




Module 3

Maximum likelihood (ML) decoding

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- Based on probability measures
 - The goal of ML decoding is to maximize the quantity $P(r|c)$; i.e., to find the codeword c so that the probability that r was received given that c was sent is maximized.

ML decoding

- ML decoder takes 'n' bits of the received codeword and returns most likely 'k' bit message among possible 2^k messages
- Simple way of implementation of ML decoder involves , considering all possible 2^k valid codewords and comparing the received vector 'r' with these codewords.
- Hamming distance between 'r' and all possible codewords are calculated.

ML decoding

- The codeword giving minimum hamming distance between 'r' and possible code words is selected as the transmitted codeword.
- For BSC, if error probability is, $\epsilon < 1/2$, then the codeword with smallest hamming distance is the ML decoder output
- If $\epsilon > 1/2$, simply swap the zeros and ones and do the decoding.

ML decoding

- For any valid codeword c , Hamming distance between r and c is d .
- $P(r|c) = \varepsilon^d (1 - \varepsilon)^{N-d}$
- Here N is the length of the received word
The logarithm of conditional probability is called the **log-likelihood** which is given below:

$$\log \mathbb{P}(r|\tilde{c}) = d \log \varepsilon + (N - d) \log(1 - \varepsilon) = d \log \frac{\varepsilon}{1 - \varepsilon} + N \log(1 - \varepsilon).$$

Maximizing loglikelihood function results in minimizing 'd',