Module. 2.

Continuous Probability Distributions.

Text Book: Probability and Statistics for Engineering and the Sciences, 8th Edition "Jay. L. Devore", Cengage, 2012.

Sections: - 4.1-4.4, 3.6, 5.1

Continuous random Variables and their Probability distributions, Expectation, mean and Varience, uniform, exponential and normal distributions, Continuous bivariate distributions, marginal distributions, independent random Variables, Expectation (multiple random Variables), Central limit Theorem (without proof).

Module. I

Discrete Random Variable

$x = 0, 1, 2, \ldots, n$

Pmf

x	0	1	2	...	n
$p(x)$	p_1	p_2	p_3	...	p_n

i) $\sum p(x) = 1$

ii) $0 \le p(x) \le 1$

$$p(x) = \begin{cases} \frac{1}{2^x} & x = 0, 1, 2, \ldots n \\ 0 & o.\omega \end{cases}$$

Mean μ $\quad E[x] = \sum x p(x)$

$$E[x^2] = \sum x^2 p(x)$$

Variance (σ^2) $\quad Var(x) = E[x^2] - E[x]^2$

$$S.D = \sqrt{Var(x)}$$

Module. II

Continuous Random Variable.

(58, 185)

(1, 2)

(a, b)

Probability density function

$$f(x) = \begin{cases} \frac{1}{2^x} & -\infty \le x \le b \\ 0 & o.\omega \end{cases}$$

i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ii) $f(x) \ge 0$

Mean $\quad E[x] = \int_{-\infty}^{\infty} x f(x)\,dx.$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx.$$

Variance (σ^2) $\quad Var(x) = E[x^2] - E[x]^2$

Standard deviation $= \sqrt{Var(x)}$

Cummulative dis fun:

$$F(x) = P[X \le x]$$

Cummulative dis: function

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

<u>Note:</u> Suppose $F(x)$ is given. To find

$$f(x) = \frac{d}{dx} F(x)$$

<u>Note:</u>

$$E[\text{constant}] = \text{Constant}$$

$$E[ax] = a\,E[x]$$

$$E[ax+b] = a\,E[x] + b$$

$$\text{Var}(\text{constant}) = 0$$

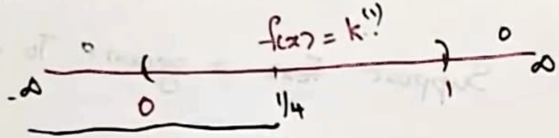
$$\text{Var}(ax) = a^2\,\text{Var}(x)$$

$$\text{Var}(ax+b) = a^2\,\text{Var}(x)$$

A continuous random Variable x has pdf.

$$f(x) = k \qquad 0 \le x \le 1$$

Determine the Constant k. find $P(x \le \frac{1}{4})$



w.k.T $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 f(x)\,dx = 1$$

$$\int_0^1 k\,dx = 1$$

$$k\,[x]_0^1 = 1$$

$$k\,[1-0] = 1$$

$$\boxed{k = 1}$$

$$f(x) = 1 \qquad 0 \le x \le 1$$

$$P(x \le \tfrac{1}{4}) = \int_{-\infty}^{1/4} f(x)\,dx.$$

$$= \int_0^{1/4} dx.$$

$$= [x]_0^{1/4}$$

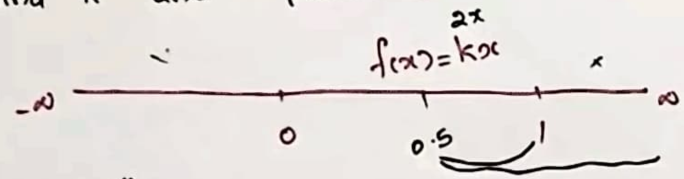
$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$

Given that the pdf of a Random Variable x is

$f(x) = kx \quad 0 < x < 1$

find k and $p(x > 0.5)$



w.k.T $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{0}^{1} f(x)\,dx = 1$

$\int_{0}^{1} kx\,dx = 1$

$k\left[\frac{x^2}{2}\right]_0^1 = 1$

$\frac{k}{2}[1 - 0] = 1$

$\frac{k}{2} = 1$

$\boxed{k = 2}$

$f(x) = 2x \quad 0 < x < 1$

$p(x > 0.5) = \int_{0.5}^{\infty} f(x)\,dx$

$= \int_{0.5}^{1} f(x)\,dx = \int_{0.5}^{1} 2x\,dx.$

$= \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_{0.5}^{1}$

$= 1 - 0.25$

$= \underline{0.75}$

If $f(x)$ $f(x) = \begin{cases} k x e^{-x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$ is the pdf of a random Variable X. find k.

u.Q

$f(x) = 0$ $\qquad$ $f(x) = k x e^{-x}$

$-\infty \qquad 0 \qquad \infty$

w.k.t $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int e^{ax} = \frac{e^{ax}}{a}$

$e^{-\infty} = 0$

$e^{\infty} = \infty$

$e^{0} = 1$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} k x e^{-x}\,dx = 1$$

$$k \int_{0}^{\infty} x e^{-x}\,dx = 1$$

$$k \left[(x)\left[\frac{e^{-x}}{-1}\right] - (1)\left[\frac{e^{-x}}{1}\right] \right]_{0}^{\infty} = 1$$

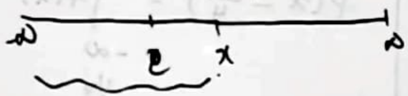
$$k \left[[0 - \overset{0}{e^{-\infty}}] - [0 - \overset{1}{e^{0}}] \right] = 1$$

$$\boxed{k = 1}$$

$$f(x) = \begin{cases} x e^{-x} & x > 0 \\ 0 & \text{elsew} \end{cases}$$

Cummulative dis fun:

$$F(x) = P[X \le x]$$



$$P(X \ge a) = 1 - P(X \le a)$$
$$= 1 - F(a)$$

Note:

$$E[\text{constant}] = \text{constant}$$

$$E[ax] = a\,E[x]$$

$$E[ax+b] = a\,E[x] + b$$

Cummulative dis: function

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

Note: Suppose $F(x)$ is given. To find

$$f(x) = \frac{d}{dx} F(x)$$

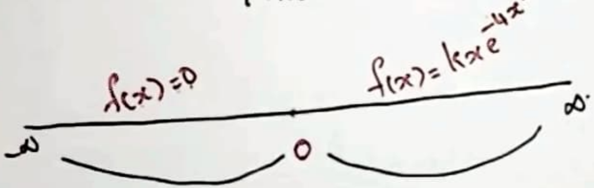
$$\text{Var}(\text{constant}) = 0$$

$$\text{Var}(ax) = a^2\,\text{Var}(x)$$

$$\text{Var}(ax+b) = a^2\,\text{Var}(x)$$

Q. Find k. So that the following is the probability density

$$f(x) = \begin{cases} 0 & x \le 0 \\ kxe^{-4x^2} & x > 0 \end{cases}$$



w.k.t $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} \overset{0}{f(x)}\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{\infty} kxe^{-4x^2}\,dx = 1$$

$$k\int_{0}^{\infty} xe^{-4x^2}\,dx = 1$$

Put $4x^2 = u$

$4(2x\,dx) = du$

$x\,dx = \frac{du}{8}$

when $x = 0$ $\quad u = 4\times 0 = 0$

$x = \infty$ $\quad u = \infty$

$\int e^{ax} = \frac{e^{ax}}{a}$

$$k\int_{0}^{\infty} e^{-u}\,\frac{du}{8} = 1$$

$$\frac{k}{8}\left[\frac{e^{-u}}{-1}\right]_{0}^{\infty} = 1$$

$e^{-\infty} = 0$

$e^{0} = 1$

$$-\frac{k}{8}\left[\overset{0}{e^{-\infty}} - \overset{1}{e^{0}}\right] = 1$$

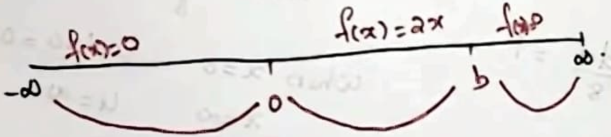
$$\frac{k}{8} = 1$$

$$\underline{\underline{k = 8}}$$

Find the value of b so that the following function is a valid PDF.

$$f(x) = \begin{cases} 2x & 0 \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Also find the CDF of x and $P(x \ge 0.5)$



$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{b} 2x\,dx + 0 = 1$$

$$2\left[\frac{x^2}{2}\right]_0^b = 1$$

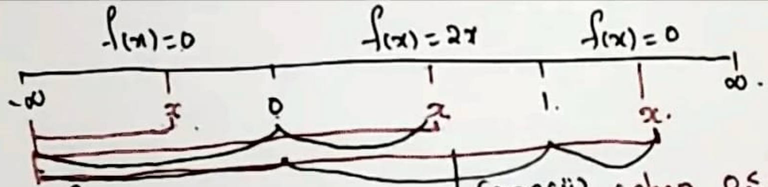
$$b^2 - 0 = 1$$

$$b^2 = 1$$

$$b = \pm 1$$

Since $b > 0$ $\boxed{b = 1}$

$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$



To find CDF

$$f(x) = \int_{-\infty}^{x} f(x)\,dx.$$

Case (i) when $x<0$

$$f(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{x} 0\,dx$$

$$= \underline{\underline{0}}$$

Case(ii) when $0 \le x < 1$

$$f(x) = \int_{-\infty}^{x} f(x)\,dx.$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$

$$= 0 + \int_{0}^{x} 2x\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^x dx$$

$$= \left[x^2 - 0\right]$$

$$= \underline{\underline{x^2}}$$

Case (iii) when $x \ge 1$

$$f(x) = \int_{-\infty}^{x} f(x)\,dx.$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{x} f(x)\,dx$$

$$= 0 + \int_{0}^{1} 2x\,dx + 0$$

$$= 2\left[\frac{x^2}{2}\right]_0^1$$

$$= [1-0]$$

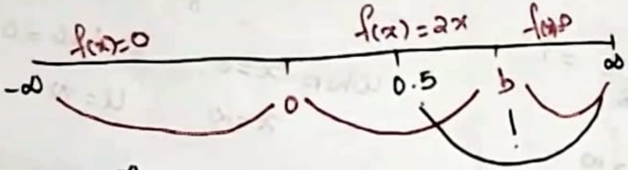
$$= 1$$

$$f(x) = \begin{cases} 0 & x<0. \\ x^2 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

Find the value of b so that the following function is a valid PDF.

$$f(x) = \begin{cases} 2x & 0 \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Also find the CDF of x and $P(x \ge 0.5)$



$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{b} 2x\,dx + 0 = 1$$

$$2\left[\frac{x^2}{2}\right]_0^b = 1$$

$$b^2 - 0 = 1$$

$$b^2 = 1$$

$$b = \pm 1$$

Since $b > 0$ $\boxed{b = 1}$

$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$P(x \ge 0.5) = \int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx.$$

$$= \int_{0.5}^{1} 2x\,dx + \int_{1}^{\infty} 0$$

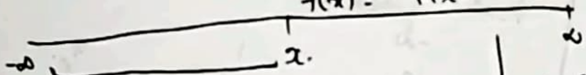
$$= 2\left[\frac{x^2}{2}\right]_{0.5}^{1}$$

$$= 1 - 0.25$$

$$= \underline{0.75}$$

Given the Probability density function $f(x) = \frac{k}{1+x^2}$ for $-\infty < x < \infty$. find k and distribution function

$$f(x) = \frac{k}{1+x^2}$$



w.k.t $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} \frac{k}{1+x^2}\,dx = 1$$

$$k\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 1$$

$$k\left[\tan^{-1}x\right]_{-\infty}^{\infty} = 1$$

$$k\left[\tan^{-1}\infty - \tan^{-1}(-\infty)\right] = 1$$

$$k\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 1$$

$$\pi k = 1$$

$$\boxed{k = \frac{1}{\pi}}$$

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty$$

when $x < \infty$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx.$$

$$= \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}x\right]_{-\infty}^{x}$$

$$= \frac{1}{\pi}\left[\tan^{-1}x - \tan^{-1}(-\infty)\right]$$

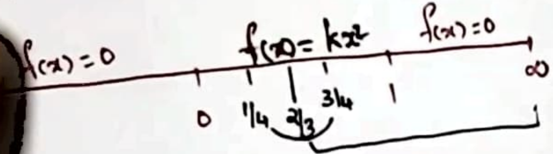
$$= \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right]$$

$$f(x) = \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right] \quad x < \infty.$$

If Probability density function of a random variable $f(x) = \begin{cases} \\ 0 & \text{otherwise.} \end{cases}$

i) find k ii) find $P\left(\frac{1}{4} < x < \frac{3}{4}\right)$ iii) $P\left(x > \frac{2}{3}\right)$

iv) find mean and Varience v) find distribution function.



w.k.t $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$

$0 + \int_{0}^{1} kx^2\,dx + 0 = 1$

$k\left[\frac{x^3}{3}\right]_0^1 = 1$

$\frac{k}{3}[1-0] = 1$

$\frac{k}{3} = 1$

$\boxed{k = 3}$

$f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$

ii) $P\left(\frac{1}{4} < x < \frac{3}{4}\right) = \int_{1/4}^{3/4} f(x)\,dx.$

$= \int_{1/4}^{3/4} 3x^2\,dx$

$= \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{1/4}^{3/4}$

$= \left(\frac{3}{4}\right)^3 - \left(\frac{1}{4}\right)^3$

$= \frac{27}{64} - \frac{1}{64}$

$= \frac{26}{64}$

iii) $P\left(x > \frac{2}{3}\right) = \int_{2/3}^{\infty} f(x)\,dx$

$= \int_{2/3}^{1} 3x^2\,dx + \int_{1}^{\infty} 0\,dx$

$= \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{2/3}^{1}$

$= 1 - \frac{8}{27} = \frac{19}{27}$

$-\infty$ $\quad f(x)=0 \quad$ 0 $\quad f(x)=3x^2 \quad$ 1 $\quad f(x)=0 \quad$ ∞

$$\text{Mean} = E[x]$$

$$= \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{-\infty}^{0} x\, f(x)\, dx + \int_{0}^{1} x\, f(x)\, dx + \int_{1}^{\infty} x\, f(x)\, dx$$

$$= \int_{0}^{1} x\,[3x^2]\, dx$$

$$= 3\int_{0}^{1} x^3\, dx = 3\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{4}[1-0] = \frac{3}{4}$$

$$\boxed{\text{Mean} = \frac{3}{4}}$$

$$\text{Variance}(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{0}^{1} x^2\,[3x^2]\, dx$$

$$= 3\int_{0}^{1} x^4\, dx$$

$$= 3\left[\frac{x^5}{5}\right]_0^1$$

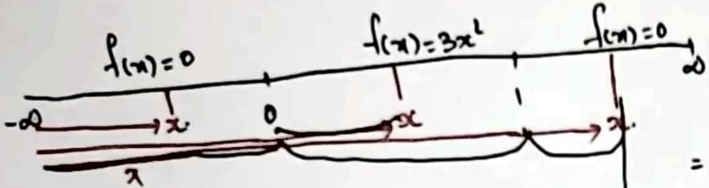
$$= \frac{3}{5}[1-0]$$

$$= \frac{3}{5}$$

$$\text{Var}(x) = \frac{3}{5} - \left(\frac{3}{4}\right)^2$$

$$= \frac{3}{5} - \frac{9}{16}$$

$$= \frac{3}{80}$$



$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

Case (i) when $x<0$

$$F(x) = \int_{-\infty}^{x} \overset{0}{f(x)}\,dx$$

$$= \underline{\underline{0}}$$

Case (ii) when $0 \le x < 1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{0} \overset{0}{f(x)}\,dx + \int_{0}^{x} f(x)\,dx$$

$$= \int_{0}^{x} 3x^2\,dx$$

$$= \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_0^x$$

$$= \left[x^3 - 0\right]$$

$$= \underline{\underline{x^3}}$$

Case (iii) when $x \ge 1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx.$$

$$= \int_{-\infty}^{0} \overset{0}{f(x)}\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{x} \overset{0}{f(x)}\,dx.$$

$$= \int_{0}^{1} 3x^2\,dx$$

$$= \cancel{3}\left[\frac{x^3}{3}\right]_0^1$$

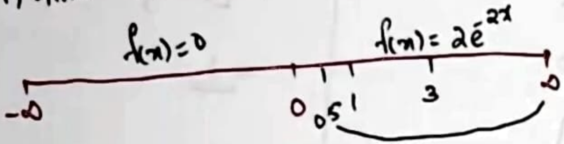
$$= \left[1 - 0\right]$$

$$= \underline{\underline{1}}$$

$$\therefore F(x) = \begin{cases} 0 & x<0 \\ x^3 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

If a random Variable has the Probability density $f(x) = \begin{cases} \dots \\ 0, & x \le 0 \end{cases}$

i) find $P(1<x<3)$ ii) $P(x>0.5)$ iii) Find distribution function iv) find mean and Varience.



$$P(1<x<3) = \int_1^3 f(x)\,dx \qquad \int e^{ax} = \frac{e^{ax}}{a}$$

$$= \int_1^3 2e^{-2x}\,dx$$

$$= \cancel{2}\left[\frac{e^{-2x}}{-\cancel{2}}\right]_1^3$$

$$= -\left[e^{-6} - e^{-2}\right]$$

$$= \underline{\underline{e^{-2} - e^{-6}}}$$

$$= \underline{\underline{0.1333}}$$

$$P(x>0.5) = \int_{0.5}^{\infty} f(x)\,dx$$

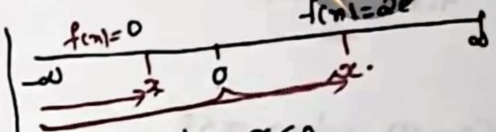
$$= \int_{0.5}^{\infty} 2e^{-2x}\,dx$$

$$= \cancel{2}\left[\frac{e^{-2x}}{-\cancel{2}}\right]_{0.5}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-1}\right]$$

$$= e^{-1}$$

$$= \frac{1}{e} = \underline{\underline{0.368}}$$



Case (i) when $x<0$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \underline{0}$$

Case (ii) when $x \ge 0$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx.$$

$$= \int_{-\infty}^{0} \underbrace{f(x)}_{0}\,dx + \int_0^x \underbrace{f(x)}_{2e^{-2x}}\,dx$$

$$= \cancel{2}\left[\frac{e^{-2x}}{-\cancel{2}}\right]_0^x = -\left[e^{-2x} - 1\right]$$

$$= \underline{1 - e^{-2x}}$$

$$F(x) = \begin{cases} 0 & x<0 \\ 1-e^{-2x}. & x \ge 0 \end{cases}$$

$f(x) = 2e^{-2x}$

$-\infty \quad \overset{0}{\longrightarrow} \quad 0 \longrightarrow \infty$

Mean $= E(x)$

$$= \int_{-\infty}^{\infty} x\, f(x)\, dx.$$

$$= \int_{-\infty}^{0} x \overset{0}{f(x)}\, dx + \int_{0}^{\infty} x f(x)\, dx$$

$$= 0 + \int_{0}^{\infty} 2x e^{-2x}\, dx.$$

$$= 2\left[(x)\left[\frac{e^{-2x}}{-2}\right] - (1)\left[\frac{e^{-2x}}{4}\right]\right]_0^{\infty}$$

$$= 2\left[\left[0 - \frac{e^{-\infty}}{4}\right] - \left[0 - \frac{e^{0}}{4}\right]\right]$$

$$= 2 \times \frac{1}{4}$$

$$\boxed{\text{Mean} = \frac{1}{2}}$$

Variance $(x) = E[x^2] - E[x]^2$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx.$$

$$= \int_{-\infty}^{0} \underbrace{x^2 \overset{0}{f(x)}} dx + \int_{0}^{\infty} x^2 \overset{2e^{-2x}}{f(x)}\, dx.$$

$$= 2 \int_{0}^{\infty} x^2 e^{-2x}\, dx$$

$$= 2\left[(x^2)\left[\frac{e^{-2x}}{-2}\right] - (2x)\left[\frac{e^{-2x}}{4}\right] + (2)\left[\frac{e^{-2x}}{-8}\right]\right]_0^{\infty}$$

$$= 2\left[[0 + 0 + 0] - \left[0 + 0 - \frac{1}{4}e^{0}\right]\right]$$

$$= 2 \times \frac{1}{4} = \frac{1}{2}$$

$$\therefore \text{Var}(x) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$S.D = \sqrt{\text{Var}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

The Cumulative distribution function CDF of a Continuous Variable X is

$$F(x) = \begin{cases} 0 & x \le 2 \\ k(x-2) & 2 < x < 6 \\ 1 & x \ge 6 \end{cases}$$

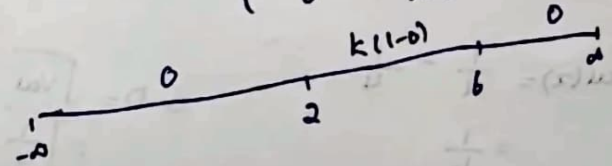
i) find PDF ii) find k iii) find $P(x > 4)$

iv) $P(3 \le x \le 5)$ v) find mean and Varience.

$$f(x) = \frac{d}{dx} F(x)$$

$$= \frac{d}{dx} \begin{cases} 0 & x \le 2 \\ k(x-2) & 2 < x < 6 \\ 1 & x \ge 6 \end{cases}$$

$$= \begin{cases} 0 & x \le 2 \\ k(1-0) & 2 < x < 6 \\ 0 & x \ge 6 \end{cases}$$



Pdf:

$$f(x) = \begin{cases} k. & 2 < x < 6 \\ 0 & \text{otherwise:} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{2} \overset{0}{f(x)}\, dx + \int_{2}^{6} \overset{k}{f(x)}\, dx + \int_{6}^{\infty} \overset{0}{f(x)}\, dx = 1$$

$$= k[x]_2^6 = 1$$

$$k[6-2] = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

$$f(x) = \begin{cases} \frac{1}{4} & 2 < x < 6 \\ 0 & \text{otherwise:} \end{cases}$$

$$P(x>4) = \int_{4}^{\infty} f(x)\,dx$$

$$= \int_{4}^{6} f(x)\,dx + \int_{6}^{\infty} f(x)\,dx.$$

$$= \frac{1}{4}\,(x)_{4}^{6}$$

$$= \frac{1}{4}\,[6-4]$$

$$= \frac{2}{4} = \frac{1}{2}$$

$$P(3 \le x \le 5) = \int_{3}^{5} f(x)\,dx = \int_{3}^{5} \frac{1}{4}\,dx$$

$$= \frac{1}{4}\,[x]_{3}^{5} = \frac{1}{4}\,[5-3] = \frac{2}{4} = \frac{1}{2}$$

Mean

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{2} x f(x)\,dx + \int_{2}^{6} x f(x)\,dx + \int_{6}^{\infty} x f(x)\,dx$$

$$= \int_{2}^{6} \frac{1}{4}\,x\,dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2}\right]_{2}^{6}$$

$$= \frac{1}{8}\,[36-4]$$

$$= \frac{32}{8}$$

$$= 4$$

$$\mathrm{Var}(x) = E[x^2] - E(x)^2$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{2}^{6} x^2 f(x)\,dx.$$

$$= \frac{1}{4}\left[\frac{x^3}{3}\right]_{2}^{6}$$

$$= \frac{1}{12}\,[216-8]$$

$$= \cancel{\frac{208}{12}}\ \cancel{\frac{104}{6}}\ \frac{52}{3}$$

$$\mathrm{Var}(x) = \frac{52}{3} - 16$$

$$= \frac{52-48}{3}$$

$$= \frac{4}{3}$$

A random Variable x has density $f(x) = \begin{cases} \frac{3}{4}(1-x^2) & \text{if } -1 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$

Find (i) mean and Varience of x and $Y = 2x-3$ (ii) mean of $Z = \sin(x)$

$f(x) = \frac{3}{4}(1-x^2)$

$-\infty \quad 0 \quad -1 \qquad\qquad 1 \quad 0 \quad \infty$

$f(-x) = -f(x)$

$$E[x] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-1}^{1} x \, \frac{3}{4}(1-x^2)\, dx$$

odd

$$= \frac{3}{4}\int_{-1}^{1} [x - x^3]\, dx = 0$$

$$= \frac{3}{4}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{1}$$

$$= \frac{3}{4}\left[\left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{4}}\right) - \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{4}}\right)\right]$$

$$= \underline{\underline{0}}$$

$$\text{Varience}(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx.$$

$$= \int_{-1}^{1} x^2 \, \frac{3}{4}(1-x^2)\, dx.$$

$$= \frac{3}{4}\int_{-1}^{1} (x^2 - x^4)\, dx$$

$$= \frac{3}{4}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{3}{4}\left[\left[\frac{1}{3} - \frac{1}{5}\right] - \left[-\frac{1}{3} + \frac{1}{5}\right]\right]$$

$$= \frac{3}{4}\left[\frac{2}{3} - \frac{2}{5}\right]$$

$$= \frac{3}{\cancel{4}}\left[\frac{\cancel{2}(5-3)}{15}\right]$$

$$= \frac{1}{5}$$

$$\therefore \text{Var}(x) = \frac{1}{5} - 0$$

$$= \underline{\underline{\frac{1}{5}}}$$

$Y = 2x - 3$ $\quad E[ax+b] = aE[x]+b$

$E[Y] = E[2x-3]$

$= 2E[x] - 3$

$= 0 - 3$

$= \underline{\underline{-3}}$

$V(ax+b) = a^2 Var(x)$

$Var(Y) = Var(2x-3)$

$= 4 Var(x)$

$= \underline{\underline{\frac{4}{5}}}$

ii) $Z = \sin x$.

$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 \\ 2\int_0^a f(x) & f(x) \text{ even} \end{cases}$

$E[Z] = E[\sin x]$

$= \int_{-\infty}^{\infty} \sin x\, f(x)\, dx.$

$(1-(-x)^2) = 1-x^2$

$f(-x) = -f(x)$

$\sin(-x) = -\sin x$

$f(-x) = -f(x)$

$= \int_{-1}^{1} \sin x \, \frac{3}{4}(1-x^2)\,dx.$

odd odd

$= \frac{3}{4}\int_{-1}^{1} \underbrace{\overset{\text{even}}{(1-x^2)} \sin x}\, dx.$

$= \underline{\underline{0}}$

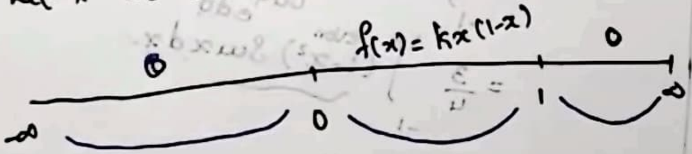
The diameter of an electric Cable is a Continuous random Variable with density function

$$f(x)=\begin{cases} kx(1-x) & 0\le x\le 1 \\ 0 & \text{otherwise.}\end{cases}$$

Find a) Value of k b) $P(x<\frac{1}{2})$ c) $P(x\le 1/2 \mid \frac{1}{3}<x<\frac{2}{3})$ d) [illegible] of x.

e) mean diameter of the Cable f) Mean Cross-Sectional area of the Cable.

Let x be diameter of an electric Cable.



$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{1} f(x)\,dx = 1$$

$$\int_{0}^{1} kx(1-x)\,dx = 1$$

$$k\int_{0}^{1}[x-x^2]\,dx = 1$$

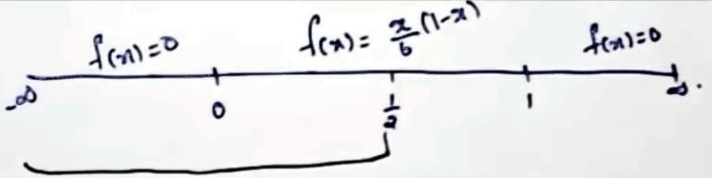
$$k\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[\left(\frac{1}{2}-\frac{1}{3}\right)-0\right]=1$$

$$\frac{k}{6}=1$$

$$\boxed{k=6}$$

$$\therefore f(x)=\begin{cases} \frac{x}{6}(1-x) & 0\le x\le 1 \\ 0 & \text{otherwise}\end{cases}$$



$$P\left(x < \frac{1}{2}\right) = \int_{-\infty}^{1/2} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1/2} f(x)\,dx$$

$$0 \qquad 6x(1-x)$$

$$= \frac{1}{6}\, 6 \int_{0}^{1/2} \left[x - x^2\right] dx$$

$$= \frac{1}{6}\, 6 \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1/2}$$

$$= \frac{1}{6}\, 6 \left[\left(\frac{1}{8} - \frac{1}{24}\right) - 0\right]$$

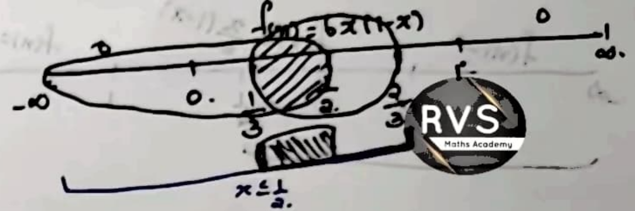
$$= \frac{1}{6}\left[\frac{(3-1)}{24}\right]$$

$$= \frac{2}{12}$$

$$= 6\left[\frac{(3-1)}{24}\right]$$

$$= \frac{1}{2}$$

c) $P\left(x \le \frac{1}{2} \,\middle|\, \frac{1}{3} < x < \frac{2}{3}\right)$ $\qquad P(A|B) = \dfrac{P(A \cap B)}{P(B)}$



$$= \frac{P\left[\left(x \le \frac{1}{2}\right) \cap \left(\frac{1}{3} < x < \frac{2}{3}\right)\right]}{P\left[\frac{1}{3} < x < \frac{2}{3}\right]}$$

$$= \frac{P\left[\frac{1}{3} < x < \frac{1}{2}\right]}{P\left[\frac{1}{3} < x < \frac{2}{3}\right]} \text{ ——(1)}$$

$$P\left[\frac{1}{3} < x < \frac{1}{2}\right] = \int_{1/3}^{1/2} 6x(1-x)\,dx.$$

$$= 6\int_{1/3}^{1/2} \left[x - x^2\right] dx$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1/3}^{1/2}$$

$$= 6\left[\left(\frac{1}{8} - \frac{1}{24}\right) - \left(\frac{1}{18} - \frac{1}{81}\right)\right]$$

$$= 6\left[\frac{1}{8} - \frac{7}{162}\right] = 6 \times \frac{13}{324} = \frac{13}{54}$$

$$P\left[\frac{1}{3} < x < \frac{2}{3}\right] = \int_{1/3}^{2/3} 6x(1-x)\,dx = 6\int_{1/3}^{2/3} (x - x^2)\,dx.$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1/3}^{2/3} = 6\left[\left(\frac{4}{18} - \frac{8}{81}\right) - \left(\frac{1}{18} - \frac{1}{81}\right)\right]$$

$$= \frac{13}{27}$$

(1) $\Rightarrow$

$$P\left[x \le \frac{1}{2} \,\middle|\, \frac{1}{3} < x < \frac{2}{3}\right] = \frac{\left(\frac{13}{54}\right)}{\left(\frac{13}{27}\right)} = \frac{1}{2}$$

e) mean diameter of cable.

v.v.imp

$$E[x] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_0^1 x[6x(1-x)]\, dx$$

$$= 6\int_0^1 [x^2 - x^3]\, dx$$

$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 6\left[\left(\frac{1}{3} - \frac{1}{4}\right) - (0)\right]$$

$$= 6\left[\frac{1}{12}\right] = \frac{1}{2}$$

Cross Sectional Area. $A = \pi r^2$

$$= \pi \frac{x^2}{4}$$

x - diameter.

$\frac{x}{2}$

$$E[ax] = aE[x]$$

$$E[A] = E\left[\pi \frac{x^2}{4}\right]$$

$$= \frac{\pi}{4} E[x^2]$$

$$= \frac{\pi}{4} \int_{-\infty}^{\infty} x^2 f(x)\, dx.$$

$$= \frac{\pi}{4} \int_0^1 x^2\, 6x(1-x)\, dx.$$

$$= \frac{3\pi}{2} \int_0^1 [x^3 - x^4]\, dx.$$

$$= \frac{3\pi}{2}\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$$

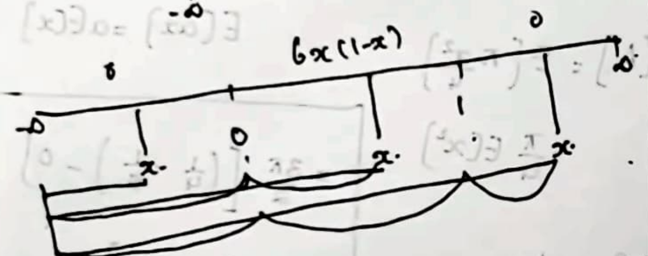
$$= \frac{3\pi}{2}\left[\left(\frac{1}{4} - \frac{1}{5}\right) - 0\right]$$

$$= \frac{3\pi}{2}\left[\frac{1}{20}\right]$$

$$= \frac{3\pi}{40}$$

CDF of x.

$$F(x) = \int_{-\infty}^{x} f(x)\,dx.$$



Case (i) when $x<0$

$$F(x) = \int_{-\infty}^{x} \overset{0}{f(x)}\,dx$$

$$= 0$$

Case (ii) $0 \le x < 1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{0} \overset{0}{f(x)}\,dx + \int_{0}^{x} \overset{6x(1-x)}{f(x)}\,dx$$

$$= 6\int_{0}^{x} (x - x^2)\,dx$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^x.$$

$$= 6\left[\left(\frac{x^2}{2} - \frac{x^3}{3}\right) - 0\right]$$

$$= \cancel{6}\left[\frac{3x^2 - 2x^3}{\cancel{6}}\right]$$

Case (iii) $x \ge 1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx.$$

$$= \int_{-\infty}^{0} \overset{0}{f(x)}\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{x} \overset{0}{f(x)}\,dx$$

$$= \int_{0}^{1} 6x(1-x)\,dx$$

$$= 6\int_{0}^{1} x - x^2$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 6\left[\left(\frac{1}{2} - \frac{1}{3}\right) - 0\right]$$

$$= 6\left[\frac{1}{6}\right] = 1$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 3x^2 - 2x^3 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

Uniform Distribution:

A continuous random Variable X with Probability density function $f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$

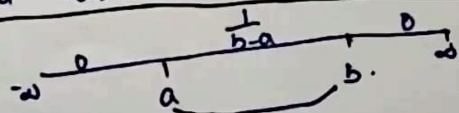
is called a uniform random Variable.

Note:

X is uniformly distributed in $[a, b]$ and is denoted as.

$X \sim U(a,b)$

Mean and Varienc of uniform Distribution.



$a^2 - b^2 = (a-b)(a+b)$

$$E[x] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_a^b x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{2(b-a)} \left[b^2 - a^2\right]$$

$$= \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}$$

$$\boxed{\text{Mean} = \frac{a+b}{2}}$$

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_a^b x^2 \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{3(b-a)}\left[b^3 - a^3\right]$$

$$= \frac{1}{3\cancel{(b-a)}}\cancel{(b-a)}(a^2+ab+b^2)$$

$$= \frac{(a^2+ab+b^2)}{3}$$

$$(a^3-b^3) = (a-b)(a^2+ab+b^2).$$

$$Var(x) = \frac{a^2+ab+b^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{(a^2+2ab+b^2)}{4}$$

$$= \frac{4a^2+4ab+4b^2-3a^2-6ab-3b^2}{12}$$

$$= \frac{a^2-2ab+b^2}{12}$$

$$Var(x) = \frac{(b-a)^2}{12} \qquad (a,b)$$

$$\boxed{Var(x) = \frac{(b-a)^2}{12}}$$

if x is uniform distribution in $(-3,3)$. find $P(x<1)$, $P(|x|>2)$, $P(|x-2|<2)$

a, b

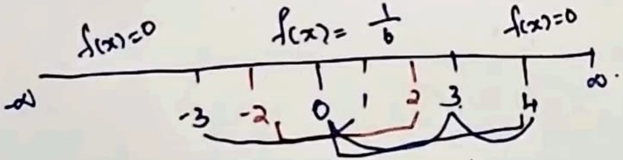
Pdf of UD is

$3-(-3)$

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$

$$\boxed{|x|<a \Rightarrow -a<x<a}$$

$$f(x) = \begin{cases} \dfrac{1}{6} & -3 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$



i) $P(x<1) = \int_{-3}^{1} f(x)\,dx = \int_{-3}^{1} \frac{1}{6}\,dx$

$= \frac{1}{6}[x]_{-3}^{1} = \frac{1}{6}[1+3] = \frac{2}{3}$

$P(|x|>2) = 1 - P(|x| \le 2)$

$= 1 - P[-2 \le x \le 2]$

$= 1 - \int_{-2}^{2} f(x)\,dx.$

$= 1 - \int_{-2}^{2} \frac{1}{6}\,dx = 1 - \frac{1}{6}[x]_{-2}^{2}$

$= 1 - \frac{1}{\cancel{6}_{3}}[\cancel{2+2}^{\cancel{4}\,2}] = \frac{1}{3}$

$P(|x-2|<2) = P(-2 < x-2 < 2)$

$= P(-2+2 < x-\cancel{2}+\cancel{2} < 2+2)$

$= P(0 < x < 4)$

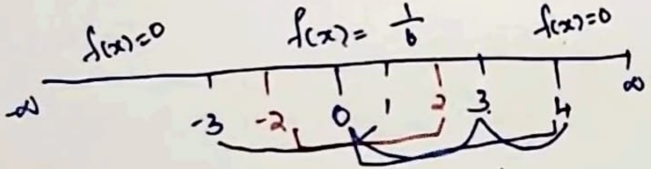
$= \int_{0}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx$

Pdf of UD is $\quad 3-(-3)$

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$

$$\boxed{|x|<a \Rightarrow -a<x<a}$$

$$f(x) = \begin{cases} \dfrac{1}{6} & -3 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$



$$P(x<1) = \int_{-3}^{1} f(x)\,dx = \int_{-3}^{1} \frac{1}{6}\,dx$$

$$= \frac{1}{6}\left[x\right]_{-3}^{1} = \frac{1}{6}\left[1+3\right] = \frac{2}{3}$$

$$P(|x|>2) = 1 - P(|x| \le 2)$$

$$= 1 - P\left[-2 \le x \le 2\right]$$

$$= 1 - \int_{-2}^{2} f(x)\,dx.$$

$$= 1 - \int_{-2}^{2} \frac{1}{6}\,dx = 1 - \frac{1}{6}\left[x\right]_{-2}^{2}$$

$$= 1 - \frac{1}{6}\left[2+2\right] = \frac{1}{3}$$

$$P(|x-2|<2) = P(-2 < x-2 < 2)$$

$$= P(-2+2 < x-2+2 < 2+2)$$

$$= P(0 < x < 4)$$

$$= \int_{0}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx$$

$$= \frac{1}{6}\left[x\right]_{0}^{3} = \frac{3}{6} = \frac{1}{2}$$

If x is a uniform distribution in $(\overset{a}{-a},\overset{b}{a})$. find a such that $P(|x|\le 1) = P(|x|>1)$

Pdf of U.D is $f(x) = \begin{cases} \frac{1}{b-a} & a\le x\le b \\ 0 & \text{otherwise} \end{cases}$ $\quad a-(-a)$ $\quad a+a$

$$f(x) = \begin{cases} \frac{1}{2a} & -a<x<b \\ 0 & \text{otherwise.} \end{cases}$$

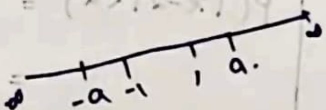
$$\boxed{\begin{array}{l} |x|\le a \\ \Rightarrow -a\le x\le a \end{array}}$$

$$P(|x|\le 1) = P(|x|>1)$$

$$= 1 - P[|x|\le 1]$$

$$2P[|x|\le 1] = 1$$

$$P(-1\le x\le 1) = \frac{1}{2}.$$



$$\int_{-1}^{1} f(x)\,dx = \frac{1}{2}.$$

$$\int_{-1}^{1} \frac{1}{2a}\,dx = \frac{1}{2}.$$

$$\frac{1}{\cancel{2}a}[x]_{-1}^{1} = \frac{1}{\cancel{2}}$$

$$1-(-1) = a.$$

$$\underline{a=2}$$

$$f(x) = \begin{cases} \frac{1}{4} & -2<x<2 \\ 0 & \text{otherwise} \end{cases}$$

x is uniformly distributed with mean 1 and Varience $\frac{4}{3}$. if 3 independent observations of x are made. what is the probability that all of them are negative

Q.

Pdf of U.D $f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$

Mean $= E[x] = 1$

$\frac{a+b}{2} = 1$

$a+b = 2$ — ①

Varience $= \frac{4}{3}$

$\frac{(b-a)^2}{12} = \frac{4}{3}$

$(b-a)^2 = 16$

$(b-a) = \pm 4$

$b-a = 4$ — ②

Solving ① and ②

$a + b = 2$

$-a + b = 4$

$0 + 2b = 6$

$\underline{\underline{b = 3}}$

① ⇒ $a + 3 = 2$

$\underline{\underline{a = -1}}$

$3-(-1) = 4$

$\therefore f(x) = \begin{cases} \frac{1}{4} & -1 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$

$P(x<0) = \int_{-1}^{0} f(x)\,dx = \int_{-1}^{0} \frac{1}{4}\,dx$

$= \frac{1}{4}[x]_{-1}^{0} = \frac{1}{4}[0-(-1)]$

$= \frac{1}{4}$

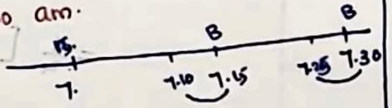
Prob: that all the three observation are negative $= \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$

$= \underline{\underline{\frac{1}{64}}}$

U.Q Buses arrives at a specified stop at 15 mts intervals starting at 7 am. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 am find the probability that he waits a) less than 5 mts for a bus.

b) atleast 10 mts for the bus   c) atleast 12 mts for the bus.

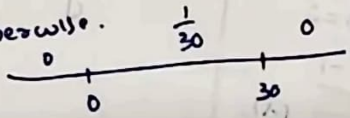
Let x denotes the waiting time of the passenger between 7 and 7.30 am.



$X \sim U(0,30)$

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & o.w \end{cases}$$

$$= \begin{cases} \frac{1}{30} & 0 \le x \le 30 \\ 0 & \text{otherwise.} \end{cases}$$



P(less than 5 mts) = P(he arrives between 7.10 am and 7.15 am or 7.25 and 7.30 am)

$= p(10 \le x \le 15) + p(25 \le x \le 30)$

$= \int_{10}^{15} \frac{1}{30}\, dx + \int_{25}^{30} \frac{1}{30}\, dx.$

$= \frac{1}{30}[x]_{10}^{15} + \frac{1}{30}[x]_{25}^{30}$

$= \frac{1}{30}[15-10] + \frac{1}{30}[30-25] = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

b) $P(\text{atleast } 10 \text{ mts})$

$= P(\text{he arrives between } 7 \text{ and } 7.05\text{am}$

$\text{or } 7.15 \text{ and } 7.20 \text{ am})$

$= P(0 \le x \le 5) + P(15 \le x \le 20)$

$$= \int_0^5 \frac{1}{30}\,dx + \int_{15}^{20} \frac{1}{30}\,dx.$$

$$= \frac{1}{30}\left[x\right]_0^5 + \frac{1}{30}\left[x\right]_{15}^{20}$$

$$= \frac{1}{\cancel{30}_{6}}\left[\overset{\cancel{5}}{5-0}\right] + \frac{1}{\cancel{30}_{6}}\left[\overset{\cancel{5}}{20-15}\right]$$

$$= \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$$

3

7. 7.5 — 7.15 7.20 — 7.30

$\underbrace{7.\ 7.5}_{7-7.03}$ $\underbrace{7.15\ 7.20}_{7.15-7.18}$

$P(\text{atleast } 12 \text{ mts})$

$= P(\text{he arrives between } 7 \text{ and } 7.03 \text{ am}$

$\text{or } 7.15 \text{ and } 7.18 \text{ am})$

$= P(0 \le x \le 3) + P(15 \le x \le 18)$

$$= \int_0^3 \frac{1}{30}\,dx + \int_{15}^{18} \frac{1}{30}\,dx.$$

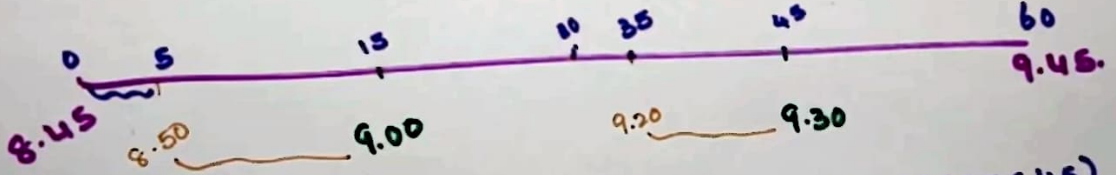
$$= \frac{1}{30}\left[x\right]_0^{3} + \frac{1}{30}\left[x\right]_{15}^{18}.$$

$$= \frac{1}{\cancel{30}_{10}}\left[\cancel{3}-0\right] + \frac{1}{\cancel{30}_{10}}\left[\overset{\cancel{3}}{18-15}\right]$$

$$= \frac{2}{10} = \underline{\underline{\frac{1}{5}}}$$

Starting at 5.00 am every half hour there is a flight from San Francisco airport to Los Angeles international airport. Suppose that none of these planes is completely sold out and they always have room for passengers. A person who wants to fly to LA arrives at airport at a random time between 8.45 am and 9.45 am. Find the probability that he waits (i) atmost 10 mins

(ii) atleast 15 mins:



$X \sim U(0,60)$

$$f(x) = \begin{cases} \frac{1}{60} & 0<x<60 \\ 0 & \text{o.w} \end{cases}$$

$p(\text{atmost } 10 \text{ min}) = p(5<x<15) + p(35<x<45) = \frac{1}{3}$

$p(\text{atleast } 15 \text{ min}) = p(15<x<30) + p(45<x<60) = \frac{1}{2}.$

find 1) $P(x \le 0)$ 2) $P(x \le \frac{\pi}{2})$ 3) $P(x > \frac{\pi}{2} \mid x > 0)$ 4) The mean phase difference and the Varience.

$x \sim U(-\pi, \pi)$

$$f(x) = \begin{cases} \frac{1}{\pi - (-\pi)} & -\pi < x < \pi \\ 0 & \text{o.w} \end{cases}$$

$$= \begin{cases} \frac{1}{2\pi} & -\pi < x < \pi \\ 0 & \text{o.w} \end{cases}$$

$$P(x \le 0) = \int_{-\pi}^{0} f(x)\,dx = \int_{-\pi}^{0} \frac{1}{2\pi}\,dx$$

$$= \frac{1}{2\pi}\left[x\right]_{-\pi}^{0}$$

$$= \frac{1}{2\pi}\left[0 - (-\pi)\right]$$

$$= \frac{1}{2}$$

2) $$P\left(x \le \frac{\pi}{2}\right) = \int_{-\pi}^{\pi/2} f(x)\,dx$$

$$= \int_{-\pi}^{\pi/2} \frac{1}{2\pi}\,dx$$

$$= \frac{1}{2\pi}\left[x\right]_{-\pi}^{\pi/2}$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{2} - (-\pi)\right]$$

$$= \frac{1}{2\pi}\cdot\frac{3\pi}{2}$$

$$= \underline{\underline{\frac{3}{4}}}$$

3) $$P\left(x > \pi/2 \,/\, x > 0\right)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P\left(\left(x > \frac{\pi}{2}\right) \cap (x > 0)\right)}{P(x > 0)}$$

$$= \frac{P\left[\frac{\pi}{2} < x < \pi\right]}{P[x > 0]} = \frac{\int_{\pi/2}^{\pi} f(x)\,dx}{\int_{0}^{\pi} f(x)\,dx}$$

$$= \frac{\int_{\pi/2}^{\pi} \frac{1}{2\pi}\,dx}{\int_{0}^{\pi} \frac{1}{2\pi}\,dx} = \frac{\cancel{\frac{1}{2\pi}}\left[x\right]_{\pi/2}^{\pi}}{\cancel{\frac{1}{2\pi}}\left[x\right]_{0}^{\pi}}$$

$$= \frac{\left[\pi - \frac{\pi}{2}\right]}{\left[\pi - 0\right]} = \frac{\frac{\pi}{2}}{\pi} = \underline{\underline{\frac{1}{2}}}$$

4) Mean $E[x] = \frac{a+b}{2}$

$$= \frac{\pi - \pi}{2}$$

$$\boxed{\text{Mean} = 0}$$

$$\cancel{Var(x) = E[x^2] - E[x]^2}$$

$$Var(x) = \frac{(b-a)^2}{12} = \frac{(-\pi - \pi)^2}{12}$$

$$= \frac{4\pi^2}{12} = \frac{\pi^2}{3}$$

Exponential Random Variable:

A continuous random variable x with probability density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

where $\lambda > 0$, is called an exponential random variable:

Note:

x follows exponential distribution with parameter λ as $x \sim \exp(\lambda)$

Mean and Variance.

Mean

$$E[x] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$e^{-\infty} = 0$

$e^{\infty} = \infty$

$e^{0} = 1$

$$= \int_{-\infty}^{0} 0 + \int_{0}^{\infty} x \lambda e^{-\lambda x}\, dx.$$

$$= \lambda \int_{0}^{\infty} x e^{-\lambda x}\, dx$$

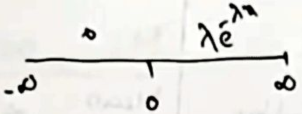
$$= \lambda \left[(x)\left[\frac{e^{-\lambda x}}{-\lambda}\right] - (1)\left[\frac{e^{-\lambda x}}{\lambda^2}\right] \right]_0^{\infty}$$

$$= \lambda \left[[0 - 0] - \left[0 - \frac{\overset{1}{e^{0}}}{\lambda^2}\right] \right]$$

$$= \lambda \left[\frac{1}{\lambda^2} \right]$$

$$\boxed{\text{Mean} = \frac{1}{\lambda}.}$$

$$Var(x) = E[x^2] - E[x]^2$$



$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$\int e^{ax} = \frac{e^{ax}}{a}$$

$$= \int_{0}^{\infty} x^2 \left[\lambda e^{-\lambda x}\right] dx$$

$$e^{-\infty} = 0$$

$$= \lambda \int_{0}^{\infty} x^2 e^{-\lambda x}\, dx$$

$$= \lambda \left[(x^2)\left[\frac{e^{-\lambda x}}{-\lambda}\right] - (2x)\left[\frac{e^{-\lambda x}}{\lambda^2}\right] + (2)\left[\frac{e^{-\lambda x}}{-\lambda^3}\right] \right]_{0}^{\infty}$$

$$= \lambda \left[[0+0+0] - \left[0+0+\frac{2e^{0}}{-\lambda^3}\right] \right]$$

$$= \lambda \left[\frac{2}{\lambda^3}\right]$$

$$E(x^2) = \frac{2}{\lambda^2}$$

$$\therefore Var(x) = E[x^2] - E[x]^2$$

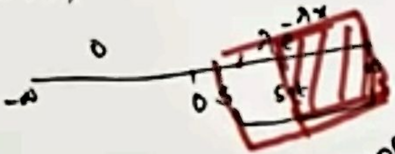
$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$\boxed{Var(x) = \frac{1}{\lambda^2}}$$

Memoryless Property of exponential distribution.

x is expontially distributed, then.

$$P(x > s+t \mid x > s) = P(x > t)$$



Proof.

$$P(x > s) = \int_s^\infty \lambda e^{-\lambda x}\,dx$$

$$= \lambda \left[\frac{e^{-\lambda x}}{-\lambda}\right]_s^\infty$$

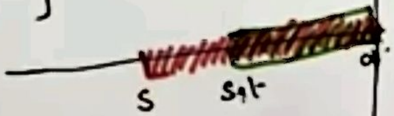
$$= -\left[\overset{0}{e^{-\infty}} - e^{-\lambda s}\right]$$

$$= e^{-\lambda s}$$

$$P(x > s+t) = e^{-\lambda(s+t)}$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$x^{m+n} = x^m \cdot x^n$$



$$P(x > s+t \mid x > s) = \frac{P((x > s+t)\cap(x > s))}{P[x > s]}$$

$$= \frac{P[x > s+t]}{P[x > s]}$$

$$= \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}}$$

$$= \frac{\cancel{e^{-\lambda s}}\, e^{-\lambda t}}{\cancel{e^{-\lambda s}}}$$

$$= e^{-\lambda t}$$

$$= P(x > t)$$