Microwaves & Antennas

Module 4: Part 1
Microwaves

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Reference Textbooks:

- Microwave Devices & Circuits by ***Samuel Y.Liao*** *(Chapter 4,9,10)*
- Microwave and Radar Engineering by ***Kulkarni*** *(Chapter 4 & 5)*

Microwaves

- The region of EM Spectrum in the range 300 MHz - 300 GHz
- Wavelength of Microwaves is in the range 100 cm - 0.1 cm

IEEE Microwave Frequency Bands

Designation	Frequency range in gigahertz
HF	0.003– 0.030
VHF	0.030– 0.300
UHF	0.300– 1.000
L band	1.000– 2.000
S band	2.000– 4.000
C band	4.000– 8.000
X band	8.000– 12.000
Ku band	12.000– 18.000
K band	18.000– 27.000
Ka band	27.000– 40.000
Millimeter	40.000–300.000
Submillimeter	>300.000

Advantages

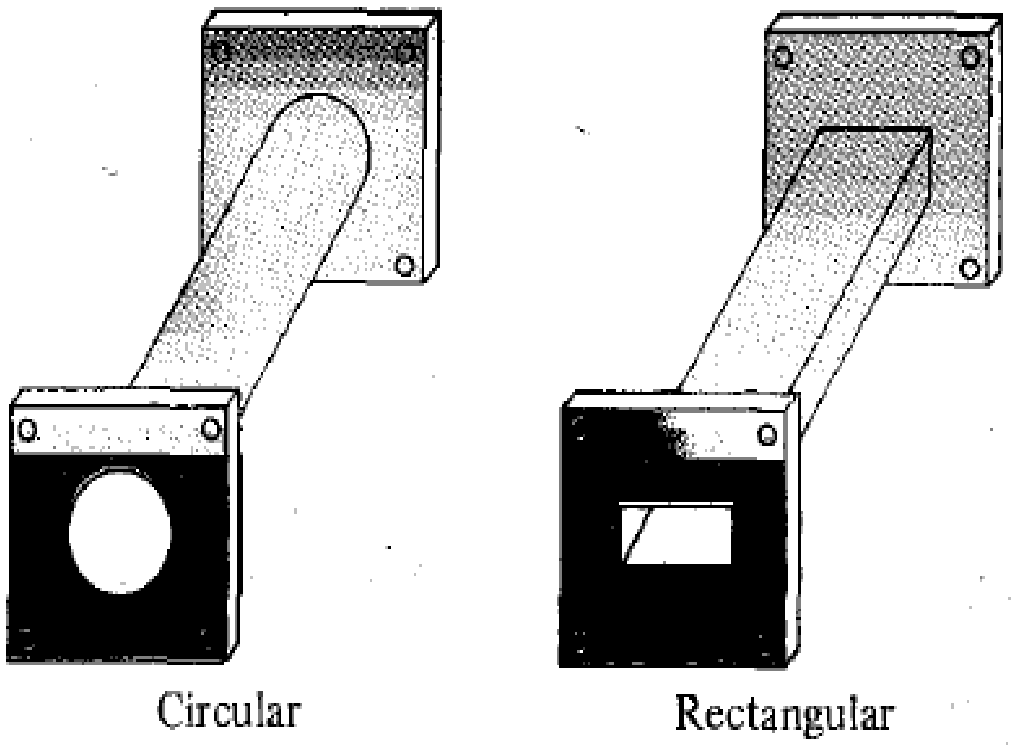
- Increased bandwidth availability
- Small size Antennas
- Improved gain & directive properties
- Fading effect & reliability
- Power requirements
- Transparency property of microwaves

Applications

- Telecommunication
- Radars
- Commercial and industrial applications
- Biomedical applications
- Electronic warfare

Waveguides

- Hollow metallic structure with uniform cross section used to guide EM waves by successive reflections from the inner walls of the tube
- Typical Shapes are Rectangular & Circular



Transmission Lines	Waveguides
1. Supports only Transverse Electromagnetic (TEM) wave.	Supports many possible field configurations (TE, TM, TEM).
2. At microwave frequencies (3-300GHz), transmission lines become inefficient as a result of skin effect and dielectric losses	It can be used at microwave range of frequencies to obtain larger bandwidth and lower signal attenuation
3. It can operate from dc (f=0) to a very high frequency → Acts like a low pass filter	Operates only above a certain frequency called the cut off frequency → Acts like a high pass filter.
	Waveguides cannot transmit dc and the waveguide dimensions becomes large at frequencies below microwave range.

Review of Electromagnetic wave Theory

Maxwell's equations,

1. $\nabla \times H = J + \frac{\partial D}{\partial t}$

2. $\nabla \times E = \frac{-\partial B}{\partial t}$

3. $\nabla . D = \rho$, where $D = \epsilon E$ and $J = \sigma E$

4. $\nabla . B = 0$, where $B = \mu H$

Wave Propagation

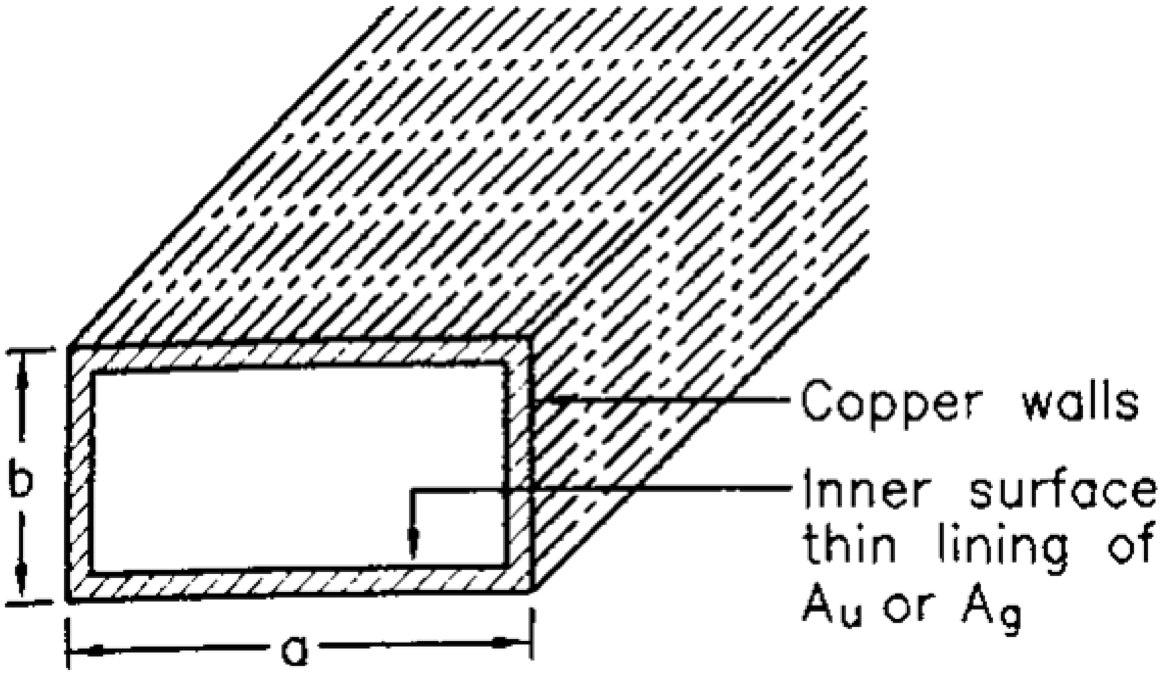
Transverse Electric (TE) wave : Electric field is transverse (perpendicular) to the direction of propagation i.e. $E_z = 0, H_z \neq 0$

Transverse Magnetic (TM) wave : Magnetic field is transverse to the direction of propagation i.e. $H_z = 0, E_z \neq 0$

Transverse Electric & Magnetic (TEM) wave : Both electric field & magnetic field are transverse to the direction of propagation i.e. $E_z = 0, H_z = 0$

Rectangular Waveguides

- Hollow metallic structure such as Copper is used with uniform rectangular cross section.
- Inner surface is coated with Gold / Silver to improve conductivity.
- Waveguide is filled with dielectric medium such as air.



$$\nabla^2 \mathrm{E} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 \mathrm{H} = \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2}$$

- These equations represent waves propagating through free space with velocity equal to that of light
- Also known as "Helmholtz wave equation"

Cut off Frequency of Rectangular Waveguide

- In a rectangular waveguide,

$$h^2 = \gamma^2 + \omega^2\mu\epsilon = A^2 + B^2$$

$$= \left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

$$\therefore \qquad \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

Propagation constant, $\gamma = \alpha + j\beta$

where α – attenuation factor, β – phase constant

- At lower frequencies,

$$\omega^2 \mu \epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

In this case, γ is real and positive, and equal to attenuation constant i.e. the wave is completely attenuated and there is no phase change.

Hence the wave cannot propagate.

- At higher frequencies,

$$\omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

In this case, γ becomes imaginary, and there will be a phase change.

Hence the wave can propagate.

- The frequency at which γ becomes zero is called cut off frequency or threshold frequency (f_c)

At $f = f_c$, $\omega = 2\pi f_c = \omega_c$ $and\ \gamma = 0$

$Propagation\ constant,$ $$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2\mu\epsilon$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

- Therefore cut off frequency, $f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$

- Since $c = 1/\sqrt{\mu\epsilon}$

$$f_c = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}$$

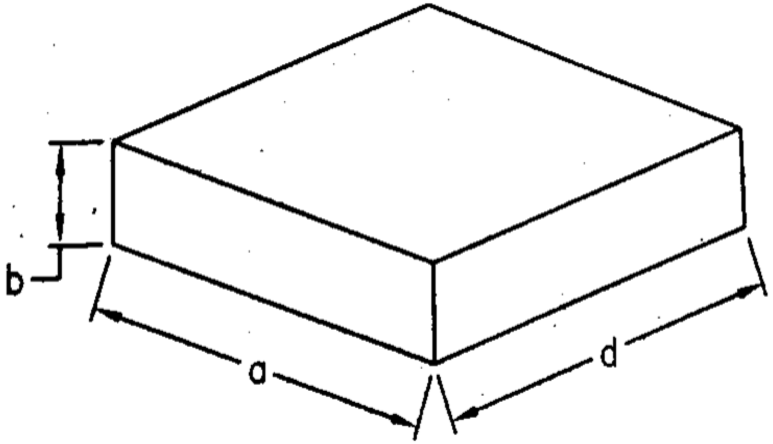
- Cut off wavelength, λ_c is given by

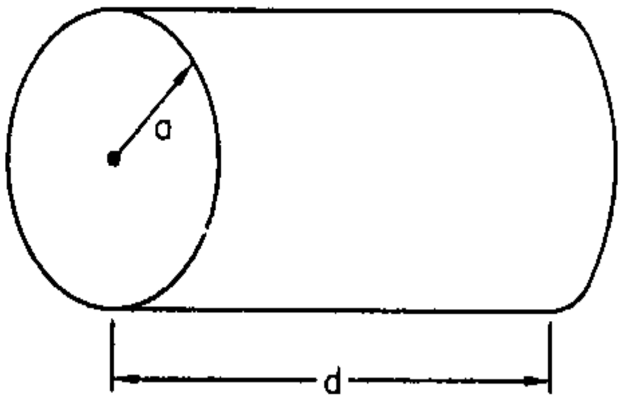
$$\lambda_c = \frac{c}{f_c} = \frac{2\pi}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}}$$

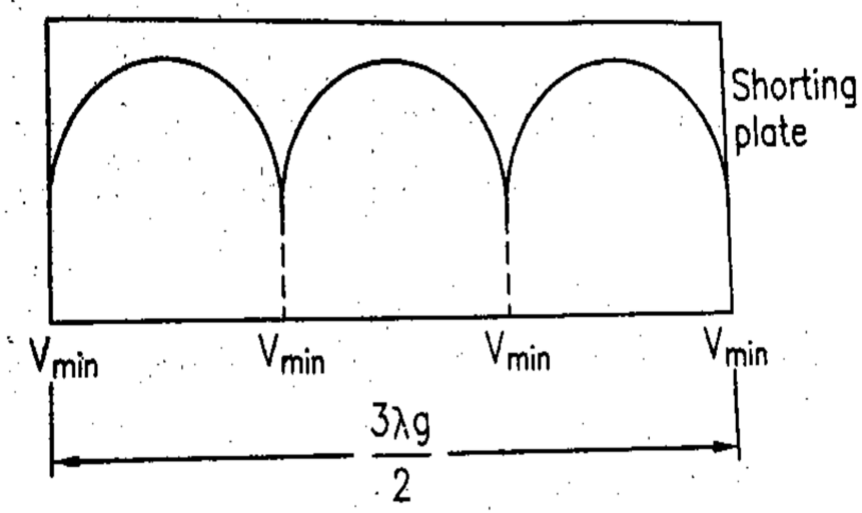
$$\lambda_c = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$$

Cavity Resonator

- If one end of waveguide is terminated in a shorting plate, there will be reflections.
- If another shorting plate is placed at a distance (integer multiple of $\lambda/2$) , the signal will bounce back and forth between the plates.
- This results in resonance and this structure is known as cavity resonator.







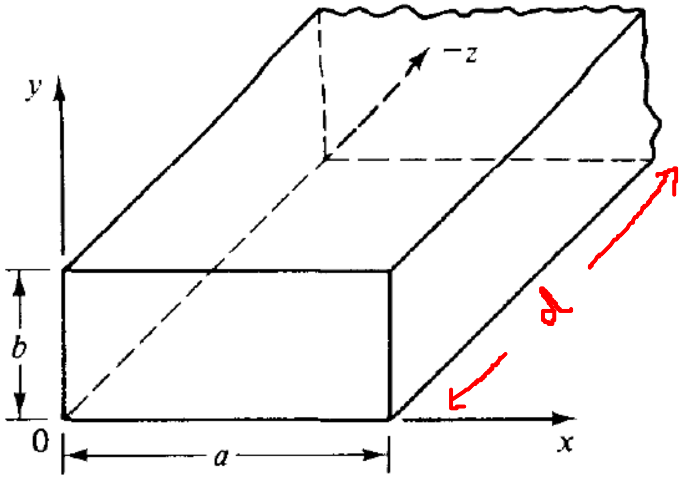
Expression for Resonance Frequency of Rectangular Cavity Resonator

- In a rectangular waveguide,

$$h^2 = \gamma^2 + \omega^2\mu\epsilon = A^2 + B^2$$

$$= \left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2$$

$$\omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2$$



For wave propagation, $\gamma = j\beta$

$$\omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$$

- If a wave has to exist in a cavity resonator, there must be a phase change corresponding to given guide wavelength (i.e. $\beta = 2\pi/\lambda_g$)

- Condition for cavity resonator to resonate is

$$\beta = \frac{p\pi}{d}$$

where p – constant (1,2,3..) d – length of resonator

At resonance frequency $f = f_o\,, \omega = 2\pi f_o = \omega_o \;\; and \; \beta = \frac{p\pi}{d}$

$$\omega_o^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\omega_o = \sqrt{\frac{1}{\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]}$$

$$f_o = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2} = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]^{1/2}$$

Example: Calculate the lowest resonant frequency of a rectangular cavity resonator of dimensions $\boldsymbol{a = 2cm, b = 1cm, and\ d = 3cm}$.

Given: a=2cm, b=1cm, d=3cm.

The lowest resonant frequency is obtained for dominant mode TE_{101}.

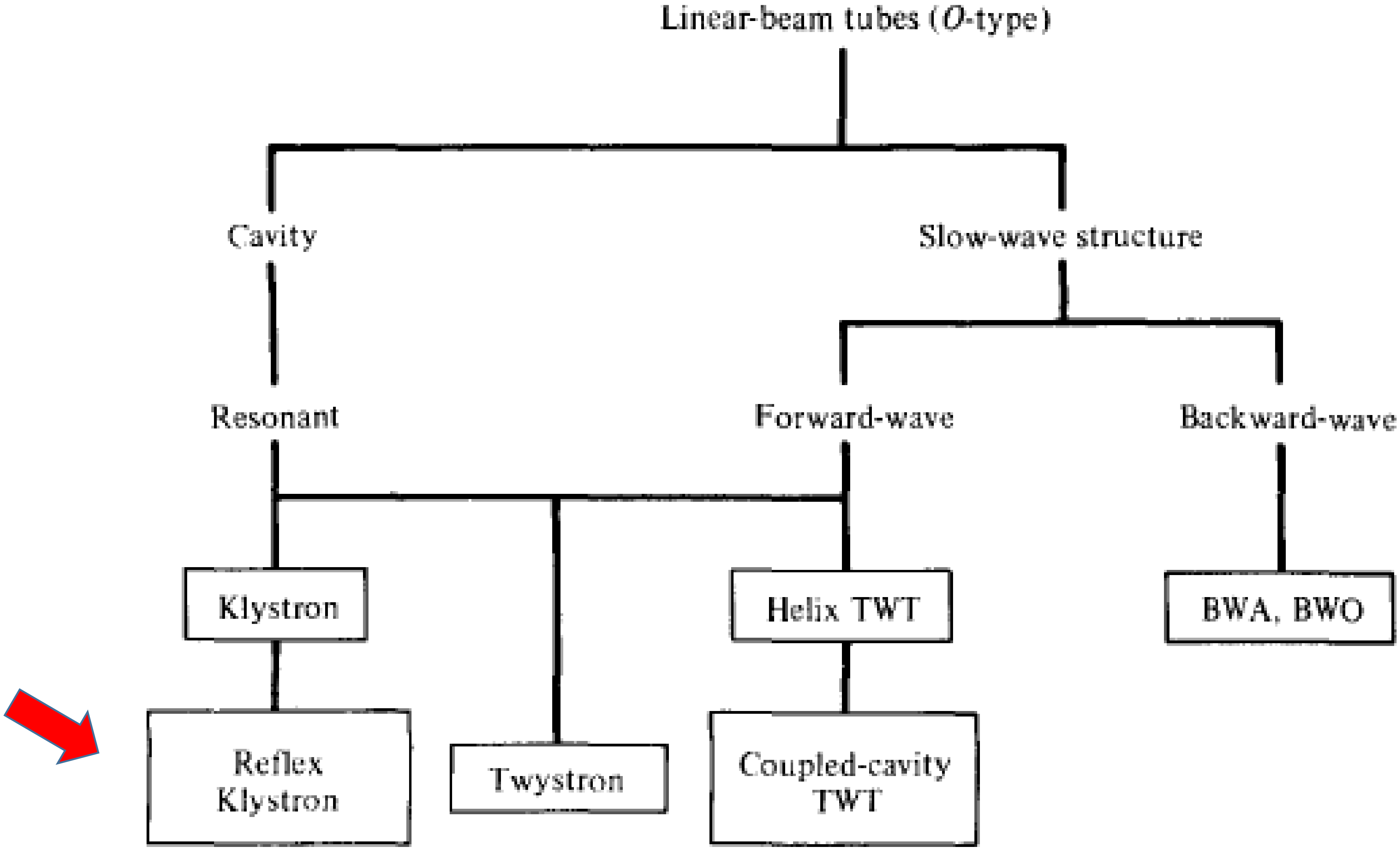
In general, TE_{mnp} m=1, n=0, p=1

$$f_o = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$\frac{1}{\sqrt{\mu\epsilon}} = 3 \times \frac{10^8 m}{s}$$

$$= \frac{3 \times 10^{10}}{2}\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{0}{1}\right)^2 + \left(\frac{1}{3}\right)^2}$$

$$= 9\text{GHz}$$



Linear-beam tubes (O-type)
Cavity
Slow-wave structure
Resonant
Forward-wave
Backward-wave
Klystron
Helix TWT
BWA, BWO
Reflex Klystron
Twystron
Coupled-cavity TWT

Linear Beam Tube (O type)

- In a linear beam tube a magnetic field whose axis coincide with that of the electron beam is used to hold the beam together as it travels the length of the tube.
- In these tubes, electrons receive potential energy from the dc beam voltage before they arrive in the microwave interaction region and this energy is converted into their kinetic energy.
- In the microwave interaction region, the electrons are either accelerated or decelerated by the microwave field and then bunched as they drift down the tube.
- The bunched electrons, in turn, induce current in the output structure. The electrons then give up their kinetic energy to the microwave fields and are collected by the collector.

There are two basic configurations of klystron tubes

1. **Two cavity klystron**
2. **Reflex Klystron** used as a low-power Microwave oscillator
3. Multi-cavity klystron used as a microwave amplifier

KLYSTRON

There are two basic configurations of klystron tubes

1. **Reflex Klystron** used as a low-power Microwave oscillator

2. **Multi cavity klystron** used as a microwave amplifier

REENTRANT CAVITY

- Reentrant cavities are designed for use in klystron and microwave triodes.
- A reentrant cavity is one in which the **metallic boundaries extend into the interior** of the cavity
  - Inductance and capacitance decreased
  - Reduced resistance losses
  - Prevents radiation losses

Types

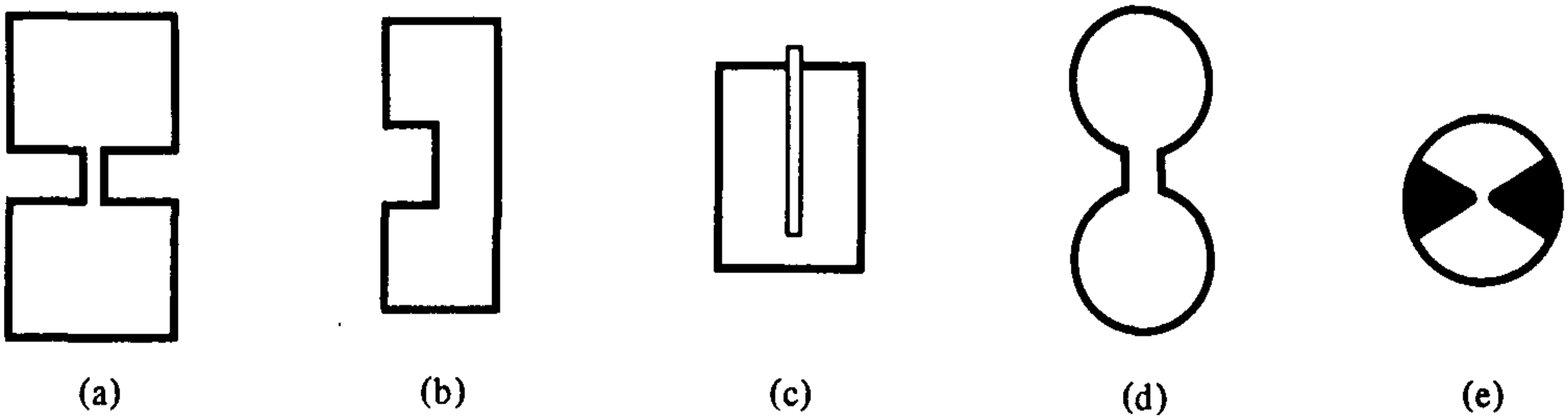


Figure 9-2-3 Reentrant cavities. (a) Coaxial cavity. (b) Radial cavity. (c) Tunable cavity. (d) Toroidal cavity. (c) Butterfly cavity.

Single Cavity Klystron

- Reflex Klystron Oscillators

Introduction

- If a fraction of the output power is fed back to the input cavity and if the loop gain has a magnitude of unity with a phase shift of multiple 2π, the klystron will oscillate.
- A two cavity klystron oscillator is usually not constructed because, when the oscillation frequency is varied, the resonant frequency of each cavity and feedback path phase shift must be readjusted for a positive feedback.

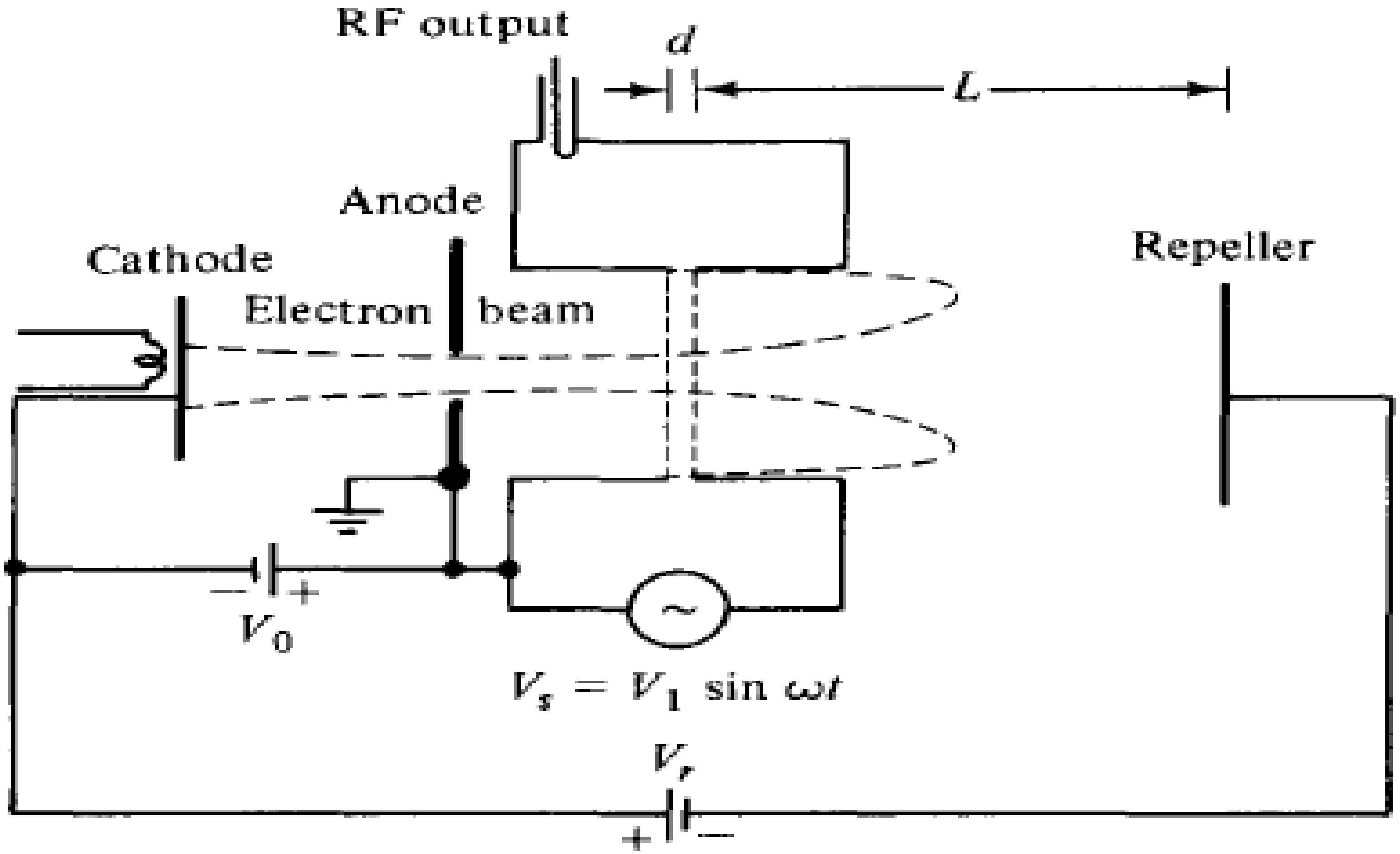
Characteristics

- Reflex Klystron oscillator is a low-power generator of 10 to 500 mW output at a frequency range of 1 to 25 GHz .
- The efficiency is about 20 to 30% .

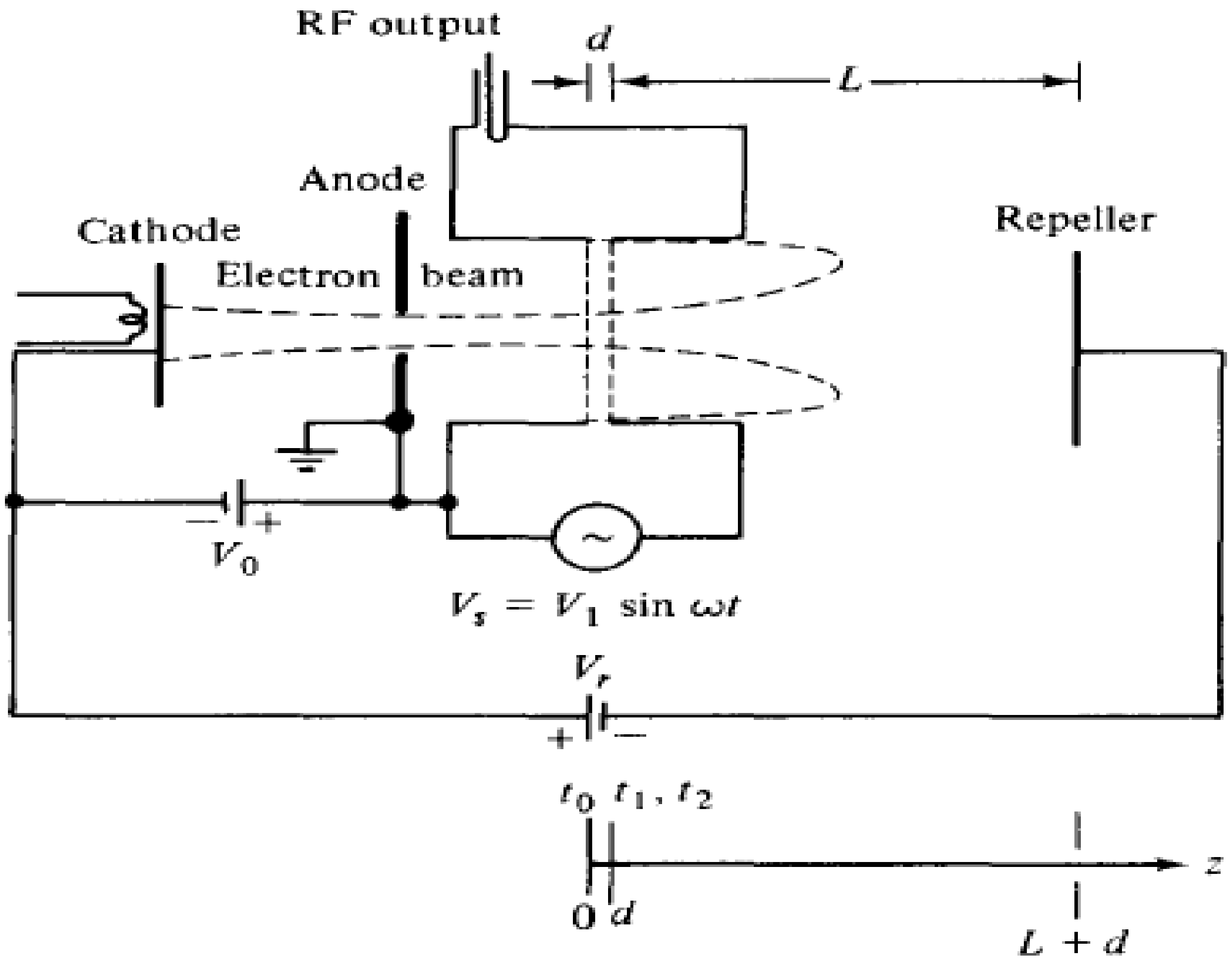
Applications

- Widely used in the laboratory for microwave measurements and in MW receivers as local oscillators in commercial, military and airborne Doppler radar as well as missiles.
- As signal source in MW generator of variable frequency.
- Portable MW links.
- Pump oscillator in parametric amplifier.

Reflex Klystron



Reflex Klystron



t_0 = time for electron entering cavity gap at $z = 0$
t_1 = time for same electron leaving cavity gap at $z = d$
t_2 = time for same electron returned by retarding field $z = d$ and collected on walls of cavity

Figure 9-4-1 Schematic diagram of a reflex klystron.

Working Principle

- **Single Re-entrant cavity** as a resonator.
- The electron beam **emitted** from the cathode is **accelerated** by the grid and passes through the cavity gap to the repeller space between the cavity anode and the repeller electrode(which is kept at a high negative potential V_r) .
- The **feedback** required to maintain the oscillations within the cavity is obtained by **reversing electron beam** emitted from the cathode towards repeller electrode and sending it back through the cavity.
- The electron beam injected from the cathode is first **velocity modulated** by the cavity-gap voltage.

Velocity modulation:

- Some electrons accelerated by the accelerating field enter the repeller space with greater velocity than those with unchanged velocity.
- Some electrons decelerated by the retarding field enter the repeller region with less velocity.
- Mechanism of Oscillation: It is assumed that the **oscillations are set up in the tube initially** due to noise or switching transients and the oscillations are sustained by device operation.
- The **electrons** passing through the cavity gap **experience this RF field** and are **velocity modulated.**

Bunching Process

- All electrons turned around by the repeller voltage then pass through the cavity gap in bunches.
- Bunches occur once per cycle centered around the reference electron and these **bunches transfer maximum energy to the gap** to get sustained oscillations.
- The electrons are finally collected by the walls of the cavity or other grounded metal parts of the tube.
- For oscillations to be sustained, the time taken by the electrons to travel into the repeller space and back to the gap (**transit time**) must have an **optimum value.**

Velocity Modulation

- The quantitative analysis of a reflex klystron can be described in under the following assumptions:

  - The electron beam is assumed to have a uniform density in the cross section of the beam.

  - Space-charge effects are negligible.

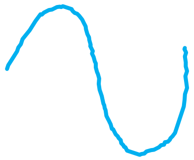
  - The magnitude of the microwave signal input is assumed to be much smaller than the dc accelerating voltage.

Velocity Modulation Process

- When electrons are first accelerated by the high dc voltage Vo before entering the cavity, their velocity is uniform:

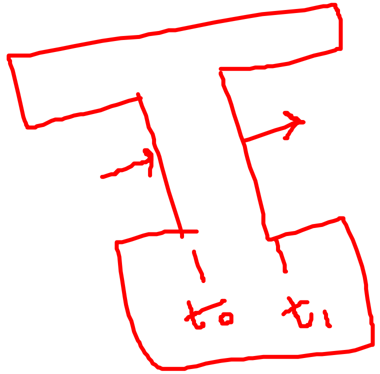
$$v_0 = \sqrt{\frac{2eV_o}{m}} = 0.593 \times 10^6 \sqrt{V_o}\ \mathrm{m/s}$$

- The gap voltage between the cavity grids appears as



$V_s = V_1 \sin(\omega t)$, where V_1 is the amplitude of the signal and $V_1 \ll V_o$ is assumed.

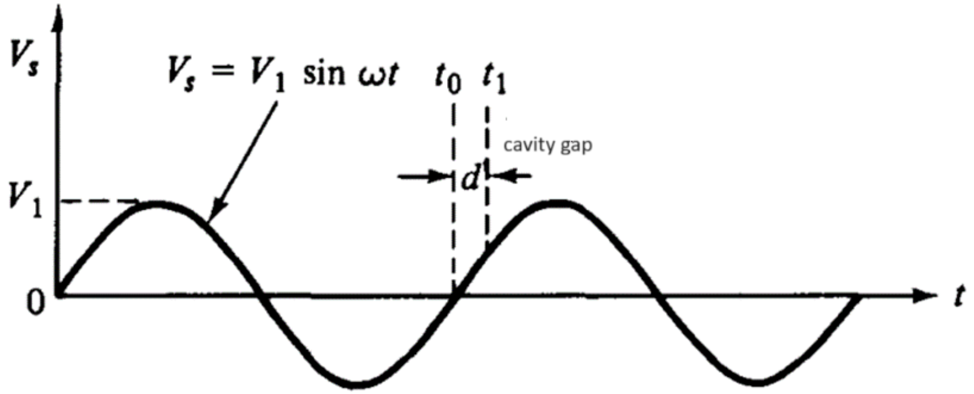
<u>To determine the average microwave voltage in the cavity gap</u>



- Since $V_1 \ll V_0$, the average transit time through the cavity gap distance d is

$$\tau \approx \frac{d}{v_0} = \mathrm{t}_1 - \mathrm{t}_0$$

Velocity Modulation Process



- The average gap transit angle can be expressed as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

$$\omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

$$\omega t_1 = \omega t_0 + \frac{\omega d}{v_0}$$

- The average microwave voltage in the cavity gap

$$\langle V_s \rangle = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t)\, dt = -\frac{V_1}{\omega\tau}[\cos(\omega t_1) - \cos(\omega t_0)]$$

$$= \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos(\omega t_0 + \frac{\omega d}{v_0})\right]$$

Velocity Modulation Process

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

- The average microwave voltage in the cavity gap

$$\langle V_s \rangle = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)\, dt = \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos(\omega t_0 + \frac{\omega d}{v_0})\right]$$

We have,

$$A - B = \omega t_0 + \frac{\theta_g}{2} - \frac{\theta_g}{2} = \omega t_0$$

$$Let, \qquad \omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A$$

$$A + B = \omega t_0 + \frac{\theta_g}{2} + \frac{\theta_g}{2} = \omega t_0 + \theta_g$$

$$\frac{\omega d}{2v_0} = \frac{\theta_g}{2} = B$$

$$\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$$

$$\cos \omega t_0 - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right) = 2\sin(\omega t_0 + \frac{\omega d}{2v_0})\sin(\frac{\omega d}{2v_0})$$

Velocity Modulation Process

$$\cos\omega t_0 - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right) = 2\sin(\omega t_0 + \frac{\omega d}{2v_0})\sin(\frac{\omega d}{2v_0})$$

- The average microwave voltage in the cavity gap

$$\langle V_s\rangle = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)\,dt = \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos(\omega t_0 + \frac{\omega d}{v_0})\right]$$

$$= \frac{V_1}{\omega\tau} 2\sin(\omega t_0 + \frac{\omega d}{2v_0})\sin(\frac{\omega d}{2v_0}) = \frac{V_1}{\frac{\omega d}{v_o}} 2\sin(\omega t_0 + \frac{\omega d}{2v_0})\sin(\frac{\omega d}{2v_0})$$

$$= \frac{V_1}{\frac{\omega d}{2v_o}}\sin(\frac{\omega d}{2v_0})\sin(\omega t_0 + \frac{\omega d}{2v_0}) = \frac{V_1\sin(\theta_g/2)}{\theta_g/2}\sin(\omega t_0 + \frac{\omega d}{2v_0})$$

$$= \beta_i V_1 \sin(\omega t_0 + \frac{\omega d}{2v_0})$$

β_i is known as the beam-coupling coefficient of the input cavity gap

*Increasing the gap transit angle decreases the coupling between the electron beam and the cavity; that is, the velocity modulation of the beam for a given microwave signal is decreased.

Velocity Modulation Process

- Immediately after velocity modulation, the exit velocity from the cavity gap is given by

$$v(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i\, V_1 \sin(\omega t_0 + \frac{\omega d}{2v_0})\right]}$$

$$v_0 = \sqrt{\frac{2eV_o}{m}}$$

$$= \sqrt{\frac{2e}{m}V_0\left[1 + \frac{\beta_i V_1}{\mathrm{V}_0}\sin(\omega t_0 + \frac{\theta_g}{2})\right]}$$

Using binomial expansion under the assumption,

$\beta_\mathrm{i}\frac{\mathrm{V}_1}{\mathrm{V}_0}$: depth of velocity modulation

$\beta_i V_1 \ll V_0$

$\mathrm{If\ x} \ll 1, (1+\mathrm{x})^\mathrm{n} = 1 + \mathrm{nx}$

$$v(t_1) = v_o[1 + \beta_\mathrm{i}\frac{\mathrm{V}_1}{2\mathrm{V}_0}\sin(\omega t_0 + \frac{\theta_g}{2})$$

$$\left(1 + \frac{\beta_i V_1}{\mathrm{V}_0}\sin(\omega t_0 + \frac{\theta_g}{2})\right)^{1/2} = \left[1 + \frac{\beta_i V_1}{2\mathrm{V}_0}\sin(\omega t_0 + \frac{\theta_g}{2})\right]$$

Velocity Modulation Process

- Velocity modulation, the exit velocity from the cavity gap is given by

$$v(t_1) = v_o[1 + \beta_i \frac{V_1}{2V_0} \sin(\omega t_0 + \frac{\theta_g}{2})$$

$$\theta_g = \omega t_1 - \omega t_0$$
$$\omega t_0 = \omega t_1 - \theta_g$$
$$\omega t_0 + \frac{\theta_g}{2} = \omega t_1 - \theta_g + \frac{\theta_g}{2}$$
$$= \omega t_1 - \frac{\theta_g}{2}$$

- Alternatively, the equation of velocity modulation can be given by

$$\boldsymbol{v(t_1)} = \boldsymbol{v_o[1 + \beta_i \frac{V_1}{2V_0} \sin(\omega t_1 - \frac{\theta_g}{2})}$$

Once the electrons leave the cavity, they drift with this velocity.

Velocity Modulation

- The electron entering the cavity gap from the cathode at z = 0 and time to is assumed to have uniform velocity

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} \qquad v_0 = \sqrt{\frac{2eV_o}{m}}$$

- The same electron leaves the cavity gap at z=d at time t_1 with velocity

$$v(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

- The same electron is forced back to the cavity z=d and time t_2 by the retarding electric field E given by,

$$E = \frac{V_r + V_0 + V_1 \ sin\omega t}{L}$$

This retarding field E is assumed to be constant in the Z direction.

Velocity Modulation

$$E = \frac{V_r + V_0 + V_1 \; sin\omega t}{L}$$

- The force equation for one electron in the repeller region is

$$\mathrm{m}\,\frac{d^2z}{dt^2} = -eE = -e\frac{V_r + V_0}{L} --- -(1)$$

$E \; is \; along \; z - direction, V_r$ is the magnitude of the repeller voltage, and $|V_1 \; sin\omega t| \ll (V_r + V_0)$ is assumed.

- Integration of equation (1) twice and substitution of electron leaving and returning time of the cavity gap yields, **the round –trip transit time in the repeller region**.

Velocity Modulation

$$v(t_1) = v_o[1 + \beta_i \frac{V_1}{2V_0} \sin(\omega t_1 - \frac{\theta_g}{2})$$

- The round trip transit time in the repeller region is given by

$$T' = t_2 - t_1 = \frac{2mL}{e\,(V_r + V_0)} v(t_1) = T_o{}'\left[1 + \frac{\beta_i V_1}{2V_0} sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$T_o' = \frac{2mL}{e\,(V_r + V_0)} v_0$$, round trip dc transit time of the center of the bunch electron.

$$\theta_0{}' = \omega T_o{}'$$

Round trip dc transit angle

- Radian frequency

$$\omega T' = \omega(t_2 - t_1) = \omega T_o{}'\left[1 + \frac{\beta_i V_1}{2V_0} sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$X' = \frac{\beta_i V_1 \theta_0{}'}{2V_0}$$

Bunching parameter of the reflex klystron oscillator

$$\omega T' = \omega(t_2 - t_1) = \theta_0{}'\left[1 + \frac{\beta_i V_1}{2V_0} sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$\omega T' = \omega(t_2 - t_1) = \theta_0{}' + X' \sin\,(\omega t_1 - \frac{\theta_g}{2})$$

Reflex Klystron

- It is assumed that the oscillations are set up in the tube initially due to noise. (Vs)
- The electron beam injected from cathode is first velocity modulated by the cavity gap voltage.
- Let us consider three electrons namely early electron (A), reference electron(B) and late electron (C).
- The **early electron** experiences a maximum positive voltage and this electron is accelerated and moves with greater velocity and penetrate deep into the repeller space.
- The **reference electron** that passes through the gap experience zero voltage gets unaffected by the gap voltage. It will move towards the repeller and gets reflected back.
- The **late electron** experiences maximum negative voltage moves with a retarding velocity which penetrate less into repeller space.

Reflex Klystron

- In order for the electron beam to generate maximum amount of energy, the returning electron beam must cross the cavity gap when the field is maximum retarding.
- Hence maximum amount of kinetic energy can be transferred from returned electrons to the cavity gaps.
- Bunch occurs once per cycle centred around reference electron.

Applegate Diagram

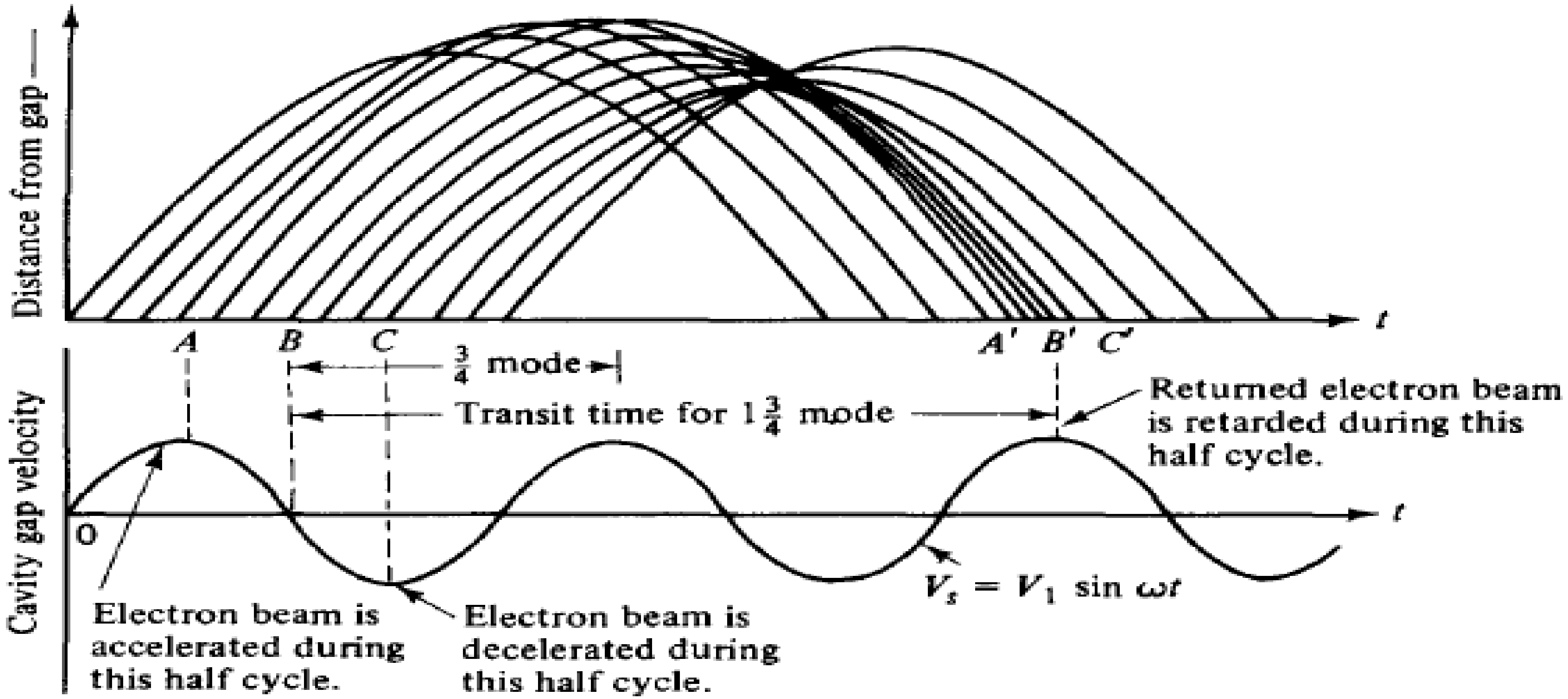


Figure 9-4-2 Applegate diagram with gap voltage for a reflex klystron.

Mode of Oscillation

- The mode of oscillation is named as N = ¾, 1 ¾ ,2 ¾ etc for modes n = 1,2,3,……

  Where N= $n-\frac{1}{4}$, n-any positive integer

- This type of a Klystron is called a **Reflex Klystron** because of the reflex action of the electron beam.

Power Output and Efficiency

- In order for the electron beam to generate a maximum amount of energy to the oscillation, the retarding electron beam must cross the cavity gap when the gap field is maximum retarding.
- So, a maximum amount of kinetic energy can be transferred from the returning electrons to the cavity walls.
- Bunch occurs once per cycle centred around reference electron.
- For maximum energy transfer, the round-trip transit angle, referring to the center of the bunch, must be given by

$$\omega(t_2 - t_1) = \omega T_0' = \left(n - \frac{1}{4}\right)2\pi = 2\pi N = 2\pi n - \frac{\pi}{2} \quad \text{-------(8)}$$

- The factor $X'J_1(X')$ reaches a maximum value of 1.25 at $X' = 2.408$ and $J_1(X') = 0.52$. In practice, the mode of n = 2 has the most power output.

- Electronic efficiency $= \frac{P_{ac}}{P_{dc}} = \frac{2X'J_1(X')}{2\pi n - \frac{\pi}{2}}$ --------- (9)

- $\text{Efficiency}_{\max} = \frac{2(2.408)J_1(2.408)}{2\pi(2)-\pi/2}$

  $=22.7\%$

- The maximum theoretical efficiency ranges from 20 to 30%.

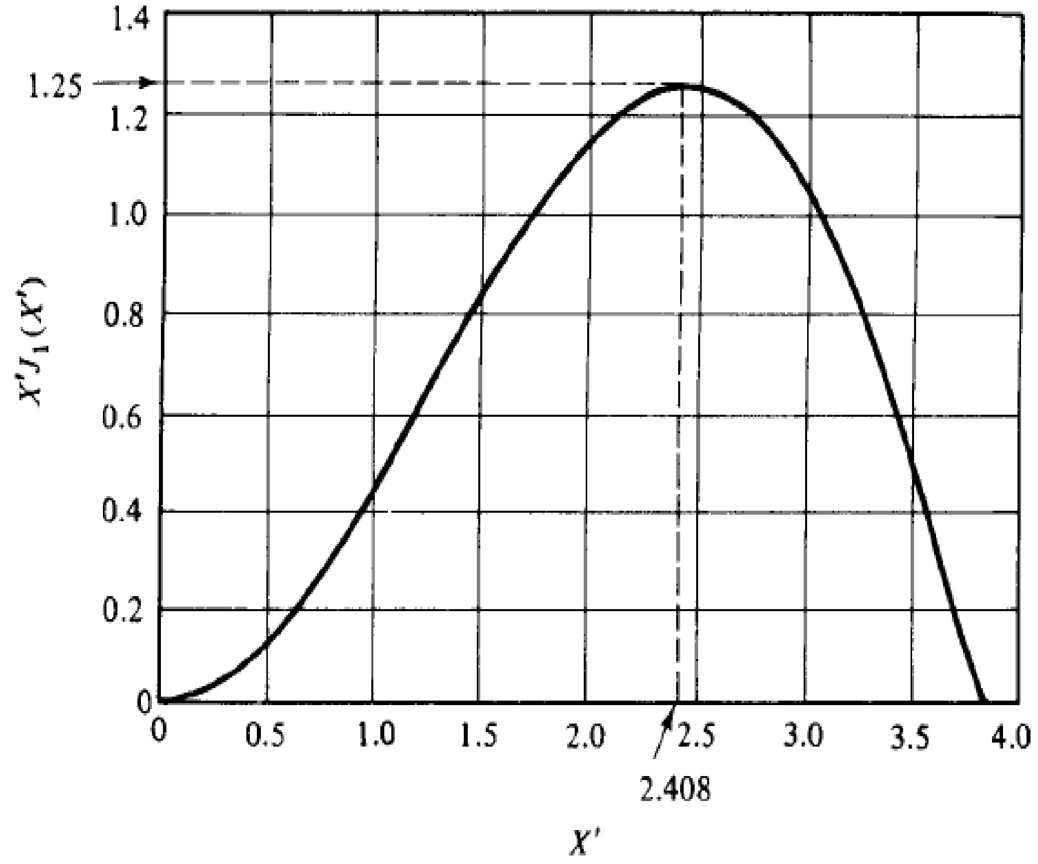


Figure 9-4-3 $X'J_1(X')$ versus X'.

Derivation of Power Output & Efficiency: Assignment

I_0 *is the DC current*

- The magnitude of fundamental component of the current induced in the cavity by the modulated electron beam is given by $I_2 = 2I_0\beta_i J_1(X')$

$$\theta_0' = \omega T_0' = 2\pi n - \frac{\pi}{2}$$

- The dc power supplied by the beam voltage V_0 is $P_{dc} = V_0 I_0$

$$X' = \frac{\beta_i V_1 \theta_0{}'}{2V_0}$$

- The ac power delivered to the load is

$$P_{ac} = \frac{V_1 I_2}{2} = V_1 I_0 \beta_i J_1(X')$$

$$\omega(t_2 - t_1) = \omega T_0' = \left(n - \frac{1}{4}\right)2\pi = N2\pi = 2\pi n - \frac{\pi}{2}$$

- The ratio of V_1 over V_o is expressed as (derived from $X' = \frac{\beta_i V_1 \theta_0{}'}{2V_0}$) ,

$$\frac{V_1}{V_0} = \frac{2X'}{\beta_i(2\pi n - \frac{\pi}{2})}$$

$$V_1 = \frac{2V_0 X'}{\beta_i \theta_0{}'} = \frac{2V_0 X'}{\beta_i \omega T_0{}'} = \frac{2V_0 X'}{\beta_i(2\pi n - \frac{\pi}{2})}$$

Power Output & Efficiency

- The ratio of V1 over Vo is expressed as,

$$P_{dc} = V_0 I_0$$

$$\frac{V_1}{V_0} = \frac{2X'}{\beta_i(2\pi n - \frac{\pi}{2})}$$

- The ac power delivered to the load is

$$P_{ac} = \frac{V_1 I_2}{2} = V_1 I_0 \beta_i J_1(X') = \frac{2X' V_0 I_0 \beta_i J_1(X')}{\beta_i(2\pi n - \frac{\pi}{2})} = \frac{2V_0 I_0 X' J_1(X')}{(2\pi n - \frac{\pi}{2})}$$

- The electronic efficiency of a reflex klystron

$$\boldsymbol{Efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{2V_0 I_0 X' J_1(X')}{V_0 I_0 (2\pi n - \frac{\pi}{2})} = \frac{\boldsymbol{2X' J_1(X')}}{\boldsymbol{2\pi n - \frac{\pi}{2}}}$$

Power Output & Efficiency

- The electronic efficiency of a reflex klystron

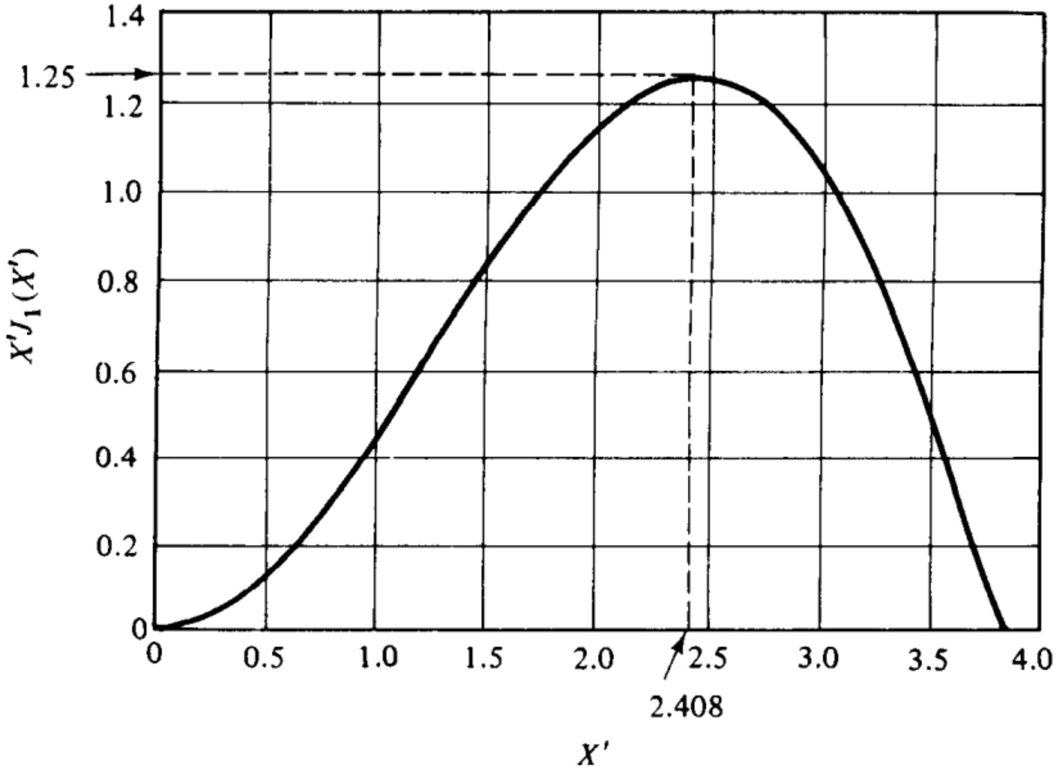
$$Efficiency = \frac{P_{ac}}{P_{dc}} = \frac{2V_0 I_0 X' J_1(X')}{V_0 I_0 (2\pi n - \frac{\pi}{2})} = \frac{2X' J_1(X')}{2\pi n - \frac{\pi}{2}}$$

- The factor $X' J_1(X')$ reaches a maximum value of 1.25 at $X' = 2.408$ and $J_1(X') = 0.52$. In practice, the mode of n = 2 has the most power output.

- If n = 2 or $1\frac{3}{4}$ mode, the maximum electronic efficiency becomes

$$Efficiency_{max} = \frac{2(2.408) J_1(2.408)}{2\pi(2) - \frac{\pi}{2}} = 22.7\%$$

The maximum theoretical efficiency of a reflex klystron oscillator ranges from 20 to 30%.



$X'J_1(X')$ versus X'.

RELATIONSHIP BETWEEN REPELLER VOLTAGE AND CYCLE NUMBER

- For a given beam voltage V_0,the relationship between the repeller voltage and cycle number n required for oscillation is found by inserting equations
- $\theta_0' = 2\pi n - \frac{\pi}{2}$ & $v_0 = 0.593 \times 10^6 \sqrt{V_0} = \sqrt{\frac{2eV_o}{m}}$

into $T_0' = \frac{2mLv_0}{e(V_r+V_0)}$

$$\theta_0' = \omega T_0' = \frac{\omega 2mLv_0}{e(V_r+V_0)}$$

$$2\pi n - \frac{\pi}{2} = \frac{\omega 2mL \times v_0}{e(V_r+V_0)}$$

Relationship between repeller voltage and cycle number

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \qquad \text{m/s}$$

$$2\pi n - \frac{\pi}{2} = \frac{\omega 2mL\times v_0}{e(V_r+V_0)}$$

Squaring on both sides will give the relation

$$\left(2\pi n - \frac{\pi}{2}\right)^2 = \frac{4\omega^2 L^2 m^2 v_0^2}{e^2(V_r + V_0)^2}$$

$$v_0^2 = \frac{2eV_0}{m}$$

$$\frac{V_0}{(V_r+V_0)^2} = \frac{\left(2\pi n - \frac{\pi}{2}\right)^2}{8\omega^2 L^2}\frac{e}{m}$$

Power and frequency characteristics of Reflex Klystron

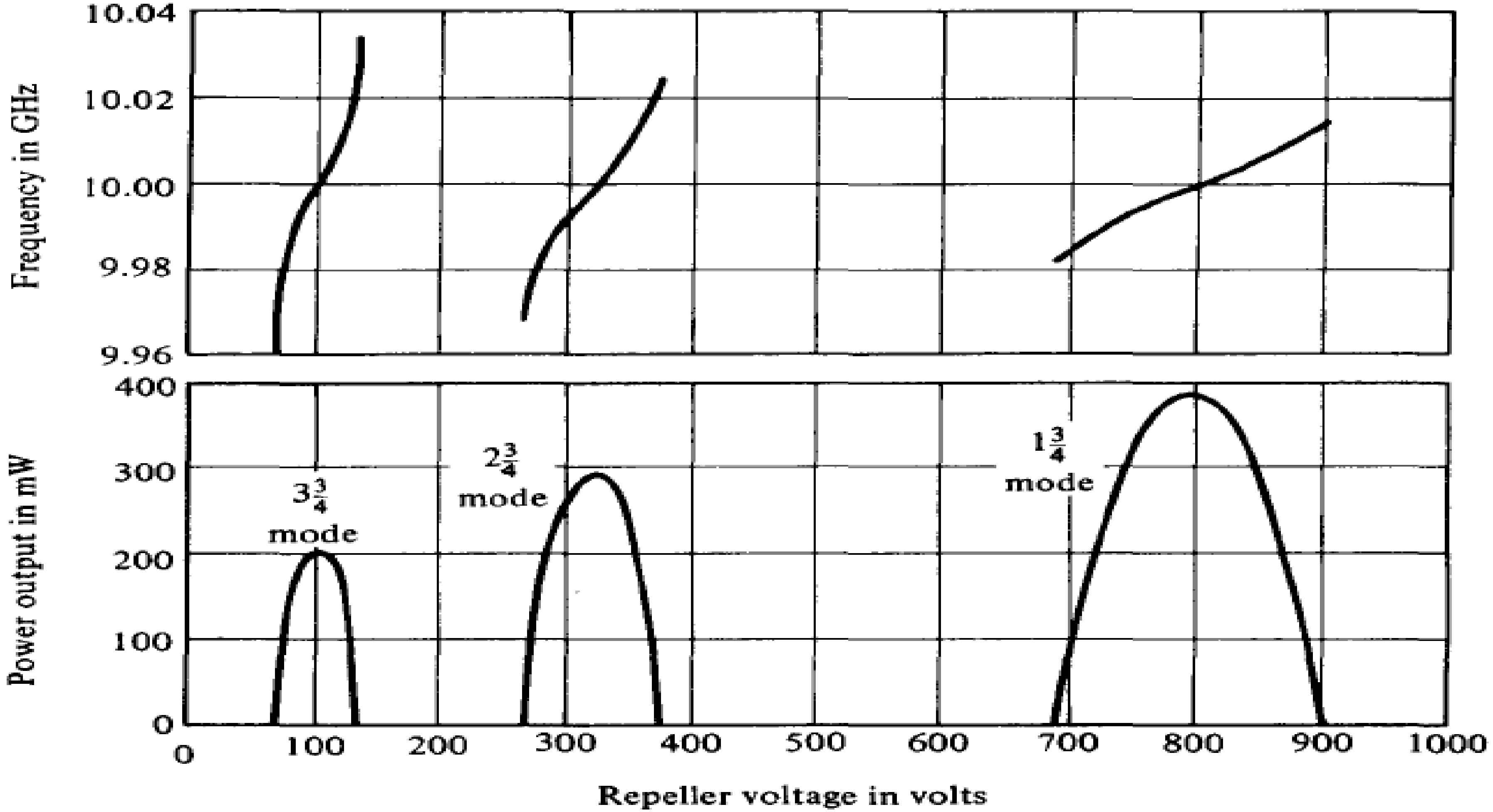


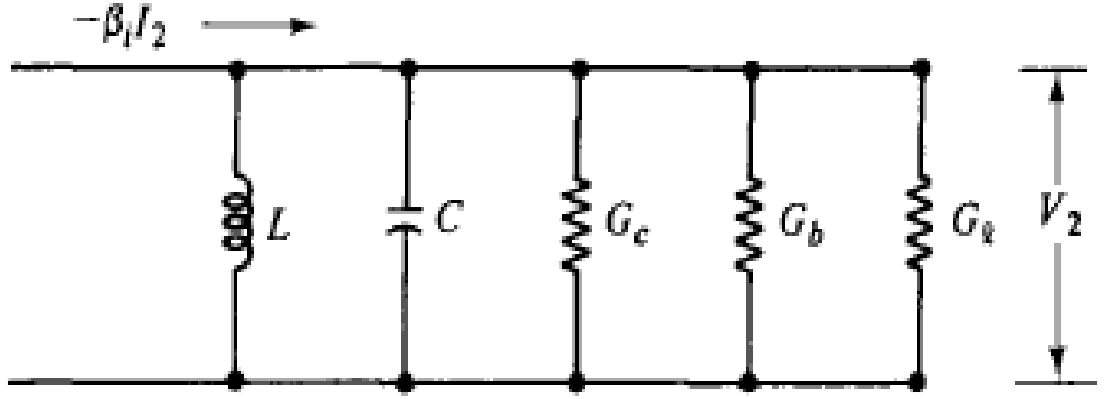
Figure 9-4-4 Power output and frequency characteristics of a reflex klystron.

Electronic Admittance

- The ratio of output current to output voltage is defined as the electronic admittance of the reflex klystron.

$$Y_e = \frac{I_0}{V_0} \frac{\beta_i^2 \theta_0'}{2} \frac{2J_1(X')}{X'} e^{j(\pi/2-\theta_0')}$$

- The equivalent circuit of a reflex klystron is shown .



- The necessary condition for oscillations is that the magnitude of the negative real part of the electronic admittance not be less than the total conductance of the cavity circuit.

$$I_2 = 2I_0\beta_i J_1(X')$$

$$|-G_e| \geq G$$

Where G =$G_C + G_b + G_l$= $\frac{1}{R_{sh}}$

R_{sh} is the effective shunt resistance

- The voltage $V_2 = I_2 R_{sh} = 2I_o J_1(X')R_{sh}$

Where I_o is the direct current.

- L & C – Energy storage elements of the cavity
- G_c – Copper losses of the cavity
- G_b – Beam loading conductance
- G_l – Load Conductance

- Electronic admittance can be rewritten in rectangular form:

  Y_e =G_e +jB_e
- The rectangular plot of the electron admittance Y_e is a spiral.
- Any value of θ_0' for which the spiral lies in the area to the left of line (-G-jb) will yield oscillation.

$$\theta_0' = \left(n - \frac{1}{4}\right)2\pi = N\ 2\pi$$

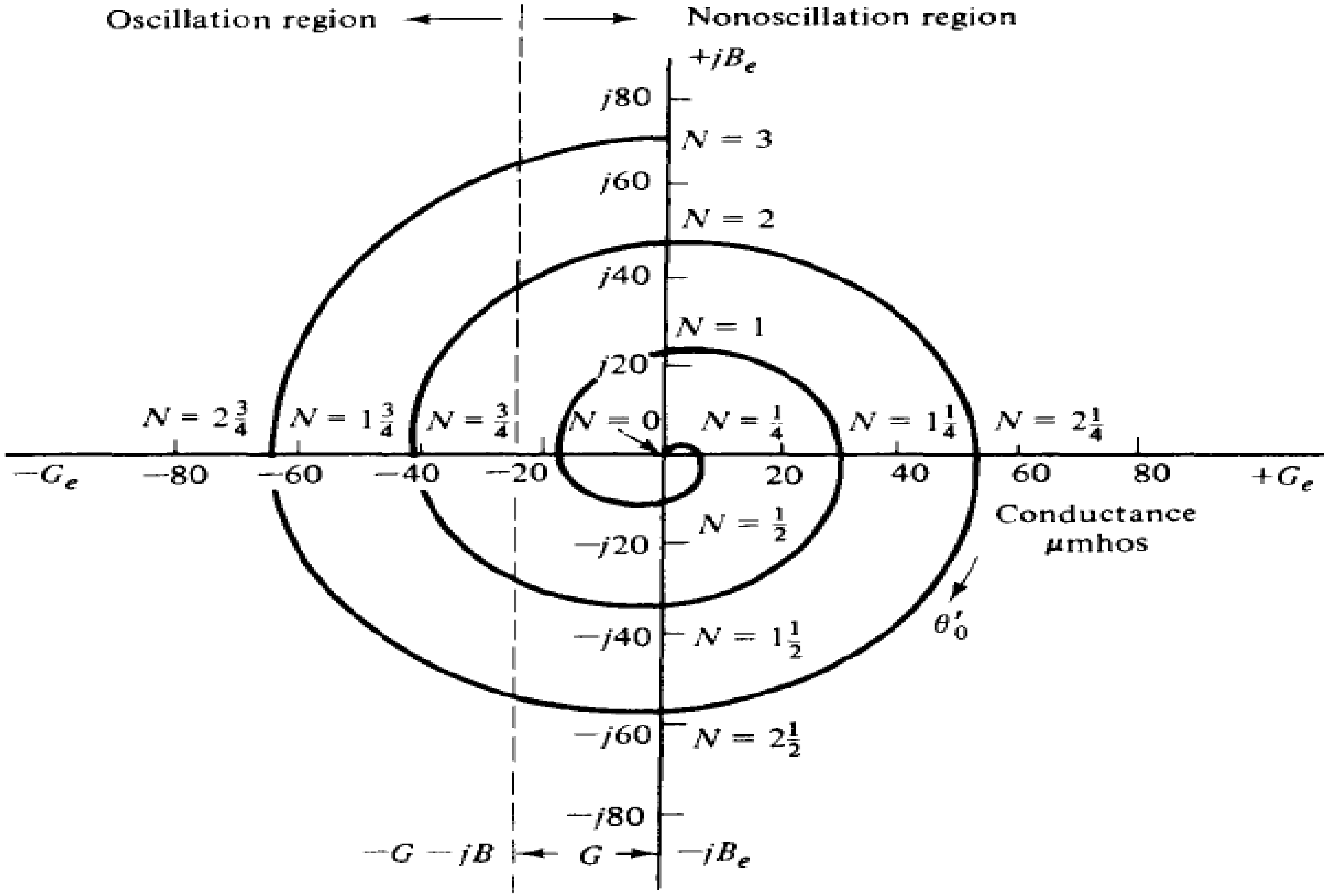


Figure 9-4-7 Electronic admittance spiral of a reflex klystron.

Problem

A reflex klystron operates under the following conditions:

$$V_0 = 600V, \qquad L = 1mm$$

$$R_{sh} = 15k\Omega, \qquad f_r = 9GHz$$

The tube is oscillating at f_r at the peak of n=2 mode or $1\frac{3}{4}$ mode. Assume that the transit time through the gap and beam loading can be neglected.

a. Find the value of repeller voltage, V_r.

b. Find the direct current necessary to give a microwave gap voltage of 200V.

c. What is the electronic efficiency under this condition?

$$Given: X' = 1.841 \; and \; J_1(X') = 0.582$$

$$\frac{V_o}{(V_r + V_o)^2} = \frac{\left(2\pi n - \frac{\pi}{2}\right)^2}{8\omega^2 L^2} \frac{e}{m}$$

$$\frac{V_o}{(V_r + V_o)^2} = \frac{\left(2\pi\, 2\; - \frac{\pi}{2}\right)^2}{8(2\pi \times 9 \times 10^9)^2 (10^{-3})^2} \frac{1.6 * 10^{-19}}{9.1 * 10^{-31}}$$

$$\frac{V_o}{(V_r + V_o)^2} = 0.832 \times 10^{-3}$$

Substitute $V_0 = 600 \text{volts}$,

$$\boldsymbol{V_r = 250 \text{ volts}}$$

Assume $\beta_0 = 1$. Since

$$V_2 = I_2 R_{sh} = 2 I_o J_1(X') R_{sh}$$

Direct current,

$$I_o = \frac{V_2}{2 J_1(X') R_{sh}}$$

Microwave gap voltage, $V_2 = 200V$

$$I_0 = \frac{200}{2 * 0.582 * 15 * 10^3} = 11.45mA$$

Electronic efficiency,

$$\text{Efficiency} = \frac{2X' J_1(X')}{2\pi n - \pi/2} = \frac{2 * 1.841 * 0.582}{2\pi(2) - \pi/2} = 19.49\%$$

Problem

Solution

$$\frac{V_0}{(V_r+V_0)^2}=\frac{\left(2\pi n-\frac{\pi}{2}\right)^2}{8\,\omega^2L^2}\times\frac{e}{m}$$

a. From Eq. (9-4-22) we obtain

$$\frac{V_0}{(V_r+V_0)^2}=\left(\frac{e}{m}\right)\frac{(2\pi n-\pi/2)^2}{8\omega^2L^2}$$

$$=(1.759\times10^{11})\frac{(2\pi 2-\pi/2)^2}{8(2\pi\times9\times10^9)^2(10^{-3})^2}=0.832\times10^{-3}$$

$$(V_r+V_0)^2=\frac{600}{0.832\times10^{-3}}=0.721\times10^6$$

$$V_r=250\text{ V}$$

b. Assume that $\beta_0=1$. Since

$$I_2=2I_0\beta_iJ_1(X')$$

$$V_2=I_2R_{sh}=2I_0J_1(X')R_{sh}$$

$$X'=\frac{\beta_iV_1\theta_0'}{2V_0}=1.841$$

$$J_1(X')=0.582$$

the direct current I_0 is

$$I_0=\frac{V_2}{2J_1(X')R_{sh}}=\frac{200}{2(0.582)(15\times10^3)}=11.45\text{ mA}$$

Problem

$$Efficiency = \frac{2X'J_1(X')}{2\pi n - \frac{\pi}{2}}$$

b. Assume that $\beta_0 = 1$. Since

$$V_2 = I_2 R_{sh} = 2I_0 J_1(X')R_{sh}$$

the direct current I_0 is

$$I_0 = \frac{V_2}{2J_1(X')R_{sh}} = \frac{200}{2(0.582)(15 \times 10^3)} = 11.45 \text{ mA}$$

c. From Eqs. (9-4-11), (9-4-12), and (9-4-20) the electronic efficiency is

$$\text{Efficiency} = \frac{2X'J_1(X')}{2\pi n - \pi/2} = \frac{2(1.841)(0.582)}{2\pi(2) - \pi/2} = 19.49\%$$

Microwave Crossed Field Tubes (Liao Ch.10)

- In a crossed field device (m-type), the dc magnetic and electric field are perpendicular to each other.
- Due to the presence of crossed fields, electrons emitted from cathode are influenced to move in curved paths.
- Only those electrons that have given up sufficient energy to the RF field can travel all the way to the anode.
- This makes m-type devices more efficient.

Magnetron Oscillator

- The generation of microwaves in such devices involves interaction of EM waves with electrons moving in presence of crossed electric and magnetic fields.
- The magnetron consists of some form of anode and cathode operated in a dc magnetic field that is normal to dc electric field between the cathode and anode.
- The electrons accelerated by the electric field and gain velocity. The greater the velocity, the more their path is bent by magnetic field.
- If the dc magnetic field is strong enough, the electrons will not arrive at the anode but return to the cathode.

Classification

- **Split Anode Magnetron:** It uses static negative resistance between the two anode segments. *It operates at frequencies below microwave region.*

- **Cyclotron frequency magnetron:** It operates under the influence of synchronism between alternating component of E field and a periodic oscillation of electrons in a direction parallel to the field. *It's output power and efficiency is very low.*

- **Travelling wave magnetron:** Based on the interaction of e^- with a travelling EM field of constant angular velocity.

  Adv: It provides oscillations of high peak power useful in radar applications

  (Eg: Cylindrical Magnetron)

Cylindrical Magnetron

- It consists of a thick cylindrical cathode at the centre and a coaxial cylindrical block of copper as anode.
- In the anode block, a number of holes and slots are cut which act as resonant anode cavities.
- The space between the cathode and anode – Interaction Space
- It is a crossed field device as electric field between anode and cathode is radial whereas the magnetic field produced by a permanent magnet is axial.
- The magnet is placed such that the magnetic lines are parallel to the vertical cathode and perpendicular to the electric field between the cathode and anode.

Cavity Magnetron

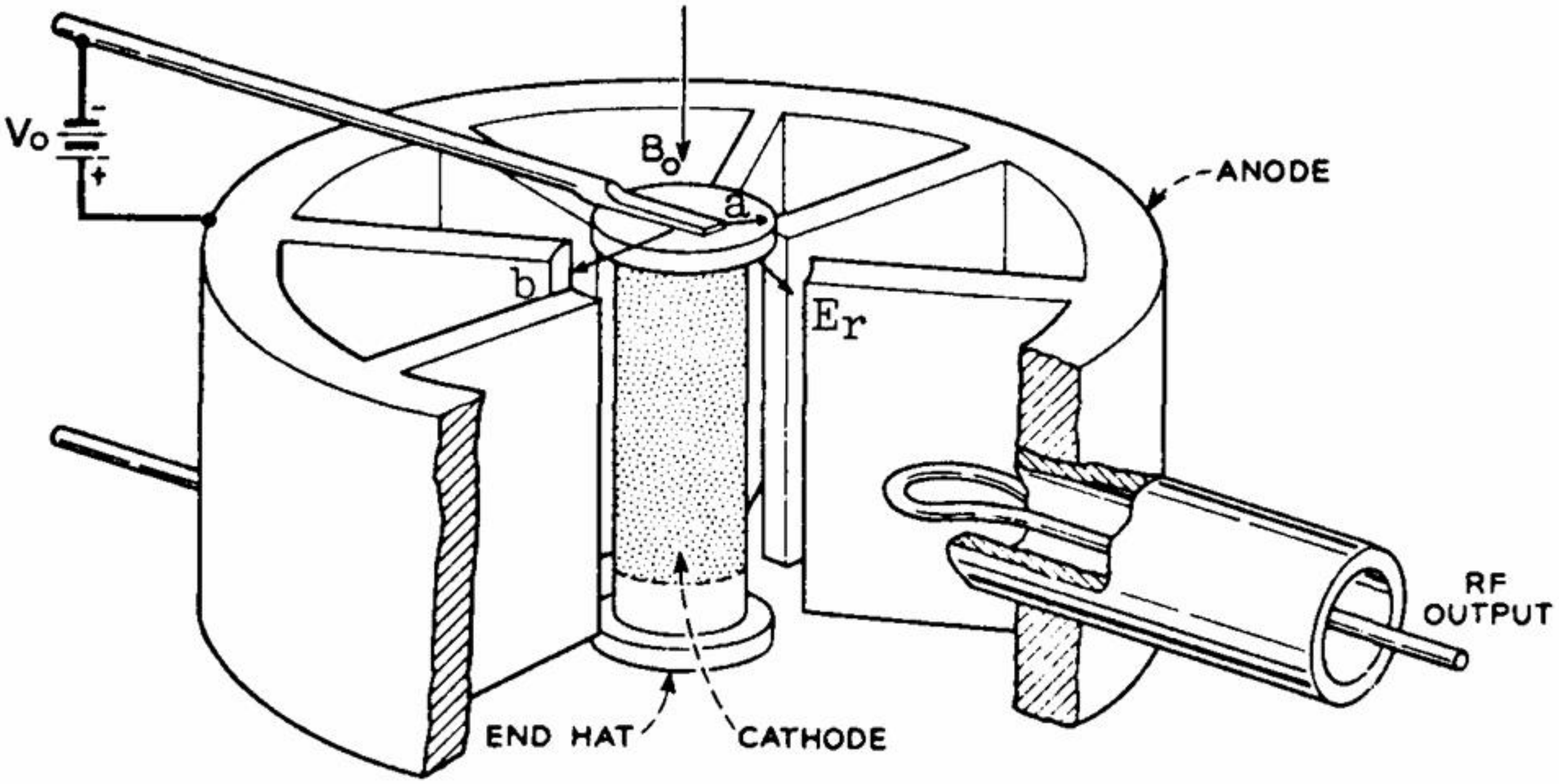
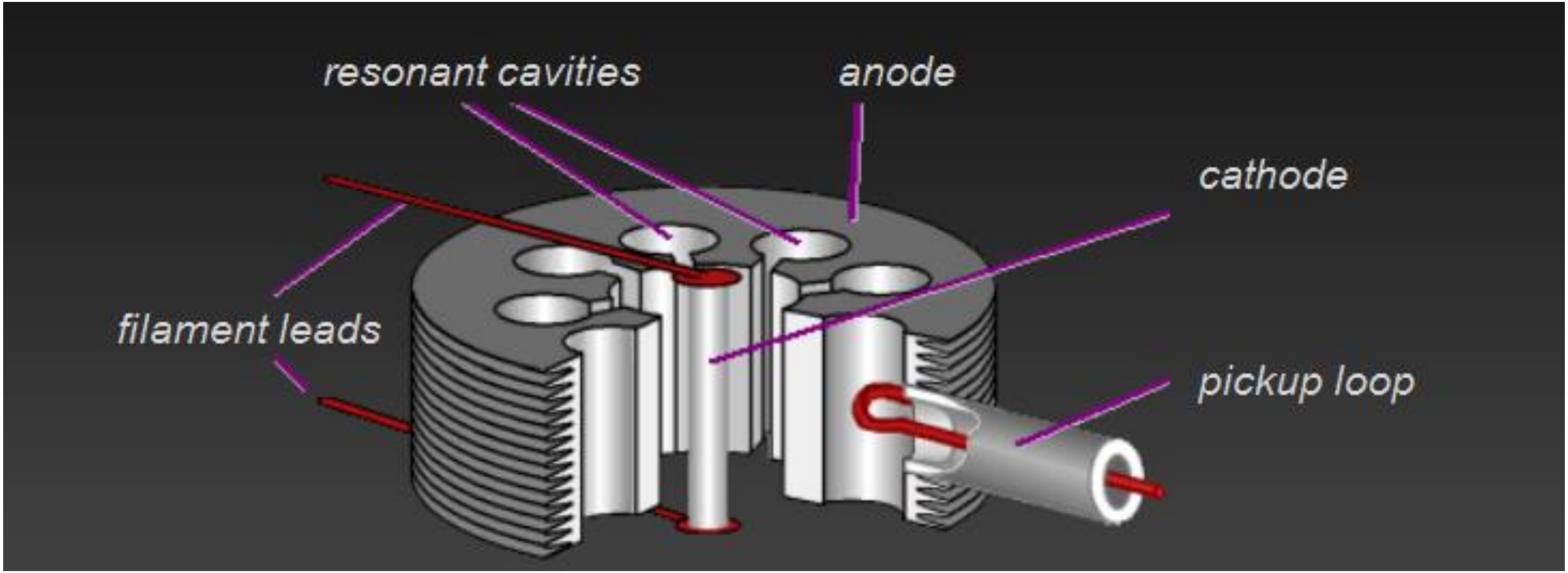
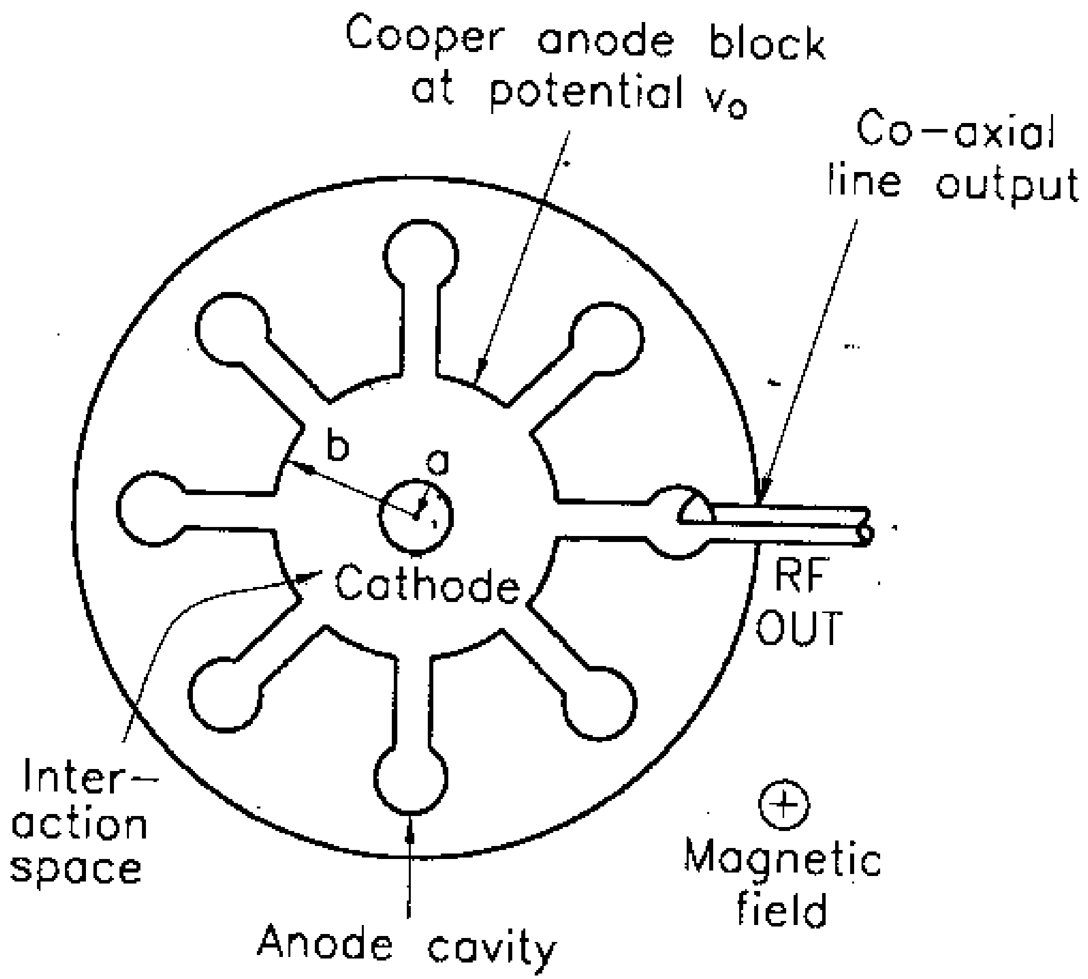


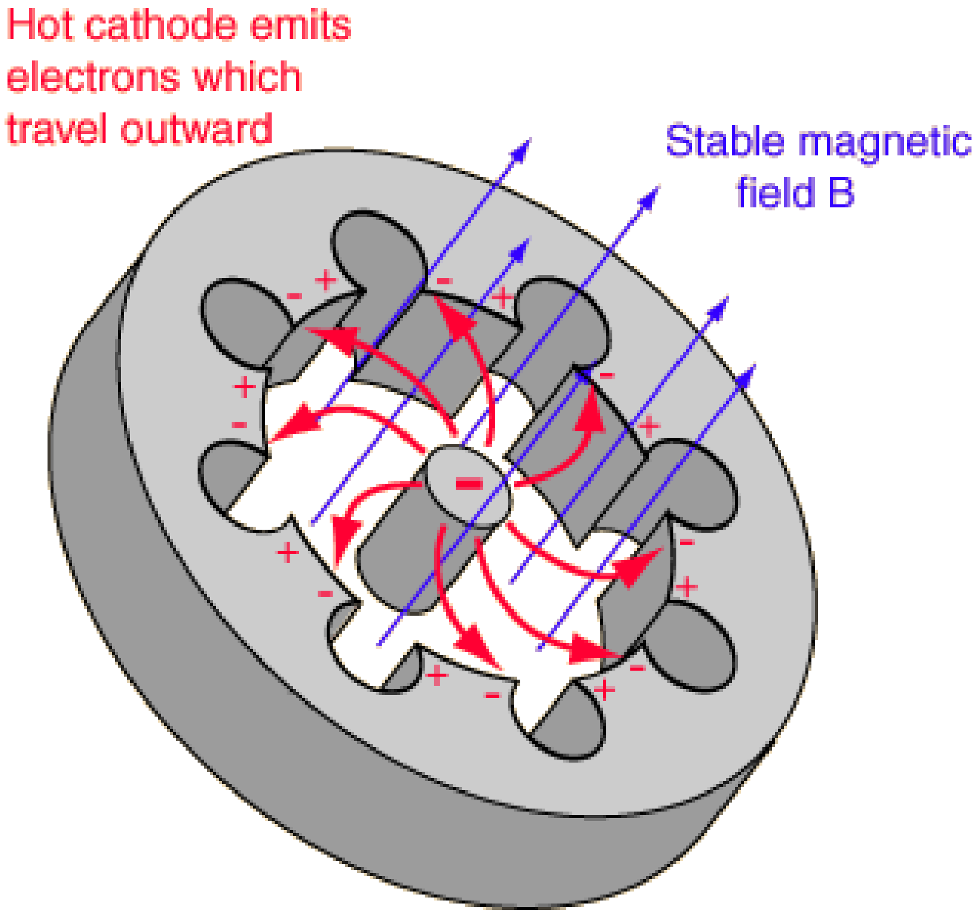
Figure 10-1-1 Schematic diagram of a cylindrical magnetron.

Cavity Magnetron





Cooper anode block
at potential v_o
Co-axial
line output
b
a
Cathode
RF
OUT
Inter-
action
space
Magnetic
field
Anode cavity



Hot cathode emits
electrons which
travel outward
Stable magnetic
field B

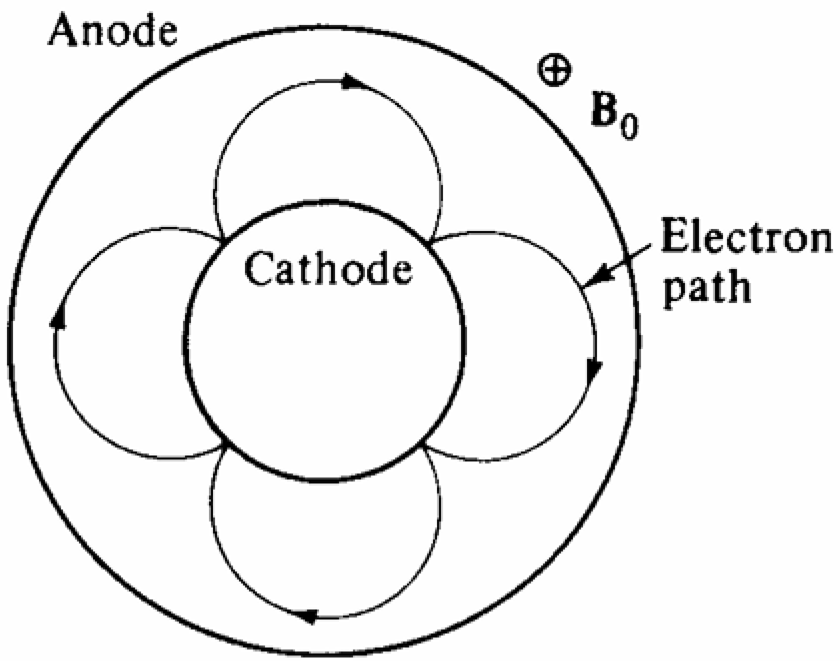
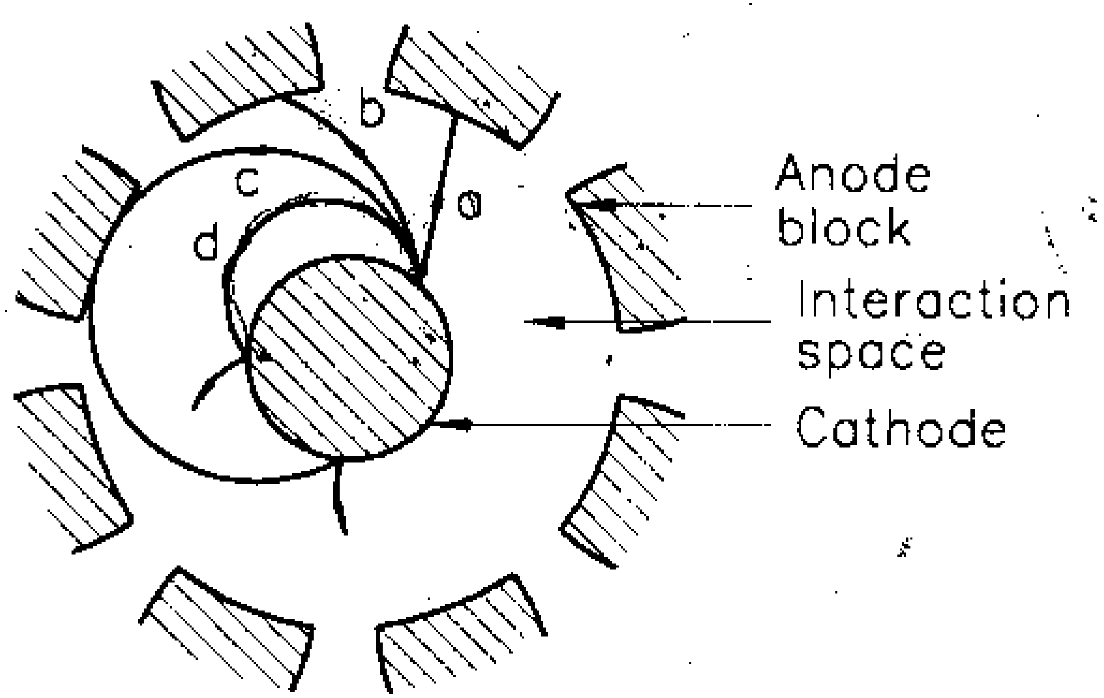


Figure 10-1-2 Electron path in a cylindrical magnetron.

Operation

- **In the absence of RF field in the cavity of magnetron:**

Depending on the relative strength of electric and magnetic field, the e^- emitted from cathode, move towards the anode through the interaction space in different paths.



a – *No magnetic field*

b – *Small magnetic field*

c – *Magnetic field* $= B_c$

d – *Excessive magnetic field*

Radius of path, $R = \frac{mv}{eB}$

- **In presence of RF field:**

Assuming that the RF oscillations are initiated by some noise transient, the oscillation will be sustained only by device operation.

- For sustained oscillation, the total phase shift around the ring of cavity resonators must be $2n\pi$, where n is an integer.
- If ϕ_v represents the relative phase change of electric field across adjacent cavities then,

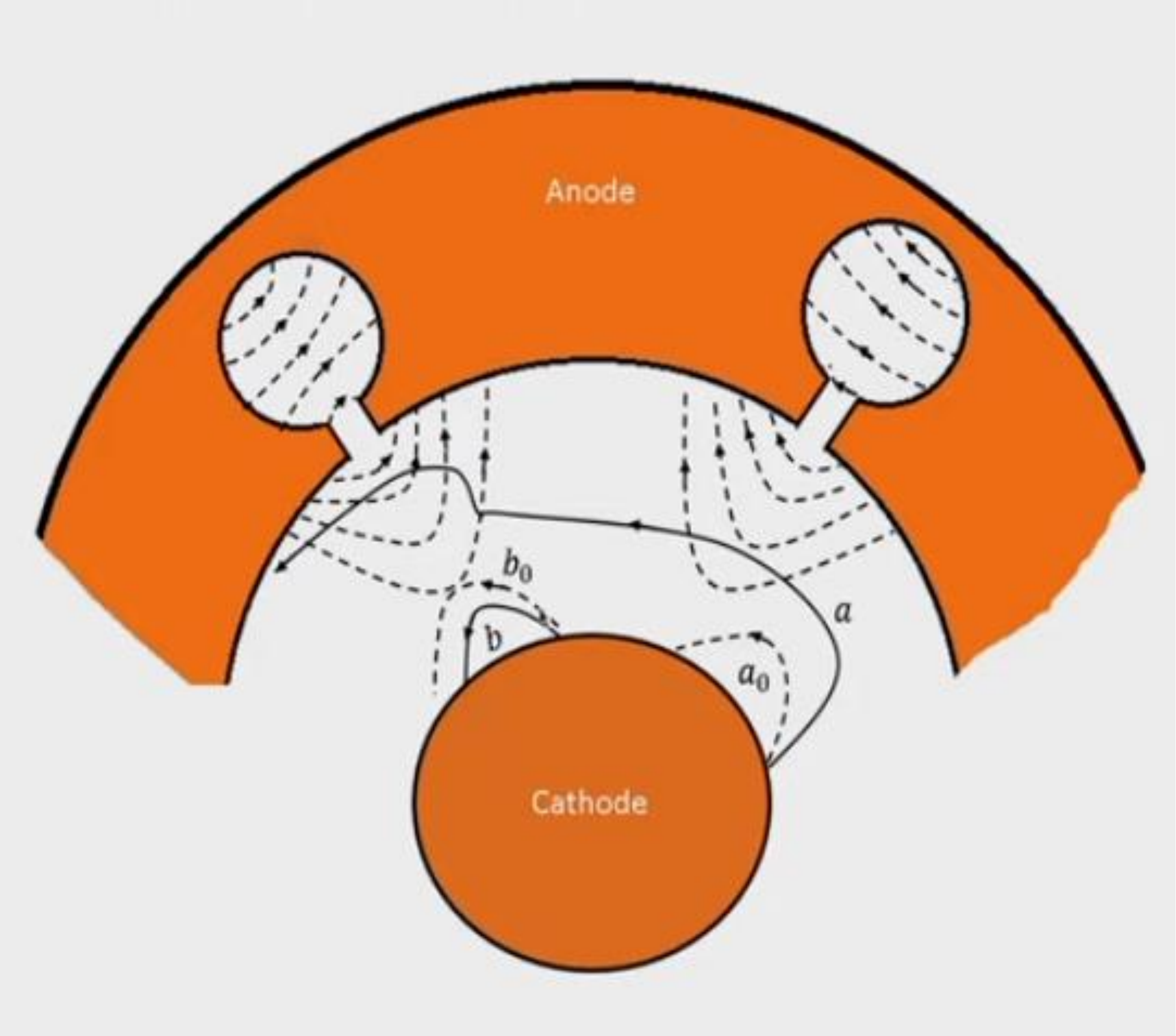
$$\phi_v = \frac{2n\pi}{N}$$

where N – no. of cavities

$$n = 0, \pm 1, \pm 2, \ldots . \pm \frac{N}{2}$$

If $n = \frac{N}{2}$, $\phi_v = \pi$ $\Rightarrow$ π- mode of resonance

π- mode of Operation



- The electron $'a'$ is slowed down by the RF field thus transferring energy to the oscillation.
- These are the favoured electrons and are responsible for bunching.
- The electron $'b'$ is accelerated by the RF field thus taking energy from the oscillation.
- Such e^- bend more sharply and return back to cathode.
- These are the unfavoured electrons that do not participate in bunching.

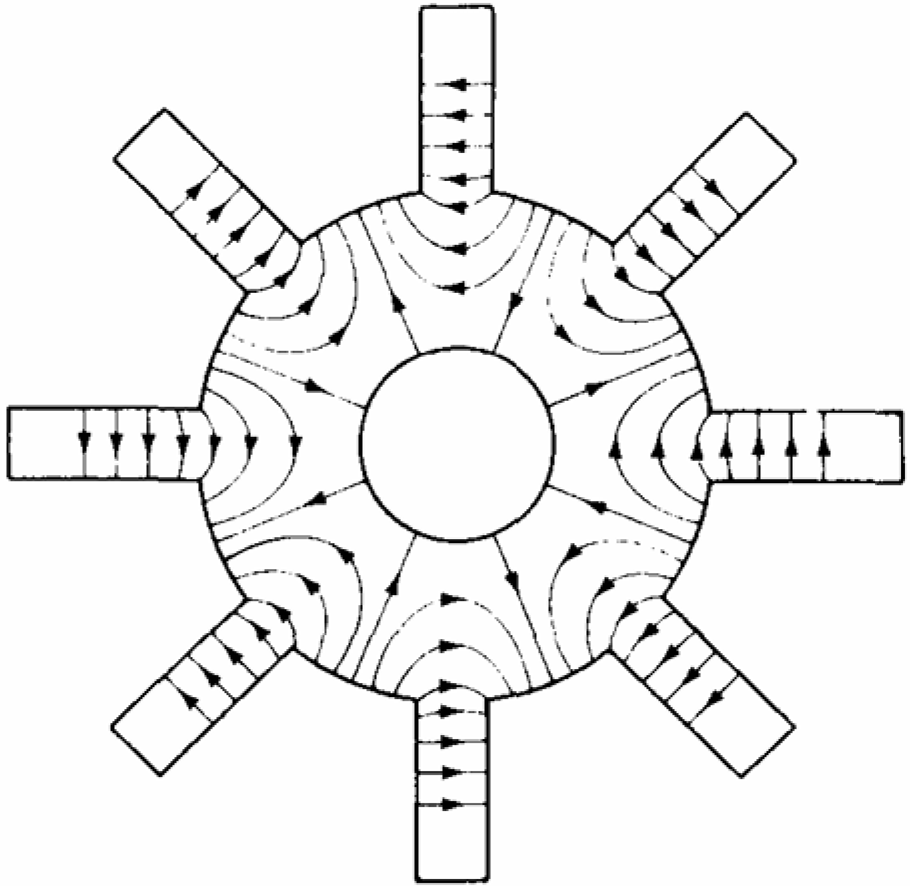
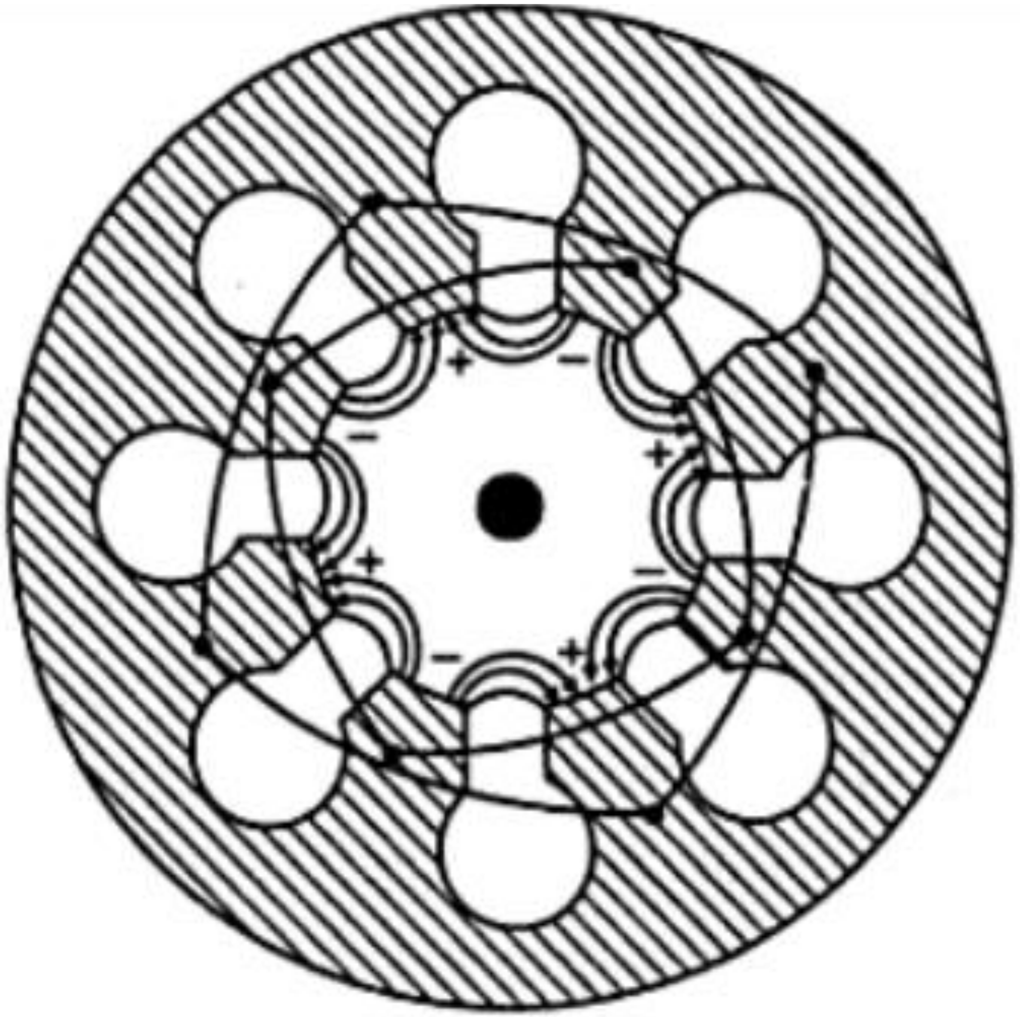


Figure 10-1-3 Lines of force in π mode of eight-cavity magnetron.

- This mechanism which leads to formation of electron bunches and by which electrons are kept in synchronism with the RF field is called **phase focussing effect**.
- The number of bunches depends on the number of cavities in the magnetron and the mode of oscillations.
- Two identical resonant cavities will resonate at two frequencies when they are coupled together; this is due to the effect of mutual coupling.
- Commonly separating the pi mode from adjacent modes is by a method called strapping. The straps consist of either circular or rectangular cross section connected to alternate segments of the anode block.

Strapping



Performance Characteristics:

1. Power output: In excess of 250 kW (Pulsed Mode), 10 mW (UHF band), 2 mW   (X band), 8 kW (at 95 GHz)
2. Frequency: 500 MHz – 12 GHz
3. Duty cycle: 0.1 %
4. Efficiency: 40 % - 70 %

Applications:

1. Pulsed radar is the single most important application with large pulse powers.
2. Voltage tunable magnetrons are used in sweep oscillators in telemetry and in missile applications.
3. Fixed frequency, CW magnetrons are used for industrial heating and microwave ovens.

Cylindrical Magnetron

- In a cylindrical magnetron, several reentrant cavities are connected to the gaps.
- The dc voltage V_0 is applied between the cathode and anode.
- The magnetic flux density B_0 is in the positive z direction.
- When the dc voltage and magnetic flux adjusted properly, the electrons will follow cycloidal paths in the cathode anode space under the combined force of both electric and magnetic field.

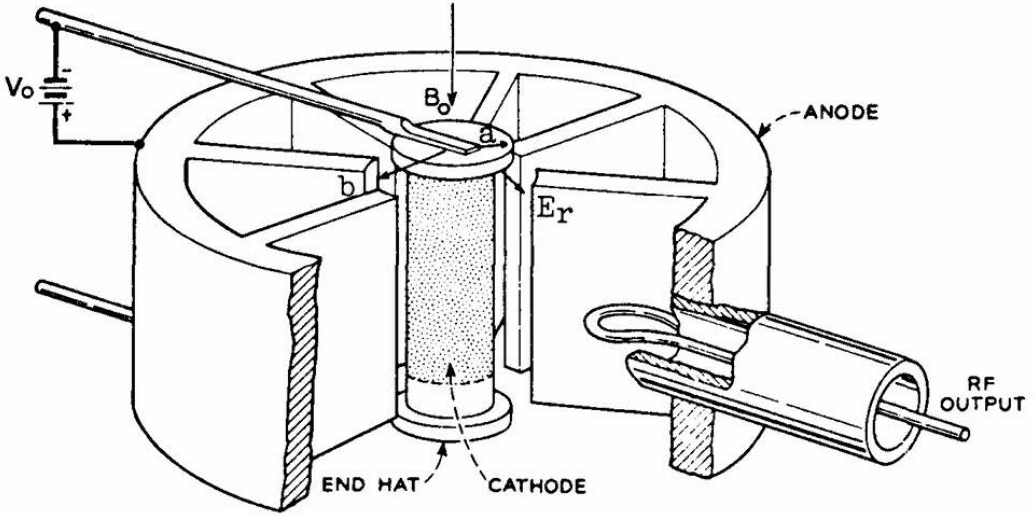


Figure 10-1-1 Schematic diagram of a cylindrical magnetron.

Equations of Electron Motion

- To describe the motion of an electron in a magnetic field:

A charged particle in motion in a magnetic field of flux density B will experience a force proportional to the charge Q, the velocity v, the flux density B and the sine of the angle between the vectors v and B.

$$F = Qv \times B = QvBsin\theta$$

$$F = -ev \times B$$

The equations of motion for electrons in a cylindrical magnetron can be written as:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_r - \frac{e}{m}rB_z\frac{d\phi}{dt} \quad .. (1)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{dr}{dt} \quad .. (2)$$

where $\frac{e}{m} = 1.759 \times 10^{11}\ C/kg$ is the charge to mass ratio of e^-

and $B_0 = B_z$ (assumed in $+ve\ z$ direction)

Rearranging terms of eq. (2)

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{dr}{dt}$$

$$\boxed{\frac{dr^2}{dt} = 2r\frac{dr}{dt}}$$

$$\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z r\frac{dr}{dt} = \frac{1}{2}\omega_c\frac{d}{dt}(r^2) \quad \text{.. (3)}$$

where $\boxed{\omega_c = \frac{e}{m}B_z}$ → ***Cyclotron Angular Frequency***

Integrating eq. (3),

$$r^2\frac{d\phi}{dt} = \frac{1}{2}\omega_c r^2 + \text{constant} \quad \text{.. (4)}$$

At $r = a$ (radius of cathode cylinder) and $\frac{d\phi}{dt} = 0$, constant$= -\frac{1}{2}\omega_c a^2$

$$r^2 \frac{d\phi}{dt} = \frac{1}{2}\omega_c \mathrm{r}^2 - \frac{1}{2}\omega_c \mathrm{a}^2$$

- Angular velocity can be expressed as

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c \left(1 - \frac{\mathrm{a}^2}{\mathrm{r}^2}\right) \quad \text{.. (5)}$$

- Since magnetic field does no work on electrons, kinetic energy of electron is given by:

$$\frac{1}{2} m \mathcal{V}^2 = eV \quad \text{.. (6)}$$

- The electron velocity has both r and ϕ components,

$$\mathcal{V}^2 = \frac{2e}{m} V = \mathcal{V}_r^2 + \mathcal{V}_\phi^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2 \quad \text{.. (7)}$$

- At $r = b$ (radius to edge of anode),

$V = V_0$ and $\frac{dr}{dt} = 0$ (electrons just graze the anode surface)

Then eq. (5) and (7) become

- Eq. (5) → $$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right)$$ .. **(8)**

Eq. (7) → $$b^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m}V_0$$ .. **(9)**

Substituting eq. (8) in (9),

$$b^2\left[\frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2e}{m}V_0$$ .. **(10)**

$$\omega_c^2 = \frac{2e}{m}V_0 \times \frac{4}{b^2\left(1-\frac{a^2}{b^2}\right)^2}$$

$$\omega_c = \sqrt{\frac{2eV_0}{m}} \times \frac{2}{b\left(1-\frac{a^2}{b^2}\right)}$$

Since, $\boxed{\omega_c = \frac{e}{m}B_{0_c}}$

Hull cutoff magnetic equation ➡ $$\boxed{B_{0c} = \frac{\left(8V_0\frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}}$$ .. **(11)**

- The voltage provided between anode and cathode so that the e^- will just graze the anode and return towards the cathode is known as **Hull's cut off voltage** (V_{0_c})
- The corresponding **cut off magnetic flux density (B_{0_c})**
- If $\boldsymbol{B_0 > B_{0_c}}$ for a given V_0, the electrons will not reach the anode.
- Conversely, the cut off voltage is given by,

$$\boxed{V_{0c} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2} \quad \text{.. (12)}$$

- Also known as **Hull cutoff voltage equation**
- If $\boldsymbol{V_0 < V_{0_c}}$ for a given B_0, the electrons will not reach the anode.

- Oscillations are possible only if the total phase shift around the structure is an integer multiple of 2π
- If there are N re-entrant cavities in the anode structure, phase shift between adjacent cavities is

$$\phi_n = \frac{2\pi m}{N}$$

- Magnetron oscillators are normally operated in the π mode

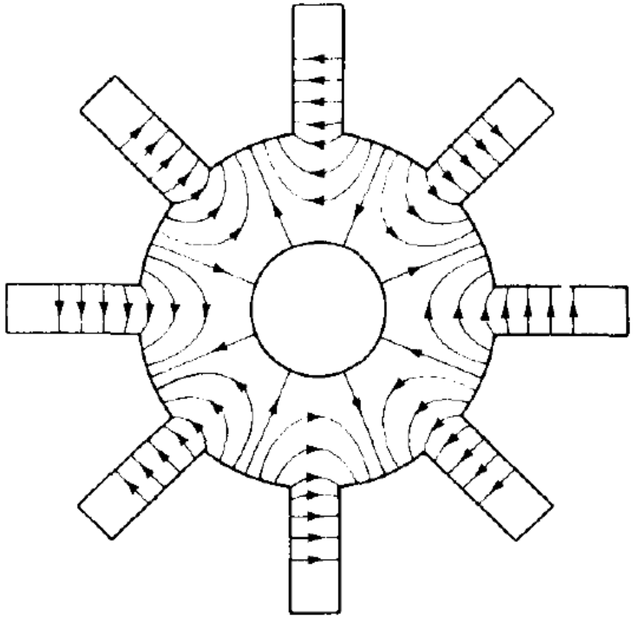


Figure 10-1-3 Lines of force in π mode of eight-cavity magnetron.

Power Output & Efficiency: Assignment

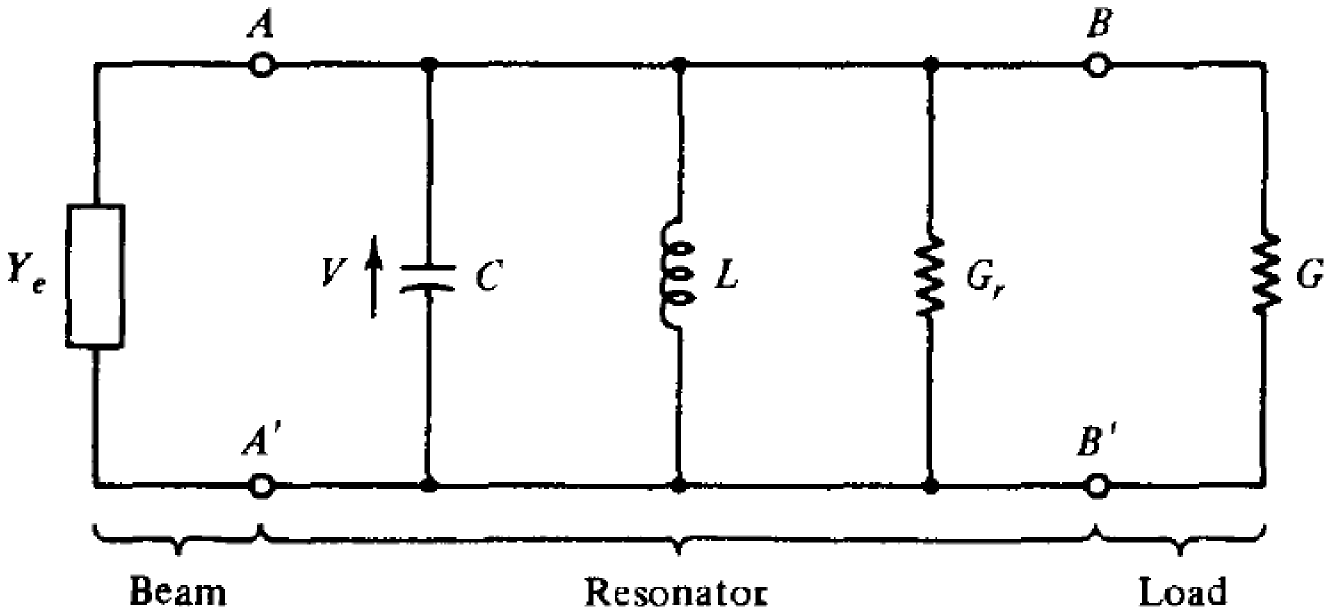


Figure 10-1-4 Equivalent circuit for one resonator of a magnetron.

Y_e = electronic admittance
V = RF voltage across the vane tips
C = capacitance at the vane tips
L = inductance of the resonator
G_r = conductance of the resonator
G = load conductance per resonator

Power Output & Efficiency

- Each resonator of the slow wave structure is considered to be a separate resonant circuit.
- Unloaded Quality factor of resonator,

$$\boxed{Q_{un} = \frac{\omega_o C}{G_r}}$$

where $\omega_o = 2\pi f_0$ is the angular resonant frequency

- The external quality factor of the load circuit is

$$\boxed{Q_{ex} = \frac{\omega_o C}{G_l}}$$

- The loaded Q_l of the resonant circuit is

$$\boxed{Q_l = \frac{\omega_o C}{G_r + G_l}}$$

- The circuit efficiency is given by

$$\eta_c = \frac{G_l}{G_r + G_l} = \frac{1}{1 + \frac{G_r}{G_l}} = \frac{1}{1 + \frac{Q_{ex}}{Q_{un}}}$$

- The maximum circuit efficiency is obtained when the magnetron is heavily loaded i.e. $G_l \gg G_r$
- The electronic efficiency is defined as:

$$\eta_e = P_{gen}/P_{dc} = \frac{V_o I_o - P_{lost}}{V_o I_o}$$

An X-band pulsed cylindrical magnetron has the following operating parameters:

Anode voltage:	$V_0 = 26$ kV
Beam current:	$I_0 = 27$ A
Magnetic flux density:	$B_0 = 0.336$ Wb/m^2
Radius of cathode cylinder:	$a = 5$ cm
Radius of vane edge to center:	$b = 10$ cm

Compute:

a. The cyclotron angular frequency

b. The cutoff voltage for a fixed B_0

c. The cutoff magnetic flux density for a fixed V_0

a. The cyclotron angular frequency is

$$\omega_c = \frac{e}{m}B_0 = 1.759 \times 10^{11} \times 0.336 = 5.91 \times 10^{10} \text{ rad}$$

b. The cutoff voltage for a fixed B_0 is

$$V_{0c} = \frac{1}{8} \times 1.759 \times 10^{11}\,(0.336)^2\,(10 \times 10^{-2})^2 \left(1 - \frac{5^2}{10^2}\right)^2$$

$$= 139.6 \times 10^5 \text{ volts}$$

c. The cutoff magnetic flux density for a fixed V_0 is

$$B_{0c} = \left(8 \times 26 \times 10^3 \times \frac{1}{1.759 \times 10^{11}}\right)^{1/2} \left[10 \times 10^{-2}\left(1 - \frac{5^2}{10^2}\right)\right]^{-1}$$

$$= 14.495 \text{ mWb/m}^2$$

Question: An X-band pulsed conventional magnetron has the following operating parameters:

Anode voltage:	$V_0 = 5.5$ kV
Beam current:	$I_0 = 4.5$ A
Operating frequency:	$f = 9 \times 10^9$ Hz
Resonator conductance:	$G_r = 2 \times 10^{-4}$ mho
Loaded conductance:	$G_\ell = 2.5 \times 10^{-5}$ mho
Vane capacitance:	$C = 2.5$ pF
Duty cycle:	$DC = 0.002$
Power loss:	$P_{\text{loss}} = 18.50$ kW

Compute:

- **a.** The angular resonant frequency
- **b.** The unloaded quality factor
- **c.** The loaded quality factor
- **d.** The external quality factor
- **e.** The circuit efficiency
- **f.** The electronic efficiency

a. The angular resonant frequency is

$$\omega_r = 2 \times 9 \times 10^9 = 56.55 \times 10^9 \text{ rad}$$

b. The unloaded quality factor is

$$Q_{un} = \frac{56.55 \times 10^9 \times 2.5 \times 10^{-12}}{2 \times 10^{-4}} = 707$$

c. The loaded quality factor is

$$Q_{\ell} = \frac{56.55 \times 10^9 \times 2.5 \times 10^{-12}}{2 \times 10^{-4} + 2.5 \times 10^{-5}} = 628$$

d. The external quality factor is

$$Q_{ex} = \frac{56.55 \times 10^{9} \times 2.5 \times 10^{-12}}{2.5 \times 10^{-5}} = 5655$$

e. The circuit efficiency is

$$\eta_c = \frac{1}{1 + 5655/707} = 11.11\%$$

f. The electronic efficiency is

$$\eta_e = \frac{5.5 \times 10^{3} \times 4.5 - 18.5 \times 10^{3}}{5.5 \times 10^{3} \times 4.5} = 25.25\%$$

Travelling Wave Tube (TWT)

- The traveling-wave tube (TWT) was invented in 1944 by Kompfner.
- For Broadband amplifier helix TWTs (proposed by Pierce and others in 1946) are widely used.
- For high average power purposes, the coupled cavity TWTs are used.
- The Traveling-Wave Tube (TWT) is an amplifier of microwave energy.
- It accomplishes this through the interaction of an electron beam and an RF circuit known as a slow wave structure.
- TWT are commonly used as amplifiers in satellite transponders, where the input signal is very weak and the output needs to be high power.
- TWT transmitters are used extensively in radar systems, particularly in airborne fire-control radar systems, and in electronic warfare and self-protection systems.

Difference between TWT & Klystron:

- In the case of the **TWT**, the microwave circuit is **non-resonant**.
- The **interaction of electron beam and RF field in the TWT is continuous** over the entire length of the circuit, but the interaction in the klystron occurs only at the gaps of a few resonant cavities.
- The wave in the **TWT is a propagating wave**; the wave in the klystron is not.
- In the coupled-cavity **TWT there is a coupling effect between the cavities**, whereas each cavity in the klystron operates independently.

Comparison of TWTA and Klystron Amplifier

Klystron Amplifier	**TWTA**
Linear beam or ‘O’ type Device	Linear beam or ‘O’ type device
Uses Resonant cavities for input and output	Uses non resonant periodic wave circuits
Narrowband device	Wideband device

Performance Characteristics:

- Frequency range : 3 GHz and higher
- Bandwidth: About 0.8 GHz
- Power output: Upto 10 kW (average)
- Power gain: upto 60 dB
- Efficiency: 20-40 %

Applications:

- Low noise RF amplifier in broadband microwave receivers
- Repeater amplifier in wide band communication links and co-axial cable
- Power output tubes in communication satellites
- CW high power TWT’s are used in troposcatter links

Slow wave structures

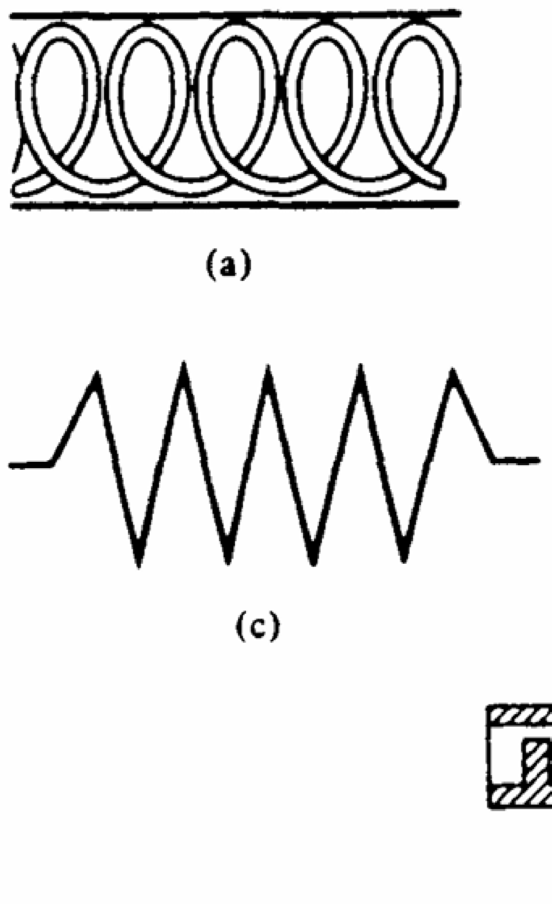
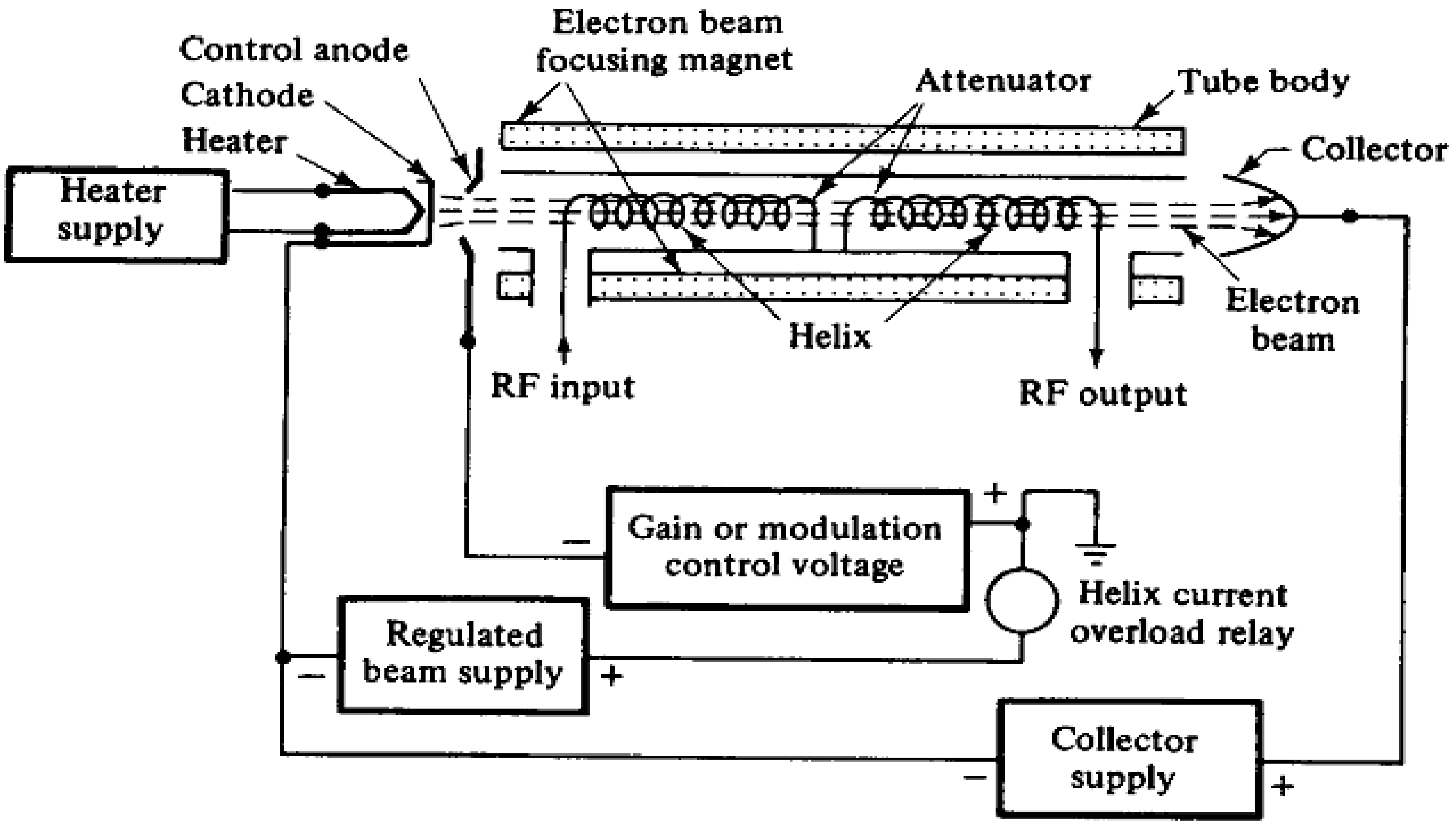


Figure 9-5-2 Slow-wave structures. (a) Helical line. (b) Folded-back line. (c) Zigzag line. (d) Interdigital line. (e) Corrugated waveguide.

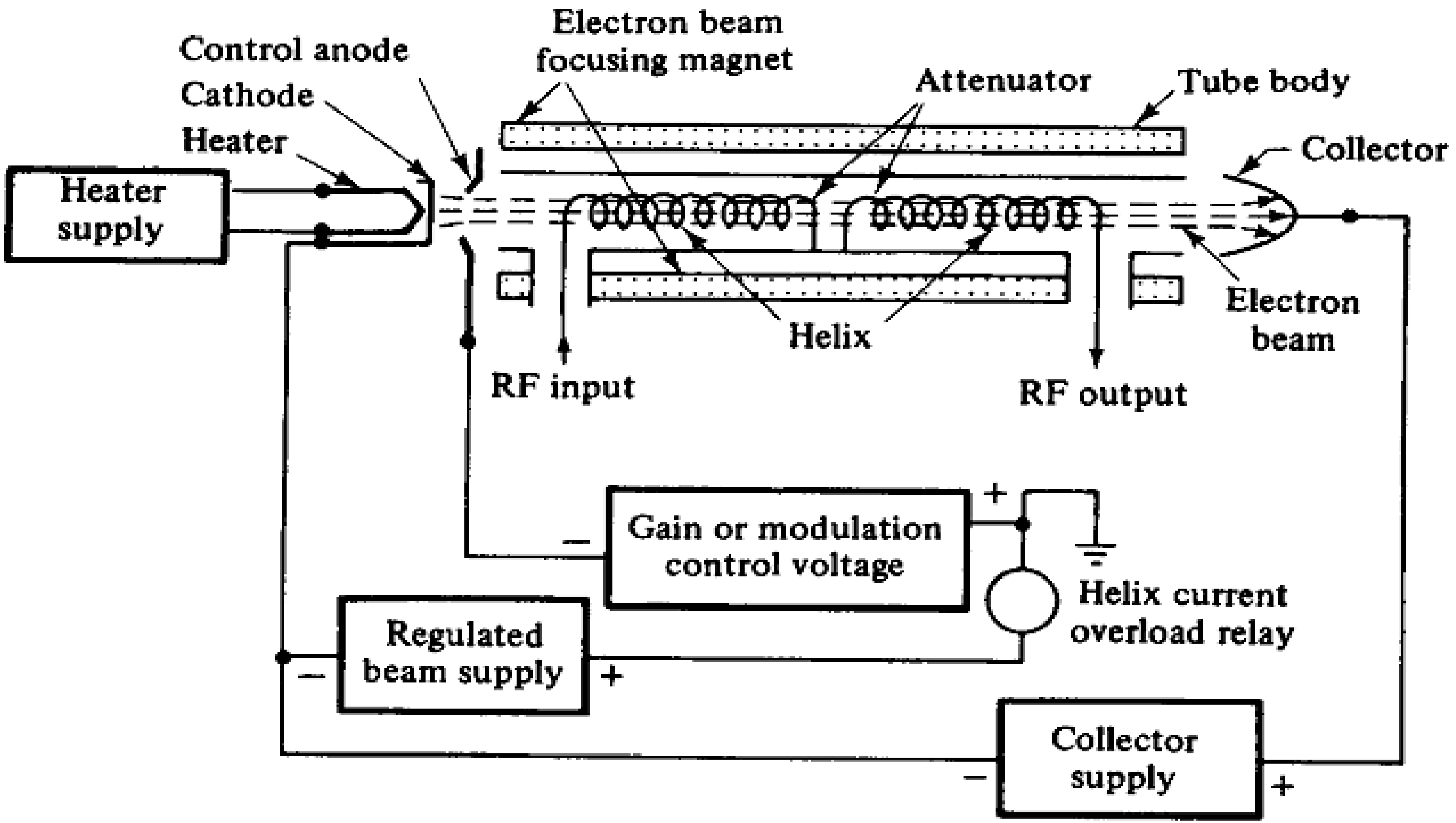
Helix Travelling Wave Tube Amplifier



Structure

- A helix traveling-wave tube consists of an electron beam and a slow-wave structure.
- The electron beam is focused by a constant magnetic field along the electron beam and the slow-wave structure.
- The commonly used slow-wave structure is a helical coil with a concentric conducting cylinder.
- Slow structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact.

Helix Travelling Wave Tube Amplifier



Principle of Operation

- Electron Gun: produces and then accelerates an electron beam along the axis of the tube.
- The surrounding static magnet provides a magnetic field along the axis of the tube to focus the electrons into a tight beam.
- A longitudinal helix slow wave non-resonant guide is placed at the centre of the tube that provides a low impedance transmission line for the RF energy within the tube.
- The TWT is designed with **helix delay structure** to slow the travelling wave down to or below the speed to the electrons in the beam.
- The RF signal wave injected at the input end of the helix travels down the helix wire at the speed of the light but the **coiled shape** causes the wave to travel a much **greater distance** than the electron beam

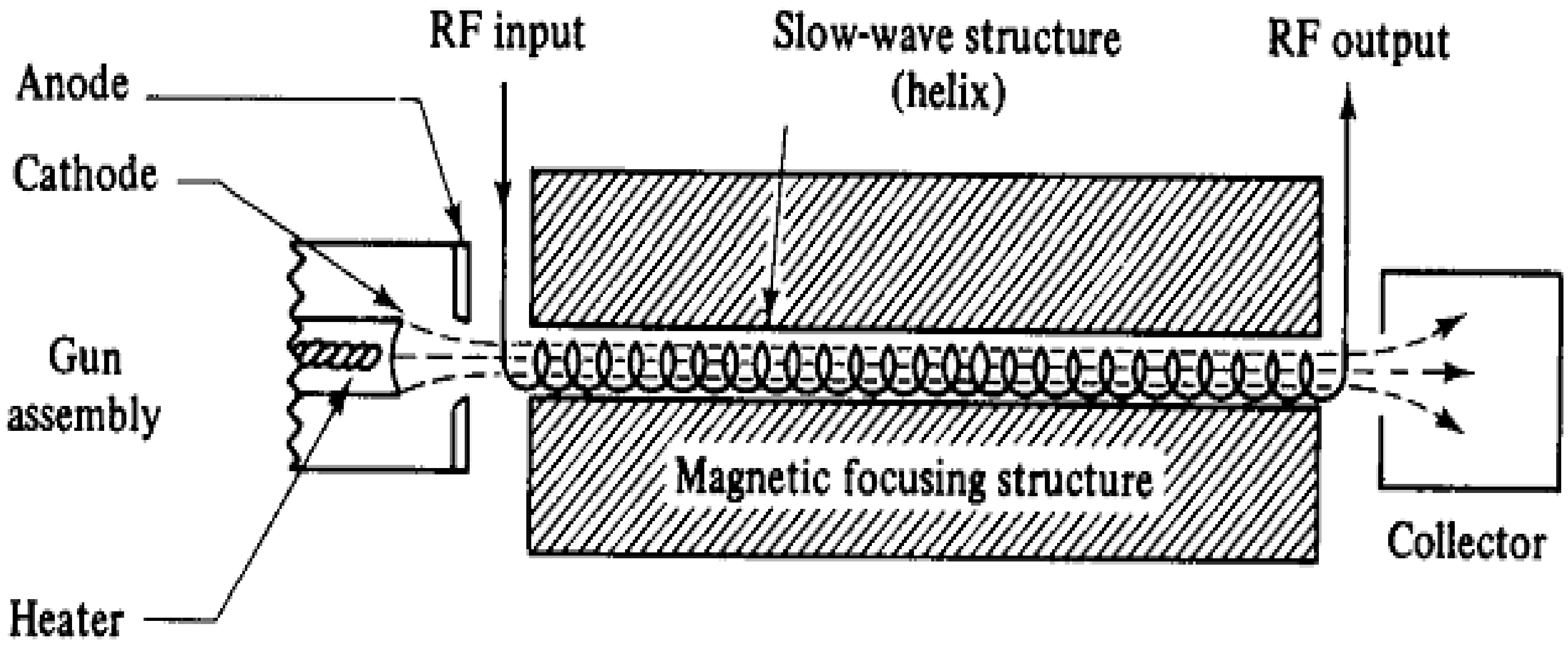
- Changing the number of turns or diameter of the turns in the helix wire, the speed at which RF signal wave travels in the form of axial E field, can be varied.
- DC beam velocity of the beam is maintained slightly greater than that of the axial field.
- The helical delay structure has the added advantage of causing a large proportion of electric fields that are parallel to the electron beam, provides **maximum interaction** between the fields and the moving electrons to form bunching.
- The electrons entering the helix at zero field are not affected by the signal wave; those electrons entering the helix at the accelerating field are accelerated, and those at the retarding field are decelerated.
- This velocity modulation causes bunching of electrons at regular intervals.

Beam velocity greater than field velocity?

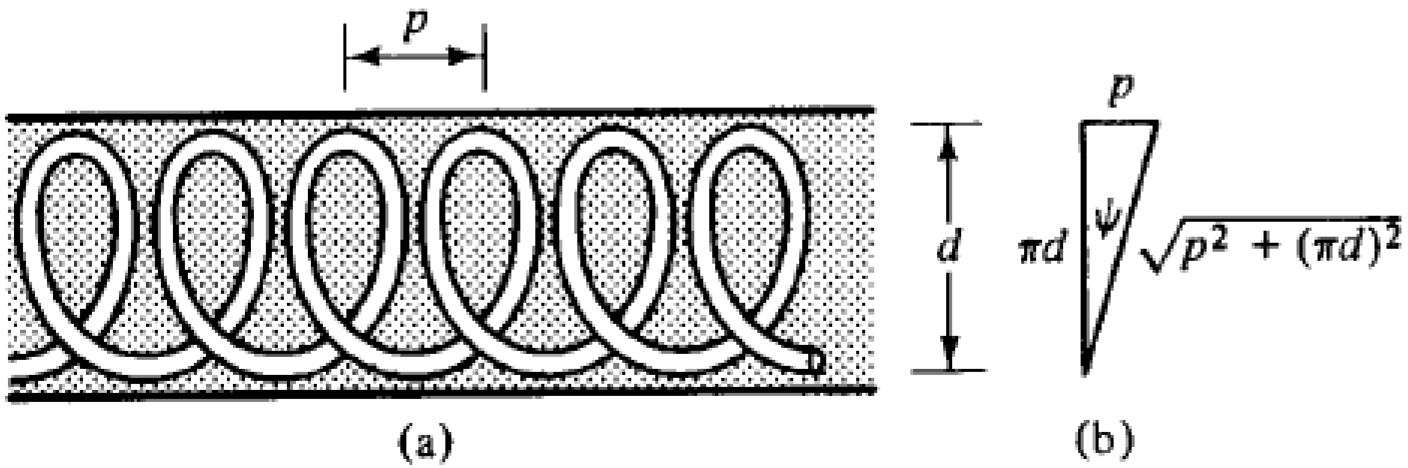
- As the dc velocity of the beam is maintained by slightly greater than the phase velocity of the travelling wave, more electrons face the retarding field than the accelerating field, and a great amount of kinetic energy is transferred from the beam to the electromagnetic field.
- Thus the field amplitude increases forming a more compact bunch and a large amplification of the signal voltage appears at the output of the helix.

Why attenuator?

- An attenuator is placed over a part of the helix on midway to attenuate any reflected waves generated due to the impedance mismatch.
- It is placed after sufficient length of the interaction region so that the attenuation of the amplified signal is insignificant compared to the amplification.



RF input
Slow-wave structure
(helix)
RF output
Anode
Cathode
Gun
assembly
Magnetic focusing structure
Collector
Heater



- It can be shown that the ratio of the phase velocity v_p along the pitch to the light velocity along the coil is given by:

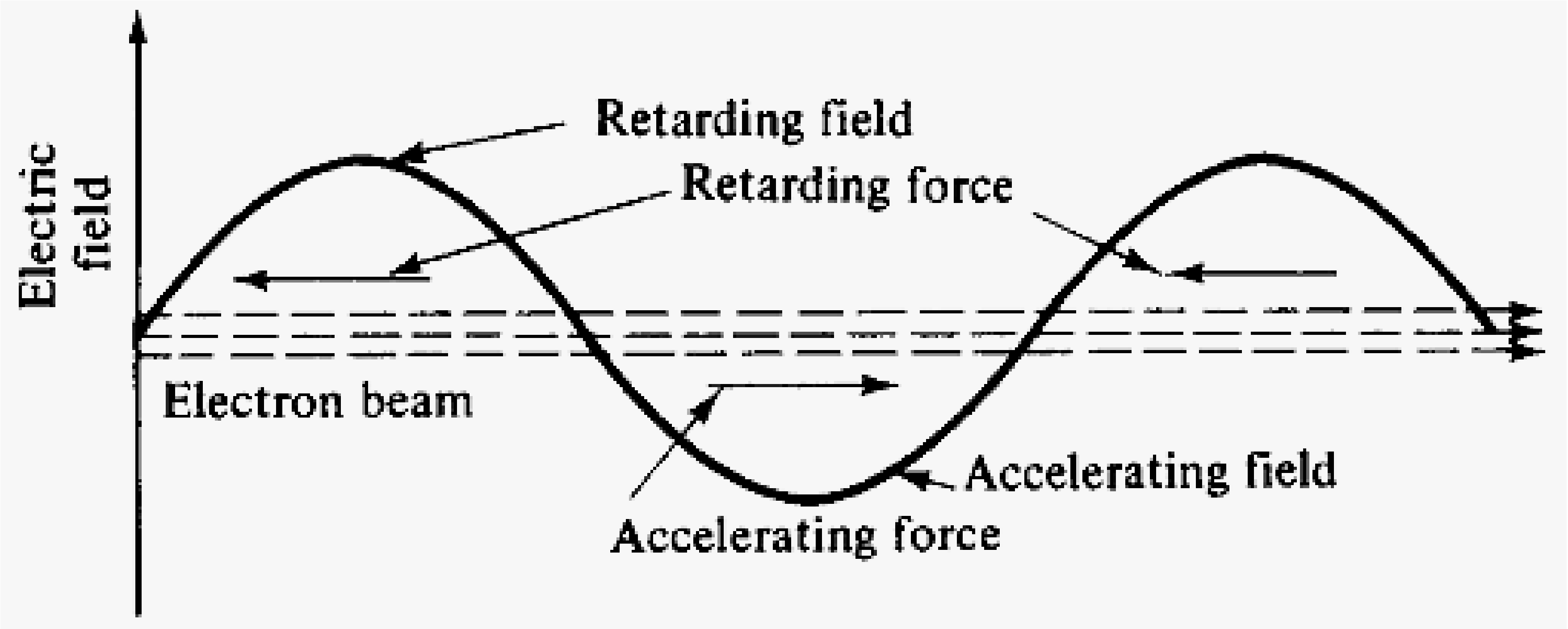
$$\frac{v_p}{c} = \frac{p}{\sqrt{p^2 + (\pi d)^2}} = \sin \psi$$

where c = 3 x 10^8 m/s is the velocity of light in free space

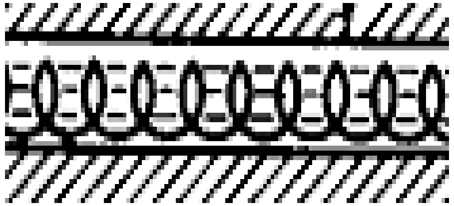
p = helix pitch

d = diameter of the helix

ψ = pitch angle


Electric field
Retarding field
Retarding force
Electron beam
Accelerating field
Accelerating force

Mathematical Analysis



- A slow wave structure must have the property of periodicity in the axial direction.

$$E(x, y, z) = \sum_{n=-\infty}^{\infty} E_n(x, y) e^{-j(2\pi n/L)z} e^{-j\beta_0 Z} = \sum_{n=-\infty}^{\infty} E_n(x, y) e^{-j\beta_n Z} \qquad .. (1)$$

- The field in a periodic structure can be expanded as an infinite series of waves, all at the same frequency but with different phase velocities

$$v_{pn} = \frac{\omega}{\beta_n} \equiv \frac{\omega}{\beta_0 + (2\pi n/L)} \qquad .. (2)$$

- The phase shift per period of fundamental wave on the structure

$$\theta_1 = \beta_0 L \qquad .. (3)$$

- where $\beta_0 = \omega/v_0$ is the phase constant of average beam velocity

- The dc transit time of the e^{-} is given by

$$T_0 = \frac{L}{v_o} \qquad \text{.. (4)}$$

- The phase constant of n^{th} harmonic is

$$\beta_n = \frac{\omega}{v_0} = \beta_0 + \frac{2\pi n}{L} \qquad \text{.. (5)}$$

- Assume that the phase velocity is synchronized with beam velocity for interactions between e^{-} beam & E field i.e.

$$v_{np} = v_0 \qquad \text{.. (6)}$$

- Practically, dc velocity of e^{-} is made slightly greater than axial velocity of EM for energy transfer.
- When a microwave signal is coupled into the helix, the axial E field exerts a force on the electron

$$\mathbf{F} = -e\mathbf{E} \quad \text{and} \quad \mathbf{E} = -\nabla V \qquad \text{.. (7)}$$

- The motion of e^- is analyzed in terms of axial E field.
- If the travelling wave is propagating in the z direction,

$$E_z = E_1 \sin(\omega t - \beta_p z) \quad \text{.. (8)}$$

At $t = t_0$ and $z = 0$ → Assume E field is max.

Here $\beta_p = \omega / v_p$ is the axial phase constant of microwave and v_p is the axial phase velocity of wave

- The equation for motion for e^- in presence of E field

$$F = ma = -eE$$

$$m \frac{dv}{dt} = -eE_1 \sin(\omega t - \beta_p z) \quad \text{.. (9)}$$

Assume that the velocity of e^- is

$$v = v_0 + v_e \cos(\omega_e t + \theta_e) \quad \text{.. (10)}$$

where v_0 = dc electron velocity

v_e = magnitude of velocity fluctuation in the velocity-modulated electron beam

ω_e = angular frequency of velocity fluctuation

θ_e = phase angle of the fluctuation

- Then,

$$\frac{dv}{dt} = -v_e \omega_e \sin(\omega_e t + \theta_e) \quad \text{.. (11)}$$

- Sub. in eq. (9)

$$m v_e \omega_e \sin(\omega_e t + \omega_e) = eE_1 \sin(\omega t - \beta_p z) \quad \text{.. (12)}$$

- For interaction between e^- and E field,

$$v \approx v_0 \quad \text{.. (13)}$$

- At any instant t, the distance travelled by the electron is given by

$$z = v_0(t - t_0) \quad \text{.. (14)}$$

- Sub. in eq. (12)

$$mv_e\omega_e \sin(\omega_e t + \theta_e) = eE_1 \sin[\omega t - \beta_p v_0(t - t_0)] \quad \text{.. (15)}$$

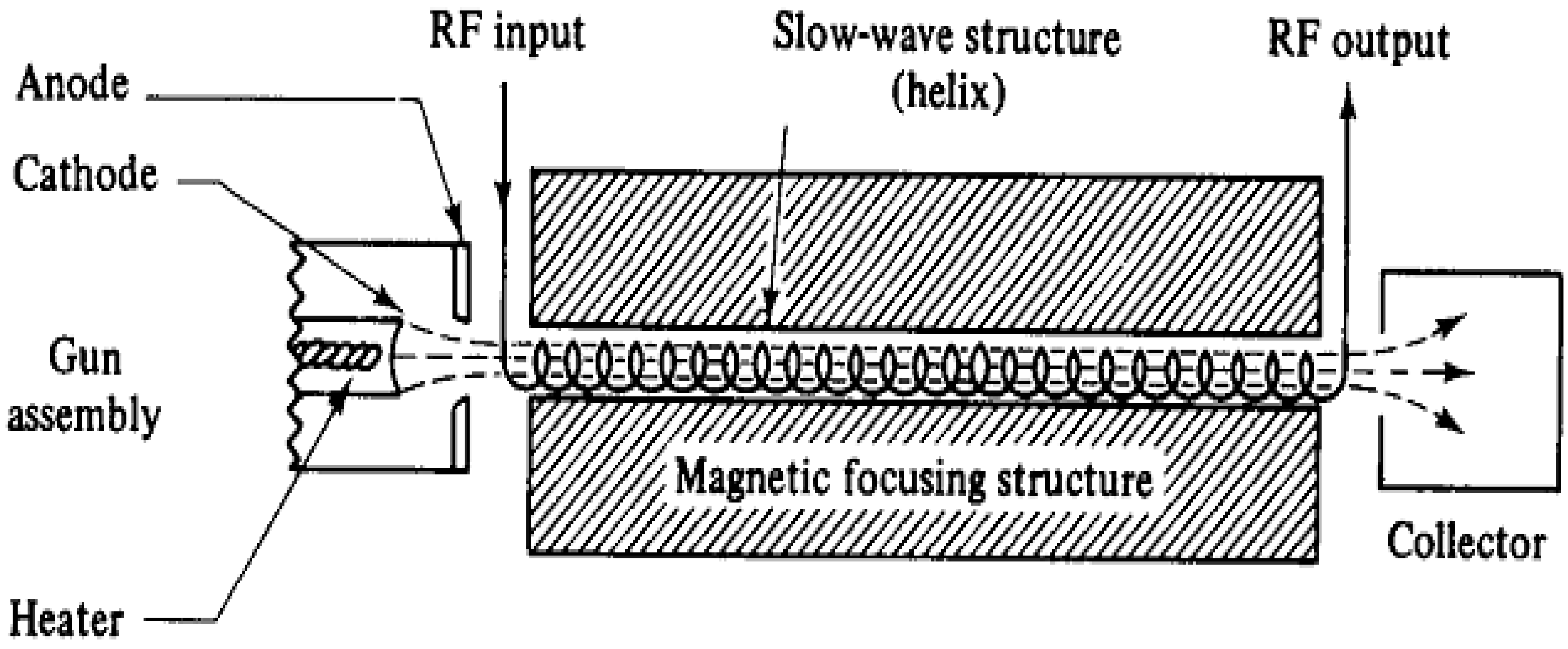
- Comparing the terms on LHS and RHS of above equation,

$$v_e = \frac{eE_1}{m\omega_e} \quad \text{.. (16)}$$

$$\omega_e = \beta_p(v_p - v_0)$$

$$\theta_e = \beta_p v_0 t_0$$

The magnitude of velocity fluctuation of electron beam is proportional to magnitude of axial E field



RF input
Slow-wave structure
(helix)
RF output
Anode
Cathode
Gun
assembly
Magnetic focusing structure
Collector
Heater

Convection Current

- To derive an expression for the convection current induced in e^- beam by axial E field.
- Considering the effect of space charge, the electron velocity , charge density , current density , and axial E field will vary about their average/dc values

$$v = v_0 + v_1 e^{j\omega t - \gamma z} \qquad \text{.. (1)}$$

$$\rho = \rho_0 + \rho_1 e^{j\omega t - \gamma z} \qquad \text{.. (2)}$$

$$\mathrm{J} = -\mathrm{J}_0 + \mathrm{J}_1 e^{j\omega t - \gamma z} \qquad \text{.. (3)}$$

$$\mathrm{E_z} = E_1 e^{j\omega t - \gamma z} \qquad \text{.. (4)}$$

$where\ \gamma = \alpha_e + j\beta_e$ is the propagation constant of the axial waves

- For a small signal, the electron beam current density is

$$J = \rho v = (\rho_0 + \rho_1 e^{j\omega t - \gamma z})(v_0 + v_1 e^{j\omega t - \gamma z})$$

$$\approx -\mathrm{J}_0 + \mathrm{J}_1 e^{j\omega t - \gamma z} \qquad \textbf{.. (5)}$$

where $-\mathrm{J}_0 = \rho_0 v_0$ and $\mathrm{J}_1 = \rho_1 v_0 + \rho_0 v_1$ (Here $\rho_1 v_1 \approx 0$)

- If an axial E field exists, it will affect the electron velocity according to the force equation.

$$F = ma = qE = -eE$$

$$m\frac{dv}{dt} = -e\left(E_1 e^{j\omega t - \gamma z}\right)$$

$$\frac{dv}{dt} = -\frac{e}{m}\left(E_1 e^{j\omega t - \gamma z}\right) \qquad \textbf{.. (6)}$$

$$\frac{d}{dt}(v_0 + v_1 e^{j\omega t - \gamma z}) = -\frac{e}{m}\left(E_1 e^{j\omega t - \gamma z}\right)$$

$$v_1\left[e^{j\omega t}\left(-\gamma\, e^{-\gamma z}\frac{dz}{dt}\right) + e^{-\gamma z}\left(j\omega\, e^{j\omega t}\right)\right] = -\frac{e}{m}\left(E_1 e^{j\omega t - \gamma z}\right)$$

$$v_1\, e^{j\omega t-\gamma z}\left[-\gamma\frac{dz}{dt}+j\omega\right]=-\frac{e}{m}\,\left(E_1 e^{j\omega t-\gamma z}\right)$$

- Here $\frac{dz}{dt}=v_0$

$$v_1\,[-\gamma v_0+j\omega]=-\frac{e}{m}\,(E_1)$$

$$v_1=\frac{-\left(\frac{e}{m}\right)(E_1)}{[-\gamma v_0+j\omega]} \qquad \textbf{.. (7)}$$

- According to the law of conservation of charge, continuity equation can be written as:

$$\nabla.J+\frac{\partial\rho}{\partial t}=0$$

$$\nabla.\left(-\mathrm{J}_0+\mathrm{J}_1 e^{j\omega t-\gamma z}\right)+\frac{\partial}{\partial t}\left(\rho_0+\rho_1 e^{j\omega t-\gamma z}\right)=0$$

$$(-\gamma J_1+j\omega\rho_1)\,e^{j\omega t-\gamma z}=0 \qquad \textbf{.. (8)}$$

$$\rho_1=\frac{-j\gamma J_1}{\omega} \qquad \textbf{.. (9)}$$

- Sub. eq.(7) and (9) in

$$\mathrm{J}_1 = \rho_1 v_0 + \rho_0 v_1 \qquad \textbf{.. (10)}$$

$$\mathrm{J}_1 = \left(\frac{-j\gamma J_1}{\omega}\right) v_0 + \rho_0 \left(\frac{-\left(\frac{e}{m}\right)(E_1)}{[-\gamma v_0 + j\omega]}\right)$$

$$\mathrm{J}_1 \left[1 + \frac{j\gamma v_0}{\omega}\right] = \frac{-\rho_0\left(\frac{e}{m}\right)(E_1)}{[-\gamma v_0 + j\omega]}$$

$$\mathrm{J}_1 \left[\frac{\omega + j\gamma v_0}{\omega}\right] = \frac{-\rho_0\left(\frac{e}{m}\right)(E_1)}{[-\gamma v_0 + j\omega]}$$

Multiplying & Dividing LHS with j

$$\mathrm{J}_1 \left[\frac{j\omega - \gamma v_0}{j\omega}\right] = \frac{-\rho_0\left(\frac{e}{m}\right)(E_1)}{[-\gamma v_0 + j\omega]}$$

$$\mathrm{J}_1 = \frac{-j\omega\rho_0\left(\frac{e}{m}\right)(E_1)}{[j\omega - \gamma v_0]^2}$$

- Multiplying & Dividing RHS with v_0,

$$\mathrm{J}_1 = \frac{-j\omega\rho_0 v_0 \left(\frac{e}{m}\right)(E_1)}{v_0[j\omega - \gamma v_0]^2}$$

$$J_1 = j\frac{\omega}{v_0}\,\frac{e}{m}\,\frac{J_0}{(j\omega - \gamma v_0)^2}E_1 \qquad \textbf{.. (11)}$$

- If the mag. of the axial electric field is uniform over the cross sectional area of the beam, the spatial ac current (i) will be proportional to the dc current I_0 with the same proportionality constant for J_1 and J_0.

- ***Convection* current** of the electron beam

$$\boxed{i = j\frac{\beta_e I_0}{2V_0(j\beta_e - \gamma)^2}E_1} \qquad \textbf{.. (12)}$$

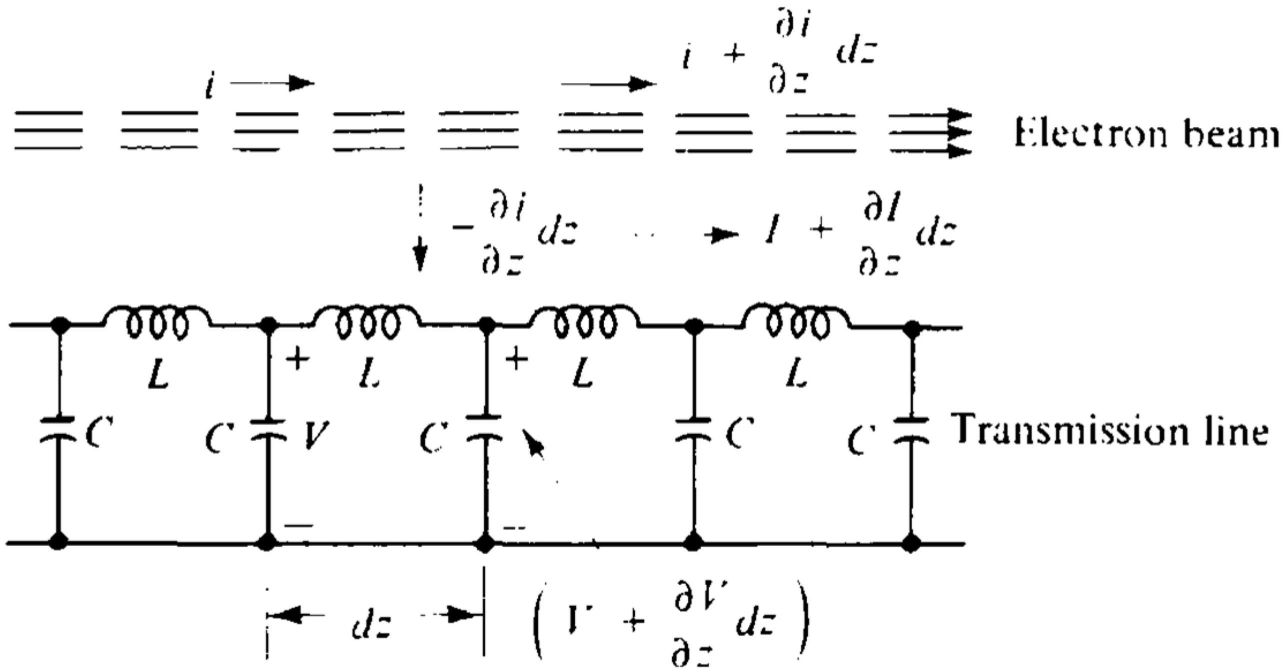
➡ ***Electronic Equation*** (determines the convection current induced by axial E field)

$where\ \beta_e \equiv \frac{\omega}{v_0}$ phase constant of velocity modulated e^- beam

$and \quad v_0 = \sqrt{(2eV_0/m)}$ (uniform velocity with which electron enters the slow wave structure)

Axial Electric Field

- The convection current in the electron beam induces an electric field in the slow wave circuit.
- This induced field adds to the field already present in the circuit and causes the circuit power to increase with distance.



- For simplicity, the slow wave helix is represented by a **distributed lossless transmission line**. The parameters are defined as follows:

L = inductance per unit length
C = capacitance per unit length
I = alternating current in transmission line
V = alternating voltage in transmission line
i = convection current

- Since the transmission line is coupled to a convection-electron beam current, a current is then induced in the line.
- The current flowing into the left-end portion of the line of length dz is $\boldsymbol{i}$ and the current flowing out of the right end of dz is $\boldsymbol{i} + \left(\frac{\partial i}{\partial z}\right) \boldsymbol{dz}$
- Since the net change of current in the length dz must be zero, the current flowing out of the electron beam into the line must be $\left[-\left(\frac{\partial i}{\partial z}\right) \boldsymbol{dz}\right]$

- Application of transmission-line theory and Kirchhoff's current law to the electron beam results , after simplification,

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} - \frac{\partial i}{\partial z} \quad \text{.. (13)}$$

Replace, $\frac{\partial}{\partial z} = -\gamma$ and $\frac{\partial}{\partial t} = j\omega$

$$-\gamma \mathrm{I} = -j\omega CV + \gamma i \quad \text{.. (14)}$$

From Kirchhoff's voltage law, the voltage equation, after simplification is

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \quad \text{.. (15)}$$

- Then,

$$-\gamma \mathrm{V} = -j\omega LI \quad \text{.. (16)}$$

- Elimination of circuit current I from eq. (14) & (16) yields

$$\gamma^2 \mathrm{V} = -V\omega^2 LC - \gamma i j\omega L \qquad \text{.. (17)}$$

If a convection-electron beam current is not present ,then eq. (17) reduces to a typical wave equation of a transmission line.

When $i = 0$, the propagation constant is defined from eq. (17) as

$$\gamma_0 \equiv j\omega\sqrt{LC} \qquad \text{.. (18)}$$

and the characteristic impedance of the line can be determined from eq(14) & (16):

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \text{.. (19)}$$

Thus,

$$\gamma_0 Z_0 = \left(j\omega\sqrt{LC}\right)\left(\sqrt{\frac{L}{C}}\right) = j\omega L \qquad \text{.. (20)}$$

- If beam current is present, eq. (17) can be rewritten using eq. (18) and (20)
- Eq. (17) ➡ $\gamma^2 V = -V\omega^2 LC - \gamma i j\omega L$

Here, $-\omega^2 LC = \gamma_0^2$ and $j\omega L = \gamma_0 Z_0$

Therefore,

$$\gamma^2 V = V\gamma_0^2 - \gamma i \gamma_0 Z_0$$

$$(\gamma^2 - \gamma_0^2)\, V = -\gamma\gamma_0 Z_0\, i$$

$$V = \frac{-\gamma\gamma_0 Z_0\, i}{(\gamma^2 - \gamma_0^2)}$$

- Since $E_z = -\nabla V = -\frac{\partial V}{\partial z} = \gamma V$

∴ Axial E field, $\boxed{E = \frac{-\gamma^2 \gamma_0 Z_0\, i}{(\gamma^2 - \gamma_0^2)}}$ ➡ **Circuit Equation**

Assignment

Power output, efficiency, admittance of reflex klystron

Power output and efficiency of magnetron

Thank You