5.14.2 *Direct form II realization*

Consider the difference equation of the form

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \tag{5.110}$$

The system function of above difference equation can be expressed as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{5.111}$$

Let $\dfrac{Y(z)}{X(z)} = \dfrac{Y(z)}{W(z)} \cdot \dfrac{W(z)}{X(z)}$ where

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

which gives us

$$W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) \ldots - a_N z^{-N} W(z) \tag{5.112}$$

and $$\frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k} \text{from which} \tag{5.113a}$$

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) + \ldots + b_M z^{-M} W(z) \tag{5.113b}$$

Now Eq.(5.112) and Eq.(5.113b) can be expressed in difference equation form i.e.,

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \ldots - a_N w(n-N) \tag{5.114}$$

and

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \ldots + b_M w(n-M). \tag{5.115}$$

From Eq. (5.114) and Eq.(5.115) we observe that the same delay terms $w(n-1), w(n-2)\ldots$ etc, are used to express $w(n)$ and $y(n)$.

The realization of Eq.(5.114) and Eq.(5.115) are shown in Fig. (5.35) and Fig. (5.36) respectively.

To obtain the realization of difference Equation (5.110) combine Fig. (5.35) and Fig. (5.36).

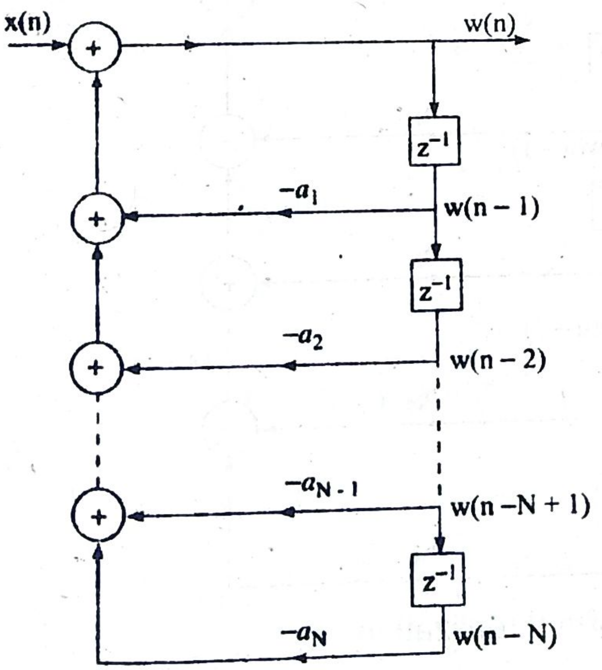


Fig. 5.35 Realization of Eq. (5.114).

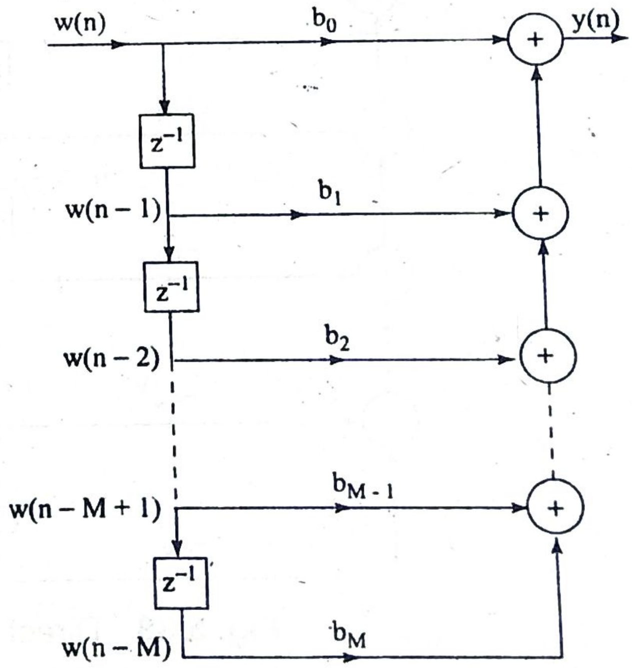


Fig. 5.36 Realization of Eq. (5.115).

From Fig. 5.37 we find that the two delay elements contain the same input $w(n)$ and hence the same output $w(n-1)$. Consequently we can merge these delays into one delay and can redraw the Fig. 5.37 as shown in Fig. 5.38.

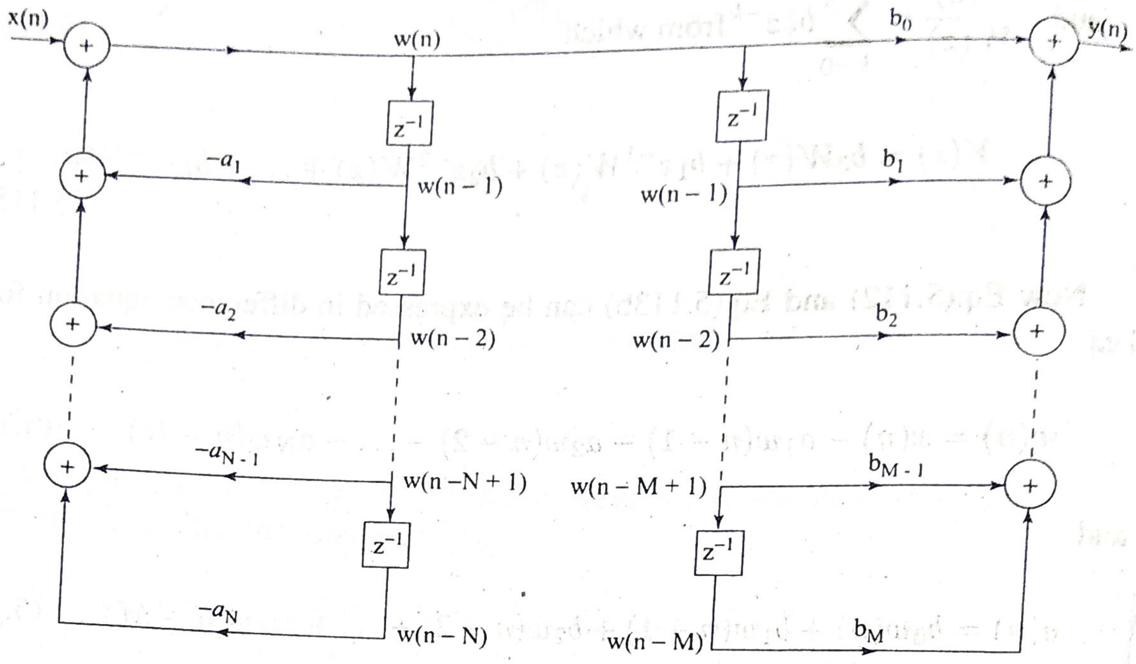


Fig. 5.37 Realization structure of Eq.(5.110)

The realization structure shown in Fig. 5.38 is called a direct form II realization. This structure requires $M+N+1$ multiplications, $M+N$ additions and the maximum of $\{M, N\}$ memory locations. Since the direct form II realization minimizes the number of memory locations, it is said to be canonic.

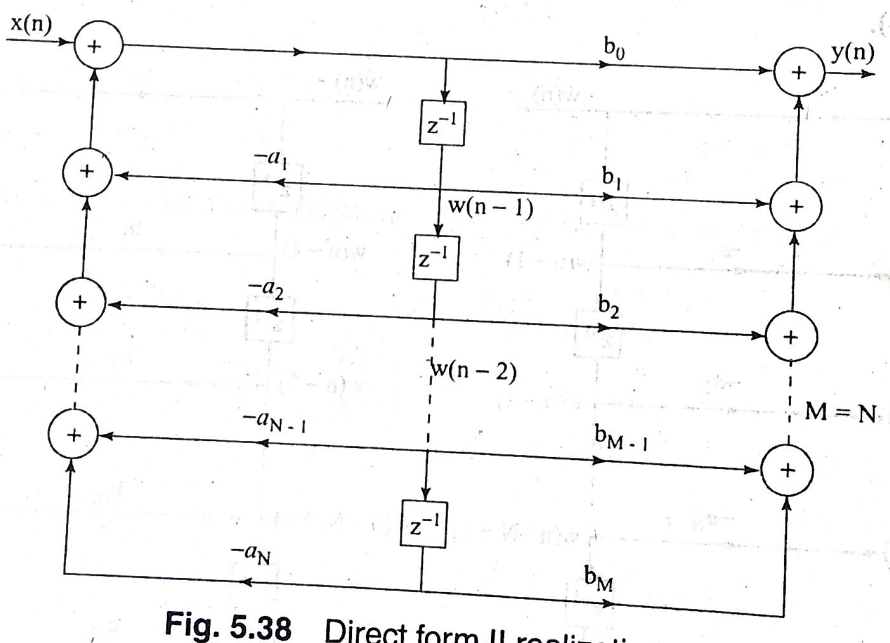


Fig. 5.38 Direct form II realization

Example 5.22 Realize the second order system $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$ in direct form II.

Solution

Given $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$

The system function

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}} \tag{5.116}$$

Let $\dfrac{Y(z)}{X(z)} = \dfrac{Y(z)}{W(z)} \cdot \dfrac{W(z)}{X(z)}$

where $$\frac{Y(z)}{W(z)} = 1 - r\cos(\omega_0)z^{-1} \tag{5.117a}$$

and $$\frac{W(z)}{X(z)} = \frac{1}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}} \tag{5.117b}$$

From Eq. (5.117a) we obtain $Y(z) = W(z) - r\cos(\omega_0)z^{-1}W(z)$ which gives us

$$y(n) = w(n) - r\cos(\omega_0)w(n-1) \tag{5.118a}$$

and from Eq. (5.117b) we have
$W(z) = X(z) + 2r\cos(\omega_0)z^{-1}W(z) - r^2z^{-2}W(z)$ which gives us

$$w(n) = x(n) + 2r\cos(\omega_0)w(n-1) - r^2w(n-2) \tag{5.118b}$$

We realize Eq. (5.118a) and Eq. (5.118b) and combine them to get the direct form II realization.

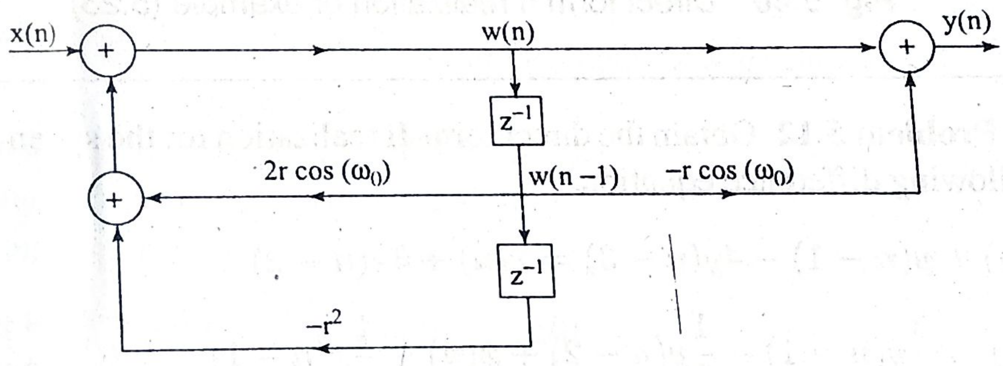


Fig. 5.39 Direct form II realization of example (5.21)

Q) Determine the direct form II realization for the following system

$y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n)$
$\quad - 0.252\, x(n-2)$

Ans:

On taking z transform.

$$Y(z) = -0.1\, z^{-1} Y(z) + 0.72\, z^{-2} Y(z) + 0.7\, X(z)$$
$$- 0.252\, z^{-2} X(z)$$

$$\Rightarrow Y(z)\left[1 + 0.1 z^{-1} - 0.72 z^{-2}\right] = X(z)\left[0.7 - 0.252 z^{-2}\right]$$

$\therefore$ The system function.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}} \quad \text{——— (1).}$$

Let $\dfrac{Y(z)}{X(z)} = \dfrac{Y(z)}{W(z)} \cdot \dfrac{W(z)}{X(z)}.$

where $\dfrac{Y(z)}{W(z)} = 0.7 - 0.252 z^{-2}$ ——— (2)

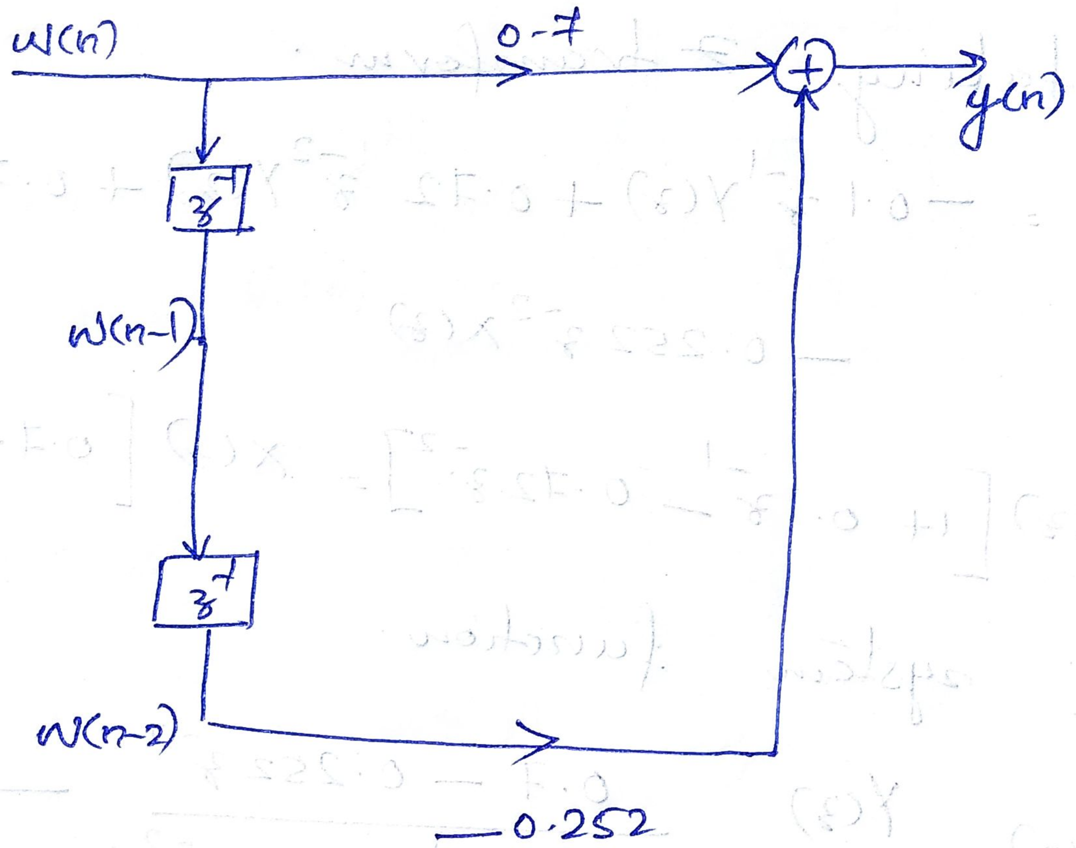
and $\dfrac{W(z)}{X(z)} = \dfrac{1}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$ ——— (3)

② $\Rightarrow$

$$Y(z) = 0.7\,W(z) - 0.252\,z^{-2}\,W(z)$$

then $y(n) = 0.7\,w(n) - 0.252\,w(n-2)$

corresponding realization



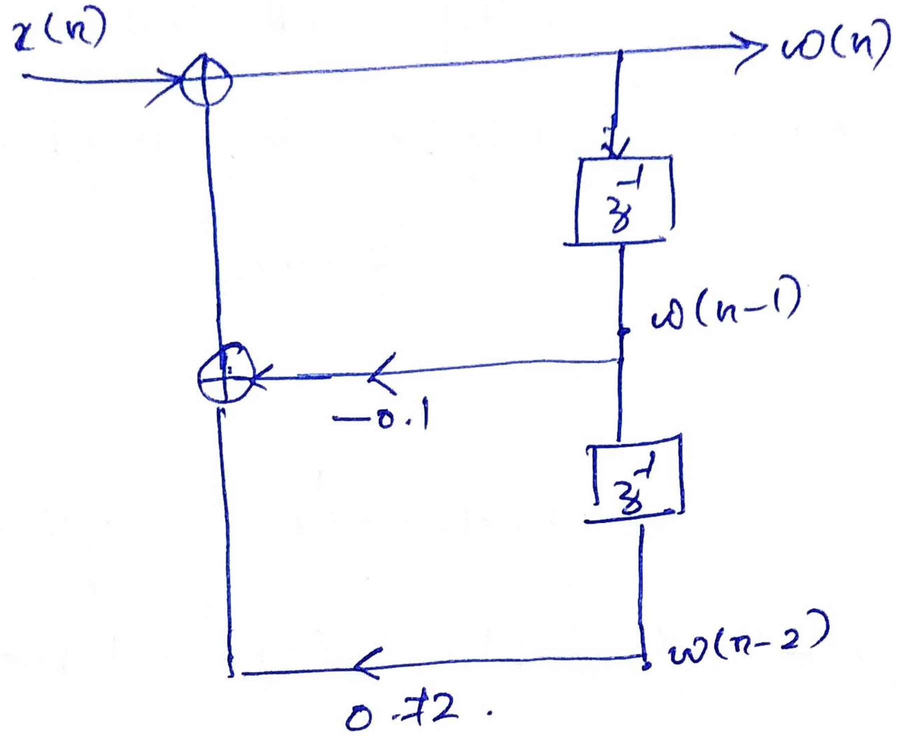
③ $\Rightarrow$

$$W(z)\left[1 + 0.1z^{-1} - 0.72z^{-2}\right] = X(z)$$

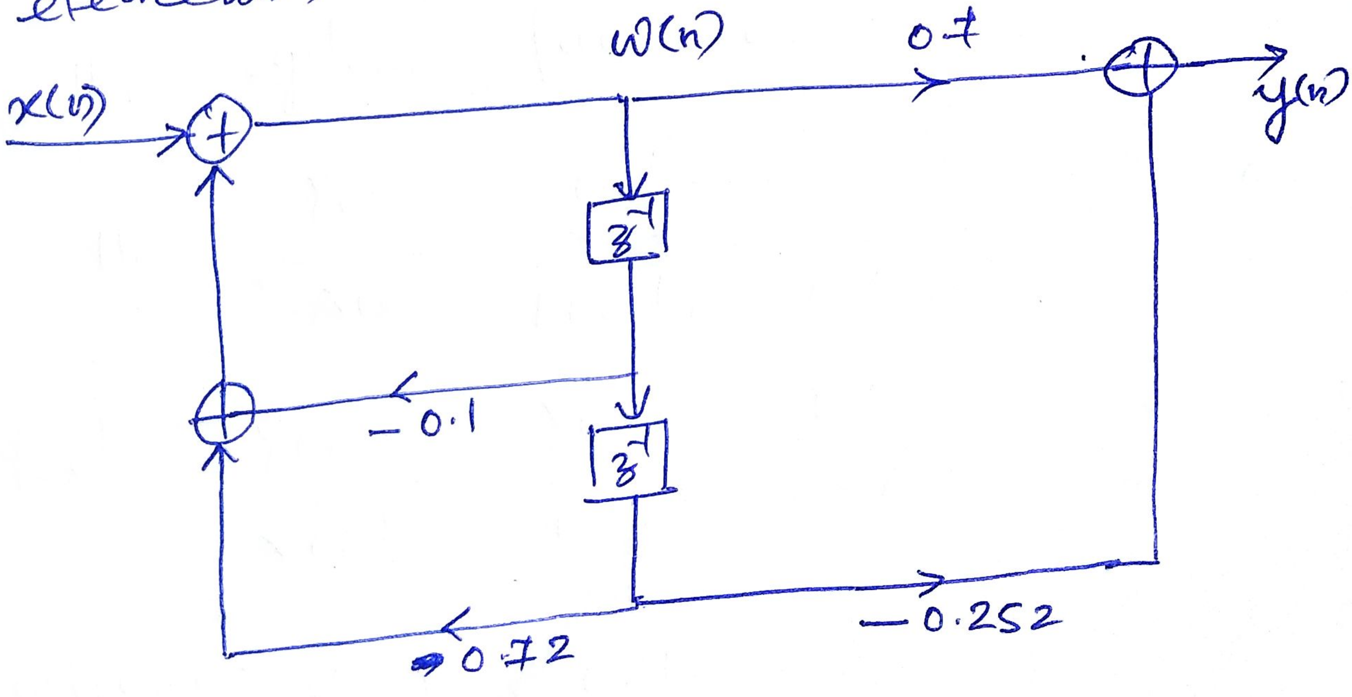
$$\therefore W(z) = X(z) - 0.1z^{-1}W(z) + 0.72z^{-2}W(z)$$

$$\therefore w(n) = x(n) - 0.1\,w(n-1) + 0.72\,w(n-2)$$

corresponding realization



Combining these two realization (we can reduce the no. of delay elements).



which is the direct form II realization.