CONTENTS

- Outline of module 4
- Course outcomes
- Error control coding
- Types of error codes

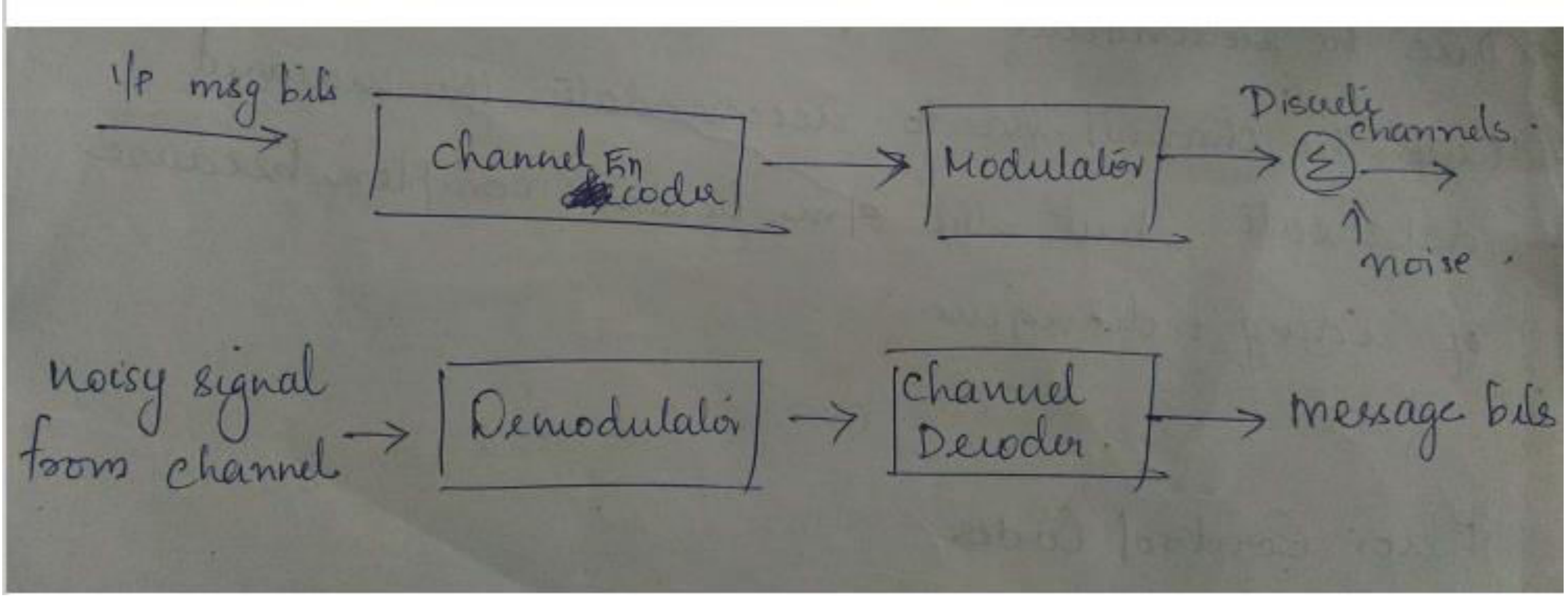
Outline

IV	Introduction to rings, fields, and Galois fields. Codes for error detection and correction – parity check coding – linear block codes – error detecting and correcting capabilities – generator and parity check matrices – Standard array and syndrome decoding

Course outcomes

- Outline of Galois fields
- Describe the linear Block codes for error detection and correction.
- Apply Parity Check matrix to design the Linear Block Code Encoder circuit
- Decoder using Syndrome Calculation circuit.

Block diagram of digital communication system



Error control coding

- Errors are introduced in the data when it is passed through the channel
- This reduces the signal power
- Error control coding is accomplished at the transmitter using channel encoder and channel decoder at the receiver
- This is done by adding “redundancy”
- Where the channel encoder adds additional bits to the message bits
- These additional bits do not carry any information-to detect & correct errors

Advantages & disadvantages

- Advantage:
- improved data quality
- Reduced bit-error rate

- Disadvantage
- Increased bandwidth
- System becomes more complex-decoding circuitry

Error correcting codes

- 2 types
- Block codes
- Convolutional codes

- Block codes consists of (n-k) redundant bits added to the ‘k’ message bits, to form n bit code word
- These check bits helps to detect and correct errors in the entire n bit code words
- Convolutional codes: the redundant bits are continuously interleaved with message bits, helps to detect and correct errors

Linear Block Codes

- It consists of n number of bits in one block or code word
- ‘k’ message bits
- (n-k) redundant bits
- Called as (n,k) block codes

Message (k bits)	Check (n-k bits)

- Structure is that message bits followed by redundant bits
- Called as Systematic codes

Illustrating the formation of linear block codes

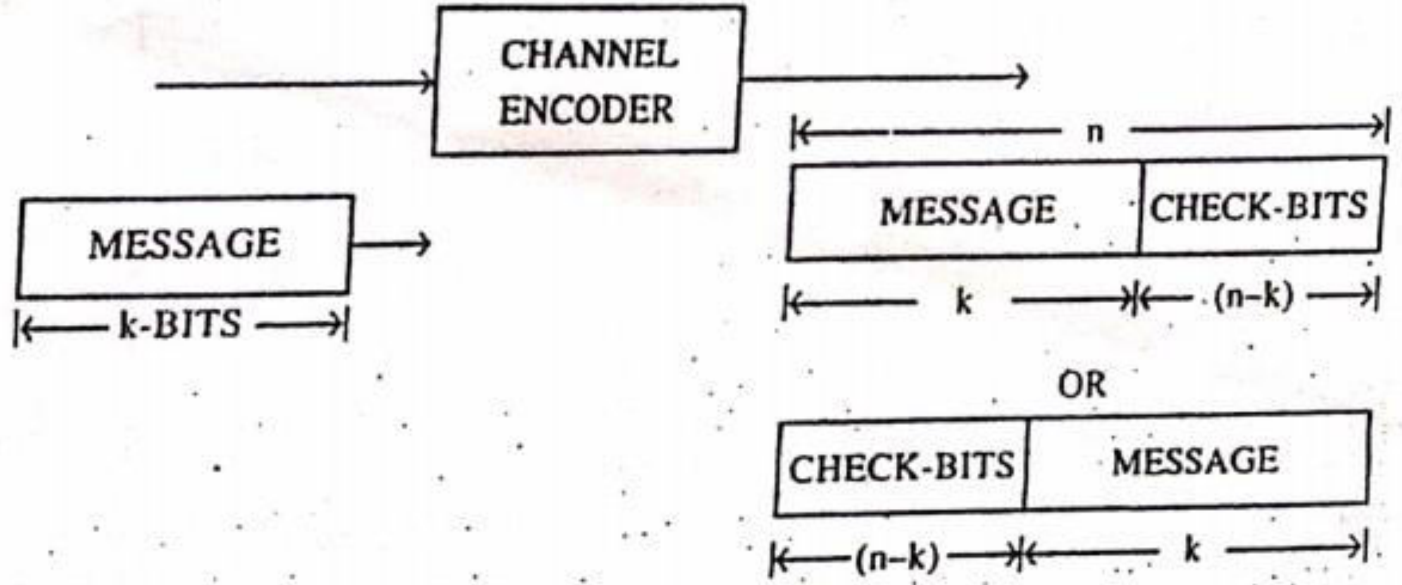


Fig. 5.2 : Illustrating the formation of linear block codes

Terms used in error control coding

Code word (n)

- Encoded block of n bits is called a code word
- Contains msg bit and check bits

Block length (n)

no: of bits 'n' after coding is called block length

Code rate (r) | Code Efficiency.

Ratio of message bits (k) and encoder o/p bits (n)

$$r = \frac{k}{n} \qquad 0 < r < 1$$

Channel data rate

Bit rate at o/p of encoder. If bit rate at encoder is R_s

$$R_0 = \frac{n}{k} \cdot R_s.$$

Code vectors.

no: of code words $= 2^k$.

add to get other.

- hamming distance between 2 code vectors is equal to no: of elements in which they differ.

eg: $X = 100$ and $Y = 110$:

$d(X,Y) = 1$

Minimum distance (d_{min})

- smallest hamming distance b/w valid code vectors

For (n,k) block codes $d_{min} \leq n-k+1$.

Weight of the code.

- no: of non-zero elements in transmitted code

two vectors $X = 01110101 \Rightarrow w(X) = 5$

CONCLUSION

- Error control codes
- Linear block codes

THANK YOU

CONTENTS

- Matrix description of linear block codes
- Formation of code words
- Encoder circuit

Encoding operations using matrices

Let message block be a row vector / k tuple given

MSB.

$D = [d_1, d_2 \ldots d_k]$ where each message bit can be 0 or 1

Thus we have 2^k distinct message blocks.

Each message block is transformed into codeword c of length n bits

$c = [c_1, c_2 \ldots c_n]$.

It may be noted that there is one unique code for each distinct message block. This set of 2^k codewords also known as <u>code vectors</u> is (n, k) block codes.

In a systematic linear block code the first k bits of code are message bits.

$$c_i = d_i \qquad i = 1, 2, 3 \ldots k.$$

The last $(n-k)$ bits in the codeword are check bits generated according to some predetermined rule.

$$C_{k+1} = P_{11}d_1 + P_{21}d_2 + P_{31}d_3 + P_{41}d_4 + \dots + P_{k1}d_k.$$

$$C_{k+2} = P_{12}d_2 + P_{22}d_2 + P_{32}d_3 + \dots P_{k2}d_k.$$

$$\vdots$$

$$C_n = P_{1(n-k)}d_1 + P_{2(n-k)}d_2 + \dots P_{k,(n-k)}d_k.$$

where coeffecients of P_{ij} are 0's and 1's and addition operation is performed using modulo 2 arithmetic operation.

$$
\begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} = \begin{bmatrix} d_1 & d_2 & \cdots & d_k \end{bmatrix}
\begin{bmatrix}
1\,0\,0\,0\,0 \ldots 0 & : & P_{11}\,P_{12} \ldots P_{1(n-k)} \\
0\,1\,0\,0\,0 \ldots 0 & : & P_{21}\,P_{22} \ldots P_{2(n-k)} \\
0\,0\,1\,0\,0 \ldots 0 & : & P_{31}\,P_{32} \ldots P_{3(n-k)} \\
0\,0\,0\,1 \ldots 0 & : & \vdots \\
\vdots & & \\
\underbrace{0\,0\,0\,0 \ldots 1}_{k \text{ terms}} & : & P_{k1}\,P_{k2} \ldots P_{k(n-k)}
\end{bmatrix}
$$

$c_1 c_2 \ldots c_k\, c_{k+1}\, c_{k+2} \ldots c_n.$

Or.

$$[C] = [D]\,[G].$$

where $[G]$ is called Generator Matrix used in Encoding operation.

$$G = \left[I_k \;\vdots\; P \right]_{k \times n}$$

Demarcation line :

I_k is the identity matrix of order k

P is the parity matrix of order k x (n-k)

Example

1. The Generator matrix for (6,3) block code is

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Find all the code vectors.

Given length $n = 6$.

message size $k = 3$.

There are $2^3 = 8$ code vectors.

000, 001, 010, 011, 101, 110, 111, 100.

$c = [d_1 d_2 d_3,\ d_2 \oplus d_3,\ d_1 \oplus d_3,\ d_1 \oplus d_2]$.

$C = D\,G$.

$$C = [d_1\ d_2\ d_3] \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right].$$

when $D = [001]$

$$C = [001]\begin{bmatrix} 100 & 011 \\ 010 & 101 \\ 001 & 110 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1\ 0]$$

when $D = 111$

$$C = [111]\begin{bmatrix} 100 & 011 \\ 010 & 101 \\ 001 & 110 \end{bmatrix}.$$

$$C = [111\ 000]$$

Message	codevector
000	000000.
001	001 110
010	010 101
011	011 011
100	100011
101	101101
110	110110

Example 2: HW

9) Generator matrix for (6,3) block code is (7).

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

Find all the code vectors

Parity Check matrix

we know that

$$G = \left[I_k \,\vdots\, P \right]_{k \times n}.$$

Parity matrix H associated with each (n,k) blockcode can be defined as

H is called Parity check matrix of order (n-k) x n which is used for error correction

$$H = \left[P^T \,\vdots\, I_{n-k} \right]_{(n-k) \times n}.$$

$$= \left[\begin{bmatrix} P_{11} & P_{21} & \cdots & P_{K1} \\ P_{12} & P_{22} & \cdots & P_{K2} \\ \vdots & & & \\ P_{1,n-k} & P_{2,n-k} & \cdots & P_{K,n-k} \end{bmatrix} \;\vdots\; \begin{matrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \end{matrix} \right]$$

K columns (n-k) columns.

The parity check matrix can be used to verify whether a codeword C is generated by the matrix $G = [I_k \mid P]$.

(3)

$$HG^T = [P^T \mid I_{n-k}] \begin{bmatrix} I_k \\ \hline P_1 \end{bmatrix}.$$

$$= P^T + P^T$$

$$= 0$$ [Since modulo 2 addition yields same nos sum as zero].

Similarly $GH^T = 0$ where 0 is a new null matrix.

WKT $C = DG$

Post multiplying H^T.

$$CH^T = DGH^T$$

$$\underline{CH^T = 0}$$

ology

Matrix H is called parity check matrix & set of ~~operations~~ equations specified by eqn (1) is called parity check equation

General eqn $C = DG$

Parity check eqn $CH^T = DGH^T = 0$.

Encoder circuit

Expanding the matrix of equation (5.9) and equating the corresponding elem
sides, we get

$$
\left.
\begin{aligned}
c_1 &= d_1 \\
c_2 &= d_2 \\
\vdots \quad & \quad \vdots \\
c_k &= d_k \\
c_{k+1} &= p_{11} d_1 + p_{21} d_2 + \dots\dots\dots + p_{k1} d_k \\
c_{k+2} &= p_{12} d_1 + p_{22} d_2 + \dots\dots\dots + p_{k2} d_k \\
\vdots \quad & \quad \vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
c_n &= p_{1,n-k} d_1 + p_{2,n-k} d_2 + \dots\dots + p_{k,n-k} d_k
\end{aligned}
\right\}
$$

The implementation of the above equation in a circuit fashion results in the encoder for (n, k) linear block code. Such a realization of encoder circuit is shown in figure 5.3 consisting of a k-bit shift register, a n-segment commutator and (n – k) number of modulo-2 adders.

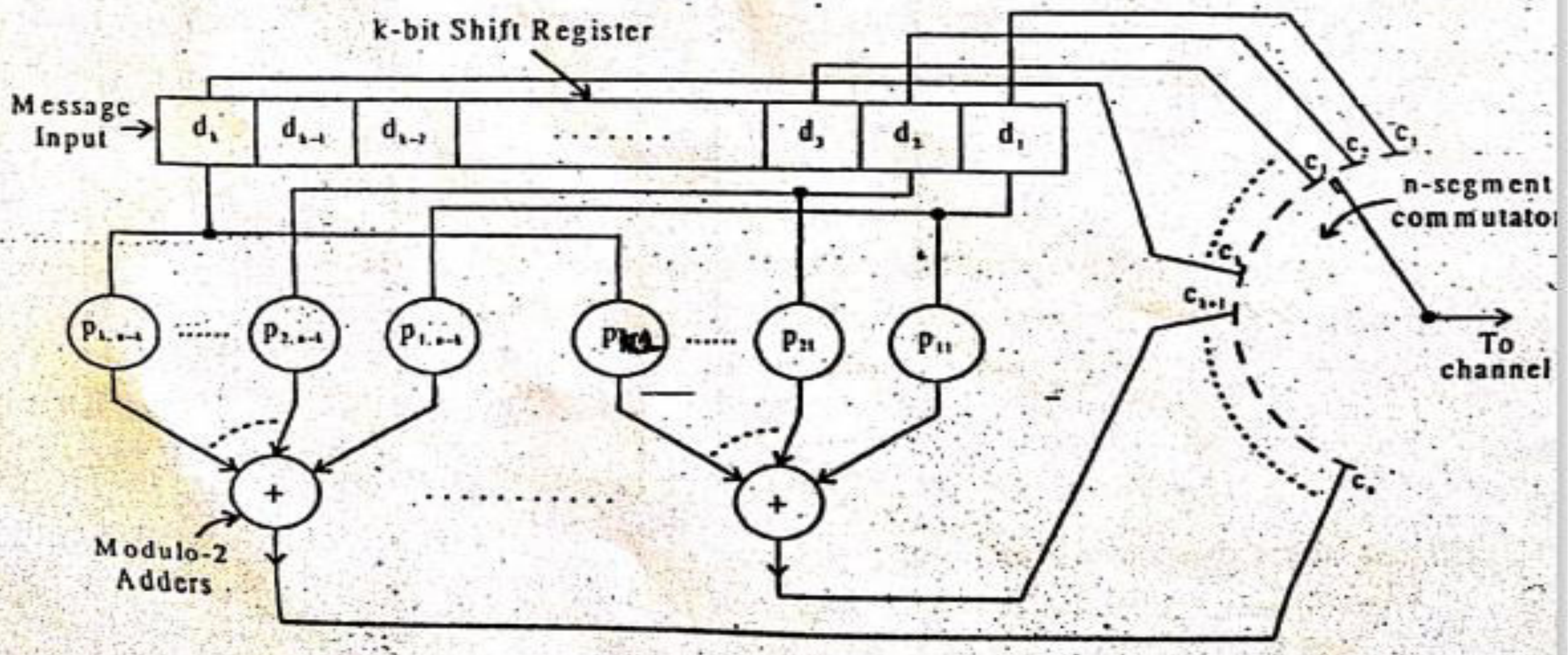


Fig. 5.3 : Encoding circuit for (n, k) linear block codes

Encoder circuit

- The entire data $d_k\, d_{k-1} \ldots\ldots d_2\, d_1$ is shifted to a k bit shift register
- The small circles $p_{11}, p_{21}, p_{k1} \ldots. p_{1,n-k}$ are either open or short circuited depending on 0 or 1
- if p11 = 0, then no connection from d1 to modulo 2 adders and if p11=1, makes a connection
- Then the message is shifted into the shift register, modulo 2 adders generate the check bits, which are fed into the commutator segments
- When the commutator brush rotates and makes contact with segments, code vector bits will be transmitted through the channel

CONCLUSION

- Matrix description of encoding operations
- Examples of code vector generation
- Encoder circuit

THANK YOU

Contents

- Quick recap
- Syndrome decoding
- Syndrome calculation circuit

Example

Eg:

For the code $c = [d_1, d_2 d_3, d_1 \oplus d_3, d_2 \oplus d_3, d_1 \oplus d_3]$

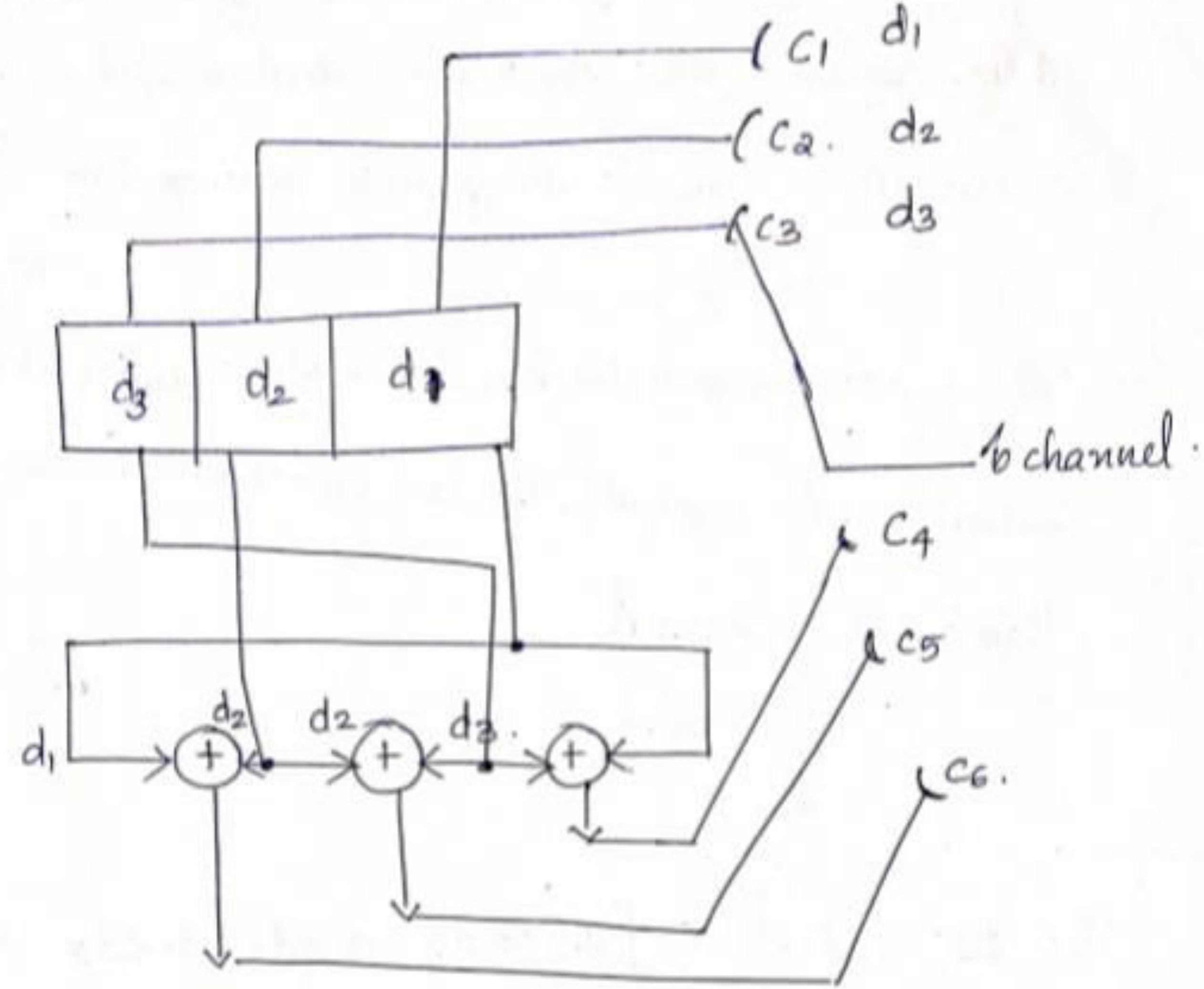
Construct the corresponding encoder circuit.

Soln.

we have $k = 3$.

→ So 3 shift registers to move message bits

→ $n = 6$. $k = 3$. $n - k = 3$, So 3 modulo 2 adders are required.



6 sequent Commutator

Syndrome

Generator matrix $G \Rightarrow$ used in encoding at transmitter

Parity check matrix $H \Rightarrow$ used in decoding at Receiver.

Let $C = [c_1\ c_2 \ldots c_n]$ be valid code vector transmitted over a communication channel belonging to (n, k) LBC

Let $R = [r_1, r_2 \ldots r_n]$ be received vector.

Due to noise in the channel, $[r_1, r_2 \ldots r_n]$ is different from $[c_1, c_2 \ldots c_n]$

The error vector or error pattern 'E' is defined as difference between R and C.

$$E = R - C$$

modulo 2 addition = modulo 2 subtraction

$$= R \oplus C$$

ie error vector e can be represented as vector E as

$$E = [e_1\ e_2\ ..\ e_n]$$

where $e_i = 1$ if $r_i \neq c_i$ and

$e_i = 0$ if $r_i = c_i$

→ 1's in the error vector represents the error caused by the noise in the channel.

→ The receiver ~~does~~ decodes code vector C from Received vector R

Receiver has knowledge about only R and not about C & E.

→ To find E and C; receiver decodes by determining a 1 by $(n-k)$ vector S called error syndrome vector / syndrome

$$S = rH^T.$$

Properties of Syndrome

1. The syndrome depends on error pattern and not in transmitted code word.

Proof

we know $r = c + e$ — (1)

$s = rH^T$ — (2).

$$S = [C+e]H^T$$

$$= CH^T + eH^T = \underline{\underline{eH^T}}$$

Hence parity matrix H of code permits to compute syndrome S, which depends only upon error pattern e.

Property 2.

All error patterns that differ by a codeword have the same syndrome.

Proof

For k message bits, there are 2^k distinct code vectors denoted as c_i, $i = 0, 1, 2 \ldots 2^k - 1$.

→ Correspondingly for any error pattern e, we define the 2^k distinct vectors e_i as $e_i = e + c_i$ — ①

$i = \{0, 1, 2 \ldots 2^k - 1\}$

i is called coset of the code.

→ In other words, coset has 2^k elements that differ atmost by a code vector. Thus an (n, k) LBC has 2^{n-k} possible cosets × eqn ① by matrix H^T

$e_i = e + c_i$

$$e_i H^T = (e + c_i) H^T$$

$$= eH^T + c_i H^T$$

$$= \underline{\underline{eH^T}}.$$ which is independent of index i

Accordingly we may state that coset of code is characterized by a unique syndrome.

Example 1

For a systematic code (6,3) LBC the parity matrix P is

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The received ~~vector~~ code word is $R = [110010]$.

Detect the and correct the single error that has occurred due to noise.

Solution

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{Then } P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

$$H = \left[P^T \,\middle|\, I_{n-k} \right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$H^T = \left[\begin{array}{c} 1\,0\,1 \\ 0\,1\,1 \\ 1\,1\,0 \\ \hline 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{array}\right] = \frac{P}{I_{n-k}}$$

Syndrome S is given by $S = [S_1 \; S_2 \; S_3]$.

$$= R \cdot H^T$$

$$= \underbrace{[1\,1\;0\,0\;1\,0]}_{1\times 6} \left[\begin{array}{c} 1\,0\,1 \\ 0\,1\,1 \\ 1\,1\,0 \\ \hline 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{array}\right]_{6\times 3}$$

$= \ [1 \quad 1{+}1 \quad 1{+}1]$

(modulo 2 addition
1+1 = 0.)

$= \underline{\underline{[1 \quad 0 \quad 0]}}$.

The syndrome [1 0 0] is present in 4th row of H^T matrix.

So 4th bit in received vector R counting from left is error.

Corrected code is [1 1 0 1 1 0].

Syndrome Calculation Circuit

Let the received vector $R = (r_1, r_2, \ldots r_n)$. syndrome vector $[S] = [s_1\ s_2\ \cdots\ s_{n-k}]$

$= RH^T$.

$$S = [r_1\ r_2\ \cdots\ r_n] \cdot \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1,n-k} \\ P_{21} & P_{22} & \cdots & P_{2,n-k} \\ \vdots & & & \\ P_{k1} & P_{k2} & \cdots & P_{k,n-k} \\ \hline 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Multiplying using Modulo 2 adder.

we get syndrome bit as

$$S_1 = r_1 P_{11} + r_2 P_{21} + r_3 P_{31} + \dots r_k P_{k1} + r_{k+1}$$

$$S_2 = r_1 P_{12} + r_2 P_{22} + \dots \dots r_k P_{k2} + r_{k+2}.$$

$$S_3 = r_1 P_{13} + r_2 P_{23} + \dots \dots r_k P_{k3} + r_{k+3}.$$

$$\vdots$$

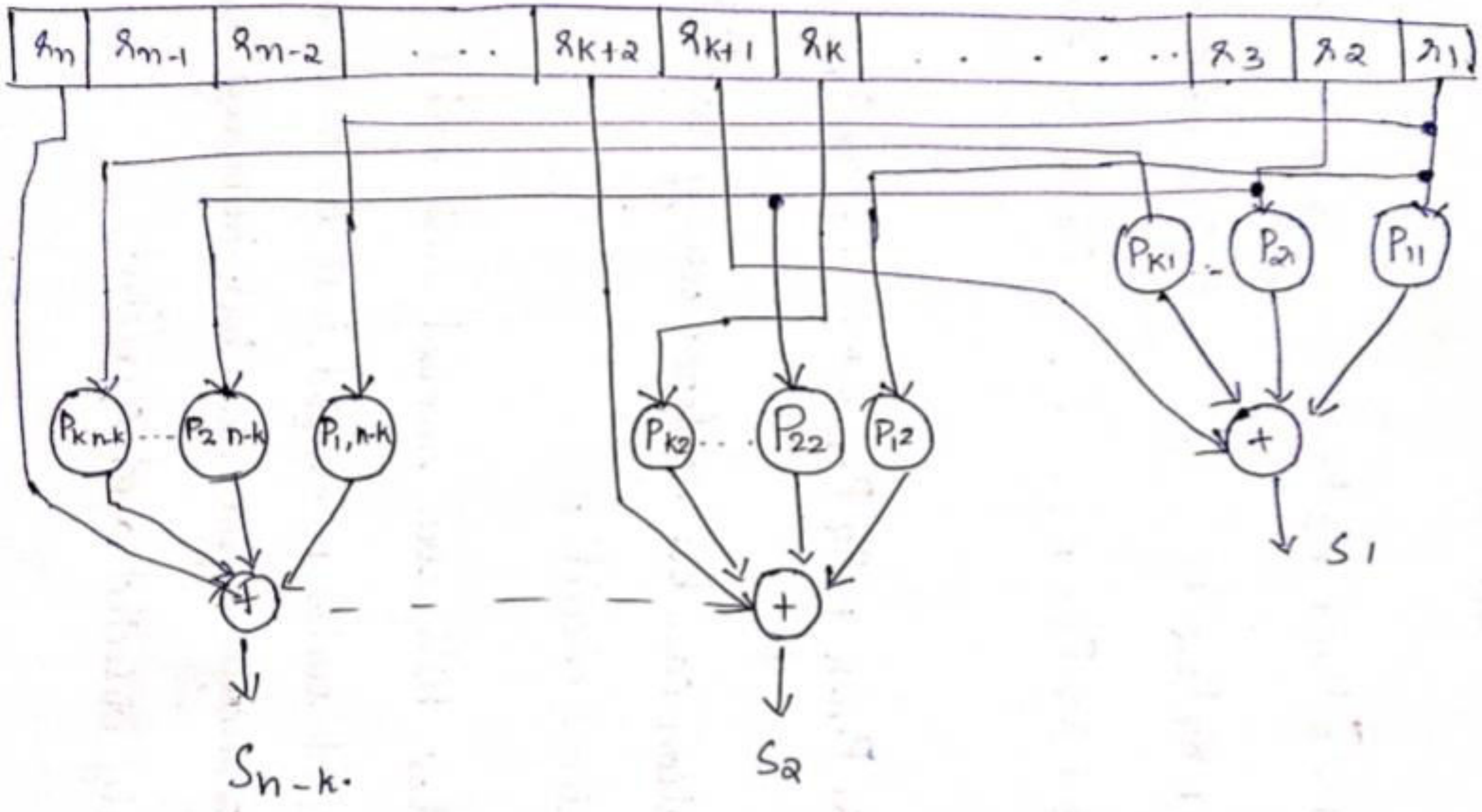
$$S_{n-k} = r_1 P_{1n-k} + r_2 P_{2n-k} + \dots r_k P_{kn-k} + r_n.$$

The above equation can be realized using circuit called syndrome calculation circuit.

The above equation can be realized using circuit called Syndrome calculation circuit.

→ The received vector bits are moved into n-bit shift register

→ As soon as received vector is shifted into the shift register, modulo 2 adder generates syndrome bit. knowing S, the error can be easily detected and corrected.


rn
rn-1
rn-2
rk+2
rk+1
rk
r3
r2
r1
Pk1
P21
P11
Pk n-k
P2 n-k
P1, n-k
Pk2
P22
P12
+
S1
S2
Sn-k.

Example 2

For a systematic ~~matrix~~ LBC (6,3) code of $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$,

The received vector is $R = [r_1\ r_2\ r_3\ r_4\ r_5\ r_6]$

Construct the corresponding syndrome calculation circuit.

$$H = \left[P^T \;\vdots\; I_{n-k} \right]$$

$$H^T = \begin{bmatrix} P \\ \hline I_{n-k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = r H^T$$

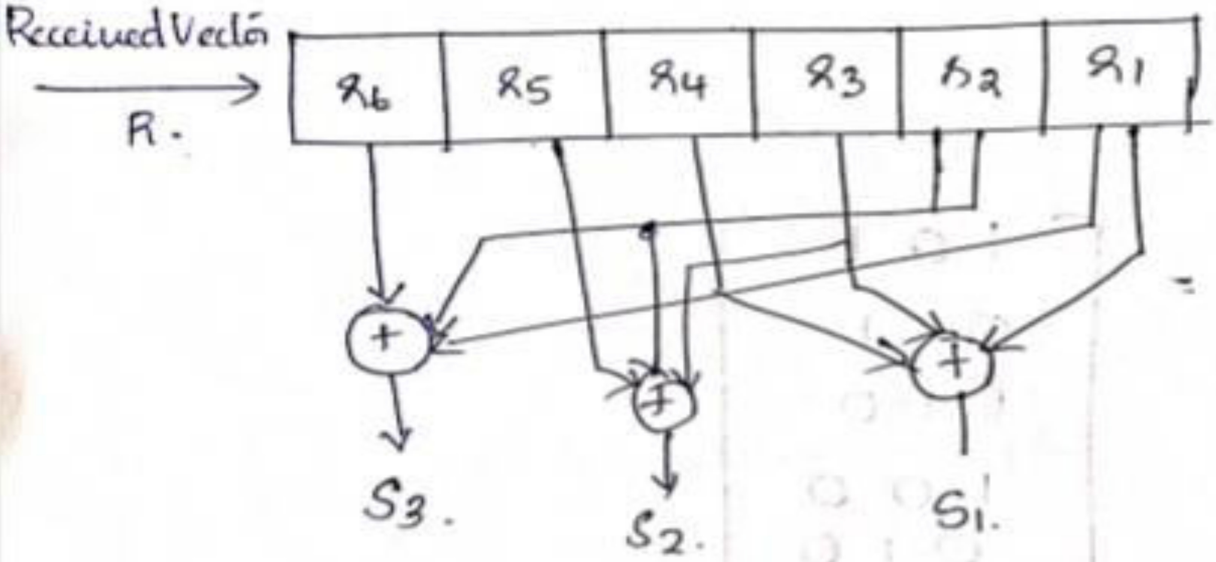
$$= \underset{1\times 6}{\left[r_1 \; r_2 \; r_3 \; r_4 \; r_5 \; r_6 \right]} \underset{6\times 3}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}$$

R1 + r3 +r4

$S_2 = r_2 \oplus r_3 \oplus r_5$.

$S_3 = r_1 \oplus r_2 \oplus r_6$.

Syndrome calculation circuit for above (6,3) LBC code.



Conclusion

- Syndrome decoding
- Syndrome calculation circuit

THANK YOU

CONTENTS

- Quick recap
- Example- syndrome calculation

Example 5.6 : For a systematic (7, 4) linear block code, the parity matrix P is given by

$$[P] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(i) Find all possible valid code-vectors.

(ii) Draw the corresponding encoding circuit.

(iii) A single error has occurred in each of these received vectors. Detect and correct those errors.

(a) $R_A = [0111110]$ (b) $R_B = [1011100]$ (c) $R_C = [1010000]$.

(iv) Draw the syndrome calculation circuit.

(i) The generator matrix [G] is given by equation (5.11) as

$$[G] = [I_k \mid P] = [I_4 \mid P]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

Using equation (5.10), the code-vectors can be found as

$$[C] = [D]\,[G]$$

As an example, let $D = [1\ 1\ 0\ 1]$

$$\therefore\ [C] = [1\ 1\ 0\ 1] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= [1\ 1\ 0\ 1\ 0\ 1\ 0]$$

For $D = [0\ 1\ 1\ 1]$

$$[C] = [0\ 1\ 1\ 1]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= [0\ 1\ 1\ 1\ 0\ 0\ 0]$$

In a similar way, the other valid code-vectors are found as given in table 5.4.

Message Vector (D)	Code-Vectors (C)	Message Vector (D)	Code-Vector (C)
0000	0000000	1000	1000111
0001	0001011	1001	1001100
0010	0010101	1010	1010010
0011	0011110	1011	1011001
0100	0100110	1100	1100001
0101	0101101	1101	1101010
0110	0110011	1110	1110100
0111	0111000	1111	1111111

(ii) The encoding circuit which requires a 4-bit shift register, 3 modulo-2 adders and a 7-segment commutator is shown in figure 5.7.

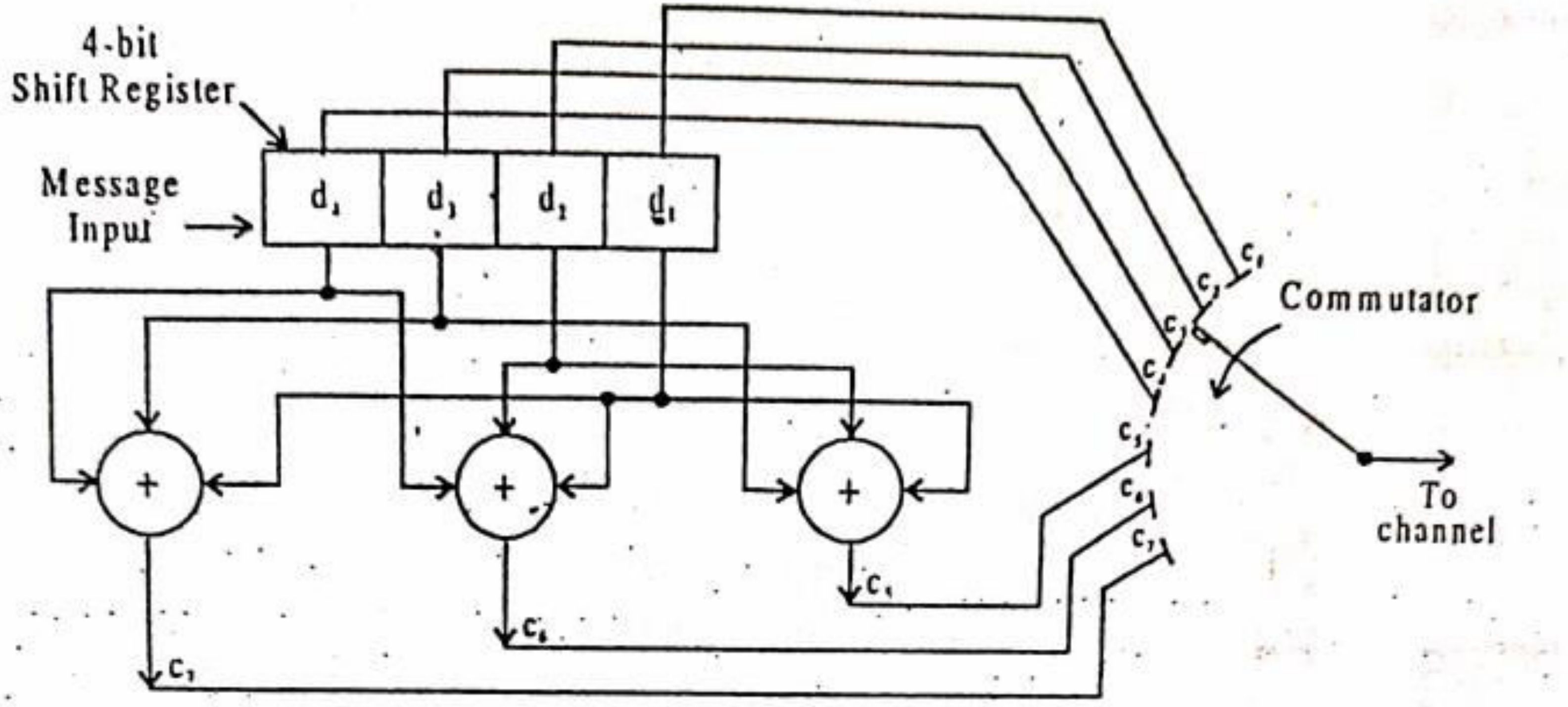


Fig. 5.7 : Encoding circuit for (7, 4) linear block code of example (5.6)

The code-vector bits in terms of the message bits are found using

$$[C] = [D]\,[G] = [d_1\ d_2\ d_3\ d_4] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad [C] = [d_1, d_2, d_3, d_4, (d_1+d_2+d_3), (d_1+d_2+d_4), (d_1+d_3+d_4)]$$

(iii) (a) Given $R_A = [0\ 1\ 1\ 1\ 1\ 1\ 0]$

The parity check matrix H is given by equation (5.14) as

$$H = \left[P^T \mid I_{n-k}\right] = \left[P^T \mid I_3\right]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

(iii) (a) Given $R_A = [0\ 1\ 1\ 1\ 1\ 1\ 0]$

The parity check matrix H is given by equation (5.14) as

$$H = \left[P^T \mid I_{n-k}\right] = \left[P^T \mid I_3\right]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

∴ The syndrome S_A is given by equation (5.22) as

$$S_A = R_A H^T$$

$$= [0\ 1\ 1\ 1\ 1\ 1\ 0] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 1\ 0]$$

→ This syndrome is located in the second row of H^T matrix. Hence the 2[nd] bit counting from left is in error. The corresponding error-vector is then given by

$$E_A = [0\ 1\ 0\ 0\ 0\ 0\ 0]$$

∴ The corrected code-vector which is the transmitted vector is given by

$$C_A = R_A + E_A = [0\ 1\ 1\ 1\ 1\ 1\ 0] + [0\ 1\ 0\ 0\ 0\ 0\ 0]$$

$= [0\ 0\ 1\ 1\ 1\ 1\ 0]$ which is the valid code-vector corresponding to the message vector 0011 as seen from table 5.4.

(b) Given $R_B = [1\,0\,1\,1\,1\,0\,0]$

$\therefore\ S_B = R_B H^T$

$$= [1\,0\,1\,1\,1\,0\,0]\begin{bmatrix}1 & 1 & 1\\1 & 1 & 0\\1 & 0 & 1\\0 & 1 & 1\\1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$= [1\ 0\ 1]$ which is located in the 3rd row of H^T.

$\therefore$ The error vector $E_B = [0\,0\,1\,0\,0\,0\,0]$

$\therefore$ The corrected code-vector $= C_B = R_B + E_B$

$= [1\,0\,1\,1\,1\,0\,0] + [0\,0\,1\,0\,0\,0\,0]$

$= [1\,0\,0\,1\,1\,0\,0]$

(c) Given $R_C = [1\,0\,1\,0\,0\,0\,0]$

$$S_C = R_C H^T = [1\,0\,1\,0\,0\,0\,0] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= [0\,1\,0] \rightarrow$ This is present in the 6^{th} row of H^T.

$\therefore$ The error vector $E_C = [0\,0\,0\,0\,0\,1\,0]$

$\therefore$ Corrected code-vector = $C_C = R_C + E_C$

$$= [1\ 0\ 1\ 0\ 0\ 0\ 0] + [0\ 0\ 0\ 0\ 0\ 1\ 0]$$

$$= [1\ 0\ 1\ 0\ 0\ 1\ 0]$$

C_C is again a valid code-vector corresponding to the message vector 1010 in table 5.4.

v) **Syndrome Calculation Circuit :**

Let the received vector be represented, in general, by

$$R = [r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7]$$

The syndrome corresponding to the above received vector R is

$$S = R\,H^T = [r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad S = [s_1, s_2, s_3] = [(r_1+r_2+r_3+r_5), (r_1+r_2+r_4+r_6), (r_1+r_3+r_4+r_7)]$$

The syndrome calculation circuit can be easily constructed as shown in figure 5.8.

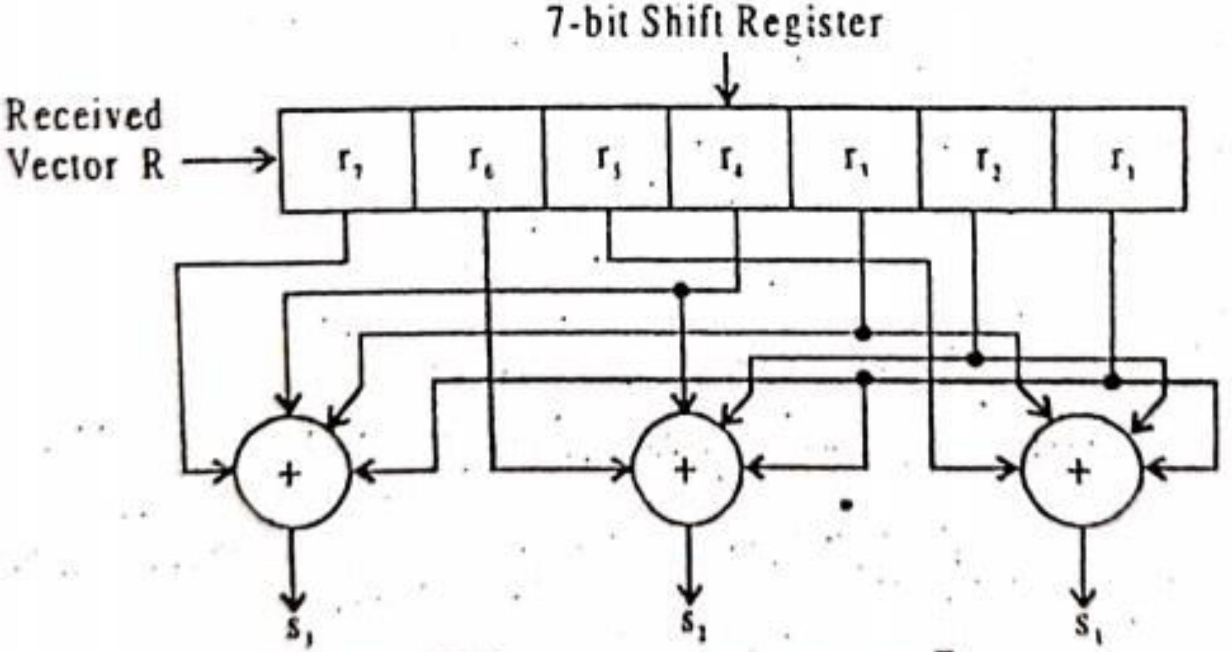


Fig. 5.8 : Syndrome calculation circuit of example 5.6

THANK YOU

CONTENTS

- Quick recap
- Syndrome decoding
- Construction of standard array
- Examples

Syndrome decoding

Let $c_1\ c_2 \dots c_{2^k}$ denotes the no:of code vectors of an (n, k) linear block code.

Let r denote the received vector, which may have one of 2^n possible values.

a possible ...

The receiver has the task of partitioning the 2^n possible received vectors into 2^k disjoint subsets D_1 D_2 D_3 ... D_{2^k} such that the i^{th} subset D_i corresponds to codevector C_i for $1 \le i \le 2^k$.

The received vector r is decoded into C_i if it is in the i^{th} subset;

The 2^k subsets described here in constitute a standard array of the linear block code.

To construct Standard array

1. The 2^k code vectors are placed in a row with all zero code vector c_1 as the left most element.

2. An error pattern e_2 is picked and placed under c_1 and a second row is formed by adding e_2 to each of the remaining code vectors in the first row.

It is important that the error pattern chosen as the first element in a row not have previously appeared in the standard array.

To construct Standard array….

3) Step 2 is repeated until all the possible error patterns have been accounted for.

Standard array for an $[n,k]$ block code:

$$\begin{bmatrix} c_1 = 0 & c_2 & c_3 & c_4 \cdots & c_i & \cdots & c_{2^k} \\ e_2 & c_2+e_2 & c_3+e_2 & c_4+e_2 \cdots & c_i+e_2 & \cdots & c_{2^k}+e_2 \\ e_3 \cdots & c_2+e_3 & c_3+e_3 & c_4+e_3 \cdots & c_i+e_3 & \cdots & c_{2^k}+e_3 \\ \vdots & & & & & & \\ e_j & c_2+e_j & c_3+e_j & c_4+e_j \cdots & c_i+e_j & \cdots & c_{2^k}+e_j \\ \vdots & & & & & & \\ e_{2^{n-k}} & c_2+e_{2^{n-k}} & c_3+e_{2^{n-k}} & \cdots & c_i+e_{2^{n-k}} & \cdots & c_{2^k}+e_{2^{n-k}} \end{bmatrix}$$

Disjoint subset D2

COSET

COSET LEADER

CO -SET

- The 2^k columns of the array represent the disjoint subsets $D_1, D_2 \ldots D_{2^k}$.

- 2^{n-k} rows of array represent cosets of the codes and their first element $e_2 \ldots e_{2^{n-k}}$ called the coset leaders

For a given channel, the probability of decoding error is minimized when most likely error patterns (those with largest prob. occurrence) are chosen as coset leader.

Decoding procedure for a linear block code

1. For the received vector r; compute the syndrome $S = rH^T$

2. within the coset characterized by the syndrome S, identify the coset leader ie error pattern with largest probability of occurrence say e_0.

NB Each coset is characterized by an unique syndrome

3. Compute the code vector

$$C = r + e_0.$$

This procedure is called <u>Syndrome Decoding</u>.

Properties of Standard array

1. Each element in the standard Array is distinct and hence different columns of array are different disjoint

2) The first n-tuple of each coset is called its coset-leader.

→ if error pattern caused by the channel coincides with a coset leader, then received vector is correctly decoded.

→ if error pattern is not a coset leader, then an incorr decoding will result. Thus coset leaders are called 'correctable error patterns'

→ when forming an array, error patterns of "smallest weight" should be chosen as coset leaders.

for eg : 7, 4 LBC the error patterns be {1000000, 0100000, 0010000 etc are taken as coset leaders}.

(14)

When this condition is satisfied, then decoding based on standard array would be

"minimum distance decoding" or

" maximum likelihood decoding.

3) All the 2^k n tuples of a coset have same syndrome and syndromes of different cosets are different.

Another way of finding the corrected code-vector.

Suppose a received vector 'r' is found in the i^{th} column; then C_i will be the corrected vector which lies on the top of that column.

Example

Construct a standard array for (6,3) linear block code, with $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

a) Let the Received vector be R=[100100]. Find the correct code.

b) R = [000011] Find the correct code .

$$C_1 = [0\ 0\ 0\ 0\ 0\ 0]$$

$$C_2 = [0\ 0\ 1\ 1\ 1\ 0]$$

$$C_3 = [0\ 1\ 0\ 0\ 1\ 1]$$

$$C_4 = [0\ 1\ 1\ 1\ 0\ 1]$$

$$C_5 = [1\ 0\ 0\ 1\ 0\ 1]$$

$$C_6 = [1\ 0\ 1\ 0\ 1\ 1]$$

$$C_7 = [1\ 1\ 0\ 1\ 1\ 0]$$

$$C_8 = [1\ 1\ 1\ 0\ 0\ 0]$$

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome S_1 S_2 S_3	Coset leader				Correct code word			
000	e_1 000000	c_2 001110	c_3 010011	c_4 011101	c_5 100101	c_6 101011	c_7 110110	c_8 111000
101	e_2 100000	$c_2 \oplus e_2$ 101110	$c_3 \oplus e_2$ 110011	111101	000101	001011	010110	011000
011	e_3 010000	$c_2 \oplus e_3$ 011110	000011	001101	110101	111011	100110	101000
110	e_4 001000	000110	011011	010101	101101	100011	111110	110000
100	e_5 000100	001010	010111	011001	100001	101111	110010	111100
010	e_6 000010	001100	010001	011111	100111	101001	110100	111010
001	e_7 000001	001111	010010	011100	100100	101010	110111	111001
111	e_8 100010	101100	110001	111111	000111	001001	010100	011010

Decoding using standard array of table 5.8 :

(i) Let the received vector R = 100100

$$\therefore \text{ Syndrome } S = R\,H^T = [1\ 0\ 0\ 1\ 0\ 0]\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [0\ 0\ 1]$$

This syndrome is present in the 7th row of the standard array for which the co-set leader is 000001. This is the correctable error pattern for R as discussed in property-2.

$$\therefore \quad E = 000001$$

$\therefore$ Corrected code vector

$$C = R + E = 100100 + 000001$$

$$\therefore \quad C = \mathbf{100101}$$

In fact, 10[illegible] is located in the 6th column and 7th row. Therefore the corrected code-vector lies on top of the [illegible]olumn, namely 100101.

(ii) Let the received vector R = 000011

$$\therefore \quad S = [0\,0\,0\,0\,1\,1]\begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\1 & 1 & 0\\1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$= [0\;1\;1]$$

This syndrome of 011 is present in the 3rd row for which the co-set leader is 010000 = E.

∴ Corrected code vector

$$C = R + E = 000011 + 010000$$

$$\therefore \quad C = \mathbf{010011}$$

This is located on top of the 4th column where the received vector is present in the 3rd row.

CONCLUSION

- Construction of standard array
- Example

CONTENTS

- Quick recap
- Standard array examples

Example 2

Q) The parity matrix of a (6, 3) linear systematic block code is given below $P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Construct the standard array.

$D_1 = [0\,0\,0] \quad C = DG.$

$$G = \left[I_k \,\vdots\, P \right]$$

$$= [0\,0\,0] \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$C_1 = [0\,0\,0\,0\,0\,0]$$

(2) $D_2 = 0\,0\,1$

$$[0\,0\,1] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$C_2 = [0\,0\,1\,1\,1\,1]$$

(3) $D_3 = 0\,1\,0$

$$[0\,1\,0] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$C_3 = [0\,1\,0\,1\,1\,0]$$

(4) $D_4 = [0\,1\,1]$ $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$$C_4 = [0\,1\,1\,1\,0\,1]$$

(5) $D_5 = [1\,0\,0]$ $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$$C_5 = [1\,0\,0\,1\,0\,1]$$

(6) $D_6 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$C_6 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

(7) $D_7 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$C_7 = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

(8) $D_8 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$C_8 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$

D	Codewords
000	000000
001	001111
010	010110
011	011101
100	100101
101	101110
110	110010
111	111000

$$H = [P^T \,\vdots\, I_{n-k}]$$

$$H = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$H^T = \left[\begin{array}{ccc} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

Syndrome	Coset leader	Codewords						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
000	000000	001111	010110	011101	100101	101110	110010	111000
101	100000	101111	110110	111101	000101	001110	010010	011000
110	010000	011111	000110	001101	110101	111110	100010	101000
011	001000	000111	011110	010101	101101	100110	111010	110000
100	000100	001111	010010	011001	100001	101010	110110	111100
010	000010	001101	010100	011111	100111	001100	110000	111010
001	000001	001111	010111	011100	100100	101111	110011	111001
111	100010	101101	110100	111111	000111	001100	010000	011010

CONCLUSION

- Practise questions

THANK YOU

CONTENTS

Problem1

Example 5.15 : The parity check bits of a (8, 4) block code are generated by

$$c_5 = d_1 + d_2 + d_4$$
$$c_6 = d_1 + d_2 + d_3$$
$$c_7 = d_1 + d_3 + d_4$$
$$c_8 = d_2 + d_3 + d_4$$

where d_1, d_2, d_3 and d_4 are the message bits

(a) Find the generator matrix and parity check matrix for this code.

(b) Find the minimum weight of this code.

(c) Show that it is capable of correcting all single error patterns and capable of detecting all double errors by preparing the syndrome table for them.

The general code vector C can be written in the form

$$\{C\} = \{d_1, d_2, d_3, d_4, (d_1 + d_2 + d_4), (d_1 + d_2 + d_3), (d_1 + d_3 + d_4), (d_2 + d_3 + d_4)\}$$

$$= [d_1 \quad d_2 \quad d_3 \quad d_4] \left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$= [D]\,[G]$$

(a) The generator matrix is given by

$$[G] = \left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right] = [I_4 \,\vdots\, P]$$

- The parity check matrix H will be of the form

$$[H] = \left[P^T \vdots I_{n-k}\right] = \left[P^T \vdots I_4\right]$$

$$\therefore \quad [H] = \left[\begin{array}{cccc:cccc} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

(b) By inspecting the H-matrix carefully, we observe that no two columns of H will add up to zero. Similarly no three columns of H will add-up to zero also. But when we add 1st, 2nd, 3rd and 6th column elements, we get zero

$$\begin{bmatrix}1\\1\\1\\0\end{bmatrix}+\begin{bmatrix}1\\1\\0\\1\end{bmatrix}+\begin{bmatrix}0\\1\\1\\1\end{bmatrix}+\begin{bmatrix}0\\1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

Hamming code is a type of Block code(7,4)

Therefore, we can conclude that $d_{min} = 4$. Hence the minimum weight $= d_{min} = 4$ from the property of linear block code.

This can also be verified by finding all the 16 valid code-vectors and then finding the Hamming weight as shown in table 5.10

Message Vector	Code-Vector	Hamming Weight	Message Vector	Code-Vector	Hamming Weight
0000	00000000	0	1000	10001110	4
0001	00011011	4	1001	10010101	4
0010	00100111	4	1010	10101001	4
0011	00111100	4	1011	10110010	4
0100	01001101	4	1100	11000011	4
0101	01010110	4	1101	11011001	5
0110	01101010	4	1110	11100101	5
0111	01110001	4	1111	11111111	8

Table 5.10 : Code-vector table for example 5.15

(c) Let us construct the syndrome table [Table 5-11] showing the single error patterns and the corresponding syndrome. For the sake of simplicity, let us consider the all-zero code-vector. Note that syndrome has $n - k = 8 - 4 = 4$ bits.

Single-error pattern	Syndrome
10000000	1110
01000000	1101
00100000	0111
00010000	1011
00001000	1000
00000100	0100
00000010	0010
00000001	0001

Table 5.11 : Syndrome - table for single error pattern.

Syndromes are all unique which means it can detect and correct single errors

The syndrome is actually calculated using $S = RH^T$

For example, $S = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [1\ 1\ 1\ 0]$

For double-error patterns, we can construct the table 5.12 given below (along with syndrome

Double-error Pattern	Syndrome	Double-error Pattern	Syndrome
11000000	0011	00101000	1111
10100000	1001	00100100	0011
10010000	0101	00100010	0101
10001000	0110	00100001	0110
10000100	1010	00011000	0011
10000010	1100	00010100	1111
10000001	1111	00010010	1001
01100000	1010	00010001	1010
01010000	0110	00001100	1100
01001000	0101	00001010	1010
01000100	1001	00001001	1001
01000010	1111	00000110	0110
01000001	1100	00000101	0101
00110000	1100	00000011	0011

Table 5.12 : Syndrome table for double-error pattern

Inference

1. The eight syndromes listed for eight single error patterns are all distinct. These syndromes are unique to single - error patterns. Hence the (8, 4) code can detect and correct single - errors.

2. The syndromes for double-error patterns are distinctively different from those for single - error patterns. However, there is no uniqueness to the syndromes for double -error patterns. For example, the double error patterns (11000000) and (00011000) have the same syndrome 0011. Hence the (8, 4) code can detect double errors but cannot correct them.

Problem2

Example 5.16 : Design a (4, 2) linear block code :

(i) Find the generator matrix for the code vector set.

(ii) Find the parity check matrix.

(iii) Choose the code-vectors to be in systematic form, with the goal of maximizing d_{min}.

(iv) Enter the sixteen 4 - tuples into a standard array.

(v) What are the error - detecting and error correcting capabilities of the code?

(vi) Make a syndrome table for the correctable error patterns.

(vii) Draw the encoding circuit.

(viii) Draw the syndrome calculating circuit.

For (4, 2) linear block code, we have $n = 4$ and $k = 2$.

Therefore the parity matrix P is of the order of $k \times (n - k) = 2 \times 2$ given by

$$[P] = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

As mentioned in example 5.10, the unit matrix rows '01' and '10' cannot be used as the rows of P and also a row of '00' which represents syndrome of no error, cannot be used. This leaves use with only choice of '11' to be used. But the parity matrix must have two distinct rows of 2 binary digits each. Hence we have no other choice than to accept either '10' or '01' as its second row. Let us choose '10' as the 2nd row of P.

(i) The generator matrix [G] is given by

$$[G] = [I_k \;\vdots\; P] = [I_2 \;\vdots\; P] = \left[\begin{array}{cc:cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

(ii) The parity check matrix [H] is given by

$$[H] = [P^T \;\vdots\; I_{n-k}] = [P^T \;\vdots\; I_2] = \left[\begin{array}{cc:cc} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$$

(iii) The systematic code-vectors are found from

$$[C] = [D]\,[G]$$

Since k = 2, we have four message-vectors given by [0 0], [0 1], [1 0] and [1 1]

$$\therefore \quad C_A = [D_A]\,[G] = [0\ 0]\begin{bmatrix}1 & 0 & \vdots & 1 & 1\\ 0 & 1 & \vdots & 1 & 0\end{bmatrix} = [0\ 0\ 0\ 0]$$

$$C_B = [D_B]\,[G] = [0\ 1]\begin{bmatrix}1 & 0 & \vdots & 1 & 1\\ 0 & 1 & \vdots & 1 & 0\end{bmatrix} = [0\ 1\ 1\ 0]$$

$$C_C = [D_C]\,[G] = [1\ 0]\begin{bmatrix}1 & 0 & \vdots & 1 & 1\\ 0 & 1 & \vdots & 1 & 0\end{bmatrix} = [1\ 0\ 1\ 1]$$

$$C_D = [D_D]\,[G] = [1\ 1]\begin{bmatrix}1 & 0 & \vdots & 1 & 1\\ 0 & 1 & \vdots & 1 & 0\end{bmatrix} = [1\ 1\ 0\ 1]$$

(iv) The standard array is constructed as shown in table 5.13 following the four steps given in section 5.11.

	Syndrome	Co-set leader			
	00	0000	0110	1011	1101
Rows of H^T	11	1000	1110	0011	0101
	10	0100	0010	1111	1001
	10	0010	0100	1001	1111
	01	0001	0111	1010	1100

Table 5.13 : Standard array for (4, 2) linear block code

From table 5.13, we can observe that there are two rows having the same syndrome '10' and therefore an error can never be corrected. This is shown clearly in part (v) below.

(v) The hamming weights can be calculated for each of the code-vectors as shown in table (5.14)

Message - vector	Code - vector	Hamming weight $[H_w]$
00	0000	0
01	0110	2
10	1011	3
11	1101	3

Table 5.14 : Code-vector table for (4, 2) linear block code

From table 5.14, we observe that the minimum Hamming weight of a non-zero code-vector is 2.

$\therefore$ For linear block code, $d_{min} = \min [H_w] = 2$

$\therefore$ Error detecting capability $= d_{min} - 1 = 2 - 1 = 1$

$\therefore$ Error correcting capability $t = \dfrac{d_{min} - 1}{2} = \dfrac{2-1}{2} = \dfrac{1}{2}$

Since 't' has to be integer $\leq \frac{1}{2}$, we find that $t = 0$. Therefore a (4, 2) linear block code is incapable of correcting any error.

For example : Let the received code-vector which has a single-error in it, be given by

$$R = [1\ 1\ 1\ 1]$$

Then the syndrome for this received vector is

$$S = RH^T$$

$$= [1\ 1\ 1\ 1]\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = [1\ 0]$$

Therefore $S = [1\ 0]$ which is present in the 2^{nd} as well as 3^{rd} row of H^T. Thus, the error vector may be respectively [0100] or [0010]. Since, we have the ambiguity, we can conclude that the given (4, 2) linear block code cannot correct any errors. However, it can detect a single error. This can be confirmed by looking at the syndrome for any received vector. If the syndrome is a non-zero vector, then an error is detected.

(vi) The syndrome table for the correctable error patterns, is shown in table 5.15, leaving out the two rows corresponding to syndrome '10' in table 5.13

Syndrome	Co-set leader			
00	0000	0110	1011	1101
11	1000	1110	0011	0101
01	0001	0111	1010	1100

Table 5.15 : Syndrome table of correctable error patterns

(vii) **Encoding circuit :** Figure 5.11 shows the encoding circuit for (4, 2) linear block code.

$$[C] = [d_1, d_2]\,[G]$$

$$= [d_1, d_2]\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$= [d_1, d_2, (d_1 + d_2), d_1]$$

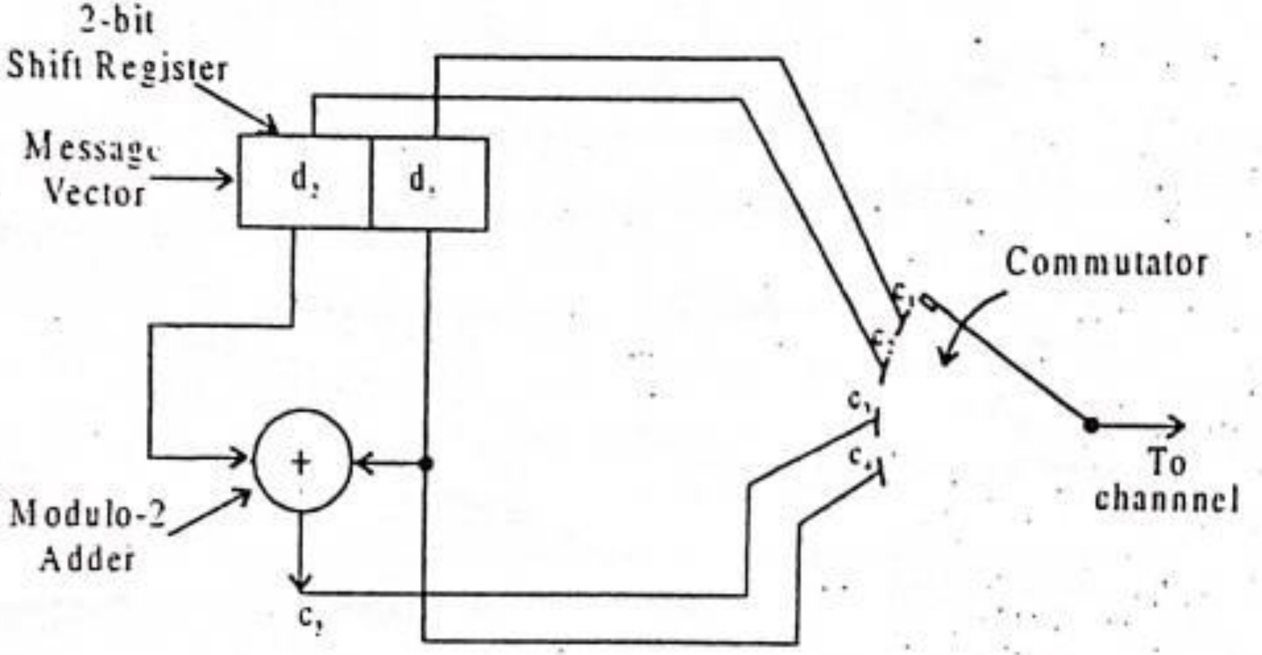


Fig. 5.11 : Encoding circuit for (4, 2) linear block code

viii) Syndrome Calculating Circuit:

The syndrome -vector S is given by

$$S = (s_1\ s_2) = RH^T$$

$$= [r_1\ r_2\ r_3\ r_4]\begin{bmatrix}1 & 1\\ 1 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}$$

$$= (r_1 + r_2 + r_3,\ r_1 + r_4)$$

The syndrome calculation circuit is drawn as shown in figure 5.12.

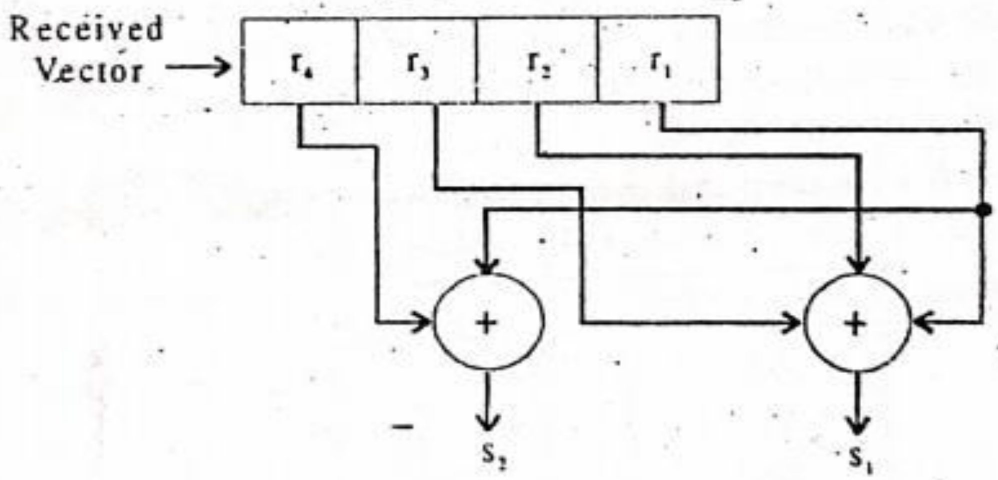


Fig. 5.12 : Syndrome calculation circuit for (4, 2) linear block code

CONCLUSION

- Practise Problems on Linear Block codes

THANK YOU