APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

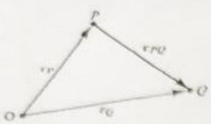
STUDY MATERIALS

KTU ASSIST

MODULE 1

- **Vector Calculus**

Unit Vector: $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$



Position Vector, $\vec{r}_P = x_P\hat{a}_x + y_P\hat{a}_y + z_P\hat{a}_z$

$\vec{r}_Q = x_Q\hat{a}_x + y_Q\hat{a}_y + z_Q\hat{a}_z$

Distance Vector: $\vec{r}_{PQ} = \vec{r}_Q - \vec{r}_P = (x_Q - x_P)\hat{a}_x + (y_Q - y_P)\hat{a}_y + (z_Q - z_P)\hat{a}_z$

Distance: $d_{PQ} = |\vec{r}_{PQ}| = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2}$

	u	v	w	h_1	h_2	h_3
Cartesian	x	y	z	1	1	1
Cylindrical	ρ	$\emptyset$	z	1	ρ	1
Spherical	r	θ	$\emptyset$	1	r	$r\sin\theta$

Differential Length	$d\vec{l} = h_1 du\, \hat{a}_u + h_2 dv\, \hat{a}_v + h_3 dw\, \hat{a}_w$
Differential Surface Area	$d\vec{S}_u = h_2 h_3 dv\, dw\, \hat{a}_u$ $d\vec{S}_v = h_1 h_3 du\, dw\, \hat{a}_v$ $d\vec{S}_w = h_1 h_2 du\, dv\, \hat{a}_w$
Differential Volume	$dV = h_1 h_2 h_3 du\, dv\, dw$
Gradient of a scalar	$\nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial V}{\partial v}\hat{a}_v + \frac{1}{h_3}\frac{\partial V}{\partial w}\hat{a}_w$
Divergence of a scalar	$\nabla . \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(A_u h_2 h_3) + \frac{\partial}{\partial v}(A_v h_1 h_3) + \frac{\partial}{\partial w}(A_w h_1 h_2)\right]$
Curl of a vector	$\nabla \times \vec{A} = \left(\frac{1}{h_1 h_2 h_3}\right)\begin{vmatrix} h_1\hat{a}_u & h_2\hat{a}_v & h_3\hat{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$
Divergence Theorem	$\oint_S \vec{A}.d\vec{S} = \int_v (\nabla . \vec{A}) dv$
Stokes' Theorem	$\oint_L \vec{A}.d\vec{l} = \int_S (\nabla \times \vec{A}) . d\vec{S}$

- **Electrostatics: Coulomb's law**

$$F = \frac{k \cdot Q_1 \cdot Q_2}{d^2}$$

- **Electric Field Intensity** **E = F/q**.

Measure of the strength of an electric field at any point. Defined as the force experienced by a unit positive charge placed at that point. Unit- NC^{-1} or Vm^{-1}.

- **Electric Flux Density**

The amount of flux passing through a defined area that is perpendicular to the direction of the flux.

$$\mathbf{D} = \varepsilon\mathbf{E}$$

Maxwell's Equations

	Point form or Differential Form	Integral Form	
1.	$\nabla . D = \rho_v$	$\oint_S D.dS = \int_v \rho_v dv$	Gauss's Law
2.	$\nabla . B = 0$	$\oint_S B.dS = 0$	Non-existence of isolated magnetic charge
3.	$\nabla \times E = 0$	$\oint_L E.dl = 0$	Conservative nature of electrostatic field
4.	$\nabla \times H = J$	$\oint_L H.dl = \int_S J.dS$	Ampere's Law

✞ Maxwell's equations for time varying fields

	Point form or Differential Form	Integral Form	
1.	$\nabla.D = \rho_v$	$\oint_S D.dS = \int_v \rho_v dv$	Gauss's Law
2.	$\nabla.B = 0$	$\oint_S B.dS = 0$	Non existence of isolated magnetic charge
3.	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_L E.dl = -\int_S \frac{\partial B}{\partial t}.dS$	Faraday's Law
4.	$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_L H.dl = \int_S \left(J + \frac{\partial D}{\partial t}\right).dS$	Modified Ampere's Law

Applications of Gauss's law

✞ Point charge, Q

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\,\mathbf{a}_r$$

✞ Line charge(line charge with uniform charge density ,ρ_l)

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o\rho}\,\mathbf{a}_\rho$$

✞ Infinite sheet (Infinite sheet of charge with uniform charge density, ρs).

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\,\mathbf{a}_n$$

Electric Potential

The potential at any point is the potential difference between that point and a chosen point at which the poential is zero.

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

Relationship between E and V

$$\mathbf{E} = -\nabla V$$

- A vector whose line integral is independent of the path is called a **conservative vector**.
- Electric field intensity is both **conservative and irrotational** (i.e.; curl=0).

Equipotential surface

An equipotential surface (or line) is one on which *V* = constant. The work done to move a charge between any two points =0

Electrostatic energy density

$$w_E = \frac{dW_E}{dv} = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{1}{2}\varepsilon_o E^2 = \frac{D^2}{2\varepsilon_o}$$

Capacitance

✞ Energy stored in a capacitor

$$W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

✞ Capacitance of coaxial cable

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}$$

Poissons and Laplace equation

✞ Poissons equation for heterogeneous medium

$$\nabla\cdot\mathbf{D} = \nabla\cdot\varepsilon\mathbf{E} = \rho_v$$

✞ Poissons equation for homogeneous medium

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

✞ Laplace equation

$$\nabla^2 V = 0$$

Continuity equation

$$\nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t}$$

- **<u>Biot – Savart's law</u>**

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

$$d\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

- **<u>Magnetic Flux Density</u>**

$$\mathbf{B} = \mu_o \mathbf{H}$$

- **<u>Magnetostatic energy density</u>**

$$w_m = \frac{1}{2}\mu H^2 = \frac{1}{2} B \cdot H = \frac{B^2}{2\mu}$$

- **<u>Inductance of coaxial cable</u>**

$$L = L_{\text{in}} + L_{\text{ext}} = \frac{\mu \ell}{2\pi}\left[\frac{1}{4} + \ln\frac{b}{a}\right]$$

- **<u>Inductance of Transmission line</u>**

$$\begin{aligned} L &= 2\,(L_{in} + L_{ext}) \\ &= \frac{\mu \ell}{\pi}\left[\frac{1}{4} + \ln\frac{d-a}{a}\right] \text{H} \end{aligned}$$

- **<u>Scalar and Vector potentials</u>**

The magnetic scalar potential V_m (ampere) $\mathbf{H} = -\nabla V_m$

The vector magnetic potential A (in Wb/m) $\mathbf{B} = \nabla \times \mathbf{A}$

<u>MODULE 2</u>

<u>Boundary conditions for eletric field (dieletric-dieletric interface)</u>

1. $E_{1t} = E_{2t}$
2. $\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$
3. $D_{1n} - D_{2n} = \rho_s$. If the region is charge free, $D_{1n} = D_{2n}$
4. $\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$

If θ_1 and θ_2 are the angle made by the electric field with the normal, then

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

<u>Boundary conditions for eletric field (dieletric-conductor interface)</u>

1. $E_{1t} = 0$
2. $D_{1t} = 0$
3. $D_{1n} = \rho_s$. If the region is charge free, $D_{1n} = 0$
4. $\varepsilon_1 E_{1n} = \rho_s$

Boundary conditions for Magnetic field

1. $B_{1n} = B_{2n}$
2. $\mu_1 H_{1n} = \mu_2 H_{2n}$
3. $H_{1t} - H_{2t} = K$. If the region is charge free, $H_{1t} = H_{2t}$
4. $\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$. If the region is charge free, $\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$

If θ_1 and θ_2 are the angle made by the electric field with the normal, then

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}}$$

EM WAVE PROPAGATION :

1. Free space ($\sigma = 0, \varepsilon = \varepsilon_o, \mu = \mu_o$)
2. Lossless dielectrics ($\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \ll \omega\varepsilon$)
3. Lossy dielectrics ($\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$)
4. Good conductors ($\sigma \simeq \infty, \varepsilon = \varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \gg \omega\varepsilon$)

$$\boxed{\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k}$$

IN LOSSY DIELECTRICS :

A lossy dielectric is a partially conducting medium in which an EM wave losses power as it propagates due to poor conduction

1. Wave Equations (Helmholtz's Equation)

$$\nabla^2 E_s - \gamma^2 E_s = 0$$
$$\nabla^2 H_s - \gamma^2 H_s = 0$$

2. Propagation Constant, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ (Unit – m^{-1})
3. Attenuation Constant, $\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma^2}{\omega^2\varepsilon^2}\right)} - 1\right]}$ (Unit – Np/m)
4. Phase Constant, $\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma^2}{\omega^2\varepsilon^2}\right)} + 1\right]}$ (Unit – rad/m)
5. Wave propagating in positive z-direction

$$\vec{E}(z,t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\ \hat{a}_x$$

Intrinsic Impedance

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle \theta_\eta = |\eta| e^{j\theta_\eta}$$

Phase constant - The no. of wavelengths required to produce a phase shift of 2pi

$$\boxed{\beta = \frac{2\pi}{\lambda}}$$

Wave velocity or **phase velocity** is the speed at which the phase of a wave travels.

$$u = \frac{\omega}{\beta},$$

Loss tangent -To determine how lossy a medium is-

$$\boxed{\tan\theta = \frac{\sigma}{\omega\varepsilon}}$$

Loss angle of the medium,

$$\theta = 2\theta_\eta$$

<u>IN LOSSLESS DIELECTRICS</u> :

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu\varepsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \qquad \lambda = \frac{2\pi}{\beta}$$

Also

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\,\angle 0°$$

and thus **E** and **H** are in time phase with each other.

<u>IN FREE SPACE :</u>

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c}$$

$$u = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c, \qquad \lambda = \frac{2\pi}{\beta}$$

$$\boxed{\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \simeq 377\ \Omega}$$

$$\mathbf{E} = E_o\cos(\omega t - \beta z)\,\mathbf{a}_x$$

$$\mathbf{H} = H_o\cos(\omega t - \beta z)\,\mathbf{a}_y = \frac{E_o}{\eta_o}\cos(\omega t - \beta z)\,\mathbf{a}_y$$

<u>IN GOOD CONDUCTORS :</u>

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu \sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}, \qquad \lambda = \frac{2\pi}{\beta}$$

Also,

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

and thus **E** leads **H** by 45°. If

$$\mathbf{E} = E_o e^{-\alpha z} \cos(\omega t - \beta z)\, \mathbf{a}_x$$

Skin depth or Depth of Penetration: The distance at which the amplitude of electric field reaches 36.8% of the maximum amplitude (0.368Eo).

Skin effect - The phenomenon whereby field intensity in a conductor rapidly decreases. Application-electromagnetic shielding.

$$E_o e^{-\alpha\delta} = E_o e^{-1}$$

$$\boxed{\delta = \frac{1}{\alpha}} \qquad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

MODULE 3

- **Poynting theorem**- used to determine the power carried by an electromagnetic wave.
- Power vector=E×H
- **Polarisation.**- Defined as the orientation of the dipole in the direction of electric field/ dipole moment per unit volume.
- **Brewsters angle** - It is the ratio of refractive index of the second medium to that of first medium or the angle of incidence at which there is no reflection ie Ero=0

*The wave transmitted at the Brewsters angle -completely transmitted without reflection.

*reflection coefficient=0 and transmission coefficient =1 *

Brewster angle is valid for parallel polarisation.

- **Snells law** -In an oblique medium,the product of the refractive index and sine of incidence Angle in medium 1 is same as that of medium 2.

$$\frac{\sin\theta i}{\sin\theta t} = \frac{n2}{n1}$$

• Polarisation of an EM	E(x,y) =Exo Cos(wt-βz+φx)ax +Eyo Cos(wt-βz+φy)ay	
1. Linear Polarisation	E(0,t) = Exo coswt ax +Eyo coswt ay	Φp = –+nπ where n=0,1,2,…
2.Circular	E(0,t) = Exo coswt ax -Eyo sinwt ay	Exo = Eyo = Eo Φp=+–(2n+1)π/2 n=0,1,2,…
3.Eliptical	E(0,t) = Exo coswt ax -Eyo sinwt ay	Φp=+–(2n+1)π/2 n=0,1,2,…
• Reflection Coefficient (Γ)	Γ=Ero/Eio = $\eta_2 - \eta_1 / \eta_2 + \eta_1$	
• Transmission Coefficient (τ)	τ = Eto /Eio = 2 $\eta_2 / \eta_2 + \eta_2$	
-Parallel Polarisation 1.Reflection coefficient (Γ) 2·Transmission coefficient (τ)	**Γ**= η2cosθt–η1cosθi / η2cosθt+η1cosθi **τ**=2 η2cosθi / η2cosθt+η1cosθi	
Brewster's Angle	Sin$\theta_{B\|\|}$ =(ε2/ε1+ε2)^1/2	

MODULE 4

Lumped Elements	**Distributed Elements**
concentrated on single point Eg:R,L,C	spread over large distance Eg:Transmission line

Transmission Line Parameters

Primary Parameter	**Secondary Parameter**
Resistance:uniformly distributed along length	**Attenuation constant**(neper/m)
Capacitance:associated with electric charges	**Phase constant**(rad/m)
Inductance:associated with flux linkage	**Propagation constant**
Conductance:leakage current	

Characteristic Impedance

$$\frac{V^+}{I^+} = Z_0 = -\frac{V^-}{I^-}$$

Lossless Transmission Line

- **c**onductors are ideal or perfect ,R=*0*
- dielectric medium is lossless ,*G=0*

ɼ=jw$\sqrt{LC}$

$\alpha = 0 \quad ß = w\ \sqrt{LC}$

$z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$

$u_p = \frac{w}{ß}$

Distortionless Transmission Line

- Attenuation is independent of frequency ɼ $= \sqrt{RG} + jw\sqrt{LC}$

$\alpha = \sqrt{RG}$ $\quad$ $\beta = w\sqrt{LC}$

$$z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$u_p = \frac{w}{\beta}$$

□ = propagation constant

α = attenuation constant

β= phase constant

Input Impedance of a transmission line

$$z_{in} = z_o \left(\frac{zL + zotanhrl}{zo + zLtanhrl}\right)$$

For a lossless transmission line

$$z_{in} = z_o \left(\frac{zL + jzotan\beta l}{zo + jzLtan\beta l}\right)$$

z_o = characteristic impedence

$z_{L\,=}$ load impedence

l = length of transmission line

$R= \alpha z_o$	$C=1/uz_o$
$G=\alpha/z_o$	$L=z_o/u$

Voltage reflection coefficient

- Reflection coefficient denotes the amount of voltage reflected by load
- It is also defined as ratio of reflected voltage to incident voltage

$$\Gamma_{L.} = \frac{ZL - Z0}{ZL + Z0}$$

Current reflection coefficient

It is the negative of voltage reflection coefficient, - Γ_L.

Voltage standing wave ratio (s)

- Standing wave is a wave which doesn't propagate ie, it is appeared as stationary.
- Standing ratio defined as maximum voltage to minimum voltage .

$$s = \frac{|Vi| + |Vr|}{|Vi| - |Vr|}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|}$$

- $0 \leq |\Gamma_L| \leq 1$ • $1 \leq s \leq \infty$

MODULE V

Transmission line as circuit elements (L and C)

- **Characteristic Impedence,** $Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$

Different cases:

1. **Short circuited transmission line($Z_L = 0$)**

$\Gamma = -1 \rightarrow s = \infty$

- $Z_{sc} = Z_{in} = jZ_0 \tan\beta l$ $\Rightarrow Z_{sc} = jX_L l$, *for short lengths*

A short-circuited line acts as a pure <u>inductor</u> for very short length.

$$l_{sc} = \frac{1}{\beta} tan^{-1}\left[\frac{X_L}{Z_0}\right]$$

2. <u>Open circuited transmission line($Z_L = \infty$)</u>

$\Gamma = 1 \rightarrow s = \infty$

- $\mathbf{Z_{oc} = -jZ_0 cot\beta l}$ $\Rightarrow Z_{oc} = (-jX_c/l), for\ short\ lengths$

An open-circuited line acts as a pure <u>capacitor</u> for very short length.

$$l_{oc} = \frac{1}{\beta} cot^{-1}\left[\frac{X_c}{Z_0}\right]$$

$$\boxed{Z_{sc}Z_{oc} = Z_o^2}$$

3. <u>Quarter wave transmission line</u>

Length of the transmission line is $\lambda/4$ $Z_{in} = \frac{Z_0^2}{Z_L}$

Quarter wave transmission line is called an **impedence transformer** - it transforms a low impedence to high impedence or high impedence to low impedence.

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4. <u>Half wave transmission line</u>

Length of the transmission line is $\lambda/2$

$$Z_{in} = Z_L$$

Stub Matching

- Stub is a section of transmission line which is either open circuited or short circuited.
- It is connected only at one end.
- Stub can be connected series or parallel.
- In single stub matching we prefer short circuited parallel stubs.
- Total input impedence, $y_{in} = y_{line} + y_{stub}$
- $y_{line} = 1 \pm jb$
- $y_{stub} = \pm jb$

Smith Chart - Frequency domain plot.

- It is the polar chart of the reflection coefficient R with respect to the normalised impedance Z_{norm}
- Used for calculating- reflection coefficient ,standing wave ratio for normalised load impedance
- The Smith chart consists of the constant resistance circles and the constant reactance circles.
- The midpoint of the Smith Chart-(1,0) point.-The resistance is unity and reactance is zero at this point.
- On moving towards the clockwise direction - traversing towards the generator.

MODULE 6 – WAVEGUIDES

- The phenomenon employed in waveguide operation – **Total Internal Reflection.**
- The waveguides are made of materials with **low bulk resistivity**
- Waveguides - carry electromagnetic waves in the **GHz** range.
- The condition for the dimensions of rectangular waveguide: **a>b**
    - a -the broad wall dimensions

- b-side wall dimensions

Transverse Electric Waves (TE Waves)	Transverse Magnetic Waves (TM Waves)
Electric field is always perpendicular to the direction of propagation	Magnetic field is always perpendicular to the direction of propagation
(or) Magnetic field is present along direction of propagation	(or) Electric field is present along direction of propagation
Also called as H-waves	Also called as E-waves
$E_z = 0 \ and \ H_z \neq 0$	$H_z = 0 \ and \ E_z \neq 0$

- **The dominant mode** is the mode which has the minimum frequency or maximum wavelength available for propagation of the waves.
- The modes TM_{mo} and TM_{on} does not exist. These modes are said to be **evanescent mode.**
- **Degenerate modes-** Two modes with same cut off frequency are called as these modes have same field distribution. Eg : TE_{mn} and TM_{mn}

TE Waves	TM Waves
TE_{10} is the dominant mode in the rectangular waveguide.	The TM_{11} mode is the dominant mode . The modes TM_{10}, TM_{01} and TM_{20} does not exist

Properties of TE &TM Waves

- **Cut-off frequency**: Frequency below which attenuation occurs and above which propagation takes place. Waveguide operates as a **high pass filter.**

$$\boxed{f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$ $$u' = \text{Phase velocity of wave} = \frac{1}{\sqrt{\mu\varepsilon}}$$

- **Cut-off Wavelength**

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}}$$

- **Phase constant**

$$\boxed{\beta = \beta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\beta' = \omega\sqrt{\mu\varepsilon} = Phase\ constant\ of\ wave\ without\ any\ waveguide$$

- **Intrinsic Impedence**

$$\boxed{\eta_{TM} = \eta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$ $$\boxed{\eta_{TE} = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

$$\boxed{\eta_{TE}\eta_{TM} = \eta'^2}$$

- **Phase velocity (up)** : The velocity with which the loci of constant phase are propagated down the waveguide.

$$\boxed{u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

- **Group velocity (ug)** : The velocity with which the resultant repeated reflected waves are travelling down the waveguide.

$$\boxed{u_g = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

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- Relation b/w group and phase velocities

- The phase velocity is always greater than the speed of light in waveguides.
- This implies the group velocity is small.

Guide Wavelength (λg) : Periodicity of wave along the waveguide is defined by the parameter guide wavelength.It is defined as the wavelength along the axis of the guide.

$$\boxed{\lambda_g = \frac{\lambda'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}} \qquad \boxed{\frac{1}{\lambda'^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}}$$

- Average power lost per unit length is equal to rate of decrease of average power per unit distance for energy to be conserved.

$$Power\ lost, P_L = -\frac{dP_{avg}}{dz} = 2\alpha P_o e^{-2\alpha z} = 2\alpha P_{avg}$$

$$\alpha = \frac{Power\ lost\ per\ unit\ length}{2\ \times Power\ transmitted}$$

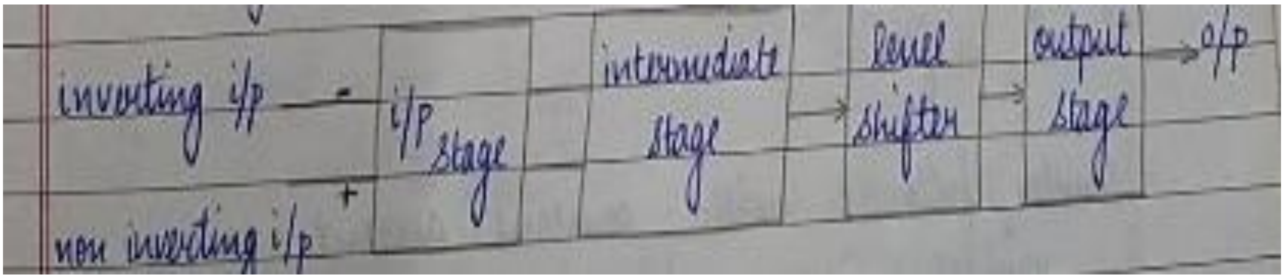
ANALOG INTEGRATED CIRCUITS

MODULE 1

OPERATIONAL AMPLIFIERS:-

- are used for mathematical operations like addition, subtraction, integration, differentiation, log, antilog,
- linear/analog IC
- multistage, direct coupled, high gain amplifier
- high input impedance (ideally infinite)
- low output impedance (ideally zero)
- can amplify the signals from the range 0Hz-1MHz

BLOCK DIAGRAM



- Input stage and intermediate stage -> differential amplifier ->amplifies the difference of the input signals
- Input stage:-
    1. Dual input
    2. Balanced output
    3. Provides high input impedance
    4. High gain

- Intermediate Stage
    1. Dual input
    2. Unbalanced output
    3. Input stage is directly coupled to intermediate stage. So the dc component gets added to it. Hence it shifts the reference voltage . to bring it back to original position, a level shifter is used.
- Level shifter
    1. Emitter follower with constant current source
- Output Stage
    1. Uses complementary symmetry push pull class AB amplifier
    2. Provides low output impedance

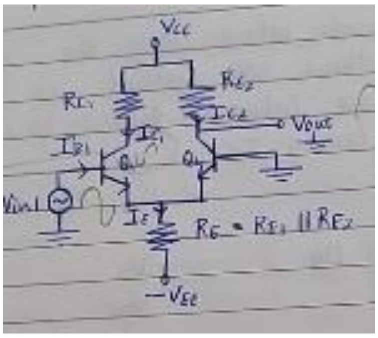
DIFFERENTIAL AMPLIFIER

- Amplifies the difference of the 2 inputs

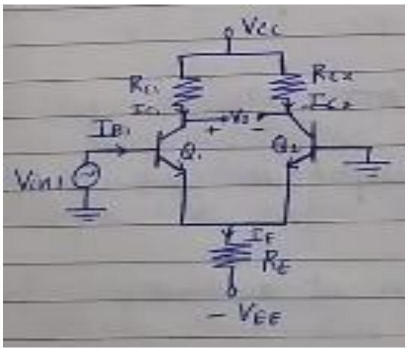
- Has 2 inputs and 2 outputs where either the 2 terminals can be used or one of them is connected w.r.t ground
- Major component of linear IC

Configuration of Differential Amplifier using BJT

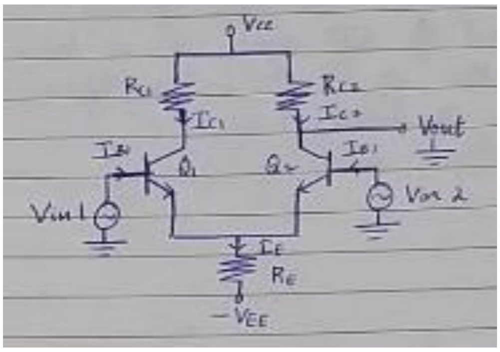
- Single input unbalanced output



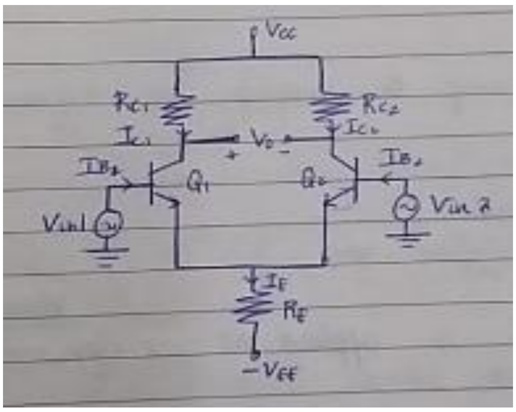
- Single input balanced output



- Dual input unbalanced output



- Dual input balanced output



- There are 2 types of gain: -
  1. Differential mode gain (Adm)
     - Ideally ∞
     - Ability of the differential amplifier to amplify the difference between the 2 inputs.

       Adm= Vo/(V1-V2)=Vo/Vid
     - Ideal value of Adm is ∞

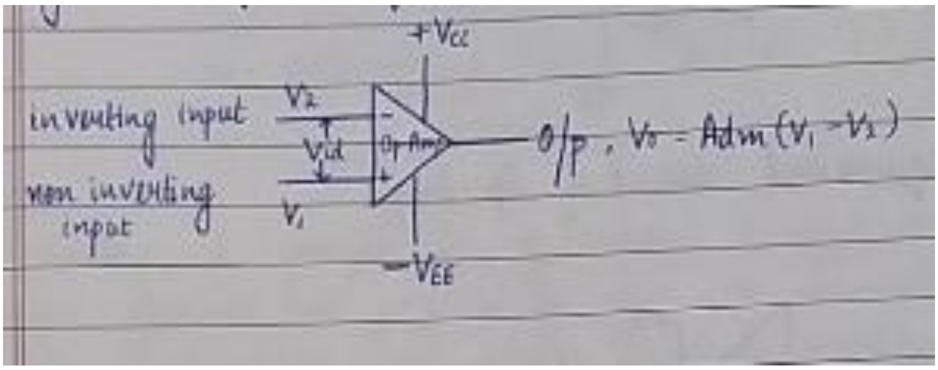
  2. Common Mode gain (Acm)
     - Vcm= (V1+V2)/2
     - Acm=Vo/Vcm
     - Vo=Adm*Vid+Acm*Vcm
     - Ideal value of Acm is 0

Common Mode Regulation Ratio

- Ability of a differential amplifier to reject common mode signals
- Ratio of differential mode gain to common mode gain

  CMRR=Adm/Acm
- Ideally it should be ∞ , practically it should be very high

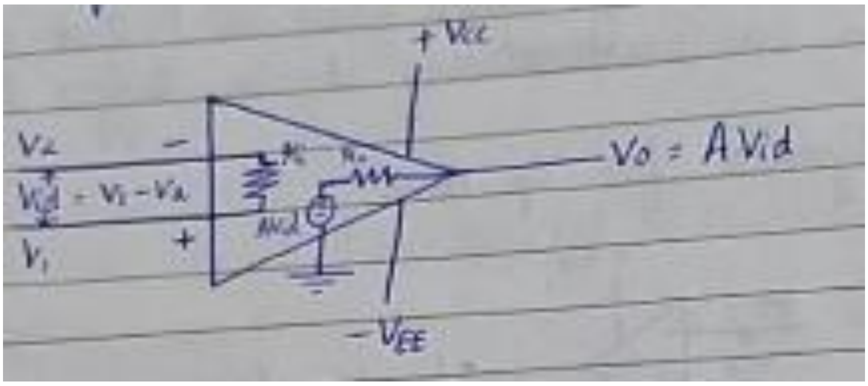
General symbol for Op Amp



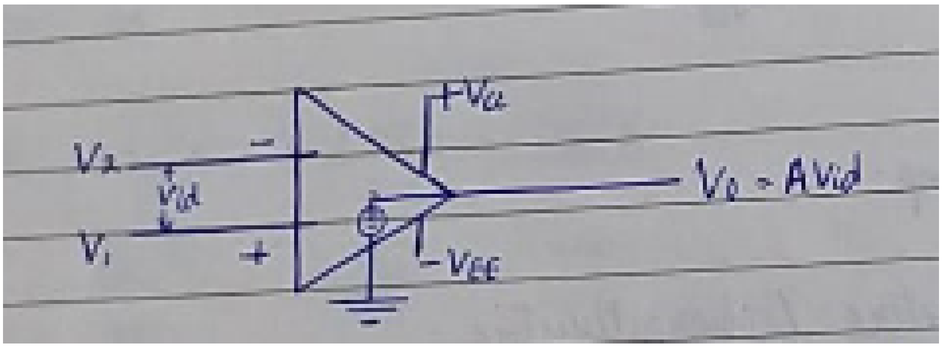
Ideal Parameters of Op Amp

- Open loop gain: ∞
- Input resistance: ∞
- Output resistance: 0
- Bandwidth: ∞
- CMRR: ∞
- Offset voltage: 0
- Slew rate: ∞

Practical Equivalent circuit of Op Amp



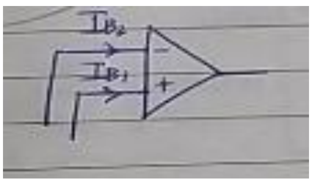
Ideal Equivalent circuit of Op Amp



DC non ideal characteristics of Op Amp

1. Input offset voltage
2. Input bias current
3. Input offset current

1. Input offset voltage:- voltage applied to the input terminals of op-amp to nullify the output voltage
2. Input bias current :- average of the 2 input current flowing into op-amp

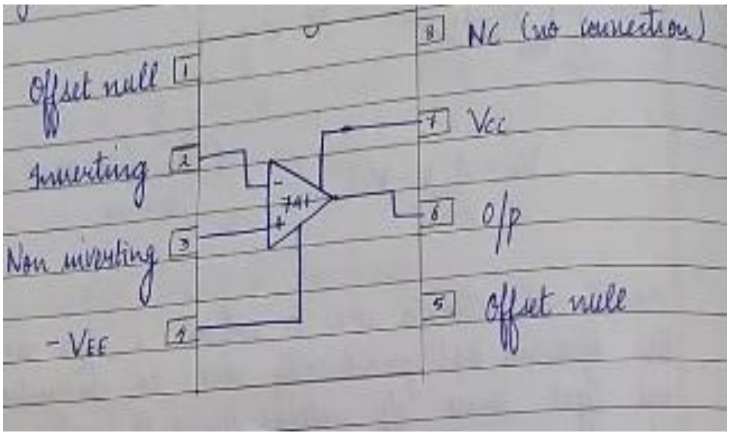


$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

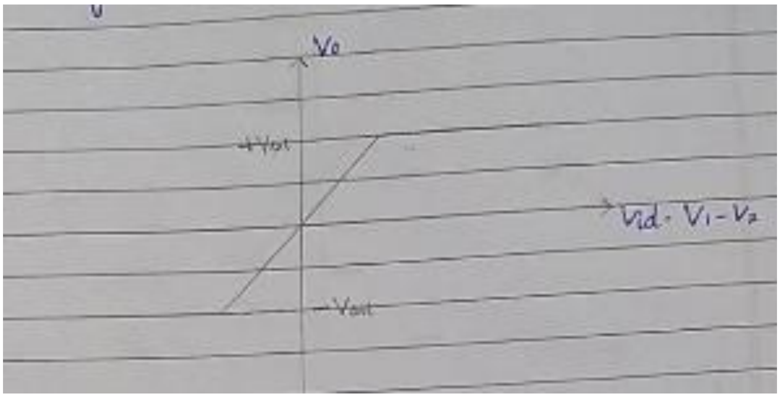
3. Input offset current:- magnitude of the difference between 2 inppt currents to the op-amp

$$I_{ios} = |I_{B1} - I_{B2}|$$

PIN DIAGRAM OF IC 741



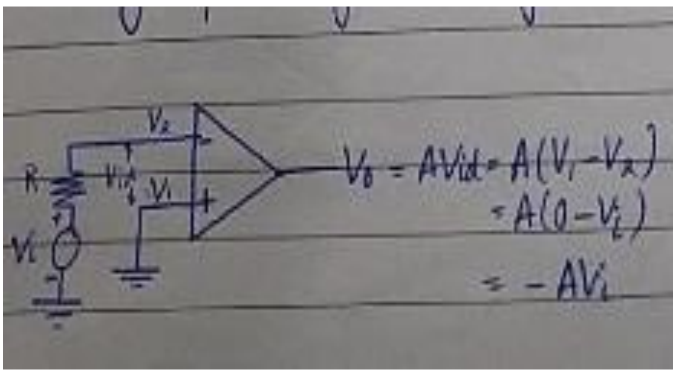
VOLTAGE TRANSFER CHARACTERSITICS OF OP-AMP

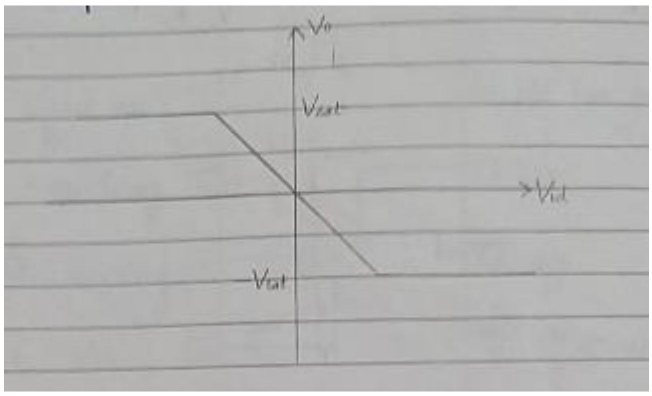


OPEN LOOP CONFIGURATION OF OP-AMP

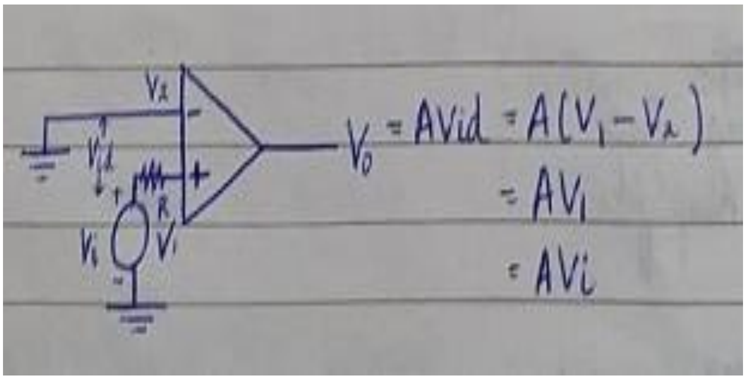
1. INVERTING AMPLIFIER

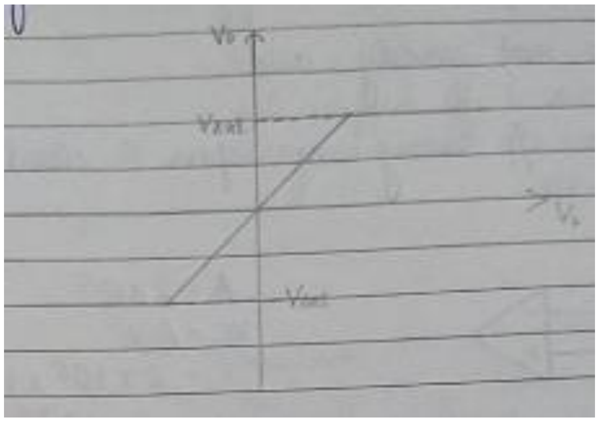
   An input voltage is applied to the inverting input terminal and the non inverting terminal is given to the grounded



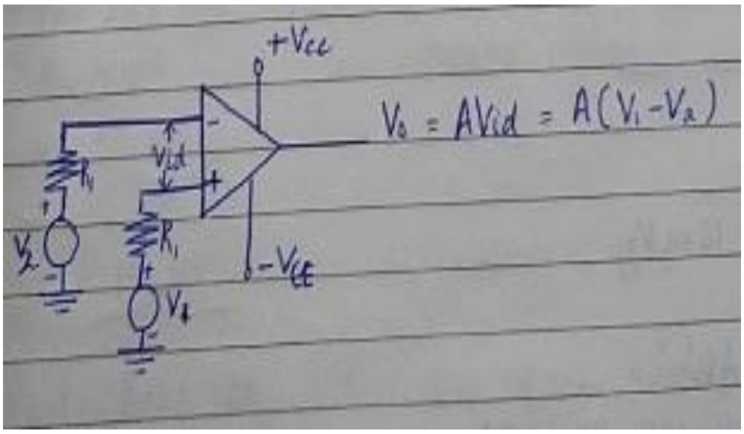


.2. Non inverting amplifier





.3. Differential amplifier



SLEW RATE

Maximum rate of change of output of an op-amp with respect to time ,i.e. maximum rate of change of output voltage w.r.t time

SR=dVout/dt

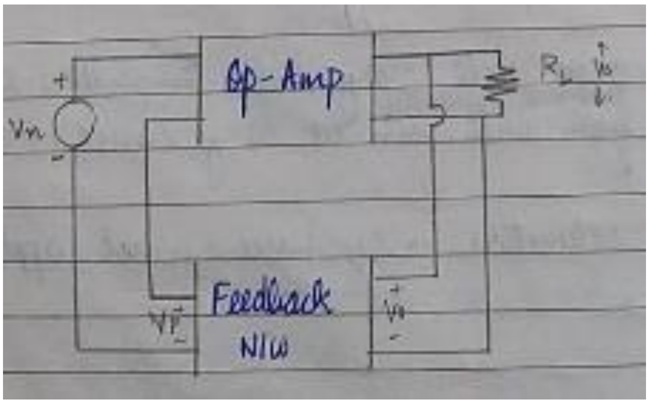
- as time decreases, slew rate increases and vice versa
- slew rate is less-> ouput shape will be distorted

Reasons for slew rate:-

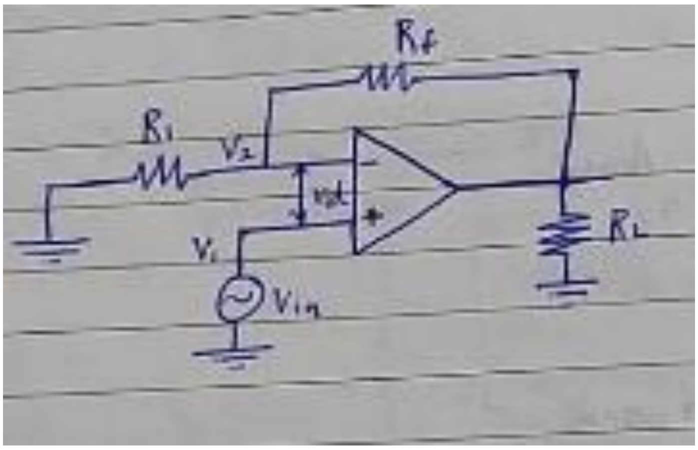
- internal compensation capacitor
- used to enhance stability of op-amp at high frequencies

FEEDBACK CONFIGURATION

- Voltage series



Voltage series feedback configuration/ Closed loop non inverting amplifier



Open loop gain, $A = \frac{V_o}{V_{id}} \rightarrow (1)$

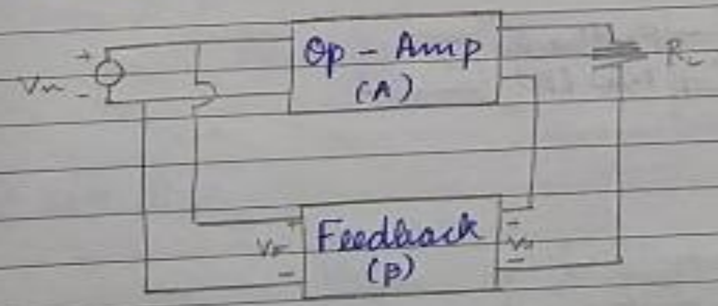
Closed loop gain, $A_f = \frac{V_o}{V_{in}} \rightarrow (2)$

Gain of feedback network, $\beta = \frac{V_f}{V_o} \rightarrow (3)$

$A_F = \frac{-Ak}{1+A\beta}$ → gain for inverting amplifier

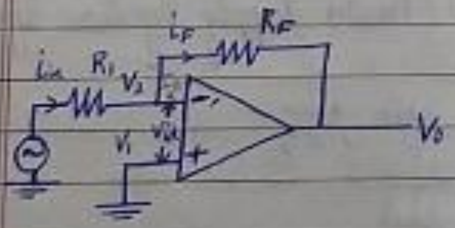
-ve sign → o/p is 180 ~~out of~~ phase shift with i/p. there is a phase reversal of 180°

$A_F = \frac{A}{1+A\beta}$ → gain for non inverting amplifier



Virtual Ground Concept in inverting amplifier configuration:

only for ⟶



Ideally the value of differential i/p voltage, $V_{id} = 0$

$$V_{id} = V_1 - V_2 = 0$$

$$V_1 = V_2$$

But $V_1 = 0$ as it is connected to ground.

$\therefore V_2 = 0$; V_2 acts as ground

This is virtual ground.

Gain equation using virtual ground concept

$$A_F = \frac{V_0}{V_{in}} = -\frac{R_F}{R_1}$$

Supply Voltage Rejection Ratio (SVRR) / Power Supply Rejection Ratio (PSRR):

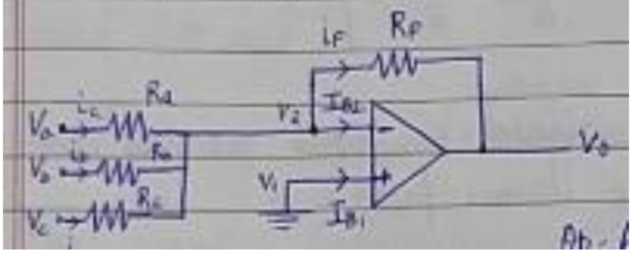
$$SVRR = \frac{\Delta V_{io}}{\Delta V} \qquad \text{unit} = \mu V/V$$

Variation in i/p offset voltage in op-amp caused by the variation in supply ~~vo~~ voltage.

Applications of op-amp

1. Summing, scaling and averaging

   Inverting configuration



Scaling amplifier :
(or weighing " ")

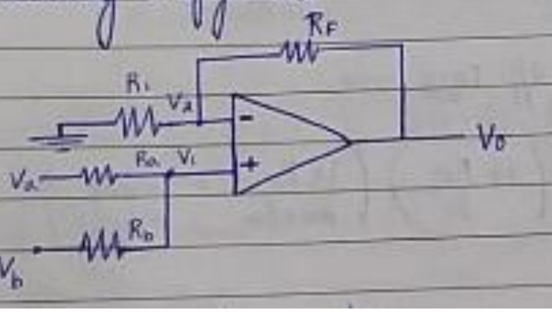
$$V_0 = -\left[\frac{R_F}{R_a}V_a + \frac{R_F}{R_b}V_b + \frac{R_F}{R_c}V_c\right]$$

$$V_0 = -\left[V_a + V_b + V_c\right]$$, hence acts as a summing amplifier

• Averaging amplifier :

$$V_0 = -\frac{R_F}{R}\left[V_a + V_b + V_c\right]$$

▸ Non inverting Configuration



$$V_0 = \left(1 + \frac{R_F}{R_1}\right)\left(\frac{V_a R_b}{R_a + R_b} + \frac{V_b R_a}{R_a + R_b}\right)$$

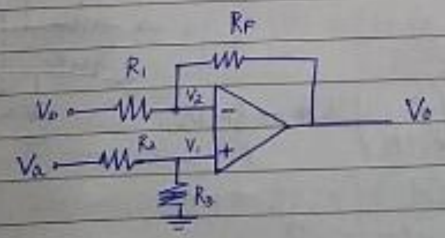
$$V_0 = \left(1 + \frac{R_F}{R_1}\right)\left(\frac{V_a + V_b}{2}\right)$$

This will act as an averaging amplifier

If $\left(1 + \frac{R_F}{R_1}\right) = 2$, the circuit will act as a summing amplifier

Differential configuration

Differential amplifier with one op-amp (subtractor):

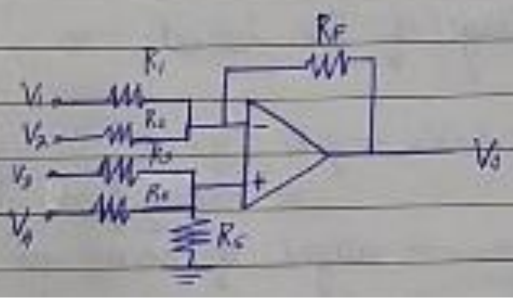


$$\boxed{V_o = \frac{R_F}{R}(V_a - V_b)}$$

Gain of differential amplifier $= \frac{R_F}{R}$

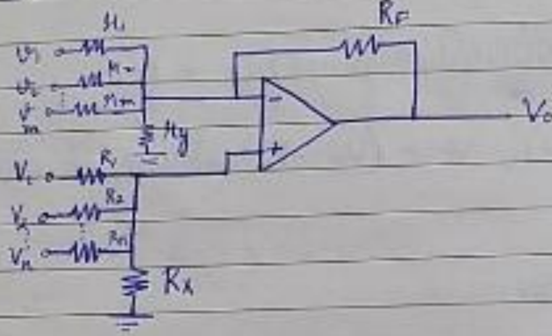
If $\frac{R_F}{R} = 1$, $V_o = (V_a - V_b)$. Hence it will act as a subtractor

Adder - Subtractor:



$= -V_1 - V_2 + V_3 + V_4$

Design for adder subtractor circuit:



Given that o/p voltage of the circuit →

$$V_o = x_1V_1 + x_2V_2 + x_nV_n - y_1v_1 - y_2v_2 \cdots - y_mv_m$$

or scaling factor

$x_1, x_2, \cdots x_n$ → weight of non inv. i/ps

$y_1, y_2, \cdots y_m$ → " " inv. i/p

$$Z = X - Y - 1$$

$X \rightarrow x_1 + x_2 + \cdots x_n$

$y \rightarrow y_1 + y_2 + \cdots y_m$

Case 1 [If $x > 0$]:

$R_x = \infty$ (no need; open)

$r_y = \frac{R_F}{x}$

$R_i = \frac{R_F}{x_i}$

$r_j = \frac{R_F}{y_j}$

Case 2 [If $x < 0$]:

$R_x = \frac{-R_F}{x}$ (-ve bcz x will be -ve)

$r_y = \infty$ (no need)

$R_i = \frac{R_F}{x_i}$

$r_j = \frac{R_F}{y_j}$

Case 3 [If $x = 0$]

$R_x = \infty$

$r_y = \infty$

$R_i = \frac{R_F}{x_i}$

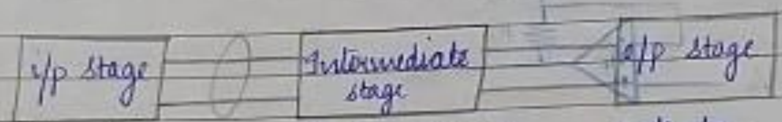
$r_j = \frac{R_F}{y_j}$

Instrumentation amplifier

Features:

1. High input impedance
2. Low output impedance
3. Very high CMRR
4. Accurate & stable gain

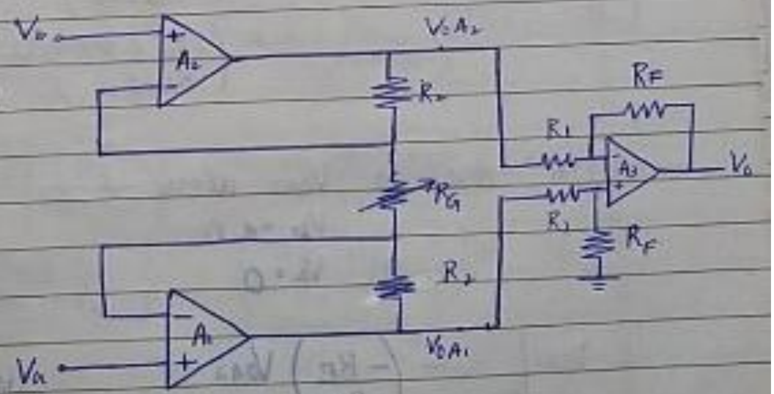
- Amplify signals & removes noise



Transducer + Pre amplifier — Instrumentation amplifier — indicator

Block diagram of instrumentation system

Superposition principle can be used to find the o/p of the amplifier voltage

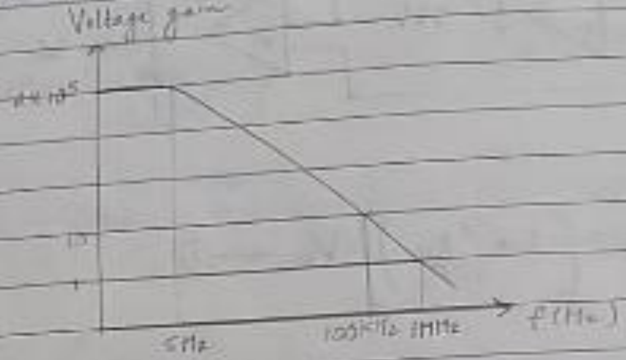


$$V_o = \frac{R_F}{R_1}\left(1 + \frac{2R_2}{R_G}\right)(V_a - V_b)$$

$R_G \rightarrow$ used to vary the gain of instrumentation amplifier without disturbing the symmetry of the circuit.

Bandwidth of Voltage series feedback amplifier [Non inverting amplifier]

Range or band of frequencies in which the gain of the amplifier is constant



Open loop op amp (741)

Unity gain Bandwidth Product (UGB)

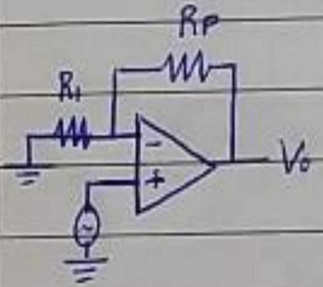
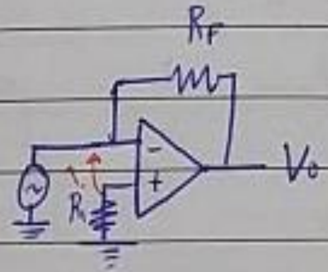
open loop $\boxed{UGB = f_0 \cdot A}$

$A \rightarrow$ gain

$f_0 \rightarrow$ bandwidth

closed loop $\boxed{UGB = f_F \cdot A_F}$

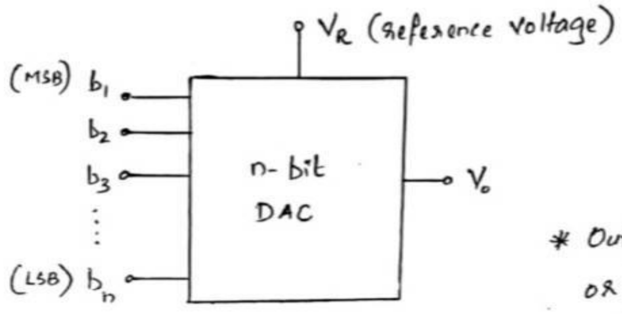
$$\boxed{f_F = f_0 (1 + A\beta)}$$

	Parameters	Non inverting amp. (Voltage series)	Inverting amp. (Voltage shunt)
1.	Gain (A_F)	$A_F = 1 + \frac{R_F}{R_1} = \frac{A}{1+A\beta}$	$A_F = \left(-\frac{R_F}{R_1}\right) = -\frac{Ak}{1+A\beta}$
2.	Feedback gain (β)	$\beta = \frac{R_1}{R_1+R_F}$	$\beta = \frac{R_1}{R_1+R_F}$
3.	Input impedance (R_{if})	$R_{if} = R_i(1+A\beta)$.	$R_{if} = R_1 + \left(R_i \| \frac{R_F}{1+A}\right)$ $\approx R_1$
4.	Output impedance (R_{oF})	$R_{oF} = \frac{R_o}{1+A\beta}$	$R_{oF} = \frac{R_o}{1+A\beta}$
5	Bandwidth		
6.	Circuit diagram.		

MODULE - 6

Digital To Analog Convertors (D/A convertor or DAC)

A DAC is a device that converts an 'n-bit' binary data to its equivalent analog data. An n-bit DAC is represented as shown below:



* Output is obtained as parts or fractions of reference voltage.

Generally, output of an 'n-bit' DAC can be written as

$$\boxed{V_o = V_R K \left(b_1 2^{-1} + b_2 2^{-2} + b_3 2^{-3} + \cdots + b_n 2^{-n}\right)}$$

MODULE - 6

Digital To Analog Convertors (D/A convertor or DAC)

data to

shown

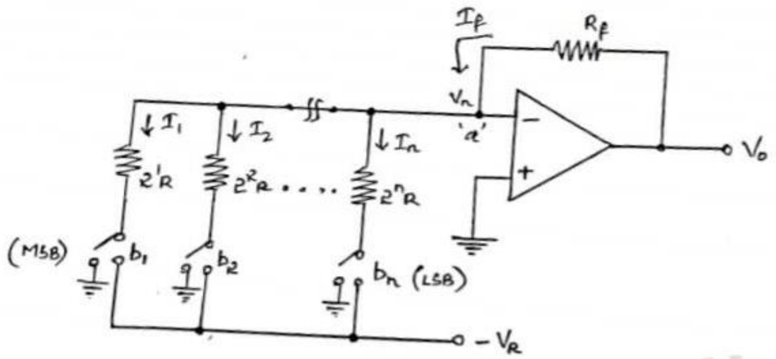
ined as parts

reference

Types of DAC:

a) Binary Weighted DAC

Binary weighted DAC is a summing amplifier circuit with input resistor values properly choosen to get the desired analog output values. Opamp is used in inverting configuration and a chain of resistors whose values are powers of '2' are connected to input of opamp. All the other ends of resistors are connected to a reference voltage through SPDT switches as shown below:



$$V_o = V_R \cdot \frac{R_f}{R}\left[\frac{b_1}{2} + \frac{b_2}{2^2} + \cdots + \frac{b_n}{\ldots}\right]$$

Eg: 3 bit binary weighted DAC

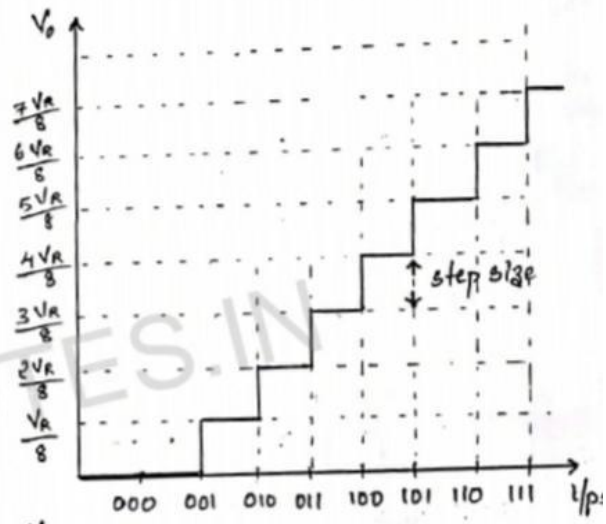
$$V_o = V_R \cdot \frac{R_f}{R}\left[\frac{b_1}{2} + \frac{b_2}{2^2} + \frac{b_3}{2^3}\right]$$

Select $\frac{R_f}{R} = 1$.

$$V_o = V_R\left[\frac{b_1}{2} + \frac{b_2}{4} + \frac{b_3}{8}\right]$$

inputs b_1	b_2	b_3	output (V_o) V_o
0	0	0	0
0	0	1	$V_R/8$
0	1	0	$2V_R/8$
0	1	1	$3V_R/8$
1	0	0	$4V_R/8$
1	0	1	$5V_R/8$
1	1	0	$6V_R/8$
1	1	1	$7V_R/8$

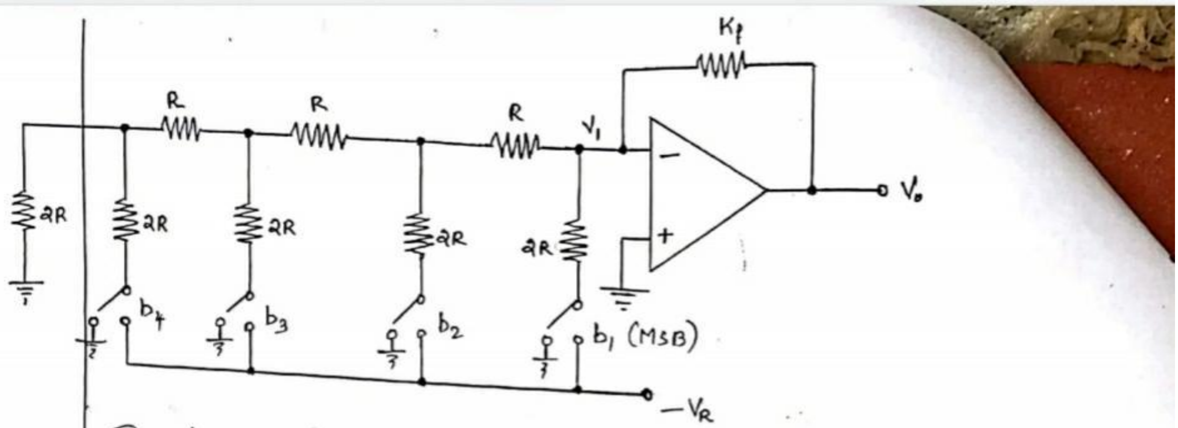
Transfer Characteristics



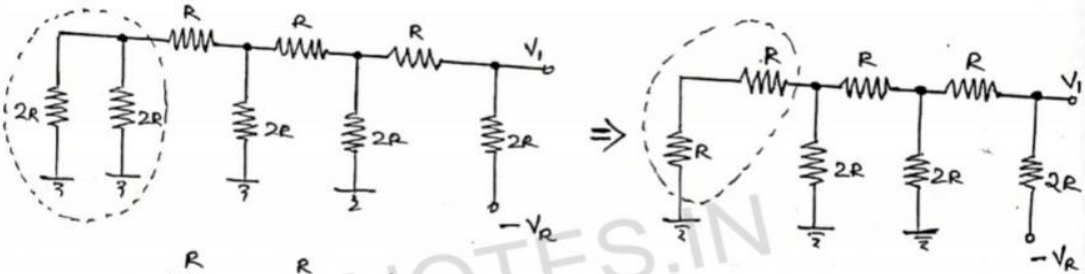
J

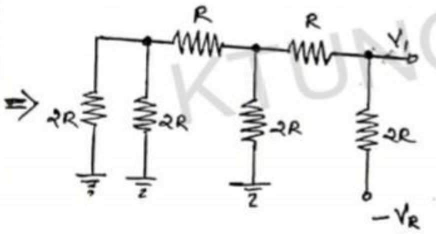
b) R – 2R Ladder DAC

A major disadvantage of binary weighted DAC is that it requires a wide range of resistors as the number of bits is increased. The R–2R ladder DAC needs only two resistor values for any number of input bits. In R-2R ladder resistors of two values R and 2R are suitably chosen. In this ladder circuit the output voltage is a weighted sum of digital inputs. An inverting opamp configuration is used and SPDT switches are used to provide inputs.

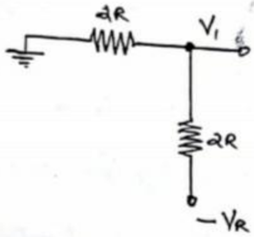


To learn the operation of the circuit, consider a 4 bit R-2R lad DAC as shown above. If the binary input $b_1 b_2 b_3 b_4$ is 1000, the equivalent ladder network can be simplified as shown below:





AND FINALLY :



Now $V_1 = \dfrac{-V_R}{2}$

Similarly if input is 0100, $V_1 = \dfrac{-V_R}{4}$.

So we can write the output equation as: (when all switches are put t

$$V_0 = -\frac{R_f}{R}\left[-\frac{V_R}{2^1} - \frac{V_R}{2^2} - \frac{V_R}{2^3} - \frac{V_R}{2^4}\right]$$

$$V_0 = V_R \cdot \frac{R_f}{R}\left[\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right]$$

Including input bits,

$$\boxed{V_0 = V_R \cdot \frac{R_f}{R}\left[\frac{b_1}{2^1} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \frac{b_4}{2^4}\right]}$$

SPECIFICATIONS OF DAC

1)Accuracy: Maximum deviation of the output from the ideal value.

2)Offset Voltage: Ideally output value of a DAC is zero when all binary inputs are zero. But there will be a small initial output voltage known as Offset voltage/ Offset Error.

3) Linearity : Maximum deviation in step size from ideal step size.

4)Monotocity: if the output value increases as the binary inputs are incremented from one value to the next.

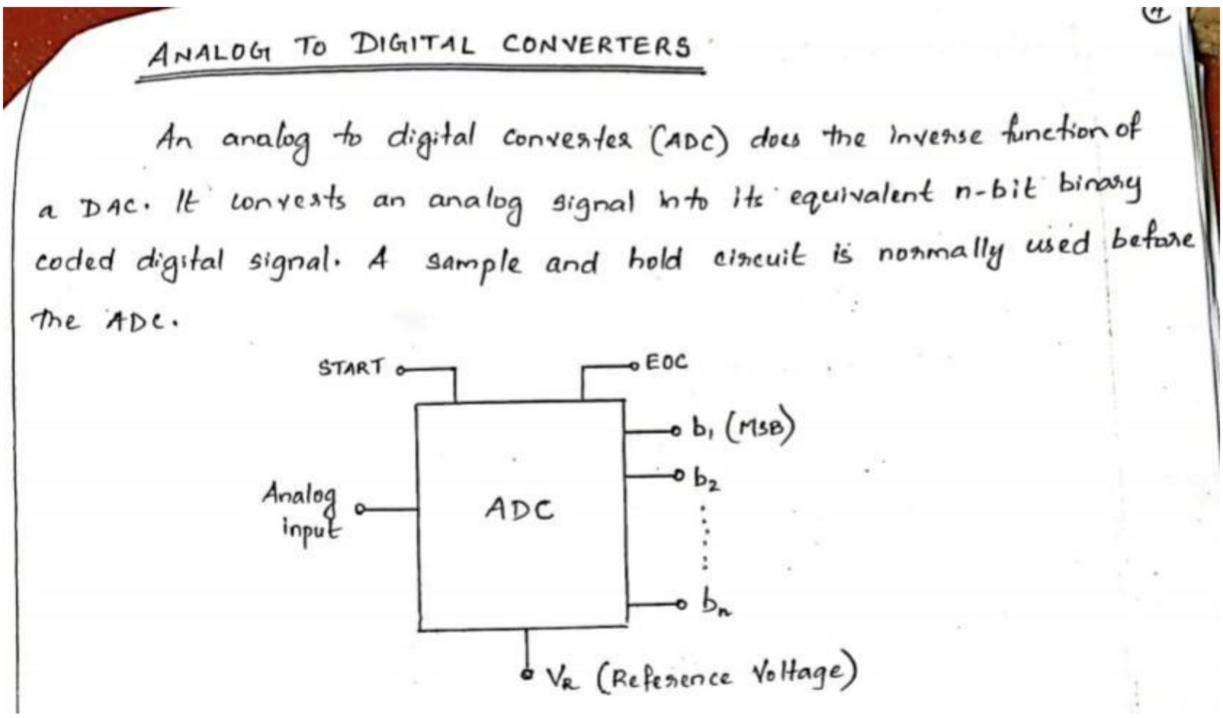
5) Resolution: Value of output when LSB set to 1 divided by step size

% Resolution=(1/Total number of steps) x 100

= (1/(2^n) -1) x100

6) Setting Time : Time required to settle down output of DAC to within half LSB of actual value for a given digital output.

7) Temperature Sensitivity: For a fixed digital input, analog output will vary



ANALOG TO DIGITAL CONVERTERS

An analog to digital converter (ADC) does the inverse function of a DAC. It converts an analog signal into its equivalent n-bit binary coded digital signal. A sample and hold circuit is normally used before the ADC.

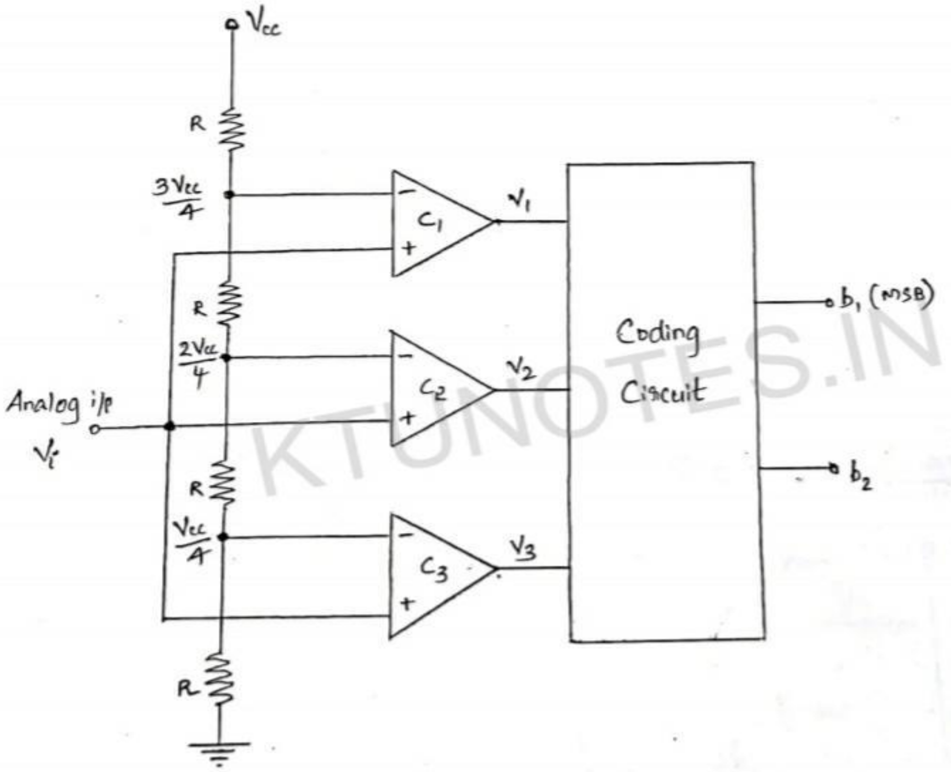
with temperature.

a) Simultaneous Type ADC / Flash ADC

Simultaneous type ADCs are based on comparing an unknown analog input voltage with a set of reference voltages. For comparing voltage we use opamps. For flash ADC the number of opamps needed for converting an analog signal to 'n' output bits is given by $2^n - 1$.

Eg: 2-bit Flash ADC

$n = 2$. No. of opamps $= 2^2 - 1 = 3$.



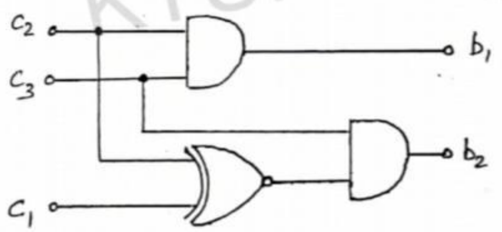
V_i	Comparator o/ps C_1	C_2	C_3	Output bits b_1	b_2
0	0	0	0	0	0
$0 \le V_i \le \frac{V_{cc}}{4}$	0	0	0	0	0
$\frac{V_{cc}}{4} < V_i \le \frac{V_{cc}}{2}$	0	0	1	0	1
$\frac{V_{cc}}{2} < V_i \le \frac{3V_{cc}}{4}$	0	1	1	1	0
$\frac{3V_{cc}}{4} < V_i \le V_{cc}$	1	1	1	1	1

Now expression for b_1 and b_2 can be written as:

$$b_1 = \bar{c}_1 c_2 c_3 + c_1 c_2 c_3 = c_2 c_3(\bar{c}_1 + c_1) = \underline{\underline{c_2 c_3}}.$$

$$b_2 = \bar{c}_1 \bar{c}_2 c_3 + c_1 c_2 c_3 = \underline{\underline{c_3(c_1 \odot c_2)}}$$

Coding Circuit:



Working: Initially a reset pulse is given to the counter making its outp bits to zero. Since inputs to DAC is zero, its output voltage $V_R = 0$. Counter will count when it gets clock pulses and this happens only when second input of AND gate is high (ie o/p of comparator is HIGH) Now let an Input voltage V_i be applied to the circuit. Since $V_R = 0$, $V_i > V_R$ and comparator output will be high. Now the counter starts getting a clock pulse. The counter increments bit by bit and the DAC converts all such digital values to analog voltage V_R. Value of V_R will gradually increase and at one stage becomes greater than or equal to V_i. At this stage output of comparator becomes low and clock pulses do not reach the counter. Counting stops and binary outpu now at counter will be the equivalent of V_i.

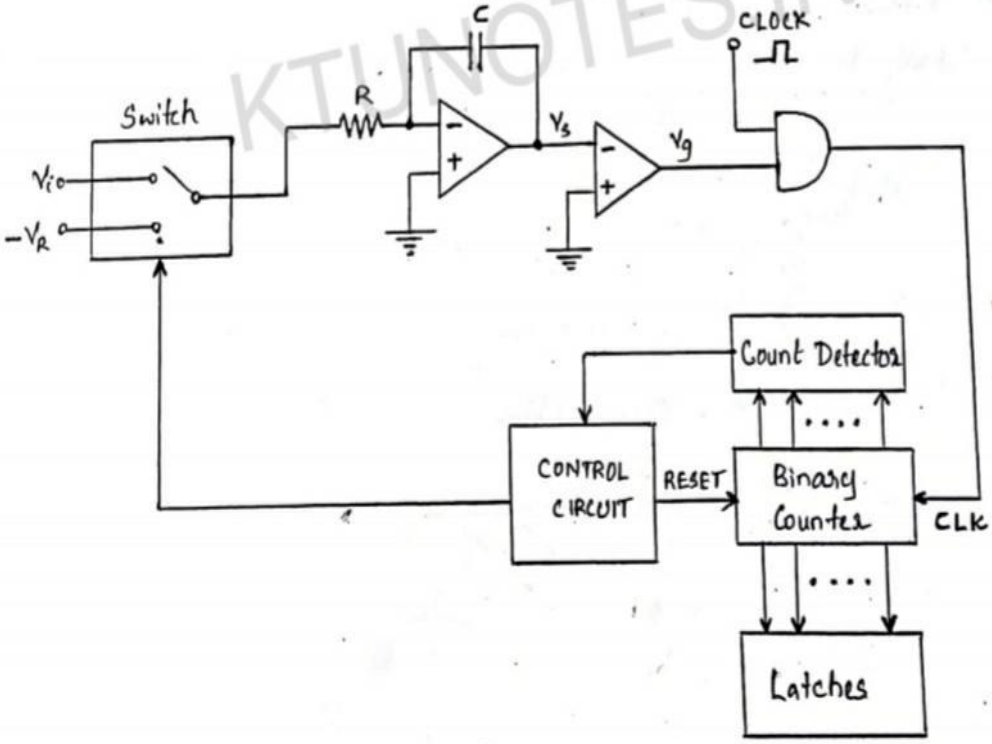
Advantages: a) Simple and needs less hardware compared to simultan ADC.

b) Suited for applications with high resolution.

Disadvantage: Here the clock speed determines the speed of conversion Conversion time is normally more than flash type ADCs.

c) Dual Slope ADC

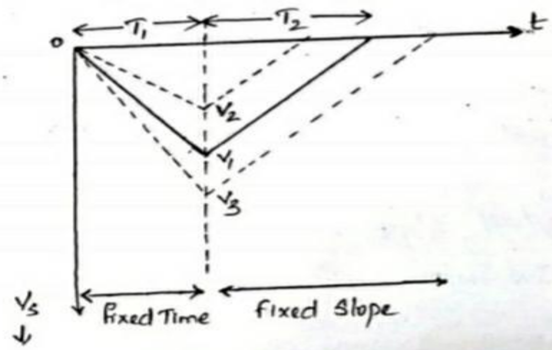
A dual slope ADC uses an integrator circuit that generates two ramp signals one with the unknown input voltage and one with a known reference voltage. Hence it is called as a 'dual slope' ADC.



In a dual slope ADC, the integrator will generate two different ramp signals. One with unknown analog input V_i and other with a reference voltage V_R.

Assume that initially counter is RESET and $V_s = 0$. When a pos[itive] voltage is applied, the integrator will produce a -ve ramp signal. The [count] detector is set to a time T_1. The -ve signal from integrator is inverted by the next opamp (V_g). Since V_g is positive AND gate will pass the clock signals to the counter. The counter will start counting and when its value becomes equal to value set in count detector, the count detector will send a message to control circuit which in turn will RESET the counter and will change the position of switch. Now when switch position is changed $-V_R$ enters to integrator input. Now integrator output will raise from -ve value to higher values. At this time V_g goes from positive value to zero. During this time V_g is positive for a short time and the AND gate sends clock pulses to the counter. When $V_g = 0$, counting stops. This count sequence is now stored in the latches.

For entire time T_2, counter increments and its final value will be the equivalent of input voltage V_i.



$V_3 > V_1 > V_2$.

4) Successive Approximation Type ADC

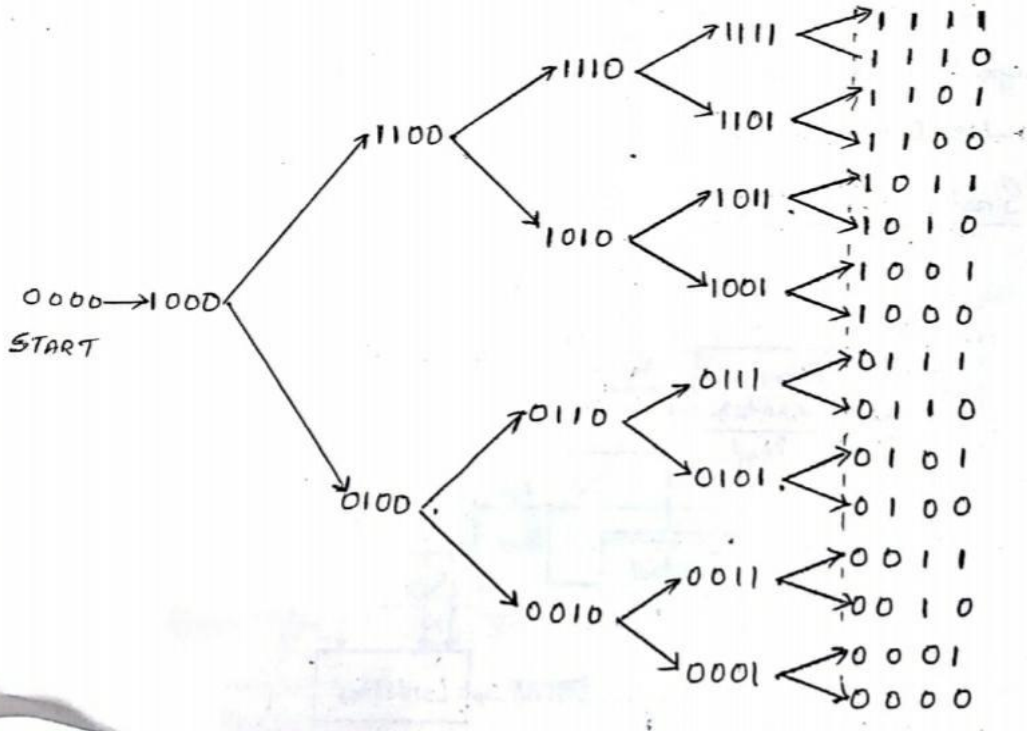
In successive approximation type ADC, an unknown input voltage is compared with an n-bit binary value by trying one bit at a time starting from MSB. In this ADC conversion time depends on number of output bits. The process is carried out through a successive approximation process as described below:

(i) MSB of the binary number is initially set to 1, with remaining bits zeroes.

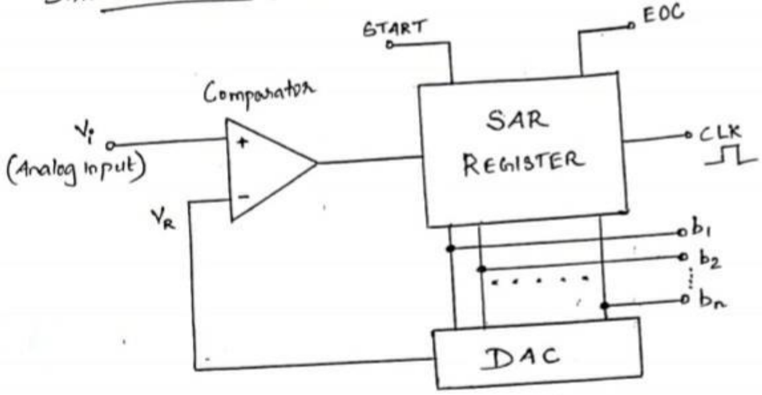
(ii) The digital equivalent of this value is compared with analog input voltage.

(iii) If the analog input voltage is equal to or greater than the digital equivalent, MSB is retained as 1 and the next bit is set to 1. Else MSB is set to '0' and next bit is set to '1'.

(iv) This comparison is continued up to LSB. The last value of digital number obtained will be the digital equivalent of input voltage.



SAR Block Diagram:

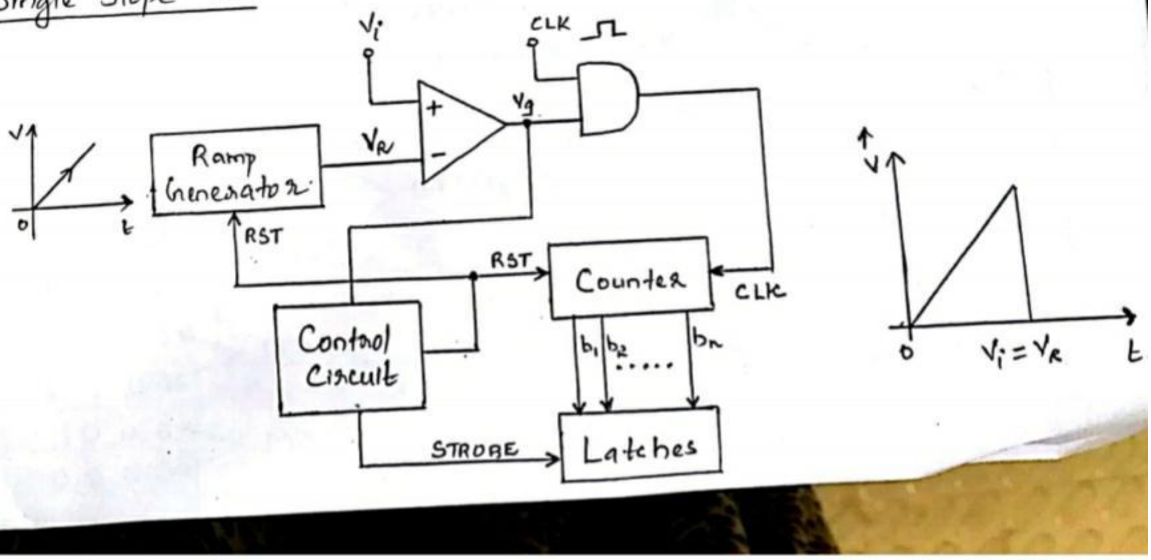


When a START signal is applied, SAR register sets MSB to 1 and all other bits to 0. The DAC converts this number to its decimal representation (equivalent) and gives it to comparator (opamp).

- If V_i is greater than or equal to V_R, MSB is set to 1 and next bit is set to 1 by SAR register.
- If V_i is less than V_R, MSB is set to 0 and next bit is set to 1 by SAR register.

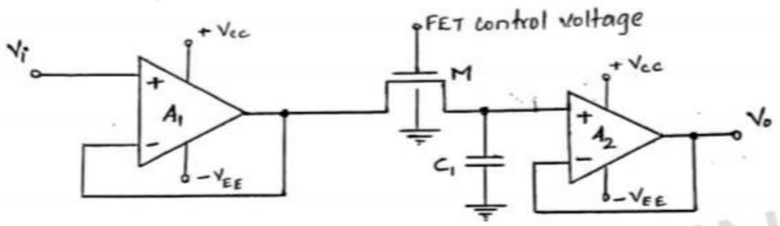
The process is completed for the remaining lower bits. An EOC signal is sent out when all the bits are scanned. The time required for n-bit conversion is equal to 'n' clock pulses.

e) Single Slope ADC



In a single slope ADC, a ramp generator voltage is given to the inverting terminal of the comparator. Voltage to be converted is given at the non-inverting terminal. As long as the input voltage is greater than the ramp generator voltage, V_g will be high and clock signal will reach the counter. Counter will count for each clock pulses and store its output in a set of latches. Once the input voltage becomes equal to ramp voltage, V_g becomes zero and the counter stops counting. At this point the control circuit will reset the ramp generator and counter.

Sample and Hold Circuit



The MOS transistor 'M' is an analog switch which is capable of switching by logic levels. It alternately connects and disconnects the capacitor C_1 to the output of opamp A_1. Opamps A_1 and A_2 acts as voltage followers.

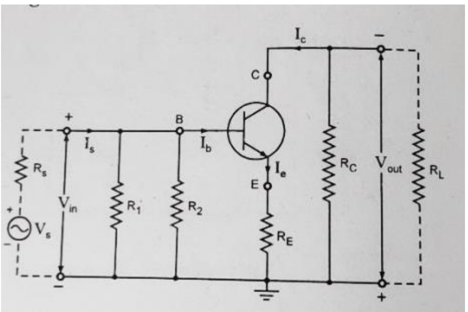
Working:

An input signal V_i is applied to opamp A_1. Transistor M is alternately switched ON and OFF by the control voltage. When transistor M is ON, capacitor C_1 charges to input voltage V_i. Now even if signal V_i changes its value capacitor C_1 will hold the initial value for a short interval of time. When M is OFF, C_1 holds the previous voltage. This value is fed as input to opamp A_2 which is again a voltage follower. More samples can be obtained by increasing

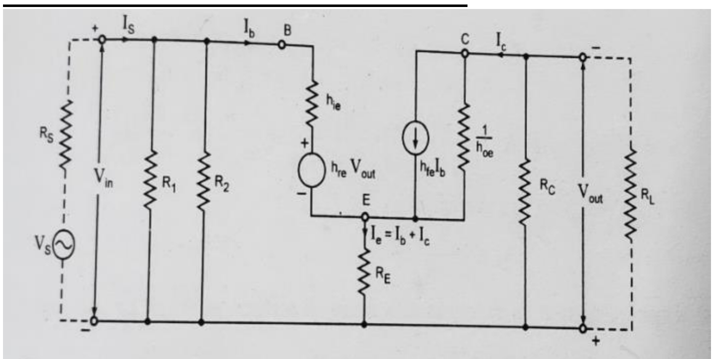
MODULE 3

A. SMALL SIGNAL ANALYSIS:

1. COMMON EMITTER- h-PARAMETERS



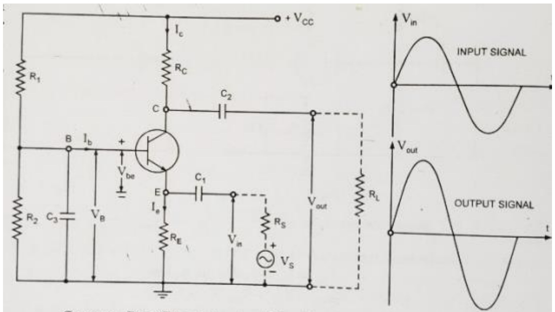
CE Amplifier AC Equivalent Circuit With an Unbypassed Emitter Resistor R_E
Fig. 12.32



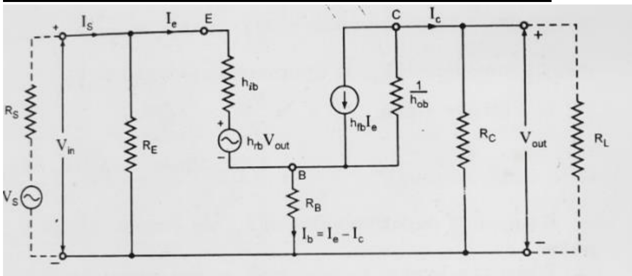
CE Amplifier h-Parameter Equivalent Circuit With R_E Included in The Circuit

- Input impedance :$Zin = R1||R2||Zb$
  :Zb=hie+Re(1+hfe)
- Output impedance:$Zout = \frac{1}{hoe}||Rc \cong Rc$
- Voltage gain :$Av \approx -\frac{Rc||RL}{RE}$
- Current gain : $Ai = -\frac{hfe*RB*Rc}{(Rc+RL)(RB+Zb)}$

2. COMMON BASE- h-PARAMETERS



Common Base Transistor Amplifier Circuit With Emitter Current Bias
Fig. 12.43

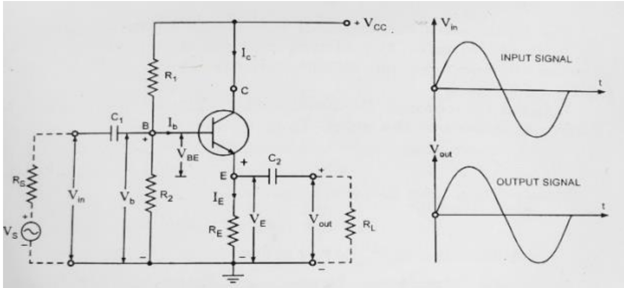


CB Amplifier h-Parameter Equivalent Circuit With Bypass Capacitor At Transistor Base Terminal Omitted
Fig.12.46

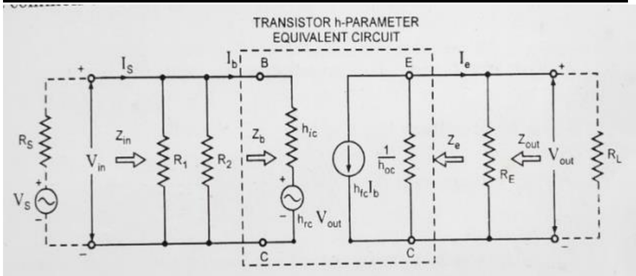
- Input impedance :$Zin = Ze||RE$

  :Ze=hib+RB(1+hfb)
- Output impedance:$Zout = \frac{1}{hob}||Rc \approx Rc$
- Voltage gain :$Av \approx -\frac{hfb(Rc||RL)}{hib+RB(1+hfb)}$
- Current gain : $Ai = -\frac{hfb*RE*Rc}{(Rc+RL)(RE+Ze)}$

3. <u>COMMON COLLECTOR- h-PARAMETERS</u>



Common Collector Transistor Amplifier Circuit With Emitter Current Biasing
Fig. 12.47



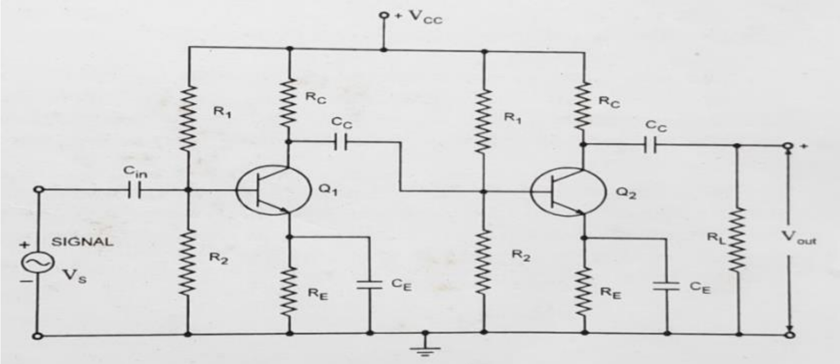
CC Amplifier h-Parameter Equivalent Circuit
Fig. 12.49

- Input impedance $:Zin = R1||R2||Zb$:Zb=hic-hfc(RE||RL)
- Output impedance:$Zout = Ze||RE \cong Ze$ $:Ze = -\frac{hic+(R1||R2||Rs)}{hfe}$
- Voltage gain $:Av \approx -\frac{hfc(RE||RL)}{hic-hfc(RE||RL)} \cong 1$
- Current gain : $Ai = -\frac{hfc*RB*RE}{(RE+RL)(RB+Zb)}$

Quantity \ Configuration	Common-emitter CE	Common-base CB	Common-collector CC
Current gain, A_i	High (– 46.5)	Low (0.98)	High (47.5)
Voltage gain, A_v	High (– 131)	High (131)	Low (0.99)
Input resistance, R_{in} for R_L = 3 kΩ	Medium (1.065 kΩ)	Low (22.5 Ω)	High (144 kΩ)
Output resistance, R_{out} for R_s = 3 kΩ	Medium (45.5 kΩ)	High (1.72 MΩ)	Low (80.5 Ω)

B. <u>CASCADE AMPLIFIER</u>:

- <u>R-C COUPLED AMPLIFIER</u>



Two-Stage R-C Coupled Transistor Amplifier
Fig. 13.5

- ✓ R1, R2, RE for biasing & stabilizing network.
- ✓ Signal @ Rc of 1st stage coupled to base of 2nd stage by Cc coupling capacitor.
- ✓ Cin-used to couple only AC signals into circuit.
- ✓ Ce-emitter bypass offers low reactance path to signal.
- ✓ Cc-dc biasing to next stage is eliminated.

a. Low frequency range: Xc$c \approx Rc$, so voltage gain decreases.

b. Mid frequency range: Xcc $\approx small$,effect of Cc neglected.

c. High frequency range:Xcc ≈ 0, presence of inter-electrode

Capacitance by depletion layers at junction.

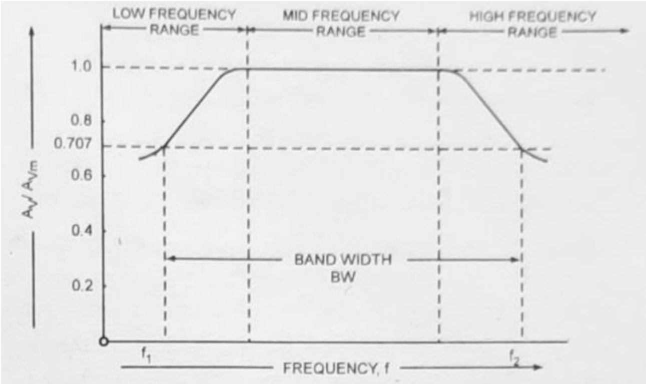
Cbc-negative feedback & gain reduces.

Cbe-low impedence path at input side and effective input signal reduced.

Presence of wiring capacitance.

As f increase, voltage gain falls off.

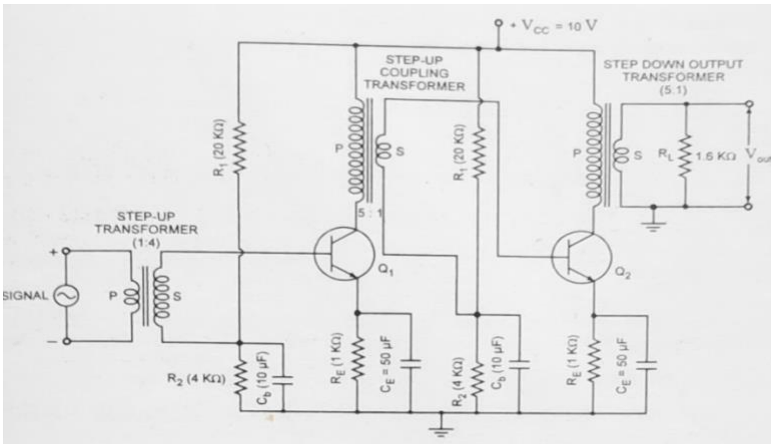
F(lower):$|Avl| = \frac{|Avm|}{\sqrt{1+(\frac{f1}{f})^2}}$ F(upper):$|Avh| = \frac{|Avm|}{\sqrt{1+(\frac{f}{f2})^2}}$



Frequency Response Curve of an R-C Coupled Amplifier
Fig. 13.14

- ✓ Constant gain over AF range, cheaper, compact circuit, low voltage and power gain, poor impedance matching. Used as voltage amps

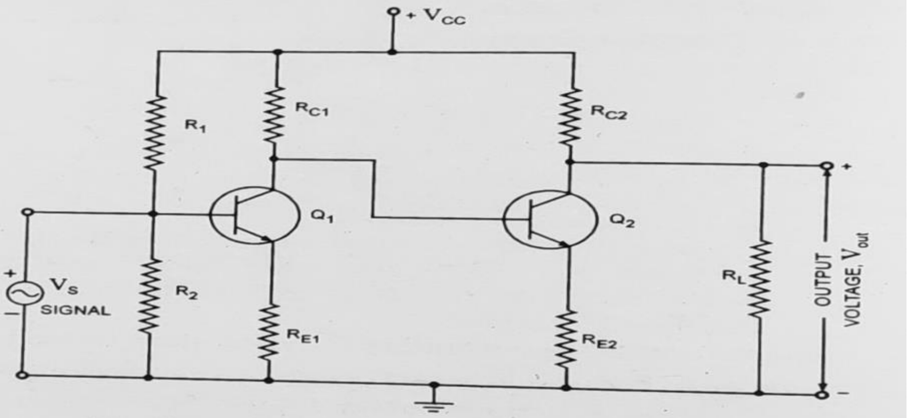
➢ <u>TRANSFORMER COUPLED AMPLIFIER</u>



Two-Stage Transformer Coupled Transistor Amplifier

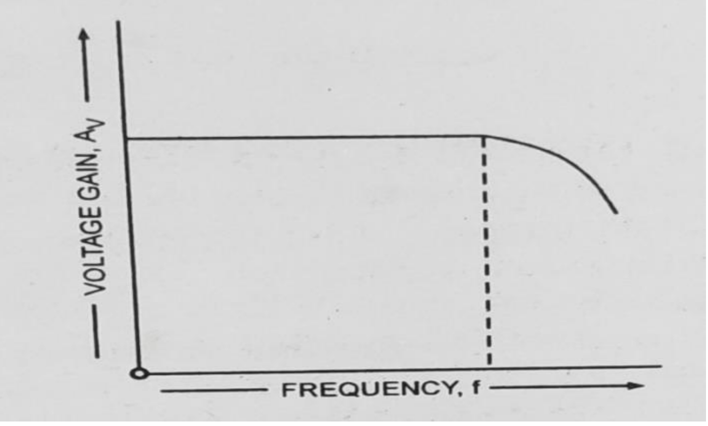
- ✓ High o/p impedance parallel to low i/p impedance of subsequent stage & effective load reduced, also called Loading effect.
- ✓ This drawback of RC cascading removed by selecting turns-ratio of transformer.
- ✓ Used for small load.
- ✓ DC isolation between stages.
- ✓ All DC volt supplied is available at collector.
- ✓ Poor frequency response, bulky, costly, inter winding capacitance. Used for RF signal amplifying.

➢ <u>DIRECT COUPLED AMPLIFIER</u>



Two-Stage Direct-Coupled Transistor Amplifier
Fig. 13.25

- ✓ Used for low frequency range and low voltage amplification.
  eg:-Thermocouple.
- ✓ Amplifier responds for small variations.
- ✓ Coupling and bypass capacitors not used, so DC levels present.



- ✓ Flat cut-off frequency, depends only on wiring & internal transistor capacitance then Gain decreases.
- ✓ Simple circuit, cheap, not suitable for high frequency signals, poor temperature stability, DC levels get amplified which shifts quiescent point.

EC MODULE 3

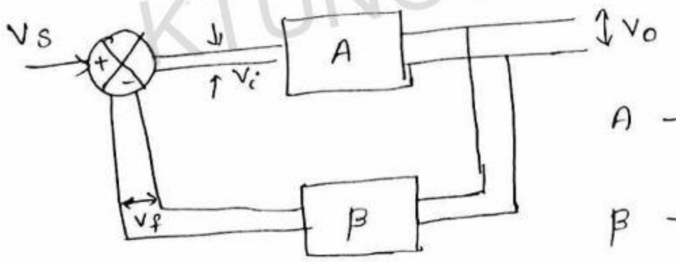
FEEDBACK AMPLIFIER

A portion of output quantity is mixed with input signal.

There are 4 types:

- series-shunt
- shunt-series
- series-series
- shunt-shunt

Terms and Definitions.



A → gain of the amplifier w/o feedback

β → feedback factor.

Positive feedback

- If the feedback signal is in such a way that it causes the overall o/p to increase, then it is referred to as positive feedback
- Regenerative feedback.

Negative feedback

- If the feedback s/g result in a decrease of overall o/p, then the feedback is referred to as negative feedback
- Degenerative feedback

Effect of Negative Feedback on Amplifier

Transfer gain with feedback,

$$Af = \frac{A}{1 + A\beta} \quad |1 + A\beta| > 1, \quad Af < A$$

gain reduces due to Negative feedback

Stability In Gain

$$|1 + A\beta| \gg 1 \Rightarrow Af = \frac{A}{A\beta} \qquad Af = \frac{1}{\beta}$$

gain depend on feedback factor (β)

Feedback factor is f^n of feedback N/w $\rightarrow$ Insensitive to temp
$\therefore$ β remains Unchanged.

Input Resistance

Case 1 : Voltage Sampling Series Mixing (Series - shunt)

Current Sampling Series Mixing (Series - Series)

$$\boxed{R_{if} = R_i(1 + A\beta)}$$ (i/p R ↑ due to feedback)

Case 2 : Voltage Sampling Shunt Mixing (Shunt - shunt)

Current Sampling Shunt Mixing (Shunt - Series)

$$\boxed{R_{if} = \frac{R_i}{1 + A\beta}}$$ (i/p R decreases due to feedback)

Output Resistance

Case 1 : Voltage Sampling Series Mixing (Series Shunt)

~~Current~~ Voltage Sampling Shunt Mixing (shunt - shunt)

$$\boxed{R_{of} = \frac{R_o}{1 + A\beta}}$$

Case 2 : Current - Sampling Series Mixing (Series - Series

Current - Sampling Shunt Mixing Shunt - Series)

$$\boxed{R_{of} = R_o(1 + A\beta)}$$

OSCILLATORS

It is a device that convert DC power from supply into AC power in the load.

According to principle involved:

- Feedback oscillator (oscillation through positive feedback mechanism)
- Negative resistance oscillator (using negative resistance effect)

According to the type of waveform produced:

- Sinusoidal oscillator
- Relaxation oscillator (other than sinusoidal waveforms)

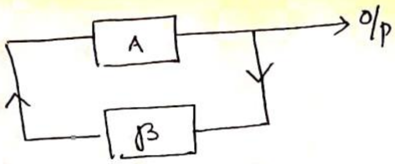
According to the feedback circuit:

- RC oscillators (f/b circuit contain resistors and capacitors)
- LC oscillators (f/b circuit contain inductors and capacitors)

According to frequency of oscillator:

- Audio frequency oscillator
- Radio frequency oscillator (upto 30MHZ)
- Very high frequency oscillator (upto 300MHZ)
- Ultra-high frequency oscillator (upto 3GHZ)
- Microwave oscillator (above 3GHZ)

FeedBack Oscillators



* An amp with +ve f/β. Network can function as an Oscillator

V_i — i/p

$V_o = A V_i$ (op)

[o/p of f/b N/w is V_i. This is i/p to the amplifier]

Now we remove the i/p. The amplifier amplifies it gives an o/p of $V_o = A V_i$

* The Circuit Oscillates even in the absence of external i/p.
* This Condition is met Only if the o/p of f/b N/w is V_i having a feedback factor β

$$\beta = \frac{V_i}{A V_i}$$

$\boxed{A\beta = 1}$ Barkhausen Criterion

It States that

i) $|A\beta| = 1$ product of gain of the amp & β of f/β N/w must be exactly Unity.

ii) $\angle A\beta = 0, 2\pi$ The total phase shift around the loop should be Zero or multiple of 2π radians

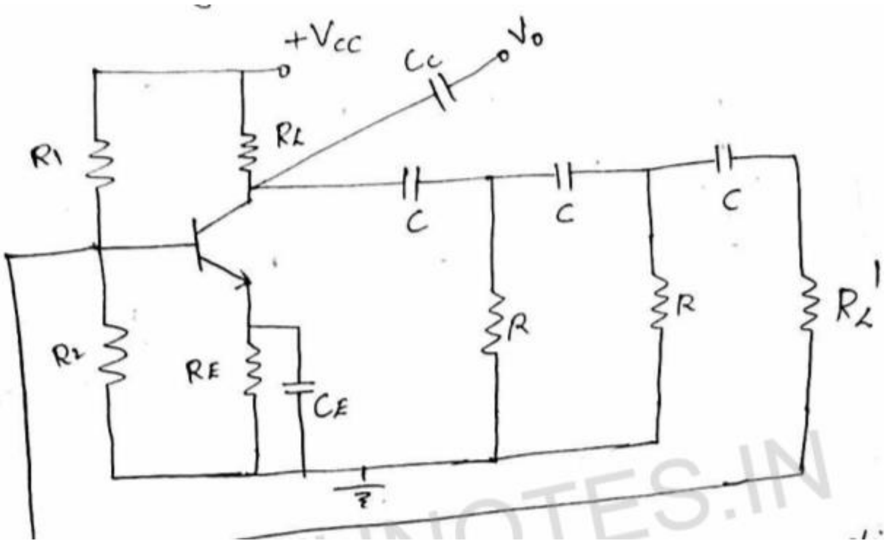
[If $|A\beta| < 1$ removal of external i/p will Cause Oscillation to damp & to die out]

Comparison

RC oscillators	LC oscillators.
1) Used for low frequency applications.	1) Used for high frequency applications.
2) Uses Resistors & Capacitors in the feedback n/w.	2) Uses Inductors & Capacitors in the feedback n/w.
3) Gives wide frequency range.	3) Cannot give wide frequency range.
4) Highly stable	4) Not stable
5) Can be easily integrated with OPAMP	5) Can't be easily integrated with OPAMP.

RC PHASE SHIFT OSCILLATOR

- AF OSCILLATOR
- USING FEEDBACK PRINCIPLE
- RC NETWORK IN FEEDBACK PATH
- GENERATE SINUSOIDAL SIGNAL



- CE AMPLIFIER WITH RC NETWORK IN FEEDBACK PATH.
- CE PROVIDE 180 DEGREE PHASE SHIFT
- TO SATISFY BARKHAUSEN CRITERION REMAINING 180 DEGREE PHASE SHIFT IS PROVIDED BY THE RC NETWORK.SUCH THAT EACH RC NETWORK PROVIDE 60 DEGREE PHASE SHIFT

Frequency of Oscillator

$$f = \frac{1}{2\pi RC\sqrt{6+4k}} \quad ; \quad k = \frac{R_L}{R}$$

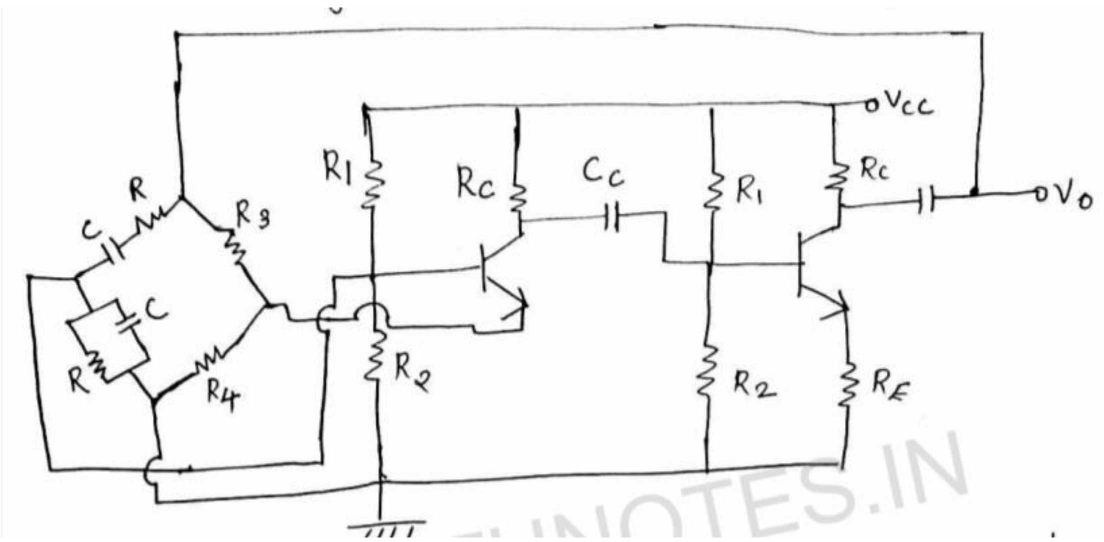
$$\boxed{\beta = 23 + 4k + \frac{29}{k}}$$ feedback factor

Minimum Value of $k = \sqrt{\frac{29}{4}} = 2.69$

$\beta_{min} = 44.5$

Weinbridge oscillator

- AF OSCILLATOR
- USING FEEDBACK PRINCIPLE
- RC NETWORK IN FEEDBACK PATH
- GENERATE SINUSOIDAL WAVEFORMS

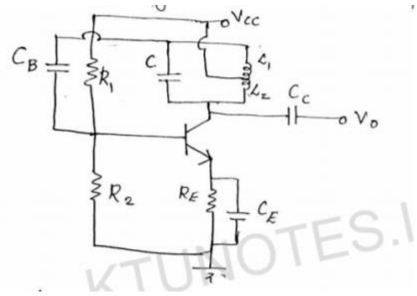


F=1/2piRC
FEEDBACK FACTOR(B)=1/3

HIGH FREQUENCY OSCILLATOR

- RF OSCILLATOR
- USING FEEDBACK PRONCIPLE
- LC NETWORK IN FEEDBACK PATH
- GIVES SINUSOIDAL WAVEFORM

- FEEDBACK PROVIDED THROUGH L1 AND OUTPUT TAKEN ACROSS L2.CE WILL PROVIDE 180 DEGREE PHASE SHIFT REMAINING IS PROVIDED BY THE F/B N/W.F=I/



F=1/2Pi((L1+L2)C)^1/2

COLPITTS OSCILLATOR

- RF OSCILLATOR
- USING FEEDBACK PRONCIPLE
- LC NETWORK IN FEEDBACK PATH
- GIVES SINUSOIDAL WAVEFORM
  F=1/2Pi(L(C1C2/C1+C2))^1/2

CRYSTAL OSCILLATOR

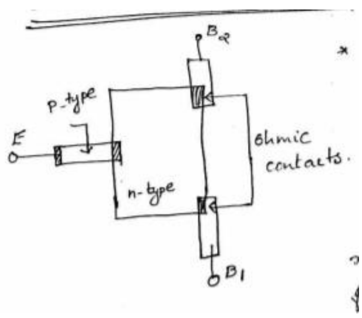
Tuned circuit using piezoelectric crystal as tuned resonant tank circuit. Used when high frequency stability is required.

PIEZOELECTRIC PROPERTY: ability to transform mechanical deformation into electrical charge and vive-versa. eg:Rochelle salt,quartz

F=1/2Pi(LC)^1/2

UJT RELAXATION OSCILLATOR

- 3 TERMINAL DEVICE
- It consist of n type semiconductor bar from the two ends of which ohmic contacts are taken.



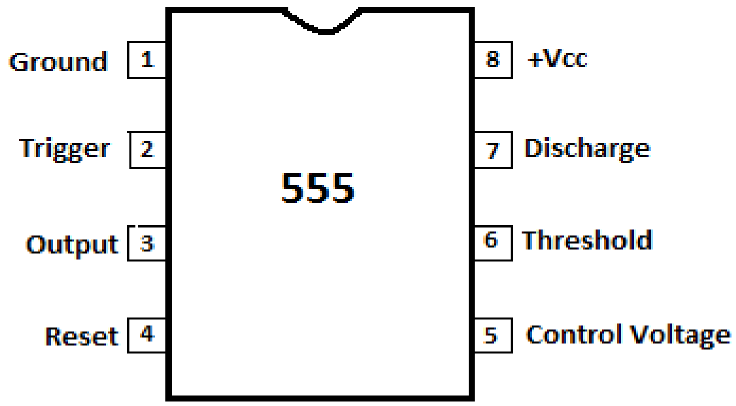
Though it is 3 terminal it has only one junction hence it is called UJT

MODULE 5

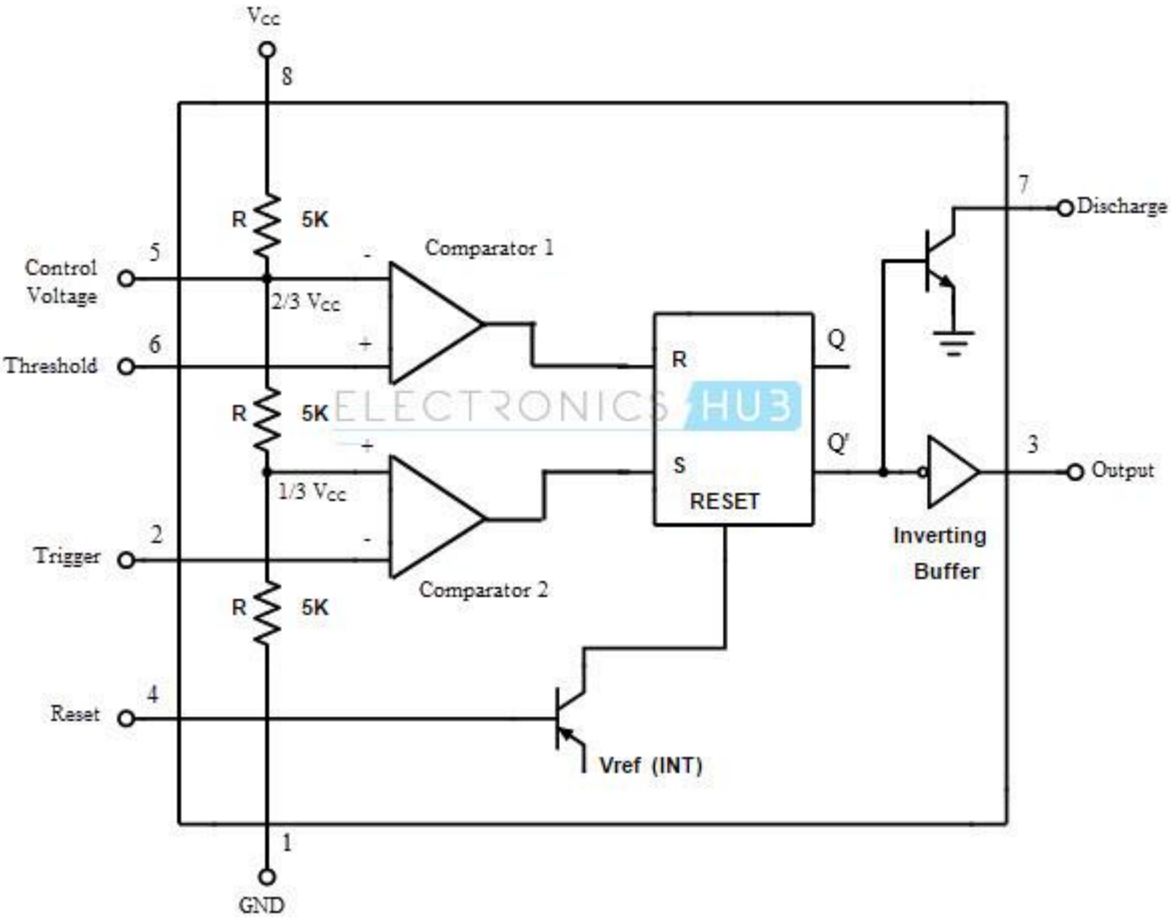
MODULE 5 AIC

555 TIMER

- Highly stable for generating accurate time delay or oscillation
- Time delay –microseconds to hours
- Supply voltage +5V to +18V
- Load up to 200Ma
- Applications:
  - Pulse generator
  - Burglar alarm
  - Traffic light control

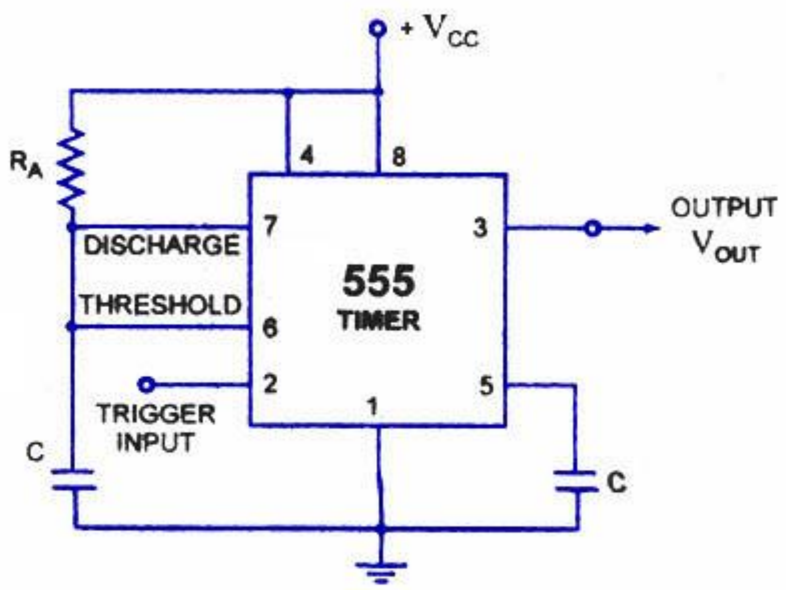


INTERNAL DIAGRAM



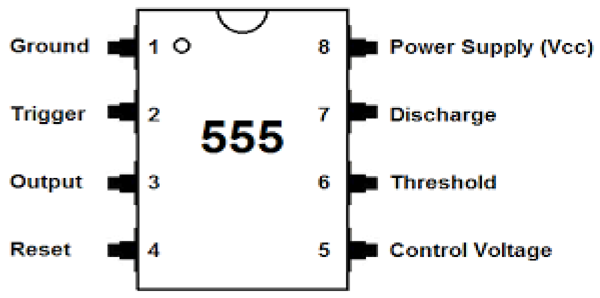
MONOSTABLE MULTIVIBRATOR

- MMV also ***c***alled as one shot mmv pulse generator circuit in which the duration of the pulse is determined by the R-C network ,connected externally to the timer
- In such a vibrator, one state of output is stable while the other is quasi-stable (unstable). For auto-triggering of output from quasi-stable state to stable state energy is stored by an externally connected capacitor C to a reference level.
- The time taken in storage determines the pulse width. The transition of output from stable state to quasi-stable state is accomplished by external triggering.



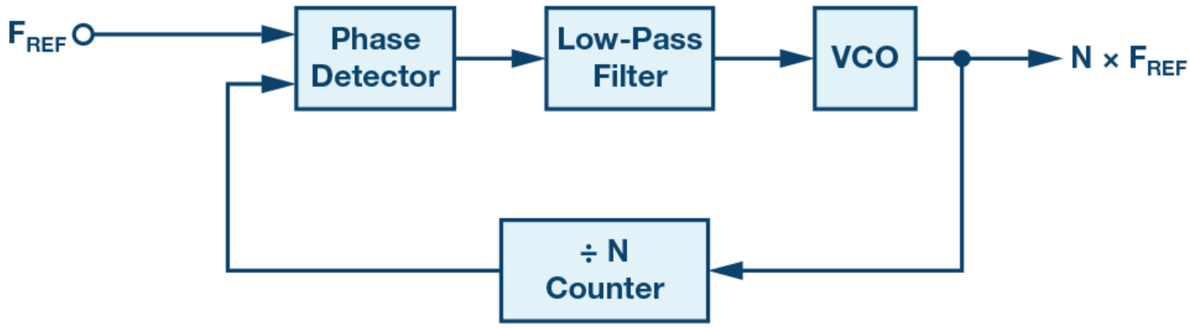
Circuit of The Timer 555 as a Monostable Multivibrator

VOLTAGE CONTROLLED OSCILLATORS



A Voltage controlled oscillator (VCO) circuit varies its frequency of the oscillation with respect to the externally applied voltage. The VCO circuit can obtain a linear variation of oscillation in relation to the input voltage.

PHASE LOCKED LOOP



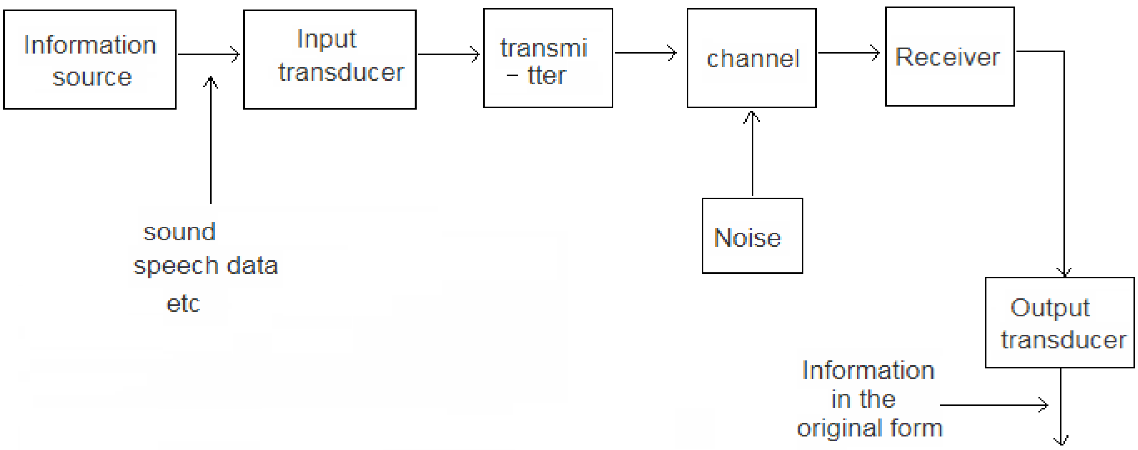
A **phase-locked loop** or **phase lock loop** (**PLL**) is a control system that generates an output whose phase is related to the phase of an input signal.

The oscillator generates a periodic signal, and the phase detector compares the phase of that signal with the phase of the input periodic signal, adjusting the oscillator to keep the phases matched.

MODULE 1

ELEMENTS OF COMMUNICATION SYSTEM

Block Diagram of communication system.



- **Information source** can be analog or digital signals. Eg: Audio signals,Vedio signals, Text messages.
- **Input transducer** converts the information into an electrical signal. Eg: Microphone (converts audio signal → Electrical signal)
- **Transmitter** converts the information to a form suitable for transmission through a channel.
  Mainly two functions : Modulation & Multiplexing.
  Modulation : Transforming the information into a high frequency signal i.e, can be transmitted over long distance.
  Multiplexing : Simultaneous transmission of no.of information signal through a single channel.
- **Channel** is a medium for carrying the information signal.
  Two types : Wired and Wireless channel.
  Wired channel: Cu Coaxial cable. Costly , hard to implement.
  Interference is less
  Wireless channel: Air or atmosphere.

- **Noise** is defined as any unwanted random signal whiched added to the original signal.
- **Distortion** is the variation in the shape of information signal due to the effect of noise.
- **Attuenation** is the loss in strength of the signal due to the noise.
- **Receiver**
- **Output transducer** converts signal back to original signal.

  Modulating signal $\longrightarrow$ Information signal

  Modulated signal $\longrightarrow$ Output signal after modulation.

Modulation

- Information signal is normally in low frequency range (20Hz-20KHz) which is impossible to transmit over large distance.
- So information is carried using a high frequency signal called as the carrier signal .
- Carrier signal is represented by **c(t)=Ac sin(2*pi*fc*t +** Φ**).**
- Three parameters of carrier signal that can be modified are Amplitude, Frequency, Phase.
- This process of modifying the parameters of the carrier signal according to the message signal or information signal is called **modulation.**

Need for modulation:

- Large antenna requirement: Antenna diameter should be atleast 1/10 th of the wavelength for efficient transmission of signal. (Standard antenna size is λ **/4** called as quater wavelength antenna).
- For efficient transmission.
- To reduce interference
- To improve signal to noise ratio : (Ratio of signal power to noise power).
- To improve multiplexing of signals.
- To improve the transmission range: Distance over which information can be transmitted over long distances.

Noise

Noise is defined as any unwanted or undesirable signal that falls in the frequency range of information.

Effects of noise on communication system are:

- It will distort the information signal and will lead to attenuation
- Bandwidth reduces.
- Due to attenuation, the range of the communication system will limited.
- It degrades the signal to noise ratio.
- The shaped of the signal is preserved , but the amplitude is altered.
- It will affect the sensitivity of the communication system. (Sensitivity is defined as the smallest signal or the weakest signal that can be properly amplified by the communication system).

Sources of noise

External noise:

- **Atmosphere noise**: Also known as static electricity and the main source of atmospheric noise is lighting discharges.
- **Extraterrestrial noise:** It is defined as the noise generated from outside the earth's atmosphere. It is called as deep space noise.
  It is divided into two :
  Solar noise : It is a noise generated due to sun's heat. Solar noise follows a cyclic pattern and it is repeated every 11years.
  Cosmic noise : it is generated due to distant stars and galaxies. It is also called blackbody noise. The magnitude of cosmic noise is relatively small.
- **Industrial manmade noise:** It is generated due to industrial or human activities.

Internal noise : Generated within any active or passive deices used in communication.

- **Thermal noise or white noise / Johnson noise:**
  - The noise which is generated within any resistive component is refered to as thermal noise.

- It is due to the random motion of electrons and holes inside any electronic component. (The free electrons inside the conductor posses the kinetic energy and it results in collision with other particles and any imperfections in the structure. The noise component generated due to this random motion is called as thermal noise).

It is called white noise because noise power is distributed uniformly over the entire frequency range. Spectrum of white noise is flat and having a constant value No independent of frequency.

Noise power is directly proportional to temperature and bandwidth.

$No \propto T$; $T \rightarrow$ Temperature

$No \propto B$; $B \rightarrow$ Bandwidth

$No \propto TB$

$No = kTB$ (watts)

k is the Boltzmann constant ($1.38*10^{-23}$J/K)

Noise power in dB= $10 \log_{10} kTB$

Noise power in dBm= $10 \log_{10} (kTB/10^{-3})$

(dBm= power in dB w.r.t one milliwatt)

(use standard temperature/ room temperature as 17^oC/290K)

RMS noise voltage for thermal noise

$$V_N^2 = 4kTBR$$

Noise power, $N= V_N^2/4R$

- **Shot noise :**

  Generated due to random arrival of electrons or holes at the output of any amplifying device such as BJT or FET . The random fluctuations or random noise that accompanies the direct current crossing the potential barrier is the shot noise. Also called as transistor noise . Active in nature.

  Shot noise current,

  $$I_n^2= 2q\, I_{dc}^2\, B \text{ (Ampere}^2\text{)}$$

  $q \rightarrow$ charge of e^- ($1.6*10^{-19}$ C)

  $B \rightarrow$ Bandwidth in Hz

- **Partition noise:**
  Partition noise occurs when current has to divide between 2 or more electrons and there occurs some fluctuations in division. Amount of partition noise less in diode than transistors . Power response noise is also flat in nature.
- **Flicker noise:**
  - At low frequencies a component of noise appears whose power decreases as frequency increases.
  - In semiconductors, flicker noise is due to variations in carrier densities (electrons and holes) which in turn gives rise to variation in conductivity.
  - The mean square voltage of flicker noise is also directly proportional to square of direct current.
- **Burst noise:**
  Low frequency noise observed in BJT and the noise is present in an audio system produces popping sound , so it is also called as popcorn noise. Power distribution of burst noise will decrease as frequency increases .

Signal to Noise Ratio (SNR)

Defined as the ratio of signal power to noise power **.**

i.e, $\mathbf{SNR = P_S/P_N}$

in dB , $\mathbf{SNR = 10 \log_{10} (P_s/P_N)}$

$\mathbf{SNR(dB) = 20 \log (V_s/V_N)}$ where $P=V^2/R$

Noise Factor (F) and Noise Figure(NF)

- Noise factor is defined as ratio of input SNR to output SNR.

Noise Factor F = input SNR / output SNR

- <u>Noise Figure is the noise factor expressed in decibels</u>.

$$NF_{(dB)} = 10 \log_{10} F$$

$$NF_{(dB)} = \text{input } SNR_{(dB)} - \text{output } SNR_{(dB)}$$

For an **ideal noiseless channel, i/p SNR = o/p SNR so F=1 and NF = 0 dB**

<u>Noise Factor for cascaded amplifier (**Friis Formula)**</u>

$F = (AN_i + N_d)/AN_i$ **Ni= input signal power**

Nd= internal noise

A= power gain

Nd=(F-1) AKTB (amount of noise added internally by the amplifier)

<u>Output noise power</u>, **No = FAKTB**

Equivalent Noise Factor ,

$$F = F_1 + (F_2-1)/A_1 + (F_3-1)/A_1A_2 + ... + (F_n-1)/A_1A_2.....A_{n-1}$$

<u>Noise figure(dB)</u>

$$NF(dB)= 10 \log F$$

$A = A_1A_2A_3........A_n$

<u>Equivalent Noise Temperature (T_e)</u>

$$F= 1 + Te/T$$

Equivalent Noise Temperature for Cascaded Amplifier

$$T_e = T_{e1} + T_{e2}/A_1 + T_{e3}/A_1A_2 + ... + T_{eN}/A_1A_2........A_{N-1}$$

MODULE 2

AMPLITUDE MODULATION

Sinusoidal Modulation:-

Signals involved in AM modulation

1. Message signal m(t)= $Amcos2\pi fmt$
2. Carrier signal c(t)=$Accos2\pi fct$

   Equation of AM signal :-

   VAm(t)=$Ac(1 + \mu cos2\pi fmt)cos2\pi fct$

   µ is the modulation index , $\mu = \frac{Am}{Ac}$

   Amplitude modulated wave consists of

   VAm(f)=Ac/2[$\delta(f - fc) + \delta(f + fc)] + \mu Ac/4[\delta(f - fc - fm) + \delta(f + fc + fm)] + \mu Ac/4[\delta(f - fc + fm) + \delta(f + fc - fm)]$

   Where fc+fm :- upper side band of the spectrum

   fc-fm :- lower side band of the spectrum

   Bandwidth of AM signal :-2fm

   Modulation index µ=(Vmax-Vmin)/(Vmax +Vmin)

   THREE TYPES OF MODULATION DEPENDING ON µ

   1. If µ<1:Undermodulation

      Gives an AM waveform without distortion , Am < Ac
   2. If µ=1: Critical modulation

      V max = Ac+Am

      =2Ac

      Vmin =Ac-Am=0
   3. µ>1: Over modulation, envelope shape get differs from the actual message signal.

      For proper modulation and demodulation , µ should be equal to1

      Power of an AM signal :-Pt = Pc$[1 + \frac{\mu^2}{2}]$

      Efficiency : - $\varphi = \frac{\mu^2}{\mu^2+2}$

      For µ=1, $\varphi \max = 33.3\%$

      Current for Am signal , It=Ic$\sqrt{1 + (\mu^2/2)}$

NON SINUSOIDAL MODULATION

Message signal:- m(t)=Am1cos2πfm1t+Am2cos2πfm2t
Carrier signal:- c(t)=Accos2πfct
VAm(t)=$Ac[1+\mu 1cos2\pi fm1t+\mu 2cos2\pi fm2t \ldots.]cos2\pi fct$

Bandwidth = 2fm(max)

Power of the signal :- $Pt=Pc[1+\frac{\mu 1^2+\mu 2^2}{2}]$
Where $\mu=\sqrt{\mu 1^2+\mu 2^2+\cdots \mu n^2}$

AMPLITUDE MODULATOR CIRCUITS

1. Low level AM modulation
2. High level AM modulation

LOW LEVEL AM MODULATION :-

- Also called as emitter modulation
- Operates in class A amplifier
- Case1: when no message signal is applied
  Act as a linear amplifier , Vo=Aq.c(t)
- Case2: when message signal is applied
  Voltage gain Av=Aq[1±μ]
  Avmax =2Aq , Avmin =0
- Operates in non linear pattern

  HIGH LEVEL MODULATION
  Also known as collector modulation
- Case1:-
  Output voltage Vo=Vcc-IcZ
  Vo(max)=Vcc , when Ic=0
  Vo =0 , when Ic is maximum
  - Operates in class C power amplifier

- Has efficiency about 80%

- Case2:-
  Output voltage Vo=2Vcc
  In this LC tank circuit is used to avoid distortion.

AM TRANSMITTERS

1. Low level AM transmitters :-Linear class A or classB push pull amplifiers are used
   - Impedance matching is achieved by coupling network
   - Used in low power low capacity system
   - Applications:- Walkie- talkie,Intercom,pagers
2. High level AM transmitters :-
   - High power modulating signals and class C power amplifiers is used

AM DEMODUALTORS

- Called as peak detector or envelop detector
- Uses germanium diode 0A79 with cut in voltage 0.3
- Therefore it is called as diode detector
- Two types of detection occurs in demodulators
  - Rectifying distortion
  - Diagonal distortion

Condition for RC time constant to avoid distortion

$$\left(\frac{1}{fc}\right) \ll RC \ll \left(\frac{1}{fm}\right)$$

The maximum frequency of modulating signal that can be demodulated using envelop detector without any distortion is fm (max)=($\sqrt{\frac{1}{\mu^2}-1})/2\pi RC$)

If μ =1, fm (max)=0

So 90% modulation occurs

If μ =$1/\sqrt{2}$, fm (max)=$\frac{1}{2\pi RC}$

In this case , 70.7% modulation occurs.

MODULE 3

- Drawbacks of conventional double sideband
- Carrier power contribution is very high ,appx 66.6% of total power if μ =1.
- $P_t = P_c * (1+\mu^2/2)$.
- Bandwidth=2fm
- since no information is present in carrier it is a wastage of power.
- Same information is present in upper and lower side band ,it uses twice as much bandwidth when compared with SSB.
- Power conservation is more since only one side band is used
- Pt=Pc(μ2/4)
- Bandwidth=fm
- Allows multiplexing of more signals because frequency spectrum is crowded.
- No selective fading occurs since no carrier is absent in ssb.
- Noise is reduced to half cause N=kTB.
- complex recievers are required for reception
- since no carrier signal, envelope detector cannot be used.

- Carrier recovery circuit is to be designed which is costly.
- Require more complex and accurate timing circuitry.
- SSB generation methods are filter method, phasing or phase shift method and weavers method.
- In filter method low freq crystal oscillator is used because high frequency side band filters are difficult to design.
- Drawback of filter method is difficulty in designing sharp cut off filters.
- Phasing method doesn't uses any filters, uses only balanced modulators and phase shift circuits.
- Drawback of phase shift method is the modulating signal contains a wide band of frequencies of varying amplitude.so balanced modulator should produce double side band supressed carrier signal without any distortion.so we need to design a wide band phase shift network which is too difficult
- In weavers method,modulating signal is not directly passed to phase shift network. It is first modulated using a low freq audio signal and then it is modulated with high freq carrier signal.

MODULE 4

Angle Modulation:It is the process in which the total phase angle of the carrier wave is varied in accordance with the instantaneous value of modulating or message signal,while keeping the amplitude of the carrier constant.

Angle Modulation is of two types:

- Phase Modulation
- Frequency Modulation

Phase Modulation:It is the process in which phase angle is varied linearly with a baseband or modulating signal.

Expression of phase modulated wave is :s(t)=Acos[wct+kpx(t)]

Kp=Proportionality constant,Phase sensitivity

Instantaneous value of phase angle:Φi=wct+kpx(t)

Frequency Modulation: Instantaneous frequency is varied linearly with a message or baseband signal about an unmodulated carrier frequency wc.

- Instantaneous frequency is given by Wi= Wc+KfX(t),Kf=frequency sensitivity.
- Expressionfor frequency modulated wave:

S(t)=Acos[wct+Kf$\int \quad x(t)dt$]

Frequency Deviation:The max change in instantaneous frequency from average frequency is called frequency deviation.

- Commercial FM broadcast=75khz
- Television broadcast=25khz
- Modulation Index=Δf/fm
- Frequency deviation depends on amplitude of modulating signal.

Power Of FM Signal

- Total power =Pcarrier+PUSB+PLSB
- Unmodulated carrier power=Pt=Ac^2/2R
- After modulation the total power is distributed across all the side bands of spectrum.
- Power of FM signal is independent of modulation index.

COMPARISON OF AM AND FM

AM

- Amplitude remains constant
- Transmitted power is independent of modulation index.
- All the transmitted power is useful.
- FM receivers are immune to noise.
- Bandwidth is large.
- BW=2(Af+Fm)
- It is possible to operate several transmitters on same frequency.

FM

- Amplitude of AM wave will change with modulating signal.
- Transmitted power is dependent on modulation index.
- Carrier power and sideband power are useless
- Not immune to noise.
- Bandwidth is much less than amplitude modulation.
- BW=2fm
- Not possible to operate more channels on same frequency.

AM RECIEVERS

Antenna

RECIVER PARAMETERS

- **SELECTIVITY**

The ability of the receiver to select a particular range of frequencies and regect all other frequencies.

High Q factor filters are used to obtain better selectivity.

- **SENSITIVITY**

The minimum signal strength that should be maintained at the input of a receiver to get a reasonable output.

Depends on gain.

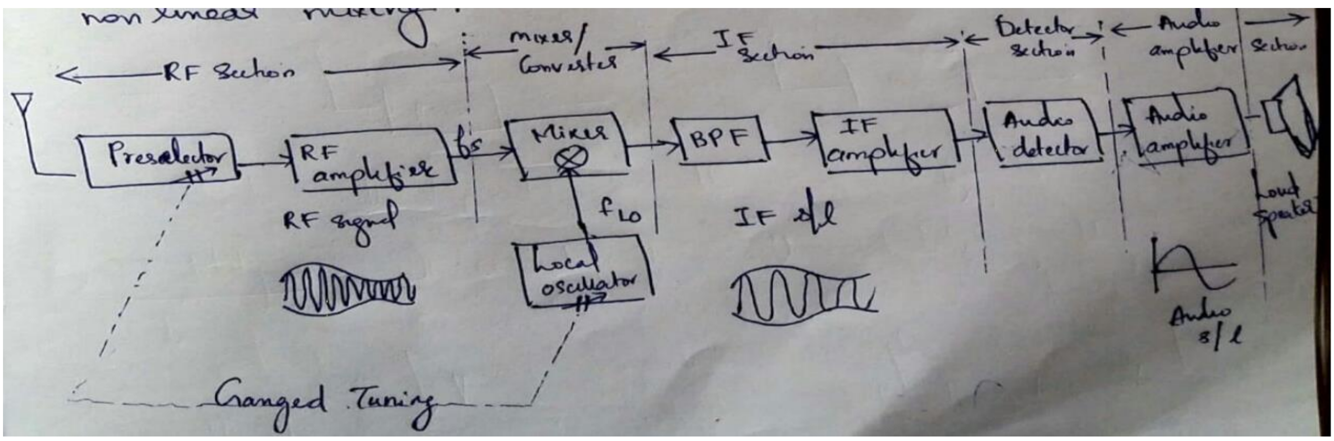
- **FIDELITY**

Defined as the ability of the receiver to produce an exact replica of original information.

- **DYNAMIC RANGE**

It is the difference in diabels between the minimum input level necessary to produce a reasonable output .

SUPER HETERODYNE RECIEVER



Superheterodyne receiver consist of 5 stages

- **RF section**: Consist of pre-selector and RF amplifier.It determines the selectivity.RF amplifies improve image frequency rejection.
- **Mixer/converter section** :Heterodyning process takes place.
- **IF section** :Consist of band pass filter and IF amplifier.
- **Detector section** :Convert IF signals back to original signal.
- **Audio amplifier section** :Consist of several cascaded amplifiers.

TUNING RANGE

Tuning of RF amplifier and oscillation is achieved by varying the capacitance or inductance of resonant circuit that act as a band pass filter.

Frequency tuning range **Rf=** $\frac{Fmax}{Fmin}$

Capacitance tuning range **Rc=**$\frac{Cmax}{Cmin}$ $,Rc = Rf^2$

TRACKING

It is the ability of local oscillator in a receiver to oscillate above or below the selected RF frequency by an amount equal to intermediate frequency throughout the entire frequency band.

Tracking error is defined as the difference between actual local oscillator frequency and desired local oscillator frequency.

PADDER TRACKING: Adding padder capacitor in series with oscillator capacitor .On adding Cp effective capacitance decreases and oscillator frequency increases.

Co=$\frac{Cp*Cosc}{Cp+Cosc}$

TRIMMER TRACKING: Adding a variable capacitor in series with the oscillator capacitor .As Co increases Flo decreases.

Co= $Cosc + Ct$

IMAGE FREQUENCY

Image frequency is frequency other than the selected RF carrier.

If Fsi is the image frequency $,Fsi = Flo + Fif$

$$Fsi = Flo - Fif$$

IMAGE FREQUENCY REJECTION RATIO

It is the numerical measure of the ability of the pre-selector to reject the image frequency.

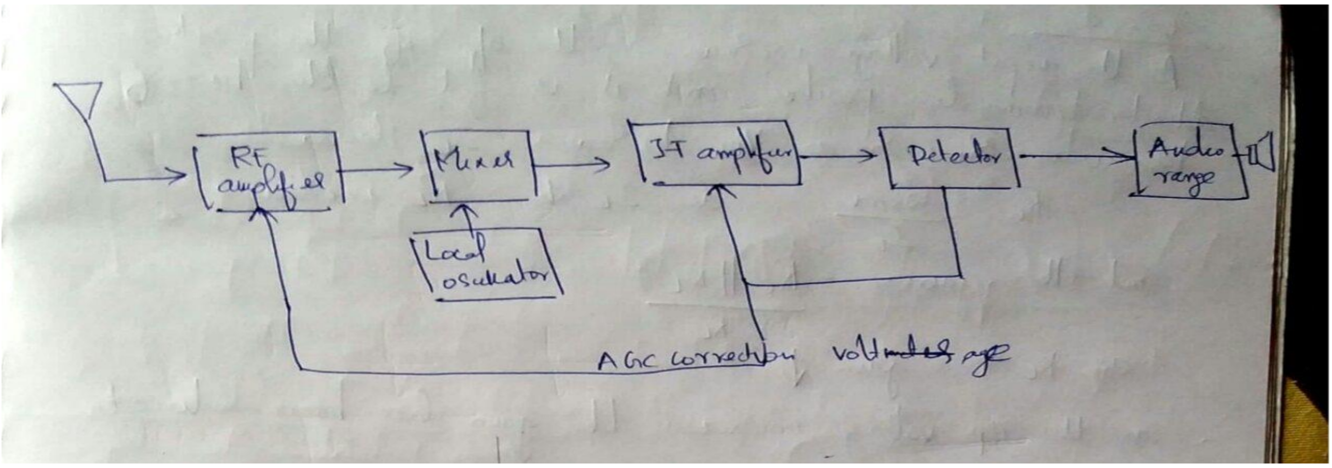
IFRR=$\sqrt{1 + Q^2 * P^2}$

P =$\frac{Fsi}{Fs} - \frac{Fs}{Fsi}$

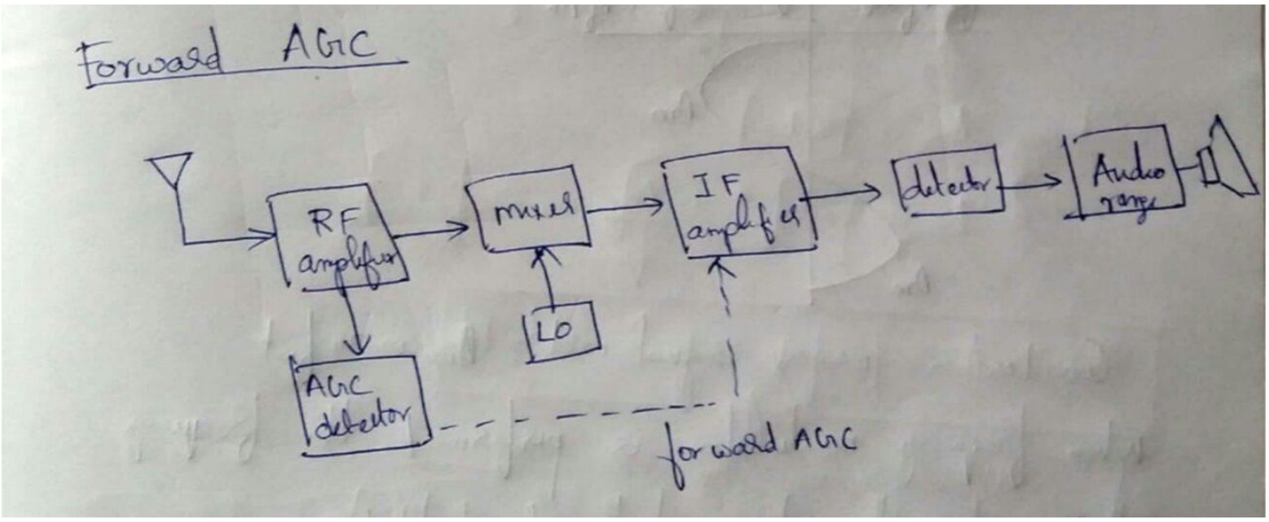
AUTOMATIC GAIN CONTROL

The AGC circuit is used for the minor variations in the received RF signals. It automatically increases the receiver gain for weak RF signals and automatically decreases the receiver gain for strong RF signal.

SIMPLE AGC



FORWARD AGC



MODULE 5

PHASE MODULATION

Phase modulation is a type of angle modulation in which phase of the carrier is varied according to variations in modulating signal.

Consider a carrier signal

$C(t) = A_c\cos(2\pi f_c t + \phi_c)$

After phase modulation,

$\Phi_{PM}(t) = \phi_c + k_p\, m(t)$

Where k_p is called as phase sensitivity.it is measured in rad/v.

The modulated signal can be represented as

$\boxed{S(t) = A_c\cos(2\pi f_c t + k_p m(t)}$

EQUIVALENCE BETWEEN FM AND PM

The angle of phase modulated signal is given by

$\theta(t) = 2\pi f_c t + k_p m(t)$

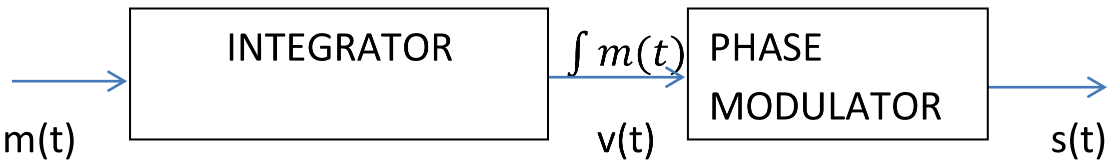
The instantaneous frequency of PM is given by

$fi_{PM} = (1/2\pi)*d\theta(t)/dt$

$fi_{PM} = f_c + (k_p/2\pi)*dm(t)/dt$

This equation implies that a PM signal is related to a FM signal.

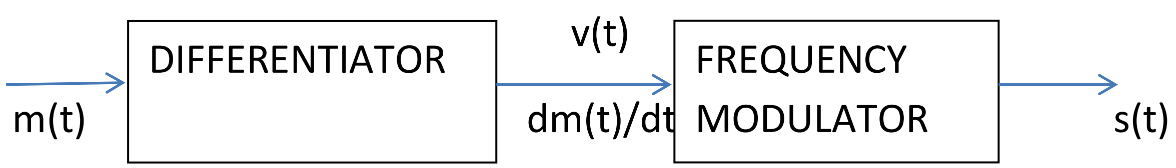
To obtain a frequency modulated signal from PM:



The general expression for a FM signal is given by

$$\boxed{S_{FM}(t) = A_c\cos 2\pi f_c t + k_p \int m(t)dt}$$

To otain a phase modulated signal from FM:



Sinusiodal Phase Modulation

$$\boxed{S(t) = A_C\cos(2\pi f_c t + \beta\cos 2\pi f_m t)}$$

where$\beta = k_p A_m$

The equqtion for FM and PM remains the same except for a phase shift of 90 degree for the modulating signal.

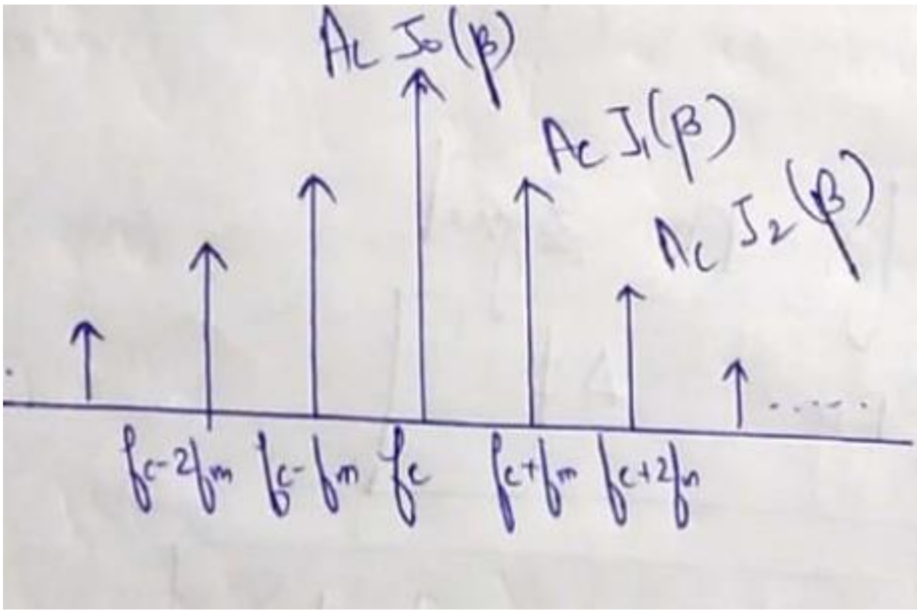
$\beta_{FM} = \Delta f / fm$

$\beta_{FM} = K_f . A_m / f_m$

Bandwidth for a PM signal is

BW = 2(β + 1)f_m.

Power, $P_C = A_C$^2/2R



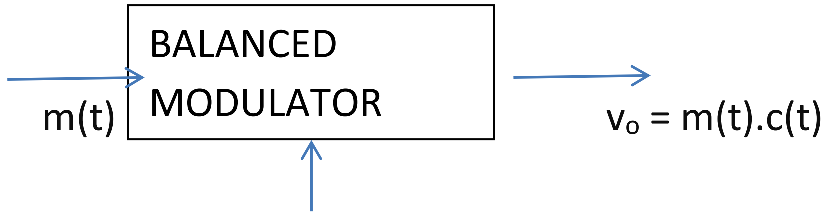
DIGITAL PHASE MODULATION

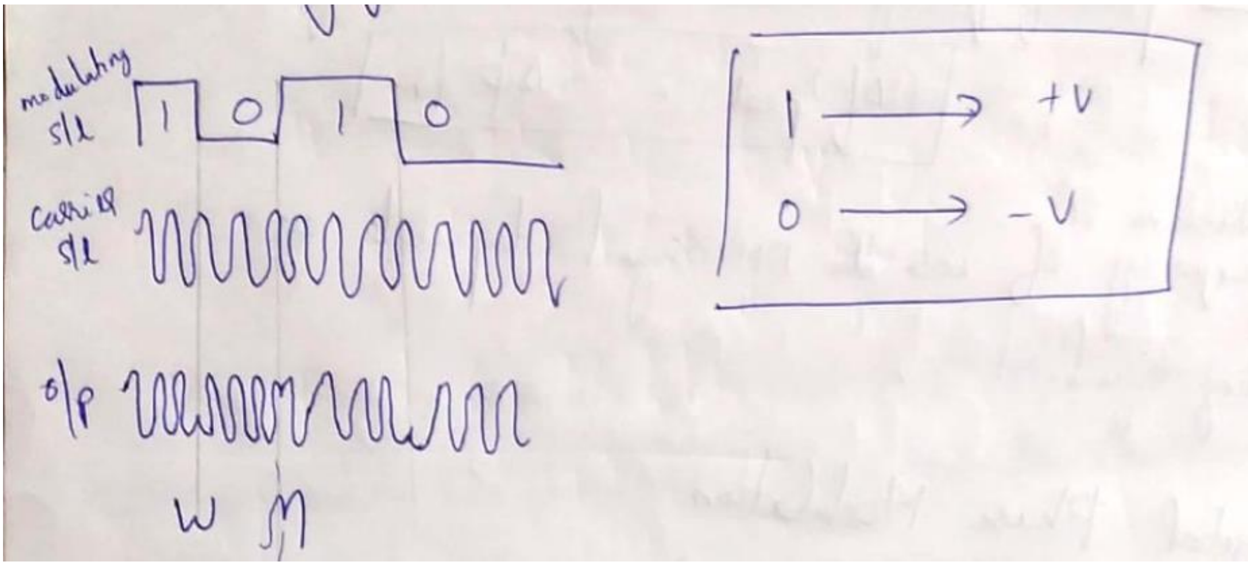
Digital phase modulation uses digital output (sequences of 1's and 0's)as modulating signal.

i.e., 1 is represented by +v volt

0 is represented by –v volt.

To obtain the phase modulated signal the digital signal and carrier signal is fed to a balanced modulator.

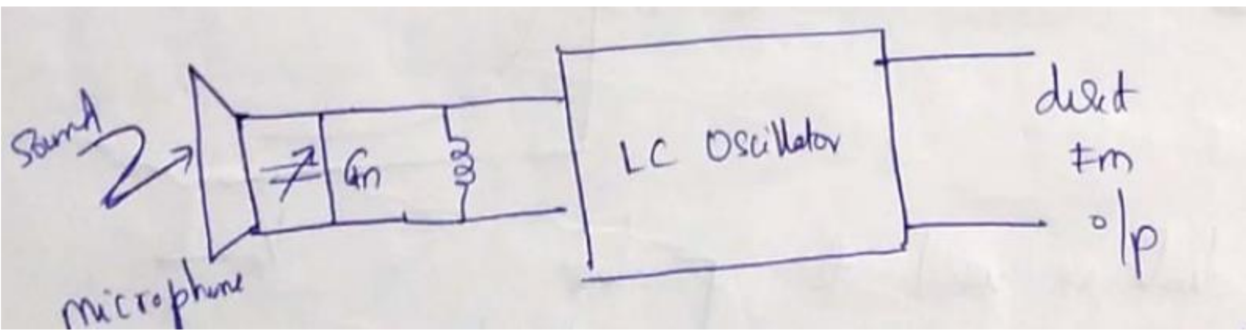




ANGLE MODULATOR CIRCUITS

The angle modulated circuits can be either direct modulators or indirect modulator circuits.Direct modulator circuits directly produces a frequency modulated signal where as an indirect frequency modulator generates FM from a PM signal.

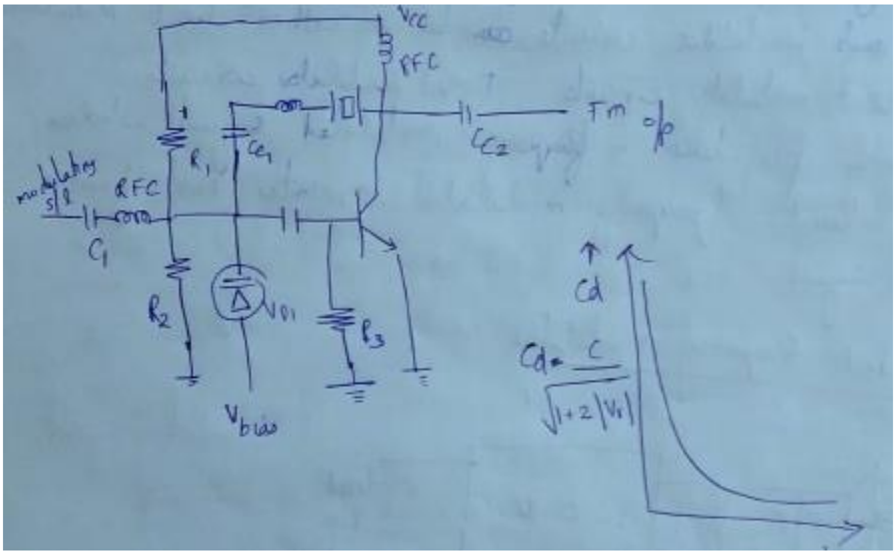
DIRECT FREQUENCY MODULATORS



- Capacitor microphone acts as the transducer to convert sound signal to electrical signal.
- As the amplitude increases the equivalent capacitor decreases $C_{eq} = \frac{\varepsilon A}{d}$.

- The frequency of the output increases
  F = ½$\pi\sqrt{LC}$
- Thus the oscillator output frequency varies direct proportion to the modulating signal.

VARACTOR DIODE MODULATOR



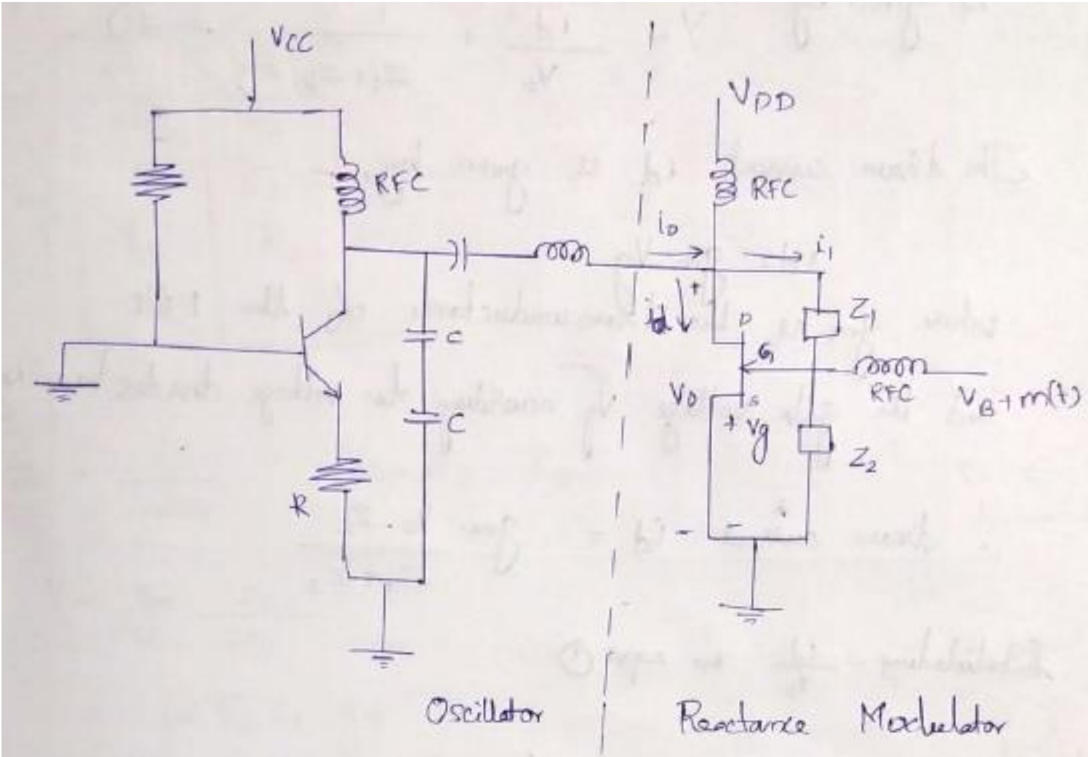
$$V_r \uparrow \longrightarrow C_d \downarrow \longrightarrow f \uparrow = \frac{1}{2\pi\sqrt{LC}\downarrow}$$

Conversly for the negative variation in modulation signal

$$V_r \downarrow \longrightarrow C_d \uparrow \longrightarrow f \downarrow = \frac{1}{2\pi\sqrt{LC}\uparrow}$$

Hence vary a varactor diode we have obtained a frequency modulated signal.

FET REACTANCE MODULATOR



- The net admittance of the reactance modulator is given by

$$Y = \frac{i_d}{V_o} + \frac{1}{Z_1 + Z_2}$$

- The drain current i_d is given by

  $I_d = g_m V_g$

  g_m is the transconductance of the FET.

  $V_g = V_o . Z_2 / Z_1 + Z_2$

$$\therefore \text{ drain current } i_d = g_m \cdot \frac{V_o \cdot Z_2}{Z_1 + Z_2}$$

- Admittance

$$Y = j\omega z \cdot g_m$$

- $C_{eq} = \tau . gm$
-

Z_1	Z_2	Z	condition	L_{eq} or C_{eq}
$\frac{1}{j\omega C_1}$	R_2	R_2C_1	$\omega\tau \ll 1$	$C_{eq} = g_m \cdot \tau$
R_1	$\frac{1}{j\omega C_2}$	R_1C_2	$\omega\tau \gg 1$	$L_{eq} = \frac{\tau}{g_m}$
R_1	$j\omega L_2$	$\frac{L_2}{R_1}$	$\omega\tau \ll 1$	$C_{eq} = g_m \cdot \tau$
R_2	$j\omega L_1$	$\frac{L_1}{R_2}$	$\omega\tau \gg 1$	$L_{eq} = \frac{\tau}{g_m}$

-

$$\boxed{f_1 = f_0 \cdot \frac{\sqrt{LC_0}}{\sqrt{(LC)_m}}}$$

The frequency after modulation is inversely proportional to the LC value of the circuit after applying modulating signal.

FM TRANSMITTER

- Transmission frequency range :-88-108Hz

- Combination of multipliers and mixers are used to reach the transmission frequency.

- For frequency multiplication, the modulation index B and instantaneous frequency deviation are multiplied by n.

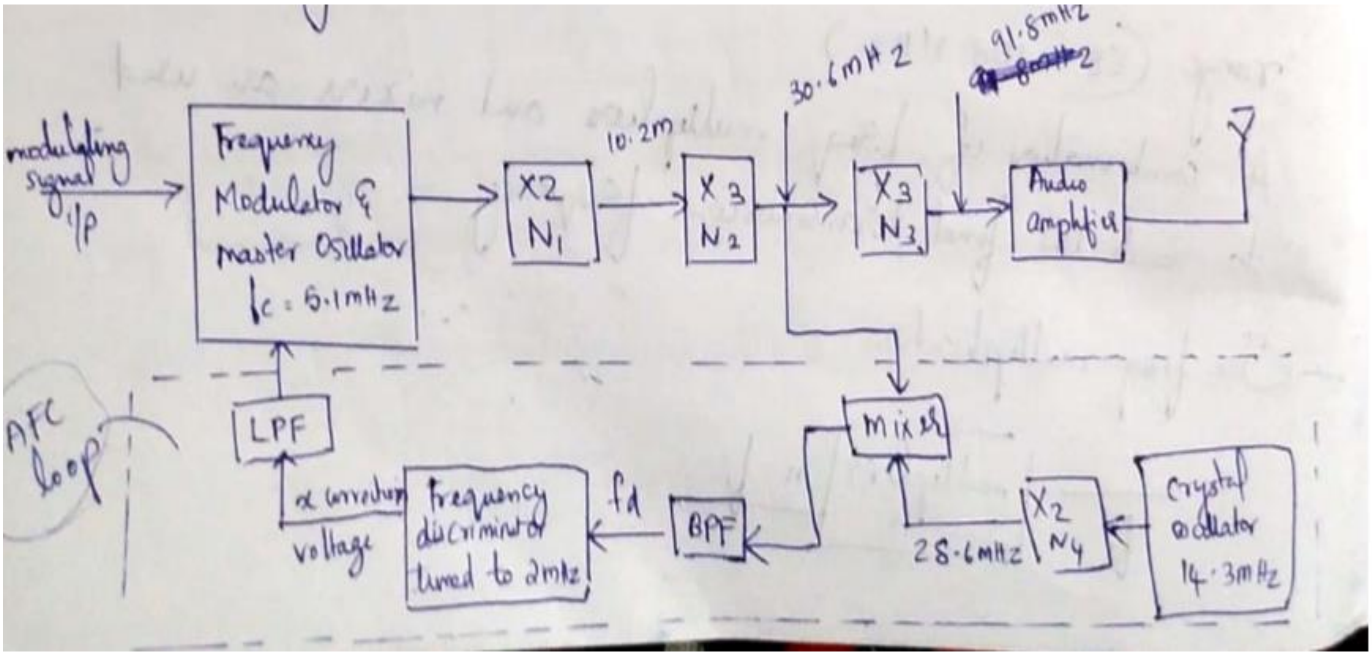
$$\beta' = \frac{\Delta f'}{fm} = n.\frac{\Delta f}{fm} = n.\beta$$

- The carrier frequency is downconverted into an intermediated frequency.so the modulation index and frequency deviation remains unchanged.

$$fc + \Delta f - fLO = fIF + \Delta f$$

$$fc - fLO = fIF$$

CROSBY DIRECT FM TRANSMITTER



- The frequency modulator circuit can be either varactor diode modulator or FET reactance modulator.
- Carrier frequency at the output of the modulator is 5.1MHz.
- Total multiplication factor is 18.
- Therefore total output frequency is 5.1*18 = 91.8MHz.
- Modulation index at the output is 18B.

AUTOMATED FREQUENCY CONTROLLER (AFC) LOOP

- Used to achieve stable transmitting frequencies.
- To perform AFC ,the carrier signal ath the output of second multiplier is fixed with a stable crystal frequency and the output of the mixer is fed to a frequency discriminator circuit.
- Frequency discrimibator is a frequency selective circuit.
- Output from the frequeny doubler circuit -28.6MHz
- It mixes with output of second multiplier to produce a difference frequency $fd == 30.6 - 28.6 = 2MHz$.

- Output of discriminator circuit is propotional to the difference between fd and the resonant frequency of discriminator.
- Discriminator responses to the low frequency changes in the carrier centre frequency .
- The low pass filter will not respond to the frequency deviation produced by modulating signal.if the discriminator responds to frequency deviation ,the feedback loop would cancel the deviation and remove the modulation from the output.this is called WIPE OFF.
- Frequency deviation for FM broadcast -75 KHz.
- Frequency deviation at the output of frequency modulator

$$\Delta f' = \frac{75KHz}{18} = 4166.7Hz.$$

$$\beta' = \frac{\Delta f'}{fm} = \frac{4166.7}{15*10^3} = 0.2778$$

$$\beta = \beta' * 18 = 5$$

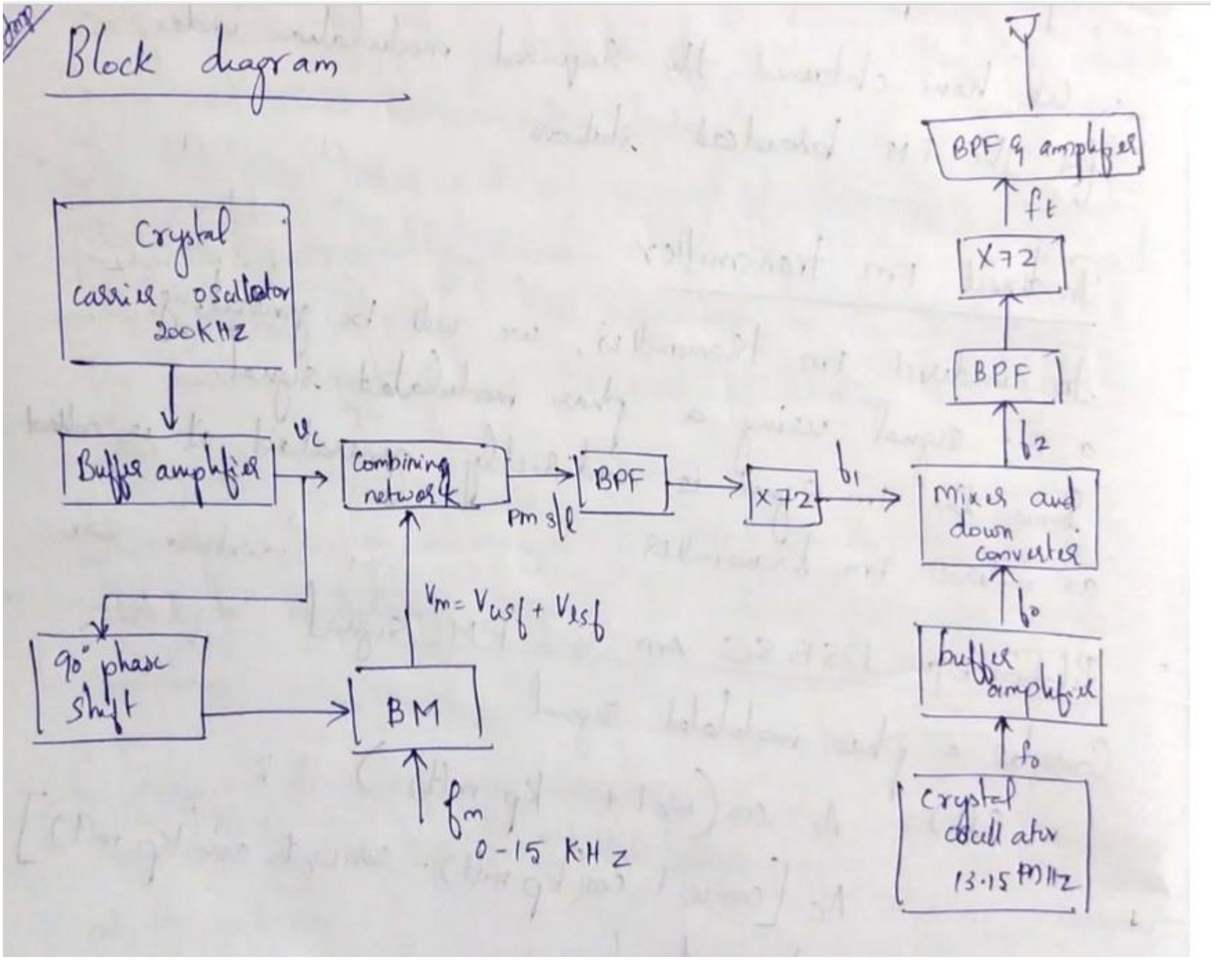
INDIRECT FM TRANSMITTER

FM signals are generated using a phase modulated signal.since the FM signal is indirectly generated ,it is called as indirect FM transmitter.

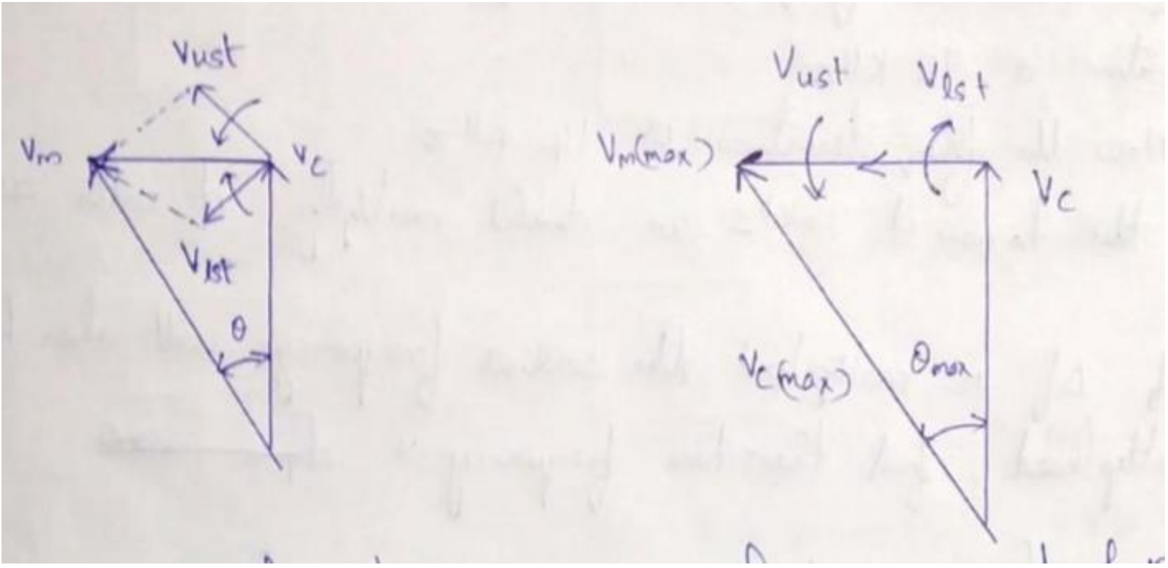
RELATION BETWEEN DSBSC AM & PM SIGNAL

$$s(t) = Ac\ [\cos \omega ct - kp\ m(t). sin\omega ct]$$

- We can obtain a phase modulated signal using a balanced modulator output.
- Carrier has a phase shift of 90 degree.

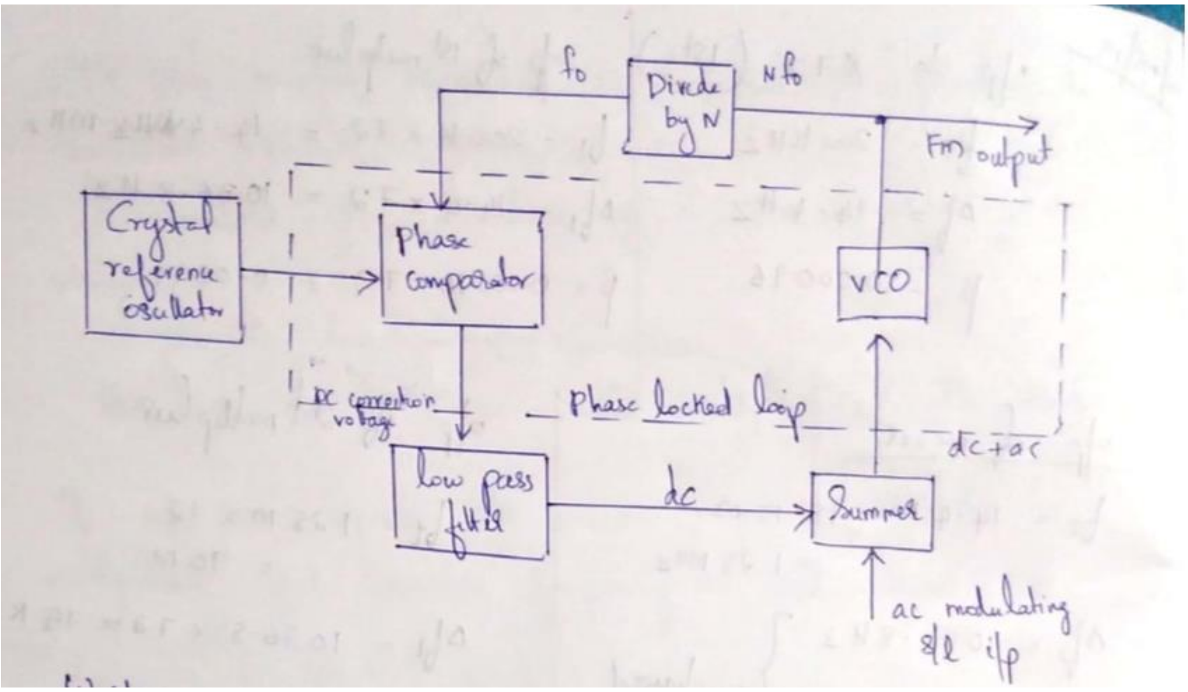


- Output of the combining network is a low modulation index phase modulated wave.



- Phase deviation=tan^-1 (vm/vc)

DIRECT FM GENERATION USING PLL



- Used to produce high modulation index.
- Output of pll is a DC voltage propotional to the difference between the two frequencies.

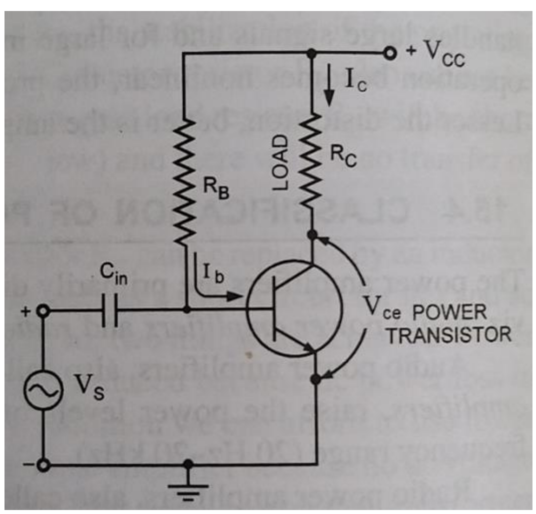
MODULE 05

Comparative study of power amplifiers:

PARAMETER	CLASS A	CLASS B	CLASS AB	CLASS C
CONDUCTION ANGLE	360°	180°	180°-360°	<180°
BIASING	ACTIVE REGION	CUT OFF	BETWEEN CLASS A & CLASS B	BELOW CUT OFF
DISTORTION	VERY LOW	HIGH	MEDIUM	VERY HIGH
CONVERSION EFFICIENCY	25-50%	78.5%	CLASS B>CLASS AB CLASS A<CLASS AB	85%

Conduction angle: The time during which the transistor conducts when an input sinusoidal signal is applied in a **power amplifier**

CLASS A SERIES FED POWER AMPLIFIER:



- Output current flows for **360°** of the input signal
- Transistor remains **forward biased** throughout the input cycle
- **Disadvantage** – efficiency is very low - 25% (remaining 75% is considered to be lost as heat)

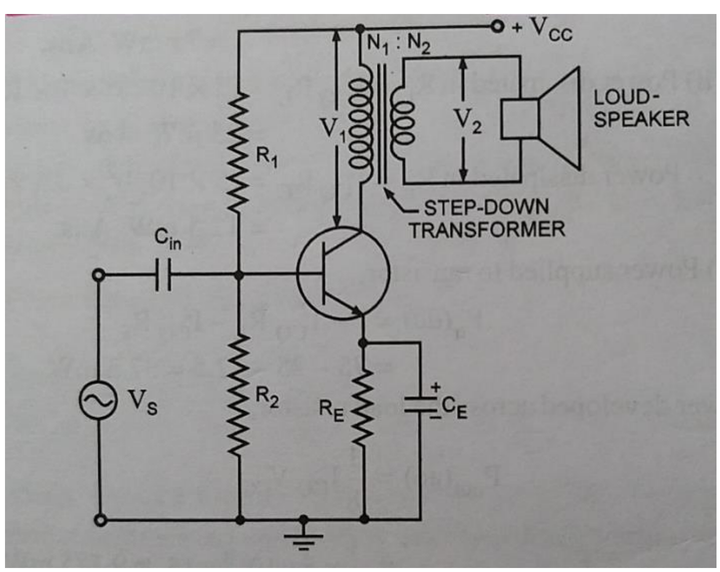
15.6.2. Collector Efficiency. The collector efficiency of a transistor is given as

$$\eta_{collector} = \frac{\text{Average ac power output, } P_{out\,(ac)}}{\text{Average dc power input to the transistor, } P_{tr\,(dc)}}$$

15.6.3. Power Efficiency. A measure of the ability of an active device to convert the dc power of supply into the ac (signal) power delivered to the load is called the *power* or *conversion* or *theoretical efficiency*. By definition the efficiency is

$$\eta = \frac{\text{AC power delivered to the load, } P_{out\,(ac)}}{\text{Total power drawn from dc supply, } P_{in\,(dc)}}$$

TRANSFORMER COUPLED CLASS A POWER AMPLIFIER:



- This arrangement permits **impedance matching** (power transferred from the power amplifier to the load will be maximum only if the amplifier output impedance equals the load impedance RL)
- Maximum efficiency-50%
- Disadvantage:
    - Low output power
    - Low collector efficiency (about 30%)

Thus the ratio of the transformer input and output resistances varies directly as the square of the transformer turn ratio :

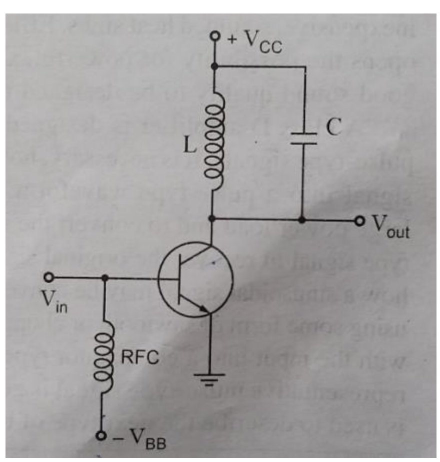
$$\frac{R'_L}{R_L} = \left(\frac{N_1}{N_2}\right)^2 = a^2$$

$$\text{or} \quad R'_L = a^2 R_L \qquad \text{...(15.16)}$$

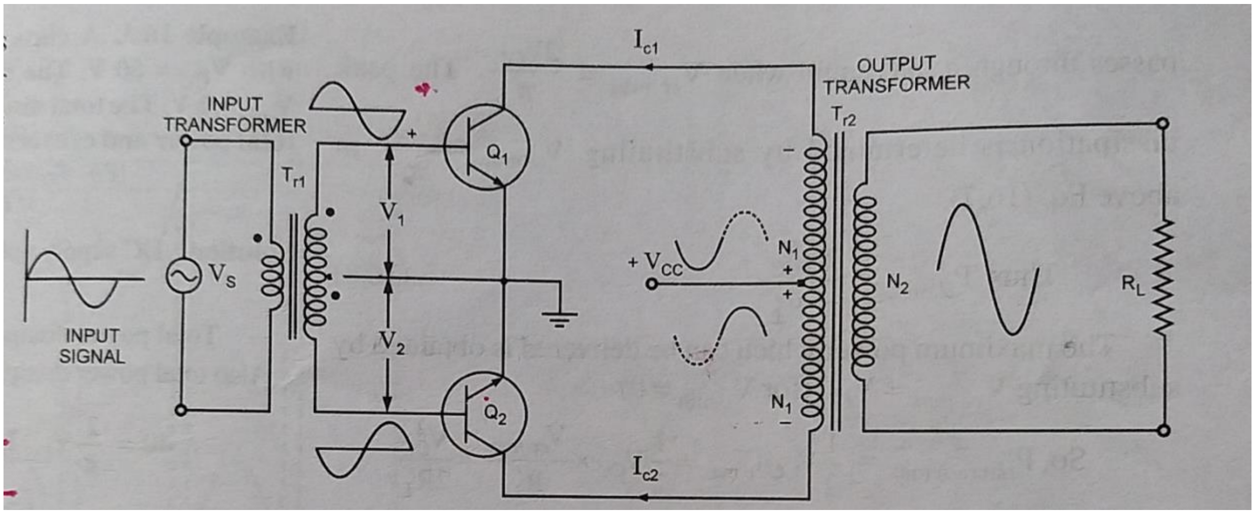
where a is ratio of primary to secondary turns of step-down transformer, R_L is the resistance of load connected across the transformer secondary and R'_L is effective resistance looking into the transformer primary.

CLASS C POWER AMPLIFIER:

- Conduction angle is less than 180° of the input signal cycle
- Power loss less compared to class A, AB power amplifiers
- Higher harmonic distortions can be eliminated by using tuned circuits
- Power conversion efficiency – 85%



CLASS B PUSH PULL AMPLIFIER:



- Consists of a complementary pair of transistors
- Conduction angle-180°
- Higher operating efficiency -78.5% (as no power is drawn by the circuit under zero signal condition)
- Preferred in systems where power supply is limited
- Automatic cancellation of all even order harmonics

DISADVANTAGES:

- Harmonic distortions higher
- Self-bias cannot be used
- Supply voltages must have good regulation

CLASS AB PUSH PULL AMPLIFIERS:

- Distortion is less than class B push pull amplifier but more than class A push pull amplifier
- Drawbacks:
  - Lower conversion efficiency
  - Wastage of standby power

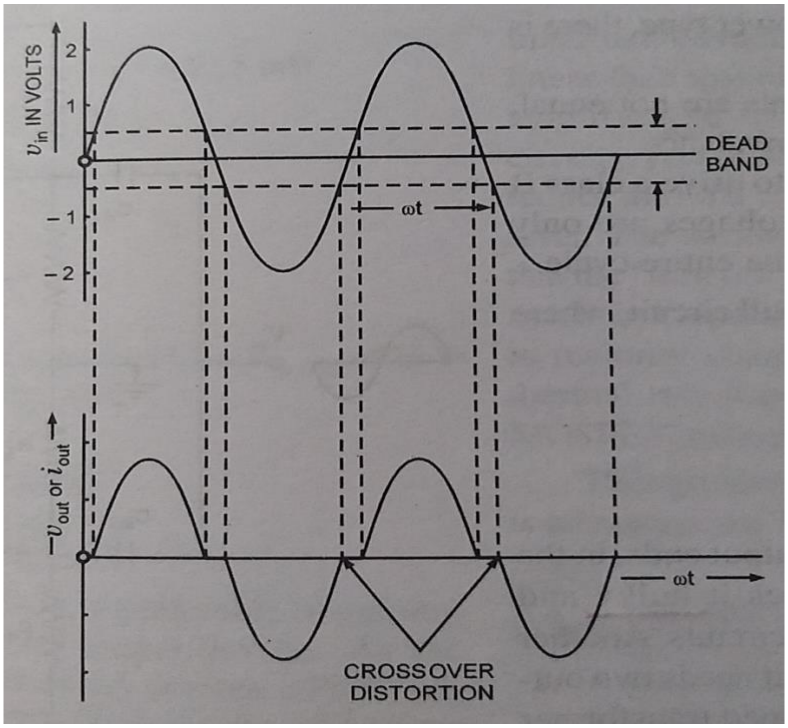
DISTORTIONS IN POWER AMPLIFIERS:

1. HARMONIC DISTORTION:

- Caused due to nonlinearity of the active device employed for amplification
- Harmonic distortion increases from class A operation to class C operation

- Distorted periodic sine wave contains other frequency components apart from the fundamental frequency called as **harmonic frequencies**

2. CROSSOVER DISTORTION:

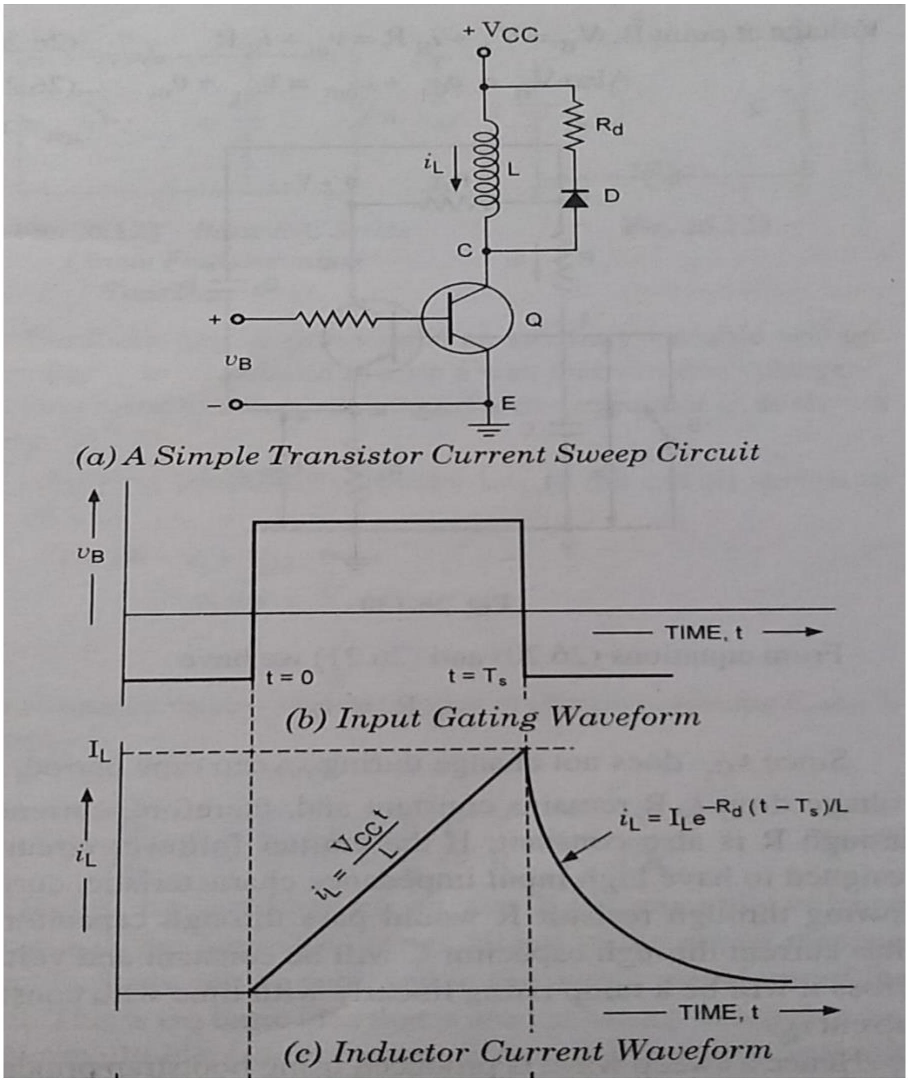


- Occurs as a result of one transistor cutting off before the other begins conducting
- It is so called because it occurs during the time operation crossover from one transistor to the other in the push pull amplifier
- To eliminate crossover distortion, it is necessary to add a small amount of forward bias to take the transistors to the average of conduction or slightly beyond.
    - This slightly lowers the efficiency of the circuit
    - There is a waste of standby power

SWEEP CIRCUITS:

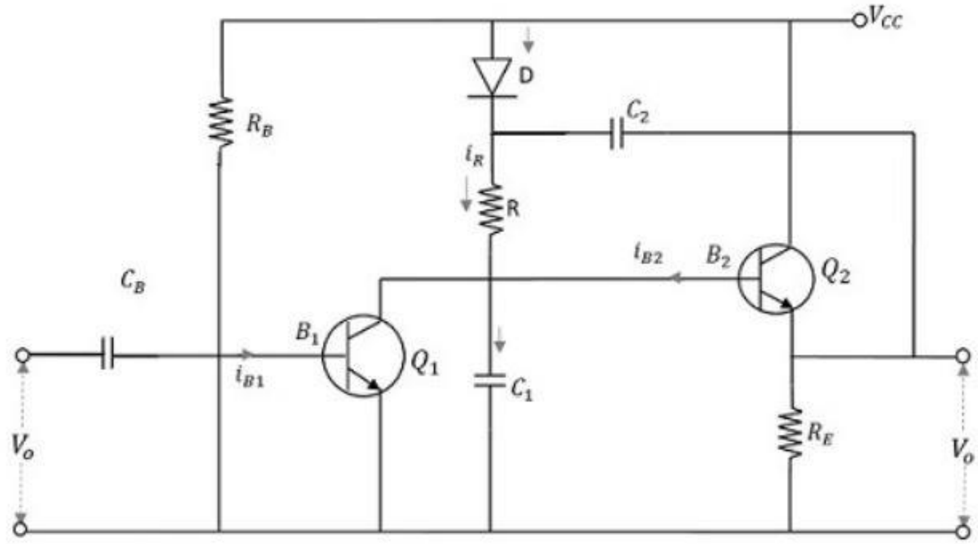
- Circuits which produce output waveform and a portion of it exhibits the linear characteristics of either voltage or current with respect to time is called as sweep circuits.
- The resulting waveform is called sweep waveform.

SIMPLE SWEEP CIRCUIT:

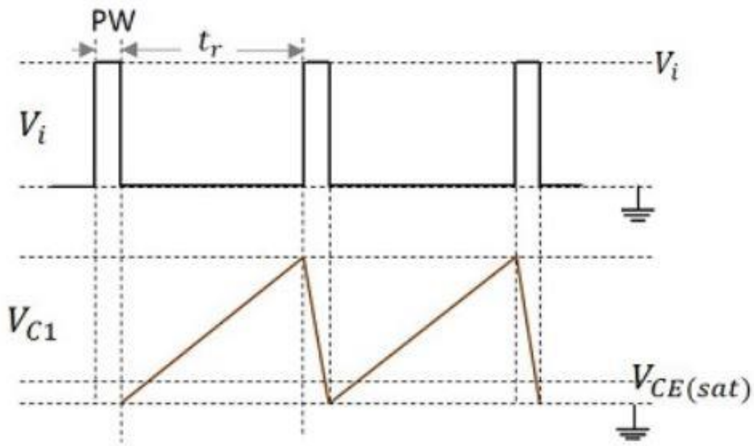


- For t<0, the transistor is in cut off
- At t=0, diode is reverse biased. The transistor conducts and goes into saturation and entire supply Vcc acts across the inductor. The inductor current increases linearly with time
- At T=Ts, the transistor is in cut off. The inductor current flows through diode and resistor until it decays to zero

BOOTSTRAP SWEEP CIRCUIT:

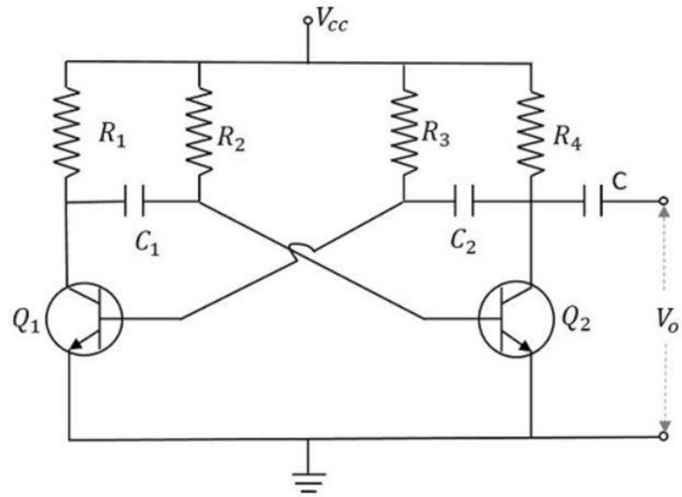


- A bootstrap sweep generator is a time base generator circuit whose output is fed back to the input through the feedback. This will increase or decrease the input impedance of the circuit. This process of **bootstrapping** is used to achieve constant charging current.
- Q_1 is ON and Q_2 is OFF. The capacitor C_2 charges to V_{CC} through the diode D.
- Then a negative trigger pulse is applied at the base of Q_1 which turns Q_1 OFF. The capacitor C_2 now discharges and the capacitor C_1 charges through the resistor R.
- The capacitor **C2** which helps in providing some feedback current to the capacitor C1 acts as a **boot strapping capacitor** that provides constant current.



ASTABLE MULTIVIBRATORS:

- Also called free running multivibrator
- Generates square wave without any external triggering
- Has 2 states; both of which are unstable
- Has specific amplitude and time period



The ON time of transistor Q_1 or the OFF time of transistor Q_2 is given by

$$t_1 = 0.69R_1C_1$$

Similarly, the OFF time of transistor Q_1 or ON time of transistor Q_2 is given by

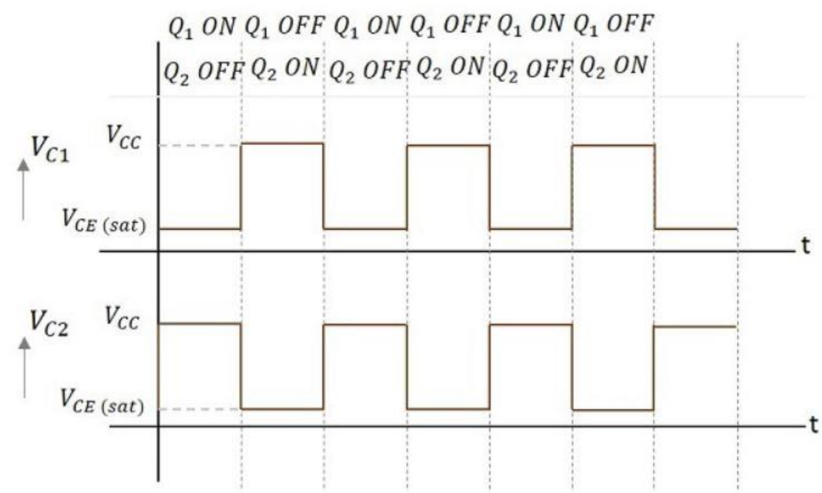
$$t_2 = 0.69R_2C_2$$

Hence, total time period of square wave

$$t = t_1 + t_2 = 0.69(R_1C_1 + R_2C_2)$$

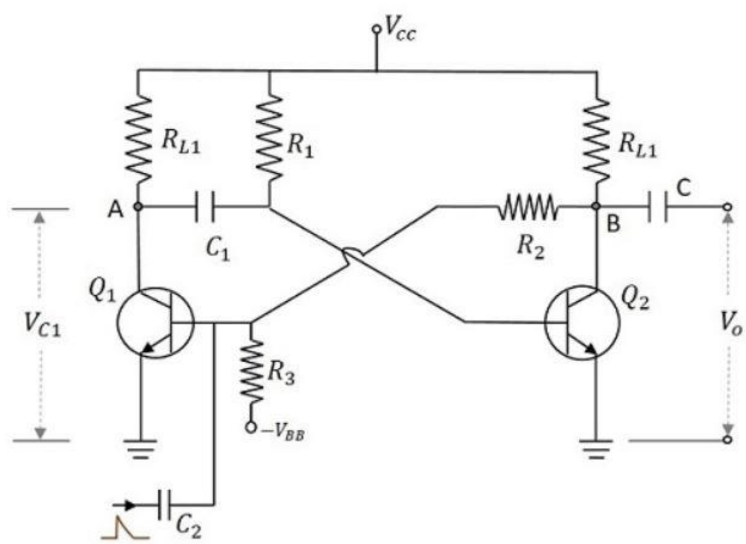
As $R_1 = R_2 = R$ and $C_1 = C_2 = C$, the frequency of square wave will be

$$f=1/t=1/1.38RC=0.7RC$$



MONOSTABLE MULTIVIBRATORS:

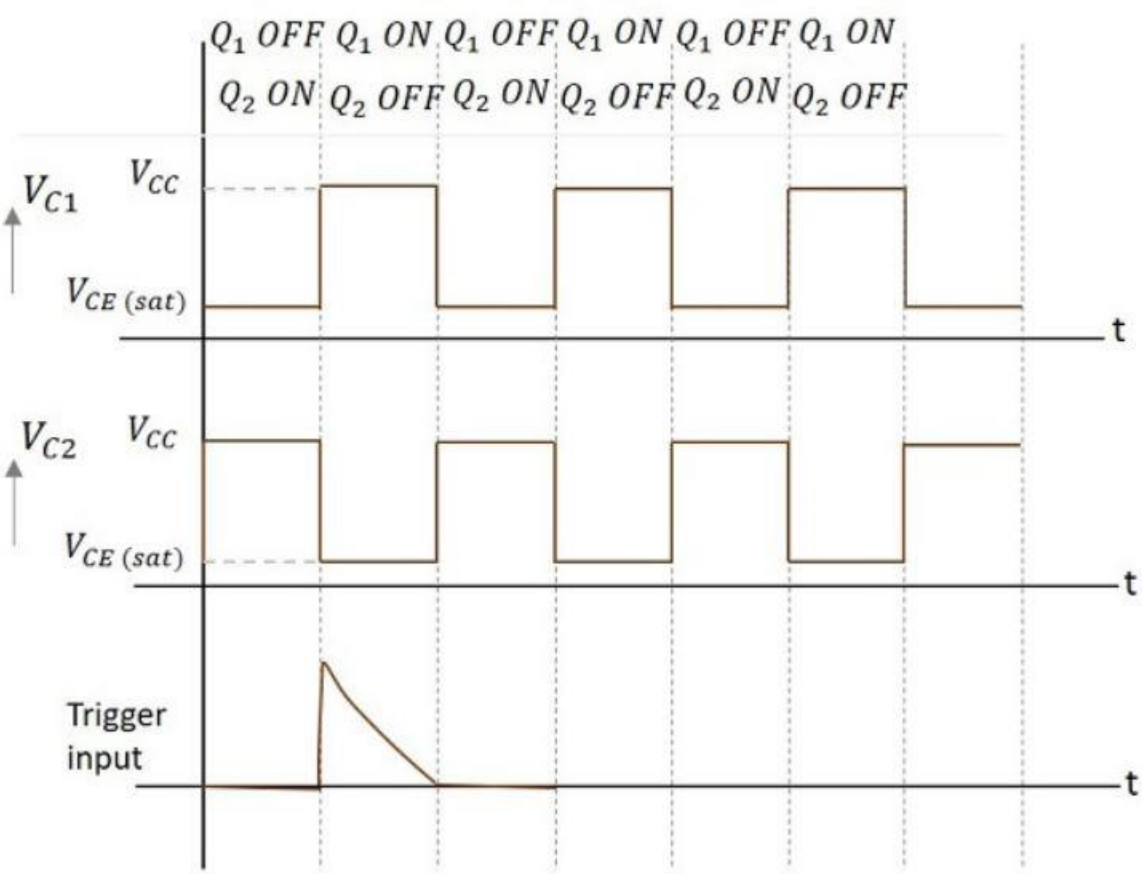
- Produce pulse waveform on application of external trigger
- Has 2 states; one of them is stable
- The major drawback is that the time between the applications of trigger pulse T has to be greater than the RC time constant of the circuit.



- When the circuit is switched ON, transistor Q_1 will be in OFF state and Q_2 will be in ON state. This is the stable state. C_1 gets charged.
- A positive trigger pulse applied at the base of the transistor Q_1 turns the transistor ON and turns OFF the transistor Q_2. The capacitor C_1 starts

discharging at this point of time. This is the **quasi-stable state or Meta-stable state.**

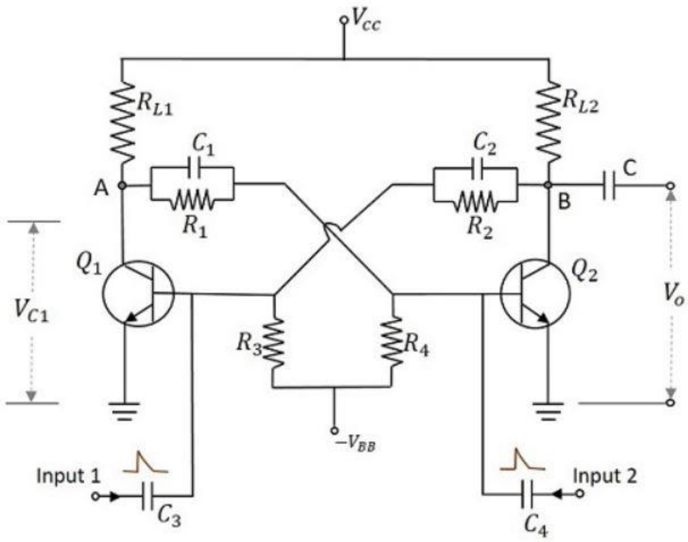
- The transistor Q_2 remains in OFF state, until the capacitor C_1 discharges completely. After this, the transistor Q_2 turns ON with the voltage applied through the capacitor discharge. This turn ON the transistor Q_1, which is the previous stable state.



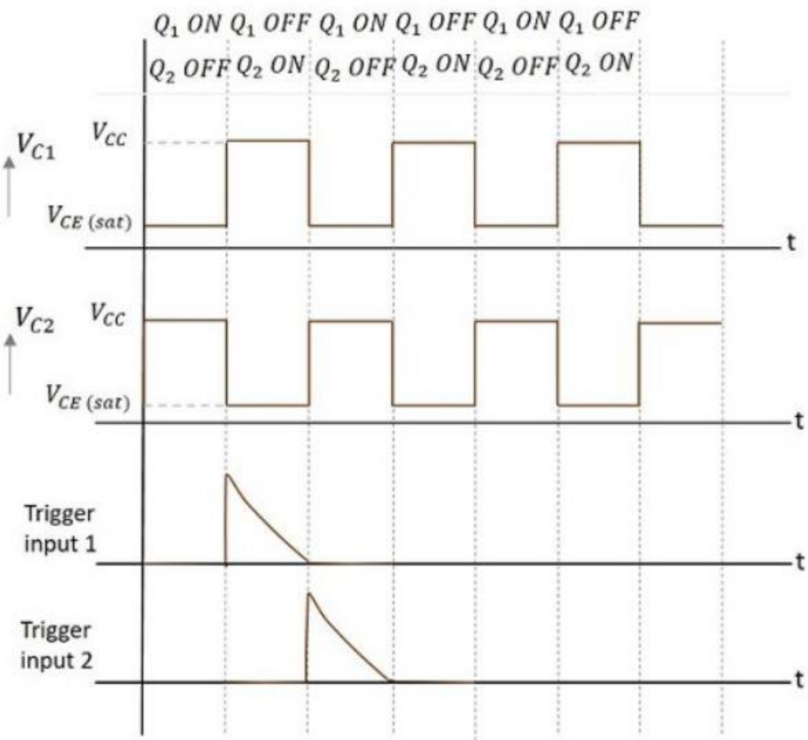
Time period, T=0.69R$_B$C

BISTABLE MULTIVIBRATORS:

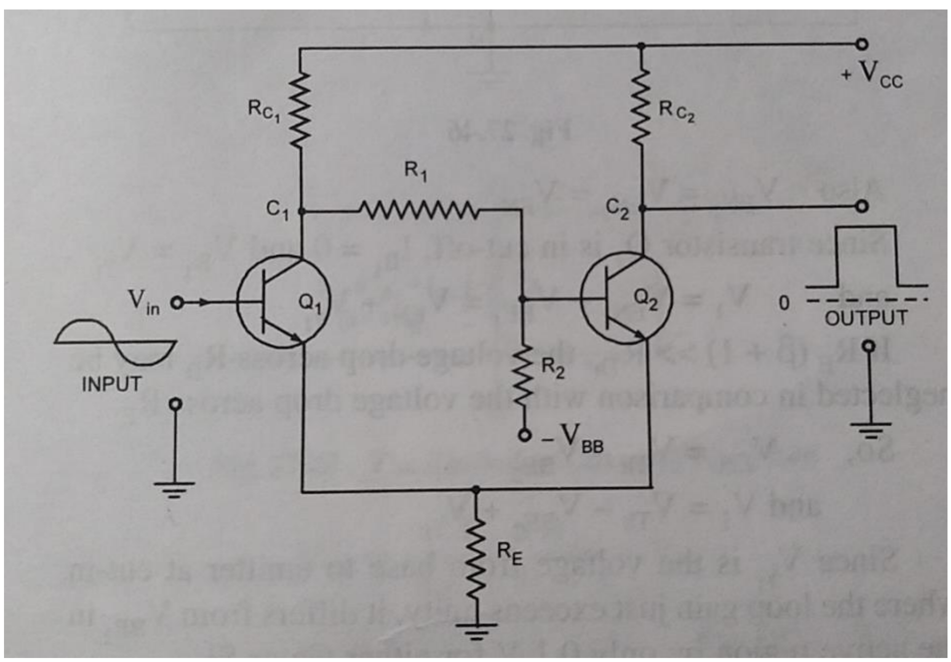
- Can be used as a flip flop
- Has 2 states both of which are stable
- To change state, trigger signal should be applied



- When the circuit is switched ON, Q_1 gets switched ON, while the transistor Q_2 gets switched OFF. This is a stable state of the Bistable Multivibrator.
- By applying a negative trigger at the base of transistor Q_1 or by applying a positive trigger pulse at the base of transistor Q_2, this stable state is unaltered.
- The collector voltage increases, which forward biases the transistor Q_2. The collector current of Q_2 as applied at the base of Q_1, reverse biases Q_1 and this cumulative action, makes the transistor Q_1 OFF and transistor Q_2 ON. This is another stable state of the Multivibrator.
- If this stable state has to be changed again, then either a negative trigger pulse at transistor Q_2 or a positive trigger pulse at transistor Q_1 is applied.



SCMITT TRIGGER:



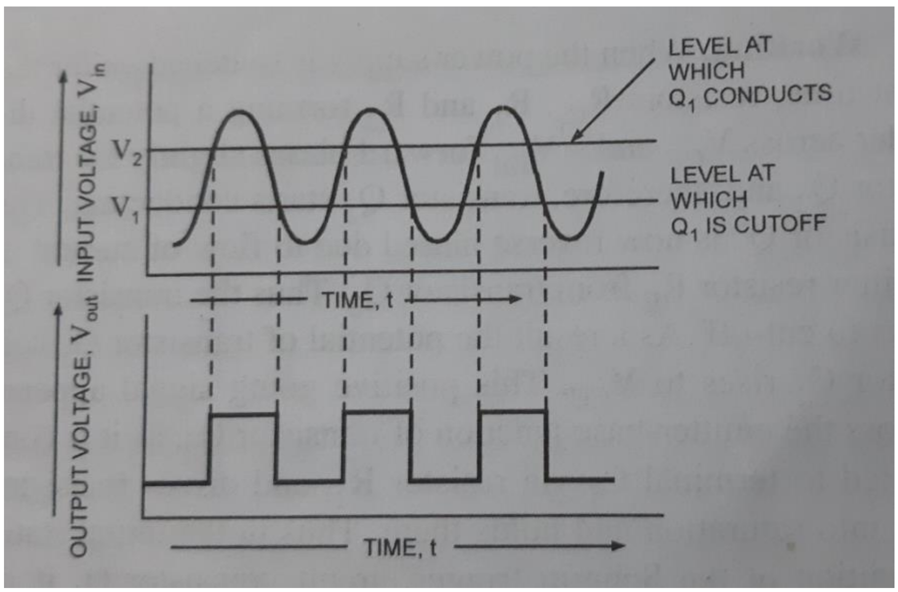
- Acts as a squaring circuit which converts any irregular waveform to square wave
- Acts as an amplitude comparator

- Acts as a flip flop circuit
- The input must surpass a low threshold voltage before the output goes low, and likewise the input must exceed a high level threshold before the output goes high.
- Circuit exhibits hysteresis i.e. to cause a change in direction we are required to go beyond the voltage at which reverse saturation takes place

Hystersis=UTP-LTP

Where UTP-Upper trigger point

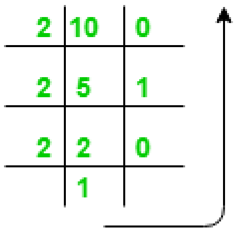
LTP-Lower trigger point



1. Decimal to Binary

$(10.25)_{10}$

Integer part :



$(10)_{10} = (1010)_2$

Fractional part
:

$0.25 \times 2 = 0.50$
$0.50 \times 2 = 1.00$

$(0.25)_{10} = (0.01)_2$

Note: Keep multiplying the fractional part with 2 until decimal part 0.00 is obtained.
$(0.25)_{10} = (0.01)_2$
Answer: $(10.25)_{10} = (1010.01)_2$

2. Binary to Decimal

$(1010.01)_2$
$1\times2^3 + 0\text{x}2^2 + 1\times2^1 + 0\text{x}2^0 + 0\text{x}2^{-1} + 1\times2^{-2} = 8+0+2+0+0+0.25 = 10.25$

$(1010.01)_2 = (10.25)_{10}$

3. Decimal to Octal

$(10.25)_{10}$
$(10)_{10} = (12)_8$
Fractional part:

$0.25 \text{ x } 8 = 2.00$

Note: Keep multiplying the fractional part with 8 until decimal part .00 is obtained.
$(.25)_{10} = (.2)_8$
Answer: $(10.25)_{10} = (12.2)_8$

4. Octal to Decimal

$(12.2)_8$
$1 \text{ x } 8^1 + 2 \text{ x } 8^0 + 2 \text{ x } 8^{-1} = 8+2+0.25 = 10.25$
$(12.2)_8 = (10.25)_{10}$

5. and Binary

To convert from Hexadecimal to Binary, write the 4-bit binary equivalent of hexadecimal.

Binary equivalent	Hexadecimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

$(3A)_{16} = (00111010)_2$

To convert from Binary to Hexadecimal, group the bits in groups of 4 and write the hex for the 4-bit binary. Add 0's to adjust the groups.

1111011011

$(001111011011\ )_2 = (3DB)_{16}$

Hexadecimal to decimal

Convert hexadecimal number 1F.01B into decimal number.

Since value of Symbols: B and F are 11 and 15 respectively. Therefore equivalent decimal number is,

$= (1F.01B)_{16}$

$= (1x16^1+15x16^0 +0x16^{-1}+1x16^{-2}+11x16^{-3})_{10}$

$= (31.0065918)_{10}$ which is answer.

2's compliment representation

The 2's compliment is the binary number that results when we add one to the 1's compliment. ie, 2's compliment = 1's compliment + 1

2's compliment form is used to represent -ve number.

1. Find the 2's compliment of $(1001)_2$

ans. 1001

0110+

1

0111

2 $(10100011)_2$

01011100+

1

01011101

Binary substraction using 2's compliment

The 2's compli is accomplished by addition. It has 2 cases

1. Substration of smaller number from larger number.

2. Subtraction of larger number from smaller number.

1. sub. of smaller number from larger number

→ Determine the 2's compliment of the smaller number

→ Add the 2's compliment to the larger number.

→ Discard the carry.

Error detection & corrections

Digital information transformed in the binary is transformed from one system to another circuit system errors may occur.

This means a signal corresponding to zero may change to one and vice versa due to the presence of noise.

To maintain the data integrity between transmitter and reciever,extra bit or more than one bit are added in the data.These

Extra bit allow the detection and sometimes correction of error in the data

Hamming code

It not only provide the dectection of bit error ,identifies which bit is in error.so that it can corrected

Number of parity bits denoted by:

2 greater than or equal to x+q+1

Even and Odd Parity

Added parity bit will make the total number of ones and even amount

In odd parity the added parity will make the total number of ones and odd amount

Alpha numeric code

Is a set elements that includes the 10 decimal digits,26 letters of the alphabet & the no:of special characters

American standard code for information interchange (ASCII)

Contain 97 graphic characters that can be printed & 34 non printing charcters used for various control functions.

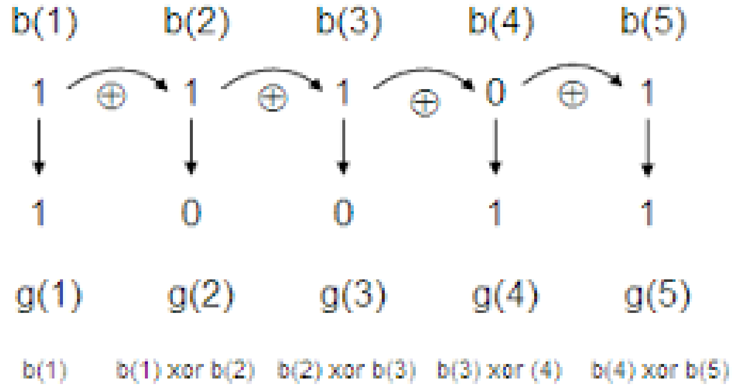
Excess code

Digit	Excess code
0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

BCD Code

BCD		Decimal
0000	=	0
0001	=	1
0010	=	2
0011	=	3
0100	=	4
0101	=	5
0110	=	6
0111	=	7
1000	=	8
1001	=	9

Gray Code(binary to gray)



MODULE 2

Properties of Gates

1). Commutative property

Addition: A+B=B+A

The order in which variables are stored makes no difference

2). Associative property

Addition.: A+(B+C) = (A+B) + C

When adding more than two variables the result is same regardless of the grouping variables.

3). Distributive property

A(B+C) = AB + AC

Law/Theorem	Law of Addition	Law of Multiplication
Identity Law	$x + 0 = x$	$x \cdot 1 = x$
Complement Law	$x + x' = 1$	$x \cdot x' = 0$
Idempotent Law	$x + x = x$	$x \cdot x = x$
Dominant Law	$x + 1 = 1$	$x \cdot 0 = 0$
Involution Law	$(x')' = x$	
Commutative Law	$x + y = y + x$	$x \cdot y = y \cdot x$
Associative Law	$x+(y+z) = (x+y)+z$	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$
Distributive Law	$x \cdot (y+z) = x \cdot y+x \cdot z$	$x+y\cdot z = (x+y) \cdot (x+z)$
Demorgan's Law	$(x+y)' = x' \cdot y'$	$(x \cdot y)' = x' + y'$
Absorption Law	$x + (x \cdot y) = x$	$x \cdot (x + y) = x$

Duality Theorem

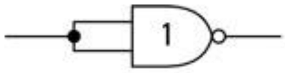
A Boolean relation can written to another relation by changing the AND operation to OR operation and vice versa

A(B+C) = AB+AC

NAND & NOR are referred to as **universal Gates.**

Using NAND gates

NOT



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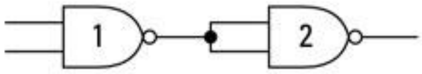

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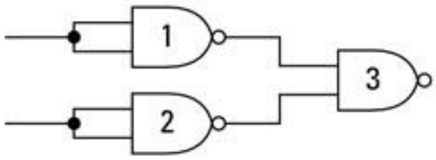
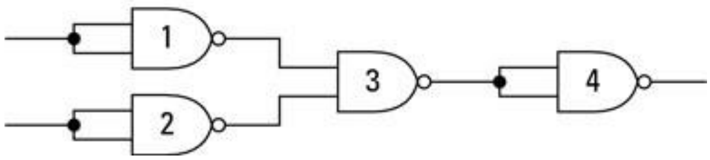
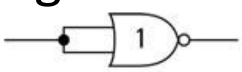


NOR



Using NOR Gates

NOT



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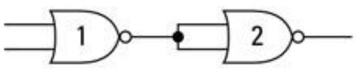

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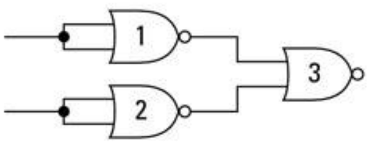
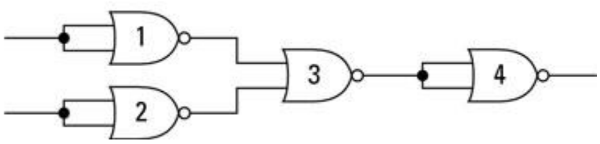


NAND



Simplification of POS expressions using K-Map

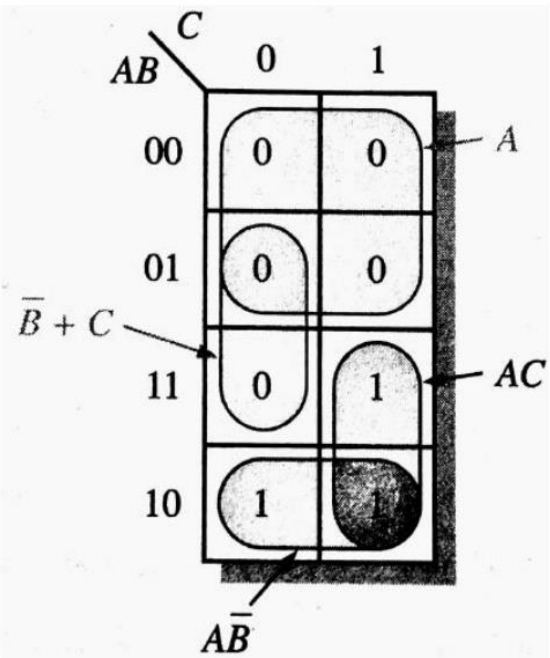
$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

The combinations of binary values of the expression are

$$(0 + 0 + 0)(0 + 0 + 1)(0 + 1 + 0)(0 + 1 + 1)(1 + 1 + 0)$$

- $\Pi M(0, 1, 2, 3, 6)$

Simplification of POS expressions



Minimum POS

$A(\overline{B} + C)$

Equivalent Minimum SOP

$AC + A\overline{B} = A(\overline{B} + C)$

Half adder

Table 4.4 Truth table for half-adder

Inputs		Outputs	
A	*B*	*Sum*	*Carry*
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

B \ A	0	1
0	0	1
1	1	0

K-map for Sum

B \ A	0	1
0	0	0
1	0	1

K-map for Carry

$S = A\overline{B} + \overline{A}B = (A \oplus B)$

$C = AB$

Full adder

Table 4.4 Truth table of full-adder

Inputs			Outputs	
A	*B*	C_{in}	*Sum*	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

C \ AB	00	01	11	10
0	0	1	0	1
1	1	0	1	0

K-map for sum *S*

C \ AB	00	01	11	10
0	0	0	1	0
1	0	1	1	1

K-map for C_{out}

Half adder

$S = A\overline{B} + \overline{A}B = (A \oplus B)$

$C = AB$

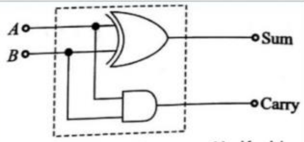
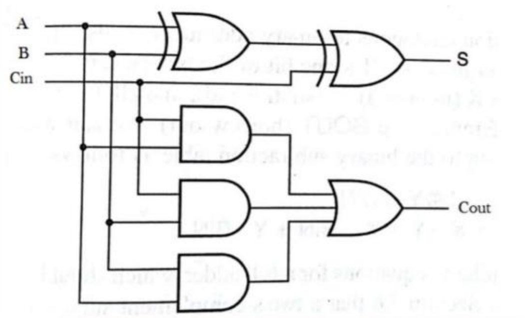


Fig. 4.4 Logic diagram of half-adder

Full adder

$$S = \overline{A}\,\overline{B}C_{in} + \overline{A}B\overline{C}_{in} + ABC_{in} + A\overline{B}\,\overline{C}_{in}$$
$$= C_{in}(\overline{A}\,\overline{B} + AB) + \overline{C}_{in}(\overline{A}B + A\overline{B})$$
$$= C_{in}(A \odot B) + \overline{C}_{in}(A \oplus B)$$
$$= C_{in}(\overline{A \oplus B}) + \overline{C}_{in}(A \oplus B)$$
$$= C_{in} \oplus A \oplus B$$
$$C_{out} = AB + AC_{in} + BC_{in}$$

Full adder



Full Subtractor

Inputs			Outputs	
A_n	B_n	C_{n-1}	D_n	C_n
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

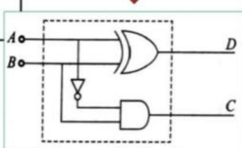
Half Subtractor

Table 4.5 Truth table of half-subtractor

Inputs		Outputs	
A	*B*	*D*	*C*
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$D = A\overline{B} + \overline{A}B = (A \oplus B)$

$C = \overline{A}B$

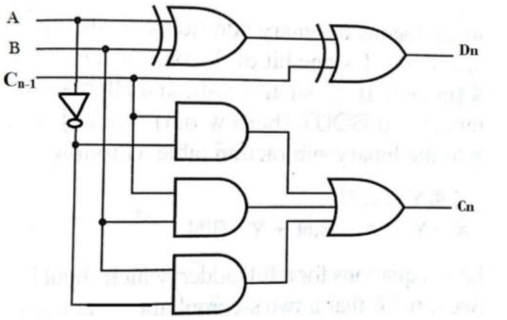


Block diagram of half-subtractor

Full Subtractor

$$D_n = \overline{A}_n\overline{B}_nC_{n-1} + A_nB_nC_{n-1} + \overline{A}_nB_n\overline{C}_{n-1} + A_n\overline{B}_n\overline{C}_{n-1}$$
$$= C_{n-1}(\overline{A}\,\overline{B}_n + A_nB_n) + \overline{C}_{n-1}(\overline{A}_nB_n + A_n\overline{B}_n)$$
$$= C_{n-1}(A_n \cdot B_n) + \overline{C}_{n-1}(A_n \oplus B_n)$$
$$= C_{n-1}(\overline{A_n \oplus B_n}) + \overline{C}_{n-1}(A_n \oplus B_n)$$
$$= C_{n-1} \oplus A_n \oplus B_n$$
$$C_n = \overline{A}_nB_n + \overline{A}_nC_{n-1} + B_nC_n$$
$$= B_nC_n + \overline{A}_nB_n + \overline{A}_nC_{n-1}$$

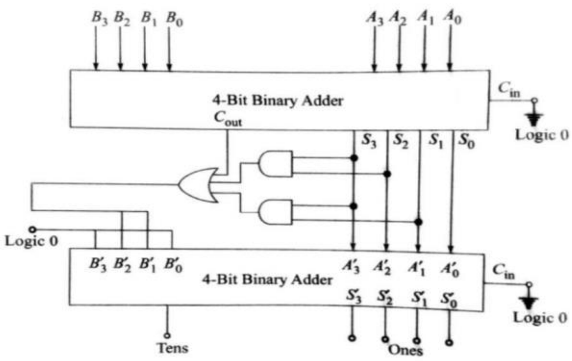
Full Subtractor



BCD Adder

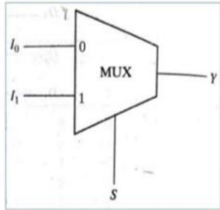
- Add two BCD numbers using ordinary binary addition
- Check whether the result is valid or invalid BCD
  - If valid(<=9), no correction needed
  - If invalid, add '6' to get the valid result

BCD Adder

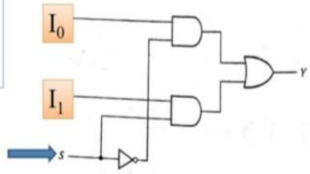


Multiplexer- Data Selector- MUX

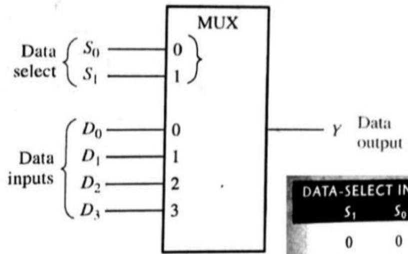
- 2:1 MUX



Data Select inputs (S)	Input Selected (Y)
0	I_0
1	I_1

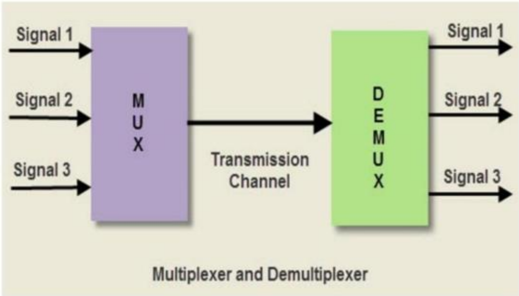


4 : 1 MUX

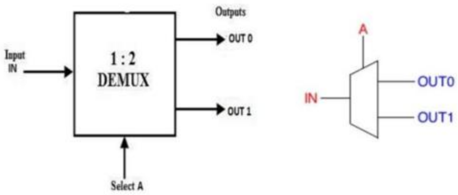


DATA-SELECT INPUTS S_1	S_0	INPUT SELECTED
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3

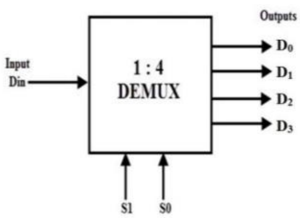
MUX and DEMUX



1-to-2 DEMUX



1-to-4 DEMUX



$$D0 = Din\ \overline{S1}\ \overline{S0}$$

$$D1 = Din\ \overline{S1}\ S0$$

$$D2 = Din\ S1\ \overline{S0}$$

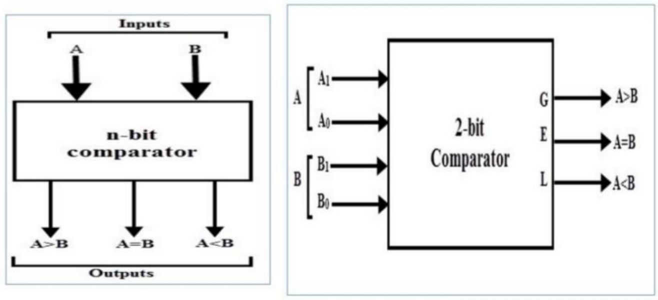
$$D3 = Din\ S1\ S0$$

1-to-4 DEMUX

Select Line		Input	Output			
S1	S0	Din	D0	D1	D2	D3
0	0	0	0	0	0	0
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	1	0
1	1	0	0	0	0	0
1	1	1	0	0	0	1

Comparator

- Check A = B A > B A < B



2 Bit Comparator

Input A		Input B		Output		
A1	A0	B1	B0	A > B (G)	A = B (E)	A < B (L)
0	0	0	0	0	1	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0

Equations of a comparator

We get the equation as f(A<B)

$$= \overline{A}_1 B_1 + \overline{A}_0 B_1 B_0 + \overline{A}_1 \overline{A}_0 B_0$$

We get the equation as f(A=B)

$$= A_1' A_0' B_1' B_0' + A_1' A_0 B_1' B_0 + A_1 A_0 B_1 B_0 + A_1 A_0' B_1 B_0'$$

$$= (A_0 \odot B_0).(A_1 \odot B_1)$$

We get the equation as f(A>B)

$$= A_1 \overline{B}_1 + A_0 \overline{B}_1 \overline{B}_0 + A_1 A_0 \overline{B}_0$$

Ex - Decimal to BCD Encoder

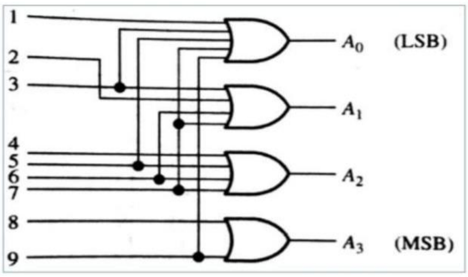
Decimal Digit	BCD Code			
	A3	A2	A1	A0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

Binary Encoder

- Example 8-to-3 line encoder

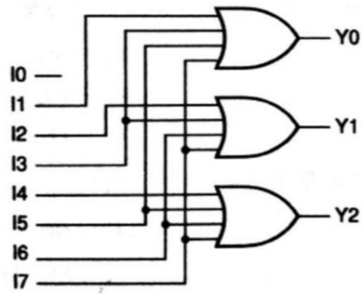
I0	I1	I2	I3	I4	I5	I6	I7	Y2	Y1	Y0
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

Ex - Decimal to BCD Encoder

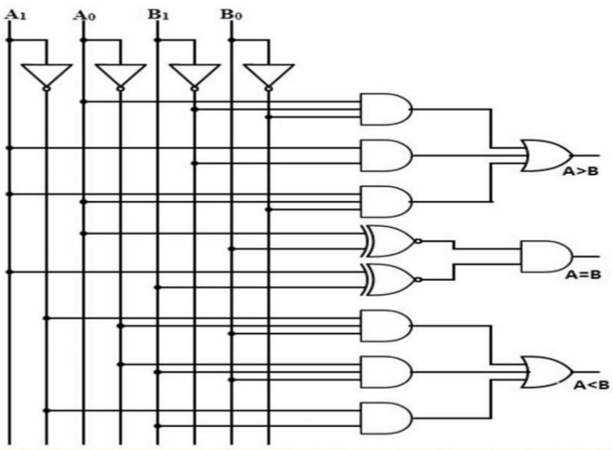


8-to-3 Encoder

$$Y0 = I1 + I3 + I5 + I7$$
$$Y1 = I2 + I3 + I6 + I7$$
$$Y2 = I4 + I5 + I6 + I7$$



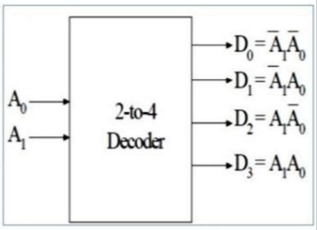
Logic Circuit for 2 bit Comparator



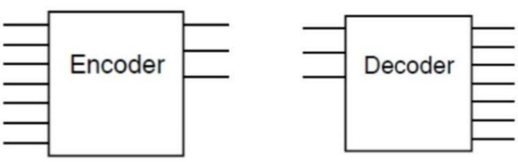
Decoder

- Example : Binary Decoders → 2 to 4 Decoder

Decimal #	Input		Output			
	A_1	A_0	D_0	D_1	D_2	D_3
0	0	0	1	0	0	0
1	0	1	0	1	0	0
2	1	0	0	0	1	0
3	1	1	0	0	0	1



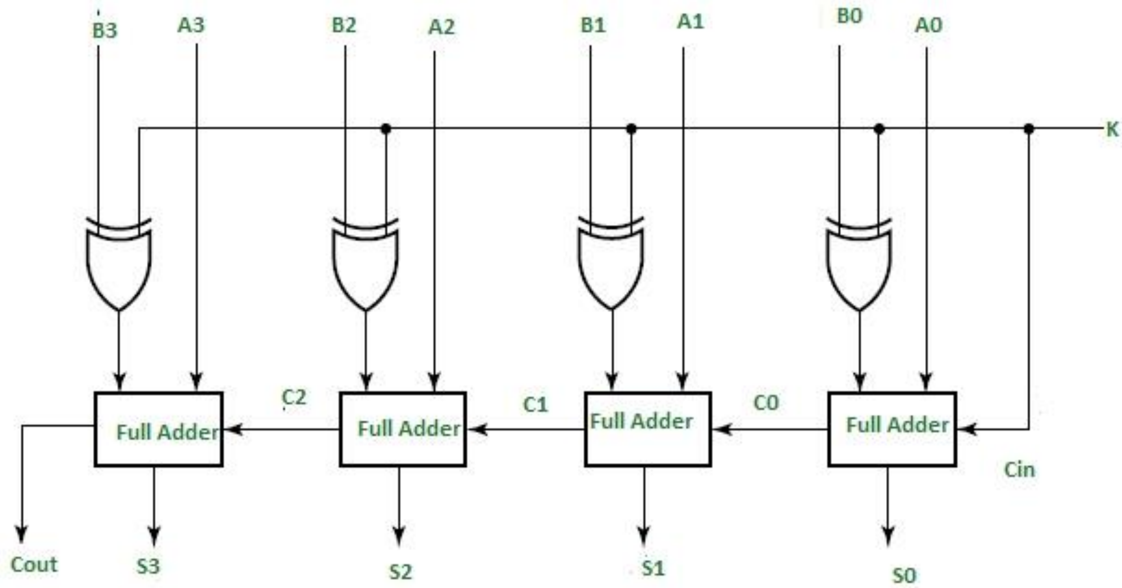
Encoders and Decoders



- Ex Binary Decoder and Encoder
  - Encoder → 2^n inputs, n outputs
  - Decoder → n inputs, 2^n outputs

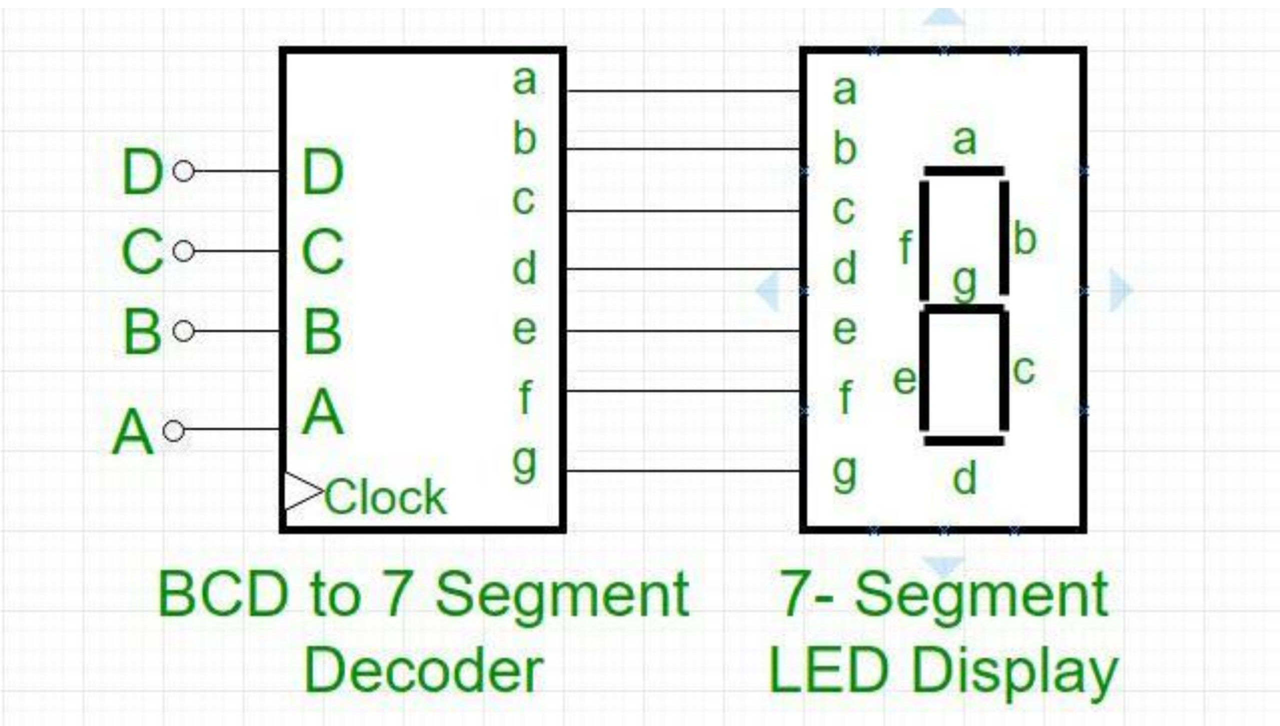
4 bit parallel adder/subtractor

The circuit consists of 4 full adders. There is a control line K that holds a binary value of either 0 or 1. When K=1, subtraction is performed and when K=0 , addition .



For an n-bit binary adder-subtractor, we use n number of full adders.

BCD to 7 segment decoder:



Logic families & its characteristics:

1. Logic levels:
    - The voltage used to represent a 1& 0 are called logic levels.
    - In a practical digital circuit, a HIGH can be any voltage between a specified minimum value & a specified maximum value.
    - A LOW can be any voltage between a specified minimum & a specified maximum.

2. Propagation delay:
    - Delay time in going from low to high logic or high to low logic (t_{pLH} & t_{pHL})
    - $t_{pd} = (t_{pLH} + t_{pHL})/2$

3. Fan in & Fan out:
    - Fan in – No. of inputs connected to gate without degradation.
    - Fan out – Max number of similar gates that gate can drive.
    - $(\text{Fan out})_H = I_{OH} / I_{IH}$
    - $(\text{Fan out})_L = I_{OL} / I_{IL}$
4. Noise Immunity:
    - It is defined as the amount of noise a logic circuit or system can tolerate, without being amplified beyond unity gain.
    - Noise margin is quantitative measure of noise immunity & is also called Margin of Safety.
    - Noise margin is defined as the difference between the operating input voltage level & input voltage level at which the circuit changes from one state to another.
5. Power dissipation:
    - Power dissipation of a logic gate is the power required by the gate to operate with 50% duty cycle at a specified frequency & is expressed in milliwatts.

TYPES OF TTL:

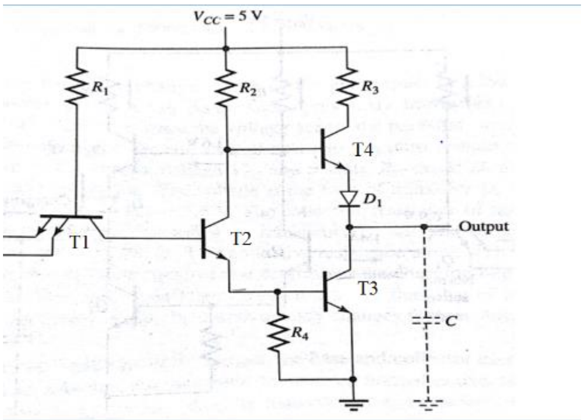
1. Standard TTL: Typical gate propagation delay of 10ns.
2. Low power TTL: Slow switching speed (33ns).
3. High speed TTL: Faster switching than standard TTL (6ns).
4. Schottky TTL: Used schottky diode clamps at gate inputs to prevent charge storage and improve switching time.
5. Low power schottky TTL: Used higher resistance values of low power TTL & Schottky diodes.

TTL:

All the families of TTL have 3 configurations for outputs.

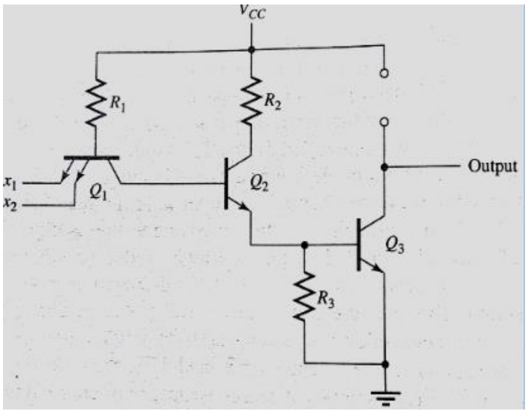
1. Totem pole output
2. Open collector output
3. Tristate output

TTL NAND Gate with Totem pole output



- 3 stages- input stage, phase splitter stage & output stage.
- T1 is a two-emitter npn transistor.
- T2 provides complementary voltages for the output transistors 3&4.
- Combination of T3 & T4 forms the output circuit often referred to as a totem pole arrangement.
- Output taken from the top of T3.

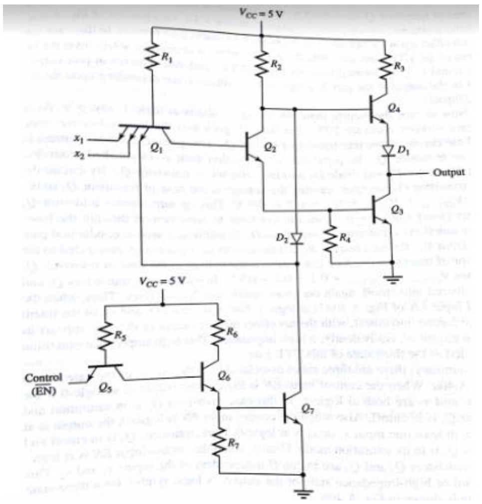
TTL with open collector output



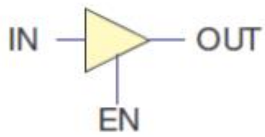
- Collector of Q3 is left open.
- Outputs of open collector TTL can be connected together.
- If 2 or more outputs are connected in parallel, then a pull up resistor is used in the open region.

Totem-pole	Open collector
	Output stage consist of *only pull-down transistor*.
External pull-up resistor is *not required*.	External pull-up resistor is *required for the operation*.
Operating speed is *high*.	Operating speed is *low*.

TRISTATE TTL



- Combines the advantages of totem pole and open collector circuits.
- 3 output states are HIGH, LOW & High Impedance (Hi-Z).



This requires two inputs:
input and enable

Truth Table

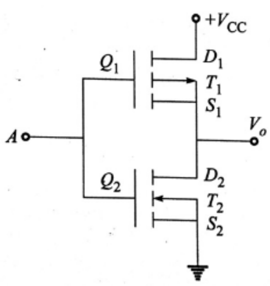
EN	IN	OUT
0	X	HI-Z
1	0	0
1	1	1

EN is to make output Hi-Z or follow input

CMOS

- P channel and N channel MOS devices are fabricated on the same chip.
- CMOS gates have equal no.of PMOS and NMOS
- Combines high speed with low power consumption
- Consumes less power (suited for battery operated systems)
- Operated at higher voltages(better noise immunity)

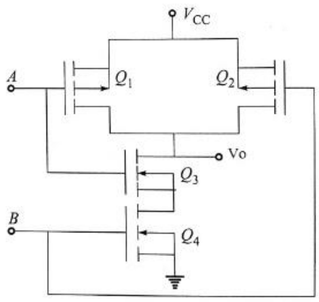
CMOS Inverter



Q_1 is p-channel
Q_2 is n-channel

A	Q_1	Q_2	Vo
0	ON	OFF	1
1	OFF	ON	0

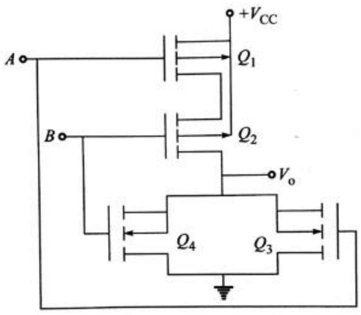
CMOS NAND



Q_1 & Q_2 is p-MOS
Q_3 & Q_4 is n-MOS

A	B	Q_1	Q_2	Q_3	Q_4	Vo
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1
1	1	OFF	OFF	ON	ON	0

CMOS NOR



Q_1 & Q_2 is p-MOS
Q_3 & Q_4 is n-MOS

A	B	Q_1	Q_2	Q_3	Q_4	Vo
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	0
1	0	OFF	ON	ON	OFF	0
1	1	OFF	OFF	ON	ON	0

Comparison of Logic families

	TTL	ECL	CMOS
Base Gate	NAND	OR/NOR	NAND/NOR
Fan-in	12-14	>10	>10
Fan-out	10	25	50 (Highest)
Power dissipation (mW)	10	175	0.001 lowest
Noise Margin	0.5V	0.16V (lowest)	1.5V (Highest)
Propagation Delay (ns)	10	<3 (High Speed) lowest	15 Highest
Noise immunity	Medium	lowest	Highest

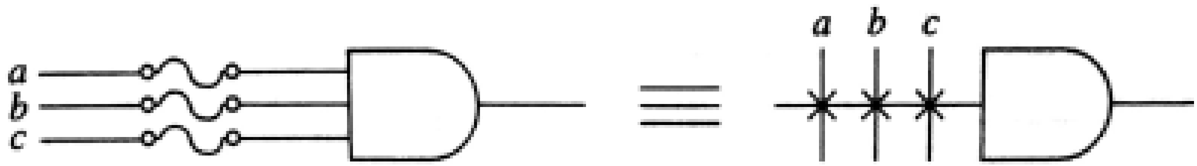
Programmable logic devices (PLD)

- Special purpose ICs
- Can be programmed as per requirement

TYPES

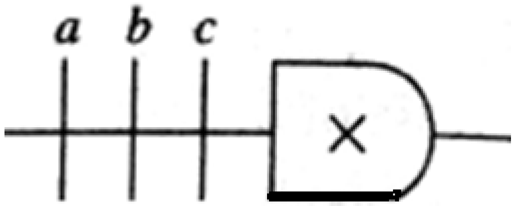
- PROM(programmable read-only memory)
- PLA (programmable logic array)
- PAL (programmable array logic)

Unprogrammed AND gate



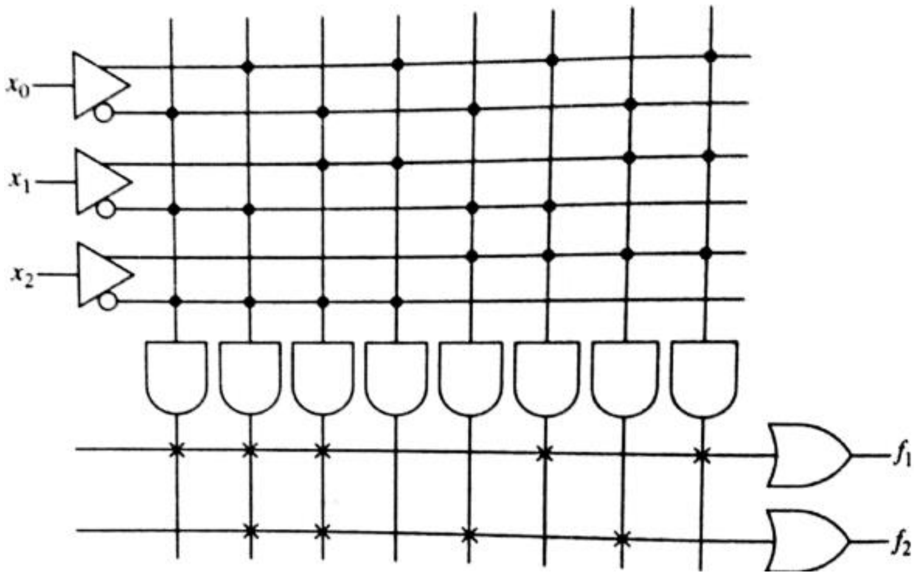
Programmed AND gate realizing the term ‘**ac**’

AND gate with all its input intact



PROM(programmable read-only memory)

- n input lines → address lines
- M output lines → bit lines
- Fixed AND array, programmable OR array
- Used for code conversion, generating bit pattern for characters, lookup tables for arithmetic functions



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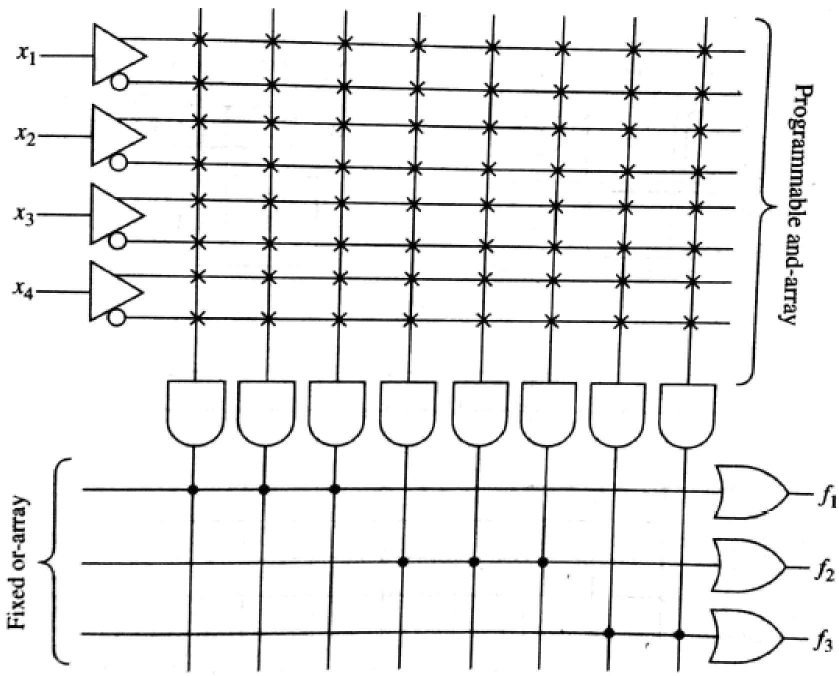
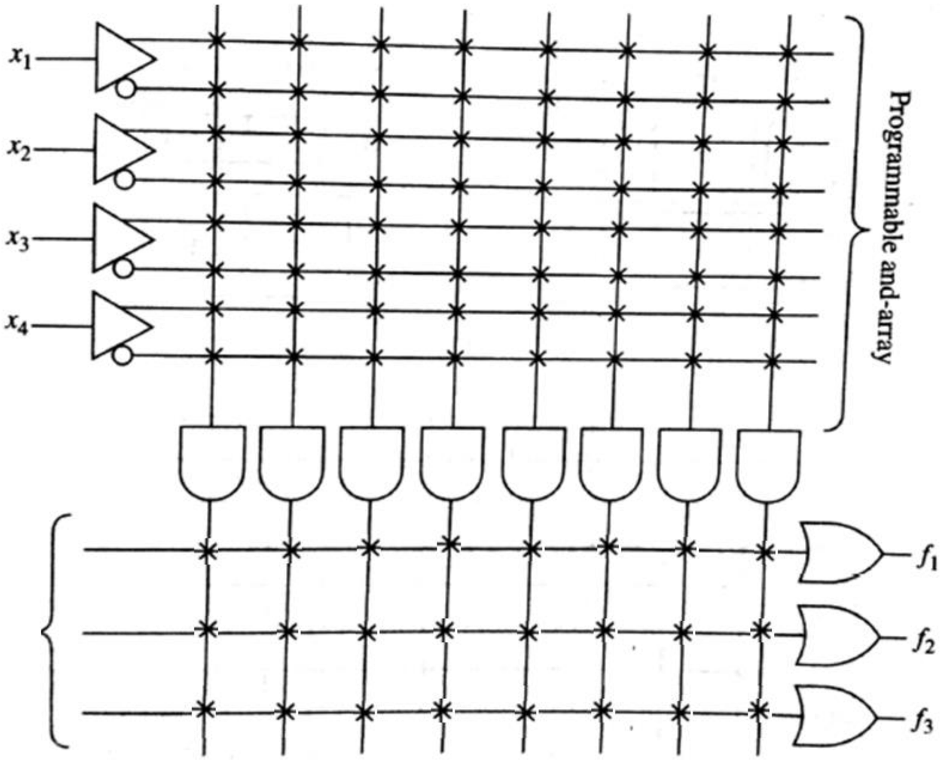
PAL (programmable array logic)

- Programmable AND array
- Fixed OR array (by the manufacturer)
- Less flexibility than PLA
- Programable AND , fixed OR



PLA (programmable logic array)

- Programmable AND & OR arrays
- n input lines
- p product lines
- m output lines



Types of PLDs

Device	And-array	Or-array
PROM	Fixed	Programmable
PLA	Programmable	Programmable
PAL	Programmable	Fixed

Implement the following using PLA

$$f_1(x,y,z) = \Sigma m(0,1,3,5)$$
$$f_2(x,y,z) = \Sigma m(3,5,7)$$

xy \ z	0	1
00	1	1
01	0	1
11	0	0
10	0	1

xy \ z	0	1
00	0	0
01	0	1
11	0	1
10	0	1

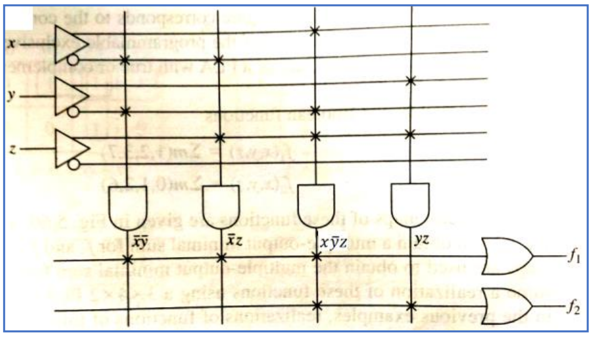
yz + xz (2 product terms yz,xz)

$$\bar{x}\bar{y} + \bar{x}z + \bar{y}z$$

The PLA provided is with 4 and gates (3x4x2)

xy \ z	0	1
00	1	1
01	0	1
11	0	0
10	0	1

xy \ z	0	1
00	0	0
01	0	1
11	0	1
10	0	1



f1 $= \bar{x}\bar{y} + \bar{x}z + x\bar{y}z$

f2 $= yz + x\bar{y}z$

So total 4 product terms

MODULE 5

Shift Registers & Shift Register Counters

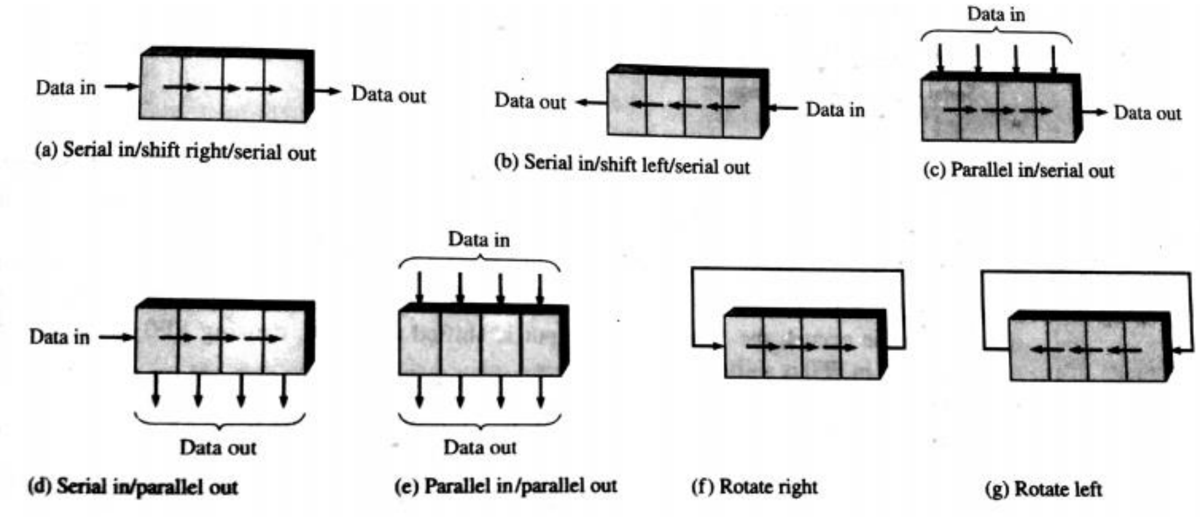
A register is a digital circuit with two basic functions :

- Data storage (memory device)
- Data movement

A shift register consists of arrangement of flip flops.

It helps in converting serial data and parallel data.

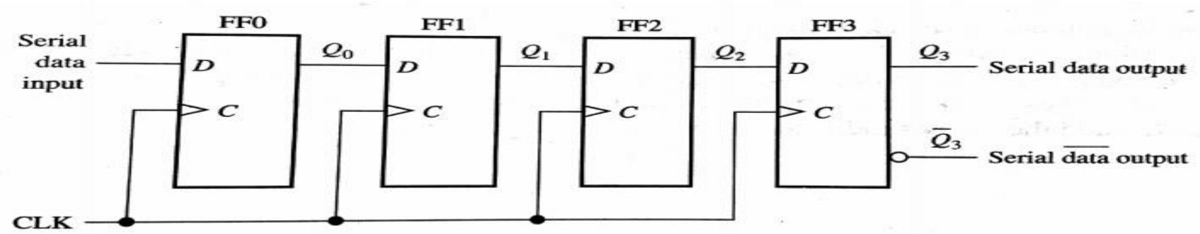
Different types of shift registers



A storage capacity of a register is the total number of digital bits it can retain. Each stage in a shift register represents one bit of storage capacity.Shifting capability of register permits the movement of data from stage to stage with in the register or in or out of the register with the help of clock pulses.

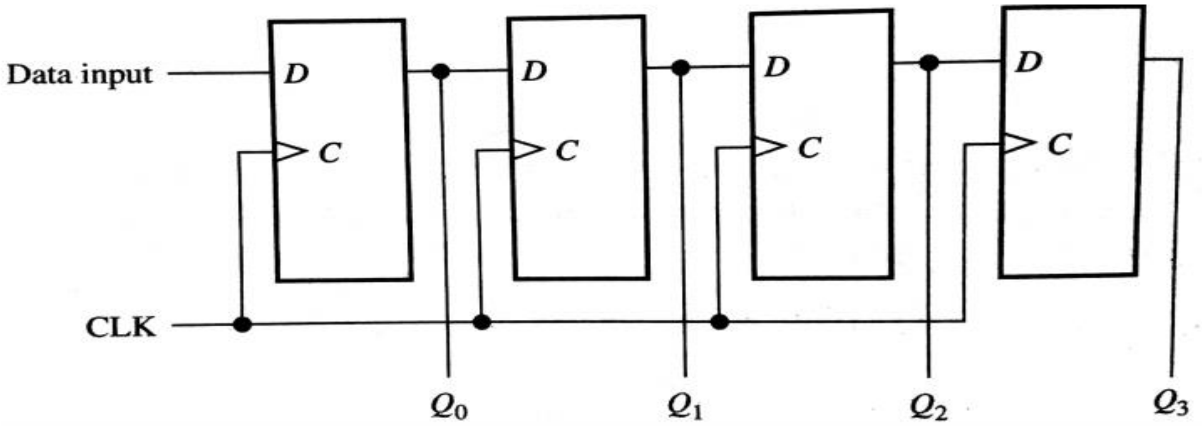
Serial In/ Serial Out Shift Registers

Accepts data serially , one bit at a time and also output data serially. When serial data is transferred into register,each new bit is clocked into first FF as positive-going edge of each clock pulse.



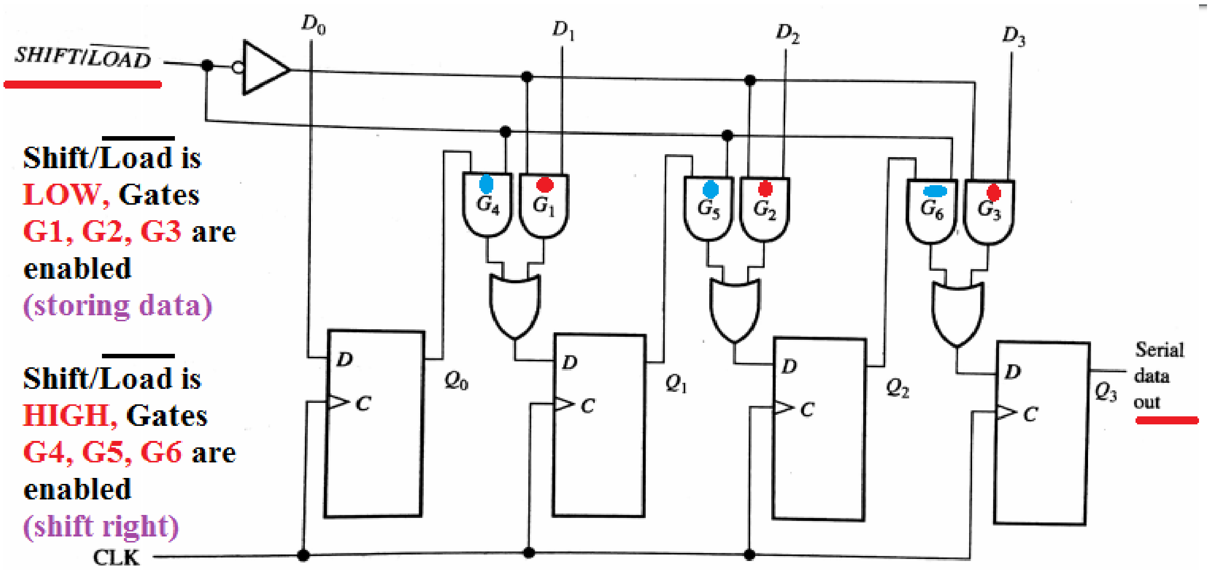
Serial In/ Parallel Out Shift Registers

The bits are entered into register serially but data stored in register is shifted out in parallel form. Once the data bit are entered into register serially, but the data stored in register is shifted out in parallel form.



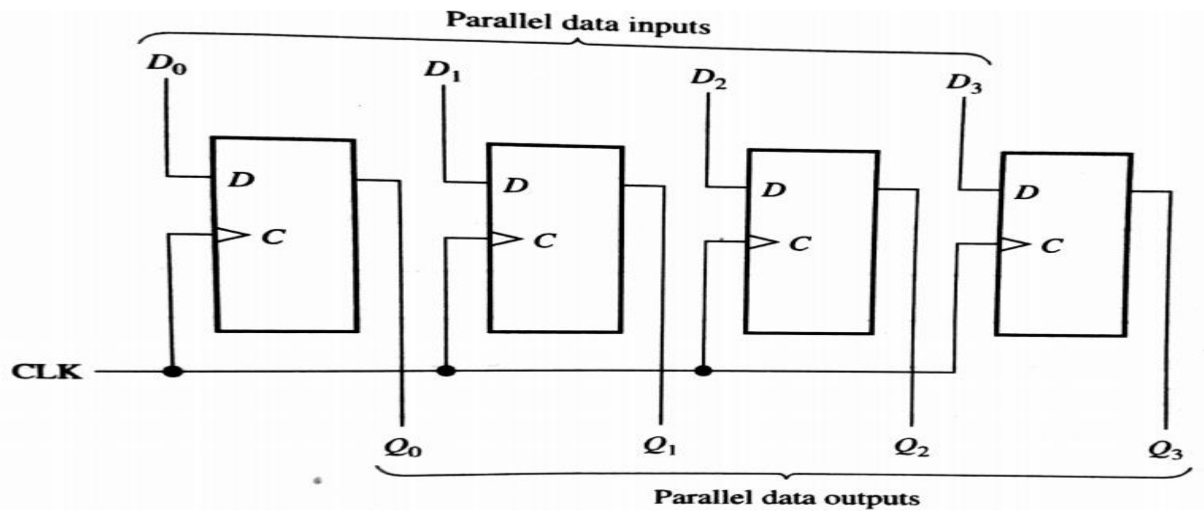
Parallel In/ Serial Out Shift Registers

The data bits are entered simultaneously into their respective stages on parallel lines, rather than on a bit-by-bit basis over a single line. When the shift/load terminal is high , gates G1,G2,G3 are disabled, but gates G4,G5 and G6 are enabled allowing the data bits to shift-right from one stage to next. When shift/load line is low gates G4,G5,G6 are disabled allowing the data inputs to appear at the D inputs of respective flip-flops.



Parallel In/ Parallel Out Shift Registers

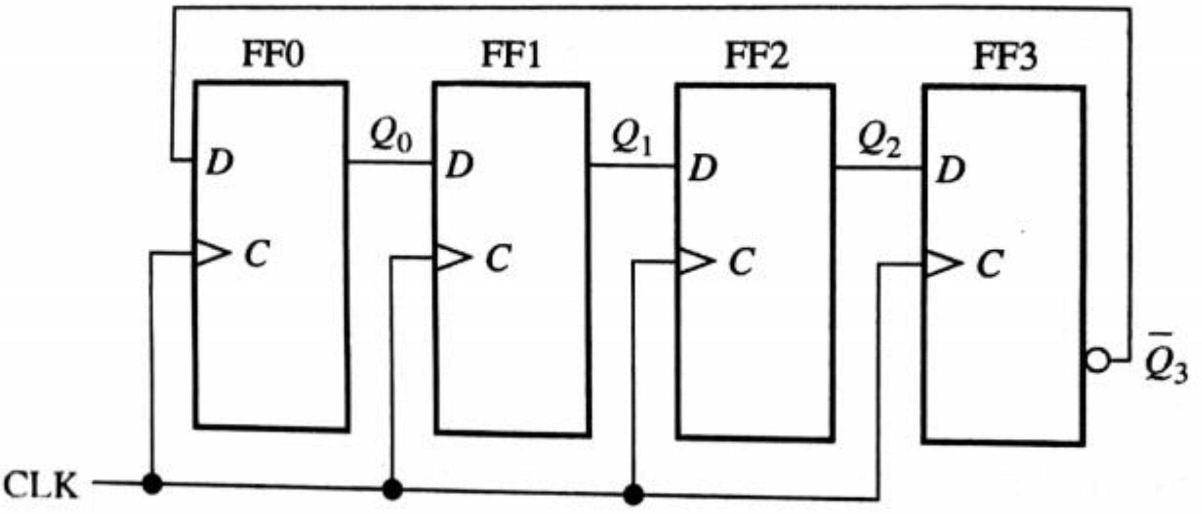
The data is entered into register in parallel form ,and also data is taken out of register parellely. Immediately following the simultaneous entry of all data bits,the bits appear on the parallel outputs.



Shift Register Counters

- **Johnson Counter**
- **Ring Counter**

JOHNSON COUNTER



(a) Four-bit Johnson counter

Johnson counter is also known as twisted ring counter. This counter is obtained from a serial-in serial out shift register by providing a feedback from the inverted output of the last FF to the D input of first FF.

Clock Pulse	Q0	Q1	Q2	Q3
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
7	0	0	0	1

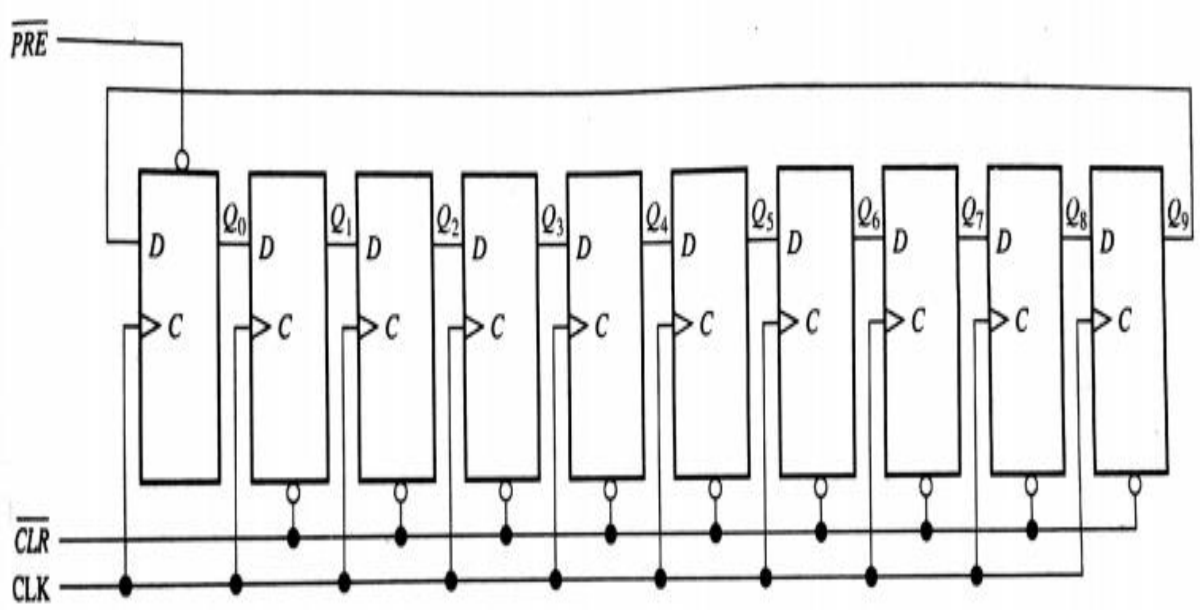
An FF Jonhson counter can have 2n unique states and can count up to 2n pulses.

More economical than ring counter.

It requires more decoding circuitry than that by the normal ring counter.

It suffers from lock-out problem.(counter finds itself in an unused state).

RING COUNTER



Simplest shift register counter. In this counter the output of each stage is connected to the D input of next stage , but output of last FF is connected back to D input of first FF. An n-bit ring counter can count only n bit.

It is very fast

It suffers from lock-out problem.(counter finds itself in an unused state).

It do not require external gates for operation.

CLOCK PULSE	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9
0	1	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	1

SHIFT REGISTER VS COUNTERS

Shift register has no specified sequence of states.

Counter has specified sequence of state.

State Machine

Synchronous Sequential Circuit Design

Mealy Model

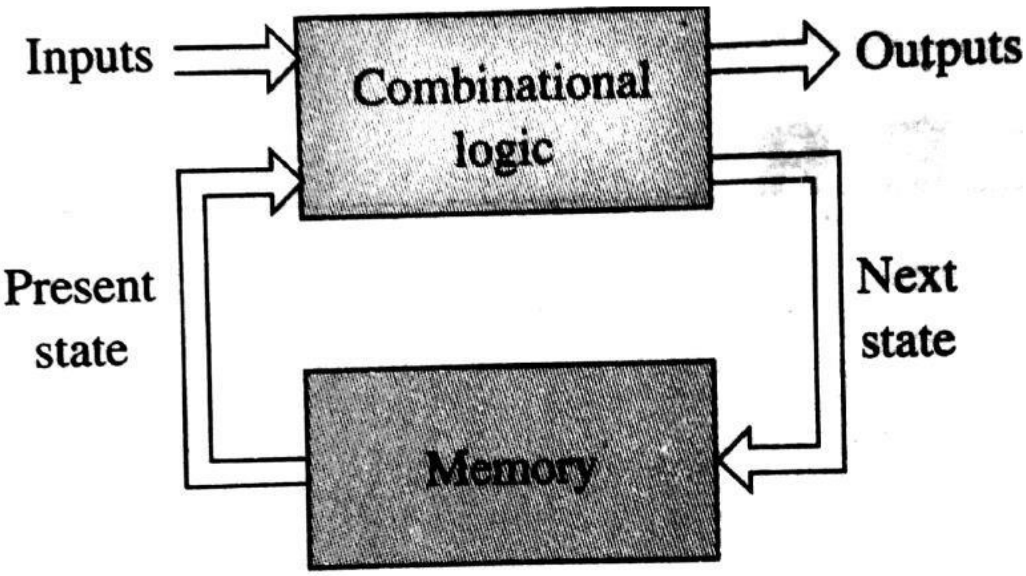
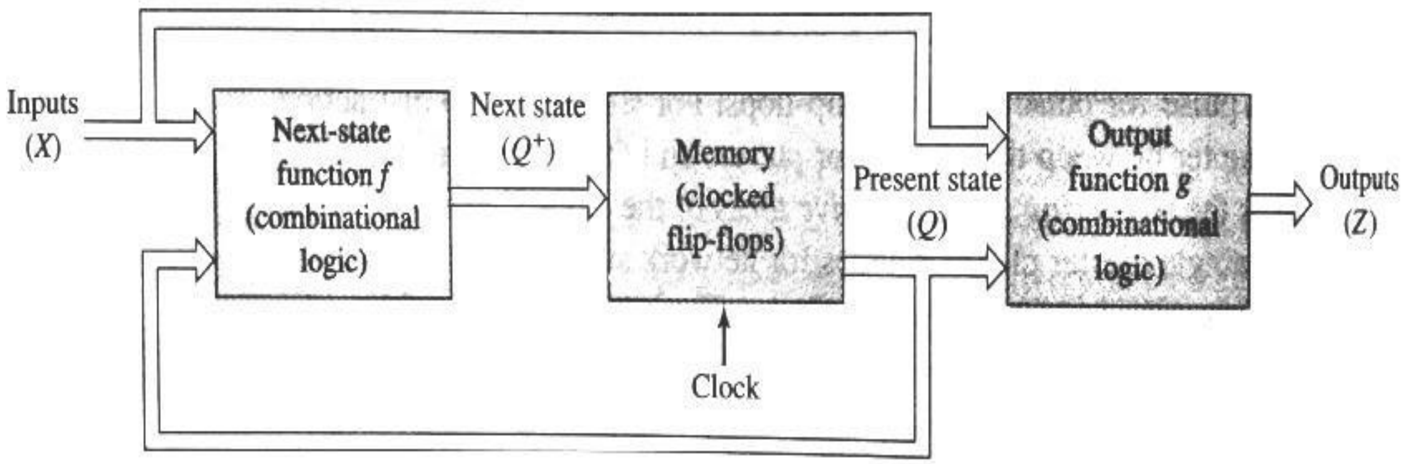


Figure 7.1 General model of a sequential network.

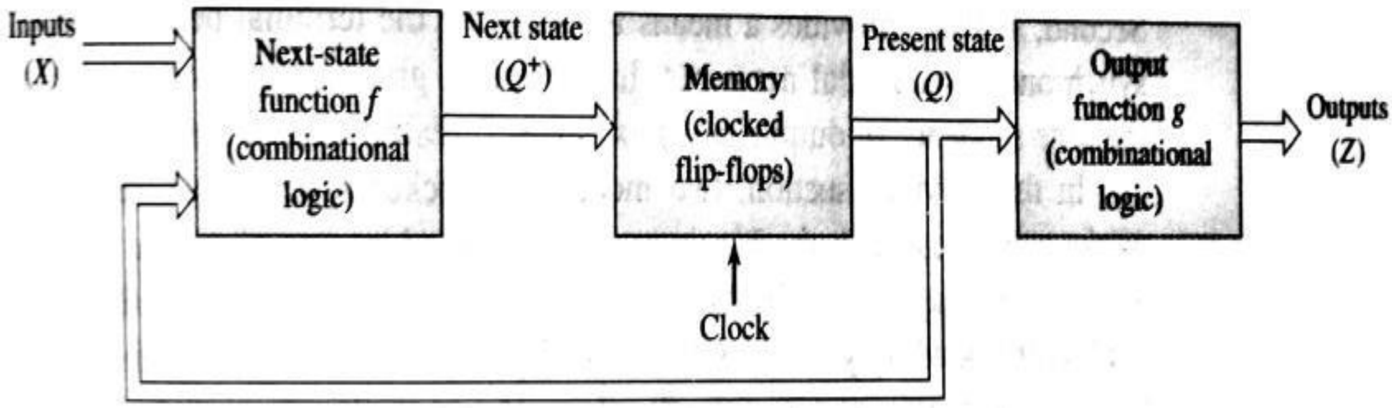
Mealy Model



Next State Function $\Rightarrow$ $Q^+ = f(X,Q)$

Output Function $\Rightarrow$ $Z = g(X,Q)$

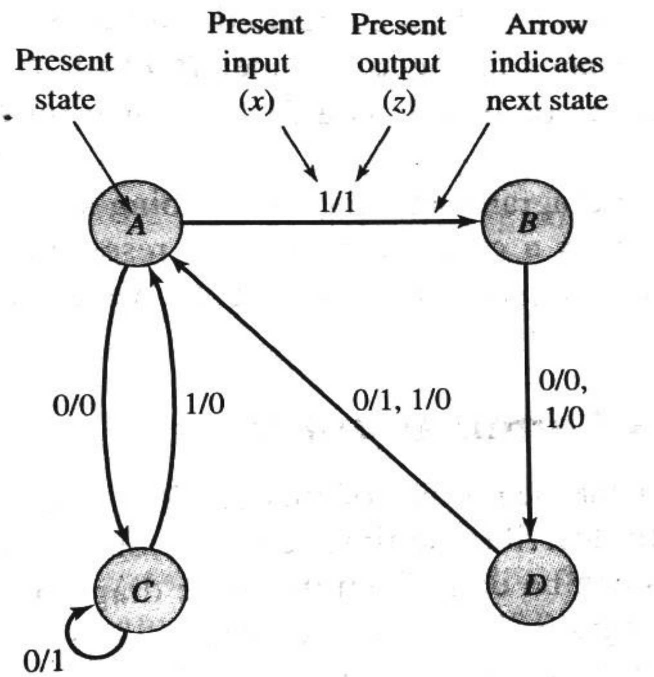
Moore Model



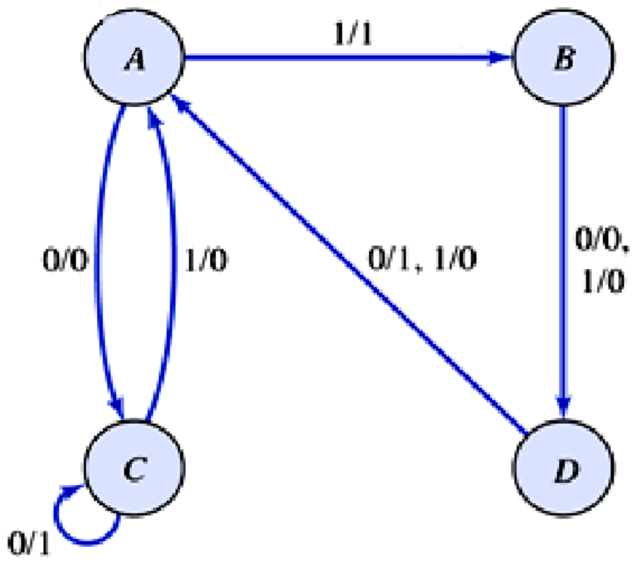
Next State Function → $Q^+ = f(X,Q)$

Output Function → $Z = g(Q)$

State Diagram



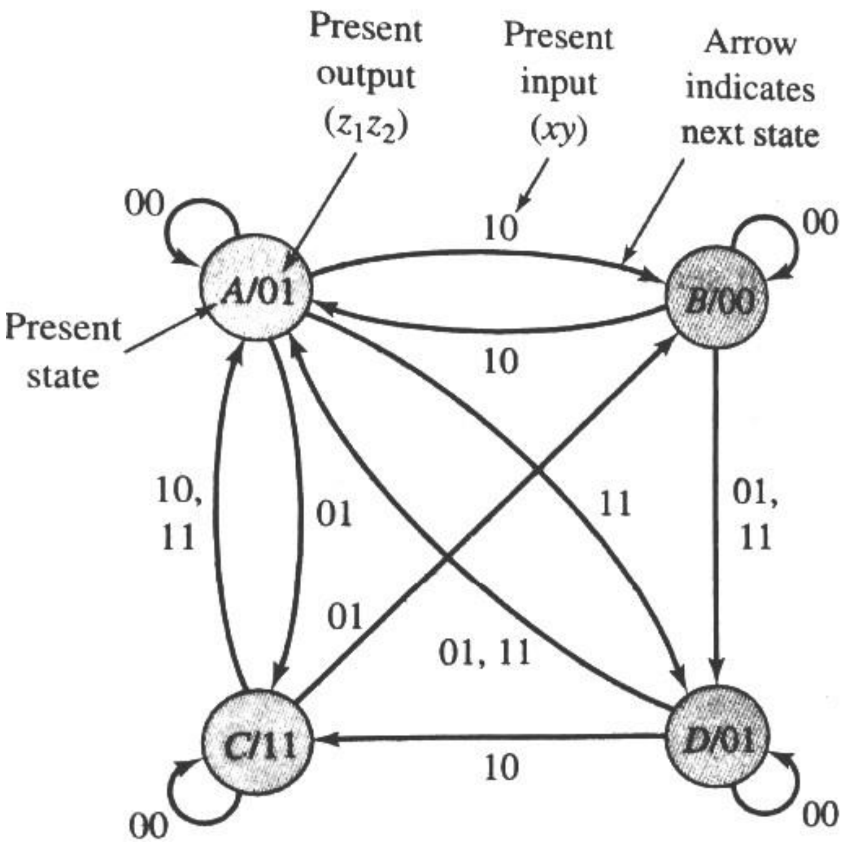
Mealy Model State Diagram



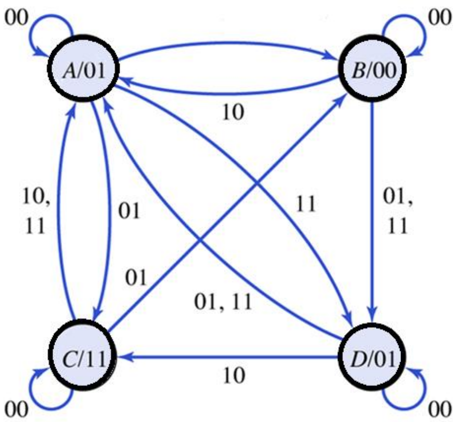
If initial state is 00(A) & input sequence applied is:

x = 0011011101

Input:	**0**	**0**	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**1**
State:	**A →**	**C →**	**C →**	**A →**	**B →**	**D →**	**A →**	**B →**	**D →**	**A → B**
Output:	**0**	**1**	**0**	**1**	**0**	**0**	**1**	**0**	**1**	**1**



Moore Model State Diagram



If A is the initial state & the applied input sequence is

x = 0 0 0 0 1 0 1

y = 0 1 1 0 1 1 1

00	**01**	**01**	**00**	**11**	**01**	**11**	
↓	↓	↓	↓	↓	↓	↓	
A →	**A** →	**C** →	**B** →	**B** →	**D** →	**A** →	**D**
↓	↓	↓	↓	↓	↓	↓	↓
01	**01**	**11**	**00**	**00**	**01**	**01**	**01**

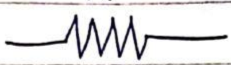
MODULE - 1

Circuit variables & Circuit Elements

Resistor (R)

Passive component that opposes the flow of current. Unit is ohm (Ω)

R



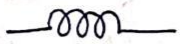
$V = IR$ $\quad$ $R = \frac{\rho l}{A}$

ρ - resistivity
l - length
A - Area of cross section

Inductor (L)

Passive component that opposes the rate of change of current. Unit is Henry (H)

L



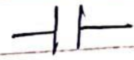
$V = L\frac{di}{dt}$ $\quad$ $L = \mu \frac{N^2 A}{l}$

μ - magnetic premiability
N - no. of turns

Capacitor (C)

Passive component that stores electrical energy in an electric field.

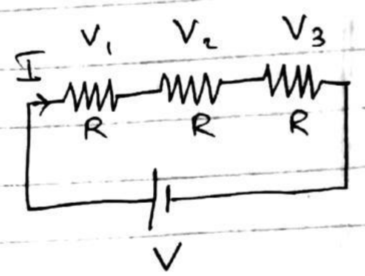
C



$V = \frac{1}{C}\int i\,dt$ $\quad$ $C = \frac{\varepsilon A}{d}$

ε - premittivity
A - area of plates

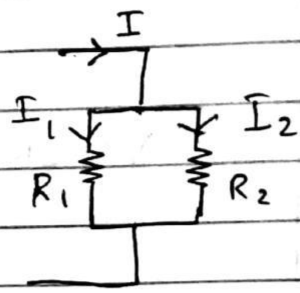
Voltage Divider Rule



$$\boxed{V_1 = V \times \frac{R_1}{R}}$$

$R = R_1 + R_2 + R_3$

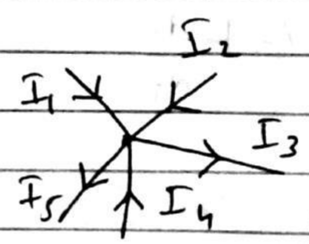
Current Divider Rule



$$\boxed{I_1 = I \times \frac{R_2}{R_1 + R_2}}$$

Kirchoff's Current law (KCL)

The algebraic sum of the branch currents at a node is zero at all instants of time.



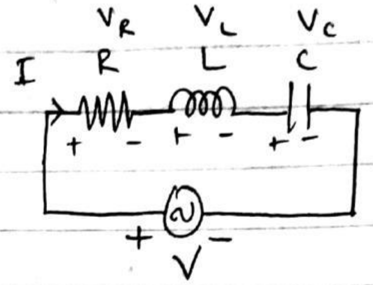
$I_1 + I_2 - I_3 + I_4 - I_5 = 0$

OR

$I_1 + I_2 + I_4 = I_3 + I_5$

Kirchoff's Voltage Law (KVL)

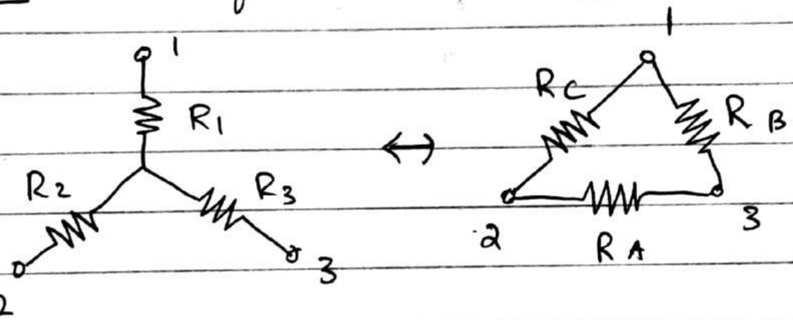
The algebraic sum of all branch voltages around any closed loop of a network is zero.



By KVL

$$V = V_R + V_L + V_C$$

Star to Delta transformation



$$R_A = R_2 + R_3 + \frac{R_2 R_3}{R_1} \qquad R_B = R_1 + R_3 + \frac{R_1 R_3}{R_2} \qquad R_C = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

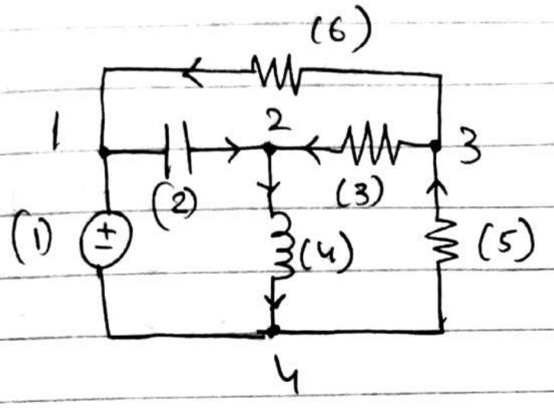
Delta to Star transformation

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \qquad R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

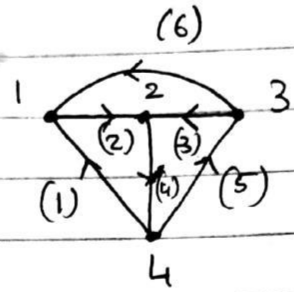
$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Network Graphs

To construct a graph from a given network, all passive elements and ideal voltage sources are replaced by short circuit and all ideal current sources are replaced by open circuits.

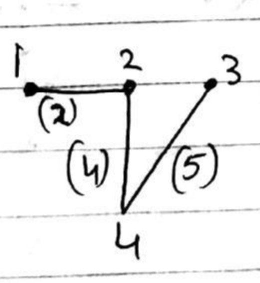


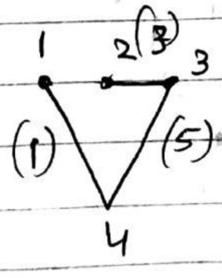
⟷



Rank of graph: If there are 'n' nodes in a graph then rank is $(n-1)$.

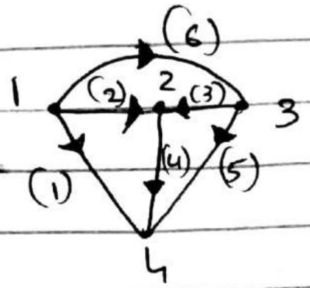
Tree: A tree is a set of branches with every node connected to every node such that removal of any branch destroys this property.





Incidence matrix (A_a)

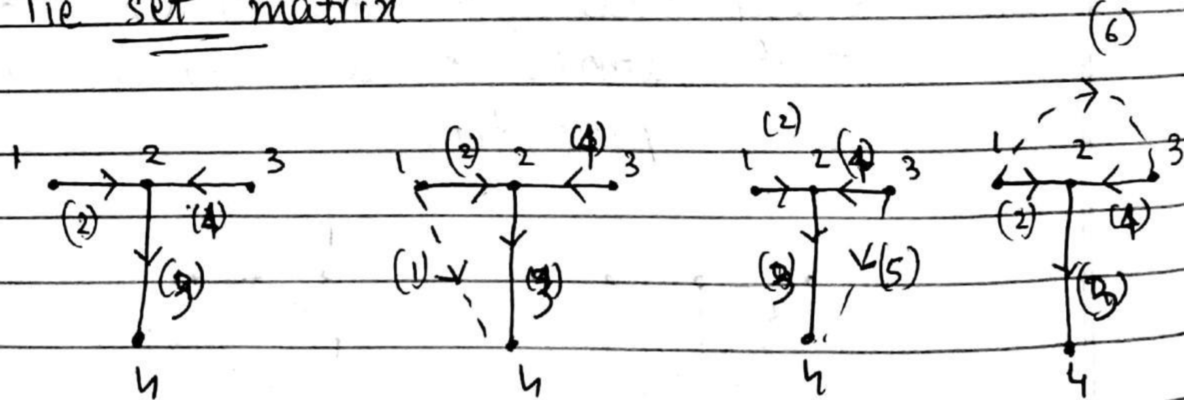
It is a matrix which shows which branch is incident on which node. Each row of the matrix represents corresponding node and each column represents corresponding branch.



Node \ Branch	1	2	3	4	5	6
1	1	1	0	0	0	1
2	0	-1	-1	1	0	0
3	0	0	1	0	1	-1
4	-1	0	0	-1	-1	0

$a_{ij} = 1$, away from node
$= -1$, toward node
$= 0$, not connected node

Tie set matrix



(1) (5) (6)

Tieset 1: [1, 2, 3]
Tieset 5: [5, 3, 4]
Tieset 6: [6, 2, 4]

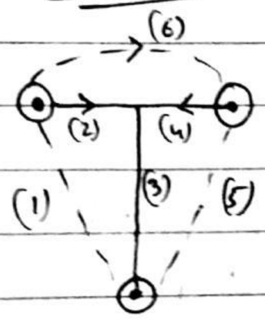
no. of tieset $= b - n + 1$ $\quad$ b: branches

$= 6 - 4 + 1$ $\quad$ n: nodes

$= \underline{\underline{3}}$

$$\begin{array}{c|cccccc} \text{Tieset} \backslash \text{Branch} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & -1 & -1 & 0 & 0 & 0 \\ 5 & 0 & 0 & -1 & -1 & 1 & 0 \\ 6 & 0 & -1 & 0 & 1 & 0 & 1 \end{array}$$

Cutset matrix



Cutset 2: {2, 1, 6}

Cutset 3: {3, 1, 5}

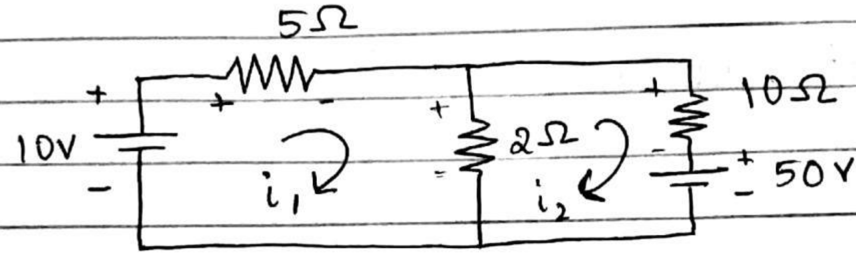
Cutset 4: {4, 5, 6}

$$\begin{array}{c|cccccc} \text{cutset} \backslash \text{Branches} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 2 & 1 & 1 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 & 1 & -1 \end{array}$$

Mesh Analysis

A mesh is defined as a loop which does not contain any other loops within it. It is applicable to only planar networks. (N/w that can be drawn on plane).
It uses KVL to determine unknown mesh currents.

eg:



$i_1 5 + (i_1 - i_2)2 = 10$ $\qquad$ $2(i_2 - i_1) + 10 i_2 = -50$

$7i_1 - 2i_2 = 10 \quad -(1)$ $\qquad$ $-2i_1 + 12 i_2 = -50 \quad -(2)$

Solving (1) & (2)

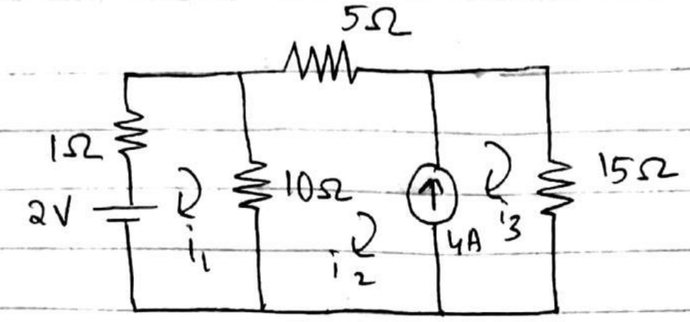
$i_2 = -4.125A$

~~i_2~~ = $i_1 = 0.25A$

Supermesh analysis

Meshes that share a current source with other meshes, no of which contains a current source in the outerloop, form a supermesh.

eg:



Here meshes 2 & 3 form a supermesh

$\therefore$ $\quad i_3 - i_2 = 4A$ — ① (Dire. of ↑)

Mesh I: $2 - I_1 - 10(I_1 - I_2) = 0$

$11I_1 - 10I_2 = 2$ — ②

Mesh II & III:

$-10(I_2 - I_1) - 5I_2 - 15I_3 = 0$

$10I_1 - 15I_2 - 15I_3 = 0$ — ③

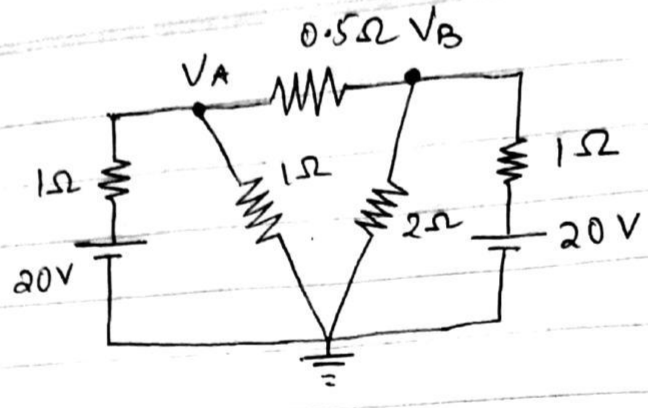
Solving ①, ② & ③

$I_1 = -2.35A \quad I_2 = -2.78A \quad I_3 = 1.22A$

Nodal Analysis

It is based on KCL. Every junction where two or more branches meet is regarded as a node.

eg:



Node A:

$$\frac{V_A - 20}{1} + \frac{V_A - 0}{1} + \frac{V_A - V_B}{0.5} = 0$$

$$4V_A - 2V_B = 20 \quad \text{—} (1)$$

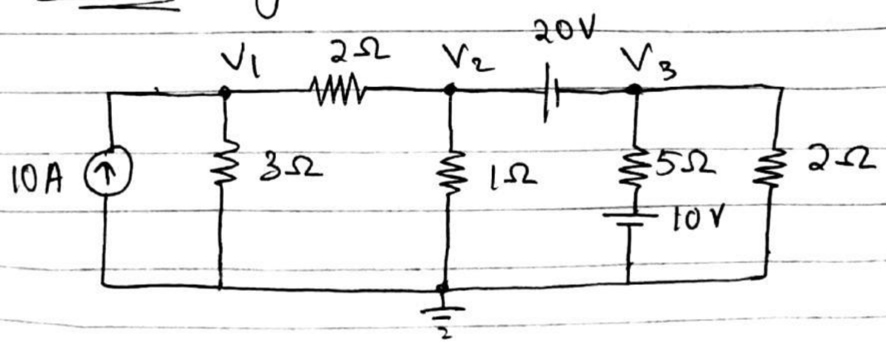
Node B:

$$\frac{V_B - V_A}{0.5} + \frac{V_B - 20}{1} + \frac{V_B - 0}{2} = 0$$

$$-2V_A + 3.5V_B = 20 \quad \text{—} (2)$$

$V_A = 11V$ $\quad$ $V_B = 12V$ $\quad$ Solving (1) & (2)

Supernode Analysis



V_2 & V_3 are supernode

Node 1:

$$\frac{V_1 - 0}{3} + \frac{V_1 - V_2}{2} = 10$$

$$0.83V_1 - 0.5V_2 = 10 \quad -(1)$$

Node 2 & (3) form supernode

$$V_2 - V_3 = 20 \quad -(2)$$

And

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$-0.5V_1 + 1.5V_2 + 0.7V_3 = 2 \quad -(3)$$

Solving (1), (2) & (3)

$V_1 = 19.04V \quad V_2 = 11.6V \quad V_3 = -8.4V$

MODULE 2

NETWORK THEOREMS APPLIED TO DC CIRCUITS AND PHASOR CIRCUITS

THEVENIN'S THEOREM (DC CIRCUITS)

It states that 'any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance ,i.e., short circuit)and constant current source replaced by infinite resistance , i.e., open circuit.'

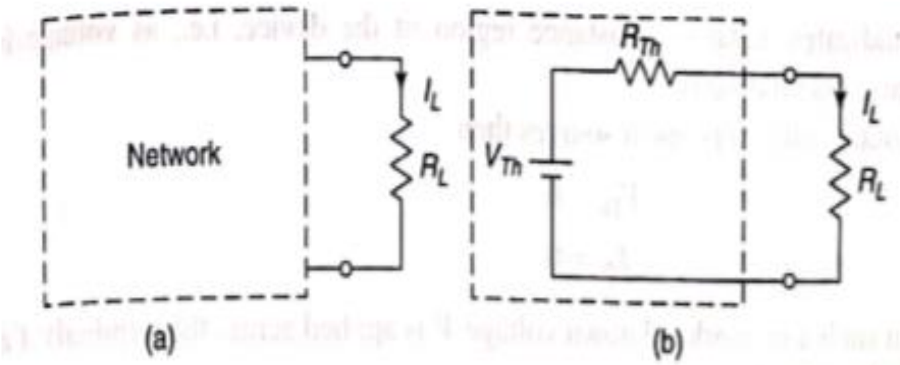


Fig. 3.109 *Network illustrating Thevenin's theorem*

EXPLANATION

Explanation Consider a simple network as shown in Fig. 3.110.

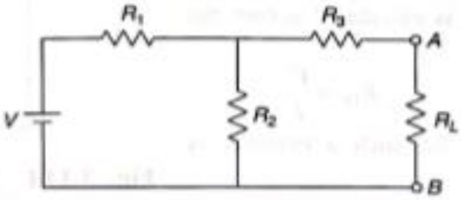


Fig. 3.110 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate open circuit voltage V_{Th} across points A and B as shown in Fig. 3.111.

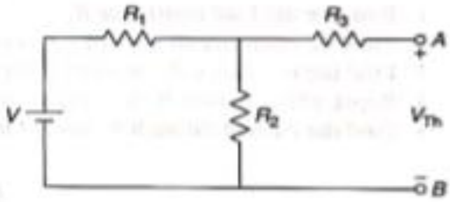


Fig. 3.111 *Calculation of* V_{Th}

$$V_{Th} = \frac{R_2}{R_1 + R_2} V$$

For finding series resistance R_{Th}, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 3.112.

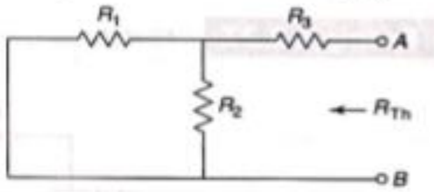


Fig. 3.112 *Calculation of* R_{Th}

$$R_{Th} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Thevenin's equivalent network is shown in Fig. 3.113.

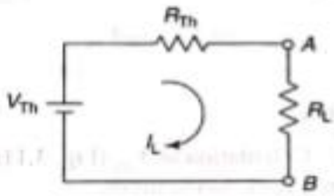


Fig 3.113 *Thevenin's equivalent network*

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

If the network contains both independent and dependent sources, Thevenin's resistance R_{Th} is calculated as,

$$R_{Th} = \frac{V_{Th}}{I_N}$$

where I_N is the short-circuit current which would flow in a short circuit placed across the terminals A and B. Dependent sources are active at all times. They have zero values only when the control voltage or current is zero. R_{Th} may be negative in some cases which indicates negative resistance region of the device, i.e., as voltage increases, current decreases in the region and vice-versa.

If the network contains only dependent sources then

$$V_{Th} = 0$$
$$I_N = 0$$

For finding R_{Th} in such a network, a known voltage V is applied across the terminals A and B and current is calculated through the path AB.

$$R_{Th} = \frac{V}{I}$$

or a known current source I is connected across the terminals A and B and voltage is calculated across the terminals A and B.

$$R_{Th} = \frac{V}{I}$$

Thevenin's equivalent network for such a network is shown in Fig. 3.114.

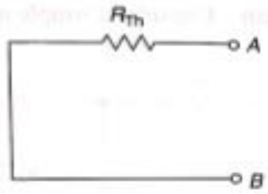


Fig. 3.114 *Thevenin's equivalent network*

STEPS TO BE FOLLOWED IN THEVENIN/S THEOREM

1. Remove the load resistance R_L.

2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through RL using Ohm's law.

$$I_L=V_{Th}/(R_{Th}+R_L)$$

THEVENIN'S THEOREM (PHASOR CIRCUITS)

In Thevenin's theorem, any linear network can be replaced by a voltage source V_{Th} in series with an impedance Z_{Th}.

NORTON'S THEOREM (DC CIRCUITS)

It states that 'any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.' The constant current source is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances.

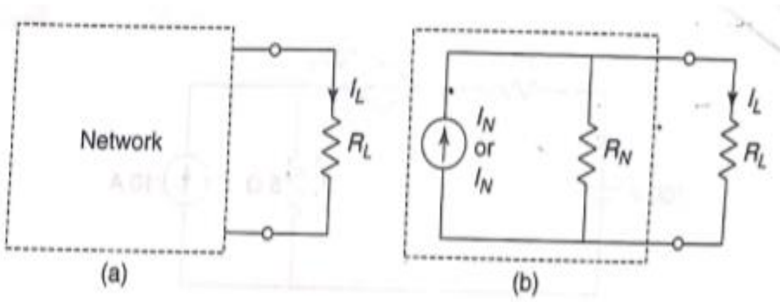


Fig. 3.251 *Network illustrating Norton's theorem*

EXPLANATION

Explanation Consider a simple network as shown in Fig.3.252

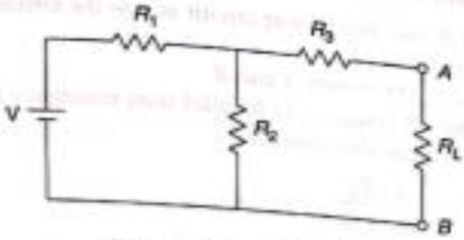


Fig. 3.252 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate short circuit current I_{SC} or I_N which would flow in a short circuit placed across terminals A and B as shown in Fig. 3.253.

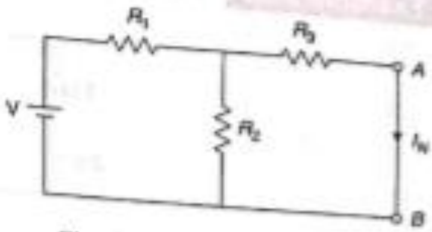


Fig. 3.253 *Calculation of I_N*

For finding parallel resistance R_N, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 3.254.

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

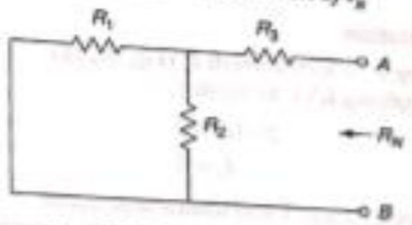


Fig. 3.254 *Calculation of R_N*

Norton's equivalent network is shown in Fig.3.255.

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

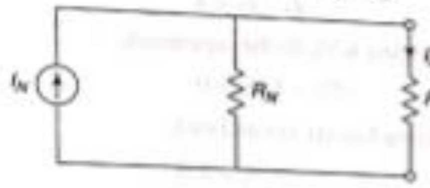


Fig. 3.255 *Norton's equivalent network*

If the network contains both independent and dependent sources, Norton's resistances R_N is calculated as

$$R_N = \frac{V_{Th}}{I_N}$$

where V_{Th} is the open-circuit voltage across terminals A and B. If the network contains only dependent sources, then

$$V_{Th} = 0$$

$$I_N = 0$$

To find R_{Th} in such network, a known voltage V or current I is applied across the terminals A and B, and the current I or the voltage V is calculated respectively.

$$R_N = \frac{V}{I}$$

Norton's equivalent network for such a network is shown in Fig. 3.256.

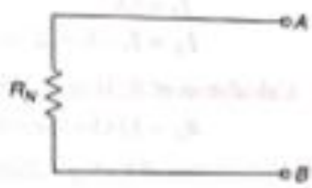


Fig. 3.256 *Norton's equivalent network*

STEPS TO BE FOLLOWED IN NORTON'S THEOREM

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{SC} OR I_N.
3. Find the resistance R_N as seen from points A and B.
4. Replace the network by a current source I_N in parallel with resistance R_N.

5. Find current through R_L by current division rule.

$$I_L = I_N R_N / (R_N + R_L)$$

NORTON'S THEOREM (PHASOR CIRCUITS)

Norton's theorem states that any linear network can be replaced by a current source I_N parallel with an impedance Z_N where I_N is the current flowing through the short-circuited path placed across the terminals.

SUPERPOSITION THEOREM (DC CIRCUITS)

It states that 'in a linear network containing more than one independent source and dependent source , the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.'

NOTE:

Independent voltage source = short circuit

Independent current source = open circuit

A dependent source has zero value only when its control voltage or current is zero.

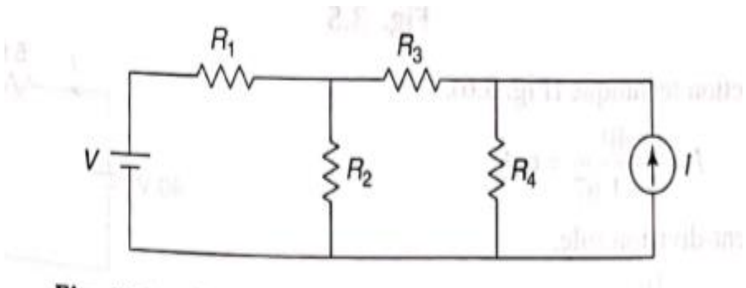


Fig. 3.1 *Network to illustrate superposition theorem*

EXPLANATION

Consider the above network we have to find current I4 through resistor R4.

The current flowing through resistor R_4 due to constant voltage source V is found to be say I_4' (with proper direction), representing constant current source with infinite resistance, i.e., open circuit.

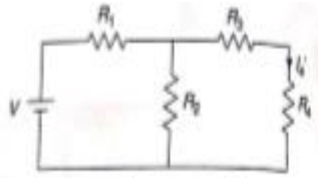


Fig. 3.2 When voltage source V is acting alone

The current flowing through resistor R_4 due to constant current source I is found to be say I_4'' (with proper direction), representing the constant voltage source with zero resistance or short circuit.

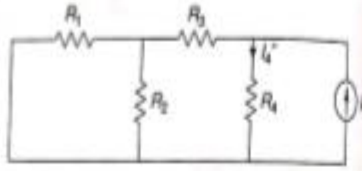


Fig. 3.3 When current source I is acting alone

The resultant current I_4 through resistor R_4 is found by superposition theorem.

$$I_4 = I_4' + I_4''$$

STEPS TO BE FOLLOWED IN SUPERPOSITION THEOREM:

1. Find the current through the resistance when only one independent source is acting, replacing all other sources by respective internal resistances.
2. Find the current through the resistance for each of the independent sources.
3. Find the resultant current through the resistance by the superposition theorem considering magnitude and direction of each current.

SUPERPOSITION THEOREM (PHASOR CIRCUITS)

It states that in a network containing more than one voltage source or current source, the total current or voltage in any branch of the network is the phasor sum of currents or voltages produced in that branch by each source acting separately. As each source is considered, all the other sources are replaced by their internal impedances. This theorem is valid only for linear systems.

RECIPROCITY THEOREM (DC CIRCUITS)

It states that 'in a linear , bilateral, active, single source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged.'

In other words ,it may be stated as 'if a single voltage source Va in the branch 'a' produces a current Ib in the branch 'b' then if the voltage source Va is removed and inserted in the branch 'b' ,it will produce a current Ib in branch 'a'.

EXPLANATION

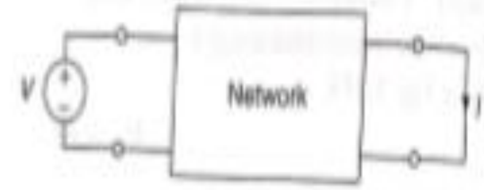


Fig. 3.443 *Network*

Explanation Consider a network shown in Fig. 3.443.

When the voltage source V is applied at the port 1, it produces a current I at the port 2. If the positions of the excitation (source) and response are interchanged, i.e., if the voltage source is applied at the port 2 then it produces a current I at the port 1.

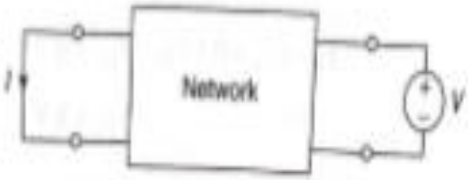


Fig. 3.444 *Network when excitation and response are interchanged*

The limitation of this theorem is that it is applicable only to a single-source network. This theorem is not applicable in the network which has a dependent source. This is applicable only in linear and bilateral networks. In the reciprocity theorem, position of any passive element (R, L, C) do not change. Only the excitation and response are interchanged.

STEPS TO BE FOLLOWED IN RECIPROCITY THEOREM

1. Identify the branches between which reciprocity is to be established.
2. Find the current in the branch when the excitation and response are not interchanged.
3. Find the current in the branch when the excitation and response are interchanged.

RECIPROCITY THEOREM (PHASOR CIRCUITS)

It states that 'in a linear, bilateral, active, single source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged.'

MILLMAN'S THEOREM (DC CIRCUITS)

It states that 'if there are n voltage sources V1,V2,…..,Vn with internal resistances R1,R2,…..,Rn respectively connected in parallel then these voltages can be replaced by a single voltage source Vm and a single series resistance Rm.

Vm=V1G1+V2G2+…+VnGn/(G1+G2+…+Gn)

Rm=1/Gm=1/ (G1+G2+…+Gn)

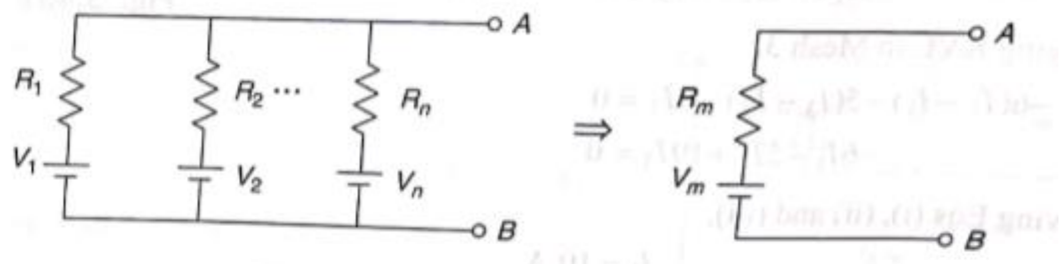


Fig. 3.455 *Millman's network*

EXPLANATION

Explanation By source transformation, each voltage source in series with a resistance can be converted to a current source in parallel with a resistance as shown in Fig. 3.456.

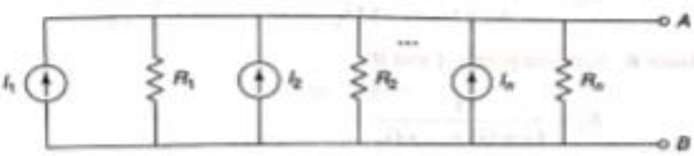


Fig. 3.456 *Equivalent network*

Let I_m be the resultant current of the parallel current sources and R_m be the equivalent resistance as shown in Fig. 3.457.

$$I_m = I_1 + I_2 + \dots + I_n = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n} = V_1 G_1 + V_2 G_2 + \dots V_n G_n$$

$$\frac{1}{R_m} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

$$G_m = G_1 + G_2 + \dots + G_n$$

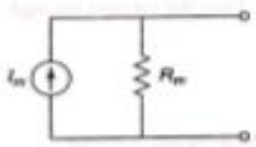


Fig. 3.457 *Equivalent network*

By source transformation, the parallel circuit can be converted into a series circuit as shown in Fig. 3.458.

$$V_m = I_m R_m = \frac{I_m}{G_m} = \frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n}$$

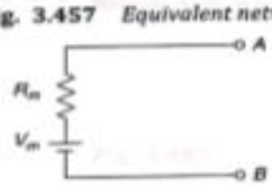


Fig. 3.458 *Millman's equivalent network*

Dual of Millman's Theorem

STEPS TO BE FOLLOWED IN MILLMAN'S THEOREM

1. Remove the load resistance RL.
2. Find Millman's voltage across points A and B.

Vm=V1G1+V2G2+…+VnGn/ (G1+G2+…+Gn)

3. Find the resistance Rm between points A and B.

$$Rm=1/Gm=1/(G1+G2+\ldots+Gn)$$

4. Replace the network by a voltage source Vm in series with the resistance Rm.
5. Find the current through R_L using ohm's law.

$$I_L=Vm/(Rm+R_L)$$

MILLMAN'S THEOREM (PHASOR CIRCUITS)

It states that 'if there are n voltage sources V1,V2,.....,Vn with internal impedances Z1,Z2,.....,Zn respectively connected in parallel then these voltages can be replaced by a single voltage source Vm and a single series impedance Zm.

$$Vm=V1Y1+V2Y2+\ldots+VnYn/(Y1+Y2+\ldots+Yn)$$

$$Zm=1/Ym=1/(Y1+Y2+\ldots+Yn)$$

MAXIMUM POWER TRANSFER THEOREM (DC CIRCUITS)

It states that 'the maximum power is delivered from a source to a load when the load resistance is equal to source resistance

PROOF

From Fig 3.363,

$$I=V/(R_S+R_L)$$

Power delivered to the load RL,

$$P=(I^2)R_L=(V^2)R_L/(R_S+R_L)^2$$

To determine the value of R_L for maximum power to be transferred to the load,

$$dP/dR_L=0$$

$$dP/dR_L=(d/dR_L)((V^2)/((R_S+R_L)^2)R_L$$

$$=[(V^2)[((R_S+R_L)^2)-(2R_L)(R_S+R_L)]]/((R_S+R_L)^4)$$

$$(R_S+R_L)^2-2R_L(R_S+R_L)=0$$

$$R_S^2+R_L^2+2R_SR_L-2R_LR_S-2R_L^2=0$$

$$R_S=R_L$$

STEPS TO BE FOLLOWED IN MAXIMUM POWER TRANSFER THEOREM

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Find the resistance R_L for maximum power transfer.

$R_L = R_{Th}$

5. Find the maximum power (fig 3.364).

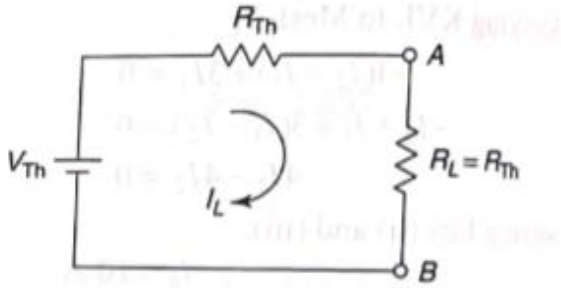


Fig. 3.364 *Thevenin's equivalent network*

$I_L = V_{Th}/(R_{Th}+R_L) = V_{Th}/2R_{Th}$

$P_{max} = I_L^2 R_L = [V_{Th}^2/(4 R_{Th}^2)]*R_{Th} = V_{Th}^2/4R_{Th}$

MAXIMUM POWER TRANSFER THEOREM (PHASOR CIRCUITS)

This theorem is used to determine the value of load impedance for which the source will transfer maximum power.

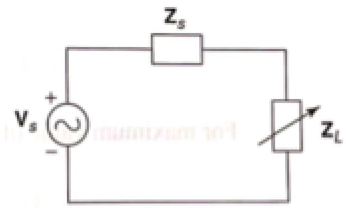


Fig. 6.143

Consider a simple network as shown in Fig.6.143.

There are three possible cases for load impedance Z_L

Case (i) When the load impedance is variable resistance (Fig.6.144)

$$I_L = V_S/(Z_S + Z_L) = V_S/(R_s + jX_s + R_L)$$

$$|I_L| = |V_s|/((R_s + R_L)^2 + X_s^2)^{(1/2)}$$

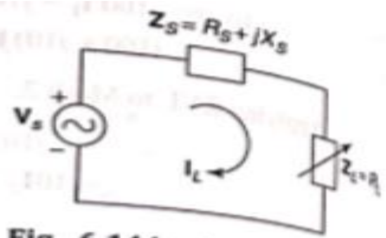


Fig. 6.144 *Purely resistive load*

The power delivered to the load is

$P_L = |I_L|^2 R_L = |V_s|^2 R_L/(R_s + R_L)^2 + X_s^2)$

For power to be maximum,

$$\frac{dP_L}{dR_L}=0$$

$$|\mathbf{V}_s|^2\left[\frac{\{(R_s+R_L)^2+X_s^2\}-2R_L(R_s+R_L)}{[(R_s+R_L)^2+X_s^2]^2}\right]=0$$

$$(R_s+R_L)^2+X_s^2-2R_L(R_s+R_L)=0$$

$$R_s^2+2R_sR_L+R_L^2+X_s^2-2R_LR_s-2R_L^2=0$$

$$R_s^2+X_s^2-R_L^2=0$$

$$R_L^2=R_s^2+X_s^2$$

$$R_L=\sqrt{R_s^2+X_s^2}=|Z_s|$$

Hence load resistance RL should be equal to the magnitude of the source impedance for maximum power transfer.

Case (ii) When the load impedance is a complex impedance with variable resistance and variable reactance (Fig. 6.145)

$$\mathbf{I}_L=\frac{\mathbf{V}_s}{\mathbf{Z}_s+\mathbf{Z}_L}$$

$$|\mathbf{I}_L|=\frac{|\mathbf{V}_s|}{\sqrt{(R_s+R_L)^2+(X_s+X_L)^2}}$$

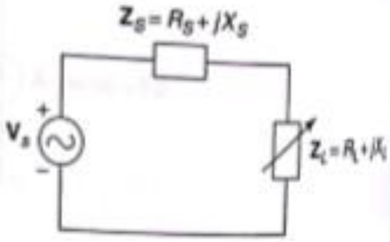


Fig. 6.145 *Complex impedance load*

The power delivered to the load is

$$P_L=|\mathbf{I}_L|^2R_L=\frac{|\mathbf{V}_S|^2R_L}{(R_s+R_L)^2+(X_s+X_L)^2}$$

For maximum value of P_L, denominator of the equation should be small, ie. $X_L=-X_s$.

$$P_L=\frac{|\mathbf{V}_S|^2R_L}{(R_s+R_L)^2}$$

Differentiating the above equation w.r.t. R_L and equating to zero,

$$\frac{dP_L}{dR_L} = |\mathbf{V}_s|^2 \left[\frac{(R_s+R_L)^2 - 2R_L(R_s+R_L)}{(R_s+R_L)^2} \right] = 0$$

$$(R_s+R_L)^2 - 2R_L(R_s+R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2$$

$$R_L = R_s$$

Hence, load resistance R_L should be equal to source resistance R_s and load reactance X_L should be equal to negative value of source reactance for maximum power transfer.

$$\mathbf{Z}_L = \mathbf{Z}_s^* = R_s - jX_s$$

i.e. load impedance should be a complex conjugate of the source impedance.

Case (iii) When the load impedance is a complex impedance with variable resistance and fixed reactance (Fig. 6.146)

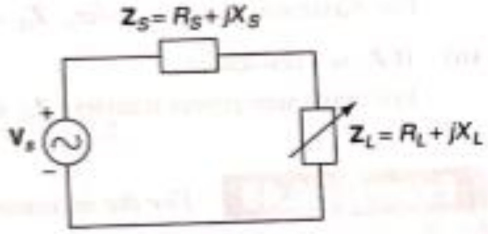


Fig. 6.146 *Complex impedance load*

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s+R_L)^2 + (X_s+X_L)^2}}$$

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_s|^2 R_L}{\sqrt{(R_s+R_L)^2 + (X_s+X_L)^2}}$$

For maximum power,

$$\frac{dP_L}{dR_L} = 0$$

$$|\mathbf{V}_s|^2 \left[\frac{(R_s+R_L)^2 + (X_s+X_L)^2 - 2R_L(R_s+R_L)}{\{(R_s+R_L)^2 + (X_s+X_L)^2\}^2} \right] = 0$$

$$(R_s+R_L)^2 + (X_s+X_L)^2 - 2R_L(R_s+R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + (X_s+X_L)^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + (X_s+X_L)^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + (X_s+X_L)^2$$

$$R_L = \sqrt{R_s^2 + (X_s+X_L)^2}$$

$$= |R_s + j(X_s+X_L)|$$

$$= |R_s + jX_s + jX_L|$$

$$= |Z_s + jX_L|$$

Hence, load resistance RL should be equal to the magnitude of the impedance Zs + jXL for maximum power transfer

LAPLACE TRANSFORM

- Converts signal in time domain to S domain
- $L[f,t] = \int_0^T e^{-st} f(t)dt = F(s)$
- Inverse Laplace Transform converts signal in s domain back to time domain
- $L^{-1}[F(s)] = f(t)$

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12	$[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13	$[e^{-\alpha t}\cos\omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin\omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$

Q) Laplace Transform of function e^{at}

$$L\left(f(t)=e^{at}\right)=\int_0^\infty e^{at}e^{-st}dt=\int_0^\infty e^{(a-s)t}dt=$$

$$\frac{1}{a-s}e^{(a-s)t}\Big|_0^\infty=(0)-\left(\frac{1}{a-s}\right)=\frac{1}{s-a}$$

$$\boxed{L\left(f(t)=e^{at}\right)=\frac{1}{s-a}}$$

Inverse Laplace Transform

$$\boxed{L^{-1}\left(\frac{1}{s+a}\right)=e^{at}}$$

GENERAL PROPERTIES OF LAPLACE TRANSFORM

	Property/Theorem	*Time Domain*	*Complex Frequency Domain*
1	Linearity	$c_1 f_1(t) + c_2 f_2(t) + \ldots + c_n f_n(t)$	$c_1 F_1(s) + c_2 F_2(s) + \ldots + c_n F_n(s)$
2	Time Shifting	$f(t-a)u_0(t-a)$	$e^{-as}F(s)$
3	Frequency Shifting	$e^{-as}f(t)$	$F(s+a)$
4	Time Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
5	Time Differentiation See also (2.18) through (2.20)	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
6	Frequency Differentiation See also (2.22)	$tf(t)$	$-\frac{d}{ds}F(s)$
7	Time Integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{F(s)}{s} + \frac{f(0^-)}{s}$
8	Frequency Integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(s)ds$
9	Time Periodicity	$f(t+nT)$	$\frac{\int_0^T f(t)e^{-st}dt}{1-e^{-sT}}$
10	Initial Value Theorem	$\lim_{t \to 0} f(t)$	$\lim_{s \to \infty} sF(s) = f(0^-)$
11	Final Value Theorem	$\lim_{t \to \infty} f(t)$	$\lim_{s \to 0} sF(s) = f(\infty)$

11.7 INVERSE LAPLACE TRANSFORM

If $L\{f(t)\} = F(s)$ then $f(t)$ is called inverse Laplace transform of $F(s)$ and symbolically written as

$$f(t) = L^{-1}\{F(s)\}$$

where L^{-1} is called the inverse Laplace transform operator.

Inverse Laplace transform can be found by the following methods:

(i) Standard results
(ii) Partial fraction expansion
(iii) Convolution theorem

11.7.2 Partial Fraction Expansion

Any function $F(s)$ can be written as $\frac{P(s)}{Q(s)}$ where $P(s)$ and $Q(s)$ are polynomials in s. For performing partial fraction expansion, the degree of $P(s)$ must be less than the degree of $Q(s)$. If not, $P(s)$ must be divided by $Q(s)$, so that the degree of $P(s)$ becomes less than that of $Q(s)$. Assuming that the degree of $P(s)$ is less than that $Q(s)$, four possible cases arise depending upon the factors of $Q(s)$.

Case I Factors are linear and distinct,

$$F(s) = \frac{P(s)}{(s+a)(s+b)}$$

By partial-fraction expansion,

$$F(s) = \frac{A}{s+a} + \frac{B}{s+b}$$

Case II Factors are linear and repeated,

$$F(s) = \frac{P(s)}{(s+a)(s+b)^n}$$

By partial-fraction expansion,

$$F(s) = \frac{A}{s+a} + \frac{B_1}{s+b} + \frac{B_2}{(s+b)^2} + \ldots + \frac{B_n}{(s+b)^n}$$

Case III Factors are quadratic and distinct,

$$F(s) = \frac{P(s)}{(s^2+as+b)(s^2+cs+d)}$$

By partial-fraction expansion,

$$F(s) = \frac{As+B}{s^2+as+b} + \frac{Cs+D}{s^2+cs+d}$$

Case IV Factors are quadratic are repeated,

$$F(s) = \frac{P(s)}{(s^2+as+b)(s^2+cs+d)^n}$$

By partial-fraction expansion,

$$F(s) = \frac{As+B}{s^2+as+b} + \frac{C_1s+D_1}{s^2+cs+d} + \frac{C_2s+D_2}{(s^2+cs+d)^2} + \ldots + \frac{C_ns+D_n}{(s^2+cs+d)^n}$$

Example 11.61 *Find the inverse Laplace transform of* $\dfrac{s+2}{s(s+1)(s+3)}$.

Solution

$$F(s)=\frac{s+2}{s(s+1)(s+3)}$$

By partial-fraction expansion,

$$F(s)=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+3}$$

$$A=sF(s)\big|_{s=0}=\left.\frac{s+2}{(s+1)(s+3)}\right|_{s=0}=\frac{2}{3}$$

$$B=(s+1)F(s)\big|_{s=-1}=\left.\frac{s+2}{s(s+3)}\right|_{s=-1}=-\frac{1}{2}$$

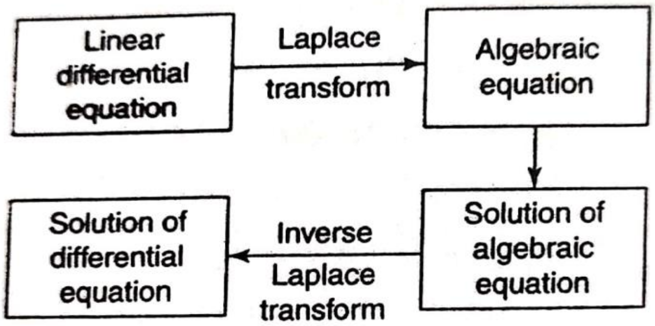
$$C=(s+3)F(s)\big|_{s=-3}=\left.\frac{s+2}{s(s+1)}\right|_{s=-3}=-\frac{1}{6}$$

$$F(s)=\frac{2}{3}\cdot\frac{1}{s}-\frac{1}{2}\cdot\frac{1}{s+1}-\frac{1}{6}\cdot\frac{1}{s+3}$$

$$L^{-1}\{F(s)\}=\frac{2}{3}L^{-1}\left\{\frac{1}{s}\right\}-\frac{1}{2}L^{-1}\left\{\frac{1}{s+1}\right\}-\frac{1}{6}L^{-1}\left\{\frac{1}{s+3}\right\}=\frac{2}{3}-\frac{1}{2}e^{-t}-\frac{1}{6}e^{-3t}$$

11.8 SOLUTION OF DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

The Laplace transform is useful in solving linear differential equations with given initial conditions by using algebraic methods. Initial conditions are included from the very beginning of the solution.



Example 11.71 Solve $\frac{dy}{dt} + 2y = e^{-3t}$, $y(0) = 1$.

Solution: Taking Laplace transform of both the sides,

$$sY(s) - y(0) + 2Y(s) = \frac{1}{s+3}$$

$$sY(s) - 1 + 2Y(s) = \frac{1}{s+3} \qquad [\because y(0) = 1]$$

$$(s+2)Y(s) = \frac{1}{s+3} + 1 = \frac{s+4}{s+3}$$

$$Y(s) = \frac{s+4}{(s+2)(s+3)}$$

By partial-fraction expansion,

$$Y(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = (s+2)Y(s)\Big|_{s=-2} = \frac{s+4}{s+3}\Big|_{s=-2} = 2$$

$$B = (s+3)Y(s)\Big|_{s=-3} = \frac{s+4}{s+2}\Big|_{s=-3} = -1$$

$$Y(s) = \frac{2}{s+2} - \frac{1}{s+3}$$

Taking inverse Laplace transform of both the sides.

$$y(t) = 2e^{-2t} - e^{-3t}$$

Laplace transforms.

1) Constant function K

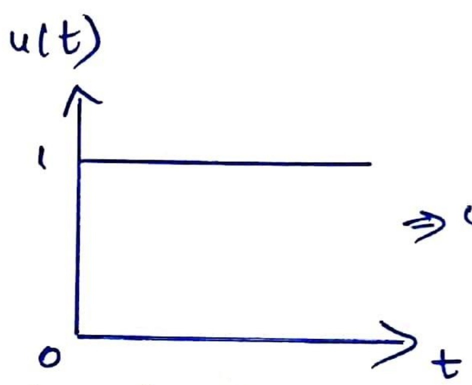
$$L\{K\} \Rightarrow \frac{K}{s}$$

2) Function t^n

$$L[t^n] \Rightarrow \frac{n!}{s^{n+1}}$$

3) Unit step function

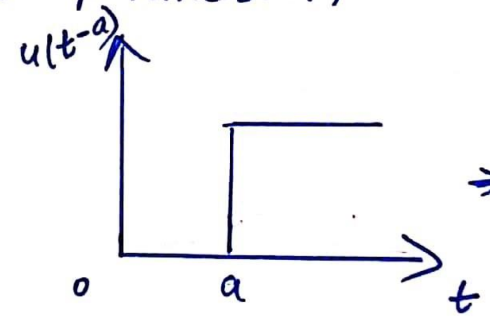
$$L[u(t)] = \frac{1}{s}$$



⇒ unit step fn

4) Delayed or shifted unit step function

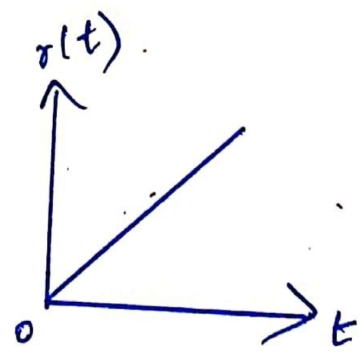
$$L[u(t-a)] = \frac{e^{-as}}{s}$$



⇒ delayed unit step fn

5) Unit - Ramp function

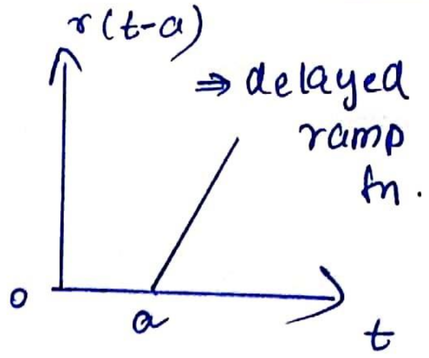
$$L(r(t)) \Rightarrow \frac{1}{s^2}$$



⇒ Ramp fn.

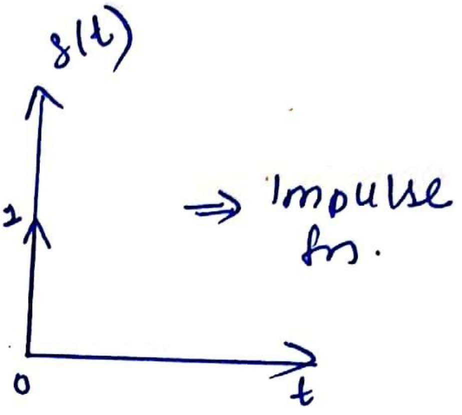
6) Delayed - Unit Ramp function

$$L(r(t-a)) \Rightarrow \frac{e^{-as}}{s^2}$$



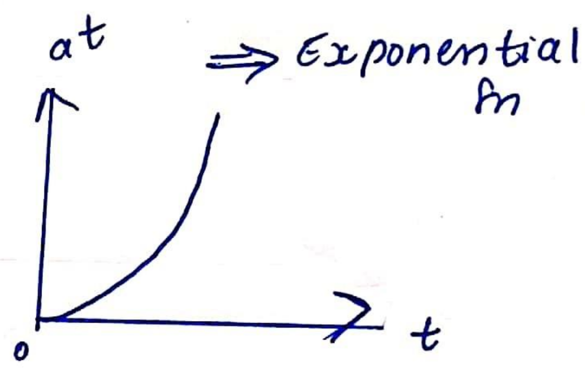
7) Unit - Impulse function

$$L[\delta(t)] = 1$$



8) Exponential Function (e^{at})

$$L[e^{at}] = \frac{1}{s-a}$$



9) Sine function

$$L[\sin \omega t] = \frac{\omega}{s^2+\omega^2}$$

10) Cosine function

$$L[\cos \omega t] = \frac{s}{s^2+\omega^2}$$

11) Hyperbolic sine function

$$L[\sinh \omega t] = \frac{\omega}{s^2-\omega^2}$$

12) Hyperbolic cosine function

$$L[\cosh \omega t] = \frac{s}{s^2-\omega^2}$$

13) Exponetially Damped function

$$L[e^{-at}\sin\omega t] = \frac{\omega}{(s+a)^2+\omega^2}$$

$$L[e^{-at}\sinh\omega t] = \frac{\omega}{(s+a)^2-\omega^2}$$

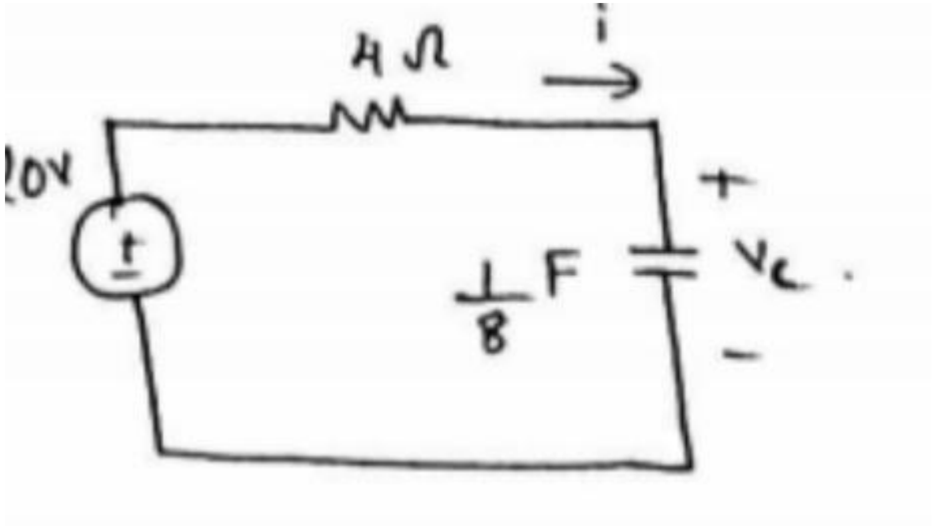
$$L[e^{-at}\cos\omega t] = \frac{s+a}{(s+a)^2+\omega^2}$$

$$L[e^{-at}\cosh\omega t] = \frac{s+a}{(s+a)^2-\omega^2}.$$

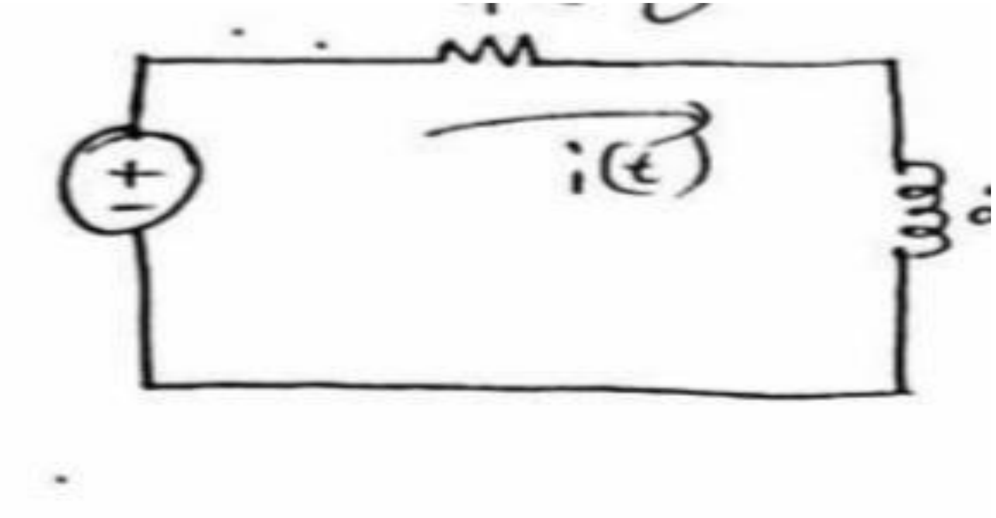
TRANSIENT ANALYSIS

- Transient response / natural response is the response 0f a system t0 a change fr0m equilibrium.
- Impulse response and step response are transient response t0 a specific input.
- Laplace transform can be used as a remarkable way t0 find system response, subject t0 any arbitrary input functi0ns

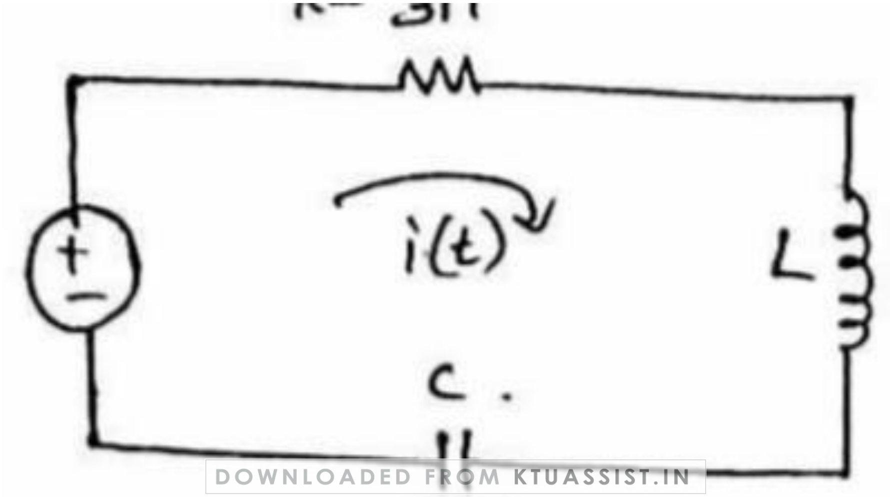
ANALYSIS 0F RC CIRCUIT



ANALYSIS 0F RL CIRCUIT



ANALYSIS 0F RLC CIRCUT



MODULE 4

- A network function gives the relation between currents or voltages at different parts of the network.
- Any network can be represented schematically by a rectangular box.

  Terminals are needed to connect any network to any other network or for taking some measurements . Two such associated terminals are called **terminal pair or port.**
  If there is only one pair of terminals in the network,it is called **one-port network.**
  If there are two pairs of terminals, it is called a **two-port network.**
  The port to which energy source is connected is called the **input port.**
  The port to which load is connected is known as the **output port.**

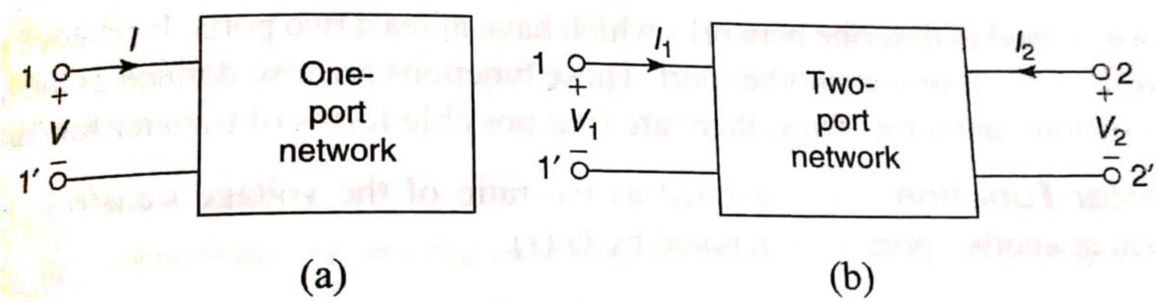


Fig. 12.1 *(a) One-port network (b) Two-port network*

  A voltage and current are assigned to each of the two ports.
  V1 and I1 are assigned to input port whereas V2and I2 are assigned to the output port.
  It is also assumed that current I1 and I2 are entering into the network at the upper terminal 1 and 2 respectively.
- A network function is broadly classified into:
  Driving point function and transfer function.

- **Driving point function:**
  If excitation and response are measured at the same ports, the network function is known as the driving point function.

1. ***Driving-point Impedance Function*** It is defined as the ratio of the voltage transform at one port to the current transform at the same port. It is denoted by $Z(s)$.

$$Z(s) = \frac{V(s)}{I(s)}$$

2. ***Driving-point Admittance Function*** It is defined as the ratio of the current transform at one port to the voltage transform at the same port. It is denoted by $Y(s)$.

$$Y(s) = \frac{I(s)}{V(s)}$$

For a two-port network, the driving-point impedance function and driving-point admittance function at port 1 are

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

Similarly, at port 2,

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

$$Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

- **Transfer function:**
  The transfer function is used to describe network which have at least two ports. It relates a voltage or current at one port to the voltage or current at another port. These functions are also defined as the ratio of a response transform to an excitation transform.Thus, there are four possible form of transfer functions.

1. ***Voltage Transfer Function*** It is defined as the ratio of the voltage transform at one port to the voltage transform at another port. It is denoted by $G(s)$.

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)}$$

$$G_{21}(s) = \frac{V_1(s)}{V_2(s)}$$

2. ***Current Transfer Function*** It is defined as the ratio of the current transform at one port to the current transform at another port. It is denoted by $\alpha(s)$.

$$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)}$$

$$\alpha_{21}(s) = \frac{I_1(s)}{I_2(s)}$$

3. ***Transfer Impedance Function*** It is defined as the ratio of the voltage transform at one port to the current transform at another port. It is denoted by $Z(s)$.

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

$$Z_{21}(s) = \frac{V_1(s)}{I_2(s)}$$

4. ***Transfer Admittance Function*** It is defined as the ratio of the current transform at one port to the voltage transform at another port. It denoted by $Y(s)$.

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)}$$

$$Y_{21}(s) = \frac{I_1(s)}{V_2(s)}$$

- **POLES AND ZERO OF NETWORK FUNCTION**

The network function F(s) can be written as ratio of two polynomials

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

Where $a_0, a_1, \dots a_n$ and $b_0, b_1, \dots, b_m$ are the coefficients of the polynomials N(s) and D(s).

Let $N(s) = 0$ have n roots as $z_1, z_2, \dots z_n$ and

$D(s) = 0$ have m roots as $p_1, p_2, \dots p_m$.

Then $$F(s) = H \frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

Where $H = \frac{a_n}{b_m}$ is a constant and is called scale factor

$z_1, z_2, \dots z_n, p_1, p_2 \dots p_m$ are complex frequencies.

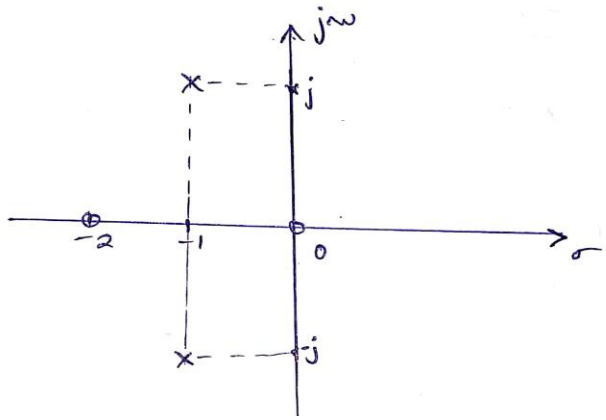
- When variable s has values z,z1,…,zn , the network function becomes zero,such complex frequencies are called **zeros of the network function**
- When variable s has values p1,p2,…pm the network function becomes infinite and has complex frequencies known as **poles of the network function.**
- A network function is completely specified by its **poles,zeros,and the scale factor.**
- If the poles or zeros are not repeated, then network function is said to be having **simple poles or simple zeros**
- If the poles or zeros are repeated, then network function is said to be having **multiple poles or multiple zeros.**

- **POLE-ZERO PLOT OF A NETWORK FUNCTION**

$$F(s) = \frac{s(s+2)}{(s+1+j)(s+1-j)}$$

The function $F(s)$ has zeros at $s=0$ and $s=-2$

The function $F(s)$ has poles at $s=-1-j$ and $s=-1+j$



- **RESTRICTIONS ON POLE AND ZERO LOCATIONS FOR DRIVING POINT FUNCTION**
-

The coefficients in the polynomials $N(s)$ and $D(s)$ must be real and positive.

The poles and zeros, if complex or imaginary, must occur in conjugate pairs.

The real part of all poles and zeros, must be negative or zero, i.e., the poles and zeros must lie in left half of s plane.

If the real part of pole or zero is zero, then that pole or zero must be simple.

The polynomials $N(s)$ and $D(s)$ may not have missing terms between those of highest and lowest degree, unless all even or all odd terms are missing.

The degree of $N(s)$ and $D(s)$ may differ by either zero or one only. This condition prevents multiple poles and zeros at $s = \infty$.

The terms of lowest degree in $N(s)$ and $D(s)$ may differ in degree by one at most. This condition prevents multiple poles and zeros at $s = 0$.

- **RESTRICTIONS ON POLE AND ZERO LOCATIONS FOR TRANSFER FUNCTION**

The coefficients in the polynomials $N(s)$ and $D(s)$ must be real, and those for $D(s)$ must be positive.

The poles and zeros, if complex or imaginary, must occur in conjugate pairs.

The real part of poles must be negative or zero. If the real part is zero, then that pole must be simple.

The polynomial $D(s)$ may not have any missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.

The polynomial $N(s)$ may have terms missing between the terms of lowest and highest degree, and some of the coefficients may be negative.

The degree of $N(s)$ may be as small as zero, independent of the degree of $D(s)$.

For voltage and current transfer functions, the maximum degree of $N(s)$ is the degree of $D(s)$.

For transfer impedance and admittance functions, the maximum degree of $N(s)$ is the degree of $D(s)$ plus one.

- **STABILITY OF NETWORK FUNCTION**

Stability of the network is directly related to the location of poles in the s-plane.

(i) When all the poles lie in the left half of the s-plane, the network is said to be stable.

(ii) When the poles lie in the right half of the s-plane, the network is said to be unstable.

(iii) When the poles lie on the $j\omega$ axis, the network is said to be marginally stable.

(iv) When there are multiple poles on the $j\omega$ axis, the network is said to be unstable.

(v) When the poles move away from $j\omega$ axis towards the left half of the s-plane, the relative stability of the network improves.

- **TIME DOMAIN BEHAVIOUR OF POLE-ZERO PLOT**

(i) When pole is at origin, i.e., at $s = 0$, the function $f(t)$ represents steady-state response of the circuit i.e., dc value. (Fig. 12.80)

Fig. 12.80 *Pole at origin*

(ii) When pole lies in the left half of the s-plane, the response decreases exponentially. (Fig. 12.81)

Fig. 12.81 *Pole in left half of the s-plane*

(iii) When pole lies in the right half of the s-plane, the response increases exponentially. A pole in the right-half plane gives rise to unbounded response and unstable system. (Fig. 12.82)

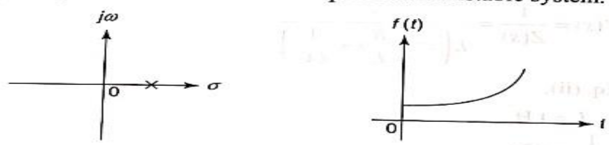


Fig. 12.82 *Pole in right half of the s-plane*

(iv) For $s = 0 + j\omega_n$, the response becomes $f(t) = Ae^{\pm j\omega_n t} = A(\cos \omega_n t \pm j \sin \omega_n t)$. The exponential response $e^{\pm j\omega_n t}$ may be interpreted as a rotating phasor of unit length. A positive sign of exponential $e^{j\omega_n t}$ indicates counterclockwise rotation, while a negative sign of exponential $e^{-j\omega_n t}$ indicates clockwise rotation. The variation of exponential function $e^{j\omega_n t}$ with time is thus sinusoidal and hence constitutes the case of sinusoidal steady state. (Fig. 12.83)

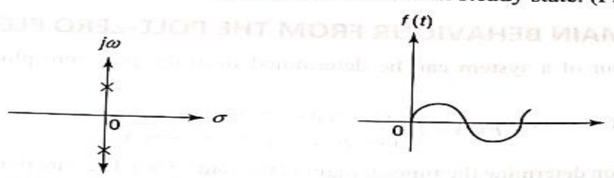


Fig. 12.83 *Poles on $j\omega$-axis*

(v)

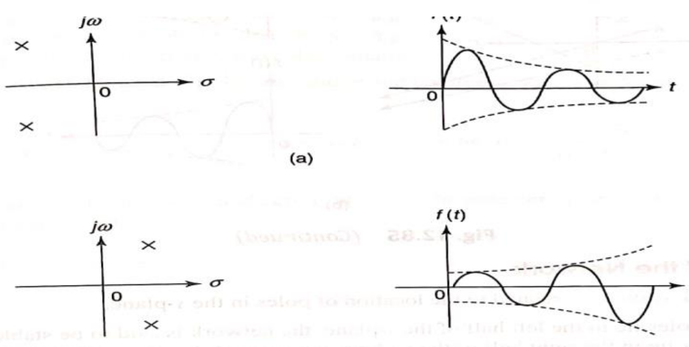


Fig. 12.84 *(a) Complex conjugate poles in left half of the S-plane*
(b) Complex conjugate poles in right half of the S-plane

- **GRAPHICAL METHOD OF DETERMINATION OF RESIDUE**

Consider a network function,

$$F(s) = H\frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

By partial fraction expansion,

$$F(s) = \frac{K_1}{(s-p_1)} + \frac{K_2}{(s-p_2)} + \cdots + \frac{K_m}{(s-p_m)}$$

The residue K_i is given by

$$K_i = (s-p_i)F(s)\big|_{s\to p_i} = H\frac{(p_i-z_1)(p_i-z_2)\cdots(p_i-z_n)}{(p_i-p_1)(p_i-p_2)\cdots(p_i-p_m)}$$

Each term $(p_i - z_i)$ represents a phasor drawn from zero z_i to pole p_i.
Each term $(p_i - p_k)$, $i \neq k$, represents a phasor drawn from other poles to the pole p_i.

$$K_i = H\frac{\text{Product of phasors (polar form) from each zero to } p_i}{\text{Product of phasors (polar form) from other poles to } p_i}$$

The residues can be obtained by graphical method in the following way:

(1) Draw the pole-zero diagram for the given network function.
(2) Measure the distance from each of the other poles to a given pole.
(3) Measure the distance from each of the other zeros to a given pole.
(4) Measure the angle from each of the other poles to a given pole.
(5) Measure the angle from each of the other zeros to a given pole.
(6) Substitute these values in the required residue equation.

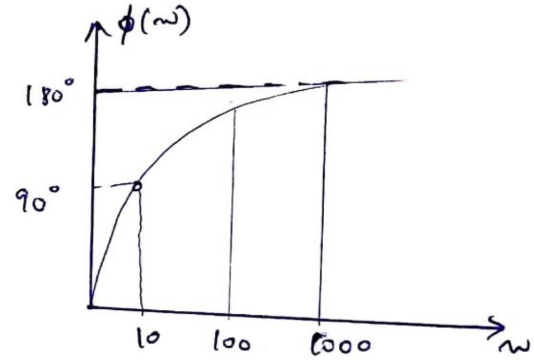
The graphical method can be used if poles are simple and complex. But it cannot be used when there are multiple poles.

- **MAGNITUDE AND PHASE RESPONSE**

$$F(s) = \frac{s+10}{s-10}$$

Magnitude of the function $F(s) = |F(j\omega)|$,

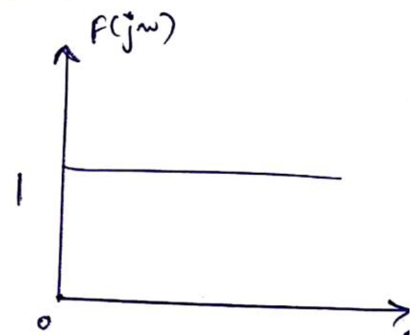
where $s = j\omega$



$$F(j\omega) = \frac{j\omega+10}{j\omega-10}$$

$$|F(j\omega)| = \frac{\sqrt{\omega^2+100}}{\sqrt{\omega^2+100}} = 1$$

phase response of the function $F(s) = \phi(\omega)$



$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(-\frac{\omega}{10}\right)$$

$$= 2\tan^{-1}\left(\frac{\omega}{10}\right)$$

if $\dfrac{a+ib}{c+id} = F(s)$

then, phase response $= \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{d}{c}\right)$

magnitude response $= \dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

MODULE 5

Z parameters

We will get the following set of two equations by considering the variables V_1 & V_2 as dependent and I_1 & I_2 as independent. The coefficients of independent variables, I_1 and I_2 are called as **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

The **Z parameters** are

$$Z_{11} = \frac{V_1}{I_1}, \ when \ I_2 = 0$$

$$Z_{12} = \frac{V_1}{I_2}, \ when \ I_1 = 0$$

$$Z_{21} = \frac{V_2}{I_1}, \ when \ I_2 = 0$$

$$Z_{22} = \frac{V_2}{I_2}, \ when \ I_1 = 0$$

Z parameters are called as **impedance parameters** because these are simply the ratios of voltages and currents. Units of Z parameters are Ohm (Ω).

We can calculate two Z parameters, Z_{11} and Z_{21}, by doing open circuit of port2. Similarly, we can calculate the other two Z parameters, Z_{12} and Z_{22} by doing open circuit of port1. Hence, the Z parameters are also called as **open-circuit impedance parameters**.

Y parameters

We will get the following set of two equations by considering the variables I_1 & I_2 as dependent and V_1 & V_2 as independent. The coefficients of independent variables, V_1 and V_2 are called as **Y parameters**.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

The **Y parameters** are

$$Y_{11} = \frac{I_1}{V_1}, \; when \; V_2 = 0$$

$$Y_{12} = \frac{I_1}{V_2}, \; when \; V_1 = 0$$

$$Y_{21} = \frac{I_2}{V_1}, \; when \; V_2 = 0$$

$$Y_{22} = \frac{I_2}{V_2}, \; when \; V_1 = 0$$

Y parameters are called as **admittance parameters** because these are simply, the ratios of currents and voltages. Units of Y parameters are mho.

We can calculate two Y parameters, Y_{11} and Y_{21} by doing short circuit of port2. Similarly, we can calculate the other two Y parameters, Y_{12} and Y_{22} by doing short circuit of port1. Hence, the Y parameters are also called as **short-circuit admittance parameters**.

T parameters

We will get the following set of two equations by considering the variables V_1 & I_1 as dependent and V_2 & I_2 as independent. The coefficients of V_2 and $-I_2$ are called as **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

The **T parameters** are

$$A = \frac{V_1}{V_2},\ when\ I_2 = 0$$

$$B = -\frac{V_1}{I_2},\ when\ V_2 = 0$$

$$C = \frac{I_1}{V_2},\ when\ I_2 = 0$$

$$D = -\frac{I_1}{I_2},\ when\ V_2 = 0$$

T parameters are called as transmission parameters or **ABCD parameters**. The parameters, A and D do not have any units, since those are dimension less. The units of parameters, B and C are ohm and mho respectively.

We can calculate two parameters, A and C by doing open circuit of port2. Similarly, we can calculate the other two parameters, B and D by doing short circuit of port2.

T ' parameters

We will get the following set of two equations by considering the variables V_2 & I_2 as dependent and V_1 & I_1 as independent. The coefficients of V_1 and $-I_1$ are called as **T' parameters**.

$$V_2 = A'V_1 - B'I_1$$

$$I_2 = C'V_1 - D'I_1$$

The **T' parameters** are

$$A' = \frac{V_2}{V_1},\ when\ I_1 = 0$$

$$B' = -\frac{V_2}{I_1},\ when\ V_1 = 0$$

$$C' = \frac{I_2}{V_1},\ when\ I_1 = 0$$

$$D' = -\frac{I_2}{I_1},\ when\ V_1 = 0$$

h-parameters

We will get the following set of two equations by considering the variables V_1 & I_2 as dependent and I_1 & V_2 as independent. The coefficients of independent variables, I_1 and V_2, are called as **h-parameters**.

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

The h-parameters are

$$h_{11} = \frac{V_1}{I_1}, \; when \; V_2 = 0$$

$$h_{12} = \frac{V_1}{V_2}, \; when \; I_1 = 0$$

$$h_{21} = \frac{I_2}{I_1}, \; when \; V_2 = 0$$

$$h_{22} = \frac{I_2}{V_2}, \; when \; I_1 = 0$$

h-parameters are called as **hybrid parameters**. The parameters, h_{12} and h_{21}, do not have any units, since those are dimension-less. The units of parameters, h_{11} and h_{22}, are Ohm and Mho respectively.

g-parameters

We will get the following set of two equations by considering the variables I_1 & V_2 as dependent and V_1 & I_2 as independent. The coefficients of independent variables, V_1 and I_2 are called as **g-parameters**.

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

The **g-parameters** are

$$g_{11} = \frac{I_1}{V_1},\ when\ I_2 = 0$$

$$g_{12} = \frac{I_1}{I_2},\ when\ V_1 = 0$$

$$g_{21} = \frac{V_2}{V_1},\ when\ I_2 = 0$$

$$g_{22} = \frac{V_2}{I_2},\ when\ V_1 = 0$$

g-parameters are called as **inverse hybrid parameters**. The parameters, g_{12} and g_{21} do not have any units, since those are dimension less. The units of parameters, g_{11} and g_{22} are mho and ohm respectively.

PARAMETER CONVERSIONS

Z parameters to Y parameters

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}}{\Delta Z}$$

Z parameters to T parameters

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

Y parameters to Z parameters

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}}{\Delta Y}$$

Y parameters to T parameters

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$$

T parameters to h-parameters

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

h-parameters to Z parameters

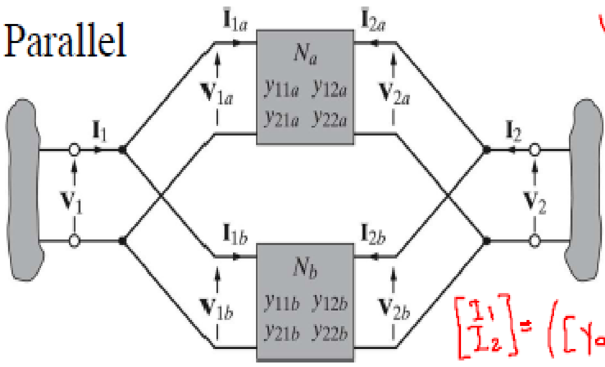
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{h_{11}h_{22}-h_{12}h_{21}}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

SERIAL AND PARALLEL CONNECTION OF TWO PORT NETWORK

When two or more two-port networks are connected, the two-port parameters of the combined network can be found by performing matrix algebra on the matrices of parameters for the component two-ports.

When two-port networks are connected
(a) in series, their impedance parameters add
(b) in parallel, their admittance parameters add

Parallel



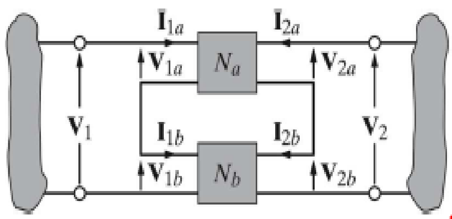
$$V_1 = V_{1a} = V_{1b} \;;\; V_2 = V_{2a} = V_{2b}$$

$$I_1 = I_{1a} + I_{1b} \;;\; I_2 = I_{2a} + I_{2b}$$

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = [Y_a]\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} ; \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = [Y_b]\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left([Y_a] + [Y_b]\right)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Series



$$I_1 = I_{1a} = I_{1b} \;;\; I_2 = I_{2a} = I_{2b}$$

$$V_1 = V_{1a} + V_{1b} \;;\; V_2 = V_{2a} + V_{2b}$$

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = [Z_a]\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} ; \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = [Z_b]\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left([Z_a] + [Z_b]\right)\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

RECIPROCAL TWO PORT NETWORK

A network is said to be reciprocal if the voltage appearing at port 2 due to a current applied at port 1 is the same as the voltage appearing at port 1 when the same current is applied to port 2. Exchanging voltage and current results in an equivalent definition of reciprocity.

SYMMETRICAL TWO PORT NETWORK

A two-port network is said to be symmetrical if the ports of the two-port network can be interchanged without changing the port voltages and currents.

Characteristic Imp (Z_0)

Chara. imp is the imp that should be terminated at a distance, so that the i/p & o/p imp are equal

$$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$$

Image impedance applies to the impedance seen looking into the ports of a n/w.

Definition of image imp. for a 2-port n/w is the imp. Z_{i1} seen looking into port 1 when port 2 is terminated with the image imp. Z_{i2}, for port 2.

In general, the image imp. of ports 1 & 2 will not be equal unless the n/w is symmetrical w.r to the ports.

Propogation Constant

By definition prop. const 'γ' of a n/w is given by,

$$\gamma = \log_e \left(\frac{I_1}{I_2} \right).$$

$$I_2 = e^{-\gamma} I_1$$

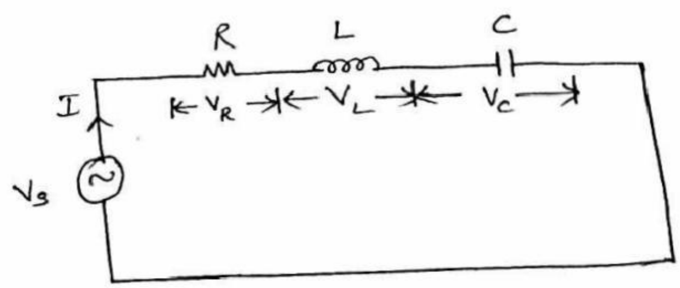
$I_2 < I_1$ for an attenuator.

MODULE 6

RESONANCE

SERIES RESONANCE

- In RLC circuit the current lags or leads the voltage is determined by Xl and Xx.
- Xl causes the total current to lags behind the applied voltage.
- Xc causes the total current to leads the applied voltage.
- Xl > Xc , the circuit is inductive
- Xc > Xl , the circuit is capacitive
- If one of the parameters in the series RLC circuit is varied in such a way that the current is in phase with the applied voltage, then the circuit is said to be in RESONANCE.
- Series Resonance occurs when Xl= Xc
- Frequency at which resonance occurs is called resonant frequency (fr).
- Impedance in a series RLC circuit is purely resistive.



Total impedance for the series RLC circuit is;

$$Z = R + j\omega L - \frac{j}{\omega C} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

$$I = \frac{V_s}{Z}.$$

At fr, the voltage across inductance and capacitance are equal in magnitude.

- They are 180° out of phase and they cancel each other and hence 0 voltage appear across LC combination.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

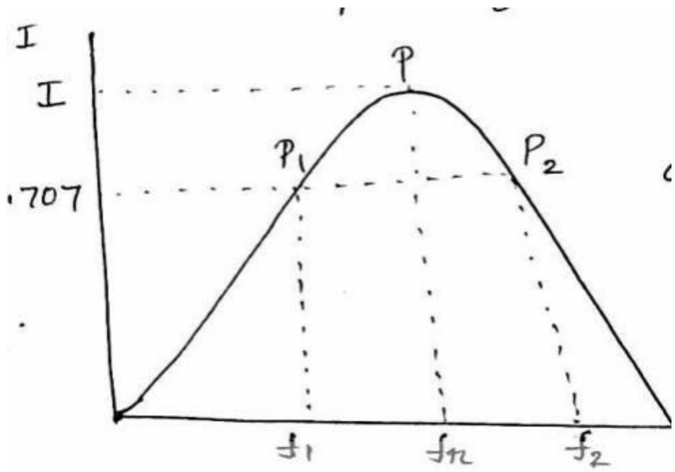
IMPEDANCE AND PHASE ANGLE OF SERIES RESONANT CIRCUIT

The impedance of series RLC circuit is given by;

$$|Z| = \sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}$$

BANDWIDTH OF RLC CIRCUIT

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to **70.7%** it's value at the resonant frequency.



The response curve is called **Selectivity curve.**
Selectivity indicates how well a resonant circuit responds to a certain frequency and eliminates all other frequencies.
BW = f2 – f1
Unit of BW = Hertz

BW = R / 2πl
Yo
f1 = fr – R/4πl
f2 = fr + R/4πl

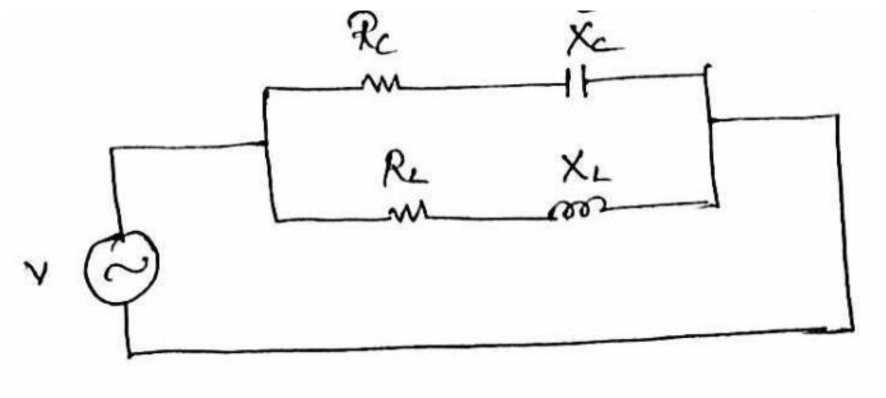
The ratio of reactance of the coil to its resistance is called as **quality factor.**

$$Q = \frac{f_r}{BW} = \frac{f_2 - f_1}{f_r} = \frac{1}{Q}.$$

Higher value of quality factor causes smaller bandwidth and vice versa.
The upper and lower cut off frequencies are also called as half power frequencies.

PARALLEL RESONANCE

- Occurs when Xc = Xl
- Frequency at which resonance occurs is called resonant frequency.
- At Xc = Xl the currents are equal in magnitude and 180° out of phase.
- Therefore they cancel each other and total current is 0.



This is the condition for resonant frequency ;

$$\therefore \omega_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

As a special case,

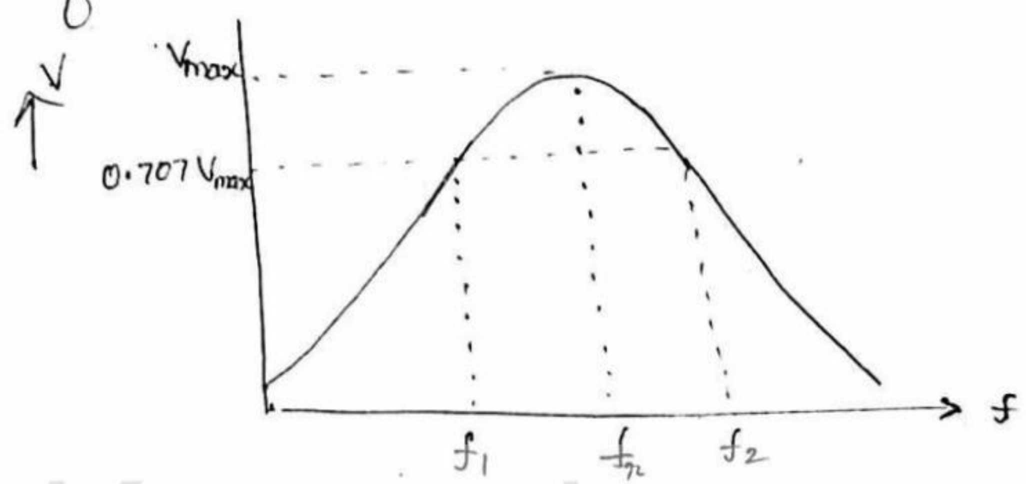
if $R_L = R_C$,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

QUALITY FACTOR IN PARALLEL RESONANCE

The voltage variation with frequency is as follows:



Bandwidth = f2 - f1

Lower half frequency ,

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Upper half frequency ,

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth w2 - w1 = 1/ RC
Quality factor ;
<u>COUPLED CIRCUITS</u>

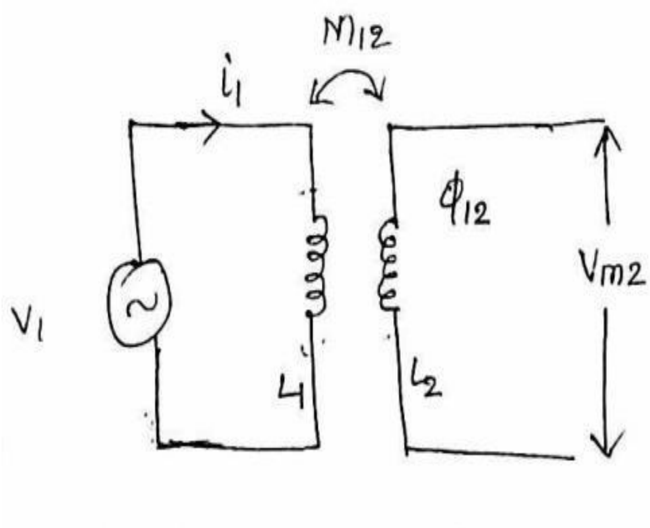
$$Q_r = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{1/RC} = \omega_r RC.$$

Any circuit involving elements with magnetic coupling.
<u>Self inductance</u>
Time rate of change in current through a coil induces a voltage in it.

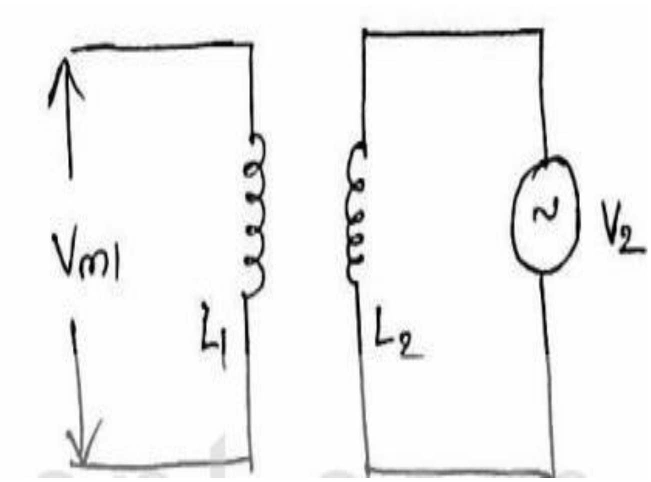
$$v(t) = L\frac{di(t)}{dt}$$

<u>Mutual inductance</u>
Mutual inductance results from a current flowing in one coil establishing a magnetic flux about the coil and also about a secondary coil which is sufficiently close to this coil.
The time varying flux surrounding the secondary coil produces a voltage across the terminals of this 2° coil.

The voltage induced is proportional to the rate of change of current flowing through the primary coil.



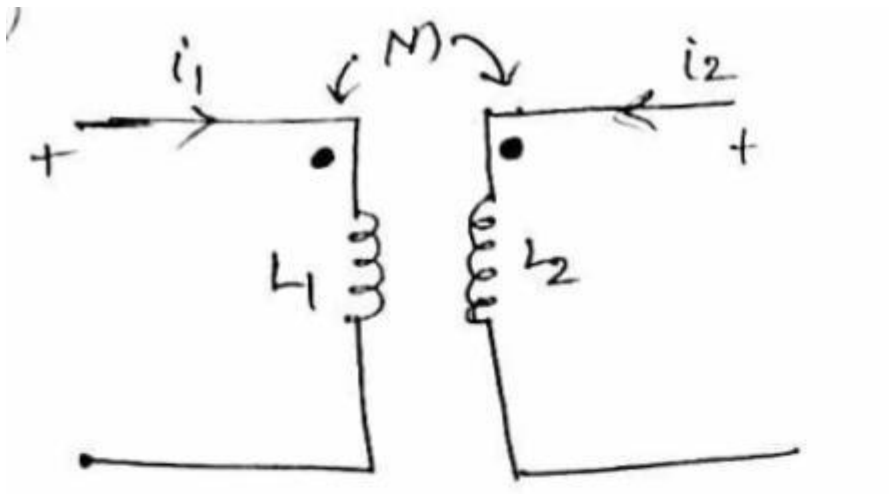
$$V_{m2} = M_{12}\frac{di_1}{dt}$$



$$V_{m1} = M_{21}\frac{di_2}{dt}$$

DOT CONVENTION FOR COUPLED COILS

Mutually induced voltages can be either positive or negative depending upon the direction of the winding of the coil and can be decided by the presence of dots placed at one end of the 2 coils.



The mutually induced voltage is positive when the currents i1 and i2 both enter the windings at the dotted terminal.

$$V_1 = L_1 \frac{di_1}{dt} \pm M\frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} \pm M\frac{di_1}{dt}.$$

COEFFICIENT OF COUPLING

The amount of coupling between the inductively coupled coils is expressed in terms of the coefficient of coupling.

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

K = coefficient of coupling
M = mutual inductance between coils
L1 , L2 = self inductance of first and second coil respectively

Case 1

If k= 1

All the flux produced in one coil is linked to other. Such a coil is called tightly coupled

Case 2

If k=0

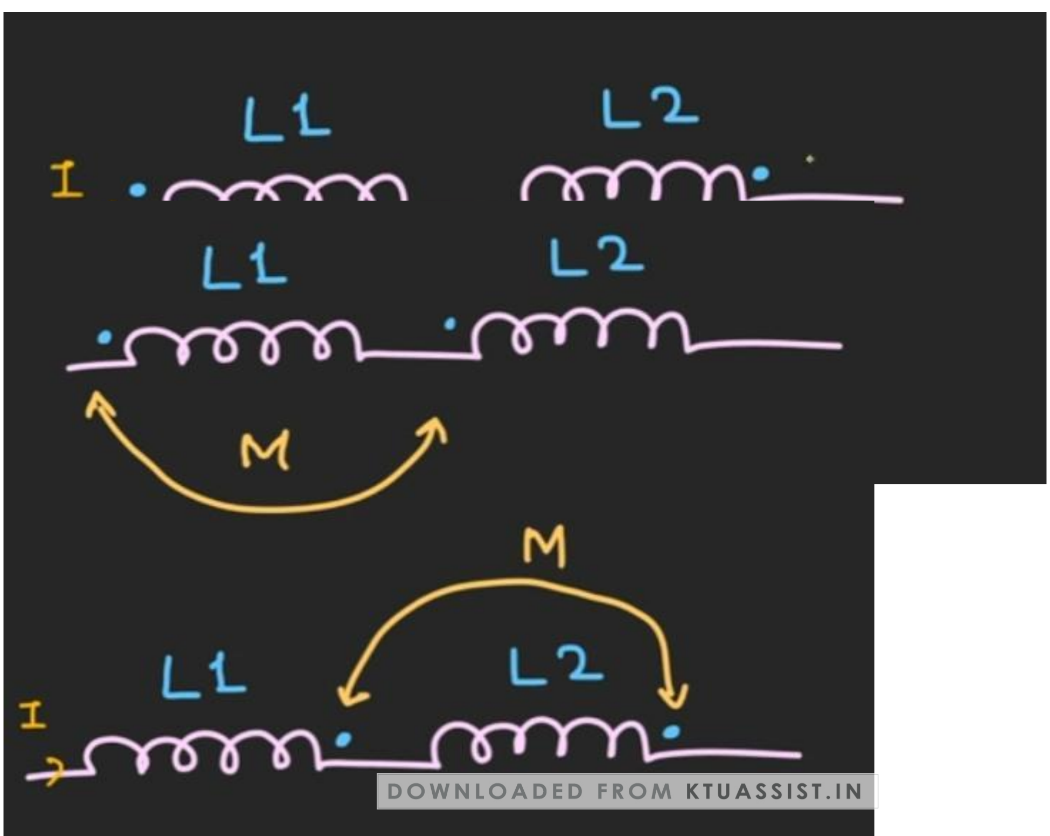
No flux is linked between the 2 coils and the coil is said to be magnetically isolated.

Case 3

If k is having a very low value;

Flux linkage will be low and the coils are said to be loosely coupled.

- **When both the currents are entering or leaving the dots then the mutual inductance is positive.**
- **When current through one coil is entering and in the other the current is leaving the dot then the mutual inductance is negative.**
- **In series coupled circuit if the current is entering the dot for both the inductors then the equation becomes Leq = L1+L2+2M**
- **In series coupled circuit if current is entering the dot for one inductor and for another inductor it is leaving then the equation is Leq = L1+L2 -2M**
- **In the parallel coupled circuit if the current is entering the dot for both inductors then the equation is Leq = (L1*L2 - M^2) / (L1+ L2 – 2M)**
- **In the parallel coupled circuit if the current is entering the dot for one inductor and current is leaving the dot for another inductor the the Leq = (L1*L2 - M^2) / (L1+ L2 + 2M)**



TUNED CIRCUIT

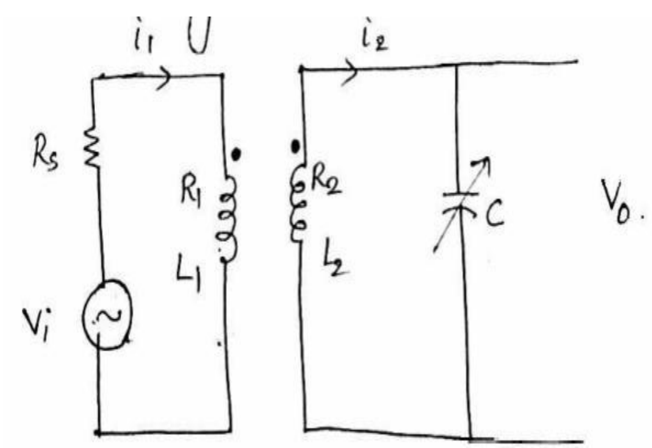
An LC Circuit also called as resonant , tank circuit or tuned circuit consists of an inductor and a capacitor. LC circuits are used either for generating signals at a particular frequency, or picking out a signal at a particular frequency. They are key component in many electronic devices , particularly radio equipment used in circuits like oscillator, filters, tuners and mixers.

TUNED circuits are of 2 types ;

- Single tuned
- Double tuned

SINGLE TUNED CIRCUITS

- Capacitor is connected to secondary only.
- Tuning is done only in the secondary.
- Primary inductance value is very low.



$$\boxed{M = \frac{\sqrt{R_s R_2}}{\omega_n}}$$ → condition for maximum output voltage.

maximum output voltage Vom,

$$\frac{V_i}{2\omega_n C\sqrt{R_s R_2}}$$

Maximum amplification factor , Am

$$A_m = \frac{1}{2\omega_n C\sqrt{R_s R_2}}$$

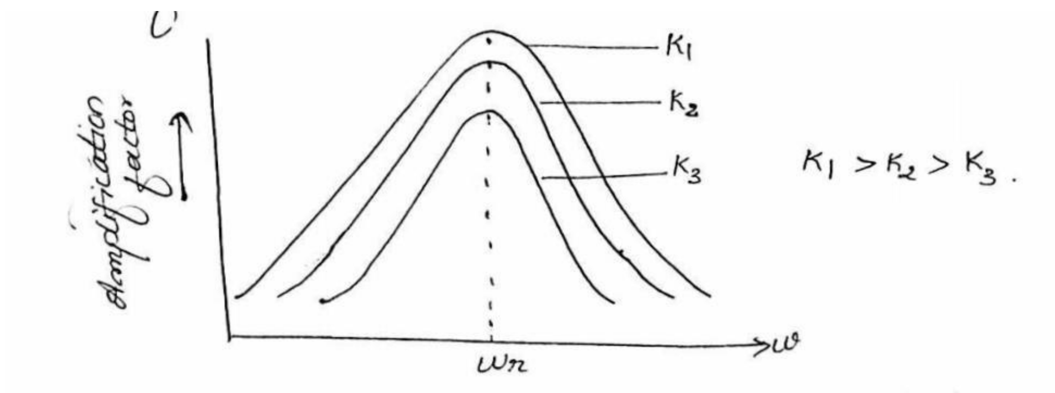
Output voltage Vo,

$$V_o = \frac{M V_i}{C\left[R_s R_2 + \omega_n^2 M\right]}$$

Resonant frequency,

$$\omega_n = \frac{1}{\sqrt{L_2 C}}$$

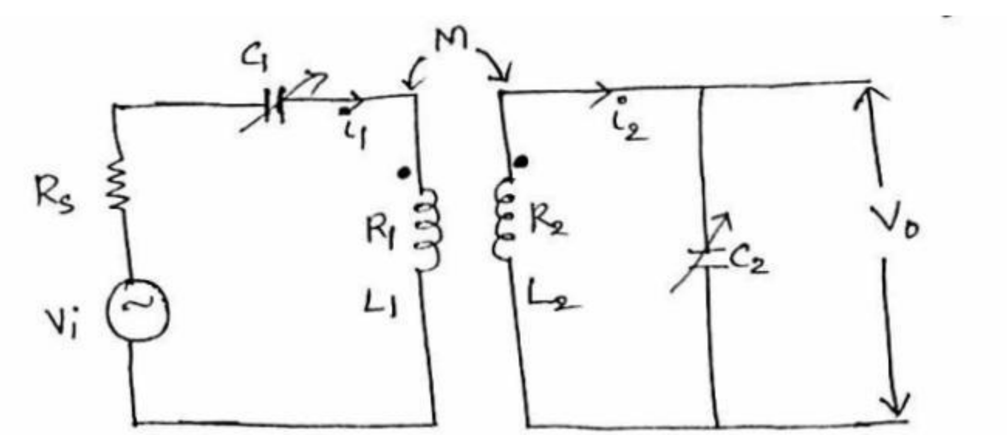
VARIATION OF AMPLIFICATION FACTOR



If the coefficient of coupling is 1 there will be maximum flux linkage.

DOUBLE TUNED CIRCUITS

- Capacitor is connected to both primary and secondary.
- The 2 capacitors will be tuned at the same frequency.
- If frequencies are not same then it is called stagger tuned circuit.



Resonant frequncy,

$$\omega_r = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}$$

Amplification factor, A= Vo/Vi

$$\text{Amplification factor,} \quad A = \frac{V_0}{V_i} = \frac{M}{C_2\left[(R_s+R_1)R_2 + \omega_r^2 M^2\right]}$$

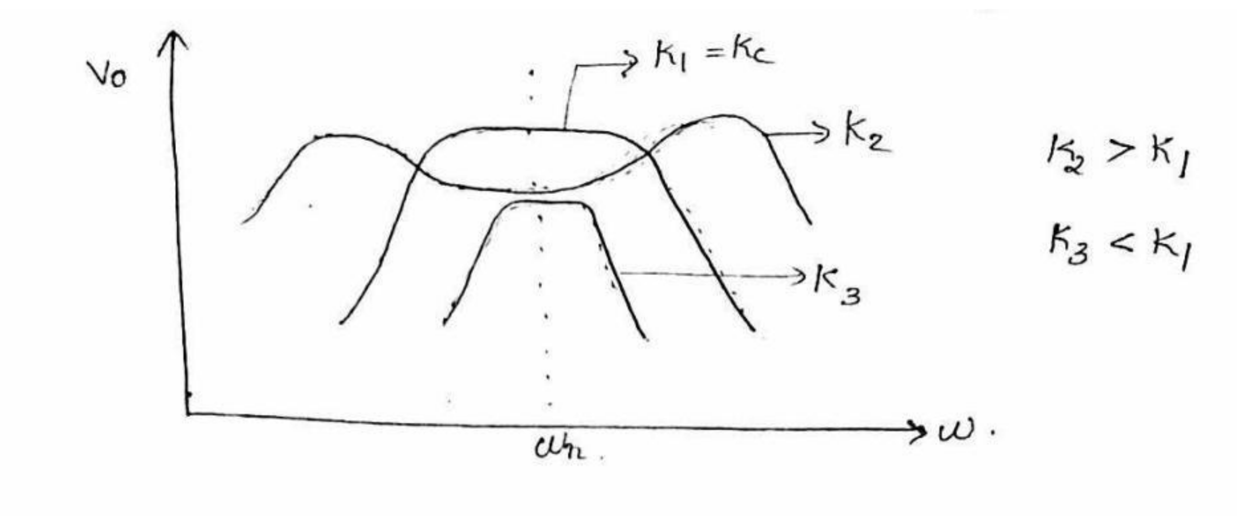
Critical mutual inductance ,Mc ,

$$\boxed{M_c = \frac{\sqrt{(R_s+R_1)R_2}}{\omega_r}}$$

Maximum output voltage, Vom,
Vom = Vi/ 2*wr* Cr *√[(Rs+R1)*R2]
Critical coefficient of coupling Kc,

$$K_c = \frac{M_c}{\sqrt{L_1 L_2}}$$

VARIATION OF OUTPUT VOLTAGE WITH FREQUENCY FOR DIFFERENT COUPLING COEFFICIENTS



Signals and systems:

signal :

- Anything that carries information
- A fn. of one or more variables, which conveys info. on the basis of a physical phenomenon.
- can be represented mathematically or graphically.
- varying or constant form of energy w.r.t an independant variable

signals classification :

- Nature of occurance
  - Basic signals
  - continuous and discrete
  - Analog and digital
- Based on Properties
  - odd and even
  - Periodic and non-periodic
  - Energy and Power

continuous and discrete s/gs :

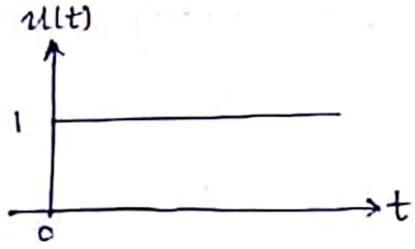
continuous:
- defined for all values of indep. variable 't'

discrete:
- Not defined for all values of independant variable, only for discrete values
- Do not exist in nature
- less energy as there is no continious consumption or change in amplitude.

- Basic signals:

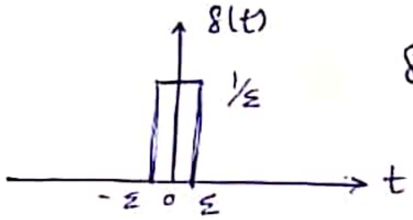
  - Speech signals are a combination of impulse, exponential and random sigs.

1. Step signal: Amplitude suddenly increases and remains constant forever. $u(t) = 1,\ t \geq 0$

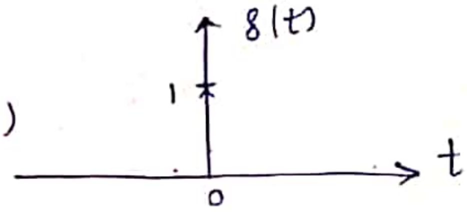


- Response of a s/m for $u(t)$ reveals how quickly s/m responds to an i/p change.

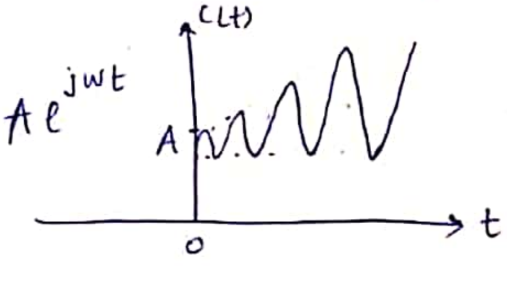
2. Impulse: Derivative of step fn. w.r.t time

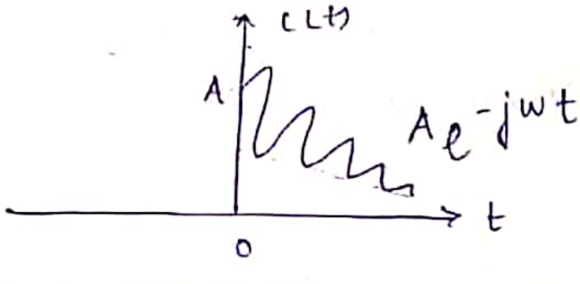


$\delta(t) = 1/\varepsilon$
$(-\varepsilon \leq t \leq \varepsilon)$



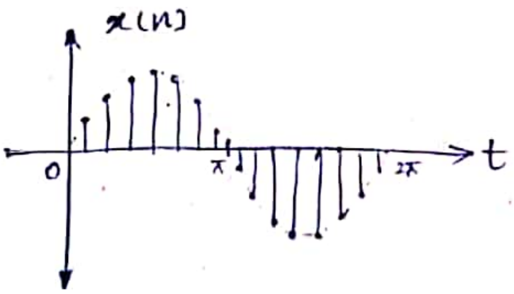
3. Complex: $c(t) = A e^{\pm j\omega t} = A[\cos\omega t \pm j \sin\omega t]$





4. Sinusoidal: $x(t) = A \sin(\omega t + \phi)$

- For a discrete sinusoidal s/g to be zero, ω must be a rational multiple of 2π; $x[n] = A \sin \omega n$

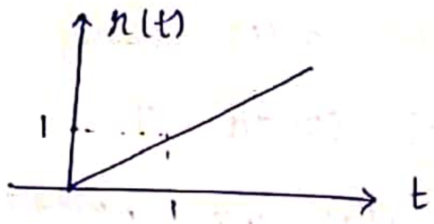


- $\sin n\pi = 0 \quad ; n \in z$
- $\cos\left[(2n+1)\frac{\pi}{2}\right] = 0 \quad ; n \in z$
- $\cos 2n\pi = 1 \quad ; n \in z$
- $\sin\left[(2n+1)\frac{\pi}{2}\right] = 1 \quad ; n \in z$

5. Ramp fn : $r(t) = \int u(t)\,dt$; integral of step fn.

$= t\,u(t)$ → Unit ramp fn.

$$r(t) = \begin{cases} t \ ; & t \geq 0 \\ 0 \ , & t < 0 \end{cases}$$



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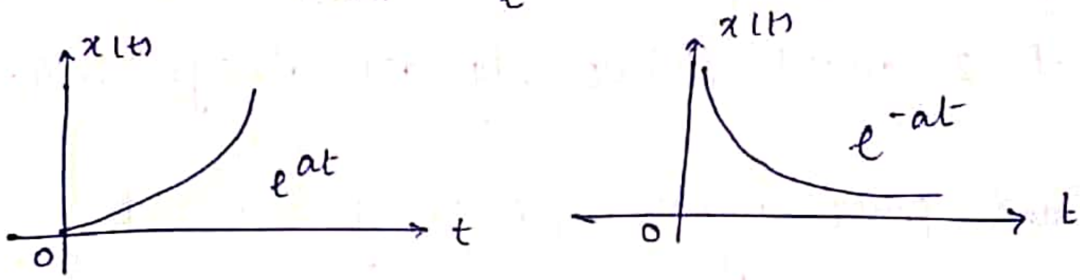
6. Exponential : $x(t) = e^{\pm at}$



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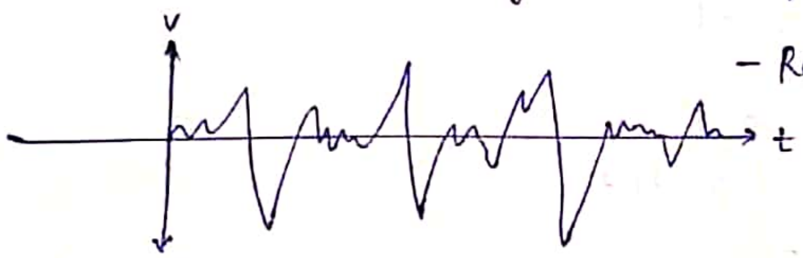
7. Random : cannot predict magnitude. eg. Noise, ECG.



— Random process

• Even and Odd signals : $x(t) = x_e(t) + x_o(t)$

$x(t) = x(-t)$ → Even signals → Symmetric

$x[n] = x[-n]$

eg. cosine → $x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$x(t) \Rightarrow x(-t) = -x(t)$ → Odd signals → odd symmetry

$x[-n] = -x[n]$

eg. sine → $x_o(t)$

$x_o(t) = \dfrac{x(t) - x(-t)}{2}$

- Most of the energy is in the even component.

- Periodic and Non-Periodic s/gs.

if $x(t+T) = x(t)$, then $x(t)$ is periodic with fundamental period T.

- Sum of 2 s/gs are periodic if $\frac{T_1}{T_2}$ = rational no. and fundamental period ⇒ LCM (T_1, T_2).

★ Sum of 2 discrete periodic s/gs are always even.

- Energy and Power signals: [Mutually Exclusive]

$$E_\infty = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} |x^2(t)|\,dt$$

$$E_{avg} = \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} x^2(t)\,dt,$$

if $P_\infty < \infty$ → it is a power signal.

★ If s/g is periodic and period is infinite, it can be a power signal. ∴ $[E = 0]$

★ If non-periodic or short-term periodic, then energy is infinite. ∴ $[P = 0]$.

- Operations on signals:

(on dependant variable) | (on independant variable)

- scaling → $y(t) = a\,x(t)$
  - $a > 1 \Rightarrow$ amplify
  - $a < 1 \Rightarrow$ attenuate
- addition → $y(t) = x_1(t) + x_2(t)$
- multiply → $y(t) = x_1(t) \times x_2(t)$
- time scale → $y(t) = x(at)$
  - $a > 1$, compression
  - $a < 1$, expansion
- time shift → $y(t) = x(t \pm T)$
- Time reversal → $y(t) = x(-t)$

★ compression causes reduction in edges which causes loss of data. when expanding a compr^d sig, you dont retrieve original.

★ signals at retrieving end are retrieved using negatives; multiplied to find maximum energy occurance.

- In applications, first selection of BW (shifting) and then compression is done (scaling).

- Systems:
  - Lumped and distributed
  - Linear and Non-linear
  - Static and Dynamic
  - Time variant and Invariant
  - Causal and Non-causal
  - FIR and IIR

SIGNALS AND SYSTEMS

MODULE 1 & 2

1. Lumped and distributed :
   - Lumped s/ms are a fn of space, ODE. Made of standard components (R, L and C). can be analysed using Laplace
   - Distributed s/ms are a fn of time and space, PDE. eg. Transmission lines

2. Static and dynamic
   - Static s/ms require no memory. o/p depends on present i/p only, no past or future value required
   - Dynamic s/ms require memory. o/p depends on past and future i/ps

3. Linear and non-linear:
   - If a change occurs in the i/p side, the corresponding change should occur in o/p side.
     ∴ It is linear. (Obeys superposition and homogenity)
   -

4. Time variant and time invariant.
   - Behaviour of s/m should be independant of time. We cannot predict the outcome of a s/m it is time variant.
   - Multiplication with an independant variable makes it variant

• Time domain → frequency domain } fourier transformation

⇒ Fourier series representation → periodic

⇒ Fourier transform → periodic & aperiodic signals

~~Continuous.~~

$x(t+T) = x(t)$

Eqn of CTFS is

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

where $\omega_0 = 2\pi F_0 = \frac{2\pi}{T}$ → fundamental frequency in rad/sec

F_0 = fundamental frequency in Hz.

$n\omega_0$ = harmonic frequencies.

C_n = Fourier coeffecients of exponential form of fourier series

$$\boxed{C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt} \quad \text{or} \quad \boxed{C_n = \frac{1}{T}\int_{0}^{T} x(t)\, e^{-jn\omega_0 t}\, dt}$$

Conditions for existance of fourier series (Dirichlet's condn)

Convergence of Fourier series

1) The signal $x(t)$ is well defined and single valued, except possibly at a finite number of points.

2) $x(t)$ absolutely integrable ⇒ $\int |x(t)|\, dt < \infty$.

$x(t) \Rightarrow$ possess only finite no. of discontinuity in period T

$x(t) \rightarrow$ must have finite no of +ve and -ve max in period T

eg: $\sin 2\pi/t$, $0 < t \le 1$

operties of CTFS

$Ax(t) + By(t) = A x(k) + By(k) \Rightarrow$ linearity

me shifting

$x(t) \leftrightarrow x(k)$

$x(t-t_0) \xleftrightarrow{FS} e^{-jk\omega_0 t_0} x(k)$

me reversal

$x(t) \xleftrightarrow{FS} x(k)$

$x(-t) \leftrightarrow x(-k)$

Time differensiation

$x(t) \xleftrightarrow{FS} x(k)$

$\frac{d}{dt} x(t) \leftrightarrow j\omega_0 k \, x(k)$

$\frac{d^n}{dt^n} x(t) \longleftrightarrow (j\omega_0 k)^n x(k)$

Time Scaling

$x(t) \xleftrightarrow{FS} x(k)$

$x(\alpha t) \xleftrightarrow{FS} x(k)\Big|_{T \to T/\alpha}$

6) **Frequency Shifting**

$x(t) \xrightarrow{FS} x(k)$

$e^{jk_0\omega_0 t} x(t) \xleftrightarrow{FS} x(k-k_0)$

7) **Multiplication property**

$x(t) \xleftrightarrow{FS} x(k)$

$y(t) \xleftrightarrow{FS} Y(k)$

then.

$x(t)\, y(t) \xleftrightarrow{FS} x(k) \circledast Y(k)$

circular convolution

8) **Convolution Property**

$x(t) \xleftrightarrow{FS} x(k)$

$y(t) \xleftrightarrow{FS} y(k)$

then

$x(t) \circledast y(t) \longleftrightarrow T\, x(k)\, y(k)$

9) **Power of a signal**

Parseval's theorem

$\frac{1}{T}\int |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |x(k)|^2$

CTFT

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega}$$

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

Properties of CTFT

1. Linearity

$x(t) \xleftrightarrow{F.T} X(j\omega)$

$y(t) \xleftrightarrow{F.T} Y(j\omega)$

$a\,x(t) + b\,y(t) \xleftrightarrow{F.T} a\,X(j\omega) + b\,Y(j\omega)$

2. Time shifting

$x(t) \longleftrightarrow X(j\omega)$

$x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(j\omega)$

3. Frequency shifting

$x(t) \xleftrightarrow{F.T} X(j\omega)$

$e^{j\omega_0 t} x(t) \xleftrightarrow{F.T} X(j(\omega-\omega_0))$

Time scaling

$x(t) \xleftrightarrow{F.T} X(j\omega)$

then $x(\alpha t) \xleftrightarrow{F.T} \frac{1}{|\alpha|} X(j\omega/\alpha)$

5. Time reversal

$x(t) \xleftrightarrow{F.T} X(j\omega)$

$x(-t) \xleftrightarrow{F.T} X(-j\omega)$

6. Time Differensiation

$x(t) \xleftrightarrow{F.T} X(j\omega)$

then $\frac{d}{dt} x(t) \longleftrightarrow j\omega X(j\omega)$

7. Differensiation in frequency

$x(t) \xleftrightarrow{F.T} X(j\omega)$

$-jt\, x(t) \longleftrightarrow \frac{d}{d\omega} X(j\omega)$

8. Convolution in Time

$x_1(t) \longleftrightarrow X_1(j\omega)$

$x_2(t) \longleftrightarrow X_2(j\omega)$

then

$x_1(t) * x_2(t) \longleftrightarrow X_1(j\omega) \cdot X_2(j\omega)$

9. Parseval's Theorem

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

LAPLACE TRANSFORMS

Laplace transform of a time domain x(t) s given by:

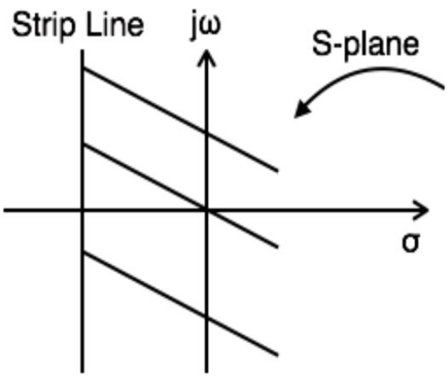
Laplace transform of $x(t) = X(S) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$. , Where s is a complex variable and is equal to s=σ+jω. Here the operator L is Laplace operator which transforms the time domain function x(t) in to frequency domain function X(S).

REGION OF CONVERGENCE:

The range variation of σ for which the Laplace transform converges is called region of convergence

Properties of ROC of Laplace Transform

- ROC contains strip lines parallel to jω axis in s-plane.

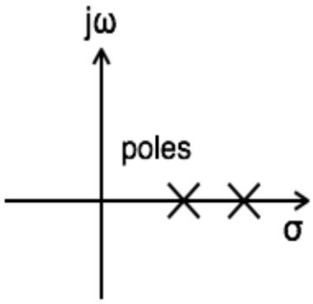


- If x(t) is absolutely integral and it is of finite duration, then ROC is entire s-plane.
- If x(t) is a right sided sequence then ROC : Re{s} > σ_o.
- If x(t) is a left sided sequence then ROC : Re{s} < σ_o.
- If x(t) is a two sided sequence then ROC is the combination of two regions.

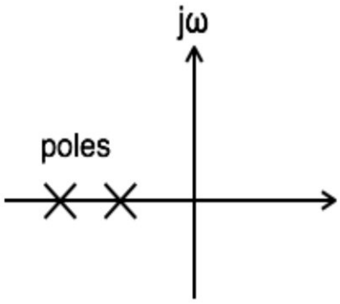
Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	Re{s} > 0
3	$u(-t)$	$\frac{1}{s}$	Re{s} < 0
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	Re{s} < 0
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	Re{s} < 0
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	Re{s} > -α
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	Re{s} < -α
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	Re{s} > -α
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	Re{s} < -α
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos\omega_0 t]\,u(t)$	$\frac{S}{S^2+\omega_0^2}$	Re{s} > 0
12	$[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{S^2+\omega_0^2}$	Re{s} > 0
13	$[e^{-\alpha t}\cos\omega_0 t]\,u(t)$	$\frac{S+\alpha}{(S+\alpha^2)+\omega_0^2}$	Re{s} > -α

Causality and Stability

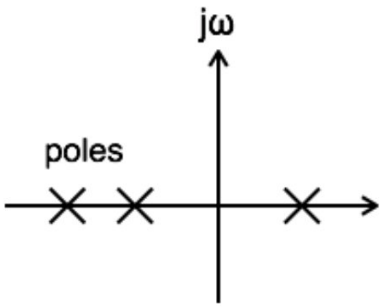
- For a system to be causal, all poles of its transfer function must be right half of s-plane.



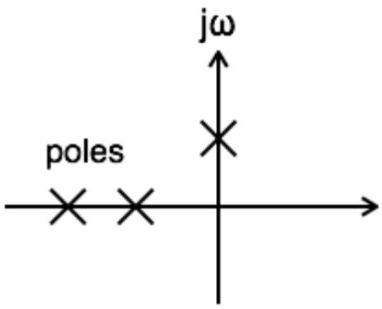
- A system is said to be stable when all poles of its transfer function lay on the left half of s-plane.



- A system is said to be unstable when at least one pole of its transfer function is shifted to the right half of s-plane.



- A system is said to be marginally stable when at least one pole of its transfer function lies on the jω axis of s-plane.



Relation between Laplace and Fourier transforms

Laplace transform of $x(t) = X(S) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

Substitute s= σ + jω in above equation.

$$\rightarrow X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma+j\omega)t}dt$$

$$= \int_{-\infty}^{\infty} [x(t)e^{-\sigma t}]e^{-j\omega t}dt$$

$$\therefore X(S) = F.T[x(t)e^{-\sigma t}] \ldots \ldots (2)$$

$$X(S) = X(\omega) \qquad for\ s = j\omega$$

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R [i.e., s is in the ROC if $(s - s_0)$ is in R]
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	"Scaled" ROC (i.e., s is in the ROC if (s/a) is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$

Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re e\{s\} > 0\}$

Initial- and Final Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

INVERSE LAPLACE TRANSFORM:

$$x(t) = \frac{1}{2\pi j}\int_{-\infty}^{\infty} X(s)e^{st}ds\,..$$

MODULE 5 & 6

	Time domain	Frequency domain
CTFS	CP	DA
CTFT	CA	CA
DTFS	DP	DP
DTFT	DA	CP

Discrete Time Fourier Series (DTFS)

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\omega_0 n} \longrightarrow \text{Synthesis eqn.}$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n} \longrightarrow \text{Analysis eqn.}$$

PROPERTIES.

1) Linearity

If $A x_1[n] \xrightarrow{FS} A X_1[k]$

$B x_2[n] \xrightarrow{F.S} B X_2[k]$

then $A x_1[n] + B x_2[n] \xrightarrow{FS} A X_1[k] + B X_2[k]$

2) Time Shifting

If $x[n] \xrightarrow{FS} X[k]$

then $x[n-p] \xrightarrow{FS} e^{-j\omega_0 pk} X[k]$

3) Time Reversal

If $x[n] \xrightarrow{FS} X[k]$

then $x[-n] \xrightarrow{FS} X[-k]$.

4) Frequency Shifting

$x[n] \xrightarrow{FS} X[k]$

$e^{j\omega_0 mn} x[n] \xrightarrow{FS} X[k-m]$

5) Multiplication

$x_1[n] \longrightarrow X_1[k]$

$x_2[n] \longrightarrow X_2[k]$

$x_1[n] . x_2[n] \xrightarrow{F.S} X_1[k] \circledast X_2[k]$

6) CONVOLUTION

$$x_1[n] \circledast x_2[n] \xrightarrow{FS} N X_1[k] X_2[k]$$

7) Difference Property

$$x[n] - x[n-1] \xrightarrow{FS} X[k] - e^{-j\omega_0 k} X[k]$$

8) Parseval's Theorem

$$\frac{1}{N} \sum_{n=0}^{N-1} x[n]x^*[n] = \sum_{k=0}^{N-1} |X[k]|^2$$

Discrete Time Fourier Transform (DTFT)

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \longrightarrow \text{Synthesis eqn.}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \longrightarrow \text{Analysis eqn}$$

PROPERTIES

1. Linearity

$$A x_1[n] \xrightarrow{FT} A X(e^{j\omega})$$
$$B x_2[n] \xrightarrow{FT} B X(e^{j\omega})$$

$$A x_1[n] + B x_2[n] \xrightarrow{F.T} A X(e^{j\omega}) + B X(e^{j\omega})$$

2. Time shifting

If $y[n] = x[n-n_0]$

then $$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[n-n_0] e^{-j\omega n}$$

3. Frequency shifting

If $x[n] \xrightarrow{FT} X(e^{j\omega})$

$$e^{j\omega_0 n} x[n] \xrightarrow{FT} X(e^{j(\omega-\omega_0)})$$

4. Time Reversal

$$x[n] \xrightarrow{FT} X(e^{j\omega})$$
$$x[-n] \xrightarrow{FT} X(e^{-j\omega})$$

5. Difference eqn

$$x[n] \xrightarrow{FT} X(e^{j\omega})$$

$$x[n] - x[n-1] \xrightarrow{F.T} X(e^{j\omega}) - e^{-j\omega} X(e^{j\omega})$$
$$\longrightarrow X(e^{j\omega})[1 - e^{-j\omega}]$$

6. Differentiation

If $x[n] \xrightarrow{FT} X(e^{j\omega})$

then $x[n] \xrightarrow{FT} j \frac{d}{d\omega} X(e^{j\omega})$

7. Multiplication.

If $x[n] \xrightarrow{FT} X(e^{j\omega})$

$y[n] \xrightarrow{FT} Y(e^{j\omega})$

$x[n].y[n] \xrightarrow{FT} \frac{1}{2\pi} X(e^{j\omega}) \circledast Y(e^{j\omega})$

8. Convolution

$x[n] \xrightarrow{FT} X(e^{j\omega})$

$y[n] \xrightarrow{FT} Y(e^{j\omega})$

$x[n] * y[n] \longrightarrow X(e^{j\omega}) . Y(e^{j\omega})$

9. Parseval's theorem

If $x[n] \xrightarrow{F.T} X(e^{j\omega})$

then $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int |X(e^{j\omega})|^2 d\omega$

- Z- transform :

$$X[z] = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \quad ; \quad z = re^{jw}$$

$$x(n) = \frac{1}{2\pi j} \int X(z)\, z^{n-1}\, dz$$

- Relation b/w DTFT and z-transform :

$$X[z] = FT\{x(n)\, r^{-n}\}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, r^{-n}\, e^{-jwn}$$

- ROC [Region of convergence] :
  - Defines the region where z- transform exists.
  - ROC for a given $x(n)$ is defined as the range of z for which the z-transform converges.
  - condⁿ for convergence : $\sum_{n=-\infty}^{\infty} |x(n)\, z^{-n}| < \infty$

* Properties :

- consists of a ring in the z-plane, centered about origin
- Doesn't contain any poles
- if $x(n)$ is of ∞ duration, the ROC is entire z-plane except $z=0$ and $z=\infty$

- if $x(n)$ is a right sided s/g and if circle $|z| = r_0$ is in the ROC, then all the finite values of z for which $|z| > r_0$ will also be in ROC.

- if $x(n)$ is left sided signal and if the circle $|z| = r_0$ is in the ROC, then all finite values of z for which $0 < |z| < r_0$ will also be in the ROC.

- if $x(n)$ is a two sided s/g and if the circle $|z| = r_0$ is in the ROC, then the ROC will consist of a ring in the z-plane that includes the circle $|z| = r_0$.

* Causality:
  - If $x(n)$ is causal, then ROC includes $z = \infty$
  - if $x(n)$ is non-causal, the ROC includes $z = 0$.

* Stability:
  - $x(n)$ is stable if and only if ROC includes unit circle.
  - $x(n)$ is stable and causal if all poles lie inside the unit circle.

- Properties :

1. Linearity : if $x_1(n) \rightarrow X_1[z]$, $x_2(n) \rightarrow X_2[z]$

   then $A x_1(n) + B x_2(n) \rightleftharpoons A X_1[z] + B X_2[z]$

2. Time shift :

   $$x(n-n_0) \rightleftharpoons z^{-n_0} . X[z]$$

3. Time reversal

   $$x(-n) \rightleftharpoons X\left[\frac{1}{z}\right] \quad , \text{ROC} = \frac{1}{R}$$

4. Time scale :

   $$\alpha^n x(n) \rightleftharpoons X\left[\frac{z}{\alpha}\right] \quad , \text{ROC} = |\alpha| R$$

5. diff^n in z- domain :

   $$n\, x(n) \rightleftharpoons -z \frac{d}{dz} X[z] .$$

6. convolution :

   $$x_1(n) * x_2(n) \rightleftharpoons X_1(z) X_2(z)$$

7. First difference property :

   $$x(n) - x(n-1) \rightleftharpoons (1 - z^{-1}) X(z)$$

Unilateral z-transform

- The unilateral or one-sided z-transform is evaluated using the portion of a signal associated with non-negative values of the time index $(n \geq 0)$.

$$X[z] = \sum_{n=0}^{\infty} x[n] z^{-n}$$

Note: For causal signals, unilateral and bilateral z-transforms are equivalent.

Frequency Response	Transfer function
In continuous case, $F.T\{h(t)\} = H(j\omega)$	- In continuous case, $L.T\{h(t)\} = H(s)$
In discrete case, $F.T\{h[n]\} = H(e^{j\omega})$	- In discrete case; $Z.T\{h[n]\} = H[z]$
In continuous, $Y(j\omega) = X(j\omega) \cdot H(j\omega)$, $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$	- In continuous case $Y(s) = X(s) \cdot H(s)$, $H(s) = \dfrac{Y(s)}{X(s)}$
In discrete case; $Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$, $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})}$	- In discrete case; $Y(z) = H(z) \cdot X(z)$, $H(z) = \dfrac{Y(z)}{X(z)}$

SAMPLING

The process of converting a continuous time signal into a discrete time signal is called *Sampling.* After sampling, the signal is defined at discrete instants of time and the time interval between two successive sampling instants is called *sampling period* or *sampling interval.*

SAMPLING THEOREM

Sampling Theorem states that the rate at which the samples must be taken to reconstruct the original analog signal with less distortion is

$$f_s >= 2f_m$$

where f_s -> sampling frequeny

f_m -> highest frequency component

NYQUIST RATE OF SAMPLING

Nyquist rate of sampling is the theoretical minimum sampling rate at which a signal can be sampled and still be reconstructed from its samples without any distortion.

A signal sampled at greater than Nyquist rate is said to be *over sampled* and a signal sampled at less than its Nyquist rate is said to be under sampled.

NYQUIST INTERVAL

Nyquist interval is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

$$\text{Nyquist rate } f_N = 2f_m \text{ Hz}$$

$$\text{Nyquist interval} = 1 / f_N = 1 / 2f_m \text{ sec}$$

ALIASING

When $w_s < 2w_m$, i.e. when the signal is under sampled , $X(w)$, the spectrum of $x(t)$ is no longer replicated in $X_s(w)$ and thus is no longer recoverable by low pass filtering. This effect is reffered to as *Aliasing.*

Aliasing is defined as the phenomenon in which a high frequency spectrum of signal takes identity of a lower frequency conponent in the spectrum of the sampled signal.

Aliasing can occur if either of the following conditions exists :

1. The signal is not band-limited to a finite range.
2. The sampling rate is too low.

To avoid aliasing , it should be ensured that :

1. x(t) is strictly band-limited.
2. F_s is greater than $2f_m$.

SAMPLING TECHNIQUES

Sampling of a signal is done in several ways. Basically there are

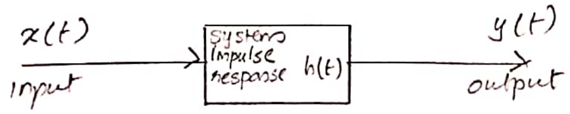
three types of sampling techniques :

1. Instantaneous sampling or impulse sampling
2. Natural sampling
3. Flat Top sampling

SIGNALS AND SYSTEMS

SYSTEMS

SYSTEM — an entity that acts on an input signal & transforms it into an output signal



Systems broadly classified as	i) Continous - time sys ii) discrete - time sys
Continous time systems	transforms Continous time i/p → Continous time o/p
discrete time system	discrete time i/p → discrete time o/p signals
These are further classified into	1. Lumped Parameter & Distributed Para. 2. Static & Dynamic 3. Causal & non Causal 4. Linear & non linear 5. Time - invariant & time varying
Lumped	· system in which each component is lumped at one point · described by ordinary diff. eq.
distributed	· signals are functions of space and time · described by partial diff eq.

Static / memoryless	response is due to present i/p alone eg. $y(t) = x(t)$ $y(t) = x^2(t)$ $y(t) = x(n)$
dynamic	response depends on past or future inputs eg. $y(t) = x(t-1)$ $y(t) = \frac{d^2x(t)}{dt^2} + x(t)$
Causal	output at any time t depends only on present and past values, but not on future values eg: $x(t-2) + 2x(t) + x(t)$
non-causal	o/p depends on future i/p eg $y(t) = x(t+2) + x(t)$
Linear	system obeys principle of superposition and homogeneity eg $y(t) = x(t^2)$
Non linear	does not obey principle of superposition & homogeneity $y(t) = 2x^2(t)$

Homogeneity property means a system which produces an o/p $y(t)$ for i/p $x(t)$ must produce an output $a\,y(t)$ for input $a\,x(t)$

Superposition principle means a system which produces an o/p $y_1(t)$ for an i/p $x_1(t)$ and an o/p $y_2(t)$ for i/p $x_2(t)$ must produce an o/p $y_1(t) + y_2(t)$ for i/p $x_1(t) + x_2(t)$

i.e $T[a\,x_1(t) + b\,x_2(t)] = a\,T[x_1(t)] + b\,T[x_2(t)]$

Linear system

Time invariant

- behaviour of systems is independent of time
- behaviour of systems does not depend on the time at which i/p is applied

i,e $y(t,T) = y(t-T)$

eg: $y(t) = e^{2x(t)}$

Time varying

- a system not satisfying the above requirements.

eg: $y(t) = t^2 x(t)$

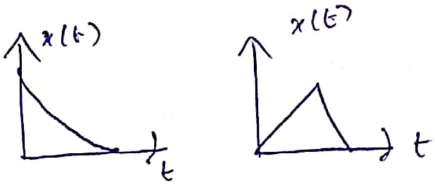
BIBO Stable

a system is said to be bounded input bounded output stable ; iff every bounded input produces a bounded output

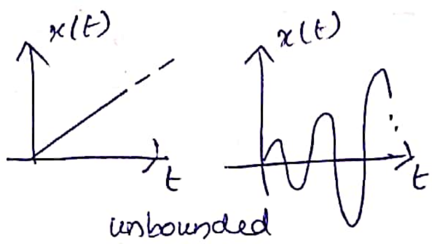
necessary condition:

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

$h(t) \rightarrow$ impulse response



bounded



unbounded

Invertible and Non-invertable

a system is known as invertable only if an inverse system exists which when cascaded with original system produces an output equal to the input of the first system

FIR and IIR systems

If the impulse response sequence is of finite duration, the system is called a finite impulse response (FIR) system and if the impulse response is of infinite duration, the system is called an infinite impulse response (IIR) system.

eg FIR

$$h(n) = \begin{cases} -2 & n = 2, 4 \\ 2 & n = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

eg IIR

$h(n) = 2^n u(n)$

EXAMPLE 2.6 Check whether the following systems are:

1. Static or dynamic
2. Linear or non-linear
3. Causal or non-causal
4. Time-invariant or time-variant

(e) $y(n) = x(n)\, x(n-2)$

(g) $y(n) = a^n u(n)$

(e) Given $$y(n) = x(n)\, x(n-2)$$

1. The output depends on past values of input. So it requires memory. Hence the system is dynamic.
2. The only term contains the product of input and delayed input. So the system is non-linear. This can be proved.

   Let an input $x_1(n)$ produce an output $y_1(n)$.

   Then $$y_1(n) = x_1(n)\, x_1(n-2)$$

   Let an input $x_2(n)$ produce an output $y_2(n)$.

   Then $$y_2(n) = x_2(n)\, x_2(n-2)$$

   The weighted sum of outputs is:

   $$ay_1(n) + by_2(n) = ax_1(n)\, x_1(n-2) + bx_2(n)\, x_2(n-2)$$

   The output due to weighted sum of inputs is:

   $$y_3(n) = T[ax_1(n) + bx_2(n)] = [ax_1(n) + bx_2(n)]\,[ax_1(n-2) + bx_2(n-2)]$$

   $$y_3(n) \neq ay_1(n) + by_2(n)$$

   Hence the system is non-linear.
3. The output depends only on the present and past values of input. It does not depend on future values of input. So the system is causal.
4. Given $$y(n) = x(n)\, x(n-2)$$

   The output due to input delayed by k units is:

   $$y(n, k) = y(n)\big|_{x(n)=x(n-k)} = x(n-k)\, x(n-2-k)$$

   The output delayed by k units is:

   $$y(n-k) = y(n)\big|_{n=n-k} = x(n-k)\;\; x(n-k-2)$$

   $$y(n, k) = y(n-k)$$

   Hence the system is time-invariant.

   So the given system is dynamic, non-linear, causal and time-invariant.

(g) Given $$y(n) = a^n x(n)$$

1. The output at any instant depends only on the present values of input. Hence the system is static.

2. Given $$y(n) = a^n x(n)$$

   For an input $x_1(n)$,

   $$y_1(n) = a^n x_1(n)$$

   For an input $x_2(n)$,

   $$y_2(n) = a^n x_2(n)$$

   The weighted sum of outputs is:

   $$py_1(n) + qy_2(n) = pa^n x_1(n) + qa^n x_2(n) = a^n [px_1(n) + qx_2(n)]$$

The output due to weighted sum of inputs is:

$$y_3(n) = T[px_1(n) + qx_2(n)] = a^n[px_1(n) + qx_2(n)]$$

$$y_3(n) = py_1(n) + qy_2(n)$$

Hence the system is linear.

3. The output depends only on the present input. It does not depend on future inputs. Hence the system is causal.

4. Given $$y(n) = T[x(n)] = a^n x(n)$$

The output due to input delayed by k units is:

$$y(n, k) = T[x(n-k)] = y(n)\big|_{x(n)=x(n-k)} = a^n x(n-k)$$

The output delayed by k units is:

$$y(n-k) = y(n)\big|_{n=n-k} = a^{n-k} x(n-k)$$

$$y(n, k) \neq y(n-k)$$

Hence the system is time-variant.

So the given system is static, linear, causal and time-variant.