Example 5.27 Obtain the direct form I, direct form II, cascade and parallel form realization for the system $y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

Solution

Direct form I

Let $3x(n) + 3.6x(n-1) + 0.6x(n-2) = w(n)$

$y(n) = -0.1y(n-1) + 0.2y(n-2) + w(n)$

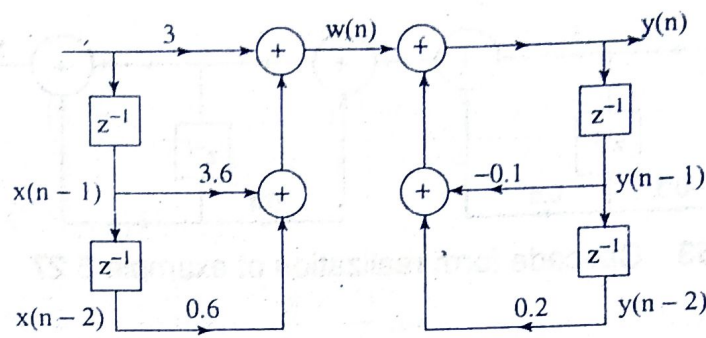By inspection, The direct form I realization is shown in Fig. 5.51.

Fig. 5.51 Direct form I realization of example 5.27

Direct form II

From the given difference equation we have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

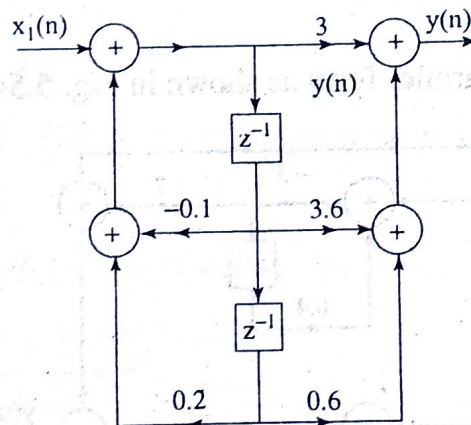The above system function can be realized in direct form II as shown in Fig. 5.52.

Fig. 5.52 Direct form II realization of example 5.27

Cascade form

we have $\dfrac{Y(z)}{X(z)} = \dfrac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$

$$= \frac{(3 + 0.6z^{-1})(1 + z^{-1})}{(1 + 0.5z^{-1})(1 - 0.4z^{-1})}$$

Let $H_1(z) = \dfrac{3 + 0.6z^{-1}}{1 + 0.5z^{-1}}$ and

$$H_2(z) = \frac{1 + z^{-1}}{1 - 0.4z^{-1}}$$

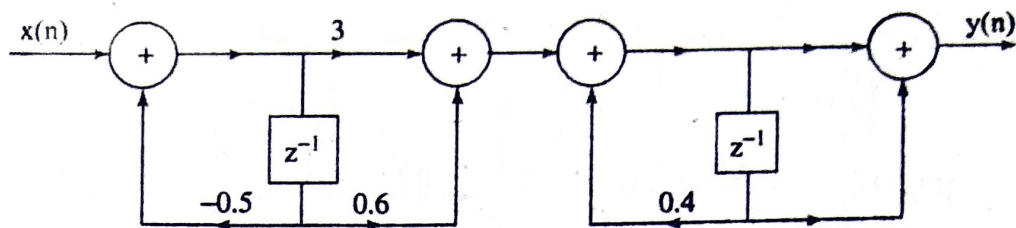Now we realize $H_1(z)$ and $H_2(z)$ and cascade both to get realization of $H(z)$

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Fig. 5.53 Cascade form realization of example 5.27

Parallel form

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= -3 + \frac{7}{1 - 0.4z^{-1}} - \frac{1}{1 + 0.5z^{-1}}$$

$$= c + H_1(z) + H_2(z)$$

$$\begin{array}{r} -3 \\ -0.2z^{-2} + 0.1z^{-1} + 1 \overline{\smash{\big)}\, 0.6z^{-2} + 3.6z^{-1} + 3} \\ 0.6z^{-2} - 0.3z^{-1} - 3 \\ \hline 3.9z^{-1} + 6 \end{array}$$

$$\rightarrow \quad H(z) = -3 + \frac{3.9z^{-1} + 6}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= -3 + \frac{A}{1 - 0.4z^{-1}} + \frac{B}{1 + 0.5z^{-1}}$$

where $A = 7, B = -1$

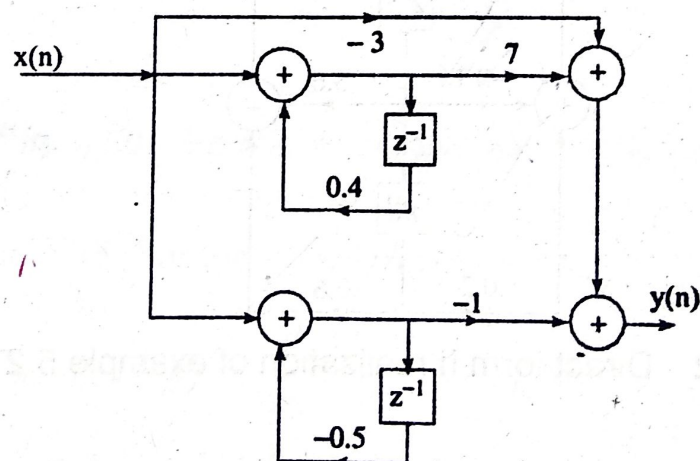Now we realize $H(z)$ in parallel form as shown in Fig. 5.54.

Fig. 5.54 Parallel form realization of example 5.27