Example 6.10 Design a filter with

$$H_d(e^{j\omega}) = e^{-j3\omega} \qquad \frac{-\pi}{4} \leq \omega \leq \frac{\pi}{4}$$
$$= 0 \qquad \frac{\pi}{4} < |\omega| \leq \pi$$

Using a Hamming window with $N = 7$ (AU EEE'07)

Solution

Given $H_d(e^{j\omega}) = e^{-j3\omega}$

The frequency response is having a term $e^{-j\omega(N-1)/2}$ which gives $h(n)$ symmetrical about $n = \frac{N-1}{2} = 3$, i.e., we get a causal sequence.

We have

$$h_d(n) = \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{-j3\omega} e^{j\omega n} d\omega$$
$$= \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{j(n-3)\omega} d\omega$$
$$= \frac{\sin\frac{\pi}{4}(n-3)}{\pi(n-3)}$$

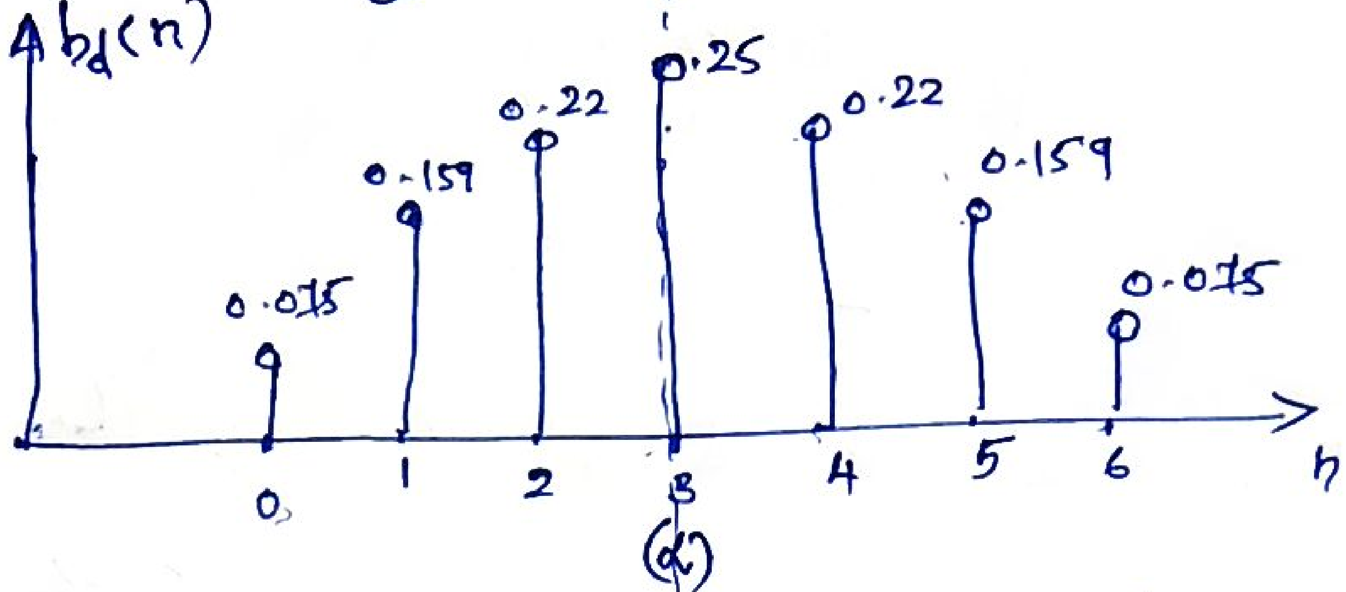
For $N = 7$ we have

$$h_d(0) = h_d(6) = 0.075$$
$$h_d(1) = h_d(5) = 0.159$$
$$h_d(2) = h_d(4) = 0.22$$
$$h_d(3) = 0.25$$

$h_d(n) \longrightarrow$ is a (causal sequence)

which is symmetric about $\alpha = \frac{N-1}{2}$.

ie about $\alpha = 3$.



$\longrightarrow$ centre of symmetry.

The non-causal window sequence is

$$
\begin{aligned}
w_{Hn}(n) &= 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \quad \text{for} \quad -(N-1)/2 \le n \le (N-1)/2 \\
&= 0 \quad \text{otherwise}
\end{aligned}
$$

For $N = 7$

$$
\begin{aligned}
w_{Hn}(n) &= 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \quad \text{for} \quad -3 \le n \le 3 \\
&= 0 \quad \text{otherwise}
\end{aligned}
$$

$$w_{Hn}(0) = 0.5 + 0.5 = 1$$

$$w_{Hn}(-1) = w_{Hn}(1) = 0.5 + 0.5 \cos \frac{\pi}{3} = 0.75$$

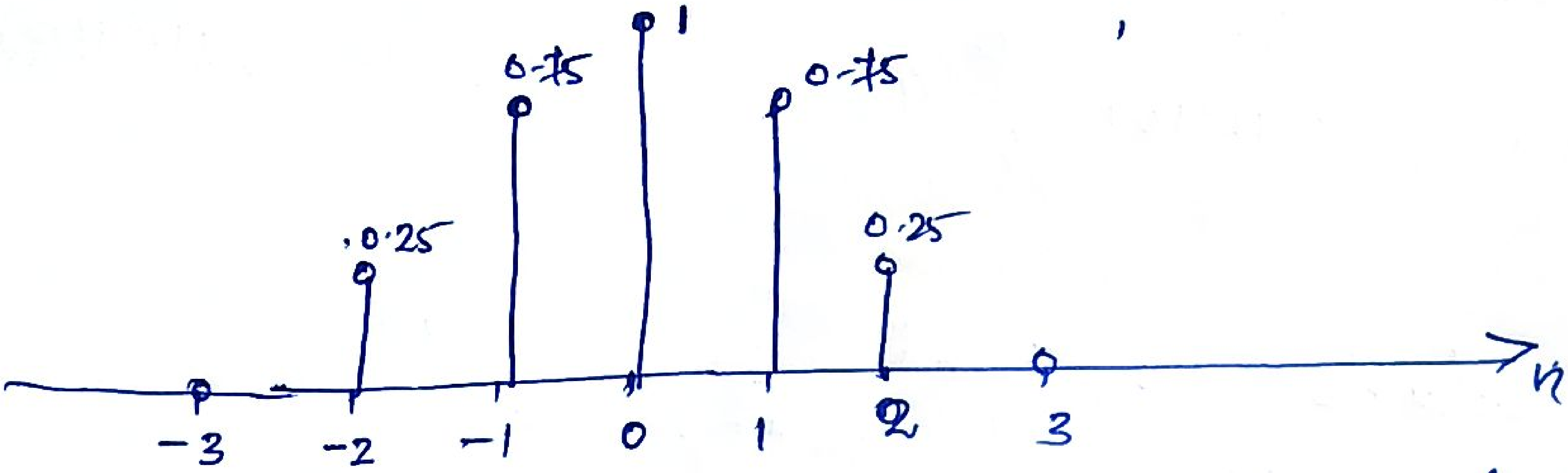
$$w_{Hn}(-2) = w_{Hn}(2) = 0.5 + 0.5\cos\frac{2\pi}{3} = 0.25$$

$$w_{Hn}(-3) = 0.5 + 0.5\cos\pi = 0$$

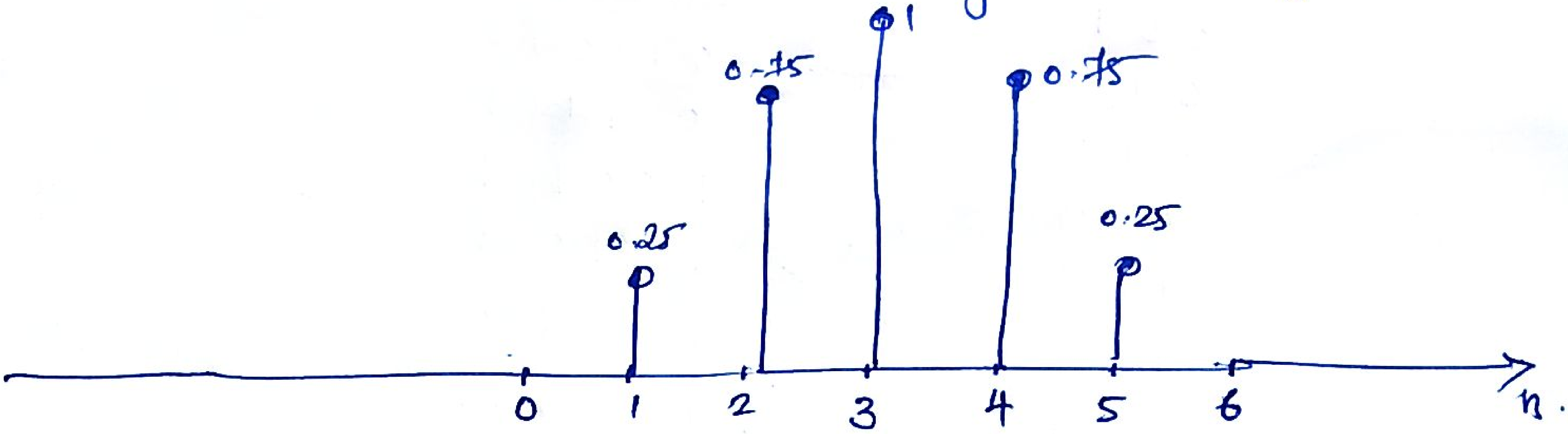
The causal window sequence can be obtained by shifting the sequence $w_{Hn}(n)$ to right by 3 samples, i.e.,

$$w_{Hn}(0) = w_{Hn}(6) = 0;\ w_{Hn}(1) = w_{Hn}(5) = 0.25$$
$$w_{Hn}(2) = w_{Hn}(4) = 0.75\ \&\ w_{Hn}(3) = 1$$

Non causal window sequence (Hanning)



Corresponding causal Hanning window sequence
(right shift by 3 units).



The filter coefficients using Hanning window are

$$h(n) = h_d(n)w_{Hn}(n) \quad \text{for} \quad 0 \le n \le 6$$
$$h(0) = h(6) = h_d(0)w_{Hn}(0) = (0.075)(0) = 0$$
$$h(1) = h(5) = h_d(1)w_{Hn}(1) = (0.159)(0.25) = 0.03975$$
$$h(2) = h(4) = h_d(2)w_{Hn}(2) = (0.22)(0.75) = 0.165$$
$$h(3) = h_d(3)w_{Hn}(3) = (0.25)(1) = 0.25$$

The corresponding causal transfer function

$$H(z) = \sum_{n=0}^{6} h(n) z^{-n}.$$

$$= h(0) \cdot z^{0} + h(1) z^{-1} + \dots \quad h(6) z^{-6}$$

$$= 0.03975 \left[z^{-1} + z^{-5} \right] + 0.165 \left[z^{-2} + z^{-4} \right]$$

$$+ 0.25 z^{-3}$$