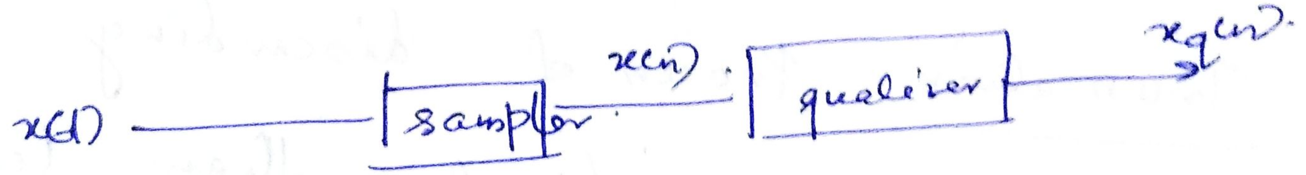
ADC Qualization noise :-

The process of converting analog to digital signal.



quantization noise of A/D conversion noise)

$$e(n) = x_q(n) - x(n).$$

- If a sinusoidal signal vary b/w +1 and −1 with dynamic range 2. and if ADC used $(b+1)$ bits excluding sign bit

- then the no. of levels available for quantizing $x(n)$ is 2^{b+1}

∴ ~~step~~ The interval between successive levels.

$$q = \frac{2}{2^{b+1}} = 2^{-b}.$$

~~Two types of~~

Two common methods of quantization are

① Truncation

② Rounding

Truncation: Process of discarding all bits less significant than least significant bit that is retained.

0.00110011 $\longrightarrow$ 0.0011

8 bits $\qquad$ 4 bits.

1.01001001 $\longrightarrow$ 1.0100

8 bits. $\qquad$ 4 bits.

Rounding:

Rounding of a number of b bits is accomplished by choosing the rounded result as b bits number closest to the original number unrounded.

0.11010 $\longrightarrow$ 0.110

or

~~0.110 11 1111~~ 0.111

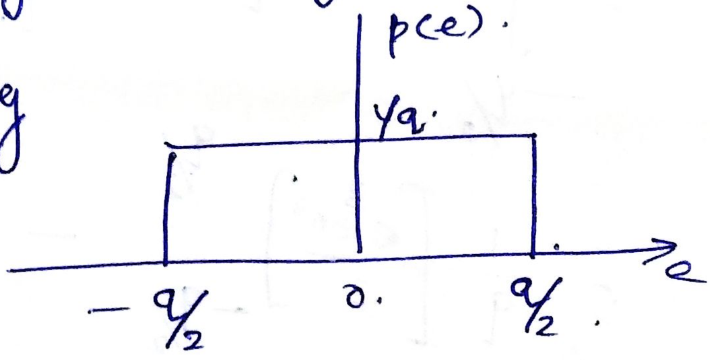
0.110 11 11 11 $\longrightarrow$ 0.111 00 000

shifts. $\hookrightarrow$ 0.110 1111.

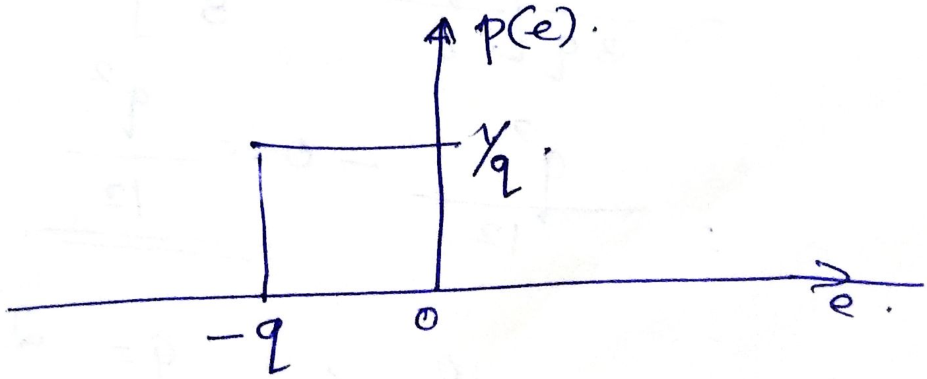
Steady state input noise power (variance of error). σ_e^2

probability density function of error

rounding



pdf of truncation



For rounding:

quantisation error sequence.

$$e(n) = x_q(n) - x(n).$$

variance of $e(n)$ is

$$\sigma_e^2 = E\left[e^2(n)\right] - E^2\left[e(n)\right]$$

$$= \int_{-\infty}^{\infty} e^2(n)\, p(e)\, de - \left[\int_{-\infty}^{\infty} e(n)\, p(e)\, de\right]^2$$

$$= \int_{-q/2}^{q/2} e^2(n)\, 1/q\, de - \left[\int_{-q/2}^{q/2} e(n) \frac{1}{q}\, de\right]^2$$

$$= \frac{1}{q}\left[\frac{e^3(n)}{3}\right]_{-q/2}^{q/2} - \frac{1}{q}\left[e^2(n)\right]_{-q/2}^{q/2}$$

$$= \frac{1}{3q}\left[\frac{q^3}{8} + \frac{q^3}{8}\right] - \frac{1}{q}\left[\frac{q^2}{4} - \frac{q^2}{4}\right]$$

$$= \frac{q^2}{12} - 0 = \frac{q^2}{12}.$$

we have $q = 2^{-b}$

$$\therefore \sigma_e^2 = \frac{(2^{-b})^2}{12} = \frac{2^{-2b}}{12}.$$

In the case of truncation.

$$\sigma_e^2 = E\left[e^2(n)\right] - E^2\left[e(n)\right].$$

$$= \int_{-\infty}^{\infty} e^2(n)\, p(e)\, de - \left[\int_{-\infty}^{\infty} e(n)\, p(e)\, de\right]^2$$

$$= \int_{-q}^{0} e^2(n)\, \tfrac{1}{q}\, de - \left[\int_{q}^{0} e(n)\, \tfrac{1}{q}\, de\right]^2$$

$$= \frac{1}{q}\left[\frac{e^3(n)}{3}\right]_{-q}^{0} - \left[\frac{1}{q}\left[\frac{e^2}{2}\right]_{-q}^{0}\right]^2$$

$$= \frac{1}{3q}\left[q^3 + 0\right] - \left[\frac{1}{q}\left[\frac{q^2}{2} - 0\right]\right]^2$$

$$= \frac{q^2}{3} - \left[\frac{q}{2}\right]^2 = \frac{q^2}{3} - \frac{q^2}{4}.$$

$$= \frac{4q^2 - 3q^2}{12} = \frac{q^2}{12}$$

$$\therefore \sigma_e^2 = \frac{(2^{-b})^2}{12} = \frac{2^{-2b}}{12}.$$

i.e. In both cases the value of $\sigma_e^2 = \frac{2^{-2b}}{12}$

~~which is also known as the~~

steady state noise power due to

input qualization is variance of noise $= \sigma_e^2 = 2^{-2b}/12$.

- If input signal is $x(n)$ and its variance is σ_x^2.

then SNR for rounding $= \dfrac{\text{signal power}}{\text{noise power}}$

$$= \frac{\sigma_x^2}{\sigma_e^2}$$

$$= \frac{\sigma_x^2}{2^{-2b}/12} = \underline{12 \times 2^{2b} \times \sigma_x^2}$$

Log scale SNR in dB

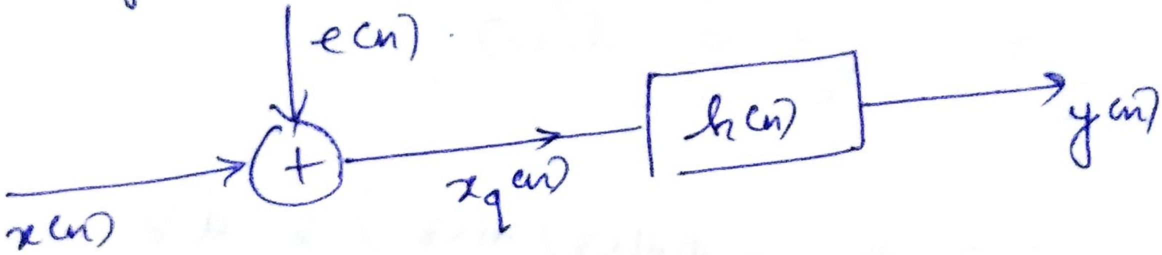
$$= 10 \log_{10} \frac{\sigma_x^2}{\sigma_e^2} = 10 \log \left[12 \times 2^{2b} \times \sigma_x^2 \right]$$

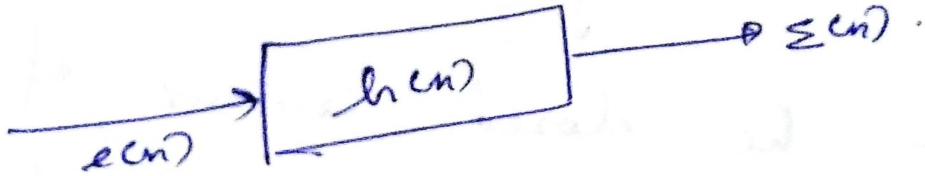
$$= \underline{10.79 + 6.02b + 10 \log \sigma_x^2}$$

ie SNR increases approximately 6 dB for each bit added to register length.

Steady state output noise power

Due to A/D conversion noise one can represent the quantized input to a digital system with impulse response $h(n)$ can be represented as.





Let $\varepsilon(n)$ be the output noise due to quantization

$$\varepsilon(n) = e(n) * h(n)$$

The variance of any term in the above sum is $\sigma_e^2 h^2(n)$

– If quantization errors are independent at different sampling ~~rate~~

instants then variance of o/p

$$\sigma_{\varepsilon}^{2}(n) = \sigma_e^2 \sum_{n=0}^{\infty} h^2(n).$$

using Parseval's theorem the steady state output noise variance due to the quantization error is

$$\sigma_{\varepsilon}^{2} = \sigma_e^2 \sum_{n=0}^{\infty} h^2(n)$$

$$= \frac{\sigma_e^2}{2\pi j} \oint H(z)\, H(z^{-1})\, z^{-1}\, dz.$$

where the closed contour of integration is around the unit circle $|z|=1$ in which case only the poles that lie inside the unit circle are evaluated using residue theorem.