Example 6.16 Determine the coefficients of a linear phase FIR filter of length $M = 15$ has a symmetric unit sample response and a frequency response that satisfies the conditions

$$H\left(\frac{2\pi k}{15}\right) = 1 \quad k = 0, 1, 2, 3$$
$$= 0 \quad k = 4, 5, 6, 7$$

Solution

$$|H(k)| = 1 \quad \text{for} \quad 0 \leq k \leq 3 \quad \text{and} \quad 12 \leq k \leq 14$$
$$= 0 \quad \text{for} \quad 4 \leq k \leq 11$$

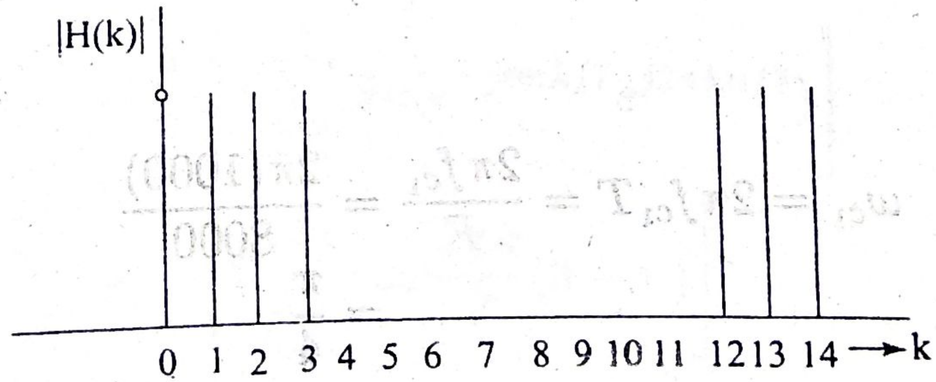

Fig. 6.60 Ideal magnitude response with samples for example 6.16

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$$

$$= \frac{-14}{15}\pi k \quad 0 \le k \le 7$$

and

$$\theta(k) = 14\pi - \frac{14\pi k}{15} \quad \text{for } 8 \le k \le 14$$

$$\begin{aligned} H(k) &= e^{-j14\pi k/15} && \text{for} \quad k = 0, 1, 2, 3 \\ &= 0 && \text{for} \quad 4 \le k \le 11 \\ &= e^{-j14\pi(k-15)/15} && \text{for} \quad 12 \le k \le 14 \end{aligned}$$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left(H(k)e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} Re\left(e^{-j14\pi k/15}e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{3}\cos\frac{2\pi k(7-n)}{15}\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(7-n)}{15} + 2\cos\frac{4\pi(7-n)}{15} + 2\cos\frac{6\pi(7-n)}{15}\right]$$

$h(0) = h(14) = -0.05$; $h(1) = h(3) = 0.041$ $h(4) = h(10) = -0.1078$

$h(2) = h(12) = 0.0666$; $h(3) = h(11) = -0.0365$ $h(5) = h(9) = 0.034$

$h(6) = h(8) = 0.3188$ $h(7) = 0.466$