# SIGNALS AND SYSTEMS.

## ASSIGNMENT - 2.

Submitted by:
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ECE - B
Roll No: 50

1) Determine the fourier series representation of the signal $x(t) = \sin 4t + \cos 6t$ plot magnitude & space spectrum.

Ans) $x(t) = \sin 4t + \cos 6t$

$$= \frac{e^{j4t} - e^{-j4t}}{2j} + \frac{e^{6jt} + e^{-6jt}}{2}$$

$$= \frac{1}{2j} e^{4jt} - \frac{1}{2j} e^{-4jt} + \frac{1}{2} e^{6jt} + \frac{1}{2} e^{-6jt}$$

Fourier transform Representation is $= \sum_{k=-\infty}^{\infty} x[k] e^{jk w_0 t}$

Fundamental frequency of $x(t) \Rightarrow x(t) = x_1(t) + x_2(t)$

$$T_1 = \frac{2\pi}{w_1} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$T_2 = \frac{2\pi}{w_2} = \frac{2\pi}{6} = \pi/3$$

$\therefore$ Fundamental time period $= T = \lambda_1 T_1 = \lambda_2 T_2$

ie, $\lambda_1 T_1 = \lambda_2 T_2 \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1} = \frac{2}{3} \Rightarrow \lambda_1 = 2$

$\therefore$

$$T = \lambda_1 T_1 = 2 \times \frac{\pi}{2} = \pi$$

$\therefore$ Fundamental frequency $w_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

$$w_0 = 2 \Rightarrow x(t) = \sum_{k=-\infty}^{\infty} x[k] \cdot e^{2jkt}$$

$$\Rightarrow x[2] = \frac{1}{2j} = \frac{-j}{2}, \quad x[-2] = \frac{-1}{2j} = \frac{j}{2}, \quad x[3] = x[-3] = \frac{1}{2}$$

magnitude $|x[k]| = \sqrt{(\frac{1}{2})^2} = \frac{1}{2}$ for $k = \pm 2$

$$|x[k]| = \sqrt{(\frac{1}{2})} = \frac{1}{2} \text{ for } k = \pm 3$$

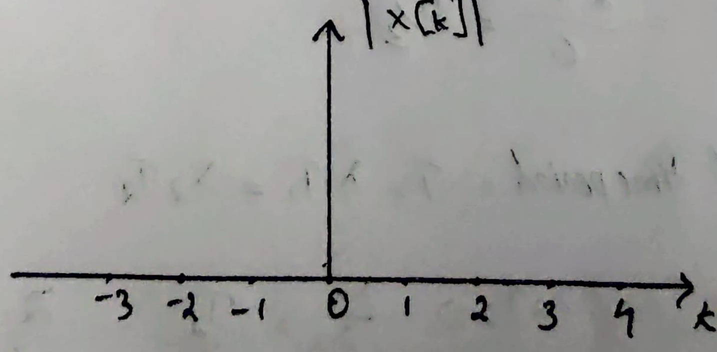
phase angle

$$\angle x[k] = \frac{-j}{2} = \tan^{-1}\left(\frac{1}{2}\right) = -\pi/2 \text{ at } k = 2$$

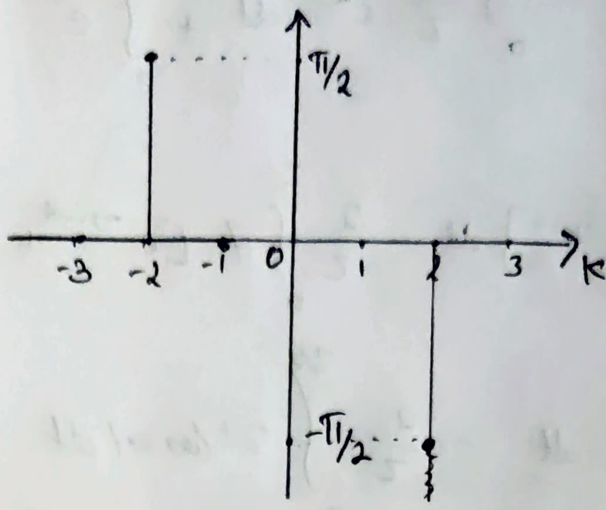
$$\frac{j}{2} = \tan^{-1}\left(\frac{1/2}{0}\right) = \pi/2 \text{ at } k = -2$$

$$1/2 = \tan^{-1}(0) = 0 \text{ for } k = 3, -3$$

magnitude spectrum



phase spectrum




2) find the Fourier transform of the signal defined by

$$x(t) = \begin{cases} 1 - \frac{2(t)}{\tau} & \text{for } |t| < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

A)

$$x(t) \xrightarrow{FT} \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \, dt.$$

The given signal is of a triangular function.

$$x(\omega) = \int_{-\frac{\tau}{2}}^{0} \left(1 + \frac{2t}{\tau}\right) e^{-j\omega t} \, dt + \int_{0}^{\tau/2} \left(1 - \frac{2t}{\tau}\right) \cdot e^{-j\omega t} \, dt.$$

$$= \int_{0}^{\tau/2} \left(1 - \frac{2t}{\tau}\right) e^{j\omega t} \, dt + \int_{0}^{\tau/2} \left(1 - \frac{2t}{\tau}\right) e^{-j\omega t} \, dt.$$

$$= \int_0^{\tau/2} e^{j\omega t}\,dt - \int_0^{\tau/2} \frac{2t}{\tau} e^{j\omega t}\,dt + \int_0^{\tau/2} e^{j\omega t}\,dt - \int_0^{\tau/2} \frac{2t}{\tau} e^{-j\omega t}\,dt$$

$$= \int_0^{\tau/2} \left[e^{j\omega t} + e^{-j\omega t}\right] dt - \frac{2}{\tau}\int_0^{\tau/2} t\left[e^{-j\omega t} + e^{-j\omega t}\right] dt$$

$$= \int_0^{\tau/2} 2\cos\omega t\,dt - \frac{2}{\tau}\int_0^{\tau/2} 2t\cos\omega t\,dt$$

$$= 2\left[\frac{\sin\omega t}{\cancel{\omega}}\right]_0^{\tau/2} - \frac{4}{\tau}\left[\frac{t \cdot \sin\omega t}{\omega}\right]_0^{\tau/2} - \int_0^{\tau/2} \frac{\sin\omega t}{\omega}\,dt$$

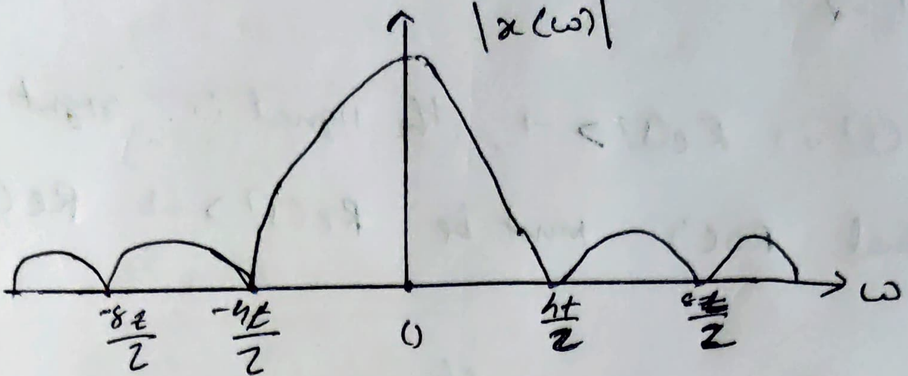
$$= \frac{2}{\omega}\left[\sin\omega\,\tau/2\right] - \frac{4}{\tau\omega}\left[\frac{\tau}{2} \cdot \sin\tau/2 - \frac{4}{\omega^2\tau}\left[\frac{\cos\omega t}{2}\right]_0^{\tau/2}\right.$$

$$= \frac{4}{\omega^2\tau}\left[1 - \cos\omega\,\tau/2\right]$$

$$= \frac{4}{\omega^2\tau}\left[2\sin^2\omega\,\tau/4\right]$$

$$= \frac{8}{\omega^2\tau}\left(\frac{\omega\tau}{4}\right)^2 \frac{\sin^2\left[\omega\tau/4\right]}{(\omega\tau/4)^2}$$

$$= \frac{\tau}{2}\,\mathrm{sinc}^2\,\frac{\omega\tau}{4}$$




3) find the Inverse laplace transform $X(s) = \frac{8-(s-2)(4s+10)}{(s+1)(s^2+4s+4)}$

ROC $Re(s) < -1$

Ans) given $X(s) = \frac{8-(s-2)(4s+10)}{(s+1)(s^2+4s^2+4)}$

$$= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$\Rightarrow 8-(s-2)(4s+10) = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

when $s=-2$

$$8-(-4)(2) = -C \Rightarrow C = -16$$

when $s = -1$

$$8-(-3)(6) = A \Rightarrow A = 26.$$

when $s = 0$

$$28 = 4A + 2B + C$$

$$2B = -60 \qquad B = -30$$

$\therefore$ poles at $\Rightarrow -1, -2$

ROC of $X(s)$ is $Re(s) > -1$, the signal is right sided &

the individual ROC's must be $Re(s) > -1$ $Re(s) > -2$

$$\Rightarrow X(s) = \frac{26}{s+1} - \frac{30}{s+2} - \frac{16}{(s+2)}2.$$

$$x(t) = L^{-1}\{X(s)\} = L^{-1}\left\{\frac{26}{s+1}\right\} =$$

$$- L^{-1}\left\{\frac{30}{s+2}\right\} - L^{-1}\left\{\frac{16}{(s+2)^2}\right\}$$

$$= 26e^{-t}u(t) - 30e^{-2t}.u(t) - 16te^{-2t}.u(t).$$

$$x(t) = 26e^{-t}(u(t)) - 30.e^{-2t}.u(t).16te^{-2t}u(t)$$

=

4) The transfer function of an LTI system is given by $H(s) = \dfrac{2s^2+9s-11}{(s+1)(s^2+5-6)}$ find the impulse response of the system if i) stable ii) is causal. will the system be both stable and causal.

0.

A)

$$H(s) = \frac{2s^2+9s-11}{(s+1)(s^2+s-6)}$$

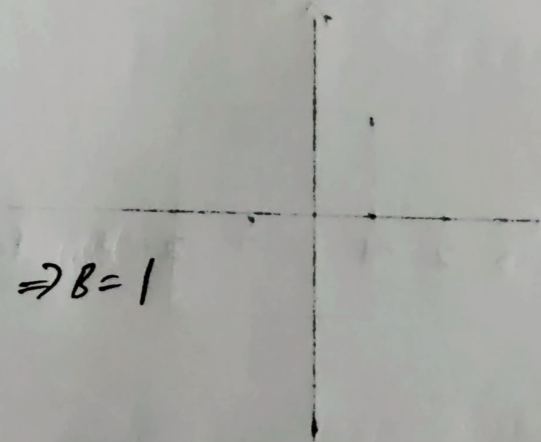
$$= \frac{2s^2+9s-11}{(s+1)\quad(s-2)(s+3)}$$

$$\frac{2s^2+9s-11}{(s+1)(s-2)(s+3)} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{s+3}$$

$$2s^2+4s-11 = A\,(s-2)(s+3) + B(s+2)(s+3) + C\,(s+1)(s-2)$$

at $s=-1$

$$-18 = -6A$$
$$\Rightarrow A = 3.$$



at $s=2$

$$15B = 15 \Rightarrow B=1$$

at $s=-3$

$$-20 = 10C \Rightarrow C=-2$$

$$H(s) \Rightarrow \frac{3}{s+1} + \frac{1}{s-2}\ \frac{-2}{s+3}$$

poles are at $s = -1, 2, -3.$

i) $h(t) = 3e^{-t}u(t) + e^{-2t}u(t) - 2e^{-3t}u(t)$.

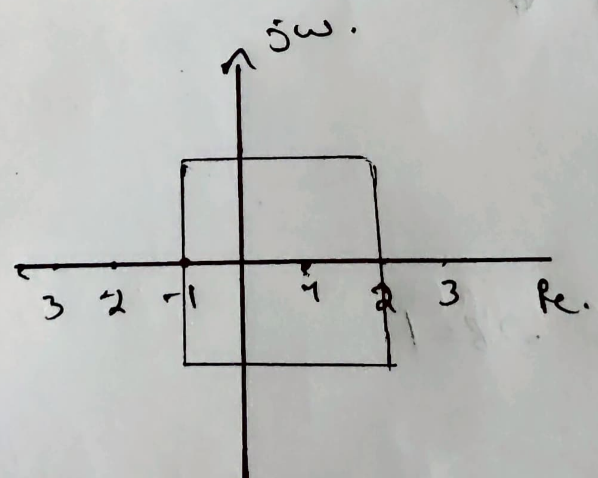
ie, for h(t) to be stable. the ROC must contain jω axis and ROC should not contain any pole

∴ Individual ROC must be

$Re(S) > -3$, $Re(S) > -1$ & $Re(S) < 2$

ie, $2 > Re(S) > -1$

∴ $h(t) = 3e^{-t}u(t) + e^{2t}u(-t) - 2e^{-3t}u(t)$.




Hence. The given system is stable 10

ii) ∴ the ROC is not at the right most pole

hence it is not causal

∴ One pole lies in the half of the s-plane

the system is both (stable & causal)