Module.I

MA1202, 204, 208

Discrete Probability Distribution

Text :- Jay . L . Devore, 'Probability and Statistics for Engineering and the Sciences', 8^{th} Edi:

Sections: 3.1 – 3.4, 3.6, 5.1

Discrete Random Variables and their probability distributions, Expectation, mean and Varience, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Discrete bivariate distributions, marginal distributions, independent random Variables, Expectation (multiple random Variables)

Experiment

Deterministic Exp:

outcome can be predicted in advance.

Random Exp:

More than one outcome.

Students in ECE class:

Abin, Adil, Athulya, Arjun, Devanarth, Sathya, Arathy, Krishnapriya, Sreeni, Priya.

185 185 165.3 180 170.4 175.3

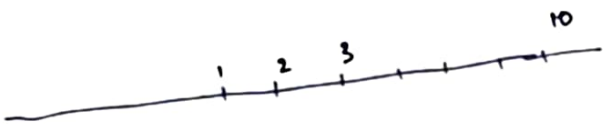
Sample space:

outcome / event.

{Abin, Adil, Athulya, ... }

$x \in S$

Random Variable:



(164, 185)

Discrete Random Variable

x: 1, 2, 3, ... , 10

Continous Random Variable:

(a, b)

Random Variable

Discrete Random Variable

eg:

Countable finite

No: of Students in class

No: of Books in Shelf

Countably infinite.

· Stars in sky.

· Rice in a Sack.

$x: 0, 1, 2, \ldots, n.$

x:	1	2	3	4	5	6	7	8	9	10
$p(x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

} pmf

Prob: Mass fun:

$p(x) = p(x=0)$

$$\boxed{\sum p(x) = 1}$$

$= 1$

Continuous Random Variable:

$0 \le p(x) \le 1$

$p(A) = \frac{\text{fav: no: of cases}}{\text{Total no: of cases}}$

$0 \le p(A) \le 1$

Given that $f(x) = \frac{k}{2^x}$ is a probability distribution for a random variable that can take values $x = 0,1,2,3,4$, find

x	0, 1, 2, 3, ..., n.
$p(x)$	

$\sum p(x) = 1$

$\frac{16}{31} \cdot \frac{k}{31} \cdot \frac{1}{2}$

i) k ii) $p(x \le 1)$ iii) $p(x > 1)$ iv) $p(x \ge 3)$ v) $p(2 \le x < 4)$

$x:$	0	1	2	3	4
$f(x) = \frac{k}{2^x}$	k	$\frac{k}{2}$	$\frac{k}{2^2}$	$\frac{k}{8}$	$\frac{k}{16}$

w.k.t $\sum p(x) = 1$

$$k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} + \frac{k}{16} = 1$$

$$k\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right] = 1$$

$$k\left[\frac{31}{16}\right] = 1$$

$$\boxed{k = \frac{16}{31}}$$

$p(A) = 1 - p(\bar{A})$

Proof

$x:$	0	1	2	3	4
$p(x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

ii) $p(x \le 1) = p(x=0) + p(x=1)$

$$= \frac{16}{31} + \frac{8}{31} = \frac{24}{31}$$

iii) $p(x > 1) = p(x=2) + p(x=3) + p(x=4)$

$$= \frac{4}{31} + \frac{2}{31} + \frac{1}{31} = \frac{7}{31}$$

OR.

$$p(x > 1) = 1 - p(x \le 1) = 1 - \frac{24}{31} = \frac{7}{31}$$

$$P(x \geq 3) = P(x=3) + P(x=4)$$

$$= \frac{2}{31} + \frac{1}{31}$$

$$= \frac{3}{31}$$

$$P(2 \leq x < 4) = P(x=2) + P(x=3)$$

$$= \frac{4}{31} + \frac{2}{31}$$

$$= \frac{6}{31}$$

...ity distribution for a.

x	$0, 1, 2, 3, \ldots, n$
$P(x)$	

...es $x = 0, 1, 2, 3, 4$, find

$\sum p(x) = 1$

iv) $P(x \geq 3)$

v) $P(2 \leq x < 4)$

$P(A) = 1 - P(\bar{A})$

Proof

x:	0	1	2	3	4
$p(x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

ii) $P(x \leq 1) = P(x=0) + P(x=1)$

$$= \frac{16}{31} + \frac{8}{31} = \frac{24}{31}$$

iii) $P(x > 1) = P(x=2) + P(x=3) + P(x=4)$

$$= \frac{4}{31} + \frac{2}{31} + \frac{1}{31} = \frac{7}{31}$$

OR.

$$P(x > 1) = 1 - P(x \leq 1) = 1 - \frac{24}{31} = \frac{7}{31}$$

Discrete Random Variable

* **Probability mass function (Pmf)**

x:	0	1	2	...	n
$p(x)$	p_1	p_2	p_3	...	p_n

* $\sum p(x) = 1$

* **Mean** (μ) Expectation of x.

$$E[x] = \sum x\, p(x)$$

* Varience (σ^2) Varience of x

$$Var(x) = E[x^2] - E[x]^2$$

$$\boxed{\begin{array}{l} E[x^2] = \sum x^2 p(x) \\ E[x^3] = \sum x^3 p(x) \end{array}}$$

* Standard deviation (σ)

$$S.D = \sqrt{Var(x)}$$

* **Distribution function [Cummulative distr: fun].**

$$f(x) = P[X \leq a]$$

* **Properties.**

$E[ax] = a\,E[x]$

$E[ax+b] = a\,E[x] + b$

$E[ax] + E[b]$

$a\,E[x] + b$

$E[\text{constant}] = \text{Constant}$

$Var(\text{constant}) = 0$, $Var(ax) = a^2 Var(x)$

$V(ax+b) = a^2 Var(x) + 0$

4) A random variable x has the following probability distribution

x:	-2	-1	0	1	2	3
$P(x)$	$\frac{1}{10}$	k	$\frac{2}{10}$	$2k$	$\frac{3}{10}$	$3k$

find (i) Value of k (ii) $p(x<2)$ and $P(-2<x<2)$

(iii) find mean and variance

(iv) find distribution function.

$2 \cdot \frac{1}{15}$

$3 \cdot \frac{1}{}$

w.k.T

$\Sigma p(x) = 1$

$\frac{1}{10} + k + \frac{2}{10} + 2k + \frac{3}{10} + 3k = 1$

$\frac{6}{10} + 6k = 1$

$6k = 1 - \frac{6}{10}$

$6k = \frac{4}{10}$

$k = \frac{\cancel{4}^{2}}{\cancel{10}_{5} \cdot \cancel{6}_{3}}$

$\boxed{k = \frac{1}{15}}$

Pmf

x:	-2	-1	0	1	2	3
$p(x)$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{2}{10}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{3}{15}$

ii) $P(x<2) = P(x=-2) + p(x=-1) + p(x=0) + p(x=1)$

$= \frac{1}{10} + \frac{1}{15} + \frac{2}{10} + \frac{2}{15} = \frac{3}{10} + \frac{\cancel{3}^{1}}{\cancel{15}_{5}} = \frac{15+10}{50}$

$= \frac{25}{50} = \frac{1}{2}$

OR.

$P(x<2) = 1 - p(x \geq 2) = 1 - [P(x=2) + p(x=3)]$

$= 1 - \left[\frac{3}{10} + \frac{1}{5}\right] = 1 - \left[\frac{1}{2}\right] = \frac{1}{2}$

$$P(-2<x<2) = P(x=-1) + P(x=0) + P(x=1)$$

$$= \frac{1}{15} + \frac{2}{10} + \frac{2}{15}$$

$$= \frac{\cancel{3}^{1}}{\cancel{15}_{5}} + \frac{1}{5} =$$

$$= \frac{2}{5}$$

Mean $E[x] = \sum x\, p(x)$

$$= (-2)\times\left(\frac{1}{10}\right) + (-1)\times\left(\frac{1}{15}\right) + 0\times\left(\frac{2}{10}\right) + 1\left(\frac{2}{15}\right) + 2\times\frac{3}{\cancel{10}_{5}} + 3\times\frac{3}{\cancel{15}_{5}}$$

$$= -\frac{1}{5} - \frac{1}{15} + 0 + \frac{2}{15} + \frac{3}{5} + \frac{3}{5}$$

$$= 1 + \frac{1}{15} = \frac{16}{15}$$

Variance (σ^2)

$\cancel{E}$ $Var(x) = E[x^2] - E[x]^2$

$$E[x^2] = \sum x^2 p(x)$$

$$= 4\times\frac{1}{10} + 1\times\frac{1}{15} + 0\times\frac{2}{10} + 1\times\frac{2}{15} + 4\times\frac{3}{\cancel{10}_{5}} + 9\times\frac{3}{\cancel{15}_{5}}$$

$$= \frac{2}{5} + \frac{1}{15} + 0 + \frac{2}{15} + \frac{6}{5} + \frac{9}{5}$$

$$= \frac{17}{5} + \frac{\cancel{3}^{1}}{\cancel{15}_{5}} = \frac{18}{5}$$

$$Var(x) = E[x^2] - E[x]^2$$

$$= \frac{18}{5} - \left(\frac{16}{15}\right)^2$$

$$= \frac{554}{225}$$

$$S.D = \sqrt{Var} = \sqrt{\frac{554}{225}}$$

x:	-2	-1	0	1	2	3
$p(x)$:	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{2}{10}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{3}{15}$
$F(x) = P(X \le x)$	$\frac{1}{10}$	$\frac{1}{6}$	$\frac{11}{30}$	$\frac{1}{2}$	$\frac{4}{5}$	1

$\frac{11}{30} + \frac{2}{15}$

$\frac{1}{6} + \frac{1}{5}$

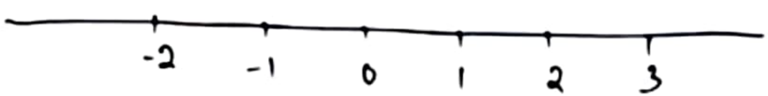
$= \frac{11}{30}$

$$F(x) = P(X \le -2) = P(X = -2) = \frac{1}{10}$$

$$F(x) = P(X \le -1) = P(X = -2) + P(X = -1) = \frac{1}{10} + \frac{1}{15} = \frac{25}{150} = \frac{1}{6}$$

$$F(x) = P(X \le 0) = P(X = -2) + P(X = -1) + P(X = 0) = \frac{1}{6} + \frac{2}{10} = \frac{11}{30}$$

$\frac{4}{5} + \frac{3}{15} = \frac{5}{5} = 1$



$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{10} & -2 \le x < -1 \\ \frac{1}{6} & -1 \le x \le 0 \\ \frac{11}{30} & 0 \le x < 1 \\ \frac{1}{2} & 1 \le x < 2 \\ \frac{4}{5} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

$\cancel{2 \le x <}$

Given that $f(x) = \frac{k}{2^x}$ is a probability distribution for a

u.a/ Can take Values $x = 0, 1, 2, 3, 4$ find i) k ii) mean and Varience

iii) Cummulative distribution function

iv) Probability that x is even.

$$k = \frac{16}{31}$$

$$E[x] = \frac{26}{31}$$

$Var(x)$ $$E[x^2] = \frac{58}{31}$$

$$Var(x) = \underline{\underline{1.167}}$$

cdf

x:	0	1	2	3	4
$p(x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$
$F(x)$	$\frac{16}{31}$	$\frac{24}{31}$	$\frac{28}{31}$	$\frac{30}{31}$	1

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{16}{31} & 0 \le x < 1 \\ \frac{24}{31} & 1 \le x < 2 \\ \frac{28}{31} & 2 \le x < 3 \\ \frac{30}{31} & 3 \le x < 4 \end{cases}$$

$$P(x \text{ is even}) = p(x=2) + p(x=4)$$
$$= \frac{4}{31} + \frac{1}{31} = \frac{5}{31}$$

Q.6/ Can take Values $x = 0, 1, 2, 3, 4$ find i) k ii) mean and (Variance)

iii) Cummulative distribution function.

iv) Probability that x is even.

$k = \frac{16}{31}$

$E[x] = \frac{26}{31}$

cdf

x :	0	1	2	3	4
$p(x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$
				$\frac{30}{31}$	1

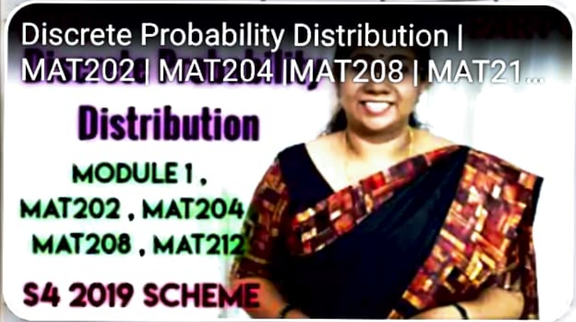
Var(x) $E[x^2]$

Var(x) =

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{16}{31} & 0 \le x < 1 \\ \frac{24}{31} & 1 \le x < 2 \\ \frac{28}{31} & 2 \le x < 3 \\ \frac{30}{31} & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

$P(x \text{ is even}) = p(x=2) + p(x$

$= \frac{4}{31} + \frac{1}{31}$



Probability Mass function

x:	0	1	2	3	....	n
$p(x)$	P_0	P_1	P_2	P_3	...	P_n.

1) $\sum P(x) = 1$

2) Mean, Expectation of x.

$$E[x] = \sum_{i=0}^{n} x_i p_i(x)$$

3) Varience of x, $Var(x)$, σ^2

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \sum x^2 p(x)$$

$$E[x^3] = \sum x^3 p(x)$$

Standard deviation $= \sqrt{\text{Variance}}$.
(σ)

4) Distribution fun
(Cumulative distribution fun)

$$F(x) = P[x \leq a]$$

Properties.

$E[\text{constant}] = \text{Constant}$

$E[ax + by] = aE[x] + bE[y]$

$Var(\text{constant}) = 0$

$Var(ax + b) = a^2 Var(x) + Var(b)$

A random Variable x has following Probability mass function.

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Discrete:

i) find k

ii) find $P(x<2)$

iii) find $P(2<x\le 5)$, $P(1<x<4)$, $P(x>6)$

iv) find distribution fun: of x.

v) find mean and Varience.

w.k.T $\sum P(x)=1$

$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$10k^2+9k=1$

$10k^2+9k-1=0$

$a=10$
$b=9$
$c=-1$

$$k=\frac{-9\pm\sqrt{81+40}}{20}$$

$$k=\frac{-9\pm\sqrt{121}}{20}$$

$$=\frac{-9\pm 11}{20}$$

$$=-\frac{20}{20},\ \frac{2}{20}$$

$$=-1,\ \frac{1}{10}$$

$0\le P(x)\le 1$

$\boxed{k=\frac{1}{10}}$

x	0	1	2	3	4	5	6	7
$P(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

ii) $P(x<2) = P(x=0) + P(x=1)$

$$= 0 + \frac{1}{10}$$

$$= \frac{1}{10}$$

iii) $P(2 < x \leq 5) = P(x=3) + P(x=4) + P(x=5)$

$$= \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$$

$$= \frac{51}{100}$$

$P(1 < x < 4) = P(x=2) + P(x=3)$

$$= \frac{2}{10} + \frac{2}{10} = \frac{4}{10}$$

$$= \frac{2}{5}$$

$P(x>6) = P(x=7)$

$$= \frac{17}{100}$$

$P(x<6) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5)$

OR.

$$= 1 - P[x \geq 6]$$

$$= 1 - [P(x=6)+P(x=7)]$$

$$= 1 - \left[\frac{2}{100} + \frac{17}{100}\right]$$

$$= \frac{81}{100}$$

x	0	1	2	3	4	5	6	7
$p(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$
$F(x) = P(x \le a)$	0	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{8}{10}$	$\frac{81}{100}$	$\frac{83}{100}$	1

<u>cdf</u>

$$F(x) = \begin{cases} 0 & x < 0 \\ 0 & 0 \le x < 1 \\ \frac{1}{10} & 1 \le x < 2 \\ \frac{3}{10} & 2 \le x < 3 \\ \frac{5}{10} & 3 \le x < 4 \\ \frac{8}{10} & 4 \le x < 5 \\ \frac{81}{100} & 5 \le x < 6 \\ \frac{83}{100} & 6 \le x < 7 \\ 1 & x \ge 7 \end{cases}$$

$$E[x] = \sum x\,p(x)$$

$$= 0 + \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{17}{10} + \frac{5}{100} + \frac{12}{100} + \frac{119}{100}$$

$$= \frac{366}{100}$$

$$= \underline{\underline{3.66}}$$

$$Var(x) = E[x^2] - E[x]^2.$$

$$E[x^2] = \sum x^2 p(x)$$

$$= 0 + \frac{1}{10} + \frac{8}{10} + \frac{18}{10} + \frac{48}{10} + \frac{25}{100} + \frac{72}{100} + \frac{833}{100}$$

$$= \frac{168}{10} = \underline{\underline{16.8}}$$

$$Var(x) = 16.8 - (3.66)^2$$

$$= \underline{\underline{3.41}}$$

$$S.D = \sqrt{Var.}$$

$$= \sqrt{3.41}$$

$$= \underline{\underline{\quad}}$$

A random variable x takes values 1, 2, 3 and 4 such that

$$2p(x=1) = 3p(x=2) = p(x=3) = 5p(x=4)$$

find PMF and CDF of x.

$$2P(x=1) = 3P(x=2) = P(x=3) = 5p(x=4) = k.$$

$2P(x=1) = k$ | $3p(x=2) = k$ | $P(x=3) = k$ | $5P(x=4) = k$

$p(x=1) = \frac{k}{2}$ | $p(x=2) = \frac{k}{3}$ | $p(x=3) = k$ | $p(x=4) = \frac{k}{5}$

x	1	2	3	4
$p(x)$	$\frac{k}{2}$	$\frac{k}{3}$	k	$\frac{k}{5}$

To find k. $\Sigma p(x) = 1$

$$\frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$$

$$k\left[\frac{5}{6} + \frac{6}{5}\right] = 1$$

$$k\left[\frac{61}{30}\right] = 1$$

$$k = \frac{30}{61}$$

x :	0	1	2	3	4
$P(x)$		$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$
$F(x)$ $= P(x \le a)$		$\frac{15}{61}$	$\frac{25}{61}$	$\frac{55}{61}$	1

$$\frac{\frac{30}{61}}{5}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{15}{61} & 1 \le x < 2 \\ \frac{25}{61} & 2 \le x < 3 \\ \frac{55}{61} & 3 \le x < 4 \\ 1 & \cancel{4 \le x <} \; x \ge 4 \end{cases}$$

V.Imp

Q) If the random Variable x takes the Values ~~1, 2, 3 and 4~~ -3, -2, -1, 0, 1, 2, 3

Such that $P(x=0) = P(x>0) = P(x<0)$: $P[x=-3] = P[x=-2] = P[x=-1] = P[x=1] = P[x=2] = P[x=3]$. Obtain the Probability distribution and distribution function of x.

x :	-3	-2	-1	0	1	2	3
$p(x)$		$\frac{1}{3}$		$\frac{1}{3}$		$P(x>0)$	

$= 1$

$P[x=0] = P[x>0] = P[x<0] = \frac{1}{3}$

$P[x=0] = \frac{1}{3}$ $\qquad$ $P[x>0] = \frac{1}{3}$ $\qquad$ $P[x<0] = \frac{1}{3}$.

$P(x=1) + P(x=2) + P(x=3) = \frac{1}{3}$ $\qquad$ $P(x=-3) + P(x=-2) + P(x=-1) = \frac{1}{3}$

$P(x=1) + P(x=1) + P(x=1) = \frac{1}{3}$

$3P(x=1) = \frac{1}{3}$

$P(x=1) = \frac{1}{9}$.

$P(x=-3) = \frac{1}{9}$.

$P(x=-2) = \frac{1}{9}$

$P(x=-1) = \frac{1}{9}$ $\qquad$ $P(x=3) = \frac{1}{9}$

$P(x=2) = \frac{1}{}$

Pmf

x	-3	-2	-1	0	1	2	3
$P(x)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$F(x) = P(X \le x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	1

$$F(x) = \begin{cases} 0 & x < -3 \\ \frac{1}{9} & -3 \le x \le -2 \\ \frac{2}{9} & -2 \le x \le -1 \\ \frac{3}{9} & -1 \le x < 0 \\ \frac{6}{9} & 0 \le x < 1 \\ \frac{7}{9} & 1 \le x < 2 \\ 8/9 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

The CDF of a random Variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.5 & 0 \le x < 2 \\ 0.75 & 2 \le x < 3 \\ 0.90 & 3 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

i) find PMF of x.

ii) find $P[X \le 3]$ and $P(2 \le X < 5)$

x	0	2	3	5
$F(x)$	0.5	0.75	0.90	1
$p(x)$	0.5	0.75-0.5 0.25	0.15	0.1

$P(X \le 3) = P(0) + P(2) + P(3)$

or

$= 1 - P(X=5)$

$= 1 - 0.1$

$= \underline{0.9}$

$P(2 \le X < 5) = P(X=2) + P(X=3)$

$= 0.25 + 0.15$

$= \underline{0.4}$

If x has the following probability distribution

x:	0	1	2	3	4	5	6	7	8
$f(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i) find a, $P(x<3)$, $P(2 \leq x \leq 4)$, $P(1 < x \leq 4)$

ii) Find mean and Variance.

iii) Find distribution function.

$a = \frac{1}{81}$

mean $= 5.48$

Var $= 4.47$.