Statical Averages of Random Variables.

Probability density function (PDF) & CDF gives some kind of information about the random variable. There are some other measures or numbers which give more useful and quick information about the random variable. Collectively these characteristic numbers or measures are known as statistical averages. These are special characteristics of the PDF.

Mean or Average

the mean or average of any random variable is expressed by the summation of the values of random variables X weighted by their probabilities

Mean value is also known as expected value of random variable X.

it is denoted by m_x or μ

$$m_x \text{ or } \mu = E[X]$$

where $E[\ ]$ represents the expectation operator

Mean value of a discrete random variable

$$m_x = \mu = E(X) = \sum_{i=1}^{n} x_i \, P(x_i)$$

x_i → possible values of random variable X

$P(x_i)$ → probabilities of x_i

$[\because \text{mean} = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + \cdots x_n P(x_n)$

Mean value of a Continuous Random Variable

$$\mu \text{ or } m_x = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

$f(x)$ → Pdf

Moments

n^{th} moment of any random variable X may be defined as the mean value of X^n

i.e. $E[X^n]$

In case of ~~discrete~~ continuous random variable

$$E[X^n] = \int_{-\infty}^{\infty} x^n \underbrace{f_X(x)}_{\text{PDF}} \, dx$$

In case of Discrete random variable

$$E[X^n] = \sum_{i=1}^{\infty} x_i^n \underbrace{P(x_i)}_{\text{probabilities}}$$

if $n=1$,

$$E[X] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

$\therefore$ 1^{st} moment of random variable X will be same as its mean value

if $n=2$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \, f_X(x) \, dx$$

$E[X^2]$ is known as the mean square value of random variable X.

Similarly the central moments are the moments of the difference between random variable X and its mean m_x.

Therefore the n^{th} central moment may be given as

$$E\left[(X - m_x)^n\right] = \int_{-\infty}^{\infty} (x - m_x)^n f_X(x)\, dx$$

The central moment for $n = 2$ is known as variance of random variable X

Variance:- Variance provides an indication about the randomness of the random variable.

$$E\left[(X - m_x)^2\right]$$

variance can be denoted as σ^2

$$\sigma^2 = Var[X]$$

$$\sigma^2 = E(X^2) - m_x^2$$

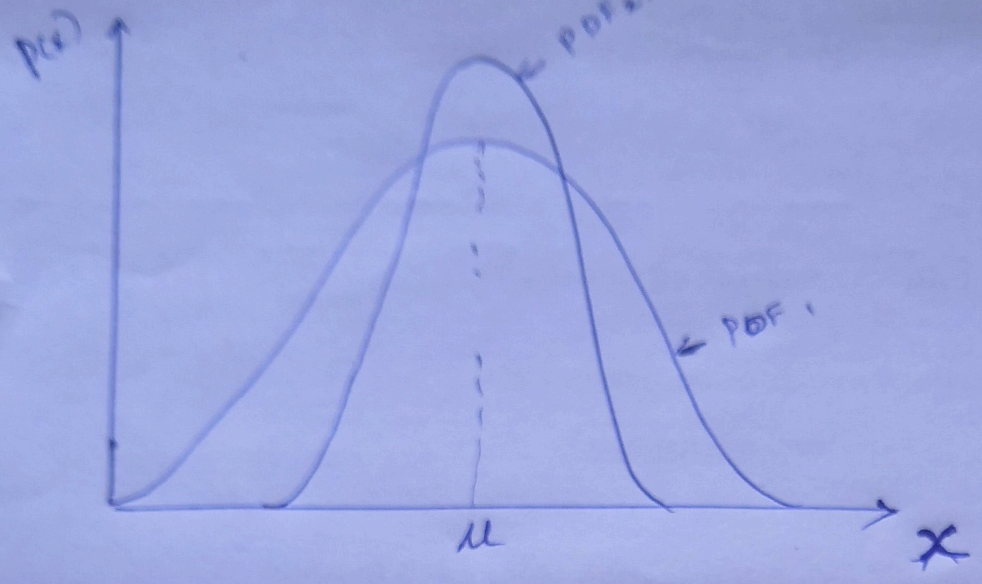
For discrete random variable $\sigma^2 = \sum P_{(x)} x^2 - m_x^2$

↳ possible values of X

└ Probabilities of x

For continuous random variable $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - m_x^2$

$$\boxed{\begin{aligned} \mu \text{ or } m_x &= \int_{-\infty}^{\infty} x\, f(x)\, dx \\ \sigma^2 &= \int_{-\infty}^{\infty} x^2 f(x)\, dx - m_x^2 \end{aligned}}$$



PDF1 has broader no: of random variables

So we can say that PDF1 has high variance

PDF2 has less no: of random variables

So PDF2 has low variance

Standard deviation $= \sqrt{\text{variance}}$

$= \sqrt{\sigma_x^2} = \sigma_x$

Thus the standard deviation $= \sigma_x = \sqrt{E(x^2) - m_x^2}$

*. The random variable X has the probability density function given by

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2) & 0 < x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find the mean and variance of X.

$$\text{Mean} = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^2 x \times \frac{3}{10}(3x - x^2)\, dx$$

$$= \int_0^2 \frac{9}{10}x^2 - \frac{3}{10}x^3\, dx$$

$$= \left[\frac{3}{10}x^3 - \frac{3}{40}x^4\right]_0^2$$

$$= \left(\frac{3}{10} \times 2^3 - \frac{3}{40} \times 2^4\right) - 0$$

$$= \frac{6}{5}$$

Variance $= E(X^2) - m_x^2$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_0^2 x^2 \times \frac{3}{10}(3x - x^2)\, dx$$

$$= \int_0^2 \frac{9}{10}x^3 - \frac{3}{10}x^4\, dx$$

$$= \left[\frac{9}{40}x^4 - \frac{3}{50}x^5\right]_0^2$$

$$= \left(\frac{9}{40} \times 2^4 - \frac{3}{50} \times 2^5\right) - 0$$

$$= \underline{\underline{\frac{42}{25}}}$$

Variance $\sigma^2 = E(X^2) - \mu^2$

$$= \frac{42}{25} - \left(\frac{6}{5}\right)^2$$

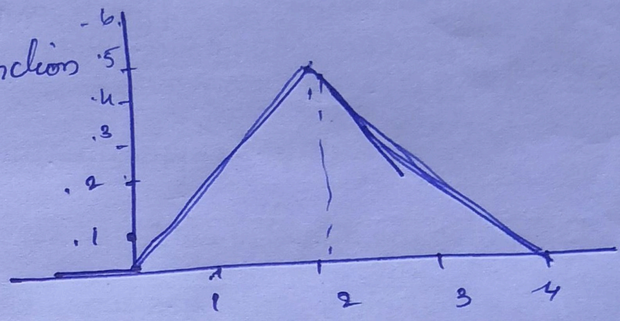
$$= \underline{\underline{\frac{6}{25}}}.$$

2. The probability density function for the random variable X is given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \le x \le 2 \\ \frac{1}{4}(4-x) & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and standard deviation of X.

Because it is a symmetrical function the mean must be 2 from the graph.



$m_x = \underline{\underline{2}}$.

Variance $= E(X^2) - m_x^2$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_0^2 x^2 \times \frac{1}{4}x\,dx + \int_2^4 x^2 \times \frac{1}{4}(4-x)\,dx$$

$$= \int_0^2 \frac{1}{4}x^3\,dx + \int_2^4 x^2 - \frac{1}{4}x^3\,dx$$

$$= \left[\frac{1}{16}x^4\right]_0^2 + \left[\frac{1}{3}x^3 - \frac{1}{16}x^4\right]_2^4$$

$$= \frac{1}{16}\times 2^4 - 0 + \left(\frac{1}{3}\times 4^3 - \frac{1}{16}\times 4^4\right) - \left(\frac{1}{3}\times 2^3 - \frac{1}{16}\times 2^4\right)$$

$$= 1 + \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 1\right)$$

$$= 1 + \frac{11}{3} = \frac{14}{3}$$

$$\sigma^2 = E(X^2) - \mu^2 = \frac{14}{3} - 2^2 = \underline{\underline{\frac{2}{3}}}$$

Standard deviation $= \sqrt{\sigma^2}$

$$= \sqrt{\frac{2}{3}} = \underline{\underline{.816}}$$