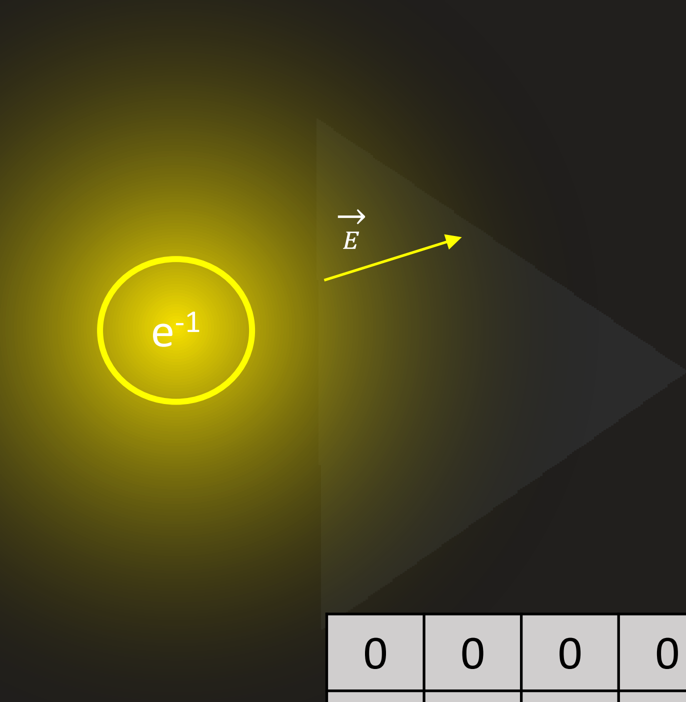
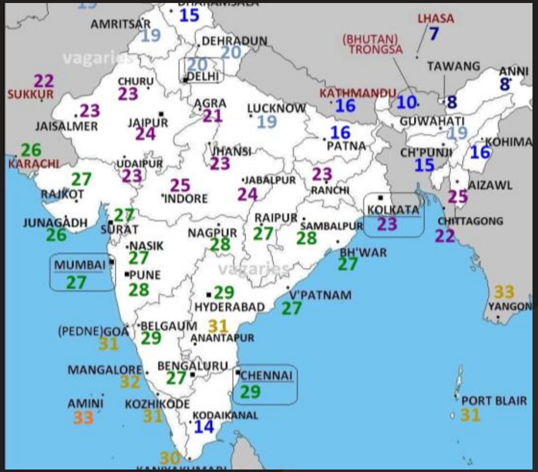
INTRODUCTION TO ELECTROMAGNETICS

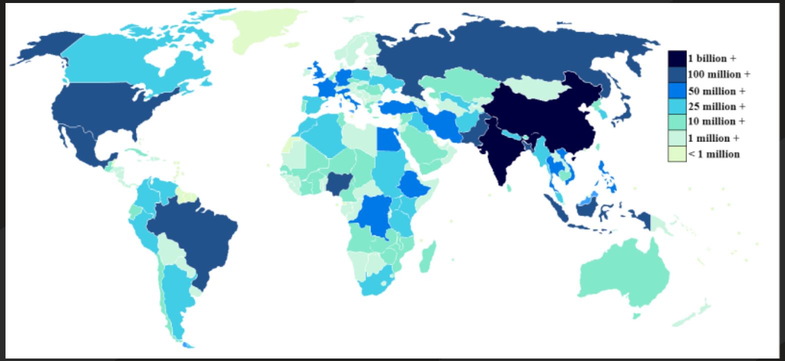
- Electromagnetics (EM) is a branch of physics in which electric and magnetics phenomenon are studied
- Electromagnetics may be regarded as the study of the interactions between electric charges at rest and in motion
- It entails the analysis, synthesis, physical interpretation and application of electric and magnetic field

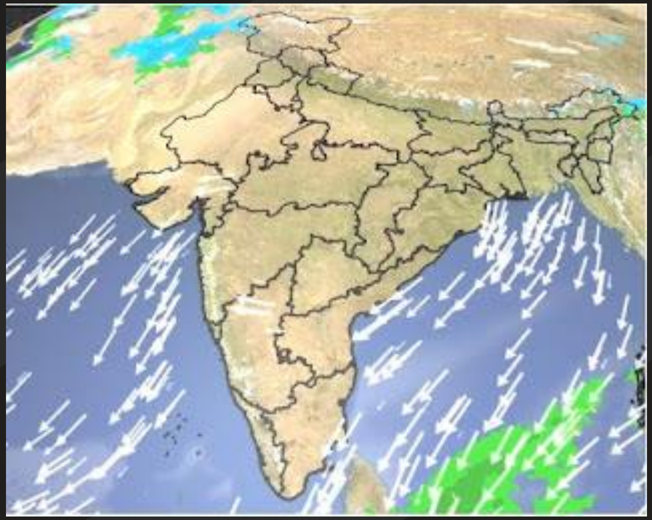
Field

A field is a region of space for which each point is associated with a quantity



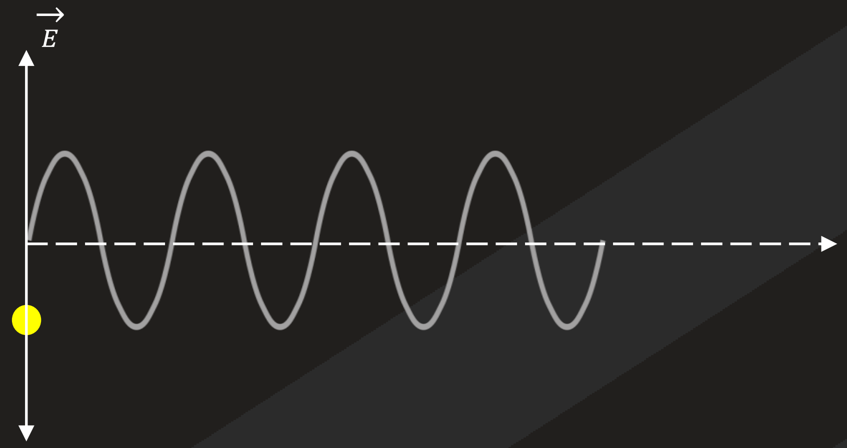






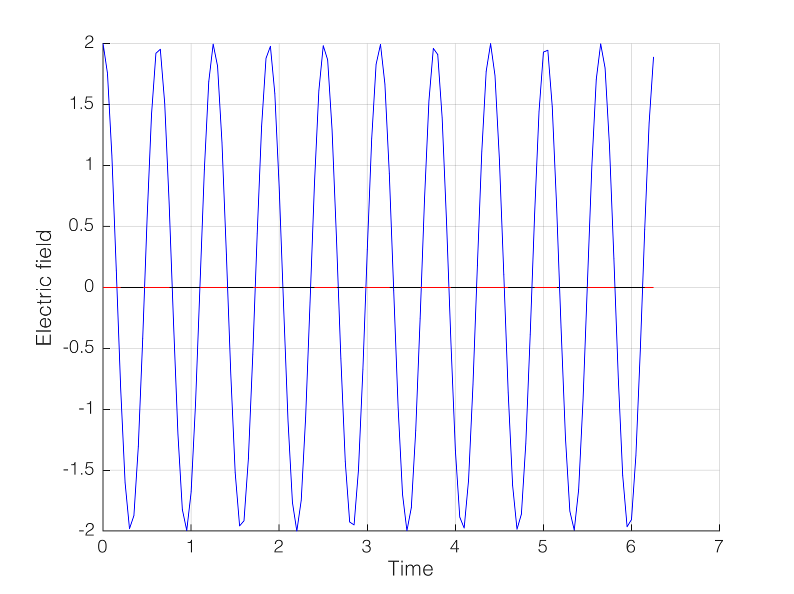
e-1

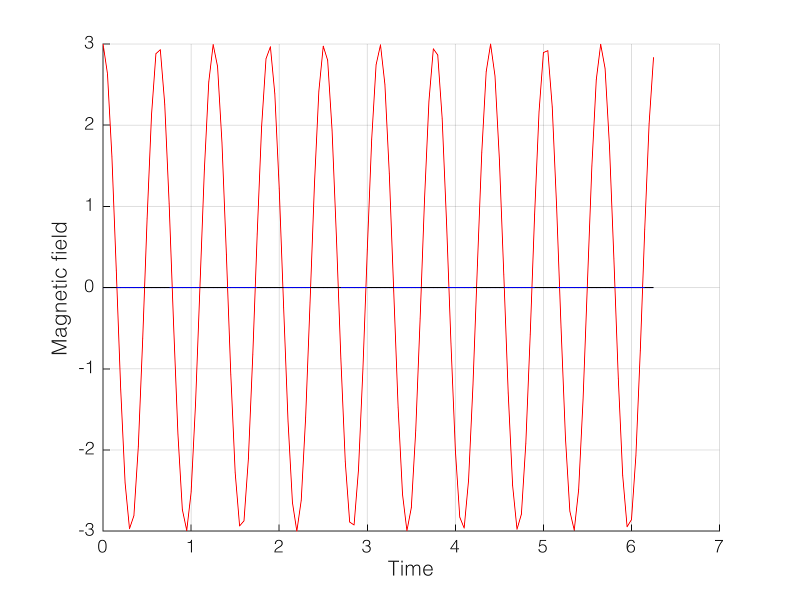
0	0	0	0	0	1
0	0	0	0	0	0
0	1	0	0	1	0
0	0	0	0	1	0
0	1	0	0	0	0
0	0	0	0	1	0

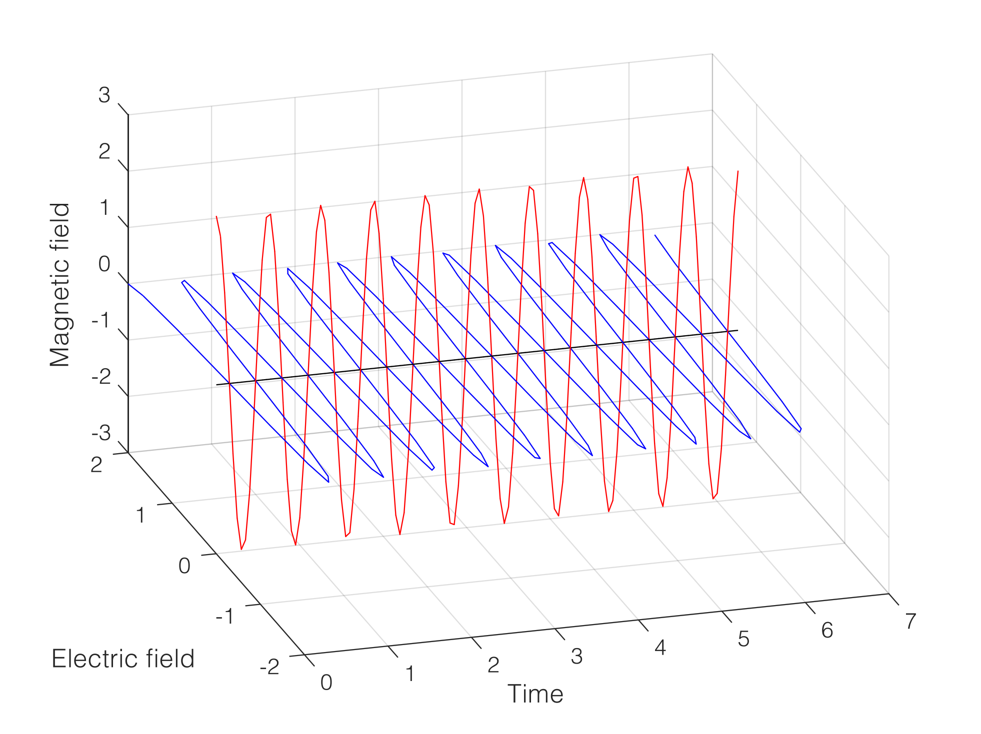


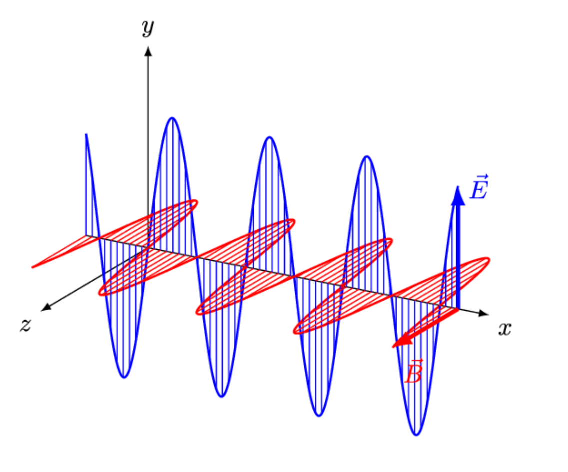
J C Maxwell

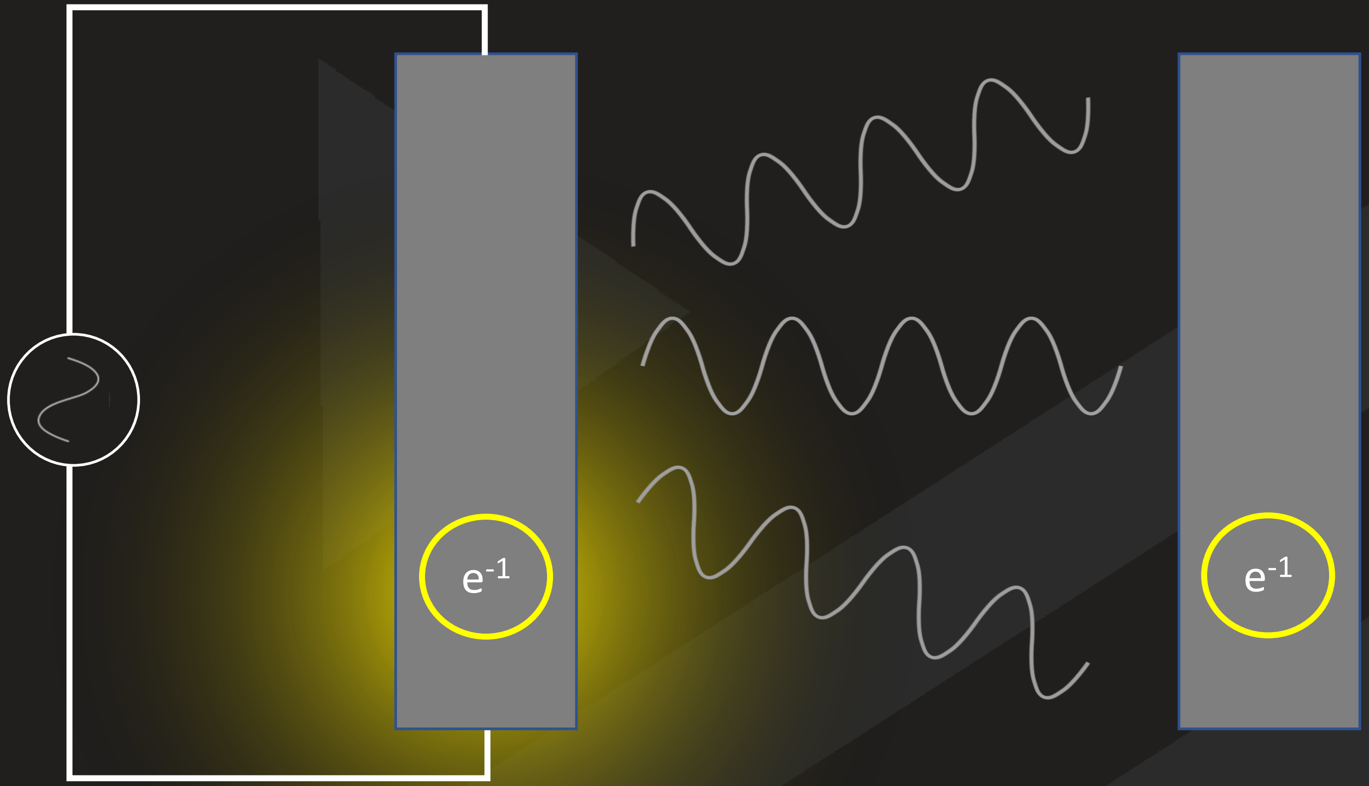
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$


Electric field
Time


Magnetic field
Time


Magnetic field
Electric field
Time


y
z
x
$\vec{E}$
$\vec{B}$



$\vec{F} = -e\vec{E}$

REVIEW OF VECTOR CALCULUS

- Vector analysis is a powerful mathematical tool in expressing analysing and understanding concepts that involve vector quantities.
- A quantity can be either scalar or vector

SCALAR

- A scalar is a quantity that has only magnitude
- Example :- Time, Mass, Distance, Temperature, etc;

VECTOR

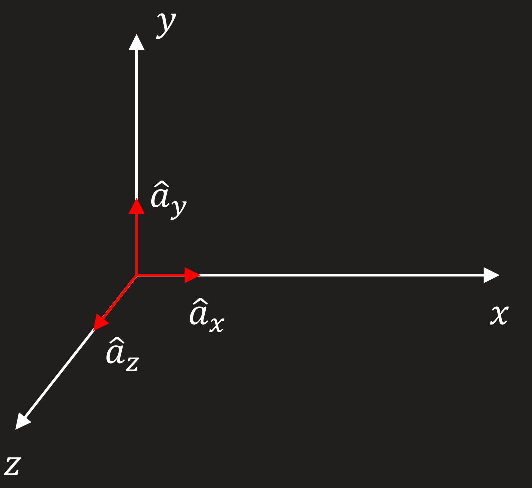
- A vector is a quantity that has both magnitude and direction
- Example :- Velocity, Force, Displacement, electric field, etc;

$\vec{A}$ $\vec{B}$

$\boldsymbol{A}$ $\boldsymbol{B}$

UNIT VECTOR

$$\vec{A} = |A|\hat{a}_n$$

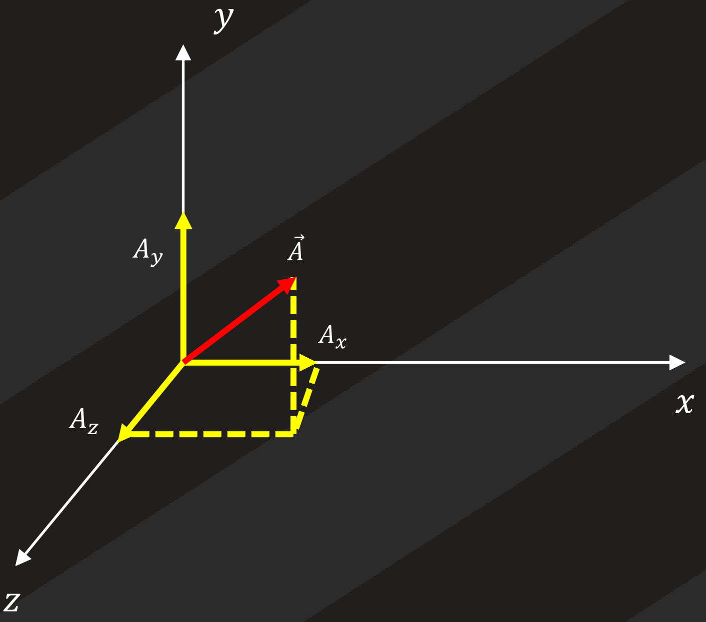


$$\hat{a}_n = \frac{\vec{A}}{|A|}$$

Unit vector

$$\vec{A} = (A_x, A_y, A_z)$$

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$



$$\hat{a}_A = \frac{\vec{A}}{|A|}$$

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{a}_A = \frac{A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

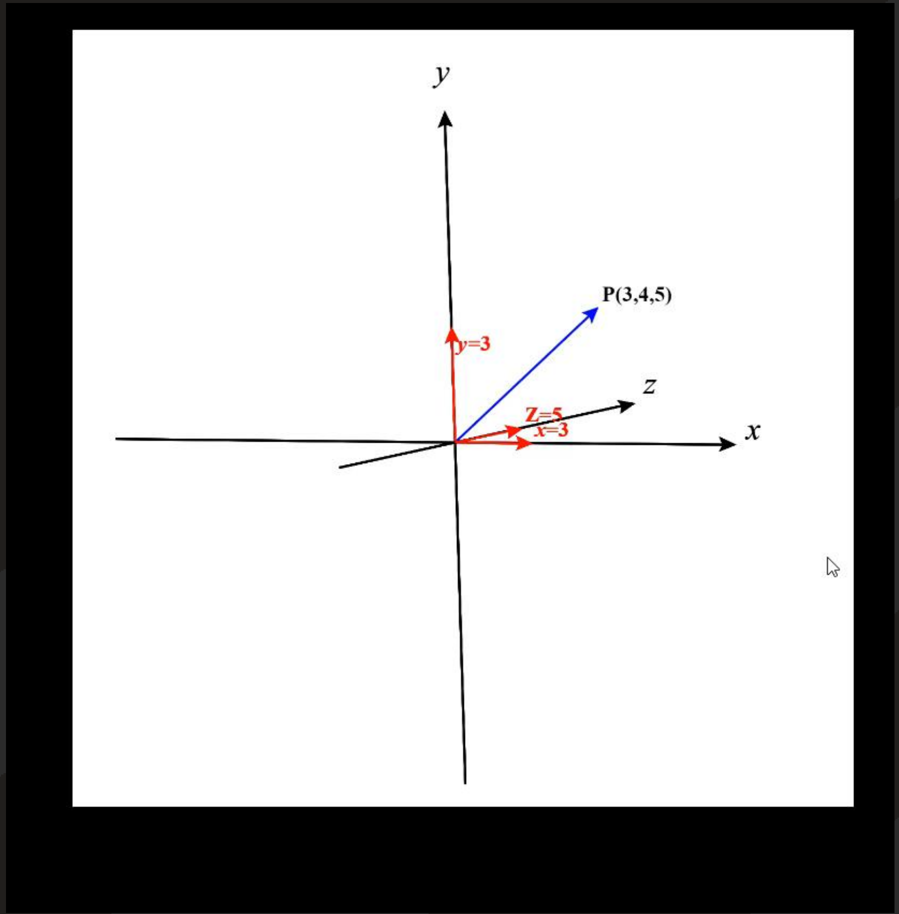
Unit vector

<u>POSITION VECTOR</u>

Position vector of point P is defined as the directed distance from the origin to P

$$\vec{P} = (3,4,5)$$

$$\vec{P} = 3a_x + 4a_y + 5a_z$$



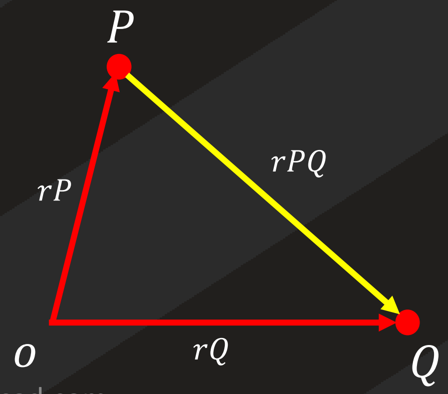
<u>DISTANCE VECTOR</u>

Distance vector (separation vector) is the displacement from one point to another

$$r_P = (P_x, P_y, P_z) \qquad r_Q = (Q_x, Q_y, Q_z)$$

$$r_{PQ} = r_Q - r_P$$

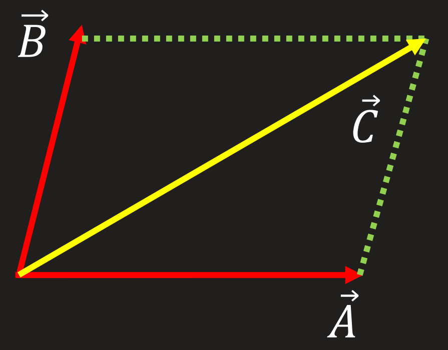
$$= (Q_x - P_x)a_x + \left(Q_y - P_y\right)a_y + (Q_z - P_z)a_z$$



VECTOR ADDITION

$$\vec{A} = (A_x, A_y, A_z)$$

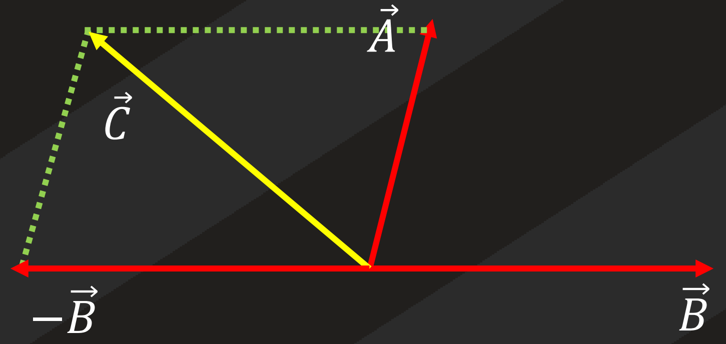
$$\vec{B} = (B_x, B_y, B_z)$$



$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = (A_x + B_x)\hat{a}_x + \left(A_y + B_y\right)\hat{a}_y + (A_z + B_z)\hat{a}_z$$

VECTOR SUBTRACTION



$$\vec{C} = \vec{A} - \vec{B}$$

$$\vec{A} = (A_x, A_y, A_z)$$

$$\vec{B} = (B_x, B_y, B_z)$$

$$\vec{C} = (A_x - B_x)\hat{a}_x + \left(A_y - B_y\right)\hat{a}_y + (A_z - B_z)\hat{a}_z$$

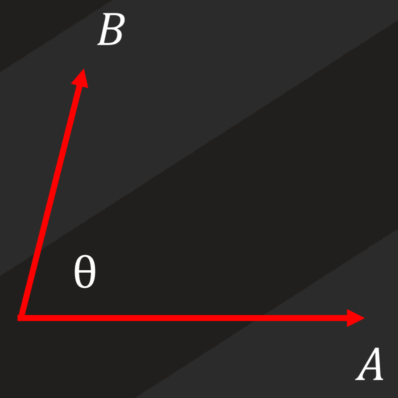
VECTOR MULTIPLICATION

1. *Scalar (or dot) product : A.B*
2. *Vector (or cross) product : A×B*

DOT PRODUCT

*The dot product of two vectors **A** & **B** as the product of the magnitude of A and B and the cosine of the angle between them*

$$\boxed{A \cdot B = AB\cos\theta_{AB}}$$



$$\vec{A} = (A_x, A_y, A_z) \qquad \vec{B} = (B_x, B_y, B_z)$$

$$\boxed{A \cdot B = A_xB_x + A_yB_y + A_zB_z}$$

Properties

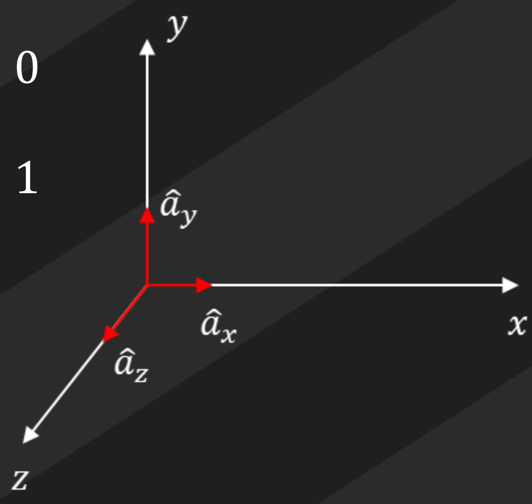
Commutative : $A \cdot B = B \cdot A$

Distributive : $A \cdot (B + C) = A \cdot B + A \cdot C$

Also note that

$$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$$

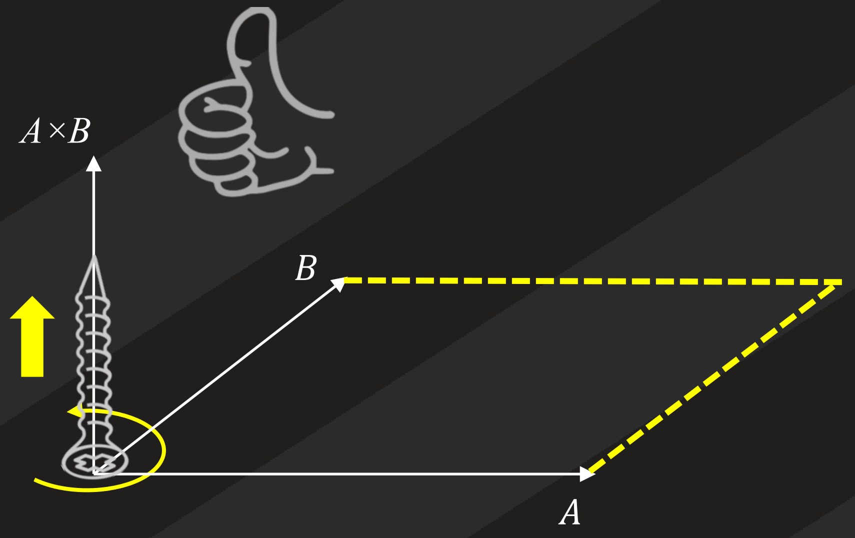
$$a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$$



CROSS PRODUCT

*The cross product of two vectors **A** & **B** is a vector quantity whose magnitude is the area of the parallelogram formed by A and B and is in the direction of advance of a right-handed screw as A is turned into B*

$$\boxed{A \times B = AB \sin\theta_{AB} a_n}$$



$$\vec{A} = (A_x, A_y, A_z) \qquad \vec{B} = (B_x, B_y, B_z)$$

$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\hat{a}_x + (A_z B_x - A_x B_z)\hat{a}_y + (A_x B_y - A_y B_x)\hat{a}_z$$

Properties

Not Commutative : $A \times B \neq B \times A$ $\quad A \times B = -B \times A$

Not Associative : $A \times (B \times C) \neq (A \times B) \times C$

Distributive : $A \times (B + C) = A \times B + A \times C$

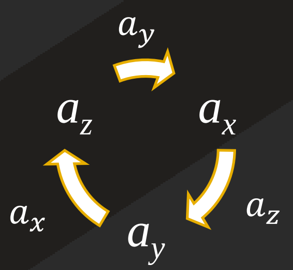
Also note that

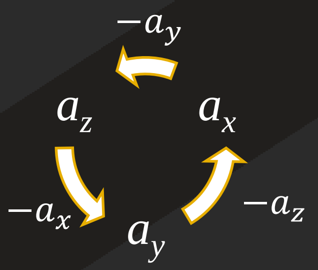
$$A \times A = 0$$

$$a_x \times a_y = a_z$$

$$a_y \times a_z = a_x$$

$$a_z \times a_x = a_y$$





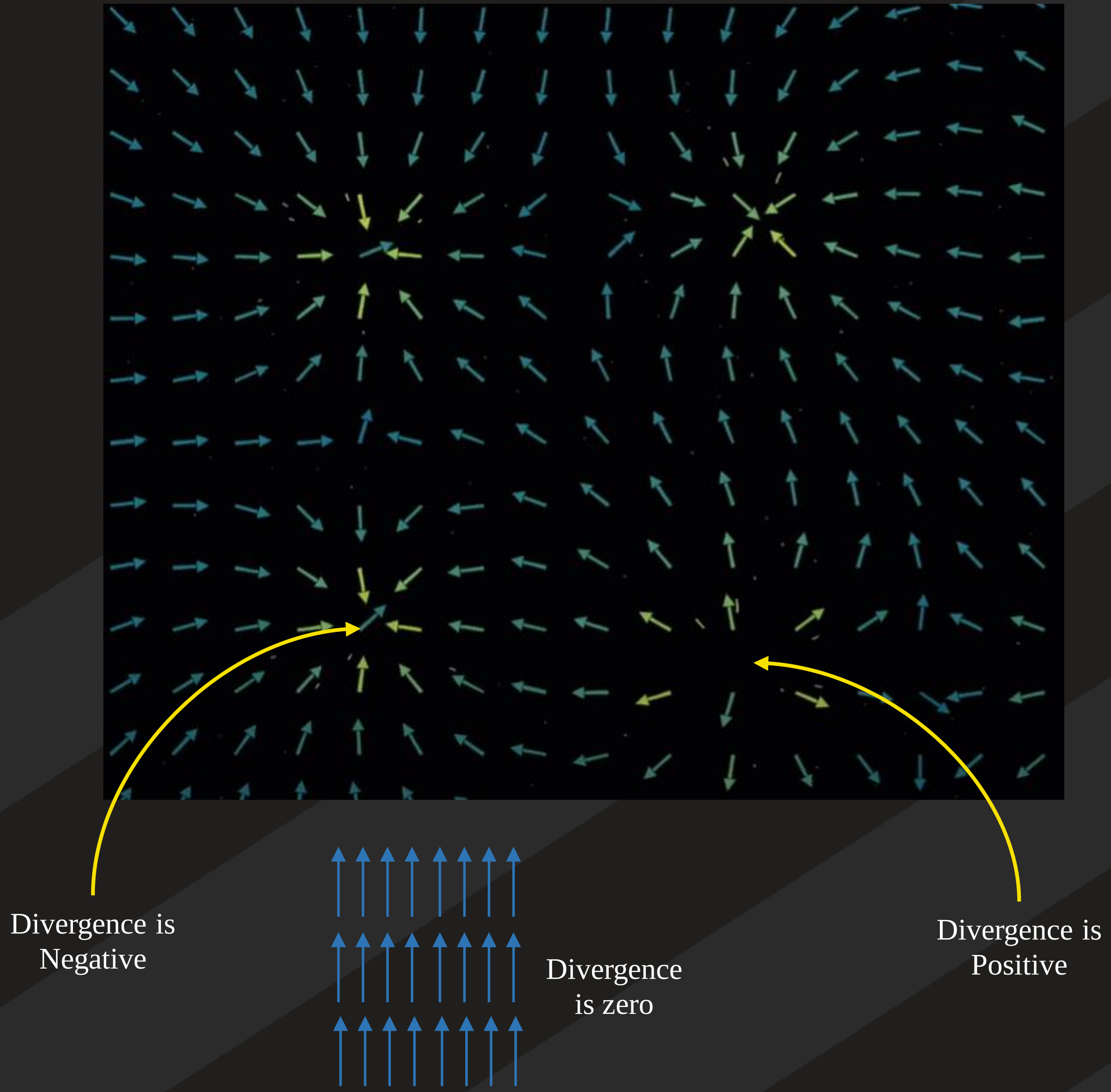
DIVERGENCE

The divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.

"The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P"

Representation

$$\boxed{div\ A = \nabla \cdot A}$$



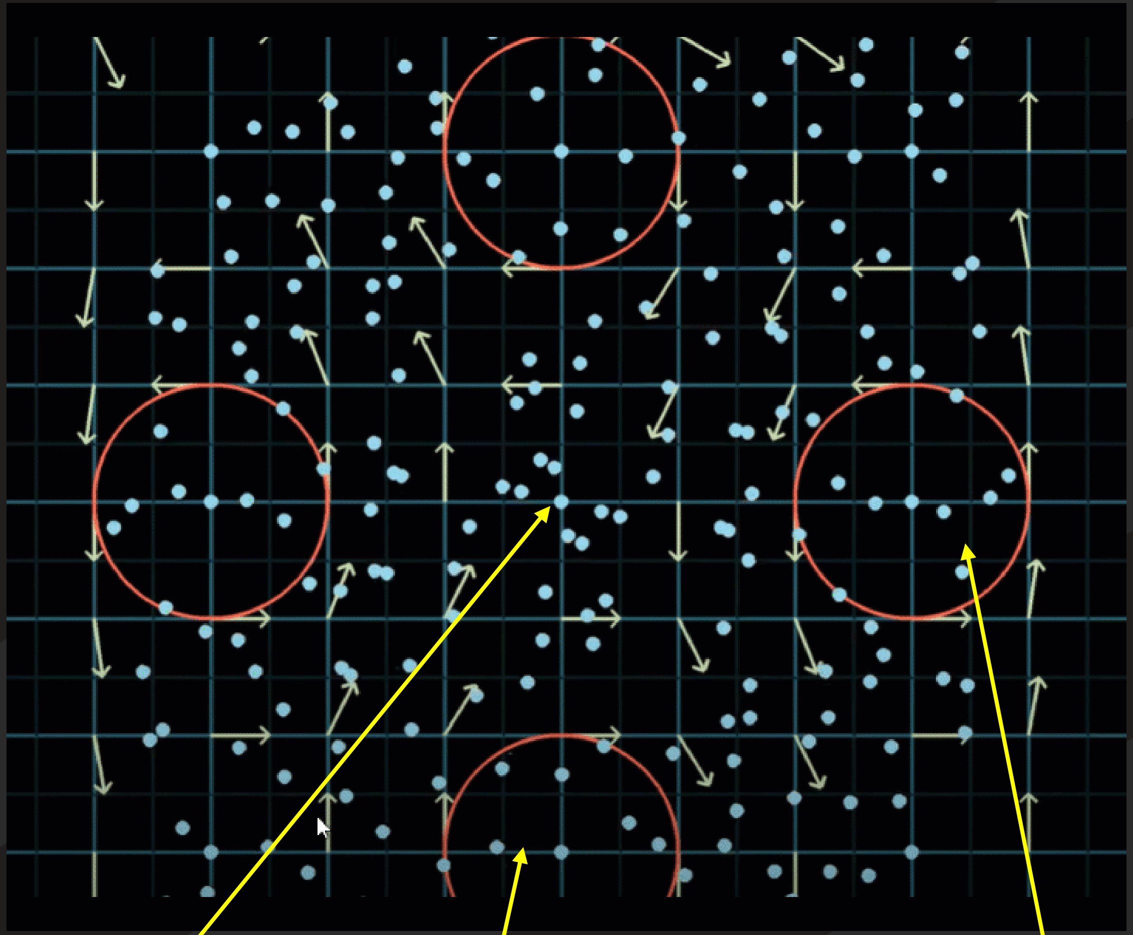
CURL

The curl at a point in the field is represented by a vector whose length and direction denote the magnitude and axis of the maximum circulation. The curl of a field is formally defined as the circulation density at each point of the field.

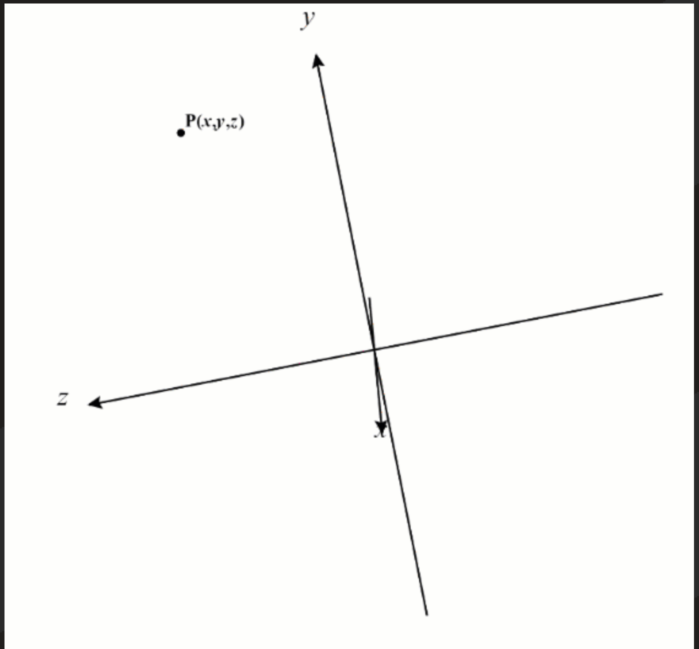
"Curl of a vector A is the axial vector whose magnitude is the maximum circulation of A per unit area as the area tends to zero and the direction is normal"

Representation

$$\boxed{curl\ A = \nabla \times A}$$



CO-ORDINATE SYSTEM

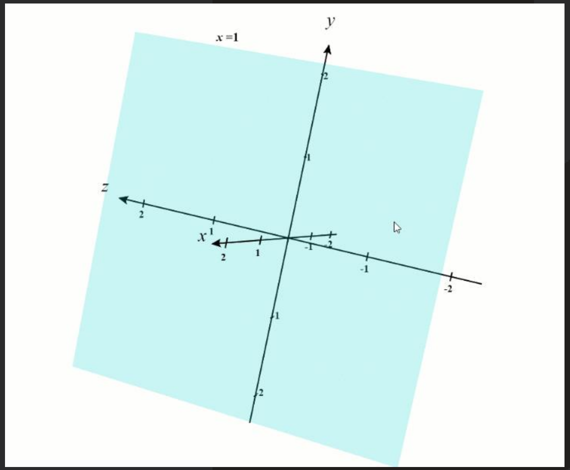


CARTESIAN COORDINATE SYSTEM

A point P in cartesian system can be represented (x,y,z)

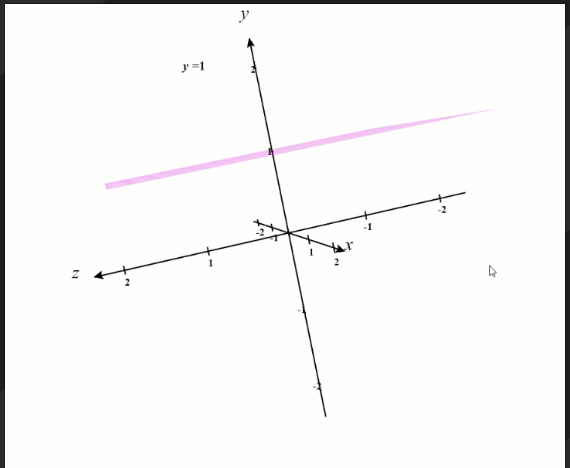
*A vector **A** in cartesian system can be represent as*

$$(A_x, A_y, A_z) \qquad A_x a_x + A_y a_y + A_z a_z$$



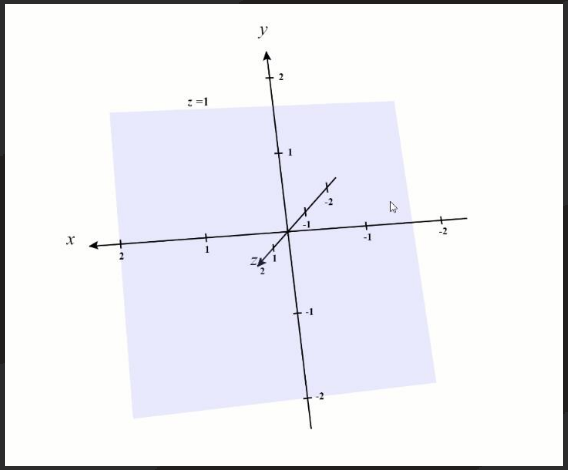
x is constant

(yz – plane)



y is constant

(xz – plane)



z is constant

(xy – plane)

CO-ORDINATE SYSTEM

CYLINDRICAL COORDINATE SYSTEM

Convenient whenever we are dealing with problems having cylindrical symmetry

A point P in cylindrical system can be represented (ρ,φ,z)

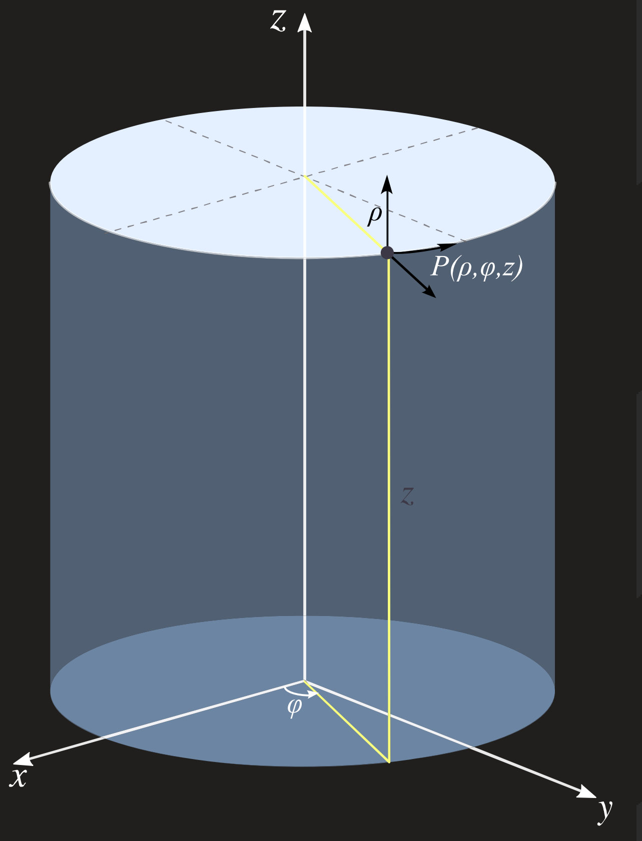
*A vector **A** in cylindrical system can be represent as*

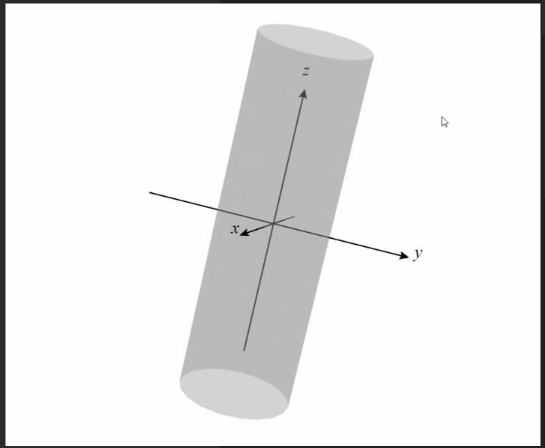
$$\boxed{(A_\rho, A_\phi, A_z) \qquad A_\rho a_\rho + A_\phi a_\phi + A_z a_z}$$

$0 \leq \rho < \infty$

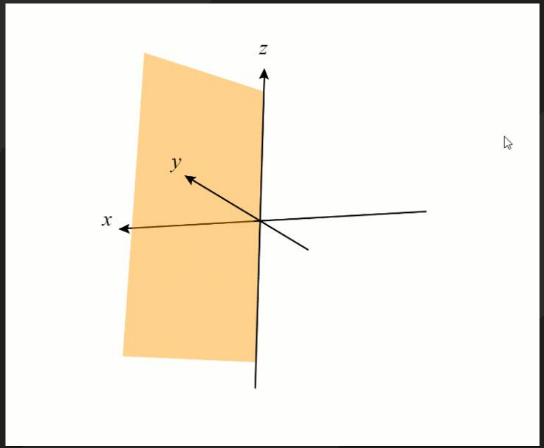
$0 \leq \phi < 2\pi$

$-\infty \leq z < \infty$

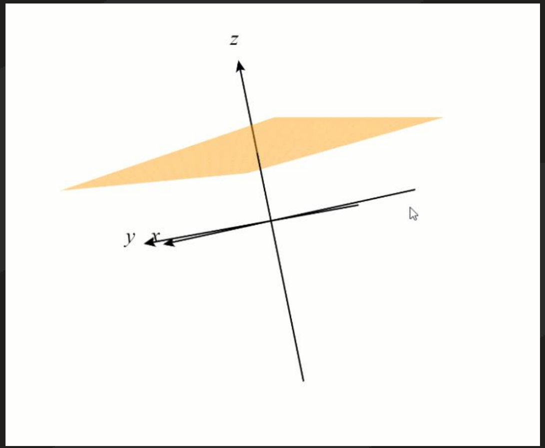




ρ is constant



φ is constant



z is constant

CO-ORDINATE SYSTEM

SPHERICAL COORDINATE SYSTEM

Convenient whenever we are dealing with problems having spherical symmetry

A point P in spherical system can be represented (r,θ,φ)

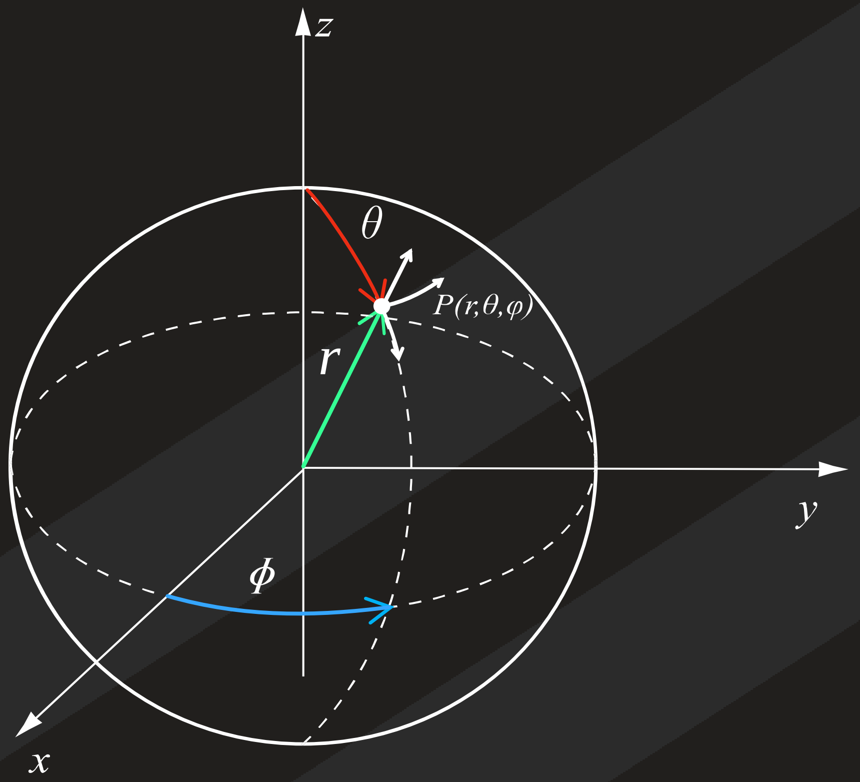
*A vector **A** in spherical system can be represent as*

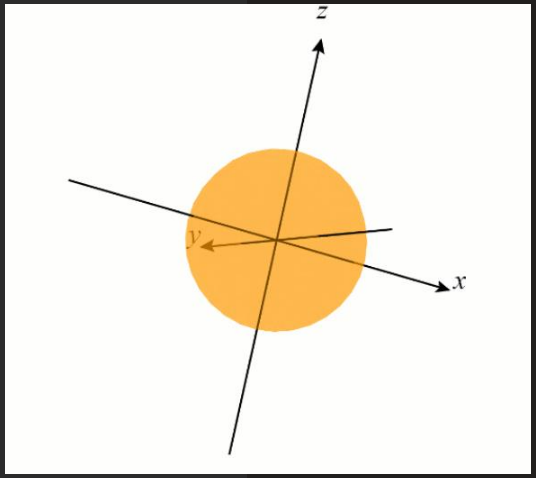
$$(A_r, A_\theta, A_\phi) \qquad A_r a_r + A_\theta a_\theta + A_\phi a_\phi$$

$0 \le r < \infty$

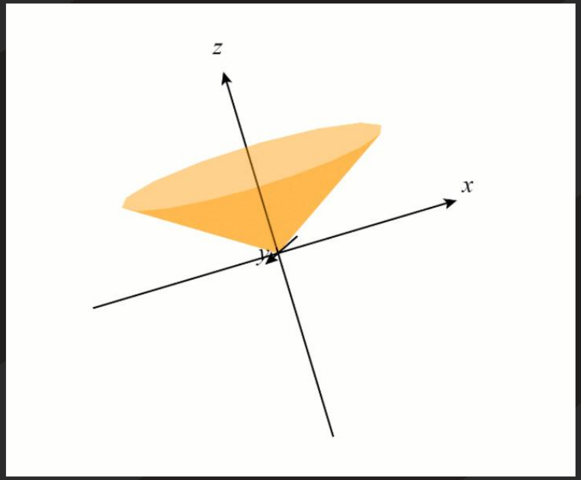
$0 \le \theta < \pi$

$0 \le \phi < 2\pi$

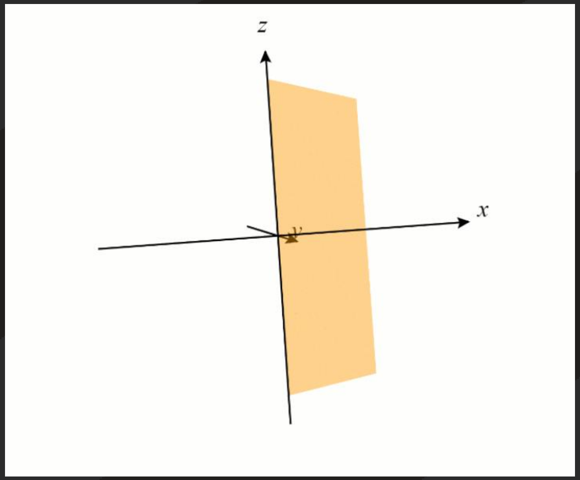




r is constant



θ is constant

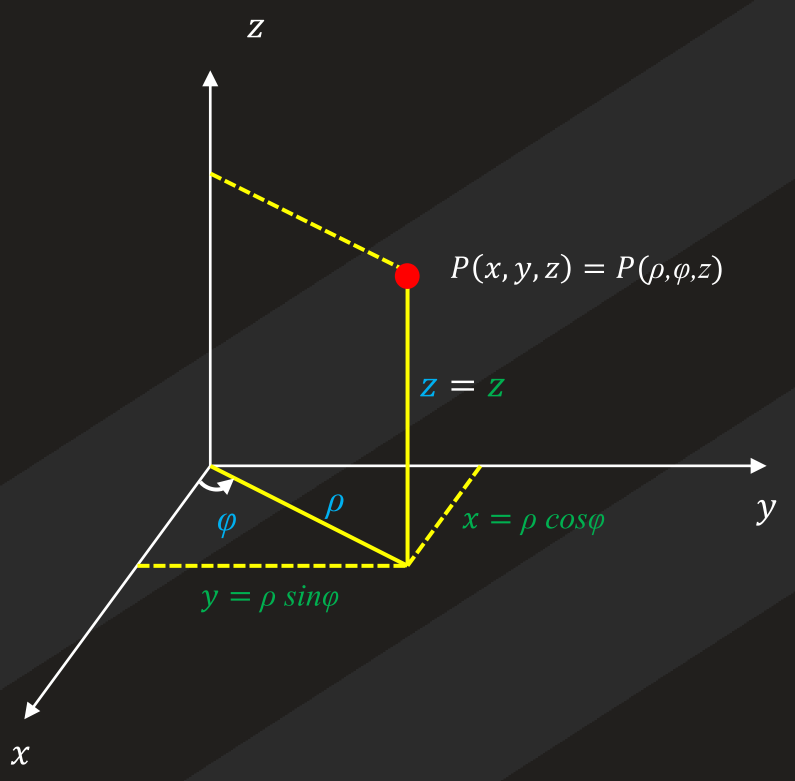


φ is constant

RELATIONSHIP BETWEEN CARTESIAN COORDINATE SYSTEM AND CYLINDRICAL COORDINATE SYSTEM

$$\rho = \sqrt{x^2 + y^2} \qquad \phi = \tan^{-1}\frac{y}{x} \qquad z = z$$

$$x = \rho\ cos\varphi \qquad y = \rho\ sin\varphi \qquad z = z$$



Relationship between unit vectors (a_x, a_y, a_z) & (a_ρ, a_ϕ, a_z)

$$a_x = cos\phi a_\rho - sin\phi a_\varphi \qquad a_\rho = cos\phi a_x + sin\varphi a_y$$

$$a_y = sin\phi a_\rho + cos\phi a_\varphi \qquad a_\varphi = -sin\phi a_x + cos\phi a_y$$

$$a_z = a_z \qquad a_z = a_z$$

Relationship between (A_x, A_y, A_z) and (A_ρ, A_φ, A_z)

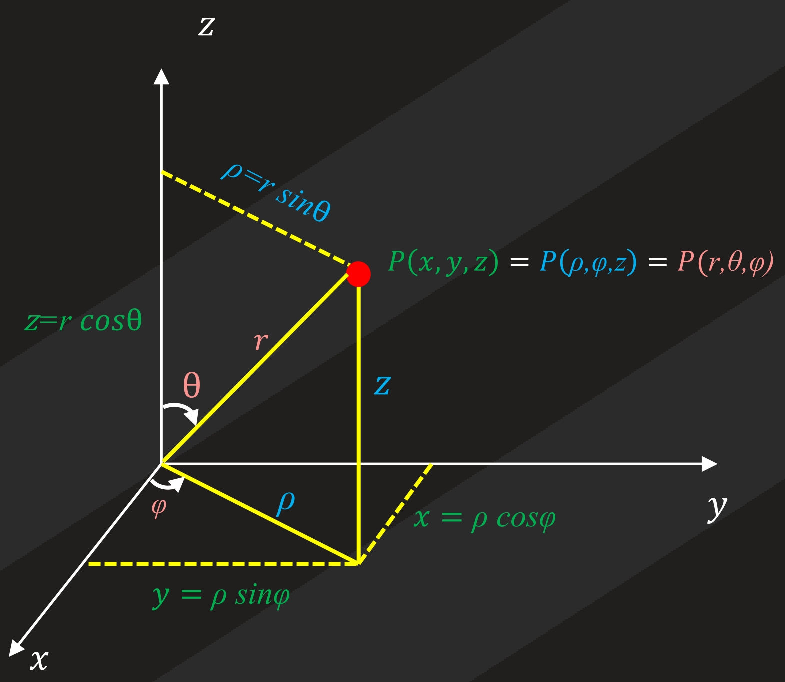
$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} cos\phi & sin\phi & 0 \\ -sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

RELATIONSHIP BETWEEN CARTESIAN COORDINATE SYSTEM AND SPHERICAL COORDINATE SYSTEM

$$r = \sqrt{x^2 + y^2 + z^2} \qquad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\,\sin\theta\,\cos\phi \qquad y = r\,\sin\theta\,\sin\phi \qquad z = r\,\cos\theta$$



Relationship between unit vectors (a_x,a_y,a_z) & (a_r,a_θ,a_φ)

$$a_x = \sin\theta\,\cos\phi\,a_r + \cos\theta\,\cos\phi\,a_\theta - \sin\phi\,a_\phi$$

$$a_y = \sin\theta\,\sin\phi\,a_r + \cos\theta\,\sin\phi\,a_\theta + \cos\phi\,a_\phi$$

$$a_z = \cos\theta\,a_r - \sin\theta\,a_\theta$$

$$a_r = \sin\theta\,\cos\phi\,a_x + \sin\theta\,\sin\phi\,a_y + \cos\theta\,a_z$$

$$a_\theta = \cos\theta\,\cos\phi\,a_x + \cos\theta\,\sin\phi\,a_y - \sin\theta\,a_z$$

$$a_\varphi = -\sin\phi\,a_x + \cos\phi\,a_y$$

Relationship between (A_x,A_y,A_z) and (A_ρ,A_φ,A_z)

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta\,\cos\phi & \sin\theta\,\sin\phi & \cos\theta \\ \cos\theta\,\cos\phi & \cos\theta\,\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\,\cos\phi & \cos\theta\,\cos\phi & -\sin\phi \\ \sin\theta\,\sin\phi & \cos\theta\,\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

VECTOR CALCULUS

DIFFERENTIAL LENGTH, AREA AND VOLUME

Cartesian coordinate system

Differential displacement

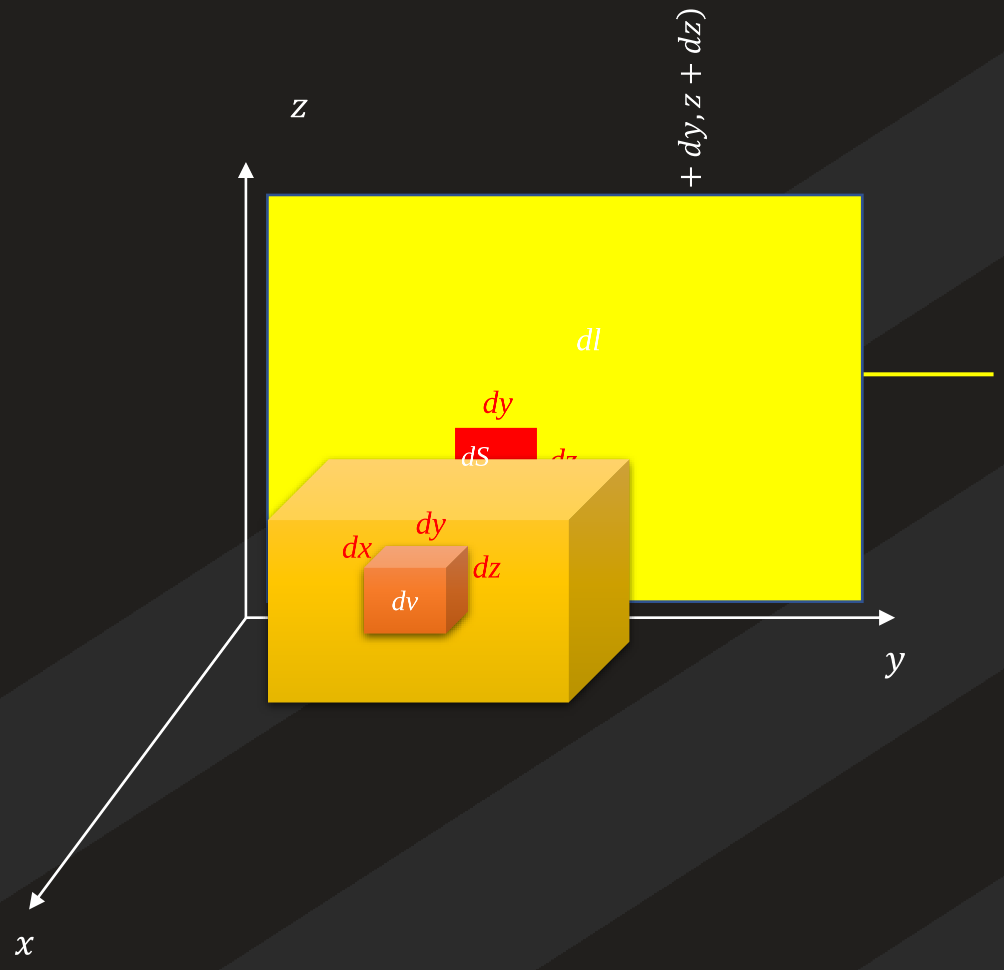
$$dl = dx\, a_x + dy\, a_y + dz\, a_z$$

Differential surface area

$$\begin{aligned} dS &= dydza_x \\ &= dxdza_y \\ &= dxdya_z \end{aligned}$$

Differential volume

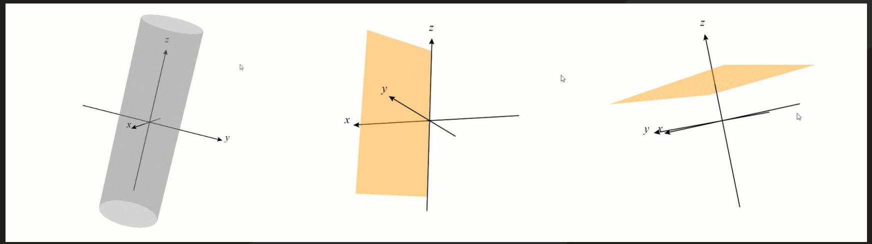
$$dv = dxdydz$$

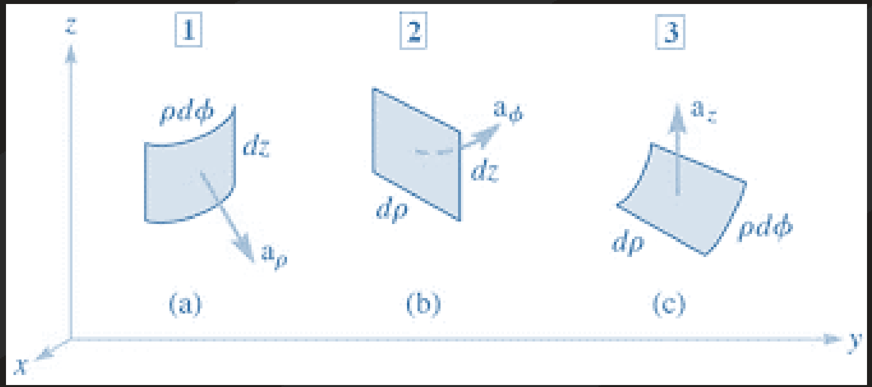


VECTOR CALCULUS

DIFFERENTIAL LENGTH, AREA AND VOLUME

Cylindrical coordinate system





Differential displacement

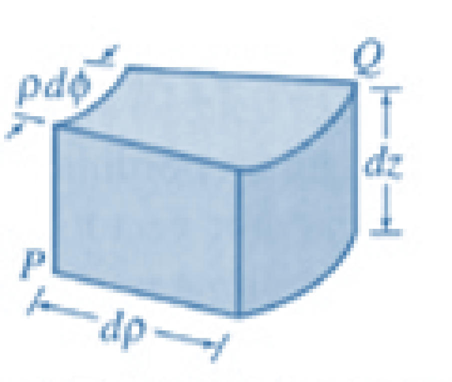
$$dl = d\rho\ a_\rho + \rho\ d\phi a_\varphi + dz\ a_z$$

Differential surface area

$$\begin{aligned} dS &= \rho\ d\phi dz\ a_\rho \\ &= d\rho\ dz\ a_\phi \\ &= \rho\ d\rho\ d\phi a_z \end{aligned}$$

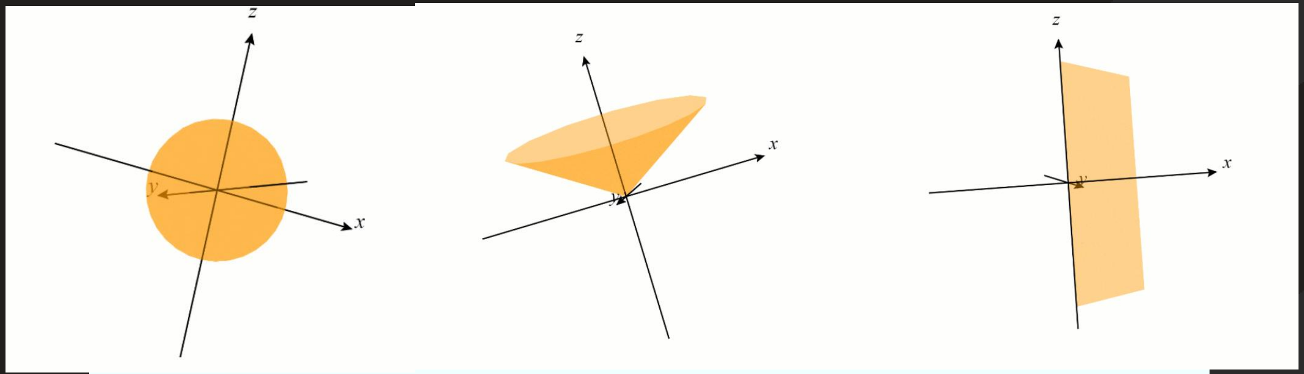
Differential volume

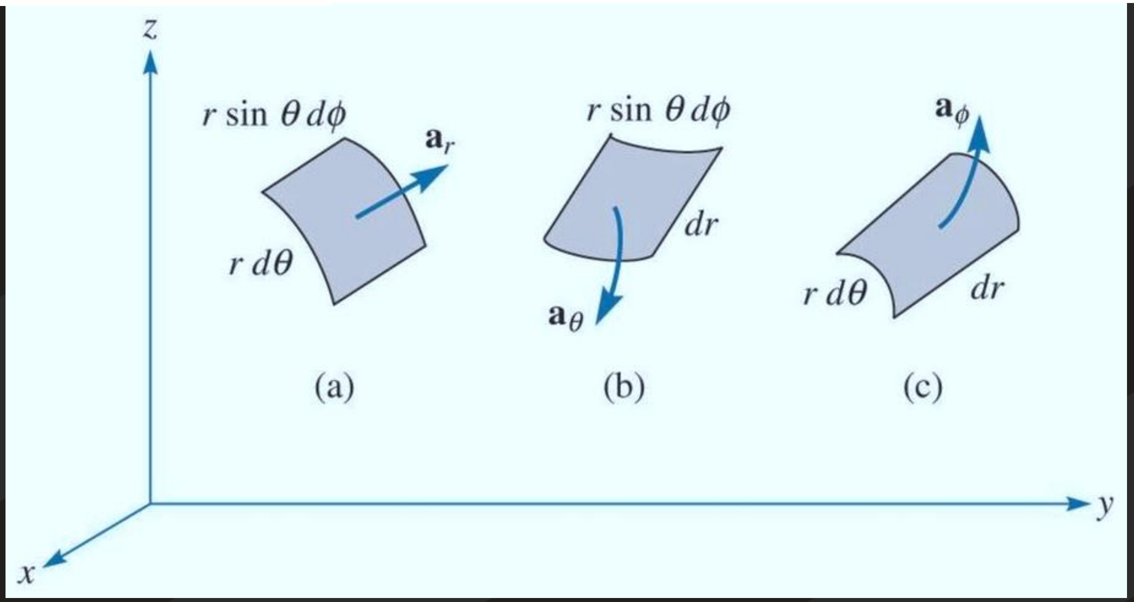
$$dv = \rho\ d\rho\ d\phi dz$$



DIFFERENTIAL LENGTH, AREA AND VOLUME

Spherical coordinate system





Differential displacement

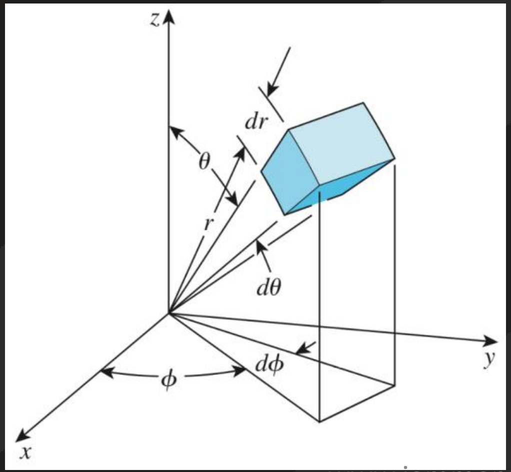
$$\boxed{dl = dr\, a_\rho + r\, d\theta\, a_\theta + r sin\theta\, d\phi a_\phi}$$

Differential surface area

$$\boxed{\begin{aligned} dS &= r^2\, sin\theta\, d\theta\, d\phi\, a_r \\ &= r\, sin\theta\, dr\, d\phi a_\theta \\ &= r\, dr\, d\theta\, a_\phi \end{aligned}}$$

Differential volume

$$\boxed{dv = r^2\, sin\theta\, dr\, d\theta\, d\phi}$$



LINE,SURFACE AND VOLUME INTEGRAL

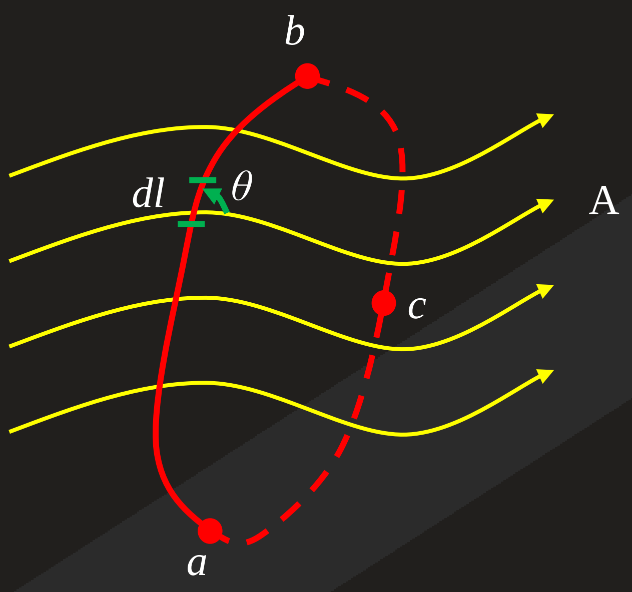
Line integral

$$\boxed{\int_L A.dl = \int_a^b |A| cos\theta}$$

If the path of integration is a closed curve

$$\boxed{\oint_L A.dl}$$

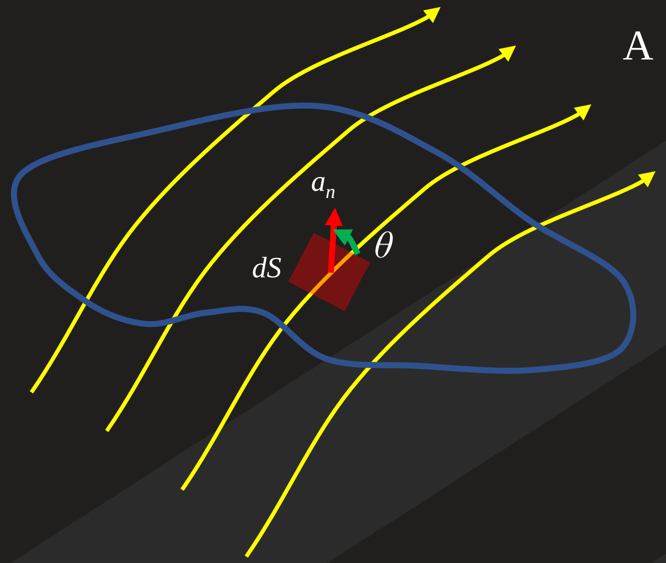
A closed contour integral



LINE,SURFACE AND VOLUME INTEGRAL

Surface integral or flux

Given a vector field A, continuous in a region containing the smooth surface S, the surface integral or flux of A through S is



$$\boxed{\int_s A.a_n\, dS \;=\; \int_s |A|\, cos\theta\, dS}$$

or

$$\boxed{\Psi = \int_s A.dS}$$

For closed surface

$$\boxed{\oint_s A.dS}$$

Net outward flux of A from S

Volume integral

We saw closed path define an open surface where as a closed surface define a volume

$$\boxed{\int_v \rho_v \, dv}$$

DEL OPERATOR

- Del operator is one of the important mathematical tool of vector algebra
- The del operator, written in ∇, is the vector differential operator
- Also known as *gradient operator*
  - Gradient of a scalar V → ∇V
  - The divergence of a vector A, → $\nabla \cdot A$
  - The curl of a vector A, → $\nabla \times A$
  - The Laplacian of a scalar V → $\nabla^2 V$

Cartesian coordinate system

$$\boxed{\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z}$$

Cylindrical coordinate system

$$\boxed{\nabla = \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z}$$

Spherical coordinate system

$$\boxed{\nabla = \frac{\partial}{\partial r} a_r + \frac{1}{r}\frac{\partial}{\partial \theta} a_\theta + \frac{1}{r\,\sin\theta}\frac{\partial}{\partial \phi} a_\phi}$$

GRADIENT OF A SCALAR

- *Generally gradient of a scalar, is an association of a del operator with a scalar quantity*
- *When a scalar V is associated with del operator ∇ given by ∇V and called as gradient of V*
- *The component ∇V in any direction gives the rate of change of V w.r.t. distance along respective direction*

Cartesian coordinate system

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

Cylindrical coordinate system

$$\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z$$

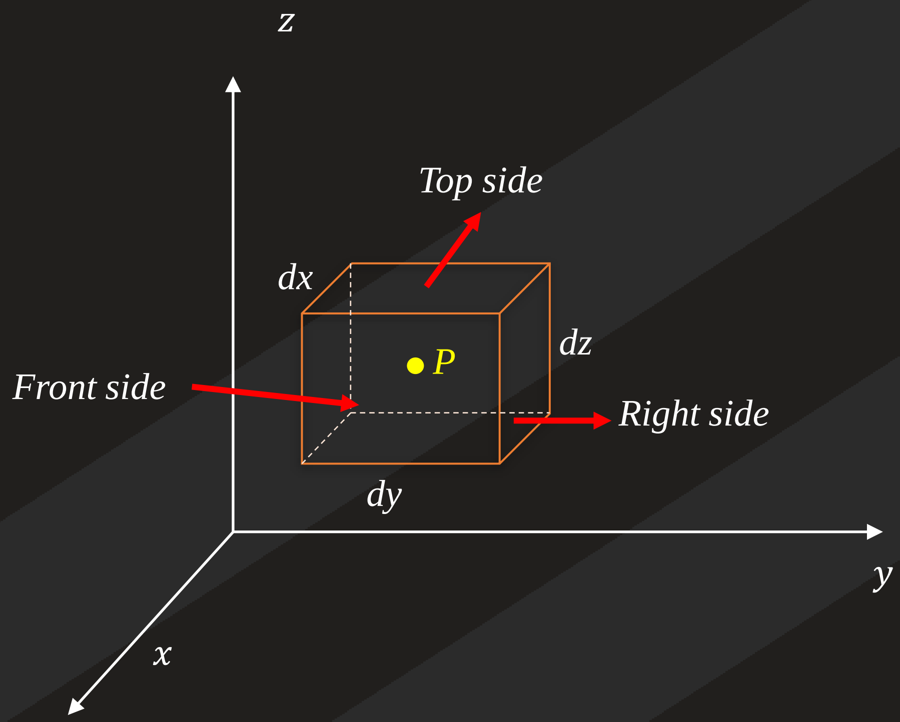
Spherical coordinate system

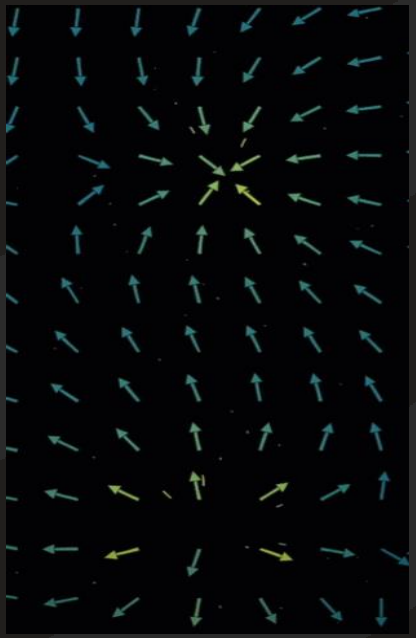
$$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\,sin\theta}\frac{\partial V}{\partial \phi} a_\phi$$

<u>DIVERGENCE OF A VECTOR</u>

The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P

$$div\ A = \nabla \cdot A = \lim_{\Delta V \to 0} \frac{\oint_S A.dS}{\Delta V}$$





We already saw that the net flow of flux of a vector field A from a closed surface S is obtained from the integral

$$\boxed{\oint_s A.dS}$$

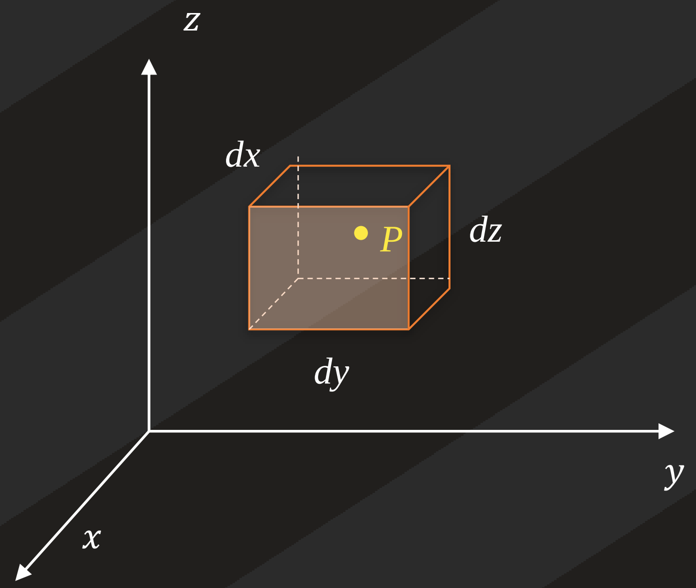
$$\oint_s A.dS = \left(\iint_{front} + \iint_{back} + \iint_{left} + \iint_{right} + \iint_{top} + \iint_{bottom}\right) A.dS \quad \text{-------} \ (1)$$

Taylor series expansion of a function about a point

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \cdots + (x-a)^n\frac{f^{(n)}(a)}{n!}$$

3D Taylor series expansion of A_x about P is

$$A_x(x,y,z) = A_x(x_0,y_0,z_0) + (x-x_0)\left.\frac{\partial A_x}{\partial x}\right|_P + (y-y_0)\left.\frac{\partial A_x}{\partial y}\right|_P + (z-z_0)\left.\frac{\partial A_x}{\partial z}\right|_P + higher\ order\ terms \quad \text{------- (2)}$$



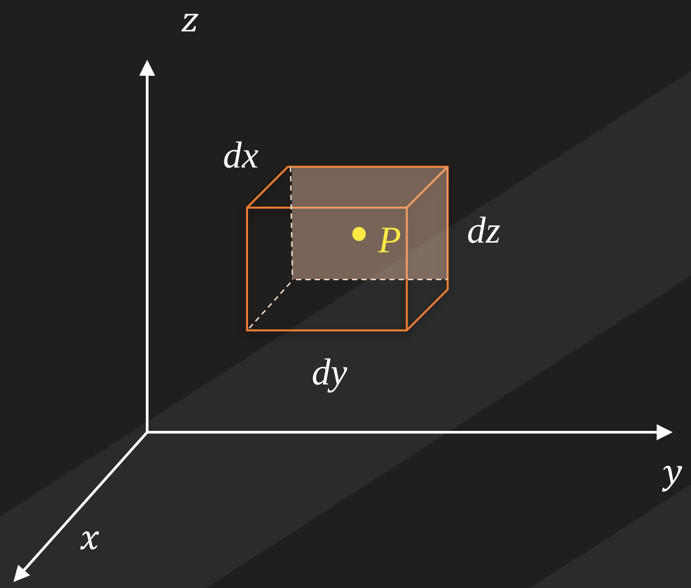
For the Front side

$$x - x_0 = \frac{dx}{2} \qquad ds = dydz\,\hat{a}_x$$

$$\iint_{front} A.\,dS = \left[A_x(x_0,y_0,z_0) + \frac{dx}{2}\left.\frac{\partial A_x}{\partial x}\right|_P\right] dydz\,\hat{a}_x + higher\ terms$$

For the back side

$$x - x_0 = -\frac{dx}{2} \quad ds = dydz \ -\hat{a}_x$$



$$\iint_{back} A.dS = \left[A_x(x_0, y_0, z_0) - \frac{dx}{2}\frac{\partial A_x}{\partial x}\bigg|_P\right] - dydz\,\hat{a}_x + higher\ terms$$

For front + back side

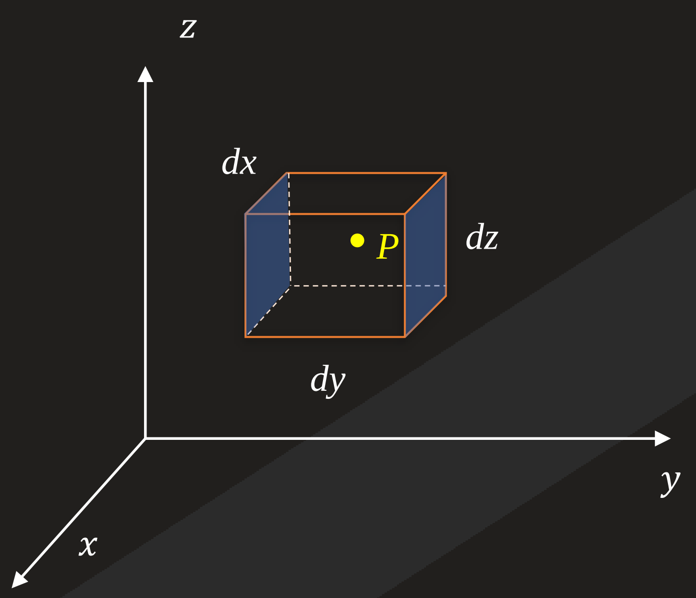
$$\iint_{front} A.dS + \iint_{back} A.dS$$

$$\iint_{front} A.dS = \left[A_x(x_0, y_0, z_0) + \frac{dx}{2}\frac{\partial A_x}{\partial x}\bigg|_P\right] dydz\,\hat{a}_x + higher\ terms \qquad \iint_{back} A.dS = \left[-A_x(x_0, y_0, z_0) + \frac{dx}{2}\frac{\partial A_x}{\partial x}\bigg|_P\right] dydz\,\hat{a}_x + higher\ terms$$

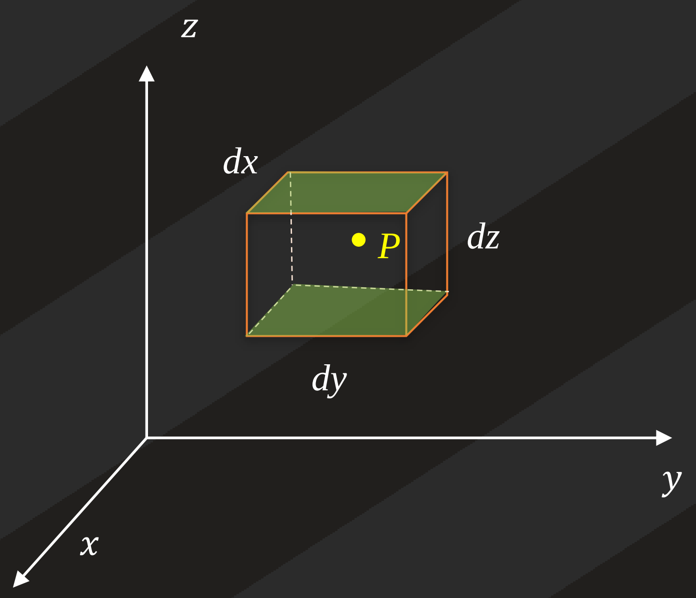
$$= dxdydz\frac{\partial A_x}{\partial x}\bigg|_P + higher\ terms \qquad \text{-------}\ (3)$$

By taking similar steps for left, right and top, bottom

$$\iint_{left} A.dS + \iint_{right} A.dS = dxdydz \left.\frac{\partial A_y}{\partial y}\right|_P + higher\ terms \quad \text{------- (4)}$$



$$\iint_{top} A.dS + \iint_{bottom} A.dS = dxdydz \left.\frac{\partial A_z}{\partial z}\right|_P + higher\ terms \quad \text{------- (5)}$$



Substituting the equation (3),(4) & (5) in (1) we get

$$\oint_S A.dS = \left(\iint_{front} + \iint_{back} + \iint_{left} + \iint_{right} + \iint_{top} + \iint_{bottom}\right) A.dS$$

$$\iint_{front} A.dS + \iint_{back} A.dS = dxdydz \left.\frac{\partial A_x}{\partial x}\right|_P + higher\ terms$$

$$\iint_{left} A.dS + \iint_{right} A.dS = dxdydz \left.\frac{\partial A_y}{\partial y}\right|_P + higher\ terms$$

$$\iint_{top} A.dS + \iint_{bottom} A.dS = dxdydz \left.\frac{\partial A_z}{\partial z}\right|_P + higher\ terms$$

The higher order terms will vanishes when ΔV→0

$$\lim_{\Delta V \to 0} \oint_S A.dS = \left.\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)\right|_P \Delta V$$

$$\boxed{\lim_{\Delta V \to 0} \frac{\oint_S A.dS}{\Delta V} = \nabla \cdot A}$$

Cartesian coordinate system

$$\boxed{\nabla A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

Cylindrical coordinate system

$$\boxed{\nabla A = \frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}}$$

Spherical coordinate system

$$\boxed{\nabla A = \frac{1}{r^2}\frac{\partial r^2 A_x}{\partial r} + \frac{1}{r sin\theta}\frac{\partial A_\theta sin\theta}{\partial \theta} + \frac{1}{r\ sin\theta}\frac{\partial A_\phi}{\partial \phi}}$$

DIVERGENCE THEOREM

"The divergence theorem states that the total outward flux of a vector field A through the closed surface S is the same as the volume integral of the divergence of A"

$$\boxed{\oint_S A.dS = \int_V \nabla \cdot A \, dv}$$

Subdivide volume v into a large number of small cells. If the kth cell has volume Δv_k and is bounded by surface S_k

$$\oint_S A.dS = \sum_k \oint_{S_k} A.dS = \sum_k \frac{\oint_{S_k} A.dS}{\Delta v_k} \Delta v_k$$

$$\boxed{\lim_{\Delta V \to 0} \frac{\oint_S A.dS}{\Delta V} = \nabla \cdot A}$$

$$\oint_S A.dS = \int_V \nabla \cdot A \, dv$$

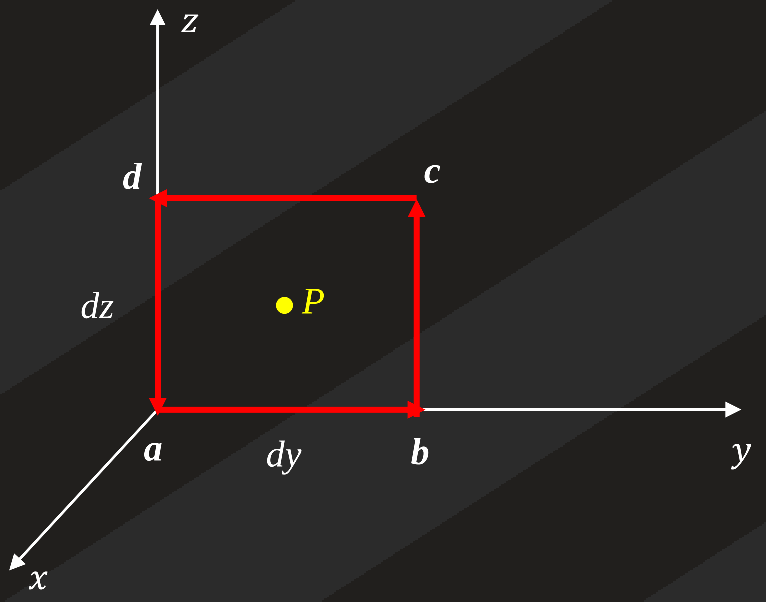
This theorem applies to any volume v bounded by the closed surface S, provided that A and $\nabla.A$ are continuous in the region

CURL OF A VECTOR

"The curl of A is an axial (or rotational) vector whose magnitude is the maximum circulation of A per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum"

$$curl\ A = \nabla \times A = \lim_{\Delta S \to 0} \left(\frac{\oint_L A.dl}{\Delta S} \right) a_n$$

Circulation of a vector field across a closed path is equivalent to the line integral of vector field over that path



$$Circulation\ of\ a\ vector\ field\ A = \oint_L A.dl$$

$$\oint_L A.dl = \left(\int_{ab} + \int_{bc} + \int_{cd} + \int_{da}\right) A.dl \qquad \text{------- (1)}$$

Taylor series expansion of a function about a point

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \cdots + (x-a)^n\frac{f^{(n)}(a)}{n!}$$

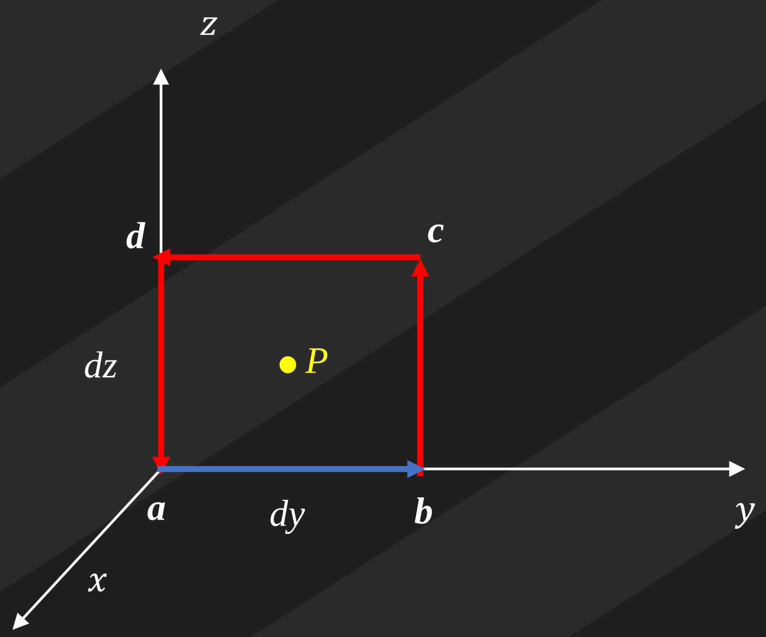
3D Taylor series expansion of A_x about P is

$$A_x(x,y,z) = A_x(x_0,y_0,z_0) + (x-x_0)\left.\frac{\partial A_x}{\partial x}\right|_P + (y-y_0)\left.\frac{\partial A_x}{\partial y}\right|_P + (z-z_0)\left.\frac{\partial A_x}{\partial z}\right|_P + higher\ order\ terms$$

<u>Section ab</u>

$$z - z_0 = \frac{dz}{2} \qquad dl = dy\hat{a}_y$$

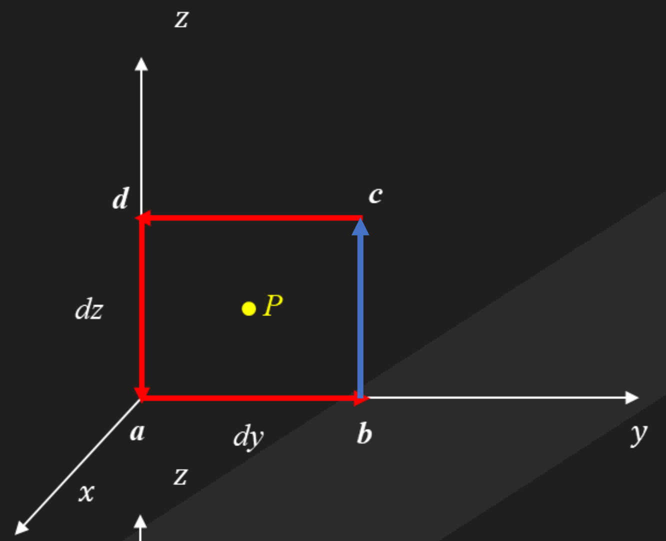
$$\int_{ab} A.dl = \left[A_y(x_0,y_0,z_0) + \frac{dz}{2}\left.\frac{\partial A_y}{\partial z}\right|_P\right]dy \qquad \text{------- (2)}$$



Section bc

$$y - y_0 = \frac{dy}{2} \qquad dl = dz\hat{a}_z$$

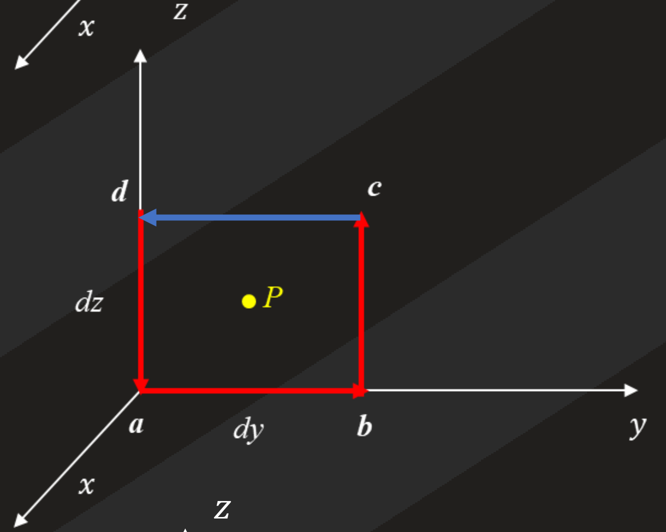
$$\int_{bc} A.dl = \left[A_z(x_0, y_0, z_0) + \frac{dy}{2}\frac{\partial A_z}{\partial y}\bigg|_P\right] dz \qquad \text{------- (3)}$$



Section cd

$$z - z_0 = -\frac{dz}{2} \qquad dl = -dy\hat{a}_y$$

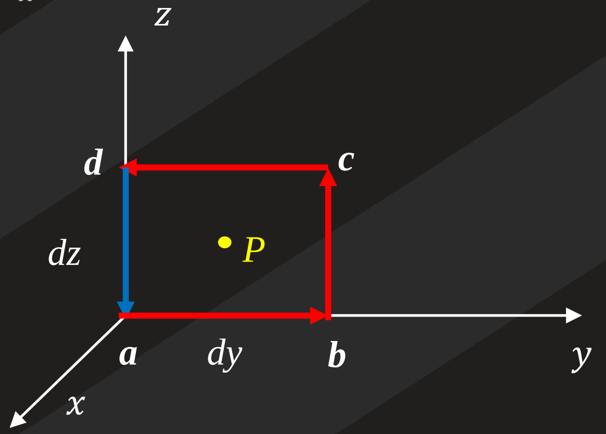
$$\int_{cd} A.dl = \left[A_y(x_0, y_0, z_0) - \frac{dz}{2}\frac{\partial A_y}{\partial z}\bigg|_P\right] - dy \qquad \text{------- (4)}$$



Section da

$$y - y_0 = -\frac{dy}{2} \qquad dl = -dz\hat{a}_z$$

$$\int_{bc} A.dl = \left[A_z(x_0, y_0, z_0) - \frac{dy}{2}\frac{\partial A_z}{\partial y}\bigg|_P\right] - dz \qquad \text{------- (5)}$$



$$\oint_L A.dl = \left(\int_{ab} + \int_{bc} + \int_{cd} + \int_{da}\right) A.dl \quad \text{------- (1)}$$

$$\int_{ab} A.dl = \left[A_y(x_0, y_0, z_0) + \frac{dz}{2} \frac{\partial A_y}{\partial z}\Big|_P \right] dy \quad \text{------- (2)}$$

$$\int_{bc} A.dl = \left[A_z(x_0, y_0, z_0) + \frac{dy}{2} \frac{\partial A_z}{\partial y}\Big|_P \right] dz \quad \text{------- (3)}$$

$$\int_{cd} A.dl = \left[A_y(x_0, y_0, z_0) - \frac{dz}{2} \frac{\partial A_y}{\partial z}\Big|_P \right] - dy \quad \text{------- (4)}$$

$$\int_{bc} A.dl = \left[A_z(x_0, y_0, z_0) - \frac{dy}{2} \frac{\partial A_z}{\partial y}\Big|_P \right] - dz \quad \text{------- (5)}$$

Since $\Delta S = dydz$ and $\Delta S \to 0$

$$\lim_{\Delta S \to 0} \oint_L A.dl = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right] \Delta S$$

$$(curl\ A)_x = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right]$$

Similarly the y- and z components of the curl of A can be found in the same way

$$(curl\ A)_y = \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right] \qquad (curl\ A)_z = \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right]$$

$$curl\ A = \nabla \times A = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right] a_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right] a_y + \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right] a_z$$

$$\boxed{\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}}$$

Cartesian coordinate system

Cartesian coordinate system

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Cylindrical coordinate system

$$\nabla \times A = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

Spherical coordinate system

$$\nabla \times A = \frac{1}{r^2 \sin\theta}\begin{vmatrix} a_r & r a_\theta & a_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & r A_\theta & r sin\theta A_\phi \end{vmatrix}$$

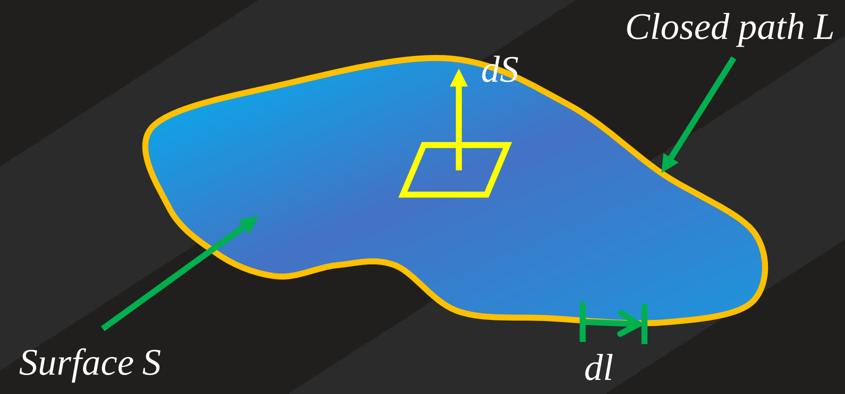
Properties of Curl

- The curl of a vector field is another vector field
- $\nabla \times (A + B) = \nabla \times A + \nabla \times B$
- $\nabla (A \times B) = A (\nabla . B) – B (\nabla . A) + (B . \nabla A) – (A . \nabla)B$
- $\nabla \times (V A) = \nabla V \times A + \nabla V \times A$
- The divergence of the curl of a vector field vanishes; i.e. $\nabla . (\nabla \times A) = 0$
- The curl of the gradient of a scalar field vanishes; i.e. $\nabla \times \nabla V = 0$

STOKES'S THEOREM

"Stokes's theorem states that the circulation of a vector filed A around a closed path L is equal to the surface integral of the curl of A over the open surface S bounded by L, provided A and ∇×A are continuous on S"

$$\oint_L A.dl = \int_S (\nabla \times A)\, dS$$



The surface S is subdivided into a large number of cells. If k^{th} cell has surface area Δs_k and bounded by path L_k

$$\oint_L A.dl = \sum_k \oint_{L_k} A.dl = \sum_k \frac{\oint_{L_k} A.dl}{\Delta S_k} \Delta S_k$$

$$\oint_L A.dl = \int_S (\nabla \times A)\, dS$$

Note: the Divergence theorem relates a surface integral to a volume integral, where as Stokes's theorem relates a line integral to a surface integral

LAPLACIAN (∇^2)

The Laplacian of a scalar field V, written as $\nabla^2 V$, is the divergence of the gradient of V

Cartesian coordinate system

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate system

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Cylindrical coordinate system

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

A scalar field is said to be harmonic in a regular manner if its Laplacian vanishes in that region

$$\nabla^2 V = 0$$

Laplace's Equation

The vector Laplacian is given by $\nabla^2 A$, it works on vector and gives the result in vector form also and can be find as

$$\nabla^2 A = \nabla(\nabla . A) - \nabla \times \nabla \times A$$

ELECTROSTATICS

- The branch of science under which we study the effects of electric charge at rest is called electrostatics
- Whole mechanism of electrostatics is arises only due to the force applied by the electric charges on each other

Electric charge

- Charge is a scalar quantity
- Charge is always considered with mass
- A body said to be positive or negatively charged either it has lack of or excess of electrons
- Charges can be transferred from one body to another depending upon their charging status
- The phenomenon of charge transfer in two bodies in contact is called as conduction.
- Charge represented by ‘q’
- SI unit of charge is ‘Coulomb’ (1 Coulomb = 1 amp-sec)

COULOMB'S LAW

Coulomb's law *states that the force F between two point charges Q_1 & Q_2 is*

1. *Along the line joining them*
2. *Directly proportional to the product Q_1Q_2 of the charges*
3. *Inversely proportional to the square of the distance R between them*

$$\boxed{F = \frac{kQ_1Q_2}{R^2}}$$

Q_1 Q_2 in coulombs (C)

R in meters (m)

F in Newtons (N)

$$\boxed{F = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}}$$

Where k is the proportionality constant

$$k = \frac{1}{4\pi\varepsilon_0} \qquad \varepsilon_0 \rightarrow permittivity\ of\ free\ space\left(\frac{Farads}{meter}\right)$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \approx \frac{10^{-9}}{36\pi} F/m$$

$$\boxed{k = \frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9 m/F}$$

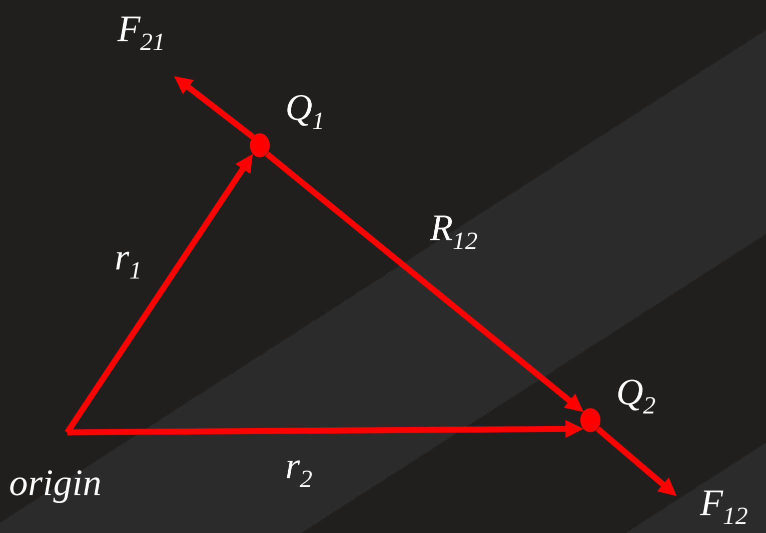
COULOMB'S LAW

$$\boxed{F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} a_{R_{12}}}$$

$$\boldsymbol{R}_{12} = r_2 - r_1$$

$$R = |R_{12}|$$

$$a_{R_{12}} = \frac{\boldsymbol{R}_{12}}{R}$$



$$F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \frac{\boldsymbol{R}_{12}}{R} = \frac{Q_1 Q_2 \boldsymbol{R}_{12}}{4\pi\varepsilon_0 R^3}$$

$$F_{12} = \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi\varepsilon_0 |r_2 - r_1|^3}$$

If we have more than two point charge, we can use principle of superposition to determine the force on a particular charge, if there are N charges,

$$F = \frac{Q Q_1 (r - r_1)}{4\pi\varepsilon_0 |r - r_1|^3} + \frac{Q Q_2 (r - r_2)}{4\pi\varepsilon_0 |r - r_2|^3} + \cdots + \frac{Q Q_N (r - r_N)}{4\pi\varepsilon_0 |r - r_N|^3}$$

$$\boxed{F = \frac{Q}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k (r - r_k)}{|r - r_k|^3}}$$

Electric field intensity or electric filed strength (E)

Electric field intensity E is the force per unit charge when placed in an electric field

$$E = \lim_{Q\to 0} \frac{F}{Q} \; or \; E = \frac{F}{Q}$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} a_R = \frac{Q(r - r')}{4\pi\varepsilon_0 |r - r'|^3}$$

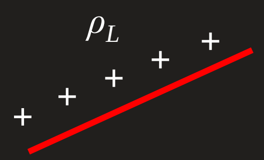
For N point charges

$$E = \frac{Q_1(r - r_1)}{4\pi\varepsilon_0 |r - r_1|^3} + \frac{Q_2(r - r_2)}{4\pi\varepsilon_0 |r - r_2|^3} + \cdots + \frac{Q_N(r - r_N)}{4\pi\varepsilon_0 |r - r_N|^3}$$

$$E = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k(r - r_k)}{|r - r_k|^3}$$

Electric field due to continuous charge distribution

Point charge

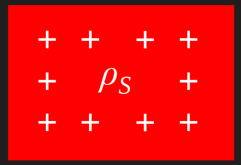


Line charge

$$dQ = \rho_L dl$$

$$Q = \int_L \rho_L dl$$

$$E = \int_L \frac{\rho_L dl}{4\pi\varepsilon_0 R^2} a_R$$

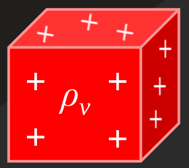


Surface charge

$$dQ = \rho_s dS$$

$$Q = \int_S \rho_S dS$$

$$E = \int_S \frac{\rho_S dS}{4\pi\varepsilon_0 R^2} a_R$$



Volume charge

$$dQ = \rho_v dv$$

$$Q = \int_v \rho_v dv$$

$$E = \int_v \frac{\rho_v dv}{4\pi\varepsilon_0 R^2} a_R$$

Electric flux density (D)

The flux due to the electric field E can be calculated by using the general flux equation we saw before

$$\Psi = \int_s A.\,dS$$

But for practical reasons this quality is not usually considered to be most useful

Electric field intensity is depending on the medium in which the charge is placed

The vector field D is defined as

$$\boxed{D = \varepsilon_0 E}$$

And the electric flux Ψ in terms of D

$$\boxed{\Psi = \int_S D.\,ds}$$

Electric flux density (D)

$$\boxed{D = \varepsilon_0 E}$$

GAUSS'S LAW

Gauss's law states that the total electric flex Ψ through any closed surface is equal to the total charge enclosed by the surface

$$\Psi = Q_{enc}$$

$$\Psi = \int_S D.ds$$

$$\Psi = \oint_s d\Psi = \oint_s D.ds$$

$$Q_{enc} = \int_v \rho_v \, dv$$

where ρ_v is the volume charge density

$$\oint_s D.ds = \int_v \rho_v \, dv$$

Applying divergence theorem

$$\oint_S A.dS = \int_V \nabla \cdot A \, dv$$

$$\int_V \nabla \cdot D \, dv = \int_v \rho_v \, dv$$

$$\boxed{\nabla \cdot D = \rho_v}$$

first Maxwell's equation

Maxwell's equation states that the volume charge density is the same as the divergence of the electric flux density

AMPERE'S CIRCUIT LAW

Amper's circuit law states that the line integral of the tangential components of H around a closed path is the same as the net current I_{enc} by the path

$$\oint_L H.dl = I_{enc}$$

$$I_{enc} = \int_s J\,ds$$

where J is the volume current density

Applying Stokes's theorem in LHS

$$\oint_L H.dl = \int_s (\nabla \times H)\,ds$$

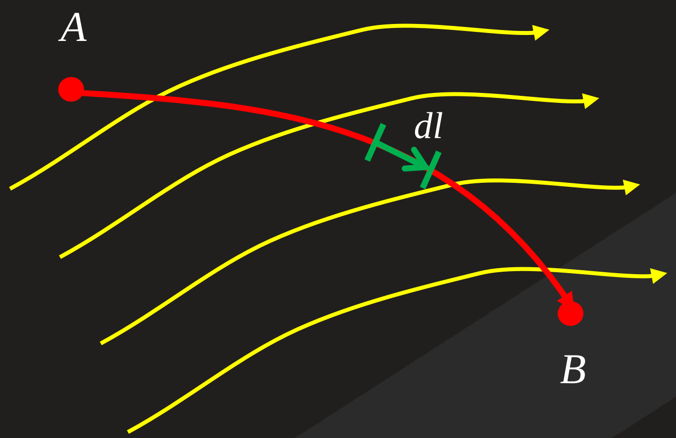
$$\boxed{\oint_L A.dl = \int_S (\nabla \times A)\,dS}$$

$$\int_s (\nabla \times H)\,ds = \int_s J\,ds$$

$$\boxed{\nabla \times H = J}$$

Third Maxwell's equation

POTENTIAL DIFFERENCE (V)



From Coulomb's law, the force on Q is **F**=**QE** *so the work done in displacing the charge by dl is*

$$dw = -\boldsymbol{F}.\,d\boldsymbol{l} = -Q\boldsymbol{E}.\,d\boldsymbol{l}$$

The total work done

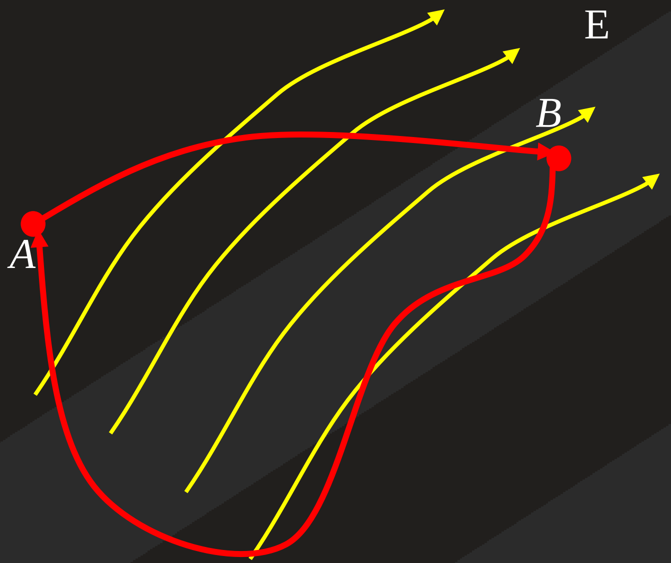
$$\boxed{W = -Q\int_A^B E.\,dl}$$

Dividing W by Q gives the potential energy per unit charge

$$\boxed{V_{AB} = \frac{W}{Q} = -\int_A^B E.\,dl}$$

RELATIONSHIP BETWEEN E & V

The potential difference between point A & B is independent of path taken



$$V_{BA} = -V_{AB}$$

$$V_{BA} + V_{AB} = 0$$

$$\boxed{\oint_L E.dl = 0}$$

Applying Stokes's theorem

$$\oint_L E.dl = \int_S (\nabla \times E)\, dS = 0$$

$$\boxed{\nabla \times E = 0}$$

Second Maxwell's equation for static electric filed

$$\boxed{\oint_L A.dl = \int_S (\nabla \times A)\, dS}$$

From the way we define potential

$$V = -\int_L E.dl$$

Differentiate this equation we get

$$dV = -E.dl$$

$$= -E_x dx - E_y dy - E_z dz$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

Equating the two equation we get

$$\boxed{E = -\nabla V}$$

POISSON'S & LAPLACE'S EQUATIONS

Easily derived from Gauss's law (for a linear isotropic medium)

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot \varepsilon E = \rho_v$$

$$\nabla \cdot (-\varepsilon \nabla V) = \rho_v$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

Poisson's equation.

A special case of this equation occurs when ρ_v=0

$$\boxed{\nabla^2 V = 0}$$

Laplace's equation.

$$\boxed{D = \varepsilon_0 E}$$

$$\boxed{E = -\nabla V}$$

Cartesian coordinate system

$$\boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

Cylindrical coordinate system

$$\boxed{\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

Spherical coordinate system

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2 \theta}\frac{\partial^2 V}{\partial \phi^2} = 0}$$

APPLICATION OF POISSON'S & LAPLACE'S EQUATIONS

Using Laplace's or Poisson's equation we can obtain

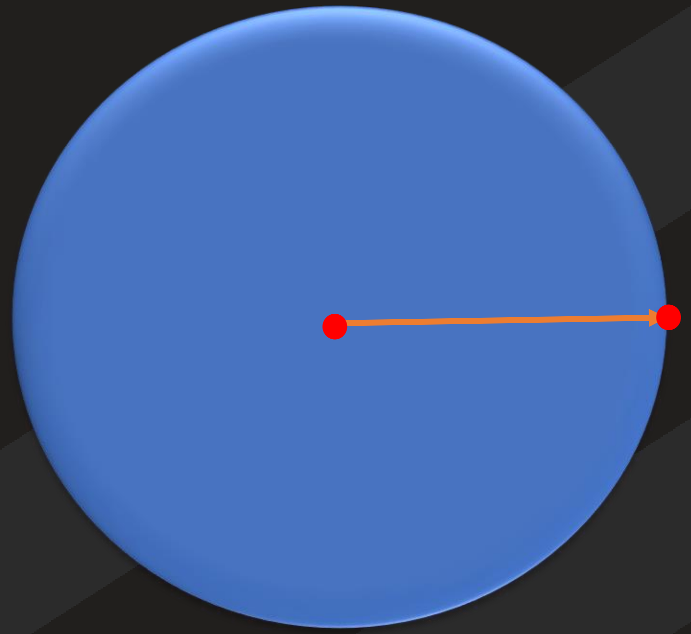
- *Potential at any point in between two surface when potential at two surface are given*
- *We can also find the capacitance between these two surface*

Coulomb's Law from Gauss's Law

Gauss's law states that the total electric flex Ψ through any closed surface is equal to the total charge enclosed by the surface

$$\Psi = Q$$

$$\boxed{\Psi = \int_S D.ds}$$



$$\int_S D.ds = Q$$

$$\int_S \varepsilon_0 E.ds = Q$$

$$\boxed{D = \varepsilon_0 E}$$

$$\varepsilon_0 E \int_S ds = Q$$

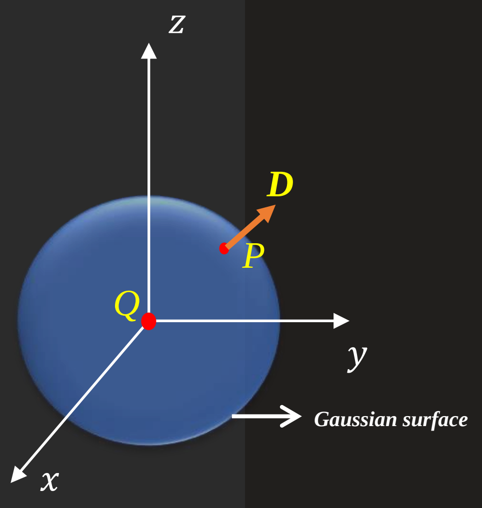
$$\varepsilon_0 E\, 4\pi r^2 = Q$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

$$\oint_s D.ds = \int_v \rho_v \, dv$$

APPLICATIONS OF GAUSS'S LAW

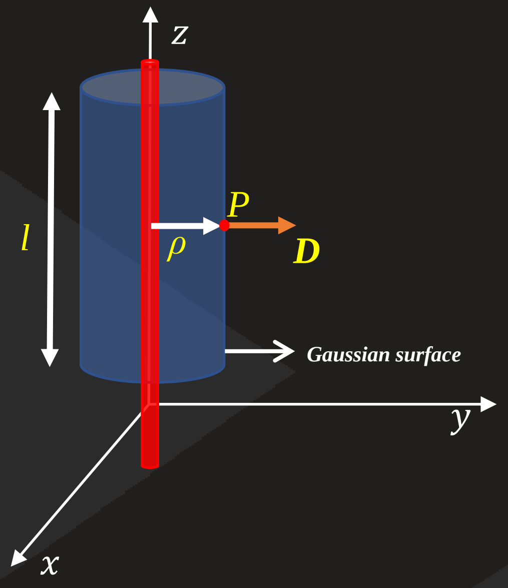
Point charge



$$\oint_s d\mathbf{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta d\phi$$

$$\boxed{D = \frac{Q}{4\pi r^2} a_r}$$

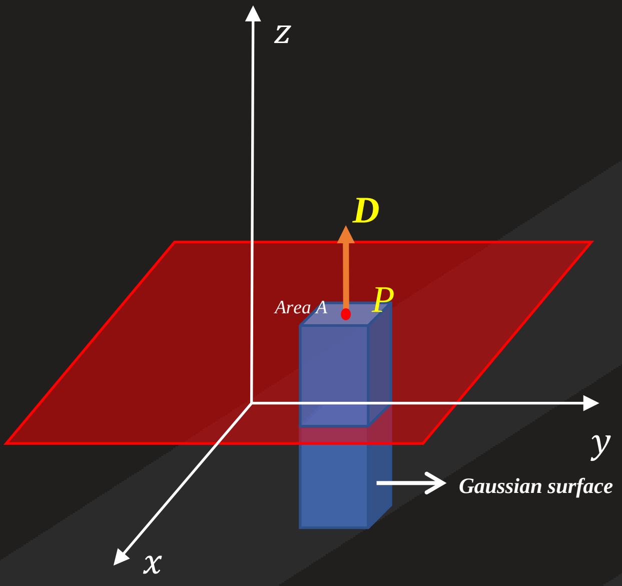
Infinite line charge



$$\oint_s d\mathbf{S} = 2\pi\rho l$$

$$\boxed{D = \frac{\rho_L}{2\pi\rho} a_\rho}$$

Infinite surface charge

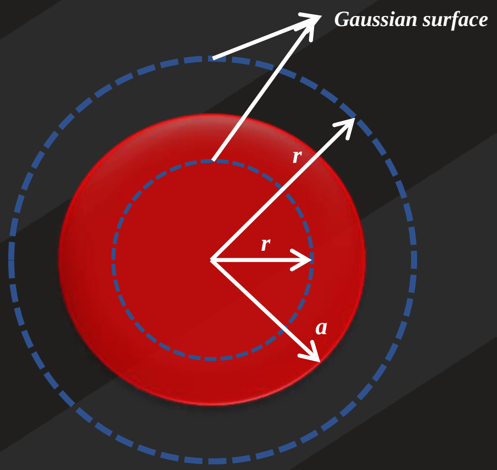


$$\oint_s d\mathbf{S} = 2A$$

$$\rho_S A = D_Z(A + A)$$

$$\boxed{D = \frac{\rho_S}{2} a_z}$$

Uniformly charged sphere



$$\oint_s d\mathbf{S} = 4\pi r^2$$

$$\boxed{D = \begin{cases} \frac{r}{3}\rho_0 a_r & 0 < r \leq a \\ \frac{a^3}{3r^2}\rho_0 a_r & r \geq a \end{cases}}$$

DIVERGENCE OF CURL OF A VECTOR

$$\boxed{\nabla \cdot \nabla \times \mathrm{A} = 0}$$

$$\boxed{\nabla . A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

$$\boxed{\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}}$$

$$\nabla \cdot \nabla \times \mathrm{A} = \left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]a_x - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]a_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]a_z$$

$$= \frac{\partial}{\partial x}\left(\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\right) - \frac{\partial}{\partial y}\left(\left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\right) + \frac{\partial}{\partial z}\left(\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\right)$$

$$= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial x \partial y} + \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_x}{\partial y \partial z}$$

$$= 0$$