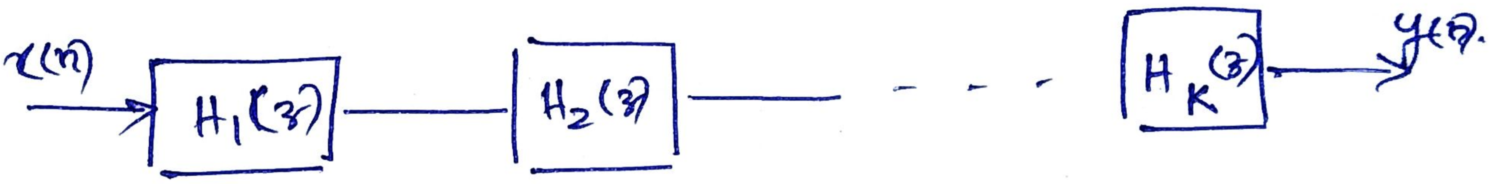
Cascade form structure:

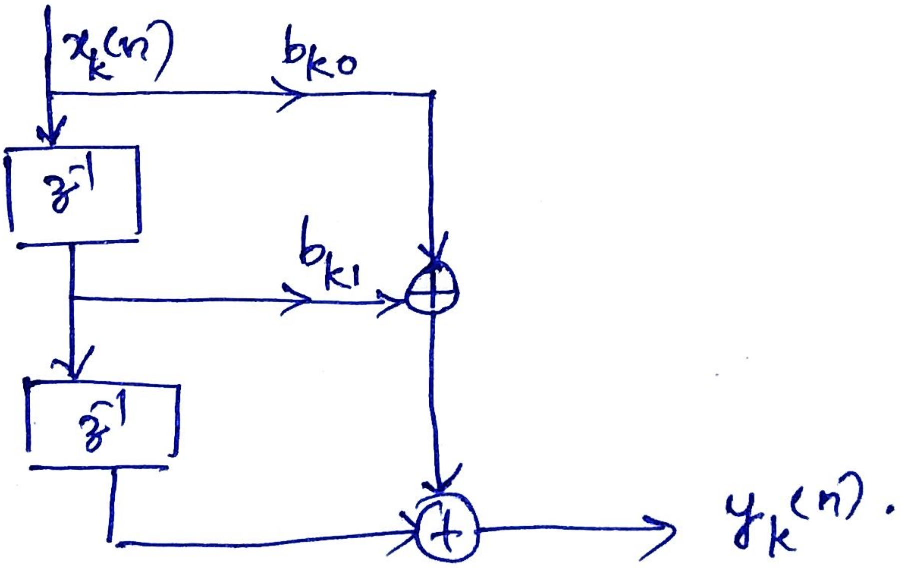
Cascade form realization is obtained from the system function by factorizing it into second order FIR systems.

$$H(z) = \prod_{k=1}^{K} \cancel{B_k}\, H_k(z).$$

corresponding cascade form is



where single $H_k(z)$.



Cascade realization

The Eq. (6.155) can be realized in cascade form from the factored form of $H(z)$. For N odd

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$H(z) = \prod_{k=1}^{\frac{N-1}{2}} \left(b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}\right)$$

$$= (b_{10} + b_{11}z^{-1} + b_{12}z^{-2})(b_{20} + b_{21}z^{-1} + b_{22}z^{-2}) \ldots$$
$$\times \left(b_{\frac{(N-1)}{2}0} + b_{\frac{(N-1)}{2}1}z^{-1} + b_{\frac{(N-1)}{2}2}z^{-2}\right) \tag{6.157}$$

For N odd, $N-1$ will be even and $H(z)$ will have $(N-1)/2$ second order factors. Each second order factored form of $H(z)$ is realized in direct form and is cascaded to realize $H(z)$ as shown in Fig. 6.69.

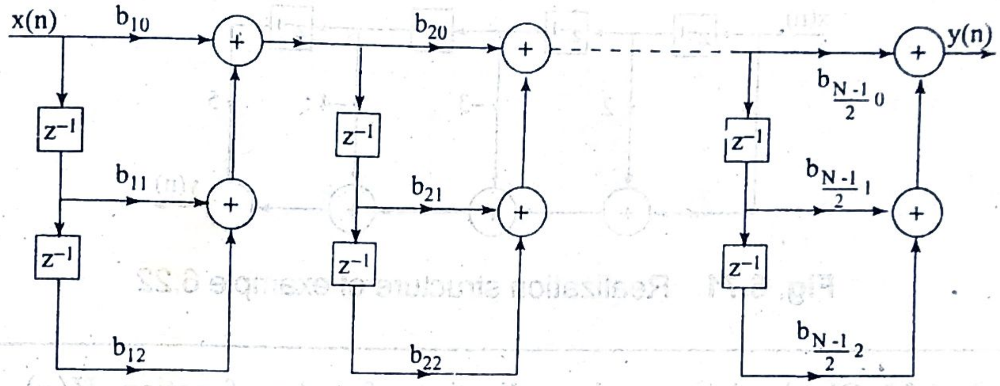


Fig. 6.69 Cascade realization of Eq. (6.157)

For N even

$$H(z) = (b_{10} + b_{11}z^{-1}) \prod_{k=2}^{N/2} (b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}) \tag{6.158}$$

when N is even, $N-1$ is odd and $H(z)$ will have one first order factor and $\frac{(N-2)}{2}$ second order factors.

$$H(z) = (b_{10} + b_{11}z^{-1})(b_{20} + b_{21}z^{-1} + b_{22}z^{-2})(b_{30} + b_{31}z^{-1} + b_{32}z^{-2})$$
$$\times \left(b_{\frac{N}{2}0} + b_{\frac{N}{2}1} + b_{\frac{N}{2}2}z^{-2}\right) \tag{6.159}$$

Now each factored form in $H(z)$ is realized in direct form and is cascaded to obtain the realization of $H(z)$ as shown in Fig. 6.70.

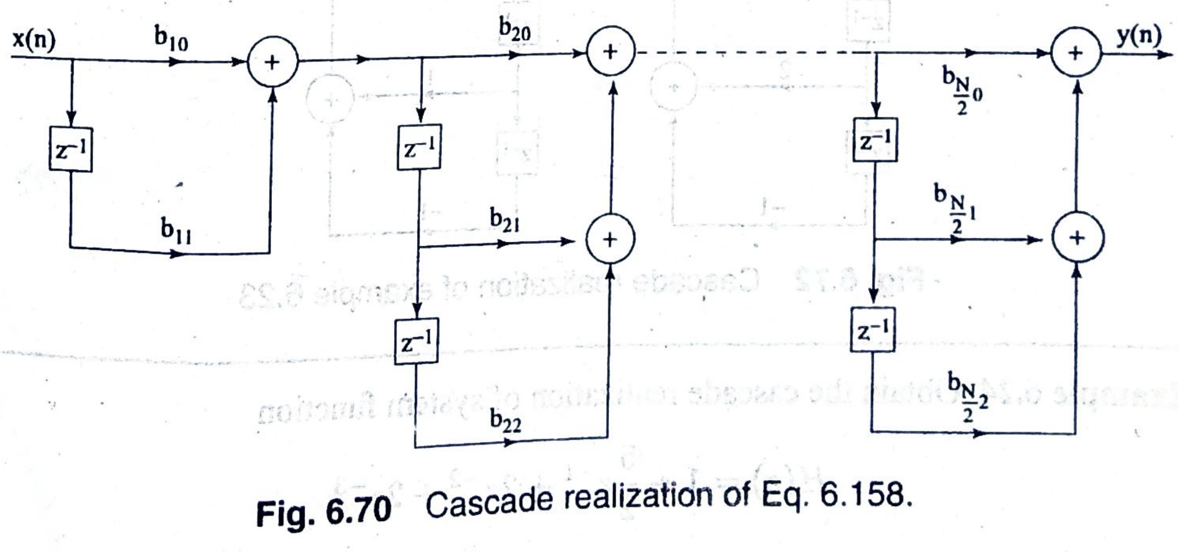


Fig. 6.70 Cascade realization of Eq. 6.158.

Example 6.23 Obtain the cascade realization of system function $H(z) = (1 + 2z^{-1} - z^{-2})(1 + z^{-1} - z^{-2})$.

Solution

$$H(z) = H_1(z)H_2(z)$$

where $H_1(z) = 1 + 2z^{-1} - z^{-2}$ and $H_2(z) = 1 + z^{-1} - z^{-2}$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} \Rightarrow Y_1(z) = X_1(z) + 2z^{-1}X_1(z) - z^{-2}X(z) \quad (6.161)$$

$$H_2(z) = \frac{Y_2(z)}{X_2(z)} \Rightarrow Y_2(z) = X_2(z) + z^{-1}X_2(z) - z^{-2}X(z) \quad (6.162)$$

The Eq. (6.161) and Eq. (6.162) can be realized in direct form and can be cascaded as shown in the Fig. 6.72.

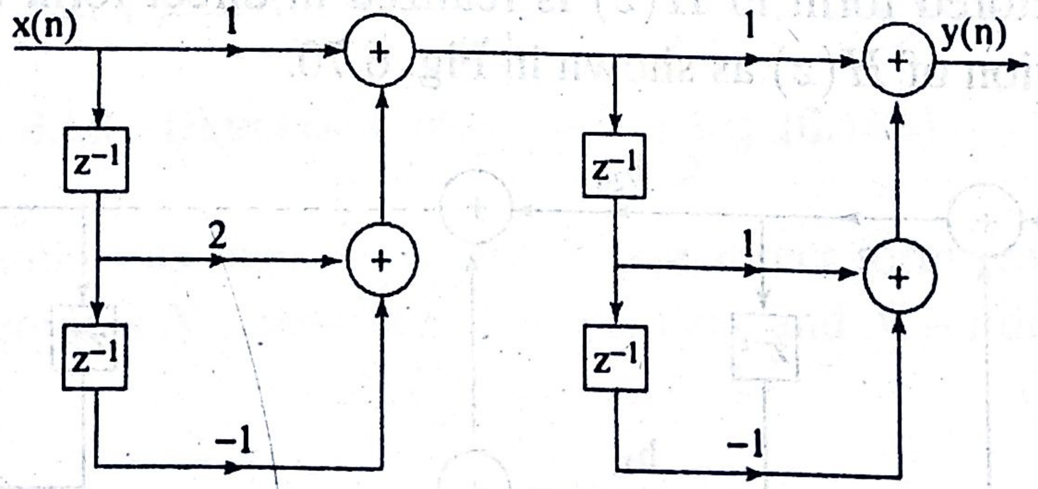


Fig. 6.72 Cascade realization of example 6.23

Q) Obtain the cascade form realization of system function.

$H(z) = 1 + \frac{5}{2}z^{-1} + 2z^{-2} + 2z^{-3}$

order $N = 4$ (even).

one — first order term

one — second order term

$H(z^{-1} = -2) = 1 + \frac{5}{2} \times -2 + 2 \times 4 + 2 \times -8$

$= 1 - 5 + 8 - 16 \neq 0.$

$H(z^{-1} = -1/2) = 1 + \frac{5}{2} \times \frac{-1}{2} + 2 \times \frac{1}{4} + 2 \times \frac{-1}{8}.$

$= 1 - \frac{5}{4} + \frac{2}{4} - \frac{1}{4} = 0$

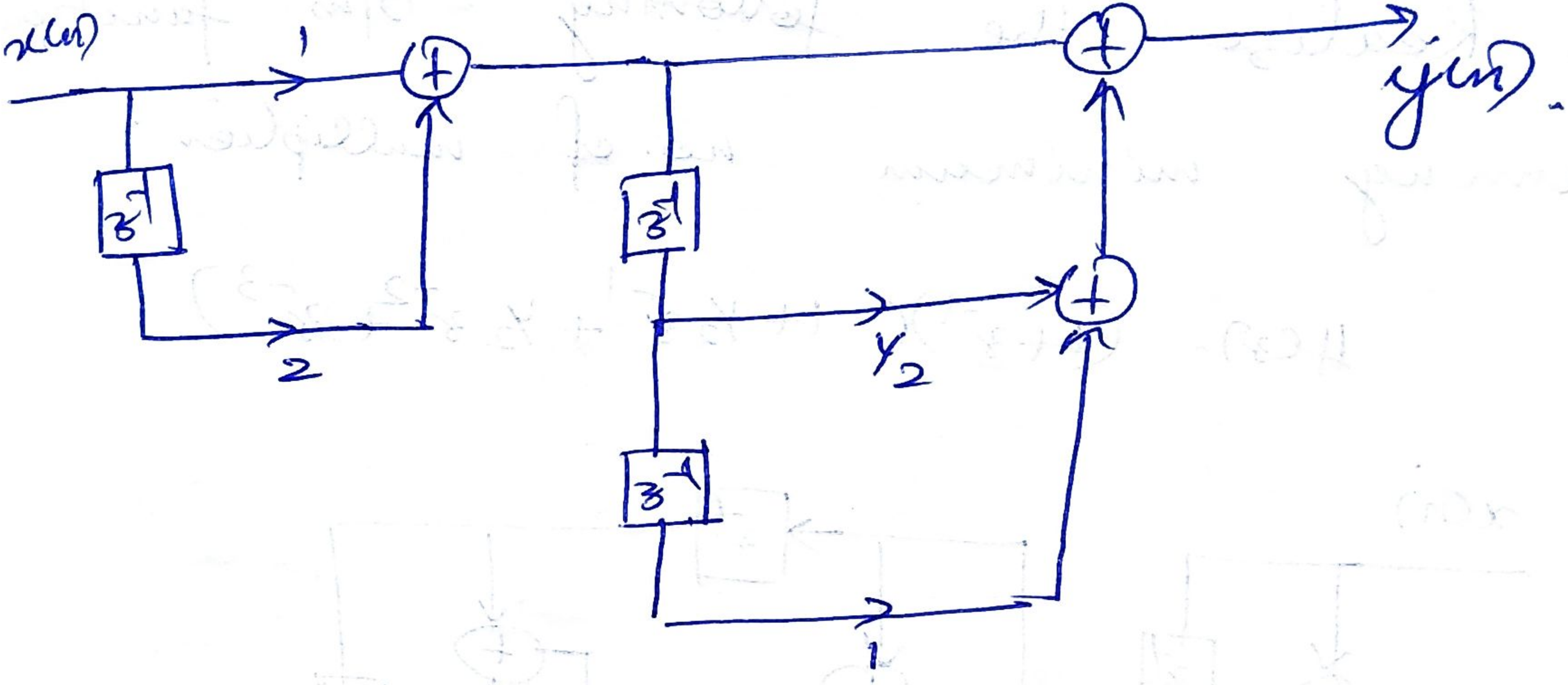
$\therefore z^{-1} = -1/2$ is a factor.

$(z^{-1} + 1/2) = 0 \Rightarrow (1 + 2z^{-1})$ is a factor

To get the second order terms.

$$\begin{array}{r|l} & \cancel{2z^{-3} + 2^{0}} \\ & z^{-2} + 1/2 z^{-1} + 1 \\ \hline 2z^{-1} + 1 & 2z^{-3} + 2z^{-2} + 5/2 z^{-1} + 1 \\ & 2z^{-1} + z^{-2} \\ \hline & z^{-2} + 5/2 z^{-1} \\ & z^{-2} + 1/2 z^{-1} \\ \hline & 2z^{-1} + 1 \\ & 2z^{-1} + 1 \\ \hline & 0 \end{array}$$

$H(z) = \underbrace{(1 + 2z^{-1})}_{H_1(z)} \underbrace{(1 + 1/2 z^{-1} + z^{-2})}_{H_2(z)}.$



x(n)
1
2
z^-1
z^-1
z^-1
1/2
1
+
+
+
y(n)