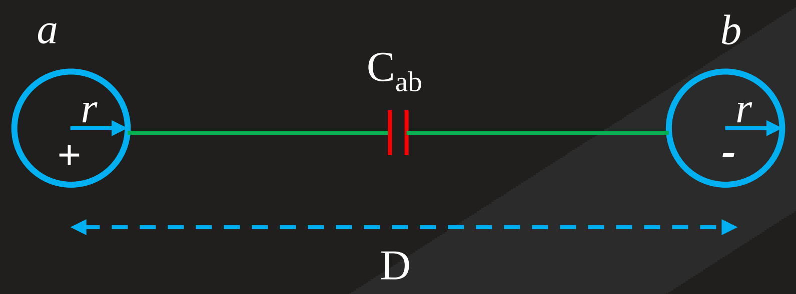
IMPLearn

CAPACITANCE OF TWO-WIRE LINE



The potential difference between the conductors a and b is

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[Q_a^+ \ln\frac{D_{ab}}{D_{aa}} + Q_b^- \ln\frac{D_{bb}}{D_{ba}}\right]$$

V_{ab} = *potential difference between conductor a and b*

q_a , q_b = *charge on conductor a & b*

D_{ab}, D_{ba} = *Distance between a&b, b&a*

D_{aa}, D_{bb} = *Radius of the conductor*

Here, $Q_a^+ = -Q_b^-$, $D_{ab} = D_{ba} = D$ *and* $D_{aa} = D_{bb} = r$

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[Q_a^+ \ln\frac{D}{r} - Q_a^+ \ln\frac{r}{D}\right] \quad = \frac{1}{2\pi\varepsilon} Q_a^+ \ln\left(\frac{D}{r}\right)^2$$

$$\boxed{\ln m - \ln n = \ln\frac{m}{n}}$$

$$\boxed{n \ln m = \ln m^n}$$

$$V_{ab} = \frac{Q_a^+}{\pi\varepsilon} \ln\frac{D}{r}$$

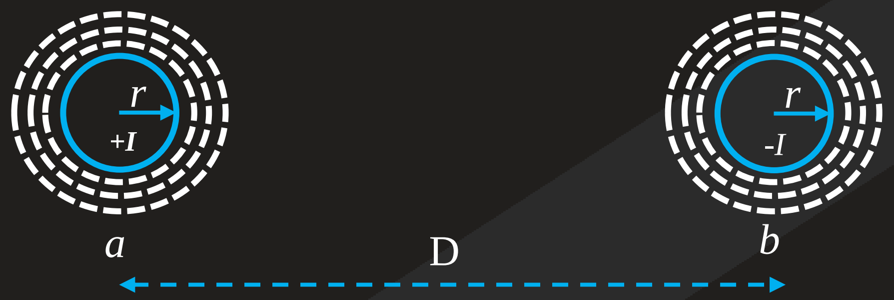
The capacitance between the conductor is

$$C_{ab} = \frac{Q_a^+}{V_{ab}} = \frac{Q_a^+}{\frac{Q_a^+}{\pi\varepsilon}\ln\frac{D}{r}}$$

$$\boxed{C_{ab} = \frac{\pi\varepsilon}{\ln\frac{D}{r}}}$$

INDUCTANCE OF A TWO-WIRE LINE

The current flow in the conductors are opposite in direction so that one becomes return path for the other.



Ψ_a = *flux linkage conductor a*

I_a , I_b = *current flow on conductor a & b*

D_{ab}, D_{ba} = *Distance between a&b, b&a*

D_{aa}, D_{bb} = *Radius of the conductor a & b*

The flux linkages of conductor 'a' is given by the formula

$$\Psi_a = \frac{\mu_0}{2\pi}\left[I_a \ln\frac{1}{D_{aa}} + I_b \ln\frac{1}{D_{ab}}\right]$$

Here, $I_a = +I$, $I_b = -I$, $D_{aa} = r$ *and* $D_{ab} = D$

$$= \frac{\mu_0}{2\pi}\left[I \ln\frac{1}{r} - I \ln\frac{1}{D}\right]$$

$$\Psi_a = \frac{\mu_0}{2\pi} I \ln\frac{D}{r}$$

The inductance of the conductor 'a'

$$L_a = \frac{\Psi_a}{I} = \frac{\mu_0}{2\pi}\ln\frac{D}{r}\,H/m$$

The flux linkages of conductor 'b' is given by the formula

$$\Psi_b = \frac{\mu_0}{2\pi}\left[I_b \ln\frac{1}{D_{bb}} + I_a \ln\frac{1}{D_{ba}}\right]$$

Here, $I_a = +I$, $I_b = -I$, $D_{bb} = r$ *and* $D_{ba} = D$

$$= \frac{\mu_0}{2\pi}\left[I \ln\frac{1}{r} - I \ln\frac{1}{D}\right]$$

$$\Psi_b = \frac{\mu_0}{2\pi} I \ln\frac{D}{r}$$

The inductance of the conductor 'b'

$$L_b = \frac{\Psi_a}{I} = \frac{\mu_0}{2\pi}\ln\frac{D}{r}\,H/m$$

$$\boxed{\ln m - \ln n = \ln\frac{m}{n}}$$

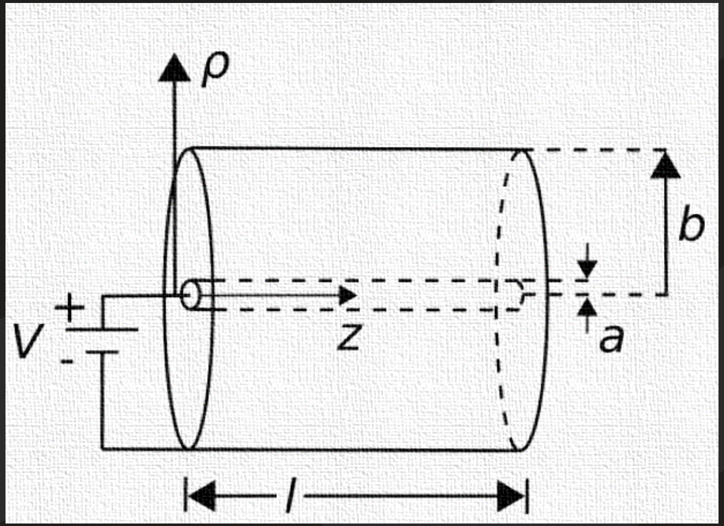
The inductance of both conductor (loop inductance)

$$\boxed{L_a + L_b = \frac{\mu_0}{\pi}\ln\frac{D}{r}\,H/m}$$

$$\frac{\mu_0}{\pi} = \frac{4\pi \times 10^{-7}}{\pi} = 4 \times 10^{-7}$$

CAPACITANCE OF A COAXIAL CABLE

The electric field inside a coaxial structure comprised of concentric conductors and having uniform charge density on the inner conductor is identical to the electric field of a line charge in free space having the same charge density.



$$V = -\int_b^a E.dl$$

$$= -\int_{\rho=b}^{a} \frac{\rho_L}{2\pi\varepsilon\rho}\hat{a}_\rho . d\rho\hat{a}_\rho \quad = -\frac{\rho_L}{2\pi\varepsilon}\int_{\rho=b}^{a}\frac{d\rho}{\rho} \quad = \frac{\rho_L}{2\pi\varepsilon}\int_{\rho=a}^{b}\frac{d\rho}{\rho}$$

$$V = \frac{\rho_L}{2\pi\varepsilon}\ln\frac{b}{a}$$

Electric Field Due to an Infinite Line Charge using Gauss' Law,

$$\boxed{E = \frac{\rho_L}{2\pi\varepsilon\rho}\hat{a}_\rho}$$

Capacitance

$$C \triangleq \frac{Q}{V} \qquad = \frac{\rho_L l}{\frac{\rho_L}{2\pi\varepsilon}\ln\frac{b}{a}}$$

$$\boxed{C = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)}}$$

INDUCTANCE OF A COAXIAL CABLE

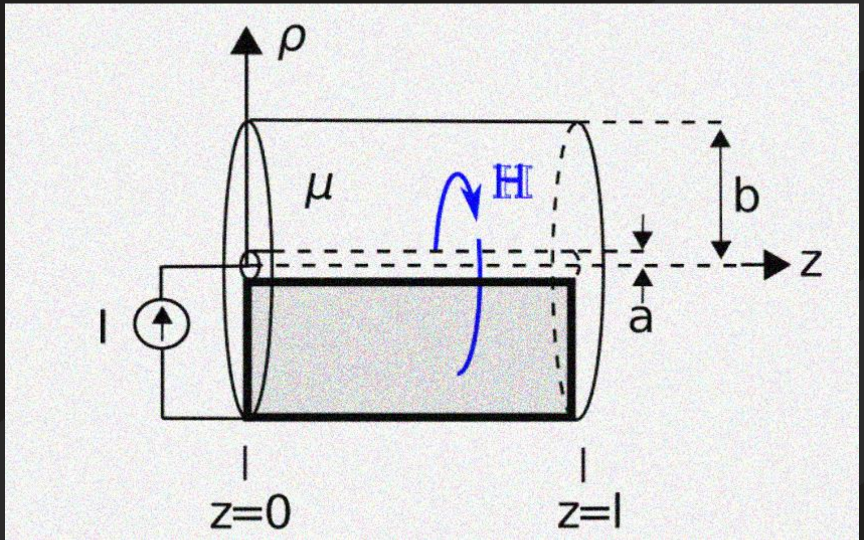
The magnetic field inside a coaxial structure comprised of concentric conductors bearing current I is identical to the magnetic field of the line current I in free space.

Magnetic flux through a surface

$$\Psi = \int_S B.dS$$

$$= \int_{\rho=a}^{b} \int_{z=0}^{l} \frac{\mu I}{2\pi\rho} \hat{a}_\phi . \hat{a}_\phi d\rho dz \quad = \frac{\mu I}{2\pi} \int_{z=0}^{l} dz \int_{\rho=a}^{b} \frac{d\rho}{\rho}$$

$$\Psi = \frac{\mu I l}{2\pi} \ln\left(\frac{b}{a}\right)$$



Magnetic field due to an Infinite current carrying conductor

$$\boxed{H = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

$$\boxed{B = \mu H}$$

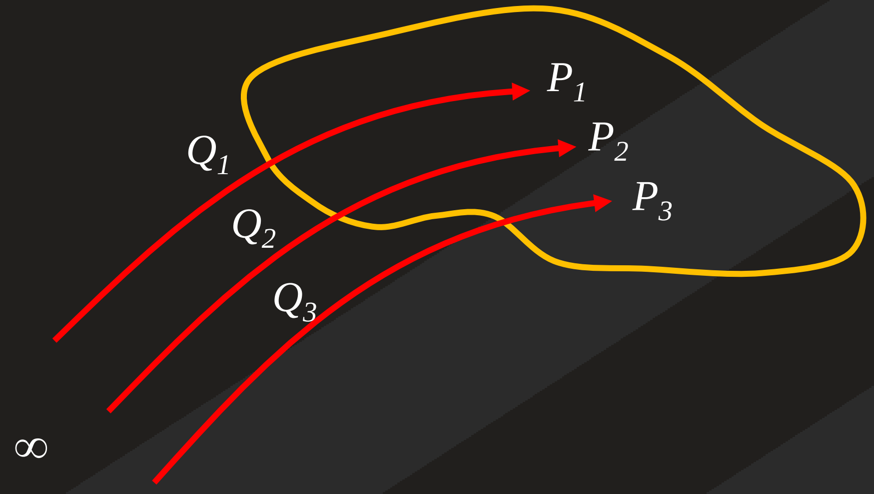
Inductance

$$L \triangleq \frac{\Psi}{I}$$

$$= \frac{\frac{\mu I l}{2\pi} \ln\left(\frac{b}{a}\right)}{I}$$

$$\boxed{L = \frac{\mu l}{2\pi} \ln\left(\frac{b}{a}\right)}$$

ENERGY STORED IN ELECTRIC FIELD



The total work done in positioning the three charges is

$$W_E = W_1 + W_2 + W_3$$

$$= 0 + Q_2V_{21} + Q_3V_{31} + Q_3V_{32}$$

$$= 0 + Q_2V_{21} + Q_3(V_{31} + V_{32}) \quad \text{----------} \ (1)$$

If the charges were positioned in reverse order

$$W_E = W_3 + W_2 + W_1$$

$$= 0 + Q_2V_{23} + Q_1V_{12} + Q_1V_{13}$$

$$= 0 + Q_2V_{23} + Q_1(V_{12} + V_{13}) \quad \text{----------} \ (2)$$

(1) + (2)

$$2W_E = Q_1(V_{12} + V_{13}) + Q_2(V_{21} + V_{23}) + Q_3(V_{31} + V_{32})$$

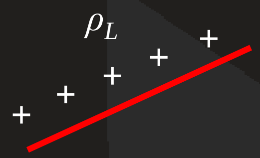
$$= Q_1V_1 + Q_2V_2 + Q_3V_3$$

$$W = \frac{Q_1V_1 + Q_2V_2 + Q_3V_3}{2}$$

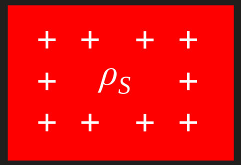
$$\boxed{W = \frac{1}{2}\sum_{k=1}^{N} Q_kV_k}$$

ENERGY STORED IN ELECTRIC FIELD

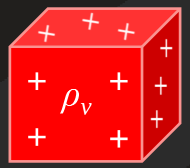
If the region has continuous charge distribution the summation becomes integration



Line charge



Surface charge



Volume charge

$$W_E = \frac{1}{2}\int_L \rho_L V dl$$

$$W_E = \frac{1}{2}\int_S \rho_S V dS$$

$$W_E = \frac{1}{2}\int_v \rho_v V dv$$

ENERGY STORED IN ELECTRIC FIELD

$$W_E = \frac{1}{2}\int_v \rho_v V dv$$

$$= \frac{1}{2}\int_v (\nabla \cdot D) V dv$$

$$= \frac{1}{2}\int_v [(\nabla \cdot VD) - D.\nabla V] dv$$

$$= \frac{1}{2}\int_v (\nabla \cdot VD) dv - \frac{1}{2}\int_v D.\nabla V dv$$

Applying Divergence theorem to the first term

$$= \frac{1}{2}\oint_S (VD).dS - \frac{1}{2}\int_v (D.\nabla V) dv$$

As the surface S becomes large the first term tends to zero

$$W_E = -\frac{1}{2}\int_v (D.\nabla V) dv \quad = \frac{1}{2}\int_v D.E dv \quad = \frac{1}{2}\int_v \varepsilon_0 E^2 dv$$

$$\boxed{\nabla \cdot D = \rho_v}$$

$$\nabla \cdot VA = A.\nabla V + (\nabla.A)V$$

From this electrostatics energy density w_E

$$(\nabla.A)V = \nabla \cdot VA - A.\nabla V$$

$$w_E = \frac{dW_E}{dv} = \frac{1}{2}\varepsilon_0 E^2$$

$$\boxed{\oint_S A.dS = \int_V \nabla \cdot A \, dv}$$

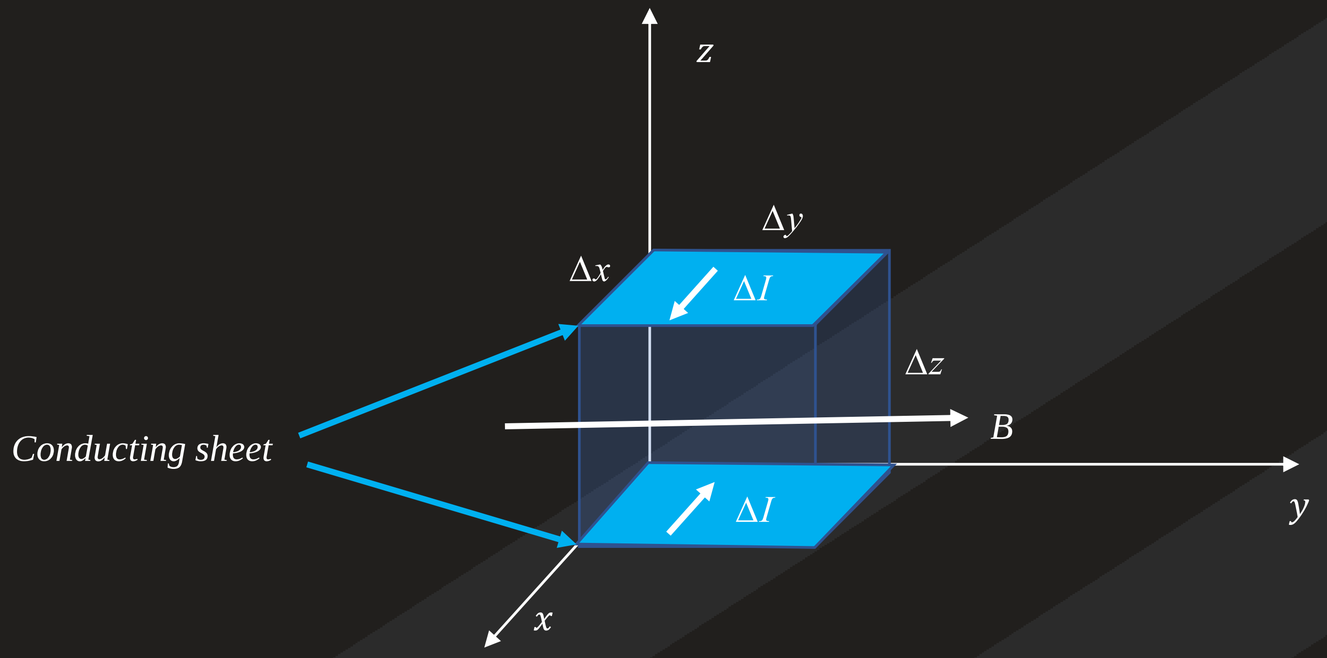
$$\boxed{w_E = \frac{D^2}{2\varepsilon_0}}$$

$$\boxed{D = \varepsilon_0 E} \qquad \boxed{E = -\nabla V}$$

ENERGY STORED IN MAGNETIC FIELD

Magnetic energy in the field of an inductor

$$W_m = \frac{1}{2} L I^2 \qquad \text{----------- (1)}$$



Consider a differential volume in a magnetic field

Each volume has an inductance

$$\Delta L = \frac{\Delta\Psi}{\Delta I} \qquad = \frac{\mu H \Delta x \Delta z}{H \Delta y}$$

Magnetic flux density → $B = \mu_0 H$

Magnetic flux through a surface S → $\Psi = \int_S B.dS$

Substitute the above equation in (1)

$$\Delta W_m = \frac{1}{2}\Delta L \Delta I^2 \;= \frac{1}{2}\frac{\mu H \Delta x \Delta z}{H \Delta y}(H\Delta y)^2 \quad = \frac{1}{2}\mu H^2 \Delta x \Delta z \Delta y \quad = \frac{1}{2}\mu H^2 \Delta v$$

$$w_E = \lim_{\Delta v \to 0} \frac{\Delta W_m}{\Delta v} \qquad = \frac{1}{2}\mu H^2 \qquad = \frac{1}{2} B.H$$

$$W_m = \frac{1}{2}\int \boldsymbol{B}.\boldsymbol{H} dv = \frac{1}{2}\int \mu H^2 \, dv$$

CONTINUITY EQUATION

Principle of charge conservation

The time rate of decrease of charge within a given volume must be equal to the net outward current flux through the surface of the volume

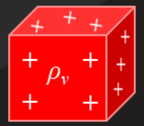
$$I_{out} = \oint J.ds \qquad = -\frac{dQ_{in}}{dt}$$

Where Q_{in} is the closed charge enclosed by the surface

$$\boxed{\oint_S A.dS = \int_V \nabla \cdot A\, dv}$$

Applying Divergence theorem

$$\oint_S J.dS = \int_V \nabla \cdot J\, dv$$



Volume charge

But

$$-\frac{dQ_{in}}{dt} = -\frac{d}{dt}\int_v \rho_v dv \qquad = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

$$dQ = \rho_v dv$$

$$Q = \int_v \rho_v dv$$

$$\int_V \nabla \cdot J\, dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

$$\boxed{\nabla . J = -\frac{\partial \rho_v}{\partial t}}$$

Continuity of current equation

$$E = \int_v \frac{\rho_v dv}{4\pi\varepsilon_0 R^2} a_R$$

DISPLACEMENT CURRENT DENSITY

From Ampere's circuit law for a static electromagnetic field

$$\nabla \times H = J \quad \text{----------} \; (1)$$

Third Maxwell's equation

Divergence of a curl of any vector filed is zero

$$\nabla . (\nabla \times H) = \nabla . \mathrm{J} = 0 \quad \text{----------} \; (2)$$

But from Continuity equation

$$\nabla . J = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad \text{----------} \; (3)$$

$$\boxed{\nabla . J = -\frac{\partial \rho_v}{\partial t}}$$

Continuity equation

To satisfy the above condition, we modify the equation

$$\nabla \times H = J + J_d \quad \text{----------} \; (4)$$

$$\nabla . (\nabla \times H) = \nabla . \mathrm{J} + \nabla . \mathrm{J_d} = 0$$

$$\nabla . \mathrm{J_d} = -\nabla . \mathrm{J}$$

$$\nabla . \mathrm{J_d} = -\nabla . \mathrm{J}$$

$$\nabla . \mathrm{J_d} = \frac{\partial \rho_v}{\partial t}$$

$$= \frac{\partial}{\partial t}(\nabla \cdot D)$$

$$\nabla . \mathrm{J_d} = \nabla . \frac{\partial D}{\partial t}$$

$$\boxed{\mathrm{J_d} = \frac{\partial D}{\partial t}}$$

Displacement current density

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

Which is the Maxwell's equation for a time varying filed , where J is the conduction current

$$J = \sigma E$$

$$\boxed{\nabla . J = -\frac{\partial \rho_v}{\partial t}}$$

Continuity of current equation

$$\boxed{\nabla \cdot D = \rho_v}$$

first Maxwell's equation

$$\nabla \times H = J + J_d \quad \text{---------- (4)}$$

Based on displacement current density we define displacement current as

$$\boxed{\mathrm{I_d} = \int J_d . ds = \int \frac{\partial D}{\partial t} . ds}$$

Conduction current

$$\boxed{J = \sigma E}$$

At low frequency J_d is usually neglected compared with J, however at high frequency the two terms are comparable

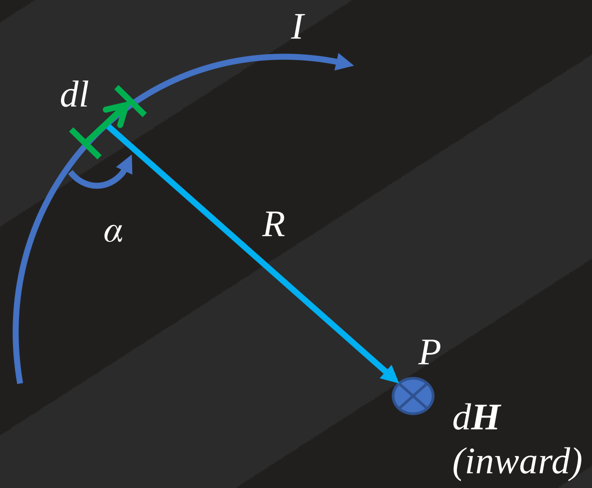
BIOT-SAVART'S LAW

Biot-Savart's Law states that the differential magnetic field intensity dH produced at a point P, by the differential current element Idl is proportional to the product of Idl and the sine of the angle α between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element

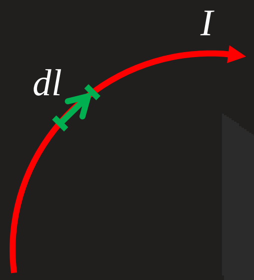
$$dH \; \alpha \; \frac{Idl\, sin\alpha}{R^2}$$

$$dH = \frac{k\, Idl\, sin\alpha}{R^2} \qquad k = \frac{1}{4\pi}$$

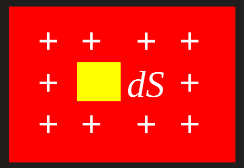
$$dH = \frac{Idl\, sin\alpha}{4\pi R^2} = \frac{Idl \times \hat{a}_R}{4\pi R^2} = \frac{Idl \times \vec{R}}{4\pi R^3}$$



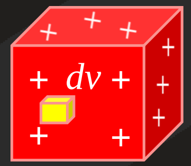
Magnetic field due to continuous current distribution



Line current



Surface current



Volume current

$$H = \int_L \frac{Idl \times \hat{a}_R}{4\pi R^2}$$

$$H = \int_S \frac{\boldsymbol{K}dS \times \hat{a}_R}{4\pi R^2} a_R$$

$$H = \int_v \frac{\boldsymbol{J}dv \times \hat{a}_R}{4\pi R^2} a_R$$

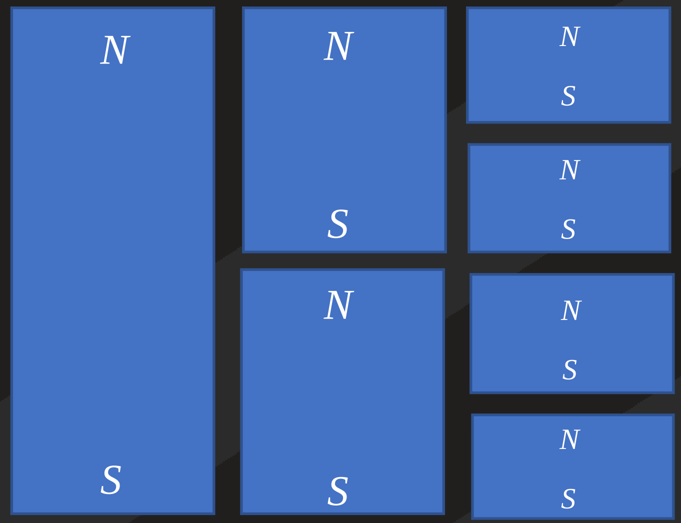
MAGNETIC FLUX DESITY

*The magnetic flux density **B** is similar to the electric flux density D.*

$$\boxed{B = \mu_0 H}$$

$\mu_0 \rightarrow permiability\ of\ free\ space\left(\frac{Henrys}{meter}\right)$

$\mu_0 = 4\pi \times 10^{-7} H/m$



*Magnetic flux through surface **S** is given by*

$$\boxed{\Psi = \int_S B.ds}$$

Unit → Webers (Wb)
Magnetic flux density → Wb/m²

$$\boxed{\oint_S A.dS = \int_V \nabla \cdot A\, dv}$$

Divergence theorem

An isolated magnetic charge does not exist

$$\oint_S B.d\mathbf{S} = \int_V \nabla \cdot B\, dv = 0$$

$$\boxed{\oint_S B.d\mathbf{S} = 0}$$

$$\boxed{\nabla \cdot B = 0}$$

Which is the fourth Maxwell's equation

MAGNETIC SALAR & VECTOR POTENTIAL

The magnetic potential could be a scalar V_m *or vector* **A.** *To define* V_m *and A we can use two important identities*

$$\nabla \times \nabla V = 0 \qquad \text{----------} \ (1)$$

$$\nabla \cdot (\nabla \times A) = 0 \qquad \text{----------} \ (2)$$

Just as $E = -\nabla V$, we define magnetic scalar potential V_m *(in amperes)* is related to H

$$\boxed{H = -\nabla V_m} \qquad \textit{If } J=0$$

From Third Maxwell's equation

$$J = \nabla \times H \qquad \text{----------} \ (3)$$

$$= \nabla \times -\nabla V_m = 0$$

$$= \nabla^2 V_m = 0 \qquad (J = 0) \qquad \text{----------} \ (4)$$

But we know $\nabla \cdot B = 0$ *(fourth Maxwell's equation) from equation (2) we can define the vector magnetic potential A*

$$\boxed{B = \nabla \times A}$$

$$\Psi = \int_S (\nabla \times A) \cdot d\boldsymbol{S}$$

$$\boxed{\Psi = \int_S B \cdot d\boldsymbol{S}}$$

$$\boxed{\Psi = \oint_L A \cdot dl}$$

$$\boxed{\oint_L A \cdot dl = \int_S (\nabla \times A)\, dS}$$

MAXWELL'S EQUATION FROM FARADAY'S LAW

Faraday figured out that a changing Magnetic Flux within a circuit (or closed loop of wire) produced an induced EMF, or voltage within the circuit.

$$EMF = -\frac{d\phi}{dt} \qquad \phi = \int_S B.d\boldsymbol{S}$$

The total EMF around the circuit is equal to summing up the small contributions at each EMF at the point around the entire circuit (KVL)

$$EMF = \oint_L E.dl$$

Note that Faraday's Law agrees with KVL in the magnetostatic case. If magnetic flux is constant, then Faraday's Law says V=0 . However, Faraday's Law is very clearly not consistent with KVL if magnetic flux is time-varying. The correction is simple enough; we can simply set these expressions to be equal.

$$\oint_L E.dl = -\frac{d}{dt}\int_S B.d\boldsymbol{S}$$

$$= -\int_S \frac{\partial B}{\partial t} d\boldsymbol{S}$$

Applying Stokes's theorem on LHS

$$\boxed{\oint_L A.dl = \int_S (\nabla \times A)\, dS}$$

$$\int_S (\nabla \times E)\, dS = -\int_S \frac{\partial B}{\partial t} d\boldsymbol{S}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

Maxwell's third equation (dynamic)

MAXWELL'S EQUATIONS

First Maxwell's equation

$$\boxed{\nabla \cdot D = \rho_v}$$

Differential or point form

$$\boxed{\oint_S \boldsymbol{D} . d\boldsymbol{S} = \int_v \rho_v \, dv}$$

Integral form

GAUSS'S LAW

$$\Psi = Q_{enc}$$

$$\Psi = \oint_s d\Psi = \oint_s D . ds \qquad Q_{enc} = \int_v \rho_v \, dv$$

$$\oint_s D . ds = \int_v \rho_v \, dv$$

Applying divergence theorem

$$\int_V \nabla \cdot D \, dv = \int_v \rho_v \, dv$$

$$\boxed{\nabla \cdot D = \rho_v}$$

first Maxwell's equation

MAXWELL'S EQUATIONS

Second Maxwell's equation

Nonexistence of Magnetic monopole

$$\boxed{\nabla \cdot B = 0}$$

Differential or point form

$$\boxed{\oint_S \boldsymbol{B}.\,d\boldsymbol{S} = 0}$$

Integral form

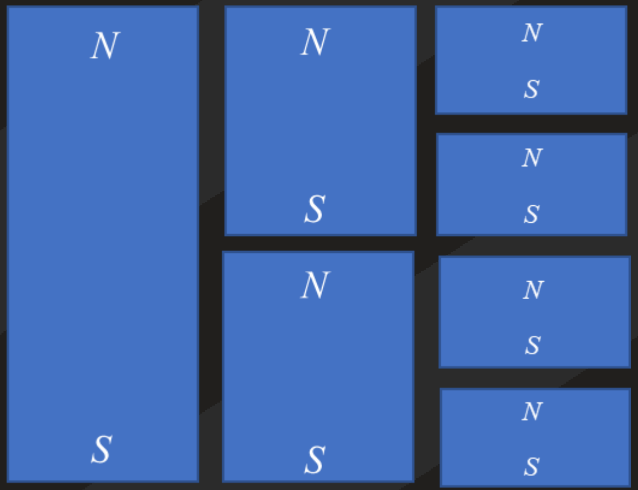
An isolated magnetic charge does not exist

$$\boxed{\oint_S B.\,d\boldsymbol{S} = 0}$$

From Divergence theorem

$$\oint_S B.\,d\boldsymbol{S} = \int_V \nabla \cdot B \; dv = 0$$

$$\boxed{\nabla \cdot B = 0}$$



MAXWELL'S EQUATIONS

Third Maxwell's equation

Conservative nature of electrostatic field

$$\boxed{\nabla \times E = 0}$$

Differential or point form

$$\boxed{\oint_L \mathbf{E}.dl = 0}$$

Integral form

The potential difference between point A & B is independent of path taken

$$\boxed{\oint_L E.dl = 0}$$

Applying Stokes's theorem

$$\oint_L E.dl = \int_S (\nabla \times E)\, dS = 0$$

$$\boxed{\nabla \times E = 0}$$

MAXWELL'S EQUATIONS

Fourth Maxwell's equation

$$\boxed{\nabla \times H = J}$$

Differential or point form

$$\boxed{\oint_L \boldsymbol{H}.dl = \int_S \boldsymbol{J}.d\boldsymbol{S}}$$

Integral form

AMPERE'S CIRCUIT LAW

$$\oint_L H.dl = I_{enc}$$

$$I_{enc} = \int_s J\,ds$$

Applying Stokes's theorem in LHS

$$\oint_L H.dl = \int_s (\nabla \times H)\,ds$$

$$\int_s (\nabla \times H)\,ds = \int_s J\,ds$$

$$\boxed{\nabla \times H = J}$$

MAXWELL'S EQUATIONS FOR STATIC FIELD

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \boldsymbol{D} = \boldsymbol{\rho_v}$	$\oint_S \boldsymbol{D}.d\boldsymbol{S} = \int_v \rho_v \, dv$	*Gauss's Law*
$\nabla \cdot \boldsymbol{B} = \boldsymbol{0}$	$\oint_S \boldsymbol{B}.d\boldsymbol{S} = 0$	*Nonexistence of Magnetic monopole*
$\nabla \times \boldsymbol{E} = \boldsymbol{0}$	$\oint_L \boldsymbol{E}.dl = 0$	*Conservative nature of electrostatic field*
$\nabla \times \boldsymbol{H} = \boldsymbol{J}$	$\oint_L \boldsymbol{H}.dl = \int_S \boldsymbol{J}.d\boldsymbol{S}$	*Ampere's law*

MAXWELL'S EQUATIONS IN FINAL FORMS

Differential Form	Integral Form	Remarks
$\nabla \cdot \boldsymbol{D} = \boldsymbol{\rho_v}$	$\oint_S \boldsymbol{D}.\,d\boldsymbol{S} = \int_v \rho_v \, dv$	*Gauss's Law*
$\nabla \cdot \boldsymbol{B} = \boldsymbol{0}$	$\oint_S \boldsymbol{B}.\,d\boldsymbol{S} = 0$	*Nonexistence of Magnetic monopole*
$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}$	$\oint_L \boldsymbol{E}.\,dl = -\int_S \frac{\partial \boldsymbol{B}}{\partial t} d\boldsymbol{S}$	*Faraday's Law*
$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$	$\oint_L \boldsymbol{H}.\,dl = \int_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right).\,d\boldsymbol{S}$	*Ampere's law*

ELECTRIC FILED : BOUNDARY CONDITIONS

If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called *boundary conditions*

To determine the boundary condition we need two standard equation

$$\oint \boldsymbol{E}.dl = 0$$

Maxwell's Equation

$$\oint \boldsymbol{D}.d\boldsymbol{s} = Q_{enc}$$

Gauss's Law

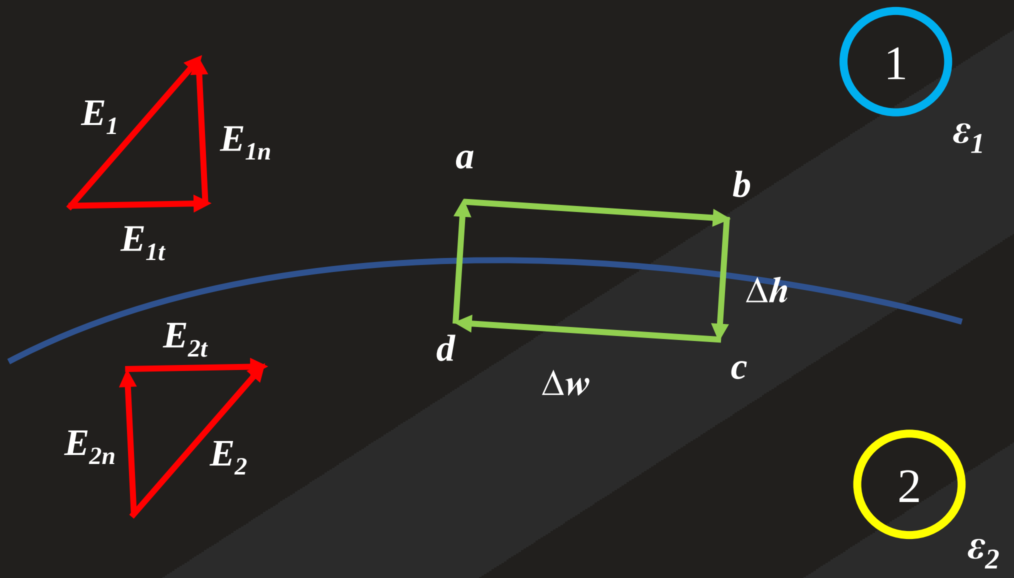
Also we consider electric field intensity E in to its two orthogonal component

$$E = E_t + E_n$$

$$E_t \rightarrow Tangential\ component$$

$$E_n \rightarrow Normal\ component$$

In Dielectric - Dielectric medium



$$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1} \qquad \varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$$

$$E_1 = E_{1t} + E_{1n} \qquad E_2 = E_{2t} + E_{2n}$$

Apply Maxwell's equation in *abcd* $\oint \boldsymbol{E}.dl = 0$

$$E_{1t}\Delta w + \left[E_{1n}\frac{\Delta h}{2} + E_{2n}\frac{\Delta h}{2}\right] - E_{2t}\Delta w - \left[E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2}\right] = 0$$

As $\Delta h \rightarrow 0$

$$0 = E_{1t} - E_{2t}$$

$$D = \varepsilon_0 E$$

$$\boxed{E_{1t} = E_{2t}}$$

Thus the tangential component of E are same on the two sides of the boundaries

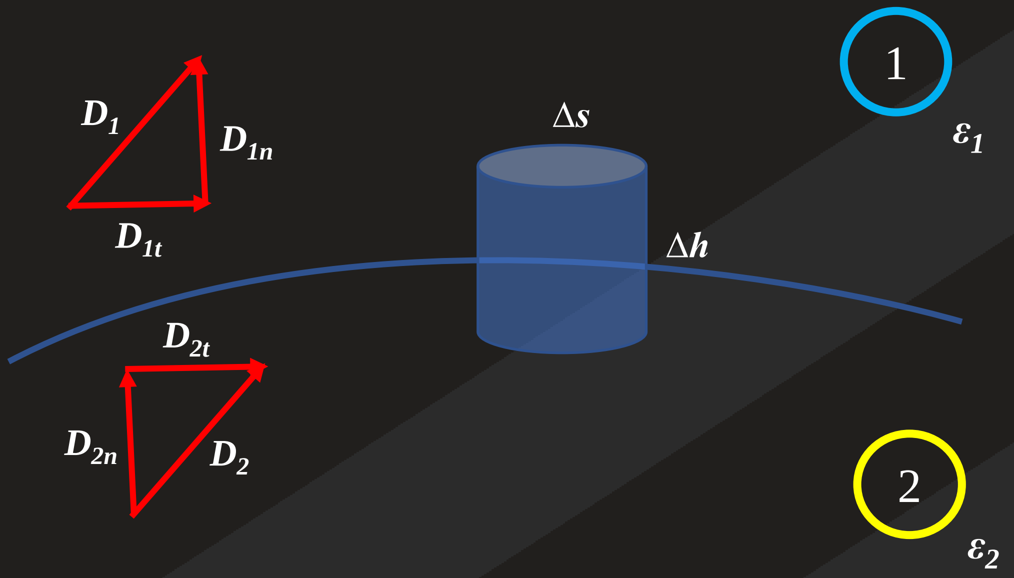
$$\boxed{\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}}$$

D undergoes some changes across the interface hence D_t is discontinuous

In Dielectric - Dielectric medium

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$ $\qquad$ $\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$

$D_1 = D_{1t} + D_{1n}$ $\qquad$ $D_2 = D_{2t} + D_{2n}$



From Gauss's Law

$$\oint_s D.ds = Q_{enc}$$

$$\Delta Q = \rho_s \Delta s = D_{1n}\Delta s - D_{2n}\Delta s$$

As $\Delta h \rightarrow 0$

$$\rho_s = D_{1n} - D_{2n}$$

If free charge exists at the interface (ρ_s=0)

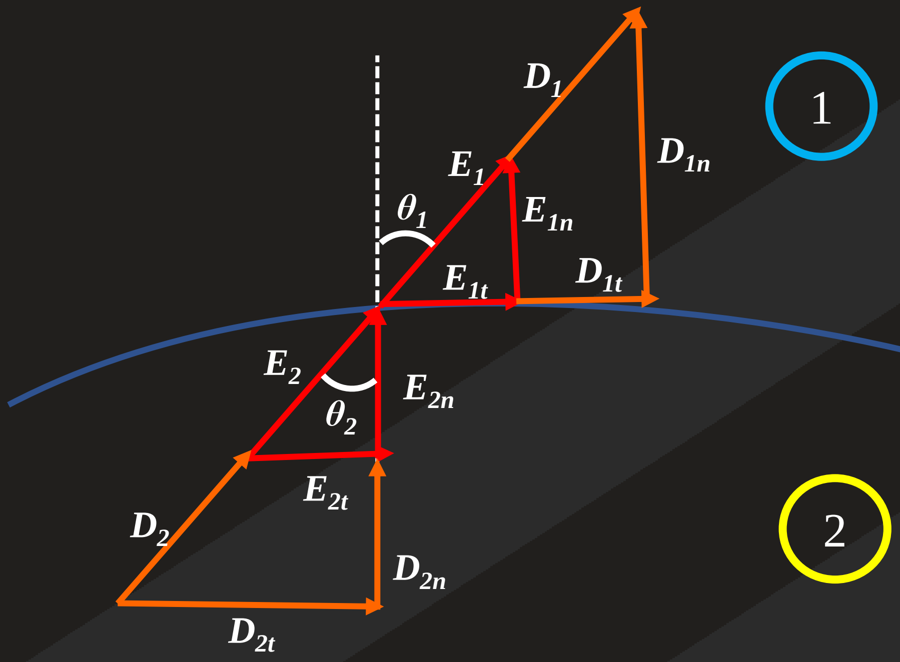
$$\boxed{D_{1n} = D_{2n}}$$

The normal component of D is continuous at boundaries

$$\boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}}$$

Thus the normal component of E discontinuous at boundaries

We can also use boundary conditions to determine the refraction of the electric field across the interface



From

$$\boxed{E_{1t} = E_{2t}}$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

From

$$\boxed{D_{1n} = D_{2n}}$$

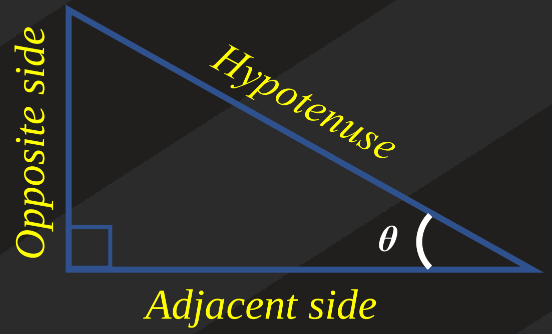
$$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

Law of refraction of electric filed
at a boundary free charge



$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

MAGNETIC FIELD : BOUNDARY CONDITIONS

Magnetic boundary conditions are the condition that H(or B) field must satisfy at the boundary between two different media

To determine the boundary condition we need two standard equation

$$\oint H.dl = I$$

Ampere's circuit law

$$\oint B.d\boldsymbol{s} = 0$$

Gauss's Law for magnetic field

Also we consider magnetic field intensity B in to its two orthogonal component

$$B = B_t + B_n$$

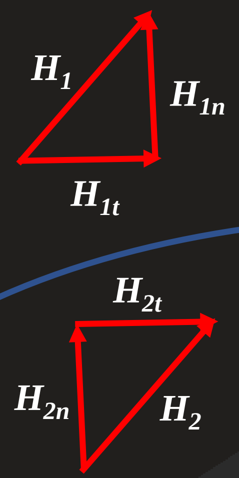
$$B_t \rightarrow Tangential\ component$$

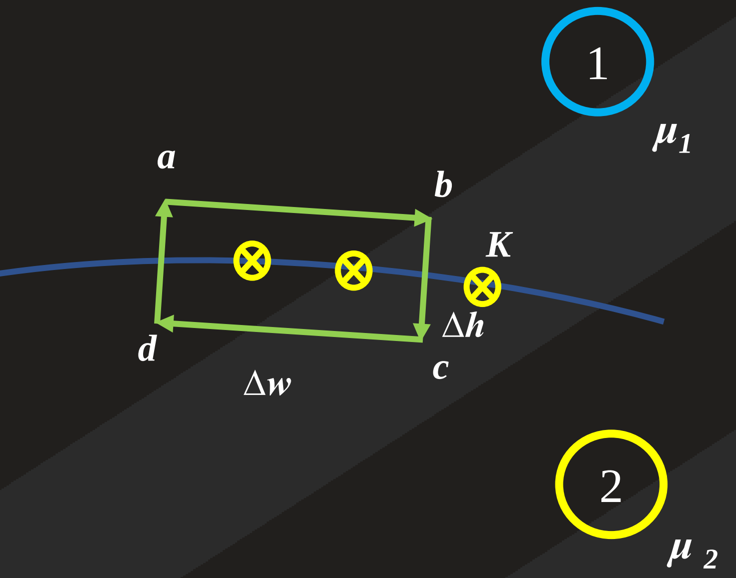
$$B_n \rightarrow Normal\ component$$

Boundary condition between two medium

$$\mu_1 = \mu_0\mu_{r1} \qquad \mu_2 = \mu_0\mu_{r2}$$

$$H_1 = H_{1t} + H_{1n} \qquad H_2 = H_{2t} + H_{2n}$$





Apply Ampere's circuit law in *abcd* $\oint \boldsymbol{H}.dl = I$

$$H_{1t}\Delta w + \left[H_{1n}\frac{\Delta h}{2} + H_{2n}\frac{\Delta h}{2}\right] - H_{2t}\Delta w - \left[H_{2n}\frac{\Delta h}{2} + H_{1n}\frac{\Delta h}{2}\right] = K\Delta w$$

As $\Delta h \rightarrow 0$

$$\mathrm{K} = H_{1t} - H_{2t}$$

When K=0

$$\boxed{H_{1t} = H_{2t}}$$

Thus the tangential component of H are continuous on the two sides of the boundaries

$$B = \mu_0 H$$

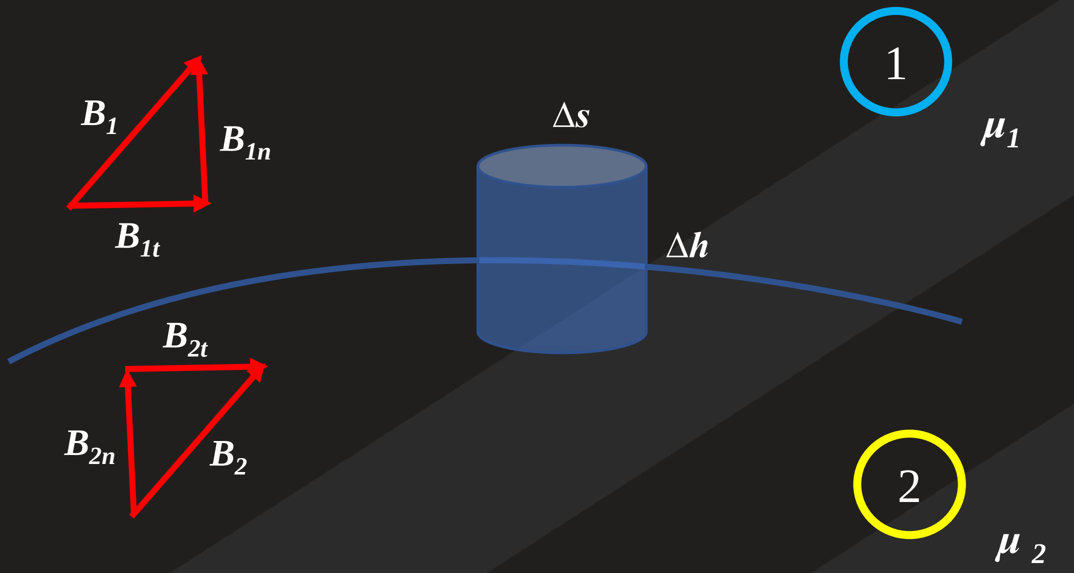
$$\boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

B is discontinuous at boundaries

Boundary condition between two medium

$\mu_1 = \mu_0\mu_{r1}$ $\mu_2 = \mu_0\mu_{r2}$

$B_1 = B_{1t} + B_{1n}$ $B_2 = B_{2t} + B_{2n}$



From Gauss's Law

$$\oint_s B.ds = 0$$

$$B_{1n}\Delta s - B_{2n}\Delta s = 0$$

As $\Delta h \rightarrow 0$

$$B_{1n} - B_{2n} = 0$$

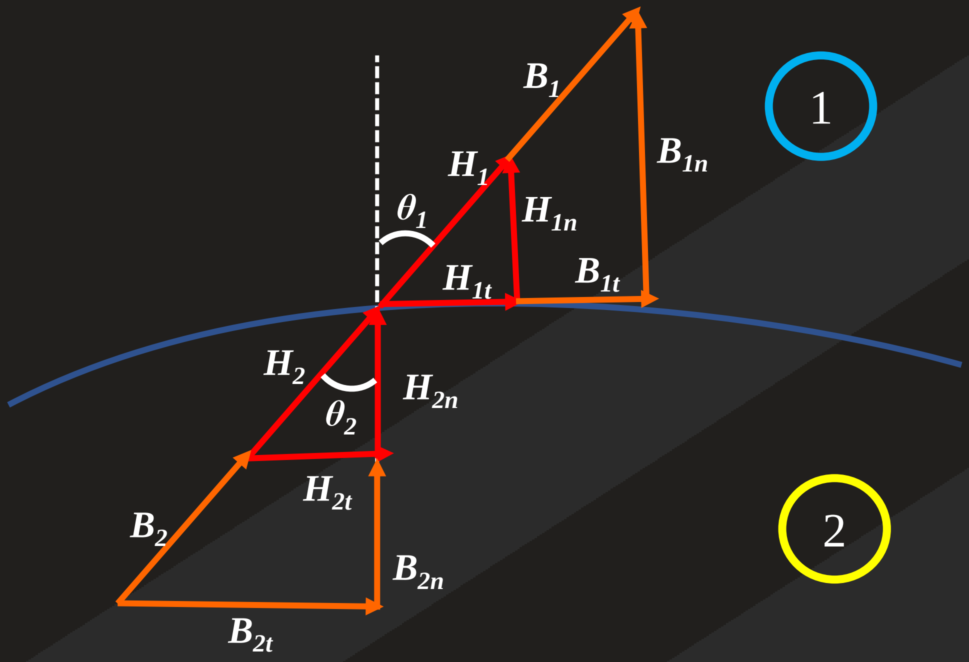
$$\boxed{B_{1n} = B_{2n}}$$

The normal component of B is continuous at boundaries

$$\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$$

Thus the normal component of H discontinuous at boundaries

If the field make an angle θ with the normal to the surface



From

$$\boxed{H_{1t} = H_{2t}}$$

From

$$\boxed{B_{1n} = B_{2n}}$$

$$H_1 \sin\theta_1 = H_2 \sin\theta_2$$

$$B_1 \cos\theta_1 = B_2 \cos\theta_2$$

$$\frac{B_1}{\mu_1}\sin\theta_1 = \frac{B_2}{\mu_2}\sin\theta_2$$

$$\frac{\tan\theta_1}{\mu_1} = \frac{\tan\theta_2}{\mu_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

Law of refraction of magnetic flux *lines at boundary with no surface current*

ELECTROMAGNETIC WAVE EQUATION

We can define a wave as a physical phenomenon in which some modification of a medium occurs at one place at a given time and an identical modification occurs at another place at a latest time in such away that the time delay is proportional to the space separation between the two place

Application of Maxwell's equation gives wave equation, assume magnetic & electric field that are present in a medium varying with time

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \quad \text{----------} \ (1)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}} \quad \text{----------} \ (2)$$

ELECTROMAGNETIC WAVE EQUATION

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad \text{---------- (1)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{---------- (2)}$$

$$\nabla \times H = \sigma E + \frac{\partial D}{\partial t} \quad \text{---------- (3)}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad \text{---------- (4)}$$

Taking curl of (4)

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t}(\nabla \times H) \quad \text{---------- (5)}$$

Substitute (3) in (5)

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\partial D}{\partial t}\right)$$

$$\nabla(\nabla . E) - \nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\varepsilon \partial E}{\partial t}\right)$$

At charge free medium $\nabla . E = 0$

$$-\nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\varepsilon \partial E}{\partial t}\right)$$

$$\boxed{\nabla^2 E = \sigma\mu \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}}$$

Similarly for magnetic field

$$\boxed{\nabla^2 H = \sigma\mu \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}}$$

For free space σ=0

$$\boxed{\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad \nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2}}$$

$$\boxed{\begin{aligned} \nabla^2 E_s &= j\omega\mu\,(\sigma + j\omega\varepsilon)E_s \\ \nabla^2 H_s &= j\omega\mu\,(\sigma + j\omega\varepsilon)H_s \end{aligned}}$$

$$\boxed{J = \sigma E}$$

$$\boxed{B = \mu H}$$

$$\boxed{\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A}$$

This is the general wave equation in terms of E & H

Wave equation in phasor from

SOLUTION OF WAVE EQUATION

- *To solve the wave equation we have to decide some constraints and condition under which whole system is working. in general we solve the wave equation for the electric and magnetic field or can be said that solved for vector potentials in both field.*
- *Firstly we go for electric field intensity **E** and magnetic field intensity **H** in a lossless medium*
- *Choose one dimensional wave equation in which electric field intensity **E** or magnetic field intensity **H** have a single component in spaces*
- *There are some conditions under which solution of wave equation is obtained*

a) *Considering that both fields are time harmonic*
b) *Electric field intensity **E** is directed in the x direction but it changes along the z direction, perpendicular to the direction of propagation*
c) *The medium of propagation of wave is lossless (σ=0)*
d) *There is no source present or source free wave equations (J=0, ρ_v=0)*