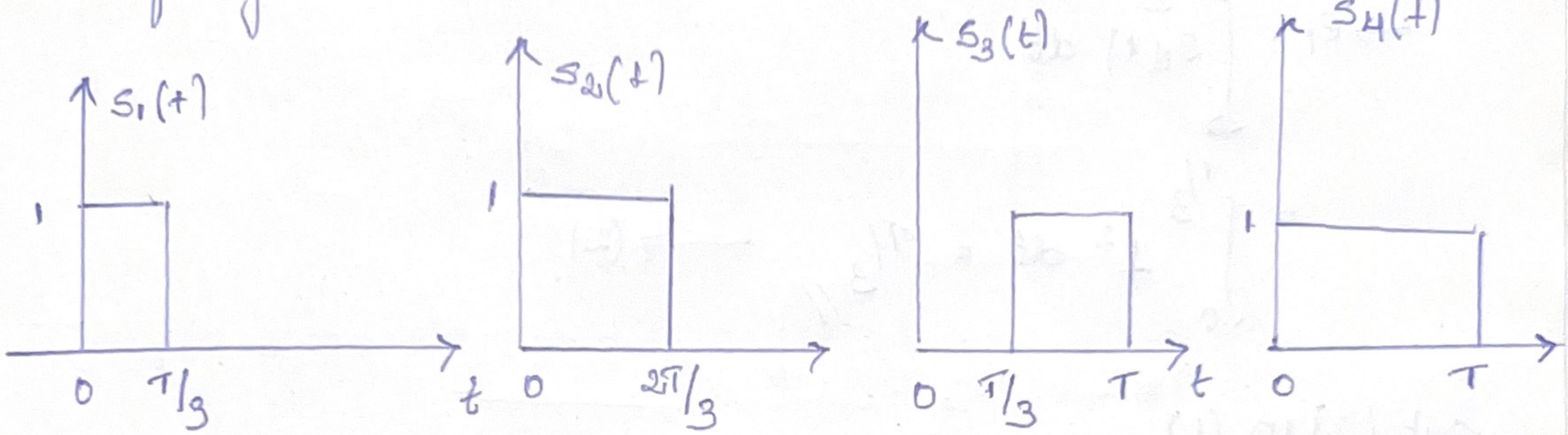
Q) Consider the signals $s_1(t)$, $s_2(t)$, $s_3(t)$ and $s_4(t)$ as shown below. Use Gram-Schmidth orthogonalization procedure to find an orthonormal basis function for this set of signals.



Ans:→ Stage 1: To find the set of linearly independent signals

We can find that $s_4(t) = s_1(t) + s_3(t)$. Therefore $s_4(t)$ is a dependent signal.

The remaining signals, we find that they form a linearly independent signal set.

No. of orthonormal basis function = No. of linearly independent signals = N = 3.

Stage 2

1) $\phi_1(t) = \dfrac{s_1(t)}{\|s_1\|} = \dfrac{s_1(t)}{\sqrt{E_1}} \longrightarrow (1)$

$\rightarrow E_1 = \int_0^T s_1^2(t)\,dt$

$= \int_0^{T/3} 1^2\,dt = T/3 \longrightarrow (2)$

Sub (2) in (1)

$\therefore \phi_1(t) = \dfrac{s_1(t)}{\sqrt{T/3}}$

$\rightarrow \phi_1(t) = \begin{cases} \sqrt{3/T}, & \text{for } 0 \le t \le T/3 \\ 0 & \text{elsewhere.} \end{cases}$

2) To find the second basis function, we define an intermediate signal $g_2(t)$ as

$g_2(t) = s_2(t) - s_{21}\phi_1(t) \longrightarrow (3)$

$\phi_2(t) = \dfrac{g_2(t)}{\|g_2(t)\|} \longrightarrow (4)$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^{T} g_2^2(t) \cdot dt}}$$

sub (3) in above equation

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \longrightarrow (5)$$

$$s_{21} = \int_0^T s_2(t)\phi_1(t) \cdot dt$$

$s_2(t) \rightarrow 0$ to $2T/3$
$\phi_1(t) \rightarrow 0$ to $T/3$

$$= \int_0^{T/3} s_2(t)\phi_1(t) \cdot dt + \int_{T/3}^{2T/3} s_2(t)\phi_1(t) \cdot dt$$

$$= \int_0^{T/3} 1 \times \sqrt{3/T} \cdot dt + \int_{T/3}^{2T/3} (1 \times 0) \cdot dt$$

$$= \sqrt{3/T} \times T/3$$

$$= \sqrt{T/3}$$

$$E_2 = \int_0^T s_2^2(t) \cdot dt = \int_0^{2T/3} 1^2 \cdot dt = 2T/3$$

Sub s_{21} and ϵ_2 in eq (5)

$$\phi_2(t) = \frac{s_2(t) - \sqrt{T/3}\,\phi_1(t)}{\sqrt{2T/3 - T/3}}$$

$$\phi_2(t) = \frac{s_2(t) - \sqrt{T/3}\,\phi_1(t)}{\sqrt{T/3}} \longrightarrow (6)$$

$\phi_2(t)$ here $s_2(t)$ is defined over 0 to $2T/3$

$\phi_1(t)$ is defined over 0 to $T/3$

$$\phi_2(t) = \begin{cases} \dfrac{1-1}{\sqrt{T/3}} = 0 & \text{for } 0 \le t \le T/3 \\ \dfrac{1-0}{\sqrt{T/3}} = \sqrt{3/T} & \text{for } T/3 \le t \le 2T/3 \\ 0 \quad\quad 0 & \text{for } t > 2T/3 \end{cases}$$

(3) To find the third basis function, define an intermediate function $g_3(t)$

$$g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$$

$$\phi_3(t) = \frac{g_3(t)}{\| g_3(t) \|} = \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t).dt}}$$

$$= \frac{s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)}{\sqrt{E_3 - s_{31}^2 - s_{32}^2}} \longrightarrow (7)$$

$$E_3 = \int_0^T s_3^2(t).dt = \int_{T/3}^{T} 1^2.dt = T - T/3 = 2T/3$$

$$s_{31} = \int_0^T s_3(t)\phi_1(t).dt = 0$$

$$s_{32} = \int_0^T s_3(t)\phi_2(t).dt = \int_{T/3}^{2T/3} s_3(t)\phi_2(t).dt + \int_{2T/3}^{T} s_3(t)\phi_2(t).dt$$

$s_3(t) \rightarrow T/3$ to T

$\phi_2(t) \rightarrow T/3$ to $2T/3$

$$= \int_{T/3}^{2T/3} 1 \times \sqrt{3/T}.dt + \int_{2T/3}^{T} 1 \times 0.dt$$

$$= \sqrt{T/3}$$

$$\therefore \phi_3(t) = \frac{s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)}{\sqrt{E_3 - s_{31}^2 - s_{32}^2}}$$

$s_3(t) \rightarrow T/3$ to T

$\phi_1(t) \rightarrow 0$ to $T/3$

$\phi_2(t) \rightarrow T/3$ to $2T/3$

$$\phi_3(t) = \begin{cases} 0 & 0 \le t \le T/3 \\ \dfrac{1 - 0 - \sqrt{T/3} \times \sqrt{3/T}}{\sqrt{2T/3 - 0 - T/3}} = 0 & T/3 \le t \le 2T/3 \\ \dfrac{1 - 0 - 0}{\sqrt{2T/3 - 0 - T/3}} = \sqrt{3/T} & 2T/3 \le t \le T \end{cases}$$

$$\phi_3(t) = \begin{cases} 0 & 0 \le t \le T/3 \\ 0 & T/3 \le t \le 2T/3 \\ \sqrt{3/T} & 2T/3 \le t \le T \end{cases}$$

Conclusion

By analyzing the basis function, we find that the given i/p signals $s_1(t)$, $s_2(t)$, $s_3(t)$ & $s_4(t)$ can be expressed as a linear combination of the same.

$$s_1(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t) + s_{13}\phi_3(t)$$

$$= \sqrt{T/3}\,\phi_1(t) + 0\cdot\phi_2(t) + 0\cdot\phi_3(t)$$

$$s_1 \longrightarrow \left(\sqrt{T/3}, 0, 0\right)$$

$$s_2(t) \longrightarrow s_2 = \left(\sqrt{T/3}, \sqrt{T/3}, 0\right)$$

$$s_3(t) \longrightarrow s_3 = \left(0, \sqrt{T/3}, \sqrt{T/3}\right)$$

$$s_4(t) \rightarrow s_4 = \left(\sqrt{T/3}, \sqrt{T/3}, \sqrt{T/3}\right).$$