Central Limit Theorem:

Let $x_1, x_2, x_3, \ldots, x_n$ be a Sequence of Independent and identically distributed random Variables each having mean μ and Variance σ^2 and let $S_n = x_1 + x_2 + \ldots + x_n$ then under Certain general Conditions, S_n follows a normal distribution with mean $n\mu$ and Variance $n\sigma^2$ $[N(n\mu, n\sigma^2)]$ as $n \to \infty$

$S.D = \sqrt{n}\,\sigma$

$$\boxed{\therefore Z_n = \frac{S_n - n\mu}{\sqrt{n}\,\sigma}}$$

$Z = \frac{X - \mu}{\sigma}$

i) Sum

ii) Average:

Note: $\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$

mean $= \mu$ Variance $= \frac{\sigma^2}{n}$

$N\left(\mu, \frac{\sigma^2}{n}\right)$

$$\boxed{Z_n = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}$$

	Mean	S.D	Z_n
i) S_n = Sum	$n\mu$	$\sigma\sqrt{n}$	$Z_n = \frac{S_n - n\mu}{\sigma\sqrt{n}}$
ii) $\bar{x}$ = Average.	μ	$\frac{\sigma}{\sqrt{n}}$	$Z_n = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

A Computer generates 20 random numbers w... e uniformly distributed between 0 and 1. Find approximately the prob... that i) their Sum is atleast 10

ii) their average is between 0.4 and 0.6

Let $x_1, x_2, \ldots, x_{20}$ be random num. gen. by Computer.

$$X_i \sim U(\overset{a}{0}, \overset{b}{1})$$

$$\mu : \text{mean} = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$$\sigma^2 = \text{Variance} = \frac{(b-a)^2}{12} = \frac{(1-0)^2}{12} = \frac{1}{12}$$

i) $S_n = x_1 + x_2 + x_3 + \cdots + x_{20}$ $\boxed{n = 20}$

$$\text{mean} = n\mu = 0.5 \times 20 = 10$$

$$\text{Variance} = n\sigma^2 = \frac{20}{12} = 1.67 \qquad S.D = \sqrt{1.67}$$

i) $P(S_n \geq 10) = P\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \geq \frac{10 - n\mu}{\sigma\sqrt{n}}\right)$

$$= P\left(Z_n \geq \frac{10-10}{\sqrt{1.67}}\right)$$

$$= P(Z \geq 0)$$

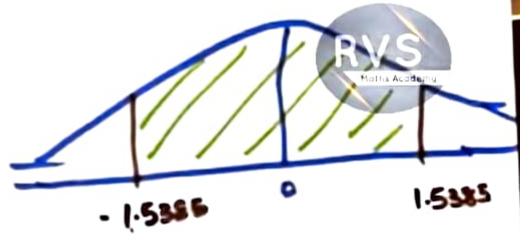
$$= 0.5$$

ii) Let $\bar{x} = \frac{x_1 + x_2 + \cdots + x_{20}}{n}$

Mean: $\mu = 0.5$ Variance: $\frac{\sigma^2}{n} = \frac{1/12}{20} = \frac{1}{240}$

$$S.D = \sqrt{\frac{1}{240}} = 0.0645$$

$$P(0.4 \le \bar{x} \le 0.6) = \left(\frac{0.4-0.5}{0.0645} \le \frac{\bar{x}-0.5}{0.0645} \le \frac{0.6-0.5}{0.0645}\right)$$



$$= (-1.5385 \le Z_n \le 1.5385)$$

$$= \cancel{(-1.54 \le Z_n \le 1.54)}$$

$$= P(-1.5385 \le Z_n \le 0) + P(0 \le Z_n \le 1.5385)$$

$$= P(0 \le Z_n \le 1.5385) + P(0 \le Z_n \le 1.5385)$$

$$= 2 \times P(0 \le Z_n \le 1.5385)$$

$$= 2 \times 0.4370$$

$$= \underline{\underline{0.8761}}$$

The lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1200 hrs and standard deviation 250 hrs. Find the probability, using Central Limit Theorem, that the average life time of 60 bulbs exceeds 1250 hrs.

Var: $\frac{\sigma^2}{n}$

$\mu = 1200$ $\qquad$ $\sigma^2 = 250$

$\bar{x} \to$ denote Av: lifetime of electric bulb.

$n = 60$

mean $= \mu = 1200$ $\qquad$ S.D $= \frac{\sigma}{\sqrt{n}} = \frac{250}{\sqrt{60}}$

$$z = \frac{\bar{x} - 1200}{\frac{250}{\sqrt{60}}}$$

$$P(\bar{x} > 1250) = \left(\frac{\bar{x} - 1200}{\frac{250}{\sqrt{60}}} > \frac{1250 - 1200}{\frac{250}{\sqrt{60}}} \right)$$

$= (Z_n > 1.549)$

$= 0.5 - P[0 < Z_n < 1.549]$

$= 0.5 - 0.4382$

$= 0.0606$

Q4) If $x_1, x_2, \ldots, x_n$ are Poisson random variables with parameter $\lambda = 2$, use Central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = x_1 + x_2 + \ldots + x_n$ and $n = 75$.

$\cancel{S_n = x_1 + x_2 + \ldots + x_n}$

mean $= \mu = \lambda = 2$

Varian $= \sigma^2 = \lambda = 2$

$S.D = \sqrt{2}$

By CLT $S_n \sim N(n\mu, n\sigma^2)$

$n = 75$

mean $= n\mu = 75 \times 2 = 150$

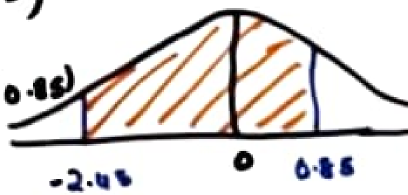
$S.D = \sigma\sqrt{n} = \sqrt{2} \cdot \sqrt{75} = \sqrt{150}$

$Z_n = \dfrac{S_n - 150}{\sqrt{150}}$

$$P(120 \le S_n \le 160) = P\left(\frac{120-150}{\sqrt{150}} \le \frac{S_n - 150}{\sqrt{150}} \le \frac{160-150}{\sqrt{150}}\right)$$

$= P(-2.45 \le Z_n \le 0.85)$

$= P(-2.45 \le Z_n \le 0) + P(0 \le Z_n \le 0.85)$



$= P(0 \le Z_n \le 2.45) + P(0 \le Z_n \le 0.85)$

$= 0.4929 + 0.3023$

$= \cancel{0.7866}$ 0.7952

	Mean	S.D.
S_n	$n\mu$	$\sigma\sqrt{n}$
$\bar{x}$	μ	$\frac{\sigma}{\sqrt{n}}$

	Mean	S.D	Z_n
i) S_n = Sum	$n\mu$	$\sigma\sqrt{n}$	$Z_n = \dfrac{S_n - n\mu}{\sigma\sqrt{n}}$
ii) $\bar{x}$ = Average.	μ	$\dfrac{\sigma}{\sqrt{n}}$	$Z_n = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

The burning time of a certain type of lamp is an exponential random variable with mean 30 hrs. What is the Probability that 144 of these lamps will provide a total more than 4500 hrs of burning time.

S_n

$\mu = 30$

$\left[\text{exp mean} = \frac{1}{\lambda} \right.$

$\text{Var} : \frac{1}{\lambda^2}$

$\sigma^2 = (30)^2 = 900$

$\text{S.D } \sigma = 30$

$n = 144$

$S_n = x_1 + x_2 + \cdots + x_{144}$

$$Z_n = \frac{S_n - n\mu}{\sqrt{n}\,\sigma}$$

$\text{mean} = n\mu = 144 \times 30 = 4320$

$\text{S.D} = \sqrt{n}\,\sigma = \sqrt{144} \times 30 = 360$

$$P[S_n > 4500] = P\left[\frac{S_n - 4320}{360} > \frac{4500 - 4320}{360}\right]$$

$= P[Z_n > 0.5]$

$= 0.5 - P[0 < z < 0.5]$

$= 0.5 - 0.1915$

$= \underline{0.3085}$

U.Q In a game involving repeated throws of a balanced die a person receives Rs 3. If the resulting number is greater than or equal to 3 and loses Rs 3 otherwise. Use Central limit theorem to find the probability that in 25 trials his total earnings exceed Rs 25.

x:	3	-3
$p(x)$:	$\frac{2}{3}$	$\frac{1}{3}$

$$E[x] = \sum x\, p(x)$$

$$= 3 \times \frac{2}{3} - 3 \times \frac{1}{3}$$

$$\mu = 1$$

$\frac{4}{6}, \frac{2}{6}$

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \sum x^2 p(x)$$

$$= 9 \times \frac{2}{3} + 9 \times \frac{1}{3}$$

$$= 9$$

$$Var(x) = 9 - 1$$

$$= 8$$

$$\sigma = \sqrt{8}.$$

$$n = 25$$

$$S_n = x_1 + x_2 + \dots + x_5$$

$$\text{mean} = n\mu = 25$$

$$S.D = \sqrt{n}\,\sigma = 5\sqrt{8}.$$

$$P\cancel{(S_n > 25)} =$$

$$Z_n = \frac{S_n - 25}{5\sqrt{8}}$$

$$P(S_n > 25) = P\left(\frac{S_n - 25}{5\sqrt{8}} > \frac{25 - 25}{5\sqrt{8}}\right)$$

$$= P(Z_n > 0)$$

$$= \underline{\underline{0.5}}$$

Q.6 A game involves a player throwing a fair die several times. In each throw, if the die shows 3 or 4 he gets Rs 5 otherwise he loses Rs 2. Use Central limit theorem to find how many times should he throw the die so that the probability is at least 0.5 that his total earnings is Rs 25 or more.

$P(S_n \geq 25) \leq 0.5$

x:	5	-2
$p(x)$:	$\frac{1}{3}$	$\frac{2}{3}$

x	5	-2
$p(x)$	$\frac{1}{3}$	$\frac{2}{3}$

$$E[x] = \Sigma x\, p(x)$$

$$= 5 \times \frac{1}{3} - 2 \times \frac{2}{3}$$

$$\mu = \frac{1}{3} = 0.333$$

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \Sigma x^2 p(x)$$

$$= 25 \times \frac{1}{3} + 4 \times \frac{2}{3}$$

$$= 11$$

$$Var(x) = 11 - (0.333)^2 = 10.989$$

$$S.D = \sigma = \sqrt{10.989} = 3.315$$

$$\boxed{n = ?}$$

$$S_n = x_1 + x_2 + \dots + x_n$$

$$\text{mean} = n\mu = n(0.333)$$

$$S.D = \sqrt{n}\,\sigma = \sqrt{n}\,(3.315)$$

$$Z_n = \frac{S_n - n\mu}{\sqrt{n}\,\sigma}$$

$$P(S_n \geq 25) \geq 0.5$$

$$P\left[\frac{S_n - n\mu}{\sqrt{n}\,\sigma} \geq \frac{25 - n\mu}{\sqrt{n}\,\sigma}\right] \geq 0.5$$

$$P\left[Z_n \geq \frac{25 - n(0.333)}{\sqrt{n}\,(3.315)}\right] \geq 0.5$$

$$\frac{25 - n(0.333)}{\sqrt{n}\,(3.315)}$$

$$\frac{25 - n(0.333)}{\sqrt{n}\,(3.315)} = 0$$

$$25 - n(0.333) = 0$$

$$25 = \frac{n}{3}$$

$$n = 75$$

Continuous two dimensional random variable.

Joint Probability density function:

Two continuous random variables X and Y are said to be jointly continuous if $P\left[x - \frac{dx}{2} \le X \le x + \frac{dx}{2},\ y - \frac{dy}{2} \le Y \le y + \frac{dy}{2}\right] = f(x,y)\,dx\,dy$.

the $f(x,y)$ is called the joint Pdf of (x,y) provided $f(x,y)$ satisfies the following conditions:

i) $f(x,y) \ge 0$

ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

pdf. $f(x) = \begin{cases} 2^x & 0<x<1 \\ 0 & \text{o.w} \end{cases}$

$f(x) = \int_{-\infty}^{x} f(x)\,dx$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

J.P.d.f

$f(x,y) = \begin{cases} x+y & 0<x<1,\ 1<y<4 \\ 0 & \text{o.w} \end{cases}$

Cumulative Distribution function

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\,dy\,dx$$

Marginal Density function of x

Marginal probability density function of x is denoted by f(x)

$$f(x) = f_x(x) = \int_{-\infty}^{\infty} f(x,y)\,dy.$$

Marginal Density function of y

Marginal probability density function of y is denoted by f(y)

$$f(y) = f_y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx.$$

Conditional Probability distribution

$$f(x/y) = \frac{f(x,y)}{f_y(y)} \quad \text{and} \quad f(y/x) = \frac{f(x,y)}{f_x(x)}$$

Independent random variables

Two random variables X and Y are said to be independent if

$$f(x,y) = f_x(x)\, f_y(y)$$

Expectation

if $g(x,y)$ be a continuous random variable x and Y. Then

$$E[g(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f(x,y)\, dx\, dy$$

$E(A) = \int A f(x)\, dx$.

$E[ax+b] = aE[x]+b$

Note: * if x and Y are independent random variables, then

$$E[xY] = E[x]E[Y]$$

* $E[ax+bY] = a E[x] + b E[Y]$

* The joint pdf of two continuous random variables X and Y is given by

$$f(x,y)=\begin{cases} kxy & 0<x<4 \quad 1<y<5 \\ 0 & \text{otherwise} \end{cases}$$

find i) k ii) $P(x \geq 3, y \leq 4)$

iii) $P(1<x<2,\ 2<y<3)$

iv) $P(x+y<3)$

v) marginal distributions of X and Y vi) check whether X and Y are independent.

w.k.t $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

$$\int_0^4 \left[\int_1^5 kxy\ dy\right] dx = 1$$

$$k\int_0^4 x\left[\frac{y^2}{2}\right]_1^5 dx = 1$$

$$\frac{k}{2}\int_0^4 x\,[25-1]\,dx = 1$$

$$\overset{6}{\cancel{12}}k\left[\frac{x^2}{2}\right]_0^4 = 1$$

$$6k[16-0]=1$$

$$96k=1$$

$$\boxed{k=\frac{1}{96}}$$

ii) $P(x \geq 3, y \leq 4) = \int_3^4 \int_1^4 \overset{\frac{xy}{96}}{f(x,y)}\, dy\, dx$

$= \frac{1}{96} \int_3^4 \int_1^4 xy \, dy\, dx$

$= \frac{1}{96} \int_3^4 x \left[\frac{y^2}{2}\right]_1^4 dx$

$= \frac{1}{96} \int_3^4 x \left[\frac{16-1}{2}\right] dx$

$= \frac{15}{192} \int_3^4 x \, dx$

$= \frac{15}{192} \left[\frac{x^2}{2}\right]_3^4$

$= \frac{15}{192 \times 2} [16 - 9] = \frac{35}{128}$

$f(x,y) = \begin{cases} \frac{xy}{96} & 0<x<4, 1<y<5 \\ 0 & \text{o.w} \end{cases}$

$P(1<x<2, 2<y<3)$

$= \int_1^2 \int_2^3 \frac{xy}{96} \, dy\, dx$

$= \frac{1}{96} \int_1^2 x \left[\frac{y^2}{2}\right]_2^3 dx$

$= \frac{1}{192} \int_1^2 x [9-4] \, dx.$

$= \frac{5}{192} \left[\frac{x^2}{2}\right]_1^2 = \frac{5}{192 \times 2} \overset{3}{[4-1]} = \frac{5}{128}$

iv) $P(x+y<3)$

$$= \int_0^2 \int_1^{3-x} \frac{xy}{96}\, dy\, dx$$

$$= \frac{1}{96} \int_0^2 x \left[\frac{y^2}{2}\right]_1^{3-x} dx$$

$9-6x+x^2-1$

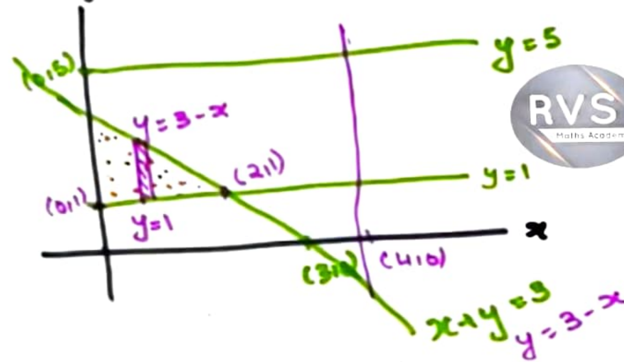
$8-6x+x^2$

$$= \frac{1}{192} \int_0^2 x\left[(3-x)^2 - 1\right] dx$$

$$= \frac{1}{192} \int_0^2 \left[8x - 6x^2 + x^3\right] dx$$

$$= \frac{1}{192} \left[\overset{4}{\cancel{8}} \frac{x^2}{\cancel{2}} - \overset{2}{\cancel{6}} \frac{x^3}{\cancel{3}} + \frac{x^4}{4} \right]_0^2$$

$$= \frac{1}{192}\left[(16 - 16 + 4) - 0\right] = \frac{1}{48}$$



$$f(x,y) = \begin{cases} \frac{xy}{96} & 0<x<4 \\ & 1<y<5 \\ 0 & \text{o.w} \end{cases}$$

$x+y=3$ — ① $y=1$ — ②

$x+1=3$

$x=2$

$(2,1)$

$x+y=3$ St. line passing thr $(3,0)$ & $(0,3)$

y=

x=   x=

y=

$y \to y=1$ to $y=3-x$.

$x \to x=0$ to $x=2$

v) marginal density function of x

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_{1}^{5} \frac{xy}{96}\,dy$$

$$= \frac{1}{96}\, x \left[\frac{y^2}{2}\right]_1^5$$

$$= \frac{1}{192}\, x\,[25-1]$$

$$= \frac{x}{8} \qquad 0<x<4$$

$$\therefore \quad f_X(x) = \begin{cases} \dfrac{x}{8} & 0<x<4 \\ 0 & \text{o.w} \end{cases}$$

marginal density fun: of y

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$= \int_{0}^{4} \frac{xy}{96}\,dx$$

$$= \frac{1}{96}\, y \left[\frac{x^2}{2}\right]_0^4$$

$$= \frac{1}{192}\, y\,[16-0]$$

$$= \frac{y}{12} \qquad 1<y<5$$

$$f_Y(y) = \begin{cases} \dfrac{y}{12} & 1<y<5 \\ 0 & \text{o.w} \end{cases}$$

vi)

$$f_X(x)\, f_Y(y)$$

$$= \frac{x}{8} \cdot \frac{y}{12}$$

$$= \frac{xy}{96}$$

$$= f(x,y)$$

$\therefore$ X and Y are independent

The joint Probability density fun: of x and y are

$$f(x,y) = \begin{cases} c & 1<x<3 \quad 2<y<4 \\ 0 & \text{otherwise} \end{cases}$$

find i) c ii) $P(1<x<2,\ 2<y<4)$

iii) $P(x+y\le 5)$ iv) marginal density fun: of x and y.

v) check whether x and y are independent

$c = \frac{1}{4}$

$$P(x+y\le 5) = \int_1^3 \int_2^{5-x} \frac{1}{4}\, dy\, dx$$

$$= \frac{1}{4} \int_1^3 \int_2^{5-x} dy\, dx$$

$$= \frac{1}{2}$$

$\left[\frac{1}{2} bh\right]$ $h=2$ $b=2$

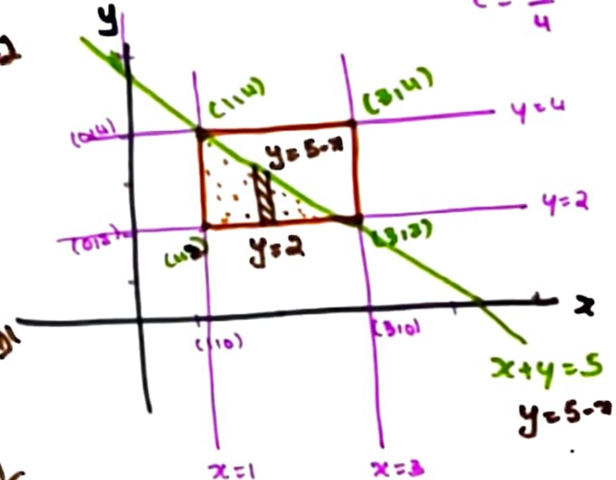
$\frac{1}{2}\cdot 2\cdot 2$

$= \iint \frac{1}{4}\, dy\, dx$

$= \frac{1}{4}$ Area of D

$= \frac{1}{4}(2)$

$= \frac{1}{4}$ $\frac{1}{2}$



$x+y=5$ (5,0) & (0,5)

$x=2$

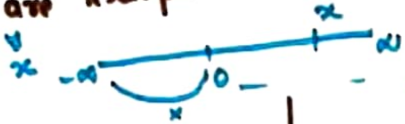
$y=2$

The density function of two dimensional random Variable (x,y) is given by

$$f(x,y) = \begin{cases} e^{-(x+y)} & \text{when } x \geq 0,\ y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

i) find joint cumulative distribution function (iii) find $P(x<1)$

ii) Check whether x and y are independent. iv) $P(x+y<1)$



i) $$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\, dy\, dx$$

$$= \int_0^x \int_0^y e^{-(x+y)}\, dy\, dx.$$

$$x^{m+n} = x^m x^n$$

$$= \int_0^x \int_0^y e^{-x} e^{-y}\, dy\, dx$$

$$= \int_0^x e^{-x} \left[\frac{e^{-y}}{-1}\right]_0^y dx$$

$$= -\int_0^x e^{-x}\left[e^{-y} - e^0\right] dx$$

$$= -\int_0^x (e^{-y}-1)\, e^{-x}\, dx.$$

$$= -(e^{-y}-1)\left[\frac{e^{-x}}{-1}\right]_0^x.$$

$$e^0 = 1$$

$$= +(e^{-y}-1)(e^{-x}-1)$$

$$f(x,y) = \begin{cases} (e^{-x}-1)(e^{-y}-1) & x \geq 0, y \geq 0 \\ 0 & o.w \end{cases}$$

To S.T x and y are independent.

we have to P.T $f_X(x)\, f_Y(y) = f(x,y)$

marginal density fun: of x

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$= \int_{0}^{\infty} e^{-(x+y)}\, dy.$$

$$= \int_{0}^{\infty} e^{-x} e^{-y}\, dy$$

$$= e^{-x}\left[\frac{e^{-y}}{-1}\right]_0^{\infty} \qquad e^{-\infty} = 0$$

$$= -e^{-x}\left[\overset{0}{e^{-\infty}} - \overset{1}{e^{0}}\right] \qquad e^{0} = 1$$

$$= \underline{\underline{e^{-x}}} \quad x \geq 0$$

$$\therefore f_X(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & o.w \end{cases}$$

marginal density fun: of y

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx.$$

||ly $\qquad = e^{-y} \quad y \geq 0$

$$f_y(y) = \begin{cases} e^{-y} & y \ge 0 \\ 0 & o.w \end{cases}$$

$$x^m x^n = x^{m+n}$$

$$\therefore f_x(x)\, f_y(y) = e^{-x}\, e^{-y}$$

$$= e^{-(x+y)}$$

$$= f(x,y)$$

$\therefore$ x and y are independent.

$$P(x<1) = \int_0^1 f(x)\,dx$$

$$f(x) = \begin{cases} e^{-x} & x \ge 0 \\ 0 & o.w \end{cases}$$

$$= \int_0^1 e^{-x}\,dx$$

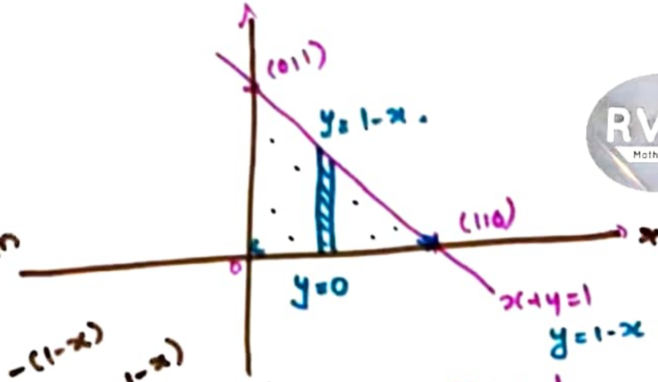
$$\int e^{ax} = \frac{e^{ax}}{a}.$$

$$= \left[\frac{e^{-x}}{-1}\right]_0^1$$

$$= -\left[e^{-1} - e^{0}\right]$$

$$= \underline{1 - e^{-1}}$$

$$P(x+y \le 1) = \int_0^1 \int_0^{1-x} \overset{e^{-(x+y)}}{f(x,y)}\, dy\, dx$$



$x+y=1$

St: line (1,0) & (0,1)

$$= \int_0^1 \int_0^{1-x} e^{-x} e^{-y}\, dy\, dx.$$

$x^m x^n = x^{m+n}$

$e^{-x} e^{-(1-x)}$

$= e^{-(x+1-x)}$

$$= \int_0^1 e^{-x} \left[\frac{e^{-y}}{-1} \right]_0^{1-x} dx.$$

$y \to y=0$ to $y = 1-x$.

$x \to x=0$ to $x=1$

$$= -\int_0^1 e^{-x} \left[e^{-(1-x)} - 1 \right] dx$$

$$= -\int_0^1 e^{-x} e^{-(1-x)} - e^{-x}\, dx$$

$$= -\int_0^1 e^{-1} - e^{-x}\, dx$$

$$= -\left[e^{-1}(x) - \left(\frac{e^{-x}}{-1} \right) \right]_0^1$$

$$= -\left[\underbrace{(e^{-1} + e^{-1})}_{2e^{-1}} - (0+1) \right]$$

$$= 1 - 2e^{-1}$$

$$= 1 - \frac{2}{e}$$

The joint cumulative function of the random Variables x and Y is given by

$$F(x,y) = \begin{cases} x^2y^2 & 0<x<1,\ 0<y<1 \\ 1 & x\geq 1,\ y\geq 1 \\ 0 & \text{otherwise} \end{cases}$$

i) find joint PDF ii) $P(x+y<1)$

iii) Are x and y are independent.

if $F(x,y)$ is given.

$$\boxed{f(x,y) = \frac{\partial^2}{\partial x \partial y} F(x,y)}$$

$$= \frac{\partial^2}{\partial x \partial y} \begin{cases} x^2y^2 & 0<x<1,0<y<1 \\ 1 & x\geq 1,\ y\geq 1 \\ 0 & \text{o.w} \end{cases}$$

$$= \frac{\partial}{\partial x} \begin{cases} 2x^2y & 0<x<1,\ 0<y<1 \\ 0 & x\geq 1,\ y\geq 1 \\ 0 & \text{o.w} \end{cases}$$

$$= \begin{cases} 2(2x)y & 0<x<1,0<y<1 \\ 0 & \text{o.w} \end{cases}$$

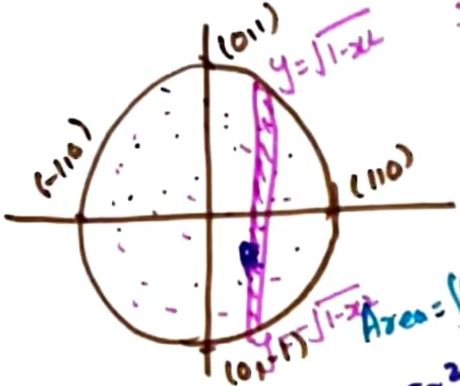
$$\boxed{f(x,y) = \begin{cases} 4xy & 0<x<1,0<y<1 \\ 0 & \text{o.w.} \end{cases}}$$

$$f(x,y) = \begin{cases} \frac{xy}{96} \end{cases}$$

$y \to y=0$ to $y=1-x$

$x \to x=0$ to $x=1$

$x+y=1$



$x^2+y^2=1$

$y^2=1-x^2$

$y=\pm\sqrt{1-x^2}$

Area $=\iint dx\,dy$

πr^2
radius $=1$

1) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy=1$$

$$\iint_R c\,dx\,dy=1$$

$$c\iint_R dx\,dy=1$$

$$c\left[\pi(1)^2\right]=1$$

$$\pi c=1$$

$$\boxed{c=\frac{1}{\pi}}$$

$$f(x,y)=\begin{cases}\frac{1}{\pi} & x^2+y^2\le 1\\ 0 & \text{o.w}\end{cases}$$

ii) marginal density fn. of X

$$f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy$$

$$\boxed{\begin{array}{l} y\to y=-\sqrt{1-x^2} \text{ to } y=\sqrt{1-x^2}\\ x\to x=-1 \text{ to } x=1\end{array}}$$

$$f_X(x)=\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{1}{\pi}\,dy$$

$$=\frac{1}{\pi}\left[y\right]_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$$

$$=\frac{1}{\pi}\left[\sqrt{1-x^2}+\sqrt{1-x^2}\right]$$

$$=\frac{2}{\pi}\sqrt{1-x^2} \quad -1\le x\le 1$$

$$f_X(x)=\begin{cases}\frac{2}{\pi}\sqrt{1-x^2} & -1\le x\le 1\\ 0 & \text{o.w}\end{cases}$$

$$f_Y(y) = \begin{cases} \frac{2}{\pi}\sqrt{1-y^2} & -1 \le y \le 1 \\ 0 & \text{o.w} \end{cases}$$

x and y are independent if

$$f_X(x)\, f_Y(y) = f(x,y)$$

$$\frac{2}{\pi}\sqrt{1-y^2}\ \frac{2}{\pi}\sqrt{1-y^2} = \frac{4}{\pi^2}\sqrt{1-x^2}\,\sqrt{1-y^2}$$

$$\neq \frac{1}{\pi}$$

$$\neq f(x,y)$$

$\therefore$ X and Y are not independent.

$$E[xy] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\, dx\, dy$$

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy\, \frac{1}{\pi}\, dy\, dx$$

$$= \frac{1}{\pi}\int_{-1}^{1} x\left[\frac{y^2}{2}\right]_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx$$

$$= \frac{1}{2\pi}\int_{-1}^{1} x\left[(\cancel{1-x^2}) - (\cancel{1-x^2})\right] dx$$

$$= \underline{\underline{0}}$$

$$E[x] = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx & f(x) \text{ even fun:} \\ 0 & f(x) \text{ is odd fun:} \end{cases}$$

$$= \int_{-1}^{1} x \frac{2}{\pi}\sqrt{1-x^2}\, dx$$

odd:fun

$$= \frac{2}{\pi}\int_{-1}^{1} x\sqrt{1-x^2}\, dx$$

$$= \underline{\underline{0}}$$

$$E[y] = \int_{-\infty}^{\infty} y\, f(y)\, dy$$

$$= \int_{-1}^{1} y\, \frac{2}{\pi}\sqrt{1-y^2}\, dy$$

odd:

$$= \frac{2}{\pi}\int_{-1}^{1} y\sqrt{1-y^2}\, dy$$

$$= \underline{\underline{0}}$$

$$E[xy] = \underline{E[x]E[y]}$$

$x>0$ $y<1$

$$\begin{cases} \frac{xy}{24} & 0<x, y<1 \end{cases}$$