Module 5

Prepared by :
Asst. Prof. POOJA P P
ECE Department
GCEK

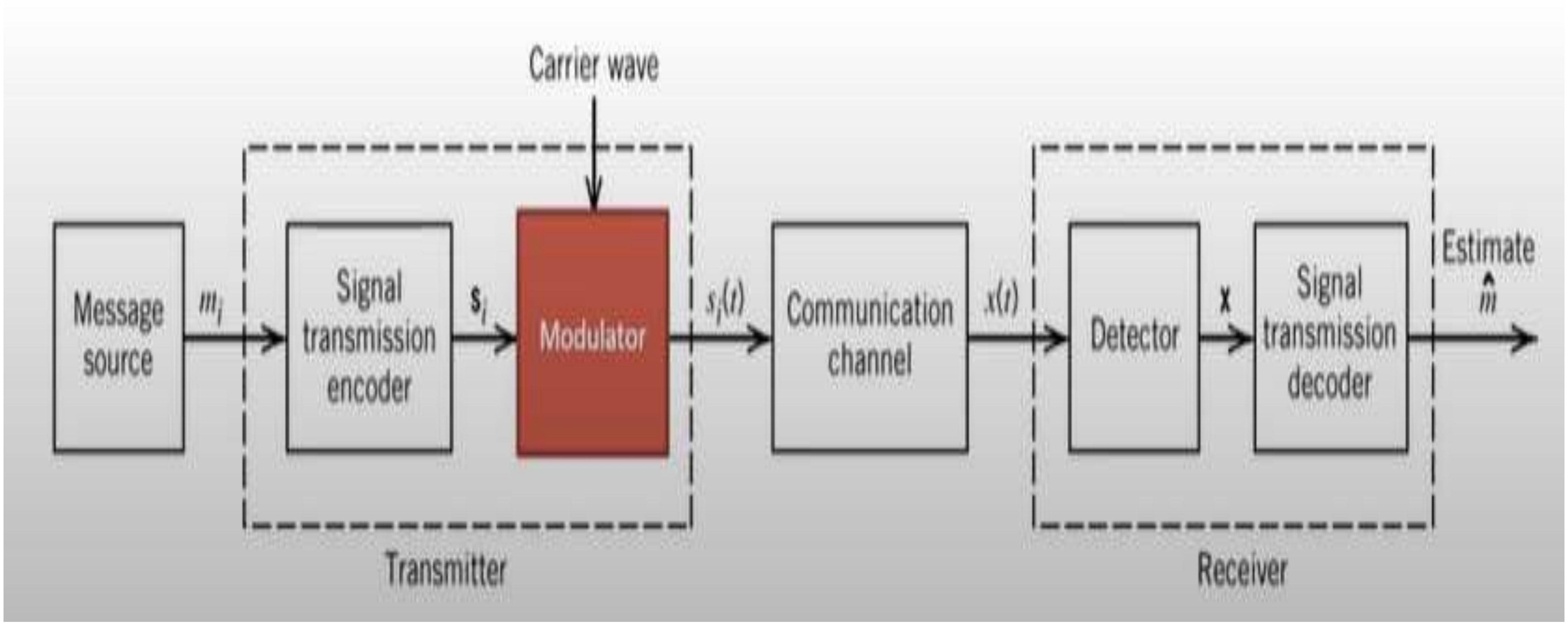
Syllabus

- Digital modulation schemes. Baseband BPSK system and the signal constellation. BPSK transmitter and receiver.

- Baseband QPSK system and Signal constellations. Plots of BER Vs SNR with analysis. QPSK transmitter and receiver.

- Quadrature amplitude modulation(QAM) and signal constellation.

Block diagram of Digital Communication System



Pass Band(Band Pass) Transmission

In digital pass band transmission, the incoming data stream is modulated onto a carrier with fixed frequency and then transmitted over a band-pass channel.

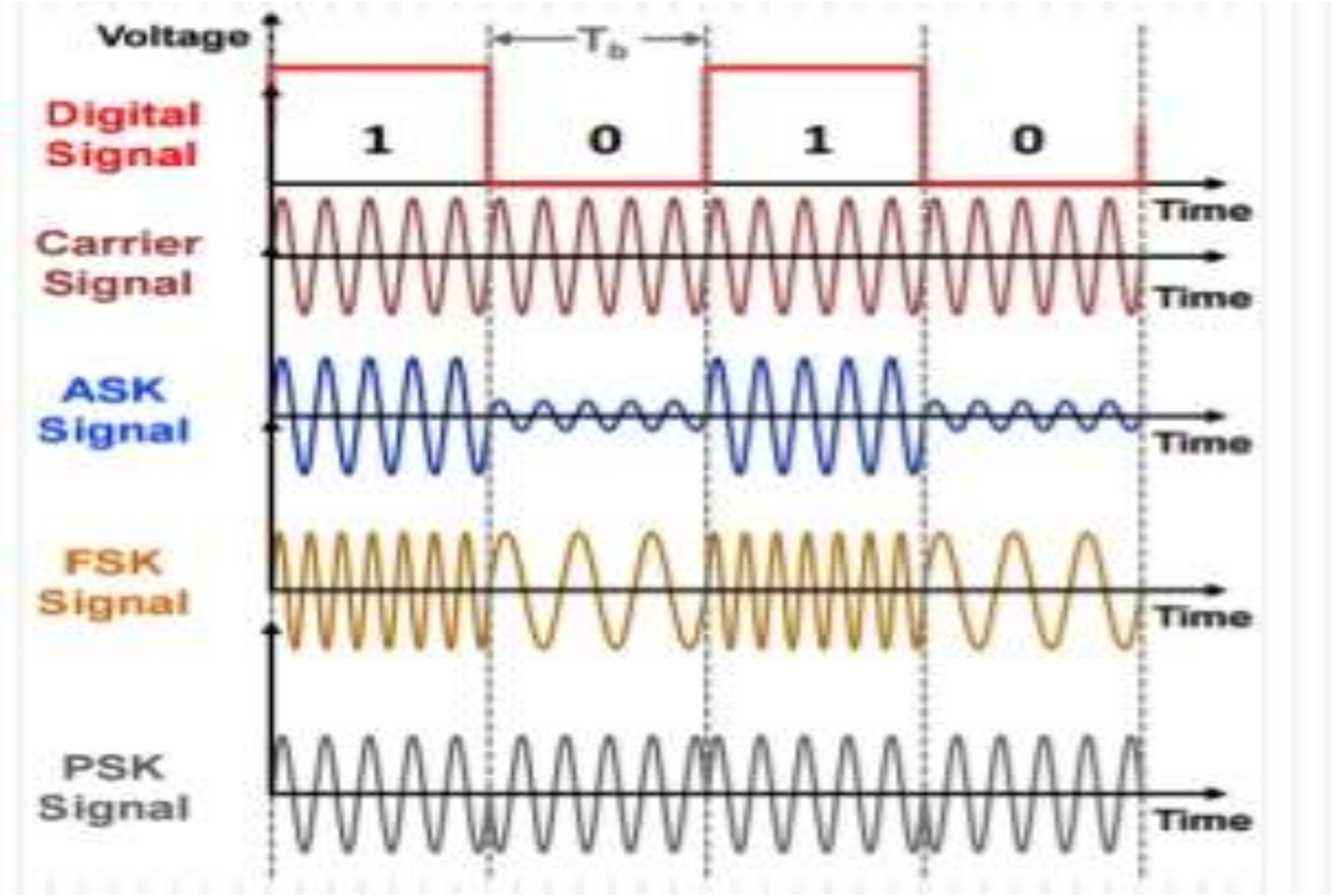
Coherent Digital Modulation Techniques

- It employs coherent detection.
- In coherent detection, the local carrier generated at the receiver is phase locked with the carrier at the transmitter.
- The coherent detection is a synchronous detection.
- Because of synchronization used at receiver the system becomes more complex.

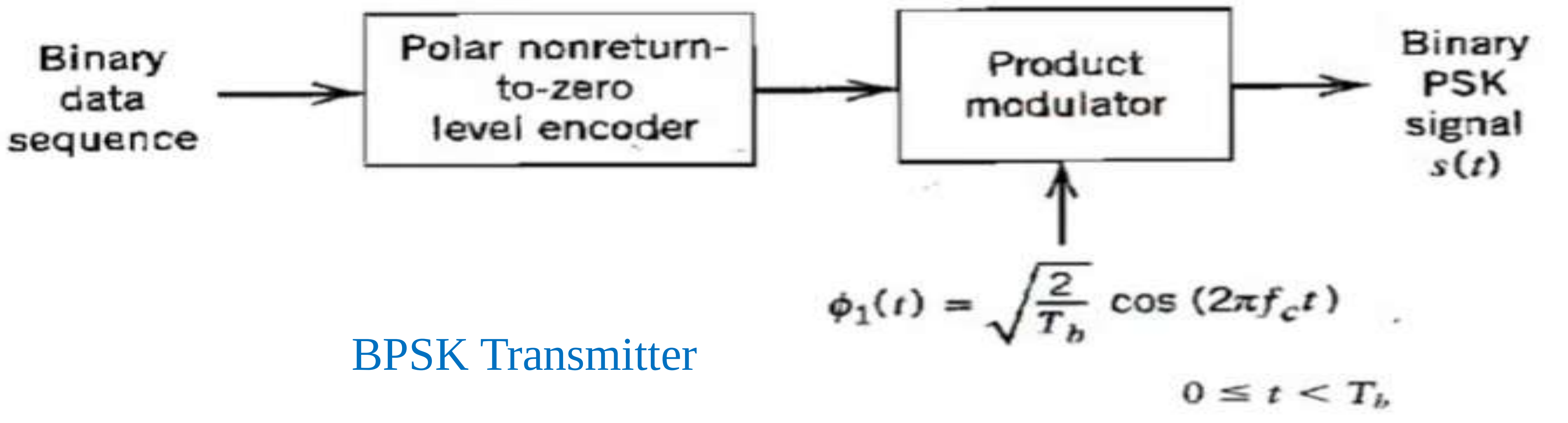
Non Coherent Digital Modulation Techniques

- The detection process does not need receiver carrier to be phase locked with the transmitter carrier.
- The system becomes simple.
- Error probability is high.

Digital modulation schemes.

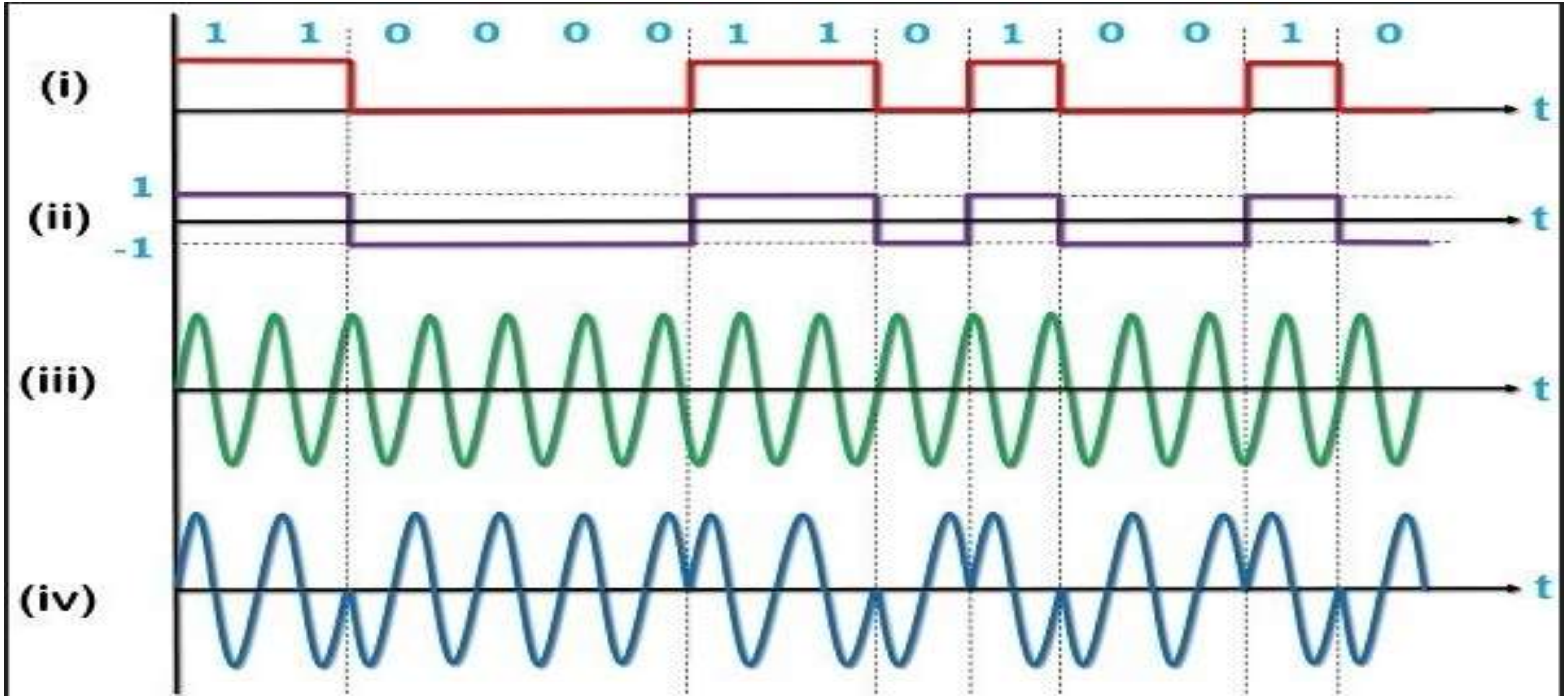


Coherent Binary Phase Shift Keying(BPSK)



BPSK Transmitter

- The BPSK transmitter performs two tasks; encoding the message bits and modulating them using BPSK modulation.
- It typically uses Non-Return-to-Zero (NRZ) coding, where a positive voltage represents a 1 and a negative voltage represents a 0.
- The message bits are then multiplied by a reference oscillator operating at the carrier frequency.



(i) Binary data sequence
(ii) Bipolar NRZ sequence
(iii) Carrier wave
(iv) BPSK waveform

- Binary Phase Shift Keying (BPSK) is a modulation technique employed in communication systems to transmit information via a communication channel.

- In BPSK the carrier signal is modified by altering its phase by 180 degrees, for each symbol.

- BPSK holds significance in communication systems like Wi-Fi, Bluetooth and Satellite Communication.

- In modulation techniques, we select a group of functions to represent the modulation scheme. These functions are usually orthogonal, to each other.

- In BPSK the modulation process involves using a Sinusoid as the Basis Function.

- By adjusting the phase of this sinusoid based on the message bits we can achieve modulation. When transmitting a 1 there is no phase shift, in the carrier signal. However, when transmitting a 0 there is a phase shift of 180 degrees in the carrier signal.

- Symbol 1 -- Transmit the carrier as it is
- Symbol 0 – Transmit the carrier by phase advancing it by 180^0

$s_1(t) = A_C Cos(2\pi f_c t)$ - for symbol '1'

$s_2(t) = A_C Cos(2\pi f_c t + \pi)$ - for symbol '0'

$s_2(t) = - A_C Cos(2\pi f_c t)$ - for symbol '0'

$0 \leq t \leq T_b$

T_b –*Bit Duration*

- Energy $E_b = \frac{A_c{}^2}{2} T_b$, therefore $A_C = \sqrt{\frac{2E_b}{T_b}}$

- Orthonormal basis function $\phi_1(t) = \sqrt{\frac{2}{T_b}} Cos(2\pi f_c t)$

Energy of signal of $s(t)$

T_b - bit duration

$$E_b = \int_0^{T_b} (A_c \cos 2\pi f_c t)^2 . dt$$

$$= A_c{}^2 \int_0^{T_b} \cos^2 2\pi f_c t . dt$$

$$= A_c{}^2 \int_0^{T_b} \left(\frac{1 + \cos 4\pi f_c t}{2}\right) . dt$$

$$= \frac{A_c{}^2}{2} \int_0^{T_b} 1\, dt + \frac{A_c{}^2}{2} \cancel{\int_0^{T_b} \cos 4\pi f_c t . dt}^{\,0}$$

Integer multiple of 2π

$$E_b = \frac{A_c{}^2 T_b}{2}$$

$$\therefore A_c = \sqrt{\frac{2E_b}{T_b}}$$

Only one orthonormal basis function

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}} = \frac{A_c \cos 2\pi f_c t}{\sqrt{A_c^2/2 . T_b}}$$

$$\phi_1(t) = \sqrt{2/T_b} \cos 2\pi f_c t$$

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \text{for symbol '1'}$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \text{for symbol '0'}$$

Where E_b= Average energy transmitted per bit

$$s_1(t) = s_{11}\phi_1(t) \qquad 0 \leq t \leq T_b$$

$$s_2(t) = s_{21}\phi_1(t) \qquad 0 \leq t \leq T_b$$

Therefore the transmitted signals are given by

$$S_1(t) = \sqrt{E_b}\varphi_1(t) \quad 0 \le t \le T_b$$

for Symbol '1'

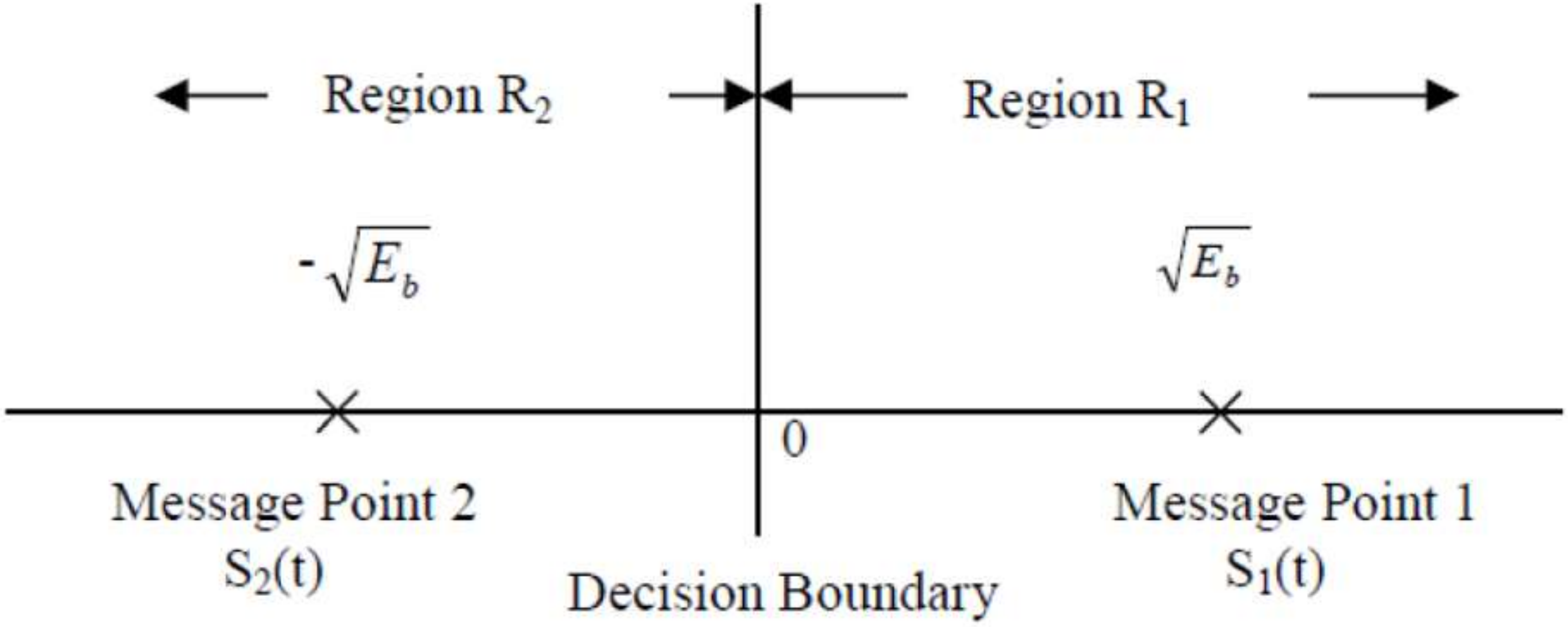
$$S_2(t) = -\sqrt{E_b}\varphi_1(t) \quad 0 \le t \le T_b$$

for Symbol '0'

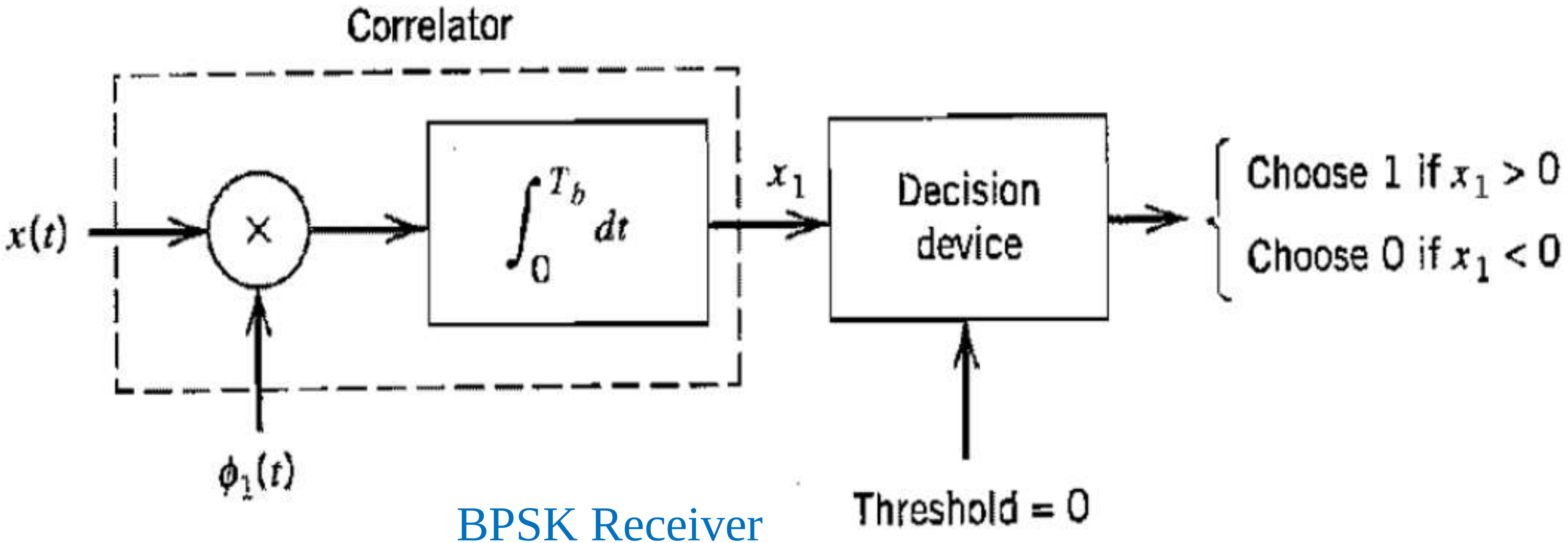
$$\begin{aligned} s_{11} &= \int_0^{T_b} s_1(t)\phi_1(t)dt \\ &= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)dt \\ &= \int_0^{T_b} \sqrt{E_b}\left(\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)\right)^2 dt \\ &= \sqrt{E_b}\int_0^{T_b} \phi_1^2(t)dt \\ &= +\sqrt{E_b} \end{aligned}$$

$$\begin{aligned} s_{21} &= \int_0^{T_b} s_2(t)\phi_1(t)dt \\ &= -\int_0^{T_b} \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)dt \\ &= -\int_0^{T_b} \sqrt{E_b}\left(\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)\right)^2 dt \\ &= -\sqrt{E_b}\int_0^{T_b} \phi_1^2(t)dt \\ &= -\sqrt{E_b} \end{aligned}$$

Signal Space Constellation of BPSK



- The constellation diagram, for BPSK, displays two constellation points positioned along the x in phase). There are no points projected onto the y-axis (quadrature) as BPSK relies on one basis function.
- The phase of the carrier wave carries all the information being transmitted.



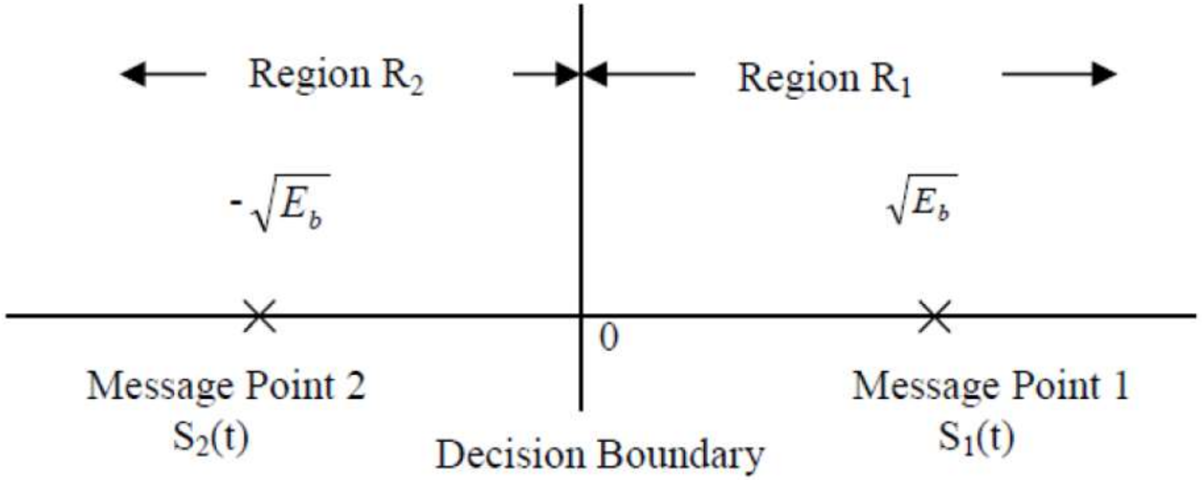
BPSK Receiver

- In a receiver, the received signal is multiplied by a reference frequency signal obtained from carrier recovery blocks.
- The resulting output is then integrated over a one-bit period using an integrator.
- A decision on each integrated bit is made by a threshold detector based on a threshold value. Since the transmitter uses NRZ signaling format the detector threshold is set to 0.

- The observation vector x_1 is related to the received signal x(t) by

$$x_1 = \int_0^T x(t)\phi_1(t)\, \mathrm{d}t$$

- If the observation element falls in the region R_1, a decision will be made in favor of symbol ‘1’. If it falls in region R_2 a decision will be made in favor of symbol ‘0’.



Signal Space Constellation of BPSK

Probability of Error or Bit Error Calculation

The error is of two types:

1) $P_e(0/1)$ ie.transmitted as '1' but received as '0'

2) $P_e(1/0)$ ie.transmitted as '0' but received as '1'

- Assume transmitted signal is affected by **AWGN noise** with power spectral density $N_0/2$, mean $\mu_w = 0$ and variance $\sigma^2 = N_0/2$.

- Received signal $x(t) = s_1(t) + w(t) = \sqrt{E_b}\phi_1(t) + w(t)$ ------- For symbol '1'

$= s_2(t) + w(t) = -\sqrt{E_b}\phi_1(t) + w(t)$ ------- For symbol '0'

Lets say,

Transmitted symbol is ‘1’

$$x_1 = \int_0^{T_b} [s_1(t) + \mathrm{w(t)}]\, \phi_1(t)\, \mathbf{dt}$$

$$= \int_0^{T_b} [\sqrt{E_b}\phi_1(t) + \mathrm{w(t)}]\, \phi_1(t)\, \mathbf{dt}$$

$$x_1 = \sqrt{E_b} + \mathbf{w}$$, where w is a random value

$$x_1 = \int_0^{T} x(t)\phi_1(t)\, \mathrm{d}t$$

$\int_0^{T_b} \phi_1(t) * \phi_1(t)$ **dt = 1**

Let : $\int_0^{T_b}$ w(t)] $\phi_1(t)$ **dt = w**

Transmitted symbol is ‘0’

$$x_1 = \int_0^{T_b} [s_2(t) + \mathrm{w(t)}]\, \phi_1(t)\, \mathbf{dt}$$

$$= \int_0^{T_b} [-\sqrt{E_b}\phi_1(t) + \mathrm{w(t)}]\, \phi_1(t)\, \mathbf{dt}$$

$$x_1 = -\sqrt{E_b} + \mathbf{w}$$, where w is a random value

Observation $x_1 = \sqrt{E_b} + \mathbf{w}$ ----- **for symbol '1'**

$x_1 = -\sqrt{E_b} + \mathbf{w}$ ----- **for symbol '0'**

AWGN noise
mean $\mu_w = 0$
variance $\sigma^2 = N_0/2$.

Mean of x_1 : $\mathbf{E}[\sqrt{E_b} + \mathbf{w}] = \mathbf{E}[\sqrt{E_b}] + \mathbf{E}[\mathbf{w}]$

$= \sqrt{E_b} + 0$

$\mathbf{E}[x_1] = \sqrt{E_b}$ ------ **for symbol '1'**

Similarly $\mathbf{E}[x_1] = -\sqrt{E_b}$ ------ **for symbol '0'**

Variance of x_1: $E[\boldsymbol{x_1} - \bar{x}_1]^2$

For bit '1' $= E[\boldsymbol{x_1} - \bar{x}_1]^2 = \mathbf{E}[\sqrt{E_b} + \mathbf{w} - \sqrt{E_b}]^2 = \mathbf{E}[w^2] = N_0/2$

For bit 'O' $= E[\boldsymbol{x_1} - \bar{x}_1]^2 = \mathbf{E}[-\sqrt{E_b} + \mathbf{w} - (-\sqrt{E_b})]^2 = \mathbf{E}[w^2] = N_0/2$

We know the pdf of Gaussian RV is

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

- x is the variable
- μ is the mean
- σ is the standard deviation

- The conditional probability density function of RV X_1, given that symbol '0' (ie. Signal $S_2(t)$) was transmitted is defined by

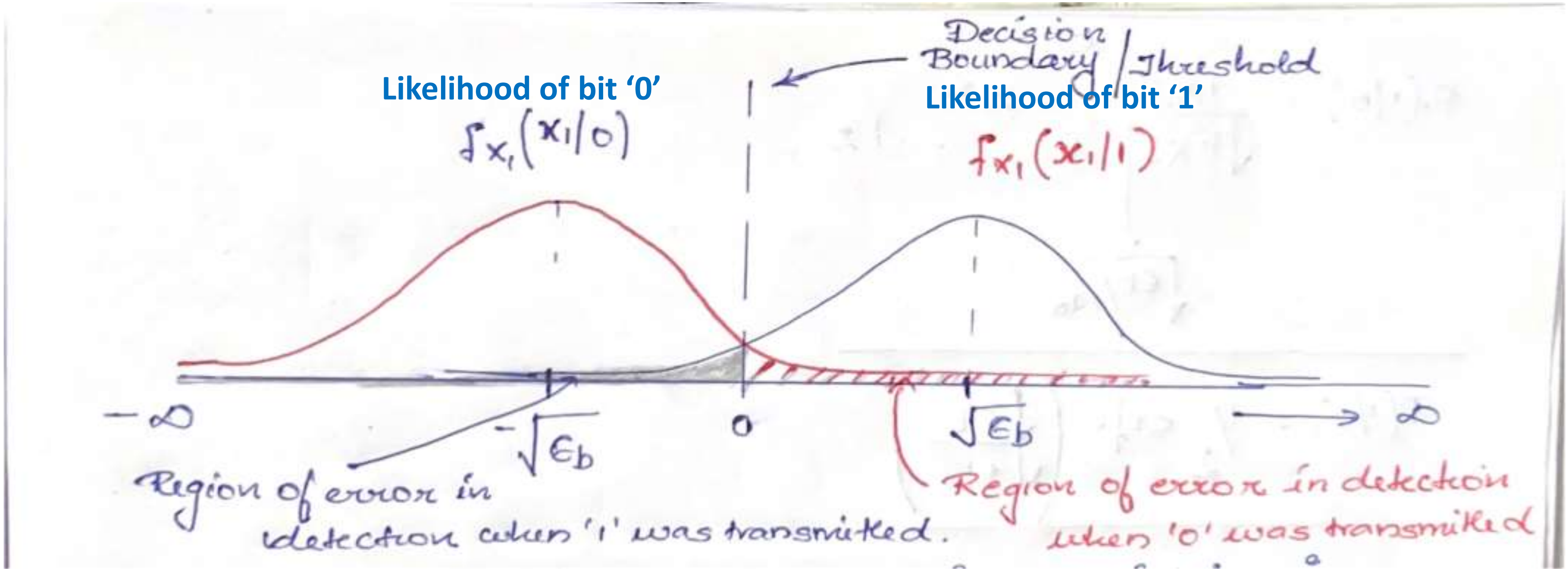
$$f_{X_1}(x_1/0) = \frac{1}{\sqrt{2\pi \times var(x_1)}} \exp\left[\frac{-1}{2 \times var(x_1)}(x_1 - m_{x_1})^2\right]$$

$$= \frac{1}{\sqrt{2\pi N_0/2}} \exp\left[\frac{-1}{2 \times N_0/2}(x_1 - (-\sqrt{E_b}))^2\right]$$

- $f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \mathrm{e}^{\frac{-(x_1+\sqrt{E_b})^2}{N_0}}$

- The conditional probability density function of RV X_1 , given that symbol ‘1’ (ie. Signal $S_1(t)$) was transmitted is defined by

- $$f_{x_1}(x_1|1) = \frac{1}{\sqrt{\pi N_0}} e^{\frac{-(x_1-\sqrt{E_b})^2}{N_0}}$$



Calculation of Probability of Error when ‘0’ was transmitted:

$$P_{e0} = P_e(1/0) = \int_0^\infty f_{x_1}(1/0)\, d_{x1}$$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

- Let,

$$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$$

$$\frac{dz}{dx_1} = \frac{d}{dx_1}\left[\frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})\right]$$

$$\frac{dz}{dx_1} = \frac{1}{\sqrt{N_0}}$$

$$dx_1 = \sqrt{N_0}\, dz$$

- when $x_1 = \infty \rightarrow z = \infty$.
- when $x_1 = 0 \rightarrow z = \sqrt{E_b/N_0}$.

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)dz\sqrt{N_0}$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)dz$$

By definition, the complementary error function equals;

$$erfc(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} \exp(-z^2)dz$$

By comparing, we get $$P_e(1/0) = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$

Similarily Calculation of Probability of Error when '1' was transmitted: $$P_e(0/1) = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$

So the **total probability of error** is given by

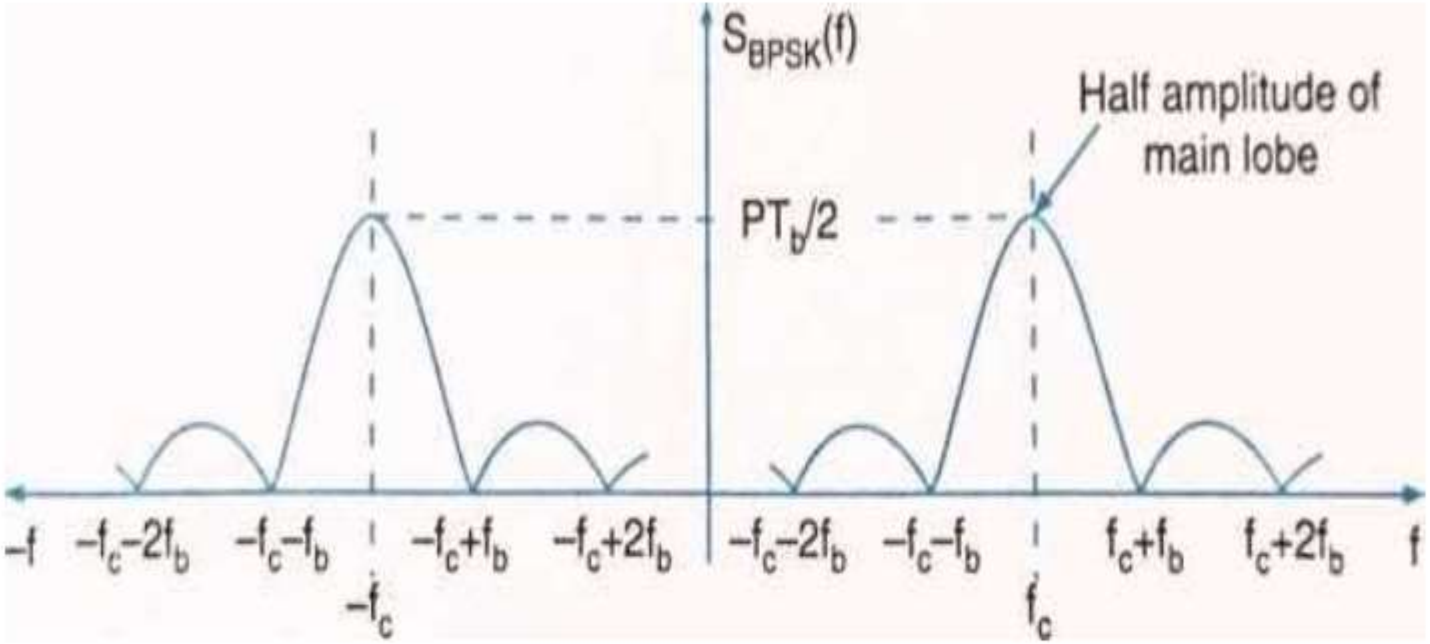
$$P_e = P_e(1/0)P_e(0) + P_e(0/1)P_e(1)$$

$$P_e(0) = P_e(1) = \frac{1}{2}$$

$$P_e = \frac{1}{2}\left[P_e\left(\frac{1}{0}\right) + P_e\left(\frac{0}{1}\right)\right]$$

$$P_e = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$

Power Spectral Density of BPSK



Bandwidth for BPSK Signal

BW = Highest frequency - Lowest frequency in the main lobe

BW = $f_c + f_b - (f_c - f_b)$ or **$BW = 2f_b$**

Advantages of BPSK

Simplicity: BPSK, which stands for Binary Phase Shift Keying is a modulation scheme. It simplifies implementation, in hardware and software by utilizing two phase states; 0 degrees and 180 degrees.

Effective Operation with Reliability: It has ability to operate effectively in the presence of noise or interference from signals ensuring reliable performance.

Less Power Consumption: BPSK consumes power compared to alternative methods making it advantageous for battery powered devices.

Easy Detection: Receivers find it easy to comprehend BPSK accurately determining the frequency and phase of the transmitted signal.

Compatible: BPSK serves as a building block for complex modulation schemes like QPSK (Quadrature Phase Shift Keying) and higher order Quadrature Amplitude Modulation (QAM).

Disadvantages of BPSK

Low Data Sending Rate: However when it comes to data transmission speed, BPSK has limitations as it can only send one piece of data at a time.

Less Efficient: It inefficiently uses the signal space, same like using a whole road for just one small car.

Limited Error Correction: BPSK does not provide as much inherent error correction capability as more complex modulation schemes, therefore, error correction needs to be added separately, which can increase system complexity.

Not for Huge Data: If you need to send a lot of data fast then BPSK might not be the best choice.