ECT305
Analog & Digital Communication

(S5 EC-B)

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Textbooks

1. “Communication Systems”, Simon Haykin, Wiley.
2. “Digital Communications: Fundamentals and Applications”, Sklar, Pearson.
3. “Digital Telephony”, John C. Bellamy, Wiley

Module 1 : Analog Communnication

- Analog Communication
- Block diagram of a communication system.
- Need for analog modulation.
- Amplitude modulation.
- Equation and spectrum of AM signal.
- DSB-SC and SSB systems.
- Block diagram of SSB transmitter and receiver.
- Frequency and phase modulation.
- Narrow and wide band FM and their spectra.
- FM transmitter and receiver.

Module 2 : Review of Random Variables and Random Processes

- Review of random variables – both discrete and continuous.
- CDF and PDF, statistical averages. (Only definitions, computations and significance)
- Entropy, differential entropy.
- Differential entropy of a Gaussian RV.
- Conditional entropy, mutual information.
- Stochastic processes, Stationarity.
- Conditions for WSS and SSS.
- Autocorrelation and power spectral density.
- LTI systems with WSS as input.

Module 3: Source Coding

- Source coding theorems I and II (Statements only).
- Waveform coding.
- Sampling and Quantization.
- Pulse code modulation, Transmitter and receiver.
- Companding. Practical 15 level A and mu-law companders.
- DPCM transmitter and receiver.
- Design of linear predictor.
- Wiener-Hopf equation.
- Delta modulation.
- Slope overload.

Module 4 : G-S Procedure and Effects in the Channel

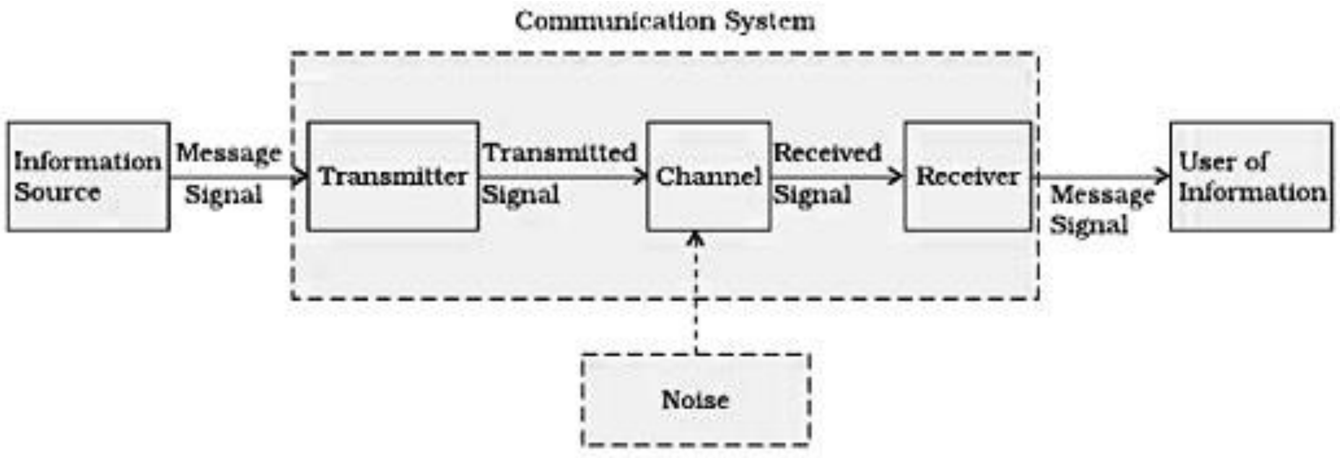
- Gram-Schmitt procedure.
- Signal space.
- Baseband transmission through AWGN channel.
- Mathematical model of ISI. Nyquit criterion for zero ISI.
- Signal modeling for ISI, Raised cosine and Square-root raised cosine spectrum, Partial response signalling and duobinary coding.
- Equalization.
- Design of zero forcing equalizer.
- Vector model of AWGN channel.
- Matched filter and correlation receivers.
- MAP receiver, Maximum likelihood receiver and probability of error.
- Capacity of an AWGN channel (Expression only) -- significance in the design of communication schemes.

Module 5 : Digital Modulation Schemes

- Digital Modulation Schemes
- Baseband BPSK system and the signal constellation.
- BPSK transmitter and receiver.
- Base band QPSK system and Signal constellations.
- Plots of BER Vs SNR with analysis.
- QPSK transmitter and receiver.
- Quadrature amplitude modulation and signal constellation.

MODULE 1

Basic Blocks of a Communication System



- Information signals generates a modulating signal which is low frequency signal i.e. our message signal (20 Hz – 20 KHz)

- In order to transmit the msg signal we need to process the info signal before sending it. All this processes takes place at Transmitter section.

- The low freq signal can't be transmitted to long distances. Hence need to increase the freq.

Modulation and its Need

Modulation : Process of superimposing a low frequency signal on a high freq signal.
Process of changing the characteristics (amp,freq,phase) of a high freq carrier s/g based on the instantaneous amplitude of the low freq msg s/g.

A modulating signal/baseband signal – represents the message.

carrier signal – depends on type of modulation.

Need for Modulation

- Baseband signals are incompatible for direct transmission over the medium so, modulation is used to transmit (baseband) signals from one place to another.
- Allows frequency translation.
- Increase range of communication .
- Reduce the antenna height .
- Avoids mixing of signals.
- Power radiated by antenna can be increased.
- Reduced noise and interference

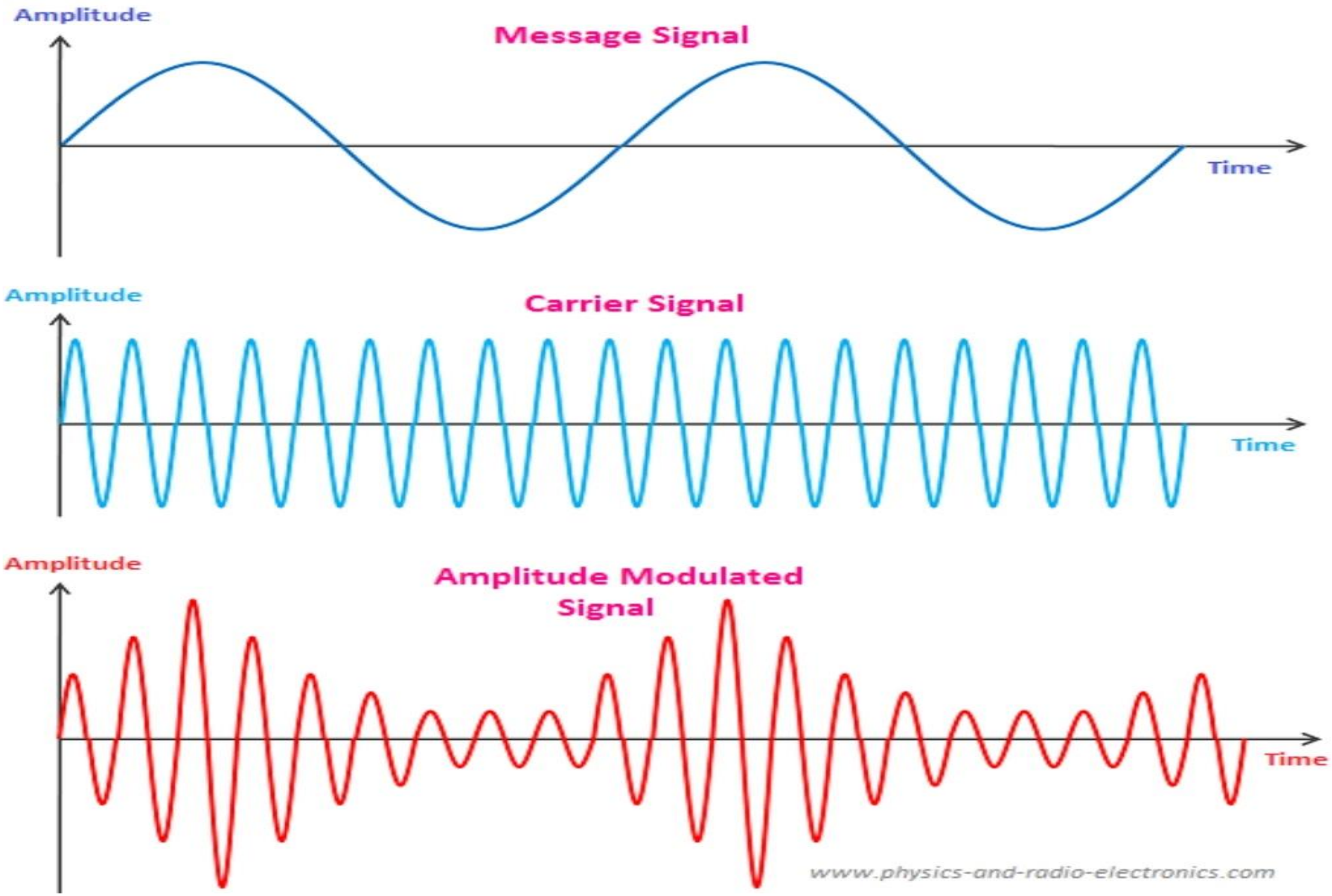
Types of Modulation

- Analog Modulation
  - Amplitude Modulation
  - Frequency Modulation
  - Phase Modulation

- Digital Modulation

Amplitude Modulation

- The process of changing the amplitude of the high freq signal according to the instantaneous amplitude of the low freq signal (Phase and Freq are kept constant)



AM signal : Mathematical Representation

$V_C(t) = V_C \sin \omega_c t$

$V_m(t) = V_m \sin \omega_m t$

$$\omega_c = 2\pi f_c$$

$$\omega_m = 2\pi f_m$$

V_c = peak carrier amplitude

V_m = peak msg signal amplitude

ω_m & ω_c = carrier signal radiating freq and modulating signal freq

Amplitude pf Modulating Signal (A)

$A = V_c + V_m(t)$

$A = V_c + V_m \sin \omega_m t$

AM signal : Mathematical Representation

$V_C\,(t) = V_C \text{ Sin } \omega_c t$

$V_m\,(t) = V_m \text{ Sin } \omega_m t$

The amplitude modulated signal (AM Signal) is

$$
\begin{aligned}
V_{am}\,(t) &= A[\text{Sin } w_c t] \\
&= [V_c + V_m \text{ Sin} w_m t][\text{Sin} w_c\, t] \\
&= V_c\,[\,1 + V_m/V_c \text{ Sin} w_m\, t]\,[\text{Sin} w_c\, t] \\
&= V_c[1 + m\text{Sin} w_m\, t][\text{Sin} w_c\, t]
\end{aligned}
$$

where m is the modulation index

$V_{am}\,(t) = V_c \text{ Sin } w_c t + mV_c \text{ Sin } w_c\, t \text{ Sin } w_m\, t$

AM signal : Mathematical Representation

Now using trigonometric eqn 2 sin A sin B = cos (A-B) – cos (A + B)

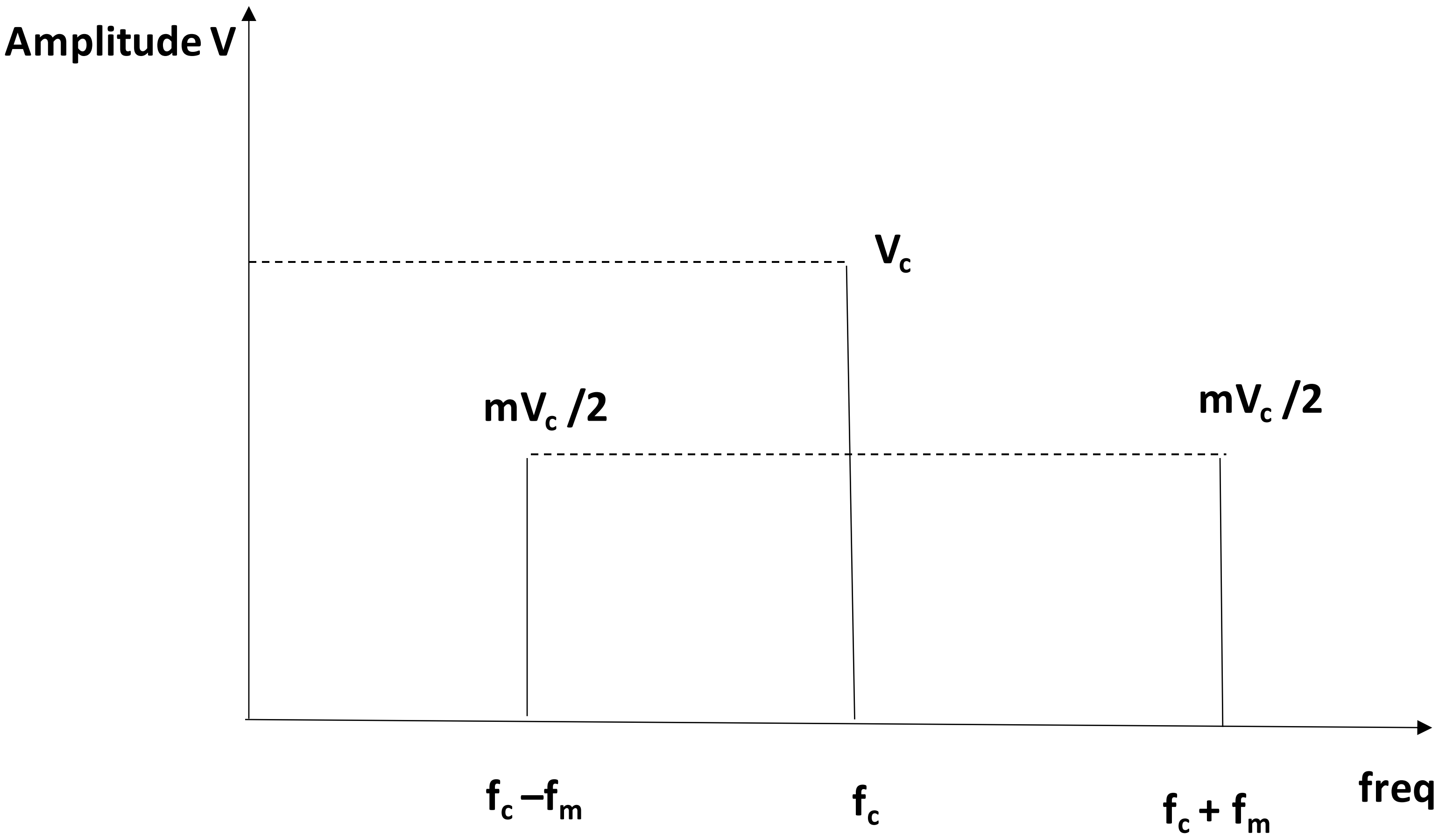
$$V_{am}(t) = V_c \text{ Sin } w_c t + mV_c \frac{\{\text{Cos }(w_c - w_m)t - \text{Cos }(w_c + w_m)t\}}{2}$$

$$= V_c \text{ Sin } w_c t + mV_c/2 \text{ Cos }(w_c - w_m)t - mV_c/2 \text{ Cos }(w_c + w_m)t$$

V_c Sin w_c t = carrier freq component

mV_c /2 Cos ($w_c - w_m$)t = lower side band freq

mV_c /2 Cos ($w_c + w_m$)t = Upper Sideband freq



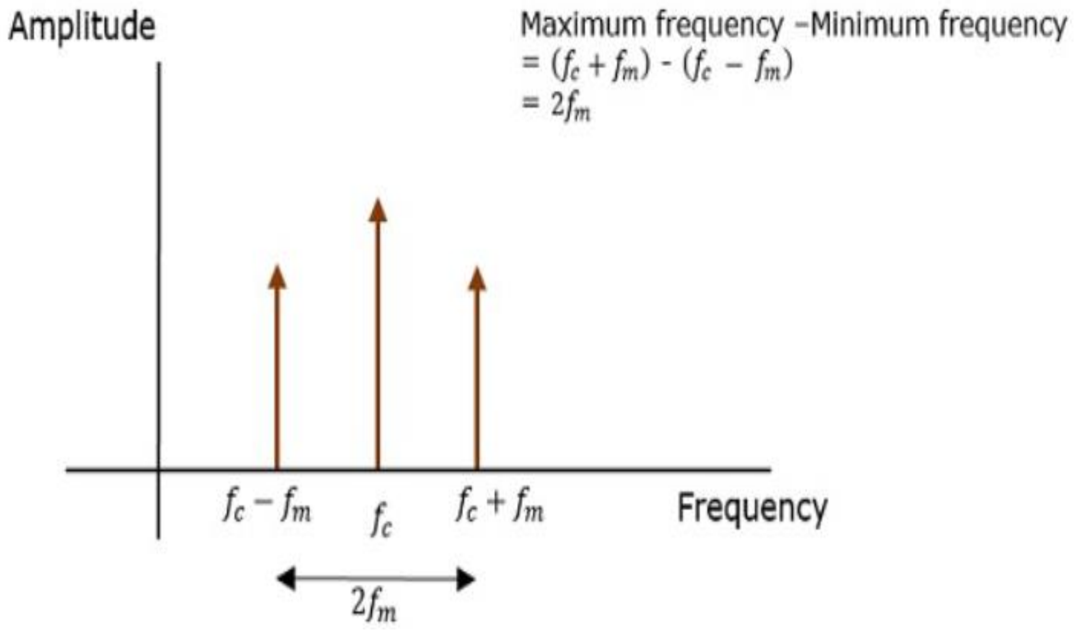
Freq spectrum of AM wave

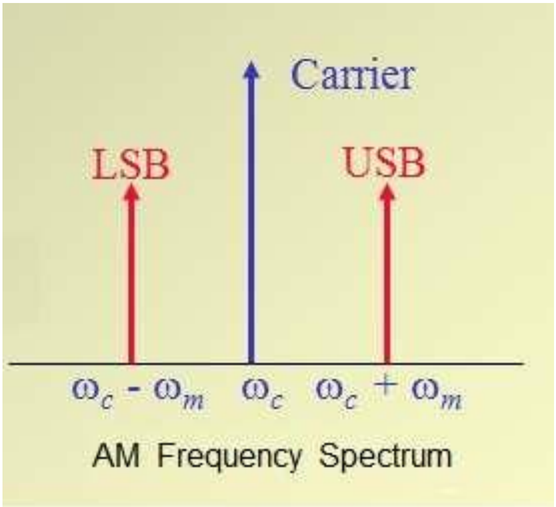
- Band width = $f_c + f_m - (f_c - f_m)$

  $= 2f_m$ (freq msg signal – f_m)

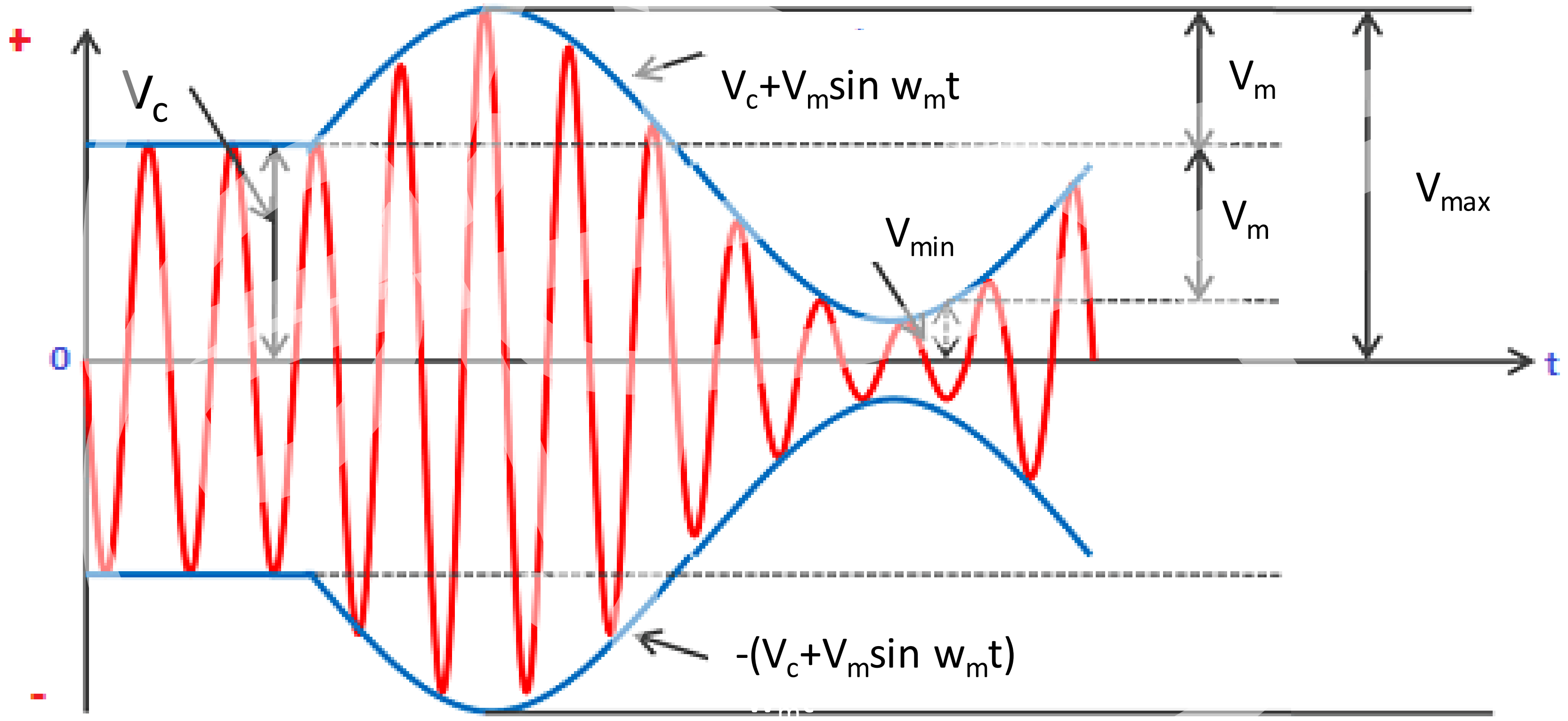
BW of an AM signal = 2 * freq of msg signal

Freq spectrum of AM wave





- Freq spectrum of a waveform consist of all the freqs contained in the wave form and their amplitude plotted in freq domain



+
-
0
t
V_c
$V_c+V_m \sin w_m t$
$-(V_c+V_m \sin w_m t)$
V_m
V_m
V_{max}
V_{min}

Modulation Index

$$m = V_m / V_c$$

$$2V_m = V_{max} - V_{min}$$

$$V_m = (V_{max} - V_{min}) / 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$V_c = V_{max} - V_m \qquad \text{Sub in (1)}$$

$$= V_{max} - (V_{max} - V_{min}) / 2$$

$$V_c = (V_{max} + V_{min}) / 2$$

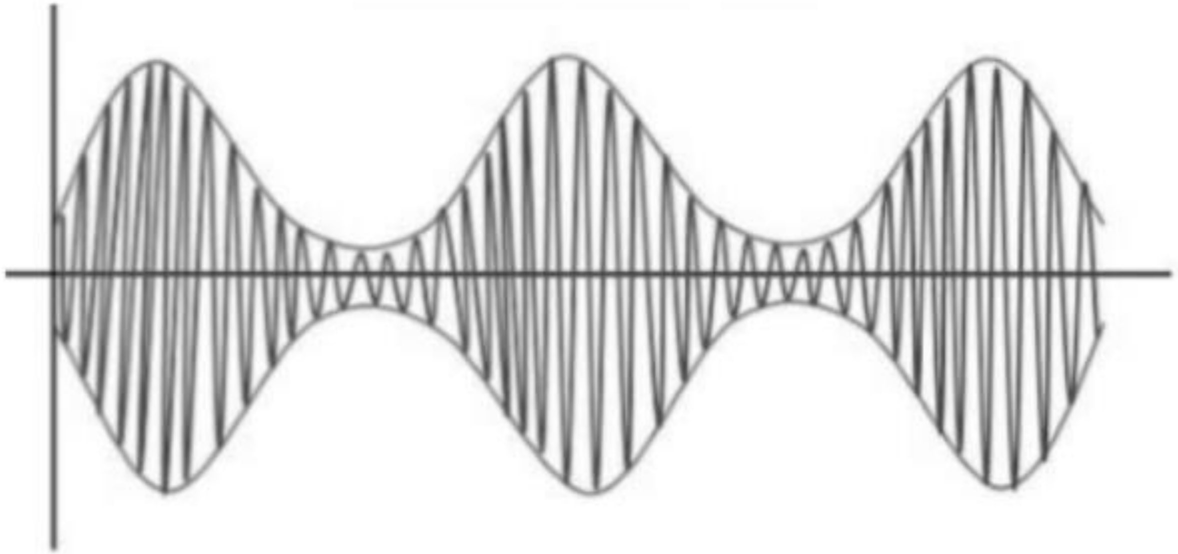
$$m = V_m / V_c = \frac{(V_{max} - V_{min}) / 2}{(V_{max} + V_{min}) / 2}$$

$$\boxed{m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

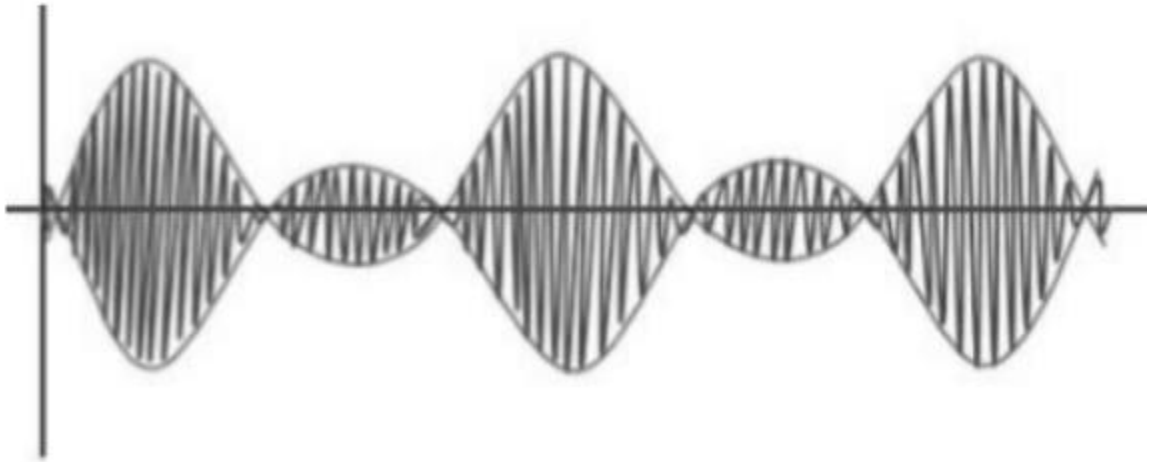
Q. Calculate m and draw the waveform.

1. Given that the msg signal amp V_m = 2V and carrier signal amp V_c = 4V
2. Given that the msg signal amp V_m = 4V and carrier signal amp V_c = 4V
3. Given that the msg signal amp V_m = 6V and carrier signal amp V_c = 4V

Under-Modulated wave



Over-Modulated wave



Power Distribution in AC Signal

- The total power of the AM waveform is the sum of carrier and side band powers.
- An AM wave consists of carrier and two sidebands. The carrier component of the modulated wave has the same amplitude as the unmodulated carrier.
- The modulated wave contains extra power in the two sideband components. The amplitude of the sidebands depends on the modulation index 'm'.
- Therefore the total power in the modulated wave will depend on the modulation index too.

The total power in the modulated , P_t = [carrier power]+ [power in LSB] + [power in USB]

$$P_t = Pc + P_{LSB} + P_{USB}$$

$$= V_{Carrier}{}^2/R + V_{USB}{}^2/R + V_{LSB}{}^2/R \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

carrier Power $P_c = V_{rms}{}^2/R$ Since V_c is peak value

$$= (V_c/\sqrt{2})^2 / R$$

$$P_c = V_c{}^2/2R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Similarly $P_{USB} = P_{LSB} = V_{SBrms}{}^2/R$

$$= \frac{[(mV_c/2)/\sqrt{2}]^2}{R} = m^2V_c^2/8R$$

Power Distribution in AC Signal

$$P_{USB} = P_{LSB} = (m^2/4)(V_c^2/2R) \quad \text{.............................} (3)$$

substituting in eqn (1)

Total power $P_t = V_C^2/2R + m^2V_c^2/8R + (m^2/4)(V_c^2/2R)$

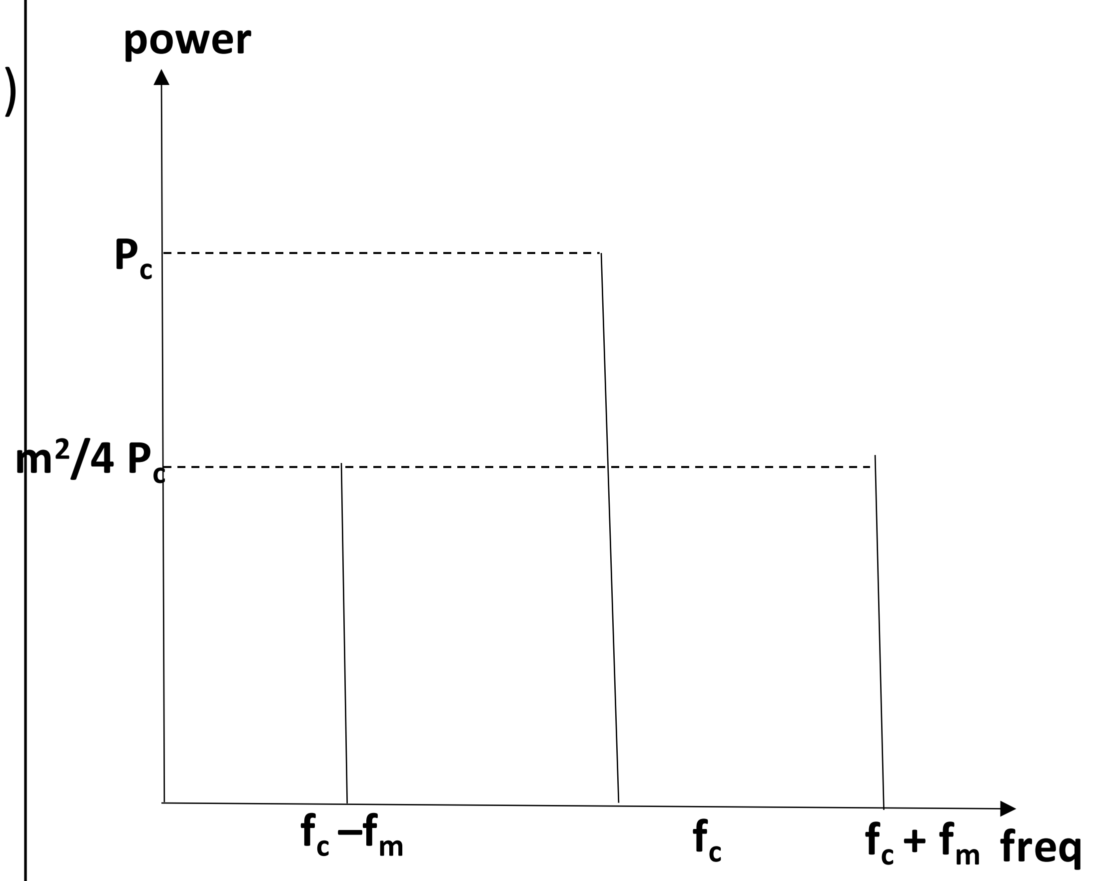
$$= P_c + m^2/4\, P_c + m^2/4\, P_c$$

$$= P_c + m^2/2\, P_c$$

$$= (1 + m^2/2)\, P_c$$

$$\boxed{P_t = P_c(1 + m^2/2)}$$

if it's 100% modulated signal , $P_t = 3P_c/2 = 1.5\, P_c$



Current Eqn of a AM Signal

$P_t = P_c\ (1 + m^2/2)$

$P_t / P_c = I_t^{\,2} R / I_c^{\,2} R \quad = (1 + m^2/2)$

$I_t = I_c \sqrt{1 + (m^2/2)}$

In case multiple signals are modulated at a certain time

Multiple modulation index = $m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + m_4^2 + m_5^2 \ldots\ldots}$

Transmission efficiency of AM wave

- Ratio of txd power which contains information to the total power
- $\eta = (P_{LSB}+P_{USB})/P_t$
- $\eta = \dfrac{2 * m^2/4\ P_c}{[1+m^2/2]P_c}$

$$\eta = m^2/(2+m^2)$$

% transmission efficiency

$$\%\ \eta = m^2/(2+m^2)\ *100\%$$

Amplitude Modulation Technique

- DSB – FC – Double Side Band Full Carrier Method
- DSB –SC – Double Sideband suppressed carrier method
- SSB
- ISB
- USB

- Double Sideband with carrier (AM): This is the most widely used type of AM modulation
- Double Sideband Suppressed Carrier (DSBSC): This is the same as the AM modulation above but without the carrier.
- Single Sideband (SSB): In this modulation, only half of the signal of the DSBSC is used.
- Vestigial Sideband (VSB): This is a modification of the SSB to ease the generation and reception of the signal.

Amplitude Modulation Technique

- In DSBC modulation, the modulated wave consists of only the upper and lower side bands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is the same as before.

- The SSBSC modulated wave consists of only the upper side band or lower side band. SSBSC is suited for transmission of voice signals. It is an optimum form of modulation in that it requires the minimum transmission power and minimum channel band width. Disadvantage is increased cost and complexity.

- In VSB, one side band is completely passed and just a trace or vestige of the other side band is retained. The required channel bandwidth is therefore in excess of the message bandwidth by an amount equal to the width of the vestigial side band. This method is suitable for the transmission of wide band signals.

DSBSC Modulation

- In the process of AM, the modulated wave consists of the carrier wave and two sidebands.
- The modulated wave has the information only in the sidebands.
- **Sideband** is nothing but a band of frequencies, containing power, which are the lower and higher frequencies of the carrier frequency.
- The transmission of a signal, which contains a carrier along with two sidebands can be termed as **Double Sideband Full Carrier** system or simply **DSBFC**.

- However, such a transmission is inefficient. Because, two-thirds of the power is being wasted in the carrier, which carries no information.

  If this carrier is suppressed and the saved power is distributed to the two sidebands, then such a process is called as **DSBSC**.

- Mathematically we can represent DSBSC as product of modulating and carrier s/gs.
- $S(t)= V_m \text{ Sin } \omega_m t * V_c \text{ Sin } \omega_c t$

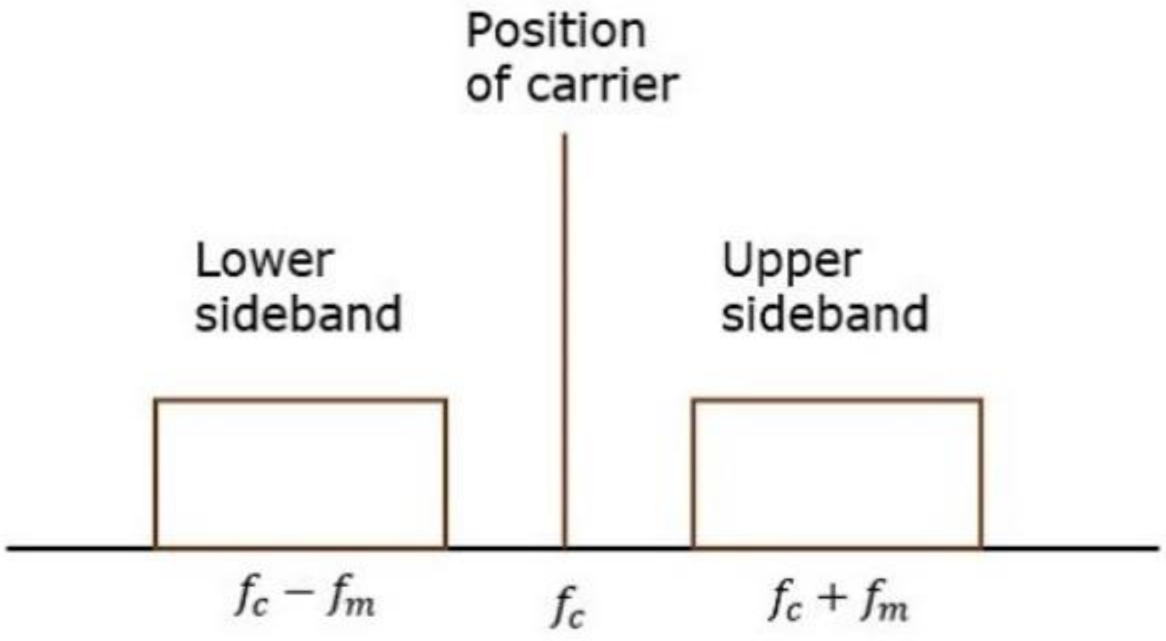
$$2 \sin A \sin B=\cos(A-B)-\cos(A+B)$$

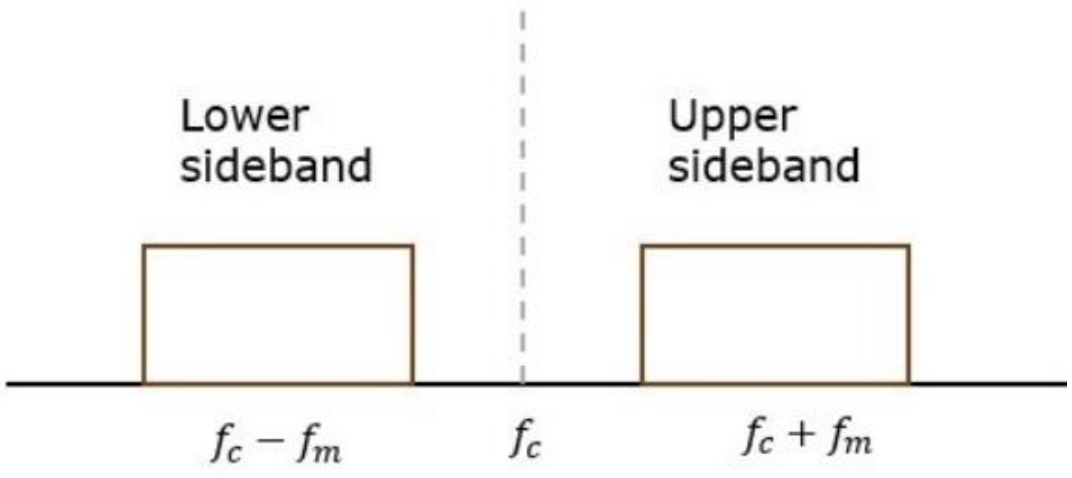
- $S(t) = V_m V_c/2 * \{\cos(\omega_c - \omega_c)t - \cos(\omega_c + \omega_c)t\}$

$$=V_m V_c /2 \text{ Cos } (w_c - w_m)t - V_m V_c /2 \text{ Cos } (w_c + w_m)t$$

DSBSC wave

- $s(t) = V_m V_c / 2 \text{ Cos } (w_c - w_m) t \; - V_m V_c / 2 \text{ Cos } (w_c + w_m) t$





Carrier is suppressed and sidebands are allowed for transmission

- bandwidth of DSBSC wave is same as that of AM wave and it is equal to twice the frequency of the modulating signal.
- BW= $2f_m$
- Power of DSBSC wave is equal to the sum of powers of upper sideband and lower sideband frequency components.

$$P_t = \frac{m^2 V_c^2}{4R}$$

- Therefore, the power required for transmitting DSBSC wave is equal to the power of both the sidebands.

Advantages of DSB-SC modulation

1. It provides 100% modulation efficiency.
2. Due to suppression of carrier, it consumes less power.
3. It provides a larger bandwidth.

Disadvantages of DSB-SC modulation

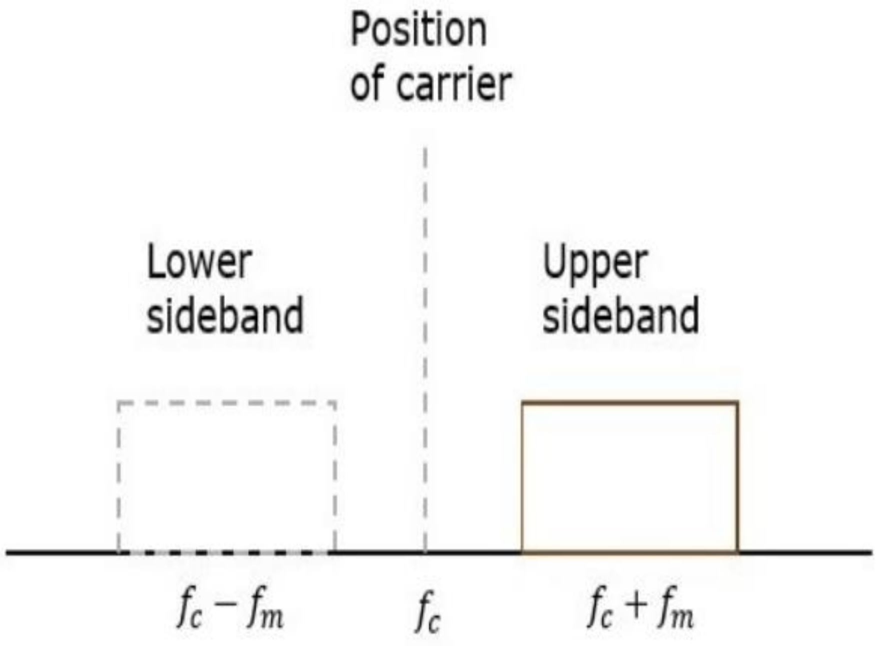
1. It involves a complex detection process.
2. Using this technique it is sometimes difficult to recover the signal at the receiver.
3. It is an expensive technique when it comes to demodulation of the signal.

Applications of DSB-SC modulation

1. During the txn of binary data, DSB-SC system is used in phase shift keying methods.
2. In order to transmit 2 channel stereo signals, DSB signals are used in Television and FM broadcasting.

Single Side Band Modulation(SSB)

- Transmission in which only one sideband is transmitted is called single-sideband transmission or SSB.
- Carrier and one sideband are completely suppressed.
- An amplitude modulated signal will have two frequency-shifted copies of the modulated signal
- These are known as sidebands.
- The DSBSC modulated signal has two sidebands. Since, the two sidebands carry the same information, there is no need to transmit both sidebands. We can eliminate one sideband.



Carrier and a sideband are suppressed and a single sideband is allowed for transmission

- The process of suppressing one of the sidebands along with the carrier and transmitting a single sideband is called as **Single Sideband Suppressed Carrier** system or simply **SSBSC**
- In this fig., the carrier and the lower sideband are suppressed. Hence, the upper sideband is used for transmission. Similarly, we can suppress the carrier and the upper sideband while transmitting the lower sideband
- This SSBSC system, which transmits a single sideband has high power, as the power allotted for both the carrier and the other sideband is utilized in transmitting this Single Sideband.

Mathematical expression for SSBSC wave

- Lower sideband $s(t) = mV_c /2 \, Cos \, (w_c - w_m)t$
- Upper sideband $s(t) = mV_c /2 \, Cos \, (w_c + w_m)t$

Bandwidth of SSBSC Wave

BW = freq of modulating s/g ie, f_m

Power Calculations of SSBSC Wave

- $P_t = P_{USB} = P_{LSB}$

$$P_t = m^2 V_c^2 \ /8R$$

Advantages

- BW or spectrum space occupied is lesser than AM and DSBSC waves. As only 1 SB is txd, the receiver BW can be reduced by half. This improves the signal to noise ratio by a factor of 2 because the narrower BW used will allow thru less noise and interference.
- Txn of more number of s/gs is allowed.
- Power is saved. As only 1 SB is transmitted, there is a further reduction in transmitter power. As the carrier is not transmitted, this enables a 50% reduction in transmitter power level for the same level of information carrying signal
- Less amount of noise is present.
- Signal fading is less likely to occur.

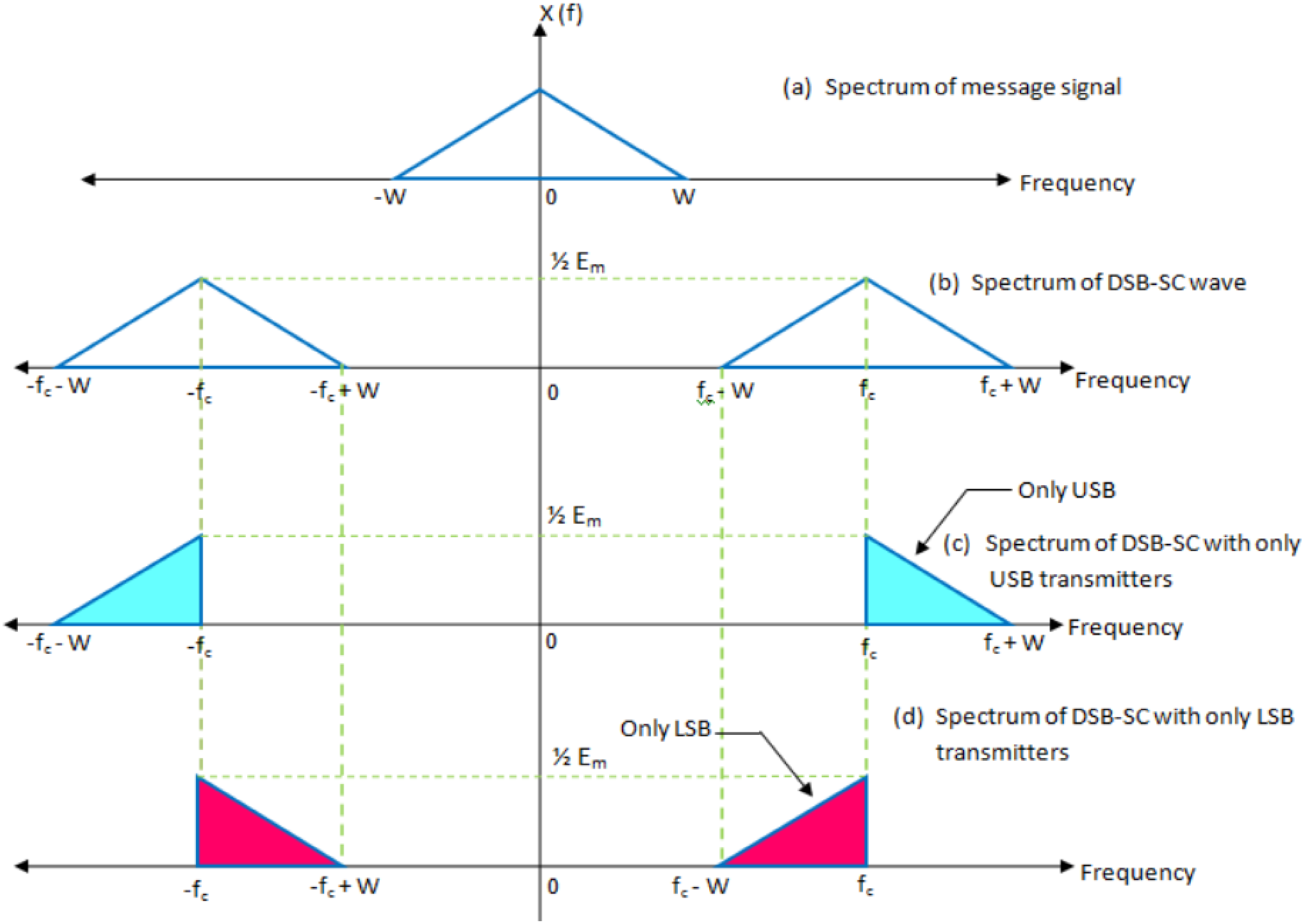
Disadvantages

- The generation and detection of SSBSC wave is a complex process. Hard to demodulate.
- The quality of the signal gets affected unless the SSB transmitter and receiver have an excellent frequency stability.

Single Side Band Modulation(SSB)

Applications

- two way radio communication.
- voice transmission.
- For power saving requirements and low BW requirements.
- In land, air, and maritime mobile communications.
- In point-to-point communications.
- In television, telemetry, and radar communications.
- In military communications

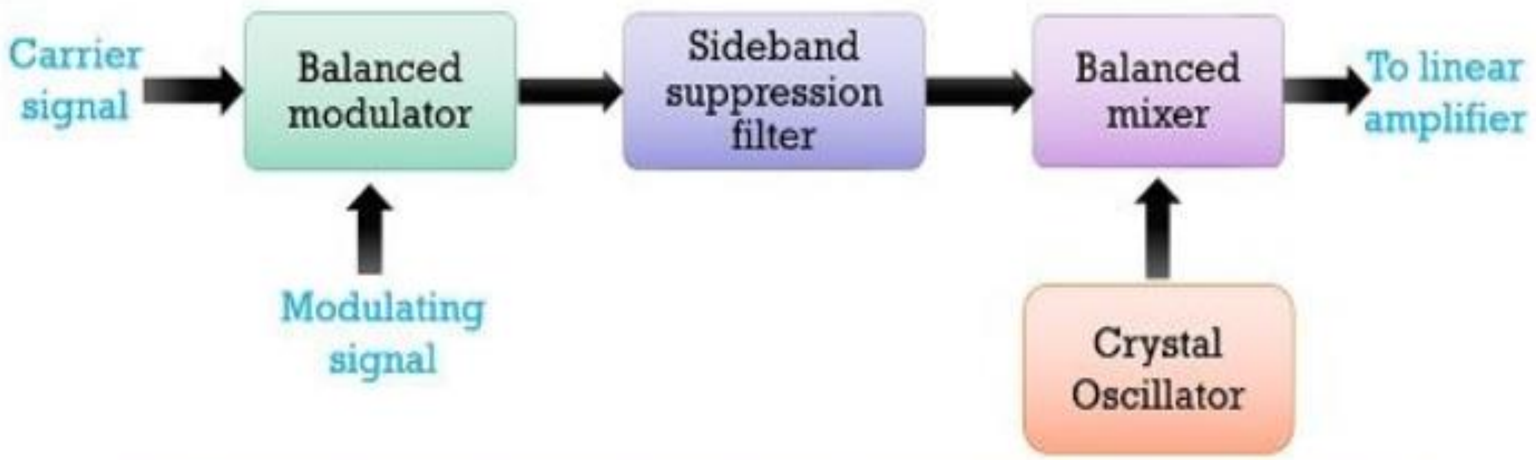


In this , replace w by f_m and E_m by V_m

Generation of SSB Signal (Transmitter)

- Filter method or Frequency discrimination method
- Phase Shift Method
- The Third method or Weaver’s method

Generation of SSB Signal : Filter Method



Block diagram of Filter method for Sideband suppression

- The balanced modulator employed here generates DSB-SC amplitude modulated wave as its output.
- As the DSB output contains the two sidebands, and only carrier component is suppressed, so sideband suppression filter is needed further in order to eliminate one of the 2 sidebands.

- The **filter characteristics** should be such that, it must have **flat passband** and should possess **high attenuation** beyond the passband. So, to have such a response, the tuned ckt must have a very **high Q factor**.
- To have such a high Q factor it is needed that the diff btw modulating freq and carrier freq. to be high. There is no any practical way to achieve such a high value.
- Thus, modulation at the initial stage is carried out at a low freq of about **100 KHz** by the balanced modulator. After this one SB is suppressed by the filter. But, as the SSB signal freq is very low in comparison to txr freq. So, a balanced mixer and crystal oscillator are employed in the circuit to boost the frequency of SSB signal up to the level of txr freq.

- Then the SSB signal is fed to a linear amplifier for further amplification. The process of frequency boosting is sometimes also termed as **Up-conversion**.
- Basically to eliminate unwanted sideband- LC, ceramic, crystal or mechanical filter are used.

➢Though ceramic or crystal filters are low in cost but provides better results at operating frequency above 1 MHz.

➢LC filter is the simplest one but cannot be used beyond 100KHz.ie, above this freq, attenuation outside the passband is insufficient.

➢Among all these, mechanical filters possess the best characteristics .Thus is widely used. Upper freq limit of this is 500KHz

Advantages of Filter method:

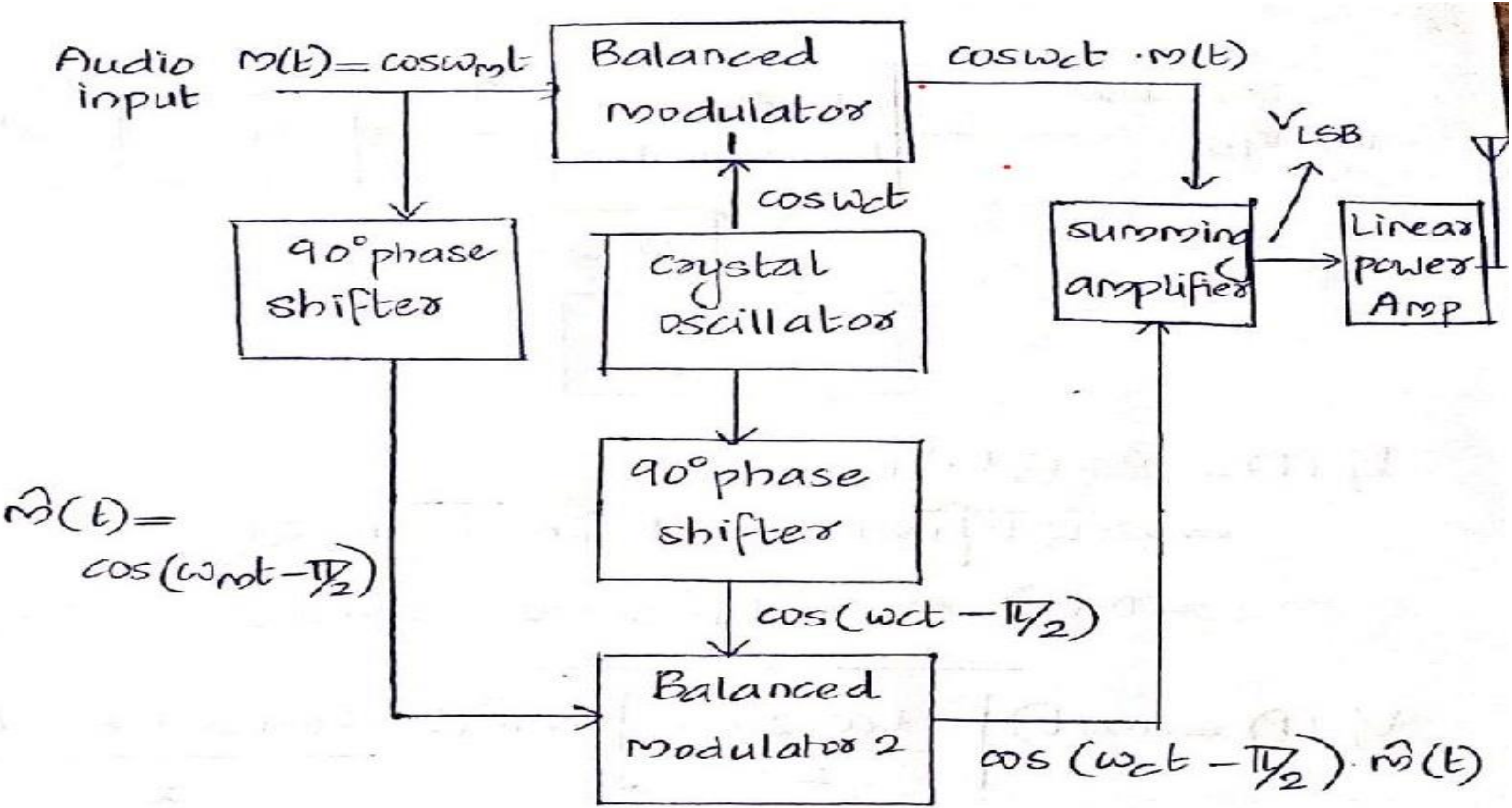
1. It provides sufficiently flat and wide bandwidth.
2. By this method, we can have suitable sideband suppression.

Disadvantages of Filter method:

1. Frequency up-conversion at the end is necessary as the system does not generate SSB at high frequencies.
2. Expensive filter increases the overall cost of the system.

Q. A carrier wave is represented by the eqn V_c (t) = 10 Sin w_c t . Draw the wave form of an AM wave for modulation index m = 0.5

Generation of SSB Signal : Phase Shift Method



Generation of SSB Signal : Phase Shift Method

V_{LSB} = [Cos w_c t.m(t) + Cos (w_ct - ∏/2) m^(t)]
(o/p of bal Mod 1) (o/p of bal Mod 2)

= Cos w_c t . Cos w_m t + [Cos (w_ct - ∏/2) Cos (w_m t - ∏/2)]

= Cos w_c t . Cos w_m t + Sin w_c t .Sin W_m t

= Cos (w_c – w_m)t

cos(A-B)= cosA cosB + sinA sinB

For V_{USB} generation, we use difference amplifier instead of summing amplifier

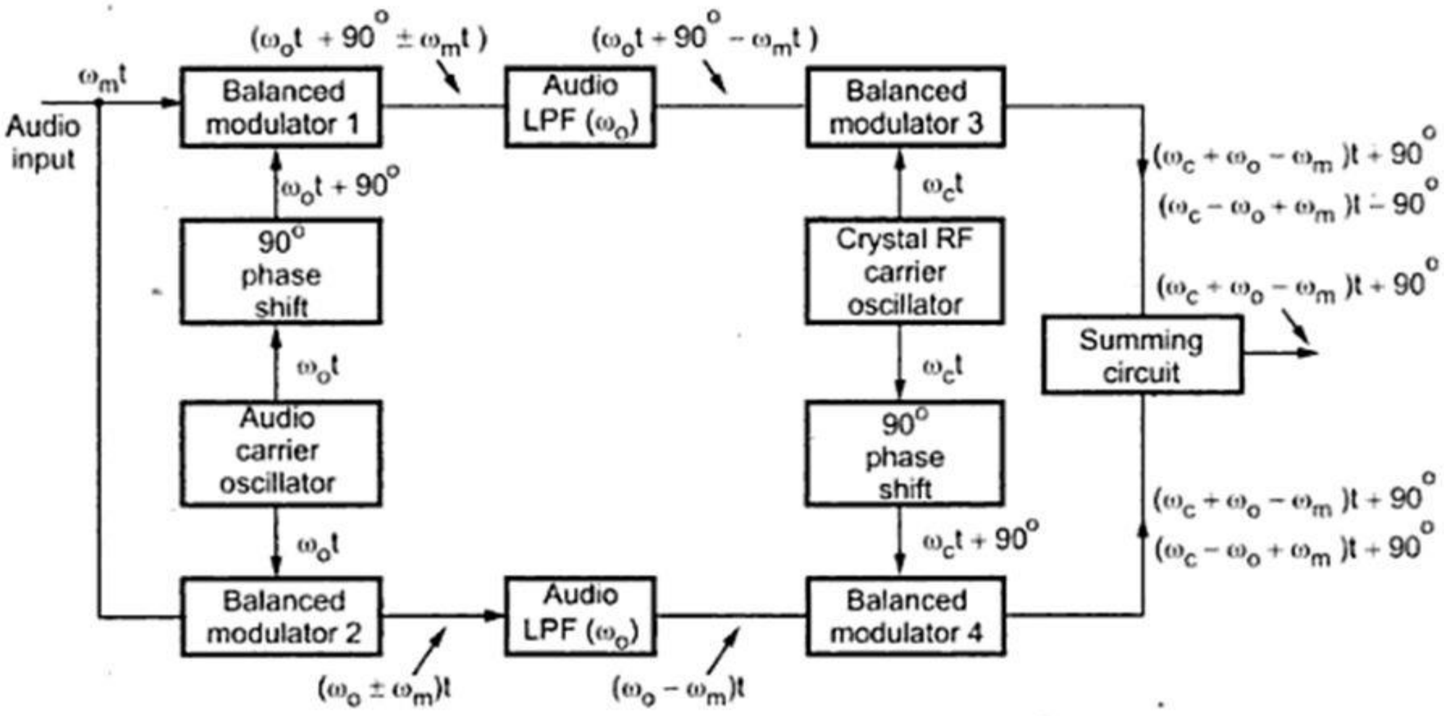
- V_{USB} = [Cos w_c t.m(t) - Cos (w_ct - ∏/2) m^(t)]
  =Cos (w_c + w_m)t

cos(A+B)= cosA cosB - sinA sinB

The Third method or Weaver's method

SUPPRESSION OF UNWANTED SIDEBAND TO GENERATE SSB-SC

Weavers method



- Similar to phase shift method but modulating s/g is first modulated on a low freq subcarrier $\boldsymbol{\omega_o}$ which is then modulated by a high freq carrier $\boldsymbol{\omega_c}$
- Balanced modulators BM1 and BM2 has unshifted modulating s/g as i/p
- BM1 also takes low freq subcarrier ω_o with a phase shift of 90 deg introduced in it
- BM2 takes low freq subcarrier s/g directly from the oscillator
- Assuming unity magnitudes and sinusoidal single freq modulation , o/p from BM1 is

- e_{BM1}= cos (ω_ot+90)cos ω_mt

2 Cos A Cos B = Cos (A + B) + Cos (A - B)

e_{BM1}=1/2 * {cos (ω_ot+ω_mt+90) + cos (ω_ot-ω_mt+90) }

Output of BM2 becomes

e_{BM2}= cosω_ot cos ω_mt

e_{BM2}=1/2 * {cos (ω_ot+ω_mt) + cos (ω_ot-ω_mt) }

- LPF with a cut off freq set at the subcarrier freq ω_o to remove the first term from each of the above s/gs (sum term) leaving only the second term(difference) as inputs to BM3 and BM4
- BM3 also takes high freq carrier ω_c directly from the oscillator
- BM4 takes high freq carrier s/g with a phase shift of 90 deg introduced in it

o/p of BM3

- $e_{BM3} = \cos \omega_c t \cos ((\omega_0 - \omega_m) t+90)$

  $e_{BM3}=1/2 * \{\cos (\omega_c t+ ((\omega_0 - \omega_m) t+90))+ \cos (\omega_c t- ((\omega_0 - \omega_m) t+90))\}$

  $e_{BM3}=1/2 * \{\cos ((\omega_c +\omega_0)t - \omega_m t+90)+ \cos ((\omega_c -\omega_0)t+\omega_m t-90)\}$

o/p of BM4

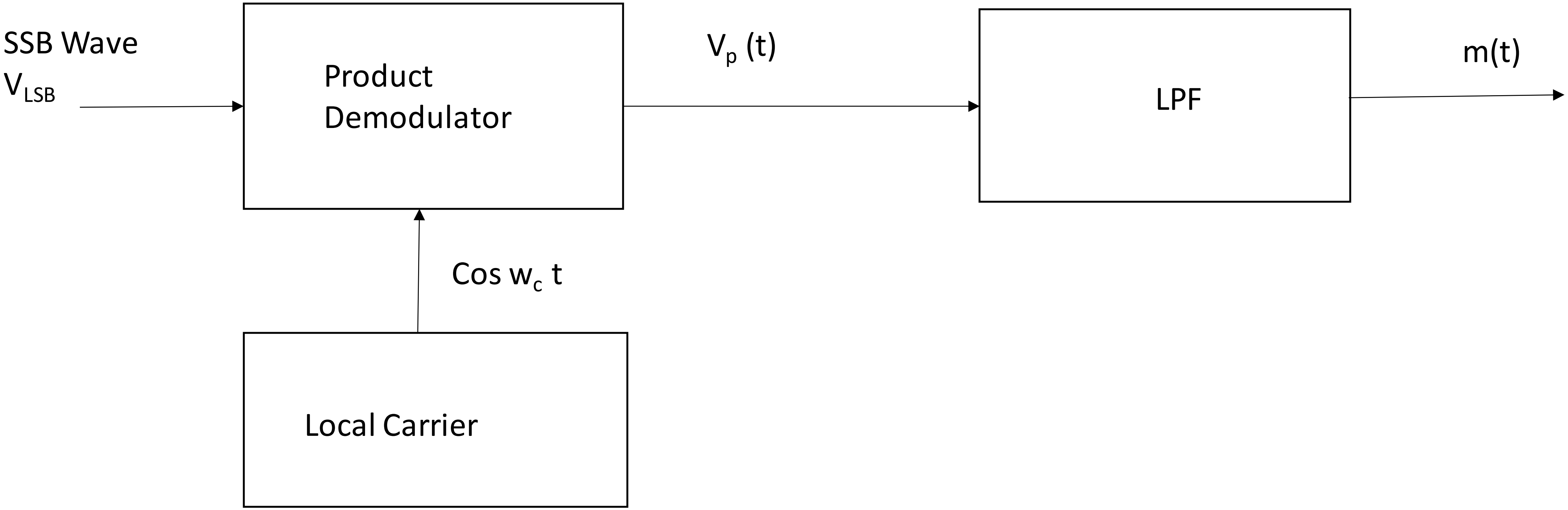
- $e_{BM4} = \cos (\omega_c t+90) \cos (\omega_0 - \omega_m) t$

  $e_{BM4}=1/2 * \{\cos (\omega_c t+90 +(\omega_0 - \omega_m) t)+ \cos (\omega_c t+90 -(\omega_0 - \omega_m) t)\}$

  $e_{BM4}=1/2 * \{\cos ((\omega_c +\omega_0)t - \omega_m t+90)+ \cos ((\omega_c -\omega_0)t+\omega_m t+90)\}$

- o/p from BM3 and BM4 are given to a summing amplifier to produce final o/p.
- $e_{out} = \cos((\omega_c + \omega_0 - \omega_m)t + 90)$ this is the lower sideband

SSBSC Demodulator: Coherent Detection Method



The process of extracting an original message signal from SSBSC wave is known as detection or demodulation of SSBSC. Coherent detector is used for demodulating SSBSC wave.

Coherent Detection Method

$$V_p\,(t) = \text{Cos}\ w_c\, t\ .\ V_{LSB}$$
$$= \text{Cos}\ w_c\, t\ [\ m(t)\ \text{Cos}\ w_c\, t + m^{\wedge}\ (t)\ \text{Sin}\ w_c\, t\ ]$$
$$= m(t)\ \cos^2 w_c\, t + + m^{\wedge}(t)\ \text{Cos}\ w_c\, t\ .\ \text{Sin}\ w_c\, t$$

$$V_p(t) = m(t)[\ (1+\text{Cos}\ 2w_c\, t)/2] + m^{\wedge}(t)\ \frac{2\text{Cos}\ w_c\, t\ .\ \text{Sin}\ w_c\, t}{2}$$

$$= m(t)/2 \quad [\ 1+\text{Cos}\ 2w_c\, t] + m^{\wedge}(t)/2\ \ (\text{Sin}\ 2w_c t)$$

$$= \underset{\downarrow \atop w_m(t)}{1/2\ m(t)} + 1/2\ [m\ (t)\text{Cos}\ \underset{\downarrow \atop 2w_c(t)}{2w_c\, t} + m^{\wedge}(t)\ \text{Sin}\ \underset{\downarrow \atop 2w_c\,(t)}{2w_c\, t})$$

Here m(t) gets separated

1. A sinusoidal carrier has amplitude of 10V and freq 30 KHz. It is amplitude modulated by a sinusoidal voltage of amp 3V and freq 1 KHz. Modulated voltage is developed across 50 ohm resistance.

- Write the eqn for modulated wave
- Plot the wave form showing the max and min Voltage.
- Determine the modulation Index.
- Draw the spectrum of the modulated wave.

2. The antenna current of an AM transmitter is 8 A only if the carrier is sent. But it increases to 8.93A if the carrier is modulated by a single sinusoidal wave. Determine the % modulation. Also find the antenna if the % of modulation changes to 0.8.

3. For an AM DSB_FC wave with a peak unmodulated carrier voltage V_c=10V and Load resistance R = 10 ohm and Modulation coefficient m=1, determine

- Power of carrier , USB & LSB.
- Total SB Power
- Total Power of Modulated Wave
- Draw power spectrum

4. A modulating signal 10 Sin (2∏ x 10^3 t) is used to modulate a carrier signal 20 Sin (2∏ x 10^4 t) . Determine

  - Mod Index
  - Percentage Modulation, freq of the SB Components , their amplitudes and Band width.

Answers

1. $V_{am}(t)= V_c[1+ m\text{Sin}w_m\ t][\text{Sin}w_c\ t]$
- f_c --------------- w_c =188.4 KHz
- f_m --------------- w_m =6.28 KHz

  $m=V_m/V_c =0.3$

  $V_{max}=13$ V

  $V_{min}=7$V

2. m=0.701

   $I_t=9.19$ A

3. $P_c = V_c^2 / 2R = 5W$

$m = 1$

$P_{LSB} = P_{USB} = m^2 V_c^2 / 8R = 1.25W$

$P_{SB} = m^2 V_c^2 / 4R = 2.5\ W$

$P_t = 1.5\ P_c$

4. $m = 0.5$

Freq of LSB $= f_c - f_m = 9KHz$

Freq of USB $= f_c + f_m = 11KHz$

Amp of SB $= mV_c/2 = 5V$

BW $= 2f_m = 2\ KHz$

Angle Modulation

- Process in which angle of carrier(either freq or phase) is varied linearly acc to msg s/g
- The standard equation of the angle modulated wave is

$$s(t)=A_c \cos\theta_i(t)$$

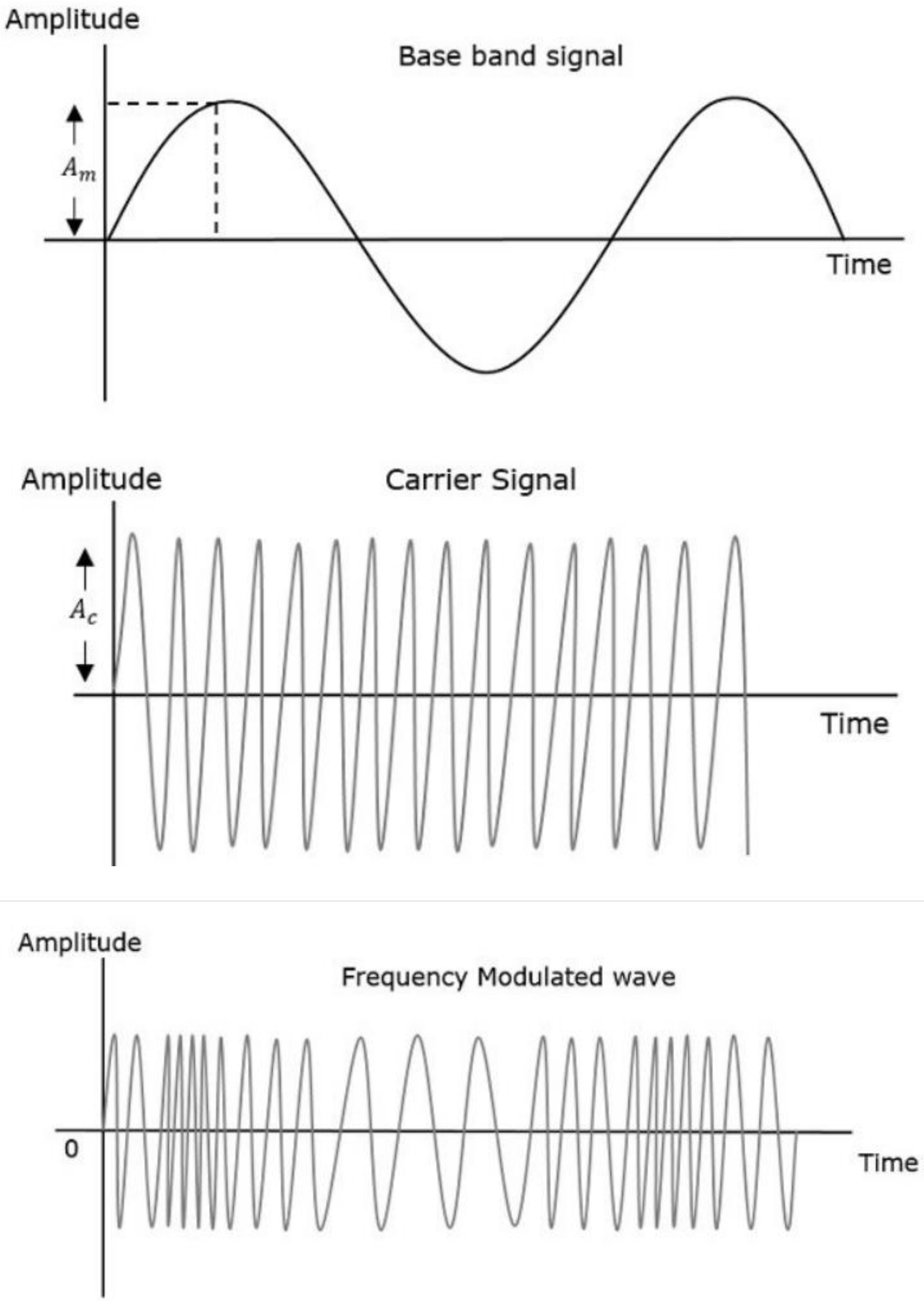
A_c is the amplitude of the modulated wave, which is the same as the amplitude of the carrier signal

θ_i is the angle of the modulated wave

- Changing the angle θ(t) of some sinusoid is the basis for the two types of angle modulation

•**Frequency Modulation** is the process of varying the frequency of the carrier signal linearly with the message signal.

•**Phase Modulation** is the process of varying the phase of the carrier signal linearly with the message signal.


Amplitude
Base band signal
A_m
Time
Amplitude
Carrier Signal
A_c
Time
Amplitude
Frequency Modulated wave
0
Time

Phase Modulation

- In this type of modulation, the phase of the carrier signal is directly changed by the message signal. The phase modulated signal will have the form

$$s_{pm}(t) = A_c \,.\, \mathrm{Cos}\,[\, w_c\, t + k_p\, m(t)\,]$$

Where,

A_c	= Carrier amplitude (constant)
w_c	= Carrier freq
$m(t)$	= message signal
k_p	= phase sensitivity of modulator(radians/volt): parameter that

specifies how much change in the angle occurs for every unit of change of m(t)

$$\phi(t) = k_p\, m(t)$$

For single tone modulation,

let $m(t)=A_m \cos w_m\, t$

$\phi(t) = k_p\, A_m \cos w_m\, t$

Phase deviation $\phi_{max} = k_p\, A_m$

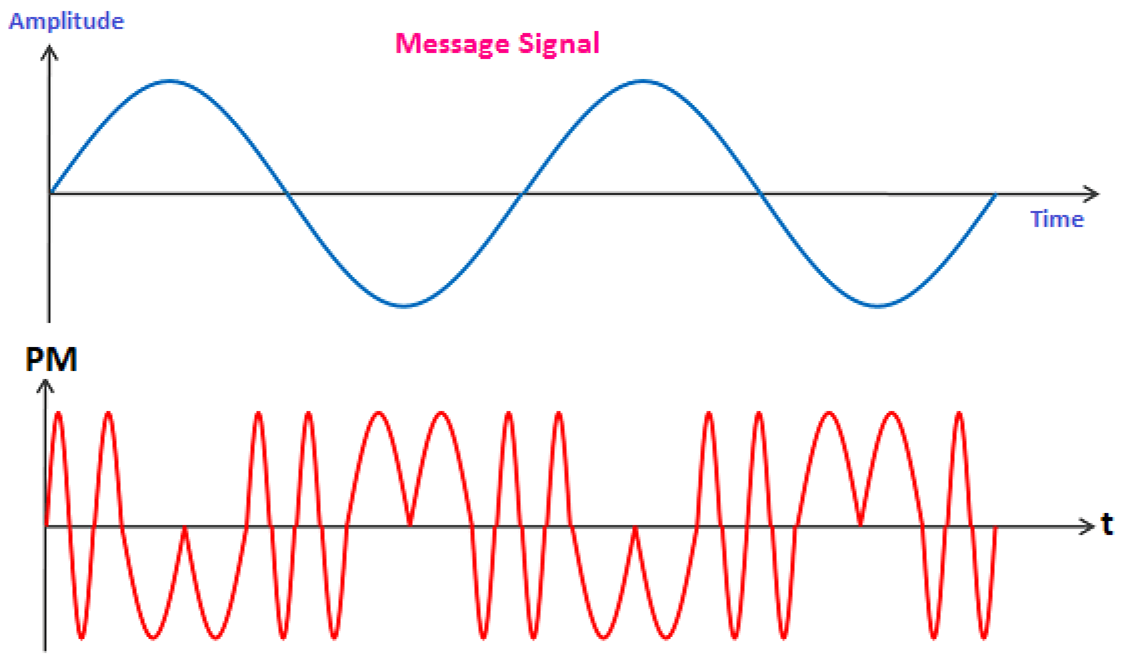
The phase and instantaneous frequency of this signal are

We know that ω(t) = $\frac{d}{dt}\theta$(t)

$$\theta_{PM}(t) = \omega_c t + k_p m(t),$$

$$\omega_i(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p \dot{m}(t).$$

So, the frequency of a PM signal is proportional to the derivative of the message signal.



Angle Modulation

- **Advantages**
  - Noise Reduction
  - Improved System Fidelity
  - more efficient use of power
- **Disadvantages**
  - Wider Band Width
  - More complex Circuits in both transmitter and receiver section

FREQUENCY MODULATION

- Modulating signal m(t) is used to vary the carrier signal. So the change in carrier freq will be proportional to the modulating signal m (t)

  $f_0 \, \alpha \, m(t)$

  $f_0 = k_f \, m(t)$ where k_f is a constant called freq variation constant or freq sensitivity

- The instantaneous carrier freq

  $f_i(t) = f_c + k_f \, m(t)$ where f_c is the freq of the carrier signal and $K_f \, m(t)$ is the Change in freq

Unit of K_f = Hz/V

- f_c is the unmodulated carrier freq hence,

  angular velocity , $w_i(t) = 2\prod f_i(t)$

For single tone modulation,

let $m(t)=A_m \cos w_m t$

$f_i(t) = f_c + k_f A_m \cos w_m t$

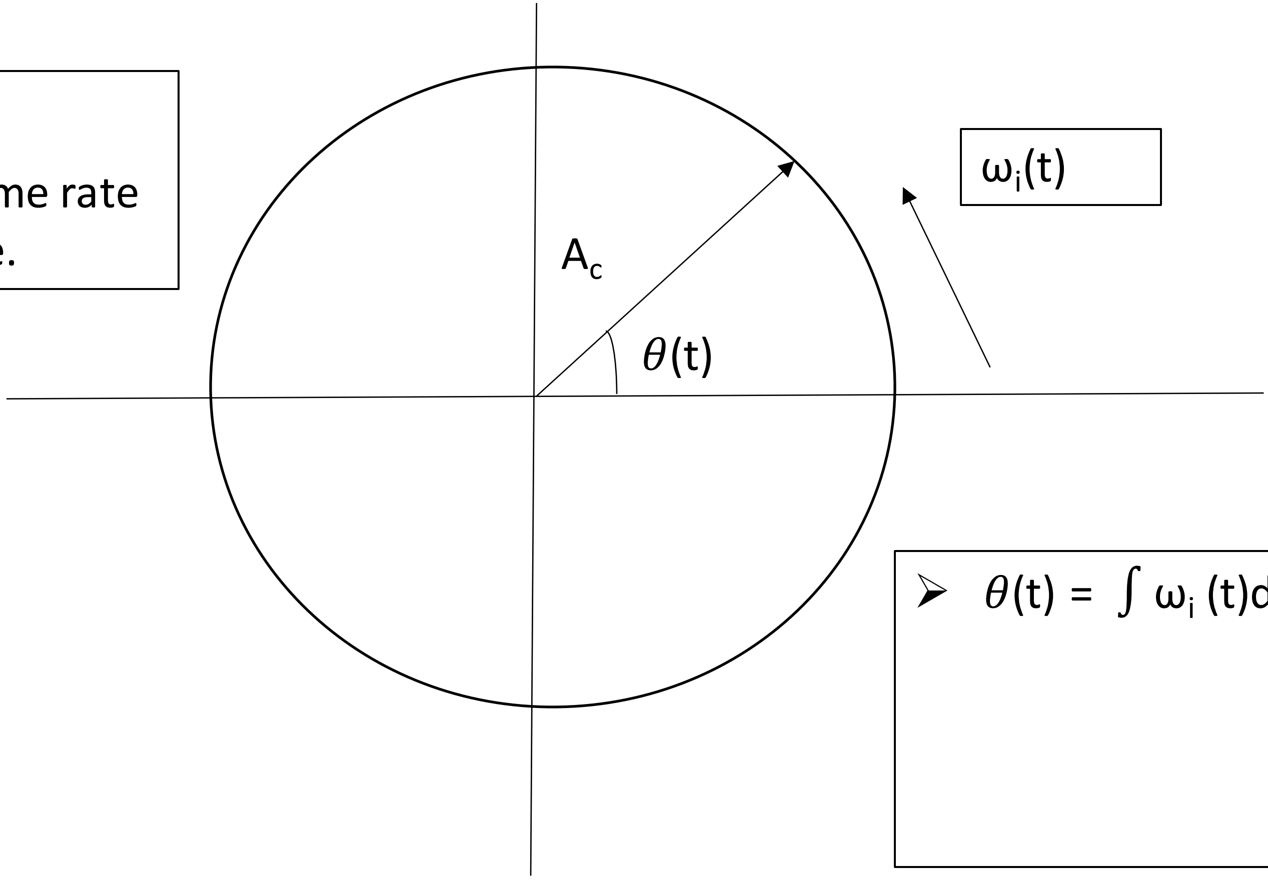
$\Delta f = k_f Am$=freq deviation

Carrier swing(total variation of carrier freq) = 2 Δf

- Generation of the modulated carrier freq can be represented graphically by means of a rotating phasor dig as shown

$\omega_i(t) = \frac{d}{dt}\theta(t)$

Angular velocity is the time rate of change of phase angle.



- $\theta(t) = \int \omega_i\,(t)dt = \int 2\pi f_i\,(t)\,dt$

$$= 2\pi \int [f_c + k_f\, m(t)]\, dt$$

$$= 2\pi\, f_c\, t + 2\, \pi k_f \int m(t)\, dt$$

- Modulating signal m(t) is contained in the angle in an indirect way
- Cosine fn representing the carrier signal is given by

$V = A_c \cos\theta(t)$ the modulated carrier signal $V_{FM}\ (t) = A_c \cos\,(2\pi\, f_c\, t + 2\, \pi k_f \int m(t)\, dt)$

Sinusoidal Freq Modulation

- for Sinusoidal Freq Mod , $m(t) = A_m \cos 2 \pi f_m t$

$$\boxed{V_{FM}(t) = A_c \cos [2\pi f_c t + 2 \pi k_f \int A_m \cos 2 \pi f_m t \, dt]}$$

- Substitute $k_f A_m = \Delta f$ where Δf = peak freq variation which is proportional to modulating signal

- $$V_{FM}(t) = A_c \cos [2\pi f_c t + 2 \pi \Delta f \int \cos 2 \pi f_m t \, dt]$$
$$= A_c \cos [2\pi f_c t + \frac{\Delta f}{fm} \sin 2 \pi f_m t]$$
where $\beta = \frac{\Delta f}{f_m}$ = Modulation index of FM signal

$$\boxed{V_{FM}(t) = A_c \cos [2\pi f_c t + \beta \sin 2 \pi f_m t]}$$

Freq Spectrum of Sinusoidal FM Modulation

- Eqn for FM signal is : $V_{FM}(t) = A_c \text{ Cos } [2\prod f_c t + \beta \text{ Sin } 2\prod f_m t]$

- The freq domain representation can be obtained by Bessel Function identity given by

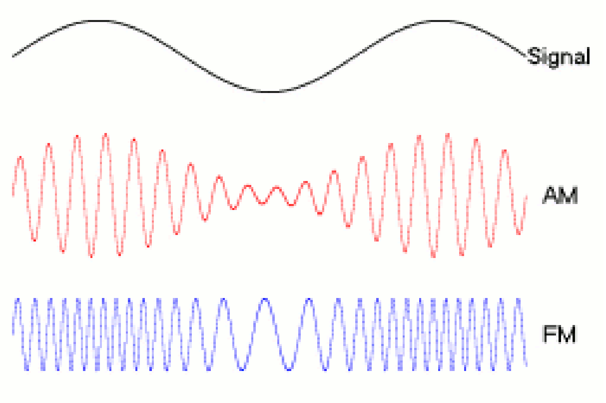
$$\text{Cos } [A + \beta \text{ Sin } B] = \sum_{n=-\infty}^{n=\infty} J_n(\beta) Cos\ (A + nB)$$

$$V_{FM}(t) = A_c [J_0(\beta) \text{ Cos } w_c t + J_1(\beta) [\text{ Cos } w_c t + w_m t] - J_1(\beta) [\text{Cos } (w_c t - w_m t) + J_2(\beta) [\text{Cos } (w_c t + 2w_m t) + J_2(\beta) [\text{Cos } (w_c t - 2 w_m t) + \ldots\ldots\ldots\ldots$$

$$J_n(\beta) = (\beta/2)^n \left[1/n! - (\beta/2)^2 / 1!(n+1)! + (\beta/2)^4 / 2!(n+2)! + \ldots\ldots\ldots\ldots\ldots \right.$$

Frequency modulation

- $V_{FM}(t) \quad = A_c \text{ Cos } [\omega_c t + \beta \text{ Sin}\omega_m t]$
- $V_{FM}(t) \quad = \quad A_c \{ J_0(\beta) \text{ Cos } w_c t + J_1(\beta) [\text{ Cos } (w_c + w_m) t - \text{Cos } (w_c - w_m)t] + J_2(\beta) [\text{Cos } (w_c + 2w_m)t + \text{Cos } (w_c - 2 w_m)t] + J_3(\beta) [\text{Cos } (w_c + 3w_m)t - \text{Cos } (w_c - 3 w_m)t] + \ldots\ldots \}$



- $J_{-n}(\beta) = (-1)^n J_n(\beta)$

n is the order , argument β is the mod index of FM s/g

$J_n(\beta)$

Modulation index	Sideband amplitude																
	Carrier	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.00	1.00																
0.25	0.98	0.12															
0.5	0.94	0.24	0.03														
1.0	0.77	0.44	0.11	0.02													
1.5	0.51	0.56	0.23	0.06	0.01												
2.0	0.22	0.58	0.35	0.13	0.03												
2.40483	0.00	0.52	0.43	0.20	0.06	0.02											
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01										
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01										
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02									
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02								
5.52008	0.00	−0.34	−0.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01							
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02							
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02						
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03					
8.65373	0.00	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02				
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01			
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01		
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

- From this table, if $\beta = 0.5$, the spectral components and corresponding Bessel coeff.values are
- 0.94 for ω_c
- 0.24 for $\omega_c \pm\omega_c$
- 0.03 for $\omega_c \pm 2\omega_c$