Random Process or Stochastic Process

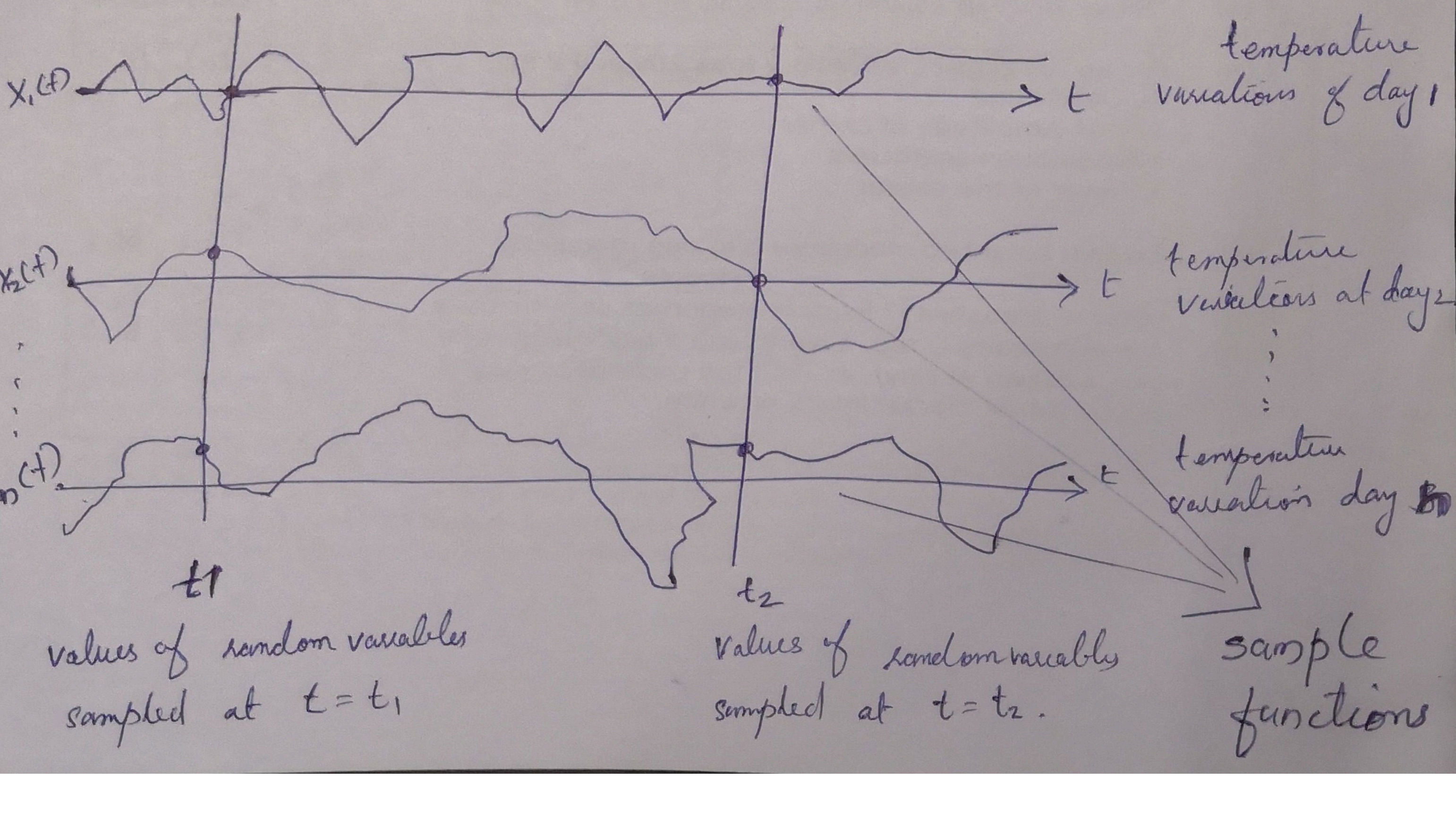
Let there be a random experiment E having outcome λ from the sample space S. This means $\lambda \in S$ (λ is the sample point).

If this λ (outcome) is associated with time ; then a function of λ and time t is formed ie $X(\lambda, t)$. then the function $X(\lambda, t)$ is known as random process. So we can say that ;

> If the random variable is a function of time it is called random process; and it can be denoted by $X(t)$

Example:

Let us consider temperature variations of different days



Now consider we are taking a sample of each sample functions at a fixed time t_1. Then we will get random variables represented by

$$\{ x(t_1, S_1), x(t_1, S_2) \; .... \; x(t_1, S_n) \} = X(t_1)$$

This ensemble of random variable or group of random variable is known as random process.

In random process statistical averages are taken along the time and they are known as time averages; Most generally statistical averages such as mean $m_x(t)$ and autocorrelation function $R_Z(t_1, t_2)$ are used to describe random process.

Mean (Ensemble mean)

if we want to find the means it is given by

$$m_x(t) = E[X(t)]$$

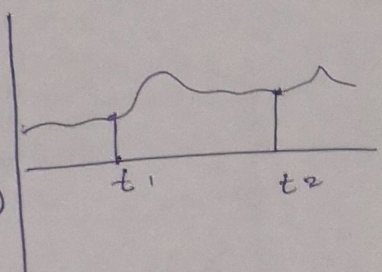
$$= \int_{-\infty}^{\infty} x \, f_{X(t)}(x) \, dx.$$

Autocorrelation

if $X(t)$ is the random process
Then we can consider the values of this random process at two time instants. $X(t_1)$ and $X(t_2)$

then the autocorrelation function is given by

$$R_{XX}(t_1, t_2) = E\left[X(t_1)\, X(t_2)\right]$$



Thus autocorrelation of the random process $X(t)$ is defined as the expectation of the product of two random variables $X(t_1)$ and $X(t_2)$.

$$R_X(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1\, x_2\, f_{X(t_1)X(t_2)}(x_1, x_2)\, dx_1\, dx_2 .$$

The difference between the two time instants taken is represented by the timedelay τ

$$\tau = t_2 - t_1$$

for convinience it can be represented as

$$R_X(\tau) = E\left[X(t)\, X(t+\tau)\right]$$

Properties of Autocorrelation function.

1. Autocorrelation function is always an even function.

ie, $R_x(\tau) = R_x(-\tau)$

2. Autocorrelation at 0 is greater than autocorrelation at any other point.

$$R_x(0) \geq R_x(\tau)$$

3) The mean square value of a random process is equal to the autocorrelation function of the random process for zero time lag.

$$R_x(\tau) = E[x(t) \cdot x(t+\tau)]$$

substituting 0 at τ

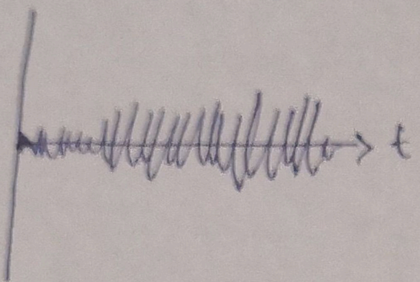
$$R_x(0) = E[x(t) \cdot x(t)]$$

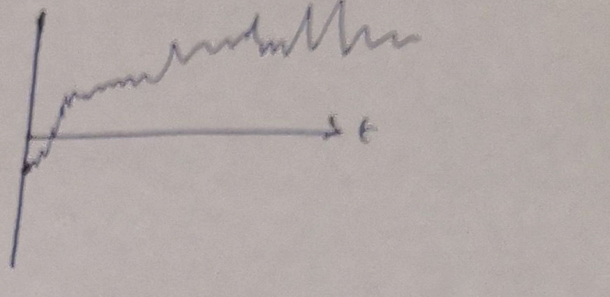
$$= E[x^2(t)]$$

$E[x^2(t)]$ is the mean square value.

Stationarity

A random process $X(t)$ is said to be stationary if its mean and variance do not change with time.





Wide sense stationary (ie; weakly stationary) Process. (WSS)

Some process may appear stationary over a certain period of time. then this will be wide sense stationary process.

Conditions for wide sense stationary

(i) The mean value should independent of time (mean = constant)

Mathematically

$m_x(t) = m_x(t_1)\ m_x(t_2) = m_x(t_3) \ldots$. Constant at all the time enstants.

(ii) the autocorelation function $R_x(t_1, t_2)$ depends only upon the time delay (time difference $t_2 - t_1$)

$$R_x(t_1, t_2) = R_x(\tau)$$

where $\tau = t_2 - t_1$.