Angle modulation:

In angle modulation total angle of the carrier signal is varied in accordance to the message signal amplitude variations

$$c(t) = A_c \cos(2\pi f_c t + \phi)$$

$$\text{Total angle} = \theta(t) = 2\pi f_c t + \phi$$

If the angle modulation occurs due to dependance of f_c on $m(t)$ then it is called frequency modulation

If the angle modulation occurs due to dependance of ϕ_c on $m(t)$; it is called phase modulation.

Phase modulation:

Carrier before modulation

$$c(t) = A_c \cos 2\pi f_c t$$

Carrier after phase modulation

$$S_{PM}(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$$

$$\underset{\uparrow \text{Radian}}{\phi(t)} = \underset{\uparrow \frac{\text{Radian}}{\text{volt}}}{k_p}\ \underset{\uparrow \text{volt}}{m(t)}$$

$k_p \longrightarrow$ phase sensitivity factor.

k_p specifies the change in phase for 1V change in the message s/l.

Frequency modulation:

- Frequency of the carrier before modulation f_c
- Frequency of the carrier after frequency modulation f_i. It is known as instantaneous frequency

$$\underset{\uparrow Hz}{f_i} = \underset{\uparrow Hz}{f_c} + \underset{\uparrow \frac{Hz}{volt}}{k_f}\ \underset{\text{volt}}{m(t)}$$

k_f = frequency sensitivity of the modulator

k_f specifies the amount of frequency change of the carrier signal per 1 volt change in the message s/l

Phase Modulation

carrier signal $c(t) = A_c \cos 2\pi f_c t$

carrier signal after phase modulation

$$S_{PM}(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

↳ phase deviation

$$\phi(t) = k_p m(t)$$

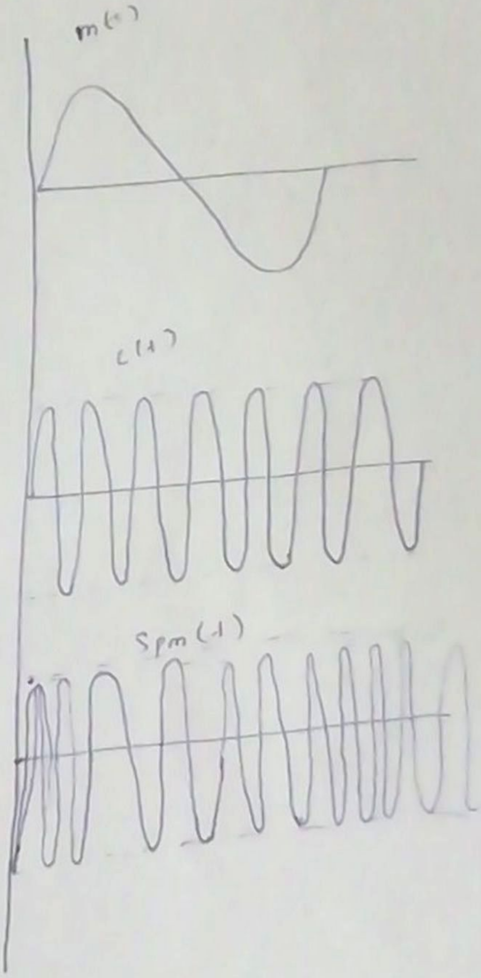
$$S_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

$$m(t) = A_m \cos 2\pi f_m t$$

Max. phase deviation $\Delta\phi = \max[\phi(t)]$

$$\Delta\phi = \max[k_p m(t)]$$

$$\boxed{\Delta\phi = k_p A_m \rightarrow \text{Rad}}$$



Single Tone phase modulation

Assume $m(t) = A_m \cos 2\pi f_m t$

$$S_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

$$= A_c \cos[2\pi f_c t + k_p A_m \cos 2\pi f_m t]$$

$$\boxed{S_{PM}(t) = A_c \cos[2\pi f_c t + \beta \cos 2\pi f_m t]}$$

$$\boxed{k_p A_m = \beta = \text{Modulation index of PM}}$$

To phase modulation max phase deviation $\Delta\phi$ and modulation index β both are equal and independant of message signal frequency variations

$$\boxed{\beta = \Delta\phi = k_p A_m}$$

General Expression for FM signal :-

Before modulation carrier can be expressed as

$c(t) = A_c \cos \theta_c t$

$= A_c \cos (2\pi f_c t + \phi_c) \rightarrow$ constant

Message signal $m(t) = A_m \cos 2\pi f_m t$

In FM the frequency of the carrier is varied linearly with message s/l $m(t)$. that is expressed as $f_i(t) = f_c + k_f m(t)$

$\hookrightarrow$ proportionality constant called frequency sensivity factor.

w.k.t in case of frequency modulation

$\theta(t) = 2\pi f_c t$

Differentiating both sides w.r.to t

$$\frac{d\,\theta(t)}{dt} = 2\pi f_c$$

If the instantaneous frequency is f_i then the above expression can be written as

$$\frac{d\,\theta_i(t)}{dt} = 2\pi f_i$$

Integrating both sides

$$\theta_i(t) = 2\pi \int_0^t f_i(t)\, dt$$

$$= 2\pi \int_0^t [f_c + k_f m(t)]\, dt$$

$$= 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt$$

w.k.t general expression for Anglemod: signal $S(t) = A_c \cos [\theta_i(t)]$

$$S_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

Single tone FM –

W.K.T general expression for FM signal

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

Assume that $m(t) = A_m \cos 2\pi f_m t$

So the equation for FM wave becomes

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t \, dt\right]$$

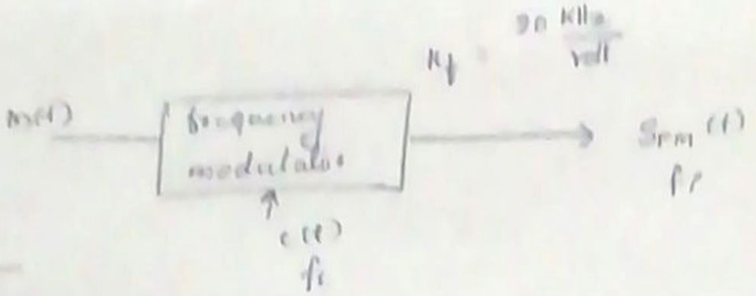
$$= A_c \cos\left[2\pi f_c t + 2\pi k_f A_m \frac{\sin 2\pi f_m t}{2\pi f_m}\right]$$

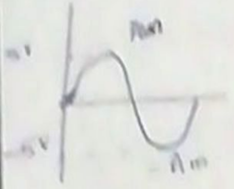
$$= A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{S_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]}$$

where $\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m}$

→ Deviation ratio or modulation index





$$f_i = f_c + k_f\, m(t)$$

$m(t) = 0$, $f_i = f_c$

$m(t) = 5V$ $\quad f_i = f_c + 20 \times 5$

$= f_c + 100\,KHz$

$m(t) = -5V$ $\quad f_i = f_c - 100\,KHz$

So we can say that

when $m(t) = 0$ $\quad f_i = f_c$

$m(t) = +ve$ $\quad f_i > f_c$

$m(t) = -ve$ $\quad f_i < f_c$

Max frequency of Resulting FM s/l

$$f_{max} = f_c + k_f A_m$$

Minimum frequency of resulting FM s/l

$$f_{min} = f_c - k_f A_m$$

Maximum frequency deviation

$$\Delta f = max\left[k_f m(t)\right]$$

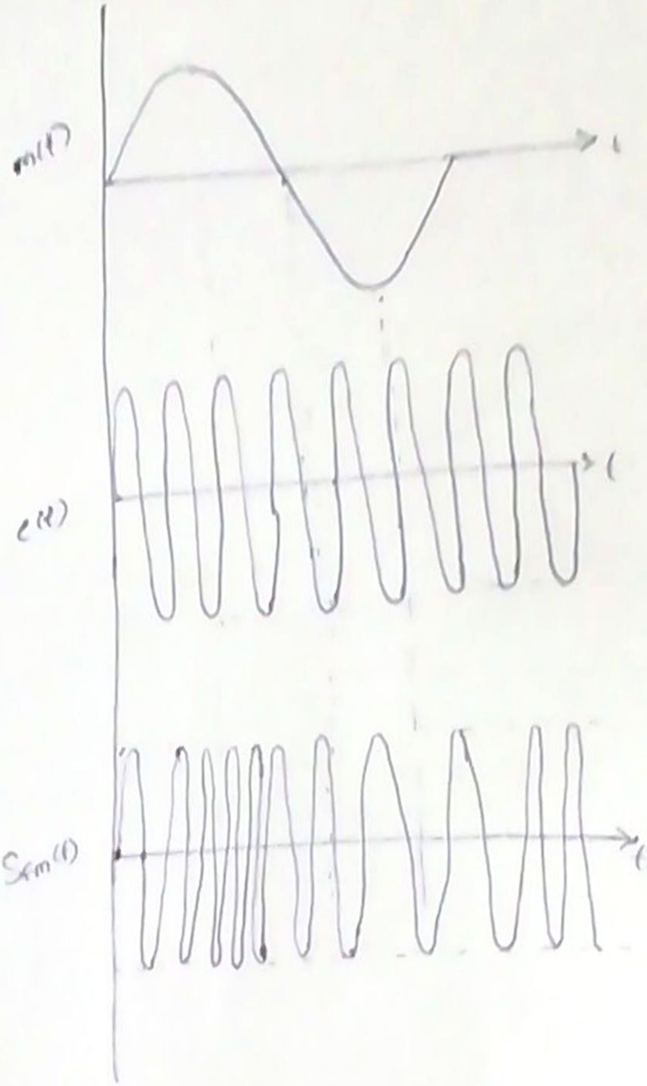
$$\Delta f = k_f A_m \text{ Hz}$$

$\therefore$ $f_{max} = f_c + \Delta f$; $f_{min} = f_c - \Delta f$

Total frequency swing of the FM signal

$$f_{max} - f_{min} = f_c + \Delta f - (f_c - \Delta f)$$

$$= 2\Delta f$$

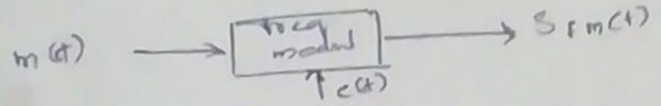


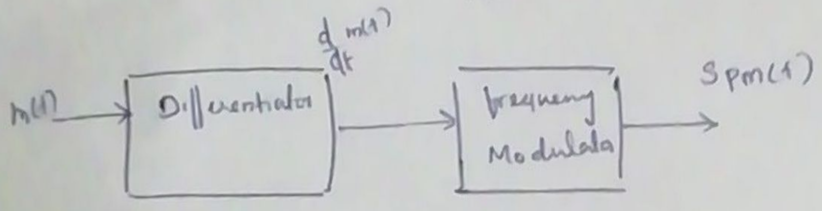
In FM the message s/l voltage variations are converted as corresponding carrier signal frequency variations. So Frequency modulation is also called as voltage to frequency conversion.

Relation b/w PM & FM

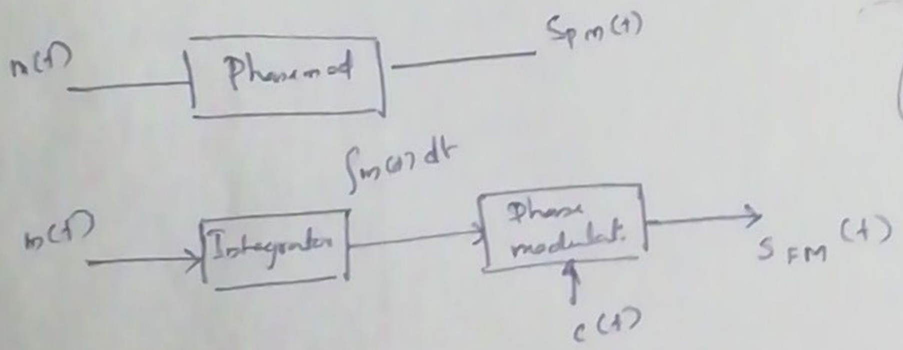
$$s_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p\, m(t)\right]$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$$





generation of PM from FM



generation of FM from PM

FM classification

Depending on the value of the modulation index (β) FM is classified in to

FM

Narrow band FM (NBFM)	Wideband FM (WBFM)
$\beta \ll 1$	$\beta \gg 1$

Narrow Band FM

General expression for single tone FM is given by

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

$\hookrightarrow$ this can be in the form of

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$S_{FM}(t) = A_c \cos 2\pi f_c t \cos(\underbrace{\beta \sin 2\pi f_m t}_{\theta}) - A_c \sin 2\pi f_c t \sin(\underbrace{\beta \sin 2\pi f_m t}_{\theta})$$

we have taken as $\cos\theta = 1 \rightarrow \cos 0 = 1$

$\sin\theta = \theta \rightarrow \sin 0 = 0$

$$S_{FM}(t) = A_c \cos 2\pi f_c t \cdot 1 - A_c \sin 2\pi f_c t \; \beta \sin 2\pi f_m t$$

$$A_c \cos 2\pi f_c t - A_c\beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$\hookrightarrow 2 \sin A \sin B$

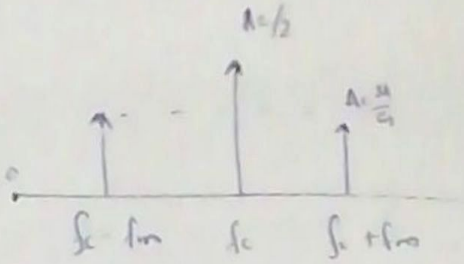
$$= A_c \cos 2\pi f_c t - \frac{A_c\beta}{2} \cos 2\pi (f_c - f_m) t + \frac{A_c\beta}{2} \cos 2\pi (f_c + f_m) t$$

w.k that expression for standard AM

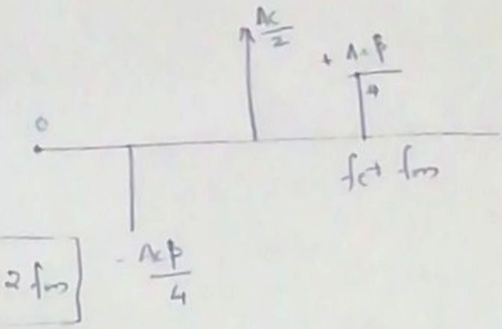
$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t$$

Expression for AM & NBFM will be same except 180° phaseshift at LSB frequency component.

$S_{AM}(t) \longleftrightarrow$



$S_{NBFM}(t) \longleftrightarrow$



$$\boxed{BW = f_c + f_m - (f_c - f_m) = 2 f_m}$$

Power of NBFM

$$P_t = P_{USB} + P_{LSB} + P_c$$

$$P_c = \frac{A_c^2}{2}$$

$$P_{LSB} = P_{USB} = \frac{\left(\frac{A_c \beta}{2}\right)^2}{2} = \frac{A_c^2 \beta^2}{8}$$

$$P_t = \frac{A_c^2}{2} + \frac{A_c^2 \beta^2}{8} + \frac{A_c^2 \beta^2}{8}$$

$$= \frac{A_c^2}{2} + \frac{A_c^2 \beta^2}{4}$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\beta^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\beta}{2}\right]$$

The bandwidth and power of the narrowband FM is Same as that of AM

So in practical case there less importance to NBFM as compared to WBFM

WideBand FM

Consider a single tone FM wave for any value of β

$s(t) = A_c \cos [\underbrace{2\pi f_c t + \beta \sin 2\pi f_m t}_{\theta}]$ ~ This is produced by a sinusoidal modulating wave $m(t)$

we know that $\cos\theta = Re[e^{j\theta}]$ $\because e^{j\theta} = \cos\theta + j\sin\theta$

$$S_{FM}(t) = A_c \, Re\left[e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\right]$$

$$= A_c \, Re\left[e^{j(2\pi f_c t)} e^{j\beta \sin 2\pi f_m t}\right]$$

$$\tilde{s}(t) = Re\left[A_c e^{j\beta \sin 2\pi f_m t}\right] \quad \text{——— (1)}$$

$\therefore S_{FM}(t) = Re[\tilde{s}(t) e^{j2\pi f_c t}]$

Assume that it is function of period $T = \frac{1}{f_m}$

i.e. having fundamental frequency f_m

→ this is known as the complex envelop of FM wave

i.e. it can retain the complete information of FM wave

it can be expand using fourier series

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t} \quad \text{——— (2)}$$

$$C_n = f_m \int_{-1/2 f_m}^{1/2 f_m} \tilde{s}(t) e^{-j2\pi n f_m t} dt$$

$$C_n = f_m A_c \int_{-1/2 f_m}^{1/2 f_m} e^{j\beta \sin 2\pi f_m t} e^{-j2\pi n f_m t} dt$$

Assume $x = 2\pi f t$ $\therefore dx = 2\pi f dt$

$$dt = \frac{dx}{2\pi f_m}$$

and also the limits changes to

$t = -\frac{1}{2f_m}$ $x = -\pi$

$t = 1/2f_m$ $x = +\pi$

(10)

$$C_n = f_m A_c \int_{-\pi}^{\pi} e^{j\beta \sin 2\pi f_m t - j 2\pi n f_m t} \frac{dx}{2\pi f_m}$$

$$= \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j\beta \sin 2\pi f_m t - j 2\pi n f_m t} \, dx$$

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j\beta \sin x - j n x} \, dx$$

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\beta \sin x - j n x} \, dx$$ — Known as Bessel function

$$C_n = A_c J_n(\beta)$$ — substitute this in eqn (2)

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j 2\pi f_m t} \quad \text{—— (3)}$$

substitute (3) in (1)

$$s(t) = Re\left[\sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j 2\pi n f_m t} e^{j 2\pi f_c t}\right.$$

$$= Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j 2\pi (f_c + n f_m) t}\right.$$

$$s(t) = \left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi f_c + n f_m) t\right.$$

this is an extended expression for FM

$$S_{WBFM}(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n f_m) t \qquad \beta \gg 1$$