In a Sampling a large no: of parts manufactured by a machine, the mean no: of defectives in a Sample of 20 is 2 out of 1000 Such Samples, how many would be expected to Contain

a) no defective b) exactly 3 defective c) not more than 3 defective

$n = 20$ $N = 1000$

$p = p(\text{defective}) = \frac{2}{20} = 0.1$

$q = 1 - p = 0.9$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x}$ $x = 0, 1, 2, \dots, n$

$= 20C_x (0.1)^x (0.9)^{20-x}$ $x = 0, 1, 2, \dots, 20$

$P(\text{no defective}) = p(x=0)$

$= 20C_0 (0.1)^0 (0.9)^{20} = \underline{0.1216}$

Required Number $= N \times p(x=0)$

$= 1000 \times 0.1216 = 121.6 \simeq \boxed{122}$

ii) $P(\text{exactly } 3 \text{ def}) = p(x=3)$

$= 20C_3 (0.1)^3 (0.9)^{17} = \underline{0.1901}$

Req: Num $= N \times p(x=3) = 1000 \times 0.1901$

$= 190.1 \simeq \boxed{190}$

iii) $p(\text{not more than } 3) = p(x \leq 3)$

$= p(0) + p(1) + p(2)$

defectives in a Sample of 20 is 2 Out of 1000 Such Samples, how many would [be]
expected to Contain

a) no defective b) exactly 3 defective c) not more than 3 defective

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$p(x) = nC_x \, p^x \, q^{n-x} \qquad x = 0,1,2,\dots,n$

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Req. Num $= N \times p(x=3) = 1000 \times 0.1901$

$= 190.1 \simeq \boxed{190}$

iii) $p(\text{not more than 3}) = p(x \le 3)$

$= p(0) + p(1) + p(2) + p(3)$

$= 20C_0 (0.1)^0 (0.9)^{20} + 20C_1 (0.1)^1 (0.9)^{19} + 20C_2 (0.1)^2 (0.9)^{18}$

$+ 20C_3 (0.1)^3 (0.9)^{17}$

$= 0.867$

Req. Num $= N \times p(x \le 3)$

$= 1000 \times 0.867$

$= 867$

U.Q. The probability that an electric Component manufactured by a firm is defective is 0.01. The produced items are sent to the market in packets of 10. In a Consignment of 1000 such packets how many can be expected to Contain (1) exactly two defectives. (ii) atmost two defectives.

$n = 10$ $N = 1000$

$p = p(\text{defective}) = 0.01$

$q = 1 - p = 1 - 0.1 = 0.99$

Prob of BD is

$p(x) = nC_x \, p^x \, q^{n-x}$ $x = 0, 1, 2, \ldots, n.$

$= 10C_x \, (0.01)^x \, (0.99)^{10-x}$ $x = 0, 1, 2, \ldots, 10$

1) $p(x = 2) =$

Req : Num = $N \times p(x=2)$

p(atmost 2 defective)

$= p(x \leq 2)$

$= p(0) + p(1) + p(2)$

$=$

Req : Num $= N \times p(x \leq 2)$

$=$

Show a 5 or 6?

$\frac{fav}{To:}$

$n = 6 \qquad N = 729$

$p = p(\text{getting } 5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

$q = 1 - p = 1 - \frac{1}{3}$

$= \frac{2}{3}$

$\therefore$ Pmf of BD is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots n$

$= 6C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x} \quad x = 0, 1, 2, \ldots 6$

P(atleast 3 dice to show 5 or 6)

$= p(x \geq 3)$

$= p(x=3) + p(x=4) + p(x=5) + p(x=6)$

OR.

$= 1 - p(x < 3)$

$= 1 - [p(x=0) + p(x=1) + p(x=2)]$

$= 1 - \left[6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 + 6C_1 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^5 + 6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4\right]$

$= \frac{233}{729}$

$\therefore$ Rq: Num $= N \times p(x \geq 3) = \cancel{729} \times \frac{233}{\cancel{729}}$

$= 233$

Find the probability that in a family of 4 children there will be a) atleast 1 boy b) 1 or 2 girls c) no girls. Out of 1000 such families chosen at random how many would you expect to have a) atleast 1 boy b) 1 or 2 girls c) no girls

$\frac{1}{2}$

$n=4$ $N=1000$

$p = p(\text{boy}) = \frac{1}{2}$

$q = 1-p = 1-\frac{1}{2} = \frac{1}{2}$

$x^m x^n = x^{m+n}$

$x+4-x = \left(\frac{1}{2}\right)^4$

Pmf of B.D is

$p(x) = nC_x \, p^x q^{n-x} \quad x = 0,1,2,\dots n.$

$= 4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$

$= 4C_x \left(\frac{1}{2}\right)^4 \quad x = 0,1,2,3,4$

$P(\text{atleast } 1 \text{ boy}) = p(x \geq 1)$

b) 1 or 2 girls c) no girls. Out of 1000 such families chosen at random how many would you expect to have a) atleast 1 boy b) 1 or 2 girls c) no girls.

$n = 4 \qquad N = 1000$

$p = p(\text{boy}) = \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$x^m x^n = x^{m+n}$

$x + 4 - x = \left(\frac{1}{2}\right)^4$

Pmf of B.D is

$p(x) = nC_x \, p^x q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$= 4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$

$= 4C_x \left(\frac{1}{2}\right)^4 \quad x = 0, 1, 2, 3, 4$

$P(\text{atleast 1 boy}) = p(x \geq 1) = 1 - p(x < 1)$

$= 1 - p(x = 0) = 1 - 4C_0 \left(\frac{1}{2}\right)^4$

$= \frac{15}{16}$

Req. Num $= N \times p(x \geq 1) = 1000 \times \frac{15}{16}$ $\frac{1}{2}$

$= \underline{936}$ 937.5

ii) $p(1 \text{ or } 2 \text{ girls}) = p(3 \text{ boys}) + p(2 \text{ boys})$

$= p(x=3) + p(x=2)$

$= 4C_3 \left(\frac{1}{2}\right)^4 + 4C_2 \left(\frac{1}{2}\right)^4 = \underline{\frac{10}{16}}$

Req. Num $= N \times \frac{10}{16} = 1000 \times \frac{10}{16} = \underline{625}$

iii) $P(\text{no girls}) = p(\text{all are boys})$

$= p(x=4) = 4C_4 \left(\frac{1}{2}\right)^4 = \frac{1}{16}.$

Req. Num $= N \times \frac{1}{16} = \frac{1000}{16} = 62.5$

$= \underline{63}$

out of 800 families with 4 children each, how many families would be expected to have i) 2 boys and 2 girls ii) atleast one boy iii) children of both sexes.

$n = 4$ $N = 800$

$p = p(\text{boy}) = \frac{1}{2}$

$q = \frac{1}{2}$.

i) $p(2 \text{ boys and } 2 \text{ boys}) = p(x=2)$

$= 4C_2 \left(\frac{1}{2}\right)^4$

$=$ ____

Req: No $= 800 \times \frac{3}{8} =$ ____

ii) $P(\text{atleast } 1 \text{ boy}) = p(x \geq 1)$

$= \frac{15}{16}$

Req: No $= 800 \times \frac{15}{16} = 750$

iii) $p(\text{children of both Sex})$

$= p(1 \text{ boy}) + p(2 \text{ boy}) + p(3 \text{ boy})$

$= p(x=1) + p(x=2) + p(x=3)$

$=$ ____

Req. No $= 800 \times$ ____ $=$

In a lot of 500 Solenoids, 25 are defective. find the probabilities of a Sample of 20 Solenoids choosen at random may have

i) no defectives ii) two defectives iii) not more than 2 defectives.

$n = 20$

$p = p(\text{defective}) = \frac{25}{500} = 0.05$

$q = 1 - p = 1 - 0.05 = 0.95$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \dots, n.$

$= 20C_x (0.05)^x (0.95)^{20-x} \quad x = 0, 1, 2, \dots, 20$

$p(\text{no defective}) = p(x=0)$

$= 20C_0 (0.05)^0 (0.95)^{20}$

$p(\text{two defective}) = p(x=2)$

$= 20C_2 (0.05)^2 (0.95)^{18}$

$= 0.3412$

$p(\text{not more than 2 defec})$

$= p(x \le 2)$

$= p(x=0) + p(x=1) + p(x=2)$

$= 20C_0 (0.05)^0 (0.95)^{20} + 20C_1 (0.05)^1 (0.95)^{19}$

$+ 20C_2 (0.05)^2 (0.95)^{18}.$

of 20 Solenoids choosen at random may have

i) no defectives ii) two defectives iii) not more than 2 defectives.

$n = 20$

$p = p(\text{defective}) = \frac{25}{500} = 0.05$

$q = 1 - p = 1 - 0.05 = \underline{\underline{0.95}}$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$= 20C_x (0.05)^x (0.95)^{20-x} \quad x = 0, 1, 2, \ldots, 20$

$p(\text{no defective}) = p(x=0)$

$= 20C_0 (0.05)^0 (0.95)^{20}$

$= \underline{\underline{0.3569}}$

$p(\text{two defective}) = p(x=2)$

$= 20C_2 (0.05)^2 (0.95)^{18}$

$= \underline{\underline{0.3412}}$

$p(\text{not more than 2 defec})$

$= p(x \leq 2)$

$= p(x=0) + p(x=1) + p(x=2)$

$= 20C_0 (0.05)^0 (0.95)^{20} + 20C_1 (0.05)^1 (0.95)^{19}$

$+ 20C_2 (0.05)^2 (0.95)^{18}.$

$= \underline{\underline{0.2823}}$

Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let x denote the number among 15 randomly selected copies that fail the test. Find the probability that i) atmost 8 fail the test. ii) exactly 8 fail the test iii) atleast 8 fail the test iv) between 4 and 7 fail the test.

$n = 15$

$n = 15$

$p = P(\text{test fail}) = \frac{20}{100} = 0.2$

$q = 1 - p = 0.8$

Pmf of B.D is

$$P(x) = nC_x\, p^x\, q^{n-x} \quad x = 0,1,2,\dots,n.$$

$$P(x) = 15C_x\,(0.2)^x\,(0.8)^{15-x} \quad x = 0,1,2,\dots,15$$

i) P(atmost 8 fail the test) $= P(x \le 8)$

$= P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$+ P(x=5) + P(x=6) + P(x=7) + P(x=8)$

$= 15C_0\,(0.2)^0\,(0.8)^{15} + 15C_1\,(0.2)(0.8)^{14} +$

$15C_2\,(0.2)^2\,(0.8)^{13} + 15C_3\,(0.2)^3\,(0.8)^{12} +$

$15C_4\,(0.2)^4\,(0.8)^{11} + 15C_5\,(0.2)^5\,(0.6)^{10} +$

$+ 15C_6\,(0.2)^6\,(0.8)^9 + 15C_7\,(0.2)^7\,(0.8)^8 +$

$15C_8\,(0.2)^8\,(0.8)^7$

$= 0.999$

ii) P(exactly 8 fail the test)

$= P(x=8)$

$= 15C_8\,(0.2)^8$

$n = 15$

$p = P(\text{test fail}) = \frac{20}{100} = 0.2$

$q = 1 - p = 0.8$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$p(x) = 15C_x (0.2)^x (0.8)^{15-x} \quad x = 0, 1, 2, \ldots, 15$

i) $P(\text{atmost 8 fail the test}) = p(x \leq 8)$

$= p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4)$

$+ p(x=5) + p(x=6) + p(x=7) + p(x=8)$

$= 15C_0 (0.2)^0 (0.8)^{15} + 15C_1 (0.2)(0.8)^{14} +$

$15C_2 (0.2)^2 (0.8)^{13} + 15C_3 (0.2)^3 (0.8)^{12} +$

$15C_4 (0.2)^4 (0.8)^{11} + 15C_5 (0.2)^5 (0.8)^{10} +$

$+ 15C_6 (0.2)^6 (0.8)^9 + 15C_7 (0.2)^7 (0.8)^8 +$

$15C_8 (0.2)^8 (\ldots$

$= 0.999$

ii) $p(\text{exactly 8 fail the test})$

$= p(x=8)$

$= 15C_8 (0.2)^8 (0.8)^7$

$= 0.003.$

iii) $p(\text{atleast 8 fail the test}) = p(x \geq 8)$

$= 1 - p(x < 8) = 1 - [p(0) + \cdots + p(7)]$

$= 1 - 0.996$

$= 0.004$

(iii) ~~P(atleast 8 fail the test)~~ $= p(x \geq 8)$

$= \cancel{1 - p[x < 8]}$

$B(x, n, p)$

$p(x \leq a)$

(iv) p(between 4 and 7 fail the test)

$= p(4 \leq x \leq 7)$

$= p(x=4) + p(x=5) + p(x=6) + p(x=7)$

$= 15C_4 (0.2)^4 (0.8)^{11} + 15C_5 (0.2)^5 (0.8)^{10} + 15C_6 (0.2)^6 (0.8)^9 + 15C_7 (0.2)^7 (0.8)^8$

$= \underline{\underline{0.348}}$

Each of six randomly selected cola drinkers is given a glass containing cola S and one containing cola F. The glasses are identical in appearance except for a code on the bottom to identify the cola. Suppose there is actually no tendency among cola drinkers to prefer one cola to the other. Find the probability that

i) exactly 3 prefer S

ii) atleast 3 prefer S.

iii) atmost one prefer S.

i) P(exactly 3 prefer S) = $p(x=3)$ $\quad 6 \begin{cases} (S)\ 6 \\ (F)\ +6 \end{cases} = 12$

$$\frac{\text{fav case}}{\text{Total case}}$$

$p = p(\text{Selected invidu pref: Cola: s})$

$= 0.5$ $\qquad \frac{6}{12}$

$q = 1-p$

$= 0.5$

Pmf of B.D is $\qquad x^m x^n = x^{m+n}$

$x + 6 - x$

$$p(x) = nC_x\, p^x q^{n-x} \quad x = 0, 1, 2, \ldots, n.$$

$$= 6C_x (0.5)^x (0.5)^{6-x} \quad x = 0, 1, 2, \ldots, 6$$

$$= 6C_x\ (0.5)^6$$

$p(\text{exactly 3 prefer Cola.s}) = p(x=3)$

$$= 6C_3\, (0.5)^3 (0.5)^3$$

$$= 0.313$$

2) $p(\text{atleast 3 prefer s}) = p(x \geq 3)$

$$= p(x=3) + p(x=4) + p(x=5) + p(x=6)$$

or

$$= 1 - p(x<3)$$

$$= 1 - [p(x=0) + p(x=1) + p(x=2)]$$

$$= 1 - [6C_0 (0.5)^6 + 6C_1 (0.5)^6 + 6C_2 (0.5)^6]$$

$\frac{6.5}{1.2}$

$$= 1 - (0.5)^6 [1 + 6 + 15]$$

$$= 0.656$$

3) $p(\text{atmost one prefer Cola. s})$

$$= p(x \leq 1)$$

$$= p(0) + p(1)$$

$$= 6C_0\, (0.5)^6 + 6C_1\, (0.5)^6$$

$$= 0.109$$

$p(y) = 5C_y \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)$

Pmf of x

$P(X=x) = P(Y = x-10) = 5C_{x-10} \left(\frac{1}{4}\right)^{x-10} \left(\frac{3}{4}\right)^{15-x}$

$x = 10, 11, 12, 13, 14, 15.$

$P($13 or more questions Correctly$) = P(X \geq 13)$

$= P(13) + P(14) + P(15)$

$= 5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 + 5C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right) + 5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0$

$= \underline{\underline{0.1035}}$

$E(x) = E[Y+10]$

$= E[Y] + 10$

$n = 5$

$p = \frac{1}{4}$

$= np + 10$

$= 5 \times \frac{1}{4} + 10$

$= \underline{\underline{\frac{45}{4}}}$

$Var(x) = V(Y+10)$

$= V(Y) + 0$

$= npq$

$= 5 \times \frac{1}{4} \times \frac{3}{4}$

$= \underline{\underline{\frac{15}{16}}}$

4.Q) In an examination a Candidate has to answer 15 multiple choice questions each of which has 4 choices for the answer. He knows the Correct answer to 10 of these questions and for the remaining 5 questions he choses an answer randomly. Let x be the total number of correct answers he gives.

$\frac{1}{4}$

(a) Find the Pmf of x

(b) what is the probabilty that he answers 13 or more questions Correctly?

(c) what is the mean and varience of the number of correct answers he gives?

Let y denote the no: of correct answers out of the 5 answers he picks up randomly.

$n = 5$

$P = p(\text{Correct answer out of 4 option}) = \frac{1}{4}$

$q = 1 - p = \frac{3}{4}$

Pmf of y $\quad P(y) = nC_y (p)^y (q)^{n-y} \quad x = 0,1,2,\dots,n$.

$= 5C_y \left(\frac{1}{4}\right)^y \left(\frac{3}{4}\right)^{5-y} \quad x = 0,1,2,\dots,5$

$\cancel{5-10-x}$

$5-(x-10)$

$15-x$

$\boxed{\begin{array}{l}\text{Total no: of Correct answer} \\ x = 10 + y \Rightarrow y = x - 10\end{array}}$

$x = 10, 11, 12, 13, 14, 15$

$p(x=10) = p(y=0) = 5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{5-0}$

$p(x=11) = p(y=1) = 5C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{5-1}$

$p(x=12) = p(y=2) = 5C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{5-2}$

$15-x$

A particular telephone number is used to receive both voice calls and fax messages. Suppose that 25% of the incoming calls involve fax message and consider a sample of 10 incoming calls.

a) what is the probability that atmost 6 of the calls involve a fax message?

b) what is the expected number of calls among the 10 that involve a fax message?

c) what is the standard deviation of the number along the 10 calls that involve a fax message?

d) what is the probability that the number of calls among the 10 that involve a fax transmission exceeds the expected number by more than 2 standard deviation.

$n = 10$

$p = P(\text{fax message}) = \frac{25}{100} = 0.25$

$q = 1 - p = 0.75$

Pmf of BD is $p(x) = nC_x\, p^x\, q^{n-x} \quad x = 0, 1, 2, \ldots, n$

$= 10C_x (0.25)^x (0.75)^{10-x} \quad x = 0, 1, \ldots, 10$

P(atmost 6 calls involve fax message)

$= P(x \le 6)$

$= 1 - P(x > 6) = 1 - [P(7) + P(8) + P(9) + P(10)]$

$= 1 - [10C_7 (0.25)^7 (0.75)^3 + 10C_8 (0.25)^8 (0.75)^2 + 10C_9 (0.25)^9 (0.75) + 10C_{10} (0.25)^{10} (0.75)^0]$

$= 0.996$

In an examination a Candidate has to answer 15 multiple choice questions each of which has 4 choices for the answer. He knows the Correct answer to 10 of these questions and for the remaining 5 questions he choses an answer

Let x

(a) find

(b) what

(c) what

(ii) $E[x] = np$

$= 10 \times 0.25$

$= 2.5$

$x = 0, 1, 2, 3, 4, 5, 6, \ldots, 10$

(iii) $S.D = \sqrt{Var(x)}$

$= \sqrt{npq}$

$= \sqrt{10(0.25)(0.75)}$

$= 1.3693$

$= P[x \geq 6]$

$= P(x=6) + P(x=7) + P(x=8) + P(x=$

$+ P(x=10)$

$= 0.0197$

Q. In an examination a Candidate has to answer 15 multiple choice questions each of which has 4 choices for the answer. He knows the Correct answer to 10 of these questions and for the remaining 5 questions he choses an answer randomly. Let x be the total number of Correct answers he gives.

$\frac{1}{4}$

(a) find the Pmf of x

(b) what is the probability that he answers 13 or more questions Correctly?

(c) what is the mean and varience of the number of correct answers he gives?

Let Y denote the no: of Correct answers out of the 5 answers he picks up randomly.

$n=5$

$p = p(\text{Correct answer out of 4 option}) = \frac{1}{4}$

$q = 1-p = \frac{3}{4}$

Pmf of Y $\quad P(Y) = nC_y\,(p)^y\,(q)^{n-y} \quad x=0,1,2,\dots,n$

$= 5C_y\left(\frac{1}{4}\right)^y\left(\frac{3}{4}\right)^{5-y} \quad x=0,1,2,\dots,5$

Total no: of Correct answer

$\boxed{X = 10+Y}$

$x = 10, 11, 12, 13, 14, 15$

$P(x=10) = P(y=0) = 5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{5-0}$

$P(x=11) = P(y=1) = 5C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{5-1}$

$P(x=12) = P(y=2) = 5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{5-2}$

$\vdots$

$\dots = P(Y=10-x)$

$p(y) = 5C_y \left(\frac{1}{4}\right)^y \left(\frac{3}{4}\right)^{5-y}$

Pmf of x

$P(X=x) = P(Y = x-10) = 5C_{x-10} \left(\frac{1}{4}\right)^{x-10} \left(\frac{3}{4}\right)^{15-x}$

$x = 10, 11, 12, 13, 14, 15.$

$P(13$ or more questions Correctly$) = P(X \geq 13)$

$= P(13) + P(14) + P(15)$

$= 5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 + 5C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right) + 5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0$

$= 0.1035$

$E[X] = E[Y+10]$

$= E[Y] + 10$

$= np + 10$

$= 5 \times \frac{1}{4} + 10$

$= \frac{45}{4}$

$Var(X) = V(Y+10)$

$= V(Y) + 0$

$= npq.$

$= 5 \times \frac{1}{4} \times \frac{3}{4}$

$= \frac{15}{16}$