SIGNALS AND SYSTEMS

ASSIGNMENT - 3.

Submitted by:

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ECE - B

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1) Obtain the response of LTI system with impulse response h = $\delta(t)$ with input signal $x(t) = e^{-at} u(t)$ using Fourier transf

$h(t) = \delta(t) \Rightarrow H(\omega) = 1$

$x(t) = e^{-at} u(t)$

$X(\omega) = \frac{1}{a+j\omega}$

$Y(\omega) = H(\omega) \cdot X(\omega)$

$= 1 \times \frac{1}{a+j\omega} = \frac{1}{a+j\omega}$

$\therefore y(t) = e^{-at} u(t).$

2) An LTI system with input $x(t)$ and output $y(t)$ is defined by the differential equation,

$$\frac{d}{dt} y(t) + 3y(t) = x(t)$$

Determine the impulse response of the system, using Laplace transform if

i) The system is causal

ii) The system is non causal.

$$x(t) = \frac{d}{dt} y(t) + 3y(t).$$

i) The system is causal.

$$L\left\{\frac{d}{dt} y(t)\right\} = s.Y(s) - y(0)$$

$$L\left\{3y(t)\right\} = 3.Y(s).$$

$$\Rightarrow X(s) = s.Y(s) + 3Y(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+3} \Rightarrow s = -3$$

For the system to be causal, the ROC must be to the right of the right most pole

ie, $Re(s) > 1$. Here the ROC is ~~greater than~~

$Re(s) > -3$

$$\therefore h(t) = e^{-3t} u(t).$$

$$\therefore \text{Impulse response} = h(t) = e^{-3t} u(t)$$

Module: 4

1) If $x[n] \xleftrightarrow{DTFS: \Omega_0} X[k]$. ($x[n]$ form the Discrete time fourier series pair with $X[k]$), the state and prove the following properties of DTFS.

i) Linearity
ii) Time shift
iii) Frequency shift.
iv) Time scaling.
v) Convolution
vi) Modulation.

i) Linearity:

If $x[n] \xleftrightarrow{DTFS; \Omega_0} X[k]$ & $y[n] \xleftrightarrow{DTFS; \Omega_0} Y[k]$

then

$$z[n] = a x[n] + b y[n] \xleftrightarrow{DTFS} Z[k] = a X[k] + b Y[k]$$

proof:

$$Z[k] = \frac{1}{N} \sum_{n=\langle N \rangle} z[n] e^{-jk\Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} [a x[n] + b y[n]] e^{-jk\Omega_0 n}$$

$$= \frac{a}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\Omega_0 n} + \frac{b}{N} \sum_{n=\langle N \rangle} y[n] \times e^{-jk\Omega_0 n}$$

$$= a X[k] + b Y[k]$$

ii) Time shift

If $x[n] \xleftrightarrow{DTFS;\,\Omega_0} X[k]$

then

$$y[n] = x[n-n_0] \xleftrightarrow{DTFS} Y[k] = e^{-jk\Omega_0 n_0} X[k]$$

proof:

$$Y[k] = \frac{1}{N} \sum_{n=\langle N \rangle} y[n] e^{-jk\Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n-n_0] e^{-jk\Omega_0 n} \quad -(1)$$

put $n - n_0 = m$ in equation (1) then

$$Y[k] = \frac{1}{N} \sum_{n=\langle N \rangle} x[m] e^{-jk\Omega_0 [m+n_0]}$$

$$= e^{-jk\Omega_0 n_0} \frac{1}{N} \sum_{n=\langle N \rangle} x[m] e^{-jk\Omega n}$$

$$= e^{jk\Omega_0 n_0} x[k]$$

iii) Frequency shift

If $x[n] \xleftrightarrow{DTFS:\,\Omega_0} X[k]$

then

$$y[n] = e^{jk_0\Omega_0 n_0} x[n] \xleftrightarrow{DTFS} Y[k] = X[k-k_0]$$

proof:

$$Y[k] = \frac{1}{N} \sum_{n=\langle N \rangle} y[n] e^{-jk\Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} e^{jk\Omega_0 n} \quad x[n] e^{-jk\Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j[k-k_0]\Omega_0 n}$$

$$= X[k-k_0]$$

Convolution:

If $x[n] \xleftrightarrow{DTFS} X[k]$ & $y[n] \xleftrightarrow{DTFS} Y[k]$

then,

$$z[n] = x[n] * y[n] \xleftrightarrow{DTFS} N\, X[k]\, Y[k]$$

Proof:

$$Z[k] = \frac{1}{N} \sum_{n=\langle N\rangle} z[n]\, e^{-jk\Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N\rangle} x[n] * y[n]\, e^{-jk\Omega_0 n}$$

Using the definition of periodic convolution

$$Z[k] = \frac{1}{N} \sum_{n=\langle N\rangle} \left[\sum_{l=\langle N\rangle} x[l]\, y[n-l] \right] e^{-jk\Omega_0 n}$$

Changing the order of summation:-

$$Z[k] = \frac{1}{N} \left[\sum_{l=\langle N\rangle} x[l] \sum_{n=\langle N\rangle} y[n-l]\, e^{-jk\Omega_0 n} \right]$$

putting $n-l=m$,

$$Z[k] = \frac{1}{N} \left(\sum_{l=\langle N\rangle} x[l] \sum_{n=\langle N\rangle} y[m]\, e^{-jk\Omega_0 m}\, e^{-jk\Omega_0 l} \right.$$

$$= \frac{1}{N} \left[\sum_{l=\langle N\rangle} x[l]\, e^{-jk\Omega_0 l} \sum_{n=\langle N\rangle} y[m]\, e^{-jk\Omega_0 m} \right]$$

$$= \frac{1}{N} \left[N\, X[k] \,.\, N\, Y[k] \right]$$

$$= X[k] . Y[k]$$

VI) ~~Modulation~~: Multiplication

If $x[n] \xleftrightarrow{DTFS} X[k]$ & $y[n] \xleftrightarrow{DTFS} Y[k]$

then

$$z[n] = x[n] \cdot y[n] \xleftrightarrow{DTFS} Z[k] = X[k] \circledast Y[k]$$

proof:

$$Z[k] = \frac{1}{N} \sum_{n=\langle N \rangle} z[n] e^{-j\Omega_0 k n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] \cdot y[n] e^{-jk\Omega_0 n}$$

The synthesis equ for x[n] is :-

$$x[n] = \sum_{l=\langle N \rangle} x[l] e^{jk\Omega_0 n}$$

$$\therefore Z[k] = \frac{1}{N} \sum_{l=\langle N \rangle} \left[\sum_{l\langle N \rangle} x[l] \sum_{n=\langle N \rangle} e^{jk\Omega_0 l} \right] y[n] e^{-jk\Omega_0 n}$$

Changing the order of summation

$$Z[k] = \frac{1}{N} \sum_{l=\langle N \rangle} x[l] \sum_{n=\langle N \rangle} y[n] e^{-j(k-l)\Omega_0 n}$$

$$= \sum_{l=\langle N \rangle} X(l) Y(k-l)$$

$$= X[k] * Y[k]$$

2) If $x[n] \xleftrightarrow{DTFT} X[e^{j\Omega}]$ ($x[n]$ forms the Discrete Time Fourier Transform with $X[e^{j\Omega}]$)

Then state and prove the following properties DTFS

vii) Linearity

viii) Time shift

ix) Frequency shift

x) Time shift

xi) Convolution

xii) Modulation.

Ans) vii) Linearity:

If $x[n] \xleftrightarrow{DTFT} X[e^{j\Omega}]$ & $y[n] \xleftrightarrow{DTFT} Y[e^{j\Omega}]$

Then

$$a\,x[n] + b\,y[n] \xleftrightarrow{DTFT} aX[e^{j\Omega}] + bY[e^{j\Omega}]$$

Proof:

$$a\,x[n] + b\,y[n] \xrightarrow{FT} \sum_{n=-\infty}^{\infty} \left[a\,x[n] + b\,y[n]\right] e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} a\,x[n]\, e^{-j\Omega n} + \sum_{n=-\infty}^{\infty} b\,y[n]\, e^{-j\Omega n}$$

$$= aX[e^{j\Omega}] + bY[e^{j\Omega}]$$

viii) Time shift:-

If $x[n] \xleftrightarrow{DTFT} X[e^{j\Omega}]$

then,

$$x[n-n_0] \longleftrightarrow X[e^{j\omega}]\, e^{-j\Omega n_0}$$

proof.

$$x[n-n_0] \overset{FT}{\Longrightarrow} \sum_{n=-\infty}^{\infty} x(n-n_0)\, e^{-jn}$$

put $n-n_0 = m$, as $n \to -\infty$ $m \to -\infty$

$$x[n-n_0] \overset{FT}{\Longrightarrow} \sum_{n=-\infty}^{\infty} x[m]\, e^{-j\Omega m}\, e^{-j\Omega n_0}$$

$$= e^{-jn_0} X(e^{j\Omega})$$

~~Frequency shift.~~

~~If x[n] DTFT~~

Ix) Frequency shift.

If $x[n] \xleftrightarrow{DTFT} X[e^{j\Omega}]$

then,

$$y[n] = x[n]\, e^{j\Omega_0 n} \xleftrightarrow{DTFT} Y[e^{j\Omega}] = X[e^{j(\Omega-\Omega_0)}]$$

proof:-

$$x[n]\, e^{j\Omega_0 n} \overset{FT}{\Longrightarrow} \sum_{n=-\infty}^{\infty} x[n]\, e^{j\Omega_0 n}\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j(\Omega-\Omega_0)n}$$

$$= X[e^{j(\Omega-\Omega_0)}]$$

x) Time scaling:

If $x[n] \xleftrightarrow{DTFT} X[e^{j\Omega}]$

then.

$$x_a[n] \xleftrightarrow{DTFT} X_a[e^{j\Omega}] = X[a\, e^{j\Omega}]$$

proof: -

$$X_a[e^{j\Omega}] = \sum_{n=-\infty}^{\infty} x_a[n]\, e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{a}\right) e^{-j\Omega n}$$

put.

$\frac{n}{a} = m$, then we get

$$X_a[e^{j\Omega}] = \sum_{m=-\infty}^{\infty} x[m]\, e^{-j[\Omega]m}$$

$$= X[a\, e^{j\Omega}]$$

xi) Convolution

If $x_1[n] \xleftrightarrow{DTFT} X_1[e^{j\Omega}]$ & $x_2[n] \xleftrightarrow{DTFT} X_2[e^{j\Omega}]$

then

$$x_1[n] * x_2[n] \xleftrightarrow{DTFT} X_1[e^{j\Omega}]\, X_2(e^{j\Omega})$$

proof

$$x_1[n] * x_2[n] \xrightarrow{FT} \sum_{n=-\infty}^{\infty} \left[x_1[n] * x_2[n]\right] e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x_1[k] \cdot x_2[n-k]\right] e^{-j\Omega n}$$

$$= \sum_{k=-\infty}^{\infty} x_1[k] \left[\sum_{n=-\infty}^{\infty} x_2(n-k)\, e^{-j\Omega n}\right]$$

of the time shifting property:

$$\sum_{k=-\infty}^{\infty} x_1[k] \cdot X_2[e^{j\Omega}]\, e^{-j\Omega k}$$

$$= X_2[e^{j\Omega}] \sum_{k=-\infty}^{\infty} x_1[k]\, e^{-j\Omega k}$$

$$= X_1[e^{j\Omega}] \cdot X_2[e^{j\Omega}]$$

xii) <u>Multiplication.</u>

If $x[n] \xleftrightarrow{DTFT} X[e^{j\Omega}]$ & $y[n] \xleftrightarrow{DTFT} Y[e^{j\Omega}]$

then

$$z[n] = x[n] \cdot y[n] \xleftrightarrow{DTFT} Z[e^{j\Omega}] = \frac{1}{2\pi}\left[X[e^{j\Omega}] * Y[e^{j\Omega}]\right]$$

proof:

$$Z[e^{j}] = \sum_{n=-\infty} z[n]\, e^{-j\Omega n} = \sum_{n=\infty}^{\infty} \left[x[n] \cdot y[n]\right] e^{-j\Omega n}$$

$$x[n] = 1/2\pi \int_{2\pi} X[\beta]\, e^{j\beta n}\, d\beta$$

$$\Rightarrow Z[e^{jw}] = \sum_{n=-\infty}^{\infty} \left[\int_{2\pi} X(\beta)\, e^{j\beta n}\, d\beta\right] y[n]\, e^{-j\Omega n}$$

By interchanging the order

$$Z[e^{j\Omega}] = \frac{1}{2\pi} \int_{2\pi} X[\beta] \sum_{n=-\infty}^{\infty} y[n]\, e^{-j(\Omega-\beta)n}\, d\beta$$

$$= \frac{1}{2\pi} X[e^{j\Omega}] * Y[e^{j\Omega}]$$