MODULE IV, MICROWAVE ENGG.

RECTANGULAR RESONATOR AND MICROWAVE TUBES

SYLLABUS OF MODULE 4

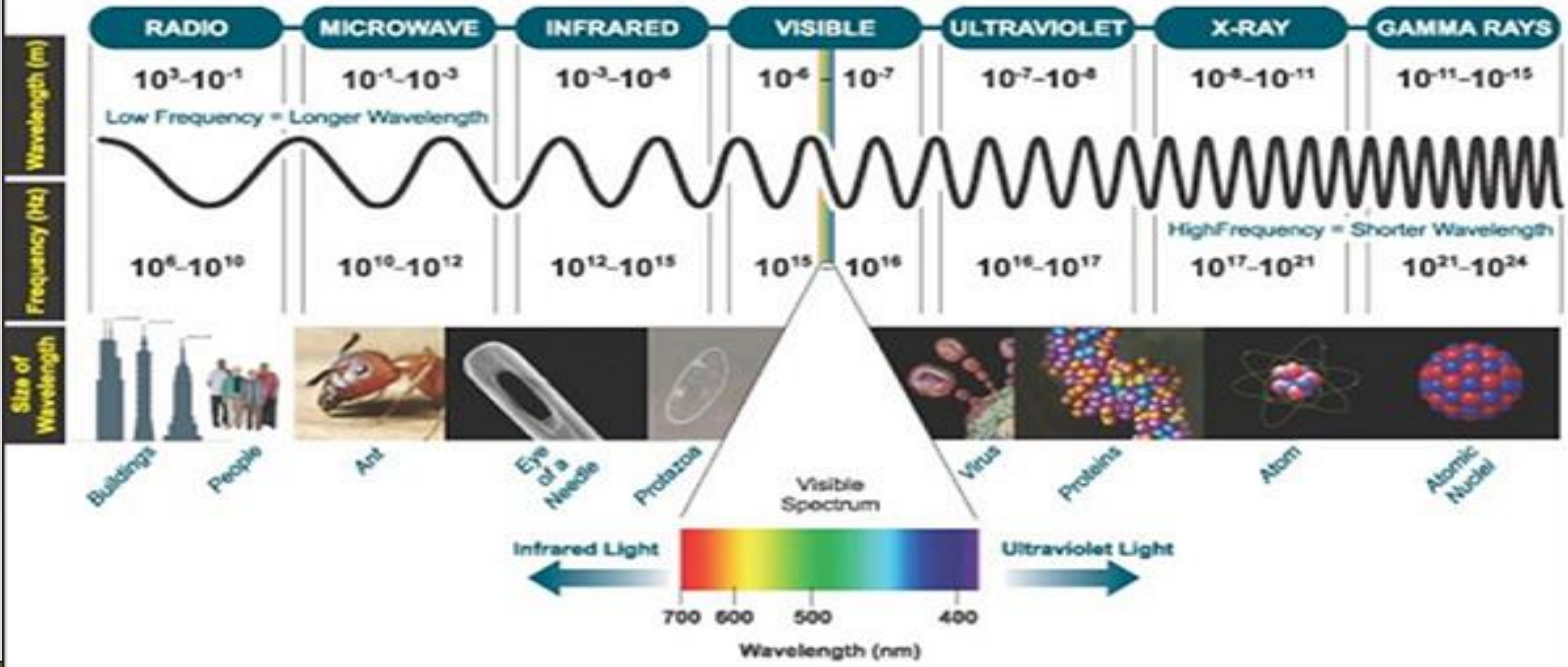
- **Microwave Introductions :** advantages, Cavity Resonators- Derivation of resonance frequency of Rectangular cavity.
- **Single cavity klystron- Reflex Klystron**
  Oscillators: Derivation of Power output, efficiency and admittance.
- **Magnetron oscillators:** Cylindrical magnetron, Cyclotron angular frequency, Power output and efficiency.
- **Travelling Wave Tube:** Slow wave structures, Helix TWT, Amplification process, Derivation of convection current, axial electric field, wave modes and gain

- Text Books
- Microwave devices and circuits, Samuel Y Liao
- Foundation of Microwave Engg, David M Pozar
- Microwave Engg , Das

INTRODUCTION

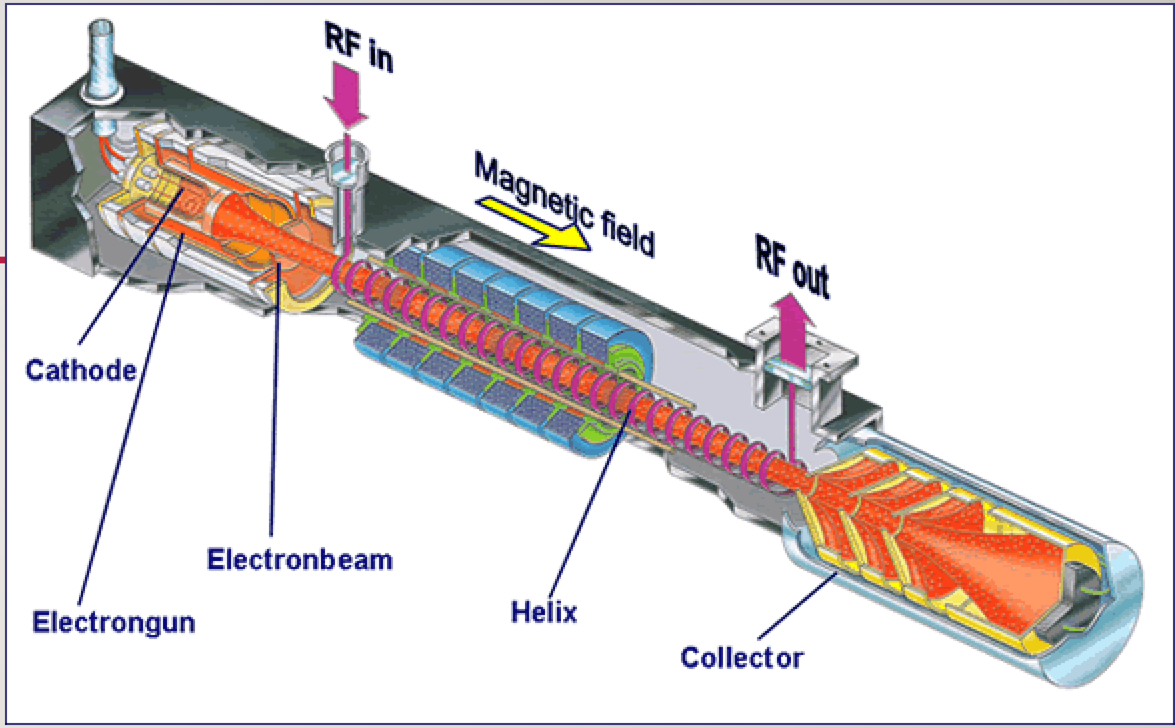
- High frequency signals, Normally GHz signals
- Micrometer wavelength waves
- Less attenuation, dispersion, scattering, diffractions
- Travels long distances without much loss

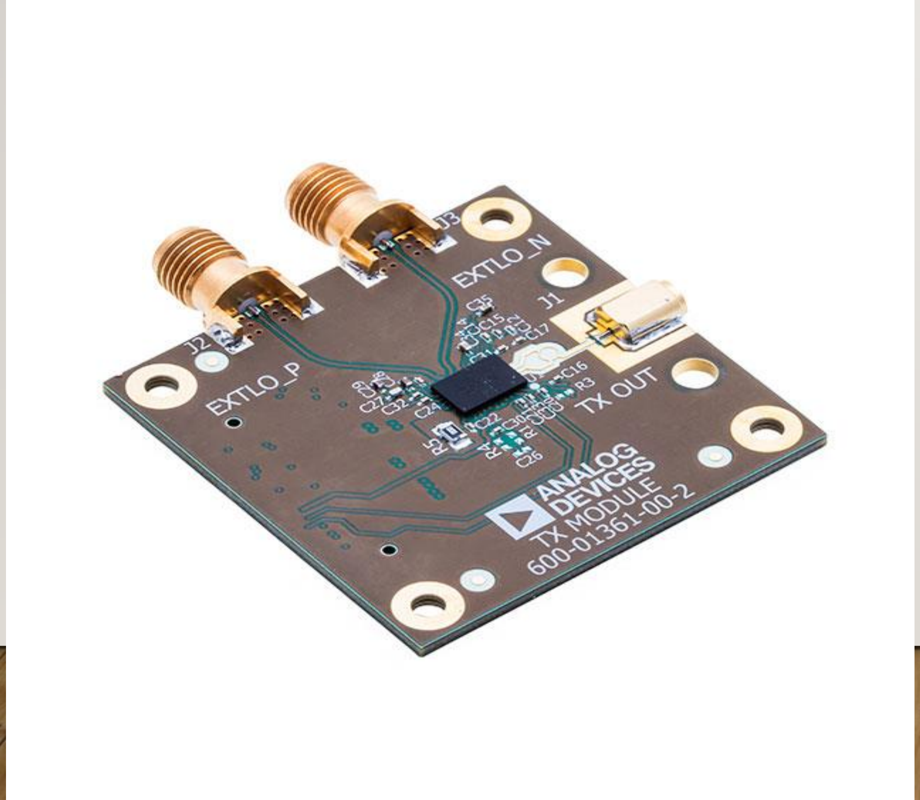
The Electromagnetic Spectrum



GENERATIONS

- MW tube devices
- MW solid state devices





FREQUENCY BANDS

Microwaves Frequency Bands

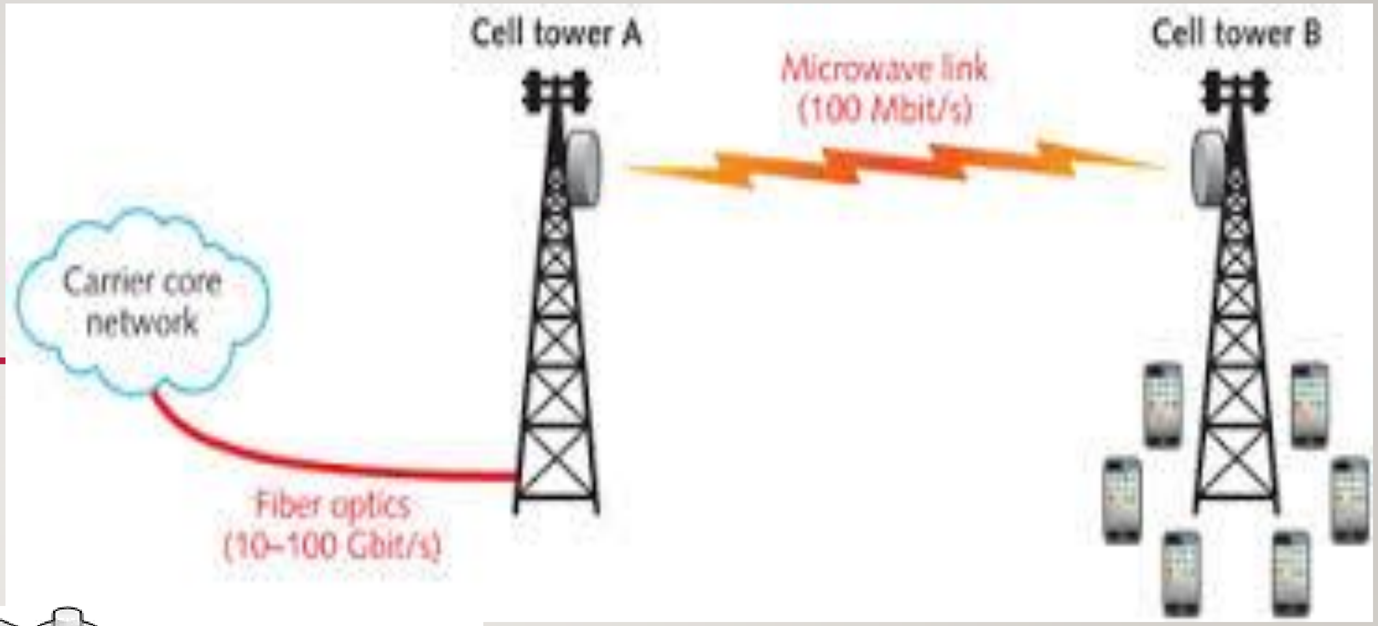
Band	Frequency range
HF Band	3 to 30 MHz
VHF Band	30 to 300 MHz
UHF Band	300 to 1000 MHz
L Band	1 to 2 GHz
S Band	2 to 4 GHz
C Band	4 to 8 GHz
X Band	8 to 12 GHz
Ku Band	12 to 18 GHz
K Band	18 to 27 GHz
Ka Band	27 to 40 GHz
V Band	40 to 75 GHz
W Band	75 to 110 GHz
mm Band	110 to 300 GHz

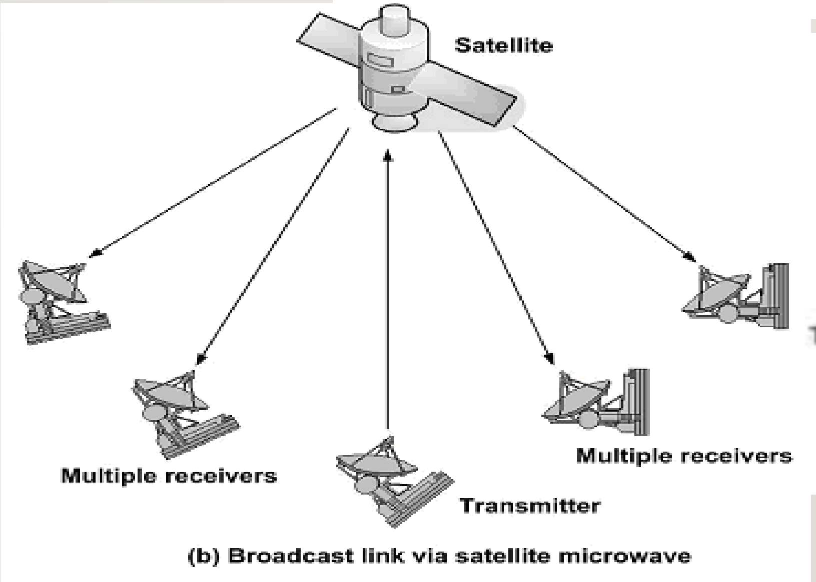
Microwave frequency bands

Designation	Frequency range
L band	1 to 2 GHz
S band	2 to 4 GHz
C band	4 to 8 GHz
X band	8 to 12 GHz
K_u band	12 to 18 GHz
K band	18 to 26.5 GHz
K_a band	26.5 to 40 GHz
Q band	30 to 50 GHz
U band	40 to 60 GHz
V band	50 to 75 GHz
E band	60 to 90 GHz
W band	75 to 110 GHz
F band	90 to 140 GHz
D band	110 to 170 GHz (Hot)

APPLICATIONS

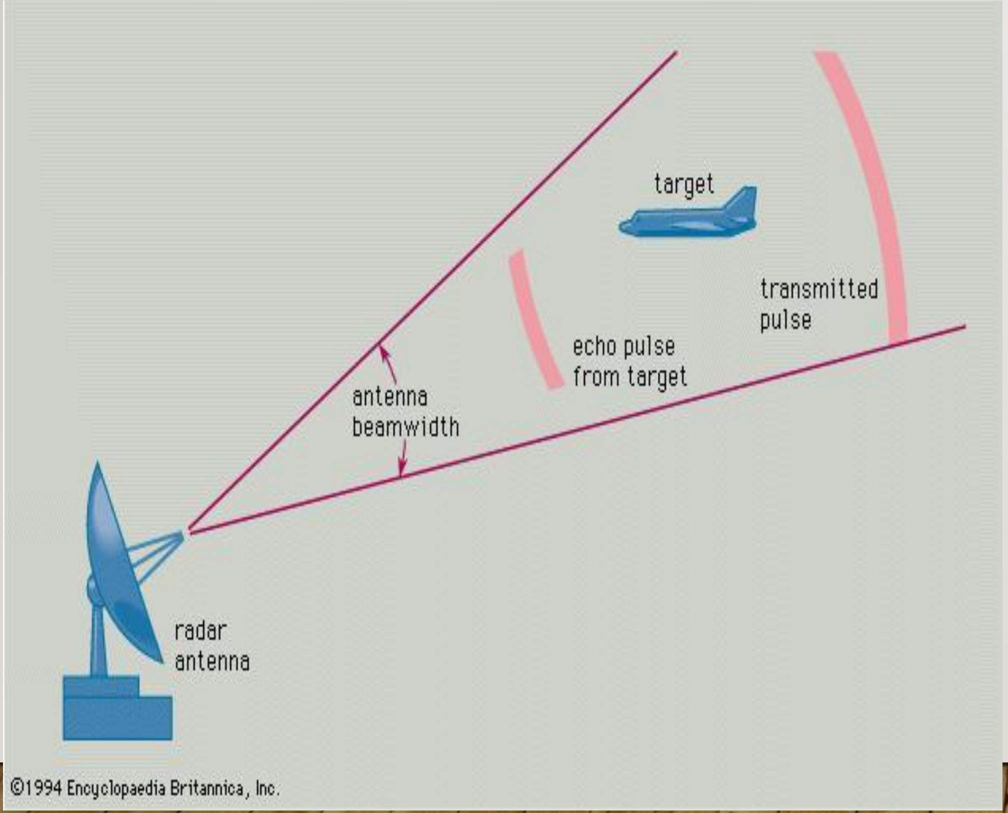
- Communications
  a) MW links
  b) Satellite links
  c) Radar
  d) Missile
  e) Defense – War Ship
  Fighter Jet , War Aircraft
  f) Medical Application
  g) Commercial Applications





(b) Broadcast link via satellite microwave

- Radar


target
transmitted
pulse
echo pulse
from target
antenna
beamwidth
radar
antenna
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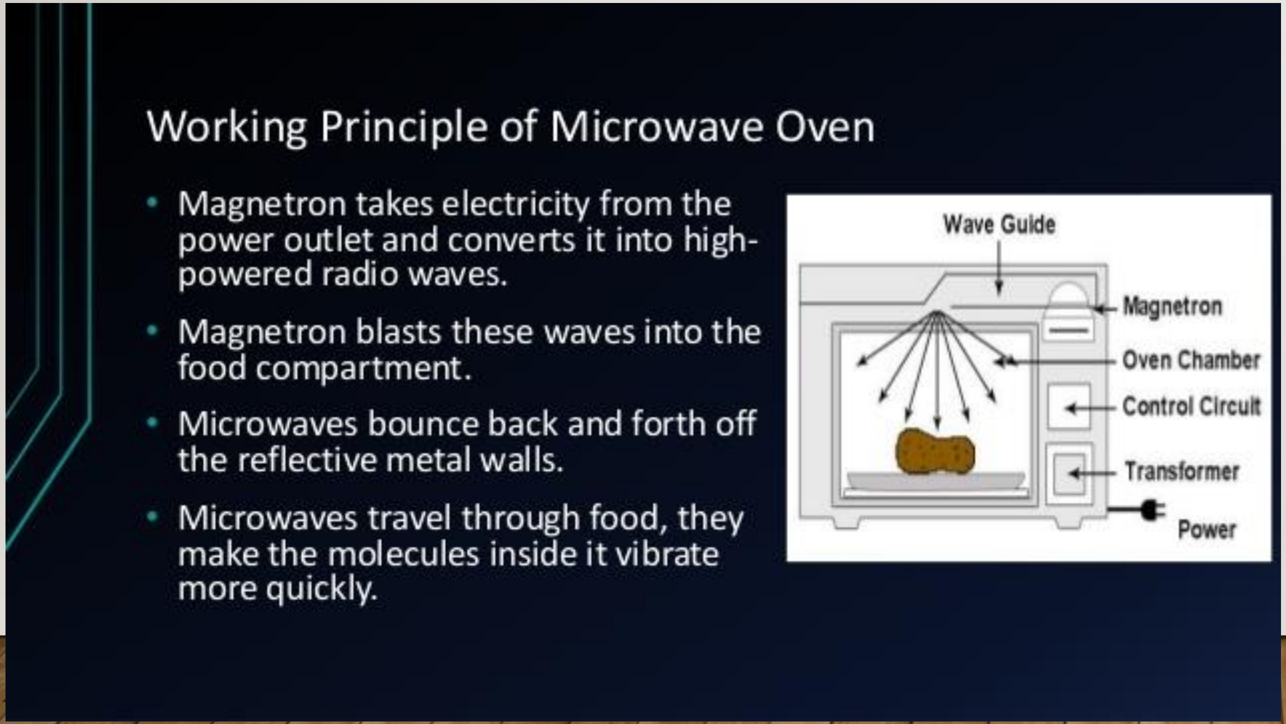
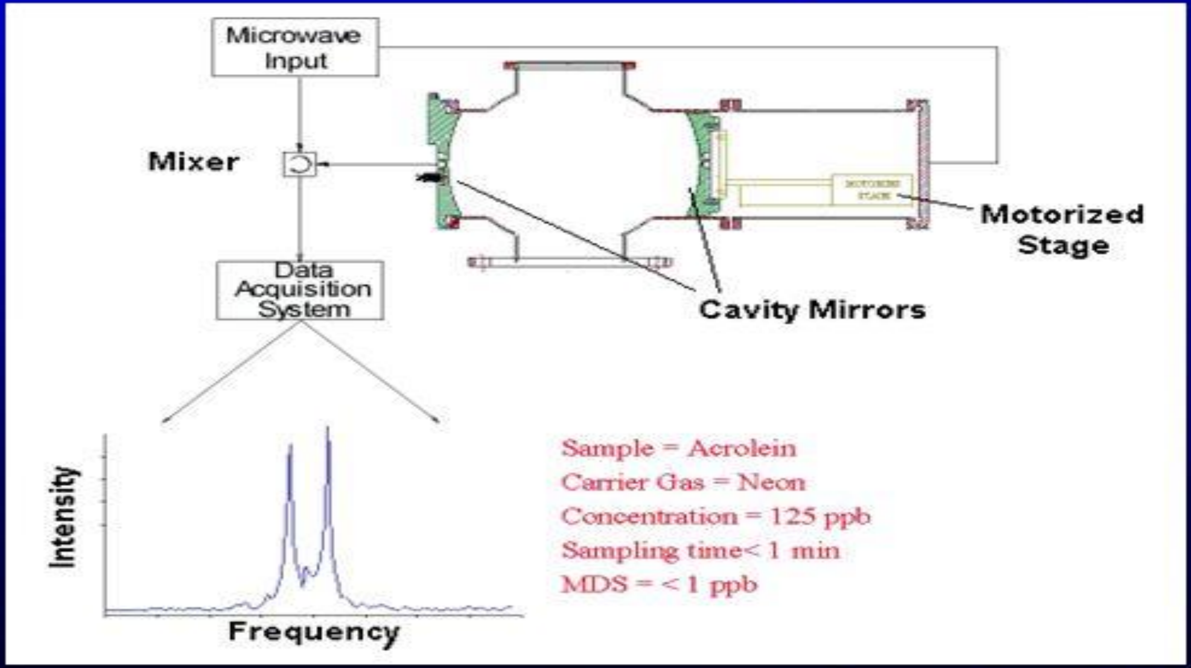
MOBILE RADAR

- Medical Applications- Diathermy
- Home appliances-oven



Applications of FT Microwave Spectroscopy

- Under development for: real-time, sensitive monitoring of gases evolved in process chemistry, plant and vehicle emissions, etc…
  - Current techniques have limits (GC, IR, MS, IMS)
  - Normally use pulsed-nozzle sources and high-precision Fabry-Perot interferometers (PNFTMW)



Compound	Detection Limit (nanomol/mol)
Acrolein	0.5
Carbonyl sulfide	1
Sulfur dioxide	4
Propionaldehyde	100
Methyl-t-butyl ether	65
Vinyl chloride	0.45
Ethyl chloride	2
Vinyl bromide	1
Toluene	130
Vinyl cyanide	0.28
Acetaldehyde	1

Diagram from http://physics.nist.gov/Divisions/Div844/facilities/ftmw/ftmw.html
For more information, see E. Arunan, S. Dev. And P. K. Mandal, *Applied Spectroscopy Reviews*, 39, 131-181 (2004).

WORKING OF SERIES RESONATOR AND PARALLEL RESONATOR

- At resonance frequency , XL=XC, cancels each other , zero reactance , impedance = R , maximum current may flow
- XL > XC inductive circuit
- XL< XC capacitive circuit
- At parallel resonance   IL=IC=IR, equal voltage
- At resonance XL=XC, only Z=R ,

6.3 Rectangular Waveguide Cavity

TE_{mn}, TM_{mn} modes :

$$E_t(x,y,z) = \underline{e}(x,y)\left(A^+e^{-j\beta_{mn}z} + A^-e^{+j\beta_{mn}z}\right)$$

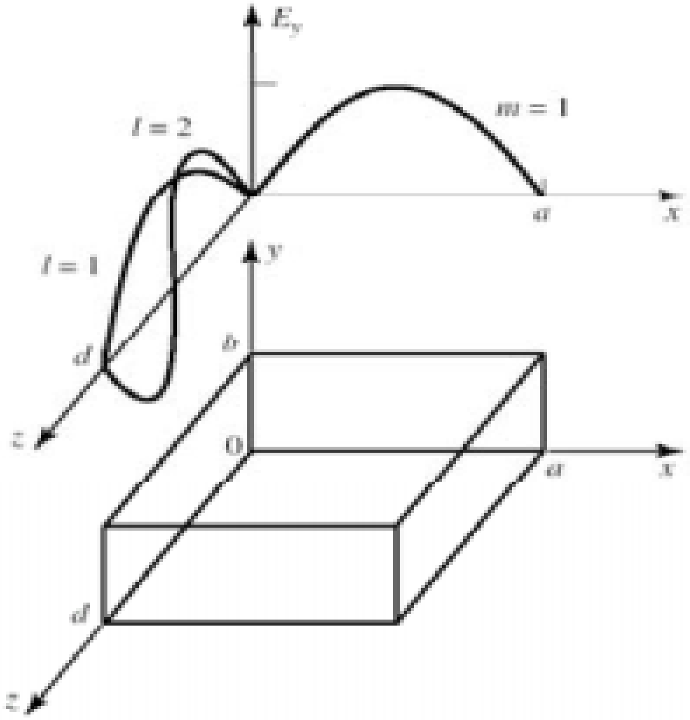
$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

For a resonant cavity, $\underline{E}_t = 0$, at $z = 0, d$

$$\beta_{mn}d = \ell\pi,\ \ \ell = 1,\ 2,\ 3,\ \ldots$$

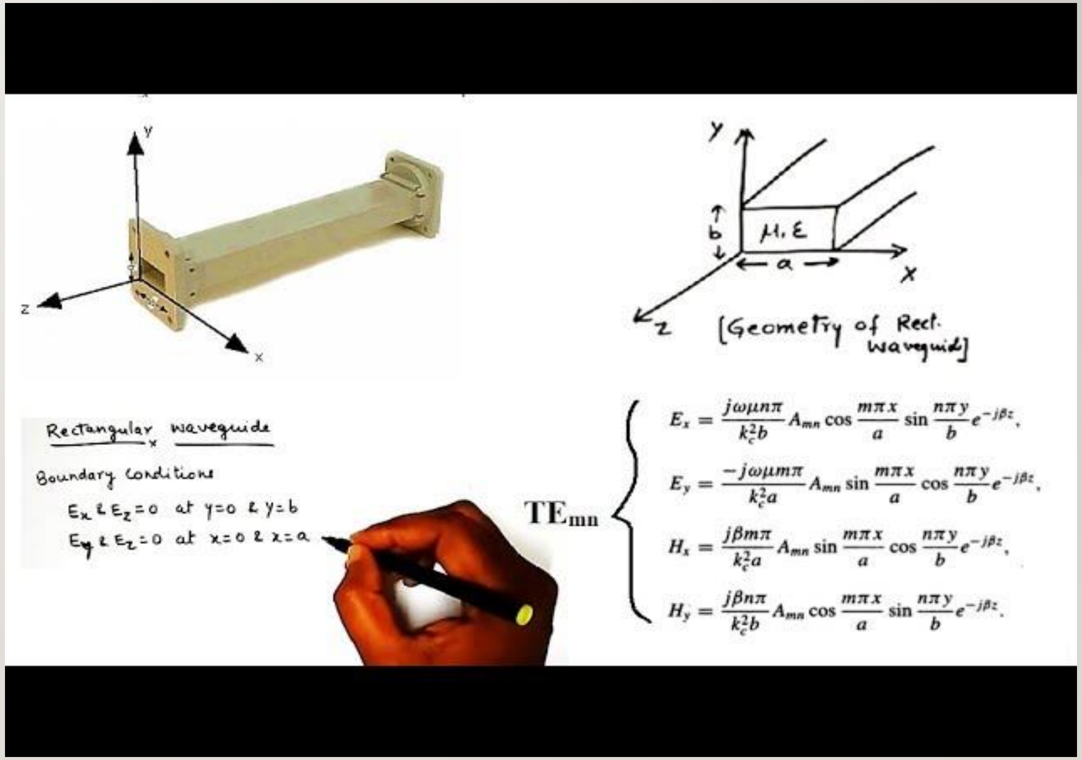
$$k_{mn\ell} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2}$$

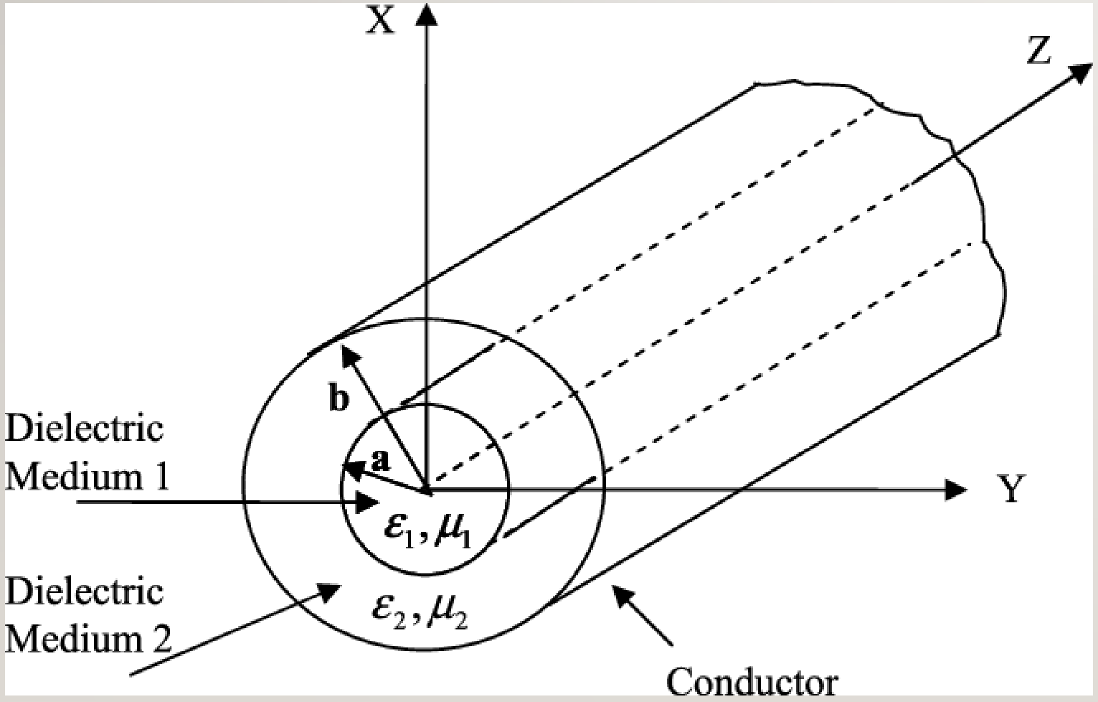
$$f_{mn\ell} = \frac{c}{2\pi}\frac{k_{mn\ell}}{\sqrt{\mu_r\varepsilon_r}}$$



WAVE GUIDES

- Rectangular Wave Guides
  - Rectangular Cavity(Give basic Equations)
  - Rectangular Resonator
- Circular Wave Guides(Give basic equations)
  - Resonator




X
Z
Y
b
a
Dielectric
Medium 1
ε_1, μ_1
Dielectric
Medium 2
ε_2, μ_2
Conductor

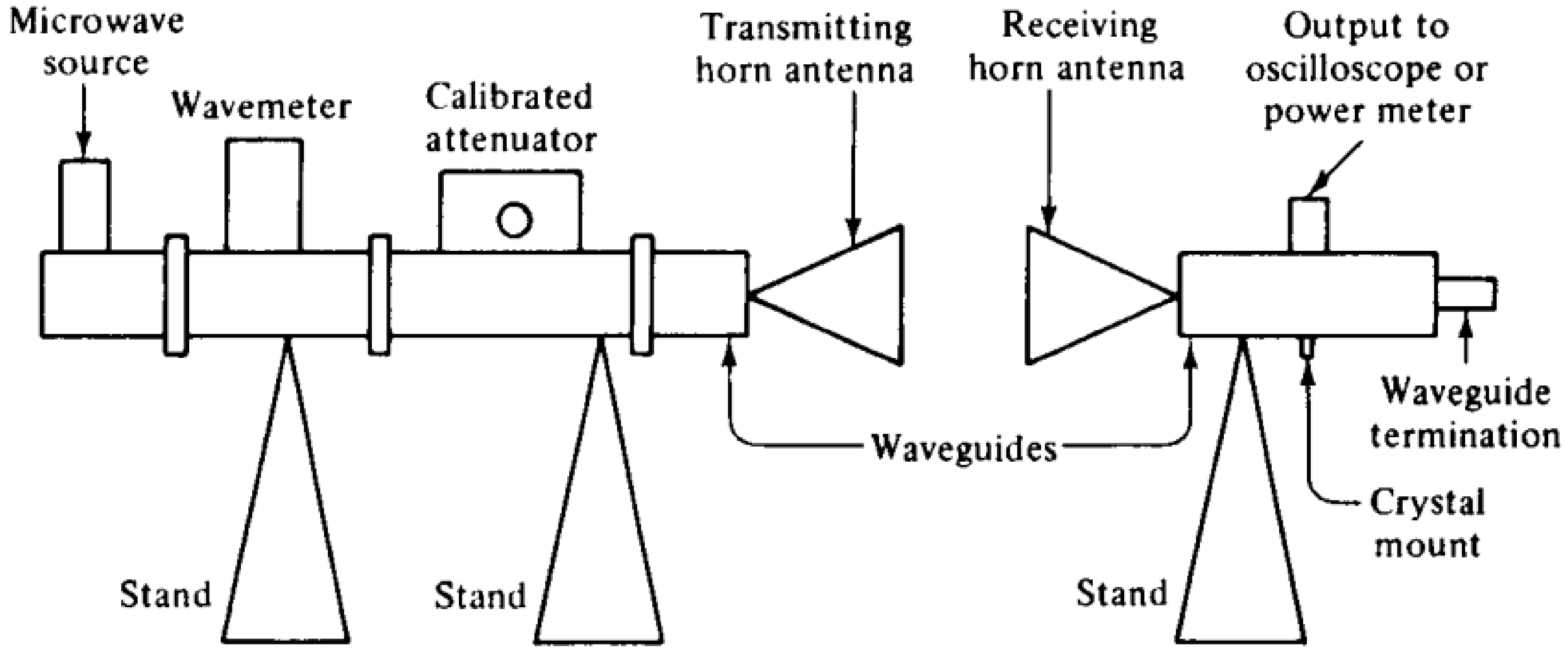
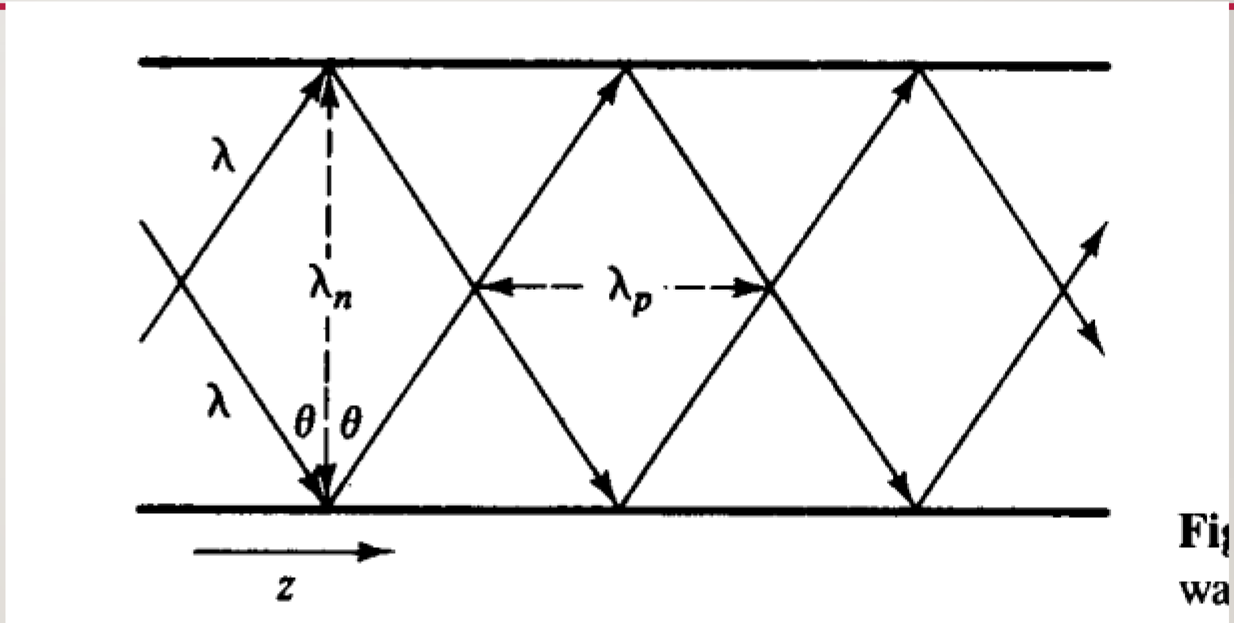


Figure 0-1 Microwave system.

Rectangular Waveguide

1. TE mn
2. TMmn



Maxwell equation

Vector wave equation

$$\nabla^2 \mathbf{E} = \gamma^2 \mathbf{E}$$

$$\nabla^2 \mathbf{H} = \gamma^2 \mathbf{H}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta.$$

MAXWELL EQUATIONS

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla = \frac{\partial}{\partial x}\mathbf{u}_x + \frac{\partial}{\partial y}\mathbf{u}_y + \frac{\partial}{\partial z}\mathbf{u}_z \qquad \text{(cartesian)}$$

$$\nabla = \frac{\partial}{\partial r}\mathbf{u}_r + \frac{\partial}{r\,\partial \phi}\mathbf{u}_\phi + \frac{\partial}{\partial z}\mathbf{u}_z \qquad \text{(cylindrical)}$$

$$\nabla = \frac{\partial}{\partial r}\mathbf{u}_r + \frac{\partial}{r\,\partial \theta}\mathbf{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\mathbf{u}_\phi \qquad \text{(spherical)}$$

$\mathbf{E}$ = *electric field intensity* in volts per meter

$\mathbf{H}$ = *magnetic field intensity* in amperes per meter

$\mathbf{D}$ = *electric flux density* in coulombs per square meter

$\mathbf{B}$ = *magnetic flux density* in webers per square meter or in tesla (1 tesla = 1 weber/m^2 = 10^4 gausses = 3×10^{-6} ESU)

$\mathbf{J}$ = *electric current density* in amperes per square meter

ρ_v = *electric charge density* in coulombs per cubic meter

he electric current density includes two components—that is,

$$\mathbf{J} = \mathbf{J}_c + \mathbf{J}_0 \qquad ($$

here $\mathbf{J}_c = \sigma \mathbf{E}$ is called the *conduction current density*

$\mathbf{J}_0$ = the *impressed current density*, which is independent of the field

- For Maxwell equation , the characteristics of the medium the field exist are needed to specify the flux interms of the fields in a specific medium ,

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{J}_c = \sigma \mathbf{E}$$

$$\epsilon = \epsilon_r \epsilon_0$$

$$\mu = \mu_r \mu_0$$

where ϵ = dielectric permittivity or capacitivity of the medium in farads per meter

ϵ_r = relative dielectric constant (dimensionless)

$\epsilon_0 = 8.854 \times 10^{-12} \approx 1/(36\pi) \times 10^{-9}$ F/m is the dielectric permittivity of vacuum or free space

μ = magnetic permeability or inductivity of the medium in henrys per meter

μ_r = the relative permeability or relative inductivity (dimensionless)

$\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of vacuum or free space

σ = conductivity of the medium in mhos per meter

If a sinusoidal time function in the form of $e^{j\omega t}$ is assumed, $\partial/\partial t$ can be replaced by $j\omega$. Then Maxwell's equations in frequency domain are given by

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{2-1-11}$$

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} \tag{2-1-12}$$

$$\nabla \cdot \mathbf{D} = \rho_v \tag{2-1-13}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2-1-14}$$

The electric wave equation is $\nabla^2\mathbf{E} = \gamma^2\mathbf{E}$ The magnetic wave equation $\nabla^2\mathbf{H} = \gamma^2\mathbf{H}$

RESONATOR

- Tunable Microwave circuits
- Used in Oscillators, Amplifiers, wave meters and filters
- At the tuned frequency impedance is real
- Average energy stored in electric field= Average energy stored in magnetic field
- Total energy = 2(electric energy stored in resonator)
- Total energy is maximum at resonant frequency
- LC circuit can be used for tuned circuit
- Time average electric field becomes equal to average energy stored in magnetic fields

RECTANGULAR CAVITY RESONATOR

$$H_z = H_{0z}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right) forTE_{mnp}$$

$$E_z = E_{0z}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{p}\right) forTM_{mnp}$$

$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 = \omega^2\mu\varepsilon$$

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \qquad (\mathrm{TE}_{mnp},\ \mathrm{TM}_{mnp})$$

$$a \succ b \prec d$$

- Resonant frequency fr at which energy of the resonator becomes maximum

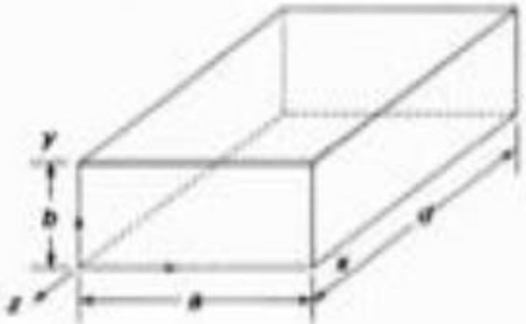
$$Q = \frac{2\pi \times \text{maximum energy stored}}{\text{energy dissipated per cycle}} = \frac{f_r}{3\,dB\ \text{Bandwidth}}$$

- Input impedance of the resonator- indicating matching performance
- Resonant frequency fr at which energy of resonator becomes maximum.

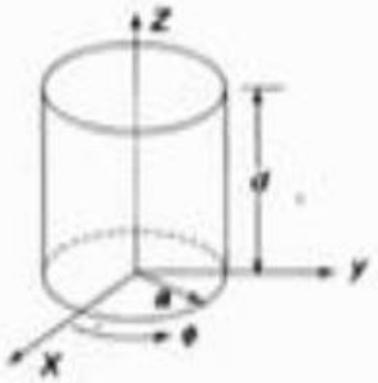
[Link for video of rectangular cavity]

For $a > b < d$, the dominant mode is the TE_{101} mode.

Waveguide Cavity Resonators



Rectangular Cavity

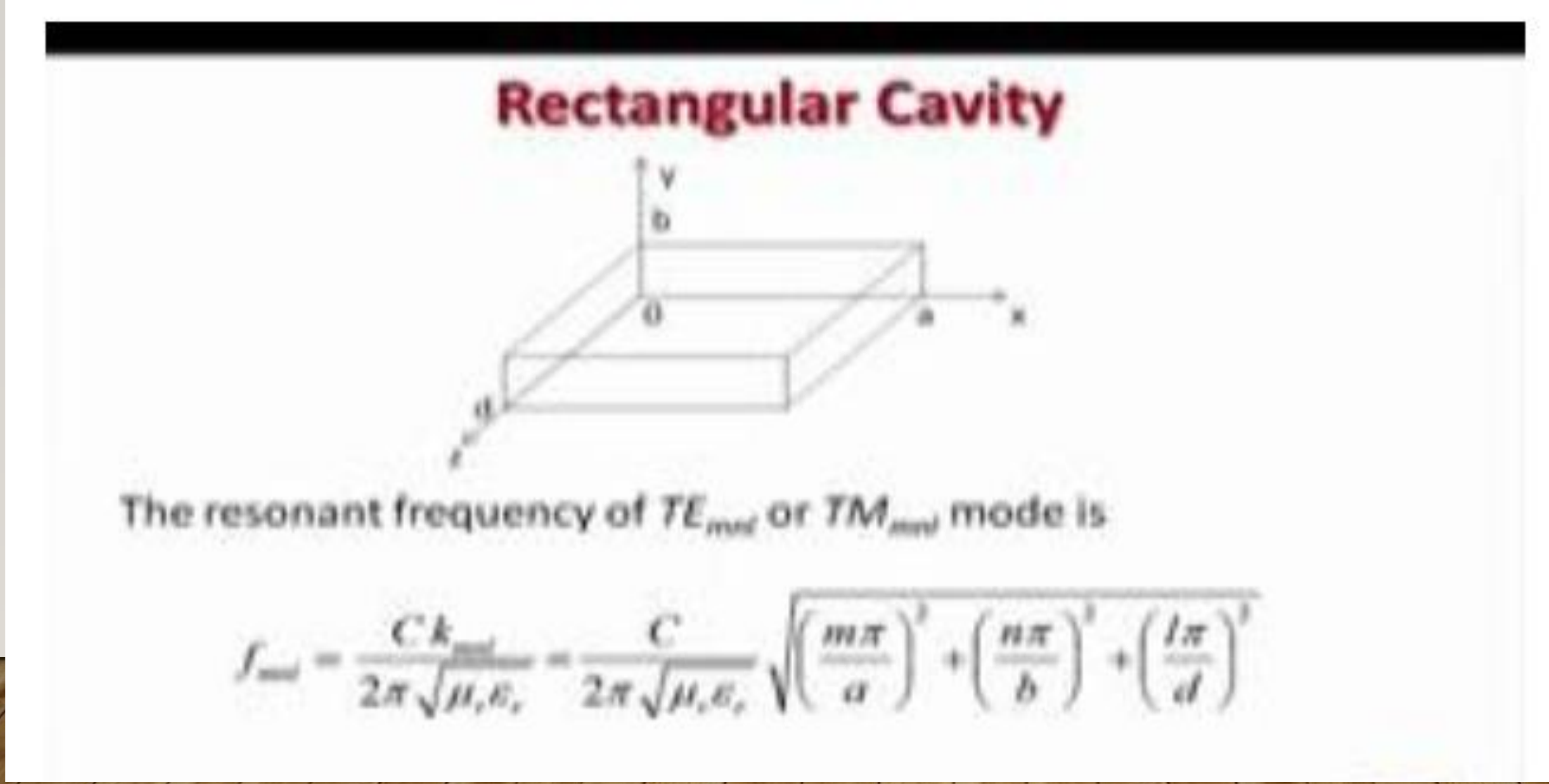


Circular Cavity

- RLC Circuit

CAVITY RESONATOR

- In rectangular resonator TE /TM dominant mode is 101
- These cavities are RLC circuits



CIRCULAR CAVITY

- For circular cavity resonator

$$H_z = H_{0z} J_n\left(\frac{X'_{np} r}{a}\right) \cos(n\phi) \sin\left(\frac{q\pi z}{d}\right) \qquad (\mathrm{TE}_{npq})$$

where $n = 0, 1, 2, 3, \ldots$ is the number of the periodicity in the ϕ direction
$p = 1, 2, 3, 4, \ldots$ is the number of zeros of the field in the radial direction
$q = 1, 2, 3, 4, \ldots$ is the number of half-waves in the axial direction
J_n = Bessell's function of the first kind
H_{0z} = amplitude of the magnetic field

$$E_z = E_{0z} J_n\left(\frac{X_{np} r}{a}\right) \cos(n\phi) \cos\left(\frac{q\pi z}{d}\right) \qquad (\mathrm{TM}_{npq})$$

where $n = 0, 1, 2, 3, \ldots$
$p = 1, 2, 3, 4, \ldots$
$q = 0, 1, 2, 3, \ldots$
E_{0z} = amplitude of the electric field

The separation equations for TE and TM modes are given by

$$k^2 = \left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2 \qquad \text{(TE mode)}$$

$$k^2 = \left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2 \qquad \text{(TM mode)}$$

$$k^2 = \omega^2 \mu \epsilon$$

$$f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \qquad \text{(TE)}$$

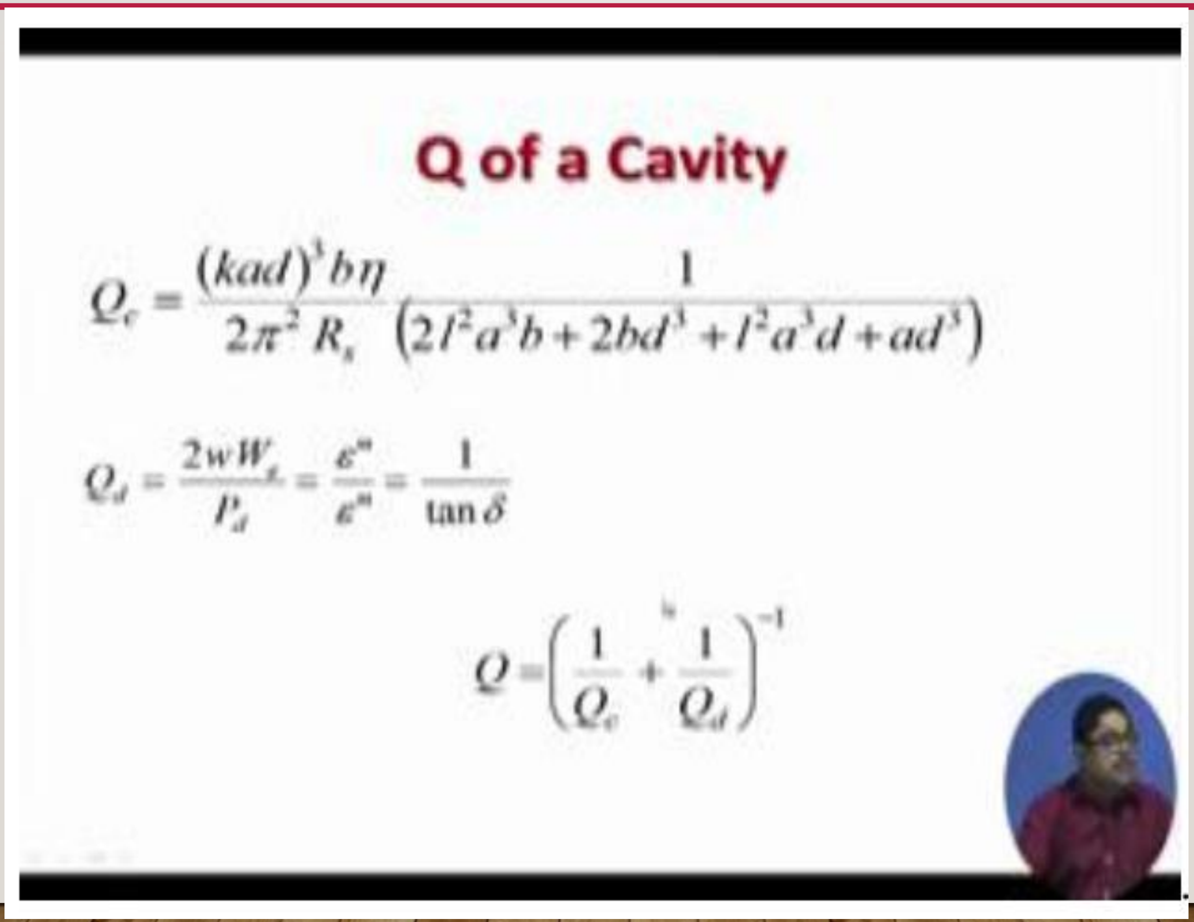
$$f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \qquad \text{(TM)}$$

TM_{110} mode is dominant where $2a > d$,

TE_{111} mode is dominant when $d \geq 2a$.

- Qc= Which is the loss due to conductor
- Qd= Loss due to dielectric

Rectangular wave guide YouTube link https://www.youtube.com/watch?v=78YmiNQuBuQ



Q of a Cavity

$$Q_c = \frac{(kad)^3 b\eta}{2\pi^2 R_s}\frac{1}{\left(2l^2a^3b + 2bd^3 + l^2a^3d + ad^3\right)}$$

$$Q_d = \frac{2wW_e}{P_d} = \frac{\epsilon'}{\epsilon''} = \frac{1}{\tan\delta}$$

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1}$$

CIRCULAR RESONATOR

Circular Waveguide Cavity

Dominant mode is TE_{111}

Used in frequency meters.

Often TE_{011} mode is used as its Q is higher than TE_{111}

Frequency resolution of the frequency meter depends on Q.

So, cavity is loosely coupled to the guide.

It's top wall is tunable.

CIRCULAR RESONATOR

$$H_z = H_{0z} J_n\left(\frac{X^{!}_{npr}}{a}\right)\cos(n\phi)\sin\left(\frac{q\pi z}{d}\right)\ for TE_{npq}$$

$$E_z = E_{0z} J_n\left(\frac{X_{np} r}{a}\right)\cos(n\phi)\cos\left(\frac{q\pi z}{d}\right)\ for TM_{npq}$$

$$k^2 = \left(\frac{X^{!}_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2\ TE \text{ mode}$$

$$\mathrm{k}^2 = \left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2\ TM \text{ mode}$$

$$\mathrm{f}_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{X^{!}_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2}\ for TE \text{ mode}$$

$$\mathrm{f}_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2}\ for TM \text{ mode}$$

QUALITY FACTOR

- Frequency selectivity of a resonant circuit
- Q=2$\prod\frac{maximum\ energy\ stored}{energy\ dissipated\ per\ cycle}$=ω$\frac{W}{P}$ or
- Q=$\frac{ωμ(volume)}{2Rs(surface\ area)}$
- Q_l = loaded Q
- Q_0 = unloaded Q
- Q_{ext} = External Q

$$Q \equiv 2\pi \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}} = \frac{\omega W}{P}$$

$$Q = \frac{\omega\mu\,(\text{volume})}{2R_s(\text{surface areas})}$$

- Draw circuit from Lio:

$$Q_l = \frac{\omega_0 L}{R(1+K)} = \frac{Q_0}{1+K}$$

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q_0 = \frac{\omega_0 L}{R}$$

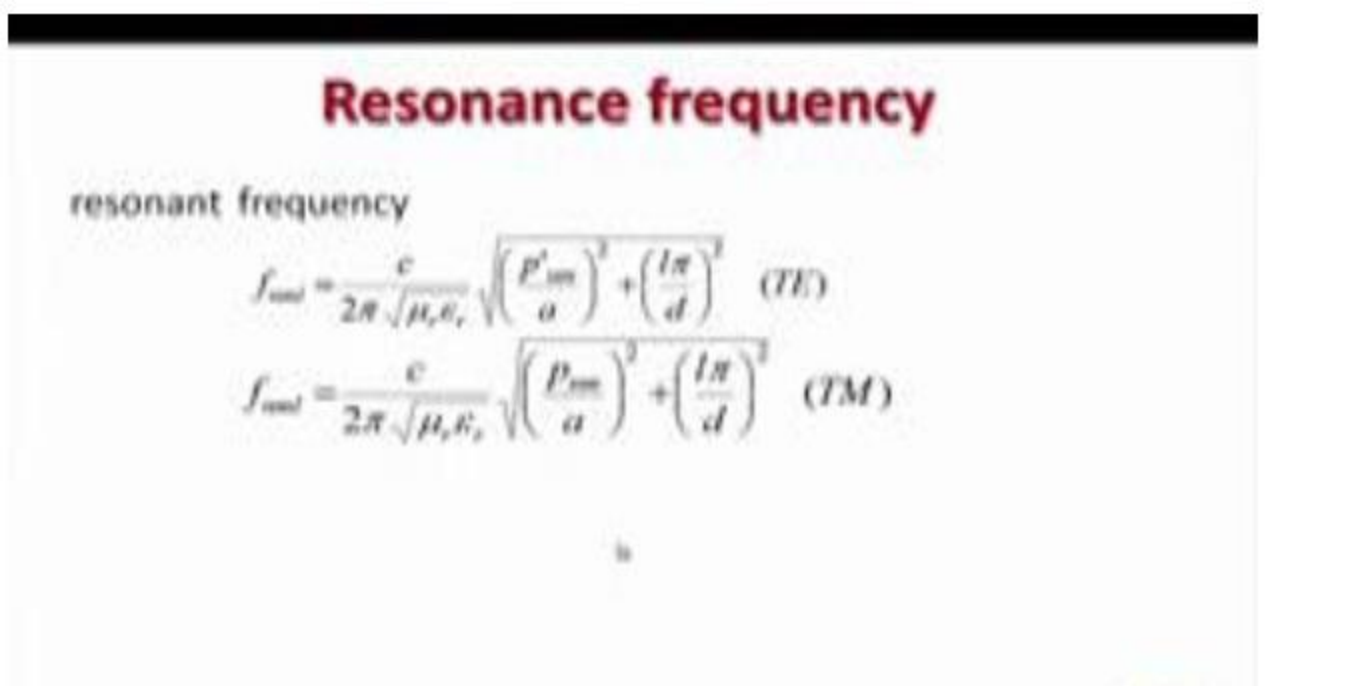
$$Q_\ell = \frac{\omega_0 L}{R(1+K)} = \frac{Q_0}{1+K}$$

- Critical Coupling K=1 ,

$$Q_\ell = \tfrac{1}{2} Q_{ext} = \tfrac{1}{2} Q_0$$

Circular wave guides yutube link
https://www.youtube.com/watch?v=p827vMMNuDY

- In circular wave guide dominant mode is TE111
- Resonance frequency of circular waveguide fr is
- Q of the circular resonator Q=



Q of the Circular Waveguide Cavity

$$Q_C = \frac{W_0 W}{P_c} = \frac{(ka)^3 \eta a d}{4 (p'_{nm})^2 R_s} \times \frac{1 - \left(\frac{n}{p'_{nm}}\right)^2}{\left\{ \frac{ad}{2}\left[1 + \left(\frac{\beta a n}{(p'_{nm})^2}\right)^2\right] + \left(\frac{\beta a^2}{p'_{nm}}\right)^2 \left(1 - \frac{n^2}{(p'_{nm})^2}\right)\right\}}$$

$$Q_d = \frac{wW}{P_d} = \frac{\varepsilon}{\varepsilon''} = \frac{1}{\tan\delta}$$

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1}$$

MICROWAVE TUBES

Reflex klystron, Magnetron, Travelling Wave Tubes(TWT)

GENERATION OF MICROWAVE SIGNAL

- Microwave Tubes – klystron, reflex klystron, magnetron and TWT.

- Diode semiconductor – Tunnel, Gunn, Impatt, Varactor diodes, PIN, LSA, Schottky barrier diode.

- **CROSSED-FIELD AND LINEAR-BEAM TUBES**
- Klystrons and Traveling-Wave tubes are examples of linear-beam tubes
  - These have a focused electron beam (as in a CRT)
- Magnetron is one of a number of crossed-field tubes
  - Magnetic and electric fields are at right angles

INTRODUCTION

- Microwave tubes are constructed to overcome the limitations with conventional tubes and UHF tubes.

Principle of Operation:-

- The basic principle of operation of microwave tube involves transfer of power from a source of DC voltage to source of AC voltage by means of a current density modulated electron beam.
- The same can be achieved by accelerating electrons in a static electric field and retarding them in an AC field.

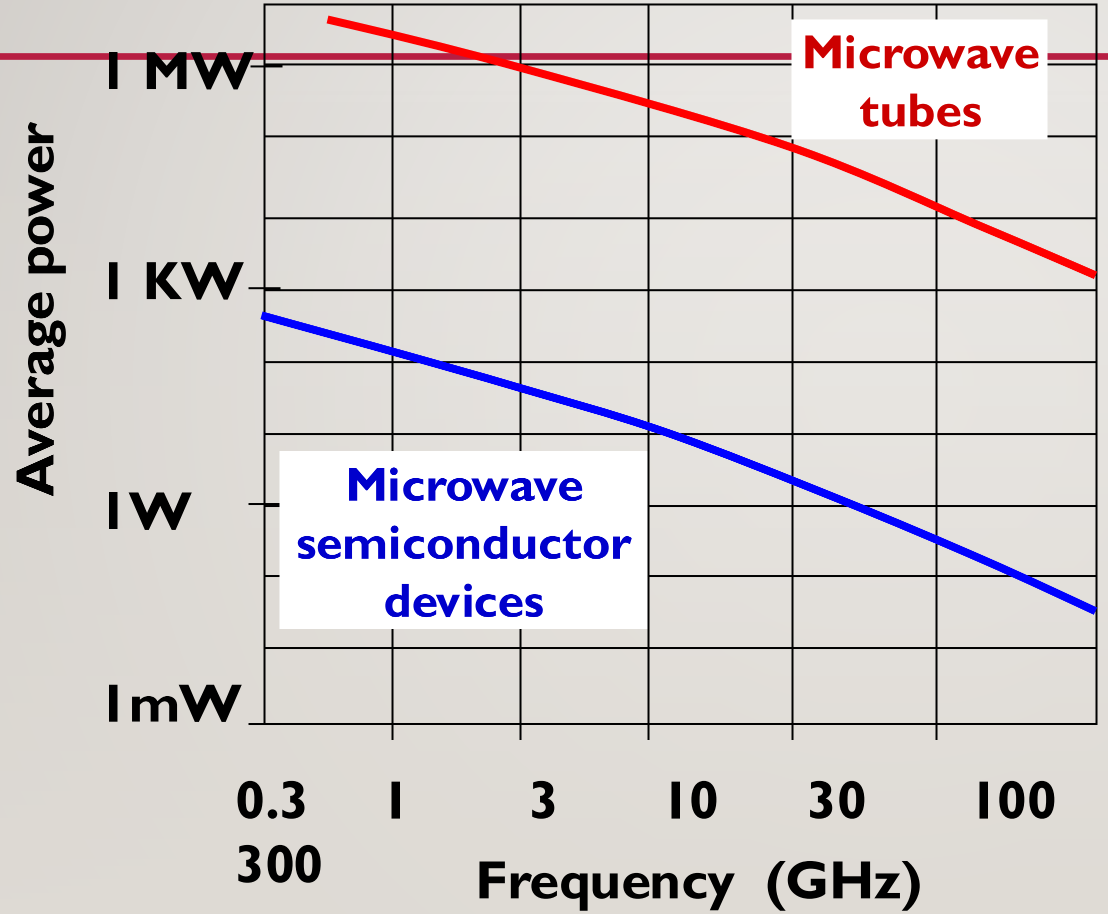
CONTD..

- The density modulation of the electron beam allows more electrons to be retarded by an AC field than accelerated by DC field, which therefore makes possible a net energy be delivered to the ac electric field.

Classification of microwave tubes:-

- The classification is based on different factors.
  - Their mechanism of producing density modulation.
  - The acceleration and retardation of electrons in the ac field.
- Important types are:-
  - Klystron amplifier (two cavity, multi cavity)
  - Klystron oscillator (reflex klystron)
  - Magnetron
  - Traveling Wave Tube(TWT)

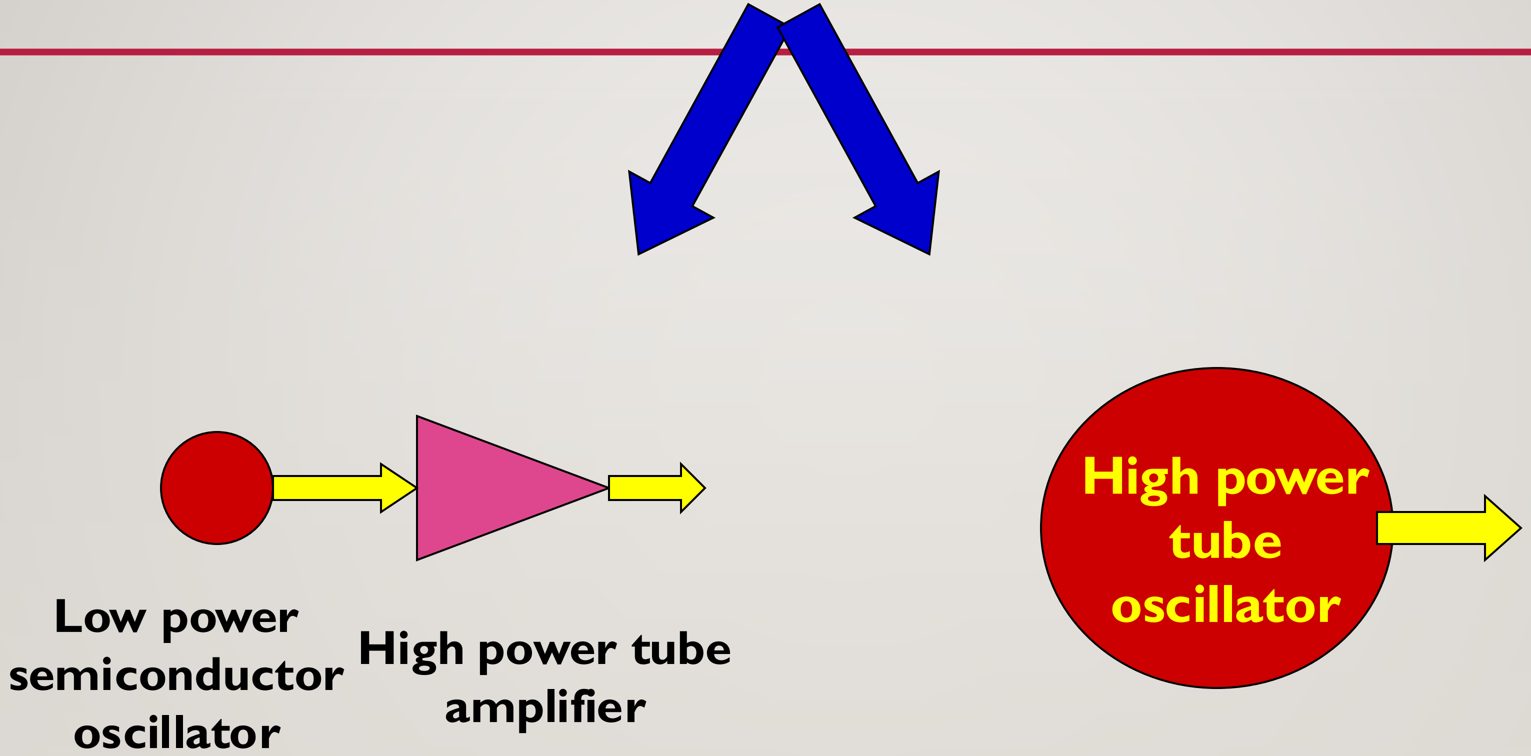
MICROWAVE TUBES



- Higher power
- Limited life time
- High vacuum
- High potential

- Lower weight
- Smaller size
- Longer life time

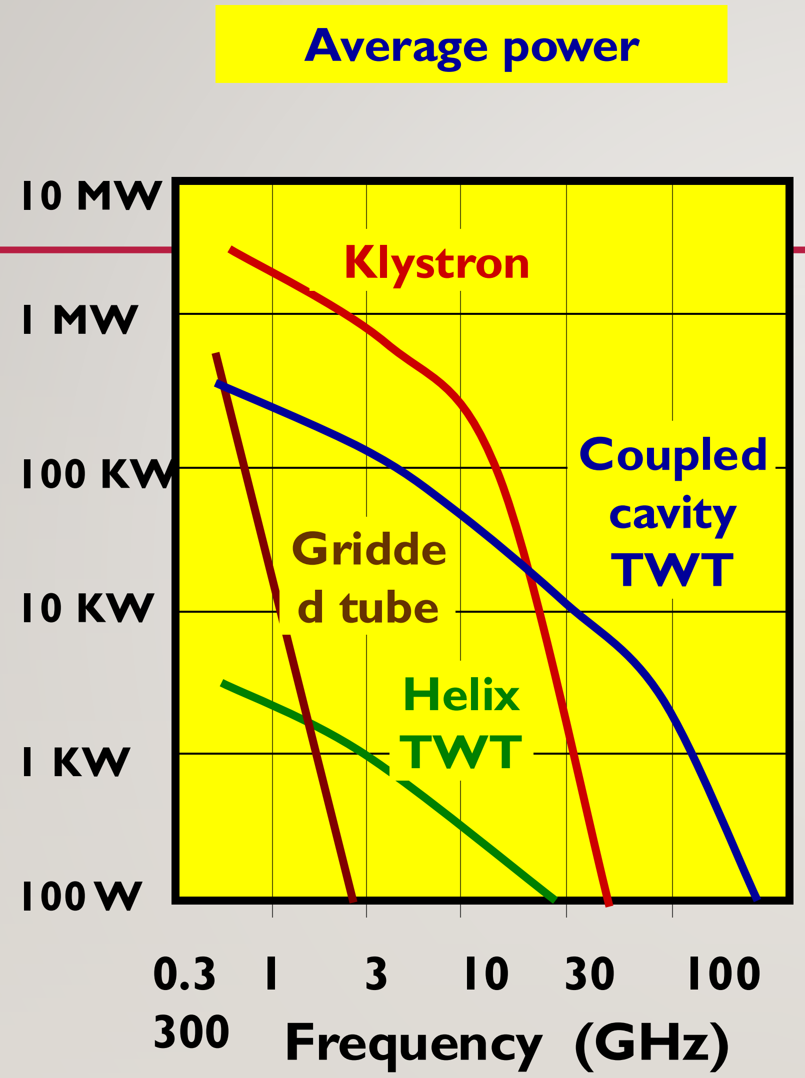
Two possible methods of achieving high output power in microwave system



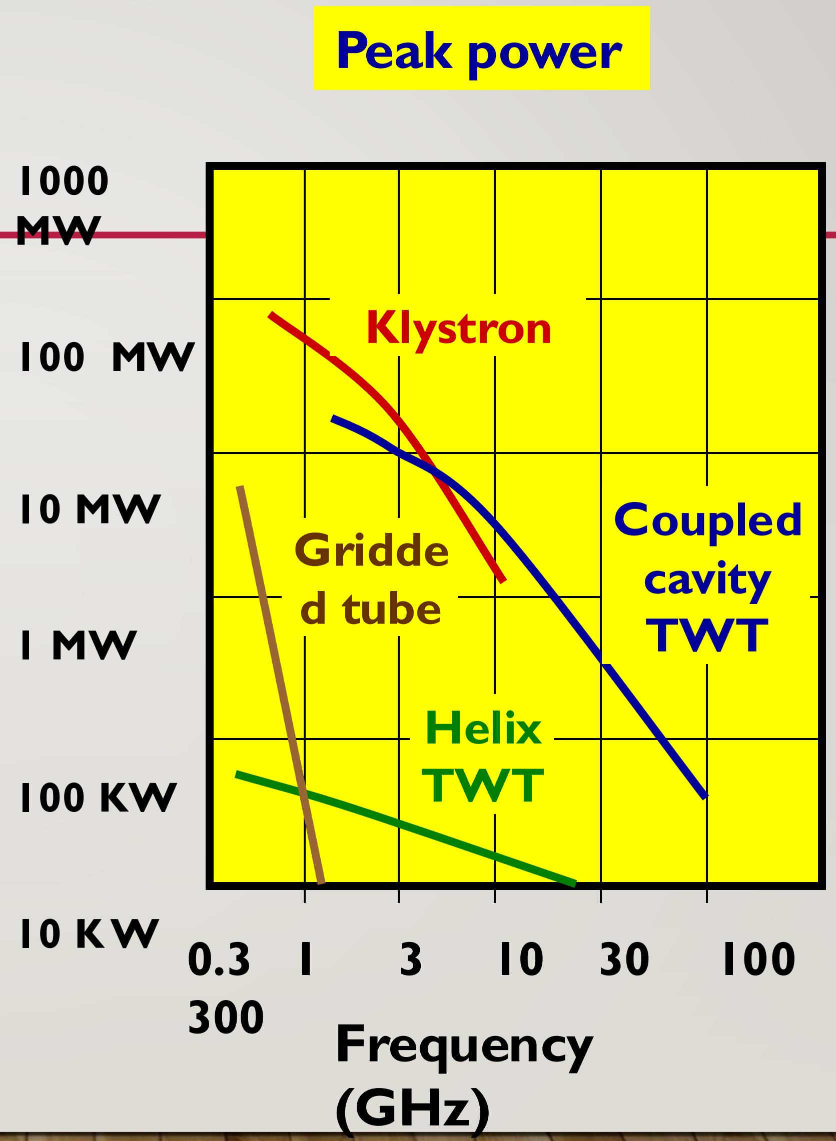
Important Parameters

- Peak power
- Average power
- Efficiency
- Gain
- Bandwidth
- Frequency
- Harmonic and spurious power
- Intermodulation products
- Manufacturability at low cost

Type	Relative BW (%)	η (%)	Gain (dB)	Relative spurious level	Relative operating voltage	Relative complexity of operation
Gridded tube	1-10	20-50	6-15	2	Low	1
Klystron	1-5	30-70	40-60	1	High	2
Helix tube	30-120	20-40	30-50	3	High	3
Coupled cavity tube	5-40	20-40	30-50	3	High	3

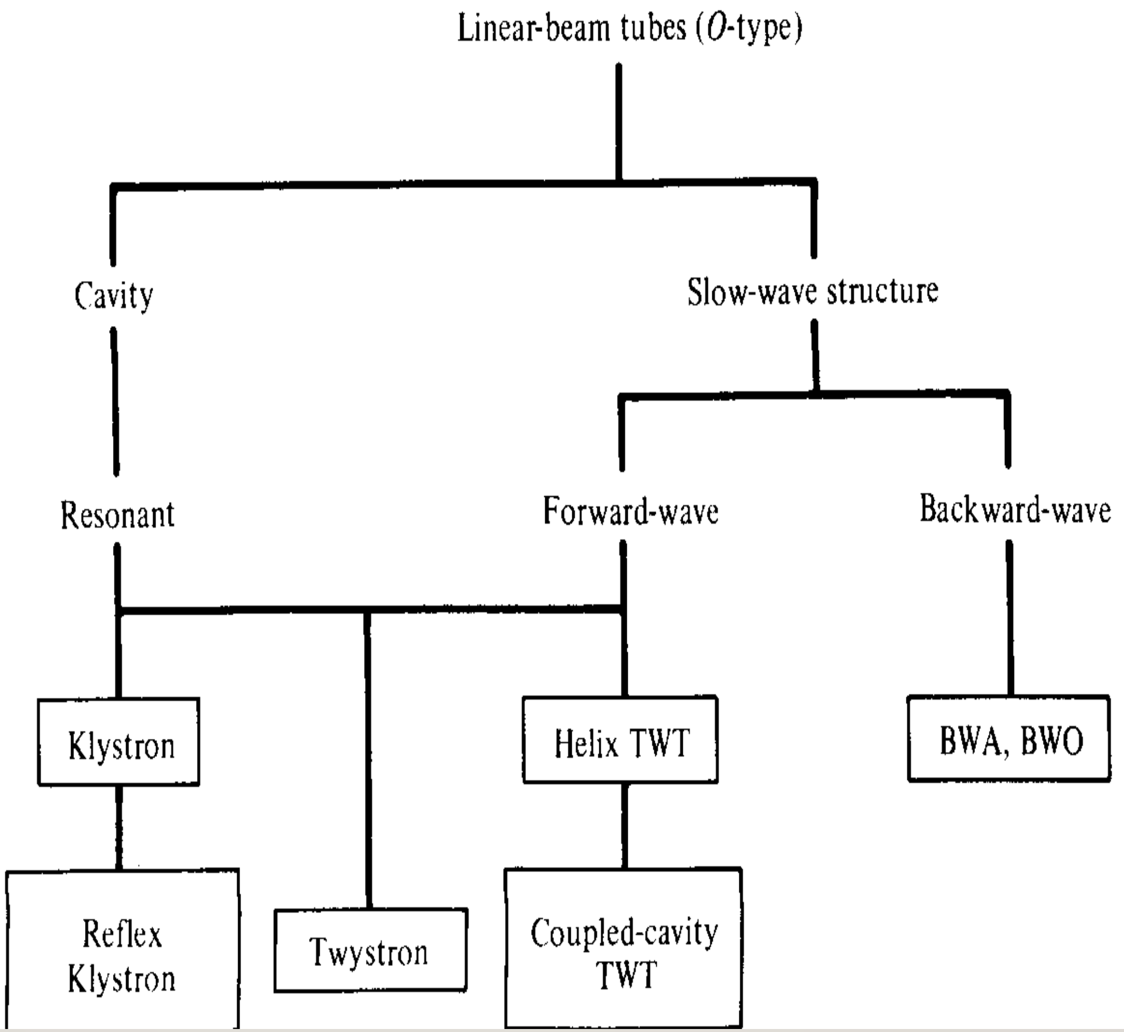


Average power
10 MW
1 MW
100 KW
10 KW
1 KW
100 W
Klystron
Coupled cavity TWT
Gridde d tube
Helix TWT
0.3
1
3
10
30
100
300
Frequency (GHz)



Peak power
1000 MW
100 MW
10 MW
1 MW
100 KW
10 KW
Klystron
Coupled cavity TWT
Gridde d tube
Helix TWT
0.3
1
3
10
30
100
300
Frequency (GHz)

TABLE 9-0-1 LINEAR BEAM TUBES (O TYPE)



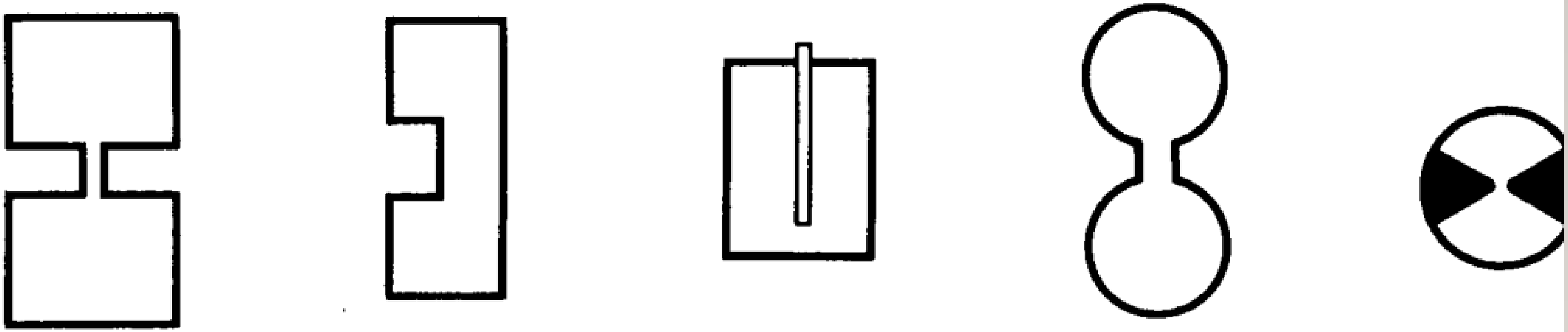
CONVENTIONAL TUBES

- Conventional Device tubes cannot be used for frequencies above 100MHz
- 1. Interelectrode capacitance
- 2. Lead Inductance effect
- 3. Transit time effect
- 4. Gain Bandwidth limitation
- 5. Effect of RF losses (Conductance, dielectric)
- 6. Effect due to radiation losses

- Efficient Microwave tubes usually operate on the theory of electron **velocity modulation** concept
- The electron **transit time** is used in the conversion of dc power to RF power

REENTRANT CAVITIES

- A rentrant cavity in which the metallic boundaries extends into the interior of the cavities.
- The shelf shielding enclosures prevents the radiation losses
- Shorted coaxial line stores more magnetic energy than electric energy
- At resonance the magnetic and electric stored energies are equal



(a) (b) (c) (d) (e)

Figure 9-2-3 Reentrant cavities. (a) Coaxial cavity. (b) Radial cavity. (c) Tunable cavity. (d) Toroidal cavity. (c) Butterfly cavity.

REFLEX KLYSRON

- A fraction of output is feed back to input with 360 degree phase shift- positive feedback .
- It is a low power oscillator
- Single cavity Klystron
- Low power generator, 10-500mw power, 1 to 25 Ghz range.
- The microwave frequencies are generated due to velocity modulation.

PERFORMANCE CHARACTERISTICS

1. Frequency: 4 – 200 GHz
2. Power: 1 mW – 2.5 W
3. Theoretical efficiency : 22.78 %
4. Practical efficiency : 10 % - 20 %
5. Tuning range : 5 GHz at 2 W – 30 GHz at 10 mW

KLYSTRON OSCILLATOR

- Klystron amplifier can be converted into oscillator by feeding back a part of catcher output into the buncher in proper phase.
- But the feed backing should be satisfy Barkhausen criterion.
  - Aβ = 1
- The schematic is same as of amplifier expect the feedback need to be added.
- The feedback must be adjusted to give correct polarity and amplitude which is basically depends on cavity tuning.
- The criterion for oscillation is given by

$$\theta + \alpha + \frac{\pi}{2} = 2\pi n \, radians$$

  - Where α+ π/2 is the phase angle difference between buncher and catcher cavity.
  - ϴ is the total phase shift between resonator and the feedback cable.

CONTD…

- If the value of Θ is zero means the oscillations are in phase.
- The maximum power output is obtained at this condition.
- Also when a small change in the dc accelerating voltage it cause change in the frequency since transit angle α varies.
- Tuning of the oscillator is done by adjusting the grid voltage, accelerating voltage and tuning the cavities.
- High frequency oscillations are obtained by controlling the temperature of the resonators.

BUNCHER

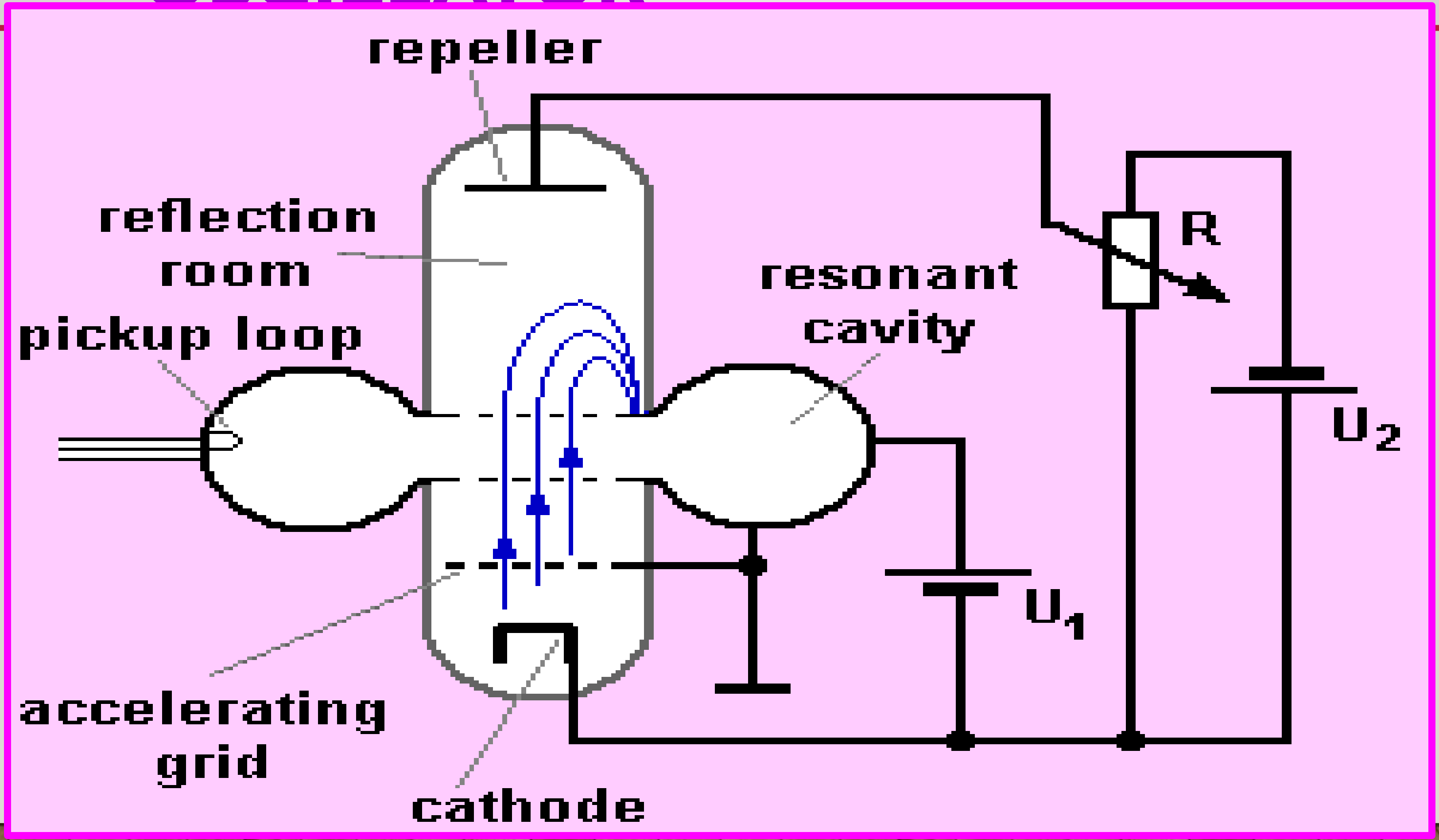
- The cathode controls the number of electrons in the electron beam and focuses the beam. The voltage between the cathode and the cavity resonators (the buncher and the catcher, which serve as reservoirs of electromagnetic oscillations) is the accelerating potential and is commonly referred to as the beam voltage. This voltage accelerates the DC electron beam to a high.

- Yutube link-reflexklystron https://www.youtube.com/watch?v=3MFHFuddLZI

REFLEX KLYSTRONS

- The reflex klystron has been the most used source of microwave power in laboratory applications.

REFLEX KLYSTRON OSCILLATOR



CONSTRUCTION

- A reflex klystron consists of an electron gun, a cavity with a pair of grids and a repeller plate as shown in the above diagram.
- In this klystron, a single pair of grids does the functions of both the buncher and the catcher grids.
- The main difference between two cavity reflex klystron amplifier and reflex klystron is that the output cavity is omitted in reflex klystron and the repeller or reflector electrode, placed a very short distance from the single cavity, replaces the collector electrode.

WORKING

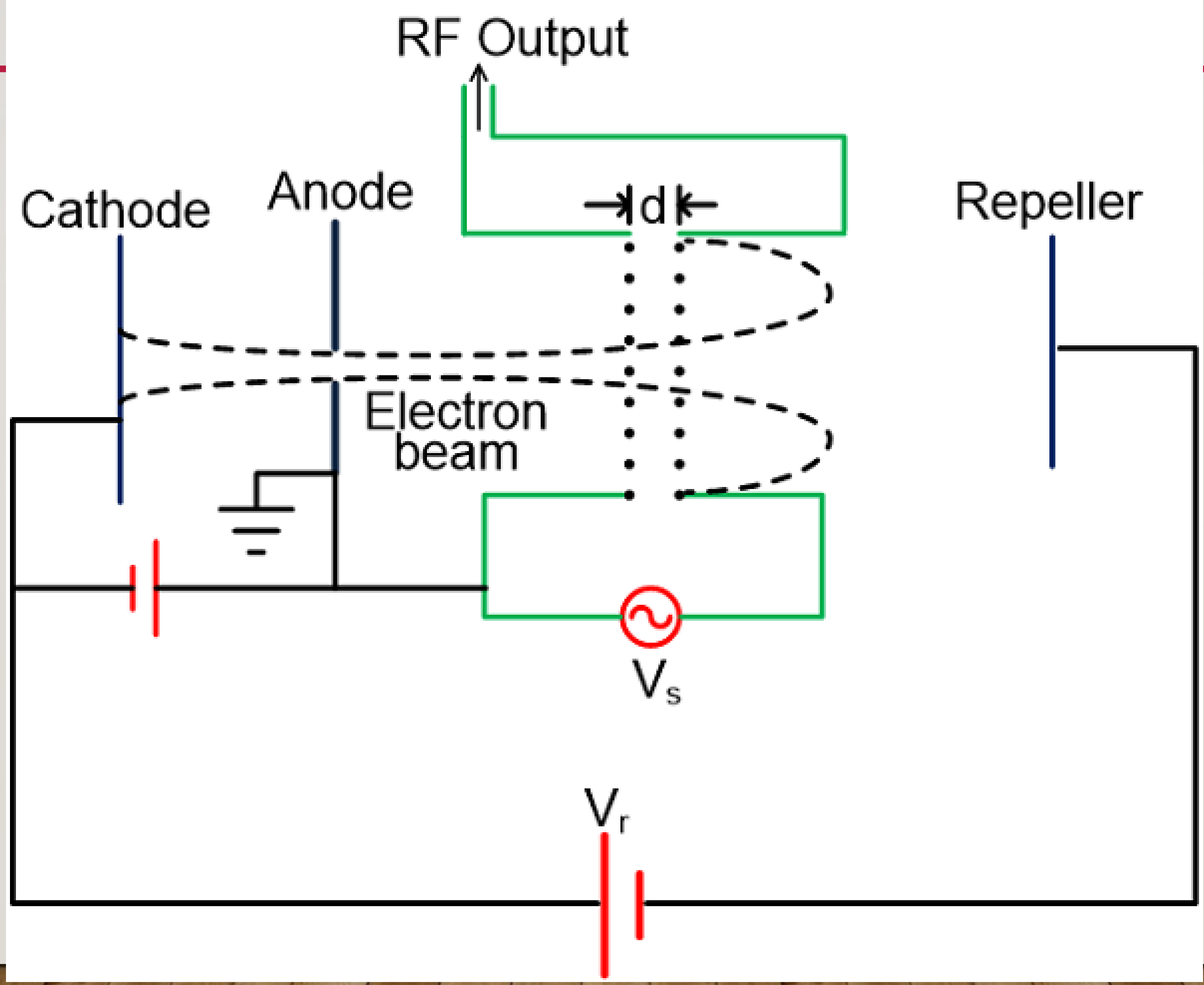
- The feedback necessary for electrical oscillations is developed by reflecting the electron beam, the velocity modulated electron beam does not actually reach the repeller plate, but is repelled back by the negative voltage.
- The point at which the electron beam is turned back can be varied by adjusting the repeller voltage.
- Thus the repeller voltage is so adjusted that complete bunching of the electrons takes place at the catcher grids, the distance between the repeller and the cavity is chosen such that the repeller electron bunches will reach the cavity at proper time to be in synchronization.
- Due to this, they deliver energy to the cavity, the result is the oscillation at the cavity producing RF frequency.

- The electron beam passes through a single resonant cavity.
- The electrons are fired into one end of the tube by an electron gun.
- After passing through the resonant cavity they are reflected by a negatively charged reflector electrode for another pass through the cavity, where they are then collected.
- The electron beam is velocity modulated when it first passes through the cavity.

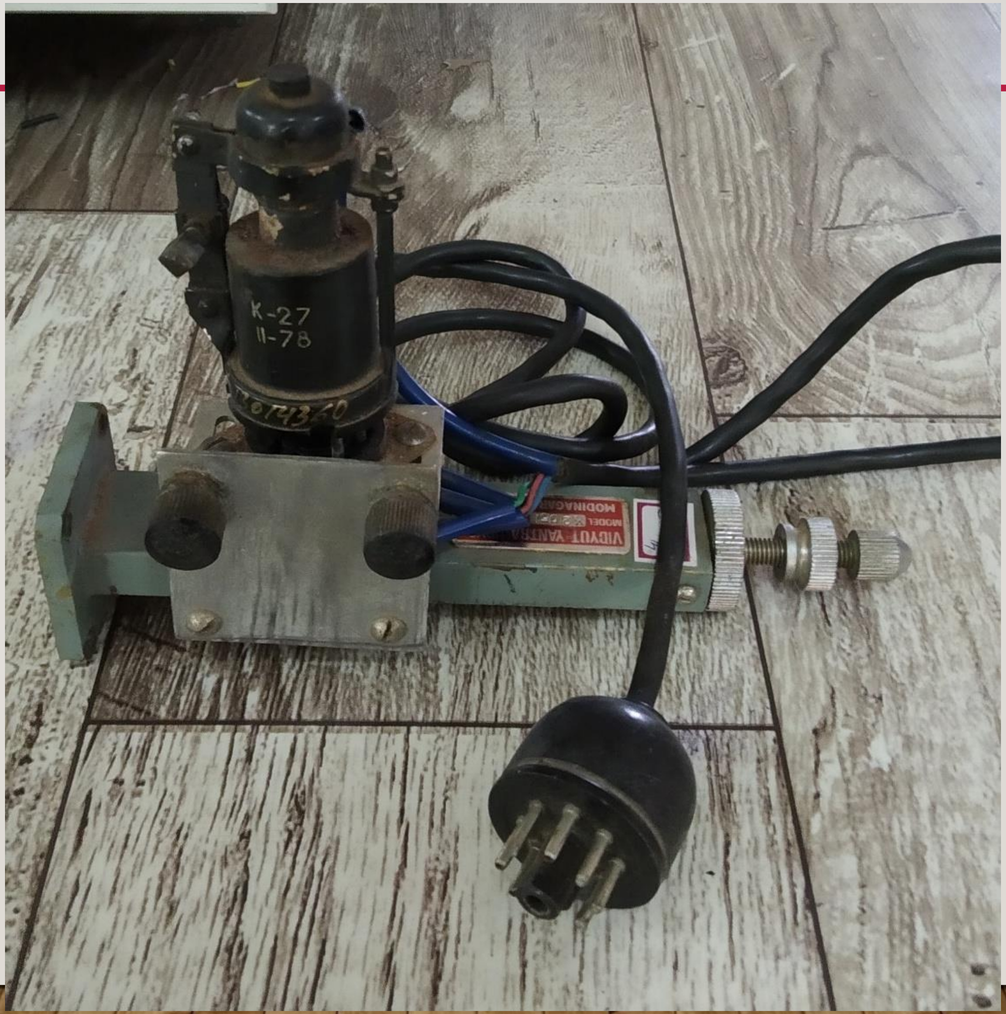
- The formation of electron bunches takes place in the drift space between the reflector and the cavity.
- The voltage on the reflector must be adjusted so that the bunching is at a maximum as the electron beam re-enters the resonant cavity, thus ensuring a maximum of energy is transferred from the electron beam to the RF oscillations in the cavity.
- The voltage should always be switched on before providing the input to the reflex klystron as the whole function of the reflex klystron would be destroyed if the supply is provided after the input.

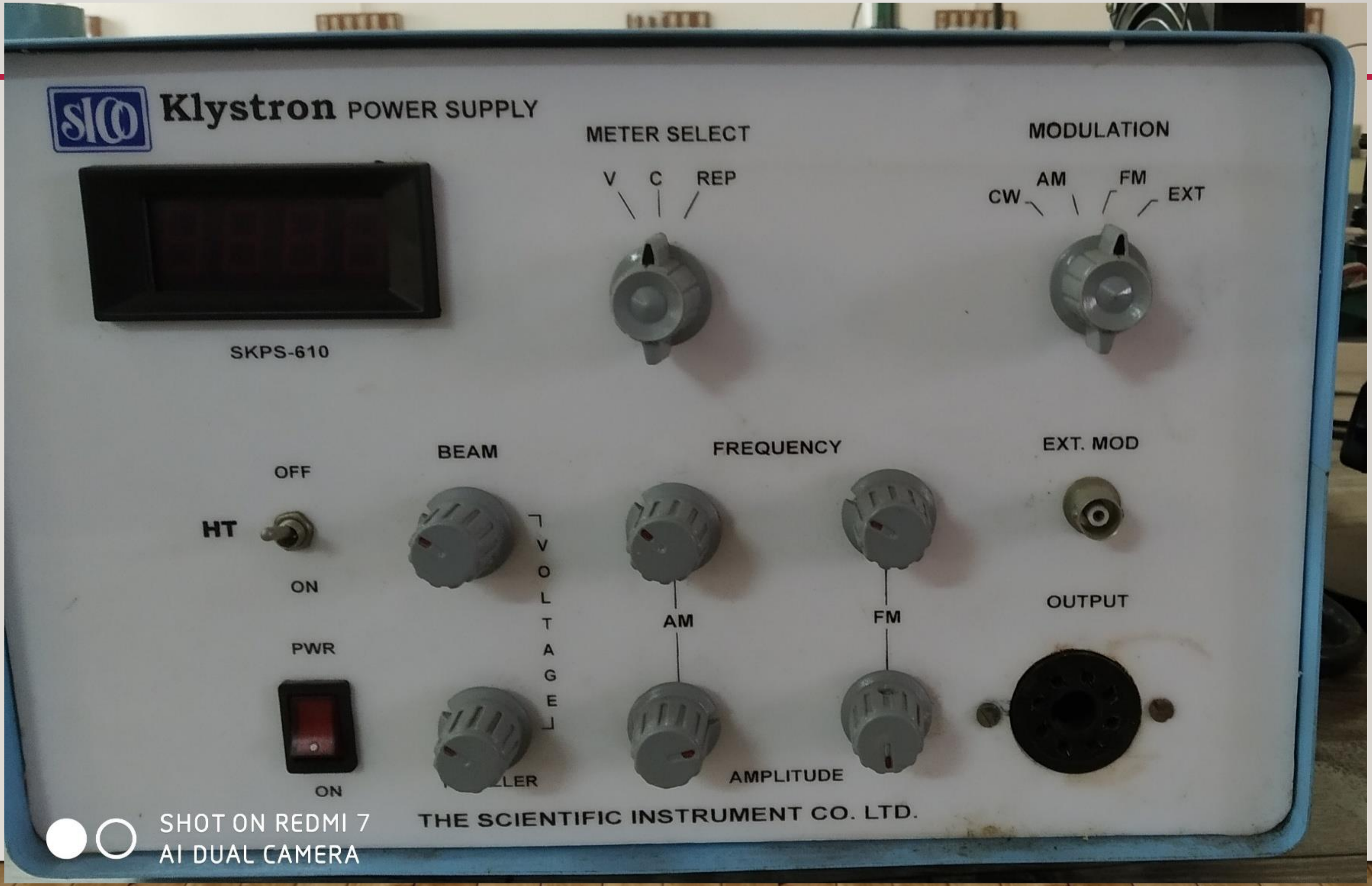
- The reflector voltage may be varied slightly from the optimum value, which results in some loss of output power, but also in a variation in frequency.
- At regions far from the optimum voltage, no oscillations are obtained at all.
- This tube is called a reflex klystron because it repels the input supply or performs the opposite function of a klystron.

- There are often several regions of reflector voltage where the reflex klystron will oscillate; these are referred to as modes.
- The frequency of oscillation is dependent on the reflector voltage, and varying this provides a crude method of frequency modulating the oscillation frequency, albeit with accompanying amplitude modulation as well.



RF Output
Cathode
Anode
d
Repeller
Electron beam
V_s
V_r


K-27
II-78


Klystron POWER SUPPLY
METER SELECT
V
C
REP
MODULATION
CW
AM
FM
EXT
SKPS-610
BEAM
FREQUENCY
EXT. MOD
OFF
HT
ON
VOLTAGE
AM
FM
OUTPUT
PWR
ON
AMPLITUDE
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APPLEGATE DIAGRAM- REFLEX KLYSTRON

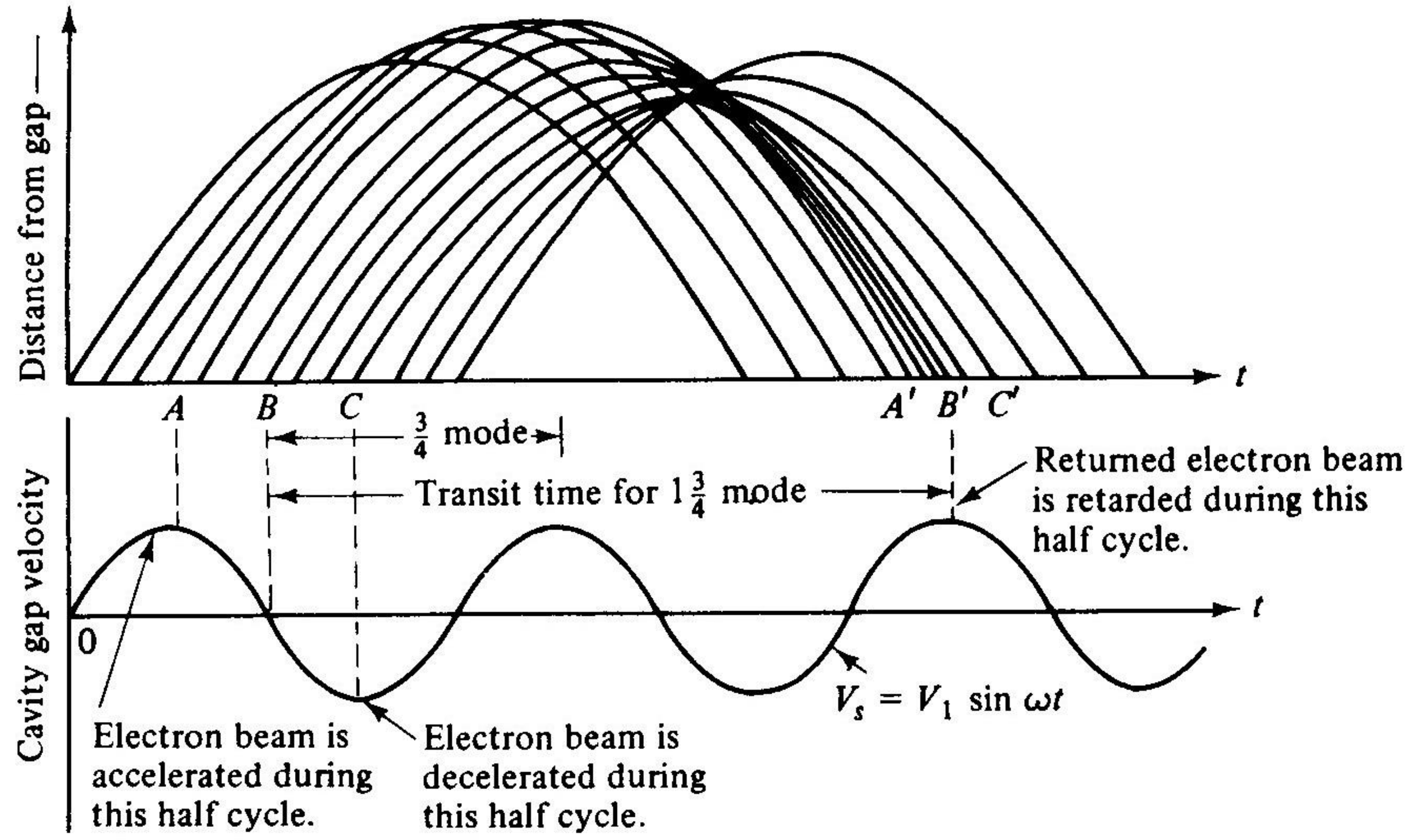
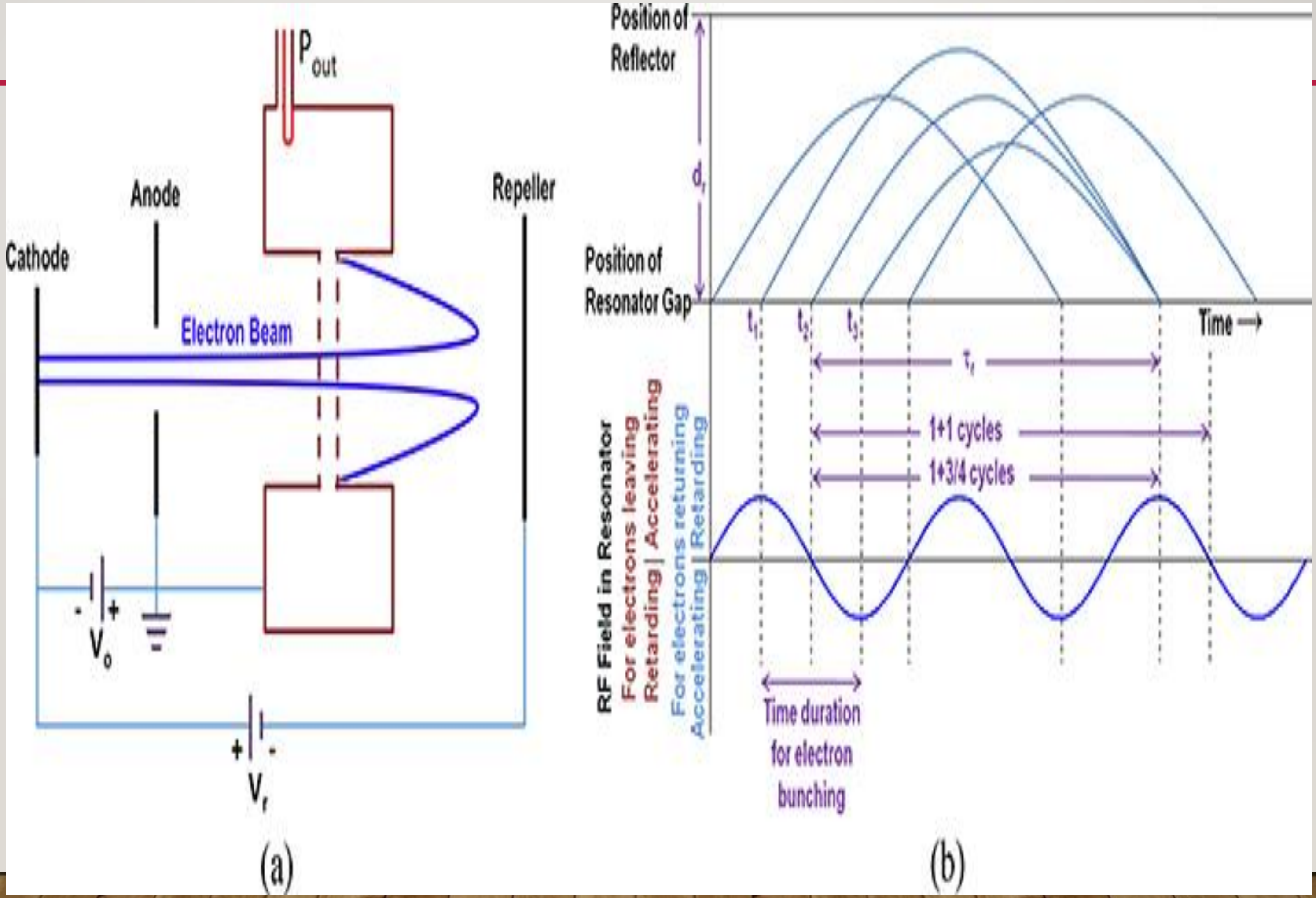
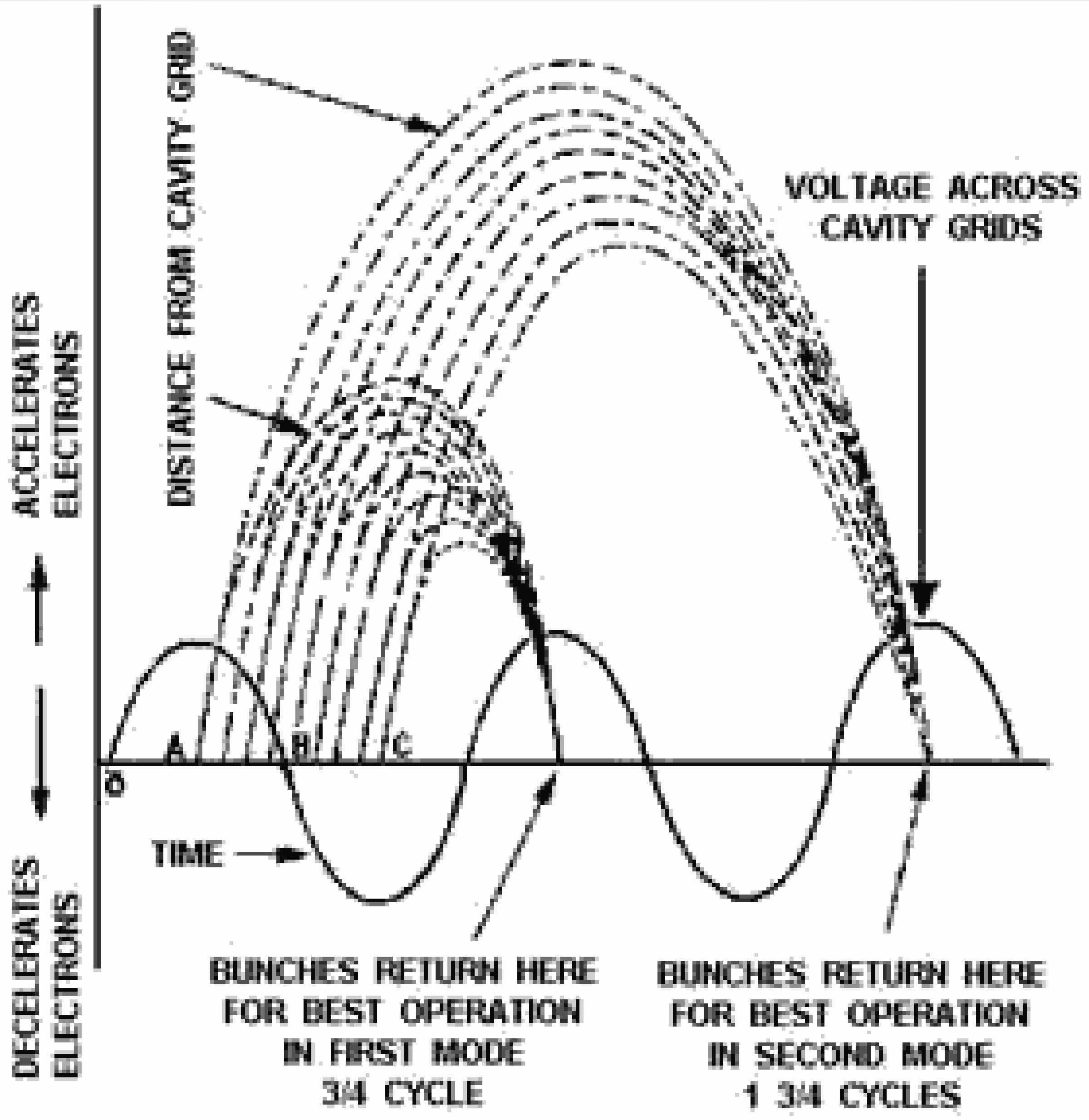


Figure 9-4-2 Applegate diagram with gap voltage for a reflex klystron.



P out
Anode
Repeller
Cathode
Electron Beam
- +
V o
+ -
V r
(a)
Position of Reflector
d r
Position of Resonator Gap
t 1
t 2
t 3
Time
τ r
1+1 cycles
1+3/4 cycles
RF Field in Resonator
For electrons leaving
Retarding | Accelerating
For electrons returning
Accelerating | Retarding
Time duration for electron bunching
(b)



DISTANCE FROM CAVITY GRID
VOLTAGE ACROSS CAVITY GRIDS
ACCELERATES ELECTRONS
DECELERATES ELECTRONS
A
B
C
O
TIME
BUNCHES RETURN HERE FOR BEST OPERATION IN FIRST MODE 3/4 CYCLE
BUNCHES RETURN HERE FOR BEST OPERATION IN SECOND MODE 1 3/4 CYCLES

VELOCITY MODULATION EXPLANATION

- The electron leaves the cavity gap at z=d, at time t1 , with velocity,

$$v(t_1) = v_0\left[1 + \frac{\beta_1 V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

- The same electron is forced back to cavity, z=d and time t2 by the retarding electric field E,

$$E = \frac{V_r + V_0 + V_1 \sin(\omega t)}{L}$$

- The force equation, for one electron in the repeller region, where Vr is the repeller voltage

$$m\frac{d^2z}{dt^2} = -eE = -e\frac{V_r + V_0}{L}$$

$$|V_1 \sin \omega t| \ll (V_r + V_0) \text{ is assumed.}$$

- At $t=t_1$, $dz/dt=v(t1)=K1$ then
- At $t=t1$, $z=d=K2$, the electron leaves the cavity with a velocity , $v(t1)$ return to the cavity , with time $t=t2$, $z=d$

- The round trip transit time in the repeller region T’=t2-t1

$$T' = t_2 - t_1 = \frac{2mL}{e(V_r + V_0)} v(t_1) = T_0'\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

- Where T0’ is $T_0' = \frac{2mLv_0}{e(V_r + V_0)}$ the round trip dc transit time of the center of the bunch electron
- The radiant frequency result in $$\omega(t_2 - t_1) = \theta_0' + X' \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

- The round trip transit angle $\theta_0' = \omega T_0'$

- The bunching parameter of the reflex klystron , $X' \equiv \frac{\beta_i V_1}{2V_0} \theta_0'$

- Power output and efficiency
- Electron beam to generate maximum energy to the oscillation
- The returning electron should cross the cavity gape , when the gap field is with maximum retarding
- The beam current of a reflex klystron is

$$i_{2t} = -I_0 - \sum_{n=1}^{\infty} 2I_0 J_n(nX') \cos\left[n(\omega t_2 - \theta_0' - \theta_g)\right]$$

- The fundamental component of the induced current is

$$i_2 = -\beta_i I_2 = 2I_0\beta_i J_1(X')\cos(\omega t_2 - \theta_0')$$

- The magnitude of the fundamental component is $$I_2 = 2I_0\beta_i J_1(X')$$
- The DC power supplied by the beam voltage V0 is $$P_{dc} = V_0 I_0$$
- The AC power supplied to the load,

$$P_{ac} = \frac{V_1 I_2}{2} = V_1 I_0 \beta_i J_1(X')$$

- So, $$\frac{V_1}{V_0} = \frac{2X'}{\beta_i(2\pi n - \pi/2)}$$

- The power output $$P_{ac} = \frac{2V_0 I_0 X' J_1(X')}{2\pi n - \pi/2}$$

- The efficiency $$\text{Efficiency} \equiv \frac{P_{ac}}{P_{dc}} = \frac{2X' J_1(X')}{2\pi n - \pi/2}$$

$$\text{Efficiency}_{max} = \frac{2(2.408)J_1(2.408)}{2\pi(2) - \pi/2} = 22.7\%$$

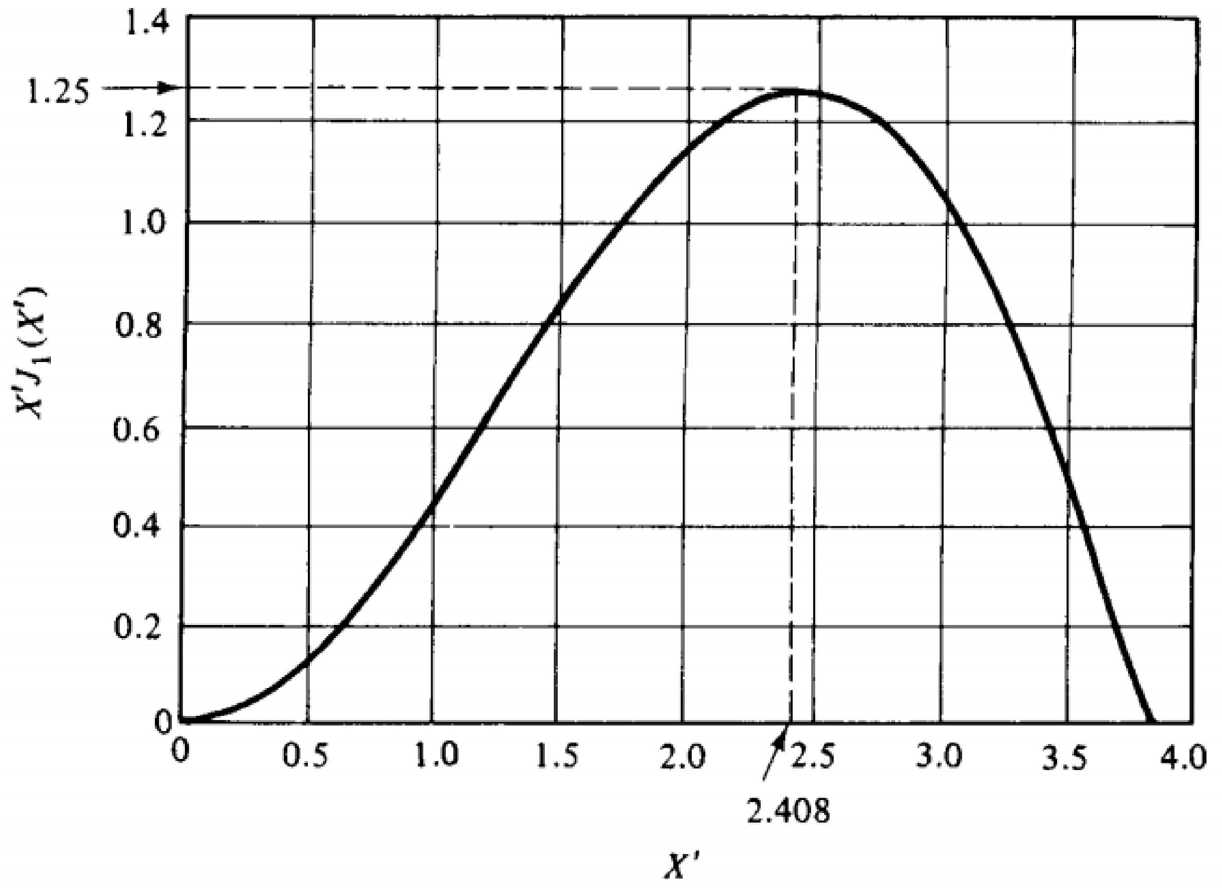


Figure 9-4-3 $X'J_1(X')$ versus X'.

- For given beam voltage, V0, the relationship between, repeller voltage and cycle number n, required for oscillation, $$\frac{V_0}{(V_r + V_0)^2} = \frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2} \frac{e}{m}$$

- Power output can be expressed in terms of repeller voltage Vr

$$P_{ac} = \frac{V_0 I_0 X' J_1(X')(V_r + V_0)}{\omega L} \sqrt{\frac{e}{2mV_o}}$$

- The frequency varies from the center frequency, and the repeller voltage, about the center voltage , the power output will vary, as bell shape
- Transition period Tr= (1+3/4)T ,(2+3/4)T, (3+3/4)T …..(n+3/4)T

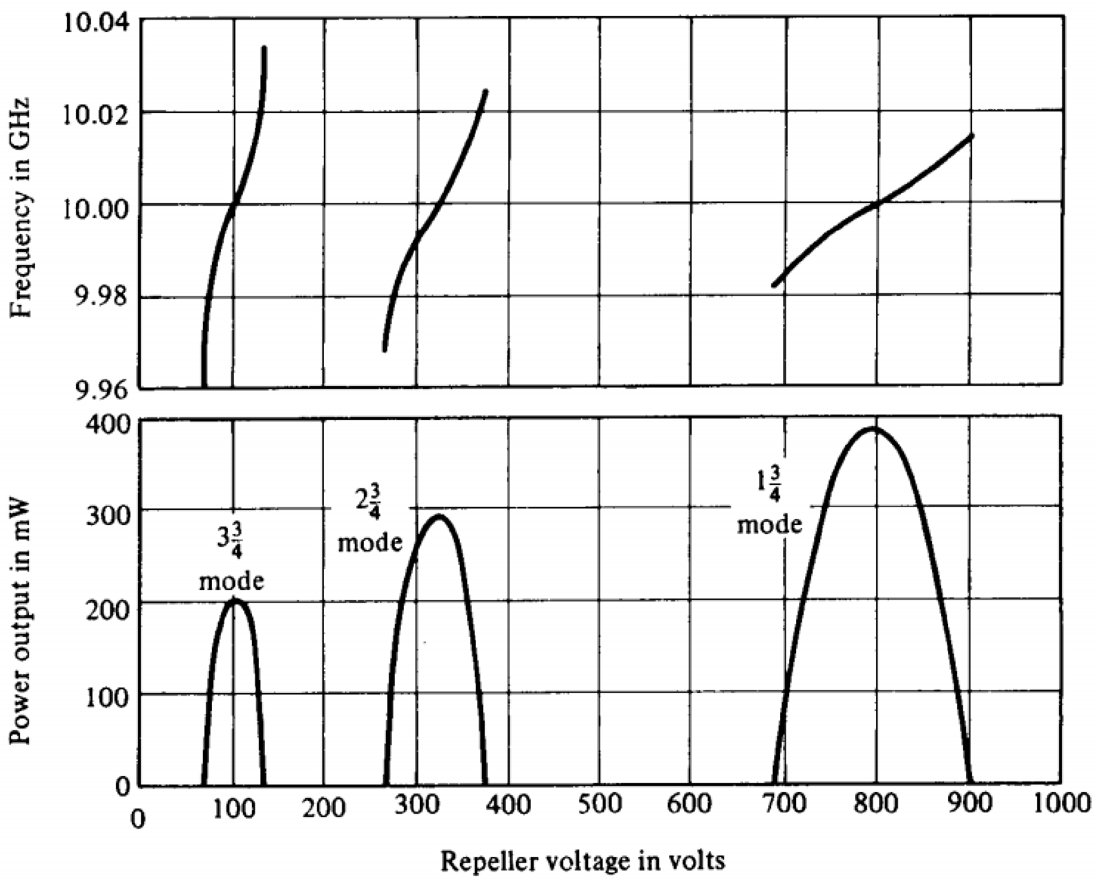
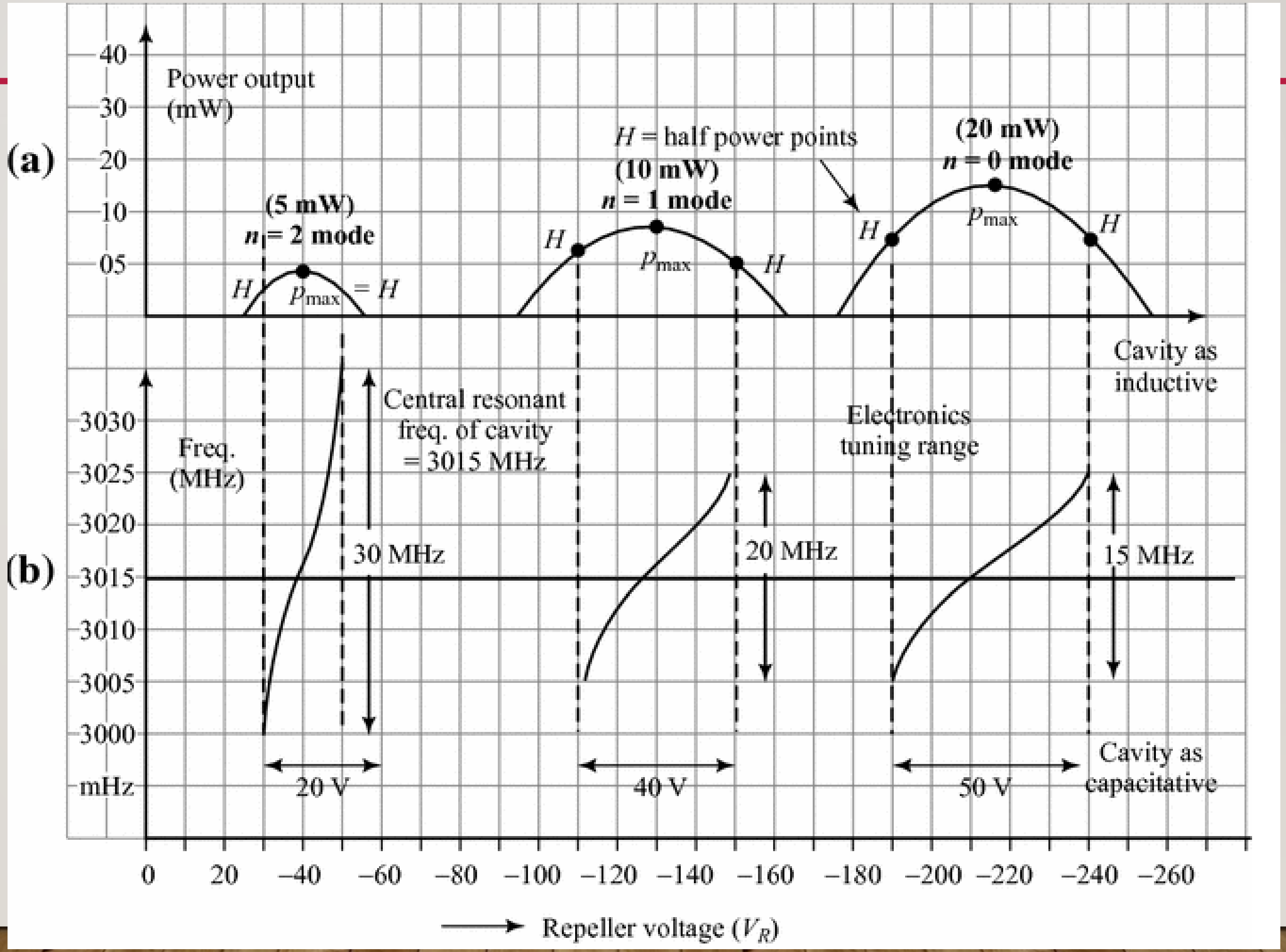


Figure 9-4-4 Power output and frequency characteristics of a reflex klystron.



(a)
40
30
20
10
05
Power output
(mW)
(5 mW)
n = 2 mode
H
pmax
= H
H = half power points
(10 mW)
n = 1 mode
H
pmax
H
(20 mW)
n = 0 mode
H
pmax
H
Cavity as
inductive
(b)
3030
3025
3020
3015
3010
3005
3000
mHz
Freq.
(MHz)
Central resonant
freq. of cavity
= 3015 MHz
30 MHz
Electronics
tuning range
20 MHz
15 MHz
20 V
40 V
50 V
Cavity as
capacitative
0
20
−40
−60
−80
−100
−120
−140
−160
−180
−200
−220
−240
−260
Repeller voltage (VR)

APPLICATIONS

➢ The reflex klystrons are used in

1. Radar receivers
2. Local oscillator in microwave receivers
3. Signal source in microwave generator of variable frequency
4. Portable microwave links
5. Pump oscillator in parametric amplifier

BUNCHING PROCESS

- The velocity modulation bunches the microwave and high velocity electron beam.
- The slower electrons and faster electrons bunches together and generates velocity modulation.
- An optimum distance between buncher cavity and catcher cavity.
- X –is the bunching parameter of klystron.

- Transit angle

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

- Average voltage in the buncher gap is V_s

$$V_1 \frac{\sin(\theta_g/2)}{\theta_g/2} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

$$\beta_i \equiv \frac{\sin[\omega d/(2v_0)]}{\omega d/(2v_0)} = \frac{\sin(\theta_g/2)}{\theta_g/2}$$

- Where β_i is beam coupling coeff. Of input cavity gap

$$\beta_i \equiv \frac{\sin[\omega d/(2v_0)]}{\omega d/(2v_0)} = \frac{\sin(\theta_g/2)}{\theta_g/2}$$

- The exit velocity from the buncher cavity

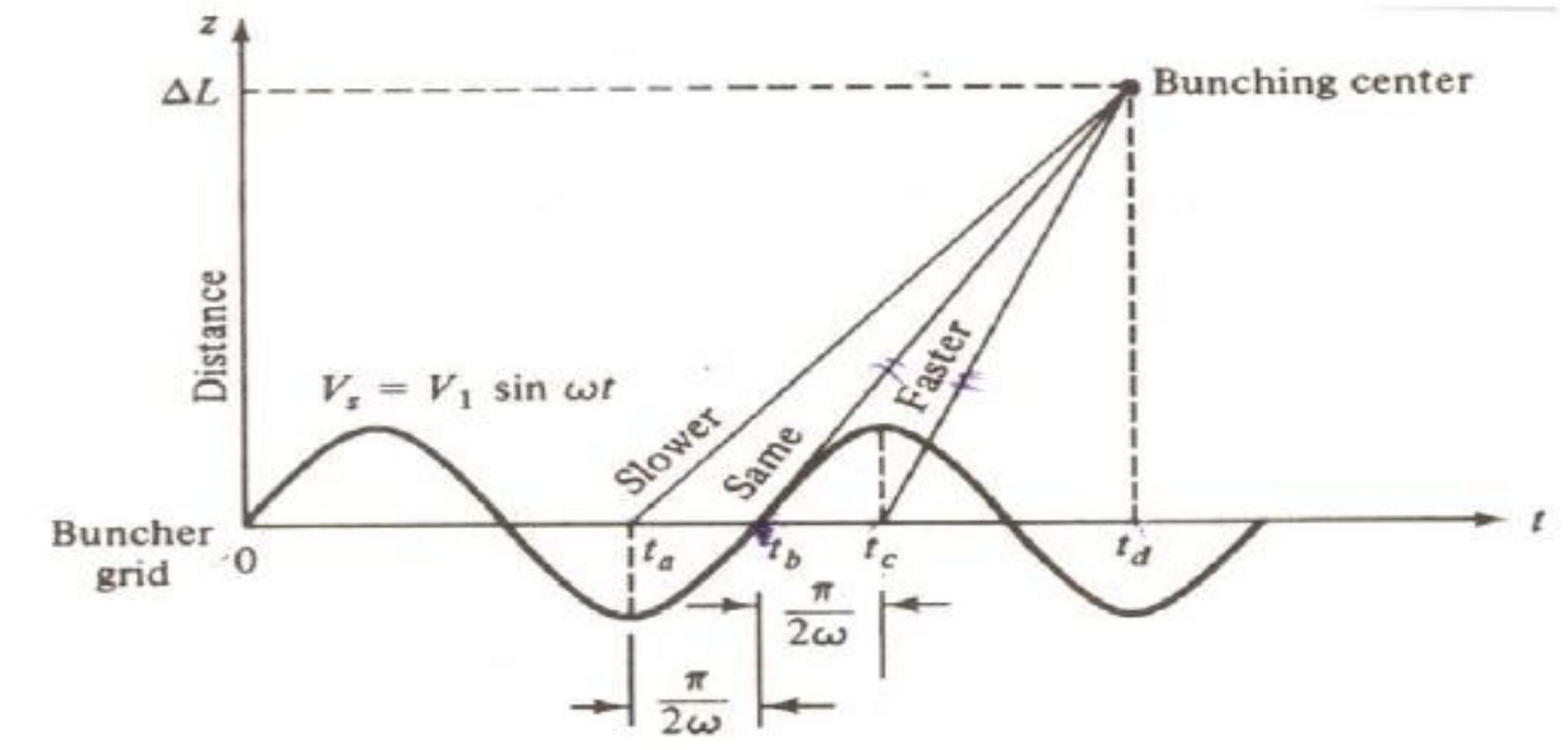
$$v(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$= \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

- Depth of velocity modulation is $\beta_i V_1 / V_0$

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

APPLEGATE DIAGRAM TWO CAVITY KLYSTRON



PROBLEM

- Q The reflex klystron operates under the condition $V0= 600V$, $L=1mm$, Rsh 15kΩ e/m = 1.759 X 10^{11} fr= 9Ghz the tube is oscillating at Fr at the peak of the n=2 mode or 13/4 mode .Assume that the transit time through the gap and beam loading can be neglected .
- A. Find the value of the Repeller voltage Vr
- B. Find the direct current necessary to give a microwave gap voltage of 200V .
- C. What is the electric efficiency under this condition

OR

8	a)	EXPLANATION AND PROOF-7 MARKS	(7)
	b)	$$\frac{V_0}{(V_r + V_0)^2} = \left(\frac{e}{m}\right)\frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$	(7)

$$= (1.759 \times 10^{11})\frac{(2\pi 2 - \pi/2)^2}{8(2\pi \times 9 \times 10^9)^2(10^{-3})^2} = 0.832 \times 10^{-3}$$

$$(V_r + V_0)^2 = \frac{600}{0.832 \times 10^{-3}} = 0.721 \times 10^6$$

$$V_r = 250 \text{ V}$$

Assume that $\beta_0 = 1$. Since

$$V_2 = I_2 R_{sh} = 2I_0 J_1(X')R_{sh}$$

the direct current I_0 is

$$I_0 = \frac{V_2}{2J_1(X')R_{sh}} = \frac{200}{2(0.582)(15 \times 10^3)} = 11.45 \text{ mA}$$

$$\text{Efficiency} = \frac{2X' J_1(X')}{2\pi n - \pi/2} = \frac{2(1.841)(0.582)}{2\pi(2) - \pi/2} = 19.49\%$$

(i) Repeller voltage Vr . - 3 mark

(ii) Direct current necessary to give a microwave gap voltage of 200 V - 2 mark

(iii) Electronic efficiency under this condition - 2 mark

Module V

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MAGNETRON

- DESCRIPTION
- WORKING
- APPLICATION

INTRODUCTION

- **A magnetron** is a high-powered vacuum tube that generates non consistent microwaves with built-in resonators or by special oscillators or solid-state devices to control the frequency.
- The electromagnetic energy created from a magnetron can travel at the speed of light and is the same type of energy used in radio and television broadcasting.

BRIEF HISTORY OF MAGNETRON

- 1920s: American Engineer Albert W. Hull invented the first magnetron while working for General Electric.
- 1940s: Two physicists, John Randall and Harry Boot, working at the University of Birmingham, England developed a much more powerful magnetron that is compact enough to fit into ships, planes and submarines.

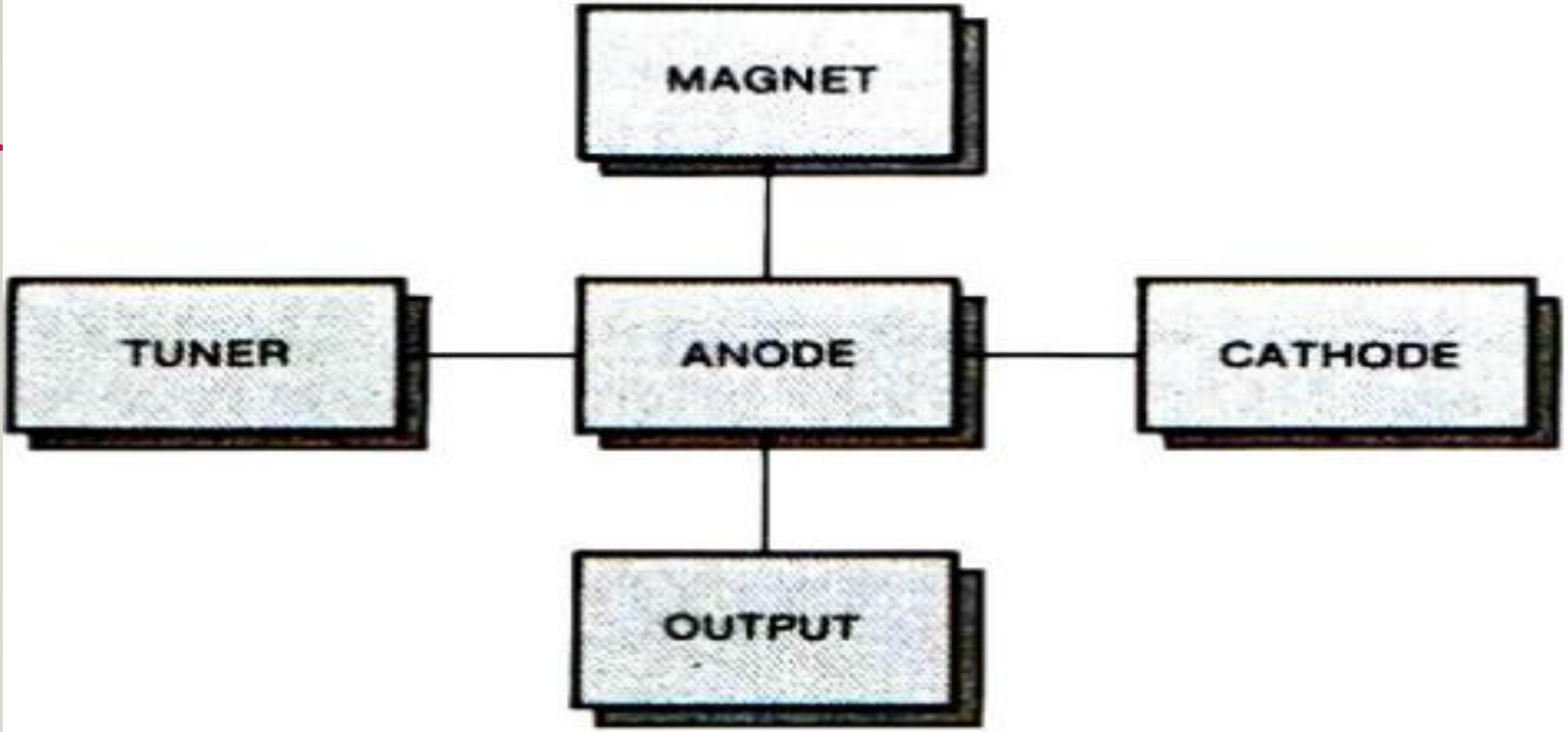
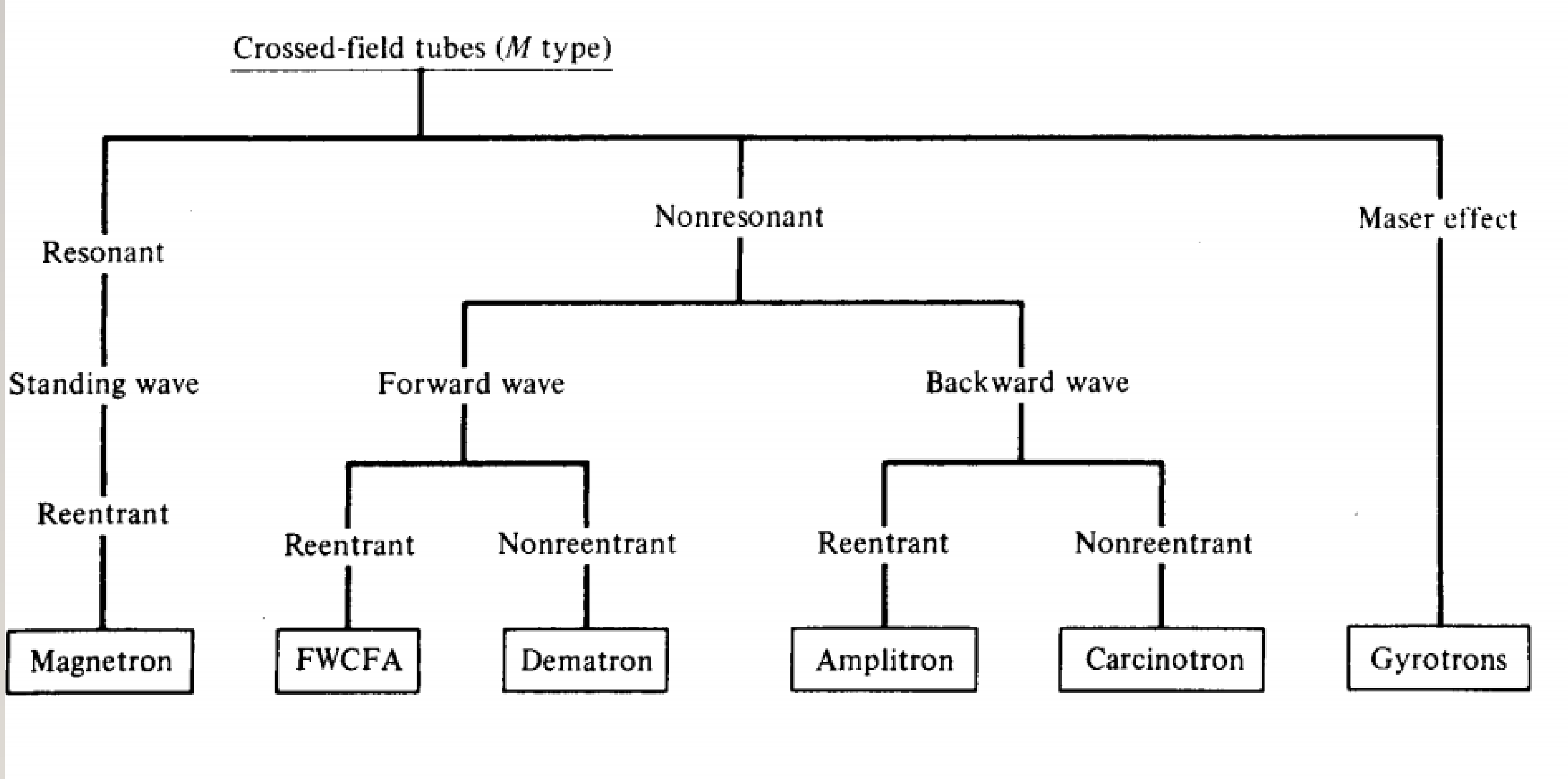


FIGURE 1.



Crossed-field tubes (M type)
Resonant
Standing wave
Reentrant
Magnetron
Nonresonant
Forward wave
Reentrant
FWCFA
Nonreentrant
Dematron
Backward wave
Reentrant
Amplitron
Nonreentrant
Carcinotron
Maser effect
Gyrotrons

MAGNETRON OSCILLATOR

- Magnetrons provide microwave oscillations of very high frequency.

Types of magnetrons

1. Negative resistance type
2. Cyclotron frequency type
3. Cavity type

DESCRIPTION OF TYPES OF MAGNETRON

Negative resistance Magnetrons

- Make use of negative resistance between two anode segments but have low efficiency and are useful only at low frequencies (< 500 MHz).

Cyclotron frequency Magnetrons

- Depend upon synchronization between an alternating component of electric and periodic oscillation of electrons in a direction parallel to this field.
- Useful only for frequencies greater than 100 MHz.

Cavity Magnetrons

- Depend upon the interaction of electrons with a rotating electromagnetic field of constant angular velocity.
- Provide oscillations of very high peak power and hence are useful in radar applications

1. *Split-anode magnetron:* This type of magnetron uses a static negative resistance between two anode segments.
2. *Cyclotron-frequency magnetrons:* This type operates under the influence of synchronism between an alternating component of electric field and a periodic oscillation of electrons in a direction parallel to the field.
3. *Traveling-wave magnetrons:* This type depends on the interaction of electrons with a traveling electromagnetic field of linear velocity. They are customarily referred to simply as *magnetrons*.

ADVANTAGES

- The magnetron is a fairly efficient device. In a microwave oven, for instance, an 1100 watt input will generally create about 700 watts of microwave energy, an efficiency of around 65%.
- The combination of the small-cavity magnetron, small antennas, and high resolution allowed small, high quality radars to be installed in aircraft.

- The electrons emitted from the cathode are influenced by the crossed field move in curved path
- Electrons will accelerate and retarded due to RF field
- The decrease in electron energy will be exactly offset by an increase in the r.f. field strength
- Since the force on an electron due to the magnetic field B is proportional to the electron velocity through the field, the retarded velocity electrons will experience less "curling force" and will therefore drift toward the anode, while the accelerated velocity electrons will curl back away from the anode .

- Maximum energy electrons give up their energy to RF field and trying to reach to anode.
- Unwilling electrons rotate cylindrically around the cavity.
- Lower cathode emission density, lower life and higher reliability.
- Operating the cavity in the TE011 mode, and slot coupling alternate anode resonators to the cavity,

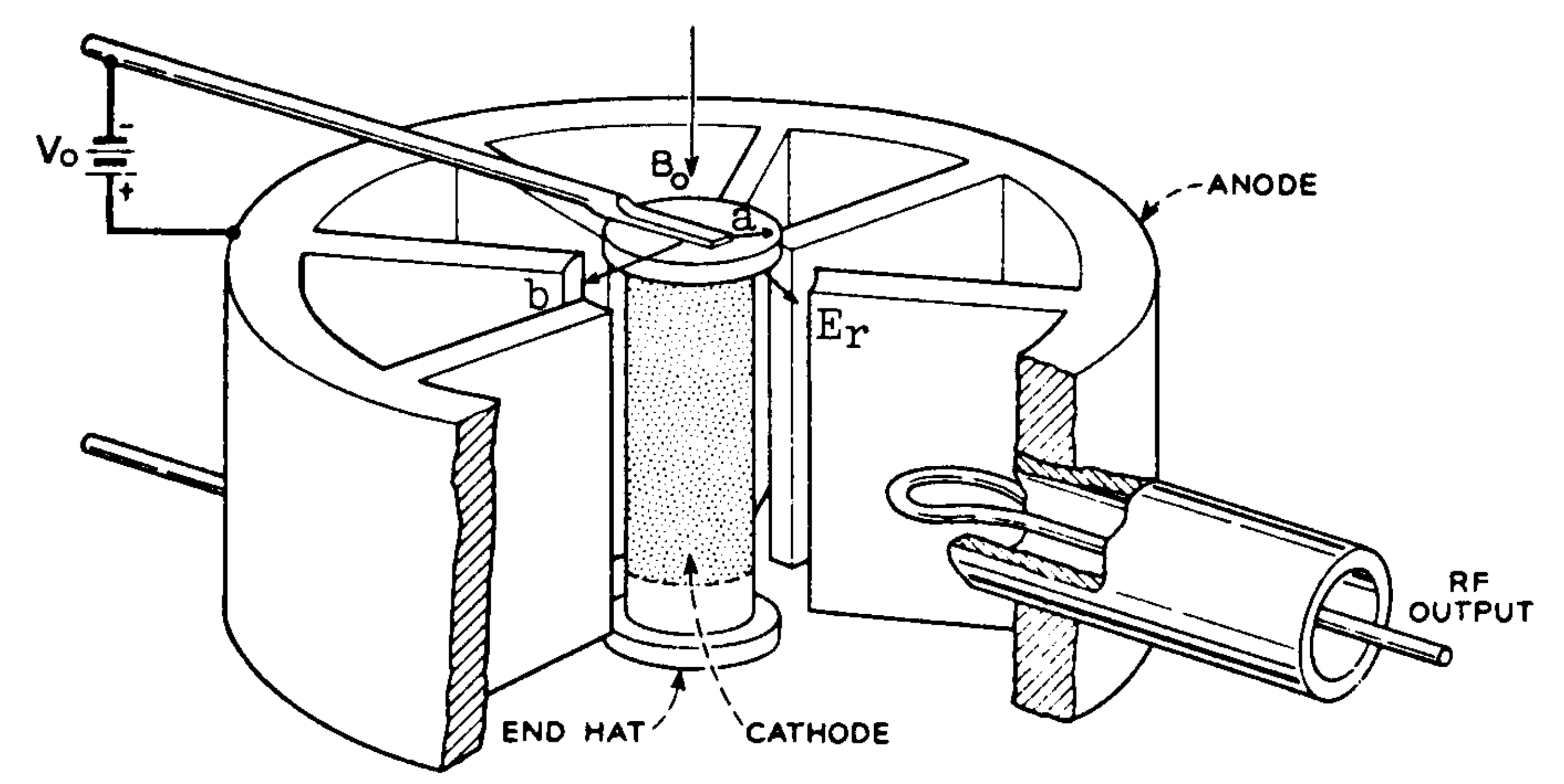
WHAT IS CYCLOTRON MAGNETRON

- Operates in microwave frequency, the power output is very small(1W for 3Ghz)
- Efficiency is very less

CYLINDRICAL MAGNETRON

- This is the conventional type magnetron.
- Several reentrant cavities are connected to the gap
- The Dc voltage $V0$ is applied between the cathode and anode
- The magnetic flux density, $B0$ is in the positive z direction
- The Dc voltage and the magnetic flux are adjusted properly
- The electron will follow cycloidal path in the anode-cathode space, under the combined force of electric and magnetic field

Dispense energy to the ac field



PHYSICAL STRUCTURE-

Anode-made of copper plates

Cathode-made of high emission material

Magnets

Cavities

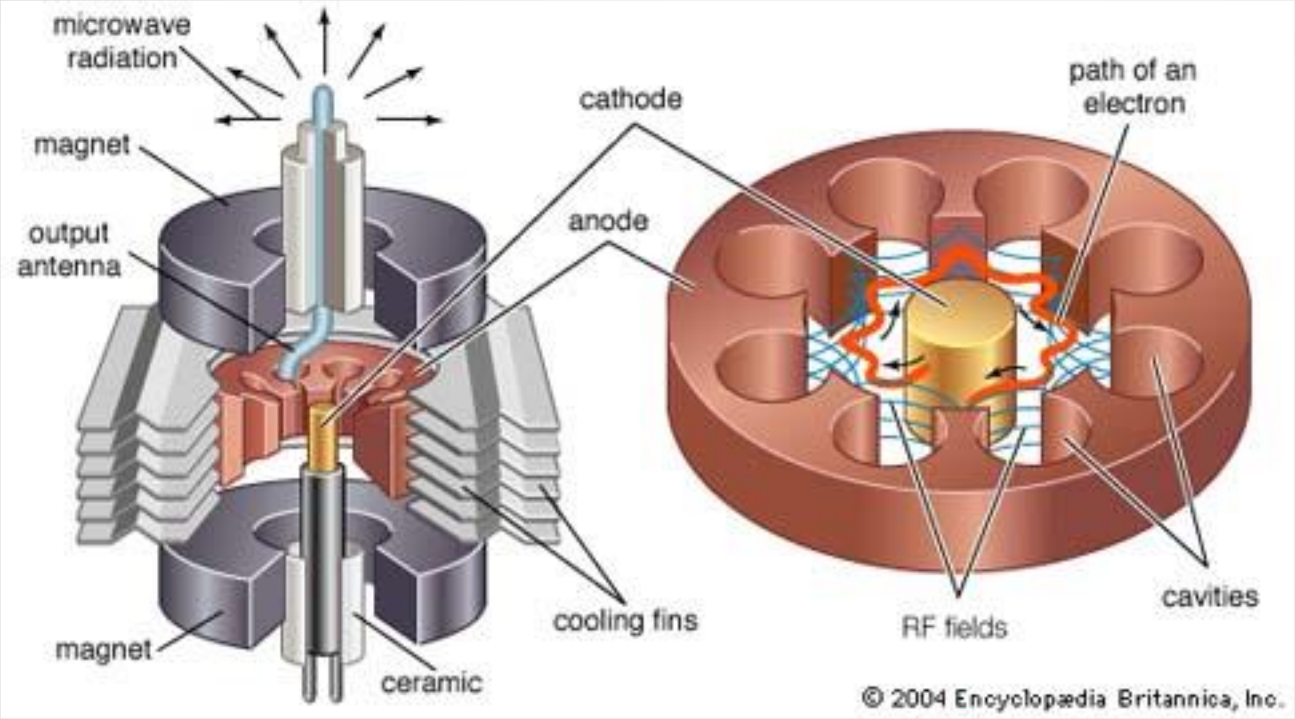
Filament Leads

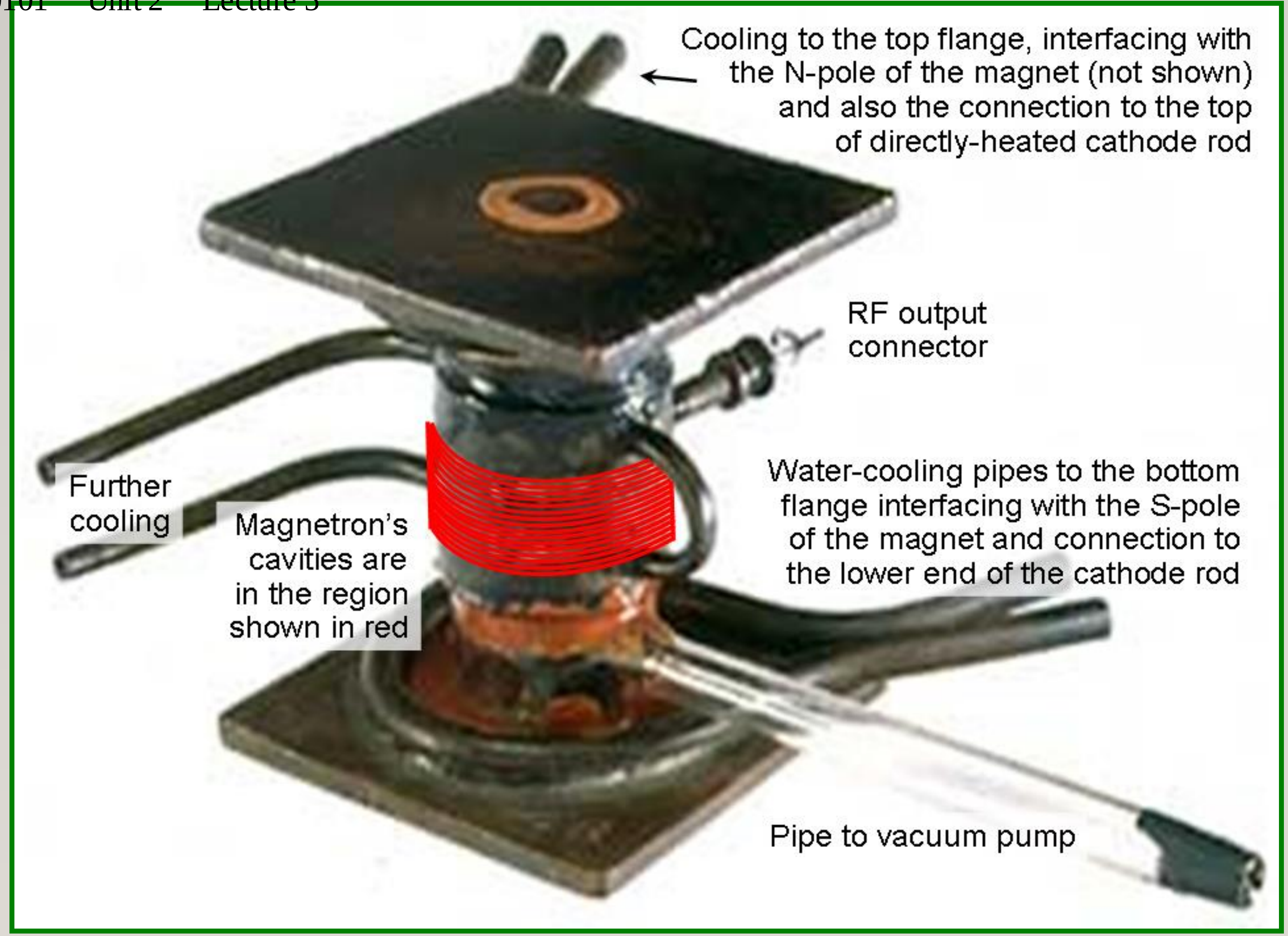
Output antenna(waveguide)

Cooling fins

DC voltage source

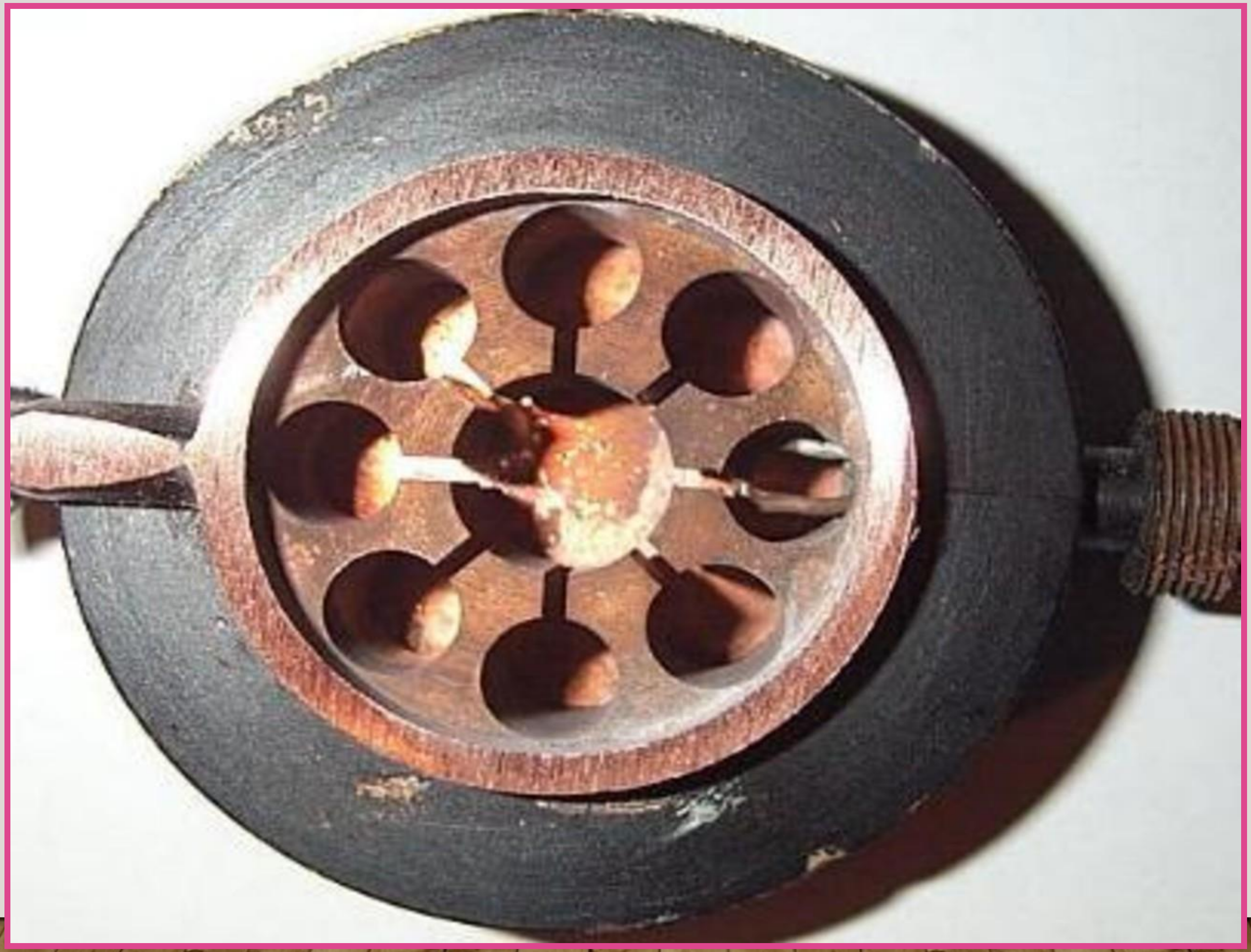
CAVITY MAGNETRON





Cavity Magnetron

Anode Assembly



CONSTRUCTION

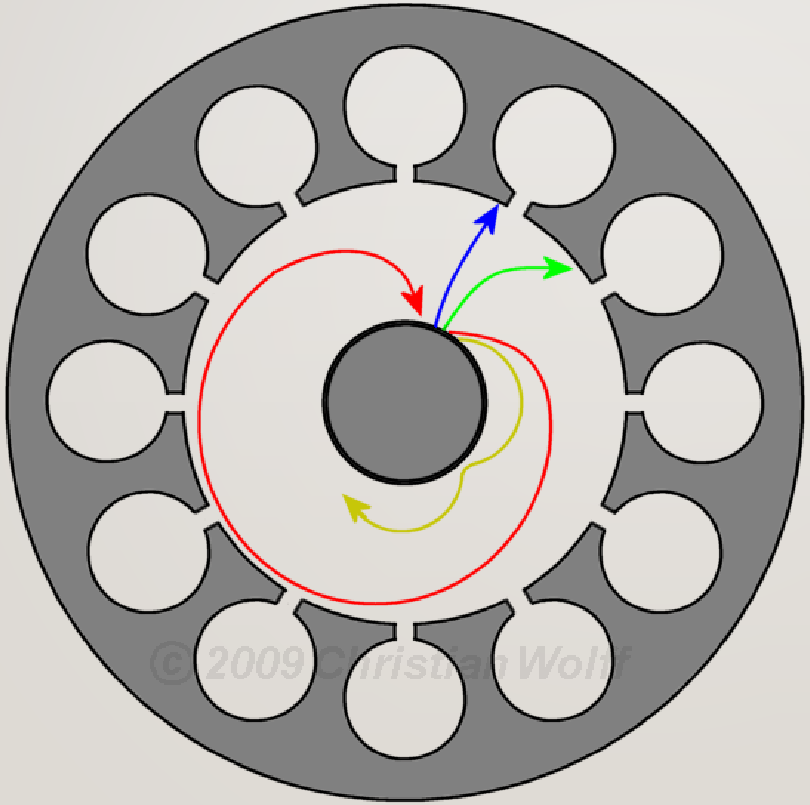
- Each cavity in the anode acts as an inductor having only one turn and the slot connecting the cavity and the interaction space acts as a capacitor.
- These two form a parallel resonant circuit and its resonant frequency depends on the value of *L* of the cavity and the *C* of the slot.
- The frequency of the microwaves generated by the magnetron oscillator depends on the frequency of the RF oscillations existing in the resonant cavities.

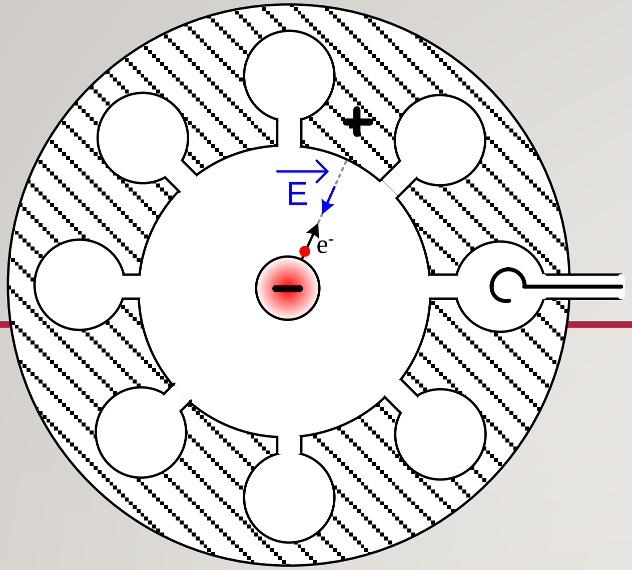
DESCRIPTION

- Magnetron is a cross field device as the electric field between the anode and the cathode is radial whereas the magnetic field produced by a permanent magnet is axial.
- A high DC potential can be applied between the cathode and anode which produces the radial electric field.
- Depending on the relative strengths of the electric and magnetic fields, the electrons emitted from the cathode and moving towards the anode will traverse through the interaction space as shown in figure.
- In the absence of magnetic field (B = 0), the electron travel straight from the cathode to the anode due to the radial electric field force acting on it.

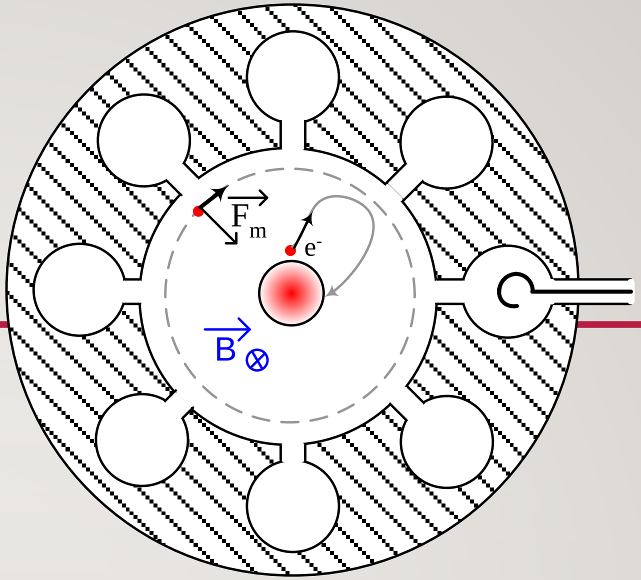
WORKING OF CAVITY MAGNETRON

1.Production and Acceleration of an electron beam in a dc field

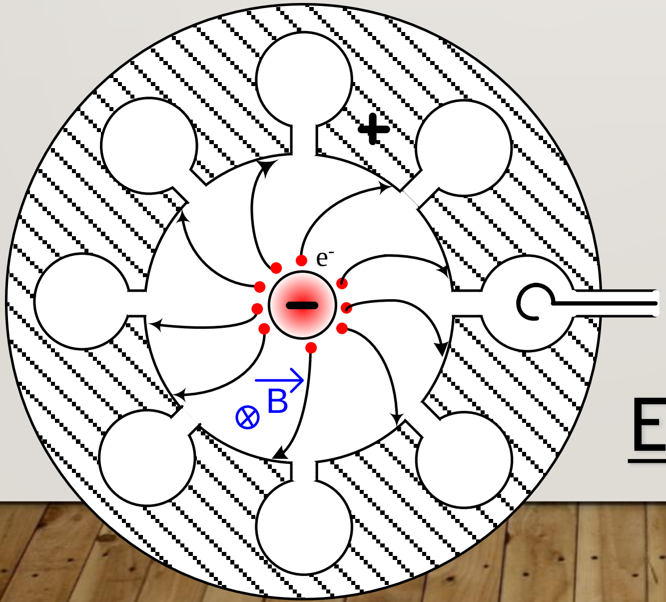




Effect of electric field

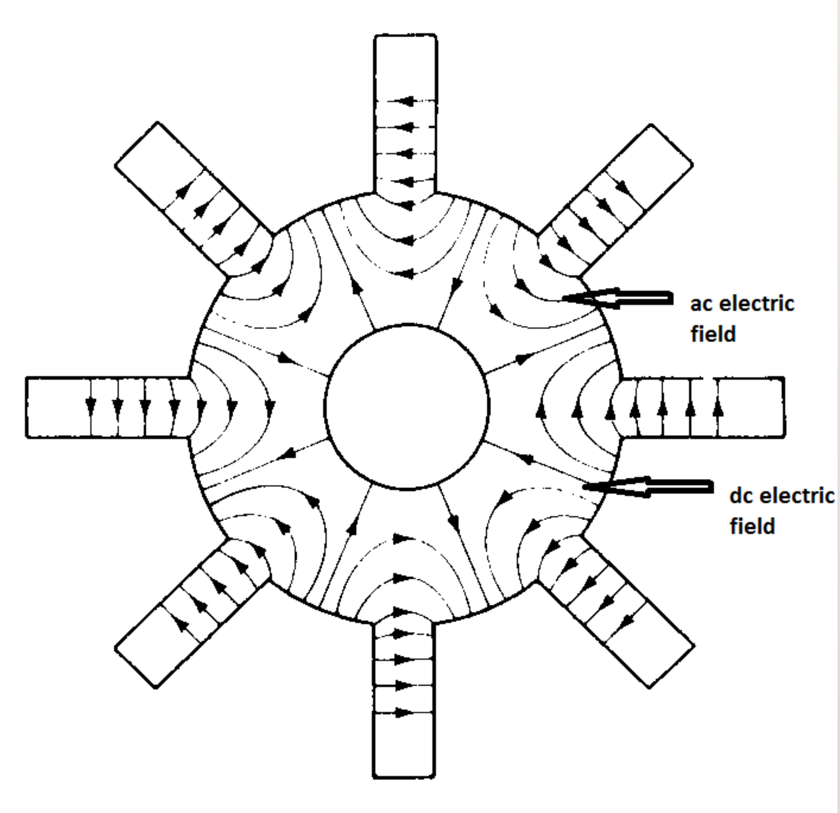


Effect of magnetic field

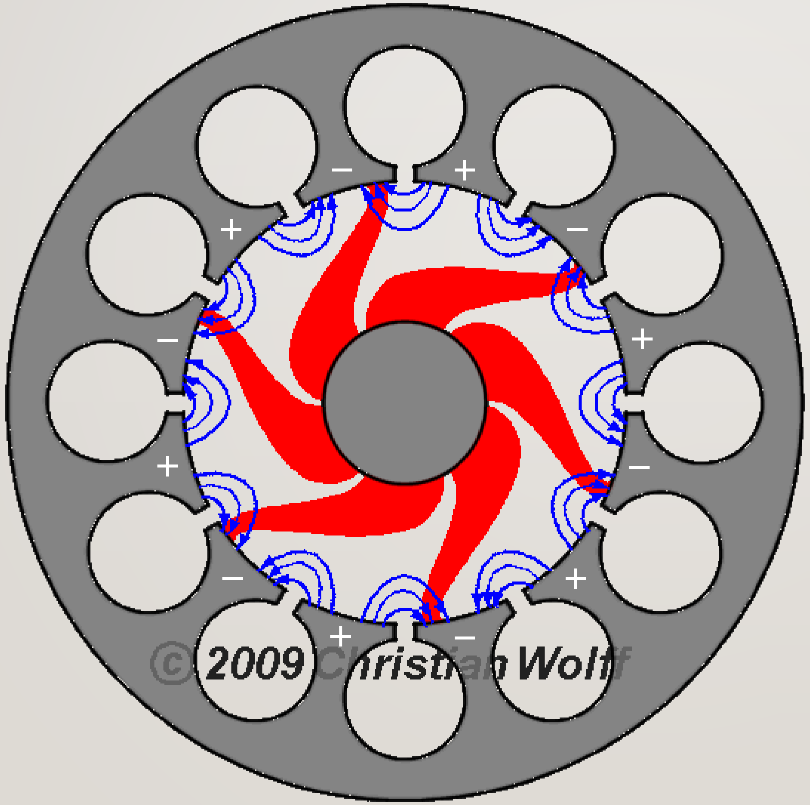


Effect of Crossed-Fields

2.Velocity-modulation of the electron beam



3.Formation of electron bunches by velocity modulation (Space-charge wheel)



CONSTRUCTION

- As shown in the figure, a cavity magnetrons consist of a hot filament (cathode) kept at, or pulsed to, a high negative potential by a high-voltage, direct-current power supply. The cathode is built into the center of an evacuated, lobed, circular chamber.
- A magnetic field parallel to the filament is imposed by an electro-magnet. The magnetic field causes the electrons, attracted to the (relatively) positive outer part of the chamber, to spiral outward in a circular path rather than moving directly to this anode.

- Spaced around the rim of the chamber are cylindrical cavities. The cavities are open along their length and connect the common cavity space. As electrons sweep past these openings, they induce a resonant, high-frequency radio field in the cavity, which in turn causes the electrons to bunch into groups.
- A portion of this field is extracted with a short antenna that is connected to a waveguide (a metal tube usually of rectangular cross section). The waveguide directs the extracted RF energy to the load, which may be a cooking chamber in a microwave oven or a high-gain antenna in the case of radar.

CYLINDRICAL MAGNETRON

- Several reentrant cavities are connected to the gap
- The DC voltage V0 is applied between the cathode and anode
- The magnetic flux density B0 is applied in the Z direction
- The equation of motion for electron in cylindrical magnetron

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_r - \frac{e}{m}rB_z\frac{d\phi}{dt}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{dr}{dt}$$

- Re arranging the above equation $\frac{d}{dt}\left(r^2 \frac{d\phi}{dt}\right) = \frac{e}{m} B_z r \frac{dr}{dt} = \frac{1}{2}\omega_c \frac{d}{dt}(r^2)$
- The cyclotron angular frequency is $\omega_c = \frac{e}{m} B_0$ $\quad r^2 \frac{d\phi}{dt} = \frac{1}{2}\omega_c r^2 + \text{constant}$
- The angular velocity is expressed as $\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{r^2}\right)$
- The kinetic energy of the electron is $\frac{1}{2} m \mathcal{V}^2 = eV$
- The electron velocity has r and ϕ components as $\mathcal{V}^2 = \frac{2e}{m} V = \mathcal{V}_r^2 + \mathcal{V}_\phi^2 = \left(\frac{dr}{dt}\right)^2 + \left(r \cdots\right.$
- Where r=a, radius of the cathode cylinder, r=b, radius from the center of the cathode to the edge of the anode, where V=V0, dr/dt=0 the electrons just graze the anode

- The electron will acquire tangential as well as radial velocity, and substituting the above equations,

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right)$$

$$b^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m}V_0$$

$$b^2\left[\frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2e}{m}V_0$$

- Ion is

$$B_{0c} = \frac{\left(8V_0\frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}$$

- The electron will graze the anode and return towards the cathode depends on the relative magnitude of V0 and B0 . So the Hull cutoff magnetic equation is

- When, B0 > B0c for a given V0, the electron will not reach the anode, just graze the anode
- The cutoff voltage is given by ,

$$V_{0c} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

- V0<V0c, for a given B0, the electrons will not reach the anode, The above equation is known as Hull cutoff voltage equation.

CYCLOTRON ANGULAR FREQUENCY

- The magnetic field is normal to the motion of electron, that travel in a cycloidal path, The outward centrifugal force is equal to the pulling force

$$\frac{m\mathcal{V}^2}{R} = e\mathcal{V}B$$

- The cyclotron angular frequency of the circular motion of the electron

$$\omega_c = \frac{\mathcal{V}}{R} = \frac{eB}{m}$$

- The period of one complete revolution is

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{eB}$$

- There are N reentrant cavities inside the anode structure, then the phase shift between two adjacent cavities

$$\phi_n = \frac{2\pi m}{N}$$

- Energy to be transferred from moving electrons to the traveling field
- The electron must be decelerated by a retarding field, when they pass through each anode cavity

- L is the mean separation between cavities, the phase constant of the fundamental mode is

$$\beta_0 = \frac{2\pi n}{NL}$$

- The travelling field of the fundamental mode travels around the structure with angular velocity, $$\frac{d\phi}{dt} = \frac{\omega}{\beta_0}$$

- The cyclotron frequency of the electron is equal to the angular frequency of the field, the interaction between the field and electron occurs and the energy is transferred ,

$$\omega_c = \beta_0 \frac{d\phi}{dt}$$

POWER OUTPUT AND EFFICIENCY(LIO)

- The unloaded quality factor of resonator $Q_{un} = \frac{\omega_0 C}{G_r}$ where $\omega_0 = 2\pi f_0$
- The external quality factor is $Q_{ex} = \frac{\omega_0 C}{G_\ell}$
- The loaded Ql of the resonant circuit is $Q_\ell = \frac{\omega_0 C}{G_r + G_\ell}$
- The circuit efficiency is given by

$$\eta_c = \frac{G_\ell}{G_\ell + G_r} = \frac{G_\ell}{G_{ex}} = \frac{1}{1 + Q_{ex}/Q_{un}}$$

- The maximum circuit efficiency is obtained when ,the magnetron is heavily loaded.

where Y_e = electronic admittance
V = RF voltage across the vane tips
C = capacitance at the vane tips
L = inductance of the resonator
G_r = conductance of the resonator
G = load conductance per resonator

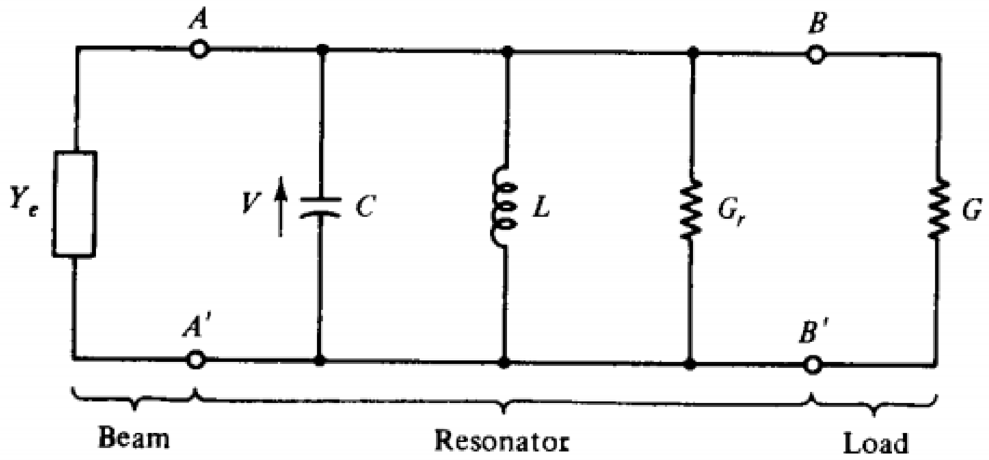


Figure 10-1-4 Equivalent circuit for one resonator of a magnetron.

- The ratio Ql/Qex is chosen as a compromise between circuit efficiency and frequency stability
- The electronic efficiency is defined as

$$\eta_e = \frac{P_{gen}}{P_{dc}} = \frac{V_0 I_0 - P_{lost}}{V_0 I_0}$$

where P_{gen} = RF power induced into the anode circuit
P_{dc} = $V_0 I_0$ power from the dc power supply
V_0 = anode voltage
I_0 = anode current
P_{lost} = power lost in the anode circuit

The RF power generated by the electrons can be written as

$$\begin{aligned} P_{gen} &= V_0 I_0 - P_{lost} \\ &= V_0 I_0 - I_0 \frac{m}{2e} \frac{\omega_0^2}{\beta^2} + \frac{E_{max}^2}{B_z^2} \\ &= \frac{1}{2} N \, |V|^2 \frac{\omega_0 C}{Q_\ell} \end{aligned}$$

where N = total number of resonators

V = RF voltage across the resonator gap

$E_{max} = M_1 |V|/L$ is the maximum electric field

$M_1 = \sin\left(\beta_n \frac{\delta}{2}\right) \Big/ \left(\beta_n \frac{\delta}{2}\right) = 1$ for small δ is the gap factor for the π-mode operation

β = phase constant

B_z = magnetic flux density

L = center-to-center spacing of the vane tips

The power generated may be simplified to

$$P_{\text{gen}} = \frac{NL^2\omega_0 C}{2M_1^2 Q_\ell} E_{\max}^2$$

The electronic efficiency may be rewritten as

$$\eta_e = \frac{P_{\text{gen}}}{V_0 I_0} = \frac{1 - \dfrac{m\omega_0^2}{2eV_0\beta^2}}{1 + \dfrac{I_0 m M_1^2 Q_\ell}{B_z e NL^2\omega_0 C}}$$

EQUATIONS OF ELECTRON MOTION

- Hull cut-off voltage equation

$V_{0c}=B_0^2b^2(1-(a/b)^2)^2e/8m$

- Hull cutoff magnetic equation

$B_{0c}=(8V_0m/e)^{1/2}/\ b(1-(a/b)^2)$

Example 10-1-1: Conventional Magnetron

An X-band pulsed cylindrical magnetron has the following operating parameters:

Anode voltage:	$V_0 = 26$ kV
Beam current:	$I_0 = 27$ A
Magnetic flux density:	$B_0 = 0.336$ Wb/m^2
Radius of cathode cylinder:	$a = 5$ cm
Radius of vane edge to center:	$b = 10$ cm

Compute:

a. The cyclotron angular frequency
b. The cutoff voltage for a fixed B_0
c. The cutoff magnetic flux density for a fixed V_0

Solution

a. The cyclotron angular frequency is

$$\omega_c = \frac{e}{m}B_0 = 1.759 \times 10^{11} \times 0.336 = 5.91 \times 10^{10} \text{ rad}$$

b. The cutoff voltage for a fixed B_0 is

$$V_{0c} = \frac{1}{8} \times 1.759 \times 10^{11}\,(0.336)^2(10 \times 10^{-2})^2\left(1 - \frac{5^2}{10^2}\right)^2$$

$$= 139.50 \text{ kV}$$

c. The cutoff magnetic flux density for a fixed V_0 is

$$B_{0c} = \left(8 \times 26 \times 10^3 \times \frac{1}{1.759 \times 10^{11}}\right)^{1/2}\left[10 \times 10^{-2}\left(1 - \frac{5^2}{10^2}\right)\right]^{-1}$$

$$= 14.495 \text{ mWb/m}^2$$

DISADVANTAGES OF MAGNETRON

- They are costly and hence limited in use.

- Although cavity magnetron are used because they generate a wide range of frequencies , the frequency is not precisely controllable.

- The use in radar itself has reduced to some extent, as more accurate signals have generally been needed and developers have moved to klystron and traveling-wave tube systems for accurate frequencies.
- NPTEL LINK
- https://freevideolectures.com/course/4367/nptel-microwave-theory-techniques/40
- Yutube Link
- https://www.youtube.com/watch?v=gnPVPfVmFwE

TRAVELLING WAVE TUBES

TRAVELLING WAVE TUBE

TRAVELING-WAVE TUBE (TWT)

- Uses a helix as a slow-wave structure
- Microwaves input at cathode end of helix, output at anode end
- Energy is transferred from electron beam to microwaves
- Coupled cavity TWT are used commonly
- Used as a High power amplifier

Frequency range: 3 GHz and higher

Bandwidth: about 0.8 GHz

Efficiency: 20 to 40%

Power output: up to 10 kW average

Power gain: up to 60 dB

TWT transmitter

kollector,	heizung,	eingang,	ausgang
collector,	heating,	input,	output

TRAVELING WAVE TUBE

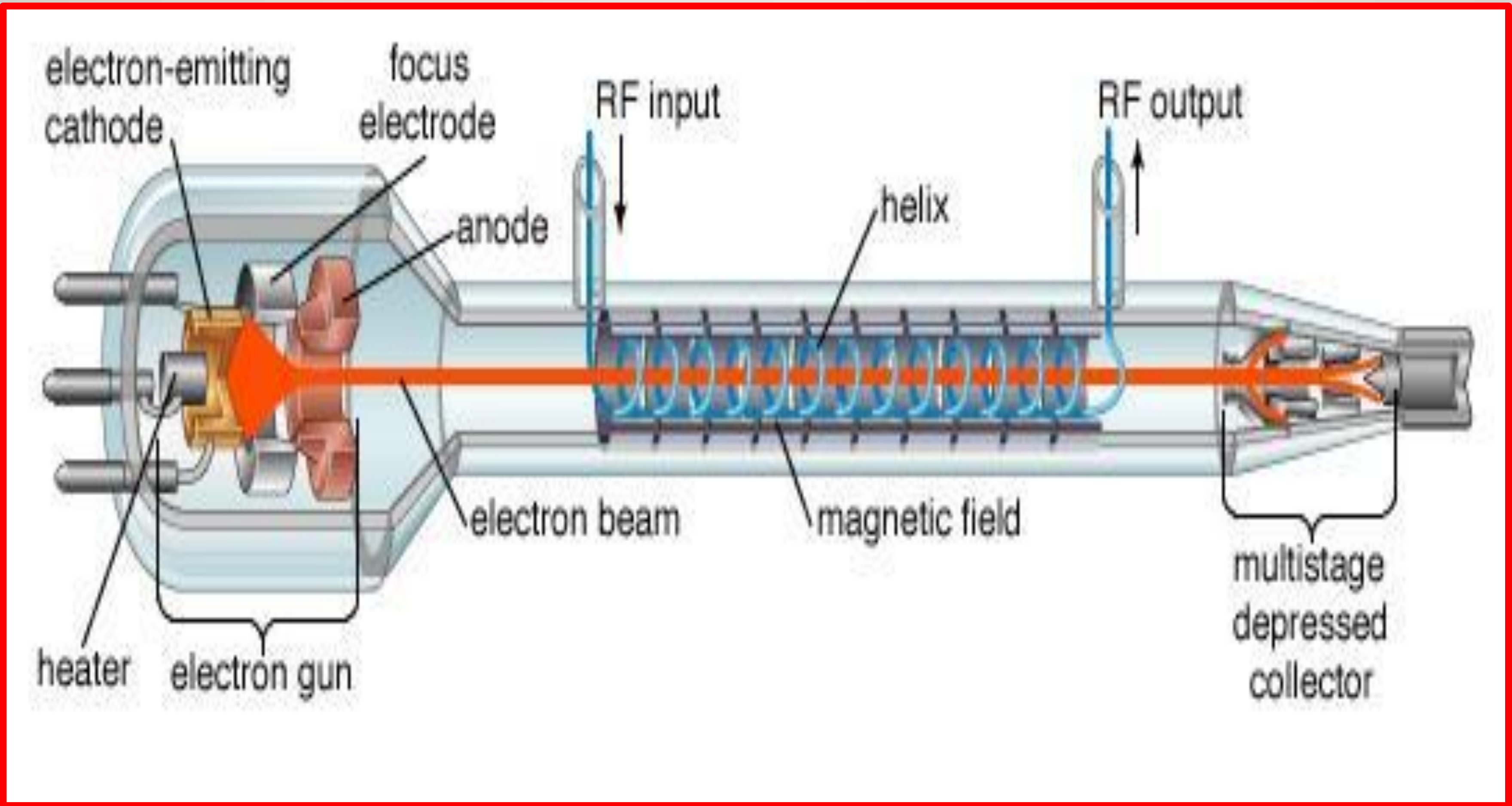
- TRAVELING WAVE TUBE (TWT) IS THE MOST VERSATILE MICROWAVE RF POWER AMPLIFIERS.

- THE MAIN VIRTUE OF THE TWT IS ITS EXTREMELY WIDE BAND WIDTH OF OPERATION.

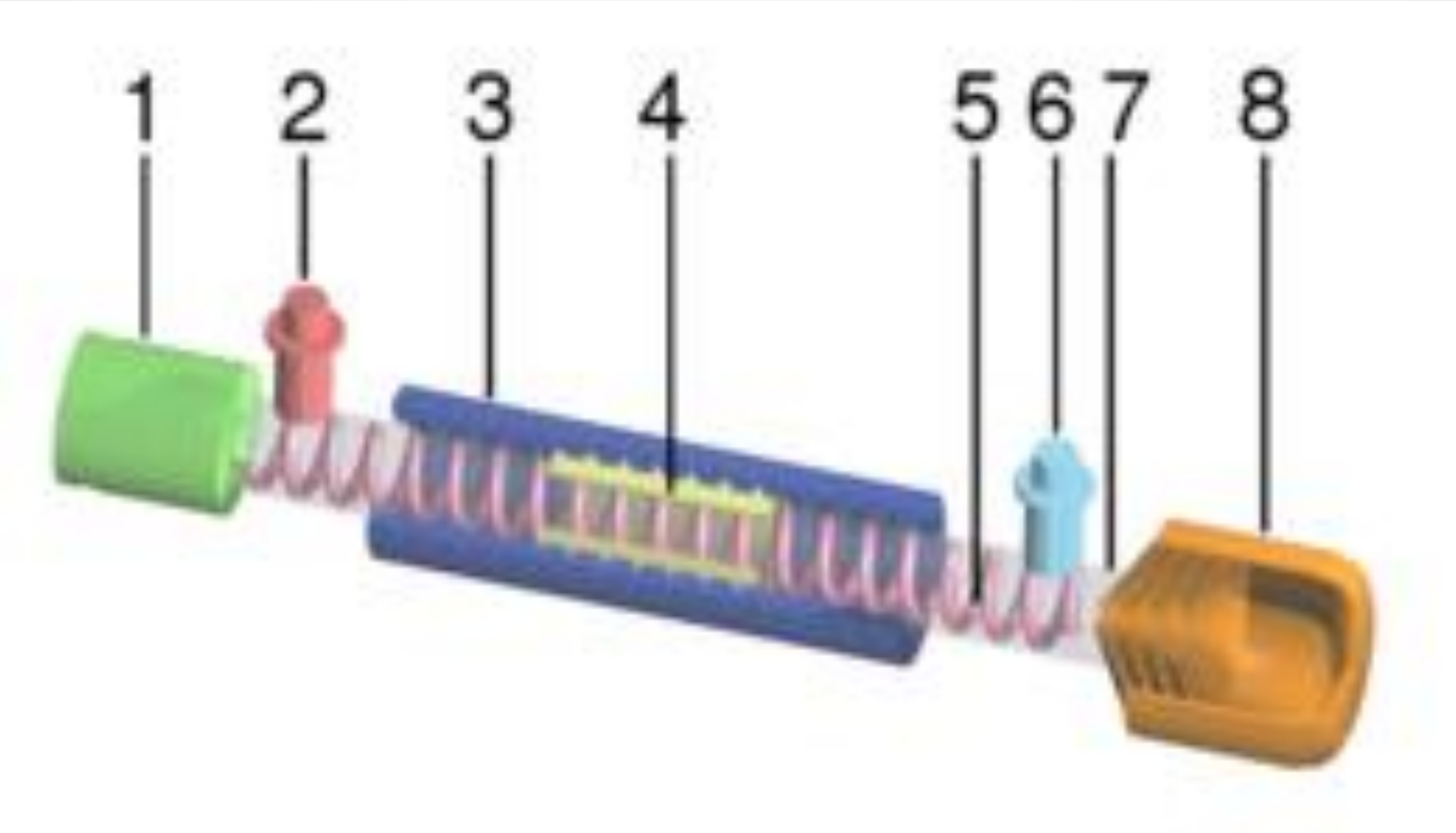
HELIX TRAVELLING WAVE TUBE

- FOR BROADBAND AMPLIFIER HELIX TWTS (PROPOSED BY PIERCE AND OTHERS IN 1946) ARE WIDELY USED
- FOR HIGH AVERAGE POWER PURPOSES THE COUPLED CAVITY TWTS ARE USED
- IN TWT, THE WAVES ARE PROPAGATING TYPE
- THE ELECTRON BEAM IS FOCUSED BY A CONSTANT MAGNETIC FIELD
- APPLIED SIGNAL PROPAGATES AROUND THE TURNS OF THE HELIX, PRODUCES AN ELECTRIC FIELD AT THE CENTER OF THE HELIX
- AN INTERACTION TAKES BETWEEN DC ELECTRIC FIELD AND AC ELECTRIC FIELD

BASIC STRUCTURE OF A TRAVELING WAVE TUBE (TWT)



HELIX TWT



1. **ELECTRON GUN**
2. **RF INPUT**
3. **MAGNETS**
4. **ATTENUATOR**
5. **HELIX COIL**
6. **RF OUTPUT**
7. **VACUUM TUBE**
8. **COLLECTOR**

ELECTRON GUN: PRODUCES AND THEN ACCELERATES AN ELECTRON BEAM ALONG THE AXIS OF THE TUBE.

THE SURROUNDING STATIC MAGNET PROVIDES A MAGNETIC FIELD ALONG THE AXIS OF THE TUBE TO FOCUS THE ELECTRONS INTO A TIGHT BEAM.

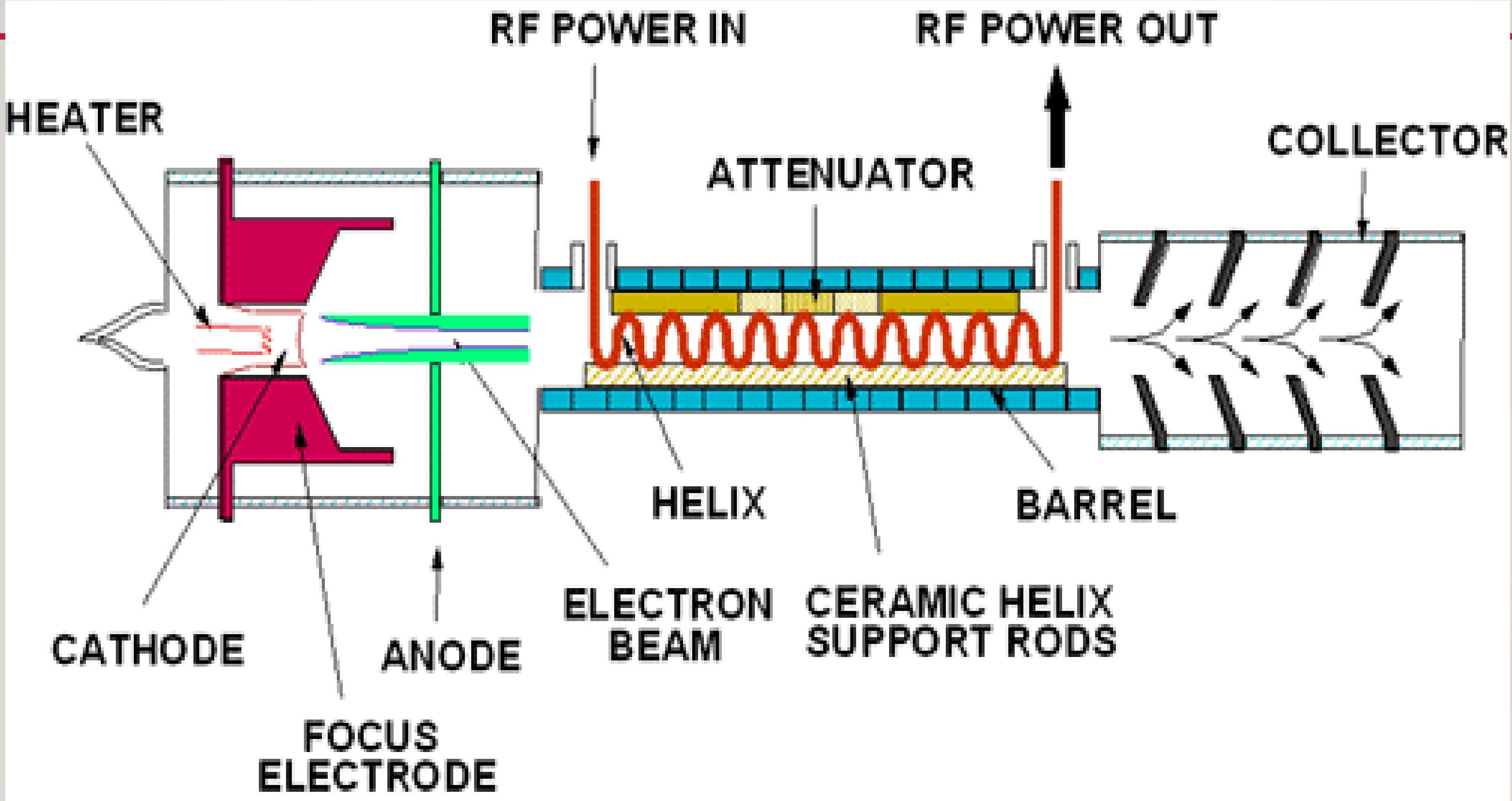
A LONGITUDINAL HELIX SLOW WAVE NON-RESONANT GUIDE IS PLACED AT THE CENTRE OF THE TUBE THAT PROVIDES A LOW IMPEDANCE TRANSMISSION LINE FOR THE RF ENERGY WITHIN THE TUBE.

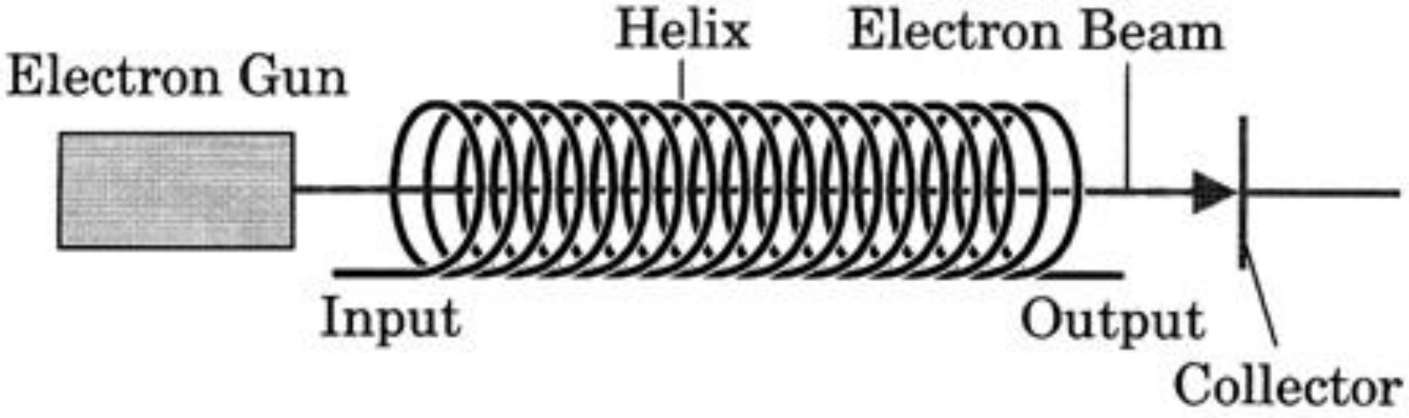
THE TWT IS DESIGNED WITH **HELIX DELAY STRUCTURE** TO SLOW THE TRAVELLING WAVE DOWN TO OR BELOW THE SPEED TO THE ELECTRONS IN THE BEAM.

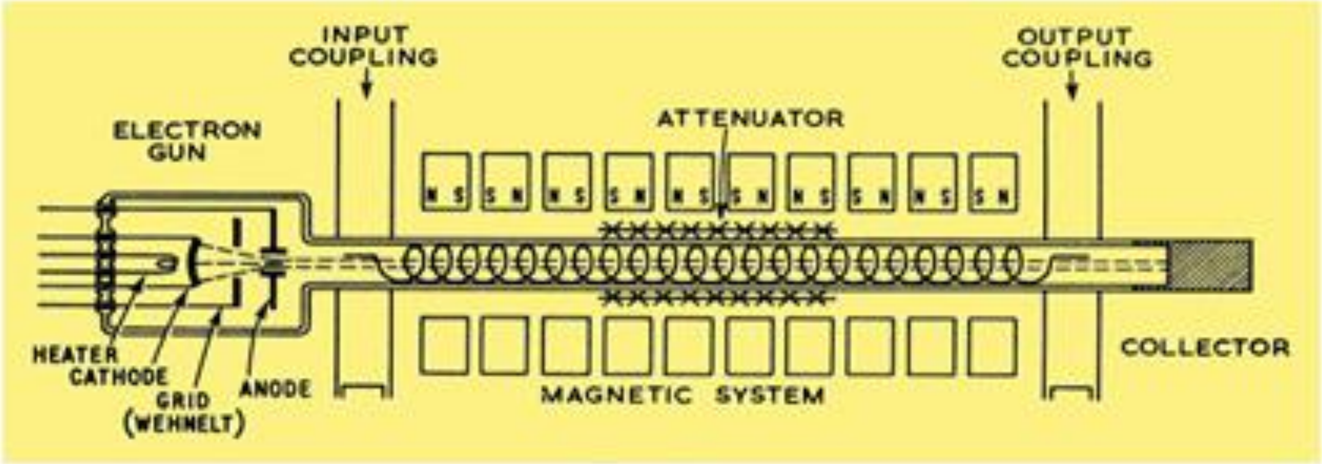
THE RF SIGNAL WAVE INJECTED AT THE INPUT END OF THE HELIX TRAVELS DOWN THE HELIX WIRE AT THE SPEED OF THE LIGHT BUT THE **COILED SHAPE** CAUSES THE WAVE TO TRAVEL A MUCH **GREATER DISTANCE** THAN THE ELECTRON BEAM.

THE AXIAL ELECTRIC FIELD (DUE TO RF SIGNAL ALONG THE HELIX) PROGRESSES WITH A VELOCITY –CLOSE TO LIGHT VELOCITY MULTIPLIED BY THE RATIO OF HELIX PITCH TO HELIX CIRCUMFERENCE.

TWT


RF POWER IN
RF POWER OUT
HEATER
COLLECTOR
ATTENUATOR
HELIX
BARREL
CATHODE
ANODE
ELECTRON BEAM
CERAMIC HELIX SUPPORT RODS
FOCUS ELECTRODE

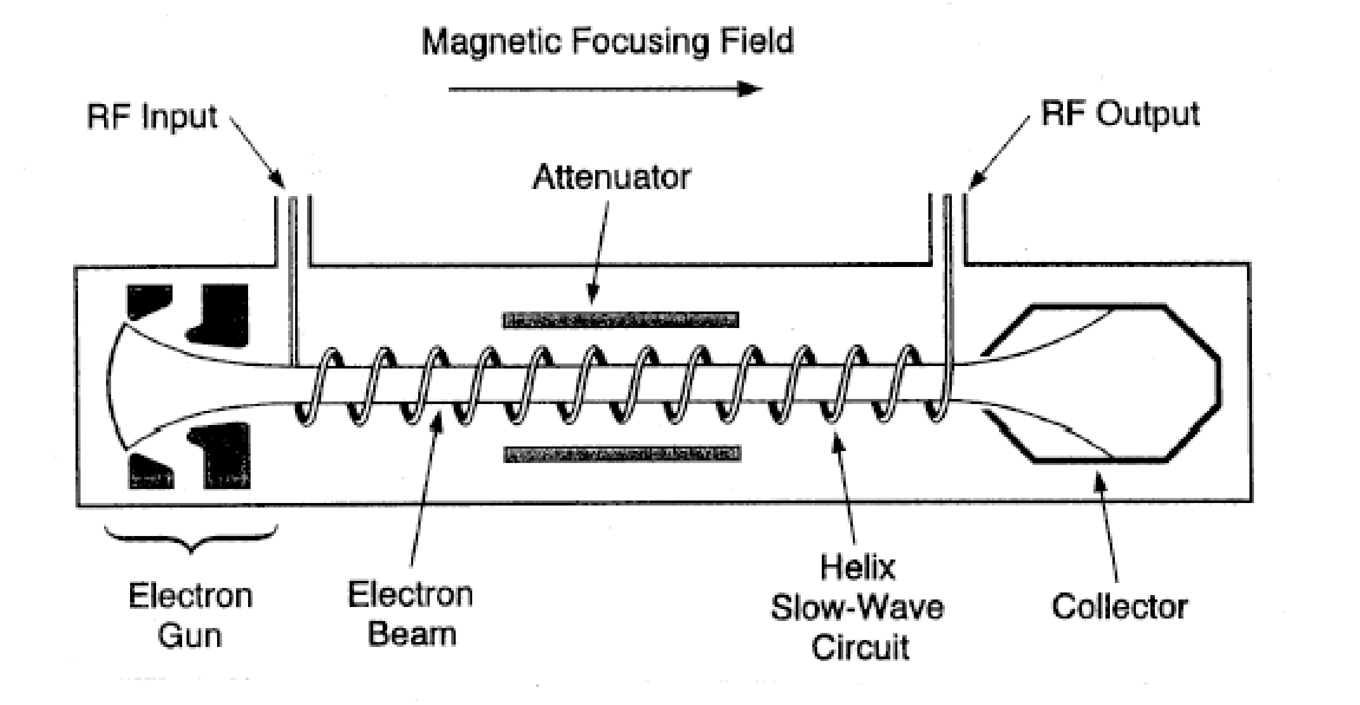

Helix
Electron Beam
Electron Gun
Input
Output
Collector


INPUT COUPLING
OUTPUT COUPLING
ATTENUATOR
ELECTRON GUN
HEATER
CATHODE
GRID (WEHNELT)
ANODE
MAGNETIC SYSTEM
COLLECTOR

Basic Helix TWT

If this were a coax line in TEM mode (not coiled), $v_p/c = 1$, independent of frequency, with unlimited bandwidth and no axial E.

By coiling the conductor, axial velocity is reduced by the amount of the increase in path length, a substantial E_z is created, and v_z remains largely independent of frequency. Gain is obtained by synchronizing the beam velocity with the axial wave velocity



(from Principles of Traveling Wave Tubes, A. S. Gilmour)

CHANGING THE NUMBER OF TURNS OR DIAMETER OF THE TURNS IN THE HELIX WIRE, THE SPEED AT WHICH RF SIGNAL WAVE TRAVELS IN THE FORM OF **AXIAL E FIELD, CAN BE VARIED**.

DC **BEAM VELOCITY** OF THE BEAM IS MAINTAINED SLIGHTLY **GREATER** THAN THAT OF THE AXIAL FIELD.

THE HELICAL DELAY STRUCTURE HAS THE ADDED ADVANTAGE OF CAUSING A LARGE PROPORTION OF ELECTRIC FIELDS THAT ARE PARALLEL TO THE ELECTRON BEAM, PROVIDES **MAXIMUM INTERACTION** BETWEEN THE FIELDS AND THE MOVING ELECTRONS TO FORM BUNCHING.

THE HELICAL COIL SLOWS DOWN THE VELOCITY OF MICROWAVE SIGNAL

INTRODUCTION:

- The traveling-wave tube (TWT) was invented in 1944 by Kompfner.
- The Traveling-Wave Tube (TWT) is an amplifier of microwave energy.
- It accomplishes this through the interaction of an electron beam and an RF circuit known as a slow wave structure.
- TWT are commonly used as amplifiers in satellite transponders, where the input signal is very weak and the output needs to be high power.
- TWT transmitters are used extensively in radar systems, particularly in airborne fire-control radar systems, and in electronic warfare and self-protection systems.

The major elements include;

- An electron beam to form and accelerate a beam of electrons
- A focusing magnet/magnetic system to focus the beam of electrons through the interaction structure
- A collector to collect the electron beam after the microwave power has been generated

- An input window where the small microwave signal to be amplified is introduced to the interaction structure
- An helix as interaction structure, where the electron beam interacts with the microwave signal to be amplified
- A microwave output window, where the microwave power is taken out of the tube
- **An internal attenuator, to absorb the power reflected back into the tube from mismatches in the output transmission line**

Missile TWTs for Active Seekers

Features that influence the design include:

- Minimal size and weight;
- Narrow-to-moderate bandwidths;.
- Off-to-fully-operational turn-on times of one second or less;
- High efficiency;
- High reliability after long inactive storage periods.

Normally, these TWTs are of the periodic-permanent-magnet (PPM) focused helix variety. They normally utilize unique cathode-heater designs to provide the very fast warm-up required. They typically have multiple stage depressed collectors with conduction cooling.

SLOW WAVE STRUCTURE

- For reducing electron velocity inside the tube
- Designed for large gain over wide bandwidth
- The phase velocity of microwave signal is greater than light velocity, hence to reduce with slow wave structure and pace with electrons
- The ratio of phase velocity, vp, along the pitch to the phase velocity is

$$\frac{v_p}{c} = \frac{p}{\sqrt{p^2 + (\pi d)^2}} = \sin \psi$$

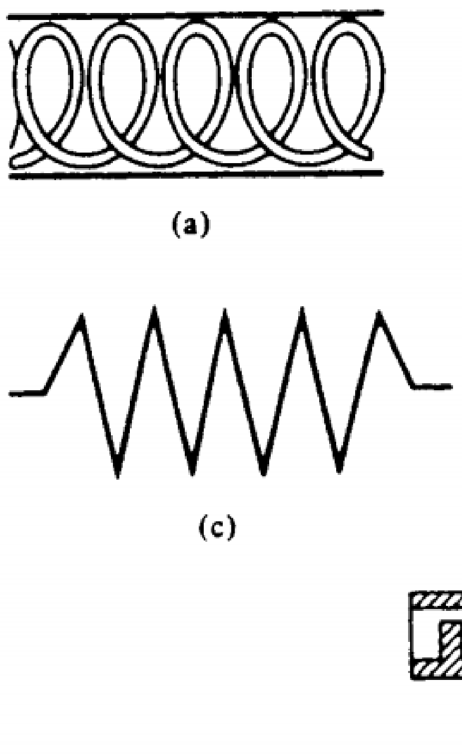


Figure 9-5-2 Slow-wave structures. (a) Helical line. (b) Folded-back line. (c) Zigzag line. (d) Interdigital line. (e) Corrugated waveguide.

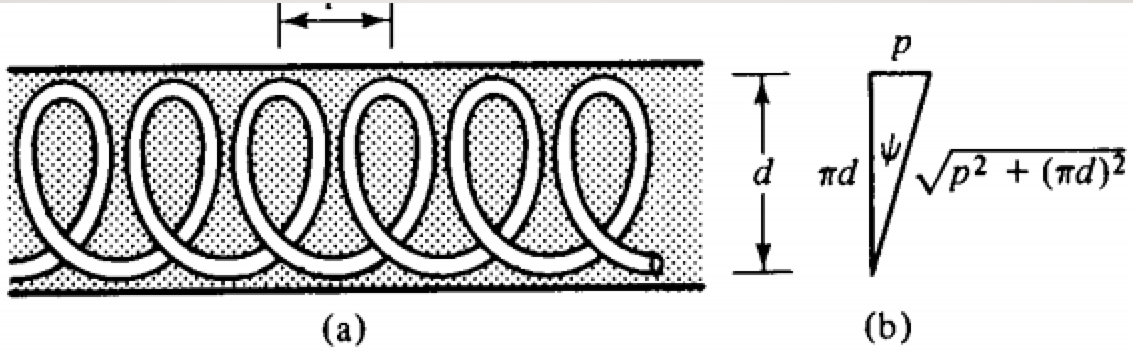


Figure 9-5-3 Helical slow-wave structure. (a) Helical coil. (b) One turn of helix.

where $c = 3 \times 10^8$ m/s is the velocity of light in free space
p = helix pitch
d = diameter of the helix
ψ = pitch angle

- The phase velocity in the axial direction is

$$v_{pe} = \frac{p}{\sqrt{\mu\epsilon[p^2 + (\pi d)^2]}}$$

- For very small pitch angle, the phase velocity is

$$v_p \approx \frac{pc}{\pi d} = \frac{\omega}{\beta}$$

$$v_{gr} = \frac{\partial\omega}{\partial\beta}$$

- The group velocity of the wave is slope of the curve

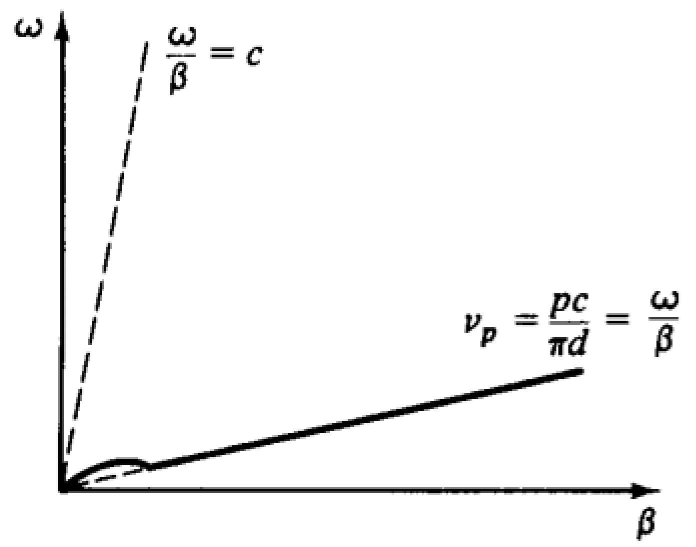


Figure 9-5-4 ω-β diagram for a helical structure.

VELOCITY MODULATION

THE ELECTRONS ENTERING THE HELIX AT ZERO FIELD ARE NOT AFFECTED BY THE SIGNAL WAVE; THOSE ELECTRONS ENTERING THE HELIX AT THE ACCELERATING FIELD ARE ACCELERATED, AND THOSE AT THE RETARDING FIELD ARE DECELERATED.

THIS VELOCITY MODULATION CAUSES BUNCHING OF ELECTRONS AT REGULAR INTERVALS OF ONE WAVELENGTH.

AS THE BUNCHES RELEASE ENERGY TO THE SIGNAL ON THE HELIX, AMPLIFICATION BEGINS.

THIS AMPLIFIED SIGNAL CAUSES A DENSER ELECTRON BUNCH WHICH IN TURN AMPLIFIES THE SIGNAL EVEN MORE.

THIS PROCESS CONTINUES AS THE RF WAVE AND THE ELECTRON BEAM TRAVEL DOWN THE LENGTH OF THE TUBE.

WHEN THE LOSS IN THE SYSTEM IS COMPENSATED BY THIS ENERGY TRANSFER, A STEADY AMPLIFICATION OF THE MICROWAVE SIGNAL APPEARS AT THE OUTPUT END.

BEAM VELOCITY GREATER THAN FIELD VELOCITY?

AS THE DC VELOCITY OF THE BEAM IS MAINTAINED BY SLIGHTLY GREATER THAN THE PHASE VELOCITY OF THE TRAVELLING WAVE, MORE ELECTRONS FACE THE RETARDING FIELD THAN THE ACCELERATING FIELD, AND A GREAT AMOUNT OF KINETIC ENERGY IS TRANSFERRED FROM THE BEAM TO THE ELECTROMAGNETIC FIELD.

THUS THE FIELD AMPLITUDE INCREASES FORMING A MORE COMPACT BUNCH AND A LARGE AMPLIFICATION OF THE SIGNAL VOLTAGE APPEARS AT THE OUTPUT OF THE HELIX.

OPERATION

- The helix acts as a delay line, in which the RF signal travels at near the same speed along the tube as the electron beam.
- The electromagnetic field due to the RF signal in the helix interacts with the electron beam, causing bunching of the electrons (an effect called velocity modulation), and the electromagnetic field due to the beam current then induces more current back into the helix (i.e. the current builds up and thus is amplified as it passes down).
- A second directional coupler, positioned near the collector, receives an amplified version of the input signal from the far end of the helix.
- An attenuator placed on the helix, usually between the input and output helices, prevents reflected wave from traveling back to the cathode.

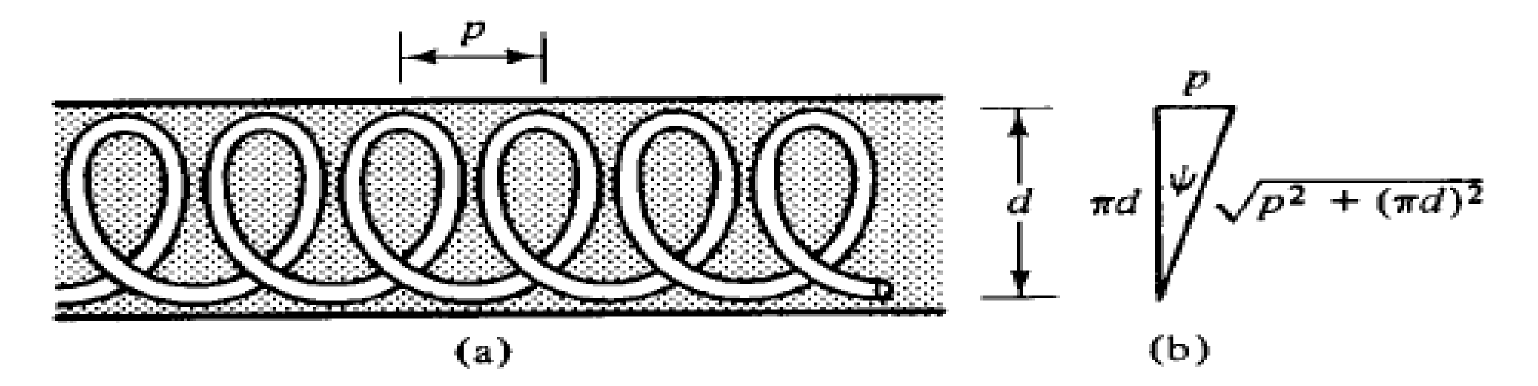
WHY ATTENUATOR?

AN ATTENUATOR IS PLACED OVER A PART OF THE HELIX ON MIDWAY TO ATTENUATE ANY REFLECTED WAVES GENERATED DUE TO THE IMPEDANCE MISMATCH.

IT IS PLACED AFTER SUFFICIENT LENGTH OF THE INTERACTION REGION SO THAT THE ATTENUATION OF THE AMPLIFIED SIGNAL IS INSIGNIFICANT COMPARED TO THE AMPLIFICATION.

HOW IT WORKS:

- A helix traveling-wave tube consists of an electron beam and a slow-wave structure. The electron beam is focused by a constant magnetic field along the electron beam and the slow-wave structure.
- The commonly used slow-wave structure is a helical coil with a concentric conducting cylinder



- The electron beam can be accelerated only to velocities that are about a fraction of the velocity of light.
- A slow-wave structure must be incorporated in the microwave devices so that the phase velocity of the microwave signal can keep pace with that of the electron beam for effective interactions.
- It can be shown that the ratio of the phase velocity v_p along the pitch to the light velocity along the coil is given by:

$$\frac{v_p}{c} = \frac{p}{\sqrt{p^2 + (\pi d)^2}} = \sin \psi$$

- Where $c = 3 \times 10^8$ m/s is the velocity of light in free space
- p = helix pitch
- d = diameter of the helix
- ψ = pitch angle

- The TWT contains an electron gun which produces and

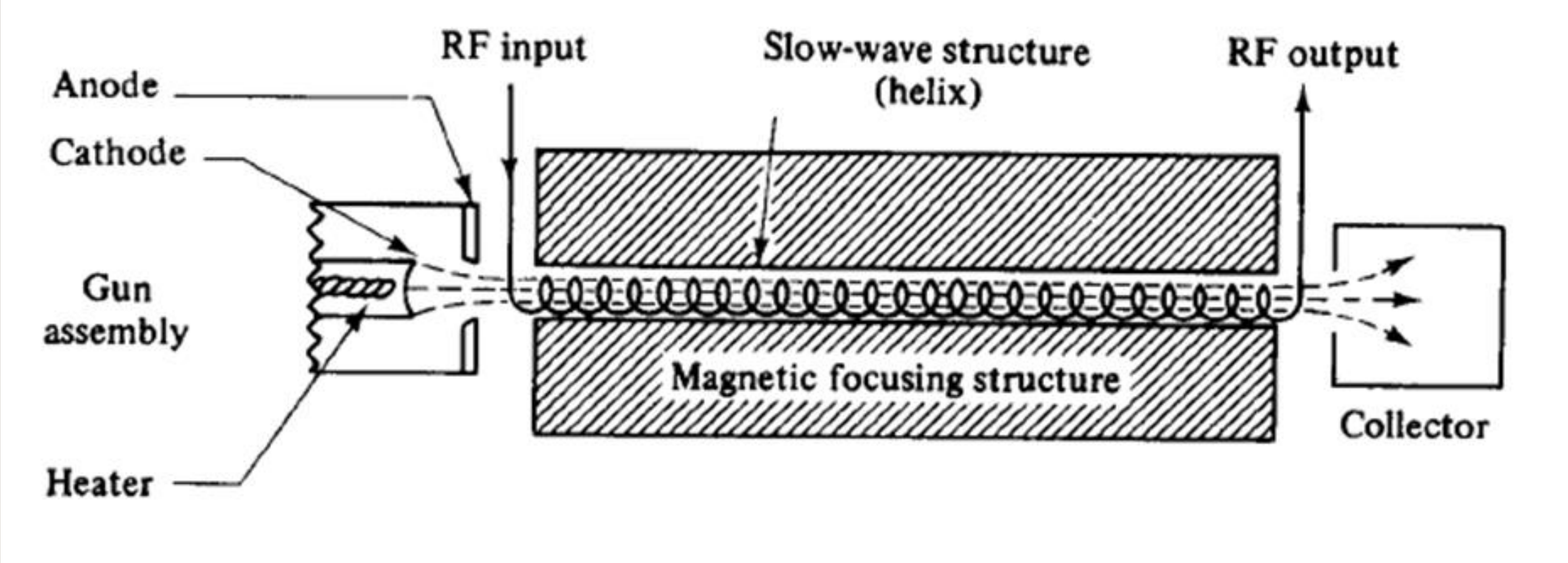
then accelerates an electron beam along the axis of the tube.

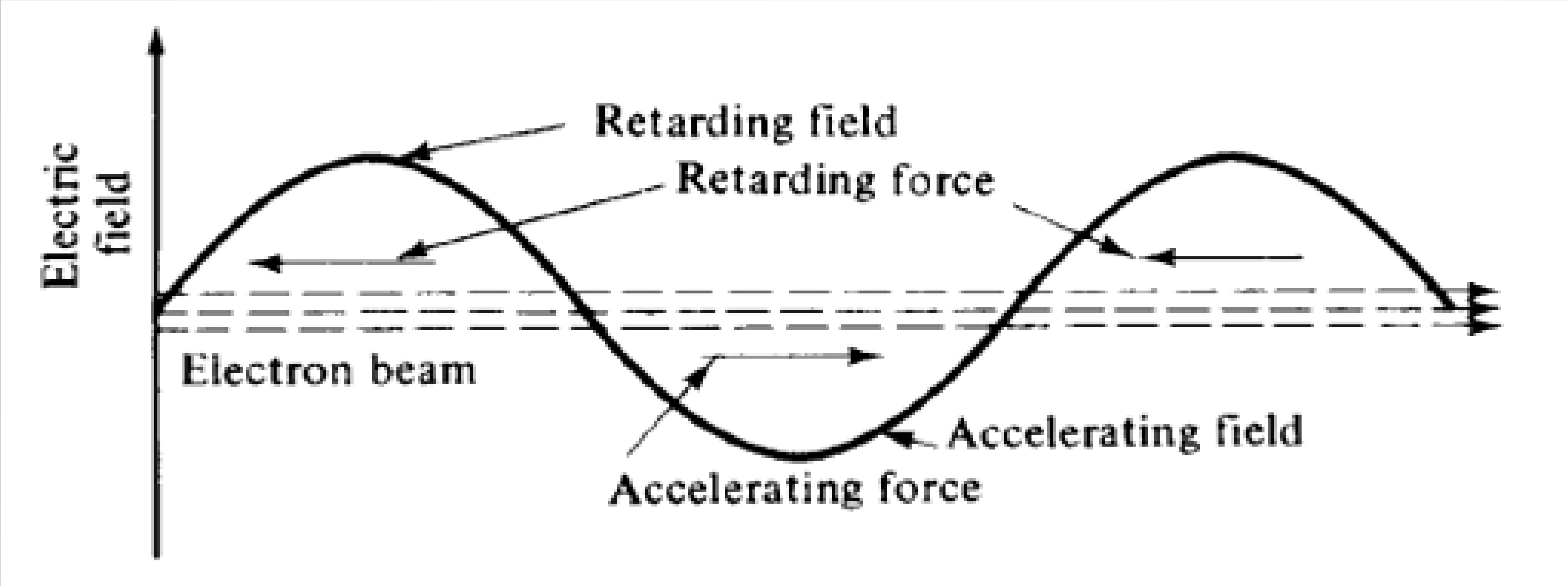
- The surrounding magnet provides a magnetic field along the axis

of the tube to focus the electrons into a tight beam.

- The helix, at the center of the tube, is a coiled wire that provides a low-impedance transmission line for the RF energy within the tube.
- The RF input and output are coupled onto and removed from the helix by waveguide directional couplers that have no physical connection to the helix.
- The attenuator prevents any reflected waves from traveling back down the helix.

- The applied signal propagates around the turns of the helix and produces an electric field at the center of the helix, directed along the helix axis.
- When the electrons enter the helix tube, an interaction takes place between the moving axial electric field and the moving electrons.
- This interaction causes the signal wave on the helix to be amplified.




Electric field
Retarding field
Retarding force
Electron beam
Accelerating field
Accelerating force

SPECIFICATIONS

FREQUENCY RANGE: 3 GHZ AND HIGHER

BANDWIDTH: ABOUT 0.8 GHZ

EFFICIENCY: 20 TO 40%

POWER OUTPUT: UP TO 10KW AVERAGE

POWER GAIN: UP TO 60DB

DIFFERENCE BETWEEN TWT & KLYSTRON:

- In the case of the TWT, the microwave circuit is non-resonant.
- The interaction of electron beam and RF field in the TWT is continuous over the entire length of the circuit, but the interaction in the klystron occurs only at the gaps of a few resonant cavities.
- The wave in the TWT is a propagating wave; the wave in the klystron is not.
- In the coupled-cavity TWT there is a coupling effect between the cavities, whereas each cavity in the klystron operates independently.

COMPARISON OF TWTA AND KLYSTRON AMPLIFIER

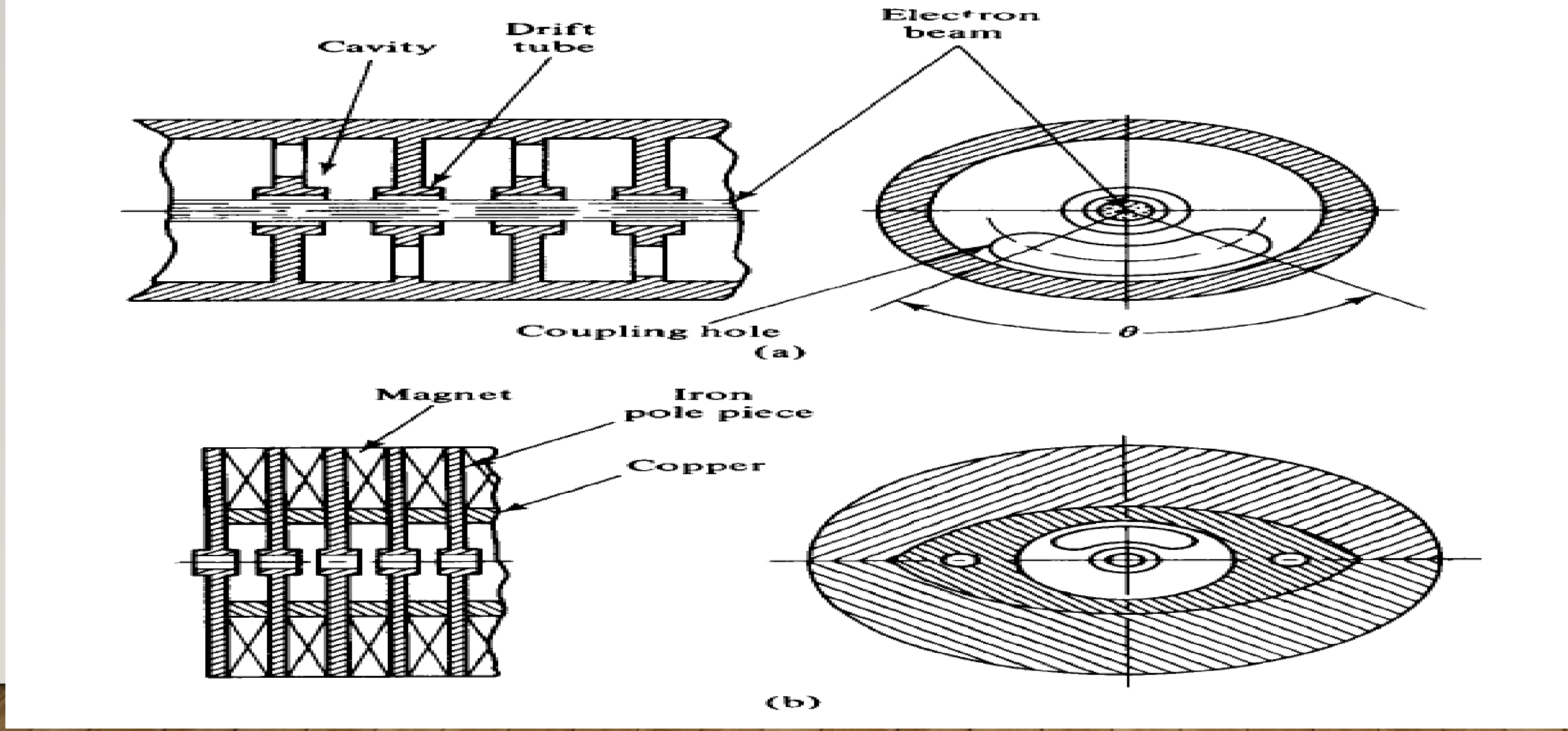
KLYSTRON AMPLIFIER	TWTA
1. LINEAR BEAM OR ‘O’ TYPE DEVICE	1. LINEAR BEAM OR ‘O’ TYPE DEVICE
2. USES RESONANT CAVITIES FOR INPUT AND OUTPUT CIRCUITS	2. USES NON RESONANT WAVE CIRCUITS
3. NARROWBAND DEVICE	3.WIDEBAND DEVICE

BASIC STRUCTURE

- The basic structure of a TWT consists of a cathode and filament heater plus an anode that is biased positively to accelerate the electron beam forward and to focus it into a narrow beam.
- The electrons are attracted by a positive plate called the collector, which has given a high dc voltage.
- The length of the tube is usually many wavelengths at the operating frequency.
- Surrounding the tube are either permanent magnets or electromagnets that keep the electrons tightly focused into a narrow beam.

TYPES OF TWT:

- **Coupled Cavity Travelling wave Tube:**



- The Coupled-cavity TWT uses a slow wave structure of a series of cavities coupled to one another.
- The resonant cavities are coupled together with a transmission line.
- The electron beam is velocity modulated by an RF input signal at the first resonant cavity.
- This RF energy travels along the cavities and induces RF voltages in each subsequent cavity.
- If the spacing of the cavities is correctly adjusted, the voltages at each cavity induced by the modulated beam are in phase and travel along the transmission line to the output, with an additive effect, so that the output power is much greater than the power input.

FEATURES

- The unique feature of the TWT is a helix or coil that surrounds the length of the tube and the electron beam passes through the centre or axis of the helix.
- The microwave signal to be amplified is applied to the end of the helix near the cathode and the output is taken from the end of the helix near the collector.
- The purpose of the helix is to provide path for RF signal.
- The propagation of the RF signal along the helix is made approximately equal to the velocity of the electron beam from the cathode to the collector

FUNCTIONING

- The passage of the microwave signal down the helix produces electric and magnetic fields that will interact with the electron beam.
- The electromagnetic field produced by the helix causes the electrons to be speeded up and slowed down, this produces velocity modulation of the beam which produces density modulation.
- Density modulation causes bunches of electrons to group together one wavelength apart and these bunch of electrons travel down the length of the tube toward the collector.

FUNCTIONING

- The electron bunches induce voltages into the helix which reinforce the voltage already present there. Due to that the strength of the electromagnetic field on the helix increases as the wave travels down the tube towards the collector.
- At the end of the helix, the signal is considerably amplified. Coaxial cable or waveguide structures are used to extract the energy from the helix.

ADVANTAGES

1. TWT has extremely wide bandwidth. Hence, it can be made to amplify signals from UHF to hundreds of gigahertz.
2. Most of the TWT's have a frequency range of approximately 2:1 in the desired segment of the microwave region to be amplified.
3. The TWT's can be used in both continuous and pulsed modes of operation with power levels up to several thousands watts.

PERFORMANCE CHARACTERISTICS

1. Frequency of operation : 0.5 GHz – 95 GHz
2. Power outputs:

   5 mW (10 – 40 GHz – low power TWT)

   250 kW (CW) at 3 GHz (high power TWT)

   10 MW (pulsed) at 3 GHz
3. Efficiency : 5 – 20 % (30 % with depressed collector)

APPLICATIONS OF TWT

1. Low noise RF amplifier in broad band microwave receivers.
2. Repeater amplifier in wide band communication links and long distance telephony.
3. Due to long tube life (50,000 hours against ¼th for other types), TWT is power output tube in communication satellite.
4. Continuous wave high power TWT's are used in troposcatter links (due to larger power and larger bandwidths).
5. Used in Air borne and ship borne pulsed high power radars.

Major applications for TWTs include:

Amplifiers:

- **Space applications**
- **Radar**
- **Electron Counter Measure**
- **Missile**

Driver for other high power RF amplifiers

AMPLIFICATION PROCESS

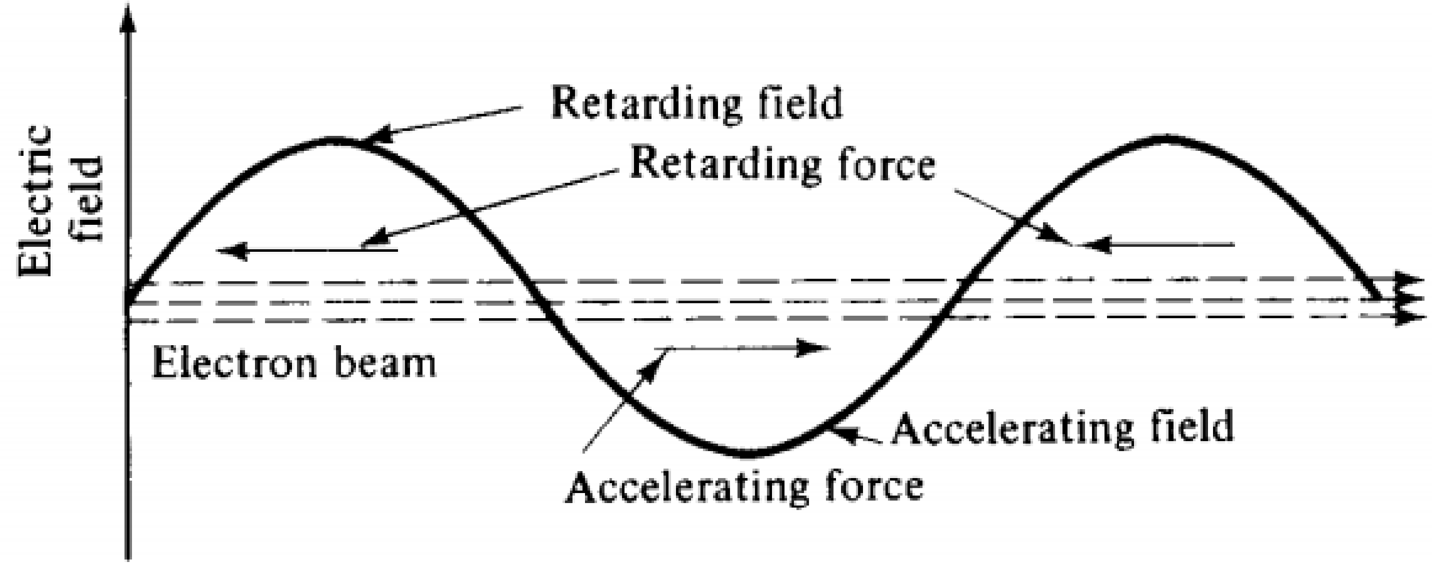


Figure 9-5-7 Interactions between electron beam and electric field.

- The dc transit time of an electron L is pitch period

$$T_0 = \frac{L}{v_0}$$

- Phase constant of nth harmonics

$$\beta_n = \frac{\omega}{v_0} = \frac{\theta_1 + 2\pi n}{v_0 T_0} = \beta_0 + \frac{2\pi n}{L}$$

- The axial harmonics phase velocity is synchronized with beam velocity for interaction between the electron beam and electric field

$$v_{np} = v_0$$

- The signal voltage is coupled to helix, the axial electric field exerts a force on electrons

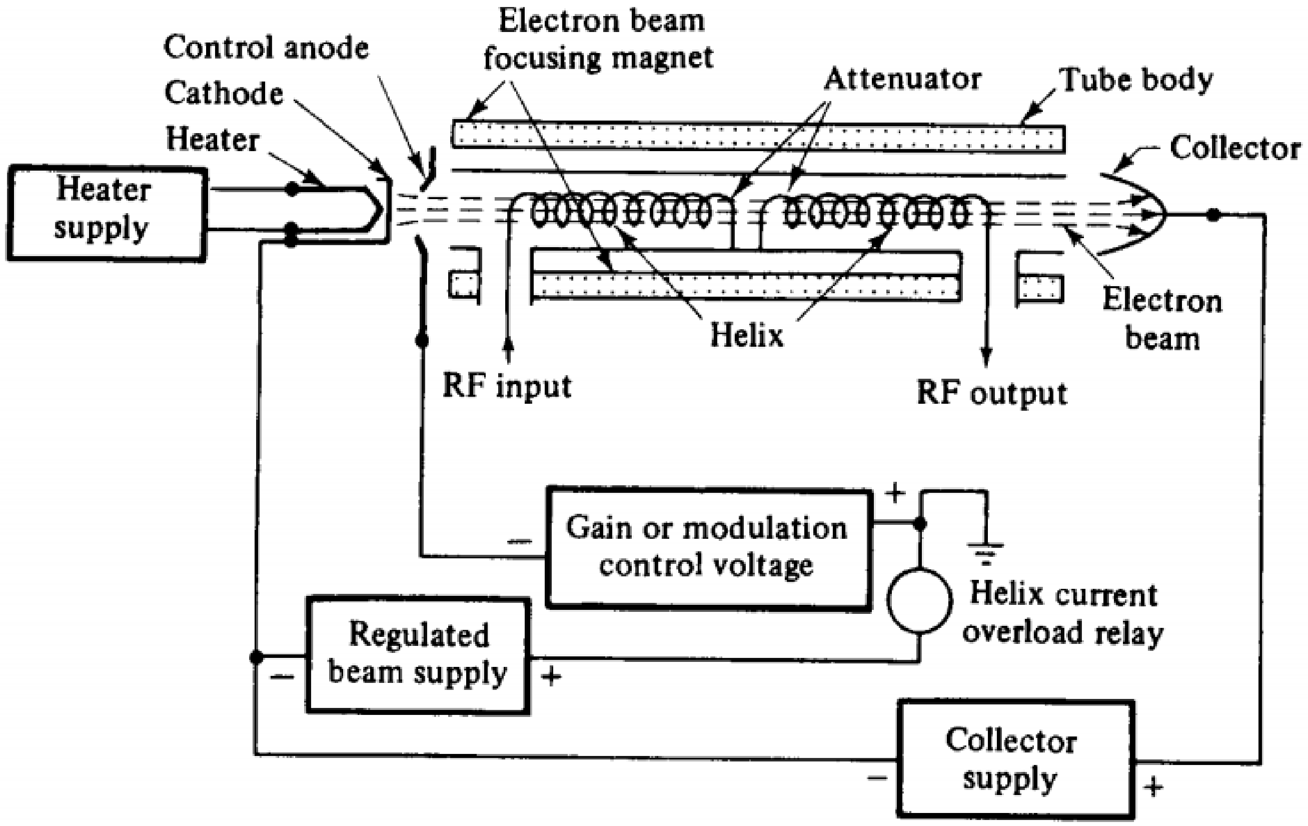
$$F = -eE, \mathrm{E} = -\nabla V$$

- The velocity of electron is

$$v = v_0 + v_e \cos(\omega_e t + \theta_e)$$

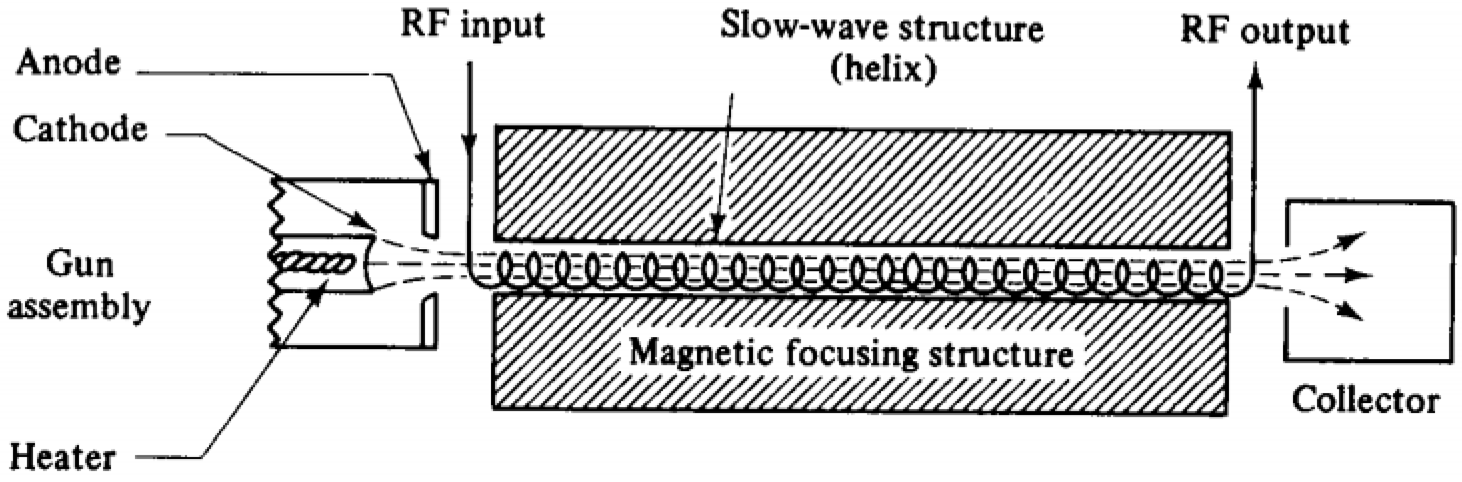
The equation of motion of the electron is given by

$$m\frac{dv}{dt} = -eE_1 \sin(\omega t - \beta_p z)$$



Control anode
Cathode
Heater
Electron beam focusing magnet
Attenuator
Tube body
Collector
Heater supply
Helix
Electron beam
RF input
RF output
Gain or modulation control voltage
Helix current overload relay
Regulated beam supply
Collector supply

(a)



(b)

Figure 9-5-6 Diagram of helix traveling-wave tube: (a) schematic diagram of helix traveling-wave tube; (b) simplified circuit.

$$mv_e\omega_e \sin(\omega_e t + \theta_e) = eE_1 \sin[\omega t - \beta_p v_0(t - t_0)]$$

$$v_e = \frac{eE_1}{m\omega_e}$$

$$\omega_e = \beta_p(v_p - v_0)$$

$$\theta_e = \beta_p v_0 t_0$$

CONVECTION CURRENT

- The space charge effect is considered, the electron velocity, the charge density, the current density, the axial electric field will perturbate the average dc value,

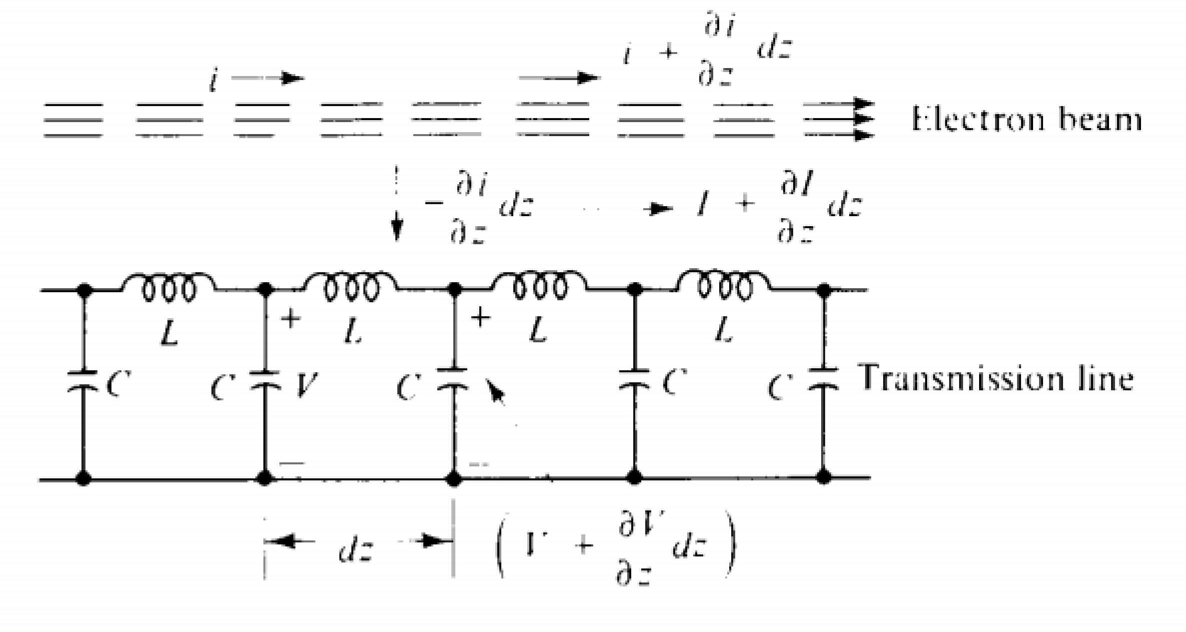
$$v = v_0 + v_1 e^{j\omega t-\gamma z}$$
$$\rho = \rho_0 + \rho_1 e^{j\omega t-\gamma z}$$
$$J = -J_0 + J_1 e^{j\omega t-\gamma z}$$
$$E_z = E_1 e^{j\omega t-\gamma z}$$

- The J_0 will be positive in the –ve Z direction.
- So the electron current density is $J = \rho v \approx -J_0 + J_1 e^{j\omega t-\gamma z}$ where

$$-J_0 = \rho_0 v_0, J_1 = \rho_1 v_0 + \rho_0 v_1 \, and \, \rho_1 v_1 \approx 0$$

AXIAL ELECTRIC FIELD

- Convection current in the electron beam induces an electric field in the slow wave circuit.
- The coupling relation between the electron beam and slow wave helix is shown.



- The convection current in the electron beam is $$i = j\frac{\beta_e I_0}{2V_0(j\beta_e - \gamma)^2}E_1$$
- Where $\beta_e = \omega/v_0$ and $v_0 = \sqrt{(2e/m)V_0}$
- The above equation is known as electronic equation, determines the convection current induced by the axial electric field.

- The propagation constant is $$\gamma_0 = j\omega\sqrt{LC}$$
- The characteristic impedance of line is $$Z_0 = \sqrt{\frac{L}{C}}$$
- The axial electric field is given by
- $$E_1 = -\frac{\gamma^2 \gamma_0 Z_0}{\gamma^2 - \gamma_0^2} i$$
- Axial electric field of the slow wave helix is affected by the spatial ac electron beam

WAVE MODES

- There are four distinct solutions for propagation constant.
- Four modes of travelling O-type tubes.
- Equating the dc electron beam velocity to the axial phase velocity of the traveling wave tube $\gamma_0 = j\beta_e$
- Then $(\gamma - j\beta_e)^3(\gamma + j\beta_e) = 2C^3\beta_e^2\gamma^2$ $C \equiv \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$ where C is the traveling wave tube gain parameter
- Three forward travelling tube corresponds to $e^{-j\beta_e z}$ and one backward wave corresponds to $e^{j\beta_e z}$

Then Eq. (9-5-53) reduces to

$$(\gamma - j\beta_e)^3(\gamma + j\beta_e) = 2C^3\beta_e^2\gamma^2 \qquad (9\text{-}5\text{-}55)$$

where C is the traveling-wave tube gain parameter and is defined as

$$C \equiv \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3} \qquad (9\text{-}5\text{-}56)$$

- The four propagation constants ϒ are

$$\gamma_1 = -\beta_e C \frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_2 = \beta_e C \frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_3 = j\beta_e(1 - C)$$

$$\gamma_4 = -j\beta_e\left(1 - \frac{C^3}{4}\right)$$

GAIN CONSIDERATION

- The Tube is perfectly matched ,there is no backward traveling wave
- The total circuit voltage is the sum of three forward voltages corresponding to the three forward traveling waves

$$V(z) = V_1 e^{-\gamma_1 z} + V_2 e^{-\gamma_2 z} + V_3 e^{-\gamma_3 z} = \sum_{n=1}^{3} V_n e^{-\gamma_n z}$$

- The input current can be found from

$$i(z) = -\sum_{n=1}^{3} \frac{I_0}{2V_0 C^2} \frac{V_n}{\delta_n^2} e^{-\gamma_n z}$$

- The input fluctuating component of velocity of the total wave is

$$v_1(z) = \sum_{n=1}^{3} j \frac{v_0}{2V_0 C} \frac{V_n}{\delta_n} e^{-\gamma_n z}$$