Module V

Implementation of discrete time systems

Consider a linear time invariant discrete time system characterized by the general linear constant coefficient difference equation

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k) \quad \text{——— (1)}$$

By mean of z- transform

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z) \quad \text{——— (2)}$$

The same system can be characterized by the rational system function

$$H(z) = \frac{Y(z)}{X(z)}$$

$$(2) \Rightarrow Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = X(z) \sum_{k=0}^{M} b_k z^{-k}$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad \text{——— (3)}$$

where $\{a_k\} = \{a_1, a_2, \ldots a_N\}$ and

$\{b_k\} = \{b_0, b_1 \cdots b_M\}$ are system parameters

<u>structures for FIR systems:</u>

In general an FIR system is described by the ~~difference~~ difference equation

$$y(n) = \sum_{k=0}^{M-1} b_k \, x(n-k) \quad \text{——— (4)}$$

or $\quad y(n) = h(n) * x(n)$

$$= \sum_{k=0}^{M-1} h(k)\, x(n-k) \quad \text{——— (5)}$$

From equ (4) and (5)

$$h(n) = \begin{cases} b_n, & 0 \le n \le M-1 \\ 0, & \text{else.} \end{cases}$$

Hence the unit sample response of the FIR system $(h(n))$ is identical to the coefficients (b_n)

$\therefore$ The system function $H(z)$ is given by

$$H(z) = \sum_{k=0}^{M-1} h(k)\, z^{-k} = \sum_{k=0}^{M-1} b_k\, z^{-k}.$$

Direct form structure

Consider the non recursive(FIR) difference equation,

$$y(n) = x(n) * h(n)$$

ie $$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

Corresponding system function

$$\therefore H(z) = \sum_{k=0}^{M-1} h(k)\, z^{-k} = \frac{Y(z)}{X(z)}$$

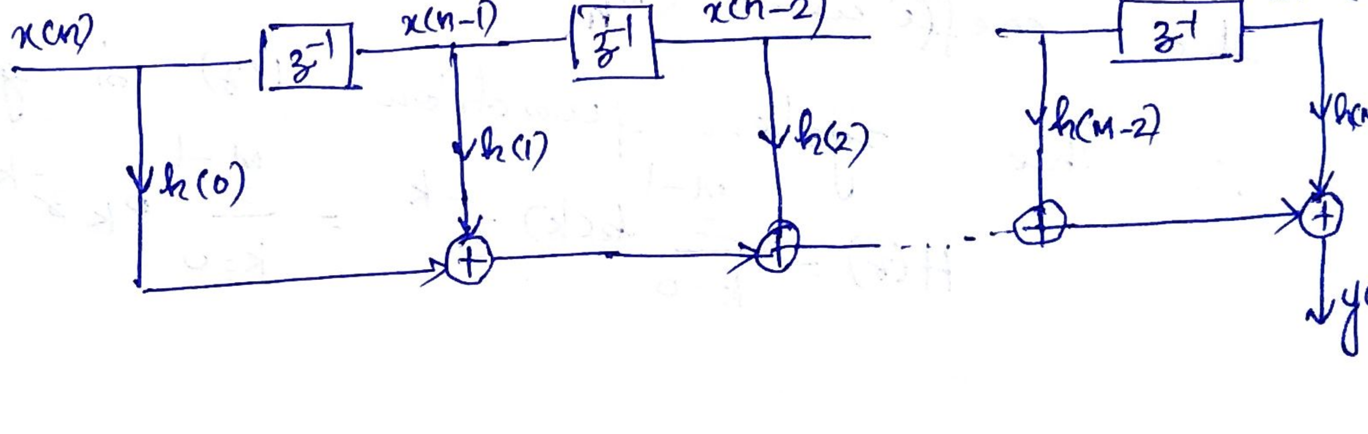
$$\frac{Y(z)}{X(z)} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \dots h(M-1) z^{-(M-1)}$$

$$Y(z) = h(0) X(z) + h(1) z^{-1} X(z) + \dots h(M-1) z^{-(M-1)} X(z)$$

$\therefore$ ~~y(n)~~ By taking inverse z-transform.

$$y(n) = h(0)\, x(n) + h(1)\, x(n-1) + \dots\dots h(M-1)\, x(n-(M-1))$$

Correspond structure can be implemented in direct form using delay element (z^{-1}) adders(+) and multipliers ($\rightarrow$)



Q) Determine the direct form realization of system function

$$H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$

Ans: Given.

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$

$$\therefore Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z) - 4z^{-3}X(z) + 5z^{-4}X(z)$$

By taking inverse z transform

$$y(n) = x(n) + 2x(n-1) - 3x(n-2) - 4x(n-3) + 5x(n-4)$$

Corresponding direct form realization