Example 1

For a (7,4) cyclic code, the received vector $z(x)$ is 1110101 and generator polynomial $g(x) = 1 + x + x^3$. Draw the syndrome calculation circuit and correct the single error in the received vector.

The generator polynomial is given by

$$g(x) = 1 + x + x^3 = g_0 + g_1 x + g_2 x^2 + g_3 x^3$$

∴ The coefficients are given by

$g_0 = 1$, $g_1 = 1$, $g_2 = 0$ and $g_3 = 1$

With these values, the circuit of figure 5.15, reduces to the one shown in figure 5.17 with only 3 flip-flops s_0, s_1 and s_2 representing the syndrome bits, two modulo-2 adders and connection from output of gate-2 to both the modulo-2 adders.

Two methods are available for correcting the error in the received vector. Both the methods are discussed below :

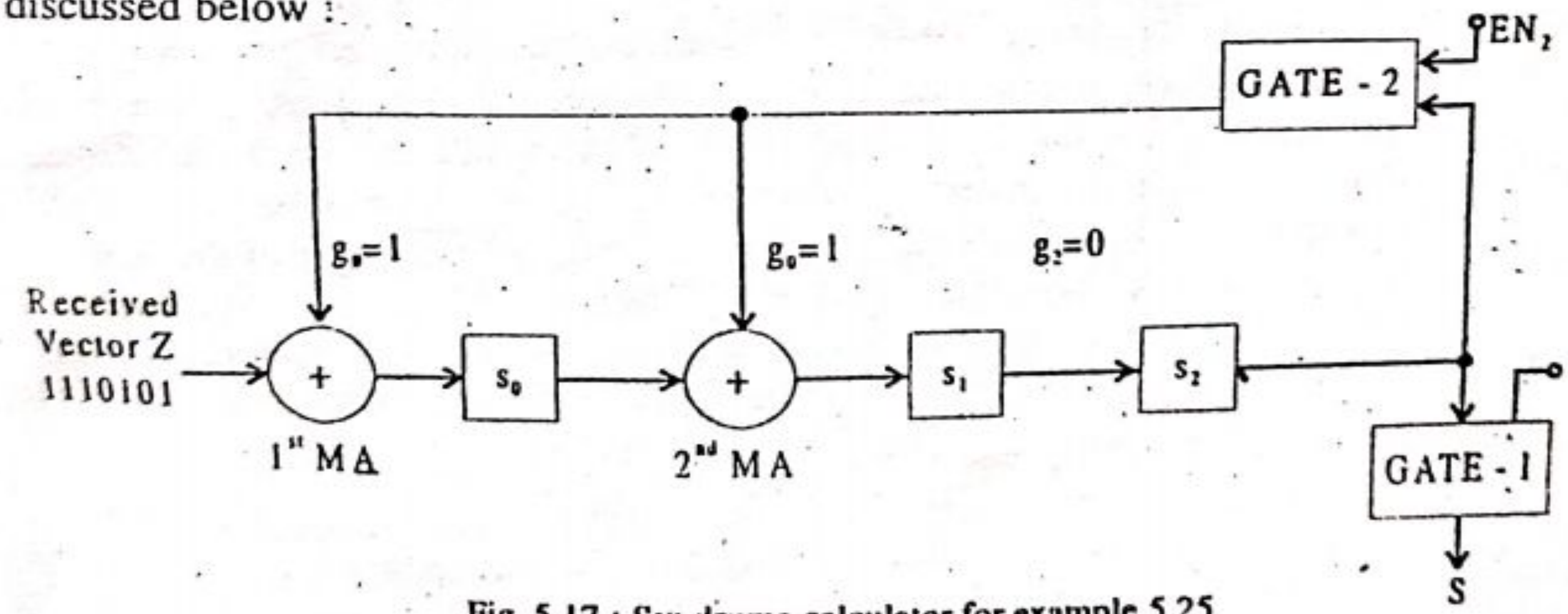

Fig. 5.17 : Syndrome calculator for example 5.25

METHOD -I :

Number of shifts	Input Z	Shift Register Contents s_0	s_1	s_2	Comments
Initialisation, gate-1 OFF and gate-2 ON		0	0	0	Shift register contents are cleared
1	1	1	0	0	
2	0	0	1	0	
3	1	1	0	1	
4	0	1	0	0	
5	1	1	1	0	
6	1	1	1	1	
7	1	0	0	1	← Indicates error

Table 5.24 : Contents of shift register in the syndrome calculator of fig. 5.17 for the received vector Z → 111010[illegible]

To correct the error, the received vector is fed into the decoder circuit of figure 5.16 and corrected vector flows out of the decoder circuit.

Knowing the syndrome $s_0 s_1 s_2 \rightarrow 0\,0\,1$ and H^T matrix, the error can be corrected analytic as shown below :

Using equation 5.72 matrix, the H^T matrix is written as

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{...... (}$$

The received vector is fed into decoder circuit and the erro[illegible]
can be corrected.

$$h(x) = \frac{x^n + 1}{g(x)} = \frac{x^7 + 1}{x^3 + x + 1}$$

By performing division

$$
\begin{array}{r|l}
 & x^4 + x^2 + x + 1 \\
\hline
x^3 + x + 1 & x^7 + 1 \\
 & x^7 + x^5 + x^4 \\
\hline
 & x^5 + x^4 + 1 \\
 & x^5 + x^3 + x^2 \\
\hline
 & x^4 + x^3 + x^2 + 1 \\
 & x^4 + x^2 + x \\
\hline
 & x^3 + x + 1 \\
 & x^3 + x + 1 \\
\hline
 & 0
\end{array}
$$

$$\underline{h(x) = x^4 + x^2 + x + 1}.$$

Reciprocal of $h(x)$.

$$x^k h(x^{-1}) = x^4 \left(1/x^4 + 1/x^2 + 1/x + 1 \right).$$

$$= 1 + x^2 + x^3 + x^4.$$

$$x^4 h(x^{-1}) = [1\;0\;1\;1\;1\;0\;0]$$

$$x^{k+1} h(x^{-1}) = x^5 h(x^{-1})$$

$$x^5 h(x^{-1}) = x^5\left(1/x^4 + 1/x^3 + 1/x + 1\right).$$

$$= x + x^3 + x^4 + x^5$$

$$= [0\;1\;0\;1\;1\;1\;0]$$

$$x^6 h(x^{-1}) = x^6\left(1/x^4 + 1/x^3 + 1/x + 1\right)$$

$$= x^3 + x^4 + x^5 + x^6 .$$

$$= [0\;0\;1\;0\;1\;1\;1]$$

$$\left.\begin{matrix} x^k \\ \\ x^{n-1} \end{matrix}\right\} H(x^{-1})$$

$$[H] = \begin{bmatrix} x^k\, h(x^{-1}) \\ x^{k+1}\, h(x^{-1}) \\ \vdots \\ x^{n-1}\, h(x^{-1}) \end{bmatrix}.$$

$$= \left[\begin{array}{ccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right].$$

First row $\leftarrow$ I^{st} row + 3^{rd} row

$$\begin{bmatrix} 1011100 \oplus \\ 0010111 \\ \\ 0101110 \\ 0010111 \end{bmatrix} = \begin{bmatrix} 1\ 0\ 0 & 1\ 0\ 1\ 1 \\ 0\ 1\ 0 & 1\ 1\ 1\ 0 \\ 0\ 0\ 1 & 0\ 1\ 1\ 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 100 \\ 010 \\ 001 \\ \hline 110 \\ 011 \\ 111 \\ 101 \end{bmatrix}$$

The syndrome $s_0 s_1 s_2 = 0\,0\,1$ obtained from the syndrome calculator circuit of figure 5.17 located in the 3rd row of H^T matrix of equation (5.79). Hence, the 3rd bit is in error.

$$\therefore \text{ Error vector } E = 0010000$$

$$\therefore \text{ The corrected vector } = Z + E$$

$$= 1110101 + 0010000$$

$$= 1100101$$

METHOD -II :

Number of shifts	Input Z(x)	Shift Register Contents s_0	s_1	s_2	Comments
Initialisation, gate-1 OFF and gate-2 ON		0	0	0	Shift register conten are cleared
1	1	1	0	0	
2	0	0	1	0	
3	1	1	0	1	
4	0	1	0	0	
5	1	1	1	0	
6	1	1	1	1	
7	1	0	0	1	←Indicates error
8	0	1	1	0	
9	0	0	1	1	
10	0	1	1	1	
11	0	1	0	1	
12	0	1	0	0	← end of shifting operation

This method is actually continuation of the previous method beyond the 7^{th} shift. When all the 7 received bits are entered into the syndrome calculator, '0's are now fed into it, from 8th shift onwards as shown in table 5.25. Each time a '0' is fed into the circuit, the fresh shift register contents are tabulated. This process is feeding '0's is continued till the shift register contents read $s_0\, s_1\, s_2 = 1\,0\,0$ [In general, for $(n-k)$ shift register, the contents should read $s_0\, s_1 \ldots s_{n-k-1} = 1\,0\,0 \ldots 0$. i.e., 1 followed by $(n-k-1)$ number of 0s]. In table 5.25, we find that, at the 12^{th} shift we get shift register contents as 100. The error is then located and corrected as given below.

The received vector Z →	1	1	1	0	1	0	1
			↑	↑	↑	↑	↑
			12^{th} shift	11^{th} shift	10^{th} shift	9^{th} shift	8^{th} shift

Since we got '100' at the 12^{th} shift, the 5^{th} bit counting from right is in error.

$\therefore$ Error vector E = 0 0 1 0 0 0 0

$$\therefore \text{Corrected vector } V = Z + E$$
$$= 1110101 + 0010000$$
$$V = 1100101 \rightarrow \text{same as before.}$$

Example 5.26 : Repeat example 5.25 for the received vector Z → 0100101.

Solution

Referring to the same syndrome calculator of figure 5.17, let us now feed the new received vector Z → 0 1 0 0 1 0 1 and list the contents of the shift register after each shift as shown in table 5.26.

Number of shifts	Input Z(x)	Shift Register Contents s_0	s_1	s_2	Comments
Initialisation, gate-1 OFF and gate-2 ON		0	0	0	Shift register contents are cleared
1	1	1	0	0	
2	0	0	1	0	
3	1	1	0	1	
4	0	1	0	0	
5	0	0	1	0	
6	1	1	0	1	
7	0	1	0	0	←Indicates error
8	0	0	1	0	
9	0	0	0	1	
10	0	1	1	0	
11	0	0	1	1	
12	0	1	1	1	
13	0	1	0	1	
14	0	1	0	0	← end of shifting operation

Table 5.26 : Contents of shift register in the syndrome calculator of figure 5.17 for example 5.26

From table 5.26, we observe that after the 14^{th} shift we get shift register contents as 100. The error is located and corrected as shown below:

The received vector Z →

0	1	0	0	1	0	1
↑		↑		↑		↑
14^{th} shift		12^{th} shift		10^{th} shift		8^{th} shift

Since we got '100' in the 14^{th} shift, the 7^{th} bit counting from right or the 1^{st} bit counting from left is in error.

∴ Error vector E = 1 0 0 0 0 0 0

$\therefore$ Corrected vector $V = Z + E$

$$= 0100101 + 1000000$$

$V = 1100101$ which is a valid code vector