5.13 Frequency Transformation in Digital Domain

A digital lowpass filter can be converted into a digital highpass, bandstop, bandpass or another digital filter. These transformations are given below.

5.13.1 *Lowpass to Lowpass*

$$z^{-1} \longrightarrow \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$$

$$\text{where } \alpha = \frac{\sin[(\omega_p - \omega_p')/2]}{\sin[(\omega_p + \omega_p')/2]} \tag{5.98}$$

ω_p = passband frequency of lowpass filter

ω_p' = passband frequency of new lowpass filter

5.13.2 *Lowpass to highpass*

$$z^{-1} = -\left[\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}}\right]$$

$$\text{where } \alpha = -\frac{\cos[(\omega_p' + \omega_p)/2]}{\cos[(\omega_p' - \omega_p)/2]} \tag{5.99}$$

ω_p = passband frequency of lowpass filter

ω_p' = passband frequency of highpass filter

5.13.3 *Lowpass to Bandpass*

$$z^{-1} \longrightarrow \frac{-\left(z^{-2} - \dfrac{2\alpha k}{1+k} z^{-1} + \dfrac{k-1}{k+1}\right)}{\dfrac{k-1}{k+1} z^{-2} - \dfrac{2\alpha k}{k+1} z^{-1} + 1}$$

where $\alpha = \dfrac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$

$$k = \cot\left[\frac{\omega_u - \omega_l}{2}\right]\tan\frac{\omega_p}{2} \tag{5.100}$$

$$\omega_u = \text{upper cutoff frequency}$$
$$\omega_l = \text{lower cutoff frequency}$$

5.13.4 *Lowpass to Bandstop*

$$z^{-1} \longrightarrow \frac{z^{-2} - \dfrac{2\alpha}{1+k}z^{-1} + \dfrac{1-k}{1+k}}{\dfrac{1-k}{1+k}z^{-2} - \dfrac{2\alpha}{1+k}z^{-1} + 1}$$

where $\alpha = \dfrac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$

$$k = \tan[(\omega_u - \omega_l)/2]\tan\frac{\omega_p}{2} \tag{5.101}$$