INFORMATION THEORY & CODING:

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Shannon’s Hartley law / Shannon’s Third Theorem

- The Theorem states that the capacity of the bandlimited Gaussian Channel with AWGN is given by
- C= B log(1+ S/N) bits/sec

B= Channel bandwidth in Hz

S= signal power in watts

N = noise power in watts , N= N0. B N=

Shannon – Hartley Law Implications

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \quad \text{bits/sec} \quad \text{——} \; (1).$$

indicates that a noiseless Gaussian channel $\frac{S}{N} = \infty$ has an infinite capacity

On the other hand, while the channel capacity does increase, it does not become infinite because with an increase in BW, noise power also increases ($N = N_0 B$).

Upper Limit for Channel Capacity

Thus for a fixed signal power and in presence of WGN; the channel capacity approaches an upper limit with increasing BW.

The limit

$N = N_0 B$ or $N = \eta B$ in eqn ① above. PSD $= \frac{N_0}{2}$.

$$C = B \left\{ \log_2 \left(1 + \frac{S}{\eta B}\right) \right\}$$

$$= \frac{S}{\eta} \cdot \frac{\eta}{S} \cdot B \left\{ \log_2 \left(1 + \frac{S}{\eta B}\right) \right\}$$

$$= \frac{S}{\eta} \cdot \log_2 \left[1 + \frac{S}{\eta B}\right]^{\frac{\eta B}{S}}$$

Log m^n

Substituli $x = \frac{S}{\eta B}$ as $B \rightarrow \infty$ $x \rightarrow 0$.

$$C = \frac{S}{\eta} \log_2 (1+x)^{1/x}.$$

$x)$

when $B \rightarrow \infty$, $x \rightarrow 0$.

$$\therefore \lim_{\substack{B \to \infty \\ x \to 0}} C = \lim_{\substack{B \to \infty \\ x \to 0}} \frac{S}{\eta} \log_2 (1+x)^{1/x}.$$

$$C_{\infty} = \frac{S}{\eta} \log_2 \lim_{x \to 0} \left[(1+x)^{1/x} \right]$$

$$= \frac{S}{\eta} \log_2 e$$

$$\boxed{C_{\infty} = 1.44 \frac{S}{\eta}}$$

This is the upper limit for channel capacity

2. Bandwidth S/N trade off

Let us consider the trade off b/w BW and S/N.

Let $\frac{S}{N} = 7$ $B = 4\,KHz$

$$C = 4KHz \cdot \log_2 (1 + \cancel{7}) = \underline{\underline{12 \times 10^3 \text{ bits/sec}}}.$$

$$\therefore \text{ Channel capacity } C_1 = B_1 \log\left(1 + \frac{S_1}{N_1}\right)$$

$$= 4 \times 10^3 \log(1 + 7)$$

$$= 12 \times 10^3 \text{ bits/sec.}$$

1.↑

Keeping the channel capacity C_2 same as C_1 and if signal-to-noise ratio is increased to 15, then

$$C_2 = C_1 = 12 \times 10^3 = B_2 \log\left(1 + \frac{S_2}{N_2}\right)$$

$$= B_2 \log(1 + 15)$$

$$\therefore \quad B_2 = 3 \text{ KHz}$$

Since the noise power $N = \eta B$, as the bandwidth gets reduced from 4 to 3 KHz, the noise also decreases indicating an increase in signal power as shown below.

We have $N_1 = \eta B_1 = (\eta)\,(4\text{ KHz})$

and $N_2 = \eta B_2 = (\eta)\,(3\text{ KHz})$

Consider $$\frac{(S_2/N_2)}{(S_1/N_1)} = \frac{15}{7}$$

$$\therefore \quad \frac{S_2}{S_1} = \frac{15N_2}{7N_1} = \frac{(15)(\eta)(3\text{KHz})}{(7)(\eta)(4\text{KHz})} = \frac{45}{28} \simeq 1.6$$

Inference

→ with 3KHz BW, noise power will be $\frac{3}{4}$ as large as with 4KHz

→ Thus S/N power will have to be increased by the factor

$$\frac{3}{4} \times \frac{15}{7} = 1.60.$$

∴ 25% reduction in bandwidth requires 60% increase in signal power

Thus to decrease the BW; the signal power has to be increased;
IIIly to decrease signal power, bandwidth must be increased

Bandwidth S/N trade off curve

BW, S/N Trade off Curve.

To draw the trade off curve from Shannon Hartley law; $C = B\log_2\left(1+\frac{S}{N}\right)$.

$C \Rightarrow$ bits/sec.

$$\frac{B}{C} = \frac{1}{\log_2(1+S/N)}.$$

$\log_2(1+50) = 5.6$

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Table for B/C for different values of S/N

S/N	0.5	1	2	5	10	20	50
B/C	1.71	1	0.63	0.37	0.289	0.23	0.176

$\log_2(1+50) = 5.6$

$\frac{S}{N} = 15$ $B = 5KHz$

$C = 20k$ bits/sec

$\frac{S}{N} = 31$ $C = 20k$ bits

$B = 4KHz$

B/c vs S/N trade off curve

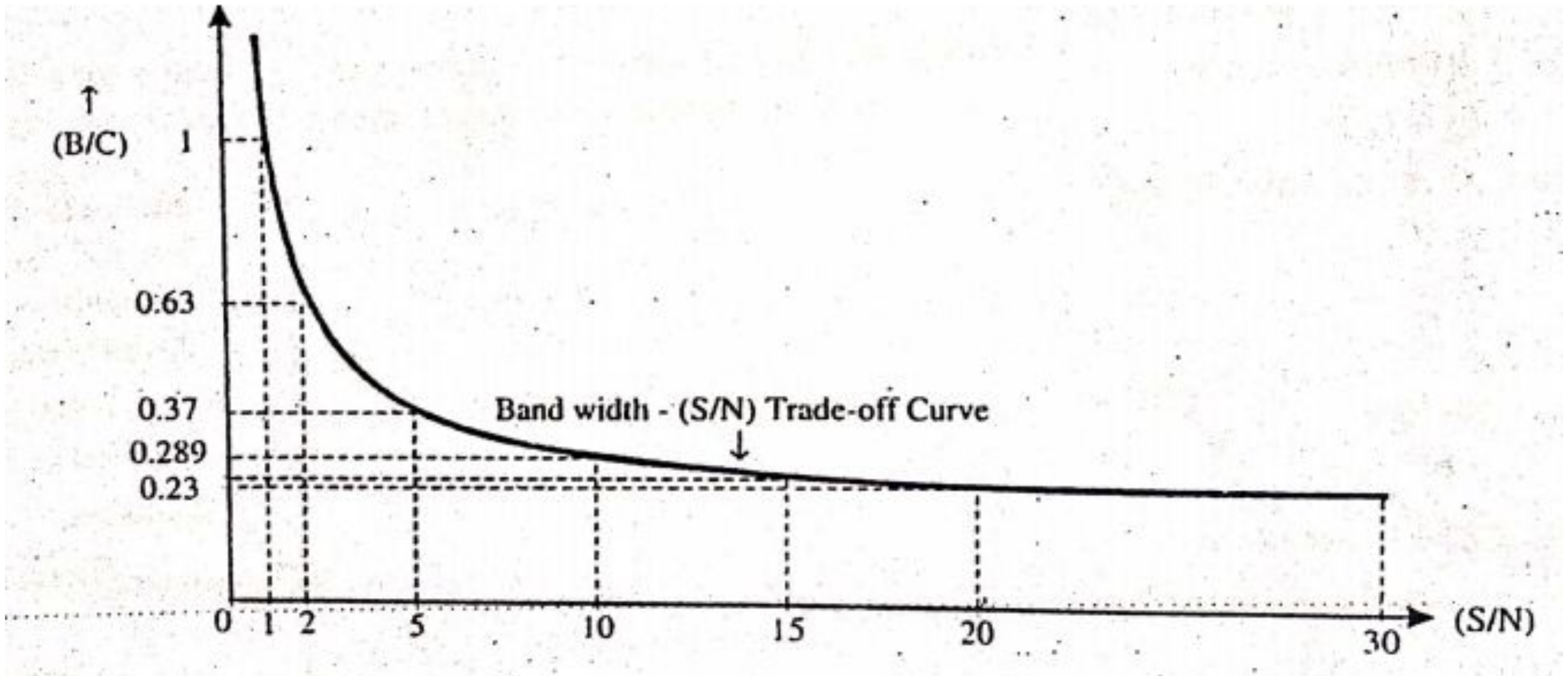

Fig. 4.24 : Bandwidth-to-(S/N) Trade-off Curve

The plot shows B/C as a function of S/C

Inference of the curve

1. Using the curve same channel capacity may be obtained by increasing bandwidth if S/N is small

There is threshold point $\frac{S}{N} = 10$ upto which exchange rate of BW with S/N is advantageous.

Beyond $\frac{S}{N} > 10$; BW reduction with larger S/N is very poor.

Conclusion

- Upper limit for Shannon’s capacity
- Trade off curve

THANK YOU