5.14.5 *Cascade Form*

Let us consider an IIR system with system function

$$H(z) = H_1(z)H_2(z)\dots H_k(z) \tag{5.122a}$$

This can be represented using block diagram as shown in Fig. 5.46.

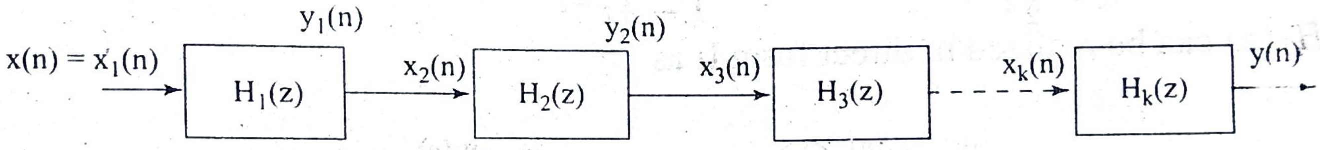


Fig. 5.46 Block diagram representation of Eq. (5.122a)

Now realize each $H_k(z)$ in direct form II and cascade all structures. For example let us take a system whose transfer function

$$\begin{aligned} H(z) &= \frac{(b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2})(b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2})}{(1 + a_{k1}z^{-1} + a_{k2}z^{-2})(1 + a_{m1}z^{-1} + a_{m2}z^{-2})} \\ &= H_1(z)H_2(z) \end{aligned} \tag{5.122b}$$

where $H_1(z) = \dfrac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$ and

$$H_2(z) = \frac{b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2}}{1 + a_{m1}z^{-1} + a_{m2}z^{-2}}$$

Realizing $H_1(z)$ and $H_2(z)$ in direct form II, and cascading we obtain cascade form of the system function.

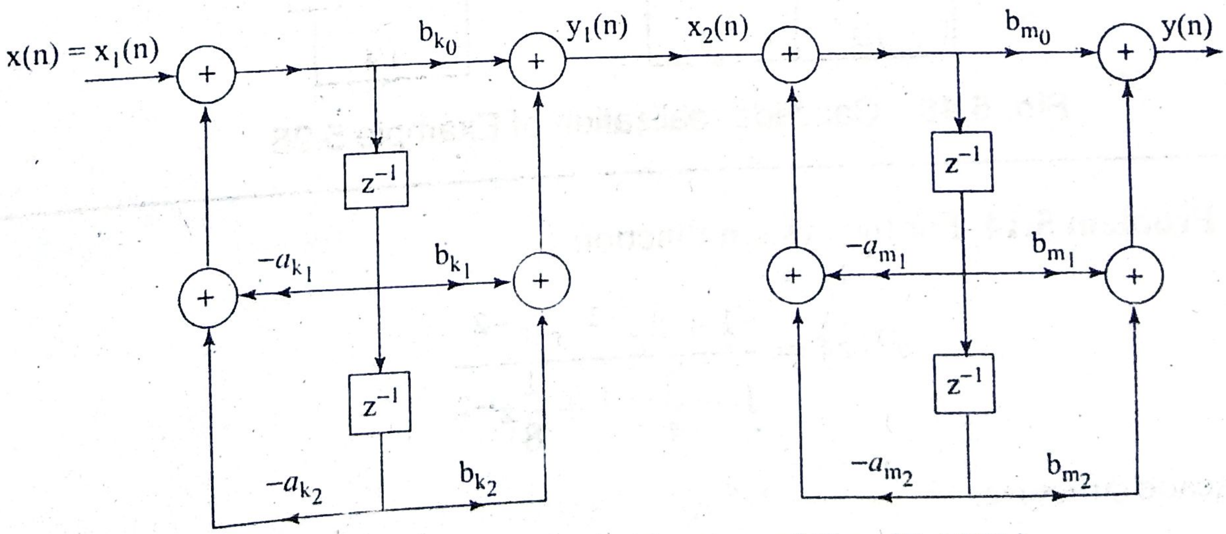


Fig. 5.47 Cascade realization of Eq. (5.122b)

Example 5.25 Realize the system with difference equation $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ in cascade form.

Solution

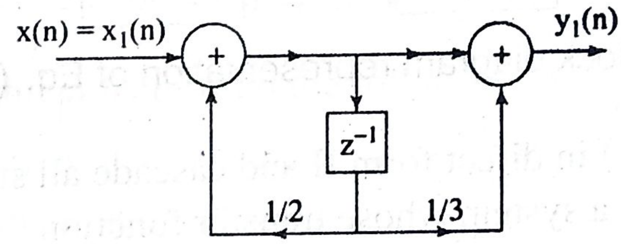
From the difference equation we obtain

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

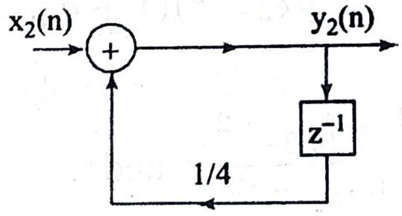
$$= \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} = H_1(z)H_2(z)$$

where $H_1(z) = \dfrac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}$ and $H_2(z) = \dfrac{1}{1 - \frac{1}{4}z^{-1}}$.

$H_1(z)$ can be realized in direct form II as



Similarly, $H_2(z)$ can be realized in direct form II as



Cascading the realization of $H_1(z)$ and $H_2(z)$ we have

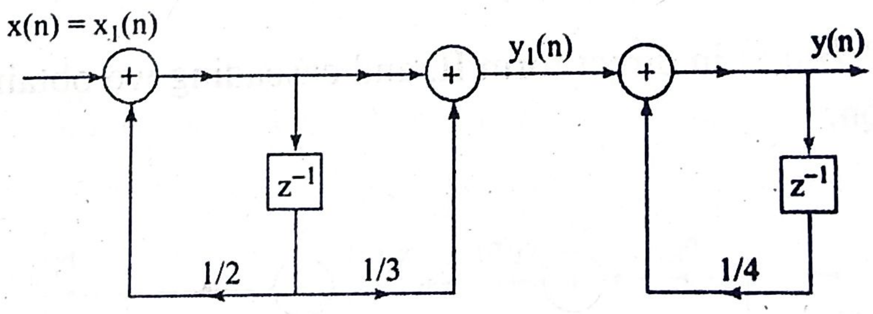


Fig. 5.48 Cascade realization of Example 5.25

(b) $H(z) = \dfrac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}{(1+\frac{1}{4}z^{-1})(1+z^{-1}+\frac{1}{2}z^{-2})(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2})}$

$= \dfrac{(1-\frac{1}{2}z^{-1})}{(1+\frac{1}{4}z^{-1})} \left(\dfrac{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}{1+z^{-1}+\frac{1}{2}z^{-2}}\right) \cdot \left(\dfrac{1}{1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}}\right)$