

ASSIGNMENT MATHS

Submitted By : Arun C.

Roll No: 50

ECE - B

Sy

2 Solve the system of equation using

a) Gauss Jacobi Method

b) Gauss Seidal method

$$x - 2y + 5z = 12, \quad 5x + 2y - z = 6, \quad 2x + 6y - 3z = 5$$

a) Gauss Jacobi Method.

$$x - 2y + 5z = 12$$

$$5x + 2y - z = 6$$

$$2x + 6y - 3z = 5$$

after re-arranging.

$$5x + 2y - z = 6$$

$$2x + 6y - 3z = 5$$

$$x - 2y + 5z = 12$$

here

$$|5| > |2| + |-1|$$

$$|6| > |2| + |-3|$$

$$|5| > |1| + |-2|$$

$$\left. \begin{aligned} \text{Then } x &= \frac{6 - 2y + z}{5} \\ y &= \frac{5 - 2x + 3z}{6} \end{aligned} \right\} (1)$$

$$z = \frac{12 - x + 2y}{5} \} \in (1)$$

Taking $x^{(0)} = 0, y^{(0)} = 0, z^{(0)}$ as initial approximation.

First Iteration

$$x^{(1)} = \frac{6 - 2y^{(0)} + z^{(0)}}{5} = 1.2$$

$$y^{(1)} = \frac{5 - 2x^{(0)} + 3z^{(0)}}{6} = \frac{5}{6} = 0.83$$

$$z^{(1)} = \frac{12 - x^{(0)} + 2y^{(0)}}{5} = \frac{12}{5} = 2.4$$

Second Iteration

$$x^{(2)} = \frac{6 - 2y^{(1)} + z^{(1)}}{5} = \frac{6 - 2 \times 0.83 + 2.4}{5} = 1.348$$

$$y^{(2)} = \frac{5 - 2x^{(1)} + 3z^{(1)}}{6} = \frac{5 - 2 \times 1.2 + 3 \times 2.4}{6} = 1.63$$

$$z^{(2)} = \frac{12 - x^{(1)} + 2y^{(1)}}{5} = \frac{12 - 1.2 + 2 \times 0.83}{5} = 2.492$$

Third Iteration

$$x^{(3)} = \frac{6 - 2y^{(2)} + z^{(2)}}{5} = \frac{6 - 2 \times 1.63 + 2.492}{5} = 1.046$$

$$y^{(3)} = \frac{5 - 2x^{(2)} + 3z^{(2)}}{6} = \frac{5 - 2 \times 1.348 + 3 \times 2.492}{6} = 1.63$$

$$z^{(3)} = \frac{12 - x^{(2)} + 2y^{(2)}}{5} = \frac{12 - (1.348) + 2 \times (1.63)}{5} = 2.71$$

Fourth Iteration

$$x^{(4)} = \frac{6 - 2y^{(3)} + z^{(3)}}{5} = \frac{6 - 2 \times 1.63 + 2.732}{5} = 1.104$$

$$y^{(4)} = \frac{5 - 2x^{(3)} + 3z^{(3)}}{6} = \frac{5 - 2 \times 1.046 + 3 \times 2.782}{6} = 1.875$$

$$z^{(4)} = \frac{12 - x^{(3)} + 2y^{(3)}}{5} = \frac{12 - 1.046 + 2 \times 1.63}{5} = 2.842$$

Fifth Iteration

$$x^{(5)} = \frac{6 - 2y^{(4)} + z^{(4)}}{5} = \frac{6 - 2(1.875) + 2.842}{5} = \underline{\underline{1.0184}}$$

$$y^{(5)} = \frac{5 - 2x^{(4)} + 3z^{(4)}}{6} = \frac{5 - 2(1.104) + 3(2.842)}{6} = \underline{\underline{1.88}}$$

$$z^{(5)} = \frac{12 - x^{(4)} + 2y^{(4)}}{5} = \frac{12 - 1.104 + 2 \times 1.875}{5} = \underline{\underline{2.92}}$$

b) Gauss Seidel method.

After rearranging the given equation.

$$5x - 2y - z = 6$$

$$2x + 6y - 3z = 5$$

$$x - 2y + 5z = 12$$

So, now

$$|a_1| > |b_1| + |c_1|$$

$$|b_2| > |a_2| + |c_2|$$

$$|c_3| > |b_3| + |a_3|$$

Here:

$$|5| > |2| + |-1|$$

$$|6| > |2| + |-3|$$

$$|5| > |1| + |-2|$$

then

$$x = \frac{6 - 2y + 2}{5}$$

$$y = \frac{5 - 2x + 3z}{6}$$

$$z = \frac{12 - x + 2y}{5}$$

taking initial approximation as $x^{(0)} = 0$, $y^{(0)} = 0$, $z^{(0)}$
then.

first Iteration

$$x^{(1)} = \frac{6 - 2y^{(0)} + 2z^{(0)}}{5} = \frac{6 - 2(0) + 0}{5} = 1.2$$

$$y^{(1)} = \frac{5 - 2x^{(1)} + 3z^{(0)}}{6} = \frac{5 - 2 \times 1.2 + 3 \times 0}{6} = 0.43$$

$$z^{(1)} = \frac{12 - x^{(1)} + 2y^{(1)}}{5} = \frac{12 - 1.2 + 2 \times 0.43}{5} = 2.332$$

Second Iteration

$$x^{(2)} = \frac{6 - 2y^{(1)} + z^{(1)}}{5} = \frac{6 - 2 \times 0.43 + 2.332}{5} = 1.494$$

$$y^{(2)} = \frac{5 - 2x^{(2)} + 3z^{(1)}}{6} = \frac{5 - 2 \times 1.494 + 3 \times 2.332}{6} \\ = 1.501$$

$$z^{(2)} = \frac{12 - x^{(2)} + 2y^{(2)}}{5} = \frac{12 - 1.494 + 2 \times 1.501}{5} = 2.701$$

Third Iteration

$$x^{(3)} = \frac{6 - 2y^{(2)} + z^{(2)}}{5} = \frac{6 - 2 \times 1.501 + 2.701}{5} = 1.13$$

$$y^{(3)} = \frac{5 - 2x^{(3)} + 3z^{(2)}}{6} = \frac{5 - 2 \times 1.139 + 3 \times 2.701}{6} = 1.304$$

$$z^{(3)} = \frac{12 - x^{(3)} + 2y^{(3)}}{5} = \frac{12 - 1.139 + 3 \times 1.304}{5} = 2.893$$

Fourth Iteration

$$x^{(4)} = \frac{6 - 2y^{(3)} + z^{(3)}}{5} =$$

$$\frac{6 - 2 \times 1.804 + 2.893}{5} = 1.057$$

$$y^{(4)} = \frac{5 - 2x^{(4)} + 3z^{(3)}}{6} = \frac{5 - 2 \times 1.057 + 3 \times 2.893}{6} = 1.927$$

$$z^{(4)} = \frac{12 - x^{(4)} + 2y^{(4)}}{5} = \frac{12 - 1.057 + 2(1.927)}{5} = 2.955$$

Fifth Iteration

$$x^{(5)} = \frac{6 - 2y^{(4)} + z^{(4)}}{5} = \frac{6 - 2 \times 1.927 + 2.959}{5} = \underline{\underline{1.021}}$$

$$y^{(5)} = \frac{5 - 2x^{(5)} + 3z^{(4)}}{6} = \frac{5 - 2 \times 1.021 + 3 \times 2.959}{6} = \underline{\underline{1.972}}$$

$$z^{(5)} = \frac{12 - x^{(5)} + 2y^{(5)}}{5} = \frac{12 - 1.021 + 2 \times 1.972}{5} = \underline{\underline{2.98}}$$