

Example 3.4 Find IDFT of the sequence

IT AU Dec'03

$$X(k) = \{5, 0, 1 - j, 0, 1, 0, 1 + j, 0\}$$

Solution

$$\text{We have } x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad n = 0, 1, \dots, N-1$$

For $N = 8$

$$x(n) = \frac{1}{8} \sum_{k=0}^{N-1} X(k) e^{j\pi kn/4} \quad n = 0, 1, \dots, 7$$

For $n = 0$

$$x(0) = \frac{1}{8} \left[\sum_{k=0}^7 X(k) \right] = \frac{1}{8} [5 + 0 + 1 - j + 0 + 1 + 0 + 1 + j + 0] = 1$$

$$\begin{aligned} x(1) &= \frac{1}{8} \left[\sum_{k=0}^7 X(k) e^{j\pi k/4} \right] = \frac{1}{8} [5 + (1 - j)(j) + 1(-1) + (1 + j)(-j)] \\ &= \frac{1}{8} [6] = 0.75 \end{aligned}$$

$$\begin{aligned} x(2) &= \frac{1}{8} \left[\sum_{k=0}^7 X(k) e^{j\pi k/2} \right] = \frac{1}{8} [5 + (1 - j)(-1) + 1(1) + (1 + j)(-1)] \\ &= \frac{1}{8} [4] = 0.5 \end{aligned}$$

3.22 Digital Signal Processing

$$\begin{aligned}x(3) &= \frac{1}{8} \left[\sum_{k=0}^7 X(k) e^{j3\pi k/4} \right] = \frac{1}{8} [5 + (1-j)(-j) + 1(-1) + (1+j)(j)] \\&= \frac{1}{8} [2] = 0.25\end{aligned}$$

$$\begin{aligned}x(4) &= \frac{1}{8} \left[\sum_{k=0}^7 X(k) e^{j\pi k} \right] = \frac{1}{8} [5 + (1-j)(1) + 1(1) + (1+j)(1)] \\&= 1\end{aligned}$$

$$\begin{aligned}x(5) &= \frac{1}{8} \left[\sum_{k=0}^7 X(k) e^{j5\pi k/4} \right] = \frac{1}{8} [5 + (1-j)(j) + 1(-1) + (1+j)(-j)] \\&= \frac{1}{8} [6] = 0.75\end{aligned}$$

$$\begin{aligned}x(6) &= \frac{1}{8} \left[\sum_{k=0}^7 X(k) e^{j3\pi k/2} \right] = \frac{1}{8} [5 + (1-j)(-1) + 1(1) + (1+j)(-1)] \\&= \frac{1}{8} [4] = 0.5\end{aligned}$$

$$\begin{aligned}x(7) &= \frac{1}{8} \left[\sum_{k=0}^7 X(k) e^{j7\pi k/4} \right] = \frac{1}{8} [5 + (1-j)(-j) + 1(-1) + (1+j)(j)] \\&= \frac{1}{8} [2] = 0.25\end{aligned}$$

$$x(n) = \{1, 0.75, 0.5, 0.25, 1, 0.75, 0.5, 0.25\}$$