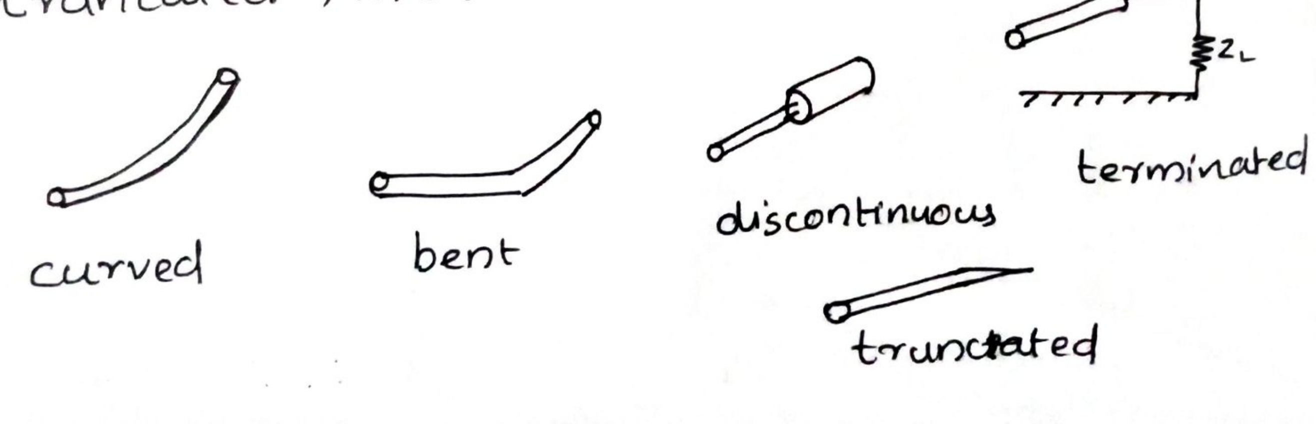
Module - I

Basic antenna parameters: gain, directivity, beam width, and effective aperture calculations, effective height, wave polarization, radiation resistance, radiation efficiency, antenna field zones, Duality and principles of reciprocity, Helmholtz theorem (derivation required), Field, directivity and radiation resistance of a short dipole and halfwave dipole (far field derivation)

Introduction:

Radiation is the study of detachment of electromagnetic energy from its sources to free space through the use of special arrangement of conductors known as antennas. By free space we mean a space that do not interfere with the normal radiation and propagation of radio waves. It does not have magnetic or gravitational fields, no solid bodies, no ionized Particles. Such a free space is ofcourse hypothetic but it simplifies the approach to wave propagation.

An electric charge is the basic source of radiation. when stationary or moving with a constant velocity, the charge does not radiate. However, if acceleration of the charge occurs, then radiation takes place. There is no radiation if the wire is straight & infinite. If the wire is curved, bent, discontinuous, terminated or truncated, then radiation occurs.



If the charge is accelerated and has a changing velocity with time, it radiates. If a charge reversing direction on reflection from the end of a wire radiates. Similarly, electric charge oscillating back and forth in simple harmonic motion along a wire undergoes periodic acceleration and radiates.

Antenna is a structure, designed for radiating electromagnetic energy efficiently in prescribed manner. In contrast to a transmission line or waveguide which is used to minimize radiation, an antenna enhance the radiation of electromagnetic waves through freespace.

An antenna may be defined as:

a) connecting link between transmission line & freespace.

b) An impedance transformer to match the txn line impedance (50 Ω) to atmosphere impedance (377 Ω)

c) A transducer that converts electrons to photons.

Types of antenna:

1) 1st generation antenna

Also known as wire antenna.

eg: monopole, Dipole etc (gain is very small)

2) 2^{nd} generation antenna.

Reflector antenna : corner, parabolic, cassegrain.

3) 3^{rd} generation antenna.

Array antenna : End fire, Broadside.

4) 4^{th} generation antenna.

microstrip antenna.

5) 5^{th} generation antenna.

eg:-smart antenna, active antenna.

Antenna parameters :-

ISOTROPIC RADIATORS :-

Isotropic radiators (isotropic source, unipole or omnidirectional radiator) are radiator which radiates uniformly. These are used as reference antenna. It does not have directional properties.

Imagine that an isotropic radiator is situated at the centre of a sphere of radius (r). Then all the energy radiated from it, must pass over the surface area of the sphere ($4\pi r^2$). Power density P at any point on the sphere gives "power radiated per unit area in any direction". Since radiated power from a isotropic source flows in radial lines, therefore the magnitude of P equal to the radial component only ($P_\theta = P_\phi = 0$), $|P| = P_r$

The total power radiated (W_t) by the source is integral over the surface of the sphere of the radial component P_r

$$W_t = \iint P_r \cdot ds = P_r \iint ds$$

$$W_t = P_r \cdot 4\pi r^2 \qquad \because \iint ds = 4\pi r^2 \text{, area of surface}$$

$$P_r = \frac{W_t}{4\pi r^2} \text{ Watts}/m^2.$$

Radiation Pattern:-

The radiated energy from an antenna is not of same strength in all directions. It is more in one direction and less or zero for other direction. The energy radiated in a particular direction by the antenna is measured in terms of field strength. For the calculation of field strength, the voltages at two points on an electric lines of force are taken and then it is divided by the distance between two points. Hence the unit of radiation pattern is volt/meter. The radiation pattern of an antenna is its most basic requirement because it determines the distribution of radiated energy in space.

It is nothing but a graph which shows the variation in actual field strength of electromagnetic field at all points which are at equal distance from the antenna. It is a 3-Dimensional graph. The radiation Patterns are different for different antennas and are affected by the location of antenna w.r.t. ground.

If the radiation from the antenna is expressed in terms of field strength E, the radiation pattern is called as "Field strength Pattern". If it is expressed in terms of power, then the resulting pattern is power pattern. However, both related to each other — a power pattern is proportional to the square of the field strength pattern.

Since radiation pattern is a three dimensional figure, it uses spherical co-ordinate system (r, θ, ϕ). The antenna is assumed to be located at origin of spherical co-ordinate system and the field strength is specified at points on the spherical surface of radius (r). The shape of radiation pattern does not depends on the radius r. The radiation field may have components E_θ and E_ϕ

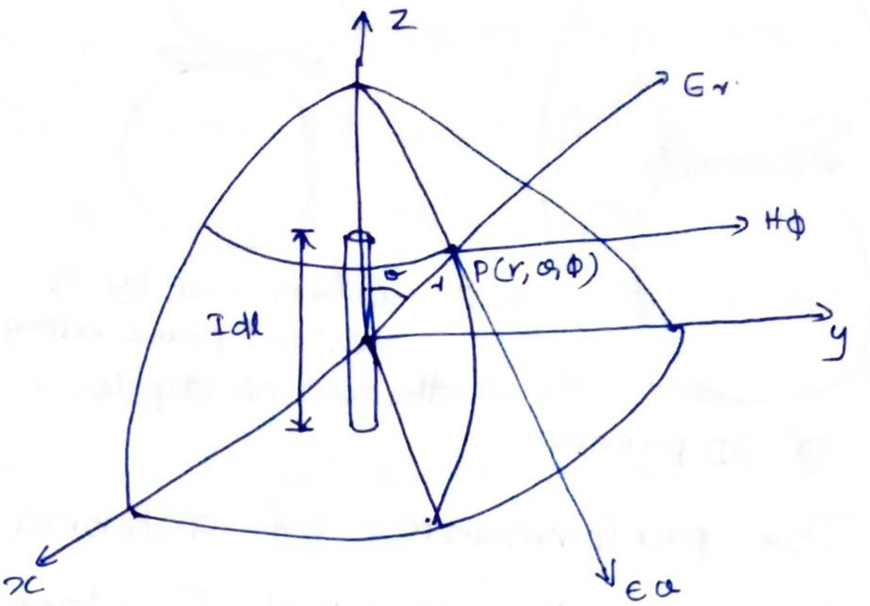
$$E = \sqrt{E_\theta^2 + E_\phi^2}.$$

E = Total electric field strength

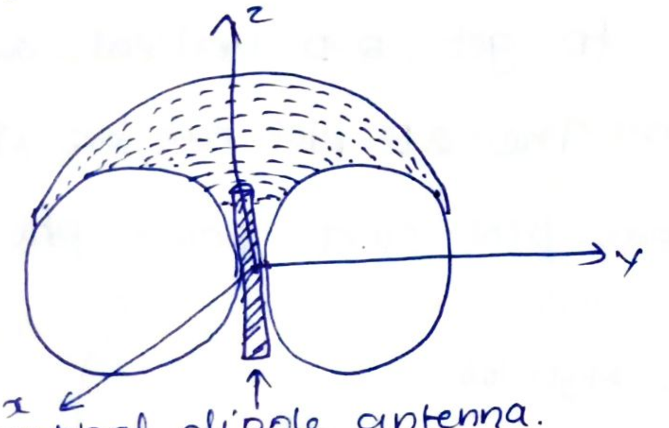
$E_\theta \rightarrow$ Amplitude of θ component

$E_\phi \rightarrow$ Amplitude of ϕ component.

A complete radiation pattern is 3-D figure and gives the radiation for all angles of θ and ϕ. To represent this on a 2-D plane a cross-section through 3D pattern is taken.

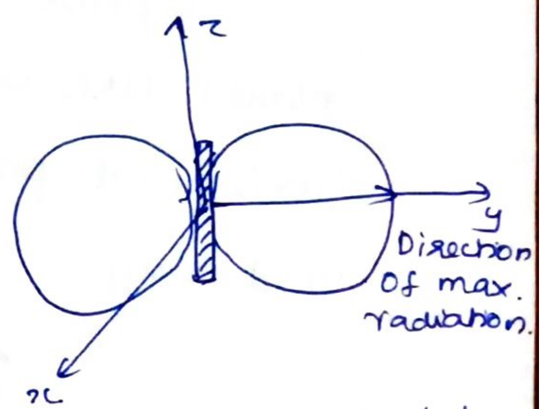


For a vertically placed dipole antenna, the three dimensional pattern is doughnut shaped. Two dimensional pattern is a figure of eight (∞), when cut by a vertical plane and a circle when cut by a horizontal plane.

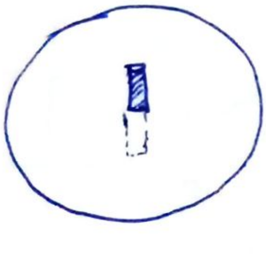


vertical dipole antenna.

Half of the 3D pattern.

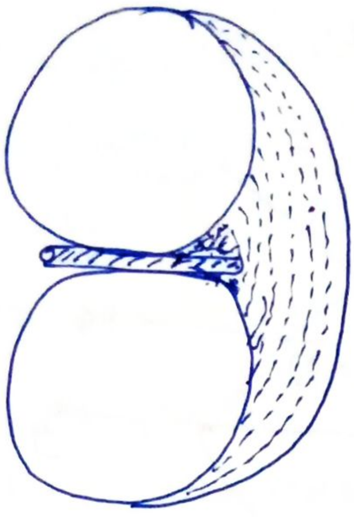


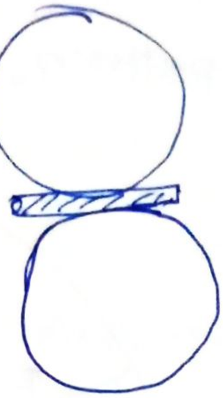
2D pattern obtained by cutting 3D pattern with a vertical plane along the axis of dipole.



2D pattern when cut by horizontal plane at the centre of the dipole.

For a horizontally placed dipole antenna, three dimensional pattern is given below (half)


Half of 3D pattern.


when cut by a Vertical plane along the axis of dipole.

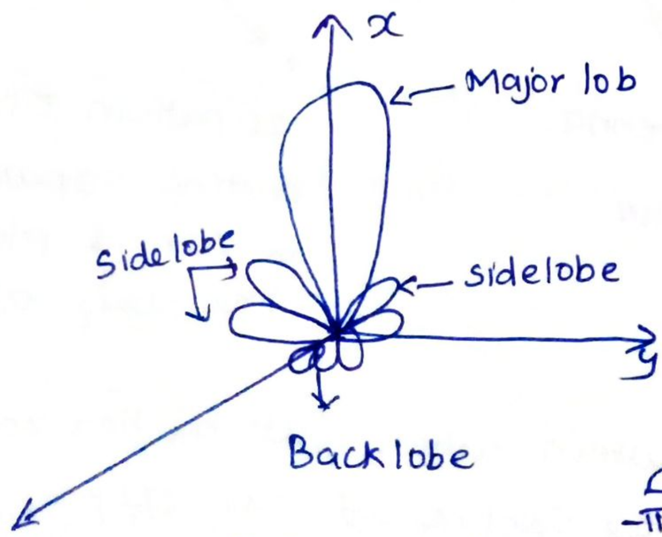

when cut by a Vertical plane at centre of dipole ⊥r to its axis.

The performance of antenna is usually described by its principal E-plane and H-plane Patterns.

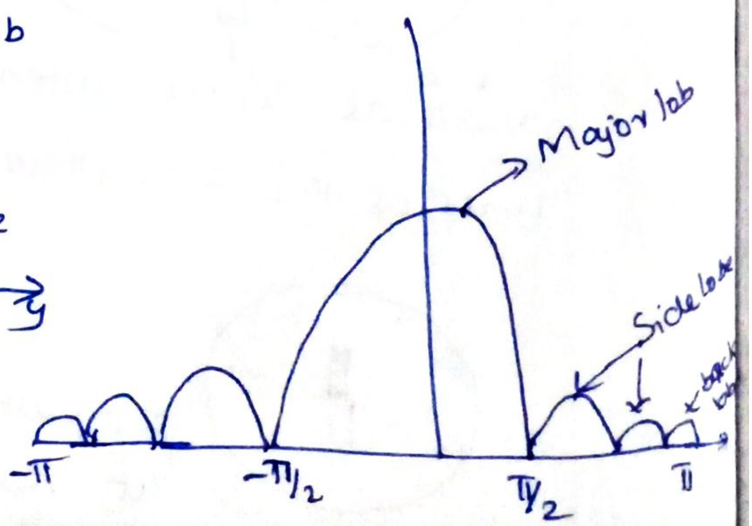
E plane → plane containing electric field vector

H plane → plane containing magnetic field vector.

Eplane (elevation plane) and H plane (Azimuth plane) are used to get 2-D Vertical and horizontal pattern. The 2D plot can be drawn in 2 ways. Linear plot and polar plot.



Polar Plot



A lobe is a portion of radiation pattern and it is sub-classified into different types. The lobe indicating maximum radiation is called major lobe, while all other lobes are called minor lobes. The lobes adjuscent to the major lobe are called side lobes, while those in a direction opposite to the major lobes are called back lobes. Minor lobes represent radiation in undesired direction and should be minimized.

Gain

since antenna is a passive device, its gain is different from the gain of an amplifier, which is the ratio of output to input.

For an antenna, there are several defenitions for gain.

~~Gain (G) = Max. radiation intensity from a reference antenna~~

$$\text{Gain (G)} = \frac{\text{Max. radiation intensity from a test antenna}}{\text{Max. radiation intensity from a reference antenna with same i/p Power}}$$

$$G = \frac{\Phi'_m}{\Phi_0}$$

Φ'_m = Max. radiation from test antenna

Φ_0 = Max. radiation from a lossless isotropic antenna.

Reference antenna :- It is a lossless isotropic radiator or antenna which radiates uniformly in all directions.

Gain is directly proportional to directivity (D)

consider an antenna with i/p power Pin and radiated power P_t. If the antenna is lose less, then $P_t = P_{in}$, then $G = D$

However, practically losses such as reflection loss, ohmic loss, dielectric loss etc, makes $P_t < P_{in}$ $\therefore G < D$.

$G = \eta D$ $\because \eta \rightarrow$ antenna efficiency.

Interms of signal Power, the gain of the antenna can be defined as,

$$\text{Gain } (G) = \frac{\text{Maximum Power received from given antenna } (P_1)}{\text{Maximum Power received from reference antenna } (P_2)}$$

$$G = \frac{P_1}{P_2}$$

Directive Gain :-

Directive Gain (G_d) in a given direction is defined as the ratio of the radiation intensity in that direction to the average radiated power

The directive gain is a function of

angles (θ and ϕ).

$\Phi(\theta, \phi)$ = Radiation intensity in a particular direction.

Φ_{av} = Average radiation Intensity in ~~a particular~~ that direction = $\frac{W_r}{4\pi}$

Directive gain (G_d) = $\frac{\text{Radiation Intensity in a Particular direction}}{\text{Average radiated Power}}$

$$G_d(\theta, \phi) = \frac{\Phi(\theta, \phi)}{\Phi_{av}} = \frac{\Phi(\theta, \phi)}{\frac{W_r}{4\pi}} = \frac{4\pi\, \Phi(\theta, \phi)}{W_r}$$

$W_r \rightarrow$ Total Power radiated.

Directivity (D)

The maximum directive gain is called as directivity of an antenna and it is denoted by D. In a particular direction the directivity D is a constant.

Directivity of an antenna is defined as the ratio of maximum radiation intensity to it average radiation intensity. ie,

$$D = \frac{\text{Maximum radiation intensity of test antenna}}{\text{Average radiation Intensity of test antenna.}}$$

$$D = \frac{\Phi(\theta, \phi)_{max}}{\Phi_{av}} \quad \text{(both of test antenna)}$$

Directivity may also be defined as the

ratio of maximum radiation intensity of the subject antenna to the radiation intensity of an isotropic antenna

$$D = \frac{\text{Maximum radiation intensity of subject (test) antenna}}{\text{Radiation intensity of isotropic antenna}}$$

$$\boxed{D = \frac{\Phi(\theta, \phi)_{max} \text{ (test antenna)}}{\Phi_0 \text{ (isotropic antenna)}}}$$

Since the average radiation intensity (Φ_{av}) is obtained by dividing total power radiated W by 4π steradiam.

$$D = \frac{\Phi(\theta, \phi)\, max}{W_r/4\pi}$$

$$D = \frac{4\pi\, \Phi(\theta, \phi)_{max}}{W_r}$$

$$\boxed{D = \frac{4\pi(\text{Maximum radiation intensity})}{\text{Total radiated power}}}$$

In general directivity may be represented by

$$D(\theta, \phi) = \frac{\Phi_{max}}{\Phi} = D$$

$$\boxed{D(\theta, \phi) = \frac{\Phi}{\Phi_{max}} D}$$

Φ_{max} = max. radiation intensity

Φ = Radiation intensity in direction of (θ, ϕ)

In db, $D(db) = 10\log_{10} D$

The numerical value of D always lies between 1 and ∞. ie, $1 \le D \le \infty$. 1 being the directivity of an isotropic antenna and hence D can not be less than 1.

Directive Gain and directivity

For a lossless isotropic antenna, directive gain and directivity is same. In this case radiation efficiency factor $k = 1$

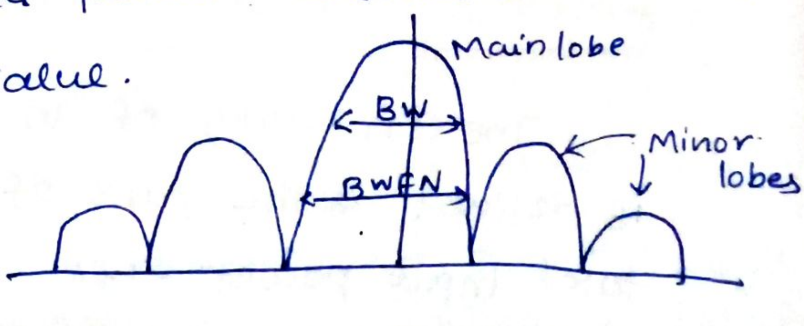
$G_d = kD$

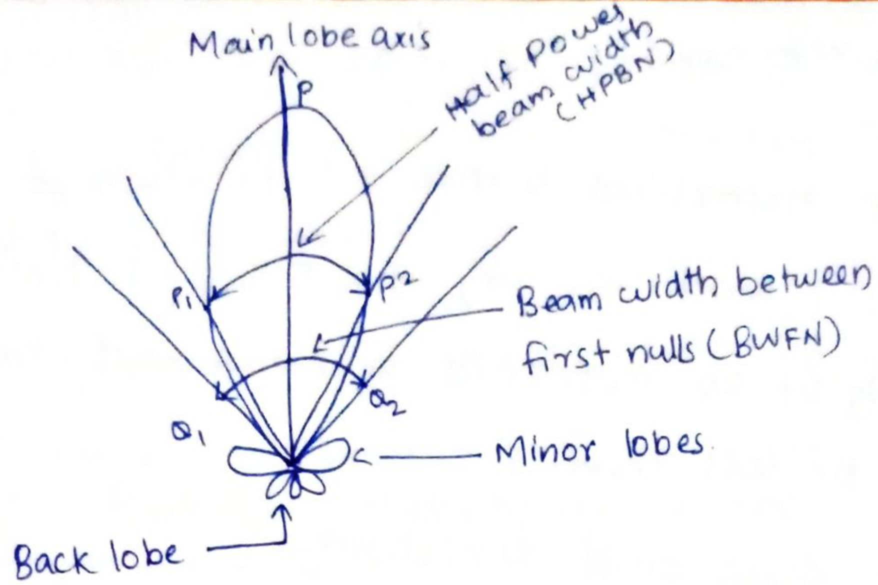
$\boxed{G_d = D}$ $\because k = 1$

Antenna beam width

Antenna beam width is the measure of the directivity of the antenna. The antenna beam width is an angular width in degrees. It is measured on a radiation pattern on major lobe

Antenna beam width is defined as the angular width in degrees between the two points on a major lobe of a radiation pattern where the radiated power decreases to half of its maximum value.





The beam width is also called Half Power Beam width (HPBW) because it is measured between two points on the major lobe where the Power is half of its maximum power. In the figure, the power is maximum at point P, while it is half at points P_1 and P_2 both. Hence the angular width between points P_1 and P_2 is nothing but antenna beam width or HPBW. It is also called 3-dB beam width.

The radiation pattern can be described interms of the angular width between first nulls or first side lobes. Then such angular beam width is called Beam width between First nulls (BWFN)

Antenna Efficiency (η)

The Efficiency of an antenna (radiation efficiency) is defined as the ratio of power radiated to the total input power supplied to the antenna and it is denoted by η or k.

$$\text{Antenna efficiency} = \frac{\text{Power Radiated } (W_r)}{\text{Total input power } (W_T)}$$

$$\eta = \frac{W_r}{W_T} = \frac{W_r}{W_r + W_l}$$

$$\eta = \frac{W_r}{W_T} \times \frac{4\pi\,\Phi(\theta,\phi)}{4\pi\,\Phi(\theta,\phi)}$$

$$= \frac{4\pi\,\Phi(\theta,\phi)}{W_T} \times \frac{W_r}{4\pi\,\Phi(\theta,\phi)}$$

$$\eta = G_p \cdot \frac{1}{G_d} = \frac{G_p}{G_d}$$

$$\boxed{\eta = \frac{G_p}{G_d} = \frac{W_r}{W_r + W_l}}$$

where, $G_p \rightarrow$ Power gain $G_d \rightarrow$ Directive gain

$W_r \rightarrow$ Power radiated W_l = Ohmic losses.

If the current flowing in the antenna is I, then

$$\eta = \frac{I^2 R_r}{I^2 (R_r + R_l)}$$

$$\boxed{\eta\,\% = \frac{R_r}{(R_r + R_l)} \times 100}$$

where, $R_r \rightarrow$ Radiation resistance

$R_l \rightarrow$ ohmic loss resistance of antenna conductor

$R_r + R_l \rightarrow$ Total effective resistance.

It is desirable to have a better radiation characteristics from the antenna, and for this loss resistances should be small as possible.

Loss resistance may consist of Ohmic loss in the antenna conductor, dielectric loss, I^2R loss in antenna and ground system, loss in earth connections, leakage loss.

Thus antenna efficiency η represents the fraction of total energy supplied to the antenna which is converted into electromagnetic waves.

Effective aperture (Effective area)

Transmitting antenna transmits EM waves and receiving antenna receives a fraction of the same. The effective area or apesture is the area over which ~~it~~ antenna extracts EM energy from the travelling EM waves. It may be defined as the ratio of power received at the antenna load terminal to the power density of the incident wave.

Effective area (effective aperture) = $\dfrac{\text{Power recieved}}{\text{Power density of the incident wave}}$

$$\boxed{A_e = \frac{W}{P} = A}$$

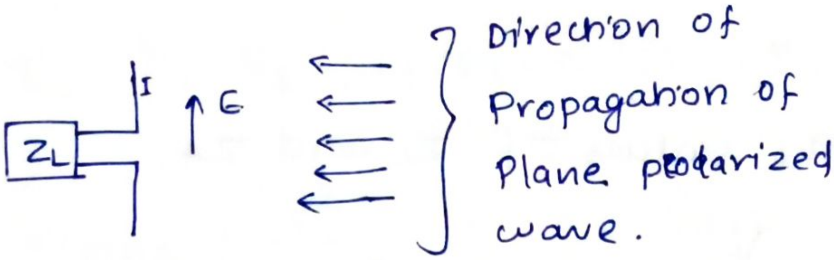
$W = PA$

$A \rightarrow$ effective area or effective apesture

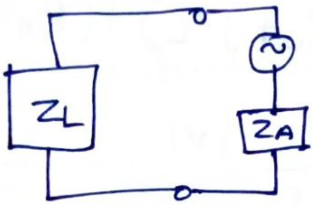
$W \rightarrow$ Power recieved (Watts)

$P \rightarrow$ Power density of the incident wave. (Watts/m^2)

Let a receiving antenna be placed in the field of plane polarized travelling waves having an effective area A and the receiving antenna is tesminated at a load impedance $Z_L = R_L + jX_L$.



(a) Receiving antenna.



(b) Equivalent circuit.

Let I be the tesminal cusseut, then received Power $W = I^2_{rms} R_L$.

$R_L \rightarrow$ Load resistance (Ω), I_{rms} = Tesminal rms current

$$\boxed{A = \frac{W}{P} = \frac{I^2_{rms} R_L}{P}}$$

From the equivalent circuit, according to thevenin's theorem, V = Equivalent Thevenin's voltage

Z_A = Equivalent thevenin's impedance.

V is the voltage induced by passing electromagnetic waves which produces cusseut I_{rms} through tesminal load impedance Z_L.

$$I_{rms} = \frac{\text{Equivalent Votage}}{\text{Equivalent Impedance}}$$

$$I_{rms} = \frac{V}{Z_L + Z_A} \text{ Amp}$$

$Z_A = R_A + jX_A$ = complex antenna impedance

$R_A = R_r + R_l$ = Radiation resistance + loss resistance

if $R_l = 0$

$R_A = R_r$

→ Putting the values of Z_L and Z_A,

$$I_{rms} = \frac{V}{(R_L + jX_L) + (R_A + jX_A)}$$

$$|I_{rms}| = \frac{|V|}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}}$$

X_L = Load reactance ; X_A = Antenna reactance.

→ power received by the terminal load impedance

$$W = I_{rms}^2 \cdot R_L$$

$$W = \frac{V^2 R_L}{(R_L + R_A)^2 + (X_L + X_A)^2}$$

→ Effective aperture $A_e = \frac{W}{P} \text{ m}^2$

$$A_e = \frac{V^2 R_L}{\left[(R_L + R_A)^2 + (X_L + X_A)^2\right] P} \text{ m}^2$$

~~if $X_L = X_A$ & $R_L = R_A$~~

→ According to maximum power transfer theorem, maximum power will be transferred from antenna to the antenna terminating load if,

$$X_L = -X_A \quad \& \quad R_L = R_A = R_r + R_l$$

$$W_{max} = \frac{V^2 \cancel{R_L}}{4 R_L^{\cancel{2}}} = \frac{V^2}{4 R_L}$$

$R_L = R_r + R_l$; if $R_l = 0$ $\quad R_L = R_r$

$$W_{max} = \frac{V^2}{4 R_r}$$

∴ Maximum effective aperture $\boxed{(A_e)_{max} = \frac{V^2}{4 R_r P} \; m^2}$

→ The ratio of effective area and max. effective area is given by effectiveness ratio (α)

$$\boxed{\alpha = \frac{A_e}{A_{e(max)}}}$$

Relation between Max. aperture and Gain or directivity

The directivity of receiving antennas are directly proportional to the maximum effective apertures.

Let there be two antennas A and B whose directivities and effective apertures are denoted by D_a, D_b and $(A_{ea})_{max}$ and $(A_{eb})_{max}$ respectively.

$$D_a \propto (A_{ea})_{max} \quad ; \quad D_b \propto (A_{eb})_{max} \quad \text{—— (1)}$$

$$\therefore \frac{D_a}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}}$$

→ The gain and directivity w.r.t. isotropic antenna is given by $G_0 = kD$ or $G_0 = \eta D$.

G_0 → Gain of transmitting or receiving antenna.

k → efficiency factor.

D → Directivity.

→ Now, k can be replaced by effectiveness ratio α

$$G_0 = \alpha D$$

$G_{oa} = \alpha_a D_a$ for Ant. A

$G_{ob} = \alpha_b D_b$ for Ant. B

from ①,

$$\frac{G_{oa}}{G_{ob}} = \frac{\alpha_a D_a}{\alpha_b D_b} = \frac{\alpha_a (A_{ea})_{max}}{\alpha_b (A_{eb})_{max}} \quad \text{—— (2)}$$

From definition, $\alpha_a = \frac{A_{ea}}{(A_{ea})_{max}}$.

$$\therefore A_{ea} = \alpha_a (A_{ea})_{max}.$$

lly $A_{eb} = \alpha_b (A_{eb})_{max}$.

From ②,

$$\frac{G_{oa}}{G_{ob}} = \frac{A_{ea}}{A_{eb}} \quad \text{—— (3)}$$

→ Assume that A is an isotropic antenna, then its directivity $D_a = 1$

$$\therefore \frac{D_a}{D_b} = \frac{1}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}}$$

$$(A_{ea})_{max} = \frac{(A_{eb})_{max}}{D_b} \quad \text{———} \; (4)$$

→ This eqn. (4) suggests that if the maximum effective aperture and directivity of antenna B (or any antenna) are known, then the ratio of two will give the maximum effective aperture of an isotropic antenna.

From (4), $\boxed{D_b = \frac{(A_{eb})_{max}}{(A_{ea})_{max}}}$

Directivity of any antenna is nothing but the ratio of its max. effective apesture to the max. effective apesture of an isotropic antenna.

→ For a short dipole antenna,

$(A_{eb})_{max} = \left[\frac{3}{8\pi}\right]\lambda^2$ and $D = \frac{3}{2}$. λ = wavelength

$$(A_{ea})_{max} = \frac{\frac{\cancel{3}\lambda^2}{4\,8\pi}}{\cancel{3}/2} = \frac{\lambda^2}{4\pi}$$

$$(A_{ea})_{max} = \frac{\lambda^2}{4\pi}$$

$$D_b = \frac{(A_{eb})_{max}}{[\lambda^2/4\pi]} = \frac{4\pi(A_{eb})_{max}}{\lambda^2}$$

In general,

$$\boxed{D = \frac{4\pi}{\lambda^2} (A_e)_{max}}$$

This is the relation between directivity and max. effective aperture of antenna.

Effective length (l_e)

The term effective length of an antenna represents the effectiveness of an antenna as radiator or collector of electromagnetic energy. In other words, effective length indicates how far an antenna is effective in transmitting or receiving the electromagnetic energy.

For a receiving antenna, the effective length may be defined in terms of induced voltage V and incident field. Effective length is the ratio of induced voltage at the terminal of receiving antenna under open circuit condition to the incident field intensity (strength) E.

$$\text{Effective length} = \frac{\text{Open circuit voltage}}{\text{Incident field strength (electric)}}$$

$$l_e = \frac{V}{E} \text{ meter of wavelength}$$

Since induced voltage V also depends on effective aperture and hence effective length and effective aperture are related to each other

$$A_e = \frac{V^2 R_L}{[(R_A+R_L)^2 + (X_A+X_L)^2] P}$$

$$V^2 = \frac{A_e [(R_A+R_L)^2 + (X_A+X_L)^2] P}{R_L}$$

$$P = E^2/Z$$

$$V = \frac{\sqrt{A_e [(R_A+R_L)^2 + (X_A+X_L)^2 E^2}}{\sqrt{Z R_L}}$$

$$l_e = \frac{V}{E} = \frac{\sqrt{A_e [(R_A+R_L)^2 + (X_A+X_L)^2}}{\sqrt{Z R_L}}$$

under conditions for maximum effective aperture,

$$X_A = -X_L$$

$$R_A = R_L = R_r + R_l \; ; \; R_A = R_r = R_L \text{ if } R_l = 0.$$

$$l_e = \frac{\sqrt{(A_e)_{max} (2R_r)^2}}{\sqrt{Z R_r}}$$

$$= \frac{2\sqrt{(A_e)_{max} R_r}}{\sqrt{Z}} \text{ m or } \lambda$$

$$\boxed{(A_e)_{max} = \frac{l_e^2 Z}{4 R_r}}$$

This is the relation between maximum effective aperture and effective length.

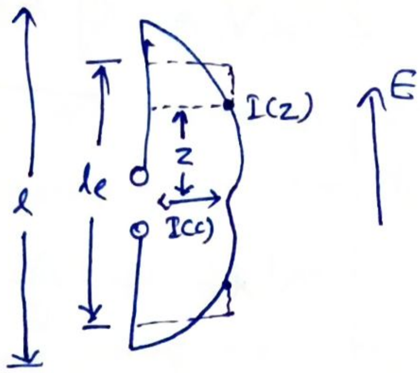
For the transmitting antenna, the effective length is that length of an equivalent linear antenna that has the same current I(c) at all the point along its length and that radiates the same field intensity E as the actual antenna.

I(c) = current at terminals of actual antenna.

I(z) = Current at any point z of the antenna

l_e = Effective length

l = Actual length



Effective length for transmitting antenna.

$$\therefore I(c)\, l_{et} = \int_{-l/2}^{+l/2} I(z)\, dz$$

$$l_{et} = \frac{1}{I(c)} \int_{-l/2}^{l/2} I(z)\, dz \quad \text{or}$$

$$\boxed{l_{et} = \frac{2}{I(c)} \int_{0}^{l/2} I(z)\, dz.}$$

Radiation Resistance (R_r)

The antenna is a radiating device, in which the power radiated into space in the form of electromagnetic waves. Hence there must be power dissipation,

$$W = I^2 R.$$

The power can be divided by square of

the current (I^2) . ie,

$R_r = \frac{W}{I^2}$ we will get a fictitous resistance called as radiation resistance. It is denoted by R_r or R_a or R_0. The radiation resistance represents a relation between total energy radiated from a transmitting antenna and the current flowing in the antenna. The radiation resistance (R_r) is thus defined as that fictious resistance which, when substituted in series with the antenna, will consume the same power as is actually radiated.

The value of radiation resistance depends on (i) configuration of antenna,

(ii) The point where the radiation resistance is considered

(iii) Location of antenna w.r.t ground and other objects

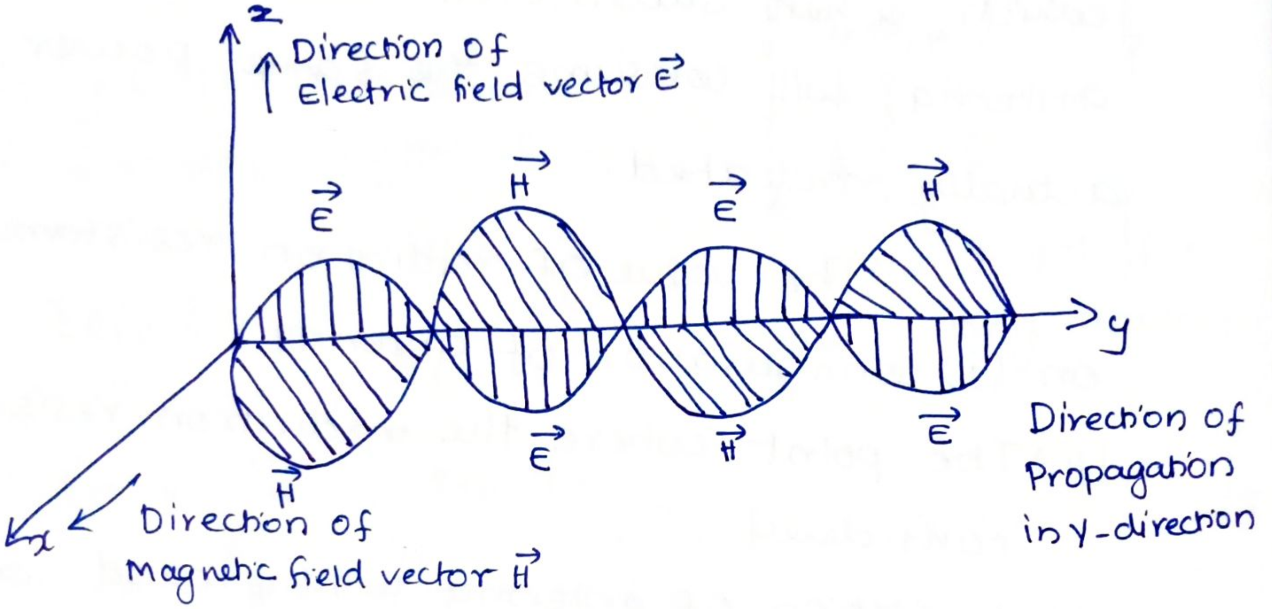
(iv) Corona discharge - a luminous discharge round the surface of antenna due to ionization of air etc.

For a half-wave dipole antenna, radiation resistance is 73 Ω. in free space. The knowledge of R_r is important because it act as load for the transmitter or for the radio frequency transmission line connecting the transmitter and antenna.

Wave Polarization:-

Polarization or plane of polarization of a radio wave can be defined by the direction in which electric vector E is aligned during the passage of atleast one full cycle.

Since electric vector E and magnetic vector H are mutually ⊥r



Polarization refers to the physical orientation of the radiated electromagnetic waves in space. An electromagnetic wave is said to linearly polarized ~~as all the electric field vectors are vertical~~ if they all have the same alignment in space. Electric field vector E is vertical or lies in the vertical plane, the wave is said to be vertically polarized. If E is in horizontal plane, the wave is said to be horizontally polarized.

The direction of antenna and polarization is alike, ie, if an antenna is vertical, it will radiate vertically polarized waves and a horizontal antenna, horizontally polarized waves.

The initial polarization of electromagnetic waves is determined by the orientation of antenna itself in the space. Hence in the design of an antenna, the type of polarization is one of the factor.

Besides, linear polarization, antenna may also radiate circularly or elliptically polarized waves. If two linearly polarized are simultaneously produce in the same direction from the same antenna provided that the two linear polarization are mutually ⊥r to each other with a phase difference of 90° then circularly polarized waves are produced. circular polarization may be right handed or left handed depending upon the sense of rotation. ie, phase difference is +ve or -ve. It results only when the amplitudes of two linearly polarized waves are equal.

If the amplitudes are not equal, then the combination of two linearly polarized waves will produce elliptically polarized wave.

Undesired radiation from an antenna is called cross polarization, for linearly polarized antennas, is ⊥r to the intended radiation.

The circular polarization is increasingly becoming popular as it is advantageous in VHF, UHF and microwave applications.

Antenna Field zones

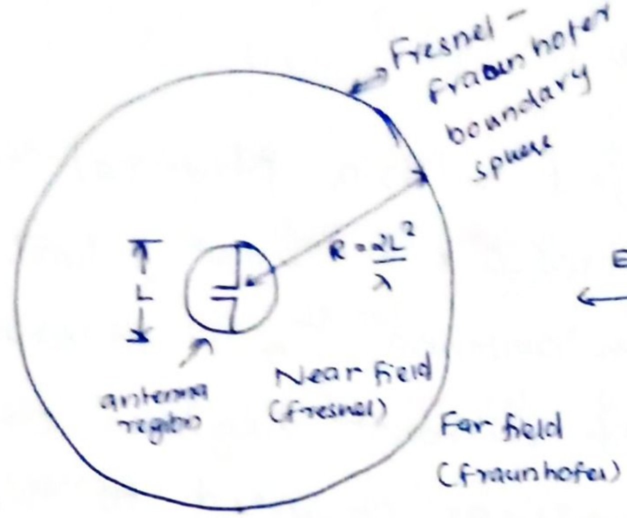
The fields around an antenna may be divided into two principal regions, one near the antenna called the near field or Fresnel zone and one at large distance called the far field or Fraunhofer zone. The boundary between two to be at a radius $R = \frac{2L^2}{\lambda}$

L = maximum dimension of the antenna, m

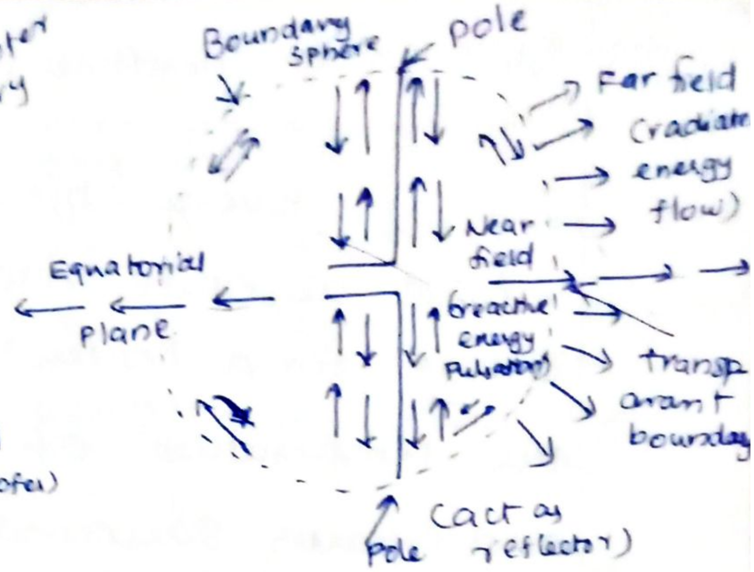
λ = wavelength, m.

In the far or fraunhofer region, the measurable field components are transverse to the radial direction from the antenna and all power flow is directed radially outward. In far field shape of the field pattern is independent of the distance.

In the near or Fresnel region, the longitudinal component of the electric field may be significant and power flow is not entirely radial. Here the shape of the field pattern depends on the distance.



(a) Near and far field



(b) Radiation field pattern

Enclosing the antenna in an imaginary sphere The region near the poles of the sphere acts as a reflector. Or the waves expanding ⊥r to the dipole in the equitorial region of the sphere result in power leakage through the sphere as if partially transparent in this region.

This's results in reciprocating (oscillating) energy flow near the antenna accompanied by outward flow in the equatorial region. The outflow accounts for the power radiated from the antenna, while the oscillating energy represents reactive power that is trapped near the antenna like in a resonator.

In reactive near field, the relationship between the strengths of E and H fields is too complex to predict. So all the measurements are done in far field.

Antenna temperature (T_A)

Every object with a physical temp above absolute zero ($0^\circ K = -273^\circ c$) radiates energy. For a lossless antenna, T_A is related to the temperature of distant regions of space and nearer surroundings coupled to the antenna via radiation resistance.

The noise power / unit B.W available at the terminals of a resistor of R and temperature T is given by

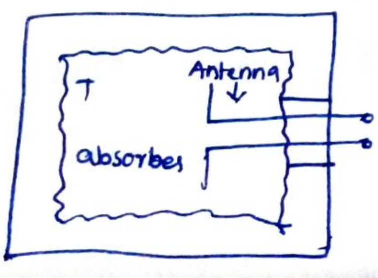
$$P = kT \text{ Watt/Hertz} \quad \text{—— (1)}$$

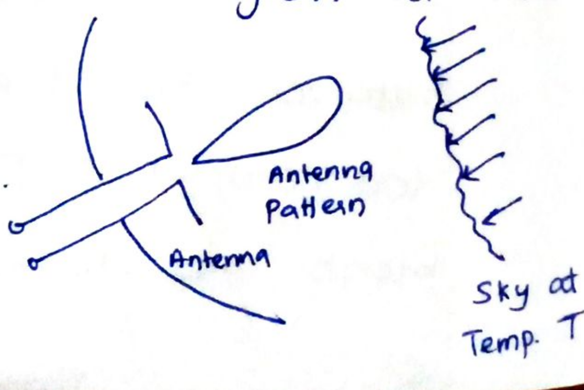
$P \rightarrow$ Power per unit BW in Watt/Hertz

$k \rightarrow$ Boltzman's constant $= 1.38 \times 10^{-23}$ J/K

$T \rightarrow$ Absolute temperature of resistor in K

If the resistor R is replaced by a lossless antenna of radiation resistor R in an anechoic (No echo) chamber at Temp. T, the noise power per unit BW available at the terminals is unchanged. However, the noise power will not be same unless an antenna is receiving from a region at the temperature T.





Now if the antenna is removed from the anechoic chamber and pointed at a sky of temp. T, the noise power at the terminals is same as that of previous cases. It is assumed that entire antenna pattern "sees" the sky temp. T.

If the power / unit BW, P is independent of frequency, the total power P is obtained by multiplying by the bandwidth (B). i.e,

$P = KTB$ watts ——② P → Total Power (Watt)

B → Band width (Hertz)

Let the antenna has an effective aperture A_e and that its beam is directed at a source of radiation which produces a power density per unit bandwidth or flux density (S) at the antenna. The power received from the source is given by

$P = S A_e B$ watts ——③.

From eqn ② and ③,

$P = S A_e B = K T B \Rightarrow S = \frac{K T_A}{A_e}$; T_A → Antenna temperature.

$$\boxed{T_A = \frac{S A_e}{k} \ {}^\circ K}$$

Reciprocity Theorem

Statement: If an e.m.f is applied to the terminals of an antenna no.1 and the current measured at the terminals of ~~the~~ another antenna no. 2, then the equal current both in amplitude and phase will be obtained at the terminals of antenna no.1 if the same emf is applied to the terminals of antenna no. 2.

or

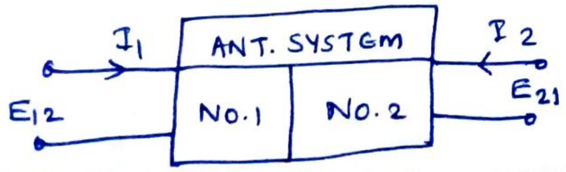
If a current I_1 at the terminals of antenna no.1 induces an emf E_{21} at the open terminals of antenna no. 2 and a current I_2 at the terminals of antenna no. 2 induces an emf E_{12} at the open terminals of antenna no.1, then $E_{12} = E_{21}$ Provided $I_1 = I_2$.

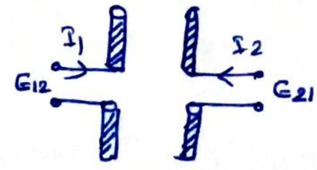
It is assumed that,

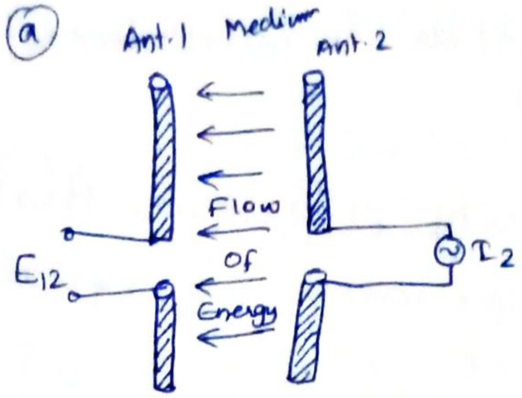
(i) emf's are of same frequency

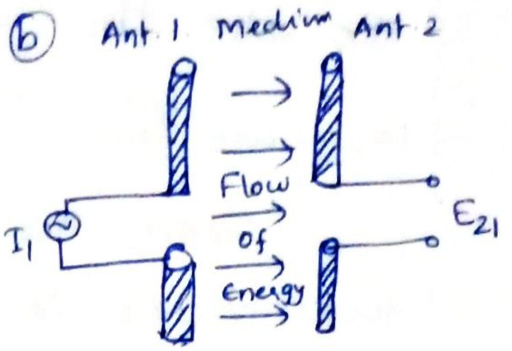
(ii) Medium between the two antennas are linear, Passive and isotropic.

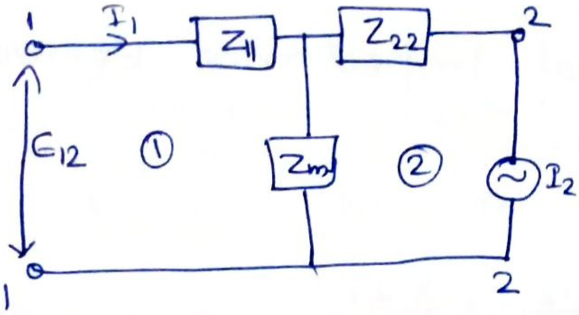
(iii) Generator producing emf and the ammeter for measuring the current have zero impedance or if not, then both the generator and the ammeter impedances are equal.

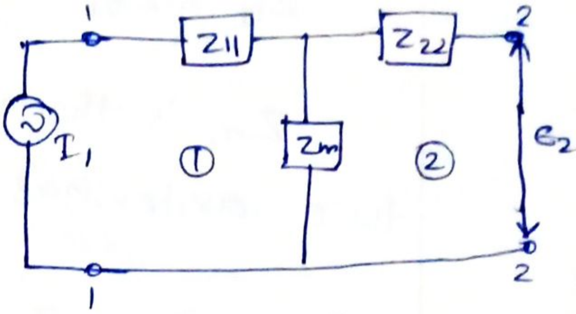












ⓒ Equivalent-T-network corresponding to 4 terminal n/w of ⓐ

ⓓ Equivalent - T-network corresponding to 4 terminal n/w of ⓑ

Let,

(1) A transmitter of frequency f and zero impedance be connected to the terminals of antenna no. 2, which is generating a current I_2 and inducing an emf E_{12} at the open terminals of ant. 1.

(2) Now the same transmitter is transferred to ant. 1 which is generating a current I_1 and inducing a voltage E_{21} at the open terminals of ant. no. 2.

Thus according to the statement of reciprocity theorem

$I_1 = I_2$ provided $E_{12} = E_{21}$

Since the ratio of an emf to current is an impedance, therefore the ratio $\frac{E_{12}}{I_2}$ is Transfer Impedance Z_{12} as in case ⓐ and

So also the ratio $\frac{E_{21}}{I_1}$ as Transfer Impedance Z_{21} in case (b).

From the reciprocity it follows that the two ratios ie, two Impedances are equal. i.e,

$$Z_{12} = Z_{21}$$

Z_m is the mutual Impedance between the two antennas.

$$Z_m = Z_{12} = Z_{21} = \frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}$$

$$\boxed{\frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}}$$

Proof :-

To prove the reciprocity theorem for antennas, the space between the antenna no. 1 and ant. no. 2 are replaced by a network of linear, passive and bilateral impedances.

Z_{11} & Z_{22} → self impedance antenna no. 1 and no. 2 respectively.

Z_m → Mutual impedance between two antennas.

1, 1 and 2, 2 → Terminals of antenna no. 1 & no. 2

Apply kirchoff's mesh law to fig (c).

From loop (2)

$$(Z_{22} + Z_m) I_2 - Z_m I_1 = 0 \quad \text{—— (1)}$$

$$\boxed{I_2 = \frac{I_1 Z_m}{(Z_{22} + Z_m)}} \quad \text{—— (2)}$$

From loop 1,

$$(Z_{11}+Z_m)I_1 - Z_m I_2 = E_{12} \quad \text{——— (3)}$$

Substitute I_2 here,

$$(Z_{11}+Z_m)\, I_1 + \frac{Z_m^2 I_1}{(Z_{22}+Z_m)} = E_{12}. \quad \text{——— (4)}$$

$$I_1 \left[\frac{(Z_{11}+Z_m)(Z_{22}+Z_m) - Z_m^2}{(Z_{22}+Z_m)} \right] = E_{12}.$$

~~or~~ $\cancel{I_1 = E_{12}(Z_{22}+Z_m)}$

$$I_1 \left[\frac{(Z_{11}Z_{22}) + (Z_{11}Z_m) + (Z_{22}Z_m) + \cancel{Z_m^2} - \cancel{Z_m^2}}{(Z_{22}+Z_m)} \right] = E_{12}.$$

$$I_1 \left[\frac{Z_{11}Z_{22} + Z_m(Z_{11}+Z_{22})}{(Z_{22}+Z_m)} \right] = E_{12}.$$

$$\text{or} \quad I_1 = \frac{E_{12}(Z_{22}+Z_m)}{Z_{11}Z_{22} + Z_m(Z_{11}+Z_{22})} \quad \text{——— (5)}$$

Substitute 5 in 2,

$$I_2 = \frac{E_{12}\cancel{(Z_{22}+Z_m)}.Z_m}{[Z_{11}Z_{22} + Z_m(Z_{11}+Z_{22})].\cancel{(Z_{22}+Z_m)}}$$

$$\boxed{I_2 = \frac{E_{12}Z_m}{[Z_{11}Z_{22} + Z_m(Z_{11}+Z_{22})]}} \quad \text{——— (6)}$$

Similarly I_1 from fig (d), can be obtained by symmetry. Suffix 2 ~~and~~ may be replaced by

1 and vice-versa in eqn (6).

$$\boxed{I_1 = \frac{E_{21} Z_m}{Z_{22} Z_{11} + Z_m (Z_{22} + Z_{11})}} \quad \text{——— (7)}$$

Thus from eqn (6) and (7). are same except the value of emf (ie, E_{12} and E_{21}). According to the theorem statement,

$E_{12} = E_{21}$ if $I_1 = I_2$.

Applying the condition, $I_1 = I_2$.

$$\frac{E_{12} Z_m}{[Z_{11} Z_{22} + Z_m (Z_{11} + Z_{22})} = \frac{E_{21} Z_m}{(Z_{22} Z_{11}) + Z_m (Z_{22} + Z_{11})}$$

or $\boxed{E_{12} = E_{21}}$ ——— (8)

Hence the theorem is proved.

Applications:-

The reciprocity theorem may be used to derive the very important properties of txting Rxing antennas. like.

(1) Equality of directional patterns

(2) Equality of Directivities

(3) Equality of Effective lengths

(4) Equality of Antenna impedances.

Duality of antennas

When two equations that describe the behaviour of two different variables are of the same mathematical form, their solutions will be identical. The variables in the two equations that occupy identical positions are known as dual quantities. The solution of one can be formed by a systematic interchange of symbols to the other. This concept is known as duality theorem.

When comparing the equations of electric current sources and magnetic current sources, it is evident that they are to each other ~~and~~ dual equations and their variables are dual quantities. Thus knowing the solutions to one set (ie, $J \neq 0$, $M = 0$ electric source), the solution of the other set (ie, $J = 0$, $M \neq 0$ magnetic sources) can be formed by a proper interchange of quantities.

Dual quantities:

(J)(Electric current source) - (M)(Magnetic current source)

E_A — H_F

H_A — $-E_F$

J — M

A — F

ε — μ

μ — ε

k — k

η — $1/\eta$

$1/\eta$ — η

Dual Equations:

$$(J \neq 0, M = 0) - (J = 0, M \neq 0)$$

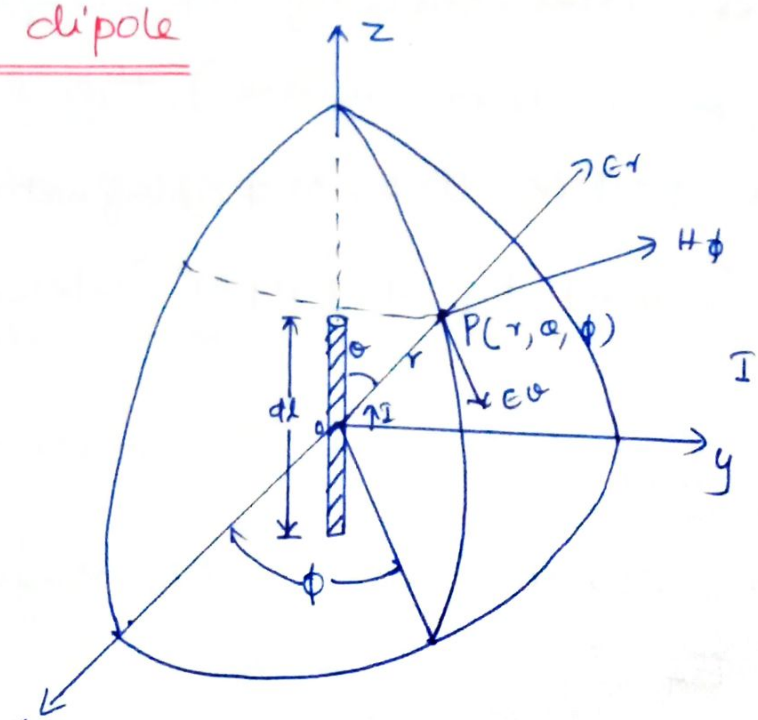
$$\nabla \times E_A = -j\omega\mu H_A \qquad \nabla \times H_F = j\omega\varepsilon E_F$$

$$\nabla \times H_A = J + j\omega\varepsilon E_A \qquad -\nabla \times E_F = M + j\omega\mu H_F$$

$$\nabla^2 A + k^2 A = -\mu J \qquad \nabla^2 F + k^2 F = -\varepsilon M$$

etc.

Field, Radiation Resistance, and directivity of a short dipole



$I = I_m \cos\omega t$

Consider an alternating current element or oscillating electric dipole possesses electromagnetic field. To find these fields everywhere around in free space, the concept of retarded vector potential is used. Let an element of length dl is placed at the origin of the spherical co-ordinate and I be the current flowing through it. The length is so short that current is constant along the length.

While ~~dealing~~ dealing with antennas, the Propagation time is important. Hence if a current is flowing in a short dipole, the effect of the current is not felt instantaneously at the Point P, but only after an interval equal to the time required for the wave to propagate over the distance r. This is nothing but the retardation effect.

$I = I_m e^{j\omega t}$ → instantaneous propagation of current.

When the propagation time or retardation time considered

$$\boxed{[I] = I_m e^{j\omega(t - \frac{r}{c})}} \quad \text{——} (1)$$

$[I]$ → Retarded current.

Electric and magnetic fields can be expressed in terms of vector and scalar potentials. If the Scalar potential V and vector potential A at the point P are known, the electric field E and magnetic field H can be determined using the equations:

$$\boxed{\begin{aligned} E &= -\nabla V - \frac{\partial A}{\partial t} \quad V/m \\ H &= \frac{1}{\mu_0}(\nabla \times A) \quad A/m \end{aligned}}$$

and —— ②

—— ③

where,

V – electric scalar potential at point P

A – Vector potential at point P

μ – Permeability of free space $= 4\pi \times 10^{-7}$ H/m

Using the retarded potentials, we have equation of E and H as

$$E = -\nabla[V] - \frac{\partial}{\partial t}[A]$$

$$E = -\nabla[V] - j\omega[A] \quad \text{—— ④} \qquad \because \frac{\partial}{\partial t} = j\omega$$

$$H = \frac{1}{\mu_0}[\nabla \times [A]] \quad \text{—— ⑤}$$

~~where,~~ The retarded vector potential of electric ~~A_z~~ current has only one component A_z ie, the current is entirely in the z-direction.

$$\boxed{[A] = A_z = \frac{\mu_0 l I_m e^{j\omega t_1}}{4\pi r}} \quad \text{and —— ⑥} \qquad \because t_1 = t - \frac{r}{c}$$

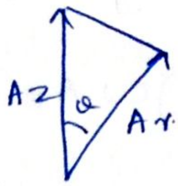
$$\boxed{V = \frac{I_m l \cos\theta \, e^{j\omega t_1}}{4\pi \varepsilon_0 c}\left[\frac{1}{r} + \frac{c}{j\omega} \cdot \frac{1}{r^2}\right]} \quad \text{—— ⑦}$$

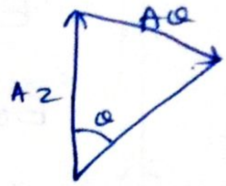
Eqns ⑥ and ⑦ are scalar and vector potentials due to a short dipole subject to the restrictions that $r >> l$ and $\lambda >> l$.

It is better to express E and H in polar coordinates. The polar coordinate components for the vector potential are

$$\boxed{A = a_r A_r + a_\theta A_\theta + a_\phi A_\phi} \quad \text{--- (8)}$$

As the vector potential of the dipole has only one z component,





$A_\phi = 0$, $A_r = A_z \cos\theta$,

$A_\theta = -A_z \sin\theta$

In polar co-ordinate, the gradiant of scalar potential is given by

$$\boxed{\nabla V = a_r \frac{\partial V}{\partial r} + a_\theta \cdot \frac{1}{r}\frac{\partial V}{\partial \theta} + a_\phi \cdot \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}} \quad \text{--- (9)}$$

→ To calculate the electric field E

$$E = -j\omega A - \nabla V$$

$$a_r E_r + a_\theta E_\theta + a_\phi E_\phi = -j\omega(a_r A_r + a_\theta A_\theta + a_\phi A_\phi)$$
$$- a_r \frac{\partial V}{\partial r} - a_\theta \frac{1}{r}\frac{\partial V}{\partial \theta} - a_\phi \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

Equating the individual components,

$$E_r = -j\omega A_r - \frac{\partial V}{\partial r} \quad \text{--- (10)}$$

$$E_\theta = -j\omega A_\theta - \frac{1}{r}\frac{\partial V}{\partial \theta} \quad \text{--- (11)}$$

$$E_\phi = -j\omega A_\phi - \frac{1}{r\sin\theta}\cdot\frac{\partial V}{\partial \phi} \quad \text{--- (12)}$$

In eqn (12), $A_\phi = 0$.

$\frac{\partial V}{\partial \phi} = 0$ as V in eqn (7) is without ϕ.

This implies that

$$\boxed{E_\phi = 0} \quad \text{—— (13)}$$

$$E_r = -j\omega A_z \cos\theta - \frac{\partial V}{\partial r} \quad \text{—— (14)}$$

$$E_\theta = j\omega A_z \sin\theta - \frac{1}{r}\frac{\partial V}{\partial \theta} \quad \text{—— (15)}$$

Now substituting the values of A_z and V after performing the desired integrations, we get the value of E_r and E_θ.

$$V = \frac{I_m l \cos\theta \, e^{j\omega t_1}}{4\pi \varepsilon_0 c}\left[\frac{1}{r} + \frac{c}{j\omega}\cdot\frac{1}{r^2}\right]$$

$$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r}\left\{\frac{I_m l \cos\theta \, e^{j\omega t_1}}{4\pi \varepsilon_0 c}\left[\frac{1}{r} + \frac{c}{j\omega}\cdot\frac{1}{r^2}\right]\right\}$$

$$= \frac{I_m l \cos\theta}{4\pi \varepsilon_0 c}\left[\frac{\partial}{\partial r}\left\{e^{j\omega t_1}\cdot\left(\frac{1}{r} + \frac{c}{j\omega r^2}\right)\right\}\right]$$

Product rule

$$= \frac{I_m l \cos\theta}{4\pi \varepsilon_0 c}\left[e^{j\omega t_1}\cdot\left(\frac{-j\omega}{c}\right)\left(\frac{1}{r} + \frac{c}{j\omega r^2}\right) + e^{j\omega t_1}\left(\frac{-1}{r^2} - \frac{2c}{j\omega r^3}\right)\right]$$

$$= \frac{I_m l \cos\theta \, e^{j\omega t_1}}{4\pi \varepsilon_0 c}\left[\frac{-j\omega}{cr} - \frac{1}{r^2} - \frac{1}{r^2} - \frac{2c}{j\omega r^3}\right]$$

$$\frac{\partial V}{\partial r} = -\frac{I_m l \cos\theta\, e^{j\omega t_1}}{4\pi\varepsilon_0 c}\left[\frac{j\omega}{cr} + \frac{2}{r^2} + \frac{2c}{j\omega r^3}\right] \longrightarrow (16)$$

Putting eqns (6) and (16) in (14), we have

$$E_r = -j\omega\left(\frac{\mu_0 l I_m e^{j\omega t_1}}{4\pi r}\right)\cos\theta + \frac{I_m l \cos\theta\, e^{j\omega t_1}}{4\pi\varepsilon_0 c}\cdot\left[\frac{j\omega}{cr} + \frac{2}{r^2} + \frac{2c}{j\omega r^3}\right]$$

$$= \frac{I_m l \cos\theta\, e^{j\omega t_1}}{4\pi}\left[\frac{-j\omega\mu_0}{r} + \frac{1}{\varepsilon_0 c}\left\{\frac{j\omega}{cr} + \frac{2}{r^2} + \frac{2c}{j\omega r^3}\right\}\right]$$

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}$$

$$E_r = \frac{I_m l \cos\theta\, e^{j\omega t_1}}{4\pi}\left[\frac{-j\omega}{r\varepsilon_0 c^2} + \frac{1}{\varepsilon_0 c}\left\{\frac{j\omega}{cr} + \frac{2}{r^2} + \frac{2c}{j\omega r^3}\right\}\right]$$

$$= \frac{I_m l \cos\theta\, e^{j\omega t_1}}{4\pi\varepsilon_0 c}\left[\cancel{\frac{-j\omega}{cr}} + \cancel{\frac{j\omega}{cr}} + \frac{2}{r^2} + \frac{2c}{j\omega r^3}\right]$$

$$= \frac{I_m l \cos\theta\, e^{j\omega t_1}}{{}_2\cancel{4}\pi\varepsilon_0 \cancel{c}} \times 2c\left[\frac{1}{cr^2} + \frac{1}{j\omega r^3}\right]$$

$$E_r = \frac{I_m l \cos\theta\, e^{j\omega t_1}}{2\pi\varepsilon_0}\left[\frac{1}{cr^2} + \frac{1}{j\omega r^3}\right]$$

$$\boxed{E_r = \frac{I_m l \cos\theta\, e^{j\omega(t - \frac{r}{c})}}{2\pi\varepsilon_0}\left[\frac{1}{cr^2} + \frac{1}{j\omega r^3}\right]} \longrightarrow (17)$$

$$\because t_1 = t - \frac{r}{c}$$

From eqn (7) on differentiating w.r.t θ, we have,

$$\frac{\partial V}{\partial \theta} = \frac{\partial}{\partial \theta}\left[\frac{I_m l \cos\theta\, e^{j\omega t_1}}{4\pi\varepsilon_0 c}\left(\frac{1}{r} + \frac{c}{j\omega r^2}\right)\right]$$

$$\frac{1}{r}\cdot\frac{\partial V}{\partial \theta} = \frac{I_m l(-\sin\theta)\, e^{j\omega t_1}}{4\pi\varepsilon_0 c}\left(\frac{1}{r^2} + \frac{c}{j\omega r^3}\right)\Bigg] \quad \text{— (18)}$$

Putting eqn (18) and (6) in to eqn (15) we get

$$E_\theta = j\omega\left(\frac{\mu_0 I_m l\, e^{j\omega t_1}}{4\pi r}\right)\sin\theta + \frac{I_m l \sin\theta\, e^{j\omega t_1}}{4\pi\varepsilon_0 c}\left(\frac{1}{r^2} + \frac{c}{j\omega r^3}\right)$$

$$= \frac{I_m l \sin\theta\, e^{j\omega t_1}}{4\pi r}\left[j\omega\mu_0 + \frac{1}{\varepsilon_0 c}\left\{\frac{1}{r} + \frac{c}{j\omega r^2}\right\}\right]$$

$$E_\theta = \frac{I_m l \sin\theta\, e^{j\omega t_1}}{4\pi}\left[\frac{j\omega}{\varepsilon_0 c^2 r} + \frac{1}{\varepsilon_0 c}\left\{\frac{1}{r^2} + \frac{c}{j\omega r^3}\right\}\right]$$

$$\mu_0 = \frac{1}{c^2\varepsilon_0}$$

$$\boxed{E_\theta = \frac{I_m l \sin\theta\, e^{j\omega t_1}}{4\pi\varepsilon_0}\left[\frac{j\omega}{c^2 r} + \frac{1}{c r^2} + \frac{1}{j\omega r^3}\right]} \quad \text{— (19)}$$

This is the electric field E in terms of its component E_r, E_θ and E_ϕ.

Let us calculate the magnetic field in terms of its component H_r, H_θ and H_ϕ.

$$\mu H = \nabla \times A = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & a_\theta r & a_\phi r \sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & rA_\phi \sin\theta \end{vmatrix}$$

But $A_\phi = 0$. From the previous equations it is seen that A_r and A_θ are independent of term ϕ. So $\frac{\partial}{\partial \phi} = 0$. Hence,

$$\mu H = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & a_\theta r & a_\phi r \sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & 0 \\ A_r & rA_\theta & 0 \end{vmatrix}$$

$a_r \mu H_r = 0$ or $H_r = 0$ —— (20)

$a_\theta \mu H_\theta = 0$ or $H_\theta = 0$ —— (21)

$$a_\phi \mu H_\phi = \frac{a_\phi \cancel{r} \cancel{\sin\theta}}{r^{\cancel{2}} \cancel{\sin\theta}} \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r) \right]$$

$$\boxed{H_\phi = \frac{1}{\mu_0 r} \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r) \right]} \quad \text{—— (22)}$$

$$A_r = A_z \cos\theta$$

$$A_r = \frac{\mu_0 I_m l \cos\theta \, e^{j\omega t_1}}{4\pi r} \quad \text{—— (23)}$$

$$\frac{\partial}{\partial \theta} A_r = - \frac{\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{4\pi r} \quad \text{—— (24)}$$

$$A_\theta = -A_z \sin\theta$$

$$A_\theta = \frac{-\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{4\pi r}$$

$$r \cdot A_\theta = \frac{-\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{4\pi}$$

$$\frac{\partial}{\partial r}(r \cdot A_\theta) = -\frac{\partial}{\partial r}\left[\frac{\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{4\pi}\right]$$

$$\frac{\partial}{\partial r}(r \cdot A_\theta) = -\frac{\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{4\pi} \cdot \left(\frac{-j\omega}{c}\right) \longrightarrow (25)$$

Now eqn (24) and (25) put into eqn (22), we get

$$H\phi = \frac{1}{\mu_0 r}\left[\frac{\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{4\pi}\left(\frac{j\omega}{c}\right) + \frac{\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{4\pi r}\right]$$

$$H\phi = \frac{\mu_0 I_m l \sin\theta \, e^{j\omega t_1}}{\mu_0 4\pi}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]$$

$$\boxed{|H| = H\phi = \frac{I_m l \sin\theta \, e^{j\omega t_1}}{4\pi}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]} \quad \text{—— (26)}$$

$$\boxed{\begin{array}{l} H_r = 0 \\ H_\theta = 0 \end{array}} \quad \begin{array}{l} \text{—— (27)} \\ \text{—— (28)} \end{array}$$

Thus the fields from the dipole have only three components E_r, E_θ and $H\phi$ and the rest of the components of E and H (E_θ, H_r & H_θ) are every where zero.

$$\boxed{\begin{aligned} E_r &= \frac{I_m l \cos\theta \, e^{j\omega(t-\frac{r}{c})}}{2\pi\varepsilon_0}\left[\frac{1}{cr^2}+\frac{1}{j\omega r^3}\right] \\ E_\theta &= \frac{I_m l \sin\theta \, e^{j\omega(t-r/c)}}{4\pi\varepsilon_0}\left[\frac{j\omega}{c^2 r}+\frac{1}{cr^2}+\frac{1}{j\omega r^3}\right] \\ E_\phi &= 0 \;;\; H_r = 0 \;;\; H_\theta = 0 \\ H_\phi &= \frac{I_m l \sin\theta \, e^{j\omega(t-r/c)}}{4\pi}\left[\frac{j\omega}{cr}+\frac{1}{r^2}\right] \end{aligned}}$$

Far field Region or Radiation zone $(|r| >> \lambda)$:-

when r is very large in comparison to λ, the higher terms of E and H ie, $\frac{1}{r^2}$ and $\frac{1}{r^3}$ can be neglected in favour of $\frac{1}{r}$. As such E_r compound is neglected ~~as~~ which has no $\frac{1}{r}$ term. Therefore, effectively only two components E_θ and H_ϕ are contributing to the far field. ie,

$$E_\theta = \frac{I_m l \sin\theta \, e^{j\omega(t-r/c)}}{4\pi\varepsilon_0}\left[\frac{j\omega}{c^2 r}\right] \quad \text{---(29)}$$

$$H_\phi = \frac{j\omega I_m l \sin\theta \, e^{j\omega(t-r/c)}}{4\pi c r} \quad \text{---(30)}$$

$$\frac{E_\theta}{H_\phi} = \frac{1}{\varepsilon_0 c} = \frac{36\pi \times 10^9}{3\times 10^8} = 120\pi \quad \therefore \boxed{\frac{E_\theta}{H_\phi} = 120\pi\,\Omega} \quad (31)$$

Radiation Resistance of a short dipole:-

If surface integral of the average Poynting vector is taken over any surface enclosing an antenna, the total power radiated by antenna is $W = \int P_{av} \cdot ds.$ ——①

The average poynting vector is

$$P_{av} = \frac{1}{2} \mathrm{Re} \left(E \times H^{*} \right) \quad \text{——②}$$

The far field components which are not zero, are E_θ and H_ϕ so that the radial component of the Poynting vector is

$$P_r = \frac{1}{2} \mathrm{Re} \left(E_\theta H_\phi^{*} \right) \quad \text{——③}$$

where E_θ and H_ϕ^* are complex quantity. and H_ϕ^* is the complex conjugate of H_ϕ. The far field components are related by the intrinsic impedance of the medium as

$$\frac{E_\theta}{H_\phi} = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\Omega$$

$$P_r = \frac{1}{2} \mathrm{Re} \left(\eta_0 H_\phi \cdot H_\phi^{*} \right) = \frac{1}{2} |H_\phi|^2 \mathrm{Re}\cdot \eta_0$$

$$\boxed{P_r = \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu_0}{\varepsilon_0}}} \quad \text{——④}$$

$\because \mathrm{Re}\, \eta_0 = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

Hence the total power radiated is

$$W = \int P_{av} \cdot ds.$$

$$= \int \frac{1}{2} Re\,(E \times H^*) \cdot ds$$

$$= \iint P_r \cdot ds$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{1}{2} Re\, E_\theta H_\phi^* \, ds. \qquad \therefore ds = r^2 \sin\theta \, d\phi$$

$$W = \frac{1}{2} \int_0^{2\pi} \int_0^{\pi} |H_\phi|^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot r^2 \sin\theta \, d\phi$$

$$= \frac{1}{2} \int_0^{2\pi} |H\phi|^2 d\phi \sqrt{\frac{\mu_0}{\varepsilon_0}} \int_0^{\pi} r^2 \sin\theta \, d\theta.$$

$$\cancel{= \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}}} \int_0^{2\pi} = \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \int_0^{2\pi} (H\phi)^2 d\phi \int_0^{\pi} r^2 \sin\theta \, d\theta.$$

$$|H\phi| = \frac{\omega I_m l \sin\theta}{4\pi c r}.$$

$$W = \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \int_0^{2\pi} d\phi \int_0^{\pi} \left(\frac{\omega I_m l \sin\theta}{4\pi c r} \right)^2 r^2 \sin\theta \, d\theta.$$

$$= \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot 2\pi \cdot \int_0^{\pi} \frac{\omega^2 I_m^2 l^2 \sin^2\theta}{16 \pi^2 c^2 \cancel{r^2}} \cdot \cancel{r^2} \sin\theta \cdot d\theta.$$

$$= \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \cancel{2\pi} \cdot \frac{\omega^2}{c^2} \cdot \frac{I_m^2 l^2}{\cancel{16}_8 \pi^{\cancel{2}}} \int_0^{\pi} \sin^3\theta \, d\theta.$$

$$W = \frac{1}{16} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{\omega^2 I_m^2 l^2}{c^2 \pi} \int_0^{\pi} \sin^3\theta \, d\theta.$$

$$\because \frac{\omega}{c} = \frac{2\pi f}{f\lambda} = \frac{2\pi}{\lambda} = \beta$$

$$W = \frac{1}{\cancel{16}_{8}}\sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{\beta^2 I_m^2 l^2}{\pi} \cdot 2\int_0^{\pi/2} \sin^3\theta \, d\theta$$

$$W = \frac{1}{8}\sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{\beta^2 I_m^2 l^2}{\pi} \cdot \frac{2}{3}$$

$$\because \int_0^{\pi/2} \sin^3\theta \, d\theta = \frac{3-1}{3} = \frac{2}{3}.$$

$$\boxed{W = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{(\beta I_m l)^2}{12\pi} \text{ Watts.}} \quad \text{—— (5)}$$

$W = I^2 R \quad \because I^2 = \left(\frac{I_m}{\sqrt{2}}\right)^2$

Power is also given by $W = \frac{1}{2} I_m^2 R$

or $R = \frac{2W}{I_m^2}$

So, Radiation resistance is given by

$$R_r = \frac{\cancel{2} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot (\beta \cancel{I_m} l)^2}{\cancel{I_m^2} \cdot \cancel{12}_{6}\pi}$$

$$R_r = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{\beta^2 l^2}{6\pi} \qquad \because \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

$$R_r = \frac{\cancel{120\pi}^{20} \beta^2 l^2}{\cancel{6\pi}} = 20\beta^2 l^2 = 20 \cdot \left(\frac{2\pi}{\lambda}\right)^2 \cdot l^2.$$

$$= 80\pi^2 \left(\frac{l}{\lambda}\right)^2 \Omega = 789.57204 \, (l/\lambda)^2 \, \Omega$$

$$\boxed{R_r \simeq 790 \, (l/\lambda)^2 \, \Omega} \quad \text{—— (6)}$$

Directivity of short dipole.

$$D = \frac{4\pi}{\Omega_A} \quad \because \Omega_A \text{ - beam solid angle}$$

$$E_\theta = \frac{I_m l \sin\theta \, e^{j\omega t_1}}{4\pi\varepsilon} \cdot \frac{j\omega}{c^2 r}$$

$$E_{max} = \frac{I_m l \, e^{j\omega t_1}}{4\pi\varepsilon} \cdot \frac{j\omega}{c^2 r}$$

$$E_n(\theta) = \frac{E_\theta}{E_{max}} = \sin\theta$$

lly $H_n(\phi) = \frac{H_\phi}{H_{max}} = \sin\theta.$

$$P_n(\theta) = E_n(\theta) \cdot \cancel{H}_n(\phi) = \sin\theta \cdot \sin\theta$$

$$P_n(\theta) = \sin^2\theta.$$

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi} P_n(\theta) \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi} \sin^3\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \cdot 2 \int_0^{\pi/2} \sin^3\theta \, d\theta.$$

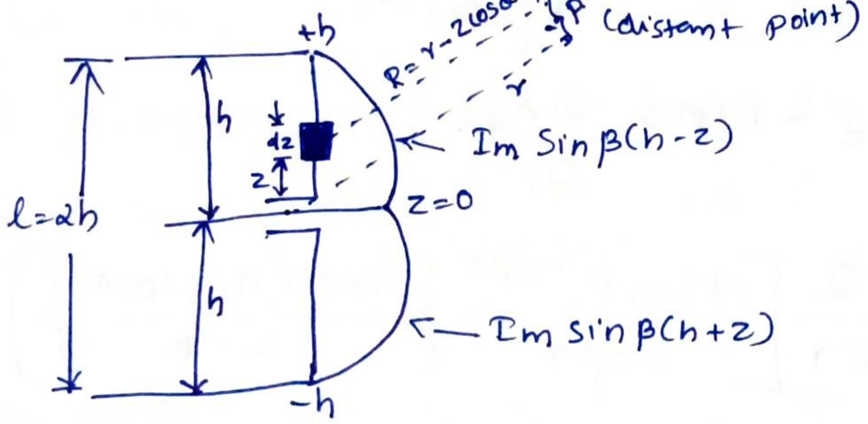
$$= 2\pi \cdot 2 \left[\frac{2}{3}\right]$$

$$\Omega_A = 4\pi \times \frac{2}{3} = \frac{8\pi}{3}.$$

$$D = \frac{4\pi}{8\pi/3} = 3/2 = 1.5 \qquad \boxed{D = 1.5}$$

Radiation from a half wave dipole ($\lambda/2$ Antenna)

A $\lambda/2$ antenna is also known as Hertz antenna or sometimes also called as half wave doublet. A dipole antenna may be defined as a symmetrical antenna in which the two ends are at equal potential relative to mid point.



The dipole is usually fed at the centre. having maximum current at the centre. i'e, maximum radiation in the plane normal to the axis. Since current is assumed sinusoidal asymtotically distributed as shown in the figure, we get,

$I = I_m \sin\beta(h-z)$ for $z>0$ ——— ①

$I = I_m \sin\beta(h+z)$ for $z<0$ ——— ②

$I_m \to$ maximum current at the current loop.

The vector potential is given by

$$\boxed{A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]} \qquad ③$$

But for a current element along the z axis, from maxwell eqn $\nabla \times A = \mu H$, we can rewrite it when only far field is considered

$$\mu H_\phi = (\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(A_\theta \cdot r)\right] \quad \text{—— (4)}$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}(-A_z \sin\theta \cdot r)\right]$$

$$\mu H_\phi = -\sin\theta \cdot \frac{\partial A_z}{\partial r}$$

$$\mu H_\phi = -\frac{\partial}{\partial r}\left[\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right\}\right]\sin\theta$$

$$= -\frac{\mu I_m e^{-j\beta r}(-j\beta)}{2\pi\beta r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right\}$$

$$H_\phi = \frac{j I_m e^{-j\beta r}}{2\pi r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right\}.$$

$$|H_\phi| = \left|\frac{I_m e^{-j\beta r}}{2\pi r} \cdot \frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right|$$

$$\boxed{H_\phi = \frac{I_m}{2\pi r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right\}} \quad \text{—— (5)}$$

This is the required magnetic field intensity expression for a halfwave dipole. The electric field expression for the radiation field can be achieved from the formula

$$\frac{E_\theta}{H_\phi} = \eta = 120\pi \quad \text{—— (6)}$$

$$|E_\theta| = 120\pi \, |H_\phi|$$

$$|E_\theta| = \cancel{120}^{60}\cancel{\pi} \cdot \frac{I_m}{\cancel{2}\cancel{\pi} r} \left\{ \frac{\cos \pi/2 \cos\theta}{\sin\theta} \right\}$$

$$\boxed{|E_\theta| = \frac{60 I_m}{r} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}} \quad \text{—— (7)}$$

This is the expression for the amplitude of the electric field intensity for the radiation field of a $\lambda/2$ antenna or a $\lambda/4$ antenna (monopole)

since E_θ and H_ϕ are in timephase. So the max. value of poynting vector is given by

$$P_{max} = |E_\theta| \, |H_\phi| \quad \text{—— (8)}$$

$$P_{av} = \frac{P_{max}}{2} = \frac{1}{2} E_\theta \cdot H_\phi.$$

$$P_{av} = \frac{1}{\cancel{2}} \cdot \frac{I_m}{\cancel{2}\pi r} \cdot \frac{\cancel{60}^{15} I_m}{r} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2$$

$$P_{av} = \frac{15 I_m^2}{\pi r^2} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2 \quad \text{—— (9)}$$

In practice, the r.m.s current is of importance. so $I_{rms} = \frac{I_m}{\sqrt{2}}$ $\therefore$ $I_m = \sqrt{2} I_{rms}$.

$$P_{av} = \frac{15 (\sqrt{2} I_{rms})^2}{\pi r^2} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2$$

$$\boxed{P_{av} = \frac{30 I_{rms}^2}{\pi r^2} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2} \quad \text{—— (10)}$$

Radiation Resistance of a half wave dipole

The elemental area of the spherical shell is given by

$$ds = 2\pi r^2 \sin\alpha \, d\alpha. \quad \text{——} (1)$$

The total Power radiated is given by

$$W = \oint_S P_{av} \, ds. \quad \text{——} (2)$$

$$= \int_0^{\pi} \frac{30 I^2_{rms}}{\pi r^2} \left\{ \frac{\cos^2(\pi/2 \cos\alpha)}{\sin\alpha^2} \right\} . 2\pi r^2 \sin\alpha \, d\alpha$$

$$= 60 I^2_{rms} \int_0^{\pi} \left\{ \frac{\cos^2(\pi/2 (\cos\alpha)}{\sin\alpha} \right\} d\alpha.$$

$$\because 2\cos^2\alpha = 1 + \cos 2\alpha.$$

$$\therefore \cos^2\alpha = \frac{1}{2}[1 + \cos 2\alpha]$$

$$W = 60 I^2_{rms} \int_0^{\pi} \frac{1}{2} \left\{ \frac{1 + \cos\pi \cos\alpha}{\sin\alpha} \right\} d\alpha.$$

$$W = 60 I^2_{rms} \cdot I. \quad \text{——} (3)$$

where, $I = \int_0^{\pi} \frac{1}{2} \left\{ \frac{1 + \cos(\pi \cos\alpha)}{\sin\alpha} \right\} d\alpha. \quad \text{——} (4)$

By numerical integration method,

$$I = 1.219. \quad \text{——} (5).$$

$$\therefore W = 60 I^2_{rms} \times 1.219$$

$$= 73.140 I^2_{rms}.$$

$$W = I^2_{rms} \cdot R_r.$$

$\therefore$ Radiation resistance, $R_r = 73.14$

$\boxed{R_r \simeq 73}$

<u>Directivity of half wave dipole :-</u>

$$D = \frac{4\pi}{\Omega_A} \quad \text{—— ①}$$

$$H_\phi = \frac{I_m}{2\pi r} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$$

$$H_{\phi max} = \frac{I_m}{2\pi r} \frac{\cos 0}{\sin 90} \qquad \because \theta = \pi/2$$

$$H_n(\phi) = \frac{H_\phi}{H_{\phi max}} = \frac{\cos \pi/2 \cos\theta}{\sin\theta} \quad \text{—— ②}$$

lly $$E_n(\phi) = \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \quad \text{—— ③}$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \left(\frac{\cos \pi/2 \cos\theta}{\sin\theta}\right)^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi} \frac{\cos(\pi/2 \cos\theta)^2}{\sin\theta} d\theta.$$

$$= 2\pi \int_0^{\pi} \frac{1+\cos 2(\pi/2 \cos\theta)}{2\sin\theta} d\theta$$

$$= 2\pi \times 1.219$$

$\boxed{\Omega_A = 7.65}$

$$D = \frac{4\pi}{7.65} = \underline{\underline{1.64}}$$

$\boxed{D = 1.64}$