Direct form realization of Linear phase FIR filter:

An FIR filter has linear phase if its unit impulse response satisfy ~~the~~ either the symmetry or asymmetry condition. ie

$$h(n) = \pm h(M-1-n) \quad ; n = 0, 1 \ldots M-1.$$

For such a system the number of multiplication is reduced from M to $M/2$ for M even and to $\frac{M-1}{2}$ for M odd.

— The direct form structure that takes the advantage of this symmetry in the case of M odd is shown below.

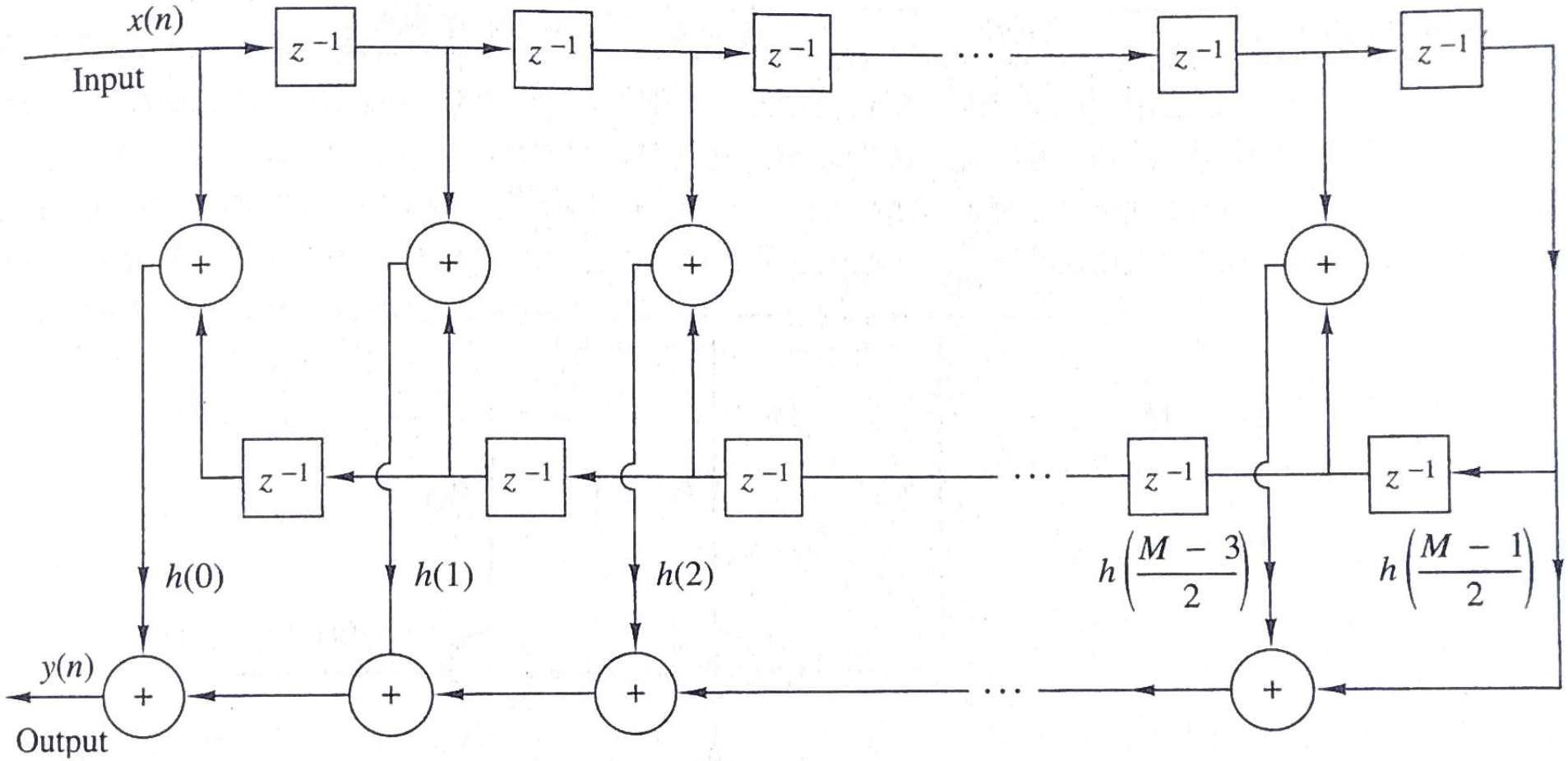


Figure 9.2.2 Direct-form realization of linear-phase FIR system (M odd).

(2) Realize the system function

$$H(z) = \tfrac{1}{2} + \tfrac{1}{3} z^{-1} + z^{-2} + \tfrac{1}{4} z^{-3} + z^{-4} + \tfrac{1}{3} z^{-5} + \tfrac{1}{2} z^{-6}$$

——— (1)

Ans:

We have

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n} \quad \text{——— (2)}$$

(1) & (2) $\Rightarrow$ $M = 7$ (odd).

$\therefore$ (2) $\Rightarrow$ $H(z) = h(0) + h(1) z^{-1} + h(2) z^{-2} +$

$$h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) z^{-6}$$

——— (3)

from (1) and (3)

$$h(0) = h(6) = \tfrac{1}{2}$$
$$h(1) = h(5) = \tfrac{1}{3}$$
$$h(2) = h(4) = 1$$
$$h(3) = \tfrac{1}{4}$$

——— (4)

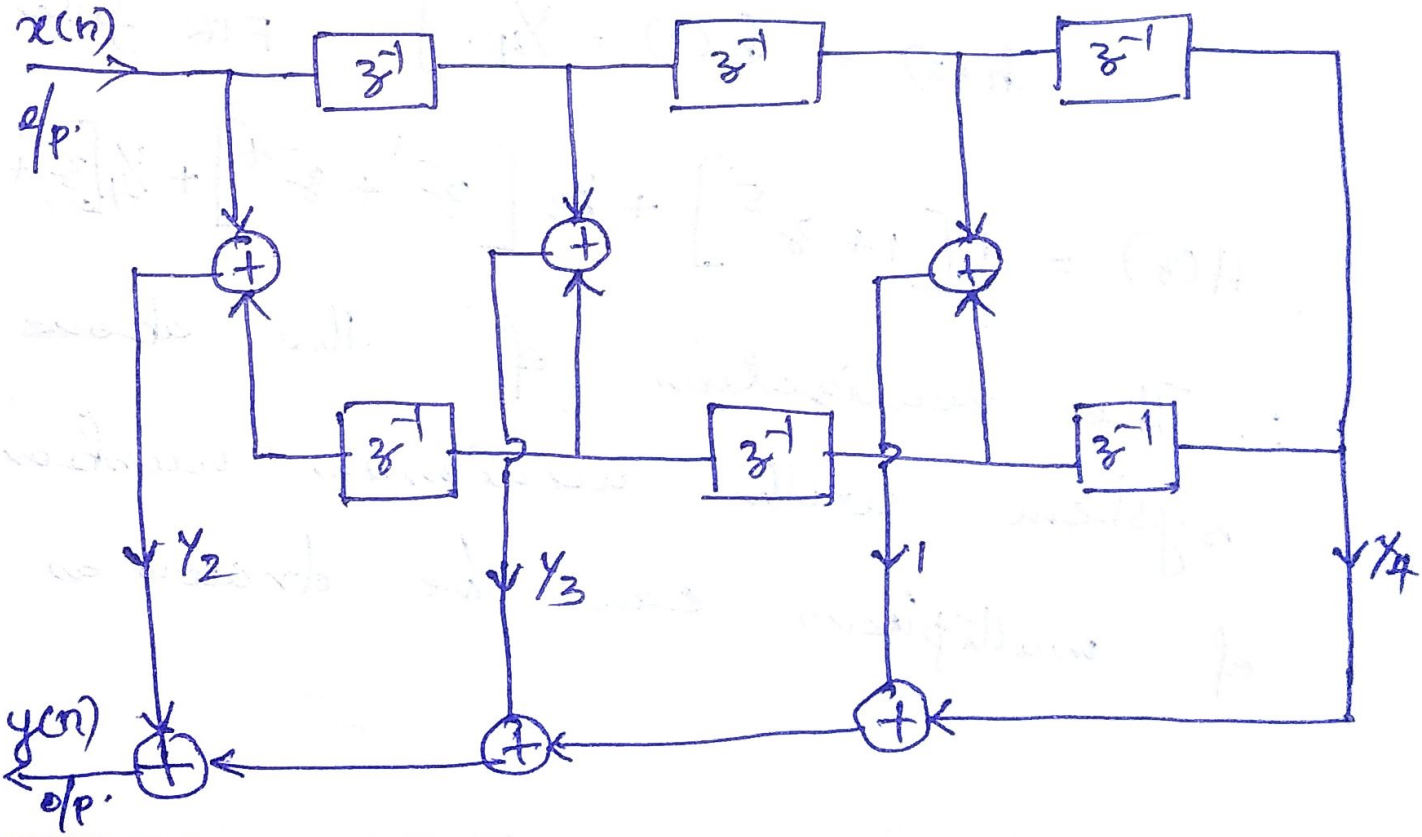
(4) $\Rightarrow$ $h(n) = h(M-1-n)$

∴ the above $H(z)$ represents a linear phase FIR filter.

∴ eqn ① ⟹

$$H(z) = \tfrac{1}{2}\left[1 + z^{-6}\right] + \tfrac{1}{3}\left[z^{-1} + z^{-5}\right] + 1\left[z^{-2} + z^{-4}\right] + \tfrac{1}{4} z^{-3}$$

∴ The direct form realization of above system can be drawn as



Q) Realize the following system function with minimum number of multipliers.

$$H(z) = 1 + \tfrac{1}{3}z^{-1} + \tfrac{1}{4}z^{-2} + \tfrac{1}{4}z^{-3} + \tfrac{1}{3}z^{-4} + z^{-5}$$

Answer:

Here $M = 6$ (even).

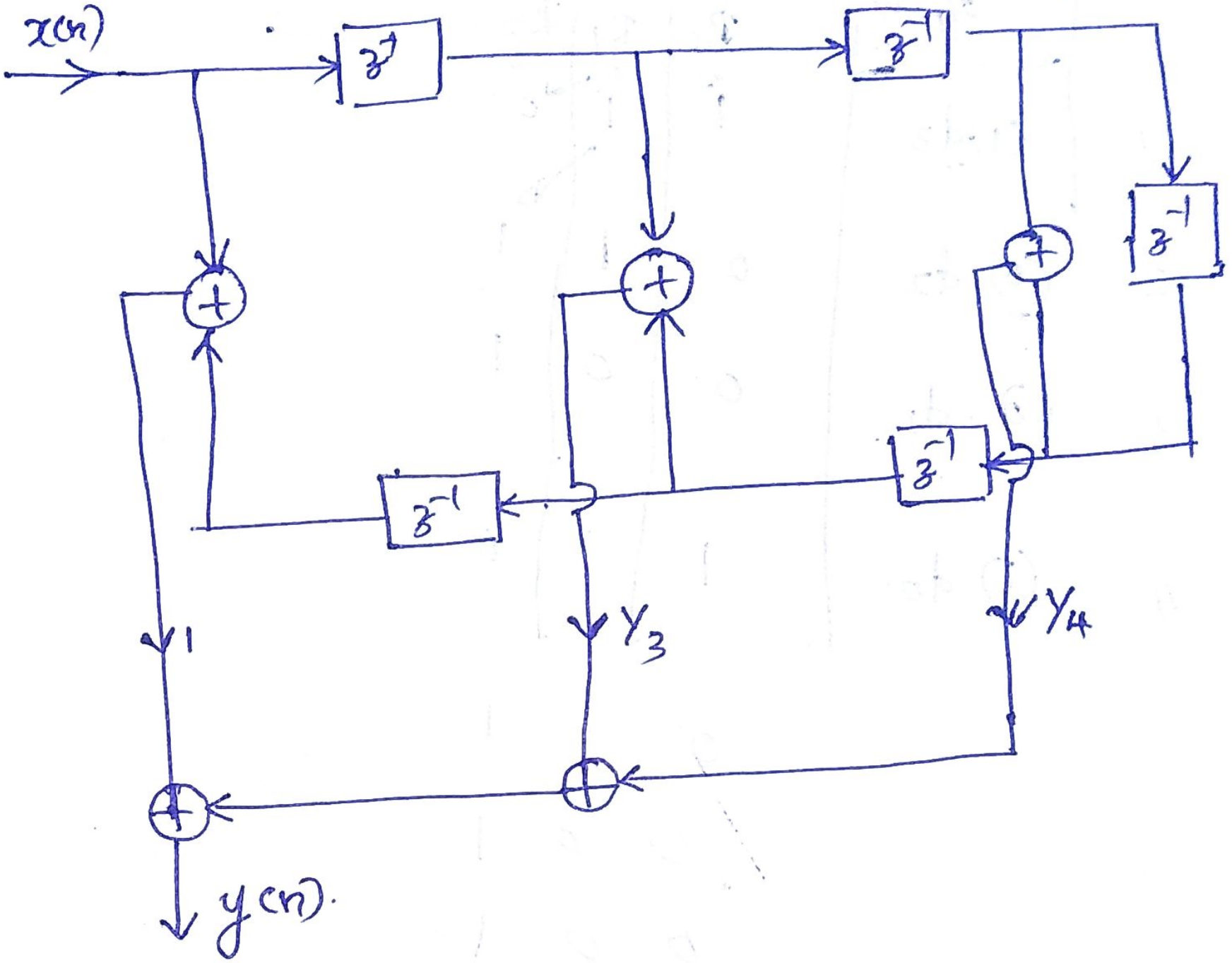
Also the coefficients of corresponding filter

$$h(0) = h(5) = 1$$
$$h(1) = h(4) = \tfrac{1}{3}$$
$$h(2) = h(3) = \tfrac{1}{4}.$$

→ The above filter is Linear phase FIR filter.

$$\therefore H(z) = 1\left[1 + z^{-5}\right] + \tfrac{1}{3}\left[z^{-1} + z^{-4}\right] + \tfrac{1}{4}\left[z^{-2} + z^{-3}\right]$$

∴ The realization of the above system with minimum number of multipliers can be drawn as


x(n)
z^{-1}
z^{-1}
z^{-1}
z^{-1}
z^{-1}
Y_1
Y_3
Y_4
y(n).