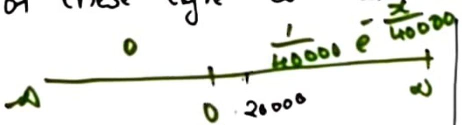
The mileage with car owners get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tyre will last i) atleast 20,000 km and ii) atmost 30,000 km.




Let x denote the mileage obtained with the tyre.

$$\text{mean} = 40000$$

$$\frac{1}{\lambda} = 40000$$

$$\lambda = \frac{1}{40000}$$

$$\int e^{ax} = \frac{e^{ax}}{a}$$

$\therefore$ Pdf of exp distribution $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & o.w \end{cases}$

$$f(x) = \begin{cases} \frac{1}{40000} e^{-\frac{x}{40000}} & x \geq 0 \\ 0 & o.w \end{cases}$$

i) P(atleast 20000 km) = $P(x \geq 20000)$

$$= \int_{20000}^{\infty} f(x)\,dx = \int_{20000}^{\infty} \frac{1}{40000} e^{-\frac{x}{40000}}\,dx.$$

$$= \frac{1}{40000}\left[\frac{e^{-\frac{x}{40000}}}{-\frac{1}{40000}}\right]_{20000}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-\frac{20,000}{40,000}}\right] = e^{-1/2}$$

$$= 0.6065$$

...dom Variable having ...

...obabilities that one of these tyre will last i) atleast ...

...) atmost 30,000 km.

Let $x$ denote the mileage obtained with the tyre.

mean = 40000

$$\frac{1}{\lambda} = 40000$$

$$\lambda = \frac{1}{40,000}$$

$$\int e^{ax} = \frac{e^{ax}}{a}$$

$$p(x>s) = e^{-\lambda s}$$

$$p(x \geq 20000) = e^{-\frac{20000}{40000}} = e^{-1/2}$$

$\therefore$ Pdf of exp distribution $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & o.w \end{cases}$

$$f(x) = \begin{cases} \frac{1}{40000} e^{-\frac{x}{40,000}} & x \geq 0 \\ 0 & o.w \end{cases}$$

i) $p(\text{atleast } 20000 \text{ km}) = p(x \geq 20000)$

$$= \int_{20000}^{\infty} f(x)\,dx = \int_{20000}^{\infty} \frac{1}{40000} e^{-\frac{x}{40000}} dx$$

$$= \frac{1}{40,000}\left[\frac{e^{-\frac{x}{40000}}}{-\frac{1}{40000}}\right]_{20000}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-\frac{20,000}{40,000}}\right] = e^{-1/2}$$

$$= 0.6065$$

ii) $p(\text{atmost } 30000 \text{ km}) = p(x \leq 30000)$

$$= 1 - p[x > 30000]$$

$$= 1 - e^{-\frac{30000}{40000}} = 1 - e^{-3/4}$$

$$= 0.5276$$

U.Q. If the distance that a car can run before its battery wears out is exponentially distributed with an average value of 5000 km. If the owner desires to take a 2000 km trip. what is the probability that he will be able to Complete the trip without having to replace the car battery.

Let $x \sim \exp(\lambda)$ denote the distance covered by the car before the battery wears out.

$$\text{Mean} = 5000 \text{ km}$$

$$\frac{1}{\lambda} = 5000$$

$$\lambda = \frac{1}{5000}$$

Pdf of exp dis

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{o.w} \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{5000} e^{-\frac{x}{5000}} & x \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

P(the car will cover atleast 2000 km)

$$\boxed{P(x>s) = e^{-\lambda s}}$$

$$= P(x \geq 2000)$$

$$= e^{-\lambda 2000}$$

$$= e^{-\frac{2000}{5000}}$$

$$= e^{-2/5}$$

Q.9 The time in hours required to repair a machine is exponentially distr... with mean 20. What is the probability that the required time


(i) exceeds 30 hrs (iii) Between 16 hrs and 24 hrs:

$$\frac{1}{\lambda} = 20$$

$$\lambda = \frac{1}{20}$$

$$P(x > 30) = e^{-\lambda 30}$$

$$P(16 \le x \le 24) = \int_{16}^{24} \frac{1}{20} e^{-\frac{x}{20}} dx = e^{-\frac{4}{5}} - e^{-\frac{6}{5}} = \underline{0.1481}$$

U.Q The amount of time that a Surveillance Camera will run without having to be reset is a random Variable having the exponential distribution with mean 50 days. Find the probability that Such a Camera will

i) have to be reset inless than 20 days.

ii) not have to be reset in atleast 60 days.

$x \sim \exp(\lambda)$


Mean = 50

$\frac{1}{\lambda} = 50$

$\lambda = \frac{1}{50}$

$p[x > s] = e^{-\lambda s}$

Pdf of exp(λ) $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & o.w \end{cases}$

$f(x) = \begin{cases} \frac{1}{50} e^{-\frac{x}{50}} & x \geq 0 \\ 0 & o.w \end{cases}$

i) p(Surveillance Camera will have to be reset inless than 20 days)

$= p(x < 20) = 1 - p[x \geq 20]$

$= 1 - e^{-\lambda 20} = 1 - e^{-\frac{20}{50}}$

$= \underline{\underline{0.3297}}$

ii) p(Sur. Camera not have to be reset in atleast 60 days).

$= p(x \geq 60)$

$= e^{-\lambda 60} = e^{-\frac{60}{50}}$

0.3012

Q.Q Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at a public telephone booth. find the probability that you will have to wait



i) more than 10 minutes (ii) between 10 and 20 minutes.

$X \sim \exp(\lambda)$

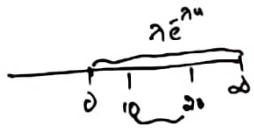
$\lambda = \frac{1}{10}$.

$\boxed{p(x>s) = e^{-\lambda s}}$

Pdf of $\exp(\lambda)$ $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & o.w \end{cases}$

$f(x) = \begin{cases} \frac{1}{10} e^{-\frac{x}{10}} & x \geq 0 \\ 0 & o.w \end{cases}$

1) P( more than 10 mnts) $= p(x \geq 10)$




$= e^{-\lambda 10}$

$= e^{-\frac{10}{10}}$

$= e^{-1} = \underline{\underline{0.368}}$

ii) p( between 10 and 20 mnts)

$= p(10 \leq x \leq 20)$

$= \int_{10}^{20} \frac{1}{10} e^{-\frac{x}{10}} dx = \frac{1}{\cancel{10}} \left[ \frac{e^{-\frac{x}{10}}}{-\frac{1}{\cancel{10}}} \right]_{10}^{20}$

$= -\left[ e^{-2} - e^{-1} \right] = e^{-1} - e^{-2} = \underline{0.2333}$

The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$.

(i) what is the probability that the repair time exceeds 2hr?

(ii) what is the conditional probability that a repair takes atleast 10 hr: given that its duration exceeds 9hrs?

$x \sim \exp(\lambda)$

$\lambda = \frac{1}{2}$.

$p(x>s) = e^{-\lambda s}$

Pdf of $\exp(\lambda)$ $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{o.w} \end{cases}$

$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & x \geq 0 \\ 0 & \text{o.w} \end{cases}$

i) P(repair time exceeds 2hr) $= p(x \geq 2)$

$= e^{-\lambda 2} = e^{-\frac{2}{2}}$

$= e^{-1} = \underline{0.3679}$

$P(x > s+t \mid x > s) = P(x > t)$

ii) $p(x \geq 10 \mid x \geq 9) = p(x > 9+1 \mid x > 9)$

(By memoryless prop:)

$= p(x > 1)$

$= e^{-\lambda(1)} = \underline{e^{-\frac{1}{2}}}$

$= \underline{0.6065}$.

The number of miles that a particular car can run before its battery wears out is exponentially distributed with an average of 10,000 miles. The owner of the car needs to take a 5000 mile trip. He wants to know the probability of the battery. Find p if he starts the journey (i) with a fresh battery (ii) with a battery that has already covered 2000 miles.

$X \sim \exp(\lambda)$

$\frac{1}{\lambda} = 10{,}000$

$\lambda = \frac{1}{10{,}000}$

$p(x>s) = e^{-\lambda s}$

$p(x>s+t \mid x>s) = p(x>t)$

Pdf of $\exp(\lambda)$

$$f(x) = \begin{cases} \frac{1}{101000} e^{-\frac{x}{101000}} & x \geq 0 \\ 0 & \text{o.w} \end{cases}$$

i) p( he completes the trip with a fresh battery

$p = p(x > 5000)$

$= e^{-\lambda 5000} = e^{-\frac{5000}{10000}} = e^{-\frac{1}{2}} = \underline{\underline{0.61}}$

ii) P( battery that has covered 2000 miles, he will complete trip if $x > 7000$)

$P = p(x > 7000 \mid x > 2000)$ (by memory less prop)

$= p(x > 2000 + 5000 \mid x > 2000)$

$= p(x > 5000) = \underline{\underline{0.61}}$

u.o Suppose a new machine is put into operation at time zero. its life time is an exponential random Variable with mean life 12 hrs:

i) what is the probability that the machine will work continuously for one day?

ii) Suppose the machine has not failed by the end of the first day, what is the probability that it will work for whole of next day.

$X \sim \exp(\lambda)$

$P(x > s) = e^{-\lambda s}$

$\frac{1}{\lambda} = 12$

$\lambda = \frac{1}{12}$.

pdf of $\exp(\lambda)$ $f(x) = \begin{cases} \frac{1}{12} e^{-\frac{x}{12}} & x \geq 0 \\ 0 & o.w. \end{cases}$

i) $P(x > 24) = e^{-\lambda 24} = e^{-\frac{24}{12}} = \underline{\underline{e^{-2}}}$

$P(x > 48 | x > 24) = P(x > 24+24 | x > 24)$

By memory less prop

$P(x > s+t | x > s) = P(x > t)$

$= P(x > 24)$

$= \underline{\underline{e^{-2}}}$

The lifetime (in years) of an electronic Component is an exponential random Variable with mean 1 year. find the lifetime L which is a typical Component is 60% certain to exceed.



$X \sim \exp(\lambda)$

Mean = 1 yr.

$\frac{1}{\lambda} = 1$

$\lambda = 1$

Pdf of $\exp(\lambda)$

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & o.w \end{cases}$$

$P(X > L) = 0.60$

$P(X > s) = e^{-\lambda s}$

$e^{-\lambda L} = 0.60$

(Since $\lambda = 1$

$e^{-L} = 0.60$

$\log a^x = x \log a$

$\log e^{-L} = \log(0.60)$

$\log e = 1$

$-L \log e = \log(0.60)$

$L = -\log(0.60)$

$= 0.51$

If $x$ follows an exponential distribution with $p(x \le 1) = p(x > 1)$. find mean and Variance of $x$.

$p(x > s) = e^{-\lambda s}$

$\log a^x = x \log a$

$\log e = 1$

$x \sim \exp(\lambda)$

$p(x \le 1) = p(x > 1)$

$1 - p(x > 1) = p(x > 1)$

$1 = 2\, p(x > 1)$

$p(x > 1) = \frac{1}{2}$

$e^{-\lambda} = \frac{1}{2}$

$\frac{1}{e^{\lambda}} = \frac{1}{2}$

$e^{\lambda} = 2$

$\log e^{\lambda} = \log 2$

$\lambda \log e = \log 2$

$\lambda = \log 2$

$\therefore$ Mean $= \frac{1}{\lambda} = \frac{1}{\log 2}$

Variance $= \frac{1}{\lambda^2} = \frac{1}{(\log 2)^2}$

V.V.imp.

In a city, the daily Consumption of milk in excess of 20,000 gallons is approximately exponentially distributed with mean 3000 gallons. The City has a daily stock of 35,000 gallons. What is the probability that of 2 days Selected at random, the Stock is insufficient for both days.



$X \to$ excess amount of milk consumed in day

$Y$

$x = Y - 20000$

$x^m \cdot x^n = x^{m+n}$

mean $= 3000$

$\frac{1}{\lambda} = 3000$

$\lambda = \frac{1}{3000}$

Pdf of $f(x) = \begin{cases} \frac{1}{3000} e^{-\frac{x}{3000}} & x \geq 0 \\ 0 & o.w \end{cases}$

$\therefore Y \to$ Total daily Consumption of milk in City.

$Y = X + 20{,}000$

$P(Y > 35000) = P(X + 20000 > 35000)$

$= P(X > 15000)$

$P(X > s) = e^{-\lambda s}$

$= e^{-\lambda 15000} = e^{-\frac{15000}{3000}} = e^{-5}$

Prob Stock is insufficient in both days

$= e^{-5} \times e^{-5} = e^{-10}$

# Normal Distribution:

'x' has a normal distribution if 'x' is a Continuous random variable with probability density function.

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

Note:

$N(\mu, \sigma^2)$

$\mu$ and $\sigma$ are parameters

with $-\infty < \mu < \infty$ and $\sigma > 0$

**Normal Distribution:** (Gaussian Distr.)

'x' has a normal distribution if 'x' is a continuous random variable with probability density function.

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

Note:

$N(\mu, \sigma^2)$

$\mu$ and $\sigma$ are parameters

with $-\infty < \mu < \infty$ and $\sigma > 0$

# Mean and Varience of Normal Distribution.

Mean = $E[x]$

$$= \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx.$$

$\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$

Put $\frac{x-\mu}{\sigma} = z$

$x = -\infty \Rightarrow z = -\infty$

$x = \infty \Rightarrow z = \infty$

$x = \sigma z + \mu$

$dx = \sigma\, dz$

odd + even = odd

e + 0 = 0

0 + 0 = e

e + e = e

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu)\, e^{-\frac{z^2}{2}}\, \sigma\, dz$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overset{\text{odd}}{z e^{-\frac{z^2}{2}}}\, dz + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overset{\text{even.}}{e^{-\frac{z^2}{2}}}\, dz.$$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & f(x) \text{ is even fun: } f(-x) = f(x) \\ 0 & f(x) \text{ is odd fun: } f(-x) = -f(x) \end{cases}$$

$$= 0 + \frac{\mu}{\sqrt{2\pi}}\, 2 \int_0^{\infty} e^{-\frac{z^2}{2}}\, dz$$

$$= \frac{\mu}{\sqrt{2\pi}} 2 \int_0^\infty e^{-u^2} \sqrt{2}\, du$$

$$\boxed{\int_0^\infty e^{-u^2} du = \frac{\sqrt{\pi}}{2}}$$

Put $\frac{z}{\sqrt{2}} = u$

$dz = \sqrt{2}\, du$

$z = 0 \Rightarrow u = 0$

$z = \infty \Rightarrow u = \infty$

$$= \frac{\mu}{\sqrt{2\pi}} \cancel{2} \sqrt{2} \frac{\sqrt{\pi}}{\cancel{2}}$$

$$\boxed{E[x] = \mu}$$

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$\boxed{\begin{aligned} &\text{Put } \frac{x-\mu}{\sigma} = z \\ &x = \sigma z + \mu \\ &dx = \sigma\, dz \end{aligned}}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma z + \mu]^2 e^{-\frac{z^2}{2}} dz.$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma^2 z^2 + 2\sigma\mu z + \mu^2] e^{-\frac{z^2}{2}} dz$$

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$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{z^2 e^{-\frac{z^2}{2}}}_{\text{even}}\, dz + \frac{2\sigma\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{z\, e^{-\frac{z^2}{2}}}_{\text{odd}}\, dz$$

$$+ \frac{\mu^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{e^{-\frac{z^2}{2}}}_{\text{even}}\, dz.$$

$$= \underline{\frac{\sigma^2}{\sqrt{2\pi}}\, 2\int_{0}^{\infty} z^2 e^{-\frac{z^2}{2}}\, dz} + 0 + \frac{\mu^2}{\sqrt{2\pi}}\, 2\int_{0}^{\infty} e^{-\frac{z^2}{2}}\, dz \quad (1)$$

$$\int_{0}^{\infty} z^2 e^{-\frac{z^2}{2}}\, dz = \int_{0}^{\infty} z\left(z e^{-\frac{z^2}{2}}\right) dz$$

$$= \left[z \int z e^{-\frac{z^2}{2}}\, dz\right]_0^{\infty} - \int_0^{\infty} (1)\left[\int z e^{-\frac{z^2}{2}}\, dz\right] dz$$

$$\int z e^{-z^2/2}\, dz$$

Put $\frac{z^2}{2} = t$

$\frac{2z}{2}\, dz = dt$

$$= \int e^{-t}\, dt$$

$$= \left[\frac{e^{-t}}{-1}\right] = -e^{-t} = \underline{-e^{-\frac{z^2}{2}}}$$

$$= \left[z\left(-e^{-\frac{z^2}{2}}\right)\right]_0^{\infty} - \int_0^{\infty} -e^{-\frac{z^2}{2}}\, dz\Big]$$

$$= [0 - 0] + \int_0^{\infty} e^{-\frac{z^2}{2}}\, dz$$

$\frac{z}{\sqrt{2}} = u$

$dz = \sqrt{2}\, du$

$$= \int_0^{\infty} e^{-u^2} \sqrt{2}\, du.$$

$$= \sqrt{2} \int_0^{\infty} e^{-u^2}\, du$$

$$= \sqrt{2}\, \frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}}{\sqrt{2}}$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} 2 \frac{\sqrt{\pi}}{\sqrt{2}} + \frac{\mu^2}{\sqrt{2\pi}} 2 \frac{\sqrt{\pi}}{\sqrt{2}}.$$



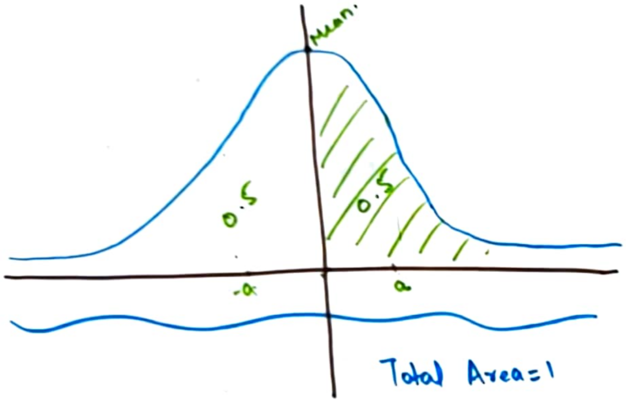
$$E[x^2] = \underline{\underline{\sigma^2 + \mu^2}}$$

$$\therefore Var(x) = \sigma^2 + \mu^2 - \mu^2$$

$$= \sigma^2$$

$$E[x] = \mu.$$

$$N(\mu, \sigma^2)$$




$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

if $x$ is a normal random Variable with mean $-3$ and Variance 4. find

(i) $P(1 \le x \le 2)$ (ii) $P(x > -1.5)$ (iii) $P(x < -3)$ (iv) $P(|x+3| < 2)$ v) $P(|x+5| > 1)$

$x \sim N(\mu, \sigma^2)$

$\mu = -3$ $\quad \sigma^2 = 4$

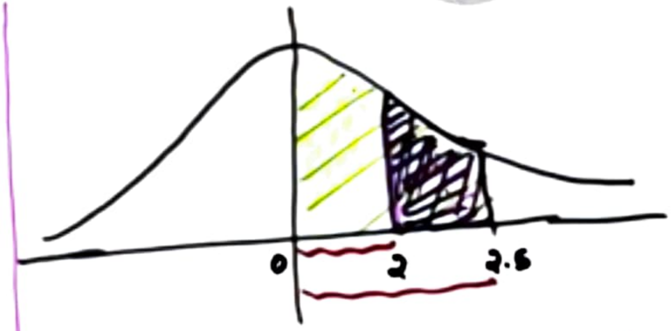
$\sigma = 2$

$$z = \frac{x-\mu}{\sigma} = \frac{x+3}{2}$$

$$P(1 \le x \le 2) = P\left(\frac{1-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{2-\mu}{\sigma}\right)$$

$$= P\left(\frac{1+3}{2} \le \frac{x+3}{2} \le \frac{2+3}{2}\right)$$

$$= P(2 \le z \le 2.5)$$

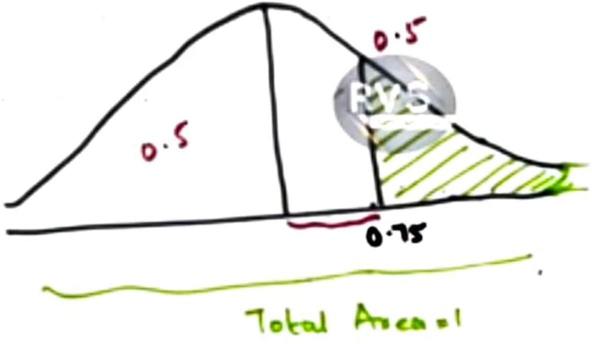



$$= P(0 \le z \le 2.5) - P(0 \le z \le 2)$$

$$= 0.4938 - 0.4772$$

$$= \underline{\underline{0.0166}}$$

ii) $P(x > -1.5) = P\left(\frac{x-\mu}{\sigma} > \frac{-1.5+3}{2}\right)$




$= P(z > 0.75)$

$= 0.5 - P(0 < z < 0.75)$

$= 0.5 - 0.2734$

$= \underline{0.2266}$

iii) $P(x < -3) = P\left(\frac{x-\mu}{\sigma} < \frac{-3+3}{2}\right)$


$= P(z < 0)$

$= \underline{0.5}$

iv) $P(|x+3|<2)$

$|x|<a \Rightarrow -a<x<a$

$= P(-2 < x+3 < 2)$

$= P(-2-3 < x < 2-3)$

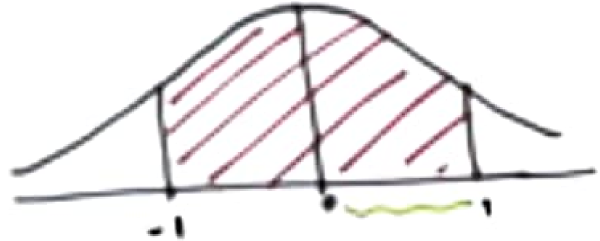
$= P(-5 < x < -1)$

$= P\left(\frac{-5+3}{2} < \frac{x+3}{2} < \frac{-1+3}{2}\right)$

$= P(-1 < z < 1)$

$= P(-1 < z < 0) + P(0 < z < 1)$

$= P(0 < z < 1) + P(0 < z < 1)$




$= 2P(0 < z < 1)$

$= 2 \times 0.3413$

$= \underline{0.6826}$

v) $P(|x+5|>1) = 1 - P(|x+5| \le 1)$ $\qquad |x|<a \Rightarrow -a<x<a$

$$= 1 - P(-1 \le x+5 \le 1)$$

$$= 1 - P(-6 \le x \le -4)$$

$$= 1 - P\left(\frac{-6+3}{2} \le \frac{x+3}{2} \le \frac{-4+3}{2}\right)$$

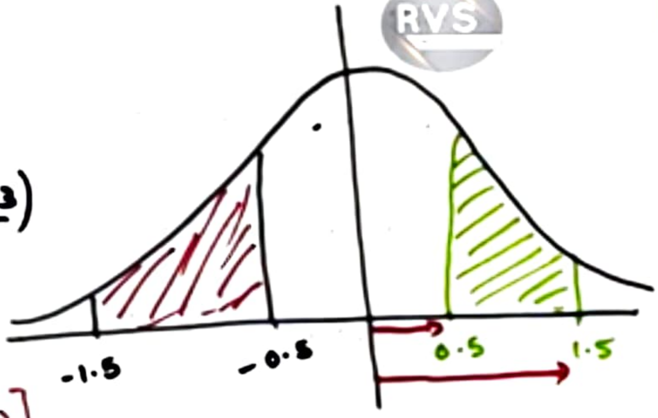
$$= 1 - P(-1.5 \le z \le -0.5)$$

$$= 1 - \left[P(0<z<1.5) - P(0<z<0.5)\right]$$

$$= 1 - \left[0.4332 - 0.1915\right]$$

$$= 1 - 0.2417$$

$$= 0.7583$$




Total Area.

if $x$ is a normal variate with mean 30 and SD 5. find

i) $P(26 \le x \le 40)$ ii) $P(x \ge 45)$ iii) $P(|x-30| > 5)$

$= 0.0013$

$$z = \frac{x-\mu}{\sigma} = \frac{x-30}{5}$$


$$P(26 \le x \le 40) = P(-0.8 \le z \le 2)$$

$$= P(-0.8 \le z \le 0) + P(0 \le z \le 2)$$

$$= P(0 \le z \le 0.8) + P(0 \le z \le 2)$$

$$= 0.2881 + 0.4772 = \underline{\underline{0.7653}}$$

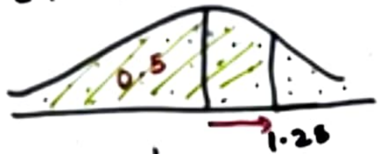
$$P(|x-30| > 5) = 1 - P[|x-30| \le 5]$$

$$= 1 - P[-5 \le x-30 \le 5]$$

$$= \underline{\underline{0.3174}}$$

Find the following standard normal probabilities a) $P(z \le 1.25)$ b) $P(z > 1.25)$

c) $P(z \le -1.25)$ and $P(-0.38 \le z \le 1.25)$





i) $P(z \le 1.25) = 0.5 + P(0 \le z \le 1.25)$

$= 0.5 + 0.3944$

$= \underline{\underline{0.8944}}$

b) $P(z > 1.25) = 1 - P(z \le 1.25)$

$= 1 - 0.8944$

$= \underline{0.1056}$


c) $P(z \le -1.25) = 0.5 - P(0 \le z \le 1.25)$

$= 0.5 - 0.3944$

$= \underline{\underline{0.1056}}$

d) $P(-0.38 \le z \le 1.25)$

$= P(-0.38 \le z \le 0) + P(0 \le z \le 1.25)$

$= P(0 \le z \le 0.38) + P(0 \le z \le 1.25)$

$= 0.1480 + 0.3944$

$= \underline{\underline{0.5424}}$

The weekly wages of 1000 workmen are ... mean of Rs 500 with a standard deviation of 50. Estimate the number of workers whose weekly wages will be i) between Rs 400 and Rs ... ii) less than Rs 400 iii) More than Rs 600

$\mu = 500 \qquad \sigma = 50$

$X \sim N(\mu, \sigma^2)$

$$z = \frac{x-\mu}{\sigma} = \frac{x-500}{50}$$

$N = 1000$

i) P( wages lies bet: Rs 400 and Rs 600)

$$= P(400 \le x \le 600)$$

$$= P\left(\frac{400-500}{50} \le \frac{x-500}{50} \le \frac{600-500}{50}\right)$$

$$= P(-2 \le z \le 2)$$


$$= P(-2 \le z \le 0) + P(0 \le z \le 2)$$

$$= P(0 \le z \le 2) + P(0 \le z \le 2)$$

$$= 2P(0 \le z \le 2)$$

$$= 2 \times 0.4772$$

$$= \underline{0.9544}$$

Requ: Num $= N \times P(x)$

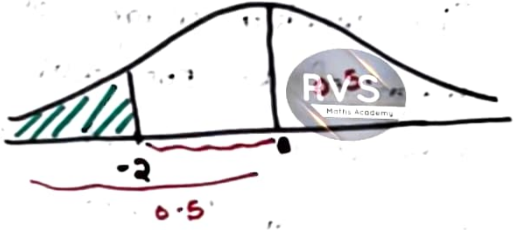
$= 1000 \times 0.9544$

$= 954.4$

$= \underline{954}$

that weekly wages is less than 400 is

$$P(x \le 400) = P\left(\frac{x-500}{50} \le \frac{400-500}{50}\right)$$




$$= P(z \le -2)$$

$$= 0.5 - P[-2 \le z \le 0]$$

$$= 0.5 - P[0 \le z \le 2] = 0.5 - 0.4772 = \underline{0.0228}$$

Requi: Num = $1000 \times 0.0228 = 22.8 \approx 23$


3) Prob: that weekly wages is more than 600

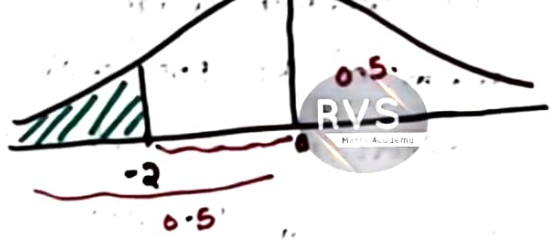
$$P(x \ge 600) = P\left(\frac{x-500}{50} \le \frac{600-500}{50}\right)$$

$$= P(z \le 2)$$

$$= 0.5 + P(0 \le z \le 2)$$

$$= 0.5 + 0.4772 =$$

$$P(x \le 400) = P\left(\frac{x-500}{50} \le \frac{400-500}{50}\right)$$

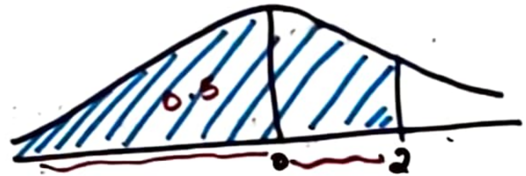



$$= P(z \le -2)$$

$$= 0.5 - P[-2 \le z \le 0]$$

$$= 0.5 - P[0 \le z \le 2] = 0.5 - 0.4772 = \underline{0.0228}$$

Requ: Num $= 1000 \times 0.0228 = 22.8 \simeq 23$




3) Prob: that weekly wages is less than 600

$$P(x \le 600) = P\left(\frac{x-500}{50} \le \frac{600-500}{50}\right)$$

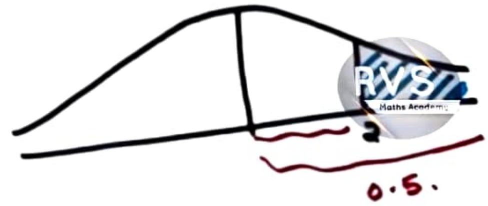
$$= P(z \le 2)$$

$$= 0.5 + P(0 \le z \le 2)$$

$$= 0.5 + 0.4772 = \underline{0.9772}$$

$\therefore$ Requ: Number $= 1000 \times 0.9772 = 977.2 \simeq \underline{977}$

3) Prob: weekly wages is more than 600




$$p(x \geq 600) = P\left(\frac{x-500}{50} \geq \frac{600-500}{50}\right)$$

$$= p(z \geq 2)$$

$$= 0.5 - P[0 \leq z \leq 2]$$

$$= 0.5 - 4772.$$

$$= \underline{\underline{0.0226}}$$

Req: Num $= 1000 \times 0.0228 = 22.8$

$$\cong \underline{\underline{23}}$$

In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average of 2040 hrs and SD of 60 hrs. Estimate the number of bulbs likely to burn for



i) more than 2150 hrs $= P(x \geq 2150) = 0.0336$ $\quad \underline{67}$

ii) less than 1950 hrs. $= P(x \leq 1950) = 0.0668$ $\quad 134$

iii) more than 1920 hrs but less than 2160 hrs:

$P(1920 \leq x \leq 2160) = P(-2 \leq z \leq 2)$

$= 0.9544$

$\underline{1909}$

$\mu = 2040$

$\sigma = 60$

$$z = \frac{x - 2040}{60}$$

ii) The time that it takes a driver to react to the break lights on a decelerating vehicle is critical in helping to avoid rear end collisions. An article suggests that reaction time for an in traffic response to break signal from standard brake lights can be modeled with a normal distribution having mean value 1.25 sec and standard deviation 0.46 sec. What is the probability that reaction time is between 1.00 sec and 1.75 sec?

$\mu = 1.25$

$\sigma = 0.46$

$$z = \frac{x-\mu}{\sigma} = \frac{x-1.25}{0.46}$$


$$P(1.00 \le x \le 1.75) = P\left(\frac{1-1.25}{0.46} \le \frac{x-1.125}{0.46} \le \frac{1.75-1.25}{0.46}\right)$$

$$= P(-0.54 \le z \le 1.09)$$

$$= P(-0.54 \le z \le 0) + P(0 \le z \le 1.09)$$

$$= P(0 \le z \le 0.54) + P(0 \le z \le 1.09)$$

$$= 0.2054 + 0.3621$$

$$= 0.5675$$

The breakdown voltage of a randomly chosen diode of a particular type is known to be normally distributed. what is the probability that a diode's breakdown voltage is within i) 1 standard deviation of its mean value ii) 2 standard deviation of its mean iii) 3 standard deviation of its mean.

$x \sim N(\mu, \sigma^2)$

$z = \dfrac{x-\mu}{\sigma}$


P(x is within 1 S.D of its mean)

$= P(\mu - \sigma \le x \le \mu + \sigma)$

$= P\left(\dfrac{\mu - \sigma - \mu}{\sigma} \le \dfrac{x-\mu}{\sigma} \le \dfrac{\mu + \sigma - \mu}{\sigma}\right)$

$= P(-1 \le z \le 1)$

$= P(-1 \le z \le 0) + P(0 \le z \le 1)$

$= P(0 \le z \le 1) + P(0 \le z \le 1)$

$= 0.3413 + 0.3413$

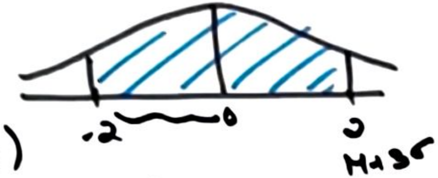
$= \underline{\underline{0.6826}}$

P(x is within 2 SD of its mean).

$= P(\mu - 2\sigma \le x \le \mu + 2\sigma)$

$= P\left(\dfrac{\mu - 2\sigma - \mu}{\sigma} \le \dfrac{x-\mu}{\sigma} \le \dfrac{\mu + 2\sigma - \mu}{\sigma}\right)$

$= P(-2 \le z \le 2)$




$= P(-2 \le z \le 0) + P(0 \le z \le 2)$

$= P(0 \le z \le 2) + P(0 \le z \le 2)$

$= 2\,P(0 \le z \le 2)$

$= 2 \times 0.4772$

$= \underline{\underline{0.9544}}$

95% $\mu \pm 2\sigma$

P( within 3 S.D of its Mean)

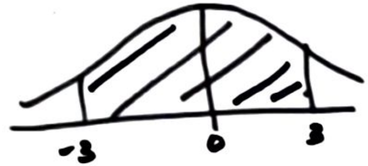
$= P(\mu - 3\sigma \le x \le \mu + 3\sigma)$

$= P\left(\dfrac{\mu - 3\sigma - \mu}{\sigma} \le z \le \dfrac{\mu + 3\sigma - \mu}{\sigma}\right)$

$= P(-3 \le z \le 3)$



$= P(-3 \le z \le 0) + P(0 \le z \le 3)$




$= 2 \times P(0 \le z \le 3)$

$= 2 \times 0.4987$

$= \underline{\underline{0.9974}}$

## Normal Approximation to Binomial:

$\mu = np \qquad \sigma = \sqrt{npq}.$

**Note:**

$np \geq 10$

$nq \geq 10$

U.Q A fair coin is tossed 900 times. Find the Probability that the number of heads is between 420 and 465

$n = 900 \qquad p = p(\text{getting head}) = \frac{1}{2}$

$q = 1 - p = \frac{1}{2}$

$np = \frac{900}{2} \geq 10 \qquad nq = \frac{900}{2} \geq 10$

$$z = \frac{x - \mu}{\sigma}$$

$\mu = np$

$= 450$

$\sigma = \sqrt{npq}$

$= \sqrt{\frac{900}{4}}$

$= 15$

$$z = \frac{x - 450}{15}$$


$P(420 \leq x \leq 465)$

$= P\left(\frac{420 - 450}{15} \leq \frac{x - 450}{15} \leq \frac{465 - 450}{15}\right)$

$= P(-2 \leq z \leq 1)$

$= P(-2 \leq z \leq 0) + P(0 \leq z \leq 1)$

$= P(0 \leq z \leq 2) + P(0 \leq z \leq 1)$

$= 0.4772 + 0.3413$

$= \underline{\underline{0.8185}}$

Q.8) What is the probability of obtaining atleast 2550 heads in tossing a Coin 5000 times?

$n = 5000$

$p = P(\text{getting head}) = \frac{1}{2}$

$q = 1 - p = \frac{1}{2}$

$np = \frac{5000}{2} = 2500 \geq 10$

$nq = 2500 \geq 10$

$\mu = np = 2500$

$\sigma = \sqrt{npq} = \sqrt{\frac{5000}{4}} = 35.36$

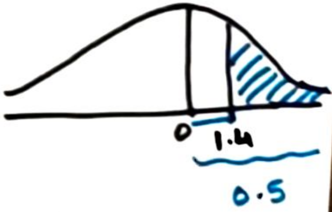
$z = \frac{x - \mu}{\sigma} = \frac{x - 2500}{35.36}$

$P(\text{atleast } 2550 \text{ heads})$

$= P(x \geq 2550)$

$= P\left(\frac{x - 2500}{35.36} \geq \frac{2550 - 2500}{35.36}\right)$

$= P(z \geq 1.4)$




$= 0.5 - P(0 \leq z \leq 1.4)$

$= 0.5 - 0.4192$

$= \underline{\underline{0.0858}}$

ii) Suppose that 25% of all students at a large public university receive financial aid. A random sample of 50 students are taken who receive financial aid.

Find the Probability that i) atmost 10 students receive aid.

ii) between 5 and 15 (inclusive) of the Selected Student receive aid.



$n = 50$

$p = P(\text{financial aid}) = 0.25$

$q = 1 - p = 0.75$

$np = 50 \times 0.25 = 12.5 \geq 10$

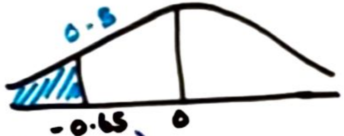
$nq = 50 \times 0.75 = 37.5 \geq 10$

$\mu = np = 12.5$

$\sigma = \sqrt{npq} = \sqrt{50 \times 0.25 \times 0.75}$

$= 3.06$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{x - 12.5}{3.06}$




i) $P(\text{atmost } 10 \text{ student}) = P(x \leq 10)$

$= P\left(\dfrac{x - 12.5}{3.06} \leq \dfrac{10 - 12.5}{3.06}\right)$

$= P(z \leq -0.65)$

$= 0.5 - P[-0.65 \leq z \leq 0]$

$= 0.5 - P[0 \leq z \leq 0.65]$

$= 0.5 - 0.2422$

$= 0.2578$

P(between 5 and 15 students)

$$= P(5 \le x \le 15)$$

$$= P\left(\frac{5-12.5}{3.06} \le \frac{x-12.5}{3.06} \le \frac{15-12.5}{3.06}\right)$$

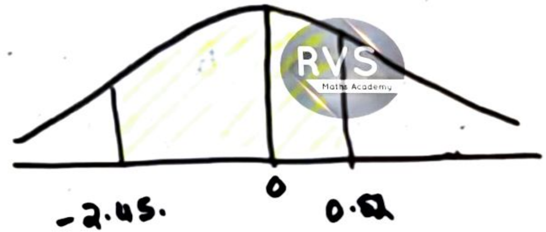
$$= P(-2.45 \le z \le 0.82)$$

$$= P(-2.45 \le z \le 0) + P(0 \le z \le 0.82)$$

$$= P(0 \le z \le 2.45) + P(0 \le z \le 0.82)$$

$$= 0.4929 + 0.2939$$

$$= \cancel{0.8320} \quad \underline{\underline{0.7868}}$$





$$P(4.5 \le x \le 15.5)$$

$$P(-2.61 \le z \le$$

$= P(5 \leq x \leq 15)$

$= P\left( \frac{5 - 12.5}{3.06} \leq \frac{x - 12.5}{3.06} \leq \frac{15 - 12.5}{3.06} \right)$

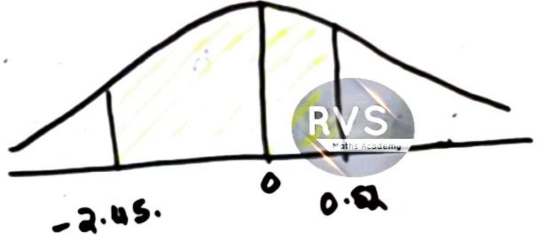
$= P(-2.45 \leq z \leq 0.82)$

$= P(-2.45 \leq z \leq 0) + P(0 \leq z \leq 0.82)$

$= P(0 \leq z \leq 2.45) + P(0 \leq z \leq 0.82)$

$= 0.4929 + 0.2939$

$= \cancel{0.8320}\ \underline{0.7868}$





$P(4.5 \leq x \leq 15.5)$

$P(-2.61 \leq z \leq 0.98)$

$P(0 \leq z \leq 2.61) + P(0 \leq z \leq 0.98)$

$= 0.4955 + 0.3365$

$= \underline{0.8320}$

if $x$ is a normal random Variable with mean 50 and Standard deviation 10.

find $\alpha$ and $\beta$ Such that $P(x < \alpha) = 0.10$ and $p(x > \beta) = 0.05$



$P(0 \le z \le \ldots) = 0.4772$

$\mu = 50 \quad \sigma = 10$

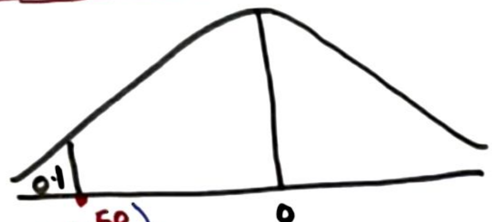
$x \sim N(\mu, \sigma^2)$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 50}{10}$$

$$P(x < \alpha) = 0.10$$

$$p\left(\frac{x - 50}{10} < \frac{\alpha - 50}{10}\right) = 0.10$$

$$P\left(z < \frac{\alpha - 50}{10}\right) = 0.10$$




$$\underbrace{-\left(\frac{\alpha - 50}{10}\right)}_{0.5}$$

$$0.5 - p\left[0 \le z \le \frac{\alpha - 50}{10}\right] = 0.1$$

$$P\left[0 \le z \le \frac{\alpha - 50}{10}\right] = 0.5 - 0.1$$

$$P\left[0 \le z \le \frac{\alpha - 50}{10}\right] = 0.4$$

$$\frac{\alpha - 50}{10} = -1.28$$

$$\alpha - 50 = -1.28 \times 10$$

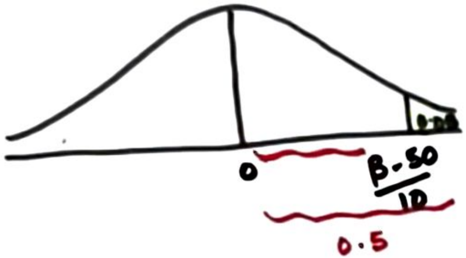
$$\alpha = -1.28 \times 10 + 50$$

$$= 37.2$$

$$P(x < 37.2) = 0.1$$

$P(x > \beta) = 0.05$

$P\left(\frac{x-50}{10} > \frac{\beta-50}{10}\right) = 0.05$






$P\left(z > \frac{\beta-50}{10}\right) = 0.05$

$0.5 - P\left(0 < z < \frac{\beta-50}{10}\right) = 0.05$

$P\left(0 < z < \frac{\beta-50}{10}\right) = 0.5 - 0.05$

$P\left(0 < z < \frac{\beta-50}{10}\right) = 0.45$

$\frac{\beta-50}{10} = 1.65$

$\beta = 1.65 \times 10 + 50$

$= \underline{\underline{66.5}}$

HW x is normally distributed.

mean = 1 and Variance = 4

i) $P(-3 < x < 3)$

ii) $P(x \leq k) = 0.9$

$P\left(z \leq \frac{k-1}{2}\right) = 0.9$

$= 0.5 + P\left(0 < z < \frac{k-1}{2}\right)$ ye


$k = 3.56$

Q.9 The marks obtained by a batch of students in a certain subject are normally distributed. 10% of students got less than 45 marks while 5% got more than 75. Find the percentage of students with score between 45 and 60.

$X \sim N(\mu, \sigma^2)$ $\qquad z = \frac{x-\mu}{\sigma}$

$P(x < 45) = 0.10$

$P\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.10$

$P\left(z < \frac{45-\mu}{\sigma}\right) = 0.10$

$0.5 - P\left[0 < z < \frac{45-\mu}{\sigma}\right) = 0.10$

$P\left(0 < z < \frac{45-\mu}{\sigma}\right) = 0.4$

$\frac{45-\mu}{\sigma} = -1.28$

$= \mu - 1.28\sigma$

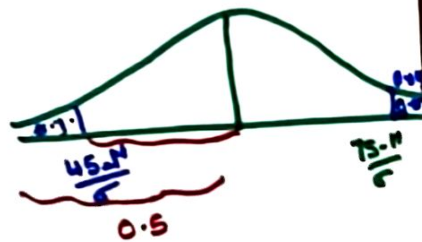
$P(x > 75) = 0.05$

$P\left(\frac{x-\mu}{\sigma} > \frac{75-\mu}{\sigma}\right) = 0.05$

$P\left(z > \frac{75-\mu}{\sigma}\right) = 0.05$

$0.5 - P\left[0 < z < \frac{75-\mu}{\sigma}\right] = 0.05$

$P\left[0 < z < \frac{75-\mu}{\sigma}\right] = 0.45$



$$(1) \Rightarrow \mu - 1.28\sigma = 45$$

$$(2) \Rightarrow \mu + 1.65\sigma = 75$$

---

$$(1)-(2) \Rightarrow \quad -2.93\sigma = -30$$

$$\sigma = \frac{-30}{-2.93}$$

$$= 10.2389$$

$$= \underline{\underline{10.24}}$$

$$(1) \Rightarrow \mu - 1.28 \times 10.24 = 45$$

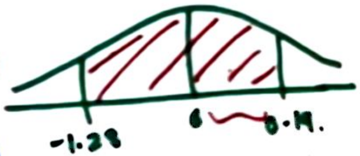
$$\mu = 45 + 13.1072$$

$$= \underline{\underline{58.10}}$$

$$P(45 < x < 60) = P\left(\frac{45 - 58.10}{10.24} < z \le \frac{60 - 58.10}{10.24}\right)$$

$$= P(-1.28 < z < 0.19)$$

$$= P(-1.28 < z < 0) + P(0 < z < 0.19)$$




$$= P(0 < z < 1.28) + P(0 < z < 0.19)$$

$$= 0.3997 + 0.0753$$

$$= \underline{\underline{0.475}}$$

$$\text{Percentage} = 0.475 \times 100$$

$$= \underline{\underline{47.5\%}}$$

Q.8 1000 light bulbs with mean length of life 120 days are installed in a factory. Their length of life is assumed to follow normal distribution with SD 20 days. How many bulbs will expire in less than 90 days? If it is decided to replace all the bulbs together. What interval should be allowed between replacements if not more than 10% should expire before replacement

$P(x < \underline{50}) = 0.10$

$X \sim N(\mu, \sigma^2)$

$\mu = 120 \qquad \sigma = 20$

$z = \dfrac{x - \mu}{\sigma}$


$$P(x < 90) = P\left(\frac{x-120}{20} < \frac{90-120}{20}\right)$$

$$= P(z < -1.5)$$

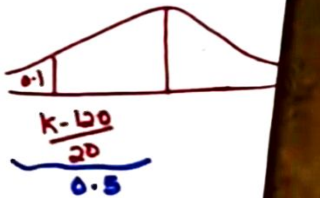
$$= 0.5 - P[0 < z < 1.5]$$

$$= 0.5 - 0.4332 = \underline{\underline{0.0668}}$$

Required Num = $1000 \times 0.0668$

$= 66.8 \cong \underline{\underline{67}}$

$$P(x < k) = 0.1$$




$$P\left(\frac{x-120}{20} < \frac{k-120}{20}\right) = 0.1$$

$$P\left(z < \frac{k-120}{20}\right) = 0.1$$

$$0.5 - P\left(0 < z < \frac{k-120}{20}\right) = 0.1$$

$$P\left(0 < z < \frac{k-120}{20}\right) = 0.4$$

$$\frac{k-120}{20} = -1.28$$

$$k = -1.28 \times 20 + 120$$

$$= \underline{\underline{94.4}}$$

$P(x < \underline{94.4}) = 0.1$

$\therefore$ atmost 94 days may be allowed