Parallel form structure of IIR filters:

A parallel form realization of an IIR system can be obtained by performing a partial fraction expansion of

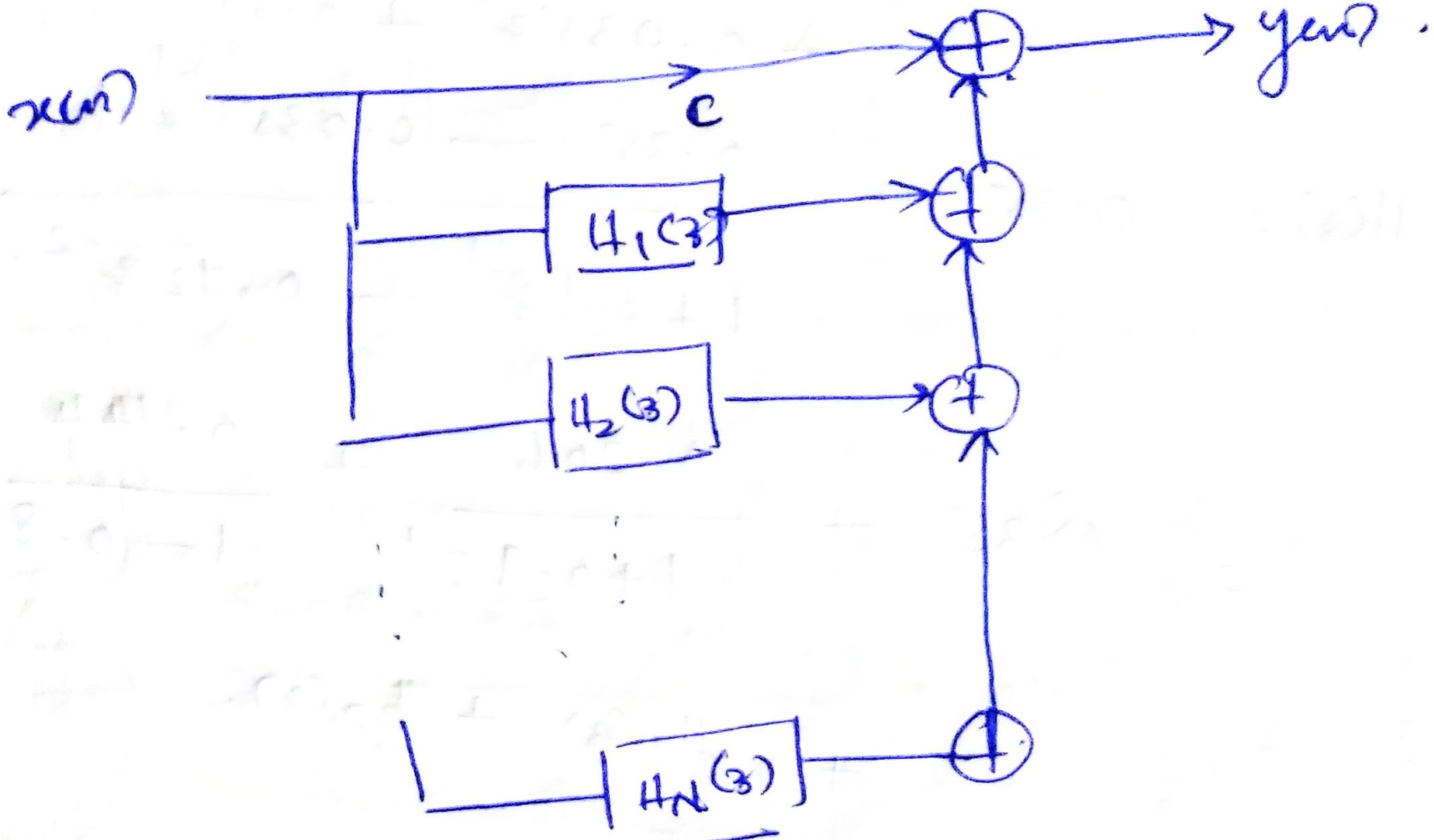
$$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1 - p_k z^{-1}}$$

where $\{p_k\}$ are the poles.

i.e. $$H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_2 z^{-1}} + \cdots + \frac{c_N}{1 - p_N z^{-1}}.$$

$= c + H_1(z) + H_2(z) + \cdots H_N(z)$.

Corresponding parallel form realization:



Q) Realize the system given by difference equation

$$y(n) = -0.1\,y(n-1) + 0.72\,y(n-2) + 0.7\,x(n) - 0.252\,x(n-2)$$ in parallel form.

Answer:

Take z transform.

$$Y(z) = -0.1 z^{-1} Y(z) + 0.72 z^{-2} Y(z) + 0.7 X(z) - 0.252 z^{-2} X(z)$$

$$Y(z)\left[1 + 0.1 z^{-1} - 0.72 z^{-2}\right] = X(z)\left[0.7 - 0.252 z^{-2}\right]$$

to get the system function, $H(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

To realize in parallel form $H(z)$ has to be factorized of the form

$$H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_2 z^{-1}}$$

to get constant 'e' perform division

$$
\begin{array}{r|l}
 & 0.35 \\
\hline
-0.72z^{-2}+0.1z^{-1}+1 & -0.252z^{-2}+0.7 \\
 & -0.252z^{-2}+0.035z^{-1}+0.35 \\
\hline
 & -0.035z^{-1}+0.35
\end{array}
$$

$\therefore$ H(z) can be re written as.

$$H(z) = 0.35 + \frac{0.35 - 0.035z^{-1}}{1+0.1z^{-1}-0.72z^{-2}}$$

Then perform partial fraction ~~of the~~ on second term.

ie $$\frac{0.35 - 0.035z^{-1}}{1+0.1z^{-1}-0.72z^{-2}} = \frac{c_1}{1-P_1z^{-1}} + \frac{c_2}{1-P_2z^{-1}}$$

$$P_1 P_2 = 0.72$$

$$-(P_1+P_2) = 0.1$$

$$P_1+P_2 = -0.1$$

$\therefore$ $P_1 = -0.9$ $P_2 = 0.8$

$$\therefore \frac{0.35 - 0.035z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}} = \frac{c_1}{1 + 0.9z^{-1}} + \frac{c_2}{1 - 0.8z^{-1}}$$

to get c_1 and c_2.

$$c_1(1 - 0.8z^{-1}) + c_2(1 + 0.9z^{-1}) = 0.35 - 0.035z^{-1}$$

$$c_1 + c_2 = 0.35 \quad \text{---(1) (equating constants).}$$

$$-0.8c_1 + 0.9c_2 = -0.035 \quad \text{---(2) (coeff of } z^{-1}\text{).}$$

$$(1) \Longrightarrow c_1 = 0.35 - c_2 \quad \text{---(3).}$$

(3) in (2) $\Longrightarrow$

$$-0.8(0.35 - c_2) + 0.9c_2 = -0.035$$

$$1.7c_2 = -0.035 + 0.28 = 0.245$$

$$c_2 = \frac{0.245}{1.7} = 0.144.$$

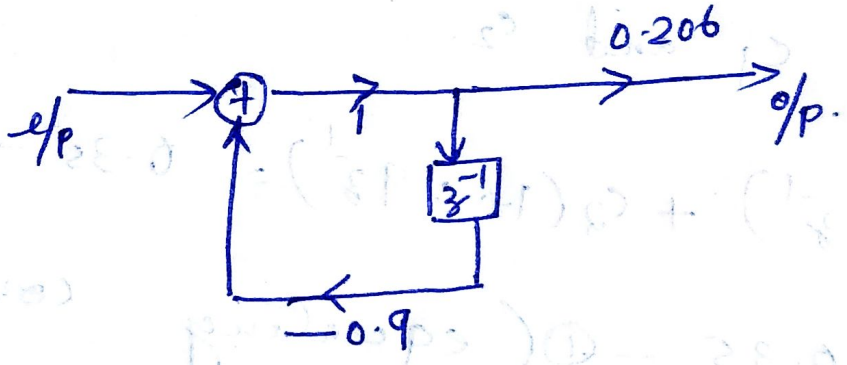
$$(1) \Longrightarrow c_1 = 0.35 - c_2 = 0.206.$$

$$\therefore H(z) = 0.35 + \frac{0.206}{1 + 0.9z^{-1}} + \frac{0.144}{1 - 0.8z^{-1}}.$$

$$= c + H_1(z) + H_2(z).$$

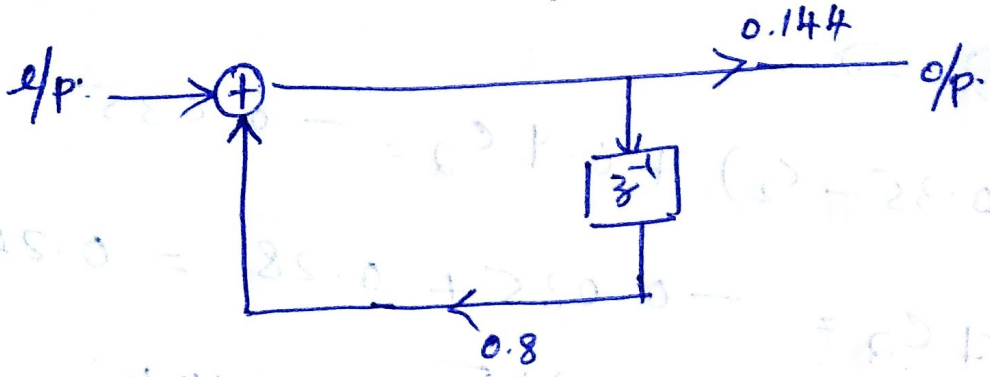
$$H_1(z) = \frac{0.206}{1+0.9z^{-1}}$$

corresponding realization

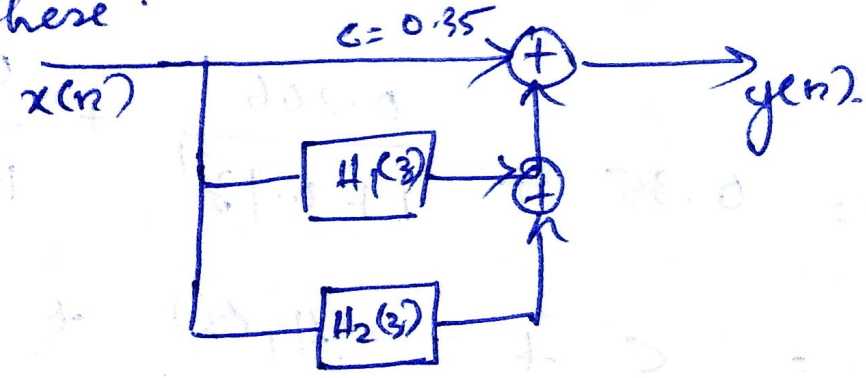


$$H_2(z) = \frac{0.144}{1-0.8z^{-1}}$$

corresponding realization



Procedure to get the final parallel realization of IIR filter Cascade there.



Corresponding parallel form realization is