SEMESTER -4

MATHEMATICS – 4 th semester

(All branches except Electrical, Electronics, Computer science, Information Technology and Applied Electronics)

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT 202	PROBABILITY,STATISTICS AND NUMERICAL METHODS	BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduces students to the modern theory of probability and statistics, covering important models of random variables and techniques of parameter estimation and hypothesis testing. A brief course in numerical methods familiarises students with some basic numerical techniques for finding roots of equations, evaluationg definite integrals solving systems of linear equations, and solving ordinary differential equations which are especially useful when analytical solutions are hard to find.

Prerequisite: A basic course in one-variable and multi-variable calculus.

Course Outcomes: After the completion of the course the student will be able to

CO 1	Understand the concept, properties and important models of discrete random variables and,using them, analyse suitable random phenomena.
CO 2	Understand the concept, properties and important models of continuous random variables and,using them, analyse suitable random phenomena.
CO 3	Perform statistical inferences concerning characteristics of a population based on attributes of samples drawn from the population
CO 4	Compute roots of equations, evaluate definite integrals and perform interpolation on given numerical data using standard numerical techniques
CO 5	Apply standard numerical techniques for solving systems of equations, fitting curves on given numerical data and solving ordinary differential equations.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2	2	2	2					2		1
CO 2	3	2	2	2	2					2		1
CO 3	3	2	2	2	2					2		1
CO 4	3	2	2	2	2					2		1
CO 5	3	2	2	2	2					2		1

Assessment Pattern

Bloom's Category	Continuous Assessment Tests(%)		End Semester Examination(%)
	1	2	
Remember	10	10	10
Understand	30	30	30
Apply	30	30	30
Analyse	20	20	20
Evaluate	10	10	10
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Let X denote the number that shows up when an unfair die is tossed. Faces 1 to 5 of the die are equally likely, while face 6 is twice as likely as any other. Find the probability distribution, mean and variance of X.

2. An equipment consists of 5 componets each of which may fail independently with probability 0.15. If the equipment is able to function properly when at least 3 of the componets are operational, what is the probability that it functions properly?

3. X is a binomial random variable $B(n,p)$with $n = 100$and $p = 0.1$. How would you approximate it by a Poisson random variable?

4. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y)

Course Outcome 2 (CO2)

1. What can you say about $P(X = a)$for any real number $a$when $X$is a (i) discrete random variable? (ii) continuous random variable?

2. A string, 1 meter long, is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twise the length of the other?

3. A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3?

4. X and Y are independent random variables with X following an exponential distribution with parameter μ and Y following and exponential distribution with parameter λ. Find $P(X+Y \leqslant 1)$

Course Outcome 3(CO3):

1. In a random sample of 500 people selected from the population of a city 60 were found to be left-handed. Find a 95% confidence interval for the proportion of left-handed people in the city population.

2. What are the types of errors involved in statistical hypothesis testing. Explain the level of risks associated with each type of error.

3. A soft drink maker claims that a majority of adults prefer its leading beverage over that of its main competitor's. To test this claim 500 randomly selected people were given the two beverages in random order to taste. Among them, 270 preferred the soft drink maker's brand, 211 preferred the competitor's brand, and 19 could not make up their minds. Determine whether there is sufficient evidence, at the 5% level of significance, to support the soft drink maker's claim against the default that the population is evenly split in its preference.

4. A nutritionist is interested in whether two proposed diets, *diet A* and *diet B* work equally well in providing weight-loss for customers. In order to assess a difference between the two diets, she puts 50 customers on diet A and 60 other customers on diet B for two weeks. Those on the former had weight losses with an average of 11 pounds and a standard deviation of 3 pounds, while those on the latter lost an average of 8 pounds with a standard deviation of 2 pounds. Do the diets differ in terms of their weight loss?

Course Outcome 4(CO4):

1. Use Newton-Raphson method to find a real root of the equation $f(x) = e^{2x} - x - 6$correct to 4 decimal places.

2. Compare Newton's divided difference method and Langrange's method of interpolation.

3. Use Newton's forward interpolation formula to compute the approximate values of the function f at $x = 0.25$ from the following table of values of x and $f(x)$

x	0	0.5	1	1.5	2
f(x)	1.0000	1.0513	1.1052	1.1618	1.2214

4. Find a polynomial of degree 3 or less the graph of which passes thorugh the points (-1,3), (0,-4), (1,5) and (2,-6)

Course Outcome 5 (CO5):

1. Apply Gauss-Seidel method to solve the following system of equations

$$4x_1 - x_2 - x_3 = 3$$
$$-2x_1 + 6x_2 + x_3 = 9$$
$$-x_1 + x_2 + 7x_3 = -6$$

2. Using the method of least squares fit a straight line of the form $y = ax + b$ to the following set of ordered pairs (x, y) :
(2,4), (3,5), (5,7), (7,10), (9,15)

3. Write the normal equations for fitting a curve of the form $y = a_0 + a_1x^2$ to a given set of pairs of data points.

4. Use Runge-Kutta method of fourth order to compute $y(0.25)$ and $y(0.5)$, given the initial value problem

$$y' = x + xy + y, y(0) = 1$$

Syllabus

Module 1 (Discrete probability distributions) **9 hours**

(Text-1: *Relevant topics* from sections-3.1-3.4, 3.6, 5.1)

Discrete random variables and their probability distributions, Expectation, mean and variance, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Discrete bivariate distributions, marginal distributions, Independent random variables, Expectation -multiple random variables.

Module 2 (Continuous probability distributions) **9 hours**

(Text-1:*Relevant topics* from sections-4.1-4.4, 3.6, 5.1)

Continuous random variables and their probability distributions, Expectation, mean and variance, Uniform, exponential and normal distributions, Continuous bivariate distributions, marginal distributions, Independent random variables, Expectation-multiple random variables, i.i.d random variables and Central limit theorem (**without proof**).

Module 3 (Statistical inference) 9 hours

(Text-1:*Relevant topics* from sections-5.4,, 3.6, 5.1,7.2, 8.1, 8.3, 9.1-9.2,9.4)

Population and samples, Sampling distribution of the mean and proportion (for large samples only), Confidence interval for single mean and single proportions(for large samples only). Test of hypotheses: Large sample test for single mean and single proportion, equality of means and equality of proportions of two populations, small sample t-tests for single mean of normal population, equality of means (**only pooled t-test, for independent samples from two normal populations with equal variance**)

Module 4 (Numerical methods -I) 9 hours

(Text 2- *Relevant topics* from sections 19.1, 19.2, 19.3, 19.5)

Errors in numerical computation-round-off, truncation and relative error, Solution of equations – Newton-Raphson method and Regula-Falsi method. Interpolation-finite differences, Newton's forward and backward difference method, Newton's divided difference method and Lagrange's method. Numerical integration-Trapezoidal rule and Simpson's 1/3rd rule (**Proof or derivation of the formulae not required for any of the methods in this module**)

Module 5 (Numerical methods -II) 9 hours

(Text 2- *Relevant topics* from sections 20.3, 20.5, 21.1)

Solution of linear systems-Gauss-Siedal and Jacobi iteration methods. Curve fitting-method of least squares, fitting staright lines and parabolas. Solution of ordinary differential equations-Euler and Classical Runge-Kutta method of second and fourth order, Adams-Moulton predictor-correction method (**Proof or derivation of the formulae not required for any of the methods in this module**)

Text Books

1. (Text-1) Jay L. Devore, *Probability and Statistics for Engineering and the Sciences*, 8th edition, Cengage, 2012
2. (Text-2) Erwin Kreyszig, *Advanced Engineering Mathematics,* 10 th Edition, John Wiley & Sons, 2016.

Reference Books

1. Hossein Pishro-Nik, *Introduction to Probability, Statistics and Random Processes*, Kappa Research, 2014 (Also available online at www.probabilitycourse.com)
2. Sheldon M. Ross, *Introduction to probability and statistics for engineers and*

scientists, 4th edition, Elsevier, 2009.

3. T. Veera Rajan, *Probability, Statistics and Random processes,* Tata McGraw-Hill, 2008
4. B.S. Grewal, *Higher Engineering Mathematics*, Khanna Publishers, 36 Edition, 2010.

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Discrete Probability distributions**	**9 hours**
1.1	Discrete random variables and probability distributions, expected value, mean and variance (discrete)	3
1.2	Binomial distribution-mean, variance, Poisson distribution-mean, variance, Poisson approximation to binomial	3
1.3	Discrete bivariate distributions, marginal distributions, Independence of random variables (discrete), Expected values	3
2	**Continuous Probability distributions**	**9 hours**
2.1	Continuous random variables and probability distributions, expected value, mean and variance (continuous)	2
2.2	Uniform, exponential and normal distributions, mean and variance of these distributions	4
2.3	Continuous bivariate distributions, marginal distributions, Independent random variables, Expected values, Central limit theorem.	3
3	**Statistical inference**	**9 hours**
3.1	Population and samples, Sampling distribution of single mean and single proportion(large samples)	1
3.2	Confidence interval for single mean and single proportions (large samples)	2
3.3	Hypothesis testing basics, large sample test for single proportion, single proportion	2
3.4	Large sample test for equality of means and equality of proportions of two populations	2

3.5	t-distribution and small sample t-test for single mean and pooled t-test for equality of means	2
4	**Numerical methods-I**	**9 hours**
4.1	Roots of equations- Newton-Raphson, regulafalsi methods	2
4.2	Interpolation-finite differences, Newton's forward and backward formula,	3
4.3	Newton's divided difference method, Lagrange's method	2
4.3	Numerical integration-trapezoidal rule and Simpson's 1/3-rd rule	2
5	**Numerical methods-II**	**9 hours**
5.1	Solution of linear systems-Gauss-Siedal method, Jacobi iteration method	2
5.2	Curve-fitting-fitting straight lines and parabolas to pairs of data points using method of least squares	2
5.3	Solution of ODE-Euler and Classical Runge-Kutta methods of second and fourth order	4
5.4	Adams-Moulton predictor-corrector methods	1

Model Question Paper

(2019 Scheme)

Reg No: .... **Total Pages: 4**

Name: ....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION

(Month & year)

Course Code: MAT

Course Name: PROBABILITY, STATISTICS AND NUMERICAL METHODS

(Common to all branches except (i) Electrical and Electronics, (ii) Electronics and Communication, (iii) Applied Electronics and Instrumentation (iv) Computer Science and Engineering (v) Information Technology)

Max Marks :100 Duration : 3 Hours

PART A

(Answer *all* questions. Each question carries 3 marks)

1. Suppose X is binomial random variable with parameters $n = 100$ and $p = 0.02$. Find $P(X < 3)$ using Poisson approximation to X. (3)

2. The diameter of circular metallic discs produced by a machine is a random variable with mean 6cm and variance 2cm. Find the mean area of the discs. (3)

3. Find the mean and variance of the continuous random variable X with probability density function $f(x) = \begin{cases} 2x-4, & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$ (3)

4. The random variable X is exponentially distributed with mean 3. Find $P(X > t + 3|X > t)$ where t is any positive real number. (3)

5. The 95% confidence interval for the mean mass (in grams) of tablets produced by a machine is $[0.56 \quad 0.57]$, as calculated from a random sample of 50 tablets. What do you understand from this statement? (3)

6. The mean volume of liquid in bottles of lemonade should be at least 2 litres. A sample of bottles is taken in order to test whether the mean volume has fallen below 2 litres. Give a null and alternate hypothesis for this test and specify whether the test would be one-tailed or two-tailed. (3)

7. Find all the first and second order forward and backward differences of y for the following set of (x, y) values: $(0.5, 1.13)$, $(0.6, 1.19)$, $(0.7, 1.26)$, $(0.8, 1.34)$ (3)

8. The following table gives the values of a function $f(x)$ for certain values of x. (3)

x	0	0.25	0.50	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

Evaluate $\int_0^1 f(x)dx$ using trapezoidal rule.

9. Explain the principle of least squares for determining a line of best fit to a given data (3)

10. Given the initial value problem $y' = y + x, \quad y(0) = 0$, find $y(0.1)$ and $y(0.2)$ using Euler method. (3)

PART B

(Answer one question from each module)

MODULE 1

11. (a) The probability mass function of a discrete random variable is $p(x) = kx, x = 1, 2, 3$ where k is a positive constant. Find (i)the value of k (ii) $P(X \leq 2)$ (iii) $E[X]$ and (iv) $\text{var}(1 - X)$. (7)

    (b) Find the mean and variance of a binomial random variable (7)

OR

12. (a) Accidents occur at an intersection at a Poisson rate of 2 per day. what is the probability that there would be no accidents on a given day? What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents? (7)

    (b) Two fair dice are rolled. Let X denote the number on the first die and $Y = 0$ or 1, according as the first die shows an even number or odd number. Find (i) the joint probability distribution of X and Y, (ii) the marginal distributions. (iii) Are X and Y independent ? (7)

MODULE 2

13. (a) The IQ of an individual randomly selected from a population is a normal distribution with mean 100 and standard deviation 15. Find the probability that an individual has IQ (i) above 140 (ii) between 120 and 130. (7)

    (b) A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find $P(X < 0)$ (7)

OR

14. (a) The joint density function of random variables X and Y is given by (7)

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, \quad y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

    Find $P(X + Y \leq 1)$. Are X and Y independent? Justify.

    (b) The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using central limit theorem, find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hours of burning time. (7)

MODULE 3

15. (a) The mean blood pressure of 100 randomly selected persons from a target population is 127.3 units. Find a 95% confidence interval for the mean blood pressure of the population. (7)

    (b) The CEO of a large electric utility claims that 80 percent of his 1,000,000 customers are very satisfied with the service they receive. To test this claim, the local newspaper surveyed 100 customers, using simple random sampling. Among the sampled customers, 73 percent say they are very satisfied. Based on these findings, do you think that the CEO is making a false claim of high satisfaction levels among his customers? Use a 0.05 level of significance. (7)

OR

16. (a) A magazine reported the results of a telephone poll of 800 adult citizens of a country. The question posed was: "Should the tax on cigarettes be raised to pay for health care reform?" The results of the survey were: Out of the 800 persons surveyed, 605 were non-smokers out of which 351 answered "yes" and the rest "no". Out of the remaining 195, who were smokers, 41 answered "yes" and the remaining "no". Is there sufficient evidence, at the 0.05 significance level, to conclude that the two populations smokers and non-smokers differ significantly with respect to their opinions? (7)

(b) Two types of cars are compared for acceleration rate. 40 test runs are recorded for each car and the results for the mean elapsed time recorded below: (7)

	Sample mean	Sample standard deviation
Car A	7.4	1.5
Car B	7.1	1.8

determine if there is a difference in the mean elapsed times of the two car models at 95% confidence level.

MODULE 4

17. (a) Use Newton-Raphson method to find a non-zero solution of $x = 2\sin x$. Start with $x_0 = 1$ (7)

(b) Using Lagrange's interpolating polynomial estimate $f(1.5)$ for the following data (7)

x	0	1	2	3
$y = f(x)$	0	0.9826	0.6299	0.5532

OR

18. (a) Consider the data given in the following table (7)

x	0	0.5	1	1.5	2
$f(x)$	1.0000	1.0513	1.1052	1.1618	1.2214

Estimate the value of $f(1.80)$ using newton's backward interpolation formula.

(b) Evaluate $\int_0^1 e^{-x^2/2}dx$ using Simpson's one-third rule, dividing the interval $[0, 1]$ into 8 subintervals (7)

MODULE 5

19. (a) Using Gauss-Seidel method, solve the following system of equations (7)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(b) The table below gives the estimated population of a country (in millions) for during 1980-1995 (7)

year	1980	1985	1990	1995
population	227	237	249	262

Plot a graph of this data and fit an appropriate curve to the data using the method of least squares. Hence predict the population for the year 2010.

OR

20. (a) Use Runge-Kutta method of fourth order to find $y(0.2)$ given the initial value problem (7)

$$\frac{dy}{dx} = \frac{xy}{1+x^2}, \quad y(0) = 1$$

Take step-size, $h = 0.1$.

(b) Solve the initial value problem (7)

$$\frac{dy}{dx} = x + y, \quad y(0) = 0,$$

in the interval $0 \leq x \leq 1$, taking step-size $h = 0.2$. Calculate $y(0.2), y(0.4)$ and $y(0.6)$ using Runge-Kutta second order method, and $y(0.8)$ and $y(1.0)$ using Adam-Moulton predictor-corrector method.

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MATHEMATICS – 4

(For Electrical, Electronics and Applied Electronics)

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT 204	PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS	BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduces students to the modern theory of probability and statistics, covering important models of random variables and analysis of random processes using appropriate time and frequency domain tools. A brief course in numerical methods familiarises students with some basic numerical techniques for finding roots of equations, evaluating definite integrals solving systems of linear equations and solving ordinary differential equations which are especially useful when analytical solutions are hard to find.

Prerequisite: A basic course in one-variable and multi-variable calculus**.**

Course Outcomes: After the completion of the course the student will be able to

CO 1	Understand the concept, properties and important models of discrete random variables and, using them, analyse suitable random phenomena.
CO 2	Understand the concept, properties and important models of continuous random variables and, using them, analyse suitable random phenomena.
CO 3	Analyse random processes using autocorrelation, power spectrum and Poisson process model as appropriate.
CO 4	Compute roots of equations, evaluate definite integrals and perform interpolation on given numerical data using standard numerical techniques
CO 5	Apply standard numerical techniques for solving systems of equations, fitting curves on given numerical data and solving ordinary differential equations.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2	2	2	2					2		1
CO 2	3	2	2	2	2					2		1
CO 3	3	2	2	2	2					2		1
CO 4	3	2	2	2	2					2		1
CO 5	3	2	2	2	2					2		1

Assessment Pattern

Bloom's Category	Continuous Assessment Tests(%)		End Semester Examination(%)
	1	2	
Remember	10	10	10
Understand	30	30	30
Apply	30	30	30
Analyse	20	20	20
Evaluate	10	10	10
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Let X denote the number that shows up when an unfair die is tossed. Faces 1 to 5 of the die are equally likely, while face 6 is twice as likely as any other. Find the probability distribution, mean and variance of X.
2. An equipment consists of 5 components each of which may fail independently with probability 0.15. If the equipment is able to function properly when at least 3 of the components are operational, what is the probability that it functions properly?
3. X is a binomial random variable $B(n, p)$with $n = 100$and $p = 0.1$. How would you approximate it by a Poisson random variable?
4. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y)

Course Outcome 2 (CO2)

1. What can you say about $P(X = a)$for any real number $a$when $X$is (i) a discrete random variable? (ii) a continuous random variable?
2. A string, 1 meter long, is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twice the length of the other?

3. A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3?

4. X and Y are independent random variables with X following an exponential distribution with parameter μ and Y following and exponential distribution with parameterλ. Find $P(X + Y \leqslant 1)$

Course Outcome 3(CO3):

1. A random process $X(t)$ is defined by $acos(\omega t + \Theta)$where a and ω are constants and Θ is uniformly distributed in$[0,2\pi]$. Show that $X(t)$ is WSS

2. How are the autocorrelation function and power spectral density of a WSS process are related to each other?

3. Find the power spectral density of the WSS random process $X(t)$, given the autocorrelation function $R_X(\tau) = 9e^{-|\tau|}$

4. A conversation in a wireless ad-hoc network is severely disturbed by interference signals according to a Poisson process of rate $\lambda = 0.01$ per minute. (a) What is the probability that no interference signals occur within the first two minutes of the conversation? (b) Given that the first two minutes are free of disturbing effects, what is the probability that in the next minute precisely 1 interfering signal disturbs the conversation? (c) Given that there was only 1 interfering signal in the first 3 minutes, what is the probability that there would be utmost 2 disturbances in the first 4 minutes?

Course Outcome 4(CO4):

1. Use Newton-Raphson method to find a real root of the equation $f(x) = e^{2x} - x - 6$correct to 4 decimal places.

2. Compare Newton's divided difference method and Lagrange's method of interpolation.

3. Use Newton's forward interpolation formula to compute the approximate values of the function $f$at $x = 0.25$from the following table of values of $x$and $f(x)$

x	0	0.5	1	1.5	2
f(x)	1.0000	1.0513	1.1052	1.1618	1.2214

4. Find a polynomial of degree 3 or less the graph of which passes through the points (-1, 3), (0,-4), (1,5) and (2,-6)

Course Outcome 5 (CO5):

1. Apply Gauss-Seidel method to solve the following system of equations
$$4x_1 - x_2 - x_3 = 3$$
$$-2x_1 + 6x_2 + x_3 = 9$$
$$-x_1 + x_2 + 7x_3 = -6$$

2. Using the method of least squares fit a straight line of the form $y = ax + b$to the following set of ordered pairs (x, y) :
(2,4), (3,5), (5,7), (7,10), (9,15)

3. Write the normal equations for fitting a curve of the form $y = a_0 + a_1x^2$to a given set of pairs of data points.

4. Use Runge-Kutta method of fourth order to compute $y(0.25)$and $y(0.5)$, given the initial value problem
$$y' = x + xy + y, y(0) = 1$$

Syllabus

Module 1 (Discrete probability distributions) 9 hours

(Text-1: *Relevant topics* from sections-3.1-3.4, 3.6, 5.1)

Discrete random variables and their probability distributions, Expectation, mean and variance, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Discrete bivariate distributions, marginal distributions, Independent random variables, Expectation (multiple random variables)

Module 2 (Continuous probability distributions) 9 hours

(Text-1:*Relevant topics* from sections-4.1-4.4, 3.6, 5.1)

Continuous random variables and their probability distributions, Expectation, mean and variance, Uniform, exponential and normal distributions, Continuous bivariate distributions, marginal distributions, Independent random variables, Expectation (multiple random variables), i. i. d random variables and Central limit theorem (without proof).

Module 3 (Random Processes) 9 hours

(Text-2: *Relevant topics* from sections-8.1-8.5, 8.7, 10.5)

Random processes and classification, mean and autocorrelation, wide sense stationary (WSS) processes, autocorrelation and power spectral density of WSS processes and their properties, Poisson process-distribution of inter-arrival times, combination of independent Poisson processes(merging) and subdivision (splitting) of Poisson processes (**results without proof**).

Module 4 (Numerical methods -I) 9 hours

(Text 3- *Relevant topics* from sections 19.1, 19.2, 19.3, 19.5)

Errors in numerical computation-round-off, truncation and relative error, Solution of equations – Newton-Raphson method and Regula-Falsi method. Interpolation-finite differences, Newton's forward and backward difference method, Newton's divided difference method and Lagrange's method. Numerical integration-Trapezoidal rule and Simpson's 1/3rd rule (**Proof or derivation of the formulae not required for any of the methods in this module**)

Module 5 (Numerical methods -II) 9 hours

(Text 3- *Relevant topics* from sections 20.3, 20.5, 21.1)

Solution of linear systems-Gauss-Seidel and Jacobi iteration methods. Curve fitting-method of least squares, fitting straight lines and parabolas. Solution of ordinary differential equations-Euler and Classical Runge-Kutta method of second and fourth order, Adams-Moulton predictor-correction method (**Proof or derivation of the formulae not required for any of the methods in this module**)

Text Books

1. (Text-1) Jay L. Devore, *Probability and Statistics for Engineering and the Sciences*, 8^{th} edition, Cengage, 2012
2. (Text-2) Oliver C. Ibe, *Fundamentals of Applied Probability and Random Processes,* Elsevier, 2005.
3. (Text-3) Erwin Kreyszig, *Advanced Engineering Mathematics*, 10 th Edition, John Wiley & Sons, 2016.

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1. Hossein Pishro-Nik, *Introduction to Probability, Statistics and Random Processes*, Kappa Research, 2014 (Also available online at www.probabilitycourse.com)
2. V.Sundarapandian, *Probability, Statistics and Queueing theory*, PHI Learning, 2009
3. Gubner, *Probability and Random Processes for Electrical and Computer Engineers*, Cambridge University Press,2006.
4. B.S. Grewal, *Higher Engineering Mathematics*, Khanna Publishers, 36 Edition, 2010.

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Discrete Probability distributions**	**9 hours**
1.1	Discrete random variables and probability distributions, expected value, mean and variance (discrete)	3
1.2	Binomial distribution-mean, variance, Poisson distribution-mean, variance, Poisson approximation to binomial	3
1.3	Discrete bivariate distributions, marginal distributions, Independence of random variables (discrete), Expected values	3
2	**Continuous Probability distributions**	**9 hours**
2.1	Continuous random variables and probability distributions, expected value, mean and variance (continuous)	2
2.2	Uniform, exponential and normal distributions, mean and variance of these distributions	4
2.3	Continuous bivariate distributions, marginal distributions, Independent random variables, Expected values, Central limit theorem.	3
3	**Random processes**	**9 hours**
3.1	Random process -definition and classification, mean , autocorrelation	2
3.2	WSS processes its autocorrelation function and properties	2
3.3	Power spectral density	2
3.4	Poisson process, inter-distribution of arrival time, merging and splitting	3
4	**Numerical methods-I**	**9 hours**
4.1	Roots of equations- Newton-Raphson, regulafalsi methods	2
4.2	Interpolation-finite differences, Newton's forward and backward formula,	3
4.3	Newton's divided difference method, Lagrange's method	2
4.3	Numerical integration-trapezoidal rule and Simpson's 1/3-rd rule	2
5	**Numerical methods-II**	**9 hours**
5.1	Solution of linear systems-Gauss-Siedal method, Jacobi iteration	2

	method	
5.2	Curve-fitting-fitting straight lines and parabolas to pairs of data points using method of least squares	2
5.3	Solution of ODE-Euler and Classical Runge-Kutta methods of second and fourth order	4
5.4	Adams-Moulton predictor-corrector method	1

Model Question Paper

(2019 Scheme)

Reg No: .... **Total Pages: 3**

Name: ....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION

(Month & year)

Course Code: MAT

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

(For (i) Electrical and Electronics, (ii) Electronics and Communication, (iii) Applied Electronics and Instrumentation Engineering branches)

Max Marks :100 Duration : 3 Hours

PART A

(Answer *all* questions. Each question carries 3 marks)

1. Suppose X is binomial random variable with parameters $n = 100$ and $p = 0.02$. Find $P(X < 3)$ using Poisson approximation to X. (3)

2. The diameter of circular metallic discs produced by a machine is a random variable with mean 6cm and variance 2cm. Find the mean area of the discs. (3)

3. Find the mean and variance of the continuous random variable X with probability density function (3)
$$f(x) = \begin{cases} 2x-4, & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

4. The random variable X is exponentially distributed with mean 3. Find $P(X > t + 3|X > t)$ where t is any positive real number. (3)

5. Give any two examples of a continuous time discrete state random processes. (3)

6. How will you calculate the mean, variance and total power of a WSS process from its autocorrelation function? (3)

7. Find all the first and second order forward and backward differences of y for the following set of (x, y) values: $(0.5, 1.13)$, $(0.6, 1.19)$, $(0.7, 1.26)$, $(0.8, 1.34)$ (3)

8. The following table gives the values of a function $f(x)$ for certain values of x. (3)

x	0	0.25	0.50	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

Evaluate $\int_0^1 f(x)dx$ using trapezoidal rule.

9. Explain the principle of least squares for determining a line of best fit to a given data (3)

10. Given the initial value problem $y' = y + x, \quad y(0) = 0$, find $y(0.1)$ and $y(0.2)$ using Euler method. (3)

PART B

(Answer one question from each module)

MODULE 1

11. (a) The probability mass function of a discrete random variable is $p(x) = kx, x = 1, 2, 3$ where k is a positive constant. Find (i)the value of k (ii) $P(X \leq 2)$ (iii) $E[X]$ and (iv) $\text{var}(1 - X)$. (7)

    (b) Find the mean and variance of a binomial random variable (7)

OR

12. (a) Accidents occur at an intersection at a Poisson rate of 2 per day. what is the probability that there would be no accidents on a given day? What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents? (7)

    (b) Two fair dice are rolled. Let X denote the number on the first die and $Y = 0$ or 1, according as the first die shows an even number or odd number. Find (i) the joint probability distribution of X and Y, (ii) the marginal distributions. (iii) Are X and Y independent ? (7)

MODULE 2

13. (a) The IQ of an individual randomly selected from a population is a normal distribution with mean 100 and standard deviation 15. Find the probability that an individual has IQ (i) above 140 (ii) between 120 and 130. (7)

    (b) A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find $P(X < 0)$ (7)

OR

14. (a) The joint density function of random variables X and Y is given by (7)

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, \quad y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

    Find $P(X + Y \leq 1)$. Are X and Y independent? Justify.

    (b) The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using central limit theorem, find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hours of burning time. (7)

MODULE 3

15. (a) A random process $X(t)$ is defined by $X(t) = Y(t)\cos(\omega t + \Theta)$ where $Y(t)$ is a WSS process, ω is a constant and Θ is uniformly distributed in $[0, 2\pi]$ and is independent of $Y(t)$. Show that $X(t)$ is WSS (7)

    (b) Find the power spectral density of the random process $X(t) = a\sin(\omega_0 t + \Theta)$, ω_0 constant and Θ is uniformly distributed in $(0, 2\pi)$ (7)

OR

16. Cell-phone calls processed by a certain wireless base station arrive according to a Poisson process with an average of 12 per minute.

    (a) What is the probability that more than three calls arrive in an interval of length 20 seconds? (7)

    (b) What is the probability that more than 3 calls arrive in each of two consecutive intervals of length 20 seconds? (7)

MODULE 4

17. (a) Use Newton-Raphson method to find a non-zero solution of $x = 2\sin x$. Start with $x_0 = 1$ (7)

(b) Using Lagrange's interpolating polynomial estimate $f(1.5)$ for the following data (7)

x	0	1	2	3
$y = f(x)$	0	0.9826	0.6299	0.5532

OR

18. (a) Consider the data given in the following table (7)

x	0	0.5	1	1.5	2
$f(x)$	1.0000	1.0513	1.1052	1.1618	1.2214

Estimate the value of $f(1.80)$ using newton's backward interpolation formula.

(b) Evaluate $\int_0^1 e^{-x^2/2}dx$ using Simpson's one-third rule, dividing the interval [0, 1] into 8 subintervals (7)

MODULE 5

19. (a) Using Gauss-Seidel method, solve the following system of equations (7)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(b) The table below gives the estimated population of a country (in millions) for during 1980-1995 (7)

year	1980	1985	1990	1995
population	227	237	249	262

Plot a graph of this data and fit an appropriate curve to the data using the method of least squares. Hence predict the population for the year 2010.

OR

20. (a) Use Runge-Kutta method of fourth order to find $y(0.2)$ given the initial value problem (7)

$$\frac{dy}{dx} = \frac{xy}{1+x^2}, \quad y(0) = 1$$

Take step-size, $h = 0.1$.

(b) Solve the initial value problem (7)

$$\frac{dy}{dx} = x + y, \quad y(0) = 0,$$

in the interval $0 \leq x \leq 1$, taking step-size $h = 0.2$. Calculate $y(0.2), y(0.4)$ and $y(0.6)$ using Runge-Kutta second order method, and $y(0.8)$ and $y(1.0)$ using Adam-Moulton predictor-corrector method.

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MATHEMATICS – 4th semester

For Computer Science and Engineering

CODE MAT 206	COURSE NAME GRAPH THEORY	CATEGORY	L	T	P	CREDIT
		BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduce fundamental concepts in Graph Theory, including properties and characterisation of Graph/Trees and Graph theoretic algorithms, which are widely used in Mathematical modelling and has got applications across computer science and Electrical Engineering.

Prerequisite: A basic course in combinatorics, set theory and strong foundations of higher secondary Mathematics

Course Outcomes: After the completion of the course the student will be able to

CO 1	Understand the basic concept in Graph theory.
CO 2	Formulate and prove fundamental theorems on Eulerian graphs and Hamiltonian graphs
CO 3	Apply theorems and algorithms on trees.
CO 4	Understand planar graphs and its properties and to detect planarity of a given graph
CO 5	Demonstrate the knowledge of fundamental concepts of matrix representation of graphs and colouring problems

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	2	1					1		2
CO 2	3	3	3	2	2							1
CO 3	3	3	3	3	2							
CO 4	3	3	3	2	1							1
CO 5	3	3	3	3	1					1		2

Assessment Pattern

Bloom's Category	**Continuous Assessment Tests**		**End Semester Examination**
	1	**2**	
Remember	5	5	10
Understand	10	10	20
Apply	10	10	20
Analyse	10	10	20
Evaluate	15	15	30
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions.

Course Outcome 1 (CO1):

(1)Differentiate a walk, path and circuit in a graph.

(2)Is it possible to construct a graph with 12 vertices such that two of the vertices have degree 3 and the remaining vertices have degree 4.Justify your answer?

(3)Prove that a simple graph with n vertices must be connected, if it has more than $\frac{(n-1)(n-2)}{2}$ edges.

(4) Prove the statement: If a graph (connected or disconnected) has exactly two odd degree, then there must be a path joining the two vertices.

Course Outcome 2 (CO2):

(1)Define Hamiltonian circuit, Euler graph. Give one example for each.

(2)Define directed graphs. Differentiate symmetric digraphs and asymmetric digraphs.

(3)Prove that a connected graph G is an Euler graph if all vertices of G are of even degree.

(4) Prove that a graph G with n vertices always has a Hamiltonian path if the sum of the degrees of every pair of verticesV_i ,$V_j$in G satisfies the condition $d(Vi) + d(Vj) \geq n-1$

Course Outcome 3(CO3):

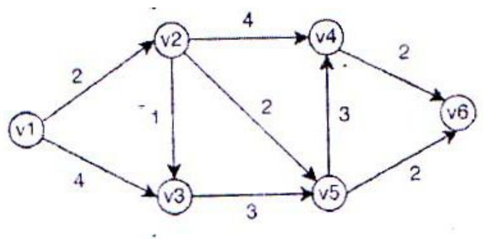
(1)(a)Discuss the centre of a tree with suitable example.

(b) Define binary tree. Then prove that number of pendant vertices in a binary tree is $\frac{(n+1)}{2}$

(2)Prove that a tree with n vertices has $n-1$ edges.

(3)Explain Floyd Warshall algorithm.

(4) Using Dijkstra's algorithm, find the shortest path between V_1 and V_6.



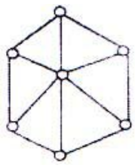
Course Outcome 4(CO4):

(1)Define edge connectivity, vertex connectivity and separable graphs. Give an example for each.

(2)Prove that a connected graph with n vertices and e edges has $e - n + 2$ edges.

(3)Prove the statement: Every cut set in a connected graph G must also contain at least one branch of every spanning tree of G.

(4)Draw the geometrical dual (G*) of the graph Ggiven below. Also check whether G and G* are self dual or not, substantiate your answer clearly?



Course Outcome 5 (CO5):

(1) Draw the graph represented by the following incidence matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(2)Find the chromatic polynomial of the graph

(3)Show that an n vertex graph is a tree iff its chromatic polynomial Pn$(\lambda) = \lambda(\lambda - 1)^{n-1}$

(4) Prove the statement:

A covering g of a graph is minimal if and only if g contains no path of length three or more:

Syllabus

Module 1 **(9 hours)**

(Relevant portions of sections 1.1, 1.2, 1.3, 1.4, 1.5, 2.1, 2.2, 2.4, 2.5)

Introduction- Basic definition – Application of graphs – finite and infinite graphs – Incidence and Degree – Isolated vertex, pendent vertex and Null graph. Paths and circuits – Isomorphism, sub graphs, walks, paths and circuits, Connected graphs, disconnect graphs and components.

Module 2 **(9 hours)**

(Relevant portions of sections 2.6, 2.7, 2.8, 2.9, 2.10, 9.1, 9.2, 9.3, 9.4)

Euler graphs, Operations on graphs, Hamiltonian paths and circuits, Travelling salesman problem. Directed graphs – types of digraphs, Digraphs and binary relation, Directed paths, Fleury's algorithm

Module 3 **(9 hours)**

(Relevant portions of sections (3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.10)

Trees – properties, pendent vertex, Distance and centres in a tree - Rooted and binary tree, counting trees, spanning trees, Fundamental circuits, Prim's algorithm and Kruskal's algorithm, Dijkstra's shortest path algorithm, Floyd-Warshall shortest path algorithm.

Module 4 **(9 hours)**

(Relevant topics from sections (4.1, 4.2, 4.3, 4.4,4.5, 5.2, 5.3, 5.4, 5.6**)**

Vertex Connectivity, Edge Connectivity, Cut set and Cut Vertices, Fundamental circuits, Planar graphs, Kuratowski'stheorem (proof not required) Different representation of planar graphs, Euler's theorem, Geometric dual.

Module 5 **(9 hours)**

(Relevant topics from sections 7.1, 7.9, 8.1, 8.3, 8.4, 8.5, 8.6 **)**

Matrix representation of graphs- Adjacency matrix, Incidence Matrix, colouring- chromatic number, chromatic polynomial, matching, covering, four colour problem and five colour problem. Greedy colouring algorithm.

Textbook:

1. NarasinghDeo, Graph theory, PHI,1979

References:

1. R. Diestel, *Graph Theory,* free online edition, 2016: diestel-graph-theory.com/basic.html.
2. Douglas B. West, Introduction to Graph Theory, Prentice Hall India Ltd.,2001
3. Robin J. Wilson, Introduction to Graph Theory, Longman Group Ltd.,2010
4. J.A. Bondy and U.S.R. Murty. Graph theory with Applications

Assignments

Assignment: Assignment must include applications of the above theory in the concerned engineering branches

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Module-I**	**9 hours**
1.	**Introduction**- Basic definition – Application of graphs – finite and infinite graphs, Incidence and Degree – Isolated vertex, pendent vertex and Null graph	3
2.	Paths and circuits – Isomorphism	2
3.	sub graphs, walks, paths and circuits	2
4.	Connected graphs, disconnect graphs and components.	2
2	**Module-II**	**9 hours**
1.	Euler graphs, Operations on graphs	3
2.	Hamiltonian paths and circuits	2
3.	Travelling salesman problem	1
4.	Directed graphs – types of digraphs, Digraphs and binary relation, Directed paths	2
5.	Fleury's algorithm	1
3	**Module-III**	**9 hours**
1	Trees – properties, pendent vertex	3

2.	Distance and centres in a tree - Rooted and binary tree, counting trees	2
3.	spanning trees, Fundamental circuits	1
4.	Prim's algorithm and Kruskal's algorithm, Dijkstra's shortest path algorithm, Floyd-Warshall shortest path algorithm	3
4	**Module-IV**	**9 hours**
1.	Vertex Connectivity, Edge Connectivity, Cut set and Cut Vertices	2
2.	Fundamental circuits	2
3.	Planar graphs, Kuratowski'stheorem	2
4.	Different representation of planar graphs, Euler's theorem, Geometric dual.	3
5	**Module-V**	**9 hours**
1.	Matrix representation of graphs- Adjacency matrix, Incidence Matrix	2
2.	Colouring- chromatic number, chromatic polynomial	2
3.	Matching, covering	2
4.	Four colour problem and five colour problem. Greedy colouring algorithm.	3

Model Question Paper

Reg.No.___________________ **Total Pages: 4**

Name:____________________

APJ ABHUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE MODEL EXAMINATION, NOVEMBER 2020

Course Code: MAT

(For Computer science and engineering and Information Technology)

2019 Scheme

Course Name: GRAPH THEORY

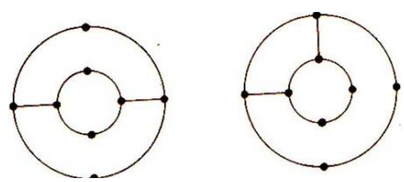
Max.Marks:100 **Duration: 3 Hours**

PART A

(Answer all questions, each carries 3 marks)

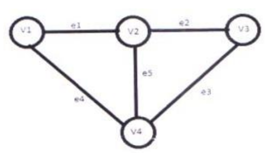
Module 1

1. Define isomorphism between two graphs. Are the following graphs isomorphic to each other? Justify your answer?



2. Prove that the number of odd degree vertices in a graph is always even.

3. Define Euler and Hamiltonian graphs. Give examples of an Euler graph which is not a Hamiltonian and vice versa.
4. Discuss Travelling Salesman Problem.
5. Prove that in a tree T(V,E) , IVI=IEI+1.
6. Define branch, chord , rank and nullity in a spanning tree with example.
7. Define cutsets ,fundamental cutsets.List any two cutsets of the following graph and find all fundamental cutsets.



8. Define a planar graph.Show that K_5 is not a planar graph.

9. Draw the adjacency graph for the following adjacency matrix.

$$X(G)=\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

10. Find the chromatic polynomial of the graph. (10x3=30)

PART B

(Answer One Full question from each module, each question carries 14 marks)

Module I

11. (a) Explain any three applications of graph theory. (7)

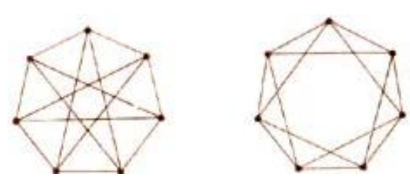
(b) (i) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. (4)

(ii) Draw a disconnected simple graph G_1 with 10 vertices and 4 components and also calculate the maximum number of edges possible in G_1. (3)

12. (a) (i) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$ (3)

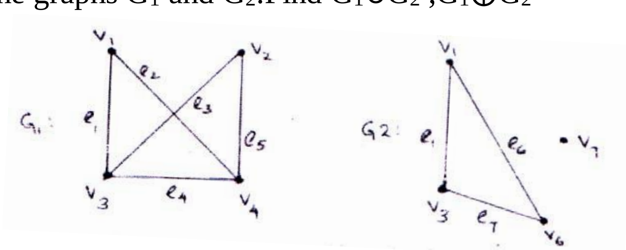
(ii) Draw all simple graphs of one, two, three and four vertices. (4)

(b) What are the basic conditions to be satisfied for two graphs to be isomorphic. Are the two graphs given below isomorphic? Explain with valid reasons. (7)



13. (a) (i) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. (5)

(ii) Consider the graphs G_1 and G_2. Find $G_1 \cup G_2$, $G_1 \oplus G_2$ (2)



(b) Explain Fleury's algorithm (5)

14. (a) Prove that an Euler graph G is arbitrarily traceable from vertex v in G iff every circuit in G contains V. (7)

(b) (i) Define Hamiltontin circuits and path with examples. Find out the number of the edge – disjoint Hamiltonian circuits possible in a complete graph with five vertices. (4)

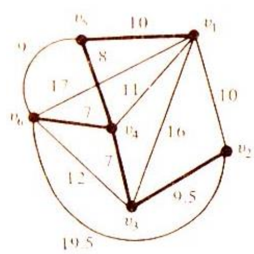
(ii) Prove that in a complete graph n vertices there are $\frac{(n-1)}{2}$ edge disjoint Hamiltonian circuits if n is odd number ≥3 (3)

Module III

15. (a)(i)Prove that all threes will have either one or two centers. (4)

(ii) Define spanning trees. Show that Hamiltonian path is a spanning tree. (3)

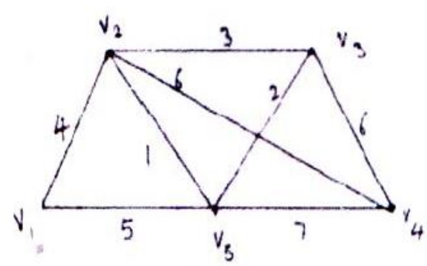
(b) Using Kruskal algorithm, find a shortest spanning tree for the following weighted graph. (7)



16. (a) (i) Prove that a graph is a tree if and only if it is minimally connected. (4)

(ii) Plot a maximum level and minimum level binary trees with 11 vertices (3)

(b) Using Prim's algorithm, find a minimal spanning tree for the following weighted graph (7)



Module IV

17. (a) Prove that the ring sum of any two cut sets in a graph is either a third cut set or an edge disjoint union of cut sets. (7)

(b) Prove that the complete graph of five vertices is nonplanar (7)

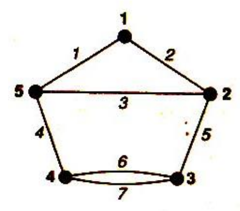
18. (a) Prove that for a connected planar graph with v vertices, e edges and r regions, v-e+r = 2. (7)

(b) (i) Prove that every cut set in a connected graph G must contain at least one branch of every spanning tree of G. (3)

(ii) Explain the term vertex connectivity. Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of G. (4)

Module V

19. (a) (i)Write down the adjacency and incidence matrix of the graph. (4)



(ii) Explain Greedy Colouring algorithm. (3)

(b) (i)Prove that a graph of n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda-1)(\lambda-2)\ldots\ldots(\lambda-n+1)$. (4)

(ii) Explain Four Color problem. (3)

20. (a) Prove that every tree with two or more vertices is 2-chromatic. (7)

(b) Let a and b be two non adjacent vertices in a graph G . Let G' be a graph obtained by adding an edge between a and b. Let G'' be a single graph obtained from G by fusing the vertex a and b together and replacing sets of parallel edges with single edge. Then $P_n(\lambda)$ of G $=P_n(\lambda)$ of G' + $P_{n-1}(\lambda)$ of G''. (7)

MATHEMATICS – (4 th semester)

(For Information Technology)

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT 208	PROBABILITY,STATISTICS AND ADVANCED GRAPH THEORY	BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduces students to the modern theory of probability and statistics, covering important models of random variables and techniques of parameter estimation and hypothesis testing. This course introduce fundamental concepts in Graph Theory, including properties and characterisation of Graph/Trees and Graph theoretic algorithms, which are widely used in Mathematical modelling and has got applications across **Information Technology**

Prerequisite: A basic course in one-variable and multi-variable calculus, knowledge of elementary set theory, matrices

Course Outcomes: After the completion of the course the student will be able to

CO 1	Understand the concept, properties and important models of discrete random variables and, using them, analyse suitable random phenomena.
CO 2	Understand the concept, properties and important models of continuous random variables and, using them, analyse suitable random phenomena.
CO 3	Perform statistical inferences concerning characteristics of a population based on attributes of samples drawn from the population
CO 4	Understand the basic concept in Graph theory, Understand planar graphs and it's properties. Demonstrate the knowledge of fundamental concepts of matrix representation of graphs, Apply fundamental theorems on Eularian graphs and Hamiltonian graphs.
CO 5	Understand the basic concept in Trees,coloring of graphs. Apply coloring of graphs, Apply algorithm to find the minimum spanning tree

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2	2	2	2					2		1
CO 2	3	2	2	2	2					2		1
CO 3	3	2	2	2	2					2		1
CO 4	3	2	2	2	2					2		1
CO 5	3	2	2	2	2					2		1

Assessment Pattern

Bloom's Category	Continuous Assessment Tests(%)		End Semester Examination(%)
	1	2	
Remember	10	10	10
Understand	30	30	30
Apply	30	30	30
Analyse	20	20	20
Evaluate	10	10	10
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Let X denote the number that shows up when an unfair die is tossed. Faces 1 to 5 of the die are equally likely, while face 6 is twice as likely as any other. Find the probability distribution, mean and variance of X.
2. An equipment consists of 5 components each of which may fail independently with probability 0.15. If the equipment is able to function properly when at least 3 of the components are operational, what is the probability that it functions properly?
3. X is a binomial random variable $B(n, p)$with $n = 100$and $p = 0.1$. How would you approximate it by a Poisson random variable?
4. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y)

Course Outcome 2 (CO2)

1. What can you say about $P(X = a)$ ↑for any real number $a$when $X$is a (i) discrete random variable? (ii) continuous random variable?
2. A string, 1 meter long, is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twice the length of the other?

3. A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3?

4. X and Y are independent random variables with X following an exponential distribution with parameter μ and Y following and exponential distribution with parameterλ. Find $P(X+Y \leqslant 1)$

Course Outcome 3(CO3):

1. In a random sample of 500 people selected from the population of a city 60 were found to be left-handed. Find a 95% confidence interval for the proportion of left-handed people in the city population.

2. What are the types of errors involved in statistical hypothesis testing? Explain the level of risks associated with each type of error.

3. A soft drink maker claims that a majority of adults prefer its leading beverage over that of its main competitor's. To test this claim 500 randomly selected people were given the two beverages in random order to taste. Among them, 270 preferred the soft drink maker's brand, 211 preferred the competitor's brand, and 19 could not make up their minds. Determine whether there is sufficient evidence, at the 5% level of significance, to support the soft drink maker's claim against the default that the population is evenly split in its preference.

4. A nutritionist is interested in whether two proposed diets, *diet A* and *diet B* work equally well in providing weight-loss for customers. In order to assess a difference between the two diets, she puts 50 customers on diet A and 60 other customers on diet B for two weeks. Those on the former had weight losses with an average of 11 pounds and a standard deviation of 3 pounds, while those on the latter lost an average of 8 pounds with a standard deviation of 2 pounds. Do the diets differ in terms of their weight loss?

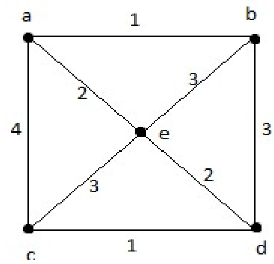
Course Outcome 4(CO4):

1. How many edges are there in a graph with ten vertices each of degree six?
2. Prove that a simple graph with n vertices must be connected, if it has more than $\frac{(n-1)(n-2)}{2}$ edges
3. Prove that a connected graph G is an Euler graph if all vertices of G are of even degree.
4. Use Kuratowski'stheorem to determine whether $K_{4,4}$ is planar.

Course Outcome 5 (CO5):

1. Prove that a tree with n vertices has $n-1$ edges.
2. Find the chromatic number of $K_{m,n}$

3. Using graph model, how can the final exam at a university be scheduled so that no student has two exams at the same time?
4. Explain Prim's algorithm and use it to find the minimum spanning tree for the graph given below



Syllabus

Module 1 (Discrete probability distributions) 9 hours

(Text-1: *Relevant topics* from sections-3.1-3.4, 3.6, 5.1)

Discrete random variables and their probability distributions, Expectation, mean and variance, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Discrete bivariate distributions, marginal distributions, Independent random variables, Expectation -multiple random variables.

Module 2 (Continuous probability distributions) 9 hours

(Text-1: *Relevant topics* from sections-4.1-4.4, 3.6, 5.1)

Continuous random variables and their probability distributions, Expectation, mean and variance, Uniform, exponential and normal distributions, Continuous bivariate distributions, marginal distributions, Independent random variables, Expectation-multiple random variables, i.i.d random variables and Central limit theorem (**without proof**).

Module 3 (Statistical inference) 9 hours

(Text-1:*Relevant topics from sections*-5.4, 3.6, 5.1, 7.2, 8.1, 8.3, 9.1-9.2,9.4)

Population and samples, Sampling distribution of the mean and proportion (for large samples only), Confidence interval for single mean and single proportions (for large samples only). Test of hypotheses: Large sample test for single mean and single proportion, equality of means and equality of proportions of two populations, small sample t-tests for single mean of normal population, equality of means (**only pooled t-test, for independent samples from two normal populations with equal variance**)

Module 4 (Advanced Graph theory -I) 9 hours

(Text-2: *Relevant topics of sections* -10.1, 10.2, 10.3, 10.4, 10.5, 10.7)

Introduction- Basic definitions, Directed graphs, pseudo graph, multigraph, Graph models, Graph terminology-vertex degree, simple graph, Complete graphs, cycles, bipartite graph, new graphs from old-union, complement, Representing graph-Adjacency matrix, Incidence Matrix , Isomorphism, Connectivity, path , cut vertices , cut edges ,connectedness in directed and undirected graphs, Counting paths between vertices-Euler paths and circuits , Fleury's algorithm(**proof of algorithm omitted)** , Hamiltonian paths and circuits. Ore's theorem, Planar graph, -Euler's formula on planar graphs, Kuratowski's theorem **(Proof of theorem omitted)**

Module 5 (Advanced Graph theory -II) (9 hours)

(Text-2: *Relevant topics* of *sections* –(10.8,11.1, 11.4, 11.5)

Graph colouring, dual graph, chromatic number, chromatic number of completegraphK_n, chromatic number of complete bipartite graph $K_{m,n}$, chromatic number of cycle C_n , Four color theorem, applications of graph colouring-scheduling and assignments

Trees-rooted trees, Properties of trees-level, height, balanced rooted tree, Spanning tree- basic theorems on spanning tree (**DFS, BFS algorithms and it's applicationsomitted**), Minimum spanning tree, Prim's algorithm and Kruskal's algorithm(**proofs of algorithms omitted)**

(9 hours)

Text Books

1. (Text-1) Jay L. Devore, *Probability and Statistics for Engineering and the Sciences*, 8th edition, Cengage, 2012
2. (Text-2) Kenneth H Rosen, *Discrete Mathematics and it's applications*, Tata Mc Graw Hill, 8 th Edition,

Reference Books

1. Hossein Pishro-Nik, *Introduction to Probability, Statistics and Random Processes*, Kappa Research, 2014 (Also available online at www.probabilitycourse.com)
2. Sheldon M. Ross, *Introduction to probability and statistics for engineers and scientists*, 4th edition, Elsevier, 2009.
3. T.Veera Rajan, *Probability, Statistics and Random processes,* Tata McGraw-Hill, 2008
4. Ralph P Grimaldi, *Discrete and Combinatorial Mathematics, An applied Introduction*, 4th edition, Pearson
5. C L Liu, *Elements of Discrete Mathematics*, Tata McGraw Hill, 4th edition,2017
6. NarasinghDeo, *Graph theory*, PHI, 1979
7. John Clark , Derek Allan Holton, *A first look at Graph Theory.*

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Discrete Probability distributions**	**9 hours**
1.1	Discrete random variables and probability distributions, expected value, mean and variance (discrete)	3
1.2	Binomial distribution-mean, variance, Poisson distribution-mean, variance, Poisson approximation to binomial	3
1.3	Discrete bivariate distributions, marginal distributions, Independence of random variables (discrete), Expected values	3
2	**Continuous Probability distributions**	**9 hours**
2.1	Continuous random variables and probability distributions, expected value, mean and variance (continuous)	2
2.2	Uniform, exponential and normal distributions, mean and variance of these distributions	4
2.3	Continuous bivariate distributions, marginal distributions, Independent random variables, Expected values, Central limit theorem.	3
3	**Statistical inference**	**9 hours**
3.1	Population and samples, Sampling distribution of single mean and single proportion(large samples)	1
3.2	Confidence interval for single mean and single proportions (large samples)	2
3.3	Hypothesis testing basics, large sample test for single mean, single proportion	2
3.4	Large sample test for equality of means and equality of proportions of two populations	2
3.5	t-distribution and small sample t-test for single mean and pooled t-test for equality of means	2
4	**Advanced Graph Theory -I**	**9 hours**
4.1	**Introduction**- Basic definition – Application of graphs Incidence	1

	and Degree – Isolated vertex, pendent vertex and Null graph	
4.2	Theorems connecting vertex degree and edges, bipartite graphs.	1
4.3	Adjacency matrix, incidence matrix, Isomorphism	1
4.4	Path, cut set, cut edges, Connectedness of directed and undirected graphs ,path isomorphism	2
4.5	Euler paths and circuits , Fleury's algorithm**(proof of algorithm omitted)** , Hamiltonian paths and circuits. Ore's theorem(proof omitted)	3
4.6	Planar graph, - Euler's theorem on planar graph , applications of Kuratowski's theorem	1
5	**Advanced Graph Theory -II**	**9 hours**
5.1	Graph colouring, dual graph	1
5.2	Chromatic number, chromatic number of K_n, $K_{m,n}$, C_n ,	2
5.3	Four colour theorem, applications of graph colouring-scheduling and assignments,	2
5.4	Trees-spanning trees-definition and example, minimum spanning tree,	2
5.5	Prim's algorithm and Kruskal's algorithm(**proofs of algorithms omitted)**	2

MODEL QUESTION PAPER (2019 Scheme)

Reg. No: .... Total Pages: 4

Name : ....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (Month & year)

Course Code: MAT208

Course Name: PROBABILITY, STATISTICS AND ADVANCED GRAPH THEORY

(For Information Technology)

Max Marks:100Duration : 3 Hours

PART A (Answer all questions. Each question carries 3 marks)

1. Suppose X is a Poisson random variable find $P(X = 1) = P(X = 2)$ .Find the mean and variance. (3)
2. The diameter of a circular metallic discs produced by a machine is a random variable with mean 6cm and variance 2cm. Find the mean area of the discs. (3)
3. If the cumulative distribution of a continuous random variable is given by

$$F(x) = \begin{cases} 0 & x \leq 1 \\ 0.5 & 1 < x < 3, \\ 1 & x \geq 3 \end{cases}$$

find$P(X \leq 2)$(3)

4. The random variable X is exponentially distributed with mean 3. Find $P(X > t + 3|X > t)$ where t is any positive real number. (3)
5. The 95% confidence interval for the mean mass (in grams) of tablets produced by a machine is [0.56 0.57], as calculated from a random sample of 50 tablets. What do you understand from this statement? (3)
6. The mean volume of liquid in bottles of lemonade should be at least 2 litres. A sample of bottles is taken in order to test whether the mean volume has fallen below 2 litres. Give a null and alternate hypothesis for this test and specify whether the test would be one-tailed or two-tailed. (3)
7. Draw the graph represented by the following adjacency matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$ (3)

8. Give an example of a graph which has a circuit that is (i) Eularian but not Hamiltonian(ii)Hamiltonian but not Eularian (iii) neither Eularian nor Hamiltonian (3)
9. Find the value of $\chi_2(K_3)$ (3)

10. How many non isomorphic spanning tree does K_3have ?. Justify your answer (3)

PART B (Answer one question from each module)

MODULE 1

11. (a) Verify that $p(x) = \left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^x, x = 1, 2, 3$is a probability distribution. Find (i) $P(X \leq 2)$ (ii) $E[X]$ and (iii) $var(X)$. (7)

(b) Find the mean and variance of a binomial random variable (7)

OR

12. (a) Accidents occur at an intersection at a Poisson rate of 2 per day. What is the probability that there would be no accidents on a given day? What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents? (7)

(b) Two fair dice are rolled. Let X denote the number on the first die and Y = 0 or 1, according as the first die shows an even number or odd number. Find (i) the joint probability distribution of X and Y, (ii) the marginal distributions. (iii) Are X and Y independent? (7)

MODULE 2

13. (a) The IQ of an individual randomly selected from a population is a normal distribution with mean 100 and standard deviation 15. Find the probability that an individual has IQ (i) above 140 (ii) between 120 and 130. (7)

(b) A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find P(X < 0) (7)

OR

14. (a) Determine the value of c so that $f(x, y) = cxy$ for $0 < x < 3$, $0 < y < 3$ and $f(x, y) = 0$ otherwise satisfies the properties of a joint density function of random variables X and Y. Also find $P(X + Y \leq 1)$. Are X and Y independent? Justify your answer (7)

(b) The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using central limit theorem, find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hours of burning time. (7)

MODULE 3

15. (a) The mean blood pressure of 100 randomly selected persons from a target population is 127.3units. Find a 95% confidence interval for the mean blood pressure of the population. (7)

(b) The CEO of a large electric utility claims that 80 percent of his 1,000,000 customers are very satisfied with the service they receive. To test this claim, the local newspaper surveyed 100 customers, using simple random sampling. Among the sampled customers, 73 percent say they are very satisfied. Based on these findings, do you think that the CEO is making a false claim of high satisfaction levels among his customers? Use a 0.05 level of significance. (7)

OR

16. (a) A magazine reported the results of a telephone poll of 800 adult citizens of a country. The question posed was: "Should the tax on cigarettes be raised to pay for health care reform?" The results of the survey were: Out of the 800 persons surveyed, 605 were non-smokers out of which 351 answered "yes" and the rest "no". Out of the remaining 195, who were smokers, 41 answered "yes" and the remaining "no". Is there sufficient evidence, at the 0.05 significance level, to conclude that the two populations smokers and non-smokers differ significantly with respect to their opinions? (7)

(b) Two types of cars are compared for acceleration rate. 40 test runs are recorded for each car and the results for the mean elapsed time recorded below:

	Sample mean	Sample Standard Deviation
Car A	7.4	1.5
Car B	7.1	1.8

Determine if there is a difference in the mean elapsed times of the two car models at 95% confidence level. (7)

MODULE 4

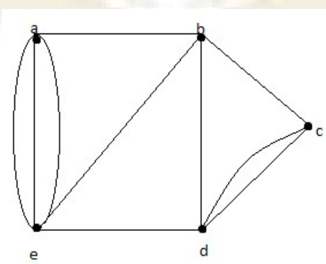
17. (a) Prove that an undirected graph has an even number of odd degree vertices (7)

(b)Show that a bipartite graph with an odd number of vertices does not have a Hamilton circuit (7)

OR

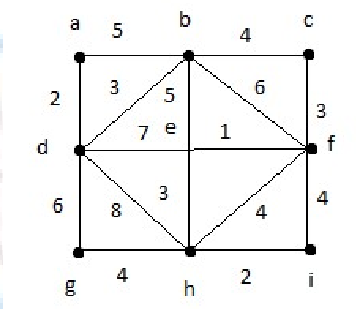
18. (a) Show that an edge in a simple graph is a cut edge if and only if this edge is not part of any simple circuit in the graph. (7)

(b) Use Fleury's algorithm to find an Euler circuit in the following graph (7)



MODULE 5

19. (a) Prove that a simple graph is a tree if and only if it is connected, but the deletion of any of it's edges produces a graph that is not connected (7)(b) Find the minimal spanning tree for the following graph by Prim's algorithm (7)



OR

20. (a)Show that a connected bipartite graph has a chromatic number of 2. (7)
(b) Prove that a full m-ary tree with l leaves has $n = \frac{ml-1}{m-1}$ vertices and $i = \frac{l-1}{m-1}$ internal vertices (7)

MAT 212	INTRODUCTION TO STOCHASTIC MODELS	CATEGORY	L	T	P	CREDIT
		BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduces students to the modern theory of probability and its applications to modelling and analysis of stochastic systems, covering important models of random variables stochastic processes. These stochastic models have important applications in engineering and are indispensible tools in reliability theory, queueing theory and decision analysis.

Prerequisite: A basic course in one-variable and multi-variable calculus**.**

Course Outcomes: After the completion of the course the student will be able to

CO 1	Develop techniques to compute probabilities of discrete distributions and selectively apply them to solve real world problems
CO 2	Develop techniques to compute probabilities of continuous distributions and selectively apply them to solve real world problems
CO 3	Analyse joint distributions, correlations and collective behaviour of multiple random variables.
CO 4	Explore stochastic phenomena using appropriate tools and models like Poisson processes
CO 5	Develop Markov chain models of selected real world phenomena and analyse them using appropriate tools

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2	2	2	2					2		1
CO 2	3	2	2	2	2					2		1
CO 3	3	2	2	2	2					2		1
CO 4	3	2	2	2	2					2		1
CO 5	3	2	2	2	2					2		1

Assessment Pattern

Bloom's Category	**Continuous Assessment Tests (%)**		**End Semester Examination (%)**
	1	**2**	
Remember	10	10	10
Understand	35	35	35
Apply	35	35	35
Analyse	10	10	10
Evaluate	10	10	10
Create			

Mark distribution

Total Marks	CIE	ESE	ESE Duration
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks
Continuous Assessment Test (2 numbers) : 25 marks
Assignment/Quiz/Course project : 15 marks

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Let X denote the number that shows up when an unfair die is tossed. Faces 1 to 5 of the die are equally likely, while face 6 is twice as likely as any other. Find the probability distribution, mean and variance of X.
2. An equipment consists of 5 componets each of which may fail independently with probability 0.15. If the equipment is able to function properly when at least 3 of the componets are operational, what is the probability that it functions properly~?
3. X is a binomial random variable $B(n,p)$with $n = 100$and $p = 0.1$. How would you approximate it by a Poisson random variable?
4. Fit a Poisson distribution to the following data which gives the number of days (f) on which $x$number of accidents have occured in an accident-prone highway for a stretch of 500 days. Fit a Poisson distribution to the data and calculate the theoretical frequencies.

x	0	1	2	3	4	5	6	7	8
f	56	156	132	92	37	22	4	0	1

Course Outcome 2 (CO2)

1. What can you say about Insert Formula P(X=a)$P(X = a)$for any real number $a$when $X$is a (i) discrete random variable? (ii) continuous random variable?
2. A string, 1 meter long, is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twise the length of the other?
3. A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3?
4. State and prove the memoryless property of exponential random variable.

Course Outcome 3(CO3):

1. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y)
2. X and Y are independent random variables with X following an exponential distribution with parameter μ and Y following and exponential distribution with parameter λ. Find $P(X + Y \leqslant 1)$
3. Random variables X and $Y$are independent with X uniformly distributed in $(-2,2)$ and Y uniformly distributed in $(-1,1)$. If $U = X + Y$ and $V = X - Y$find $\mathrm{cov}(X, Y)$.
4. A communication channel is designed to transmit a sequence of signals. But due to noise in the transmission system each signal has a probability 0.02 of being received in error. If 1000 signals are transmitted, find using Central Limit Theorem the probability that at aleast 800 of them are received without error.

Course Outcome 4(CO4):

1. A random experiment consists of observing a busy traffic intersection continuously for one hour and counting the number of cars crossing the intesection from the start of the hour upto the current time. Classify this process and plot a possible sample function (realisation) of this process.
2. A random process $X(t)$ is defined by $acos(\omega t + \Theta)$where$a$ and ω are constants and Θ is uniformly distributed in $[0,2\pi]$. Show that $X(t)$ is WSS
3. Find the mean, variance and total power of the WSS random process $X(t)$, given the autocorrelation function $R_X(\tau) = 9e^{-|\tau|}$
4. A conversation in a wireless ad-hoc network is severely disturbed by interference signals according to a Poisson process of rate $\lambda = 0.01$ per minute. (a) What is the

probability that no interference signals occur within the first two minutes of the conversation? (b) Given that the first two minutes are free of disturbing effects, what is the probability that in the next minute precisely 1 interfering signal disturbs the conversation? (c)Given that there was only 1 interfering signal in the first 3 minutes, what is the probabilty that there would be utmost 2 disturbances in the first 4 minutes?

Course Outcome 5 (CO5):

1. Consider the experiment of sending a sequence of messages across a communication channel. Due to noise, there is a small probability p that the message may be received in error. Let X_n denote the number of messages received correctly upto and including the $n-$thtransmission. Show that X_n is a homogeneous Markov chain. What are the transition probabilities~?

2. A survey conducted among consumers of two brands (A and B) of toothpastes revealed the following data; given that a person last purchased brand A, there is a 90% chance that her next purchase will be again brand A and given that a person last purchased brand B, there is an 80% chance that her next purchase will be again brand B. (i) If a person is currently a brand B purchaser, what is the probability that she will purchase brand A two purchases from now? (ii) What fraction of the consumers survayed purchase brand A? Brand B? (iii) It is estimated that a total of 1.2 crores of tooth paste units (of brand A and B combined) are purchased every year. On selling one unit of brand A tooth paste, the company earns a profit of Rs.2. For Rs.10 lakhs, an advertising firm guarantees to decrease from 10% to 5% the fraction of brand A customers who switch to brand B after a purchase. Should the company that makes brand A hire the advertising firm?

3. If $P$is the transition probability matrix of an ergodic chain, what happens to P^nas $n \to \infty$?

4. Give an example of transition probability matrix of a Markov chain in which all states are periodic of period 3.

Syllabus

Module 1 (Discrete probability distributions)

Discrete random variables and their probability distributions, Expectation, mean and variance, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Geometric distribution, Fitting binomial and Poisson distributions.

Module 2 (Continuous probability distributions)

Continuous random variables and their probability distributions, Expectation, mean and variance, Uniform distribution-mean variance, exponential distribution-mean, variance, memory less property, Normal distribution-mean, variance, use of normal tables.

Module 3 (Joint distributions)

Joint distributions- discrete and continuous, marginal distributions, expectations involving multiple random variables, independence, correlations and covariance involving pairs of random variables, central limit theorem.

Module 4 (Stochastic processes)

Stochastic processes-definition and classification, mean, autocorrelation, cross correlations, wide sense stationary processes, Poisson process-distribution of inter-arrival times, splitting and merging properties.

Module 5 (Markov chains)

Discrete time Markov chain, transition probability matrix, Chapman-Kolmogorov theorem (without proof), Computation of transient probabilities, classification of states of finite-state chains,-irreducible and ergodic chains, steady-state probability distribution,

Text Books

1. SaeedGhahramani, Fundamentals of probability with stochastic processes, Pearson Education, Third edition, 2012

2. HosseinPishro-Nik, “Introduction to Probability, Statistics and Random Processes”, Kappa Research, 2014 (Also available online at www.probabilitycourse.com)

Reference Books

1. Sheldon M Ross, “Introduction to probability models”, Elsavier.
2. Geoffrey R. Grimmett and David R. Stirzaker, “Probability and random processes”, Oxford University Press
3. Oliver C. Ibe, “Fundamentals of Applied Probability and Random Processes”, Elsevier, 2005.
4. Sundarapandian, “Probability, Statistics and Queuing Theory”, Prentice-Hall Of India.

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Discrete Probability distributions**	
1.1	Discrete random variables and probability distributions, expected value, mean and variance (discrete)	3
1.2	Binomial distribution-mean, variance, Poisson distribution-mean, variance, Poisson approximation to binomial	3
1.3	Geometricdistribution, distribution fitting	3
2	**Continuous Probability distributions**	
2.1	Continuous random variables and probability distributions, expected value, mean and variance (continuous)	3
2.2	Uniform distribution, exponential distribution, and normal distributions, mean and variance of these distributions, other properties	4
2.3	Normal distribution-mean, variance , use of normal tables	2
3	**Joint distributions**	
3.1	Discrete joint distributions, computation of probability, marginal distributions	2
3.2	Continuous joint distributions, computation of probability, marginal distributions	2
3.3	Independence of random variables, expectation involving more than one random variable	2
3.4	correlations and covariance involving pairs of random variables, central limit theorem	3
4	**Stochastic processes**	
4.1	Stochastic processes-definition and classification, mean, autocorrelation, cross correlations	3
4.2	wide sense stationary processes, properties	2
4.3	Poisson process, distribution of inter-arrival times	2

4.3	Splitting and merging of Poisson processes	2
5	**Discrete time Markov chains**	
5.1	Discrete time Markov chain, transition probability matrix, Chapman-Kolmogorov theorem	3
5.2	Computation of transient probabilities	2
5.3	classification of states of finite-state chains,-irreducible and ergodic chains	2
5.4	Steady state probability distribution of ergodic chains	2

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

MODEL QUESTION PAPER

FOURTH SEMESTER B.TECH DEGREE EXAMINATION
(Industrial Engineering)

INTRODUCTION TO STOCHASTIC MODELS

Max Marks :100 Duration : 3 Hours

PART A
(Answer *all* questions. Each question carries 3 marks)

1. Suppose X is binomial random variable with parameters $n = 100$ and $p = 0.02$. Find $P(X < 3)$ using Poisson approximation to X. (3)

2. The diameter of circular metallic discs produced by a machine is a random variable with mean 6cm and variance 2cm. Find the mean area of the discs. (3)

3. Find the mean and variance of the continuous random variable X with probability density function (3)
$$f(x) = \begin{cases} 2x - 4, & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

4. The random variable X is exponentially distributed with mean 3. Find $P(X > t + 3|X > t)$ where t is any positive real number. (3)

5. Let X denote the height (in inches) and Y denote the weight (in pounds) of a randomly chosen indivdual. If the units of X and Y are changed to centimeters and kilograms respectively, how would it affect $\text{cov}(X, Y)$ and the correlation coefficient $\rho(X, Y)$? (3)

6. State giving reasons whether the relation $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$ is true for random variables X and Y. (3)

7. Give an examle of a contiuous time discrete state random process, with non-constant mean function. (3)

8. $N(t)$ is a Poisson process with $P[N(2) = 0] = 0.1353$. Find $P[N(4) = 0]$ (3)

9. Consider the experiment of sending a sequence of messages across a communication channel. Due to noise, there is a small probability p that the message may be received in error. Let X_n denote the number of messages received correctly upto and including the n-th transmission. Is X_n a Markov chain ? Justify. (3)

10. The transition probability matrix of a Markov chain is $P = \begin{pmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{pmatrix}$. Find $P(X_3 = 2|X_1 = 1)$. (3)

PART B
(Answer one question from each module)

MODULE 1

11. (a) The probability mass function of a discrete random variable is $p(x) = kx, x = 1, 2, 3$ where k is a positive constant. Find (i)the value of k (ii) $P(X \leq 2)$ (iii) $E[X]$ and (iv) $\text{var}(1 - X)$. (7)

    (b) Find the mean and variance of a binomial random variable (7)

OR

12. (a) Accidents occur at an intersection at a Poisson rate of 2 per day. what is the probability that there would be no accidents on a given day? What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents? (7)

(b) A safety engineer feels that 35% of all industrial accidents in her plant are caused by failure of employees to follow instructions. She decides to look at the accident reports (selected randomly and replaced in the pile after reading) until she finds one that shows an accident caused by failure of employees to follow instructions. On average, how many reports would the safety engineer expect to look at until she finds a report showing an accident caused by employee failure to follow instructions? What is the probability that the safety engineer will have to examine at least three reports until she finds a report showing an accident caused by employee failure to follow instructions? (7)

MODULE 2

13. (a) Let X be a continuous random variable with density (7)

$$f(x) = \begin{cases} 0 & x < -1 \\ x & -1 \le x < 0 \\ ae^{-bx} & x \ge 0 \end{cases}$$

and expected value 1. Find the values of a and b. Also find var(X).

(b) The IQ of an individual randomly selected from a population is a normal distribution with mean 100 and standard deviation 15. Find the probability that an individual has IQ (i) above 140 (ii) between 120 and 130. (7)

OR

14. (a) A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find $P(X < 0)$ (7)

(b) Suppose that the time between customer arrivals in a store is given by an exponential random variable X, such that the average time between arrivals is 2 minutes. Suppose you walk past the store and notice its empty. What is the probability from the time you walk past the store, the store remains empty for more than 5 minutes? (7)

MODULE 3

15. (a) Two fair dice are rolled. Let X denote the number on the first die and $Y = 0$ or 1, according as the first die shows an even number or odd number. Find (i) the joint probability distribution of X and Y, (ii) the marginal distributions. (iii) Are X and Y independent ? (7)

(b) The joint density function of random variables X and Y is given by (7)

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, \quad y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(X + Y \le 1)$. Are X and Y independent? Justify.

OR

16. (a) Let X and Y be discrete random variables with joint probability mass function defined by (7)

$$f(x, y) = \begin{cases} \frac{1}{4}, & (x, y) \in \{(0, 0), (1, 1), (1, -1), (2, 0)\} \\ 0 & \text{otherwise} \end{cases}$$

Find cov(X, Y) and interpret the result. Are X and Y independent ?

(b) The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using central limit theorem, find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hours of burning time. (7)

MODULE 4

17. (a) A stochastic process is defined by $S_n = S_{n-1} + X_n \, (n = 1, 2, \ldots)$ where $S_0 = 0$ and X_i are independent random variables each taking values ± 1 with equall probability. Write any two possible realisations of this process. Also find the ensemble mean of the process. (7)

(b) A stochastic process $X(t)$ is defined by $X(t) = A\cos(\omega t) + B\sin(\omega t)$ where A and B are independent random variables with zero mean and equal variance. Show that $X(t)$ is stationary in the wide sense. (7)

OR

18. An insurance company models the arrival of insurance claims as a Poisson process with rate 60 per year.

(a) What is the probability that there are more than 3 claims in a one-month period ? What is the expected number and variance of the number of claims in a one-month period ? (7)

(b) The company estimates that the probability that an insurance claim is of more than Rs. 10 lakh is 0.2. What is the probability that there are more than 3 claims with claim amount more than Rs. 10 lakh during a 4-year period ?(Assume that the claim amounts are independent). (7)

MODULE 5

19. A survey conducted among consumers of two brands (A and B) of toothpastes reveal the following data; given that a person last purchased brand A, there is a 90% chance that her next purchase will be again brand A and given that a person last purchased brand B, there is an 80% chance that her next purchase will be again brand B,

(a) What percent of the consumers surveyed purchase brand A? brand B? (7)

(b) It is estimated that a total of 1.2 crores of tooth paste units (of brand A and B combined) are purchased every year. On selling one unit of brand A tooth paste, the company earns a profit of Rs. 2. For Rs. 10 lakhs, an advertising firm guarantees to decrease from 10% to 5% the fraction of brand A customers who switch to brand B after a purchase. Should the company that makes brand A hire the advertising firm? (7)

OR

20. (a) State the memoryless property of a Markov chain. Give one example each of a random process which is (i) a Markov chain (ii) not a Markov chain. In each case justify your claim mathematically. (7)

(b) The transition probability matrix of a discrete time Markov chain is (7)

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0.2 & 0 & 0.8 \\ 0 & 1 & 0 \end{bmatrix}$$

Classify the states as (i) periodic or aperiodic (ii) transient or recurrent. Also check whether the Markov chain is ergodic.

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