VECTOR SPACES

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- ‘V’ be a **set of elements** on which binary addition + is defined.
- ‘F’ be a **field**.
- Multiplication operation ‘.’ is also defined b/w the elements in F & V
- Let V is a vector space over the field F, if it satisfies the following conditions :

- V is commutative under addition
- For any element ‘a’ in F & any element ‘v’ in V, a.v is an element in V.
- Distributive law:

  Let elements u & v in V and a & b in F , then

  $a.(u+v)= a.u + a.v$

  $(a+b).v = a.v +b.v$
- Associative law :

  Let any v in V & any a & b in F,

  $(a.b).v = a.(b.v)$
- Let 1 be the unit element of F. Then, for any v in V, $1.v=v$

- Elements of V are called **vectors**
- Elements in F are called **scalars**
- Addition on V is called **vector addition**
- Multiplication that combines a scalar in F and vector in V into a vector in V is called **scalar multiplication**

Properties

1) Let 0 be the zero element of the field F. for any vector v in V , $0.v = 0$
2) For any scalars c in F ,$c.0 = 0$
3) For any scalar c in F & any vector v in V,

   $(-c).v = c.(-v) = -(c.v)$

Example 2.11

Let $n = 5$. The vector space V_5 of all 5-tuples over GF(2) consists of the following 32 vectors:

$$
\begin{array}{llll}
(0\ 0\ 0\ 0\ 0), & (0\ 0\ 0\ 0\ 1), & (0\ 0\ 0\ 1\ 0), & (0\ 0\ 0\ 1\ 1), \\
(0\ 0\ 1\ 0\ 0), & (0\ 0\ 1\ 0\ 1), & (0\ 0\ 1\ 1\ 0), & (0\ 0\ 1\ 1\ 1), \\
(0\ 1\ 0\ 0\ 0), & (0\ 1\ 0\ 0\ 1), & (0\ 1\ 0\ 1\ 0), & (0\ 1\ 0\ 1\ 1), \\
(0\ 1\ 1\ 0\ 0), & (0\ 1\ 1\ 0\ 1), & (0\ 1\ 1\ 1\ 0), & (0\ 1\ 1\ 1\ 1), \\
(1\ 0\ 0\ 0\ 0), & (1\ 0\ 0\ 0\ 1), & (1\ 0\ 0\ 1\ 0), & (1\ 0\ 0\ 1\ 1), \\
(1\ 0\ 1\ 0\ 0), & (1\ 0\ 1\ 0\ 1), & (1\ 0\ 1\ 1\ 0), & (1\ 0\ 1\ 1\ 1), \\
(1\ 1\ 0\ 0\ 0), & (1\ 1\ 0\ 0\ 1), & (1\ 1\ 0\ 1\ 0), & (1\ 1\ 0\ 1\ 1), \\
(1\ 1\ 1\ 0\ 0), & (1\ 1\ 1\ 0\ 1), & (1\ 1\ 1\ 1\ 0), & (1\ 1\ 1\ 1\ 1).
\end{array}
$$

The vector sum of $(1\ 0\ 1\ 1\ 1)$ and $(1\ 1\ 0\ 0\ 1)$ is

$$(1\ 0\ 1\ 1\ 1) + (1\ 1\ 0\ 0\ 1) = (1 + 1,\ 0 + 1,\ 1 + 0,\ 1 + 0,\ 1 + 1) = (0\ 1\ 1\ 1\ 0).$$

Using the rule of scalar multiplication defined by (2.28), we obtain

$$0 \cdot (1\ 1\ 0\ 1\ 0) = (0 \cdot 1, 0 \cdot 1, 0 \cdot 0, 0 \cdot 1, 0 \cdot 0) = (0\ 0\ 0\ 0\ 0),$$
$$1 \cdot (1\ 1\ 0\ 1\ 0) = (1 \cdot 1, 1 \cdot 1, 1 \cdot 0, 1 \cdot 1, 1 \cdot 0) = (1\ 1\ 0\ 1\ 0).$$

The vector space of all n-tuples over any field F can be constructed in a similar manner. However, in this book, we are concerned only with the vector space of all n-tuples over GF(2) or over an extension field of GF(2) [e.g., GF(2^m)].

V being a vector space over a field F, it may happen that a subset S of V is also a vector space over F. Such a subset is called a *subspace* of V.