Comparison of number of computation for "direct computation of DFT" Vs "FFT Algorithm"

① Number of computation for direct computation of N-point DFT

We have the equation for ~~DFT~~ N-point DFT (direct computation)

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} \quad ; k=0,1\ldots N-1$$

By expanding the equation for

$$X(0) = \cancel{x(n)\,e^{-j2}} \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}0.n}.$$

expanding.

$$X(0) = \underbrace{\underbrace{x(0).W_N^{0.0}}_{\text{complex } \times^n} + \underbrace{x(1)\,W_N^{0.1}}_{\text{complex } \times^n} + \cdots\cdots \underbrace{x(N-1)\,W_N^{0.(N-1)}}_{\text{complex } \times^n}}_{N-1 \text{ complex Addition}}$$

In direct computation of N-point DFT, considering only a single value $k=0$. we have

$N \longrightarrow$ no. of complex multiplication

$N-1 \longrightarrow$ no. of complex Addition.

Considering all the N values of k.

$N \cdot N = N^2 \longrightarrow$ no. of complex $\times^n$ and.

$N \cdot (N-1) \longrightarrow$ no. of complex $+^n$.

② No. of computation in Radix-2 FFT

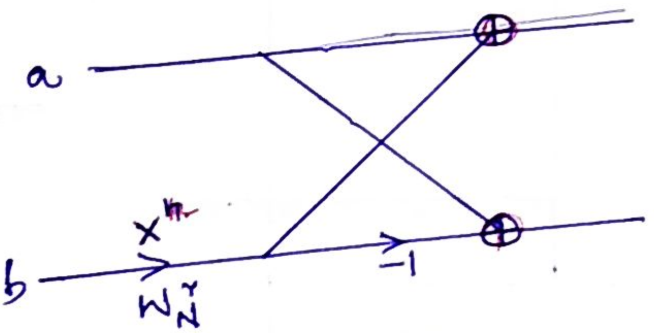
In the case of radix-2 FFT

$N = 2^m \implies m = \log_2 N$

- and there will be m - stages of computation.

- and each stage have $\frac{N}{2}$ butterfly.

Consider a single butterfly.



No. of calculation in one butterfly.

no. of complex $+^n \longrightarrow 2$.

no. of complex $\times^n \longrightarrow 1$

$\therefore$ Total no. of complex $+^n = m \times \frac{N}{2} \times 2$.

$$= [\log_2 N] \times \frac{N}{2} \times 2$$

$$= N \log_2 N$$

Total no. of complex $\times^{n} = m \times \frac{N}{2} \times 1$

$$= [\log_2 N] \times \frac{N}{2} = \frac{N}{2} \log_2 N.$$

— A comparison of the no. of complex multiplication and addition.

N	m = ($\log_2 N$)	Direct computation no. of complex $+^n$ $= N(N-1)$	no of complex $\times^n$. N^2.	Radix-2 FFT no. of complex $+^n$ $N \log_2 N$	no. of complex $\times^n$. $\frac{N}{2} \log_2 N$	Speed Improvement factor $= \frac{N^2}{\frac{N}{2} \log_2 N}$
4	2	12	16	8	4	$\frac{16}{4} = 4$
8	3	56	64	24	12	$\frac{64}{12} = 5.33$
16	4					
32	5					
64	6					
128	7					
256	8					
512	9					

Q) Find the number of complex multiplications and additions involved in the calculation of 1024 point DFT using (1) direct computation and (2) radix-2 FFT algorithm. (KTU Dec 2018)