POWER & THE POYNTING VECTOR

- According to Maxwell's theory we know that propagation of electromagnetic wave is basically flow of power from one place to another in space.
- In other words, electromagnetic wave flows the energy with itself, where power is generally describes in terms of energy per unit area surface (unit : watt/m^2)
- In summarize, whole phenomenon is expressed by some direct relationship between electric and magnetic filed magnitudes with the rate of energy transfer from source to receiver.

From Maxwell's equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\boxed{B = \mu H}$$

$$\boxed{J = \sigma E}$$

$$\boxed{D = \varepsilon E}$$

$$\nabla \times E = -\frac{\mu \partial H}{\partial t} \quad \text{----------} \ (1)$$

$$\nabla \times H = \sigma E + \frac{\varepsilon \partial E}{\partial t} \quad \text{----------} \ (2)$$

$$\nabla \times E = -\frac{\mu \partial H}{\partial t} \quad \text{----------} \ (1)$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \quad \text{----------} \ (2)$$

Doting both sides of (2) with E

$$\nabla.(A \times B) = B.(\nabla \times A) - A.(\nabla \times B)$$

$$E.(\nabla \times H) = \sigma E^2 + E.\varepsilon \frac{\partial E}{\partial t}$$

$$\nabla.(H \times E) = E.(\nabla \times H) - H.(\nabla \times E)$$

$$E.(\nabla \times H) = H.(\nabla \times E) + \nabla.(H \times E)$$

$$\frac{\partial E^2}{\partial t} = 2E$$

Applying the property

$$H.(\nabla \times E) + \nabla.(H \times E) = \sigma E^2 + E.\frac{\varepsilon \partial E}{\partial t}$$

$$= \sigma E^2 + \frac{\varepsilon}{2}\frac{\partial}{\partial t}E^2 \quad \text{----------} \ (3)$$

Doting both sides of (1) with H

$$H.(\nabla \times E) = H.\left(-\frac{\mu \partial H}{\partial t}\right)$$

$$H.(\nabla \times E) = -\frac{\mu}{2}\frac{\partial}{\partial t}H^2 \quad \text{----------} \ (4)$$

Substituting (4) in (3)

$$-\frac{\mu}{2}\frac{\partial}{\partial t}H^2 + \nabla.(H \times E) = \sigma E^2 + \frac{\varepsilon}{2}\frac{\partial}{\partial t}E^2$$

$$-\frac{\mu}{2}\frac{\partial}{\partial t}H^2 - \nabla.(E \times H) = \sigma E^2 + \frac{\varepsilon}{2}\frac{\partial}{\partial t}E^2$$

Rearranging

$$\nabla.(E \times H) = -\frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) - \sigma E^2$$

Taking volume integral on both sides

$$\int_v \nabla.(E \times H)\,dv = -\frac{\partial}{\partial t}\int_v \left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv - \int_v \sigma E^2 dv$$

Applying divergence theorem on LHS

$$\oint_s (E \times H) dS = -\frac{\partial}{\partial t}\int_v \left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv - \int_v \sigma E^2 dv$$

$$\oint_s (\boldsymbol{E} \times \boldsymbol{H}) d\boldsymbol{S} = -\frac{\partial}{\partial t} \int_v \left(\frac{\varepsilon \boldsymbol{E}^2}{2} + \frac{\mu \boldsymbol{H}^2}{2} \right) d\boldsymbol{v} - \int_v \boldsymbol{\sigma} \boldsymbol{E}^2 d\boldsymbol{v}$$

Poynting's Theorem

Total power leaving the volume = *Rate of decrease in energy stored in electric and magnetic fields* - *Ohmic power dissipation*

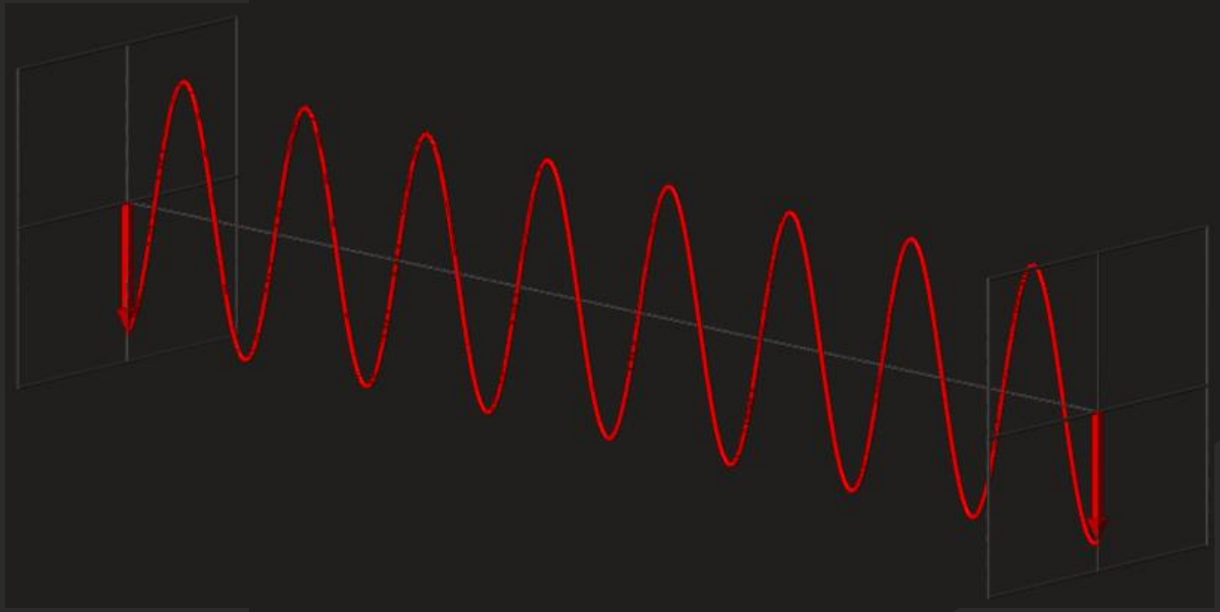
The quantity E $\times$ ***H*** *is know as the Poynting vector* $\mathscr{P}$

$$\mathscr{P} = \boldsymbol{E} \times \boldsymbol{H}$$

Poynting theorem *states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within v minus the ohmic losses*

POLARIZATION – EM WAVES

Linear Polarization

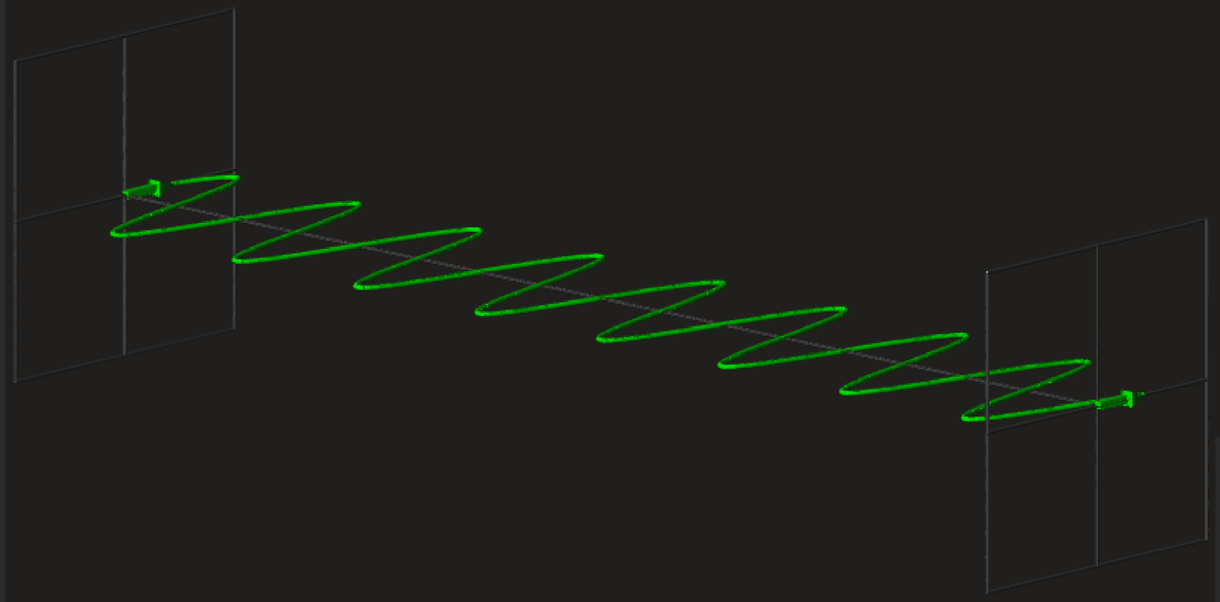


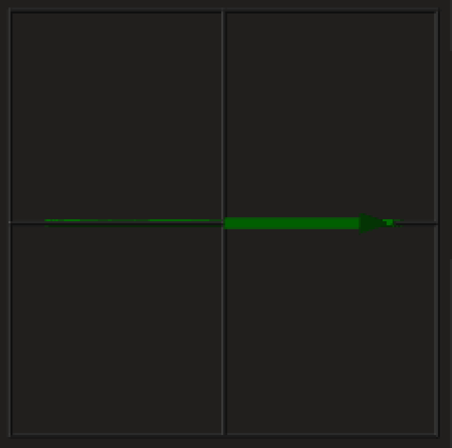
$$E_x = E_{x0} \cos\left(\omega t + \varphi_x\right) a_x$$

$$H_y = 0$$

POLARIZATION – EM WAVES

Linear Polarization



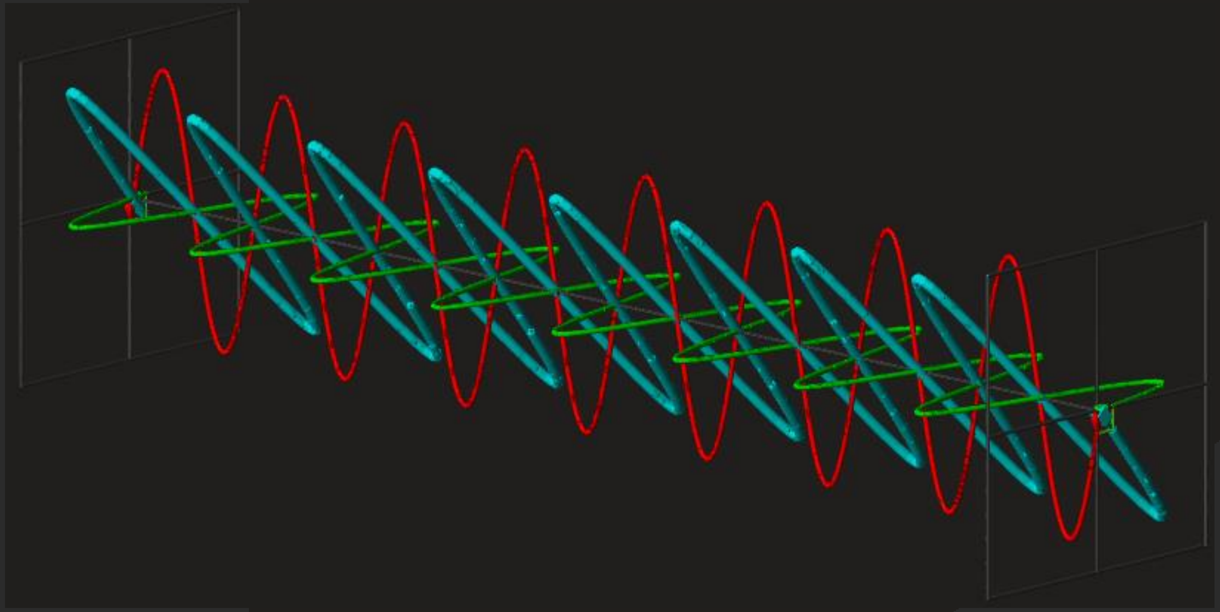


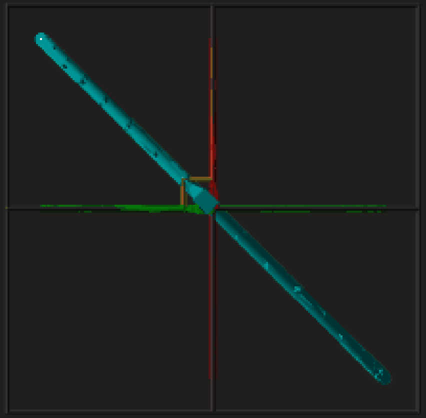
$$E_x = 0$$

$$H_y = H_{y0} cos\,(\omega t + \varphi_y) a_y$$

POLARIZATION – EM WAVES

Linear Polarization





$$\boldsymbol{E} = \boldsymbol{E_{x0} cos\ (\omega t + \varphi) a_x}$$

$$\boldsymbol{H} = \boldsymbol{H_{y0} cos\ (\omega t + \varphi) a_y}$$

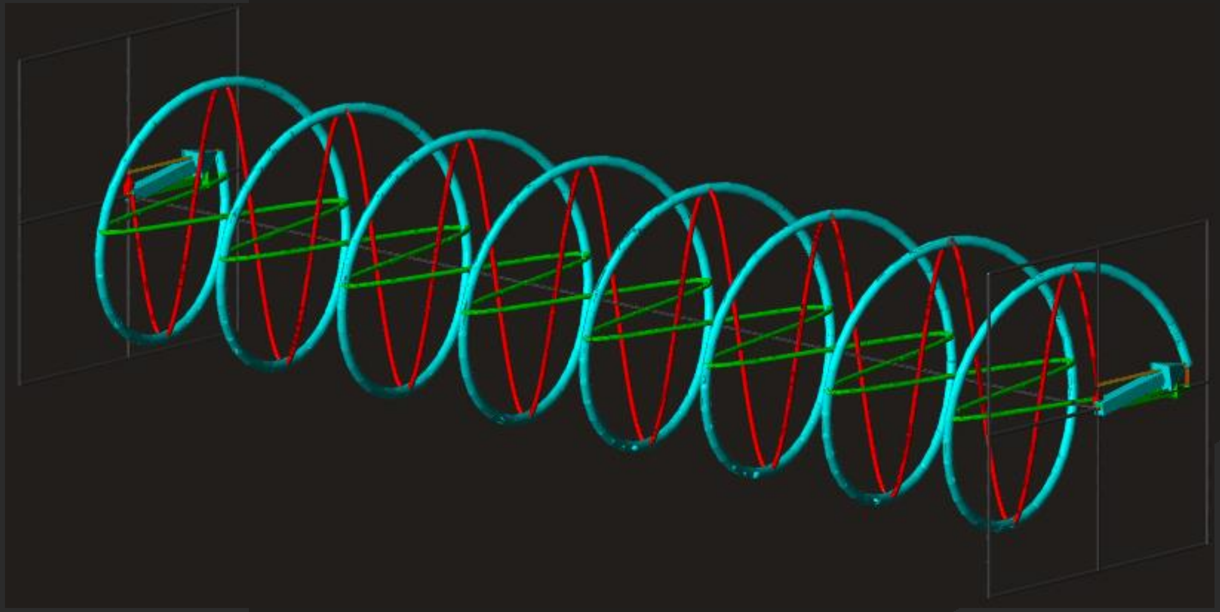
$$\boldsymbol{E_{x0} = H_{y0}}$$

These two waves are termed *linearly polarized*, since the electric field/Magnetic field vector oscillates in a straight-line

POLARIZATION – EM WAVES

Circular Polarization

Circularly polarized wave consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase.



Phase shift is +90^0

$$E_x = E_{x0} cos\,(\omega t) a_x$$

$$E_{x0} = H_{y0}$$

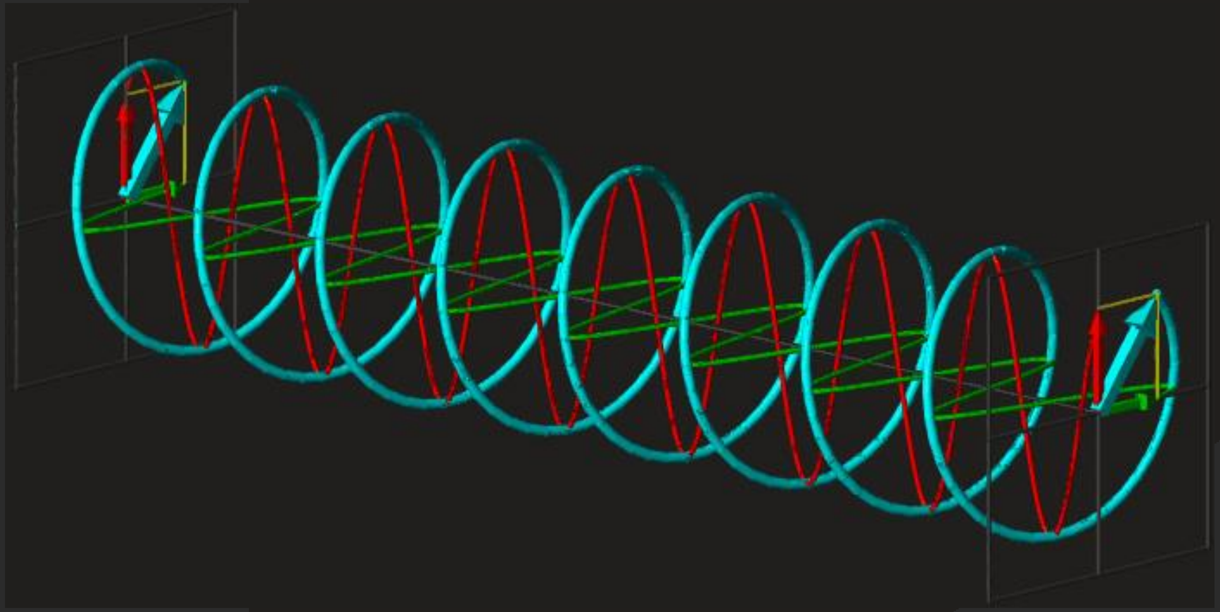
$$H_y = H_{y0} cos\left(\omega t + \frac{n\pi}{2}\right) a_y$$

n is an odd number

1. The field must have two orthogonal polarized components
2. The two components must have the same magnitude ($E_{x0} = E_{y0}$)
3. The two components must have a time-phase difference of multiples of 90 degrees.

POLARIZATION – EM WAVES

Circular Polarization



Phase shift is -90^0

$$E_x = E_{x0} cos\,(\omega t) a_x$$

$$E_{x0} = H_{y0}$$

$$H_y = H_{y0} cos\left(\omega t - \frac{n\pi}{2}\right) a_y$$

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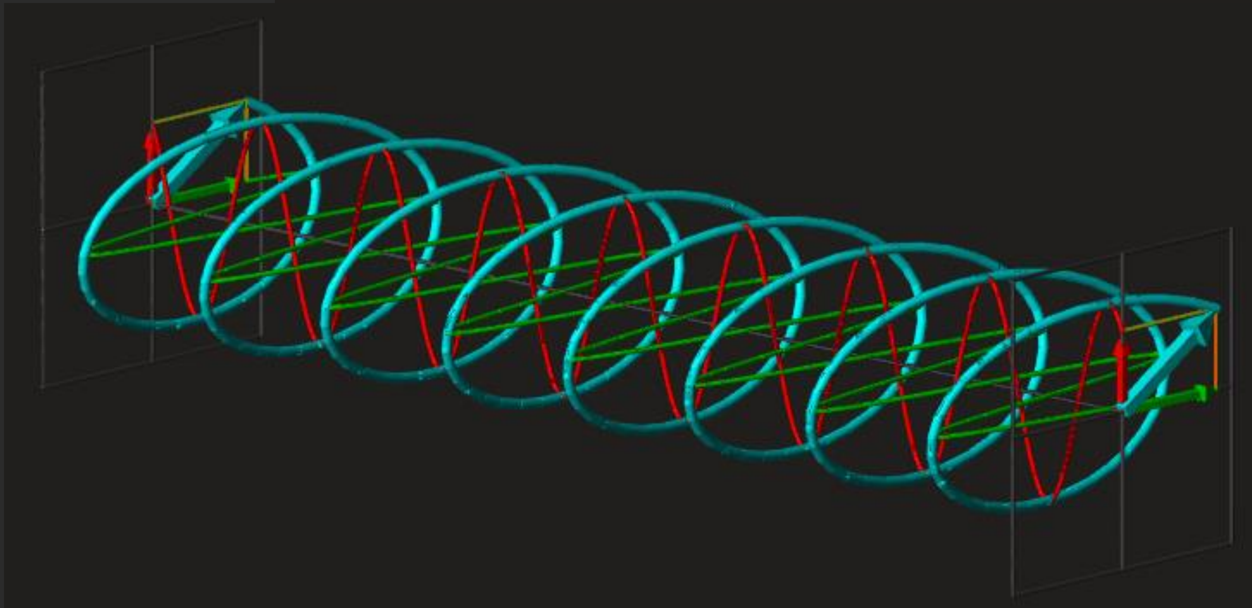
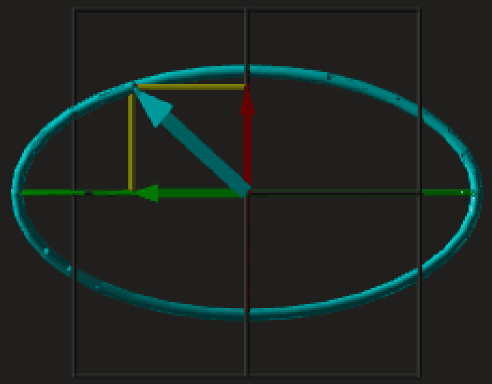
POLARIZATION – EM WAVES

Elliptical Polarization

Elliptically polarized wave consists of two perpendicular waves of unequal amplitude which differ in phase by 90°.





$$E_x = E_0 cos\,(\omega t) a_x$$

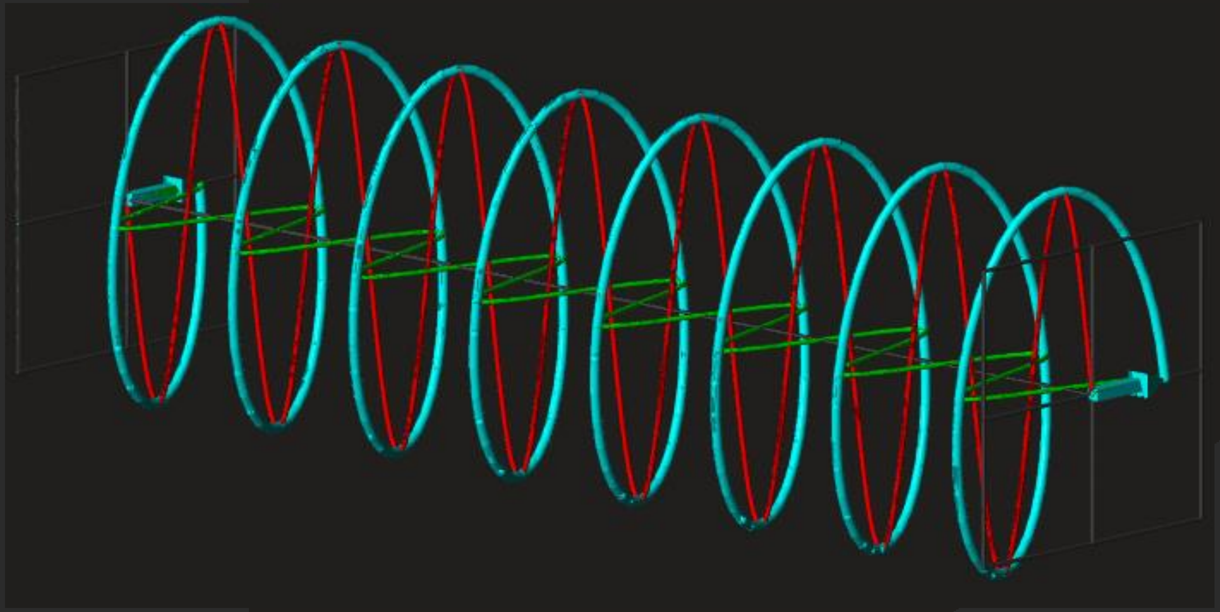
$$E_{x0} < H_{y0}$$

$$H_y = H_0 cos\left(\omega t + \frac{\pi}{2}\right) a_y$$

$$Phase\ shift\ is + 90^0$$

POLARIZATION – EM WAVES

Elliptical Polarization



$$E_x = E_0 cos\,(\omega t) a_x$$

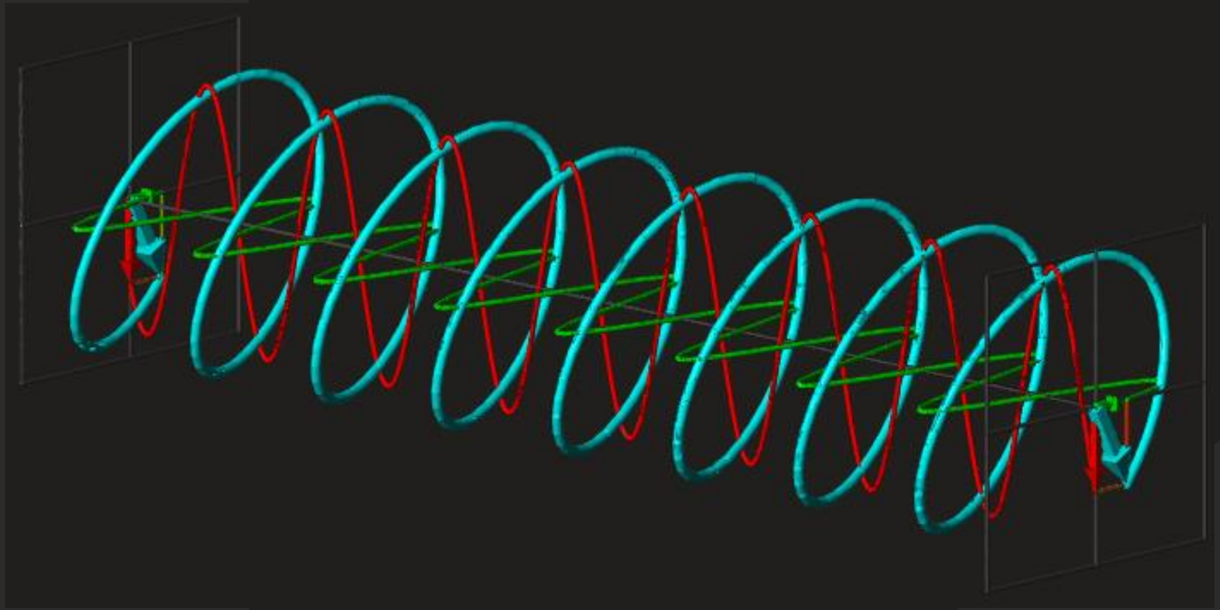
$$E_{x0} > H_{y0}$$

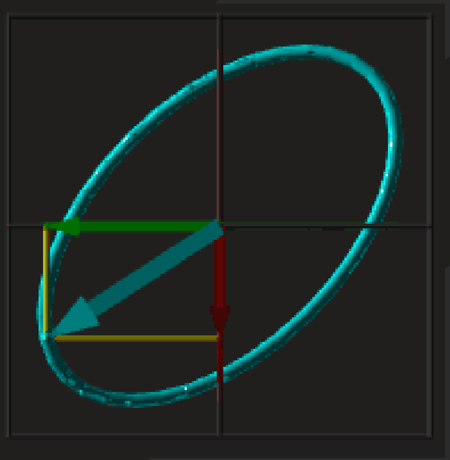
$$H_y = H_0 cos\,\left(\omega t + \frac{\pi}{2}\right) a_y$$

$$Phase\ shift\ is + 90^0$$

POLARIZATION – EM WAVES

Elliptical Polarization





$E_x = E_0 cos\,(\omega t) a_x$

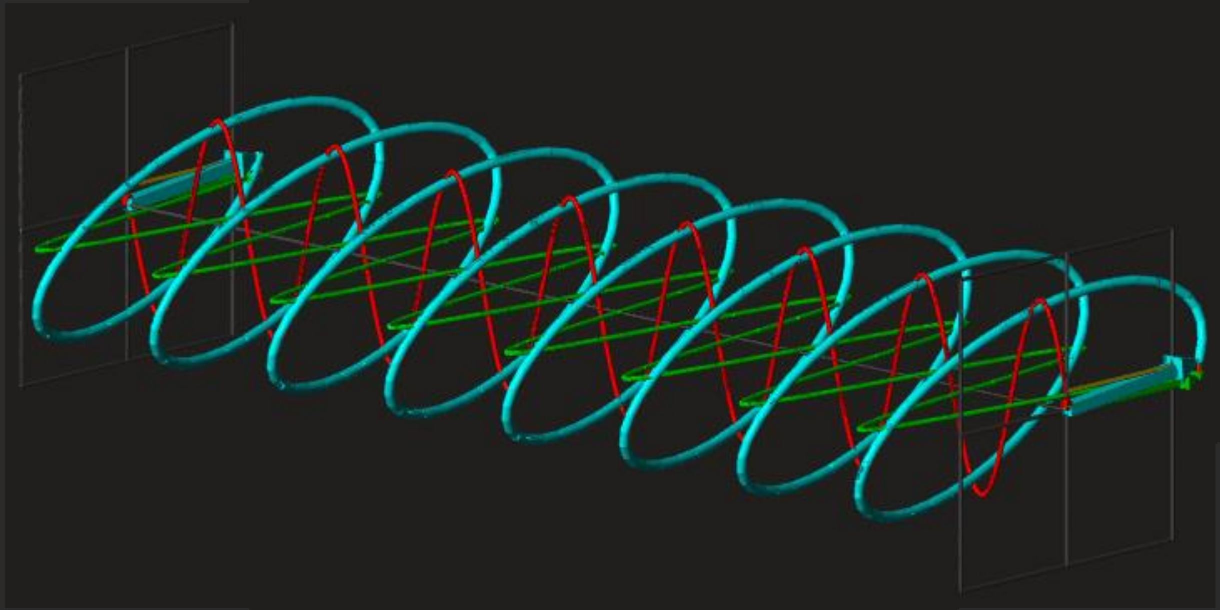
$E_{x0} = H_{y0}$

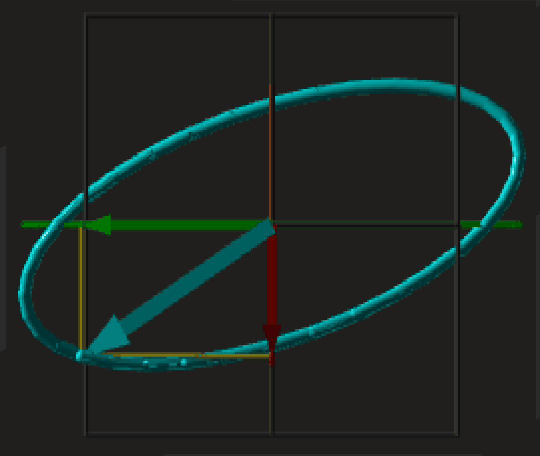
$H_y = H_0 cos\left(\omega t + \frac{\pi}{2}\right) a_y$

$Phase\ shift\ is + 120^0$

POLARIZATION – EM WAVES

Elliptical Polarization





$E_x = E_0 cos\,(\omega t) a_x$

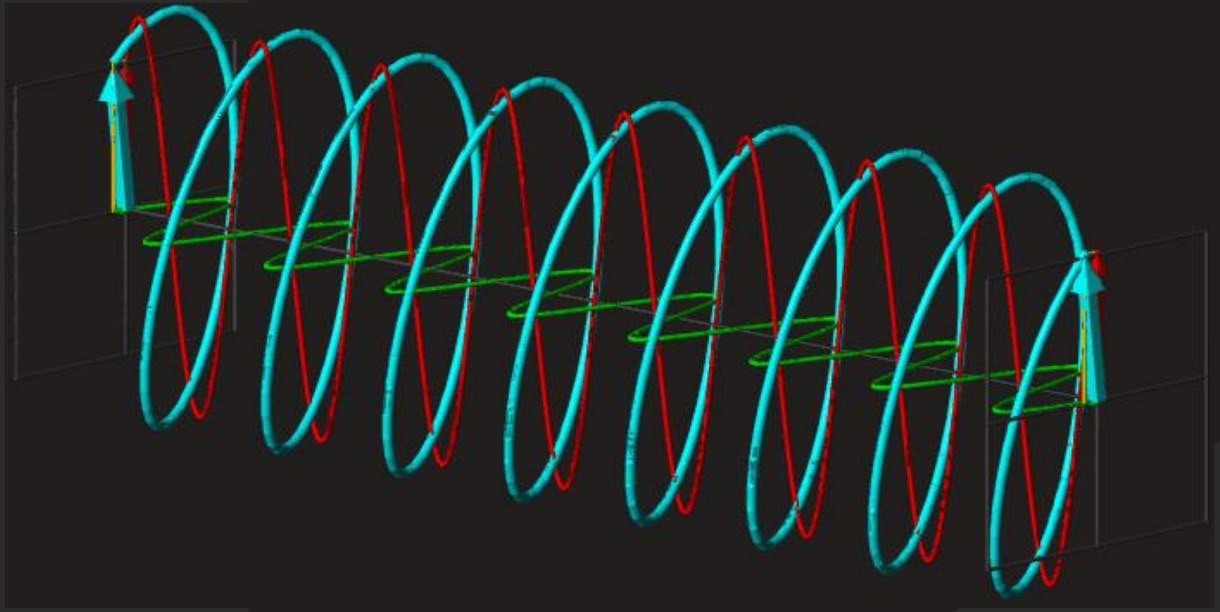
$E_{x0} < H_{y0}$

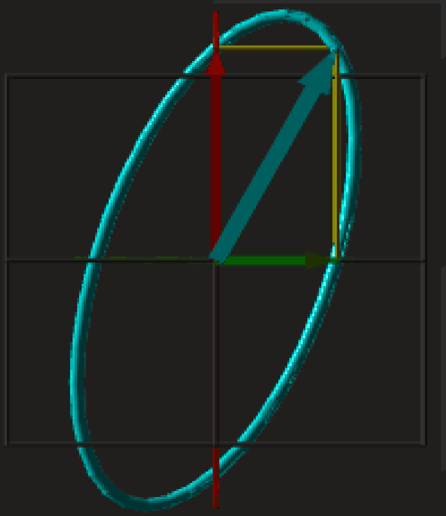
$H_y = H_0 cos\,\left(\omega t + \frac{\pi}{2}\right) a_y$

$Phase\ shift\ is + \mathbf{120^0}$

POLARIZATION – EM WAVES

Elliptical Polarization





$$E_x = E_0 cos\,(\omega t) a_x$$

$$E_{x0} > H_{y0}$$

$$H_y = H_0 cos\left(\omega t + \frac{\pi}{2}\right) a_y$$

$$Phase\ shift\ is + 120^0$$

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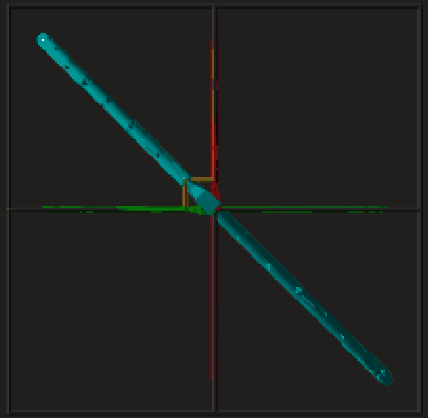
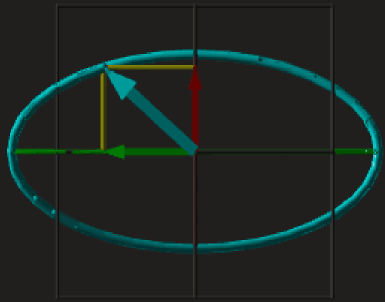


$E_{x0} = H_{y0}$
Phase shift is 0^0

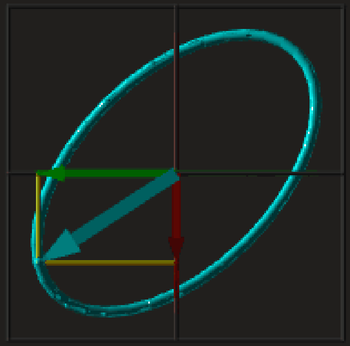
$E_{x0} = H_{y0}$
Phase shift is $+90^0$

$E_{x0} = H_{y0}$
Phase shift is -90^0

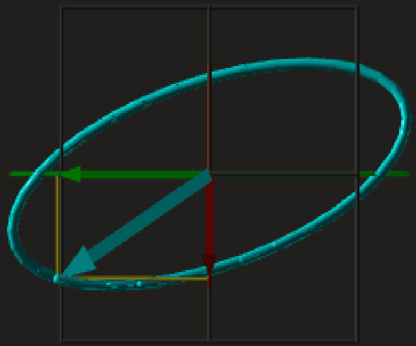


$E_{x0} < H_{y0}$
Phase shift is $+90^0$

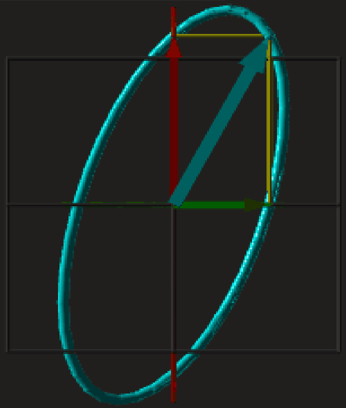
$E_{x0} > H_{y0}$
Phase shift is $+90^0$



$E_{x0} = H_{y0}$
Phase shift is $+120^0$



$E_{x0} < H_{y0}$
Phase shift is $+120^0$



$E_{x0} > H_{y0}$
Phase shift is $+120^0$

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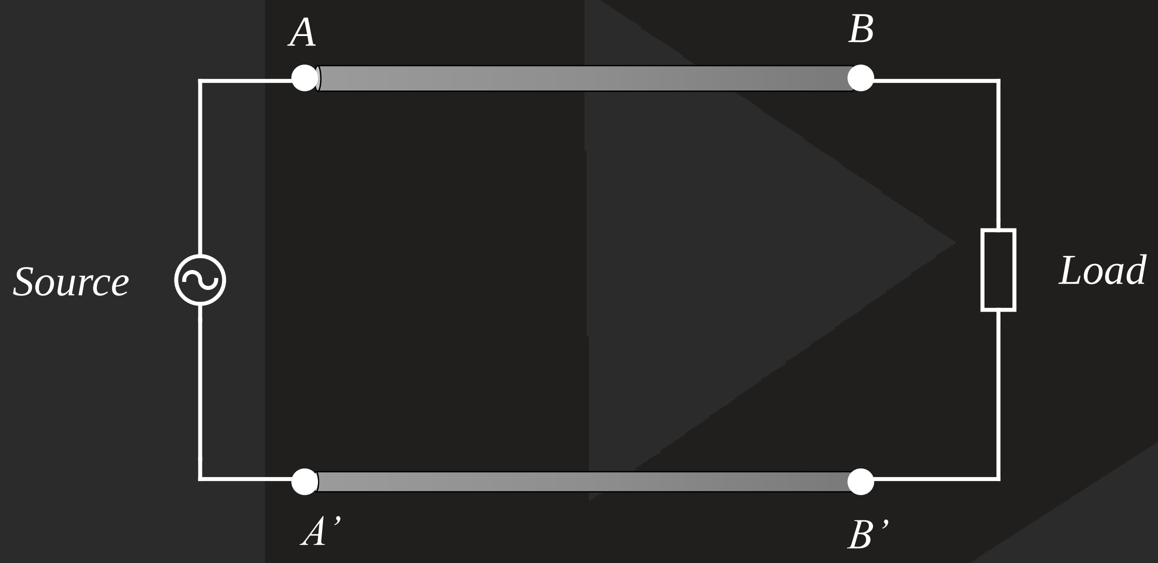
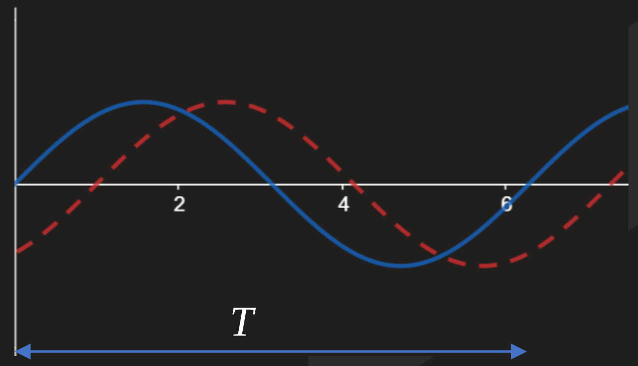
TRANSMISSION LINES

- Transmission lines are the conductors that serve as a path for transmitting (sending) electrical waves (energy) through them.
- These basically forms a connection between transmitter and receiver in order to permit signal transmission.
- It transmits the wave of voltage and current from one end to another.
- The transmission line is made up of a conductor having a uniform cross-section along the line.





- *The time delay is known as transit time (tr)* $\boldsymbol{t_r = \frac{l}{v}}$
- *Due to the transit time the voltage difference (potential difference) occurs*

To reduce the transit time effect

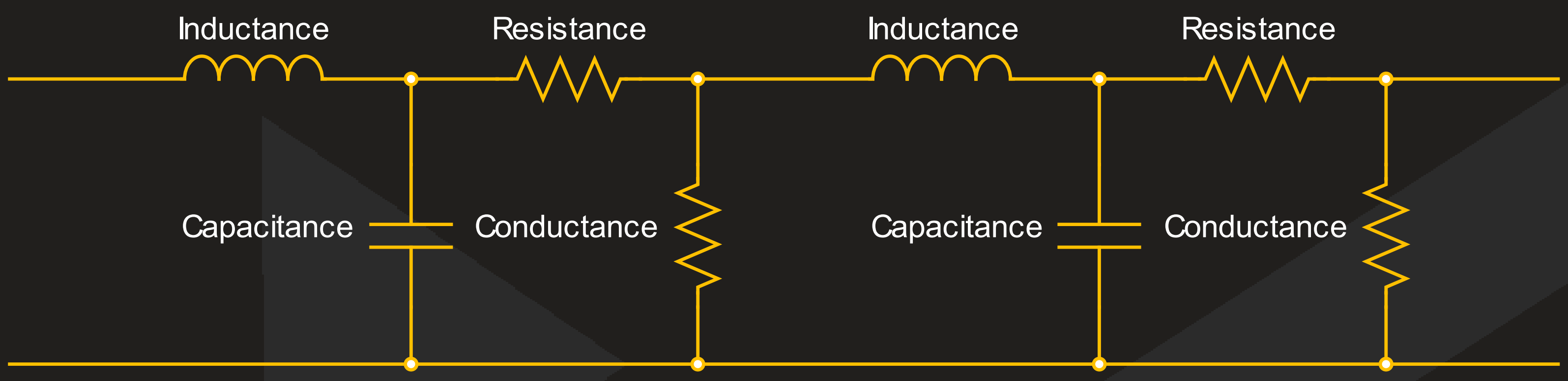
$$\boldsymbol{T \gg t_r}$$

$$\boldsymbol{\frac{1}{f} \gg \frac{l}{v}}$$

$$\boldsymbol{\frac{v}{f} \gg l}$$

$$\boldsymbol{\lambda \gg l}$$

Let us consider a parallel transmission line with all its distributive parameter



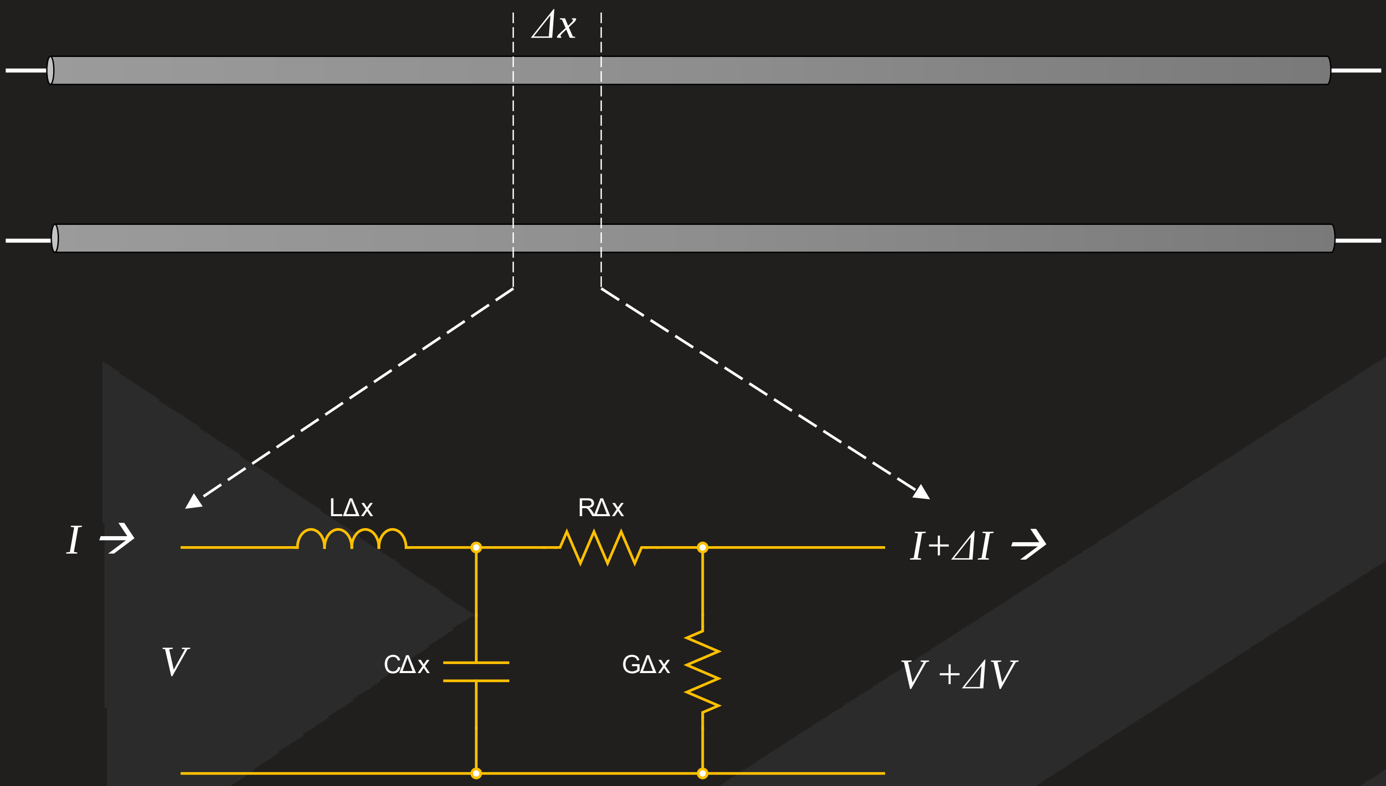
R → Resistance (Ω/m)

L → Inductance (H/m)

C → Capacitance (F/m)

G → Conductance (℧/m)

These are known as primary parameters of transmission line



The change in voltage and current

$$\Delta \boldsymbol{V} = -(\boldsymbol{R}\Delta \boldsymbol{x} + \boldsymbol{j\omega L}\Delta \boldsymbol{x})\boldsymbol{I}$$

$$\Delta \boldsymbol{I} = -(\boldsymbol{G}\Delta \boldsymbol{x} + \boldsymbol{j\omega C}\Delta \boldsymbol{x})\boldsymbol{V}$$

Negative due to reduction in voltage and current

$$\Delta V = -(R\Delta x + j\omega L\Delta x)I \quad \text{----------} \ (1)$$

$$\Delta I = -(G\Delta x + j\omega C\Delta x)V \quad \text{----------} \ (2)$$

Applying limit

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = -(R + j\omega L)I \quad \text{----------} \ (3)$$

$$\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = \frac{dI}{dx} = -(G + j\omega C)V \quad \text{----------} \ (4)$$

Differentiating (3) w.r.t. x

$$\frac{d^2V}{dx^2} = -(R + j\omega L)\frac{dI}{dx} \quad \text{----------} \ (5)$$

Substitute (4) in (5)

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C)V$$

If we substitute

$$\gamma^2 = (\boldsymbol{R} + \boldsymbol{j\omega L})(\boldsymbol{G} + \boldsymbol{j\omega C})$$

$$\boxed{\frac{\boldsymbol{d^2V}}{\boldsymbol{dx^2}} = \gamma^2 \boldsymbol{V} \qquad \frac{\boldsymbol{d^2I}}{\boldsymbol{dx^2}} = \gamma^2 \boldsymbol{I}}$$

Propagation constant: $\gamma = \sqrt{(\boldsymbol{R} + \boldsymbol{j\omega L})(\boldsymbol{G} + \boldsymbol{j\omega C})}$

CHARACTERISTICS IMPEDANCE (Z_0)

The characteristic impedance of the line is the ratio of positively travelling voltage wave to current wave at any point on line

Let

$$V = Ve^{-\gamma x} \qquad I = Ie^{-\gamma x}$$

We saw

$$\frac{dV}{dx} = -(R + j\omega L)I$$

$$\frac{d}{dx}Ve^{-\gamma x} = -(R + j\omega L)Ie^{-\gamma x}$$

$$-Ve^{-\gamma x}\gamma = -(R + j\omega L)Ie^{-\gamma x}$$

$$\frac{V}{I} = \frac{(R + j\omega L)}{\gamma}$$

$$= \frac{(R + j\omega L)}{\sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$\boxed{\boldsymbol{Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}}}$$

Characteristic impedance of transmission line

Also

$$\boxed{\boldsymbol{Y_0 = \frac{1}{Z_0}}}$$

Admittance

REFLECTION COEFFICIENT

Let forward and reverse travelling waves be

$$V(x) = V^+e^{-\gamma x} + V^-e^{\gamma x} \quad \text{----------} \ (1)$$

$$I(x) = I^+e^{-\gamma x} + I^-e^{\gamma x} \quad \text{----------} \ (2)$$

$$Z_0 = \frac{V^+}{I^+}, Z_0 = -\frac{V^-}{I^-}$$

(2) → $$I = \frac{V^+}{Z_0}e^{-\gamma x} - \frac{V^-}{Z_0}e^{\gamma x}$$

At *l=0*

$$Z_L = \frac{V}{I} = \frac{V^+e^{-\gamma x} + V^-e^{\gamma x}}{\frac{V^+}{Z_0}e^{-\gamma x} - \frac{V^-}{Z_0}e^{\gamma x}}$$

$$Z_L = \left[\frac{V^+ + V^-}{V^+ - V^-}\right]Z_0$$

$$Z_L = \left[\frac{V^+ + V^-}{V^+ - V^-}\right] Z_0$$

$$Z_L = \left[\frac{V^+(1 + \frac{V^-}{V^+})}{V^+(1 - \frac{V^-}{V^+})}\right] Z_0$$

$$Z_L = \left[\frac{(1 + \Gamma_{(0)})}{(1 - \Gamma_{(0)})}\right] Z_0$$

$$Z_L - Z_L\Gamma_{(0)} = Z_0 + Z_0\Gamma_{(0)}$$

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Reflection coefficient at the load end

The term ***voltage reflection coefficient*** *at any point on the line is the ratio of voltage reflected wave to the incident wave at the load end*

$$\Gamma_{(l)} = \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}} \qquad \Gamma_{(0)}/\Gamma_L = \frac{V^-}{V^+}$$

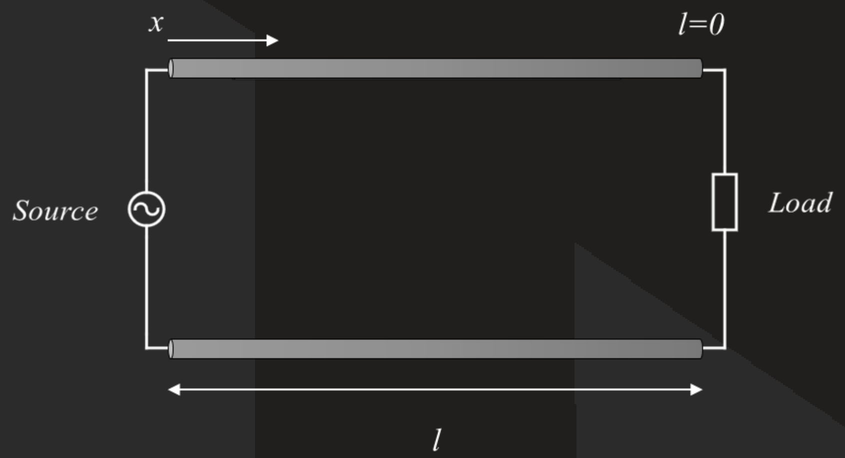
$$\Gamma_{(l)} = \Gamma_{(0)} e^{2\gamma l}$$

If $Z_L = Z_0$

$$\Gamma_L = 0$$

Which means when the load impedance is equal to characteristic impedance then there are no reflection occurs, this condition is known as ***matched load condition***

IMPEDANCE OF TRANSMISSION LINE



At point l, (x=-l)

$$V(-l) = V^+e^{+\gamma l} + V^-e^{-\gamma l}$$

$$V(-l) = V^+\left[e^{\gamma l} + \frac{V^-}{V^+}e^{-\gamma l}\right]$$

$$V(-l) = V^+[e^{\gamma l} + \Gamma_L e^{-\gamma l}] \quad \text{----------- (1)}$$

$$I(-l) = I^+e^{+\gamma l} + I^-e^{-\gamma l}$$

$$I(-l) = \frac{V^+}{Z_0}[e^{\gamma l} - \Gamma_L e^{-\gamma l}] \quad \text{----------- (2)}$$

$$Z(-l) = \frac{V(-l)}{I(-l)}$$

$$= \frac{V^+[e^{\gamma l} + \Gamma_L e^{-\gamma l}]}{\frac{V^+}{Z_0}[e^{\gamma l} - \Gamma_L e^{-\gamma l}]}$$

$$= Z_0\left[\frac{e^{\gamma l} + \Gamma_L e^{-\gamma l}}{e^{\gamma l} - \Gamma_L e^{-\gamma l}}\right]$$

$$= Z_0\left[\frac{e^{\gamma l} + \frac{Z_L - Z_0}{Z_L + Z_0}e^{-\gamma l}}{e^{\gamma l} - \frac{Z_L - Z_0}{Z_L + Z_0}e^{-\gamma l}}\right]$$

$$= Z_0\left[\frac{Z_Le^{\gamma l} + Z_0e^{\gamma l} + Z_Le^{-\gamma l} - Z_0e^{-\gamma l}}{Z_Le^{\gamma l} + Z_0e^{\gamma l} - Z_Le^{-\gamma l} + Z_0e^{-\gamma l})}\right]$$

$$= Z_0\left[\frac{Z_L(e^{\gamma l} + e^{-\gamma l}) + Z_0\,(e^{\gamma l} - e^{-\gamma l})}{Z_L(e^{\gamma l} - e^{-\gamma l}) + Z_0\,(e^{\gamma l} + e^{-\gamma l})}\right]$$

$$\Gamma_L = \frac{V^-}{V^+}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z(-l) = Z_0\left[\frac{Z_L(e^{\gamma l} + e^{-\gamma l}) + Z_0\,(e^{\gamma l} - e^{-\gamma l})}{Z_L(e^{\gamma l} - e^{-\gamma l}) + Z_0\,(e^{\gamma l} + e^{-\gamma l})}\right]$$

$$= Z_0\left[\frac{Z_L 2\cosh(\gamma l) + Z_0 2\sinh(\gamma l)}{Z_L 2\sinh(\gamma l) + Z_0 2\cosh(\gamma l)}\right]$$

$$= Z_0\left[\frac{\cosh(\gamma l)\left(Z_L + Z_0\dfrac{\sinh(\gamma l)}{\cosh(\gamma l)}\right)}{\cosh(\gamma l)\left(Z_L\dfrac{\sinh(\gamma l)}{\cosh(\gamma l)} + Z_0\right)}\right]$$

$$\boxed{Z_{in}(-l) = Z_0\left[\frac{\boldsymbol{Z_L + Z_0\tanh(\gamma l)}}{\boldsymbol{Z_0 + Z_L\tanh(\gamma l)}}\right]}$$

Impedance transformation relation or simply ***input impedance***

The impedance of the transmission line from input to the output can be characterized by this equation

$$\boldsymbol{\cosh(x) = \frac{e^{x} + e^{-x}}{2}}$$

$$\boldsymbol{\sinh(x) = \frac{e^{x} - e^{-x}}{2}}$$

$$\boldsymbol{\tanh(jx) = j\tan(x)}$$

At lossless line (α=0)

$$Z_{in}(-l) = Z_0\left[\frac{Z_L + Z_0\tanh(j\beta l)}{Z_0 + Z_L\tanh(j\beta l)}\right]$$

$$\boxed{Z_{in} = Z_0\left[\frac{\boldsymbol{Z_L + jZ_0\tan(\beta l)}}{\boldsymbol{Z_0 + jZ_L\tan(\beta l)}}\right]}$$

STANDING WAVE RATIO (SWR)

- It is defined as the ratio of maximum to minimum current or voltage on the line
- It is the measure of mismatch between the load and the transmission line

$$\boxed{s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}}$$

$$\Gamma_L = \frac{V^-}{V^+}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Voltage Standing Wave Ratio (VSWR)

$$\frac{V_{max}}{V_{min}} = \frac{V^+ + V^-}{V^+ - V^-}$$

$$= \frac{V^+\left(1 + \frac{V^-}{V^+}\right)}{V^+\left(1 - \frac{V^-}{V^+}\right)}$$

$$\boxed{VSWR = \frac{(1 + \Gamma_L)}{(1 - \Gamma_L)}}$$

We know that

$$Z_0 = \frac{V_{max}}{I_{max}} = \frac{V_{min}}{I_{min}}$$

$$I_{max} = \frac{V_{max}}{Z_0} \qquad V_{max} = \frac{I_{max}}{Z_0}$$

$$I_{min} = \frac{V_{min}}{Z_0} \qquad V_{min} = \frac{I_{min}}{Z_0}$$

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = sZ_0$$

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{s}$$