IDFT Computation from Radix-2 DIT-FFT Algorithm.

FFT Algorithm can be used to compute the inverse-DFT of an N-point sequence $X(k)$.

IDFT of an N-point sequence $X(k)$ is defined as.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk} \quad \text{---------- (1)}$$

To make the above equation a comparable with DFT equation do some rearrangements in eqn (1)

Take complex conjugate and multiply by N in eqn (1).

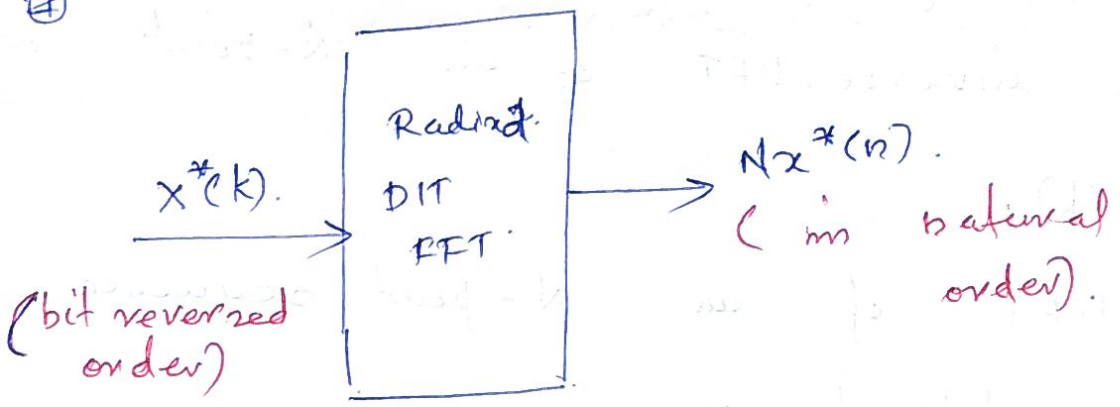
$$N x^*(n) = \sum_{k=0}^{N-1} x^*(k) W_N^{nk} \quad \text{---------- (2)}$$

RHS of eqn (2) is DFT of sequence $x^*(k)$ and can be computed using FFT algorithm with out put $N x^*(n)$.

To get $x(n)$ divide by N and take

complex conjugate.

Note: to calculate IDFT using Radix 2 DIT FFT algorithm.



① Apply i/p as $x^*(k)$ [complex conjugate of $x(k)$] in bit reversed order.

② Output is $N x^*(n)$ in natural order to get $x(n)$; divide by N, take complex conjugate.

Q) Find the IDFT of the sequence

$$X(k) = \{10, -2+2j, -2, -2-2j\}.$$

using Radix-2 DIT-FFT Algorithm.

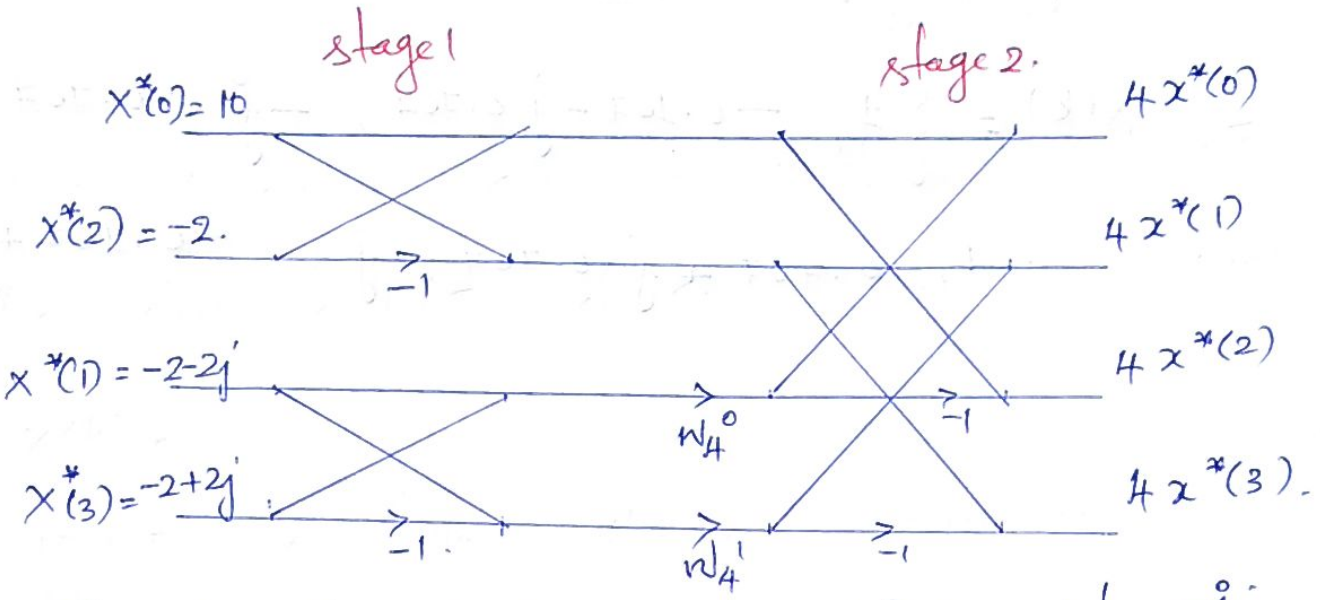
Answer

$$X(0) = 10. \Rightarrow x^*(0) = 10.$$

$$x(1) = -2+2j \Rightarrow x^*(1) = -2-2j$$

$$x(2) = -2 \Rightarrow x^*(2) = -2$$

$X(3) = -2-2j$, $X^*(3) = -2+2j$.



The twiddle factors are $W_4^0 = 1$, $W_4^1 = -j$.

Input	Output of stage 1.	Output (output of stage 2)
10	8	4
-2	12.	8
$-2-2j$	-4	12.
$-2+2j$	$-4j$	16.

The output $Nx^*(n)$ in normal order.

$\therefore x(n) = \frac{1}{4} Nx^*(n) = \{1, 2, 3, 4\}$

Since all values are real no need to take complex conjugate.

HW)

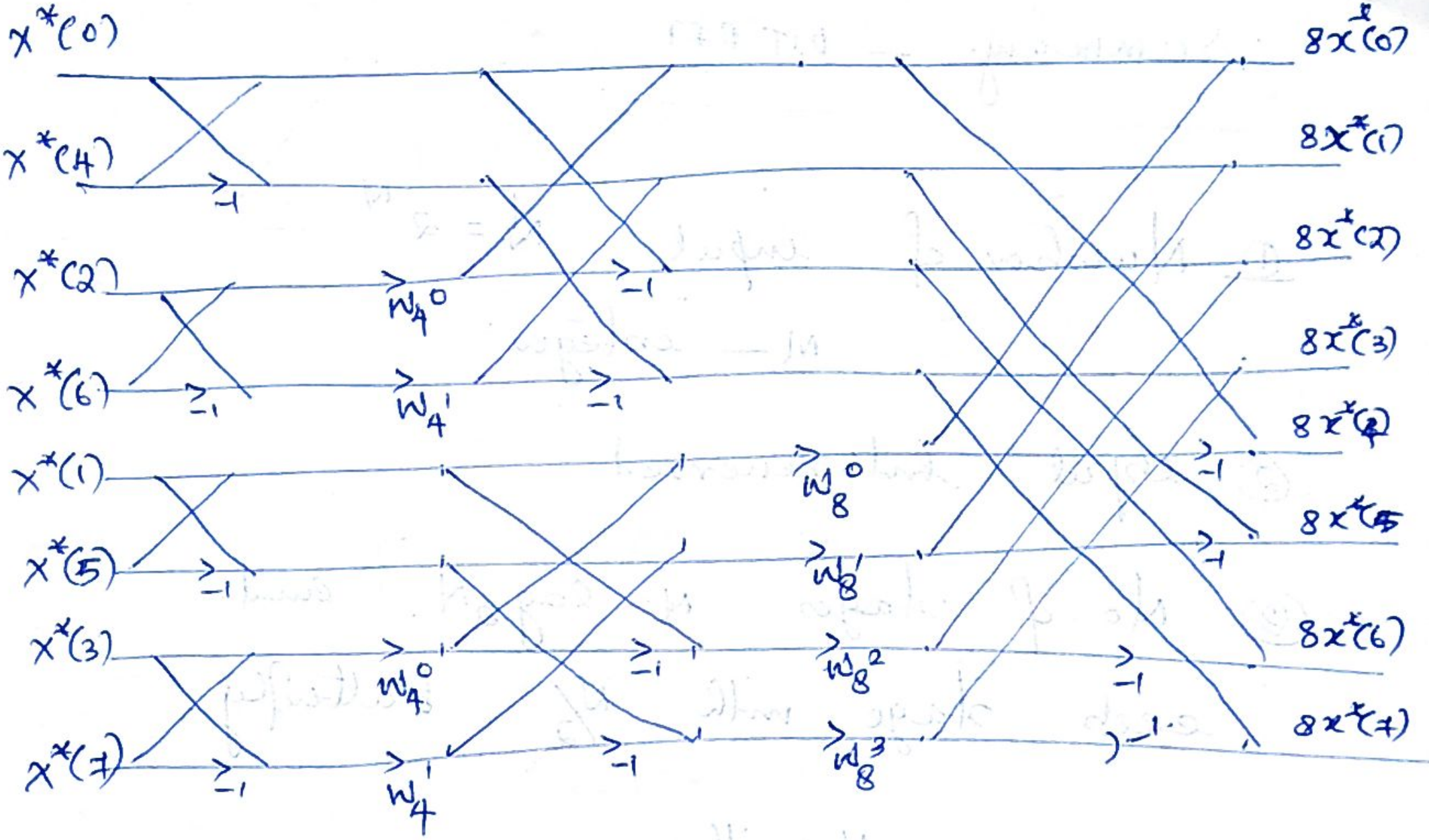
Q) Compute IDFT of the sequence

$$x(k) = \{1, -0.707 - j\,0.707, -j, 0.707 - j0.707,$$

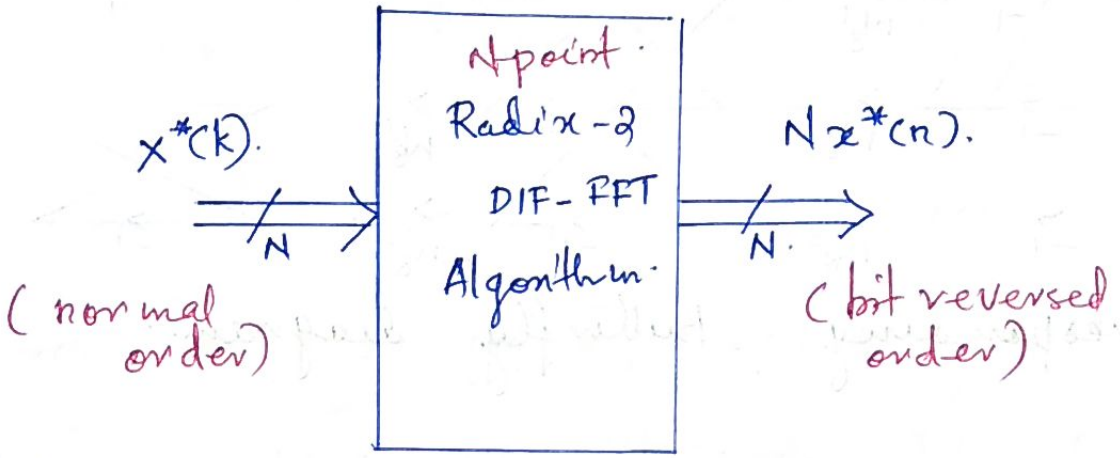
$$1, 0.707 + j\,0.707, j, -0.707 + j0.707\}$$

using Radix-2 DIT-FFT Algorithm.

A:


x*(0)
x*(4)
x*(2)
x*(6)
x*(1)
x*(5)
x*(3)
x*(7)
-1
W_4^0
W_4^1
W_8^0
W_8^1
W_8^2
W_8^3
8x*(0)
8x*(1)
8x*(2)
8x*(3)
8x*(4)
8x*(5)
8x*(6)
8x*(7)

② IDFT computation using radix-2 DIF-FFT Algorithm:



Note:

① Apply input $X^*(k)$ [complex conjugate of $X(k)$ in natural order.

② Output is $N x^*(n)$ in bit reversed order. To get $x(n)$, make it into natural order, divide by N, take complex conjugate

② Compute IDFT of the sequence

$$X(k) = \{10, -2+2j, -2, -2-2j\}$$

using radix-2 DIF- FFT Algorithm.

Answer:

step ① Input has to be applied as $X^*(k)$.

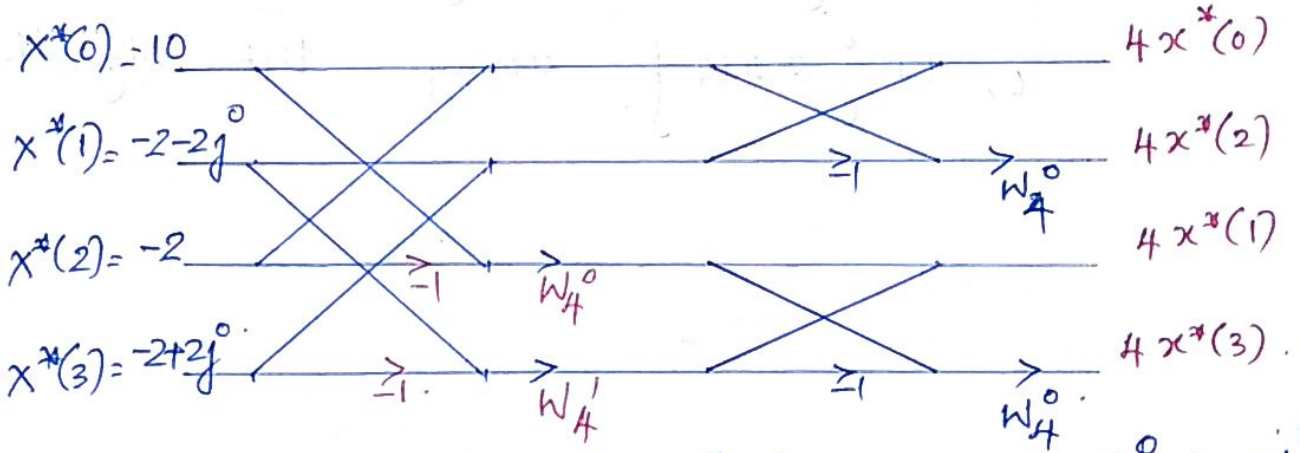
$X(0) = 10 \Rightarrow x^*(0) = 10$

$X(1) = -2+2j \Rightarrow x^*(1) = -2-2j$

$X(2) = -2 \Rightarrow x^*(2) = -2$

$X(3) = -2-2j \Rightarrow x^*(3) = -2+2j$

step ② Apply $x^*(k)$ to DIF - FFT algorithm.



Associated twiddle factors are $W_4^0 = 1$, $W_4^1 = -j$

$(X^*(k))$ input	output of stage 1 (S1).	Output of stage 2 (output). bit reversed order.
10	$10 + -2 = 8$	$8 + -4 = 4 \longrightarrow Nx^*(0)$
$-2-2j$	$(-2-2j) + (-2+2j) = -4$	$8 - -4 = 12 \longrightarrow Nx^*(2)$
-2	$(10 - -2) W_4^0 = 12$.	$12 + -4 = 8 \longrightarrow Nx^*(1)$
$-2+2j$	$[(-2-2j) - (-2+2j)](W_4^1)$ $= (-4j)(-j) = -4$	$12 - -4 = 16 \longrightarrow Nx^*(3)$

Step 3 :- The output $N x^*(n)$ in bit reversed order. Make it into natural order.

$4 x^*(n) = \{4, 8, 12, 16\}$.

$$x(n) = \frac{1}{4}\left[4 x^*(n)\right]^*$$

$x(n) = \{1, 2, 3, 4\}$

HW : Compute IDFT of the sequence

$x(k) = \{4, 1 - j2.414, 0, 1 - j0.414, 0, 1 + j0.414, 0, 1 + j2.414\}$ using DIF Algorithm.