$$X_3(3) = \frac{1}{4}\sum_{k=0}^{3} x(k)\, e^{\frac{j3\pi k}{2}}$$

$$= \frac{1}{4}\left[12e^{0} + 4e^{j3\pi}\right]$$

$$= \frac{1}{4}\left[12 + 4(\cos 3\pi + i\sin 3\pi)\right]$$

$$= \frac{12+4(-1)}{4}$$

$$= \underline{\underline{2}}$$

$$x(n) = \underline{\{4, 2, 2, 2\}}$$

MODULE 3

DESIGN OF FIR FILTERS

→ Symmetric and antisymmetrical FIR structures.

→ Design of FIR filters using window methods.
- using rectangular window
- using hamming window
- hanning window

→ Comparison of design methods for linear phase FIR filters

The main advantages of FIR filters are
i) They are stable
ii) They can be easily designed.
iii) Free of limit cycle oscillations.
iv) Various design methods are available.

One possible way of obtaining FIR filter is to truncate the infinite fourier series at $n = \pm\frac{(N-1)}{2}$ where N is the length, or period of the sequence. Sometimes oscillations will be available in the o/p signal. To reduce the oscillations, the fourier coefficients of the filter are modified by multiplying the impulse response $h_d(n)$ by a waiting sequence $w(n)$ called "window"

$h(n) = h_d(n)\, \omega(n)$.

Types of Window Functions.

1. Rectangular Window: It is given by

$$\omega_R(n) = \begin{cases} 1 & \text{for } -\frac{(N-1)}{2} \le n \le \frac{N-1}{2} \\ 0 & \text{o.w.} \end{cases}$$

The spectrum of rectangular window is given by

$$W_R(e^{j\omega}) = \sum_{n=-\frac{(N-1)}{2}}^{\frac{N-1}{2}} e^{-j\omega n}$$

The impulse response

$$h(n) = h_d(n)\, \omega_R(n)$$

where $h_d(n) \rightarrow$ desired impulse response

$\omega_R(n) \rightarrow$ Rectangular window response function

The freq. response is given by

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, W_R(e^{j\omega})\, d\omega$$

The convolution of desired response and window response give rise to ripples in both passband & stop band this effect is known as Gibbs' phenomenon

2. Hanning Window ($W_{Hn}(n)$)

$$W_{Hn}(n) = \begin{cases} 0.5+0.5\cos\frac{2\pi n}{N-1} & \text{for } -\frac{(N-1)}{2} \le n \le \frac{N-1}{2} \\ 0 & \text{o.w} \end{cases}$$

3) Hamming Window ($W_H(n)$)

$$W_H(n) = \begin{cases} 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right), & \text{for } -\frac{(N-1)}{2} \le n \le \frac{N-1}{2} \\ 0, & \text{o.w} \end{cases}$$

Since the hamming windows generates less oscillations this window is generally prepared.

Steps to Design FIR filters using Window

1. Find out the window coefficient, $\omega(n)$.
2. Obtain the desired impulse response or desired filter coefficient $h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$.
3. Obtain the impulse response or filter coefficients $h(n) = h_d(n)\, \omega(n)$.
4. The transfer function of the filter is given by
$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\, [z^{n} + z^{-n}]$$
(non realisable transfer function)
5. The realisable transfer function is given by
$$H'(z) = z^{-\frac{(N-1)}{2}} H(z)$$
6. From the realisable transfer function $H'(z)$, obtain the causal filter coefficients $h'(n)$ (ie coefficients of z^{-n})
7. Find out the freq. response

$\bar{H}(e^{j\omega})$ using the eqtn

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

where $a(0) = h'(\frac{N-1}{2})$

$a(n) = 2h'[\frac{N-1}{2} - n]$

8. Find $\bar{H}(e^{j\omega})$ for different values of ω

9. Find $|\bar{H}(e^{j\omega})|$ using the equation.

$$|\bar{H}(e^{j\omega})| = 20 \log |\bar{H}(e^{j\omega})|$$

10. Plot the graph with ω along x axis & $|\bar{H}(e^{j\omega})|$ along y axis.

28/9/17 Thursday.

Problem.

1. Design an ideal highpass filter with freq response

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } \pi/4 \le |\omega| \le \pi \\ 0 & \end{cases}$$

for N=11 using hanning window

Sol: for hanning window

$$\omega_{H}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2\pi n}{N-1} & -\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2} \\ 0 & \text{o.w.} \end{cases}$$

given that

N = 11

$$\omega_{Hn}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{\pi n}{5} & -5 \le n \le 5 \\ 0 & \text{o.w} \end{cases}$$

To find window coefficients

$\omega_{Hn}(0) = 0.5 + 0.5 \cos 0$

$= \underline{\underline{1}}$

$\omega_{Hn}(1) = 0.5 + 0.5 \cos \frac{\pi}{5}$

$= \underline{\underline{0.904}}$

$\omega_{Hn}(2) = 0.5 + \cos \cos \frac{2\pi}{5}$

$= 0.654$

$\omega_{Hn}(3) = 0.345$

$\omega_{Hn}(4) = 0.0954$

$\omega_{Hn}(5) = 0$

$\omega_{Hn}(-1) = 0.5 + 0.5 (-\pi/5)$

$= 0.9045$

$\omega_{Hn}(-2) = 0.6545$

$\omega_{Hn}(-3) = 0.345$

$\omega_{Hn}(-4) = 0.0954$

$\omega_{Hn}(-5) = 0$

To obtain the desired filter coefficients

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\pi/4} 1\, e^{j\omega n} d\omega + \int_{\pi/4}^{\pi} e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \left[\left(\frac{e^{j\omega n}}{jn} \right)_{-\pi}^{-\pi/4} + \left(\frac{e^{j\omega n}}{jn} \right)_{\pi/4}^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\pi/4 n}}{jn} - \frac{e^{-j\pi n}}{jn} + \frac{e^{j\pi n}}{jn} - \frac{e^{j\pi/4 n}}{jn} \right]$$

$\frac{e^{j\theta}-e^{-j\theta}}{2j} = \sin\theta$, $\frac{e^{j\theta}+e^{-j\theta}}{2} = \cos\theta$

$$= \frac{1}{2\pi}\left[-\frac{(\cos n\pi - j\sin n\pi)}{jn} + \frac{\cos n\pi + j\sin n\pi}{jn}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-\cancel{\cos n\pi} + j\sin n\pi + \cos n\pi + j\sin n\pi}{jn}\right]$$

$$= \frac{1}{2\pi}\frac{2j\sin n\pi}{jn}\left[(\cos \tfrac{\pi}{4}n - \sin \pi/4\, n) - + \cos \pi/4\, n - j\sin \pi/4\right]$$

$$= \frac{\sin n\pi}{n\pi}\left(-2\sin \pi/4\, n\right)$$

$$= \frac{1}{2\pi}\left[\frac{2j\sin n\pi}{jn} - \frac{j\sin \pi/4\, n}{jn}\right]$$

$$= \frac{\sin n\pi}{n\pi} - \frac{\sin \pi/4\, n}{n\pi} = \frac{1}{n\pi}\left[\sin n\pi - \sin \tfrac{\pi}{4}n\right]$$

we obtained $hd(n) = \frac{1}{n\pi}\left(\sin n\pi - \sin \frac{\pi n}{4}\right)$

$hd(1) = \frac{1}{\pi}(\sin\pi - \sin\pi/4)$

$= -0.225$

$hd(2) = -0.3182 \quad -0.159$

$hd(3) = -0.2250 \quad -0.0750$

$hd(4) = 0$

$hd(5) = 0.0450$

$hd(-1) = -0.225$

$hd(-2) = -0.159$

$hd(-3) = -0.0750$

$hd(-4) = 0$

$hd(-5) = +0.0450$

$\lim_{n\to 0} \frac{\sin\theta}{\theta} = 1$

$$hd(0) = \frac{\sin \pi n}{\pi n} - \frac{\sin \pi/4\, n}{\pi n\, 1/4}\cdot 1/4$$

$$= 1 - 1/4$$

$$= \frac{3}{4} = 0.75$$

we have

⇒ To obtain filter coefficients $h(n)$

$$h(n) = hd(n)\cdot w_{Hn}(n)$$

$h(0) = 0.75$	
$h(1) = -0.2034$	$h(-1) = -0.2034$
$h(2) = -0.1039$	$h(-2) = -0.1039$
$h(3) = -0.0258$	$h(-3) = -0.0258$
$h(4) = 0$	$h(-4) = 0$
$h(5) = 0$	$h(-5) = 0$

⇒ To obtain transfer function of the filter

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\,(z^{n} + z^{-n})$$

$$= h(0) + \sum_{n=1}^{5} h(n)\,(z^{n} + z^{-n})$$

$$= h(0) + h(1)(z^{1}+z^{-1}) + h(2)(z^{2}+z^{-2}) + h(3)(z^{3}+z^{-3}) + h(4)(z^{4}+z^{-4}) + h(5)(z^{5}+z^{-5})$$

$$= 0.75 + (-0.2034)(z^{1}+z^{-1}) + (-0.1039)(z^{2}+z^{-2}) + (-0.0258)(z^{3}+z^{-3})$$

$= 0.75 - 0.203z - 0.203z^{-1} - 0.103z^{2} - 0.103z^{-2}$
$- 0.025z^{3} - 0.025z^{-3}$

to obtain realizable transfer function

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z),$$

$$= z^{-5} H(z)$$

$$= 0.75z^{-5} - 0.203z^{-4} - 0.203z^{-6} - 0.103z^{-3} - 0.103z^{-7}$$
$$- 0.025z^{-2} - 0.025z^{-8}.$$

To obtain causal filter coefficients $h'(n)$

$h'(0) = 0 \qquad h'(2) = -0.025$

$h'(1) = 0$

$h'(3) = -0.103$

$h'(4) = -0.203$

$h'(5) = 0.75$

$h'(6) = -0.203$

$h'(7) = -0.103$

$h'(8) = -0.025$

⇒ To obtain the freq response

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$= a(0)\cos 0 + a(1)\cos \omega + a(2)\cos 2\omega$$
$$+ a(3)\cos 3\omega + a_4 \cos 4\omega + a_5 \cos 5\omega$$

To obtain the freq response $\bar{H}(e^{j\omega})$

$$a(0) = h'\left(\frac{N-1}{2}\right) = h'(5) = 0.75$$

$$a(1) = 2h'\left(\frac{N-1}{2} - n\right)$$

$$= -0.406$$

$$a(2) = 2h'\left(\frac{N-1}{2} - 1\right)$$

$$= -0.206$$

$$a(3) = -0.05$$

$$a(4) = 0$$
$$a(5) = 0$$

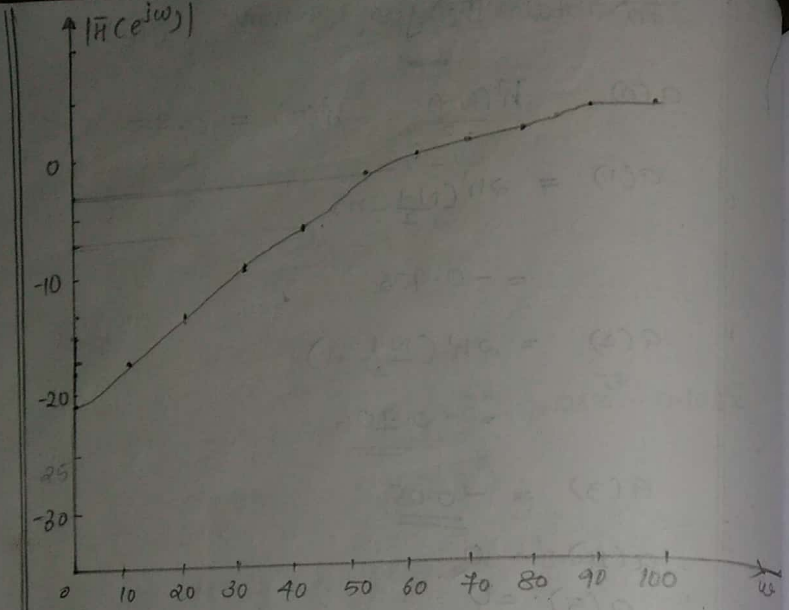
$$\bar{H}(e^{j\omega}) = 0.75\cos \omega n^{(0)} - 0.406\cos \omega - 0.206\cos 2\omega - 0.025\cos 3\omega + 0 + 0.$$

3/10/17
Tuesday

$$= 0.75 - 0.406\cos \omega - 0.206\cos 2\omega - 0.025\cos 3\omega$$

To obtain the graph

ω	$\bar{H}(e^{j\omega})$	$\lvert \bar{H}(e^{j\omega}) \rvert = 20\log \lvert \bar{H}(e^{j\omega}) \rvert$
0	0.088	-21.110
10	0.113	-18.93
20	0.185	-14.656
30	0.295	-10.603
40	0.428	-7.3711
50	0.568	-4.913
60	0.7	-3.098
70	0.812	-1.808
80	0.898	-0.934
90	0.956	-0.390
100	0.989	-0.0960



2. Design a filter with $H_d(e^{j\omega}) = \begin{cases} e^{-3j\omega} & -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0 & \text{o.w.} \end{cases}$

using hamming window for $N = 7$

Sol: for hamming window

$$W_H(n) = \begin{cases} 0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & -\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2} \\ 0 & \text{o.w.} \end{cases}$$

given that

$N = 7$

$$\therefore W_H(n) = \begin{cases} 0.54 + 0.46 \cos\left(\frac{\pi n}{3}\right) & -3 \le n \le 3 \\ 0 & \text{o.w.} \end{cases}$$

To find window coefficient

$W_H(0) = 0.54 + 0.46$

$= \underline{1}$

$W_H(1) = 0.54 + 0.46 \cos(\pi/3)$

$= \underline{0.77}$

$W_H(2) = 0.54 + 0.46 \cos\left(\frac{2\pi}{3}\right)$

$= \underline{0.31}$

$W_H(3) = 0.54 + 0.46 \cos\left(\frac{3\pi}{3}\right)$

$= \underline{0.08}$

$W_H(-1) = 0.54 + 0.46 \cos\left(\frac{-1\times\pi}{3}\right)$

$= \underline{0.77}$

$W_H(-2) = \underline{0.31}$; $W_H(-3) = \underline{0.08}$

To obtain the desired filter coefficient

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-3j\omega}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{j\omega(-3+n)}\, d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega(-3+n)}}{(-3+n)j}\right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi(n-3)j} \left[e^{j\frac{\pi}{4}(n-3)} - e^{-j\frac{\pi}{4}(n-3)}\right]$$

$$= \frac{1}{2\pi(n-3)j}\left[2j \sin \pi/4\,(n-3)\right]$$

$$= \frac{\sin \pi/4\,(n-3)}{\pi(n-3)}$$

$\therefore h_d(n) = \dfrac{\sin \pi/4 (n-3)}{\pi(n-3)}$

$h_d(0) = \dfrac{\sin \pi/4 (-3)}{\pi(-3)}$

$= \underline{0.075}$

$h_d(1) = \underline{0.159}$

$h_d(2) = 0.225$

$h_d(3) = \lim_{n \to 0} \dfrac{\sin \frac{180}{4} \times (n-3)\, 1/4}{\pi (n-3)\, 1/4}$

$= \dfrac{1}{4} = 0.25$

$h_d(-1) = 0$

$h_d(-3) = -0.053$

$h_d(-2) = -0.045$

To obtain filter coefficients

$h(n) = h_d(n)\, w(n)$

$h(0) = 0.075$

$h(1) = 0.122 \qquad h(-1) = 0$

$h(2) = 0.069 \qquad h(-2) = -0.013$

$h(3) = 0.02 \qquad h(-3) = -0.00424$

To obtain transfer function of the filter

$H(z) = h(0) + \sum_{n=1}^{\frac{N}{2}-1} h(n)\,(z^{n} + z^{-n})$

$= h(0) + \sum_{n=1}^{3} h(n)\,(z^{n} + z^{-n})$

$= h(0) + h(1)(z^{1} + z^{-1}) + h(2)(z^{2} + z^{-2}) + h(3)(z^{3} + z^{-3})$

$= h(0) + h(1) z^{1} + h(1) z^{-1} + h(2) z^{2} + h(2) z^{-2} + h(3) z^{3} + h(3) z^{-3}.$

$= 0.075 + 0.122 z^{1} + 0.122 z^{-1} + 0.069 z^{2} + 0.069 z^{-2}$
$+ 0.02 z^{3} + 0.02 z^{-3}.$

To obtain realisable transfer function

$H'(z) = z^{-(\frac{N-1}{2})} H(z)$

$= z^{-3} H(z)$

$\therefore H'(z) = 0.075 z^{-3} + 0.122 z^{-2} + 0.122 z^{-4} +$
$0.069 z^{-1} + 0.069 z^{-5} + 0.02 z^{0} + 0.02 z^{-6}$

To obtain causal filter coefficient $h'(n)$

$h'(0) = 0.02 \qquad h'(3) = 0.075 \qquad h'(6) = 0.02$

$h'(1) = 0.069 \qquad h'(4) = 0.122$

$h'(2) = 0.122 \qquad h'(5) = 0.069$

To obtain the freq response

$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a_n \cos \omega n$

$= a_0 \cos 0 + a_1 \cos \omega + a_2 \cos 2\omega + a_3 \cos 3\omega$

$a_0 = h'(\frac{N-1}{2})$

$= h'(3) = 0.075$

$a_1 = 2 h'(\frac{N-1}{2} - n)$

$= 2 h'(3-1)$

$= 2 h'(2)$

$= 2 \times 0.122$

$= 0.244$

$a_2 = 2h'\left(\frac{N-1}{2} - n\right)$

$= 2\, h'(3-2)$

$= 2\, h'(1)$

$= 2 \times 0.069$

$= 0.138$

$a_3 = 2\, h'\left(\frac{N-1}{2} - 3\right)$

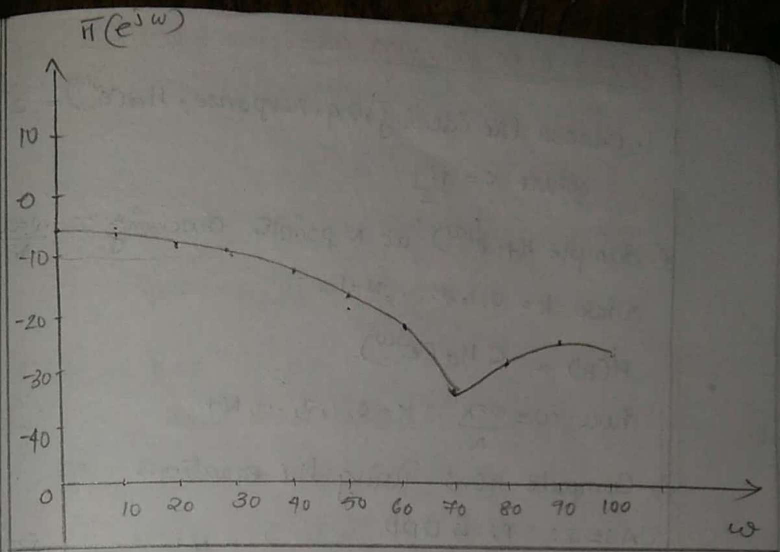
$= 2\, h'(0)$

$= 2 \times 0.02$

$= 0.04$

$$\bar{H}(e^{j\omega}) = 0.075 + 0.244 \cos\omega + 0.138 \cos 2\omega + 0.04 \cos 3\omega .$$

ω	$\bar{H}(e^{j\omega})$	$\lvert \bar{H}(e^{j\omega}) \rvert$
0	0.497	−6.07
10	0.479	−6.39
20	0.429	−7.350
30	0.355	−8.995
40	0.265	−11.535
50	0.173	−15.239
60	0.088	−21.110
70	0.018	−34.894
80	−0.032	−29.89
90	−0.063	−24.013
100	−0.077	−22.270



6/10/17 Friday

DESIGN OF FIR FILTER USING FREQUENCY SAMPLING METHOD

In this method, the ideal or desired freq response is sampled at sufficient no: of periods. These samples are the DFT coefficients of the impulse response of the filter. Hence the impulse response of the filter is determined by taking inverse DFT.

Let

$H_d(e^{j\omega}) \rightarrow$ ideal freq. response

$H(k) \rightarrow$ DFT sequence obtained by sampling $H_d(e^{j\omega})$

$h(n) \rightarrow$ Impulse response of the filter.

Steps to be followed are:

1. Choose the ideal freq. response, $H_d(e^{j\omega}) = e^{-j\alpha\omega}$
   where $\alpha = \frac{N-1}{2}$
2. Sample $H_d(e^{j\omega})$ at N points, choosing $\omega = \frac{2\pi k}{N}$
   where $k = 0, 1, 2 \dots, N-1$
   $H(k) = C\, H_d(e^{j\omega})$
   here $\omega = \frac{2\pi k}{N}$ $k = 0, 1, 2, \dots, N-1$
3. Compute h(n) using the equations

CASE I: N is ODD

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \text{Re}\, H(k)\, e^{j2\pi kn/N}\right]$$

CASE II: N is EVEN

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} \text{Re}\, H(k)\, e^{j2\pi kn/N}\right]$$

4. Take z transform of $h(n)$ using the eqn.

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

Problem.

1. Determine the coefficients of a linear phase FIR filter of length N=15 that has symmetric unit sample response and freq. response given as

$$H\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 & \text{for } k = 0, 1, 2, 3 \\ 0.4 & \text{for } k = 4 \\ 0 & \text{for } k = 5, 6, 7. \end{cases}$$

Sol Take the ideal responce

$$H_d(e^{j\omega}) = e^{-j\alpha\omega}$$

$$\alpha = \frac{N-1}{2} = \frac{15-1}{2} = 7$$

$$\therefore\ H_d(e^{j\omega}) = e^{-7j\omega}.$$

we know that

$$H(k) = C\, H_d(e^{j\omega})$$

So, H(k) can be written as.

$$H(k) = \begin{cases} e^{-7j\omega} & \text{for } k = 0, 1, 2, 3. \\ 0.4\, e^{-j7\omega} & \text{for } k = 4 \\ 0 & \text{for } k = 5, 6, 7. \end{cases}$$

h(n) is found out

here N = ODD (15)

$$\therefore\ h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \text{Re}\, H(k)\, e^{j2\pi kn/N}\right]$$

$$= \frac{1}{15}\left[H(0) + 2\sum_{k=1}^{7} \text{Re}\, H(k)\, e^{j2\pi kn/15}\right]$$

$$H(0) = e^{-j7\omega}$$

$$= e^{-j7 \cdot \frac{2\pi k}{N}}$$

$$h(n) = \frac{1}{15}\left[e^{-j2\pi 7\pi k/N} + 2\sum_{k=1}^{7} \text{Re}\, H(k)\, e^{j2k\pi n/15}\right]$$

$$= \frac{1}{15}\left[e^{-7j\frac{2\pi k}{N}} + 2\left[\sum_{k=1}^{3} e^{j2\pi k(n-7)/15}\right.\right.$$

$$\left. + 2\ 0.4\, e^{j\frac{2\pi k}{15}(7+n)}\right]$$

$$= \frac{1}{15}\left[e^{-7j\frac{2\pi k}{15}} + 2\sum_{k=1}^{3} e^{\frac{j2\pi k(n-7)}{15}} + 2\times 0.4\, e^{j\frac{2\pi k(-7+n)}{15}} \right]$$

$$= \frac{1}{15}\left[\underbrace{e^{\frac{-14j\pi k}{15}}}_{k=0} + \underbrace{2\sum_{k=1}^{3} e^{\frac{j2\pi k(n-7)}{15}}}_{k=1,2,3} + \underbrace{0.8\, e^{\frac{j2\pi k(-7+n)}{15}}}_{k=4} \right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{3} \cos\frac{2\pi k(n-7)}{15} + 0.8\cos\frac{2\pi k(n-7)}{15} \right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(n-7)}{15} + 2\cos\frac{4\pi(n-7)}{15} + 2\cos\frac{6\pi(n-7)}{15} + 0.8\cos\frac{8\pi(n-7)}{15} \right]$$

from symmetry property

$$\boxed{h(n) = \cancel{h(N-1)}\, h(N-1-n)}$$

Sub $n = 0$ to 14

$$h(0) = \frac{1}{15}\left(1 + 2\cos\frac{2\pi(0-7)}{15} + 2\cos\frac{4\pi(0-7)}{15} + 2\cos\frac{6\pi(0-7)}{15} + 0.8\cos\frac{8\pi(0-7)}{15}\right)$$

$= -0.014 = h(14)$

$h(1) = h(13) = -1.94\times 10^{-3}$

$h(2) = h(12) = 0.04$

$h(3) = h(11) = 0.01223$

$h(4) = h(10) = -0.0913$

$h(5) = h(9) = -0.0180$

$h(6) = h(8) = 0.313$

$h(7) = 0.52$

To find z transform

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{14} h(n) z^{-n}$$

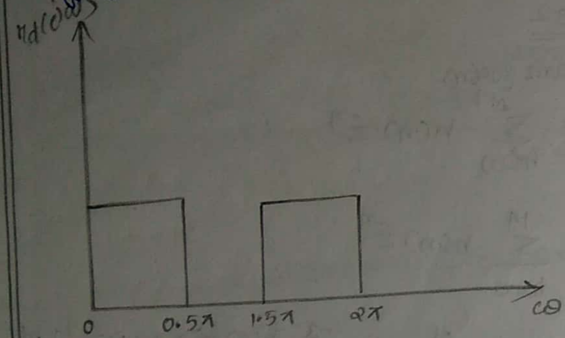
$$= h(0)z^{-0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7} + h(8)z^{-8} + h(9)z^{-9} + h(10)z^{-10} + h(11)z^{-11} + h(12)z^{-12} + h(13)z^{-13} + h(14)z^{-14}$$

$$= -0.014 + -0.0019z^{-1} + 0.04z^{-2} + 0.01223z^{-3} + -0.0913z^{-4} - 0.0181z^{-5} + 0.313z^{-6} + 0.52z^{-7} + 0.313z^{-8} + -0.0181z^{-9} - 0.0913z^{-10} + 0.01223z^{-11} + 0.04z^{-12} + -0.0019z^{-13} - 0.014z^{-14}$$

$$= -0.014 + (-3.88\times 10^{-3})(z^{-1}+z^{-13}) + 0.08(z^{-2}+z^{-12}) + 0.0244(z^{-3}+z^{-11}) +$$

$$= 0.014(1+z^{-14}) - 0.0019(z^{-1}+z^{-13}) + (0.04)(z^{-2}+z^{-12}) + 0.0122(z^{-3}+z^{-11}) + -0.0913(z^{-4}+z^{-10}) - 0.0181(z^{-5}+z^{-9}) + 0.313(z^{-6}+z^{-8}) + 0.52(z^{-7})$$

2. Design a linear phase FIR low pass filter with cutoff frequency of 0.5π rad/sec. by taking $N=11$.



$$H_d(e^{j\omega}) = \begin{cases} 1 & 0 \le \omega \le 0.5\pi \\ 0 & 0.5\pi \le \omega \le 1.5\pi \\ 1 & 1.5\pi \le \omega \le 2\pi. \end{cases}$$

we know that ideal response

$$H_d(e^{j\omega}) = e^{-j\alpha\omega}$$

$$\alpha = \frac{N-1}{2} = \frac{11-1}{2} = 5$$

$$\therefore H_d(e^{j\omega}) = e^{-5j\omega} \qquad \omega = \frac{2\pi k}{N}$$

$$= e^{\frac{-10\pi j k}{11}}$$

$$= e^{-j10\pi k/11}$$

$k=0,\ \omega=0$

$k=1,\ \omega = \frac{2\pi(1)}{11} = 0.1818\pi$

$k=2,\ \omega = \frac{2\pi 2}{11} = \frac{4\pi}{11} = 0.363\pi$

$k=3,\ \omega = \frac{2\pi \times 3}{11} = 0.545\pi$

$k=4,\ \omega = \frac{2\pi 4}{11} = 0.727$

$k=5,\ \omega = \frac{2\pi \times 5}{11} = 0.909$

$k=6,\ \omega = 1.090\pi$

$k=7,\ \omega = 1.2727\pi$

$k=8,\ \omega = 1.454\pi$.

$k=9,\ \omega = 1.636\pi$

$k=10,\ \omega = 1.818\pi$

when $k = 0$ to 2

Samples lie between $0 \le \omega \le 0.5\pi$

when $k = 3$ to 8

Samples lie between $0.5\pi \le \omega \le 1.5\pi$.

$H(k) = H_d(e^{j\omega})e$

when $k = 9, 10$

Samples lie between $1.5\pi \le \omega \le 2\pi$.

$$\therefore H(k) = \begin{cases} e^{\frac{-j10\pi k}{11}} & , k=0,1,2 \\ 0 & , k=3,4,5,6,7,8 \\ e^{\frac{-j10\pi k}{11}} & , k=9,10. \end{cases}$$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{n=1}^{\frac{N-1}{2}} Re\, H(k)\, e^{\frac{j2\pi kn}{N}}\right]$$

$$= \frac{1}{11}\left[H(0) + 2\sum_{n=1}^{5} Re\, H(k)\, e^{\frac{j2\pi kn}{11}}\right]$$

$$H(0) = e^{-j5\frac{2\pi k}{N}}$$

$$= e^{\frac{-j10k\pi}{11}}$$

$$h(n) = \frac{1}{11}\left[e^{\frac{-j10\pi k}{11}} + 2\sum_{n=}^{5} Re\, H(k)\, e^{\frac{j2\pi kn}{11}}\right]$$

$$= \frac{1}{11}\left[e^{\frac{-j10\pi k}{11}} + 2\sum_{k=0}^{2} Re\, e^{\frac{-j10\pi k}{11}}\, e^{\frac{j2\pi kn}{11}}\right]$$

$$h(n) = \frac{1}{11}\left[e^{\frac{-j10\pi k}{11}} + 2\sum_{k=0}^{2} \text{Re}\, e^{\frac{j2\pi k(n-5)}{11}}\right]$$

$$= \frac{1}{11}\left[1 + 2\cos\frac{2\pi(n-5)}{11} + 2\cos\frac{4\pi(n-5)}{11}\right]$$

from symmetry property
$h(n) = h(N-1-n)$

$$h(0) = \frac{1}{11}\left[e^{0} + 2\cos\frac{2\pi(0-5)}{11} + 2\cos\frac{4\pi(0-5)}{11}\right]$$

$= \underline{0.0694} = h(10)$

$h(1) = -0.054 = h(9)$

$h(2) = -0.109 = h(8)$

$h(3) = 0.047 = h(7)$

$h(4) = 0.3193 = h(6)$

$$h(5) = \frac{1}{11}\left[e^{0} + 2\cos\frac{2\pi(5-5)}{11} + 2\cos\frac{4\pi(5-5)}{11}\right]$$

$= \underline{0.4545}$

To find z transform

$$H(z) = \sum_{n=0}^{10} h(n) z^{-n}$$

$= h(0)z^{-0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7} + h(8)z^{-8} + h(9)z^{-9} + h(10)z^{-10}$

$= 0.0694z^{0} + -0.054z^{-1} - 0.109z^{-2} + 0.047z^{-3} + 0.3193z^{-4} + 0.454z^{-5} + \cancel{0.047}\ 0.3193z^{-6} + 0.047z^{-7} + -0.109z^{-8} - 0.054z^{-9} + 0.0694z^{-10}$

$= 0.0694(1+z^{-10}) - 0.054(z^{-1}+z^{-9}) - 0.109(z^{-2}+z^{-8}) + 0.047(z^{-3}+z^{-7}) + 0.3193(z^{-4}+z^{-6}) + 0.454z^{-5}$

1/10/17 Monday

3) Using freq sampling method design a bandpass filter with following specifications for N=7. Sampling freq $F = 8000$ Hz, cutoff frequency $f_{c1} = 1000$ Hz, $f_{c2} = 3000$ Hz

Sol: Given that $N=7$, $F = 8000$ Hz, $f_{c1} = 1000$ Hz, $f_{c2} = 3000$ Hz

we know the relation $2\pi f_c = \frac{\omega_c}{T}$

from this $\omega_{c1} = 2\pi f_{c1} T \Rightarrow \omega_{c1} = \frac{2\pi(1000)}{8000}$ $T = \frac{1}{F}$

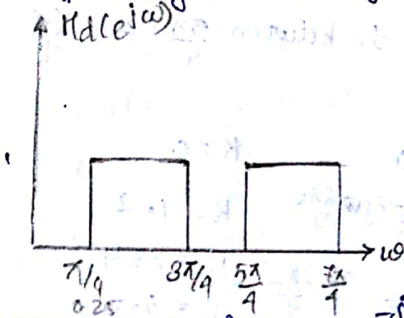
$= \underline{\frac{\pi}{4}}$

$\omega_{c2} = 2\pi f_{c2} T$

$= \frac{2\pi(3000)}{8000}$

$= \underline{\frac{3\pi}{4}}$

So the o/p waveform can be plotted as.



$$H_d(e^{j\omega}) = \begin{cases} 0 & 0 \le \omega \le \pi/4 \\ 1 & \pi/4 \le \omega \le 3\pi/4 \\ 0 & 3\pi/4 \le \omega \le 5\pi/4 \\ 1 & 5\pi/4 \le \omega \le 7\pi/4 \end{cases}$$

Ideal response $H_d(e^{j\omega}) = e^{-j\alpha\omega}$.

$\alpha = \frac{N-1}{2} = \underline{3}$

$\therefore H_d(e^{j\omega}) = e^{-3j\omega}$

$\omega = \frac{2\pi k}{N} = \frac{2\pi k}{7} = 0.285\pi k$

$k=0 \quad \omega = 0 \quad — \quad 0 \le \omega \le \pi/4$

$k=1 \quad \omega = 0.285\pi$

$k=2 \quad \omega = 0.57\pi \quad \Big\} \ \pi/4 \le 3\pi/4$

$k=3 \quad \omega = 0.855\pi$

$k=4 \quad \omega = 1.14\pi \quad \Big\} \ 3\pi/4 \le \omega \le \frac{5\pi}{4}$

$k=5 \quad \omega = 1.425\pi$

$k=6 \quad \omega = 1.71\pi \quad \Big\} \ \frac{5\pi}{4} \le \omega \le \frac{7\pi}{4}$

when $k=0$.

Sample lie between $0 \le \omega \le \pi/4$.

when $k=1,2$

Samples lie between $\pi/4 \le \omega \le \frac{3\pi}{4}$

when $k=3,4$

Samples lie between $\frac{3\pi}{4} \le \omega \le \frac{5\pi}{4}$

when $k=5,6$

Samples lie between $\frac{5\pi}{4} \le \omega \le \frac{7\pi}{4}$

$N=7$ odd

$$H(k) = \begin{cases} 0 & k=0 \\ e^{-3j\frac{2\pi k}{7}} & k=1,2 \\ 0 & k=3,4 \\ e^{-j3\frac{2\pi k}{7}} & k=5,6 \end{cases}$$

$$= \begin{cases} 0 & k=0 \\ e^{\frac{-6j\pi k}{7}} & k=1,2 \\ 0 & k=3,4 \\ e^{-j6\pi k/7} & k=5,6 \end{cases}$$

$N=7$ odd.

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} H(k)\, e^{\frac{j2\pi kn}{N}}\right]$$

$$= \frac{1}{7}\left[H(0) + \sum_{k=1}^{3} H(k)\, e^{\frac{j2\pi kn}{7}}\right]$$

$$= \frac{1}{7}\left[0 + 2\sum_{k=1}^{2} e^{\frac{-6j\pi k}{7}}\, e^{\frac{j2\pi kn}{7}}\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{2} \mathrm{Re}\, e^{\frac{2j\pi k(n-3)}{7}}\right]$$

$$= \frac{1}{7}\left[2\mathrm{Re}\, e^{\frac{2j\pi(n-3)}{7}} + 2\mathrm{Re}\, e^{\frac{4j\pi(n-3)}{7}}\right]$$

$$= \frac{1}{7}\left(2\cos 2\pi\frac{(n-3)}{7} + 2\cos\frac{4\pi(n-3)}{7}\right)$$

from symmetry property

$h(n) = h(N-1-n)$

$h(0) = -0.0792 = h(6)$

$h(1) = -0.320 = h(5)$

$h(2) = 0.114 = h(4)$

$h(3) = 0.571$

To find z transform

$$H(z) = \sum_{n=0}^{6} h(n)\, z^{-n}$$

$$H(z) = h(0)z^{-0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$= -0.0792z^{-0} + -0.320z^{-1} + 0.114z^{-2} + 0.571z^{-3} + 0.114z^{-4} + -0.320z^{-5} - 0.0792z^{-6}$$

$$H(z) = -0.0792(1+z^{-6}) - 0.320[z^{-1}+z^{-5}] + 0.114(z^{-2}+z^{-4}) + 0.57z^{-3}$$

11/10/17
wednesday

DESIGN OF FIR FILTERS - SYMMETRIC & ANTI-SYMMETRIC FILTERS

Depending on the value of N (odd or even) and type of symmetry of the filter's impulse response, there are four types of linear phase FIR filters.

i) symmetric impulse response and N is odd with center of symmetry at $\frac{N-1}{2}$

ii) Symmetric impulse response and N is even with center of symmetry at $\frac{N-1}{2}$.

iii) antisymmetric impulse response and N is odd with centre of symmetry at $\frac{N-1}{2}$.

iv) antisymmetric impulse response and N is even with centre of symmetry at $\frac{N-1}{2}$

1) Symmetric impulse response & N is odd with centre of sym

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{n=0}^{\frac{N-1}{2}} 2h(n)\cos\omega\left(\frac{N-1}{2}-n\right) + h\left(\frac{N-1}{2}\right)\right]$$

Let $\frac{N-1}{2} - n = p$

$$H(e^{j\omega}) = e^{-j\omega\frac{(N-1)}{2}} \left[\sum_{n=0}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-p\right)\cos\omega p + h\left(\frac{N-1}{2}\right)\right]$$

$$= e^{-j\omega\frac{(N-1)}{2}} \sum_{n=0}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2}-n\right]\cos\omega n + h\left(\frac{N-1}{2}\right)$$

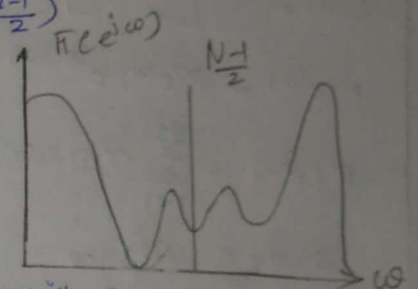
$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \cdot \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$$

where $a(0) = h\left(\frac{N-1}{2}\right)$ and $a(n) = 2h\left(\frac{N-1}{2}-n\right)$

$H(e^{j\omega}) = e^{-j\omega\frac{(N-1)}{2}}$, $\bar{H}(e^{j\omega}) = \bar{H}(e^{j\omega})e^{j\theta(\omega)}$

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$$

$$\theta(\omega) = -\omega\left(\frac{N-1}{2}\right)$$



2. Symmetric impulse Response & N is even with centre of symmetry $\frac{N-1}{2}$

The frequency response is given by

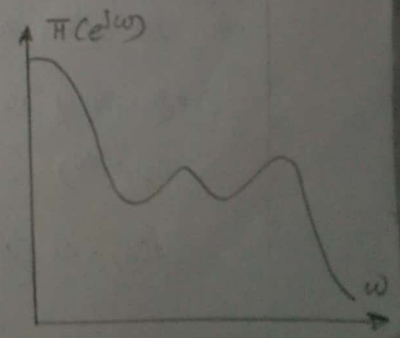
$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{N/2} b(n)\cos\left(\frac{N-1}{2}\right)\omega$$

$$b(n) = 2h\left(\frac{N}{2}-n\right)$$

$$H(e^{j\omega}) = \bar{H}(e^{j\omega})e^{j\theta(\omega)}$$

$$\bar{H}(e^{j\omega}) = \sum_{n=1}^{N/2} b(n)\cos\left(\frac{N-1}{2}\right)\omega$$

$$\theta(\omega) = -\omega\left(\frac{N-1}{2}\right)$$



3) Symmetric Impulse Response and N is odd with centre of symmetry of $\frac{N-1}{2}$

The frequency response is

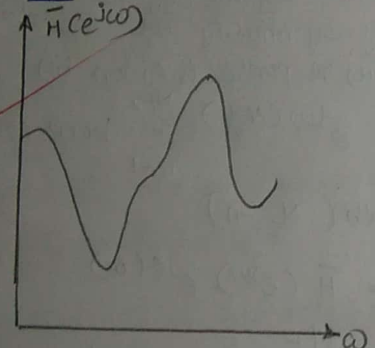
$$H(e^{j\omega}) = e^{-j\omega(N-1)/2} \, e^{j\pi/2} \sum_{n=1}^{\frac{N-1}{2}} c(n) \sin \omega n$$

$$c(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$H(e^{j\omega}) = \bar{H}(e^{j\omega}) \, e^{j\theta(\omega)}$$

$$\bar{H}(e^{j\omega}) = \sum_{n=1}^{\frac{N-1}{2}} c(n) \sin \omega n$$

$$\theta(\omega) = \frac{\pi}{2} - \left(\frac{N-1}{2}\right)\omega$$



4) Asymmetric impulse response and N is even with centre of symmetry $\frac{N-1}{2}$

The frequency response is

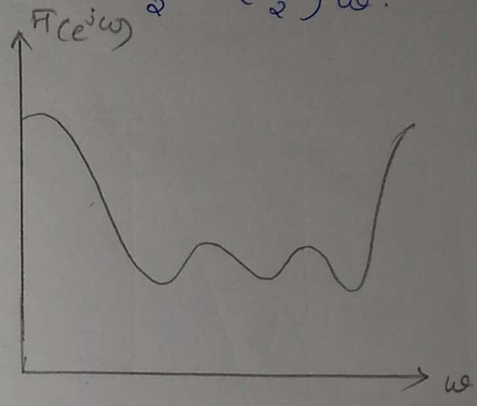
$$H(e^{j\omega}) = e^{-j\omega(N-1)/2} \, e^{j\pi/2} \left[\sum_{n=1}^{N/2} d(n) \sin \omega (n - 1/2)\right]$$

$$d(n) = 2h\left[\frac{N}{2} - n\right]$$

$$H(e^{j\omega}) = \bar{H}(e^{j\omega}) \, e^{j\theta(\omega)}$$

$$\bar{H}(e^{j\omega}) = \sum_{n=1}^{N/2} d(n) \sin \omega (n - 1/2).$$

$$\theta(\omega) = \frac{\pi}{2} - \left(\frac{N-1}{2}\right)\omega.$$



Bindu
11/10/17
Completed

26/09/2017 Tuesday.

MODULE - III

FILTERS

FIR filter → finite impulse response.
- used in linear phase application.

IIR filter → Infinite impulse response.
- denominator = 1.

Difference Equation.

eg-

$$H(z) = 1+z^{-3} \longrightarrow (FIR)$$

$$\frac{Y(z)}{X(z)} = 1+z^{-3}$$

$$Y(z) = (1+z^{-3})X(z)$$

$$Y(z) = X(z) + z^{-3}X(z)$$

Taking inverse on both sides.

Difference Eqn → $y(n) = x(n) + x(n-3)$.

eg-

$$H(z) = \frac{1+z^{-1}}{1+0.5z^{-1}+z^{-2}} \longrightarrow (IIR)$$

$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1+0.5z^{-1}+z^{-2}}$$

$$Y(z) + 0.5z^{-1}Y(z) + z^{-2}Y(z) = X(z) + z^{-1}X(z)$$

$$y(n) + 0.5y(n-1) + y(n-2) = x(n) + x(n-1)$$

DE → $y(n) = x(n) + x(n-1) - 0.5y(n-1) - y(n-2)$

$$H(z) = \frac{1+z^{-1}}{1+0.5z^{-1}+z^{-2}}$$

$\left(\frac{0.5}{2}\right)^2$

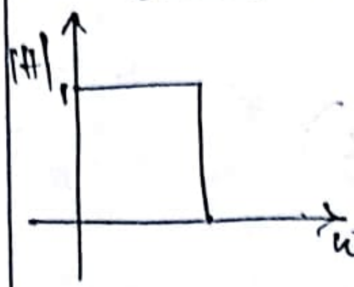
$\left(\frac{1}{4}\right)^2$

By partial fraction we can find $h(n)$.

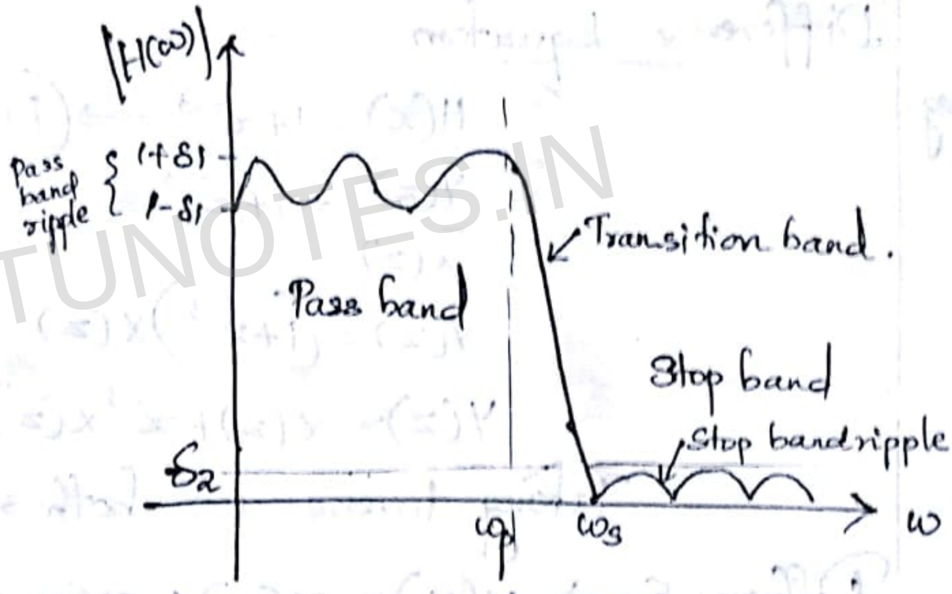
Practical Frequency Selective Filter.

- LPF, HPF, BPF, BSF.

Ideal



Practical



δ_1 – Pass band ripple.

δ_2 – Stop band ripple.

ω_p – Pass band freq.

ω_s – stop band freq.

Design a filter means to find the coefficients

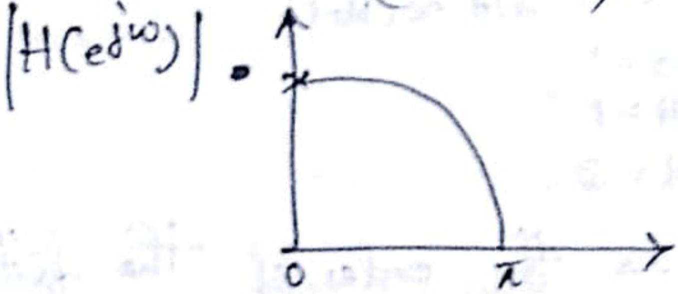
eg: $H(z) = \frac{1+①z^{-1}}{1+(0.5)z^{-1}+①z^{-2}}$

eg- $H(z) = 1 + z^{-1}$ (low pass filter).

$$H(e^{j\omega}) = 1 + e^{-j\omega}$$

$$= e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)$$

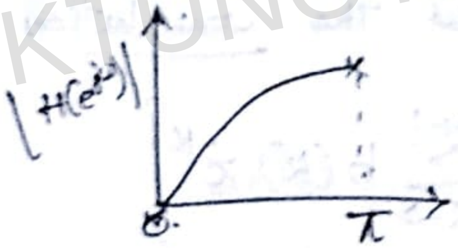
$$= e^{-j\frac{\omega}{2}}\left(2\cos\frac{\omega}{2}\right)$$



eg:- $H(z) = 1 - z^{-1}$

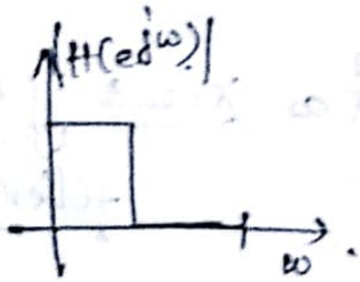
$$H(e^{j\omega}) = 1 - e^{-j\omega} = e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)$$

$$= e^{-j\frac{\omega}{2}}\left(2j\sin\frac{\omega}{2}\right)$$

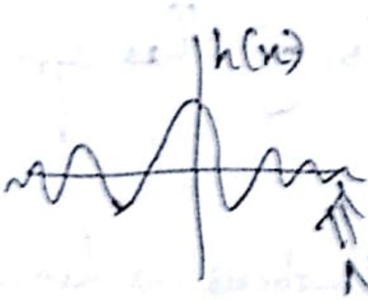


$\Rightarrow$ High pass filter.

eg:-



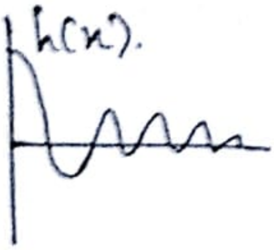
$\longleftrightarrow$



sinc function

$\Uparrow$ Non-causal function

To make



$\Rightarrow$ Thats why ripple factor occurs

* Non-causal functions are not realisable.

General Expression for the difference Equation of an FIR filter

$$\boxed{y(n) = b_0\, x(n) + b_1\, x(n-1) + \dots + b_{M-1}\, x(n-M+1)}$$

eg:- $y(n) = x(n) + x(n-1)$

$b_0 = 1$

$b_1 = 1$

$M = 2$.

M = is called as the order of the filter.

(no: of non-zero coefficients).

$$\boxed{y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)}$$

This eqn is called the Convolution form of FIR filter.

where $\boxed{H(z) = \sum_{k=0}^{M-1} h(k)\, z^{-k}}$

The roots of this eqn are called as zeros of the filter

* Audio applications → Linear

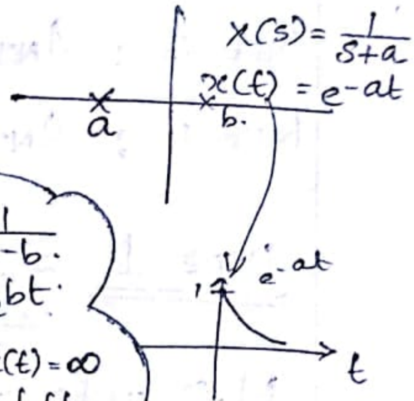
03/10/2017
Tuesday.

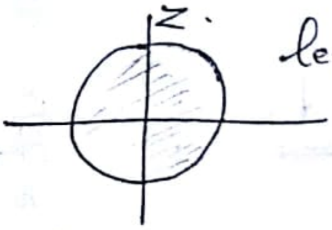
(*) FIR – always stable. – used in linear phase requirement.

eg:- $H(z) = 1 + z^{-1} = 1 + \frac{1}{z}$.

pole $\Rightarrow z = 0$ (origin)

left half of s-plane, pole's stable.





left → inside the circle.

right → outside the circle.

$X(s) = \frac{1}{s-b}$.

$x(t) = e^{bt}$.

As $t \to \infty$ $x(t) = \infty$

S/m is unstable.

(*) IIR – not stable.

* lesser no: of multiplier
* less power consumed.

LINEAR PHASE FIR FILTER

If a FIR filter is having linear phase, then this condition is satisfied:

$$\boxed{h(n) = \pm h(M-1-n)} \qquad n = 0, 1, 2, \dots, M-1$$

eg:-) $h(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$.

$= \{1, 2, 3, +4, -4, -3, -2, -1\}$.

If it is in this way then h(n) is in linear phase.

$\pm \Rightarrow$ Antisymmetry - Symmetry.

* There are 4 types

TYPE - 1 : SYMMETRY AND M is ODD.

TYPE - 2 : SYMMETRY AND M is EVEN

TYPE - 3 : ANTISYMMETRY AND M is ODD.

TYPE - 4 : ANTISYMMETRY AND M is EVEN.

TYPE 1 Symmetry & M is Odd.

$h_0 = h(0)$
$h_1 = h(1)$

M ⇒ order of the filter.

$h_n = \pm h(M-1-n)$

When M is odd,

$M-1 = \text{even}.$

Assume that $M=5$, $M-1=4$, transfer function is given as

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + h_4 z^{-4}$$

$h_0 = h(4-0) = h_4$
$h_1 = h_{4-1} = h_3$
$h_2 = h_{4-2} = h_2$
$h_3 = h_{4-3} = h_1$
$h_4 = h_{4-4} = h_0$

Take z^{-2} common from RHS side.

$$H(z) = z^{-2}\left[h_0 z^2 + h_0 z^1 + h_2 + h_3 z^{-1} + h_4 z^{-2}\right]$$

$$= z^{-2}\left[h_0(z^2 + z^{-2}) + h(z^1 + z^{-1}) + h_2\right]$$

So generally,

$$H(z) = z^{-\frac{(M-1)}{2}} \left\{ h\left(\frac{M-1}{2}\right) + \sum_{k=0}^{\frac{M-3}{2}} h(k) \left(z^{\frac{M-1-2k}{2}} - z^{-\frac{(M-1-2k)}{2}} \right) \right\}$$

Type 2 Symmetric & M is Even

let $M=6$, $M-1=5$

$$h(n) = h(5-n)$$

General expression for $M=6$ is

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + h_4 z^{-4} + h_5 z^{-5}$$

$h_0 = h_{5-0} = h_5$

$h_1 = h_{5-1} = h_4$

$h_2 = h_{5-2} = h_3$

$h_3 = h_{5-3} = h_2$

$h_4 = h_{5-4} = h_1$

$h_5 = h_{5-5} = h_0$

$$H(z) = z^{-5/2} \left[h_0 z^{+5/2} + h_1 z^{3/2} + h_2 z^{1/2} + h_3 z^{-1/2} + h_4 z^{-3/2} + h_5 z^{-5/2} \right]$$

$$= z^{-5/2} \left[h_0 \left(z^{5/2} + z^{-5/2} \right) + h_1 \left(z^{3/2} + z^{-3/2} \right) + h_2 \left(z^{1/2} + z^{-1/2} \right) \right]$$

So generally,

$$H(z) = z^{-\frac{(M-1)}{2}} \sum_{k=0} h(k) \left\{ z^{\frac{M-1-2k}{2}} + z^{-\frac{(M-1-2k)}{2}} \right\}$$

TYPE - 3

Antisymmetry And M is Odd

When M is Odd,

$$M-1 = \text{even}$$

Let us assume $M=5$

i.e, $M-1=4$.

$$h_n = -h(M-1-n)$$

Transfer Function,

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + h_4 z^{-4}$$

$$h_0 = -h(5-1-0) = -h(4)$$

$$h_1 = -h(5-1-1) = -h(3)$$

$$h_2 = -h(5-1-2) = -h(2)$$

Taking z^{-2} common from RHS side.

$$H(z) = z^{-2}\left[h_0 z^2 + h_1 z^1 + h_2 + h_3 z^{-1} + h_4 z^{-2}\right]$$

$$= z^{-2}\left[h_0 z^{+2} + h_1 z^1 + h_2 - h_1 z^{-1} - h_0 z^{-2}\right]$$

$$H(z) = z^{-2}\left[h_0\left(z^{+2} - z^{-2}\right) + h_1\left(z^{1} - z^{-1}\right) + h_2\right]$$

So generally,

$$H(z) = z^{-\frac{(M-1)}{2}}\left\{h\left(\frac{M-1}{2}\right) + \sum_{k=0}^{\frac{M-3}{2}} h(k)\left(z^{\frac{M-1-2k}{2}} - z^{-\frac{(M-1-2k)}{2}}\right)\right\}$$

TYPE-4

Antisymmetry And M is Even

When M is Even,

$M-1 = \text{Odd}$

Let us assume $M=6$

$M-1 = 5$

$$h_n = -h(M-1-n).$$

Transfer function is,

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + h_4 z^{-4} + h_5 z^{-5}$$

$h_0 = -h(6-1-0) = -h(5)$
$h_1 = -h(6-1-1) = -h(4)$
$h_2 = -h(6-1-2) = -h(3)$
$h_3 = -h(6-1-3) = -h(2)$
$h_4 = -h(6-1-4) = -h(1)$
$h_5 = -h(6-1-5) = -h(0)$.

Taking $z^{-5/2}$ common from RHS.

$$H(z) = z^{-5/2}\left[h_0 z^{5/2} + h_1 z^{3/2} + h_2 z^{1/2} + h_3 z^{-1/2} + h_4 z^{-3/2} + h_5 z^{-5/2}\right],$$

$$= z^{-5/2}\left[h_0 z^{5/2} + h_1 z^{3/2} + h_2 z^{1/2} - h_2 z^{-1/2} - h_1 z^{-3/2} - h_0 z^{+5/2}\right]$$

$$= z^{-5/2}\left[h_0\left(z^{5/2} - z^{-5/2}\right) + h_1\left(z^{3/2} - z^{-3/2}\right) + h_2\left(z^{1/2} - z^{-1/2}\right)\right]$$

So Generally,

$$H(z) = z^{-\frac{(M-1)}{2}} \sum_{k=0}^{\frac{M}{2}-1} h(k)\left\{ z^{\frac{M-1-2k}{2}} - z^{-\left(\frac{M-1-2k}{2}\right)} \right\}$$

10/2017
Tuesday.

Generally for an FIR filter

$$h(n) = \pm h(M-1-n)$$

Taking z transform,

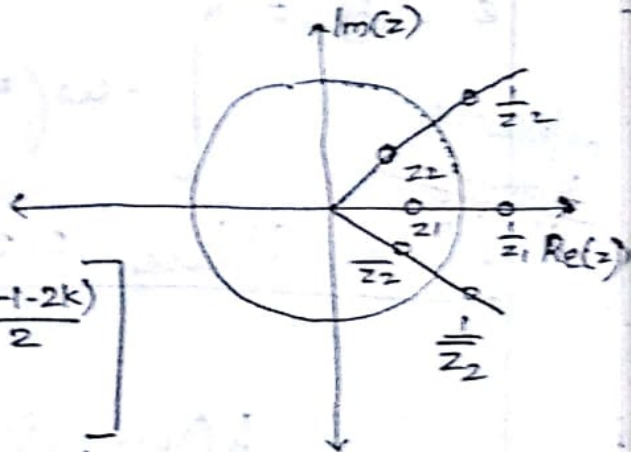
$$H(z) = \pm z^{-(M-1)} H(z^{-1})$$

* When z_1 is a root of $H(z)$, then $H(z^{-1})$ has a root $\frac{1}{z_1}$.
* So if z_1 is a root of $H(z)$, then $\frac{1}{z_1}$ is also the root of $H(z)$

⊛ If $h(n)$ is real, there are complex roots for $H(z)$ and they occur in conjugate pairs.

→ If z_2 is a complex root of $H(z)$ then $\overline{z_2}$ is also a root of $H(z)$.

$\therefore z_2, \frac{1}{z_2}, \overline{z_2}, \frac{1}{\overline{z_2}}$ are also the roots.



Generally,

$$H(z) = z^{-\frac{(M-1)}{2}} \sum_{n=0}^{\frac{M}{2}-1} h(n)\left[z^{\left(\frac{M-1-2k}{2}\right)} \pm z^{-\frac{(M-1-2k)}{2}} \right]$$

M even.

$$H(z) = z^{-\frac{(M-1)}{2}}\left[h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} h(n) \left[z^{\frac{M-1-2k}{2}} \pm z^{-\frac{(M-1-2k)}{2}} \right]\right]$$

M odd.

FREQUENCY RESPONSE

Frequency Response Of Symmetry linear phase filter

$$h(n) = +h(M-1-n)$$

General Expression, $h(\omega) = H_r(\omega)\, e^{-j\omega\left(\frac{M-1}{2}\right)}$

Magnitude Part

$$H_r(\omega) = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad M \text{ odd.}$$

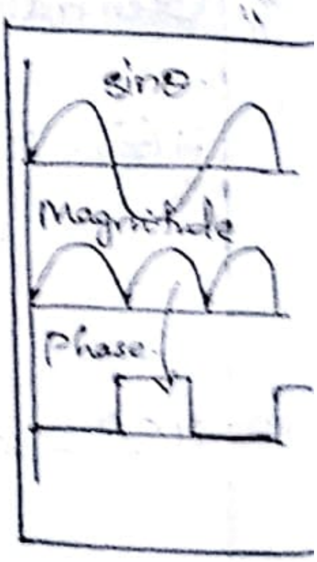
$$H_r(\omega) = 2\sum_{n=0}^{\frac{M}{2}-1} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad M \text{ even}$$

Phase part

$$e^{j\omega} = \Theta = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & \text{if } H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & \text{if } H_r(\omega) < 0. \end{cases}$$

10/2017 Monday:

Linear Phase FIR Filter with Antisymmetry.



$$h(n) = -h(M-1-n)$$

For antisymmetry, $h\left(\frac{M-1}{2}\right) = 0$

Also,

$$H(\omega) = H(e^{j\omega}) = H_r(e^{j\omega})\, e^{-j\left(\omega\left(\frac{M-1}{2}\right) + \frac{\pi}{2}\right)}$$

Magnitude part

where $H_r(e^{j\omega}) = 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin\omega\left(\frac{M-1}{2}-n\right)$ M odd

$$H_r(e^{j\omega}) = 2\sum_{n=0}^{\frac{M}{2}-1} h(n)\sin\omega\left(\frac{M-1}{2}-n\right) \quad M \text{ even}.$$

Phase part

$$\Theta = e^{j\omega} = \begin{cases} \frac{\pi}{2}-\omega\left(\frac{M-1}{2}\right), & H_r(e^{j\omega})>0 \\ \frac{3\pi}{2}-\omega\left(\frac{M-1}{2}\right), & H_r(e^{j\omega})<0 \end{cases}$$

FIR filter Design Using Windows

I Design a linear phase FIR filter using Window.

The desired frequency response is;

$$H_d(e^{j\omega}) = \sum_{n=0}^{\infty} h_d(n)\, e^{-j\omega n}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

This $h_d(n)$ is infinite in length and also non-causal.

$\therefore$ Not realisable.

So to realise, $h_d(n)$ is multiplied by another fn W_n.

where w_n is called as the window fn.

$$w_n = \begin{cases} 1 & 0 < n < M-1 \\ 0 & , \text{otherwise} \end{cases}$$

This window function is called as a <u>Rectangular Window function</u>.

Let $h(n) \triangleq h_d(n) w(n)$

$$h(n) = \begin{cases} h_d(n) & , 0 < n < M-1 \\ 0 & , \text{otherwise} \end{cases}$$

<u>Taking fourier transform</u>.

$$H(e^{j\omega}) = \text{convolution}\left(H_d(e^{j\omega}), W(e^{j\omega})\right)$$

where $W(e^{j\omega}) = \sum_{n=0}^{M-1} w(n) e^{-j\omega n}$

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) W(e^{j(\omega - v)}) dv$$

Fourier transform of the <u>Rectangular function</u>

$$W(e^{j\omega}) = \sum_{n=0}^{M-1} w(n) e^{-j\omega n}.$$

but $w(n) = \begin{cases} 1 & 0 < n < M-1 \\ 0 & \text{otherwise} \end{cases}$

$$W(e^{j\omega}) = \sum_{n=0}^{M-1} e^{-j\omega n}$$

$$= \frac{1 - e^{-j\omega M}}{1 - e^{-j\omega}}$$

$$= \frac{e^{\frac{-j\omega M}{2}}\left(e^{\frac{j\omega M}{2}} - e^{-j\frac{\omega M}{2}}\right)}{e^{\frac{-j\omega}{2}}\left(e^{\frac{j\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}$$

$$= e^{\frac{-j\omega(M-1)}{2}} \times \frac{2j\sin\left(\frac{\omega M}{2}\right)}{2j\sin\left(\frac{\omega}{2}\right)}$$

$$= e^{\frac{-j\omega(M-1)}{2}} \frac{\sin\left(\frac{\omega M}{2}\right)}{\sin \omega/2}$$

finite fn

$= \Sigma a^n$

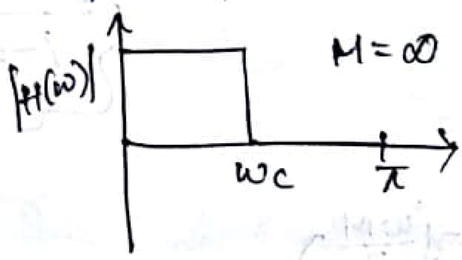
$= \frac{1-a^{\text{no. of terms}}}{1-a}$

Magnitude $|W(e^{j\omega})| = \frac{\left|\sin\left(\frac{\omega M}{2}\right)\right|}{\left|\sin(\omega/2)\right|}$ for $0<\omega<2\pi$ (or) $-\pi<\omega<\pi$

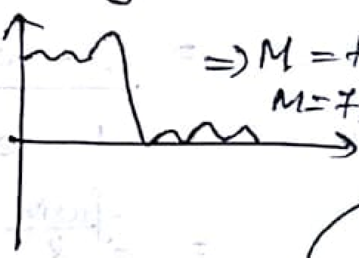
Phase part

$$\Theta(e^{j\omega}) = \begin{cases} -\omega\left(\frac{M-1}{2}\right), & \sin\left(\frac{\omega M}{2}\right)>0 \\ -\omega\left(\frac{M-1}{2}\right)+\pi, & \sin\left(\frac{\omega M}{2}\right)<0 \end{cases}$$

Ideal low pass filter



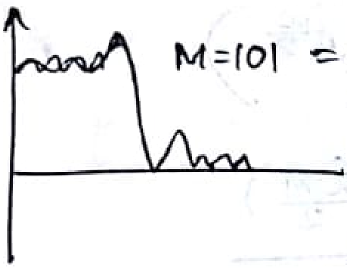
Practically Realised filter



$\Rightarrow M$ = truncated.
$M=71$

M - can be adjusted

M has to be increased.



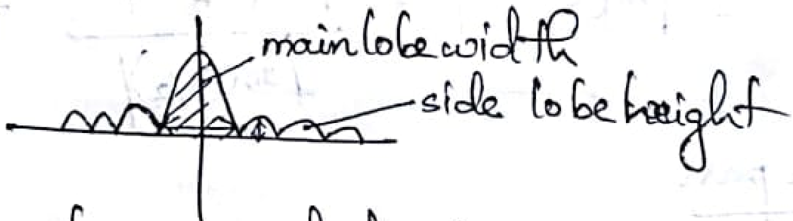
$M=101$ = No. of ripples increases.

* Discontinuity.

* ~~Discontinuity~~.

Fourier Series → truncate → Discontinuity ⇒ Gibbs phenomena.

* As M increases, the main lobe width ↓, and side lobe height ↑, this introduces ringing effect which is undesirable in the filter characteristics.



↳ This can be avoided to a certain extend by using other window functions.

eg:- <u>Let us design a low pass filter using window method.</u>

Desired response, $H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\omega\left(\frac{M-1}{2}\right)}, & 0 < |\omega| < \omega_c \\ 0, & \text{otherwise} \end{cases}$

10/10/2017 Tuesday

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega\left(n-\frac{M-1}{2}\right)} d\omega$$

$$= \frac{1}{2\pi} \frac{e^{j\omega_c\left(n-\frac{M-1}{2}\right)} - e^{-j\omega_c\left(n-\frac{M-1}{2}\right)}}{j\left(n-\frac{M-1}{2}\right)}$$

$$= \frac{1}{\cancel{2}\pi} \times \frac{\cancel{2}\cancel{j}\sin\omega_c\left(n-\frac{M-1}{2}\right)}{\cancel{j}\left(n-\frac{M-1}{2}\right)}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\omega_c\left(n-\frac{M-1}{2}\right)}{\pi\left(n-\frac{M-1}{2}\right)}, & n \neq \frac{M-1}{2} \\ \dfrac{\omega_c}{\pi}, & n = \frac{M-1}{2} \end{cases}$$

$\boxed{\text{length} = \infty}$

If Rectangular window is used,

$$h(n) = \begin{cases} \dfrac{\sin\omega_c\left(n-\frac{M-1}{2}\right)}{\pi\left(n-\frac{M-1}{2}\right)} & 0 < n < M-1 \\ \dfrac{\omega_c}{\pi}, & n = \frac{M-1}{2} \end{cases}$$

$\boxed{\omega_c = 0.3\pi}$

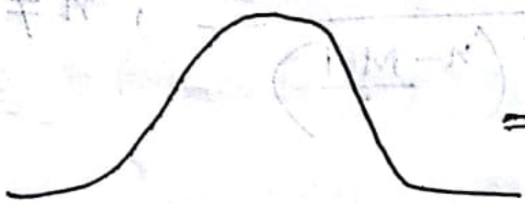
$$h(0) = \frac{\sin\omega_c\left(0-\frac{6}{2}\right)}{\pi\left(0-\frac{6}{2}\right)} = \frac{-\sin\omega_c(3)}{-3\pi} = \frac{\sin 3\omega_c}{3\pi} = 5.23\times10^{-3}$$

Hanning window

$$w(n) = 0.5\left(1 - \cos\frac{2\pi n}{M-1}\right) \quad 0<n<M-1$$

Hamming window

$$w(n) = 0.54 - 0.46\cos\frac{2\pi n}{M-1}$$



$\Rightarrow$ shape of both window

* Maximum value is 1 for all windows.

Linear Phase FIR filter using Frequency Sampling Method.

Two types of design.

1) Type 1.
2) Type 2.

TYPE - 1

Here $H(K) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}k}$ where $k : 0 \to N-1$

$$H(K) = |H(K)| \, e^{j\theta(K)}$$

For linear phase, $\theta(K) = -\alpha\omega\Big|_{\omega = \frac{2\pi}{N}k}$ α = constant

Filter coefficients $h(n)$

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(K) \, e^{\frac{j2\pi kn}{N}} \quad n : 0 \to N-1$$

There is no guarantee that $h(n)$ is real. $h(n)$ can be real if $H(N-k) = H^*(K)$ for N odd or even. In addition $H\left(\frac{N}{2}\right) = 0$, N is even.

To satisfy the phase requirements,

$$\theta(K) = -\frac{N-1}{N}\pi k \quad k = 0, 1, \dots, \frac{N-1}{2}$$

$$= (N-1)\pi - \frac{N-1}{N}\pi k \quad \text{for } k = \frac{N+1}{2}, \dots, N-1$$

for both N even and odd.

In addition to this,

$$\theta(K) = 0 \quad \text{for } k = \frac{N}{2}, \text{ N is even.}$$

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{K=1}^{(N-1)/2} Re\left[H(K)\, e^{\frac{j2\pi kn}{N}}\right]\right\} \text{ N odd.}$$

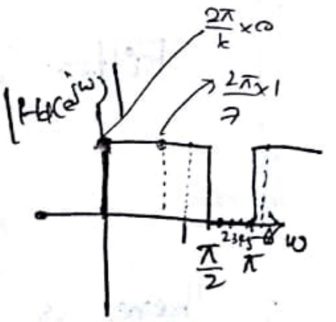
$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{K=1}^{\frac{N}{2}-1} Re\left[H(K)\, e^{\frac{j2\pi kn}{N}}\right]\right\} \text{ N even.}$$

Qn. Determine the filter coefficients obtained by sampling

$$H_d(e^{j\omega}) = \begin{cases} e^{-j(N-1)\omega/2}, & 0<|\omega|<\pi/2 \\ 0, & \frac{\pi}{2}<|\omega|<\pi \end{cases}$$

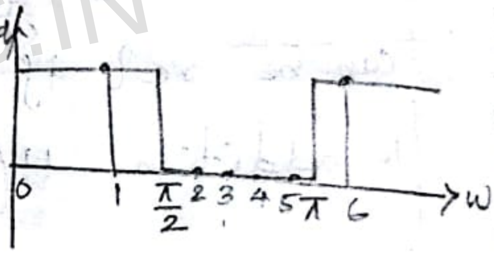
for $N=7$. $\boxed{\text{odd}}$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{7}k}.$$



$$|H_d(e^{j\omega})| = \begin{cases} 1, & 0<|\omega|<\pi/2 \\ 0, & \pi/2<|\omega|<\pi \end{cases}$$

$$|H(k)| = \begin{cases} 1, & k=0,1,6 \\ 0, & k=2,3,4,5 \end{cases}$$



$$H(k) = \begin{cases} e^{-j\left(\frac{6\pi k}{7}\right)}, & k=0,1 \\ 0, & k=2,3,4,5 \\ e^{j\frac{6\pi(7-k)}{7}}, & k=6 \end{cases}$$

Here N ⇒ odd.

$$\therefore h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \mathrm{Re}\left[H(k)e^{\frac{j2\pi kn}{N}}\right]\right\}$$

13/10/2017 Friday

$$h(n) = \frac{1}{7}\left\{1 + 2\,\mathrm{Re}\sum_{K=1}^{\frac{N-1}{2}}\left[H(K)\,e^{\frac{j2\pi Kn}{7}}\right]\right\}$$

$$= \frac{1}{7}\left\{1 + 2\,\mathrm{Re}\sum_{K=1}^{\frac{N-1}{2}}\left[e^{-\frac{j6\pi k}{7}} \cdot e^{\frac{j2\pi kn}{7}}\right]\right\}$$

$$= \frac{1}{7}\left\{1 + 2\,\mathrm{Re}\left(e^{-\frac{j6\pi k}{7} + \frac{j2\pi kn}{7}}\right)\right\}$$

$$= \frac{1}{7}\left\{1 + 2\,\mathrm{Re}\; e^{\frac{j2\pi(n-3)}{7}}\right\}$$

$$= \frac{1}{7}\left\{1 + 2\cos\left(\frac{2\pi(n-3)}{7}\right)\right\}$$

$$h(0) = \frac{1}{7}\left\{1 + 2\cos\left(\frac{-6\pi}{7}\right)\right\} = -0.11456$$

$h(1) = 0.0792$.

$h(2) = 0.3209$

$h(3) = 3/7$.

$h(4) = h(M-n-1) = h(7-4-1) = h(2) = 0.3209$

$h(5) = h(1) = 0.0792$.

$h(6) = h(0) = -0.11456$

Qn. Determine the coefficients of linear phase FIR band pass filter of length N = 7 using frequency sampling method with following specifications.

Sampling frequency $F_s = 8000\,Hz$.

Cutoff frequency $F_{c1} = 1000\,Hz$

$F_{c2} = 3000\,Hz$.

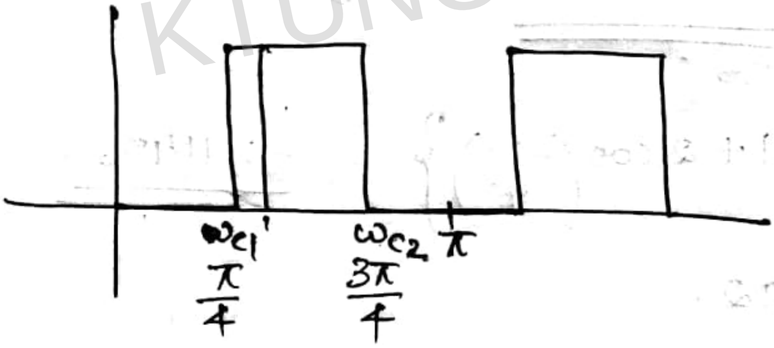
$$\omega_{c1} = \frac{2\pi F_{c1}}{F_s} = \frac{2\pi \times 1000}{8000} = \frac{\pi}{4}$$

$$\omega_{c2} = \frac{2\pi F_{c2}}{F_s} = \frac{2\pi \times 3000}{8000} = \frac{3\pi}{4}$$

If Sampling freq is not given

$F_s \geq 2 f_m$

Here $f_m = 3000\,Hz$



$\frac{2\pi k}{N}$

$k = 0, 1, 2, \ldots, 6$

$|H(k)| = 1 \quad k = 1,$

$\quad\quad\quad\; 0 \quad k = 0,$

$k=0 \quad \frac{2\pi 0}{7} = 0.$

$k=1 \Rightarrow \frac{2\pi}{7} \Rightarrow 0.89$

$k=2 \Rightarrow \frac{4\pi}{7} \Rightarrow 1.795$

$k=3 \Rightarrow \frac{6\pi}{7} = 2.69$

25/10/2017
Wednesday.

Comparison Of Design Methods in Linear phase FIR filter

Different methods of FIR filter are :

1) Truncation method / Window method.
2) Frequency Sampling method
3) Chebyshev Approximation method.

Window Design Method

Disadvantage

* Lack of control over ω_p and ω_s (critical frequency)
* Generally the design depends on the type of window and order n.

Frequency Sampling Method

* Here $H_r(e^{j\omega})$ is specified at $\omega = \frac{2\pi k}{M}$ or at $\frac{\pi(2k+1)}{M}$
$\downarrow$ even, $\downarrow$ odd.

and at ~~transient~~ transition band with is a multiple of $\frac{2\pi}{M}$. The advantage is $H_r(e^{j\omega})$ is either 0 or 1 except in the transition band

Chebyshev Approximation

* Most popular

Advantages

* Overall control of specification so this is a better design. For low pass filter the specification in terms of ω_p, ω_s, δ_1, δ_2 and M can be designed.

* In FIR filter design there is no simple formula to calculate the order.

The simplest formula available is from Kaiser, where

$$M = \frac{-2\log_{10}(\sqrt{\delta_1 \delta_2}) - 13}{14.6\,\Delta f} + 1$$

If $M = 6.48$ take it as 7

$\Delta f \Rightarrow$ transition band.

$$\Delta f = \frac{\omega_s - \omega_p}{2\pi}$$