Example 3.21 Using linear convolution find $y(n) = x(n) * h(n)$ for the sequences $x(n) = (1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1)$ and $h(n) = (1.2)$. Compare the result by solving the problem using (a) overlap-save method (b) overlap-add method.

Solution

The linear convolution of $x(n)$ and $h(n)$ is

$$y(n) = x(n) * h(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Overlap-save method

The input sequence can be divided into blocks of data as follows.

$\rightarrow M - 1$ zeros appended

$$x_1 = \{0, \underbrace{1, 2, -1,}_{\text{3 datas}}\}$$

$$x_2(n) = \{-1, \underbrace{2, 3 - 2}_{\text{3 new datas}}\}$$

$\rightarrow M - 1 = 1$ data from previous block

$$x_3(n) = \{-2, -3, -1, 1\}; \quad x_4(n) = \{1, 1, 2, -1\}; \quad x_5(n) = \{-1, 0, 0, 0\}$$

Given $h(n) = \{1, 2\}$. Appending two zeros to the sequence we obtain

$$h(n) = \{1, 2, 0, 0\}$$

$$y_1(n) = x_1(n) \; N \; h(n)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$$

$$y_2(n) = x_2(n) \; \text{(N)} \; h(n)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 7 \\ 4 \end{bmatrix}$$

Similarly $y_3(n) = \{0, -7, -7, -1\}$; $\quad y_4(n) = \{-1, 3, 4, 3\}$;
$y_5(n) = \{-1, -2, 0, 0\}$

$\boxed{-2, 1, 4, -3}$
↓
discard

$\boxed{-5, 0, 7, 4}$
↓
discard

$\boxed{0, -7, -7, -1}$
↓
discard

$\boxed{-1, 3, 4, 3}$
↓
discard

$\boxed{-1, -2, 0, 0}$
↓
discard

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Overlap-add method

In this method the sequence $x(n)$ can be divided into data blocks as shown below.

$$x_1(n) = \{1, 2, -1, 0\}$$
↓
$M - 1 = 1$ zero added

$$x_2(n) = \{2, 3, -2, 0\}; \quad x_3(n) = \{-3, -1, 1, 0\}$$
$$x_4(n) = \{1, 2, -1, 0\}; \quad h(n) = \{1, 2, 0, 0\}$$

$$y_1(n) = x_1(n) \text{ (N) } h(n) = \{1, 4, 3, -2\}$$

$$y_2(n) = x_2(n) \text{ (N) } h(n) = \{2, 7, 4, -4\}$$

$$y_3(n) = x_3(n) \text{ (N) } h(n) = \{-3, -7, -1, 2\}$$

$$y_4(n) = x_4(n) \text{ (N) } h(n) = \{1, 4, 3, -2\}$$

$$\boxed{1, 4, 3, -2}$$

$\updownarrow$ add

$$\boxed{2, 7, 4, -4} \qquad (\because M - 1 = 1)$$

$\updownarrow$ add

$$\boxed{-3, -7, -1, 2}$$

$\updownarrow$ add

$$\boxed{1, 4, 3, -2}$$

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$