

tiators.

$$H(e^{j\omega}) = \bar{H}(e^{j\omega}) e^{j\theta(\omega)} \rightarrow (6.25)$$

Example 6.1 Determine the frequency response of FIR filter defined by $y(n) = 0.25x(n) + x(n-1) + 0.25x(n-2)$. Calculate the phase delay and group delay.

Solution

Given

$$y(n) = 0.25x(n) + x(n-1) + 0.25x(n-2)$$

Taking Fourier transform on both sides

$$\begin{aligned} Y(e^{j\omega}) &= 0.25X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) + 0.25e^{-2j\omega}X(e^{j\omega}) \\ H(e^{j\omega}) &= \frac{Y(e^{j\omega})}{X(e^{j\omega})} = 0.25 + e^{-j\omega} + 0.25e^{-2j\omega} \\ &= e^{-j\omega}(0.25e^{j\omega} + 1 + 0.25e^{-j\omega}) = e^{-j\omega}(1 + 0.5\cos\omega) \\ &= e^{-j\omega}\bar{H}(e^{j\omega}) \end{aligned} \quad (6.41a)$$

Comparing Eq. (6.41a) with Eq. (6.25) we get $\theta(\omega) = -\omega$.

The phase delay $\tau_p = \frac{-\theta(\omega)}{\omega} = \frac{\omega}{\omega} = 1$.

The group delay $= -\frac{d\theta(\omega)}{d\omega} = \frac{-d}{d\omega}(-\omega) = 1$.