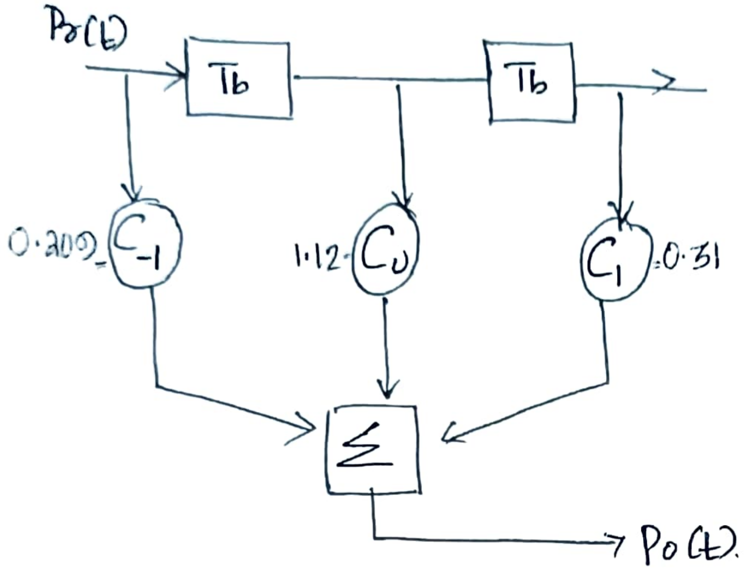
$$1 = -0.3 C_{-1} + 1 C_0 + -0.2 C_1$$

$$0 = 0.1 C_{-1} + -0.3 C_0 + 1 C_1$$

$$\Rightarrow \quad C_{-1} = 0.209$$

$$C_0 = 1.12$$

$$C_1 = 0.31$$



→ The name Zero forcing corresponds to bringing down the intersymbol interference (ISI) to zero.

→ <u>Disadvantages of</u> <u>ZF Equalizers</u>

* The zero-forcing equalizer removes all ISI, is ideal when the channel is noiseless.
* When the channel is noisy, the zero-forcing equalizer will amplify the noise greatly at frequencies f where the channel response $H(j2\pi f)$ has a small magnitude in the attempt to invert the channel completely.

Vector Model of AWGN channel

→ when a signal $s_i(t)$ corresponding to the symbol m_i is transmitted through the channel, noise $w(t)$ gets added to the signal so that the o/p signal from the channel is denoted by $x(t)$.

ie $x(t) = s_i(t) + w(t)$ $\begin{cases} 0 \le t \le T \\ i = 1, 2, 2 \dots m. \end{cases}$

→ Here $w(t)$ is a sample function of a white guassian noise process of zero mean and PSD $N_0/2$.

→ At the received side $x(t)$ becomes the input to the product integrator (correlator)

Now the o/p of the j^{th} correlator is given as

$$x_j = \int_0^T x(t)\, \phi_j(t)\, dt$$

$$= s_{ij} + w_j \qquad j = 1, 2, \dots N$$

where $s_{ij} = \int_0^T s_i(t)\, \phi_j(t)\, dt.$

$w_j = \int_0^T w(t)\, \phi_j(t)\, dt.$

→ In the presence of channel noise we need to check whether $x(t)$ can be completly represented in an N-dimensional space. For that we define a new variable $X'(t)$ whose sample function $x'(t)$ is related to the received signal $x(t)$ as follows.

i.e $$x'(t) = x(t) - \sum_{j=1}^{N} x_j \phi_j(t)$$

$$x'(t) = s_i(t) + w(t) - \sum_{j=1}^{N} (s_{ij} + w_j) \phi_j(t)$$

$$= \cancel{\sum_{j=1}^{N} s_{ij} \phi_j(t)} + w(t) - \cancel{\sum_{j=1}^{N} s_{ij} \phi_j(t)} - \sum_{j=1}^{N} w_j \phi_j(t)$$

$$\left[s_i(t) = \sum_{j=1}^{N} s_{ij} \phi_j(t) \right]$$

$$\therefore x'(t) = w(t) - \sum_{j=1}^{N} w_j \phi_j(t)$$

$$\underline{x'(t) = w'(t)}$$

i.e the sample function $x'(t)$ therefore depends only on the channel noise $w(t)$

$\therefore$ we may express the received signal as

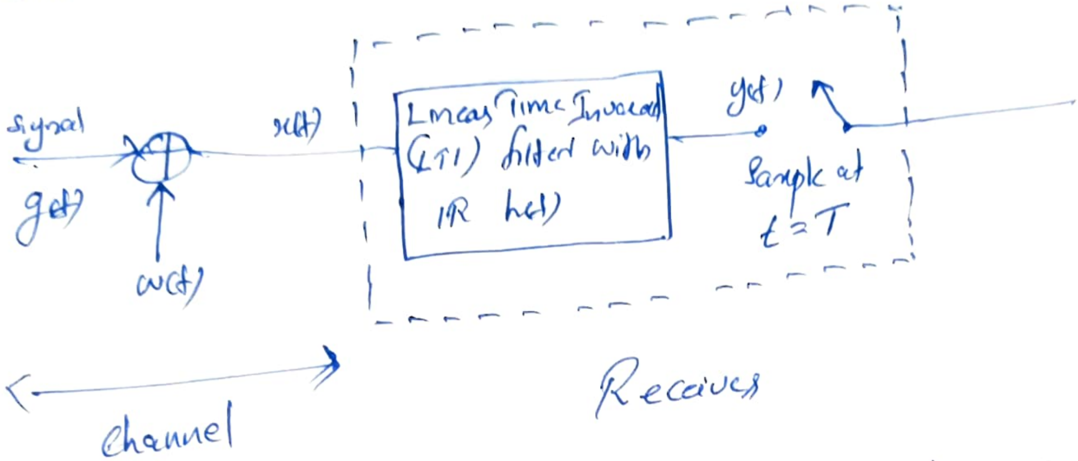
$$x(t) = \sum_{j=1}^{N} x_j \phi_j(t) + x'(t)$$

i.e $$\underline{x(t) = \sum_{j=1}^{N} x_j \phi_j(t) + w'(t)}$$

In the above equations 1st part can be represented in 'N' dimensional vector space and second part $w'(t)$ cannot be represented in N-dimensional space.

Matched filter

Consider a received model, consisting of Linear time invariant filter of impulse response $h(t)$



The filter input $x(t)$ consist of a pulse signal $g(t)$ corrupted by additive channel noise $w(t)$

i.e. $x(t) = g(t) + w(t) \quad 0 \le t \le T$

here the noise signal $w(t)$ is the sample function of a white noise process of zero mean and power spectral density of $N_0/2$

→ The function of the receiver is to detect the pulse signal $g(t)$ in an optimum manner.

Since the filter is linear, the resulting o/p $y(t)$ is $y(t) = g_0(t) + n(t)$

$g_0(t)$ → Signal corresponding to $g(t)$

$n(t)$ → noise component corresponding to $w(t)$..

The requirement of the optimum filter is to make the output signal component $g_o(t)$ considerably greater than the noise component $n(t)$, which is equal to maximizing the peak pulse SNR, which is defined as

$$SNR, \eta = \frac{|g_o(T)|^2}{E[n^2(t)]} \quad \text{——— (1)}$$

where $|g_o(T)|$ is the instantaneous power in the original signal and $E[n^2(t)]$ is a measure of average output noise power.

$$\boxed{P_{signal} = \frac{|g_o(T)|^2}{R} = |g_o(T)|^2 \quad \because R = 1\Omega}$$

~~Properties of Matched filter~~

1. → For a finite energy signal $g(t)$, the impulse response $h_{opt}(t)$ of a matched filter is the time reversed and delayed version of $g(t)$.

ie $h_{opt}(t) = k\, g(T-t)$

→ Frequency response of the matched filter is the complex conjugate of the fourier transform of the input signal, except for the factor $ke^{-j2\pi fT}$

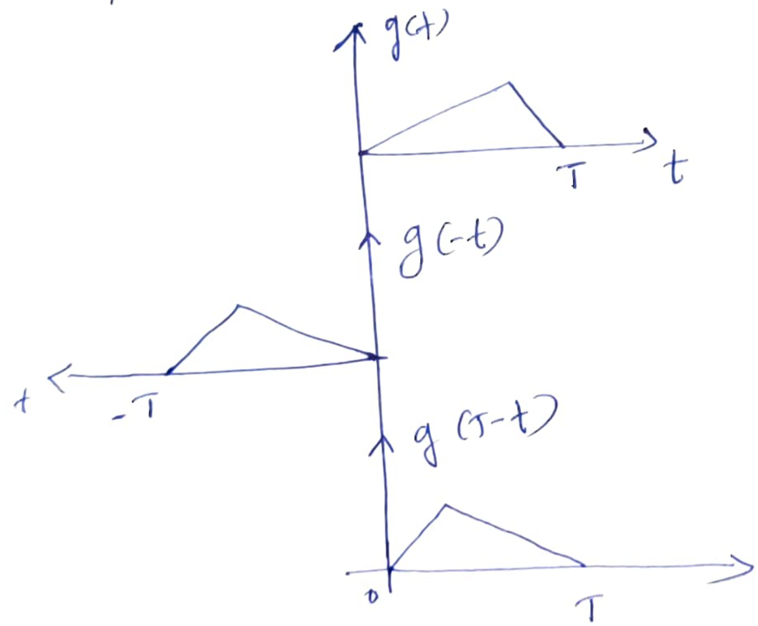
$$H_{opt}(f) = k\, G^*(f)\, e^{-j2\pi fT}$$

→ Since the impulse response of the filter is matched to the input signal, it is known as matched filter.

Properties of Matched filter

→ 1. For the signals corrupted by AWGN, the matched filter maximizes the SNR at its output.

2. The signal and the matched filter ~~imp~~ impulse response are mirror images of each other.



3. The peak pulse SNR of a matched filter depends only on the ratio of the signal energy to PSD of white noise at the filter ~~output~~ input.

$$\eta_{max} = \frac{2E}{N_0}$$

Correlator Receiver

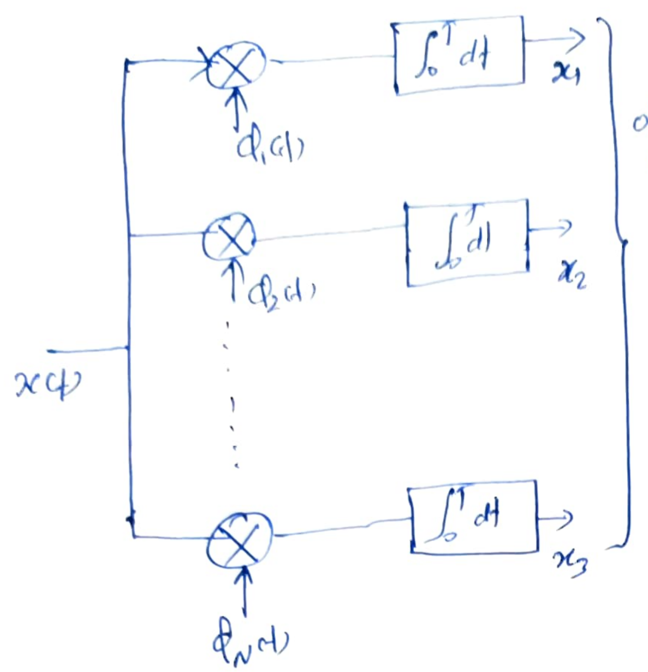
⇒ To find the best estimate of the transmitted symbol, we have to develop the block diagram for the optimum receiver. There are two steps in the decoding process.

1. The Received signal $x(t)$ is converted to the observation vector x by passing the received signal through a bank of 'N' correlators.

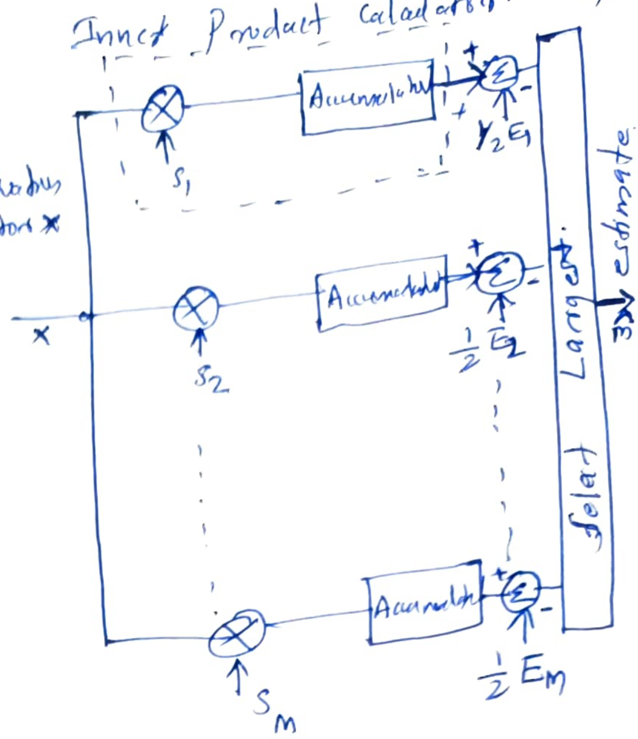
2. The second part of the receiver, which is also called signal transmission decoder computes the Euclidean distance between observation vector 'x' and all 'm' signal vectors and chooses that signal vector closest to the observation vector.

→ The optimum receiver implementing the above two steps is commonly referred to as correlation receiver.

→ Correlation receiver works on ML decision and it consist of two subsystems. Shown below.



Bank of N-correlator



Signal transmission decoder

→ The bank of N correlators is supplied with orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$ that are generated locally. This bank of 'N' correlators operates on the received signal $x(t)$, to produce observation vector $\mathbf{x}$.

→ The signal transmission decoder operates on the observation vector $\mathbf{x}$ to produce an estimate $\hat{m}$ of the transmitted symbol m_i, in a way that would minimize the average probability of symbol error

→ The N elements of the observation vector x are first multiplied by the corresponding 'N' elements of each of the 'M' signal vectors $s_1, s_2, \ldots s_M$ and the resulting product are successively summed in the accumulators to form corresponding set of inner products.

→ Thus the inner product are correlated by subtracting half the energy of the corresponding symbols. Finally the largest in the resulting set of numbers is selected; which may be the best estimate of the transmitted symbol.

Maximum Likelihood Decoding (ML decoding) :-

conditional probability density functions $f_X(X/m_i)$, $i = 1 \cdots M$ are the very characterisation of AWGN channel.

At the receiver, we are given an observation vector 'x' and we have to estimate the message 'm_i'

$$x(t) \rightarrow \boxed{Rxr} \xrightarrow{\hat{m}} \text{estimate of } m_i$$

For that we can introduce the idea of likelihood function.

$$L(m_i) = f_X(X/m_i), \quad i = 1, 2 \cdots\cdots M.$$

In practice, it is more convenient to work with log-likelihood function $l(m_i)$

$$l(m_i) = \log L(m_i)$$

log likelihood function for an AWGN, channel as

$$l(m_i) = \frac{-1}{N_0} \sum_{j=1}^{N} (x_i - s_{ij})^2, \quad i = 1, 2 \cdots M.$$

Given the observation vector x, we make the decision $\hat{m} = m_i$, the probability of error in this decision, may denoted by

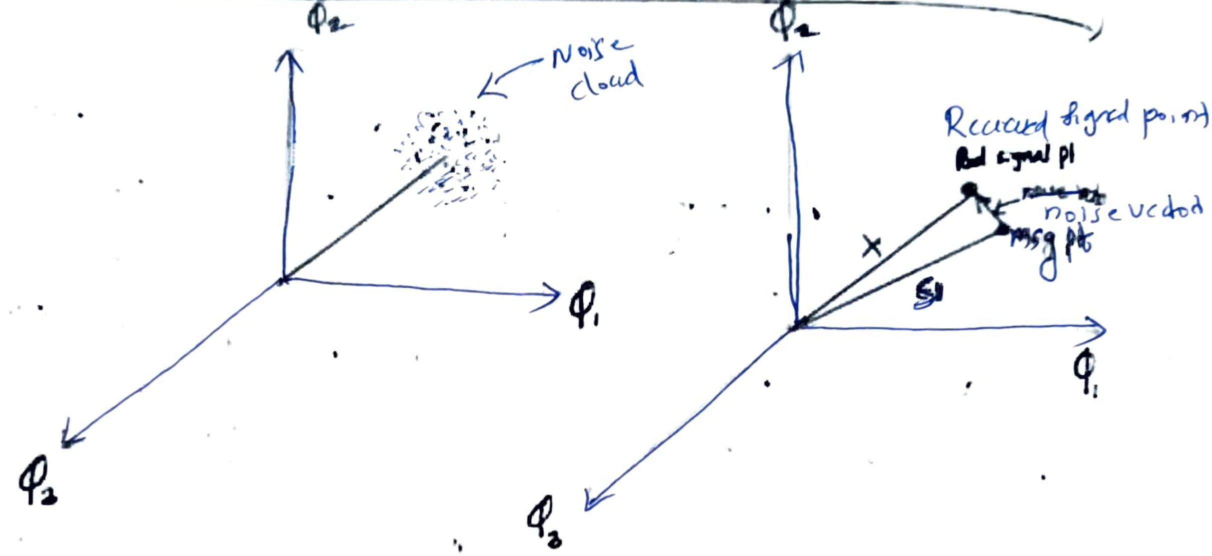
$$P_e(m_i/x) = P(m_i \text{ not set}/x)$$
$$= 1 - P(m_i \text{ sent}/x)$$

The decision making criteria is to minimize the probability of error in mapping. Therefore we can state an optimum decision rule as

MAP rule

Set $\hat{m} = m_i$ if

$$P(m_i \text{ sent}/x) \geq P(m_k \text{ sent}/x) \quad \text{for all } k \neq i$$



This Decision rule is known as <u>Maximum a Posteriori probability (MAP) rule</u>

MAP rule may be expressed in terms of a priori probabilities of the transmitted signal and in terms of likelihood function

we may restate MAP rule as follows

Set $\hat{m} = m_i$ if

$$\frac{p_k \, f_x(x/m_k)}{f_x(x)} \text{ is maximum for } k = i$$

p_k - a priori probability of transmitting symbol m_k

$f_x(x/m_k)$ - Conditional pdf of x given symbol m_k.

$f_X(x) \rightarrow$ un conditional probability ~~of~~ density function of x.

Here
1) $f_X(x) \rightarrow$ independent of transmitted symbol
2) a prion probability $P_k = P_i$ (when all symbols are equally likely)
3) $f_X(X/m_k)$ has one-to-one relationship to log likelihood function

So we can re state the rule as

$$\boxed{\begin{array}{l} \text{Set } \hat{m} = m_i \text{ if} \\ L(m_k) \text{ is maximum for } k = i \end{array}}$$ ML rule

This decision rule is known as Maximum likelihood rule and the device implemented this rule, is known as ML decoder / ML receiver

Here ML decoder computes log-likelihood function for all the M possible message symbol, compares them, and then decides in favor of the maximum"

ML decoder assumes all symbols equally likely. but MAP not

Capacity of an AWGN Channel :-

In an additive white Gaussian Noise (AWGN) Channel. the channel output Y is given by

$$Y = X + n$$ where X is the channel i/p & n is the AWGN with mean = 0 Variance =

The capacity C_s of an AWGN channel is given by

$$\boxed{C_s = \max_{\{p(x)\}} I(X;Y) = B \log_2 (1 + SNR)}$$ b/sec

Shannon- Hartely law.

$I(X;Y)$ = Mutual Information

where the maximization is over all possible input probability distributions $\{p(x_i)\}$.

$$C = B \log_2 \left(1+\frac{S}{N}\right)$$

B = Channel Bandwidth in Hz

SNR = Signal to Noise ratio

- Shannon-Hartely law underscores the fundamental role of Bandw. and SNR in communication.
- It also shows that we can exchange increased bandwidth for decreased signal power for a system with given capacity As C.

→ From the equation we can say that channel capacity is limited by the Bandwidth of the channel & the noise signal.

→ For a noiseless channel, $N=0$ & then channel capacity will be infinite, practically N is always finite, & therefore channel capacity is also finite.

Problem Given an AWGN channel with 4 kHz bandwidth & the noise power spectral density $\eta/2 = 10^{-12}$. The signal power required at the receiver is 0.1 mW. Calculate the capacity of this channel.

Given:

$B = 4000$ Hz.

$S = 0.1 \times 10^{-3}$ W

$N = \eta B = 2 \times 10^{-12} \times 4000 = 8 \times 10^{-9}$ W.

$$\therefore \frac{S}{N} = \frac{0.1 \times 10^{-3}}{8 \times 10^{-9}} = \underline{\underline{1.25 \times 10^{4}}}$$

Now, we have

$$C = B \log \left(1+\frac{S}{N}\right)$$

$$= 4000 \log \left(1+1.25 \times 10^{4}\right)$$

$$C = \underline{54.44} \times 10^{3} \text{ b/s} \quad \text{Ans}$$