Design of FIR filters by Frequency Sampling Technique:

In this method the desired frequency response $[H_d(e^{j\omega})]$ is sampled at N points. These samples are the DFT coefficients $[H(k)]$ of the impulse response of the filter. Hence the impulse response of the filter $[h(n)]$ determined by taking inverse DFT.

$H_d(e^{j\omega}) \rightarrow$ desired frequency response.

$H(k) \rightarrow$ DFT coefficients obtained by sampling $H_d(e^{j\omega})$

$h(n) \rightarrow$ Impulse response of FIR filter.

Procedure for Type 1 Design

1. Choose the desired frequency response $H_d(e^{j\omega})$

2. Sample $H_d(e^{j\omega})$ at N points by taking samples $\omega = \omega_k = \frac{2\pi}{N}k$;

$k = 0, 1 \ldots N-1.$

to get $H(k)$

ie $H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}k}$; for $k = 0, 1, \dots N-1$

③ Compute the N-samples of $h(n)$ using the following equation.

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \mathrm{Re}\left[H(k)\, e^{j\frac{2\pi}{N}kn}\right]\right\} \longrightarrow N \text{ odd.}$$

and.

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} \mathrm{Re}\left[H(k)\, e^{j\frac{2\pi}{N}kn}\right]\right\} \longrightarrow N \text{ even.}$$

④ The transfer function (System function) of FIR filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}.$$

6.9.3 *Design*

We exploit the basic symmetry property of the sampled frequency response to simplify the computations in designing an FIR filter. Based on the set of samples that we choose from the frequency response, there are two types of design.

Type 1 design

In this type of design the frequency samples of the desired response $H_d(e^{j\omega})$ are determined, using the relation

$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k} \qquad k = 0, 1, \ldots N-1 \tag{6.119}$$

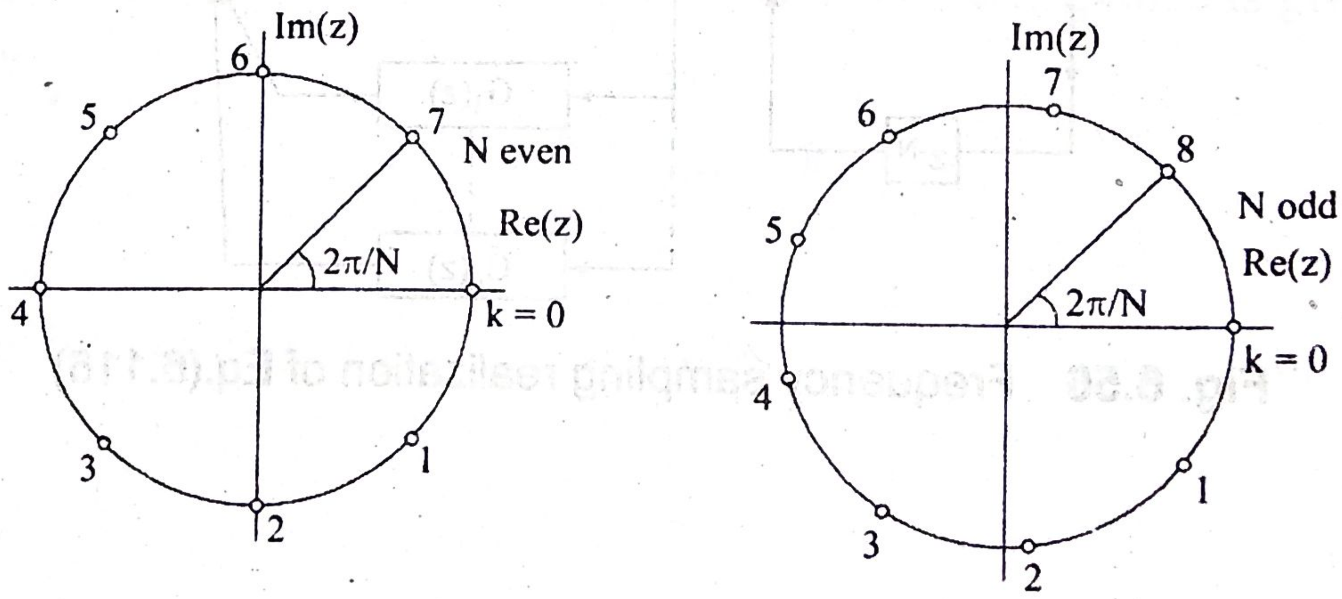


Fig. 6.57 Location of DFT samples on the unit circle for type 1 design

Example 6.15 Determine the filter coefficients $h(n)$ obtained by samping

$$H_d(e^{j\omega}) = e^{-j(N-1)\omega/2} \quad 0 \leq |\omega| \leq \frac{\pi}{2}$$
$$= \quad 0 \quad \frac{\pi}{2} \leq |\omega| \leq \pi$$

for $N = 7$.

Answer:

Since $N = 7$.

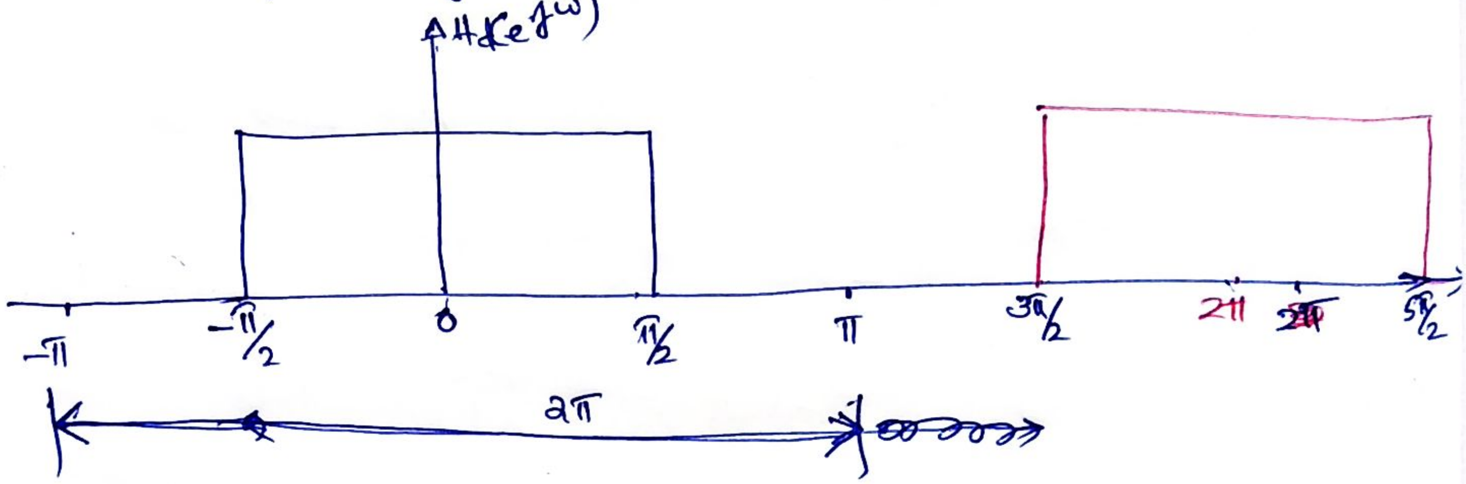
$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & 0 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi \end{cases}$$

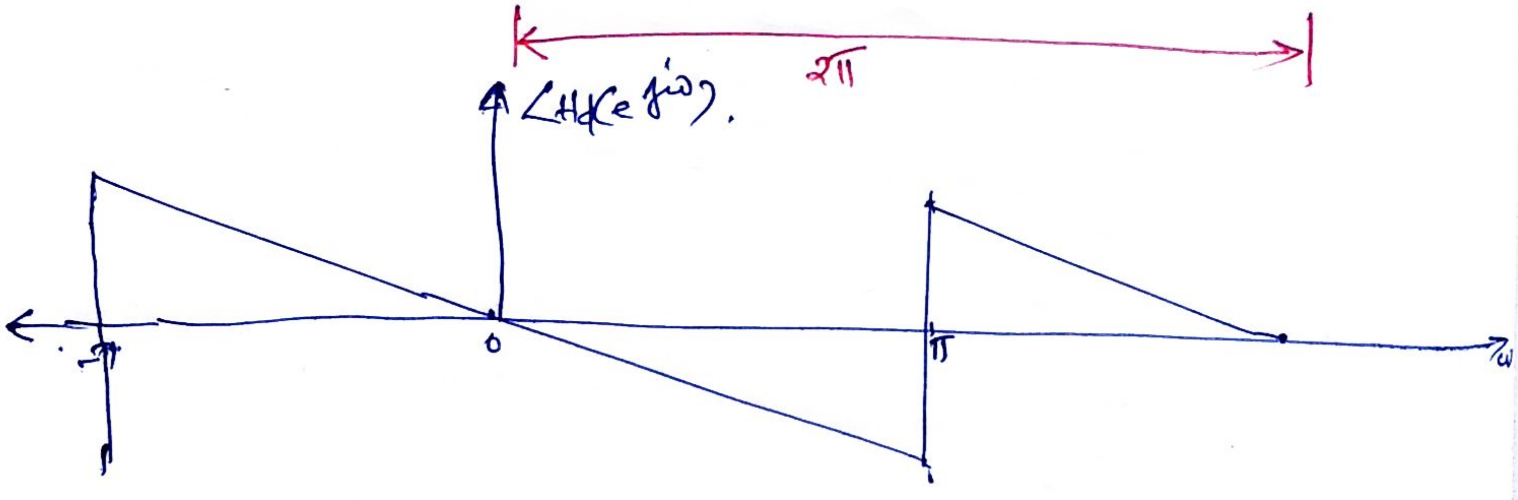
Considering magnitude and phase seperately:

$$|H_d(e^{j\omega})| = \begin{cases} 1 & 0 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi \end{cases}$$

$$\angle H_d(e^{j\omega}) = -3\omega.$$

Corresponding plot

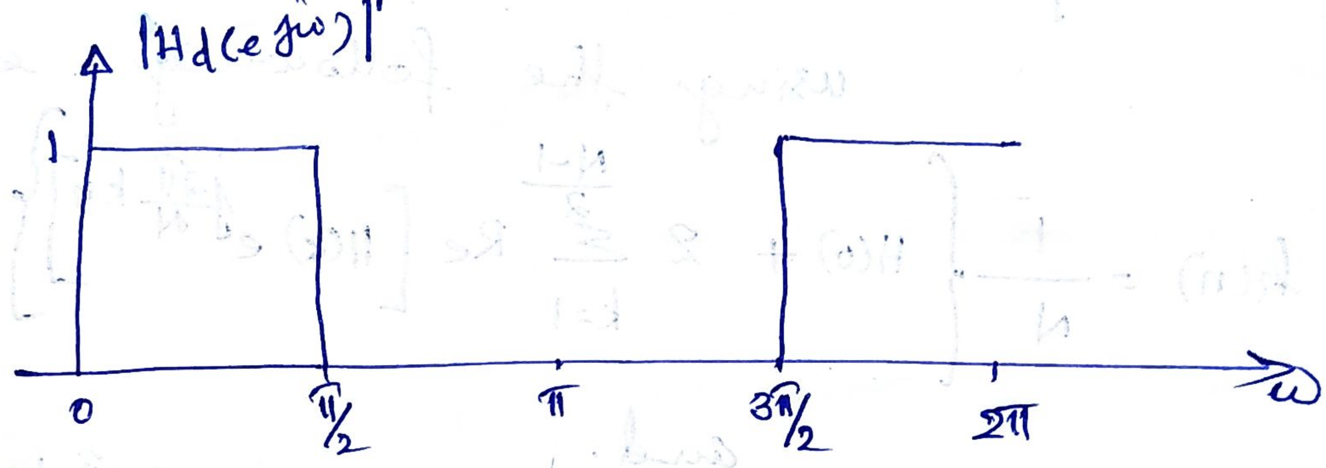




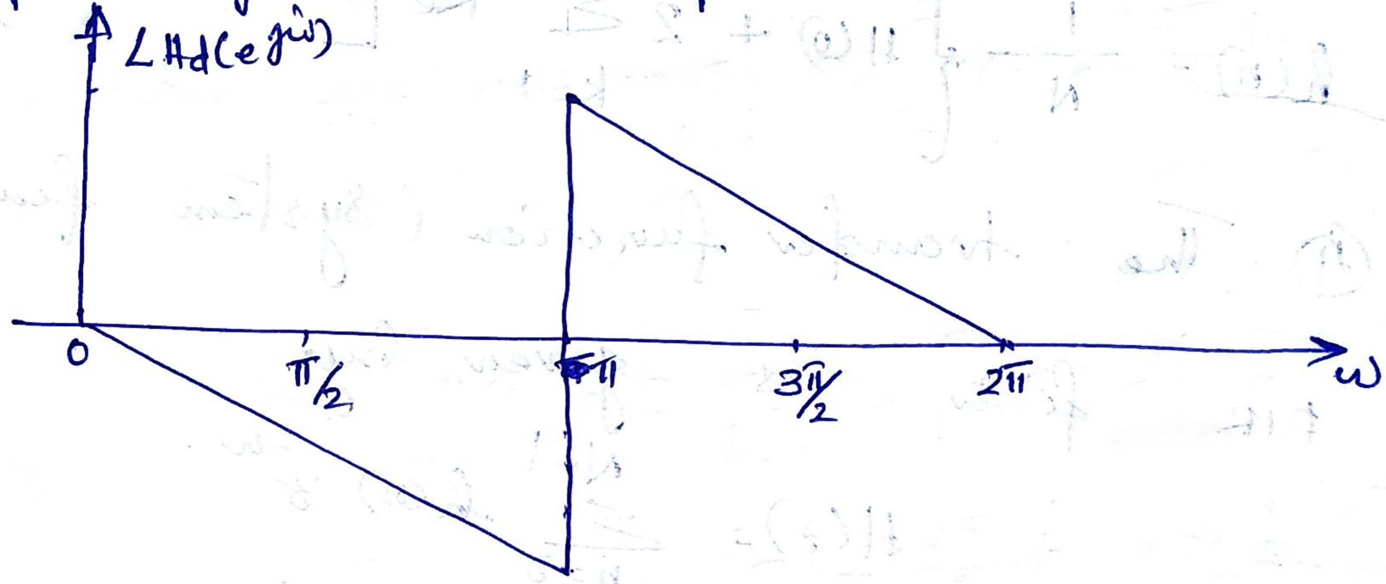
step1

Considering frequency (ω) from 0 to 2π

Magnitude response can be redrawn as.



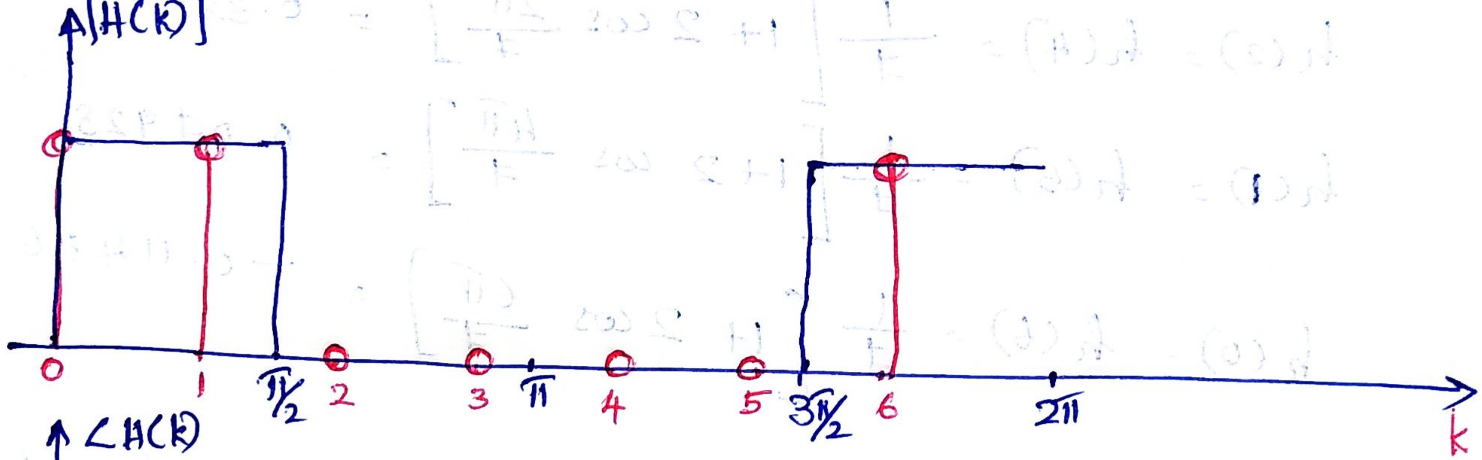
Corresponding phase response

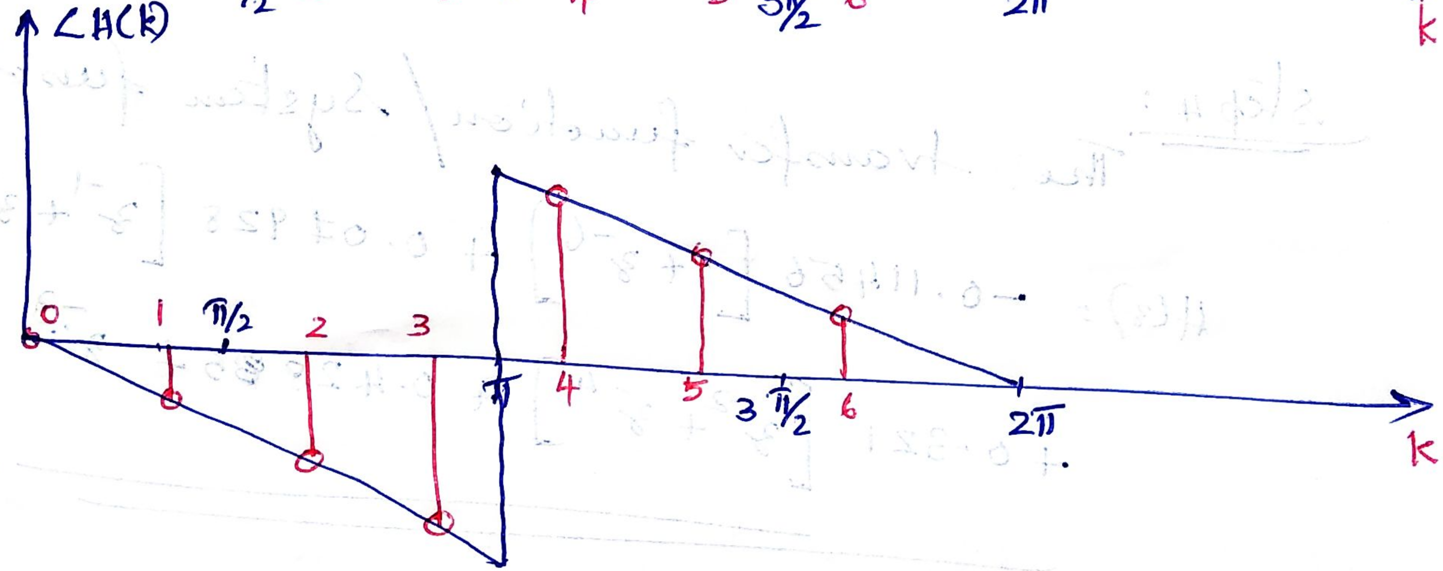


Step 2 :

Sample $H_d(e^{j\omega})$ at $N = 7$ points

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{7}k} \quad ; \quad k = 0, 1, \cdots 6.$$





$k=0 \Rightarrow w = \frac{2\pi}{7} \cdot 0 = 0 \Rightarrow |H_d(e^{jw})| = 1 \Rightarrow |H(0)| = 1$

$k=1 \Rightarrow w = \frac{2\pi}{7} \cdot 1 = \frac{2\pi}{7} \Rightarrow |H_d(e^{jw})| = 1 \Rightarrow |H(1)| = 1$

$k=2 \Rightarrow w = \frac{2\pi}{7} \cdot 2 = \frac{4\pi}{7} \Rightarrow |H_d(e^{jw})| = 1 \Rightarrow |H(2)| = 1$

$k=3 \Rightarrow w = \frac{2\pi}{7} 3 = \frac{6\pi}{7} \Rightarrow |H_d(e^{jw})| = 0 \Rightarrow |H(3)| = 0$

$k=4 \Rightarrow w = \frac{2\pi}{7} \cdot 4 = \frac{8\pi}{7} \Rightarrow |H_d(e^{jw})| = 0 \Rightarrow |H(4)| = 0$

$k=5 \Rightarrow w = \frac{2\pi}{7} 5 = \frac{10\pi}{7} \Rightarrow |H_d(e^{jw})| = 0 \Rightarrow |H(5)| = 0$

$k=6 \Rightarrow w = \frac{2\pi}{7} \cdot 6 = \frac{12\pi}{7} \Rightarrow |H_d(e^{jw})| = 1 \Rightarrow |H(6)| = 1$

ie $|H(k)| = 1$ for $k = 0, 1, 6$

$= 0$ for $k = 2, 3, 4, 5.$

To get $H(k)$.

$$H(k) = e^{-j\left(\frac{N-1}{2}\right)w} \Big|_{w = \frac{2\pi}{N} k}; \quad k = 0, 1 \cdots 6$$

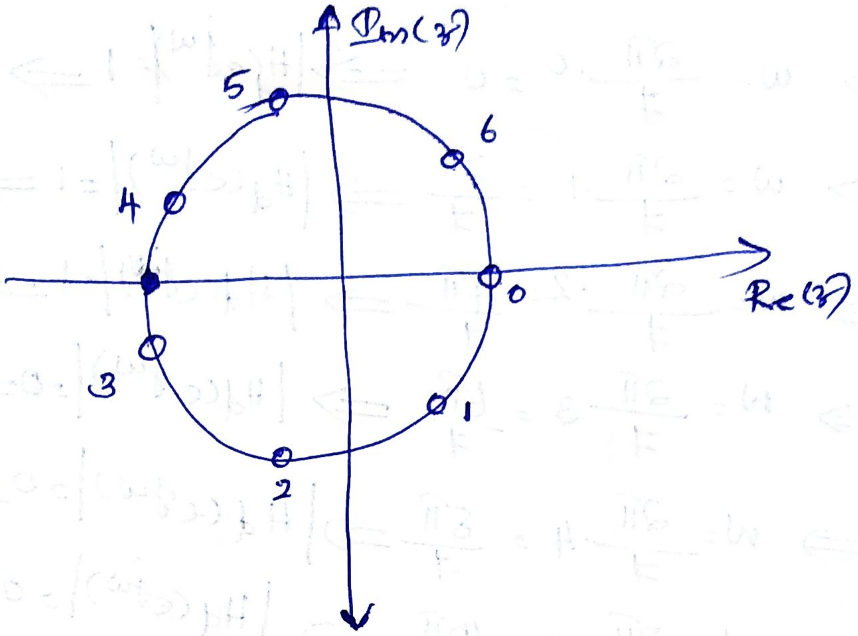
$$= e^{-j3w} \Big|_{w = \frac{2\pi}{7} k}.$$

$$= e^{-j3 \cdot \frac{2\pi}{7} k}.$$

$$H(k) = e^{-j\frac{6\pi}{7} k}.$$

ie

$$H(k) = \begin{cases} e^{-j\frac{6\pi}{7} k} ; & k = 0, 1, 6 \\ 0 & \text{else.} \end{cases}$$



step 3: To find $h(n)$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \text{Re}\left[H(k)\, e^{j\frac{2\pi}{7}kn}\right]\right]$$

$n = 0, 1 \cdots N-1.$

$$= \frac{1}{7}\left[H(0) + 2\sum_{k=1}^{3} \text{Re}\left[H(k)\, e^{j\frac{2\pi}{7}kn}\right]\right]$$

$$= \frac{1}{7}\left[H(0) + 2\,\text{Re}\left[e^{-j\frac{6\pi}{7}\cancel{k}^{1}}\, e^{j\frac{2\pi}{7}\cancel{k}^{1}n}\right]\right]$$

$$= \cancel{\frac{1}{7}\left[H(0) + 2\right.}$$

$$= \frac{1}{7}\left[e^{0} + 2\cdot\text{Re}\left[e^{j\frac{2\pi}{7}\cancel{k}(n-3)}\right]\right]$$

$$h(n) = \frac{1}{7}\left[1 + 2\cos\frac{2\pi}{7}(n-3)\right]$$

h(n) will be symmetric about $\alpha = 3$.

$$h(3) = \frac{1}{7}\left[1+2\right] = 0.42857.$$

$$h(2) = h(4) = \frac{1}{7}\left[1+2\cos\frac{2\pi}{7}\right] = 0.321$$

$$h(1) = h(5) = \frac{1}{7}\left[1+2\cos\frac{4\pi}{7}\right] = 0.07928$$

$$h(0) = h(6) = \frac{1}{7}\left[1+2\cos\frac{6\pi}{7}\right] = -0.11456$$

Step 4: The transfer function / System function.

$$H(z) = -0.11456\left[1+z^{-6}\right] + 0.07928\left[z^{-1}+z^{-5}\right] + 0.321\left[z^{-2}+z^{-4}\right] + 0.428857\, z^{-3}$$