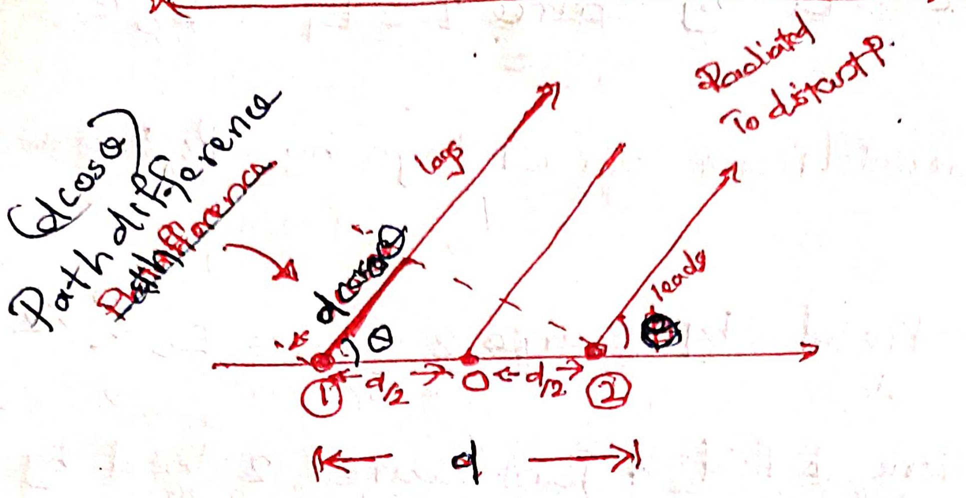
# Array of two Point Sources




⇒ Two istropic point Source of Same Amplitude and Phase. that means two isotropic Point sources ① and ② having Current Supplied with equal amplitude and Same Phase.

⇒ due to current Supplied to both is equal, electricfield magnitude generated by both elemts will be equal.

$$\boxed{E_1 = E_2 = E_0} \quad \because \text{current is Same.}$$

⇒ Path difference $= d\cos\theta$

Total Phase difference $\psi = \left(\frac{2\pi}{\lambda}\right) d\cos\theta$

$\beta = 2\pi/\lambda \Rightarrow \beta d\cos\theta$

Generally $\beta d\cos\theta + \alpha$

associated Same Phase $\therefore \alpha = 0$.

Electric field by Source 1 = $E_1 = E_0 e^{-j\psi/2}$

Here field of Point source 1 lags by $\psi/2$ Phase.

Electric field by Source 2 = $E_2 = E_0 e^{j\psi/2}$

Here E-field of Point Source 2 lead by $\psi/2$ Phase.

⇒ Total Electric field

$$E = E_1 + E_2$$

$$= E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$= E_0 \left( e^{-j\psi/2} + e^{j\psi/2} \right)$$

$$= 2E_0 \left( \frac{e^{-j\psi/2} + e^{j\psi/2}}{2} \right) \qquad \boxed{\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta}$$

$$E = 2E_0 \cos(\psi/2) \quad \text{— ①}$$

$E_n$ is normalized Electric field.

$$E_n = 2E_0$$

then ① become $E = E_n \cos(\psi/2)$ — ②

We know $\psi = \beta d \cos\theta + \alpha$

$$\boxed{E = E_n \cos\left(\frac{\beta d \cos\theta + \alpha}{2}\right)}$$

This is equation of farfield Pattern of two Isotropic point sources of Same amplitude and Phase

putting $E_n = 1$ (Pattern is Said to be normalized)

Here $\alpha = 0$,

$$E = \cos\left(\frac{\beta d \cos\theta}{2}\right)$$

if $d = \lambda/2$, $\beta = 2\pi/\lambda$

$$E = \cos\left(\frac{2\pi/\lambda \times \lambda/2 \times \cos\theta}{2}\right)$$

$$\boxed{E = \cos\left(\pi/_2 \cos\theta\right)}$$

$$\boxed{E = \cos\left(\frac{\pi}{2}\cos\theta\right)}$$

for

case (i) (broadside) - two isotropic point source radiator.

## Maxima direction

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta_{max} = \pm n\pi \quad , \text{ where } n = 0, 1, 2, \ldots n$$

$$\frac{\pi}{2}\cos\theta_{max} = 0$$

$$\cos\theta_{max} = 0$$

$$\boxed{\theta_{max} = 90^\circ \text{ and } 270^\circ}$$

## minima Direction.

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\frac{\pi}{2}\cos\theta_{min} = \pm(2n+1)\frac{\pi}{2} \quad \text{where } n = 0, 1, 2, \ldots n$$

$$\frac{\pi}{2}\cos\theta_{min} = \pm\frac{\pi}{2}$$

$$\cos\theta_{min} = \pm 1$$

$$\boxed{\theta_{min} = 0^\circ \text{ and } 180^\circ}$$

③

## Half Power Point direction [HPPD]

Power is $\frac{1}{2}$ or Voltage or current is $\frac{1}{\sqrt{2}}$ times of maximum value of Voltage or Current.

[$\frac{P_{max}}{2}$ or $\frac{V_{max} \text{ or } I_{max}}{\sqrt{2}}$ called Half Power Point]

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm \frac{1}{\sqrt{2}}$$

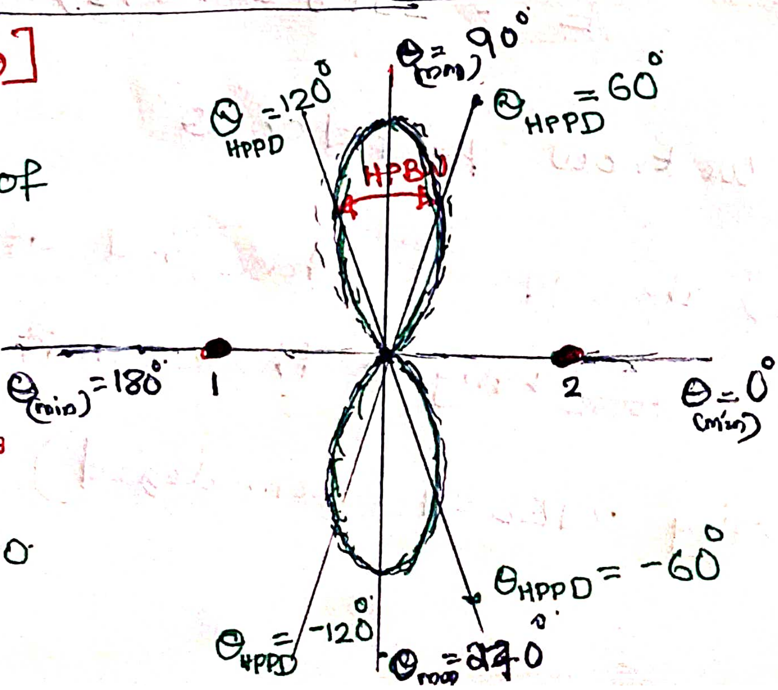
$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4} \quad ; \text{ where } n = 0, 1, \ldots$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\theta_{HPPD} = \pm\frac{1}{2}$$

$$\boxed{\theta_{HPPD} = \pm 60^\circ, \pm 120^\circ}$$

Field Pattern [E Vs Θ]




* This is simplest type of broad Side array
* also known as broadside Couplet as two isotropic antennas are in Phase.
* Here $d = \lambda/2$ & $\alpha = 0$

case (ii) : Array of Point source with

Equal amplitude and opposite Phase (end fire)

*(Same figure)

Source 1 $\Rightarrow E_1 = -E_0 e^{-j\psi/2}$.

Source 2 $\Rightarrow E_2 = E_0 e^{j\psi/2}$.

Total Electric field (farfield E)

$$E = E_1 + E_2$$

$$= -E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E = E_0 \left(e^{j\psi/2} - e^{-j\psi/2}\right)$$

$$E = 2jE_0 \sin(\psi/2)$$

$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$

$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$

We know $\psi = \beta d \cos\theta$

term $j \Rightarrow$ the opposite Phase fed at source 1 and 2, ~~cause~~ bring a Phase shift of $90^\circ$ in the total field

putting $2jE_0 = 1$ (normalized).

(2)

Let $d = \lambda/2$ & $2jE_0 = 1$, $\beta = 2\pi/\lambda$

$$E = \sin\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \frac{\cos\theta}{2}\right)$$

$$\boxed{E = \sin\left(\frac{\pi}{2}\cos\theta\right)}$$

Case (ii) (endfire) two isotropic point source Radiator).

Maxima direction

maximum value of Sine function is $\pm 1$

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta_{max} = \pm(2n+1)\frac{\pi}{2}$$

$n = 0, 1, \ldots$

$$\frac{\pi}{2}\cos\theta_{max} = \pm\frac{\pi}{2}$$

$$\cos\theta_{max} = \pm 1$$

$$\boxed{\theta_{max} = 0^\circ \text{ and } 180^\circ}$$

## minima direction

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\frac{\pi}{2}\cos\theta_{min} = \pm n\pi \quad \text{where } n = 0, 1, 2 \ldots$$

$$\cos\theta_{min} = 0$$

$$\boxed{\theta_{min} = 90^\circ \text{ and } 270^\circ}$$

## Half Power point direction

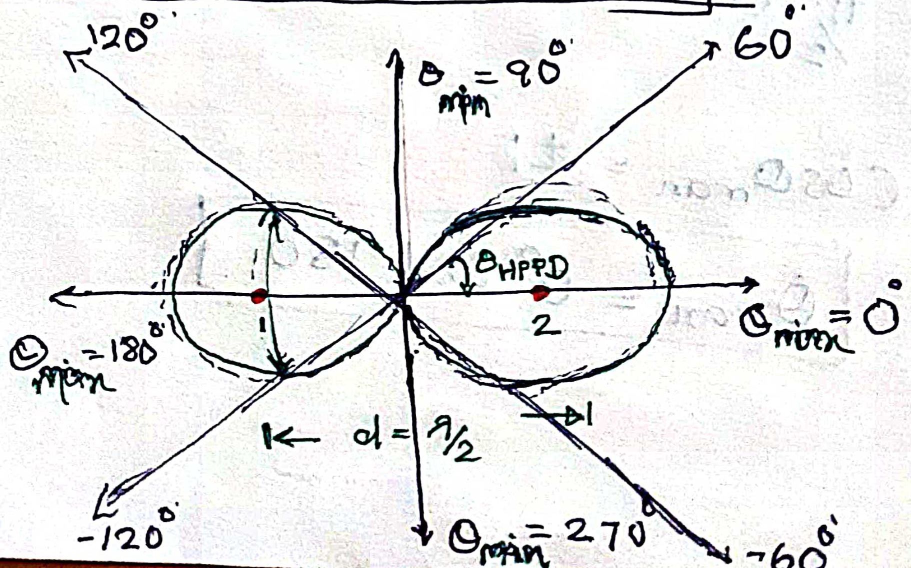
$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\theta_{HPPD} = \pm\frac{1}{2}$$

$$\boxed{\theta_{HPPD} = \pm 60^\circ, \pm 120^\circ}$$



Case I — Radiation Pattern of two istropic Point Source Radiator Seperated by Wavelength ($\lambda$)

$[d = \lambda]$

Electric field by two elements

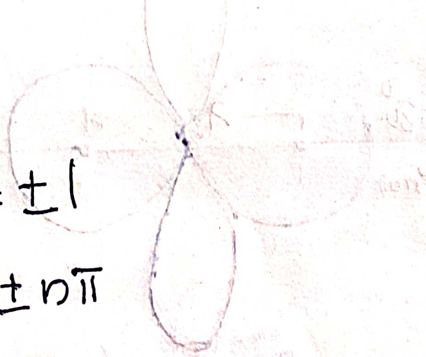
$$E = E_n \cos(\psi/2)$$

$$= E_n \cos\left(\frac{\beta d \cos\theta + \alpha}{2}\right)$$

$\beta = 2\pi/\lambda$ , $d = \lambda$ , $\alpha = 0$

$$E = E_n \cos\left(\frac{\cancel{2}\pi/\cancel{\lambda} \times \cancel{\lambda} \cos\theta + 0}{\cancel{2}}\right)$$

$$E = E_n \cos(\pi \cos\theta)$$



For Maxima

$$\cos(\pi\cos\theta) = \pm 1$$

$$\pi \cos\theta_{max} = \pm n\pi$$

$$\cos\theta_{max} = \pm n \quad , \quad n = 0, 1, 2, \ldots$$

| | |
|---|---|
| ⇒ for $n = 0$ | ⇒ for $n = 1$ |
| $\cos\theta_{max} = 0$ | $\cos\theta_{max} = \pm 1$ |
| $\theta_{max} = 90^\circ, 270^\circ$ | $\theta_{max} = 0^\circ, 180^\circ$ |

$$\boxed{\theta_{max} = 0^\circ, 90^\circ, 180^\circ, 270^\circ}$$

→ <u>for minima ( Nulls )</u>

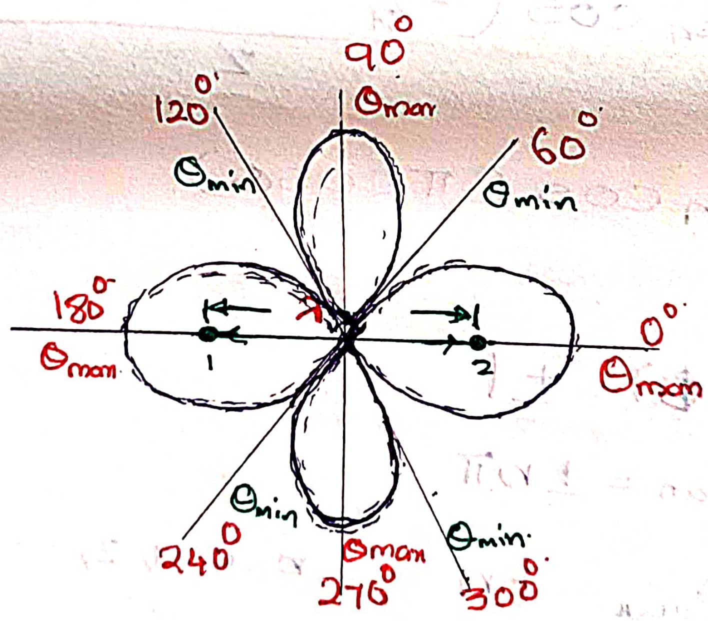
$$\cos(\pi \cos\phi) = 0$$

$$\pi \cos\phi_{min} = \pm (2n+1)\pi/2 \, , \quad n = 0, 1, 2, \cdots$$

⇒ for $n = 0$.

$$\cos\theta_{min} = \pm 1/2$$

$$\boxed{\theta_{min} = 60^\circ, 120^\circ, 240^\circ, 300^\circ}$$




This is farfield - E - pattern at $d = \lambda$

Case II $\quad d = 2\lambda, \quad \alpha = 0$

$$E = E_n \cos(2\pi \cos\theta)$$

$\Rightarrow \theta_{min}$ or Nulls at: $48^\circ, 76^\circ$

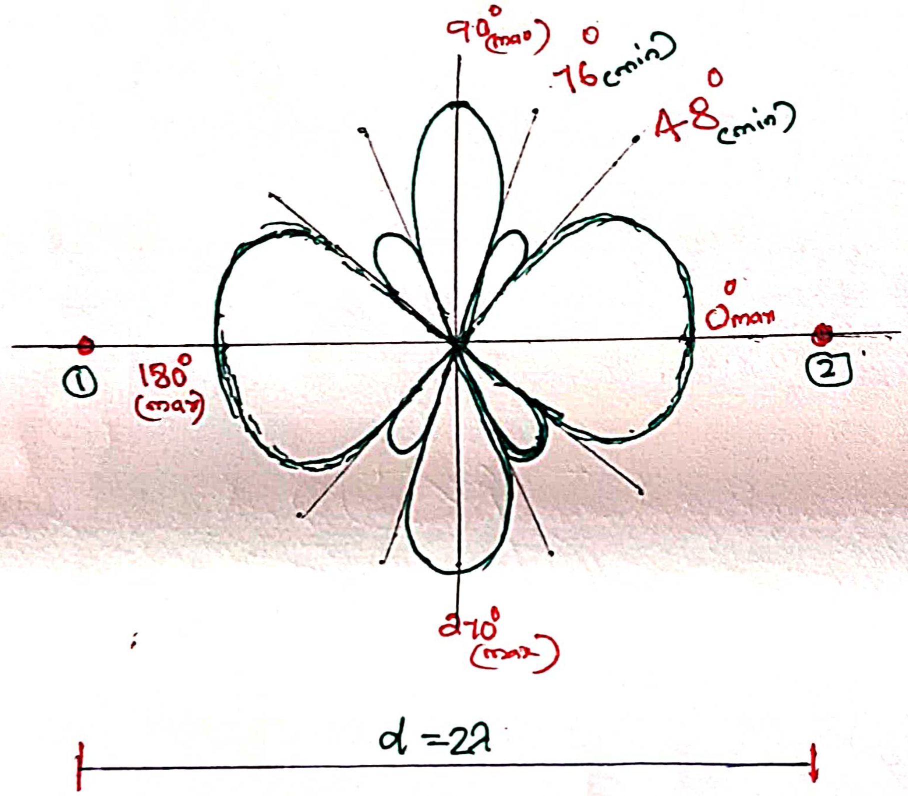



Fig. Radiation pattern of isotropic radiator spaced $2\lambda$.

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# Pattern multiplication

Principle of Pattern Multiplication states that the radiation Pattern of an array is the Product of Pattern of the Individual Pattern with array Pattern.

The array pattern is the function of the location of the antennas in the array and their relative complex excitation of amplitude.

## Advantage

* It helps to sketch the radiation pattern of array antennas rapidly from the simple product of element Pattern and array Pattern.

⇒ Generalised Expression of Principle of Pattern multiplication:

$$E = \{ E_i(\theta, \phi) \times E_a(\theta, \phi) \} \times \{ E_{pi}(\theta, \phi) + E_{pa}(\theta, \phi) \}$$

E = (multiplication of field pattern) x (addition of Phase Pattern)

$E \Rightarrow$ Total field

$E_i(\theta, \phi) \Rightarrow$ Field Pattern of Individual Source.

$E_a(\theta, \phi) \Rightarrow$ Field Pattern of array of isotropic point source [array factor of isotropic element]

$E_{pi}(\theta, \phi) \Rightarrow$ Phase pattern of individual source

$E_{pa}(\theta, \phi) \Rightarrow$ Phase pattern of array of isotropic Point source.

## Disadvantage

* The Principle is applicable only for arrays containing identical elements

## Example

① Radiation Pattern of 4-isotropic element fed in Phase, spaced $\lambda/2$ apart.

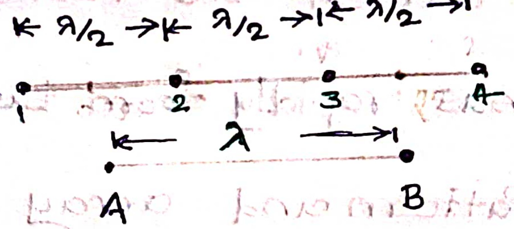
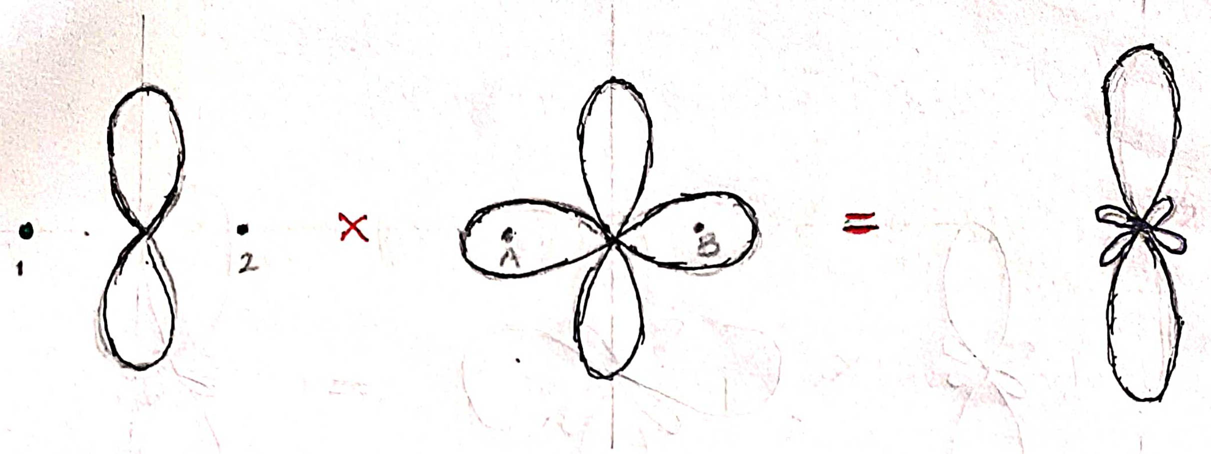



fig: Array of 4 element

* element 1 and 2 Considered as one Unit 'A'
* lly element 3 & 4 considered as one unit 'B'
* let A and B are two isotropic antennas spaced at $\lambda$

⇒ According to Pattern of 4 elements is obtained by multiplying the radiation pattern of individual element and array of two units spaced $\lambda$, as shown in below.

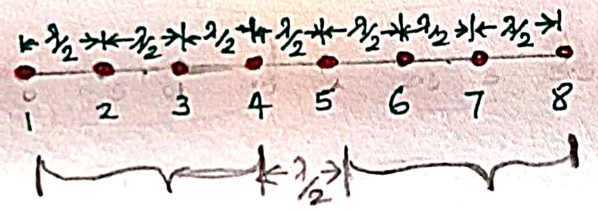


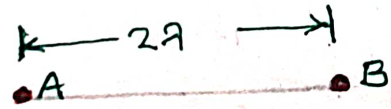

Unit pattern ($d = \lambda/2$)

Group Pattern ($d = \lambda$)

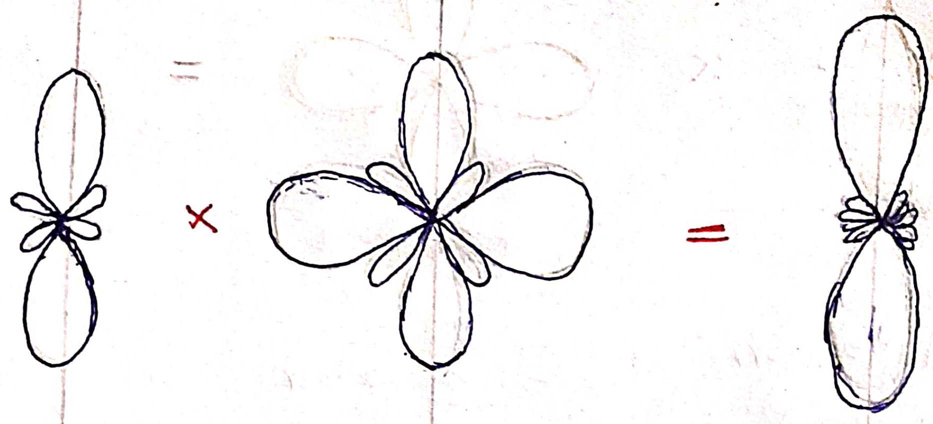
Resultant Pattern of 4 Isotropic element.

**Example 2:** Array of 8 elements spaced at $\lambda/2$




* The Radiation pattern of 8 isotropic elements is obtained by multiplying the unit pattern of 4 individual elements and group pattern of two isotropic radiators spaced $2\lambda$, as shown below.



Unit pattern [4 individual element]

Group Pattern ($d = 2\lambda$)

Resultant Pattern of 8 isotropic elements

9.

## Uniform Linear array

It is an array where the elements are spaced and excited equally along a straight line.

Linear : Equal Spacing

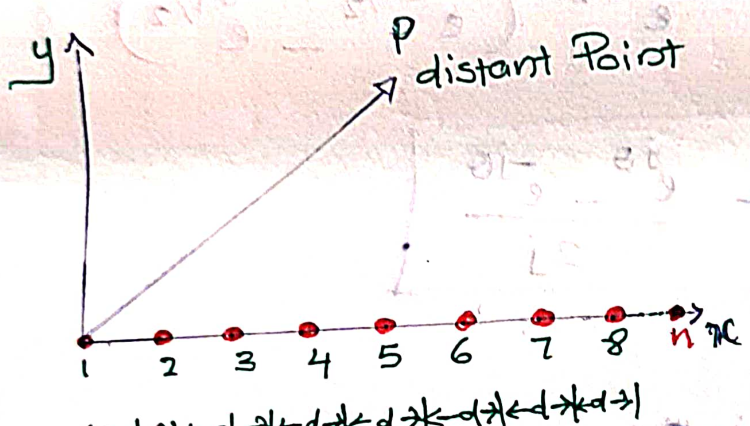
Uniform : fed with current of equal amplitude and uniform Progressive Phase shift

**Linear array with n isotropic point source of equal amplitude and spacing.**

⇒ These array are Suitable for production of narrow radiation beam

⇒ These arrays are required for point to point Communication.

⇒ These are also used in high angular resoulation radars




Linear array with n isotropic point Sources with equal amplitude and spacing

→ Total far-field pattern at distant point P, given by

$$E_t = E_0 e^{0j\psi} + E_0 e^{1j\psi} + E_0 e^{2j\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$E_t = E_0 \left(\underbrace{1 + e^{j\psi} + e^{2j\psi} + \dots + e^{j(n-1)\psi}}\right)$$

This is the form of G.P. (Sum of the first n terms of G.P) ⇒ $\boxed{S_n = \frac{a(1-r^n)}{1-r}}$ $r<1$

Here $\begin{cases} a = 1 \\ r = e^{j\psi} \end{cases}$

$$E_t = E_0 \frac{(1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

$$E_t = E_0 \frac{(1 - e^{jn\psi/2} e^{jn\psi/2})}{(1 - e^{j\psi/2} e^{j\psi/2})}$$

$$E_t = E_0 \frac{e^{jn\psi/2}(e^{-jn\psi/2} - e^{+jn\psi/2})}{e^{j\psi/2}(e^{-j\psi/2} - e^{j\psi/2})} \quad \text{— (1)}$$

We know $\boxed{\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}}$

$\cancel{E_t = E_0 e^{jn\psi/2} (e^{-}}$

(1) become,

$$E_t = E_0 e^{j(n-1)\psi/2} \left[ \frac{\left(\frac{e^{jn\psi/2} - e^{-jn\psi/2}}{2j}\right)}{\left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}\right)} \right]$$

$$E_t = E_0 e^{j(n-1)\psi/2} \frac{\sin n\psi/2}{\sin \psi/2}$$

$$E_t = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)} e^{j\phi} \qquad \text{where } \phi = \frac{(n-1)\psi}{2}$$

$$\boxed{E_t = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)} \angle \phi} \quad \text{— (2)}$$

This is the equation of total far field pattern of linear array of n-isotropic point source. with source 1 as reference point for source.

* If the reference point (source 1) is shifted to the centre of the array from the source or origin of the coordinate, then Phase angle $\phi$ is automatically eliminated,

$$\boxed{E_t = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)}} \quad \text{— (3)}$$

(Reference Source located at the centre of co-ordinate)

* If individual elements are non-isotropic but similar sources then,

1) '$E_0$' will represent individual source pattern or Primary Pattern. ~~and~~

2) $\dfrac{\sin(n\psi/2)}{\sin(\psi/2)}$ the arryfactor or secondary pattern.

for $\psi = 0$, then equation (3) becomes Indeterminant. and hence L' hospital rule must be applied to evaluate the function

$$\lim_{\psi \to 0} (E_t) = E_0 \cdot \lim_{\psi \to 0} \frac{\frac{d}{d\psi}(\sin n \psi/2)}{\frac{d}{d\psi}(\sin \psi/2)}$$

$$= E_0 \lim_{\psi \to 0} \frac{\cos n\psi/2 \cdot n/2}{\cos \psi/2 \cdot 1/2}$$

$$\boxed{E_{t\,max} = E_0 \cdot n} \quad \text{— (4)}$$

The normalised field pattern is obtained as,

(3) ÷ (4) $\Rightarrow E_{norm} = \frac{E_t}{E_{t\,max}} = \frac{E_0 \frac{\sin n\psi/2}{\sin \psi/2}}{E_0 \cdot n}$

$$\boxed{E_{norm} = \frac{\sin n \psi/2}{n \sin \psi/2} = (AF)_n}$$

AF ⇒ Array Factor.

(1) A-copy

# ARRY OF n ISOTROPIC SOURCES OF EQUAL AMPLITUDE AND SPACING [ BROADSIDE CASE ]

(i)

Maximum Radiation $\perp$re to array axis. ($90^\circ$ and $270^\circ$)

The equation of field Pattern is given by

$$E_t = \frac{E_0 \sin n\psi/2}{\sin \psi/2}$$

$$\psi = \beta d \cos\theta + \alpha$$

for broadside array individual elements are fed at same Phase (ie $\alpha = 0$, and $\psi = 0$)

$\alpha = 0$, $\beta d \cos\theta_{max} = 0$

$\psi = \beta d \cos\theta$ $\quad$ $\theta_{max} = 90^\circ$ and $270^\circ$

Direction of major lobe (Pattern maxima)

$$E_t = \frac{E_0 \sin n\psi/2}{\sin\psi/2} \quad \text{—— (1)}$$

broadside $\alpha = 0$, $\psi = 0$

$\psi = \beta d \cos\theta = 0$ $\quad$ $\cos\theta = 0$

$\theta = 90^\circ$ or $270^\circ$ [ $\underbrace{\text{Principal}}_{\text{majorlobe}}$ maxima occure in these direction]

Now with $\psi = 0$; the above equn (1) becomes indeterminant and hence D HOSPital rule must be applied to evalute the fun.

$$E_{t\,max} = \lim_{\psi \to 0} \frac{E_0 \frac{\sin n\psi}{2}}{\sin\psi/2} = \lim_{\psi \to 0} E_0 \frac{\cos n\psi/2}{\cos\psi/2} \cdot \frac{n/2}{1/2}$$

$$\boxed{E_{t\,max} = E_0 n}$$

maximum value of $E_t$ is $nE_0$ and it occurs at

$\psi = 0$ ie $\beta d \cos\theta = 0$

or

$\cos\theta = 0$

$$\boxed{\theta_{max} = 90^\circ \text{ and } 270^\circ}$$

These are major Lobe maxima.

minor lobe maxima

$$E_t = \frac{E_0 \sin n\psi/2}{\sin \psi/2}.$$

This is maximum when numerator is maximum

$$\sin n\psi/2 = 1$$

(Secondary max) or $n\psi/2 = (2N+1)\pi/2$

$N = 1, 2, 3, 4 \ldots$

$N = 0$ corresponds to major lobe maxima.

$$\psi/2 = \pm (2N+1)\pi/2 \cdot 1/n$$

$$\boxed{\psi = \pm (2N+1)\pi/n}$$

we know $\psi = \beta d \cos\theta$

$$\beta d \cos(\theta_{max})_{minor} = \pm (2N+1)\pi/n.$$

2) we know $\beta = 2\pi/\lambda$

$$\frac{2\pi}{\lambda} d \cos(\theta_{max})_{minor} = \pm (2N+1)\frac{\pi}{n}$$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left\{\pm \frac{(2N+1)\lambda}{2nd}\right\}}$$

⇒ $N = 1, 2, 3 \ldots\ldots$

⇒ $n$ = number of array elements

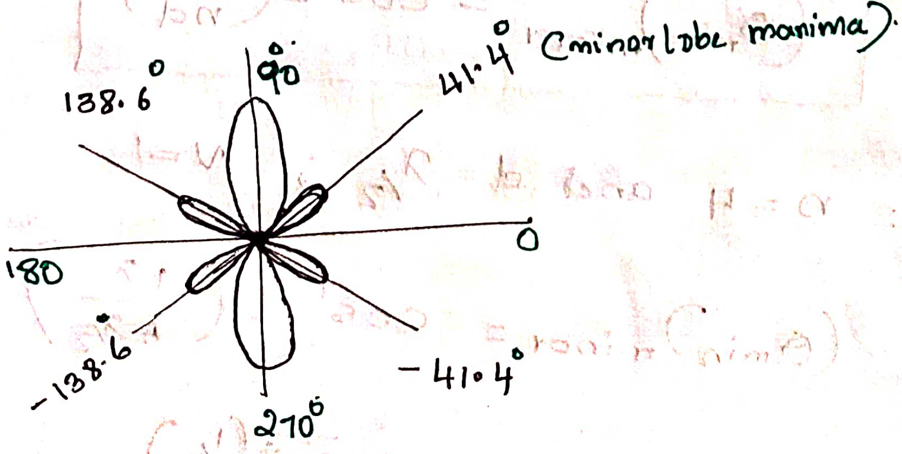
for eg. $n=4$ and $d = \lambda/2$, $N=1$

$$(\theta_{max})_{minor} = \cos^{-1}\left\{\frac{\pm 3\lambda}{2 \times 4 \times \lambda/2}\right\}$$

$$= \cos^{-1}(\pm 3/4) \text{ \& } \cos^{-1}(\pm 5/4)$$

$$(\theta_{max})_{min} = \pm 41.4^\circ \text{ or } \pm 138.6^\circ$$

are the four minor lobe maxima



## Direction of Pattern minima

$$E_t = E_0 \frac{\sin n\psi/2}{\sin \psi/2}$$

for $E_t = 0$ ; $\sin n\psi/2 = 0$ Provided $\sin \psi/2 \neq 0$

where $N = 1, 2, 3, \ldots$

$$\sin \frac{n\psi}{2} = 0$$

$$n\psi/2 = \pm N\pi$$

$$\boxed{\psi = \pm \frac{2N\pi}{n}}$$

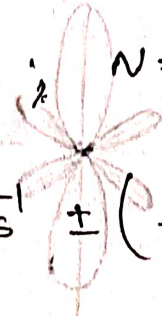
$$\beta d(\cos\theta_{min})_{minor} = \pm \frac{2N\pi}{n}$$

$$(\cos\theta_{min})_{minor} = \pm \frac{2N\cancel{\pi}}{n} \times \frac{\lambda}{2\cancel{\pi}} \times \frac{1}{d}$$

$$(\cos\theta_{min})_{minor} = \pm \frac{N\lambda}{nd}$$

$$\boxed{(\theta_{min})_{minor} = \cos^{-1} \pm \left(\frac{N\lambda}{nd}\right)}$$

eg: $n = 4$ and $d = \lambda/2$ ; $N = 1$



$$(\theta_{min})_{minor} = \cos^{-1} \pm \left(\frac{1\lambda}{4\lambda/2}\right)$$

$$(\theta_{min})_{minor} = \cos^{-1} \pm (1/2)$$

$$(\theta_{min})_{minor} = 60^\circ, 120^\circ, -60^\circ, -120^\circ.$$

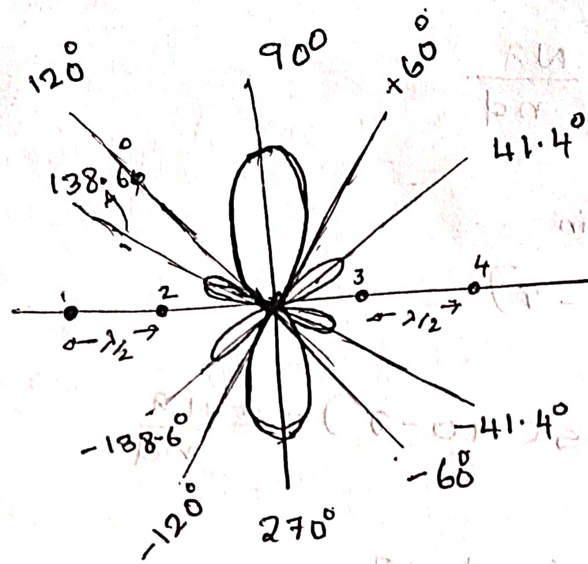
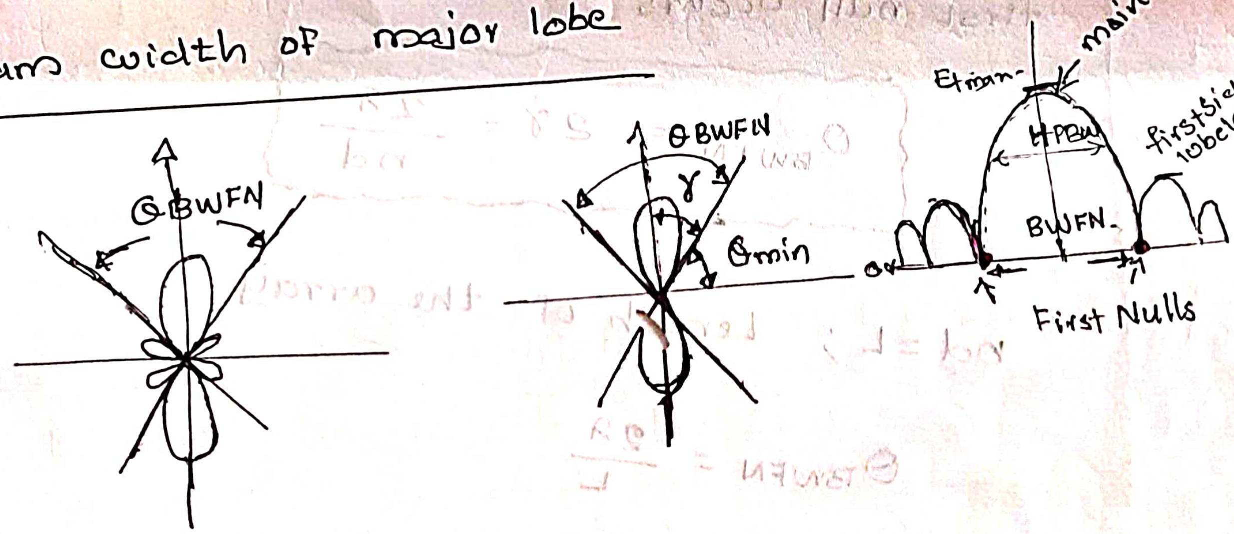



fig : four isotropic sources of equal amplitude and phase, field pattern in broadside case.

## Beam width of major lobe




it is the angle b/w first nulls

or

double angle between first null and major lobe maximum

$$\theta_{BWFN} = 2\gamma$$

$$= 2(90 - \theta_{min})$$

$\gamma = 90 - \theta_{min}$ → complementry angle.

we know $\theta_{min} = \cos^{-1}\left(\pm \frac{N\lambda}{nd}\right)$

$$\cos\theta_{min} = \pm \frac{N\lambda}{nd}$$

we know $\gamma = 90 - \theta_{min}$

$$\theta_{min} = (90 - \gamma)$$

$$\cos\theta_{min} = \cos(90 - \gamma) = \pm\frac{N\lambda}{nd}$$

$$= \sin\gamma = \pm\frac{N\lambda}{nd}$$

∵ $\gamma$ is very small; $\sin\gamma = \gamma$

$$\gamma = \pm\frac{N\lambda}{nd}$$

first null occurs when $N = 1$; ie $\gamma_1 = \frac{\lambda}{nd}$

$$\boxed{\theta_{BWFN} = 2\gamma = \frac{2\lambda}{nd}}$$

$nd = L$; Length of the array

$$\theta_{BWFN} = \frac{2\lambda}{L}$$

$$= \frac{2}{(L/\lambda)} \text{ radian}$$

radian = $\left(\frac{180}{\pi}\right)$ degree

radian = $57.3^\circ$ degree

$$= \frac{2 \times 57.3^\circ}{(L/\lambda)} \text{ degree}$$

Beam width b/w first Null

$$\boxed{BWFN = \frac{114.6^\circ}{L/\lambda} \text{ or } \frac{114.6^\circ \times \lambda}{nd}}$$

$$\boxed{\text{Half Power Beam width } HPBW = \frac{BWFN}{2}}$$

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$$\boxed{\theta_{HPBW} = \frac{57.3^\circ}{(L/\lambda)}}$$

* The Null-to-Null beamwidth of broadside array [BWFN] $= \dfrac{2\lambda}{nd}$

$$\boxed{\text{Directivity } D = 2\left(\frac{L}{\lambda}\right) = 2\left(\frac{nd}{\lambda}\right)}$$

## Array of n Sources of equal amplitude and Spacing

### [END-FIRE CASE]

We know, the General expression for $E_t$, is

$$E_t = \frac{E_0 \sin \frac{n\psi}{2}}{\sin \psi/2}$$

* for end-fire array maximum radiation occurs when $\psi = 0$ and $\theta = 0^\circ$ or $180^\circ$. (i.e The direction of maximum radiation coincides with the array axis).

when $\psi = 0$;

$$\lim_{\psi \to 0} E_t = \lim_{\psi \to 0} E_0 \frac{\cos n\psi/2 \cdot n/2}{\cos \psi/2 \cdot 1/2}$$

$$E_{t\,max} = nE_0$$

$$\boxed{\theta_{max} = 0^\circ \text{ and } 180^\circ}$$

Direction of main lobe ($\psi = 0$, $\alpha = -\beta d$)

We know $\psi = \beta d \cos\theta + \alpha$

$$0 = \beta d \cos 0 + \alpha$$

$$\boxed{\alpha = -\beta d}$$

→ This is the Condition for end-fire array

* If Spacing b/w two Sources $\lambda/2$ or $\lambda/4$ then Phase angle by which Source 2 lags behind source 1 is $\frac{2\pi}{\lambda} \cdot \lambda/2$ or $\frac{2\pi}{\lambda} \cdot \lambda/4 = \pi$ or $\pi/2$.

## Direction of minor lobe Maximum

$$E_t = E_0 \frac{\sin n\psi/2}{\sin \psi/2}$$

which is maximum when

$$\sin n\psi/2 = 1 \quad \text{and} \quad \sin \psi/2 \neq 0$$

$$n\psi/2 = \pm(2N+1)\pi/2$$

$$\psi = \frac{\pm(2N+1)\pi}{n}$$

$$\beta d \cos(\theta_{max})_{minor} + \alpha = \pm\frac{(2N+1)\pi}{n}$$

we know $\boxed{\alpha = -\beta d}$

$$\beta d \cos(\theta_{max})_{minor} - \beta d = \pm\frac{(2N+1)\pi}{n}$$

$$\beta d\left[\cos(\theta_{max})_{minor} - 1\right] = \pm\frac{(2N+1)\pi}{n}$$

$$\cos(\theta_{max})_{minor} - 1 = \pm\frac{(2N+1)\pi}{n\beta d}$$

$$\cos(\theta_{max})_{minor} = \pm\frac{(2N+1)\pi}{n\beta d} + 1$$

$$(\theta_{max})_{minor} = \cos^{-1}\left\{ \pm \frac{(2N+1)\pi}{n\beta d} + 1 \right\}$$

example.

If $n=4$, $d=\lambda/2$, $\alpha=-\beta d=-\pi$, $N=1$

$$(\theta_{max})_{minor1} = \cos^{-1}\left\{ \pm \frac{(2\times1+1)\pi}{4\times\frac{2\pi}{\lambda}\times\frac{\lambda}{2}} + 1 \right\}$$

$$\pm\left(\frac{2\times N+1}{4}+1\right)$$

$$= \cos^{-1}\left\{ \pm \frac{3}{4} + 1 \right\}$$

$$= \cos^{-1}\left\{ 7/4,\ 1/4 \right\}$$

$$= \cos^{-1}(1/4), \quad \because \cos^{-1}(7/4) \text{ doesnot exist.}$$

$$(\theta_{max})_{minor1} = 75.5^\circ$$

→ If $N=2$

$$(\theta_{max})_{minor2} = \cos^{-1}\left\{ \pm \frac{5}{4} + 1 \right\}$$

$$= \cos^{-1}\left\{ 9/4,\ -1/4 \right\}$$

$$= \cos^{-1}(-1/4) \quad \because \cos^{-1}(9/4) \text{ doesnot exist}$$

$$(\theta_{max})_{minor2} = 104.5^\circ$$

## Direction of pattern minima

$$E_t = E_0 \frac{\sin n\psi/2}{\sin \psi/2}$$

$E_t = 0$, when $\sin n\psi/2 = 0$ and $\sin \psi/2 \neq 0$

$$\sin n\psi/2 = 0$$

$$n\psi/2 = \pm N\pi$$

$$\psi = \pm \frac{2N\pi}{n}$$

We know $\psi = \beta d \cos\theta + \alpha$, $\cancel{\alpha = \beta d}$

$$\beta d \cos(\theta_{min})_{min} + \alpha = \pm \frac{2N\pi}{n}$$

here, $\alpha = -\beta d$.

$$\beta d \cos(\theta_{min})_{min} - \beta d = \pm \frac{2N\pi}{n}$$

$$\beta d \left[\cos(\theta_{min})_{min} - 1\right] = \pm \frac{2N\pi}{n}$$

$$\cos(\theta_{min})_{min} - 1 = \pm \frac{2N\pi}{n\beta d}$$

$$\beta = 2\pi/\lambda$$

$$\cos\theta_{min} - 1 = \pm \frac{2N\pi}{\frac{2\pi}{\lambda} nd} = \pm \frac{N\lambda}{nd}$$

we know $\cos\theta = 1 - 2\sin^2 \theta/2$

$$1 - 2\sin^2 \frac{\theta_{min}}{2} - 1 = \pm \frac{N\lambda}{nd}$$

$$2\sin^2 \frac{\theta_{min}}{2} = \pm \frac{N\lambda}{nd}$$

$$\sin^2 \frac{\theta_{min}}{2} = \pm \frac{N\lambda}{2nd}$$

$$\sin \frac{\theta_{min}}{2} = \pm \sqrt{\frac{N\lambda}{2nd}}$$

$$\frac{\theta_{min}}{2} = \sin^{-1}\left(\pm \sqrt{\frac{N\lambda}{2nd}}\right)$$

$$\boxed{\theta_{min} = 2\sin^{-1}\left(\pm \sqrt{\frac{N\lambda}{2nd}}\right)}$$

Example $n = 4$, $d = \lambda/2$, $N = 1$

10 8

$$(\theta_{min})_{\bullet} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$(\theta_{min})_1 = 2\sin^{-1}\left(\pm\sqrt{\frac{1\times\lambda}{\cancel{2}\times 4\times \lambda/\cancel{2}}}\right)$$

$$= 2\sin^{-1}\left(\pm\sqrt{1/4}\right) = 2\sin^{-1}\left(\pm 1/2\right)$$

$$= 2\times(\pm 30^\circ)$$

$$(\theta_{min})_1 = \pm 60^\circ$$

If $N = 2$.

$$(\theta_{min})_2 = 2\sin^{-1}\left(\pm\sqrt{2/4}\right) = 2\sin^{-1}\left(\pm\sqrt{1/\cancel{2}}\right)$$

$$(\theta_{min})_2 = 2\times(\pm 45^\circ)$$

$$(\theta_{min})_2 = \pm 90^\circ.$$

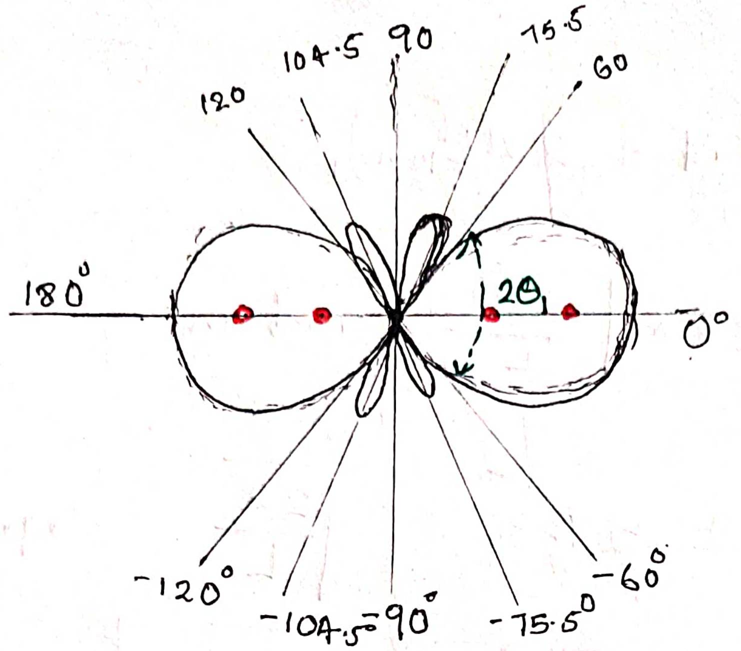
If $N = 3$

$$(\theta_{min})_3 = 2\sin^{-1}\left(\pm\sqrt{3/4}\right) = 2\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

$$= 2\times(\pm 60^\circ)$$

$$(\theta_{min})_3 = \pm 120^\circ$$

$$\boxed{\theta_{min} = 60^\circ, 90^\circ, 120^\circ, -60^\circ, -90^\circ, -120^\circ}$$




## Beam Width of Major lobe

Beam Width = 2 x angle b/w first null and manimum of major lob

$$BWFN = 2 \times \theta_1$$

$$\theta_{min} = 2 \sin^{-1}\left( \pm \sqrt{\frac{N\lambda}{2nd}} \right)$$

$$\sin \theta_{min} = 2\left( \pm \sqrt{\frac{N\lambda}{2nd}} \right)$$

if $\theta_{min}$ Very Small, $\therefore \sin \theta_{min} \cong \theta_{min}$

$$\theta_{min} = \pm \sqrt{\frac{2N\lambda}{nd}}$$

If the array is long of length L then,

$$L = (n-1)d \cong nd.$$

$$\boxed{(\theta_{min}) = \pm \sqrt{\frac{2N\lambda}{L}}}$$

→ If N=1

$$(\theta_{min})_1 = \pm \sqrt{\frac{2\lambda}{L}}$$

$$BWFN = 2 \times (\theta_{min})_1$$

$$= \pm 2 \times \sqrt{\frac{2\lambda}{L}} \text{ radian}$$

$$BWFN = \pm 57.3^\circ \times 2\sqrt{\frac{2\lambda}{L}} \text{ degree}$$

$$\boxed{BWFN = \pm 114.6 \sqrt{\frac{2}{L/\lambda}}}$$

* Null-to-Null beam width of end-fire array $= 2\sqrt{\frac{2\lambda}{nd}}$

* The directivity of end-fire array, $D = 4\left(\frac{L}{\lambda}\right)$

$L = nd$

$$\boxed{D = 4\left(\frac{nd}{\lambda}\right)}$$

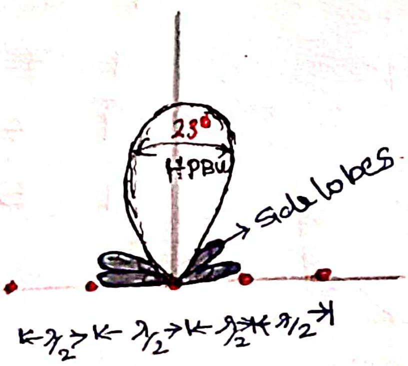
# Binomial arrays

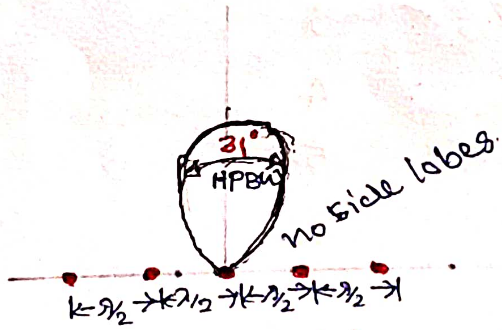
→ Binomial array is an array whose elements are excited according to the current levels determined by the binomial coefficient, $^nC_r$, $n$ being the number of element in the array.

→ It is a non-uniform array.

→ Advantage : ~~No~~ no side lobes in the resultant Pattern of array

(Note : A uniform distribution yields the maximum directivity but the minor lobes are relatively large. So ~~is~~ ~~We~~ the main objective of ~~Constructed~~ binomial array to eliminate this minor or side lobes in the resultant pattern of array.




eg:

$n = 5,$
$d = \lambda/2$

Uniform array

Binomial array

amplitude ratio 1 : 4 : 6 : 4 : 1

→ Disadvantage : 1) HPBW increases and hence the directivity decreases

2) For design of a large array, larger amplitude ratio of sources is required.

→ To reduce the sidelobe level, John stone Proposed that sources have amplitudes Proportional to the coefficients of a binomial series of the form.

$$(a+b)^{n-1} = a^{n-1} + (n-1)\, a^{n-2}\, b + \frac{(n-1)(n-2)}{2!}\, a^{(n-3)}\, b^{2} + \cdots\cdots$$

where n is the number of sources.

→ For array of 2 to 6 ~~(elements)~~ sources, the relative amplitude levels are given by

| Number of elements | Relative amplitude |
|---|---|
| n = 1 | 1 |
| 2 | 1, 1 |
| 3 | 1 2 1 |
| 4 | 1 3 3 1 |
| 5 | 1 4 6 4 1 |
| 6 | 1 5 10 10 5 1 |

→ These coefficients for any number of radiating sources can also be obtained from Pascal's triangle

Pascal's triangle.

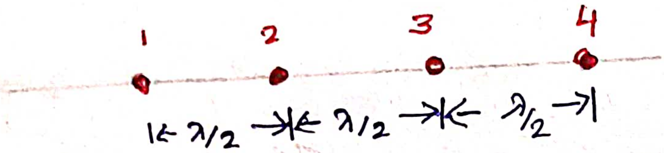
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$\rightarrow n = 3, \quad L = 2d = (3-1)d$

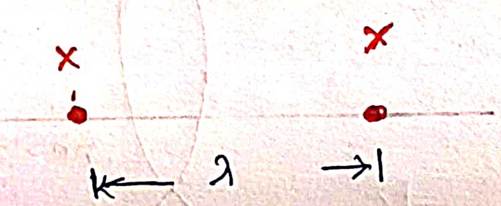
$\boxed{\text{length, } L = (n-1)d}$

eg: case I : Uniform array

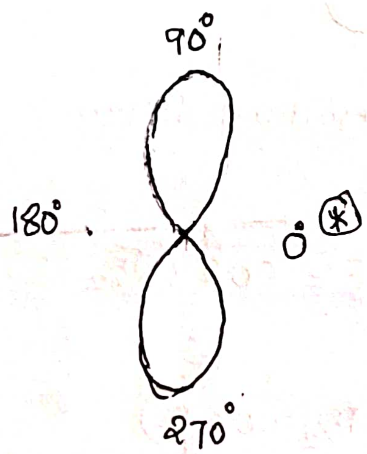
If $n = 4, \; d = \lambda/2$



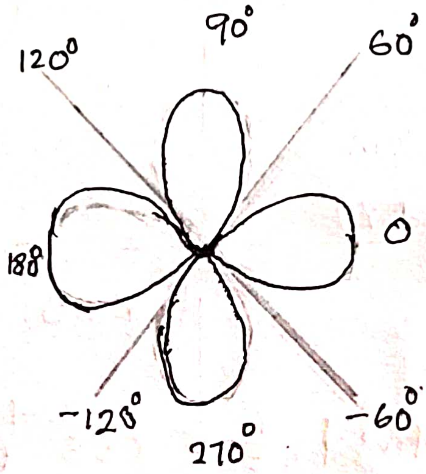

$L = (n-1)d$



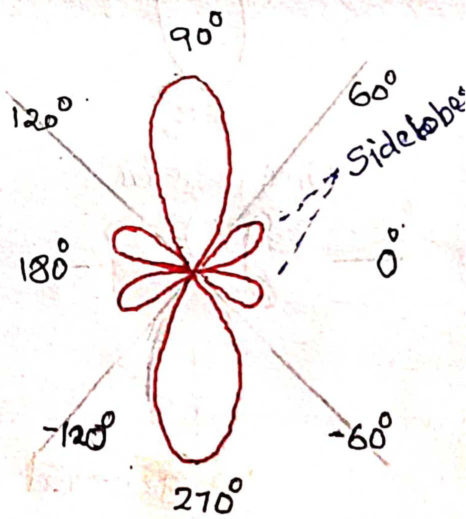

Resultant Pattern = Unit pattern of X or Y ⊛ group pattern X and Y




⊛




⇒

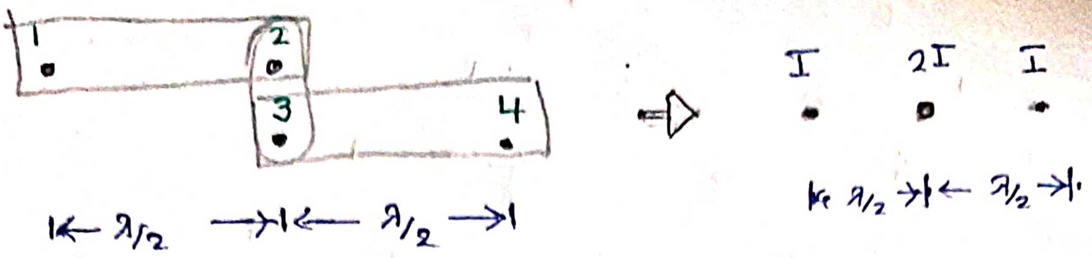



unit pattern (X or Y) ⊛ group pattern (X and Y) ⇒ Resultant Pattern.

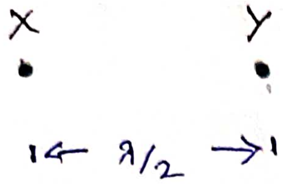
$E = \cos\left(\frac{\pi}{2}\cos\theta\right)$

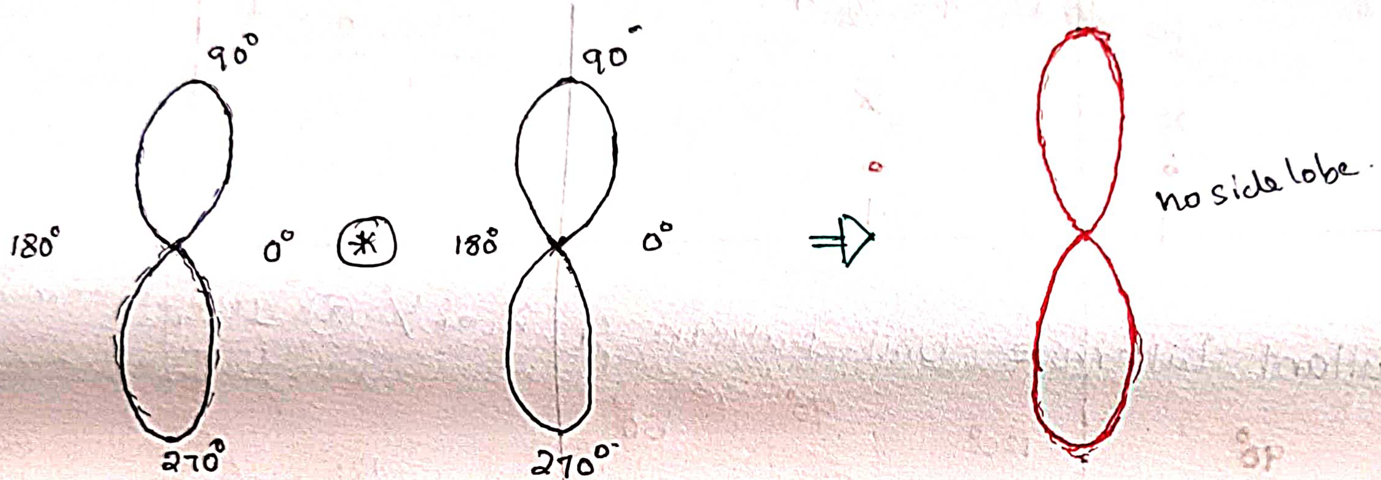
Case II :

For Binomial array




Here $n=3$




Unit pattern. (X or Y)

group pattern (X and Y)

Resultant Pattern.

if $n = 3$, $E = \cos^2(\pi/2 \cos\theta)$

In general Electric field, $E = \cos^{n-1}(\pi/2 \cos\theta)$

Length of array, $L = (n-1)\lambda/2$

$$\boxed{HPBW = \frac{1.06}{\sqrt{n-1}}}$$

$$HPBW = \frac{1.06}{\sqrt{2L/\lambda}} = \frac{0.75}{\sqrt{L/\lambda}}$$

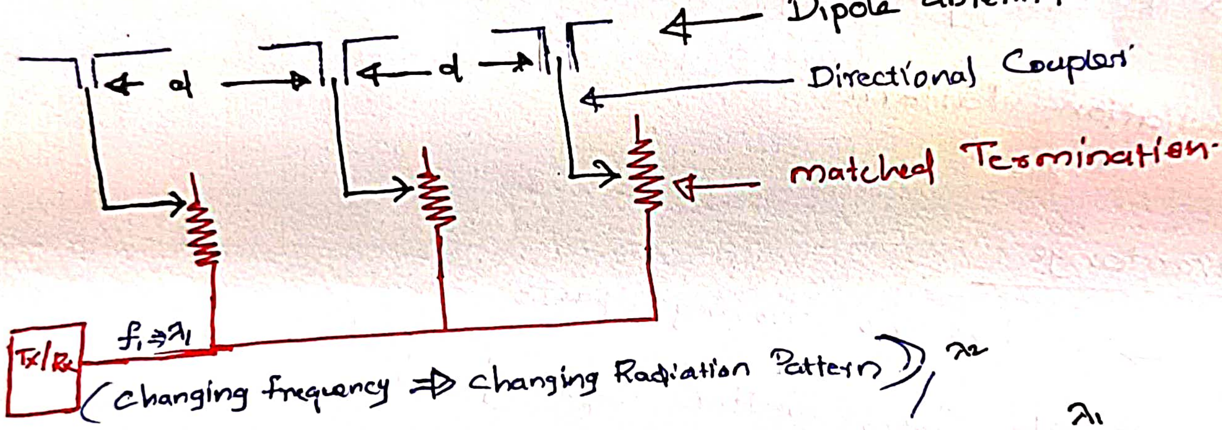
$$\boxed{D = 1.77\sqrt{n}}$$

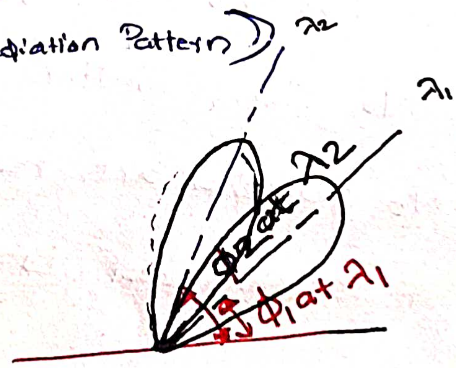
$$D = 1.77\sqrt{1+2(L/\lambda}$$

# Dolph - Tchebyshev Array

→ In this antenna array, direction of the beam of radiation is swept or scanned by changing the frequency.

→ Consider a line - fed array of uniformly spaced elements with a $Tx^r/Rx^r$ connected to the right end of the line as shown in below figure.




→ Here position of antenna and coupler is fix and direction of radiation can be changed by changing frequency.

→ Angle of radiation $\phi$ is expressed as

$$\cos\phi = \frac{c}{v} + \frac{m}{d/\lambda}$$

$\phi \Rightarrow$ Angle of Radiation.
$c \Rightarrow$ velocity of light
$v \Rightarrow$ Phase velocity
$m \Rightarrow$ mode number
$d \Rightarrow$ spacing b/w elements
$\lambda \Rightarrow$ wavelength.

~~Fa~~ For example

⇒ If $c = v$, $\lambda = d$, $m = -1$,

$$\cos\phi = 1 + \frac{(-1)}{1}$$

$$= 1 - 1$$

$$\cos\phi = 0$$

$$\phi = 90^\circ$$

⇒ If $c = v$, $\lambda = 0.9d$, $m = -1$

$$\cos\phi = 1 + \frac{(-1)}{1/0.9}$$

$$\cos\phi = 1 - 0.9$$

$$\cos\phi = 0.1$$

$$\phi = 84.26^\circ$$

→ If changing ~~in~~ frequency / wavelength, we can steer the Radiation Pattern.

## Advantange

→ It has no moving parts
→ No Phase shifter required } ⇒ Low Cost
→ No switches are required
→ It is simplest type of scanned array

## Tchebyshev Polynomial

→ on the name of Tchebyshev, the First letter T is used as symbol.

$$\Rightarrow T_m(x) = \cos(m \cos^{-1} x), \quad \text{for } |x| \leq \pm 1$$

$$T_m(x) = \cosh(m \cos^{-1} x), \quad \text{for } |x| \geq \pm 1$$

Consider $\delta = \cos^{-1} x$ $\Rightarrow x = \cos\delta$

$$T_m(x) = \cos(m\delta)$$

for $m = 0$

$$T_0(x) = \cos 0 = 1$$

For $m = 1$

$$T_1(x) = \cos\delta = x$$

For $m = 2$

$$T_2(x) = \cos(2\delta)$$

$$= 2\cos^2\delta - 1$$

$$= 2x^2 - 1$$

For $m = 3$

$$T_3(x) = \cos(3\delta)$$

$$= 4\cos^3\delta - 3\cos\delta$$

$$= 4x^3 - 3x$$

Further higher term

$\Rightarrow$ From above equation we can find the ~~Relation b/w the Polynom~~

$$T_{m+1}(x) = 2x\, T_m(x) - T_{m-1}(x) \qquad \text{—— (3)}$$

$$T_3(x) = 2x\, T_2(x) - T_1(x)$$

$$= 2x\,(2x^2 - 1) - x$$

$$= 4x^3 - 3x$$

→ For $n=4$, we must put $n=3$ in above equation ie

$$T_4(x) = 2x\,T_3(x) - T_2(x)$$

$$= 2x(4x^3 - 3x) - 2x^2 + 1$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

Similarly, the value of ~~$T_5$~~; $T_6$, $T_7$, $T_8$, ~~$T_9$~~, and $T_{10}$ also found and summarized as follows.

$$T_0(x) = 1$$

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

$$T_5(x) = 16x^5 - 20x^3 + 5x$$

$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$

$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$$

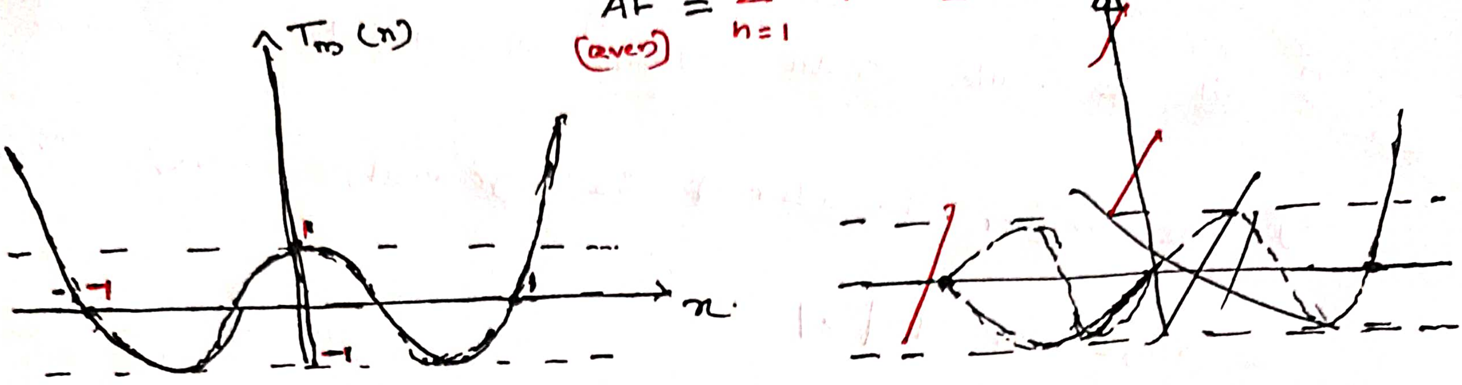
$$T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$$

$$T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$$

$$T_{10}(x) = 512x^{10} - 1280x^8 - 608x^6 + 40x^4 - 14x.$$

→ <u>General Characteristics of DCA</u>

$$\underset{(\text{even})}{AF} = \sum_{n=1}^{M} a_n \cos\left[(2n-1)u\right], \quad N = 2M \text{ even}$$

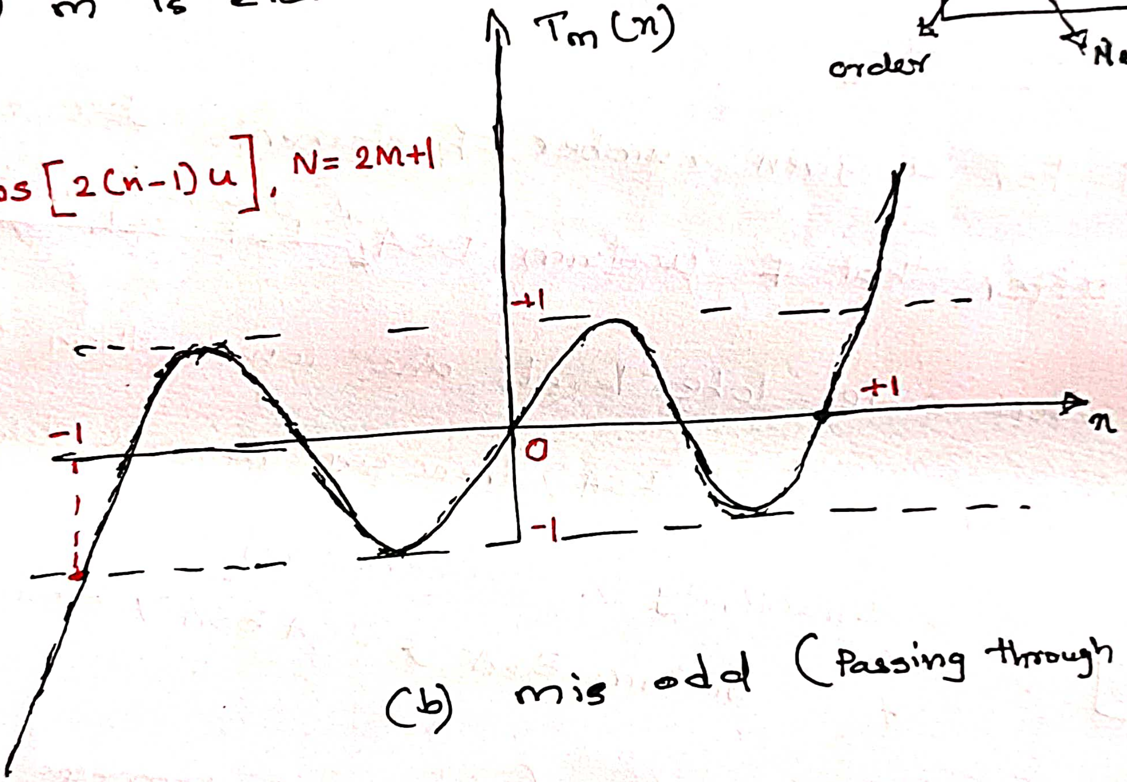



(a) m is even.

$\boxed{m = N-1}$

order

N element




$$\underset{(\text{odd})}{AF} = \sum_{n=1}^{M+1} a_n \cos\left[2(n-1)u\right], \quad N = 2M+1$$

(b) m is odd (Passing through origin)

→ Side lobes arise in the region $x < 1$

→ The main lobe extends into the range $x > 1$.

→ The value of m and the degree of Tchebyshelf Polynomial are the same.

→ <u>Properties of chebyshev Polynomials:</u>

1] All polynomials of any order 'm' pass through the point (1,1)

2] All the nulls occur within $-1 \leq n \leq 1$

3] The higher the order of the Polynomial (mainlobe), the steeper the slope for $|x| > 1$

## Advantage

1] For a given number of elements directivity next after that of uniform BSA, or (greater than Binomial BSA) ~~but side lobe levels~~ but side-lobe levels are lower than uniform BSA (for Binomial BSA, there are no side lobe at all).

2] Directivity & Side lobe level

Uniform BSA > DCA BSA > Binomial BSA

~~Lower Side lobe~~ (i.e DCA provides minimum ~~Binomial BSA~~ optimum beamwidth for a specified degree of Side-lobe level reduction)

2]. It results in side lobes are lobes are all of the same amplitudes, unlike in Uniform BSA.

3] Ratio of current between centre element and end element is small, which provides ease in feeding design.