Conversion of the Continuous AWGN Channel into a Vector Channel

- Continuous signal to vector representation

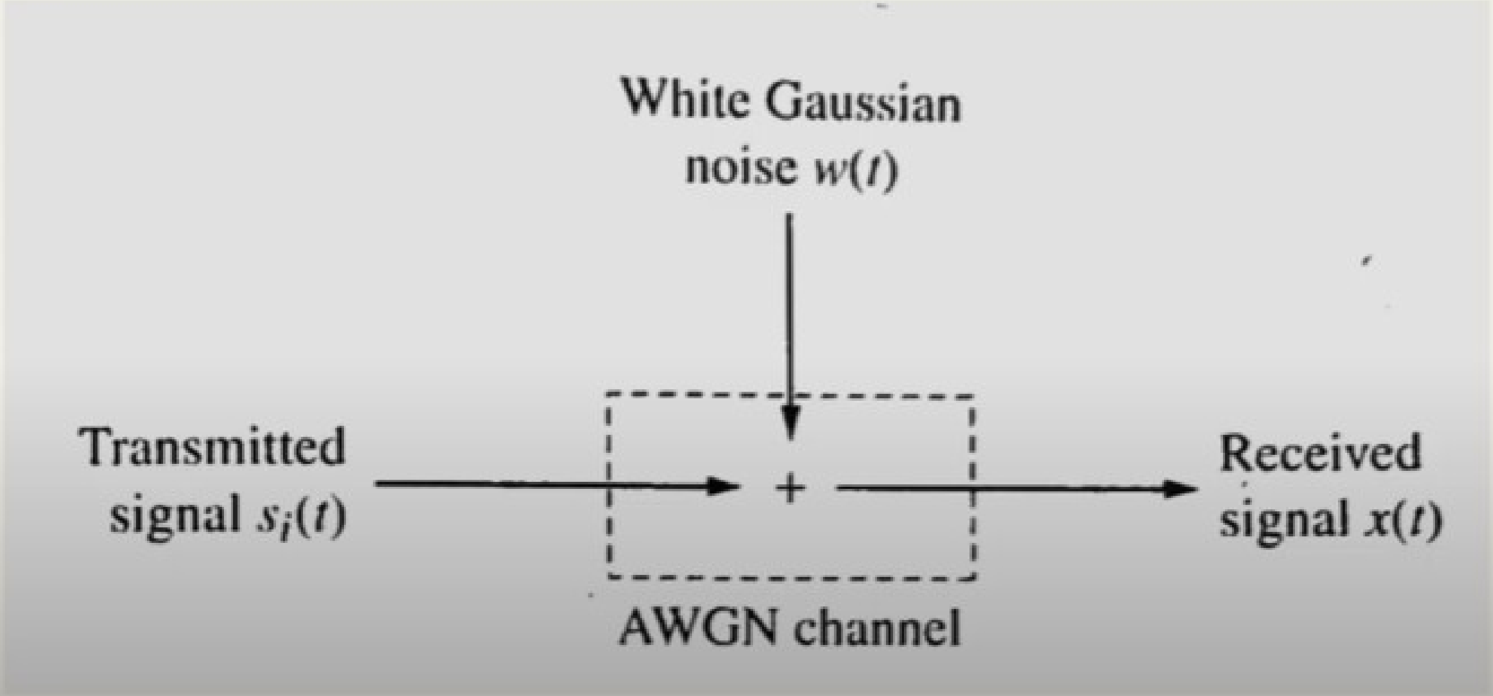
$S_i(t)$ ---------- s_i =

Similarly we convert continuous AWGN channel to its vector model.

AWGN – additive white Gaussian noise

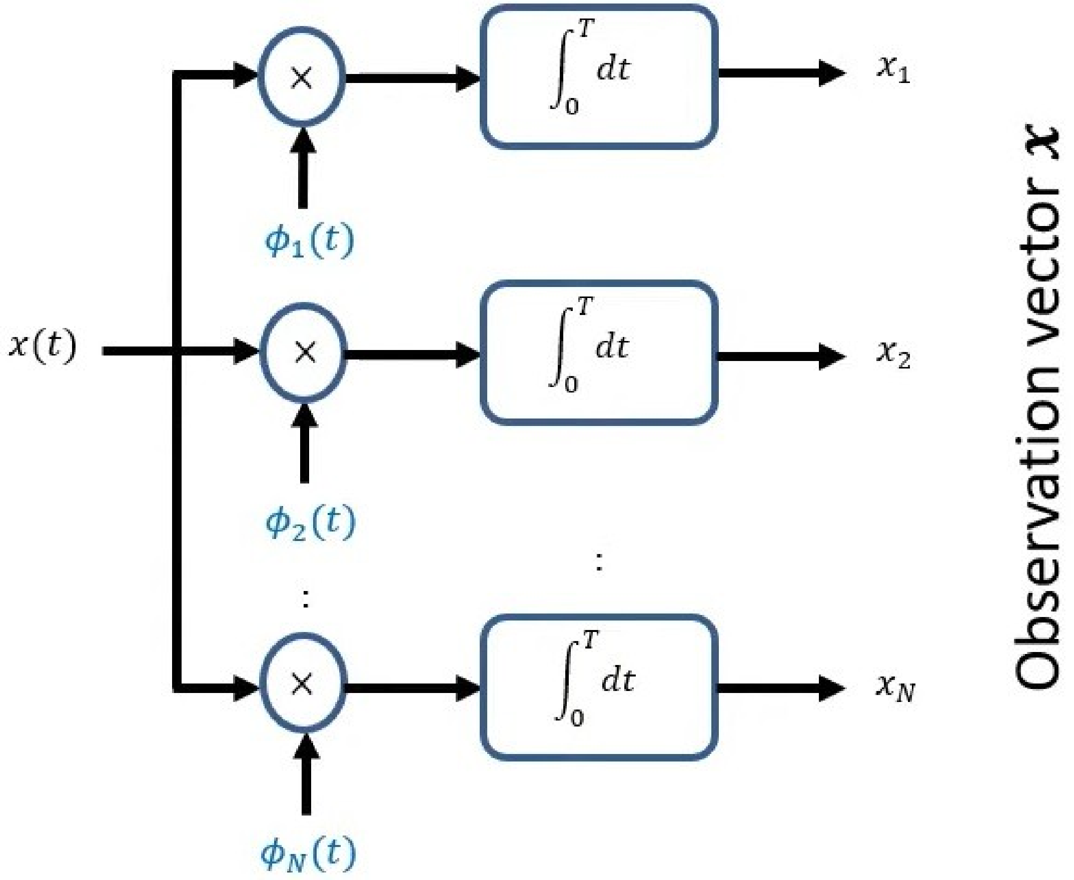
- Additive – As its name suggests, noise is added to a signal. Noise is generated randomly.
- White –noise has the same power distribution at every frequency. Therefore, white noise has a constant Power Spectral Density (the measure of a signal's power compared to frequency) across all frequencies.
- Gaussian – Due to noise source's random nature, a mathematical model is used to calculate the probability of events. Noise pdf is a Gaussian distribution

Model of AWGN channel



------(1)

- Consider a bank of N pdt integrators : correlator



- Consider j^{th} correlator from the set of N correlators. o/p of j^{th} correlator

$= {}_j(t)\ dt$

$= {}_j(t)\ dt$

$= {}_j(t)\ dt + {}_j(t)\ dt$

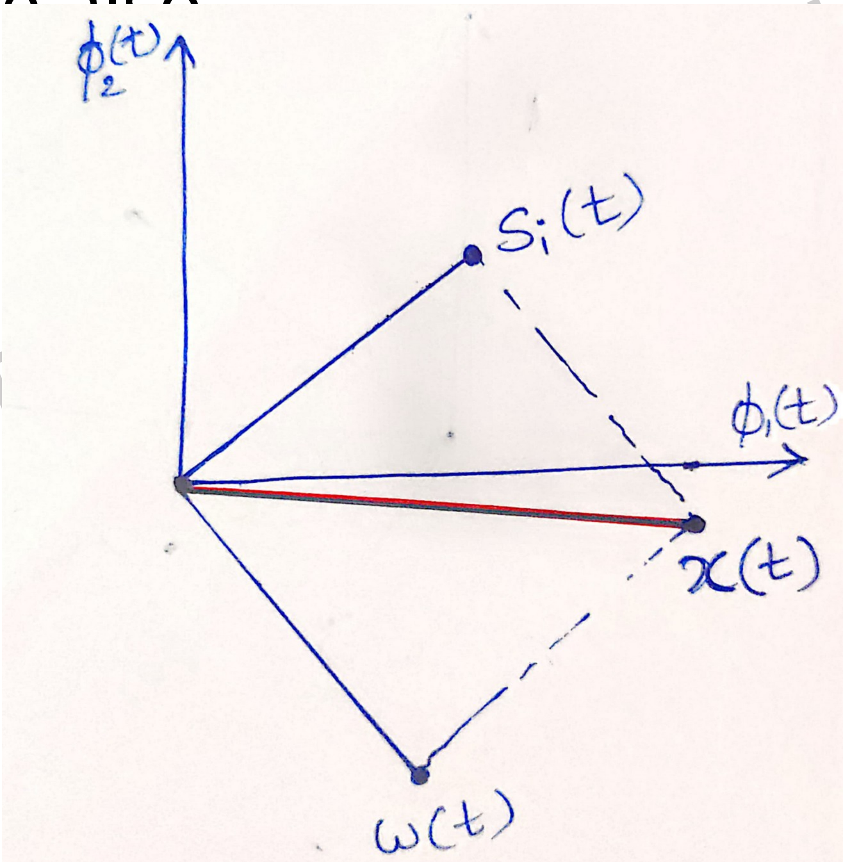
$= s_{ij} + w_j$ ----(2) where w_j is the noise component in the j^{th} correlator o/p

-deterministic s/g

w(t) - random s/g

- Let $X(t)$ be a random process
- So $x(t)$ can be considered as the sample function of $X(t)$
- Let $W(t)$ be a random process defining noise
- So $w(t)$ can be considered as the sample function of $W(t)$

- We can represent a signal vector $s_i(t)$ using orthonormal basis functions.
- But when we consider rxd s/g x(t) it becomes difficult to represent it with the same orthonormal basis functions as it includes noise also.



- Using synthesizer, we can represent s/g $s_i(t)$ from its vector elements.

If we have , then $s_i(t)$ =

But if we have , then x(t)

- Now Let’s define X′(t) be a random process
- So x’(t) can be considered as the sample function of X′(t)

x’(t) = x(t) - --------(3)

Substitute eqn (1) & (2) in (3)

x’(t) = -

= - -

= -

= w′(t)

$$x'(t) = w'(t)$$

- From eqn (3) , $x(t) = x'(t) +$

$$x(t) = \; + w'(t)$$

This is the vector representation of channel.

$w'(t)$ – remainder term.

Statistical characterization of vector channel

1. Mean
2. Variance
3. Co-variance

1. Mean

- Consider a random vector X =
- $X_1, X_2, \ldots, X_N$ random variables.
- $x_1, x_2, \ldots, x_N$ are sample values of random variables $X_1, X_2, \ldots, X_N$.

Mean of X_j =

$= E[X_j]$

From eqn (2) , $= s_{ij} + w_j$

$X = S_{ij} + W_j$

$= E[S_{ij} + W_j]$

$= E[S_{ij}] + E[W_j]$

There is no randomness in **deterministic s/g**.

So $E[S_{ij}] = S_{ij}$

Noise is **AWGN** . That is zero mean .

$E[W_j] = 0$

Mean $= S_{ij}$

2. Variance

- We know that variance $= E[(x-\mu)^2]$

$= E[(X_j -)^2]$

$= E[(X_j - \mathbf{S_{ij}})^2]$

$= E[W_j{}^2]$ and $W_j = {}_j(t)\, dt$

$= E[{}_j(t)\, dt\ {}_j(u)\, du]$

$= {}_j(u)\, dt\, du\, E[)$

E[) = $R_w(t,u)$ is the autocorrelation function of noise w(t)

- But for an AWG noise, autocorr fn =

= $_j(u)$ dt du

= dt du

We know that by time shifting property

$dt = x(t_0)$

$= \ ) \, du$

$= \ du$

- du = energy of orthonormal basis fn =1

Variance =

3. Co-variance

$$Cov\,[X_j\,X_k] = E[(X_j - \ )\ (X_k - \ )]$$

$$= E[(X_j - S_{ij})\ (X_k - S_{ik})]$$

$$= E[(W_j\ )\ (W_k)]$$

Where $W_j = \ {}_j(t)\ dt$

$W_k = \ {}_k(u)\ du$

$$Cov\,[X_j\,X_k] = E[\ {}_j(t)\ dt\ \ {}_k(u)\ du\ ]$$

- Cov $[X_j X_k] = {}_k(u)\ E[w(t)w(u)\ dt\ du$
- E[) $= R_w(t,u) =$
- Cov $[X_j X_k] = {}_k(u)\ \ dt\ du$

$$= {}_j(t)\ \ dt\ du$$

We know that by time shifting property

$dt = x(t_0)$

Cov $[X_j X_k]$ = $_j(u)$ du

) du = 0 property of orthonormal basis fn.

Therefore

Cov $[X_j X_k] = 0$

ie X_j ʻs are uncorrelated.