$x_2(3) = \frac{1}{2j}\{1-1j\} = \frac{1-1j}{2j} = -j(1-2j)x$

$= -2j = i2$ $\quad = -2j+2x = i$

$-2-2j$

Module-3

- FIR filters are having linear ~~relati~~ phase characteristics.
- FIR filters are stable, and a lot of design methodes for FIR filter. (direct design).
- Multifiltering, i.e, same system can do multiple type of filtering (LPF, HPF)

Design of FIR filter

An FIR filter of length M (finite impulse) with i/p $x(n)$ and o/p $y(n)$ is described by difference equation.

$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n)$

$y(n) = x(n) * h(n)$

$$y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_{M-1} x(n-M+1)$$

$$= \sum_{k=0}^{M-1} b_k x(n-k)$$

$$y(n) = \sum_{k=0}^{M-1} h(k) \cancel{z^{-k}} x(n-k)$$

$$H(z) = \sum_{k=0}^{M-1} h(k) z^{-k}$$

$h(n) = \pm h(M-1-n)$ $\quad n = 0, 1, 2, \dots, M-1.$

$$H(e^{j\omega}) = \sum_{n=0}^{M-1} h(n) e^{-j\omega n}$$

$$H(e^{j\omega}) = |H(e^{j\omega})| \cdot e^{j\theta\omega}.$$

$\alpha \rightarrow$ center of symmetry.

In FIR filter; $\alpha \rightarrow$ constant

$\theta(\omega) = -\alpha\omega$

(as FIR filter is linear)

$H_d(e^{j\omega})$ will be given $\xrightarrow{IDTFT} h_d(n) \rightarrow h[n] \rightarrow H[z]$
(impulse)
(based on design method)

Sufficient condition for FIR filter to have linear phase;

$h(n) = \pm h(m-1-n)$

If it is +ve → symmetrical
If it is -ve → Antisymmetrical $\Big\}\ \alpha$

$\alpha = \frac{m-1}{2}$

$$\tau_p = -\frac{\theta(\omega)}{\omega}$$
$$\tau_g = -\frac{d\theta(\omega)}{d\omega}$$
$$\alpha = \tau_p = \tau_g$$

Case:

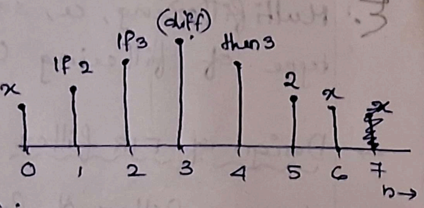
① m = odd FIR → odd symmetry.

If $m = 7$. $n = 0, 1, \cdots 6$.

$h(n) = \pm h(m-1-n)$

$h(0) = h(m-1) = h(7-1) = \underline{h(6)}$

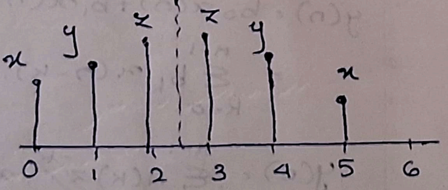
1st value same as last value.



Here center of symmetry;
$\alpha = 3$.

② When m = even.

$m = 6$.



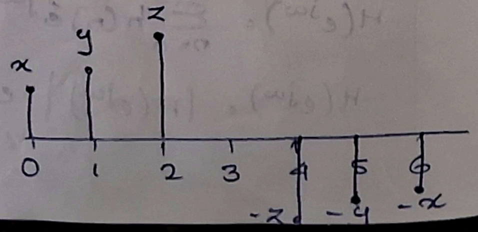
$\alpha = 2.5$

③ When antisymmetrical.

$m = 7$. (odd).

$h(0) = -h(6)$.

antisymmetrical odd in FIR then $\alpha = 0$.



always $\alpha = 0$, $h(\alpha) = 0$.

* Different methodes to design FIR filter.

Window methode.

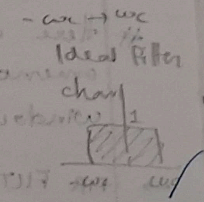
? Derive the desired impulse response of an ideal low pass digital filter (FIR).

= $h_d(n)$ (desired impulse response) $= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$.

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c}^{\omega_c} = \frac{1}{2\pi jn}\left[e^{jn\omega_c} - e^{-jn\omega_c}\right]$$

$$= \frac{1}{\pi n}\left[\frac{e^{jn\omega_c} - e^{-jn\omega_c}}{2j}\right] = \frac{1}{\pi n}\sin(n\omega_c) \quad n \neq 0$$

$$= \frac{\omega_c}{\pi}\cdot\frac{\sin(\omega_c n)}{\omega_c n}$$

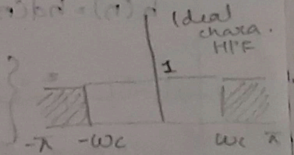


We know; $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.

When $n = 0$

$$h_d(n) = \frac{\omega_c}{\pi}$$

? Derive the desired impulse response of an ideal high pass digital filter.



= $$h_d(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} 1\, e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} 1\, e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi jn}\left[\left[e^{j\omega n}\right]_{-\pi}^{-\omega_c} + \left[e^{j\omega n}\right]_{\omega_c}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[\left(e^{-j\omega_c n} - e^{-jn\pi}\right) + \left(e^{j\pi n} - e^{j\omega_c n}\right)\right]$$

$$= \frac{1}{2\pi j n}\left[e^{-j\omega_c n} - e^{-j\pi n} + e^{j\pi n} - e^{j\omega_c n}\right]$$

$$= \frac{1}{\pi n}\left[\frac{e^{-j\omega_c n} - e^{j\omega_c n}}{2j} + \frac{e^{j\pi n} - e^{-j\pi n}}{2j}\right]$$

$$= \frac{1}{\pi n}\left[\sin(\pi n) - \sin(\omega_c n)\right] \quad n \neq 0$$

$$= \left[\frac{\sin \pi n}{\pi n} - \frac{\omega_c}{\pi}\frac{\sin(\omega_c n)}{\omega_c n}\right]$$

$$h_d(0) = 1 - \frac{\omega_c}{\pi} \quad n = 0$$

Here in Fourier Series there arises the Gibb's phenomenon so we use different method called window's methods.

→ FIR FILTER USING WINDOWS.

$$H_d(\omega) = \sum_{n=0}^{\infty} h_d(n) e^{-j\omega n}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega.$$

$$h(n) = h_d(n)\,\omega(n).$$

$$= \begin{cases} h_d(n), & n = 0,1,\dots,M-1 \\ 0, & \text{otherwise.} \end{cases}$$

$$W(\omega) = \sum_{n=0}^{M-1} w(n) e^{-j\omega n}. \quad h(n) = h_d(n)\,\omega(n) \xleftrightarrow{\text{DTF}} \frac{1}{2\pi}\left[H(j\omega) * W(j\omega)\right]$$

$$H(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(v)\, W(\omega - v)\, dv.$$

→ Rectangular window:

$$w(n) = \begin{cases} 1, & n = 0,1,\dots M-1 \\ 0, & \text{o.w.} \end{cases}$$ } time domain

Bartlett ×
Blackman ×
Haming ✓
Hanning ✓
Rectangular ✓

The FT of rectangular window is

$$W(\omega) = \sum_{n=0}^{M-1} e^{-j\omega n}.$$

$$= 1 - e$$

→ Hamming:

$$0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right)$$ } time domain

→ Hanning:

Table 6.2 ✓

$$\frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{M-1}\right)\right)$$ } time domain

Main lobe ⇒ decide transition width

Side lobe ⇒ decide noise.

Hamming better than Hanning

- Rectangular window has highest side lobe (-13) so it has high noise.
- Rectangular has low transition width (Main lobe ↓) so sharp peak.

Symmetrical linear phase FIR LPF using window.

$$H_d(\omega) = \begin{cases} 1 e^{-j\omega\left(\frac{M-1}{2}\right)}, & 0 \le |\omega| \le \omega_c \\ 0, & \text{o.w.} \end{cases} \quad \left\{ e^{-j\omega\alpha} \quad \underbrace{\alpha = \frac{m-1}{2}}_{\text{center of symmetry.}} \right\} \quad M \to \text{length}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\left(\frac{m-1}{2}\right)} e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega\left(n - \frac{m-1}{2}\right)} d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\frac{m-1}{2})}}{j(n-\frac{m-1}{2})}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi j(n-\frac{m-1}{2})}\left[e^{j(n-\frac{m-1}{2})\omega_c} - e^{-j(n-\frac{m-1}{2})\omega_c}\right]$$

$$= \frac{1}{\pi(n-\frac{m-1}{2})}\left[\frac{e^{j(n-\frac{m-1}{2})\omega_c} - e^{-j(n-\frac{m-1}{2})\omega_c}}{2j}\right]$$

$$= \frac{1}{\pi(n-\frac{m-1}{2})}\left[\sin\left(\omega_c\left(n-\frac{M-1}{2}\right)\right)\right] \quad n \neq \frac{M-1}{2}.$$

Steps: For designing of filter:

① $h_d(n)$ ② $\omega(n)$ ③ $h(n) = h_d(n)\,\omega(n)$

④ $H[z]$

Ideal case $\frac{1}{\pi n} \sin \omega_c n$

Symmetrical

? Derive the desired impulse response of an FIR filter of length M. having cutt-off frequency ω_c.

$$= \quad h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega. \qquad H_d(\omega) = 1 \cdot e^{-j\alpha\omega} = e^{-j\omega\frac{(m-1)}{2}}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega(\frac{m-1}{2})}\, e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{-j\omega(\frac{m-1}{2})} e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} e^{-j\omega(\frac{m-1}{2})} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{j\omega(n-\frac{(m-1)}{2})}\, d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\frac{(m-1)}{2})}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left(\left[\frac{e^{j\omega(n-(\frac{m-1}{2}))}}{j(n-(\frac{m-1}{2}))}\right]_{-\pi}^{-\omega_c} + \left[\frac{e^{j\omega(n-\frac{(m-1)}{2})}}{j(n-\frac{(m-1)}{2})}\right]_{\omega_c}^{\pi}\right).$$

$$= \frac{1}{2\pi j\left(n-\frac{(m-1)}{2}\right)}\left[\left(e^{-j\omega_c(n-(\frac{m-1}{2}))} - e^{-j\pi(n-\frac{(m-1)}{2})}\right) + \left[e^{j\pi(n-\frac{(m-1)}{2})} - e^{j\omega_c(n-(\frac{m-1}{2}))}\right]\right]$$

$$= \frac{1}{\pi\left(n-\frac{(m-1)}{2}\right)}\left[\frac{e^{-j\omega_c(n-\frac{(m-1)}{2})} - e^{j\omega_c(n-\frac{(m-1)}{2})}}{2j} + \frac{e^{j\pi(n-(\frac{m-1}{2}))} - e^{-j\pi(n-\frac{(m-1)}{2})}}{2j}\right].$$

$$= \frac{1}{\pi(n-\frac{(m-1)}{2})}\left[-\sin\omega_c\left(n-\frac{(m-1)}{2}\right) + \sin\pi\left(n-\frac{(m-1)}{2}\right)\right]$$

$$h_d(n) = \frac{1}{\pi(n-\frac{(m-1)}{2})}\left[\sin\pi\left(n-\frac{(m-1)}{2}\right) - \sin\omega_c\left(n-\frac{(m-1)}{2}\right)\right]$$

$$n \neq \frac{M-1}{2}.$$

When $n = \frac{m-1}{2}$; ($\alpha = 0 \Rightarrow$ then ideal case)

$$h_d(n) = 1 - \frac{\omega_c}{\pi} \Leftarrow \left(\frac{\sin\pi(n-\frac{(m-1)}{2})}{\pi(n-\frac{(m-1)}{2})} - \frac{\omega_c}{\pi}\,\frac{\sin(\omega_c(n-\frac{(m-1)}{2}))}{\omega_c(n-\frac{(m-1)}{2})}\right)$$

$(1) \qquad - \frac{\omega_c}{\pi}$

...mentation of the design:

$$H[z] = z^{-\left(\frac{m-1}{2}\right)} \left[h(0) + \sum_{n=1}^{\left(\frac{m-1}{2}\right)} z^{-n} h(n) \left[z^{-n} + z^{n} \right] \right]$$

⇒ In FIR filter general equation;

$$H(z) = \sum_{n=0}^{m-1} h(n) z^{-n} \quad — ①$$

* Location of zeroes of FIR filter in z-plane:

- In FIR filter for a zero there always exist its inverse

$z_0 \Rightarrow z_0^{-1}$

FIR is called non-recursive filter because in differential equation output depends on the present i/p and past i/p not past o/p.

IR → recursive. because in differential equation the output depends on the present input, past input and also past output i.e, there is a feedback.

Proof:

$$H[z_0] = \sum_{n=0}^{m-1} h(n) z_0^{-n} = 0 \quad — ②$$

$$= h(0) z_0^{-0} + h(1) z_0^{-1} + h(2) z_0^{-2} + \dots\dots + h(m-1) z_0^{-(m-1)} = 0 \quad ③$$

Sufficient condition:

$h(n) = h(m-1-n) \quad — ④$

④ in ③;

$$H[z_0] = h(m-1) z_0^{-(m-1)} + h(m-2) z_0^{-(m-2)} + \dots\dots + h(0) = 0$$

$$= z_0^{-(m-1)} \left[h_0 z_0^{m-1} + h(1) z_0^{m-2} + \dots + h(m-1) z_0^{0} \right] = 0.$$

$$H[z_0] = z_0^{-(m-1)} \left[\sum_{n=0}^{m-1} h(n) z_0^{n} \right] = 0$$

$$= z_0^{-(m-1)} \left[\sum_{n=0}^{m-1} h(n) (z_0^{-1})^{-n} \right] = 0 \quad — ⑤.$$

Comparing ② and ⑤.

If there exist a zero ~~and~~ its inverse exist for an FIR filter.

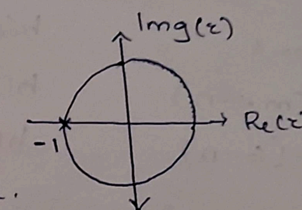
→ We can locate zeroes of FIR filter for 4 cases:

Case I: z is a real value

$z_1 = -1$ [for eg].

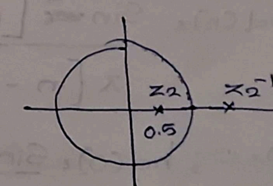
Inverse and original value ~~have~~ will ~~+~~ be same value.

Here $z_1 = -1, z_1^{-1} = -1$



~~Case~~. Case II: ~~Let~~ z_2 is real zero.

$|z_2| < 1$. [(ε − α)]

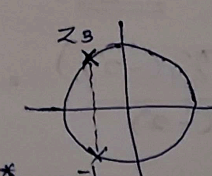


(unit circle)

Case III:

$|z_3| = 1$

Here z_3 will not be on any axis but on the circle.



$z_3^{*} = z_3^{-1}$

z_3^{-1} = conjugate of z_3.

Final output for implementation of the design:

$$H[z] = z^{-\left(\frac{m-1}{2}\right)} \left[h(0) + \sum_{n=1}^{\left(\frac{m-1}{2}\right)} z^{-n} h(n) \left[z^{-n} + z^{n} \right] \right]$$

⇒ In FIR filter general equation;

$$H(z) = \sum_{n=0}^{m-1} h(n) z^{-n} \quad — ①$$

* Location of zeroes of FIR filter in z-plane:

- In FIR filter for a zero there always exist its inverse

$z_0 \to z_0^{-1}$

Proof:

$$H[z_0] = \sum_{n=0}^{m-1} h(n) z_0^{-n} = 0 \quad — ②$$

$$= h(0) z_0^{-0} + h(1) z_0^{-1} + h(2) z_0^{-2} + \dots\dots + h(m-1) z_0^{-(m-1)} = 0 \quad └ ③$$

Sufficient condition;

$h(n) = h(m-1-n)$ — ④

④ in ③;

$m-1-(m-1)$

$m-1-m+1$

$$H[z_0] = h(m-1) z_0^{-(m-1)} + h(m-2) z_0^{-(m-2)} + \dots\dots + h(0) = 0$$

$$= z_0^{-(m-1)} \left[h_0 z_0^{m-1} + h(1) z_0^{m-2} + \dots + h(m-1) z_0^{0} \right] = 0.$$

FIR is called non-recursive filter. because In differential equation output depends on the present i/p and past i/p not past o/p.

IR → recursive. because In differential equation the output depends on present input, past input and also past output i.e, there is a feedback.

$$H[z_0] = z_0^{-(m-1)} \left[\sum_{n=0}^{m-1} h(n) z_0^{n} \right] = 0$$

$$= z_0^{-(m-1)} \left[\sum_{n=0}^{m-1} h(n) (z_0^{-1})^{-n} \right] = 0 \quad — ⑤$$

Comparing ② and ⑤.

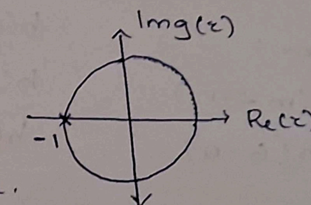
If there exist a zero ~~and~~ its inverse exist for an FIR filter.

→ We can locate zeroes of FIR filter for 4 cases:

\# Case I: z is a real value

$z_1 = -1$ [for eg].

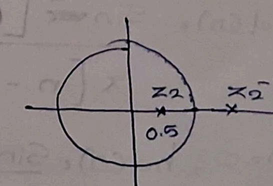
Inverse and original value will be same value.



Here $z_1 = -1$, $z_1^{-1} = -1$

\# Case II: z_2 is real zero.

$|z_2| < 1$. [(ε − α)]

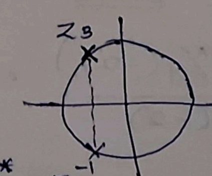


(unit circle)

\# Case III:

$|z_3| = 1$

Here z_3 will not be on any axis but on the circle.

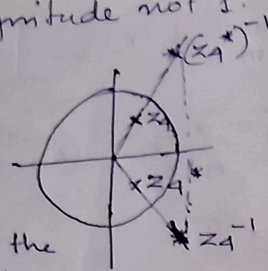


$z_3^* = z_3^{-1}$

z_3^{-1} = conjugate of z_3.

Case IV : Magnitude not 1.

$|z_4| \neq 1$.



$z_4^* \to$ in the circle.

$z_4^{-1} \to$ outside the circle.

Both are different values.

? Design an FIR LPF. of length 7 having cut of frequency $\pi/4$ using (i) Rectangular (ii) Hamming (iii) Hanning window.

= When $m = 7$

$h(n) = h(m-1-n)$

$= h(6-n)$

$h(0) = h(6)$
$h(1) = h(5)$
$h(2) = h(4)$
should be same value.

(i) $\to$ $hd(n) = \dfrac{\sin \omega_c \left[\left(n - \frac{m-1}{2}\right)\right]}{\pi\left[n - \left(\frac{m-1}{2}\right)\right]}$

For LPF $n = \infty$, $hd(n) = \dfrac{\omega_c}{\pi}$.

$\omega_c = \dfrac{\pi}{4}$, $m = 7$, $\alpha = \dfrac{m-1}{2} = \underline{\underline{3}}$.

$hd(n) = \dfrac{\sin \frac{\pi}{4}\left[(n-3)\right]}{\pi[n-3]}$, $n \neq 3$.

$n = 0$; $hd(0) = \dfrac{\sin\left[\frac{\pi}{4} \times (-3)\right]}{\pi(-3)} = \dfrac{\cancel{-}\sin(3\pi/4)}{\cancel{-}3\pi}$

$= \underline{\underline{0.075}}$

$n = 1$; $hd(1) = \dfrac{\sin(\frac{\pi}{4}(-2))}{\pi(-2)} = \dfrac{\cancel{-}\sin 2\pi/4}{\cancel{-}2\pi} = \underline{\underline{0.159}}$

$hd(2) = \dfrac{\cancel{-}\sin(\pi/4)}{\cancel{-}\pi} = 0.225$.

$hd(4) = \dfrac{\sin \pi/4}{\pi} = 0.225$.

$hd(6) = \dfrac{\sin 3\pi/4}{3\pi} = \underline{\underline{0.075}}$.

$hd(5) = \dfrac{\sin(2\pi/4)}{2\pi}$

$= \underline{\underline{0.159}}$.

$hd(3) = \dfrac{\omega_c}{\pi}$

$= \dfrac{\pi/4}{\pi}$

$= \dfrac{1}{4}$

$\to$ Next calculates the window;

$\omega n = 1$, $n \leq 0 \leq M-1$

$\to$ Final impulse response; $h(n) = hd(n) \times \omega(n)$.

$h(0) = h(6) = \underline{\underline{0.075}}$ $h(2) = h(4) = \underline{\underline{0.225}}$

$h(1) = h(5) = \underline{\underline{0.159}}$ $h(3) = \underline{\underline{\frac{1}{4}}}$.

(ii) $\to$ Desired impulse responce.

$hd(0)$, $hd(1)$, $hd(2)$, $hd(3)$, $hd(4)$, $hd(5)$, $hd(6)$

$\to$ Calculate the window; at

$\omega(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right)$.

$\omega(0) = 0.54 - 0.46\cos(0) = \underline{\underline{0.08}}$.

$\omega(1) = 0.54 - 0.46\cos\left(\frac{2\pi}{6}\right) = \underline{\underline{0.31}}$.

$\omega(2) = 0.54 - 0.46\cos\left(\frac{4\pi}{6}\right) = \underline{\underline{0.77}}$.

$\omega(3) = 0.54 - 0.46\cos(\pi) = \underline{\underline{1}}$.

$\omega(4) = 0.54 - 0.46\cos\left(\frac{8\pi}{6}\right) = \underline{\underline{0.77}}$.

$\omega(5) = 0.54 - 0.46\cos\left(\frac{10\pi}{6}\right) = \underline{\underline{0.31}}$.

$\omega(6) = 0.54 - 0.46\cos\left(\frac{12\pi}{6}\right) = \underline{\underline{0.08}}$.

→ $h(n) = hd(n)\,w(n)$

$h(0) = 0.075 \times 0.08 = \underline{\underline{6 \times 10^{-3}}}$ $\quad h(6) = \underline{\underline{6 \times 10^{-3}}}$.

$h(1) = 0.159 \times 0.31 = \underline{\underline{0.049}}$ $\quad h(5) = 0.049$.

$h(2) = 0.225 \times 0.77 = \underline{\underline{0.173}}$ $\quad h(4) = \underline{\underline{0.173}}$

$h(3) = \underline{\underline{0.25}}$.

(iii) → Desired

→ $w_n = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{M-1}\right)\right)$ $\quad w(1) = \frac{1}{2}\left(1 - \cos\frac{2\pi}{6}\right)$

$= \underline{\underline{0.25}}$.

$w(0) = \frac{1}{2}\left(1 - \cos(0)\right) = \underline{\underline{0}}$.

$w(2) = \frac{1}{2}\left(1 - \cos\left(\frac{4\pi}{6}\right)\right) = \underline{\underline{0.75}}$.

$w(3) = \frac{1}{2}\left(\left(1 - \cos(\pi)\right)\right) = \underline{\underline{1}}$.

$w(4) = \frac{1}{2}\left(\left(1 - \cos\left(\frac{8\pi}{6}\right)\right)\right) = 0.75$

$w(5) = \frac{1}{2}\left(\left(1 - \cos\frac{10\pi}{6}\right)\right) = \underline{\underline{0}}.25$.

$w(6) = \frac{1}{2}\left(\left(1 - \cos\left(\frac{12\pi}{6}\right)\right)\right) = \underline{\underline{0}}$.

→ $h(n) = hd(n)\,w(n)$

$h(0) = 0.075 \times 0 = \underline{\underline{0}}$ $\quad h(1) = 0.159 \times 0.25$

$= \underline{\underline{0.039}}$.

$h(6) = \underline{\underline{0}}$.

$h(5) = \underline{\underline{0.039}}$

$h(3) = \underline{\underline{0.25}}$

$h(2) = 0.225 \times 0.75 = \underline{\underline{0.168}}$ $\quad h(4) = \underline{\underline{0.168}}$

? Design an FIR HPF of cut off frequency $\pi/4$ rad/s. and having the desired impulse response $hd(e^{j\omega}) = e^{-j4\omega}$, $|\omega c \leq \omega \leq |\pi|$ using

$-\pi \leq \omega \leq -\omega c$.

= $\cancel{hd(n)}$: (a) Rectangular (b) Hamming (c) Hanning.

$e^{-j\alpha\omega} = hd(e^{j\omega})$. $\quad \underline{\underline{\alpha = 4}}$. $\quad \alpha = \frac{m-1}{2}$ $\quad 4 = \frac{m-1}{2}$.

$\underline{\underline{m = 9}}$.

$hd(n) = \frac{1}{\pi\left(n - \frac{m-1}{2}\right)}\left[\sin\pi\left(n - \frac{m-1}{2}\right) - \sin\frac{\pi}{4}\left(n - \frac{m-1}{2}\right)\right]$

$hd(n) = \frac{1}{\pi(n-4)}\left(\sin\pi(n-4) - \sin\frac{\pi}{4}(n-4)\right]$.

$hd(0) = \frac{1}{\pi(-4)}\left(\sin(-4\pi) - \sin\left(-\frac{4\pi}{4}\right)\right]$

$= \frac{1}{-4\pi}\left(-\sin 4\pi + \sin\pi\right) = \underline{\underline{0}}$.

$hd(1) = \frac{1}{\pi(-3)}\ \sin(-3\pi) - \sin\left(-\frac{3\pi}{4}\right)\right]$

$= -\underline{\underline{0.075}}$.

$hd(2) = \frac{1}{-2\pi}\left(-\sin 2\pi + \sin\frac{2\pi}{4}\right) = -\underline{\underline{0.159}}$.

$hd(3) = \frac{1}{-\pi}\left(-\sin(\pi) + \sin(\pi/4)\right) = -\underline{\underline{0.225}}$.

$hd(4) = 1 - \frac{\omega c}{\pi} = 1 - \frac{\pi/4}{\pi} = -\underline{\underline{0.75}}$.

$hd(5) = \frac{1}{\pi}\left(\sin(\pi) - \sin(\pi/4)\right) = -\underline{\underline{0.225}}$

$hd(6) = \frac{1}{2\pi}\left(\sin(2\pi) - \sin\frac{2\pi}{4}\right) = -\underline{\underline{0.159}}$.

$hd(7) = -\underline{\underline{0.075}}$

$hd(8) = \underline{\underline{0}}$

(i) Rectangular : $w_n = 1, \; 0 \le n \le m-1$

$h(0) = h(8) = \underline{\underline{0}}$ $\quad h(1) = h(7) = -0.075 \times 1 = \underline{\underline{0.075}}$

$h(2) = h(6) = \underline{\underline{-0.159}}$ $\quad h(3) = h(5) = \underline{\underline{-0.225}}$

$h(4) = \underline{\underline{0.75}}$.

(ii) Hamming $w(n) = 0.54 - 0.46 \cos\left(\frac{2n\pi}{m-1}\right)$

$w(0) = 0.54 - 0.46 \cos\left(\frac{2\pi}{0}\right) = \underline{\underline{0.08}}$ $\quad (m-1=8)$

$w(1) = 0.54 - 0.46 \cos\left(\frac{2\pi}{8}\right) = \underline{\underline{0.214}}$.

$w(2) = 0.54 - 0.46 \cos(\pi/4) = \underline{\underline{0.54}}$,

$w(3) = 0.54 - 0.46 \cos\left(\frac{6\pi}{8}\right) = \underline{\underline{0.86}}$.

$w(4) = 0.54 - 0.46 \cos\left(\frac{8\pi}{8}\right) = \underline{\underline{1}}$.

$w(5) = 0.54 - 0.46 \cos\left(\frac{10\pi}{8}\right) = \underline{\underline{0.86}}$.

$w(6) = \underline{\underline{0.54}}$

$w(7) = \underline{\underline{0.214}}$

$w(8) = \underline{\underline{0.08}}$.

$h(n) = h_d(n) \times w(n)$

$h(0) = h(8) = \underline{\underline{0}}$. $\quad h(1) = h(7) = -0.075 \times 0.214$
$= \underline{\underline{-0.016}}$

$h(2) = h(6) = -0.159 \times 0.54$
$= \underline{\underline{-0.085}}$

$h(3) = h(5) = -0.225 \times 0.86 = \underline{\underline{-0.193}}$.

$h(4) = \underline{\underline{0.75}}$.

(iii) Hanning. $w(n) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{m-1}\right)\right)$.

$w(0) = \frac{1}{2}(1 - \cos(0)) = \underline{\underline{0}}$.

$w(1) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi}{8}\right)\right) = \underline{\underline{0.146}}$.

$w(2) = \frac{1}{2}\left(1 - \cos\frac{4\pi}{8}\right) = \underline{\underline{0.5}}$.

$w(3) = \frac{1}{2}\left(1 - \cos\frac{6\pi}{8}\right) = \underline{\underline{0.853}}$

$w(4) = \frac{1}{2}(1 - \cos\pi) = \underline{\underline{1}}$.

$w(5) = \frac{1}{2}\left(1 - \cos\frac{10\pi}{8}\right) = \underline{\underline{0.853}}$

$w(6) = \underline{\underline{0.5}}$ $\quad w(7) = \underline{\underline{0.146}}$ $\quad w(8) = \underline{\underline{0}}$.

$h(0) = h(8) = \underline{\underline{0}}$ $\quad h(1) = h(7) = -0.075 \times 0.146$
$= \underline{\underline{-0.0109}}$.

$h(2) = h(6) = -0.159 \times 0.5$
$= \underline{\underline{-0.079}}$.

$h(3) = h(5) = -0.225 \times 0.853$ $\quad h(4) = \underline{\underline{0.75}}$.
$= \underline{\underline{-0.1919}}$.

* Design of IIR filter from Analog filters.

$$H_a(s) = \frac{B(s)}{A(s)} = \frac{\sum_{k=0}^{M} \beta_k s^k}{\sum_{k=0}^{N} \alpha_k s^k}.$$

(Design is indirect)

IIR filter first we have to design the analog counterpart.

$$H_a(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt.$$

impulse responce.

$\beta_k, \alpha_k \rightarrow$ filter coefficient.

Analog filter having the rational sys function $H(s)$ can be described by the linear constant coeff differential equation;

$$\sum_{k=0}^{N} \alpha_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} \beta_k \frac{d^k x(t)}{dt^k}$$

Taking Laplace; $\sum_{k=0}^{N} \alpha_k s^k = \sum_{k=0}^{M} \beta_k s^k$

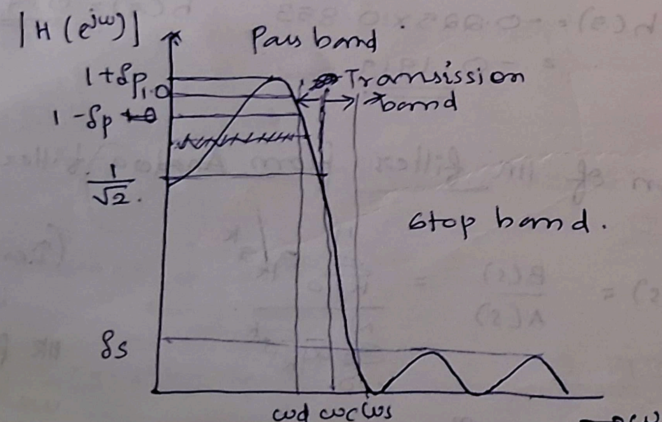
→ Indirect Design:

Step: ① Map the digital specification to those of analog filter specification.

② Derive the TF of analog LP prototype

③ ~~Design~~ Convert this analog LP prototype to given filter. (LP, HP, BP) type.

④ Final digital transformation using number of methods. (Bilinear transfer etc.) $\{H(s) \to H(z)\}$



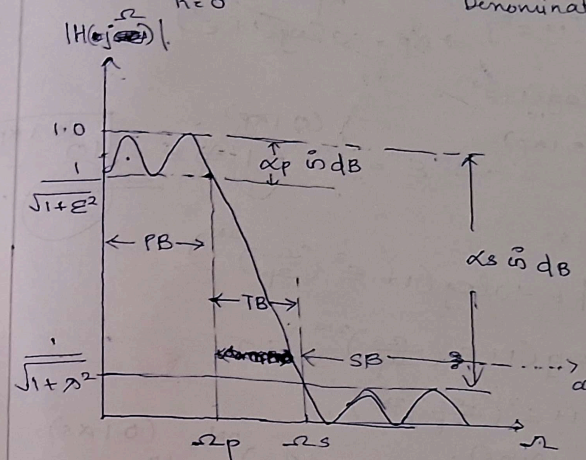
Instead of; $1-\delta_p = \frac{1}{\sqrt{1+\varepsilon^2}}$ ω_s = stopband frequency.

$\delta_s = \frac{1}{\sqrt{1+\lambda^2}}$

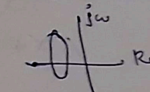
$$H(s) = \frac{\sum_{k=0}^{M} \beta^k s^k}{\sum_{k=0}^{N} \alpha_k s^k} = \frac{N(s)}{D(s)}$$ Numerator / Denominator $(N \geq M)$

mostly $N = M$

$N \to$ order of filter.



Stable filter: poles are in left side



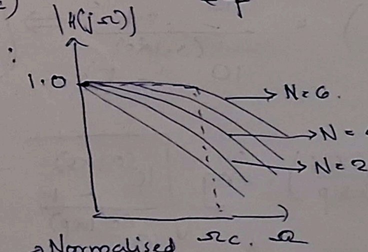
Butterworth: Flat passband and Flat stop band.

All pole filter $\left(H(s) = \frac{1}{D(s)}\right)$

For butterworth:

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} = \frac{1}{1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}} \quad \text{--- ①}$$

For diff values of N:



Steps: ① ~~#~~ N to find Butterworth filter. ② Normalised $H_N(s)|_{\Omega_c = 1 \text{rad/sec}}$ ③ Apply the diff ~~of~~ analog freq ~~TF~~ transformation (TF)
LP → LP   LP to HP   LP → BP   LP → BR.

Always first we have to find LP analog prototype

$(2\times 10\log|H(j\Omega)|)$

$$20\log|H(j\Omega)| = -10\log\left[1+\epsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}\right] \quad \text{—} \; ②$$

when $\Omega = \Omega_p$; $\underbrace{20\log|H(j\Omega_p)|}_{\alpha_p} = -10\log[1+\epsilon^2]$

$-\alpha_p = -10\log[1+\epsilon^2] \Rightarrow \alpha_p = 10\log(1+\epsilon^2)$

$0.1\alpha_p = \log_{10}(1+\epsilon^2)$.

$$(1+\epsilon^2) = 10^{(0.1\alpha_p)} \Rightarrow \epsilon^2 = 10^{(0.1\alpha_p)} - 1 \Rightarrow \epsilon = \sqrt{10^{(0.1\alpha_p)} - 1}.$$

At ② $\Omega = \Omega_s$;

$$20\log|H(j\Omega_s)| = -10\log\left(1+\epsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right).$$

$$-\alpha_s = -10\log\left(1+\epsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right).$$

$$0.1\alpha_s = \log\left(1+\epsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right).$$

$$1+\epsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = 10^{(0.1\alpha_s)}. \Rightarrow \epsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = 10^{(0.1\alpha_s)} - 1.$$

$$\epsilon^2 = \frac{10^{(0.1\alpha_s)} - 1}{\cancel{\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}}} \qquad \left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = \frac{10^{(0.1\alpha_s)} - 1}{\epsilon^2}.$$

$$\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = \frac{10^{(0.1\alpha_s)} - 1}{10^{(0.1\alpha_p)} - 1}. \Rightarrow \left(\frac{\Omega_s}{\Omega_p}\right)^{N} = \sqrt{\frac{10^{(0.1\alpha_s)} - 1}{10^{(0.1\alpha_p)} - 1}}.$$

$$N\log_{10}\left(\frac{\Omega_s}{\Omega_p}\right) = \log\sqrt{\frac{10^{(0.1\alpha_s)} - 1}{10^{(0.1\alpha_p)} - 1}}$$

$$N \geq \frac{\log\sqrt{\frac{10^{(0.1\alpha_s)} - 1}{10^{(0.1\alpha_p)} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log(A)}{\log(1/K)}. \qquad \left\{ A = \frac{\lambda}{\epsilon}, \; K = \frac{\Omega_p}{\Omega_s} \right\}$$

$$\Omega_c = \frac{\Omega_p}{\epsilon^{1/N}} = \frac{\Omega_s}{\lambda^{1/N}} \qquad \Omega_c \to \text{cutt off frequency.}$$

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} = \frac{1}{1+\epsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}}$$

$$\left(\frac{\cancel{\Omega^{2N}}}{\Omega_c^{2N}}\right) = \epsilon^2\left(\frac{\cancel{\Omega^{2N}}}{\Omega_p^{2N}}\right)$$

$$\left(\frac{\Omega_c}{\Omega_p}\right)^{2N} = \frac{1}{\epsilon^2}.$$

$$(\Omega_c)^{\frac{2N}{2N}} = \frac{(\Omega_p)^{\frac{2N}{2N}}}{\epsilon^{\frac{2}{2N}}}$$

$$\Omega_c = \frac{\Omega_p}{\epsilon^{1/N}}.$$

$$\left(\frac{\Omega_s}{\Omega_p}\right)^N = \frac{\sqrt{10^{0.1\alpha_s} - 1}}{\sqrt{10^{0.1\alpha_p} - 1}} \Rightarrow \left(\frac{\Omega_s}{\Omega_p}\right)^N = \frac{\lambda}{\epsilon}.$$

$$(\Omega_s)^N = (\Omega_p)^N \frac{\lambda}{\epsilon}.$$

$$(\Omega_s)^N = (\Omega_c\epsilon^{1/N})^N \frac{\lambda}{\epsilon} \Rightarrow (\Omega_s)^N = (\Omega_c)^N \cancel{\epsilon} \frac{\lambda}{\cancel{\epsilon}}$$

$$\Omega_s = \Omega_c \lambda^{1/N}$$

$$\Omega_c = \frac{\Omega_s}{\lambda^{1/N}}.$$