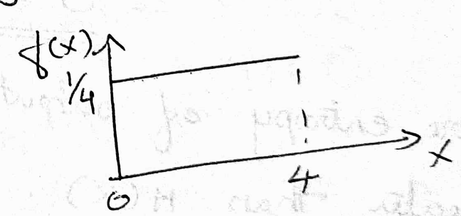
$-\infty$

<u>Ex 1</u> : Consider a RV, 'x' uniformly distributed in the interval $(0, 4)$. Find the differential entropy $H(x)$. If x is a voltage which is applied to an amplifier with gain 8, find the differential entropy of output of the amplifier.

· Uniform distribution $\Rightarrow$



$\therefore \quad f(x) = \frac{1}{4}, \quad 0 \leq x \leq 4$

$$\therefore H(X) = \int_0^4 \frac{1}{4} \log_2 4 \; dx$$

$$= \frac{1}{4} \log_2 4 \; [x]_0^4$$

$$= \log_2 4$$

$\therefore H(X) = 2$ bits / sample.

Now let Y = output of the amplifier.

$\therefore Y = 8X$ ——— (1)

pdf of Y, $f(y) = f(x) \left| \frac{dx}{dy} \right|$ ——— (2)

differentiating eqn (1) $\Rightarrow \frac{dy}{dx} = 8$

$$\therefore \left| \frac{dx}{dy} \right| = \frac{1}{8}$$

$$\therefore f(y) = f(x) \cdot \left| \frac{dx}{dy} \right|$$

$$= \frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$$

ie, pdf of y, $f(y) = \frac{1}{32}$, $0 \leq y \leq 32$

$$H(Y) = \int_0^{32} \frac{1}{32} \cdot \log_2 (32) \, dy$$

$$= \log_2 (32)$$

$= 5$ ~~sb~~ bits / sample $\quad \therefore 32 = 2^5$

Here entropy of output of amplifier is 2.5 times greater than $H(X)$.

But amplification shouldnot alter the information.

Here the discrepancy is due to the fact that the reference level were not considered.

Reference entropy of input,

$$R_X = \lim_{\Delta x \to 0} \left(-\log(\Delta x)\right)$$

Reference entropy of output,

$$R_Y = \lim_{\Delta y \to 0} \left[-\log \Delta y\right]$$

$$R_X - R_Y = \lim_{\Delta x \to 0} \left[-\log \Delta x\right] - \lim_{\Delta y \to 0} \left[-\log \Delta y\right]$$

$$= \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left[\log \Delta y - \log \Delta x\right]$$

$$= \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \log\left(\frac{\Delta y}{\Delta x}\right)$$

$$= \log\left[\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left[\frac{\Delta y}{\Delta x}\right]\right]$$

$$R_X - R_Y = \log\left[\frac{dy}{dx}\right] \quad \text{——— (3)} \quad \because \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Since, $\frac{dy}{dx} = 8$,

$$R_X - R_Y = \log_2(8)$$

$= 3$ bits/sample.

$\therefore$ Absolute entropy of X, $= R_X + H(X)$

" " " Y $= R_Y + H(Y)$

$\therefore$ Absolute entropy of X − Absolute entropy of Y

$= R_X - R_Y + H(X) - H(Y)$

$= 3 + [2 - 5]$

$= 0$