101001/EC700A: Microwave & Antennas

Module 5
Microwave Hybrid Circuits & Semiconductor Devices

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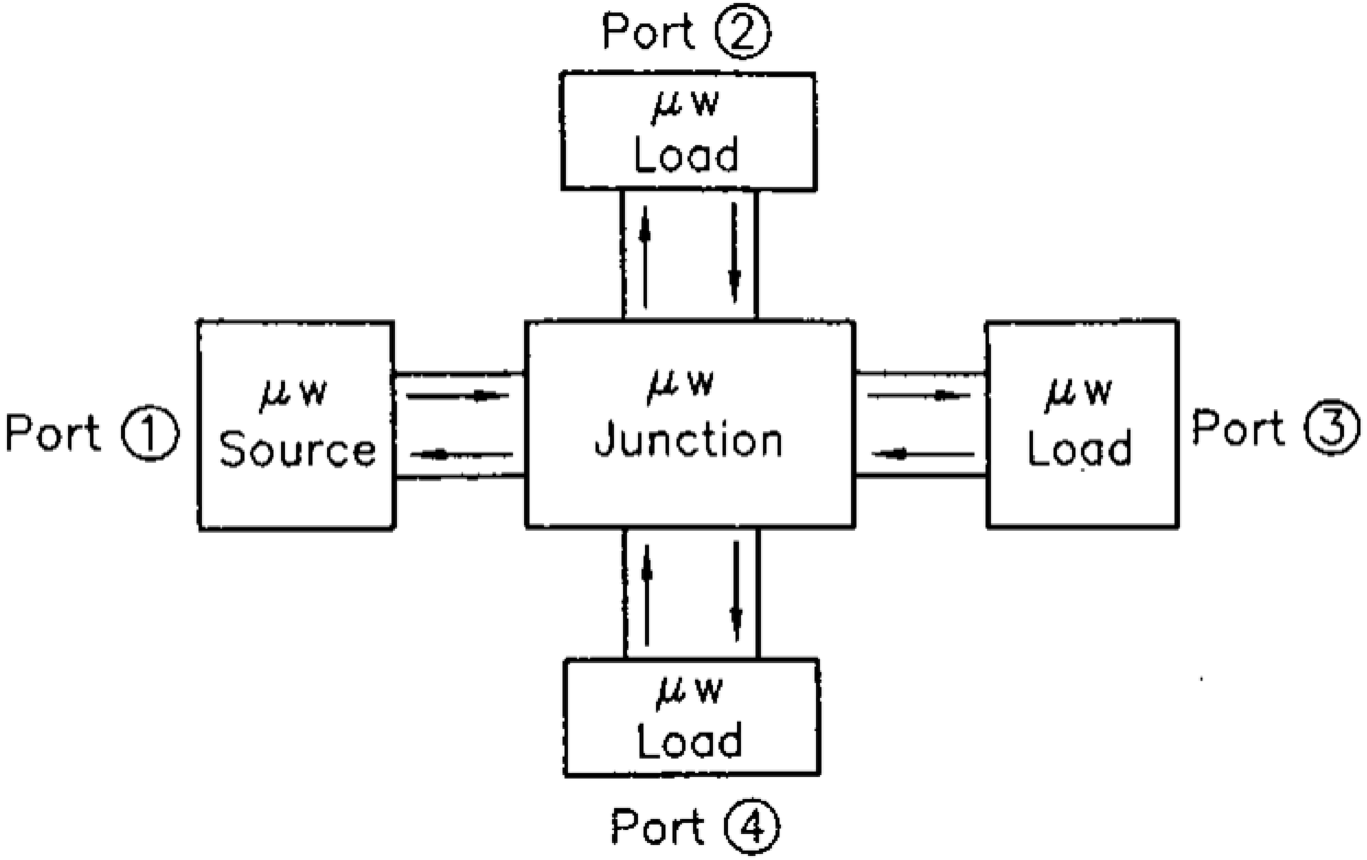
Content

- Scattering Parameters
- Magic Tee
- Hybrid Ring
- Directional Coupler
- Circulator & Isolator
- Gunn Diodes

Reference: Ch 4 Liao, Ch 6 Kulkarni

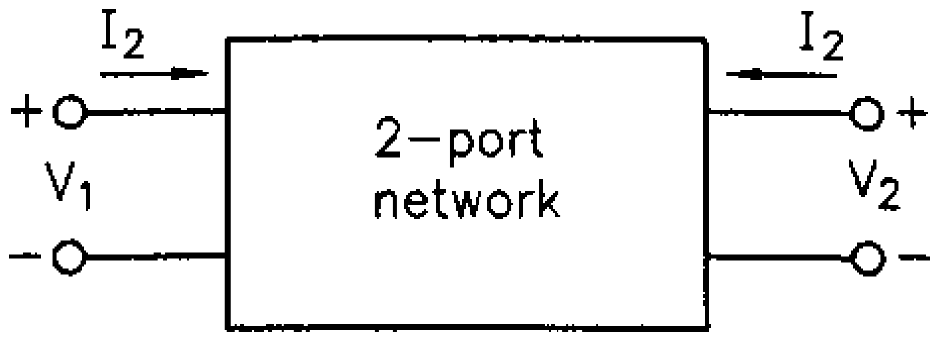
Microwave Waveguide Junctions

- A microwave junction is the interconnection of 2 or more microwave components.
- The junction can be used to combine or split power all or part of microwave energy into particular directions.
- Consider a 4 port microwave waveguide junction



Scattering (S) parameters

- Low frequency circuits can be described by 2 port network and their parameters Z, Y, H, ABCD
- The microwave junction is defined by S parameters or Scattering Parameters



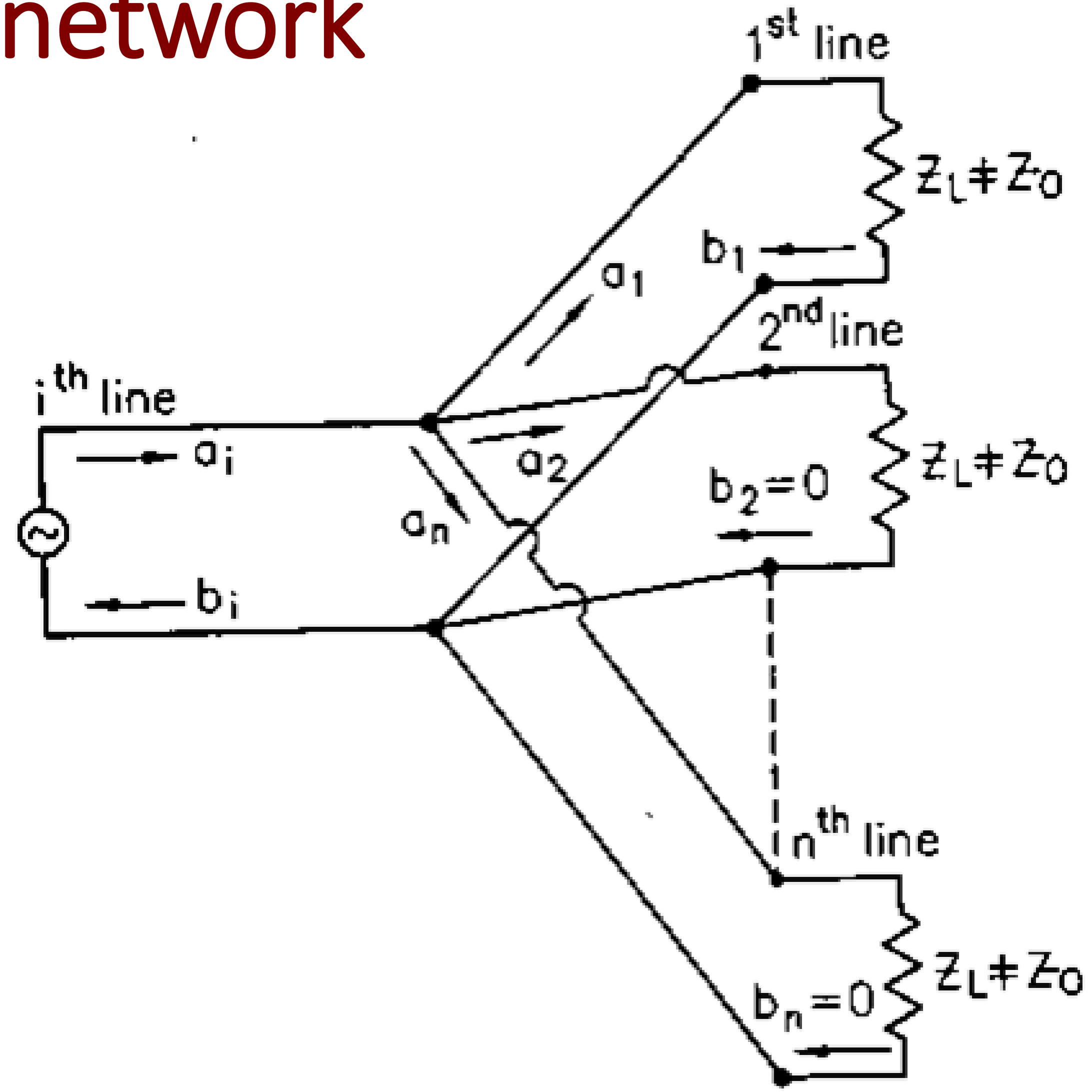
- Scattering Matrix can be defined as:

A square matrix which gives all the combinations of power relationships between the various input and output ports of a microwave junction

- The elements of this matrix are called scattering coefficients or S parameters

S Parameter of multi-port network

- Consider a junction of 'n' number of transmission lines with the i^{th} line terminated in a source



Case 1: If first line is terminated in an impedance $Z_L \neq Z_0$ and all the remaining lines in an impedance $Z_L = Z_0$

- If a_i is the incident wave at the junction due to the source, it divides itself among the $(n - 1)$ lines as a_1 , a_2 , … a_n
- Due to mismatch in the first line, there is reflected wave b_1 going back to the junction

$$b_1 = S_{i1} \, . \, a_1$$

where S_{i1} is the reflection coefficient of 1st line

1 – reflection from 1st line and

i – source connected at i^{th} line

- The contribution to the outward travelling wave in the j^{th} line is

$$b_i = S_{ij} \, . \, a_j \quad [\because b_2 = b_3 = .. = b_n = 0]$$

Case 2: Let all the (n-1) lines be terminated in an impedance other than Z_0 i.e. $Z_L \neq Z_0$

There will be reflections into the junction from every line. Hence the total contribution to the outward travelling wave is

$$b_i = S_{i1}\,.\,a_1 + S_{i2}\,.\,a_2 + S_{i3}\,.\,a_3 + \,..\, S_{in}\,.\,a_n$$

Here $i = 1$ to n

Therefore

$$\begin{aligned}
b_1 &= S_{11}\,a_1 + S_{12}\,a_2 + S_{13}\,a_3 + \ldots + S_{1n}\,a_n \\
b_2 &= S_{21}\,a_1 + S_{22}\,a_2 + S_{23}\,a_3 + \ldots + S_{2n}\,a_n \\
&\vdots \\
b_n &= S_{n1}\,a_1 + S_{n2}\,a_2 + S_{n3}\,a_3 + \ldots + S_{nn}\,a_n
\end{aligned}$$

In matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & S_{22} & \dots & S_{2n} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \dots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ \vdots \\ a_n \end{bmatrix}$$

Column Matrix [b] corresponding to Reflected waves or Output

Scattering Column Matrix [S] of order $n \times n$

Matrix [a] corresponding to Incident waves or Input

$$[b] = [S]\,[a]$$

S matrix of a multi port network.

When a junction of n number of waveguides is considered,

a's represent inputs to particular ports.

b's represent outputs out of various ports.

S_{ij} corresponds to scattering coefficients resulting due to input at ith port and output taken out of jth port.

S_{ii} denotes how much of power is reflected back from the junction into the ith port when input power is applied at the ith port itself.

Properties of S Matrix:

1. [S] is always a square matrix of order (n x n)
2. [S] is a symmetric matrix, i.e. $S_{ij} = S_{ji}$
3. [S] is a unitary matrix, i.e. $[S][S]* = [I]$

4. The sum of the products of each term of any row (column) multiplied by the complex conjugate of the corresponding terms of any other row (column) is zero

   i.e. $\sum_{i=1}^{n} S_{ik}\, S_{ij}^{*} = 0 \quad for\ k \neq j$

5. If any of the terminals (say the kth port) are moved away from the junction by an electrical distance $\beta_k l_k$, each of the coefficients S_{ij} involving k will be multiplied by the factor $e^{-j\beta_k l_k}$

6. If all the ports are perfectly matched to the junction, the diagonal elements of the matrix will be zero.

Microwave T Junction

- A T junction is an intersection of 3 waveguides in the form of a letter T
- There are several types of Tee junctions:

  H Plane Tee

  E Plane Tee

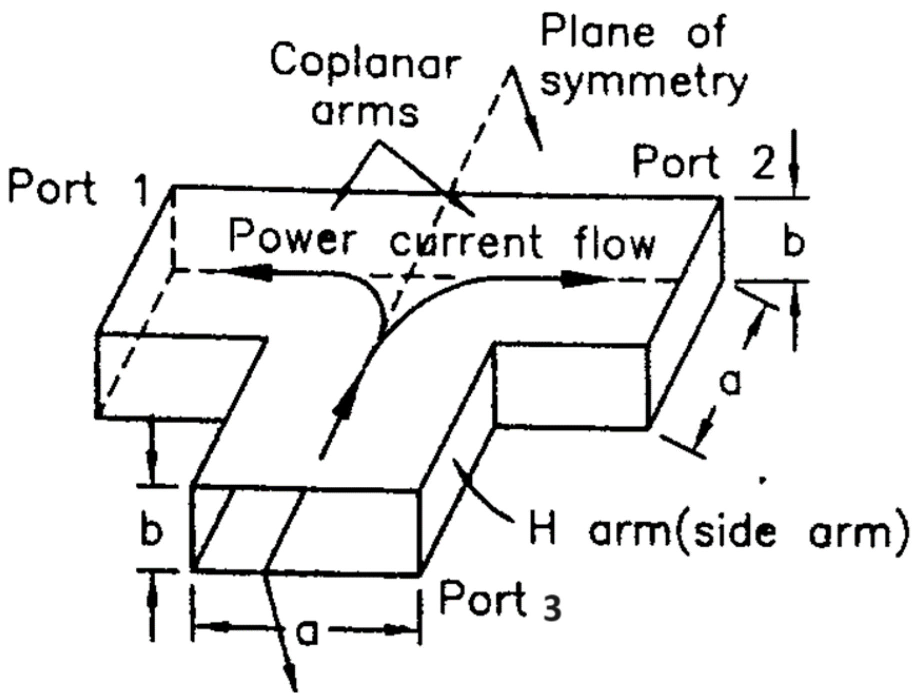
  Magic Tee (E - H Plane Tee)

  Hybrid Ring (Rat – Race Junction)

H Plane Tee

- It is formed by cutting a rectangular slot along the width of the main waveguide and attaching another waveguide
- If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in the same magnitude.

$$S_{13} = S_{23}$$



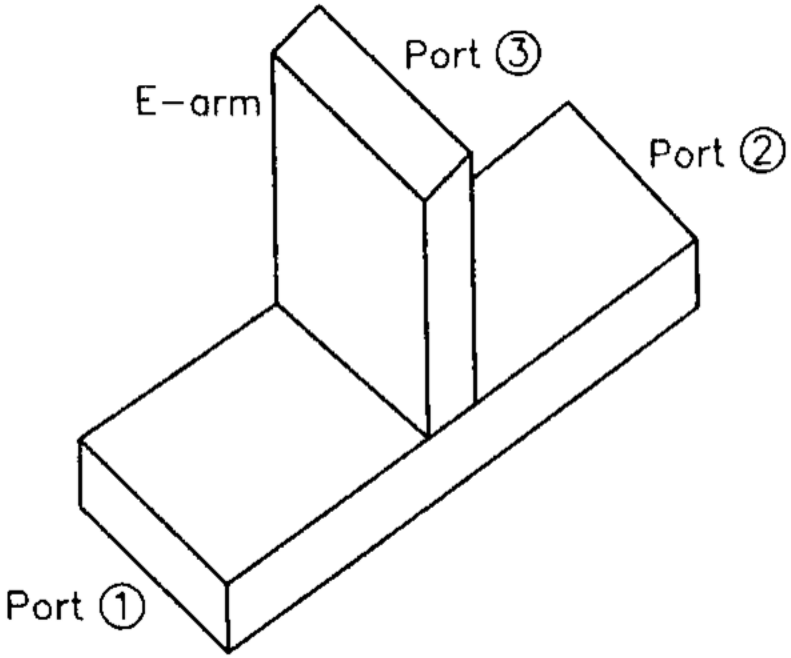
Ports (1) and (2) of main waveguide – collinear ports

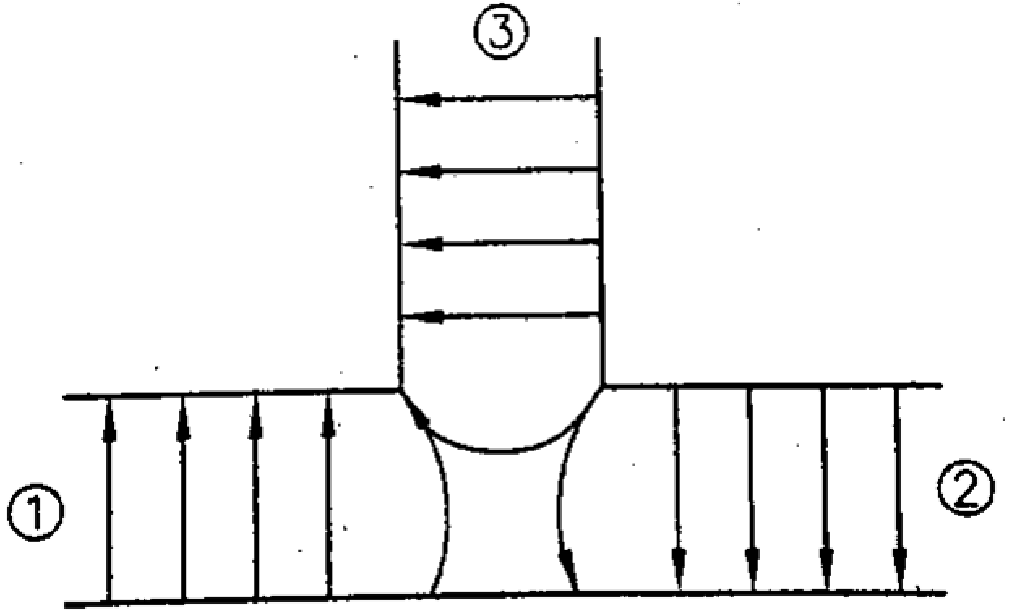
port (3) – H arm or Side arm

E Plane Tee

- It is formed by cutting a rectangular slot along the broader dimension of the main waveguide and a side arm is then attached.
- When the waves fed into the side arm, the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in same magnitude.

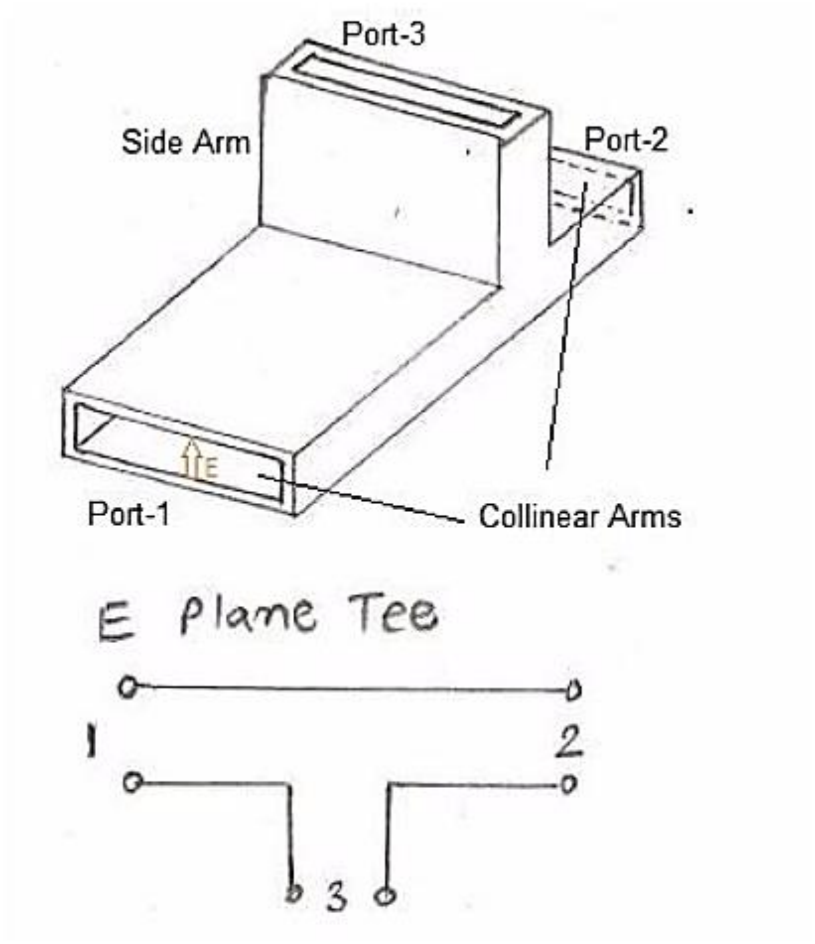
$$S_{23} = -S_{13}$$

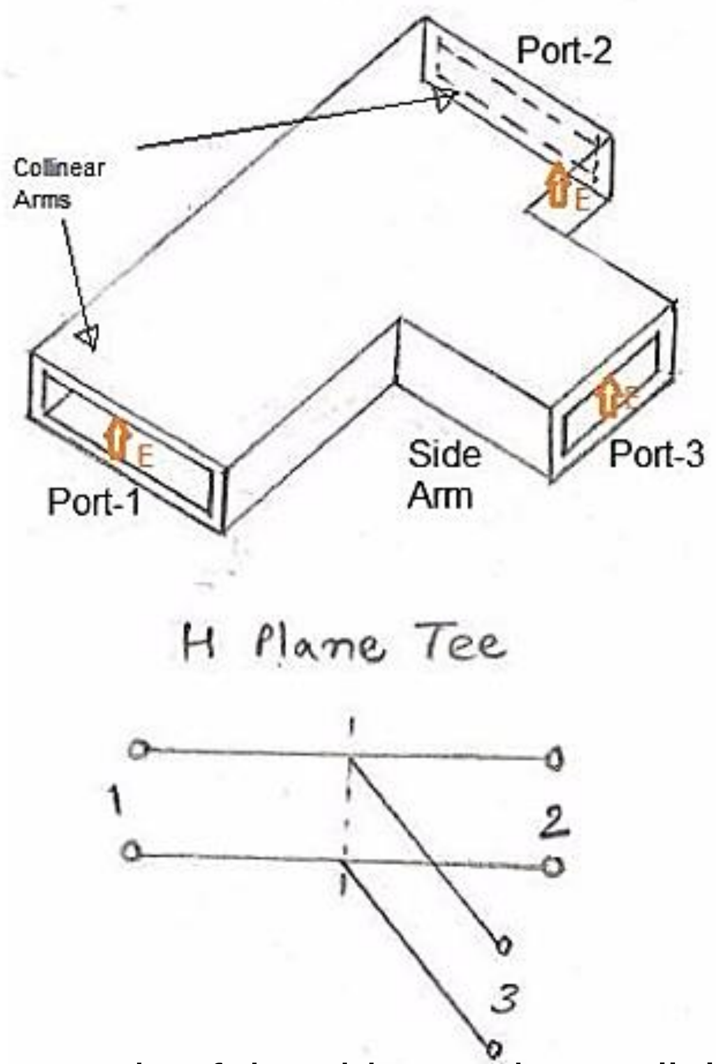




E-plane Tee(Series Tee)

H-plane Tee(Shunt Tee)



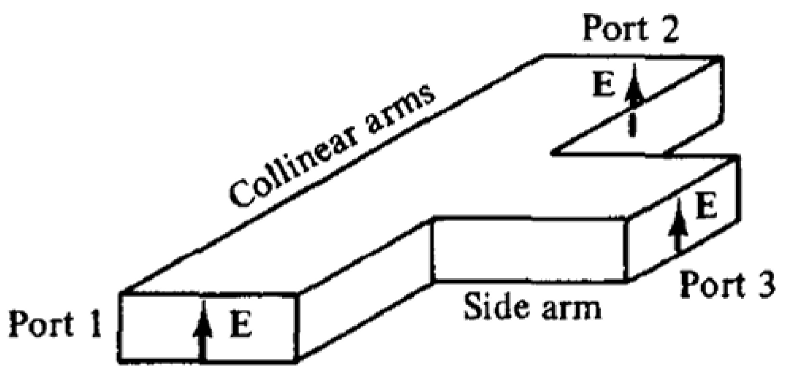


In E-plane tee, axis of the side arm is parallel to the E field

In H-plane tee, axis of the side arm is parallel to the H field

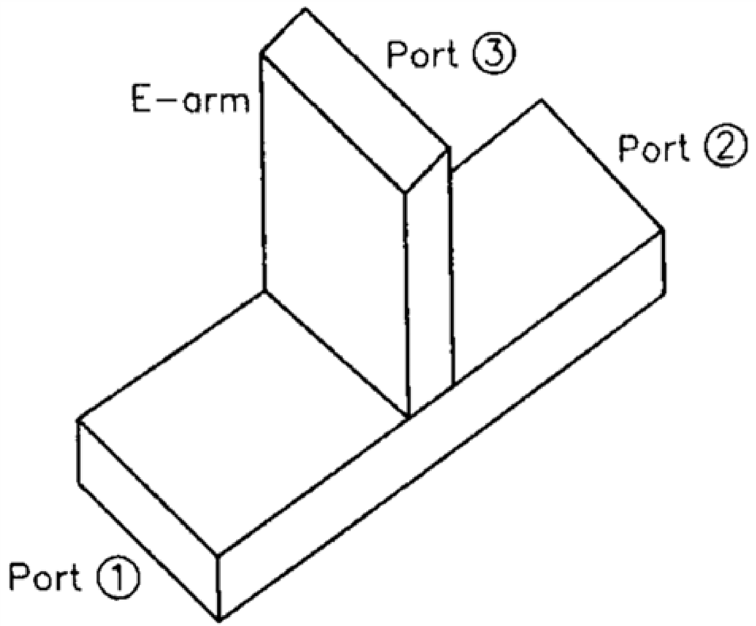
H plane tee

- $S_{13} = S_{23}$
- Input to port 3→ power divides equally between port 1 and port 2 (same magnitude and phase)
- Input to port 1 & port 2 → power addition in port 3 as output



E plane tee

- $S_{13} = -S_{23}$
- Input to port 3→ power divides equally between port 1 and port 2, but with opposite phase (same magnitude and 180 phase shift)
- Input to port 1 & port 2 → power difference in port 3 as output

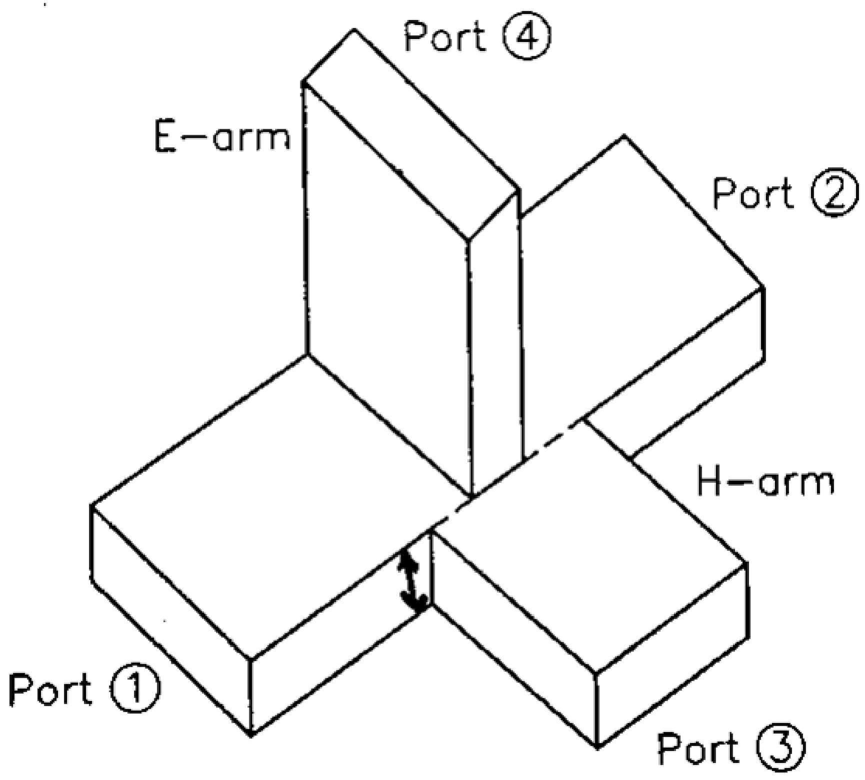


In E-plane tee, axis of the side arm is parallel to the E field

In H-plane tee, axis of the side arm is parallel to the H field

E-H Plane Tee (Hybrid or Magic Tee)

- It is formed by cutting rectangular slots along the width and breadth of a long waveguide and attaching side arms



Characteristics of E-H Plane Tee (Hybrid or Magic Tee)

- If two waves of equal magnitude and the same phase are fed into port 1 and port 2, the output will be zero at port 4 and additive at port 3.
- If a wave is **fed into port 3**, it will be divided equally between port 1 and port 2 of the collinear arms **and will not appear at port 4**. $S_{23} = S_{13}$
- If a **wave is fed into port 4**, it will produce an output of equal magnitude and opposite phase at port 1 and port 2. $S_{24} = -S_{14}$

  The output at port 3 is zero.

$$S_{43} = S_{34} = 0$$

- If a wave is fed into one of the collinear arm at port 1 or port 2, it will not appear in the other collinear arm at port 2 or port 1 because the E arm causes a phase delay while H arm causes a phase advance.

$$S_{12} = S_{21} = 0$$

The ports (1) and (2) of main waveguide – collinear arms and

port (3) – H arm port (4) – E arm

To define the S matrix,

- As there are 4 ports, the order of the scattering matrix is 4 X 4

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{.. (1)}$$

- Due to the H plane Tee Section,

$$S_{23} = S_{13} \quad \text{.. (2)}$$

- Due to the E plane Tee Section,

$$S_{24} = -S_{14} \quad \text{.. (3)}$$

- Because of geometry of the junction, ports 3 and 4 are isolated

$$S_{34} = S_{43} = 0 \qquad \text{.. (4)}$$

- From the property of symmetry,

$$\begin{aligned} S_{12} &= S_{21}, \quad & S_{13} &= S_{31} \quad and \quad & S_{23} &= S_{32} \\ S_{34} &= S_{43}, \quad & S_{24} &= S_{42} \quad and \quad & S_{41} &= S_{14} \end{aligned} \qquad \text{.. (5)}$$

- Port 3 & 4 are perfectly matched to the junction

$$S_{33} = S_{44} = 0 \qquad \text{.. (6)}$$

- Now S matrix becomes

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \qquad \text{.. (7)}$$

- From the unitary property

$$[S]\,[S]^* = [I]$$

$$i.e., \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ..\ (8)$$

- Multiplying, we get

$$R_1C_1: \quad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad ..\ (9)$$

$$R_2C_2: \quad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad ..\ (10)$$

$$R_3C_3: \quad |S_{13}|^2 + |S_{13}|^2 = 1 \quad ..\ (11)$$

$$R_4C_4: \quad |S_{14}|^2 + |S_{14}|^2 = 1 \quad ..\ (12)$$

- From eqs. (11) and (12)

$$\boxed{\begin{aligned} S_{13} &= \frac{1}{\sqrt{2}} \\ S_{14} &= \frac{1}{\sqrt{2}} \end{aligned}}$$

.. (13)

.. (14)

- Comparing eqs. (9) & (10)

$$|S_{11}|^2 = |S_{22}|^2$$

$$S_{11} = S_{22}$$

.. (15)

- Sub. S_{13} and S_{14} in eq. (9)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$\boxed{S_{11} = S_{12} = 0}$$

.. (16)

From eq. (15) → $\boxed{S_{22} = 0}$ .. **(17)**

Such a junction where all the 4 ports are perfectly matched to the junction is called a *Magic Tee*

- The [S] of Magic Tee is obtained by substituting the scattering parameters

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \quad \text{.. (18)}$$

Now we know that $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\left.\begin{array}{l} b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4); \quad b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2) \\ b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4); \quad b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2) \end{array}\right\} \quad \text{.. (19)- (22)}$$

Case 1: If input is given to port 3 and no input given to port 1, 2 and 4

i.e. $a_3 \neq 0 \;\; and \; a_1 = a_2 = a_4 = 0$

Sub. in eqs. (19)-(22),

$$b_1 = \frac{a_3}{\sqrt{2}} ; b_2 = \frac{a_3}{\sqrt{2}} ; b_3 = b_4 = 0$$

This is the ***property of H Plane Tee***

Case 2: If input is given to port 4 and no input given to port 1, 2 and 3

i.e. $a_4 \neq 0 \;\; and \; a_1 = a_2 = a_3 = 0$

Sub. in eqs. (19)-(22),

$$b_1 = \frac{a_4}{\sqrt{2}} ; b_2 = -\frac{a_4}{\sqrt{2}} ; b_3 = b_4 = 0$$

This is the ***property of E Plane Tee***

Case 3: If input is given to port 1 and no input given to port 2, 3 and 4

i.e. $\boldsymbol{a_1 \neq 0 \;\; and \; a_2 = a_3 = a_4 = 0}$

Sub. in eqs. (19)-(22),

$$b_1 = 0\,; b_2 = 0\,; b_3 = \frac{a_1}{\sqrt{2}}\,; b_4 = \frac{a_1}{\sqrt{2}}$$

i.e. When power is fed into port 1, nothing comes out of port 2 even though they are collinear ports (Magic!!).

Hence ports 1 and 2 are called ***isolated ports***

Case 4: If equal inputs are given to port 3 and 4 ($\boldsymbol{a_3 = a_4, a_1 = a_2 = 0}$)

Then, $b_1 = \frac{1}{\sqrt{2}}(2a_3); \quad b_2 = 0, b_3 = b_4 = 0$

Case 5: If equal inputs are given to port 1 and 2 ($\boldsymbol{a_1 = a_2, a_3 = a_4 = 0}$)

Then, $a_1 = a_2, a_3 = a_4 = 0;$

$$b_1 = 0 = b_2 = b_4; b_3 = \frac{1}{\sqrt{2}}(2a_1)$$

4-4-2 Magic Tees (Hybrid Tees)

A magic tee is a combination of the E-plane tee and H-plane tee (refer to Fig. 4-4-7). The magic tee has several characteristics:

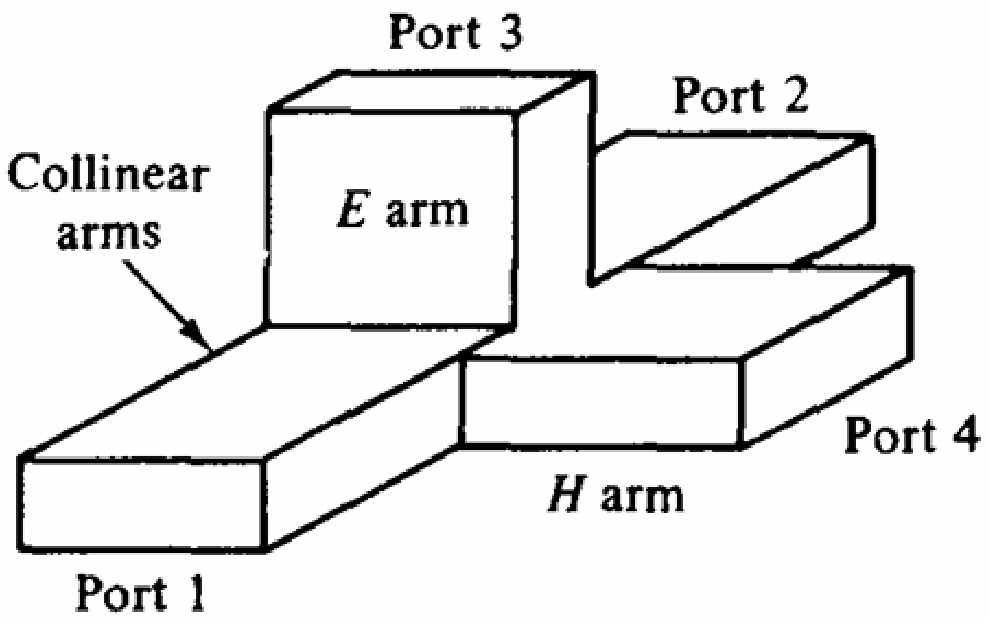


Figure 4-4-7 Magic tee.

1. If two waves of equal magnitude and the same phase are fed into port 1 and port 2, the output will be zero at port 3 and additive at port 4.
2. If a wave is fed into port 4 (the *H* arm), it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3 (the *E* arm).
3. If a wave is fed into port 3 (the *E* arm), it will produce an output of equal magnitude and opposite phase at port 1 and port 2. The output at port 4 is zero. That is, $S_{43} = S_{34} = 0$.
4. If a wave is fed into one of the collinear arms at port 1 or port 2, it will not appear in the other collinear arm at port 2 or port 1 because the *E* arm causes a phase delay while the *H* arm causes a phase advance. That is, $S_{12} = S_{21} = 0$.

Therefore the **S** matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

The magic tee is commonly used for mixing, duplexing, and impedance measurements. Suppose, for example, there are two identical radar transmitters in equipment stock. A particular application requires twice more input power to an antenna than either transmitter can deliver. A magic tee may be used to couple the two transmitters to the antenna in such a way that the transmitters do not load each other. The two transmitters should be connected to ports 3 and 4, respectively, as shown in Fig. 4-4-8. Transmitter 1, connected to port 3, causes a wave to emanate from port 1 and another to emanate from port 2; these waves are equal in magnitude but opposite in phase. Similarly, transmitter 2, connected to port 4, gives rise to a wave at port 1 and another at port 2, both equal in magnitude and in phase. At port 1 the two opposite waves cancel each other. At port 2 the two in-phase waves add together; so double output power at port 2 is obtained for the antenna as shown in Fig. 4-4-8.

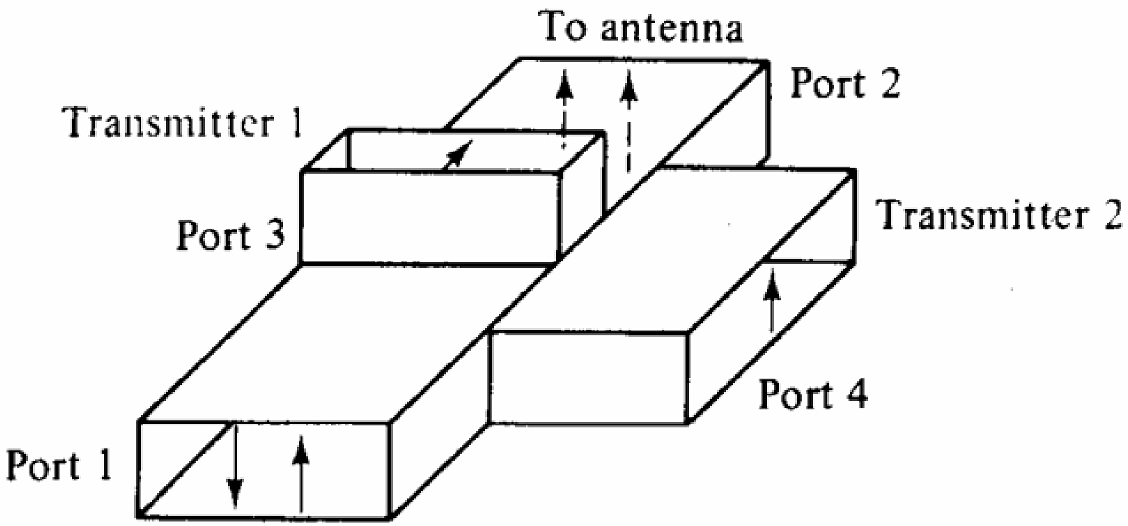
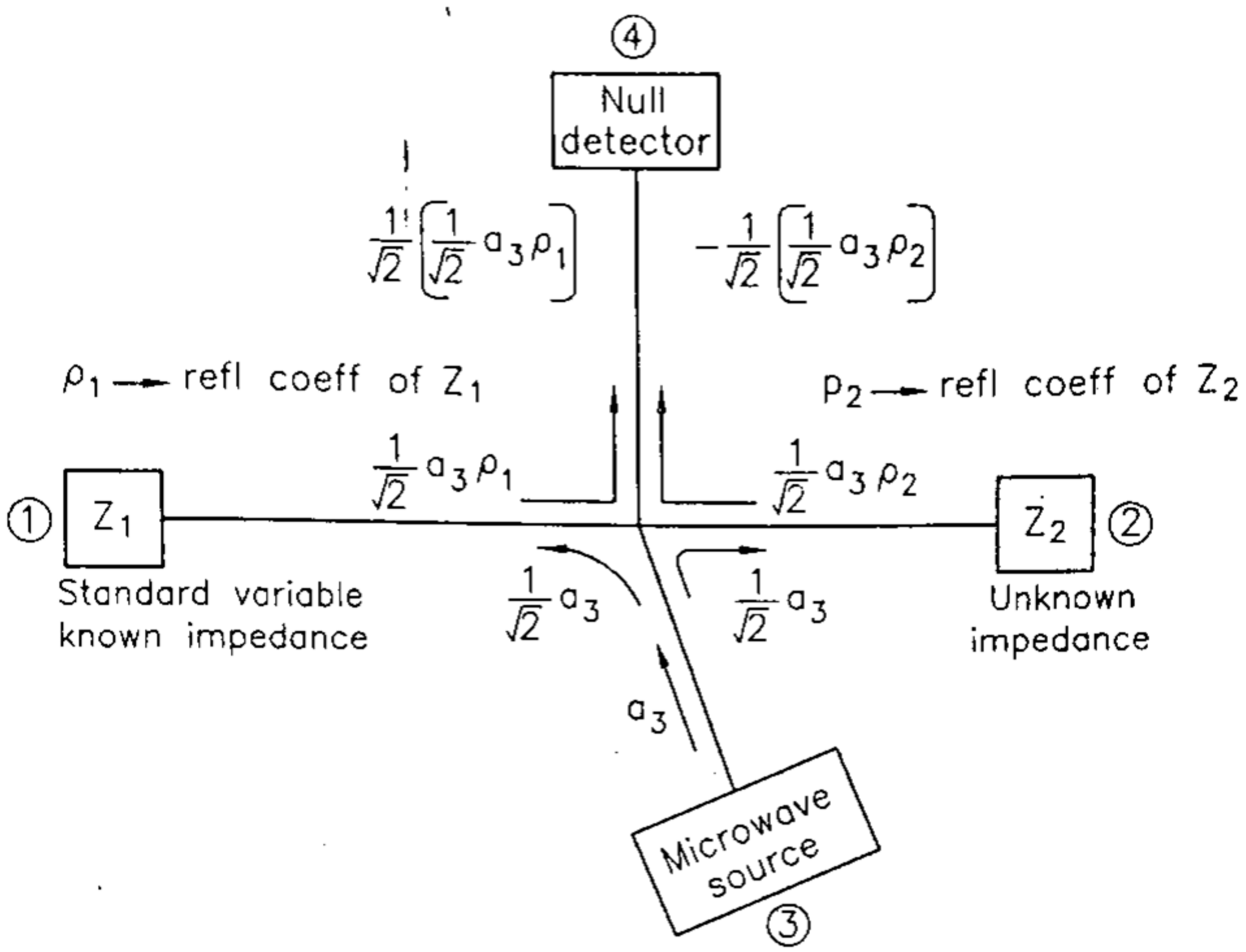


Figure 4-4-8 Magic tee-coupled transmitters to antenna.

Application of Magic Tee

- Measurement of Impedance

- Thus the unknown impedance can be measured by adjusting the std. variable impedance till the bridge is balanced and both the impedances become equal



- The power from microwave source at port 3 (a_3) gets equally divided between the arms 1 and 2.
- The impedances at ports 1 and 2 are not equal to characteristic impedance. Hence there will be reflections from arms 1 and 2
- The powers entering the Magic Tee junction from arms 1 and 2 are $\frac{\rho_1 a_3}{\sqrt{2}}$ and $\frac{\rho_2 a_3}{\sqrt{2}}$
- The resultant wave into the arm 4 (i.e. the null detector)

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 \rho_1\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 \rho_2\right) = \frac{1}{2} a_3 (\rho_1 - \rho_2)$$

- For perfectly balancing the bridge, net wave reaching the null detector equated to zero

$$\frac{1}{2} a_3 (\rho_1 - \rho_2) = 0$$

$$\rho_1 - \rho_2 = 0 \quad \text{or} \quad \rho_1 = \rho_2$$

$$\frac{Z_1 - Z_z}{Z_1 + Z_z} = \frac{Z_2 - Z_z}{Z_2 + Z_z}$$

$$Z_1 = Z_2$$

$$\rho = \frac{V_{reflected}}{V_{incident}} = \frac{Z_t - Z_o}{Z_t + Z_o}$$

where,

ρ =reflection coefficient

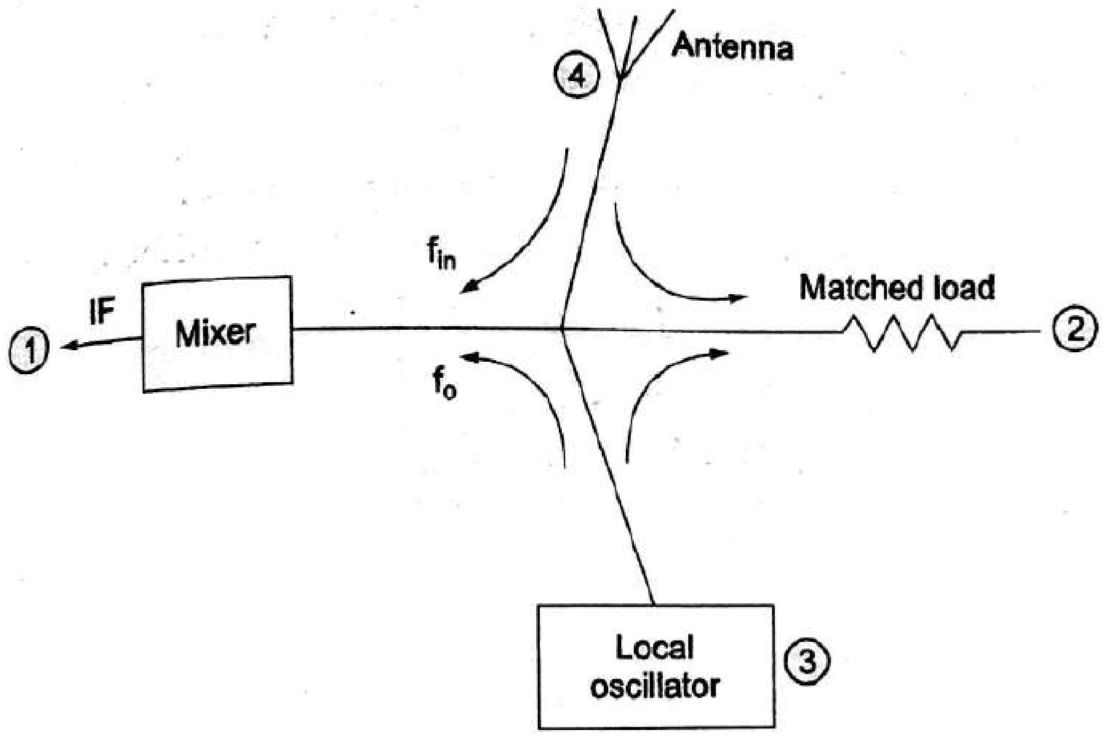
Zo = impedance of the transmission line on which signal travels

Zt = impedance of transmission line or the component immediately after the junction

- Zz is characteristic impedance.
- Thus the unknown impedance can be measured by adjusting the std. variable impedance till the bridge is balanced and both the impedances become equal

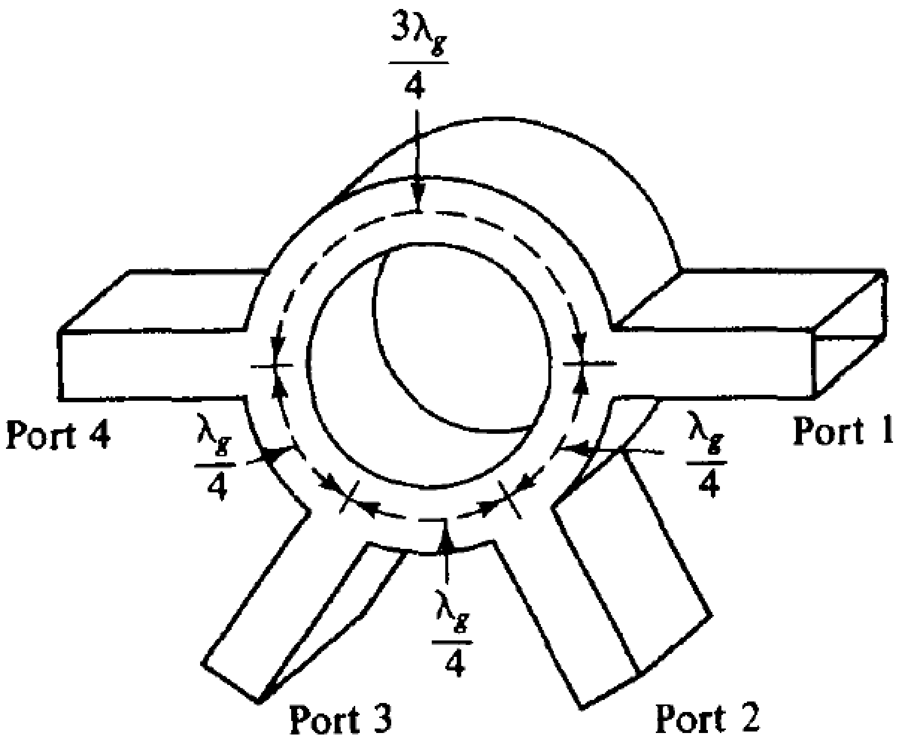
Magic Tee as mixer

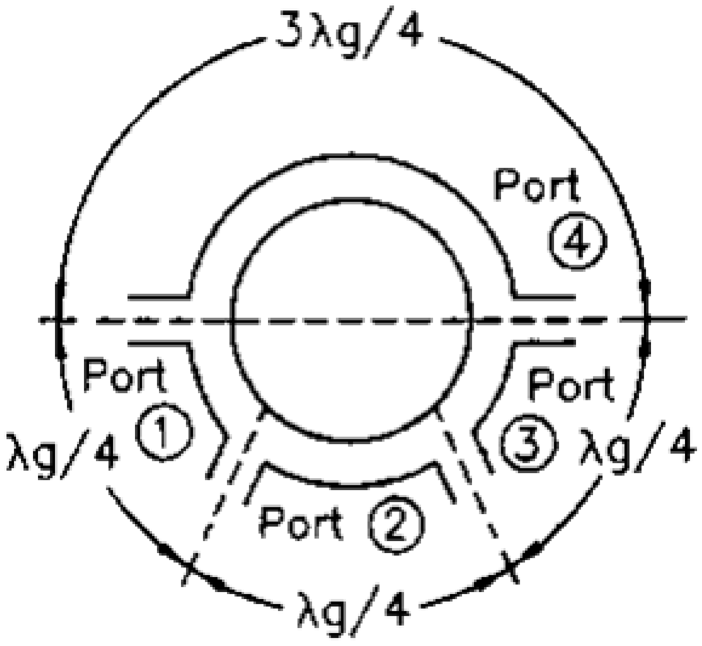
- Magic tee can be used as a mixer where the signal and local oscillator signals are fed into E and H arms.
- They are mixed together to get the intermediate frequency.



Rat Race Junction (Hybrid Ring)

- A four port junction
- The 4 ports/arms are connected in the form of an angular ring at proper intervals.





- For proper operation, the mean circumference of total race must be $1.5\lambda_g$ and each of the ports is separated from its neighbor by a distance $\frac{\lambda_g}{4}$
- When power is fed to port 1, it splits equally (in clockwise and anticlockwise directions) into ports 2 and 4 but nothing enters at port 3.
- At ports 2 and 4, the powers combine in phase but at port 3, cancellation occurs due to $\lambda_g/2$ path difference
- If 2 unequal signals are applied at port 1, an output proportional to their sum will emerge from port 2 and 4 while a differential output will appear at port 3
- Scattering matrix of rat-race junction

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

4-4-3 Hybrid Rings (Rat-Race Circuits)

A hybrid ring consists of an annular line of proper electrical length to sustain standing waves, to which four arms are connected at proper intervals by means of series or parallel junctions. Figure 4-4-9 shows a hybrid ring with series junctions.

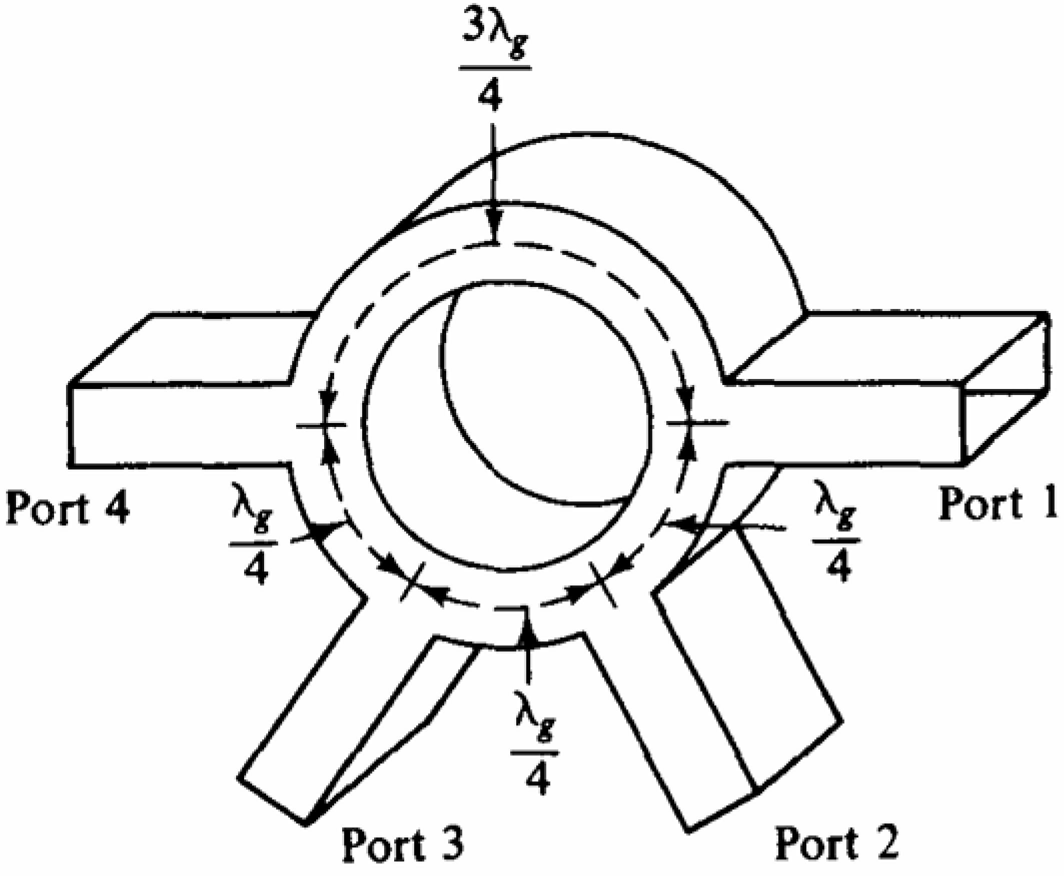


Figure 4-4-9 Hybrid ring.

The hybrid ring has characteristics similar to those of the hybrid tee. When a wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves traveling in the clockwise and counterclockwise directions is 180°. Thus the waves are canceled at port 3. For the same reason, the waves fed into port 2 will not emerge at port 4 and so on.

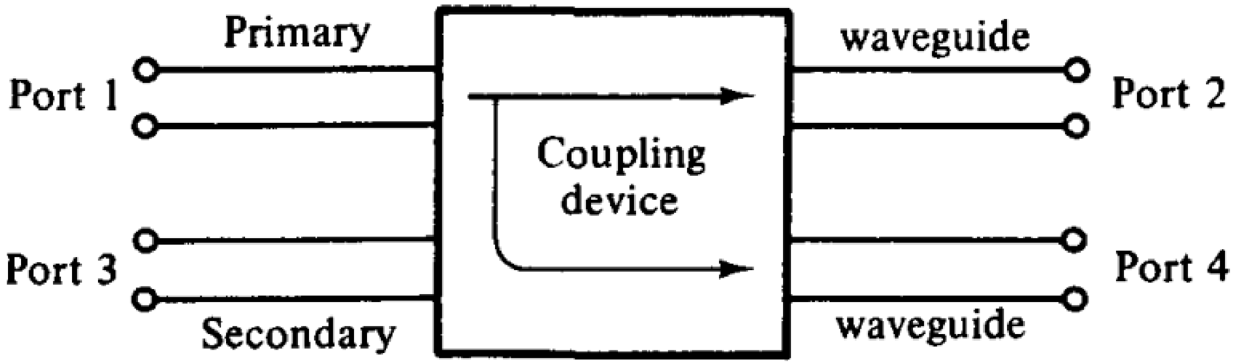
The **S** matrix for an ideal hybrid ring can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \tag{4-4-24}$$

It should be noted that the phase cancellation ocurs only at a designated frequency for an ideal hybrid ring. In actual hybrid rings there are small leakage couplings, and therefore the zero elements in the matrix of Eq. (4-4-24) are not quite equal to zero.

Directional Coupler

- A directional coupler is a four-port waveguide junction
- It consists of a primary waveguide 1-2 and a secondary waveguide 3-4.
- When all ports are terminated in their characteristic impedances, there is free transmission of power, without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports.
- The degree of coupling between port 1 and port 4 and between port 2 and port 3 depends on the structure of the coupler.
- The characteristics of a directional coupler can be expressed in terms of its coupling factor and its directivity.



- The coupling factor is a measure of the ratio of power levels in the primary and secondary lines.
- Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1.
- This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line.
- The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide.
- An ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and port 4 are perfectly matched. Actually, well-designed directional couplers have a directivity of only 30 to 35 dB.

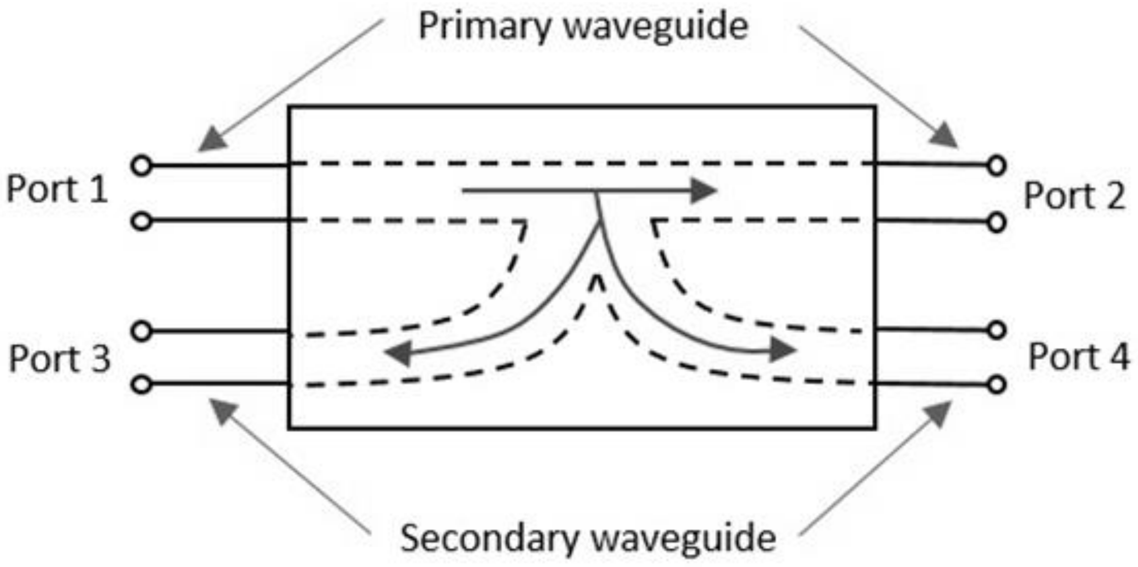
$$\text{Coupling factor} = 10\log_{10}\frac{P_1}{P_4}$$

$$\text{Directivity} = 10\log_{10}\frac{P_4}{P_3}$$

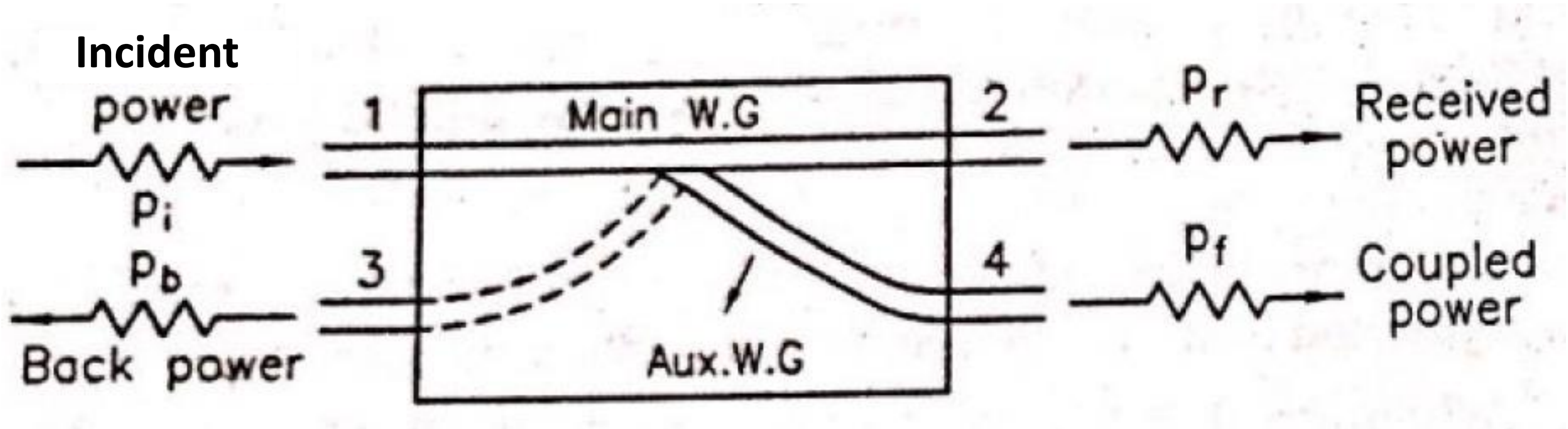
where P_1 = power input to port 1
P_3 = power output from port 3
P_4 = power output from port 4

Directional Coupler

- Directional couplers are flanged, built in waveguide assemblies which can sample a small amount of microwave power for measurement purposes.
- They can be unidirectional or bi-directional
- In its most common form, the directional coupler is a four port waveguide junction consisting of a primary main waveguide and a secondary auxiliary waveguide.



Directional Coupler Indicating Powers



P_i = Incident power at port 1

P_r = Forward Received power at port 2

P_f = Forward Coupled power at port 4

P_b = Back / Reflected power at port 3

$$C = 10\log_{10}\frac{P_i}{P_f} \quad dB$$

$$D = 10\log_{10}\frac{P_f}{P_b} \quad dB$$

$$I = 10\log_{10}\frac{P_i}{P_b} \quad dB$$

- Coupling factor is a measure of how much of the incident power is being sampled
- Directivity is a measure of how well the directional coupler distinguishes between the forward and reverse traveling powers.

Parameters of Directional Coupler

The performance of a directional coupler is usually defined in terms of:

- **Coupling Factor, C** : defined as the ratio of the incident power to the forward coupled power measured in dB

$$C = 10\log_{10}\frac{P_i}{P_f} \ \text{dB} \quad \text{or} \quad C = 10\log_{10}\frac{P_1}{P_4}$$

- **Directivity, D** : defined as the ratio of forward coupled power to the back power

$$D = 10\log_{10}\frac{P_f}{P_b} \ \text{dB} \quad \text{or} \quad D = 10\log_{10}\frac{P_4}{P_3}$$

- **Isolation, I** : defined as the ratio of incident power to the back power

$$I = 10\log_{10}\frac{P_i}{P_b} \ \text{dB} \quad \text{or} \quad I = 10\log_{10}\frac{P_1}{P_3}$$

Isolation, I (dB) = Coupling Factor, C (dB) + Directivity, D (dB)

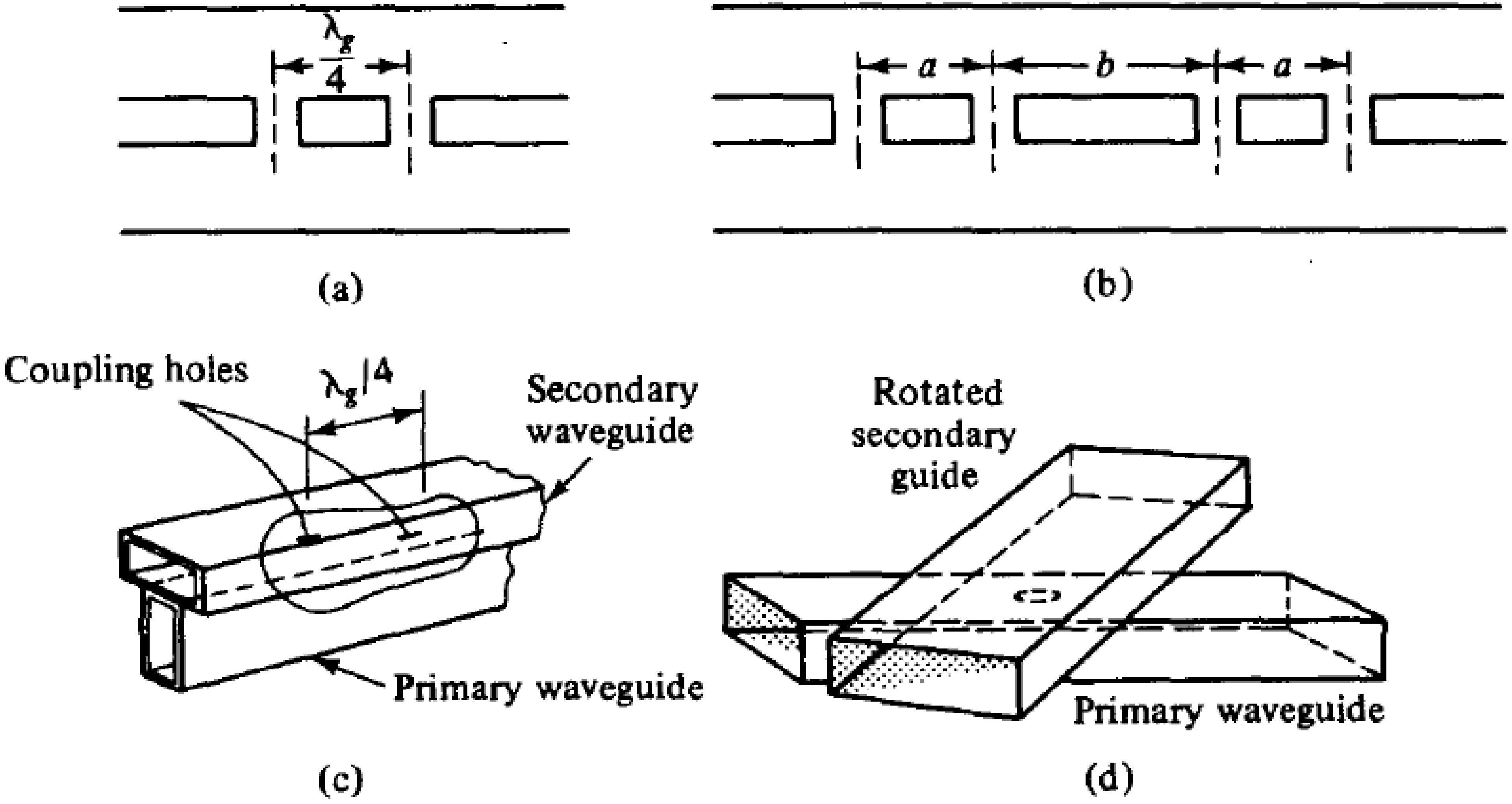


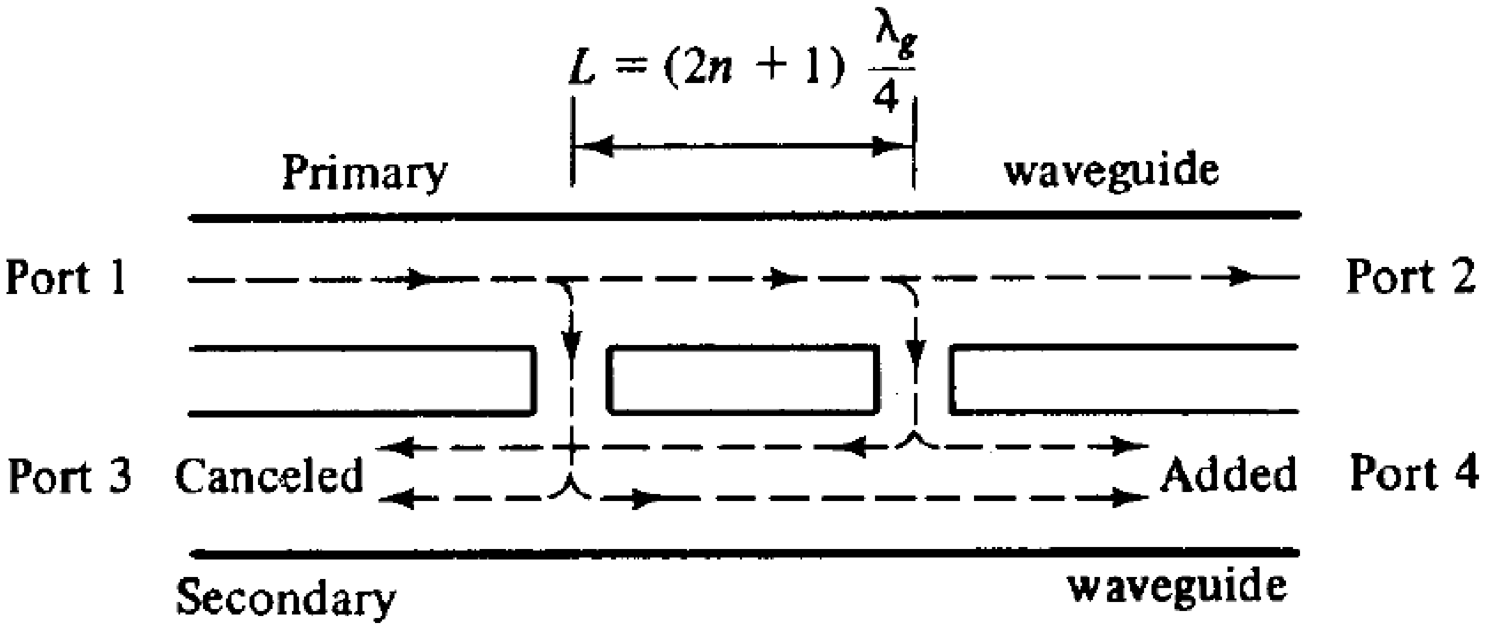
Figure 4-5-2 Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

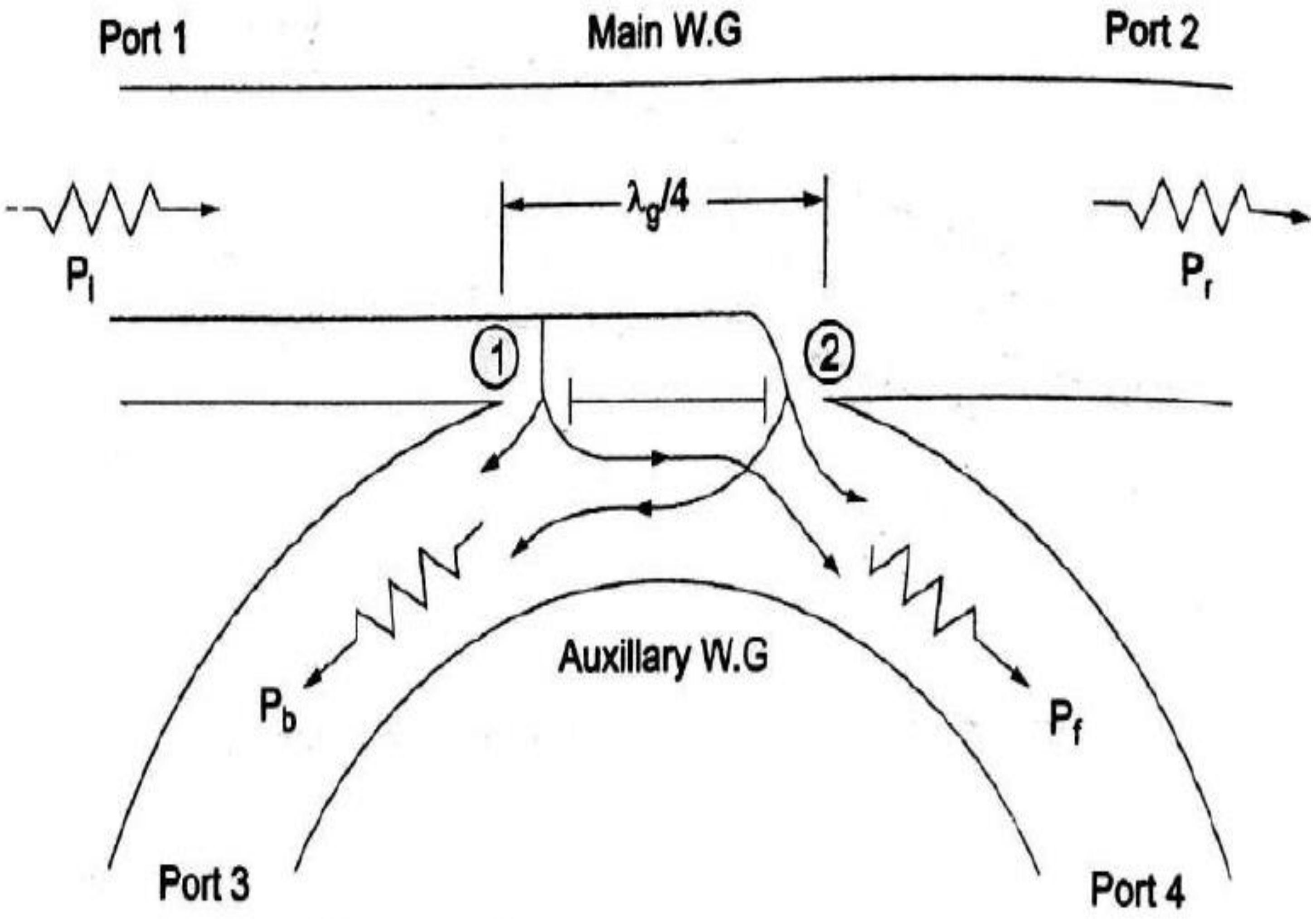
Two – Hole Directional Coupler

- It consist of 2 guides, the main and auxiliary waveguide with two tiny holes common between them.

- The center of the two holes are separated by a distance of $\mathrm{L} = (2n+1)\frac{\lambda_g}{4}$

  where λ_g is the guide wavelength.





- Leakages out of holes 1 and 2 are both in phase at the position of hole 2 – Adds up to produce the P_f
- But the two leakages are out of phase by 180 at the position of hole 1 and they cancel each other (ideally $P_b = 0$)
- Magnitude of power coming out of the two holes depends upon the dimensions of the two holes
- Since the distance between two holes is $\frac{\lambda_g}{4}$, the incident power will have to travel $\frac{\lambda_g}{4} + \frac{\lambda_g}{4} = \frac{\lambda_g}{2}$, when it comes back from hole 2 resulting in 180 phase shift

- A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as slot antennas.
- The forward waves in the secondary guide are in the same phase, (regardless of hole space) and are added at port 4.
- The backward waves in the secondary guide are out of phase by $\frac{2\pi}{\lambda_g}(2L)$ and are cancelled at port 3

S Matrix of Directional Coupler

As the directional coupler is a 4 port network, the order of the scattering matrix is 4 X 4

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{.. (1)}$$

- All the 4 ports are perfectly matched to the junction. Hence

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \quad \text{.. (2)}$$

- From the symmetric property,

$$S_{12} = S_{21}\,, \qquad S_{13} = S_{31} \quad and \qquad S_{23} = S_{32}$$
$$S_{34} = S_{43}\,, \qquad S_{24} = S_{42} \quad and \qquad S_{41} = S_{14} \quad \text{.. (3)}$$

- Ideally back power is zero, i.e. there is no coupling between ports 1 and 3

$$S_{13} = S_{31} = 0 \qquad \text{.. (4)}$$

- Also, there is no coupling between ports 2 and 4

$$S_{24} = S_{42} = 0 \qquad \text{.. (5)}$$

- Now S matrix becomes

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}. \qquad \text{.. (6)}$$

- Using the unitary property of S matrix

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1: \quad |S_{12}|^2 + |S_{14}|^2 = 1$ .. (7)

$R_2C_2: \quad |S_{12}|^2 + |S_{23}|^2 = 1$ .. (8)

$R_3C_3: \quad |S_{23}|^2 + |S_{34}|^2 = 1$ .. (9)

$R_1C_3: \quad S_{12} S_{23}^* + S_{14} S_{34}^* = 0$ .. (10)

- Comparing eqs. (7) and (8)

$$S_{14} = S_{23}$$ .. (11)

- Comparing eqs. (8) and (9)

$$S_{12} = S_{34}$$ .. (12)

- Assume that S_{12} is real and positive $=$ 'P'

$$S_{12} = S_{34} = P = S_{34}^*$$ .. (13)

- Eq. (10) → $\quad PS_{23}^* + S_{23}P = 0$

$$P(S_{23}^* + S_{23}) = 0$$

- Since $P \neq 0$, $(S_{23}^* + S_{23}) = 0$
- This condition is possible only if $S_{23} = jq$ and $S_{23}^* = -jq$
- Therefore,

$$S_{23} = S_{14} = jq \quad \text{.. (14)}$$

$$S_{12} = S_{34} = P$$

- From eq. (7), relation between P and q can be obtained as

Eq. (7) → $|S_{12}|^2 + |S_{14}|^2 = 1 \Rightarrow P^2 + q^2 = 1$

Sub. these S parameter values, [S] matrix of directional coupler is reduced to

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix} \quad \text{.. (15)}$$

Question

- Determine the coupling, directivity and isolation (in dBs) of a lossless directional coupler carrying the following: Incident power: 40mW, power at the coupling port: 10mW, and power at the decoupled port: 0.1mW.

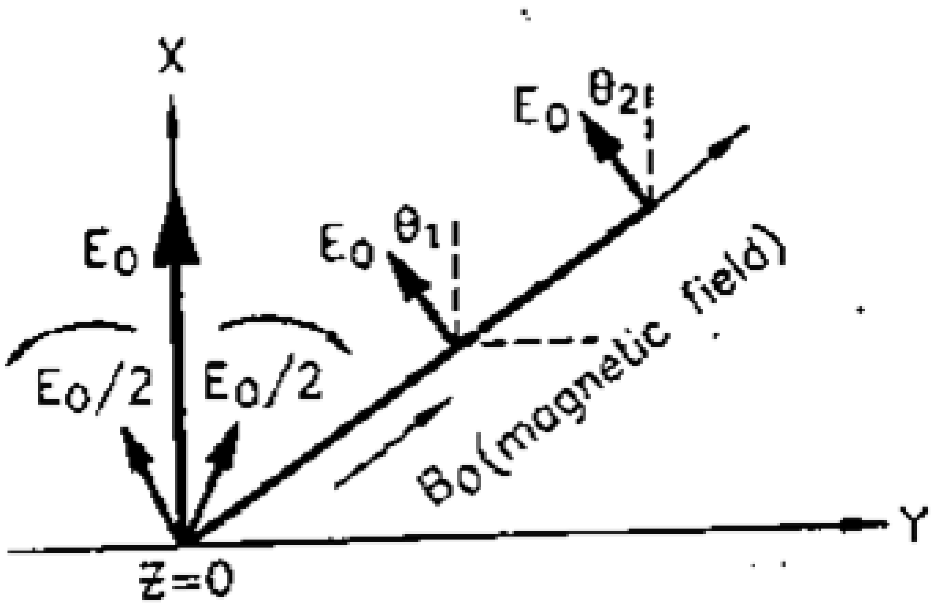
$$C = 10\log_{10}\frac{\mathrm{P_i}}{\mathrm{P_f}}\ \ \mathrm{dB} = 10\log_{10}\frac{40mW}{10mW}\ \ \mathrm{dB} = 6.02\mathrm{dB}$$

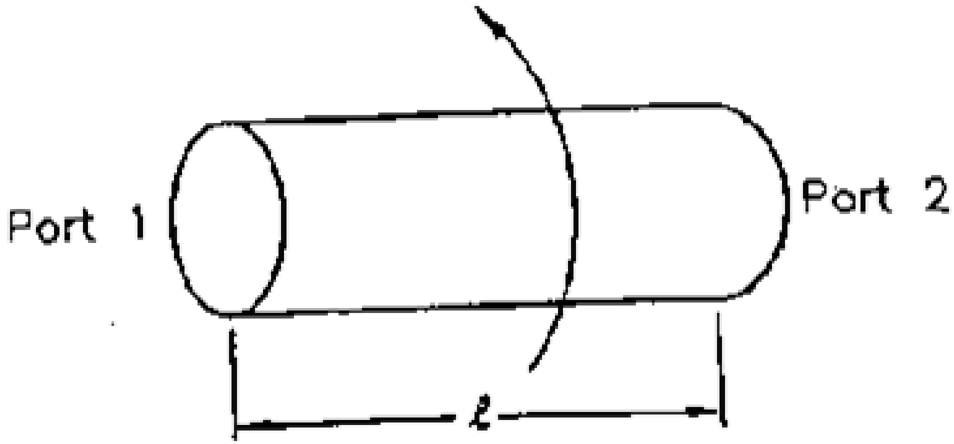
$$D = 10\log_{10}\frac{\mathrm{P_f}}{\mathrm{P_b}}\ \ \mathrm{dB} = 10\log_{10}\frac{10mW}{0.1mW}\ \ \mathrm{dB} = 20\mathrm{dB}$$

$$I = 10\log_{10}\frac{\mathrm{P_i}}{\mathrm{P_b}}\ \ \mathrm{dB} = 10\log_{10}\frac{40mW}{0.1mW}\ \ \mathrm{dB} = 26.02\mathrm{dB}$$

Faraday Rotation in Ferrite Devices

- Ferrites are non metallic materials with high resistivities nearly 10^{14} times greater than metals with ε_r around 10-15 and μ_r of the order of 1000.
- Its general composition of the form $MeO.Fe_2O_3$ i.e. a mixture of metallic oxide and ferric oxide.
- At microwave frequencies, ferrites exhibit non reciprocal property.
- When a piece of ferrite is affected by a dc magnetic field, the ferrite exhibits Faraday Rotation.
- If a plane TEM (linearly polarized along the x-axis) is made to propagate through the ferrite material, its plane of polarization will rotate with distance

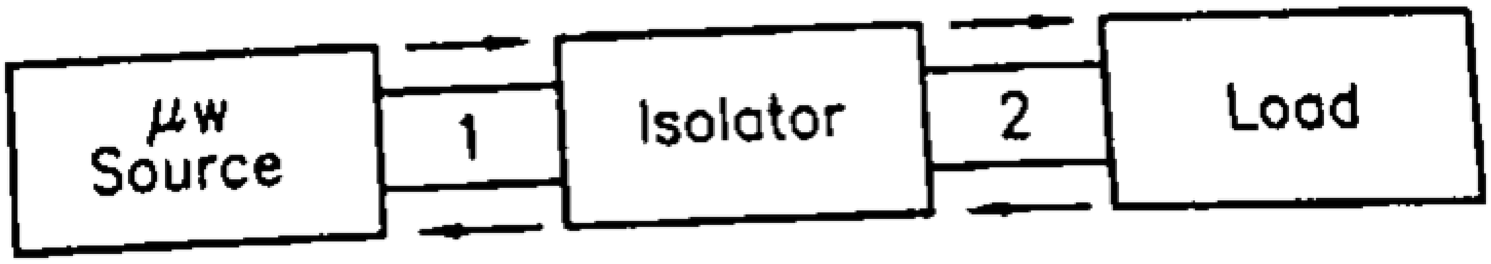




The direction of rotation of linearly polarized wave is independent of the direction of propagation of the wave.

Isolator

- A two port device which provides a very small attenuation for transmission from port 1 to port 2 but maximum attenuation for transmission from port 2 to port 1.
- It is used to match a source with a variable load. Due to mismatch, the reflections from the load are completely absorbed by the isolator.
- This ensures that there is no change in frequency and output power due to variation in load.



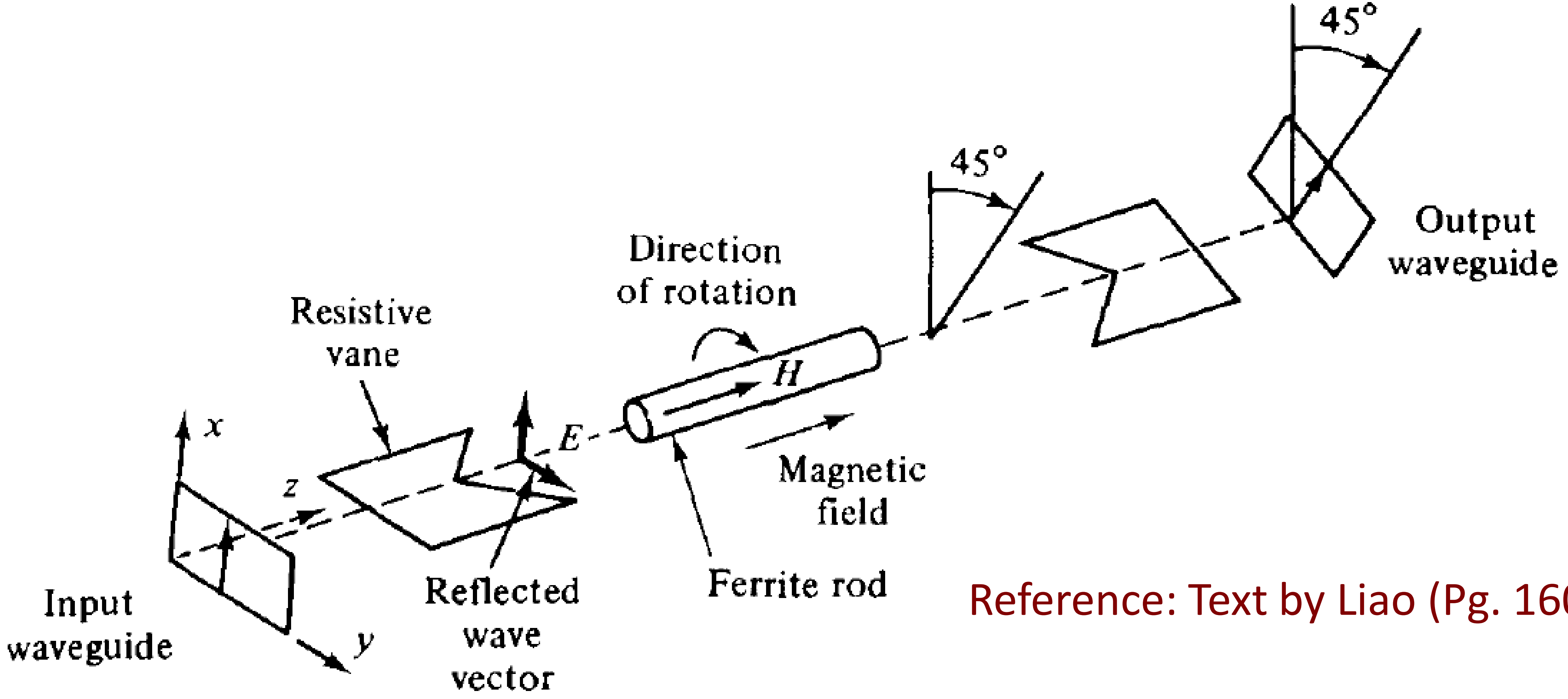
4-6-2 Microwave Isolators

An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus the isolator is usually called *uniline*. Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator.

- Isolators can be constructed in many ways. They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads. On the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide.

- The input resistive card is in the y-z plane, and the output resistive card is displaced 45° with respect to the input card. The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by 45°. The degrees of rotation depend on the length and diameter of the rod and on the applied dc magnetic field.
- An input TE_{10} dominant mode is incident to the left end of the isolator. Since the TE_{10} mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation. The wave in the ferrite rod section is rotated clockwise by 45° and is normal to the output resistive card. As a result of rotation, the wave arrives at the output end without attenuation at all.
- On the contrary, a reflected wave from the output end is similarly rotated clockwise 45° by the ferrite rod. However, since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card.
- The typical performance of these isolators is about 1-dB insertion loss in forward transmission and about 20- to 30-dB isolation in reverse attenuation.

Faraday Rotation Isolator



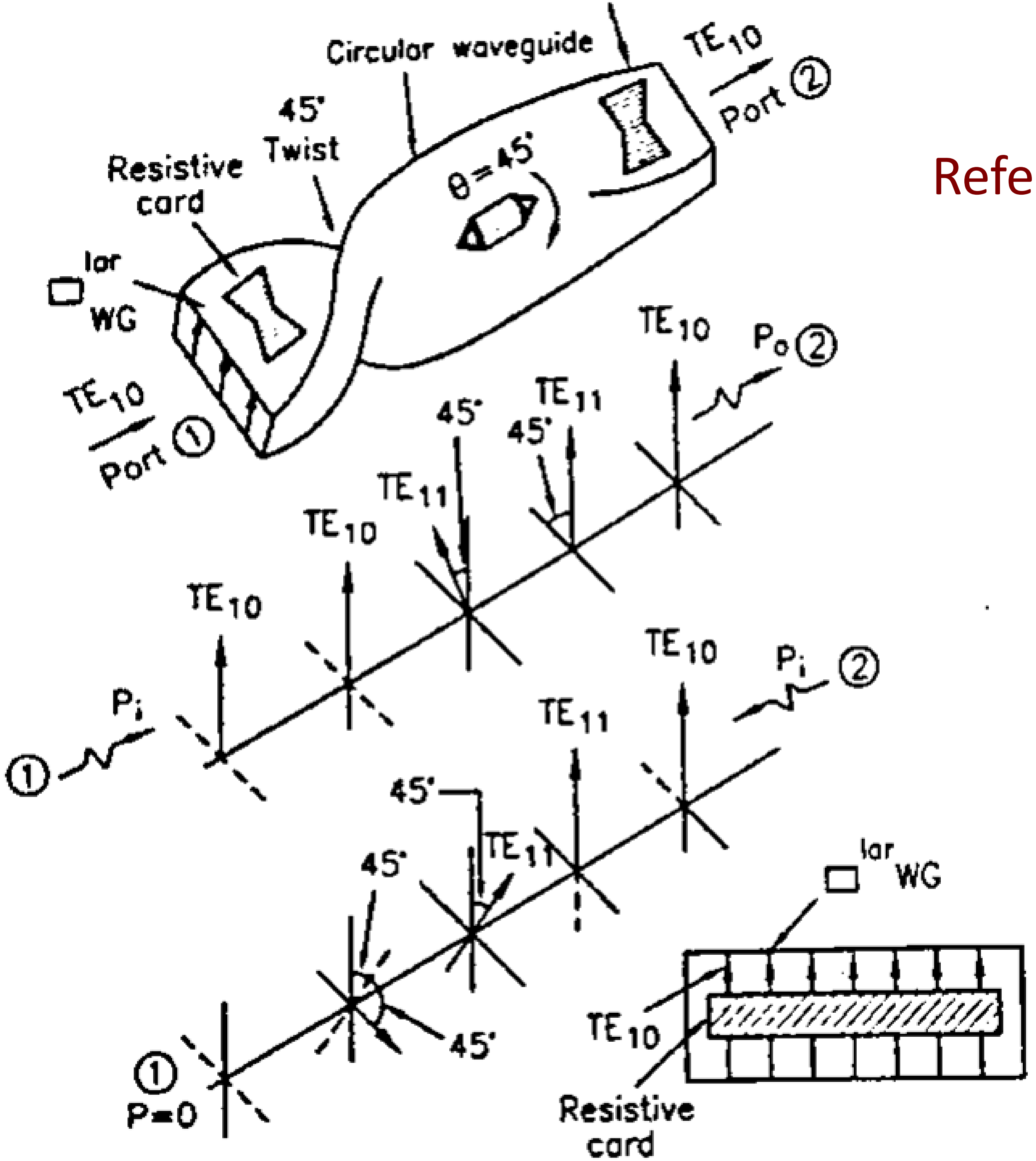
Reference: Text by Liao (Pg. 160)

S-matrix of 2-port isolator

$$s = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$Output\ at\ port\ 1, b_1 = 0$$
$$Output\ at\ port\ 2, b_2 = a_1$$

- When an input is given to port 1, an output is obtained at port 2.
- But if an input is given to port 2, no output should be obtained at port 1.



Reference: Text by Kulkarni (Pg. 222)

(b) Isolator

An isolator is a 2 port device which provides very *small* amount of attenuation for transmission from port ① to port ② but provides *maximum* attenuation for transmission from port ② to port ①. This requirement is very much desirable when we want to match a source with a variable load.

In most microwave generators, the output amplitude and frequency tend to fluctuate very significantly with changes in load impedance. This is due to mismatch of generator output to the load resulting in reflected wave from load. But these reflected waves should not be allowed to reach the microwave generator, which will cause amplitude and frequency instability of the microwave generator.

When isolator is inserted between generator and load, the generator is coupled to the load with zero attenuation and reflections if any from the load side are completely absorbed by the isolator without affecting the generator output. Hence the generator appears to be matched for all loads in the presence of isolator so that there is no change in frequency and output power due to variation in load. This is shown in Fig. 6.34.

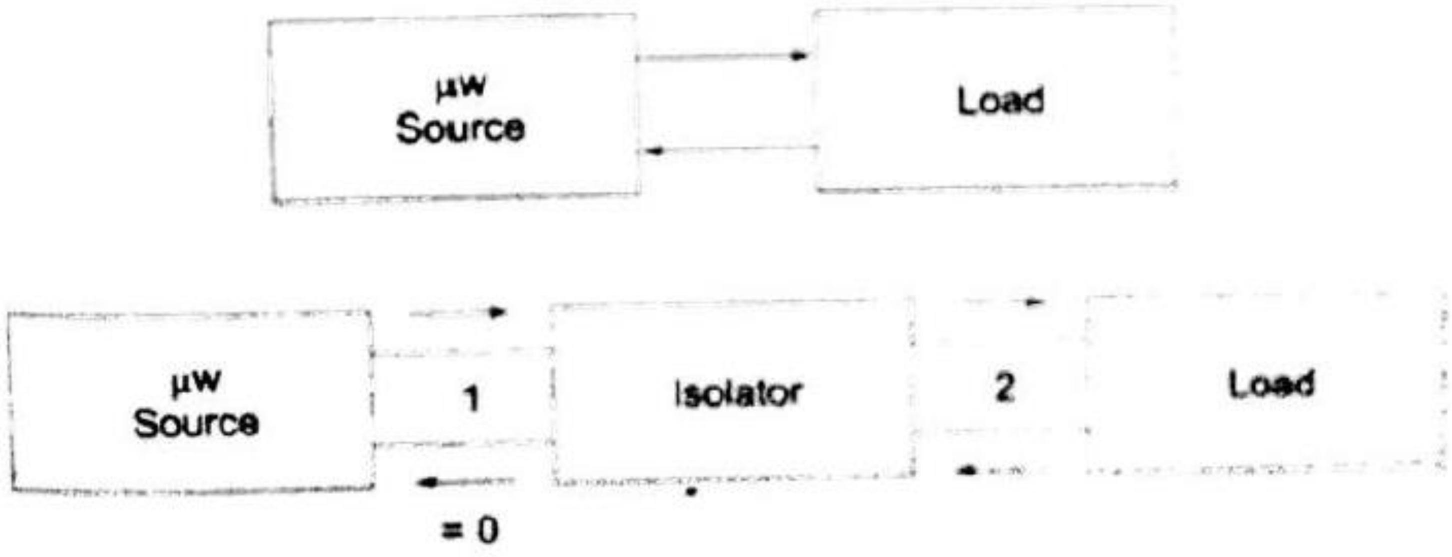


Fig. 6.34

of

Construction : The construction of isolator (Fig. 6.35) is similar to gyrator except that an isolator makes use of 45° twisted rectangular waveguide (instead of 90° twist) and 45° faraday rotation ferrite rod (instead of 90° in gyrator), a resistive card is placed along the larger dimension of the rectangular waveguide, so as to absorb an wave whose plane of polarisation is parallel to the plane of resistive card. The resistive card does not absorb any wave whose plane of polarization is perpendicular to its own plane.

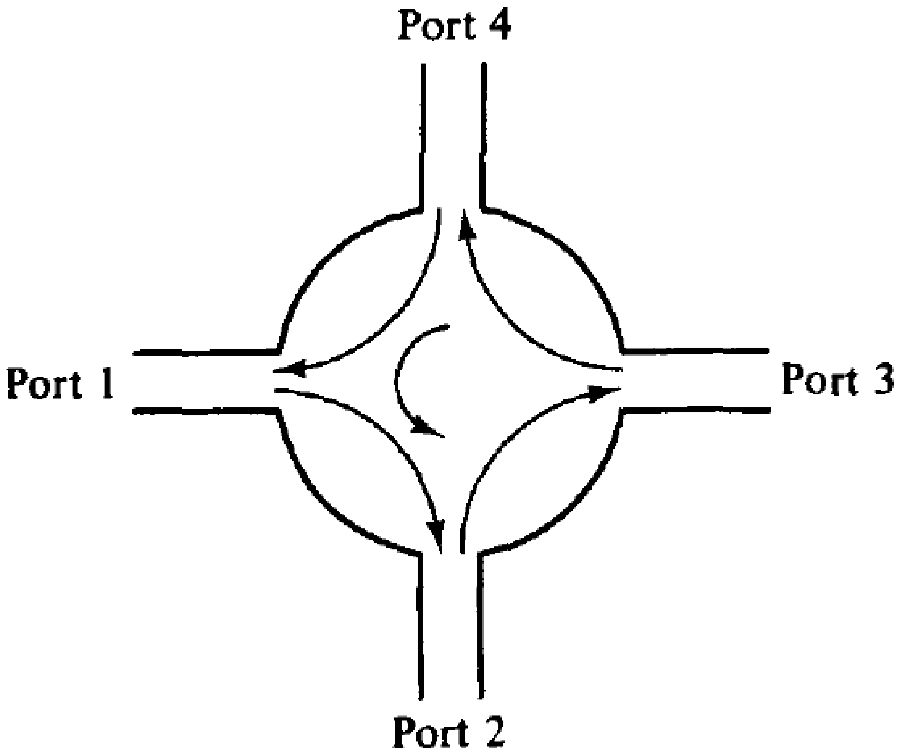
Operation : A TE_{10} wave passing from port ① through the resistive card and is not attenuated. After coming out of the card, the wave gets shifted by 45° because of the twist in anticlockwise direction and then by another 45° in clockwise direction because of the ferrite rod and hence comes out of port ② with the same polarization as at port ① without any attenuation.

But a TE_{10} wave fed from port ② gets a pass from the resistive card placed near port ② since the plane of polarization of the wave is perpendicular to the plane of the resistive card. Then the wave gets rotated by 45° due to Faraday rotation in clockwise direction and further gets rotated by 45°

in clockwise direction due to the twist in the waveguide. Now the plane of polarization of the wave will be parallel with that of the resistive card and hence the wave will be completely absorbed by the resistive card and the output at port ① will be zero. This power is dissipated in the card as heat. In practice 20 to 30 dB isolation is obtained for transmission from port ② to port ①.

Circulator

- A microwave circulator is a multiport waveguide junction in which the wave can flow only from the nth port to the $(n\ +1)th$ port in one direction
- The four-port microwave circulator is the most commonly used.



- One type of four-port microwave circulator is a combination of two 3-dB side-hole directional couplers and a rectangular waveguide with two nonreciprocal phase shifters as shown

Reference: Text by Liao (Pg. 158)

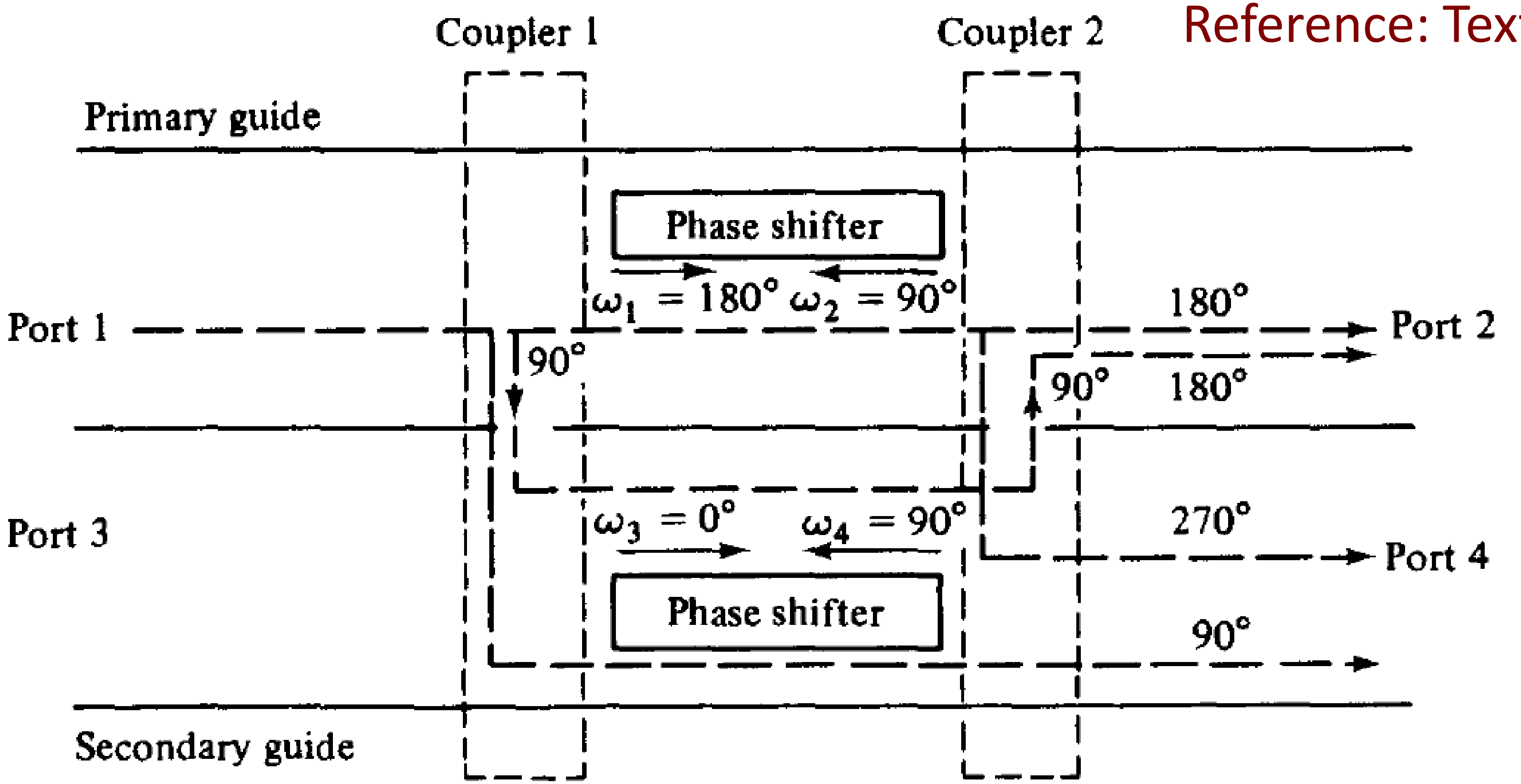


Figure 4-6-3 Schematic diagram of four-port circulator.

- In general, in a waveguide containing ferrite phase shifters, differential propagation phase in the two directions of propagation should be

$$\omega_1 - \omega_3 = (2m + 1)\pi \qquad \text{rad/s}$$

$$\omega_2 - \omega_4 = 2n\pi \qquad \text{rad/s}$$

where m and n are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on. As a result, the sequence of power flow is designated as $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$.

Note: An isolator can be constructed using a 4 port circulator & terminating ports 3 and 4 with matched loads

A perfectly matched, lossless, and nonreciprocal four-port circulator has an **S** matrix of the form

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

Using the properties of S parameters

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(c) Circulator

A circulator is a four port microwave device which has a peculiar property that each terminal is connected only to the next clockwise terminal. *i.e.*, port ① is connected to port ② only and not to port ③ and ④ and port ② is connected only to port ③ etc. This is shown in Fig. 6.36. Although there is no restriction on the number of ports, four ports are most commonly used. They are useful in parametric amplifiers, tunnel diode, amplifiers and duplexer in radars.

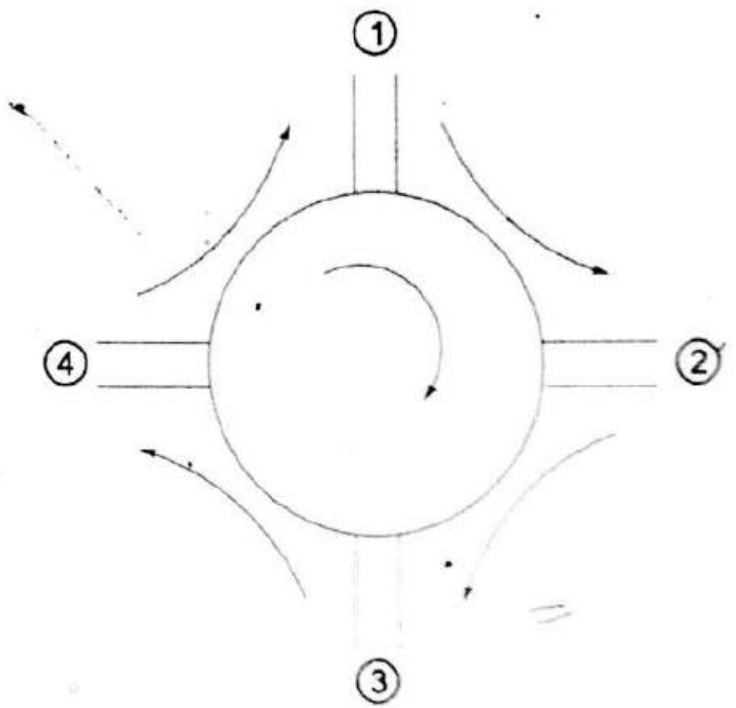


Fig. 6.36

Construction : A four port Faraday rotation circulator is shown in Fig. 6.37. The power entering port ① is TE_{10} mode and is converted to TE_{11} mode because of gradual rectangular to circular transition. This power passes port ③ unaffected since the electric field is not significantly cut and is rotated through 45° due to the ferrite, passes port ④ unaffected (for the same reason as it passes port ③) and finally emerges out of port ②. Power from port ② will have plane of polarization already tilted by 45° with respect to port ①. This power passes port ④ unaffected because again the electric field is not significantly cut. This wave gets rotated by another 45° due to ferrite rod in the clockwise direction. This power whose plane of polarization is tilted through 90° finds port ③ suitably aligned and emerges out of it. Similarly port ③ in coupled only to port ④ and port ④ to port ①.

Reference: Text by Kulkarni (Pg. 223)

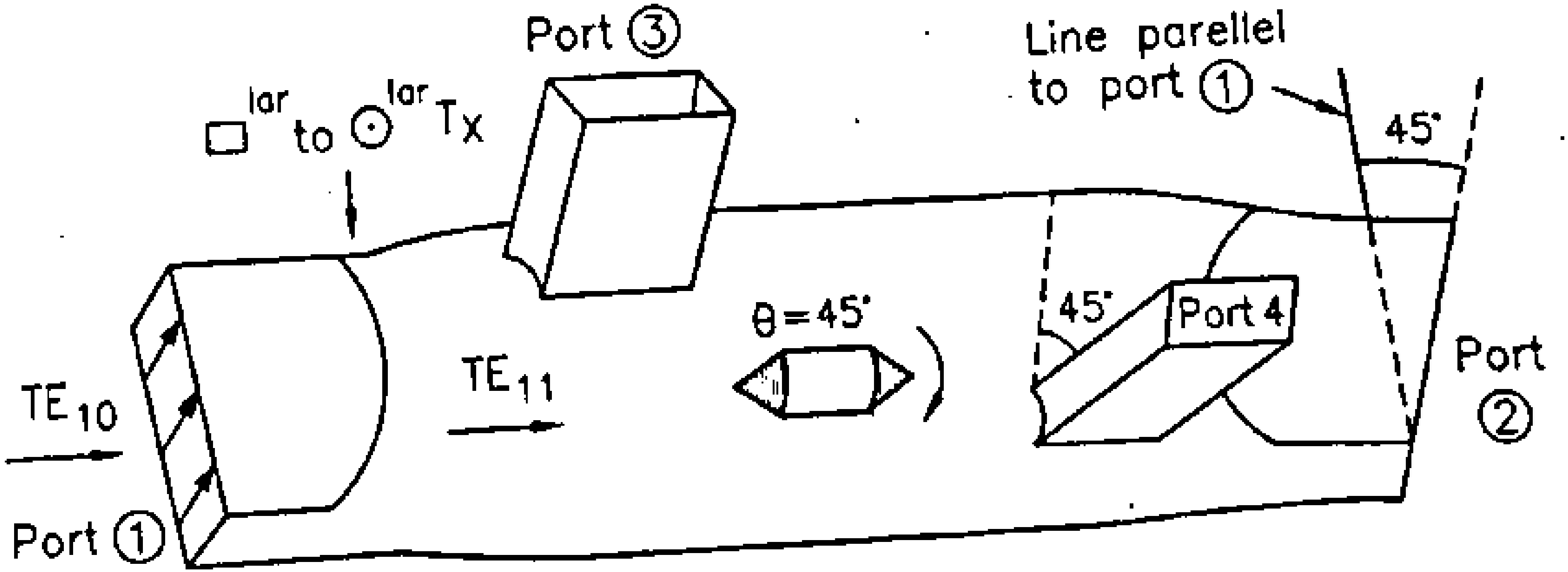


Fig. 6.37 *Four port circulator.*

101001/EC700A: Microwave & Antennas

Module 5-Part 2

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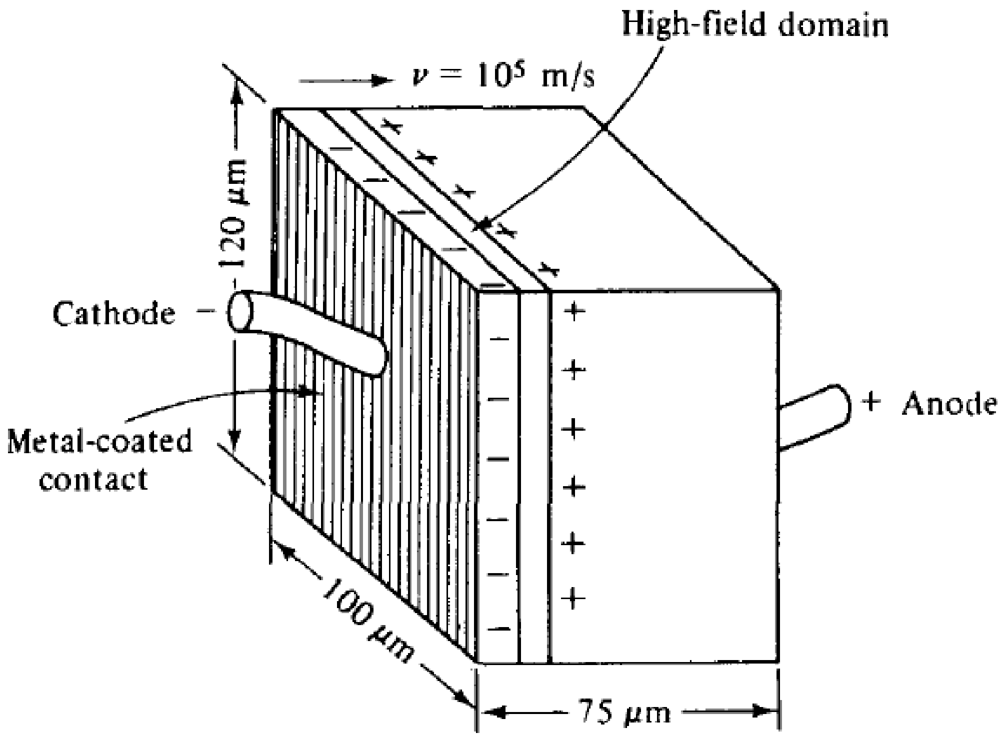
Gunn Diode (Transferred Electron Devices – TED)

- Named after J.B Gunn (1963), who discovered periodic fluctuations of current passing through the n-type GaAs specimen when the applied voltage exceeded a critical value.



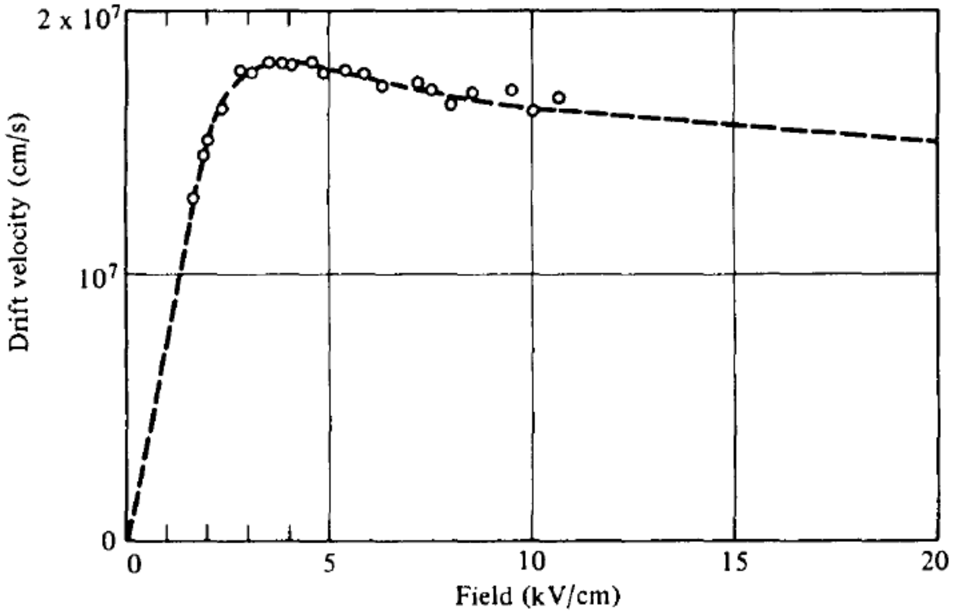
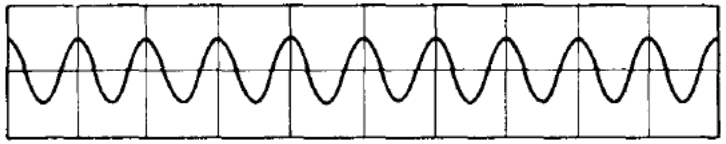


Figure 7-1-2 Drift velocity of electrons in *n*-type GaAs versus electric field. (*After J. B. Gunn [8]; reprinted by permission of IBM, Inc.*)



Differential Negative Resistance

- Mathematically, the negative resistance of a sample in a particular region expressed as

$$\frac{dI}{dV} = \frac{dJ}{dE} = \text{negative resistance}$$

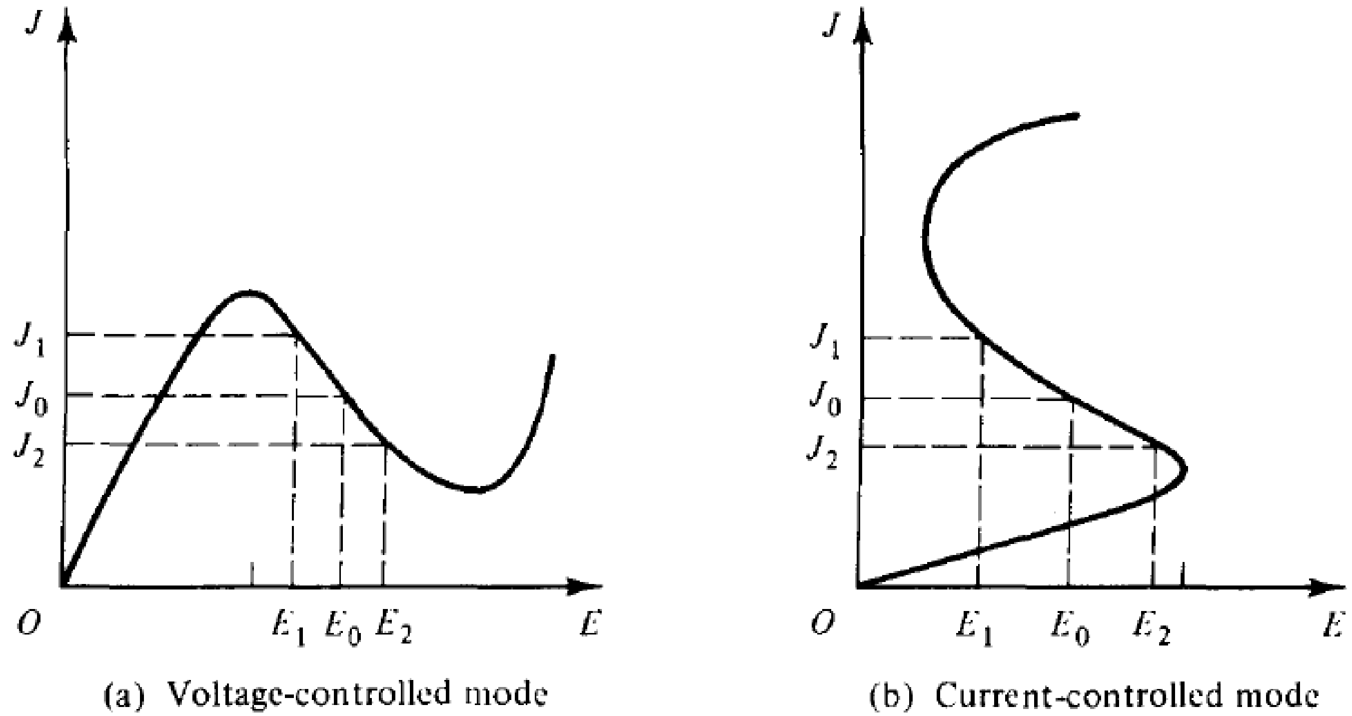
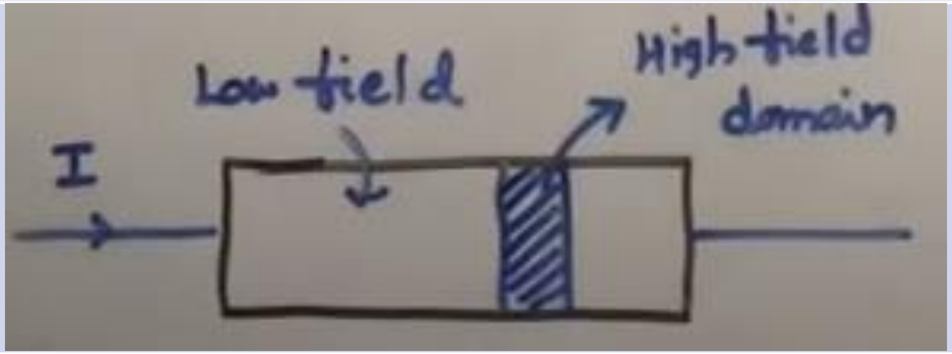
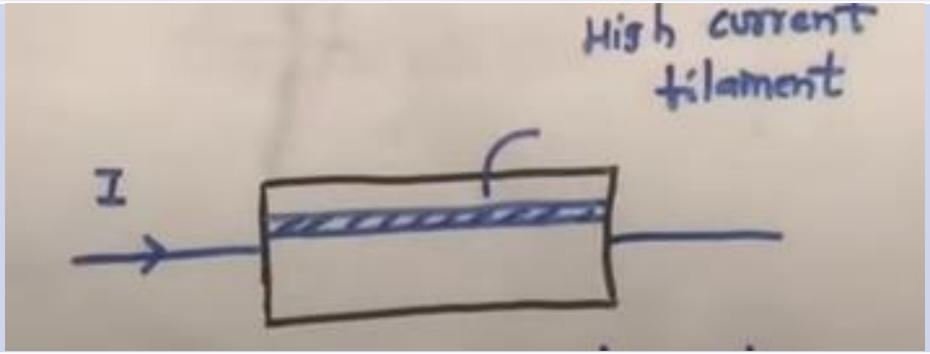
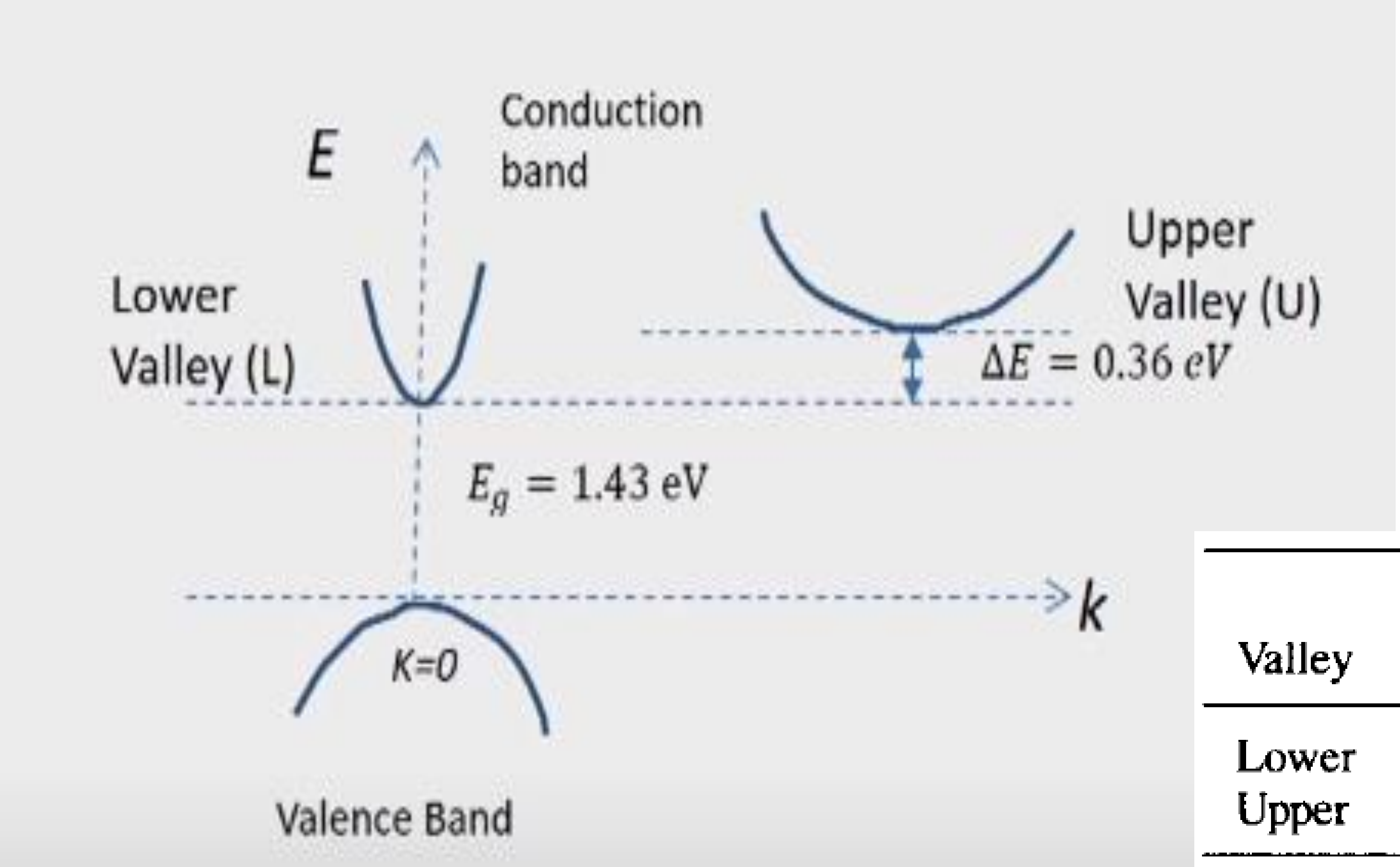


Figure 7-2-3 Multiple values of current density for negative resistance. (*From B. K. Ridley [5]; reprinted by permission of the Institute of Physics.*)

Modes

Voltage Controlled Mode	Current Controlled Mode
The current densities can be multivalued	Voltage can be multivalued
High field domain are formed separating two low field regions	High current filaments running along the field direction are formed.
	

Ridley Watkins-Hilsum (RWH) theory (2 Valley Model)



Basic mechanism in operation of n-type GaAs device is the transfer of e^- from Lower conduction Valley (L) and Upper Valley (U)

Valley	Effective Mass M_e	Mobility μ
Lower	$M_{e\ell} = 0.068$	$\mu_\ell = 8000\ cm^2/V\text{-sec}$
Upper	$M_{eu} = 1.2$	$\mu_u = 180\ cm^2/V\text{-sec}$

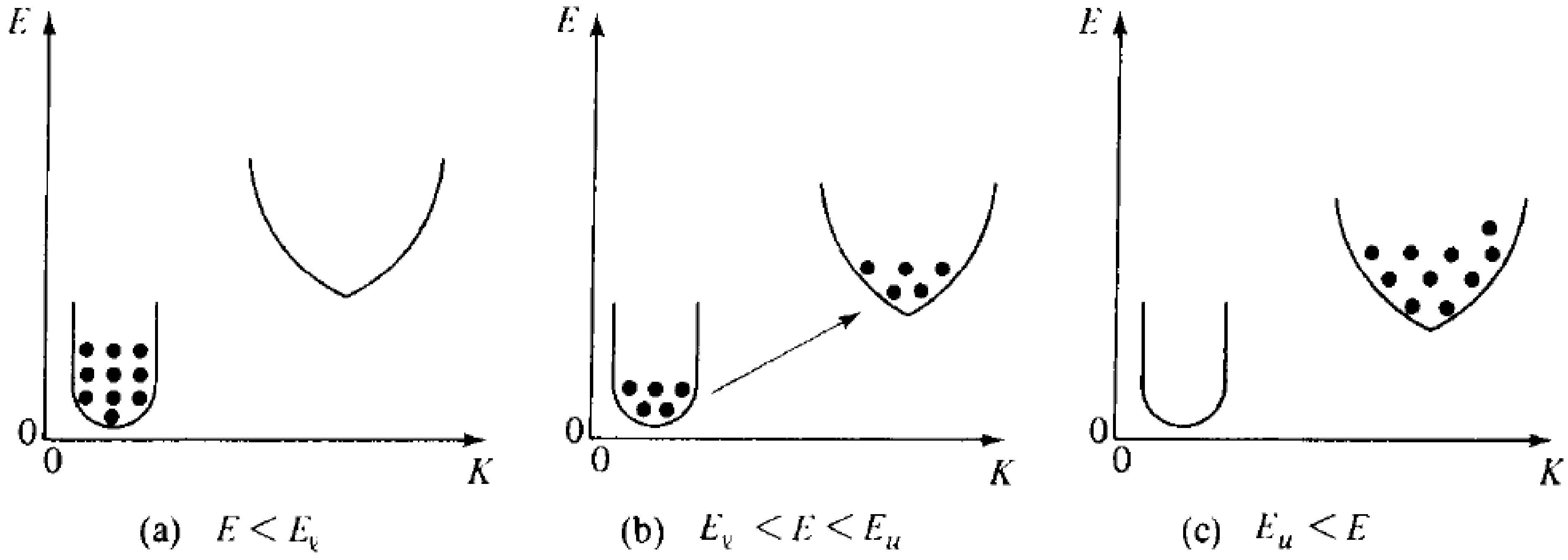


Figure 7-2-5 Transfer of electron densities.

If electron densities in the lower and upper valleys are n_ℓ and n_u, the conductivity of the n-type GaAs is

$$\sigma = q\mu n$$

$$\sigma = e(\mu_\ell n_\ell + \mu_u n_u)$$

where e = the electron charge

μ = the electron mobility

Example 7-2-1: Conductivity of an *n*-Type GaAs Gunn Diode

Electron density:	$n = 10^{18}\ \text{cm}^{-3}$
Electron density at lower valley:	$n_\ell = 10^{10}\ \text{cm}^{-3}$
Electron density at upper valley:	$n_u = 10^{8}\ \text{cm}^{-3}$
Temperature:	$T = 300°\text{K}$

Determine the conductivity of the diode.

Solution From Eq. (7-2-2) the conductivity is

$$
\begin{aligned}
\sigma &= e(\mu_\ell n_\ell + \mu_u n_u) \\
&= 1.6 \times 10^{-19}(8000 \times 10^{-4} \times 10^{16} + 180 \times 10^{-4} \times 10^{14}) \\
&\simeq 1.6 \times 10^{-19} \times 8000 \times 10^{-4} \times 10^{16} \qquad \text{for } n_\ell \gg n_u \\
&= 1.28 \text{ mmhos}
\end{aligned}
$$

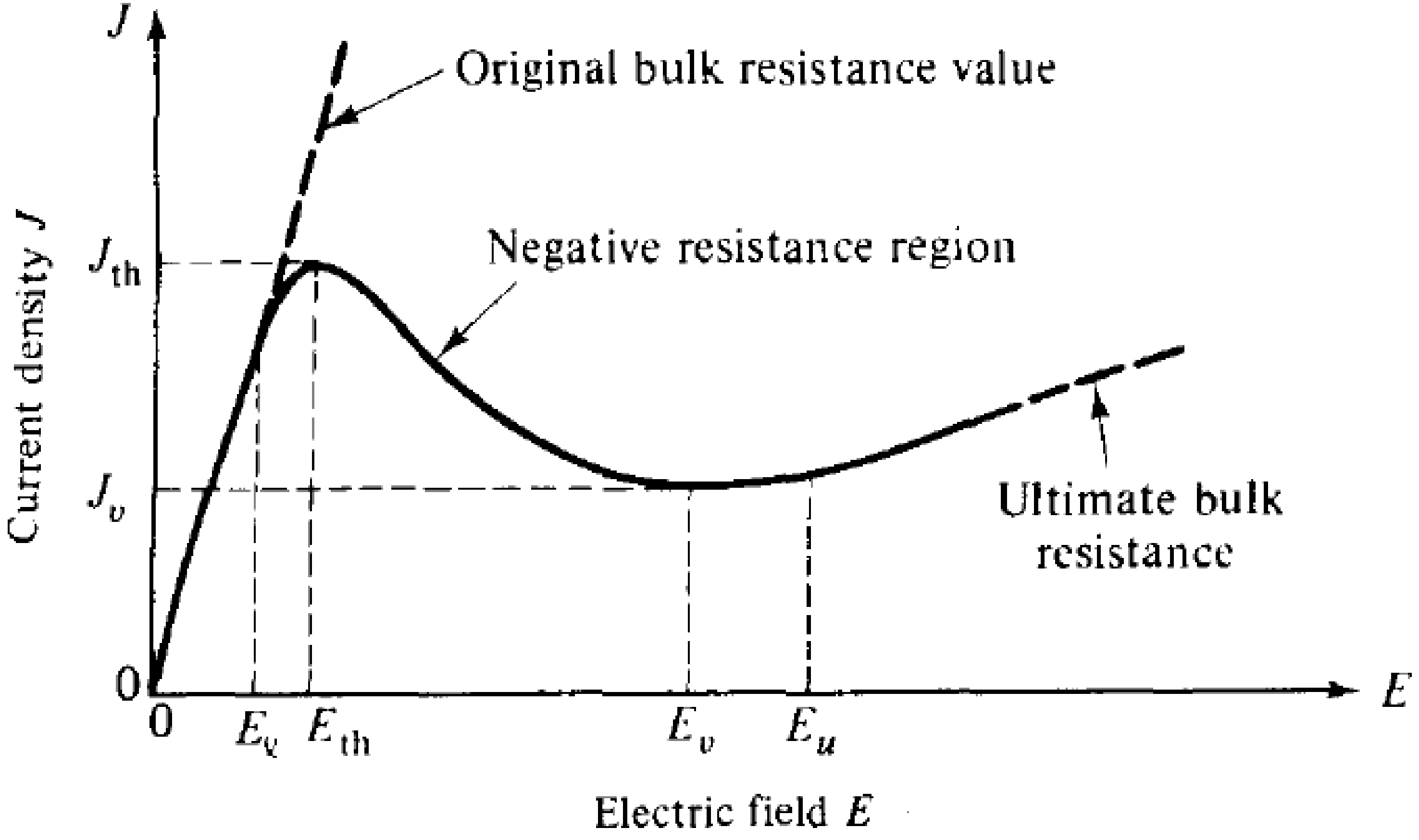


Figure 7-2-6 Current versus field characteristic of a two-valley semiconductor.

Condition for negative conductance

- In a semiconductor, magnitude of drift current density is given by,

$$J = \sigma E = ne\mu E \qquad \textbf{...(1)}$$

- Differentiating (1) w.r.t. E

$$\frac{dJ}{dE} = \sigma + E\frac{d\sigma}{dE}$$

Dividing throughout by σ

$$\frac{1}{\sigma}\frac{dJ}{dE} = 1 + \frac{E}{\sigma}\frac{d\sigma}{dE} = 1 + \frac{d\sigma/dE}{(\sigma/E)} \qquad \textbf{...(2)}$$

- Instability and oscillations will build up in a gunn diode system only when conductivity becomes negative

i.e. $\frac{1}{\sigma}\frac{dJ}{dE} < 0$

- Eq. (2) ➡ $1 + \frac{d\sigma/dE}{(\sigma/E)} < 0 \quad \rightarrow \quad 1 < -\frac{(d\sigma/dE)}{(\sigma/E)}$

$$\boxed{-\frac{(d\sigma/dE)}{(\sigma/E)} > 1} \qquad \text{...(3)}$$

This is the condition required for negative resistance.

- Conductivity $\sigma = e(\mu_l n_l + \mu_u n_u)$
- Differentiating w.r.t. E

$$\frac{d\sigma}{dE} = e\left(\mu_l \frac{dn_l}{dE} + n_l \frac{d\mu_l}{dE}\right) + e\left(\mu_u \frac{dn_u}{dE} + n_u \frac{d\mu_u}{dE}\right) \qquad \text{...(4)}$$

Assuming that the total electron density is a constant

$$\mathrm{n} = \mathrm{n_l} + \mathrm{n_u} = \text{constant}$$

$$\frac{\mathrm{dn}}{\mathrm{dE}} = \frac{\mathrm{d(n_l + n_u)}}{\mathrm{dE}} = 0$$

$$\frac{dn_\ell}{dE} = -\frac{dn_u}{dE} \qquad \textbf{...(5)}$$

- Sub. in eq. (4)

$$\frac{d\sigma}{dE} = e\left(\mu_\ell \frac{dn_\ell}{dE} + \mu_u \frac{dn_u}{dE}\right) + e\left(n_\ell \frac{d\mu_\ell}{dE} + n_u \frac{d\mu_u}{dE}\right)$$

$$\frac{d\sigma}{dE} = e\left((\mu_l - \mu_u)\frac{dn_l}{dE}\right) + e\left(n_l \frac{d\mu_l}{dE} + n_u \frac{d\mu_u}{dE}\right) \qquad \textbf{...(6)}$$

Assume that μ_l and μ_u are proportional to E^p where p is a constant

$$\frac{d\mu}{dE} \propto \frac{dE^p}{dE} = pE^{p-1} = p\frac{E^p}{E} \propto p\frac{\mu}{E} = \mu\frac{p}{E} \qquad \text{...(7)}$$

Sub. in eq. (6)

$$\frac{d\sigma}{dE} = e(\mu_\ell - \mu_u)\frac{dn_\ell}{dE} + e(n_\ell\mu_\ell + n_u\mu_u)\frac{p}{E} \qquad \text{...(8)}$$

Dividing by $\sigma = e(\mu_l n_l + \mu_u n_u)$

$$\frac{1}{\sigma}\frac{d\sigma}{dE} = \left(\frac{\mu_l - \mu_u}{\mu_l n_l + \mu_u n_u}\right)\frac{dn_l}{dE} + \frac{p}{E}$$

Multiplying throughout by E and assuming $n_u/n_l = f$; a constant

$$\frac{1}{(\sigma/E)}\frac{d\sigma}{dE} = \left(\frac{\mu_l - \mu_u}{\mu_l + \mu_u f}\right)\frac{E}{n_l}\frac{dn_l}{dE} + p \qquad \text{...(9)}$$

Comparing eq. (3) and (9)

Eq. (3) $\Rightarrow$ $$-\frac{(d\sigma/dE)}{(\sigma/E)} > 1$$

Therefore, $$-\left[\left(\frac{\mu_l - \mu_u}{\mu_l + \mu_u f}\right)\frac{E}{n_l}\frac{dn_l}{dE} + p\right] > 1$$

$$\boxed{\left[\left(\frac{\mu_l - \mu_u}{\mu_l + \mu_u f}\right)\left(-\frac{E}{n_l}\frac{dn_l}{dE}\right) - p\right] > 1} \qquad \text{...(10)}$$

This is the condition required to produce negative conductance in semiconducting materials.

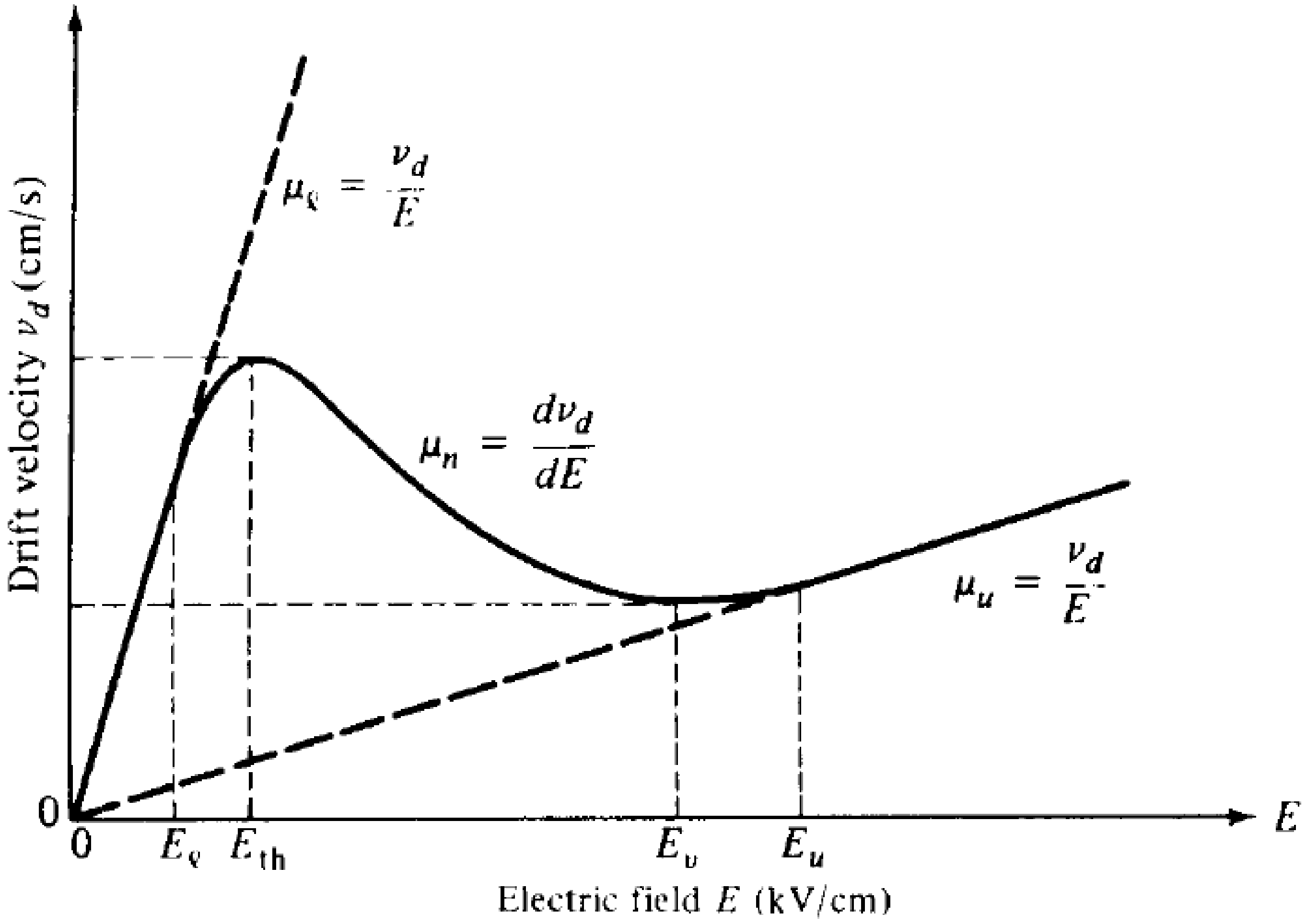


Figure 7-2-7 Electron drift velocity versus electric field.

- In a semiconductor, magnitude of current density is given by,

$$J = \sigma E = nq\mu E$$

$$J = nqv$$

where q = electric charge
n = electron density, and
v = average electron velocity.

- Differentiating J w.r.t. E yields,

$$\frac{dJ}{dE} = qn\frac{dv}{dE}$$

- The condition for negative differential conductivity is

$$\frac{dv_d}{dE} = \mu_n < 0$$

Example 7-2-2: Characteristics of a GaAs Gunn Diode

A typical n-type GaAs Gunn diode has the following parameters:

Threshold field	$E_{th} = 2800$ V/cm
Applied field	$E = 3200$ V/cm
Device length	$L = 10\ \mu$m
Doping concentration	$n_0 = 2 \times 10^{14}$ cm^{-3}
Operating frequency	$f = 10$ GHz

a. Compute the electron drift velocity.
b. Calculate the current density.
c. Estimate the negative electron mobility.

a. The electron drift velocity is

$$v_d = 10 \times 10^9 \times 10 \times 10^{-6} = 10^5 \text{ m/sec} = 10^7 \text{ cm/sec}$$

b. From Eq. (7-2-12) the current density is

$$J = qnv = 1.6 \times 10^{-19} \times 2 \times 10^{20} \times 10 \times 10^9 \times 10^{-5}$$

$$= 3.2 \times 10^6 \text{ A/m}^2$$

$$= 320 \text{ A/cm}^2$$

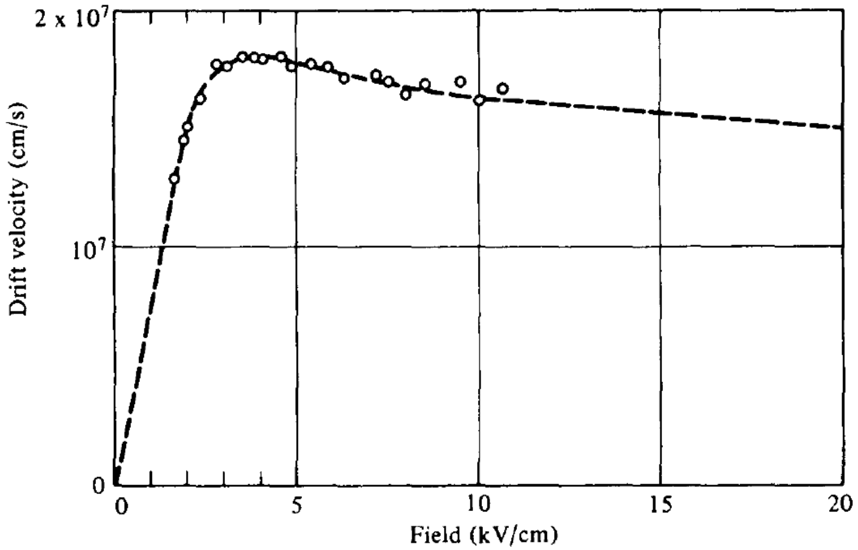
c. The negative electron mobility is

$$\mu_n = -\frac{v_d}{E} = -\frac{10^7}{3200} = -3100 \text{ cm}^2/\text{V}\cdot\text{sec}$$

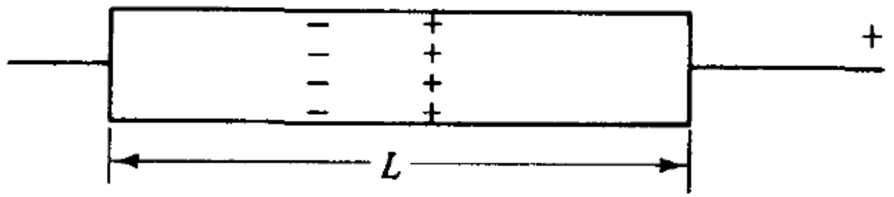
High Field Domain

- In the n-type GaAs diode, the majority carriers are electrons.
- When a small voltage is applied to the diode, the electric field and conduction current density are uniform throughout the diode.
- At low voltage the GaAs is ohmic, since the drift velocity of the electrons is proportional to the electric field.
- The conduction current density in the diode is given by

$$\mathbf{J} = \sigma \mathbf{E}_x = \frac{\sigma V}{L}\mathbf{U}_x = \rho v_x \mathbf{U}_x$$



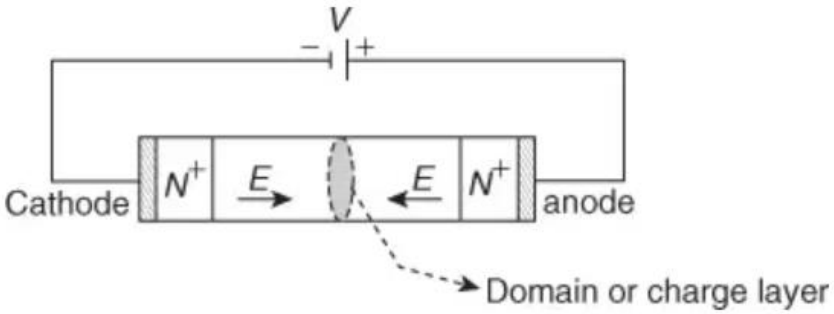
J = conduction current density
a = conductivity
Ex = electric field in the *x* direction
L = length of the diode
V = applied voltage
p = charge density
v = drift velocity
U = unit vector



- When applied E field exceeds the threshold value, the charge densities and electric field within the sample become non uniform creating domains.
- The electric field inside the dipole domain would be greater than the fields on either side of the dipole.
- This high field domain is formed near the cathode and it move towards the anode.

$$V = -\int_0^L E_x \, dx$$

- For a constant voltage, V an increase in the electric field within the specimen must be accompanied by a decrease in electric field in rest of the diode.
- High field domain drifts with the carrier stream across the electrodes and disappears at the anode contact.
- When electric field increases, electron drift velocity decreases and thus GaAs has negative resistance
- The next domain starts forming at the cathode when the field value exceeds the threshold value and the process is repeated.



Domain formation in a Gunn diode

- The time taken by the dipole domain to travel from the cathode to the anode – **'Transit Time'**
- The fundamental frequency in MHz is given by $\boldsymbol{f = \frac{V_d}{L}}$

  where V_d - drift velocity

  and L - Device Length in μm
- The length of the domain is inversely proportional to the doping level.
- The concentration-length product $(n_0 L)$ along with the frequency of operation determines the mode of operation of device.

Modes of Oscillation

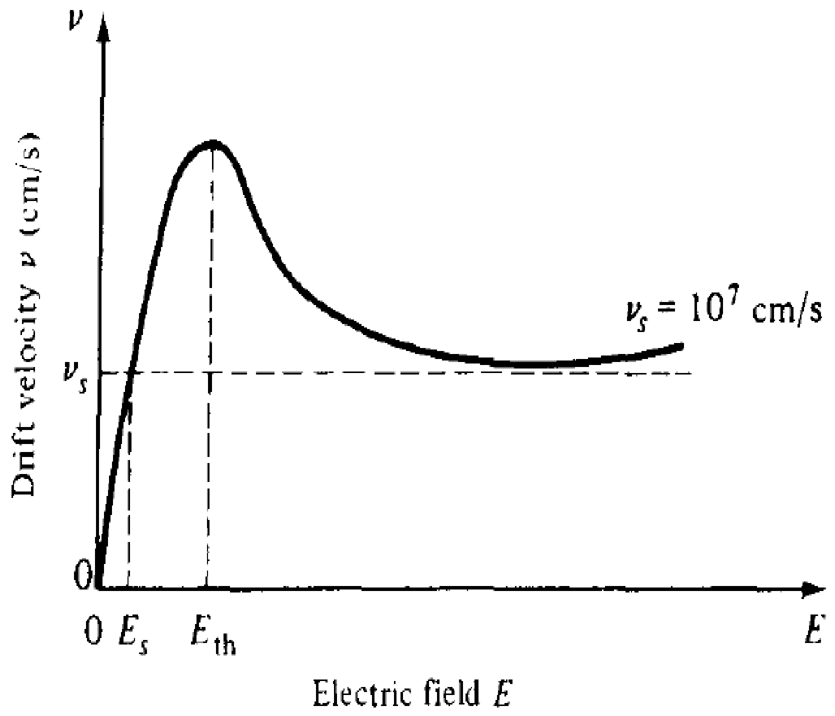
- Transit Time Domain Mode
- Delayed / Inhibited Domain Mode
- Quenched Domain Mode
- Limited Space Charge Accumulation (LSA) Mode

Transit Time Domain Mode

- Here $\mathrm{fL} = 10^7\ \mathrm{cm/s} = \mathbf{drift\ velocity}, \mathbf{v_d}$
- When electron drift velocity = saturation velocity $= \boldsymbol{v_s}$, the high field domain is stable.
- In this case, oscillation period = transit time i.e. $\boldsymbol{\tau_0 = \tau_t}$
- Efficiency is below 10 %
- The operating frequency $(\mathbf{f} = \mathbf{v_d}/\mathrm{L})$ in this mode is slightly sensitive to applied voltage

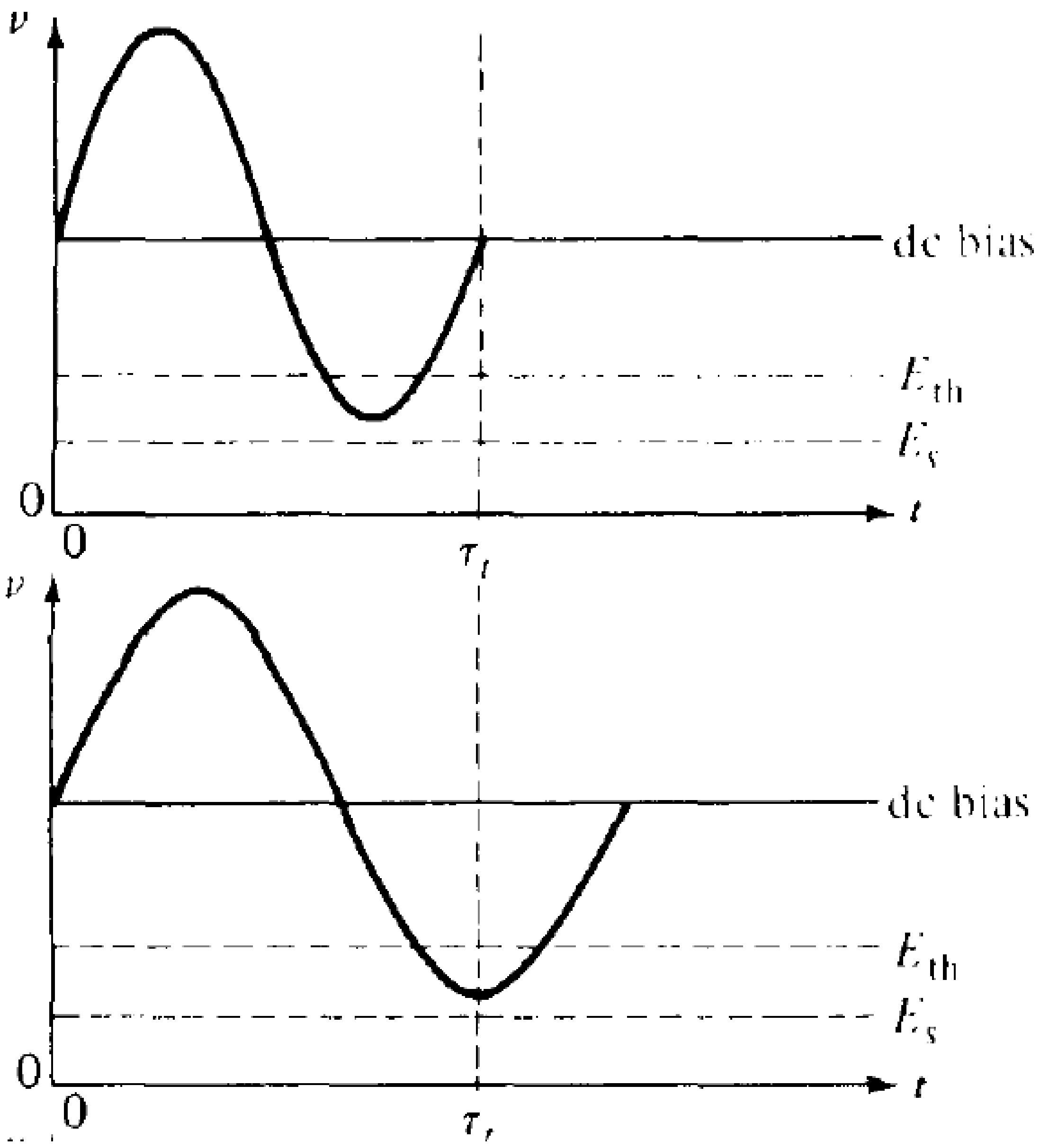
Delayed / Inhibited Domain Mode

- Here $10^6\ \mathrm{cm/s} < \mathrm{fL} < 10^7\ \mathrm{cm/s}$
- The transit time is chosen such that the domain is collected while $E < E_{th}$
- The oscillation period > transit time i.e. $\boldsymbol{\tau_0 > \tau_t}$
- Efficiency is approx. 20 %



(a) Transit-time mode
$\tau_0 = \tau_t$

(b) Delayed mode
$\tau_0 > \tau_t$

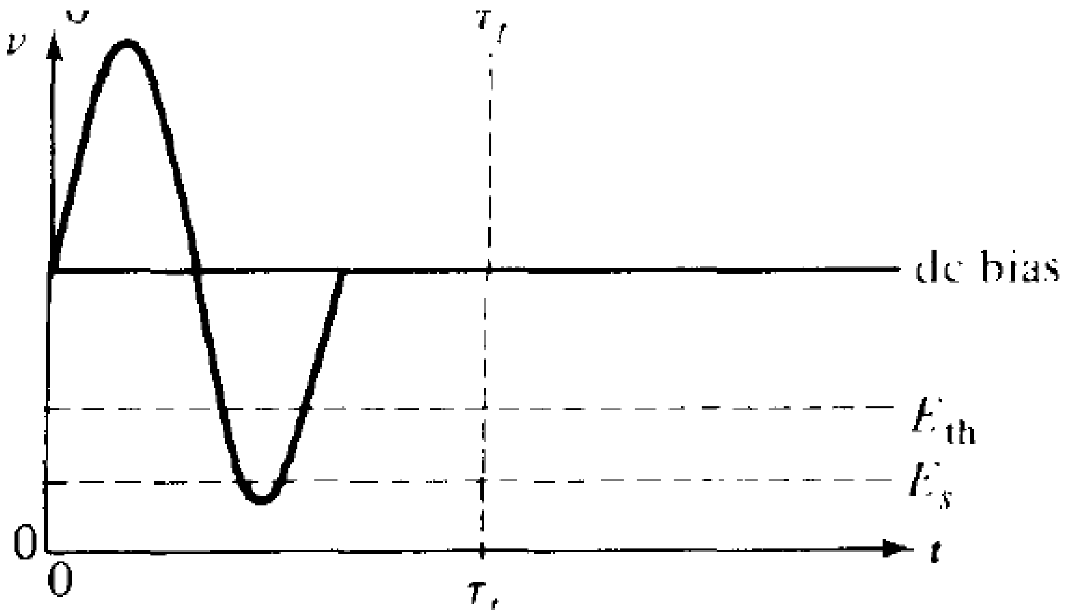


Quenched Domain Mode

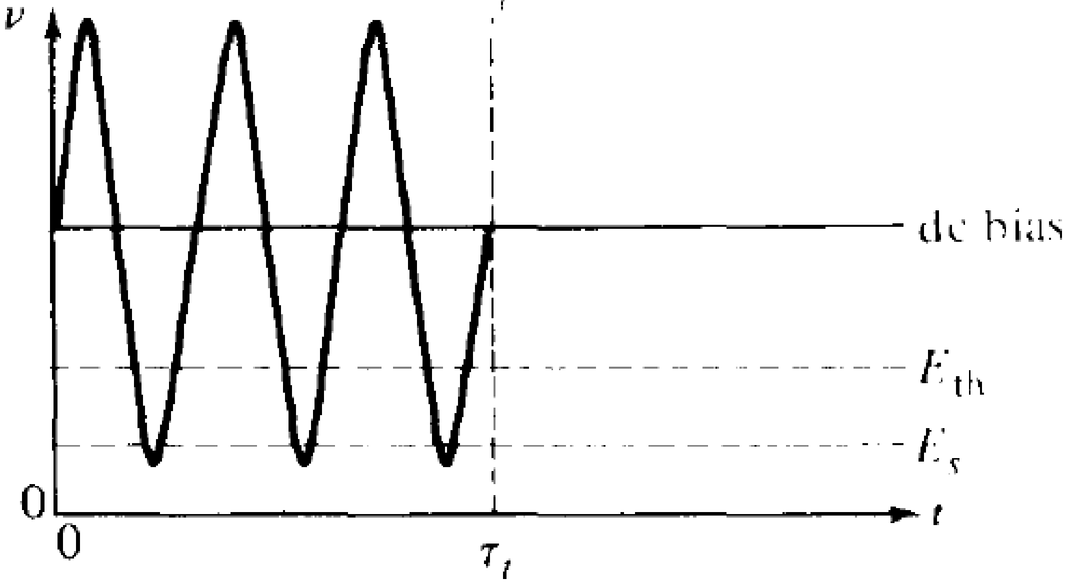
- Here $fL > 2 \times 10^7\ cm/s$
- If the bias field drops below the sustaining field *Es* during the negative half-cycle, the domain collapses before it reaches the anode.
- Therefore the oscillations occur at the frequency of the resonant circuit rather than at the transit-time frequency.
- Theoretical Efficiency approx. 13 %

Limited Space Charge Accumulation (LSA) Mode ($fL > 2 \times 10^7\ cm/s$)

- When the frequency is very high, the domains do not have sufficient time to form while the field is above threshold. As a result, most of the domains are maintained in the negative conductance state during a large fraction of the voltage cycle.
- Any accumulation of electrons near the cathode has time to collapse while the signal is below threshold.



(c) Quenched mode
$\tau_0 < \tau_t$



(d) LSA mode
$\tau_0 < \tau_t$
$\tau_0 = 3\tau_d$

- The frequency of oscillation in the LSA mode is independent of the transit time of the carriers and is determined solely by the circuit external to the device. (High Q resonator needed for LSA mode)
- The efficiency of the LSA mode can reach 20%.

Thank You