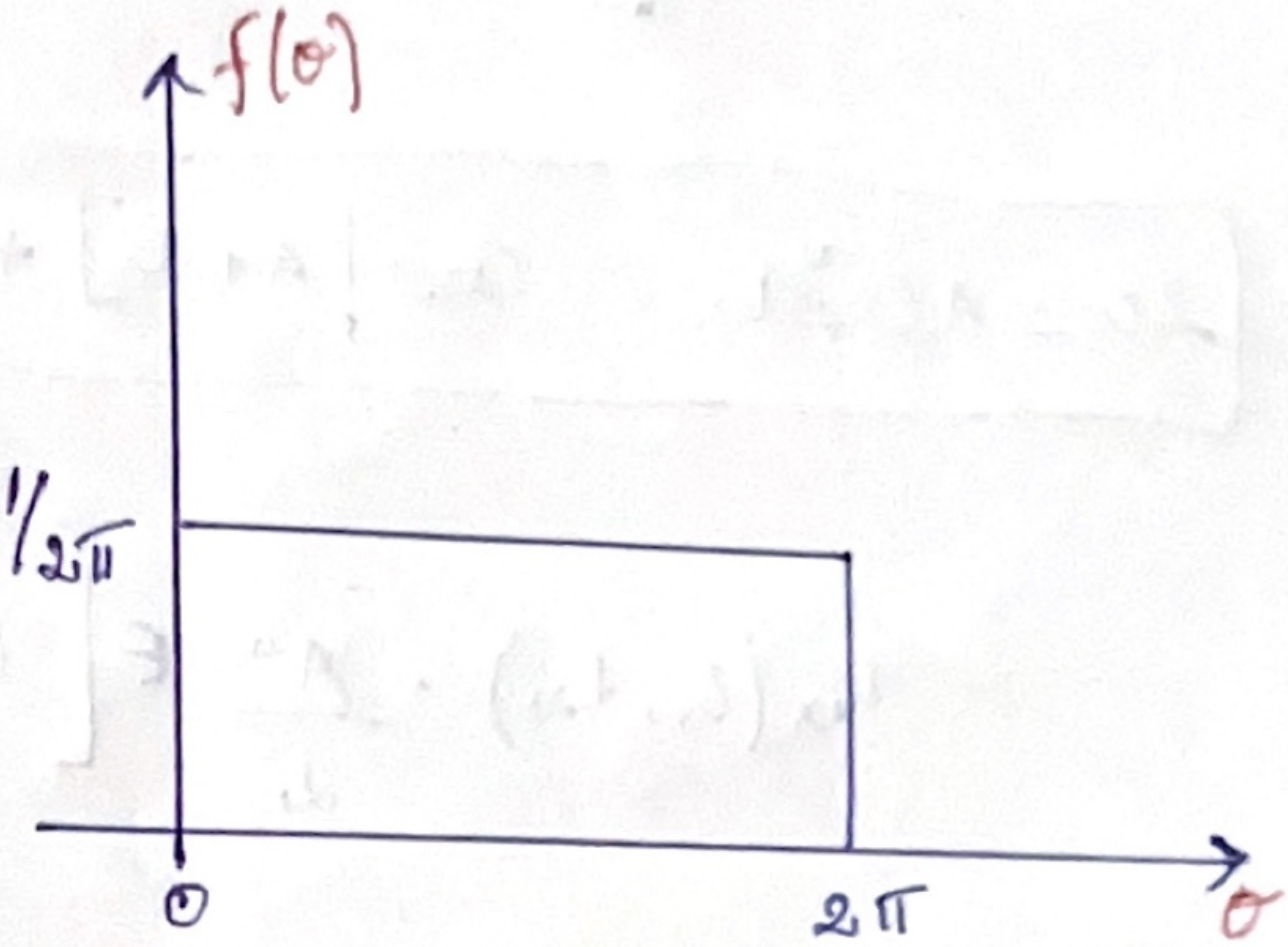
Q) Consider a random process $X(t) = A\cos(\omega_0 t + \theta)$ where A and ω_0 are constants and θ is a random variable uniformly distributed in the interval $0 \to 2\pi$. Determine the mean and autocorrelation function.

Ans) $x(t) = A\cos(\omega_0 t + \theta)$

here θ is varying



(1) Mean = $E[x]$

$$= E[A\cos(\omega_0 t + \theta)]$$

$$= \int_0^{2\pi} A\cos(\omega_0 t + \theta) \cdot f(\theta) \cdot d\theta$$

$$= \int_0^{2\pi} A \cdot \cos(\omega_0 t + \theta) \times 1/2\pi \cdot d\theta$$

$$= A/2\pi \int_0^{2\pi} \cos(\omega_0 t + \theta) \cdot d\theta$$

$$= A/2\pi \times 0$$

$$= \underline{\underline{0}}$$

Autocorrelation function (ACF)

$$R_X(t_1,t_2) = E[x(t_1) . x(t_2)]$$

$$= E\left[A\cos(\omega_0 \overset{B}{t_1} + \theta) . A\cos(\omega_0 \overset{A}{t_2} + \theta)\right]$$

$$\boxed{2\cos A \cos B = \cos[A+B] + \cos[A-B]}$$

$$\therefore R_X(t_1,t_2) = \frac{A^2}{2} E\left[\cos(\omega_0 t_2 + \omega_0 t_1 + 2\theta) + \cos(\omega_0(t_2 - t_1))\right]$$

$$= \frac{A^2}{2} E\left[\cos(\omega_0 t_2 + \omega_0 t_1 + 2\theta)\right] + \frac{A^2}{2} E\left[\cos(\omega_0(t_2 - t_1))\right]$$

$$= \frac{A^2}{2} \int_0^{2\pi} \cancelto{0}{\cos(\omega_0 t_2 + \omega_0 t_1 + 2\theta) \times f(\theta)\, d\theta} + \frac{A^2}{2} . \cos \omega_0 (t_2 - t_1)$$

$$R_X(t_1,t_2) = \frac{A^2}{2} \cos \omega_0 (t_2 - t_1)$$

$$R_X(\tau) = \frac{A^2}{2} \cos \omega_0 \tau$$

Q) Consider the random process $x(t) = A\cos\omega_0 t + B\sin\omega_0 t$. ω_0 is a constant, A and B are independent RVs, both having zero mean and same variance. Determine the mean and autocorrelation function.

Ans) $x(t) = A\cos\omega_0 t + B\sin\omega_0 t$

given $E[A] = 0$ $\qquad$ Variance $= \sigma^2 = E[A^2] - [E[A]]^2$

$E[B] = 0$ $\qquad$ $\sigma^2 = E[A^2] - 0$

$\therefore E[A] = \sigma^2$

Similarly $E[B] = \sigma^2$.

mean $\rightarrow$ $E[x(t)] = E[A\cos\omega_0 t + B\sin\omega_0 t]$

$= E[A]\cos\omega_0 t + E[B]\sin\omega_0 t$

$= 0$ //

ACF $\rightarrow$ $E[x(t_1) \cdot x(t_2)]$

$= E[(A\cos\omega_0 t_1 + B\sin\omega_0 t_1)(A\cos\omega_0 t_2 + B\sin\omega_0 t_2)]$

$= E[A^2\cos\omega_0 t_1 \cdot \cos\omega_0 t_2 + \cancelto{0}{AB\cos\omega_0 t_1\sin\omega_0 t_2} +$

$\cancelto{0}{AB\sin\omega_0 t_1\cos\omega_0 t_2} + B^2\sin\omega_0 t_1\sin\omega_0 t_2]$

$E[AB] = E[A] \cdot E[B]$

$= 0$.

$= \underbrace{E[A^2]}_{\sigma^2}\cos\omega_0 t_1\cos\omega_0 t_2 + \underbrace{E[B^2]}_{\sigma^2}\sin\omega_0 t_1\sin\omega_0 t_2$

$= \sigma^2[\cos\omega_0 t_1\cos\omega_0 t_2 + \sin\omega_0 t_1\sin\omega_0 t_2]$

$= \sigma^2\cos\omega_0(t_2 - t_1)$

$= \sigma^2\cos\omega_0\tau$ //