Example 5.5 For the given specifications design an analog Butterworth filter. $0.9 \leq |H(j\Omega)| \leq 1$ for $0 \leq \Omega \leq 0.2\pi$. $|H(j\Omega)| \leq 0.2$ for $0.4\pi \leq \Omega \leq \pi$.

Solution

From the data we find $\Omega_p = 0.2\pi$; $\Omega_s = 0.4\pi$; $\frac{1}{\sqrt{1+\varepsilon^2}} = 0.9$ and $\frac{1}{\sqrt{1+\lambda^2}} = 0.2$ from which we obtain

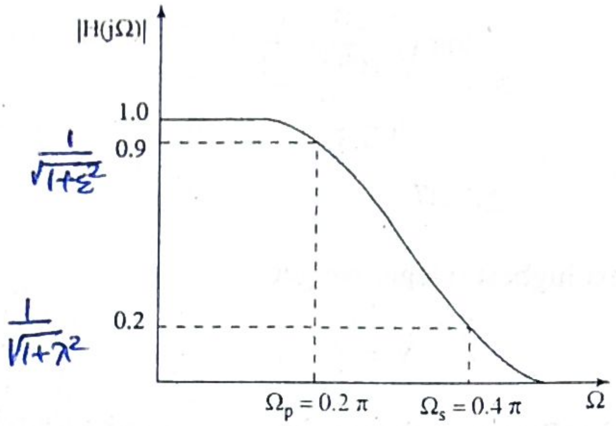

Fig. 5.8 Magnitude response of example 5.5

$$\varepsilon = 0.484 \text{ and } \lambda = 4.898$$

$$N \geq \frac{\log\left(\dfrac{\lambda}{\varepsilon}\right)}{\log\dfrac{\Omega_s}{\Omega_p}} = \frac{\log\dfrac{4.898}{0.484}}{\log\left(\dfrac{0.4\pi}{0.2\pi}\right)} = 3.34$$

i.e., $N = 4$

From the table 5.1, for $N = 4$, the transfer function of normalised Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)}$$

we know $\Omega_c = \dfrac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \dfrac{\Omega_p}{\varepsilon^{1/N}} = \dfrac{0.2\pi}{(0.484)^{1/4}} = 0.24\pi.$

$H(s)$ for $\Omega_c = 0.24\pi$ can be obtained by substituting $s \rightarrow \dfrac{s}{0.24\pi}$ in $H(s)$ i.e.,

$$H(s) = \frac{1}{\left\{\left(\dfrac{s}{0.24\pi}\right)^2 + 0.76537\left(\dfrac{s}{0.24\pi}\right) + 1\right\}}$$

$$\times \frac{1}{\left(\dfrac{s}{0.24\pi}\right)^2 + 1.8477\left(\dfrac{s}{0.24\pi}\right) + 1}$$

$$= \frac{0.323}{(s^2 + 0.577s + 0.0576\pi^2)(s^2 + 1.393s + 0.0576\pi^2)}$$

Practice Problem 5.1 For the given specifications find the order of the Butterworth fitter

$$\alpha_p = 3\,\text{dB}; \quad \alpha_s = 18\,\text{dB}; \quad f_p = 1\,\text{kHz}; \quad f_s = 2\,\text{kHz}.$$

Practice Problem 5.2 Design an analog Butterworth filter that has

$$\alpha_p = 0.5\,\text{dB}; \quad \alpha_s = 22\,\text{dB}; \quad f_p = 10\,\text{kHz}; \quad f_s = 25\,\text{kHz}.$$
