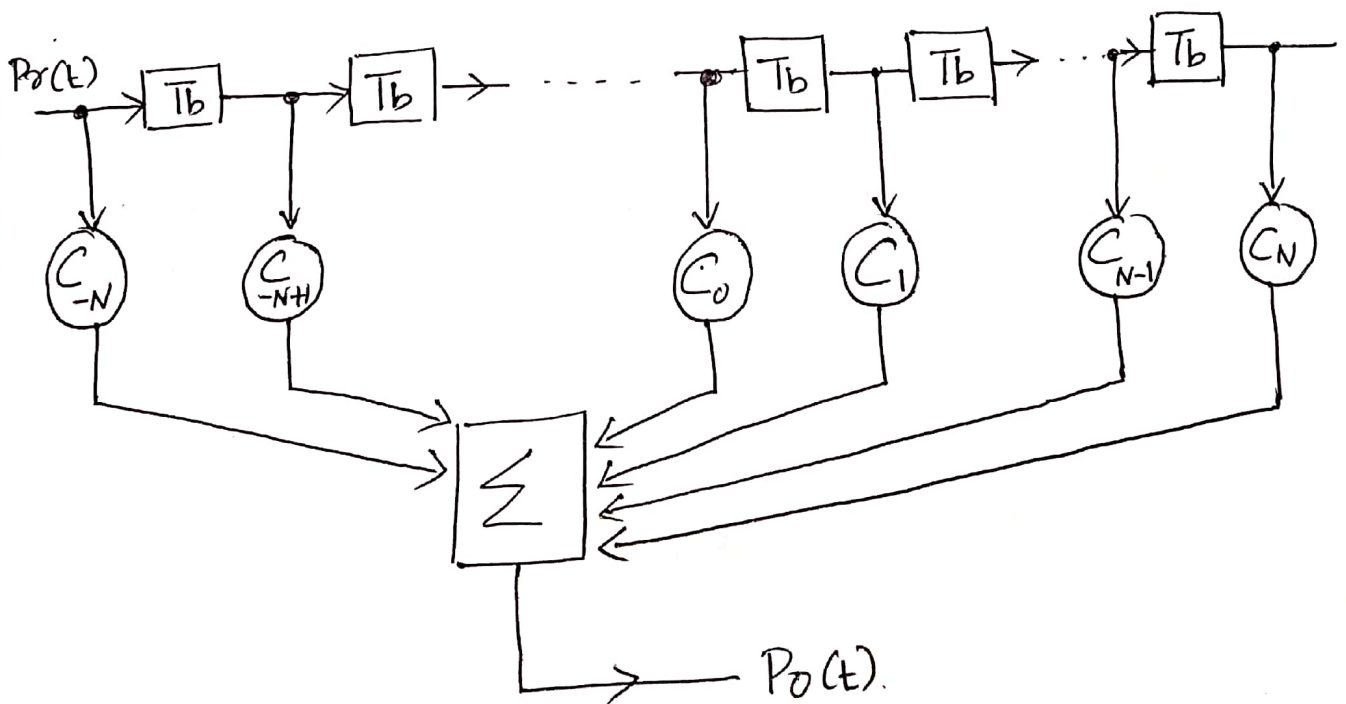


# Zero forcing Equalization.

- When we transmit Nyquist criterion pulse for zero ISI through communication channel, then because of channel non-linearities, distortions in the pulse occurs.
- In other words, pulse which is present at the receiver input is a distorted pulse with non zero values at sampling instants.
- In order to compensate these distortions at the receiver, we use  $E_e$  to get back the Nyquist criterion pulse, we use equalizer.
- One such equalizer is a Zero-forcing Equalizer as shown as,



Fig(1)  $\Rightarrow$  Zero forcing Equalizer

→ where  $P_i(t)$  = input pulse

$P_o(t)$  = output pulse

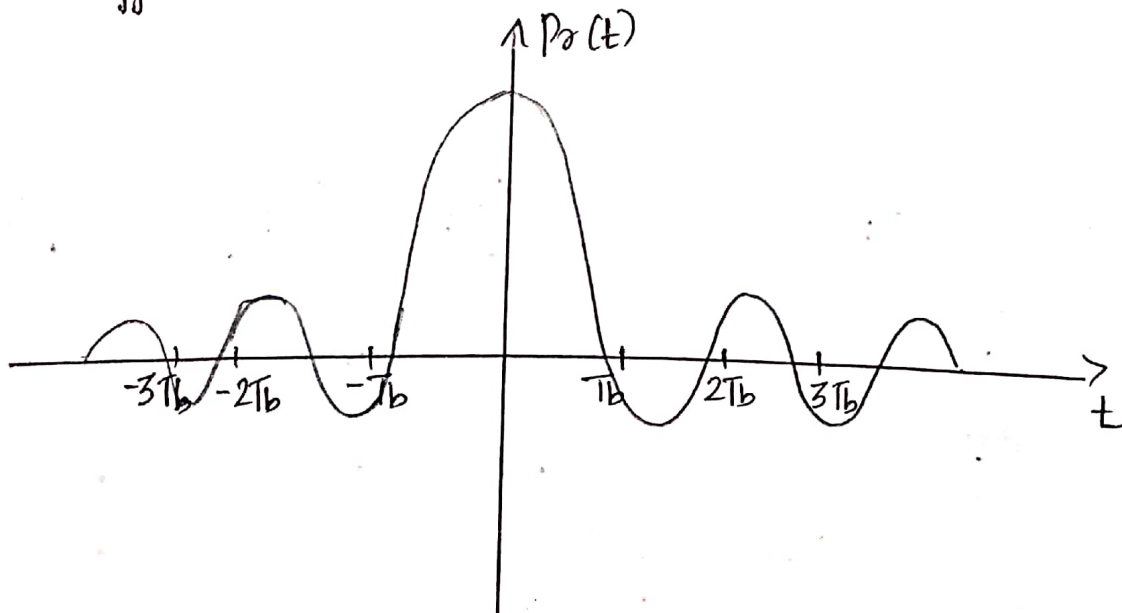
$T_b$  = Delay Elements

$C_{-N}, C_{-N+1}, \dots, C_N$  = Coefficients

→ For Zero ISI pulse, we need to select these coefficients in such a way that, the output pulse must satisfy Nyquist criteria.

→ The structure shown above is <sup>like</sup> a tap-delay line filter or FIR filter. It has  $-N$  to  $+N$  coefficients or taps. i.e.  $2N+1$  taps  $\Rightarrow 2N+1$  tap equalizers.

→ Here the number of delay elements are  $2N$  while the coefficients are  $2N+1$ .



Fig(2)  $\Rightarrow$  Received Pulse

→ Consider a pulse  $P_a(t)$  shown above, we can infer that <sup>at</sup> the sampling instant  $T_b$  is not zero, at  $2T_b$  is not zero, at  $3T_b$  it is not zero & so on... It is also on negative side also.

→ This implies that,  $P_a(t)$  is not a Nyquist criterion pulse at the receiver input.

→ So we need to design the equalizer in such a way that, at these sampling instants, the values must be zero. So the effect of ISI can be removed.

→ From the figure (1), the o/p pulse  $P_o(t)$  can be written as,

$$P_o(t) = \sum_{n=-N}^{+N} C_n P_a(t - nT_b) \quad \text{--- (1)}$$

→ At sampling instant,  $t = kT_b$ , where  $k$  is an integer, where  $k = 0, \pm 1, \pm 2, \dots$

$$P_o(kT_b) = \sum_{n=-N}^{+N} C_n P_a(kT_b - nT_b) \quad \text{--- (2)}$$

→ If  $kT_b = k$  &  $nT_b = n$ , we can rewrite eqn (2) as,

$$P_o(k) = \sum_{n=-N}^{+N} C_n P_a(k - n) \quad ; \quad k = 0, \pm 1, \pm 2, \dots \quad \text{--- (3)}$$

where  $n =$  lap values  $[-N \text{ to } N]$

$k =$  Sampling instants

→ The aim is to choose the value of coefficient  $c_n$  in such a way that the d/p pulse  $p_0(k)$  satisfy the Nyquist criteria.

→ The Nyquist criteria is defined by,

$$\left. \begin{aligned} p_0(k) &= 1 & ; k=0 \\ &= 0 & ; k \neq 0 \end{aligned} \right\} \text{--- (4)}$$

→ Expanding the eqn. (3) for different values of  $k$  from  $-N$  to  $+N$ ,

$$p_0(-N) = c_{-N} p_x(0) + c_{-N+1} p_x(-1) + \dots + c_N p_x(-2N)$$

$$p_0(-N+1) = c_{-N} p_x(1) + c_{-N+1} p_x(0) + \dots + c_N p_x(-2N+1)$$

(5)

→ The above eqns. (5), can be written in matrix form as,

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} p_x(0) & p_x(-1) & \dots & p_x(-2N) \\ p_x(1) & p_x(0) & \dots & p_x(-2N+1) \\ \vdots & \vdots & \ddots & \vdots \\ p_x(2N) & \dots & \dots & p_x(0) \end{bmatrix} \begin{bmatrix} c_{-N} \\ c_{-N+1} \\ \vdots \\ c_0 \\ \vdots \\ c_N \end{bmatrix}$$

(6)



→ Eqn (6) shows  $P_r$  matrices, coefficients matrices &  $P_o$  matrices.

→ The condition  $P_o$  for the given values of  $P_r$ , we can easily find out the values of coefficients from  $C_N$  to  $C_1$ .  
& then condition of Nyquist criterion will satisfied.

### Example

a) Design a three tap zero forcing equalizer if following is given :

$$P_r[0] = 1$$

$$P_r[2] = 0.1$$

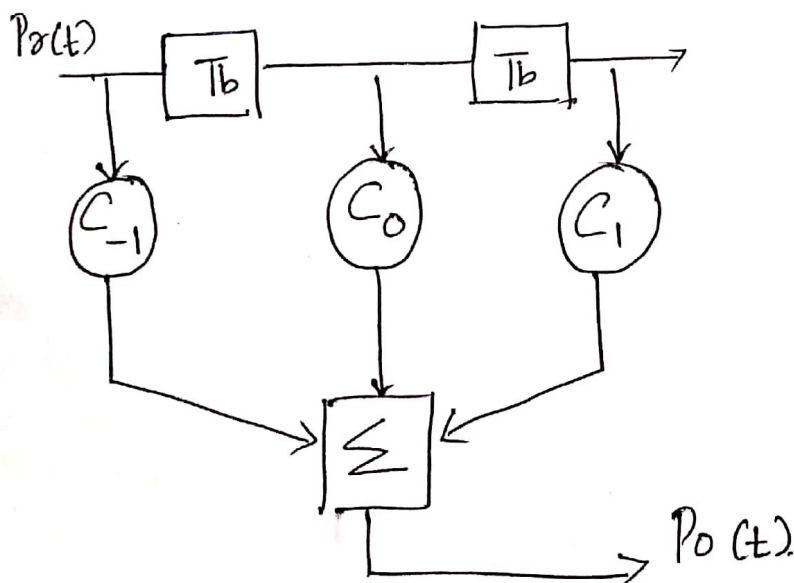
$$P_r[-1] = -0.2$$

$$P_r[-2] = 0.05$$

$$P_r[1] = -0.3$$

∴ Here  $2N+1 = 3 \Rightarrow N = 1 \Rightarrow 2N = 2$  Delay Elements.

First step: Structure of a 3-Tap Equalizer



Step 2 : Put the values or parameters into the matrices of the form,

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} P_x(0) & P_x(-1) & P_x(-2) \\ P_x(1) & P_x(0) & P_x(-1) \\ P_x(2) & P_x(1) & P_x(0) \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix}$$

Putting the values,

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -0.2 & 0.05 \\ -0.3 & 1 & -0.2 \\ 0.1 & -0.3 & 1 \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix}$$

⇒ Solving these equations:

$$0 = 1 \cdot C_{-1} + -0.2C_0 + 0.05C_1$$

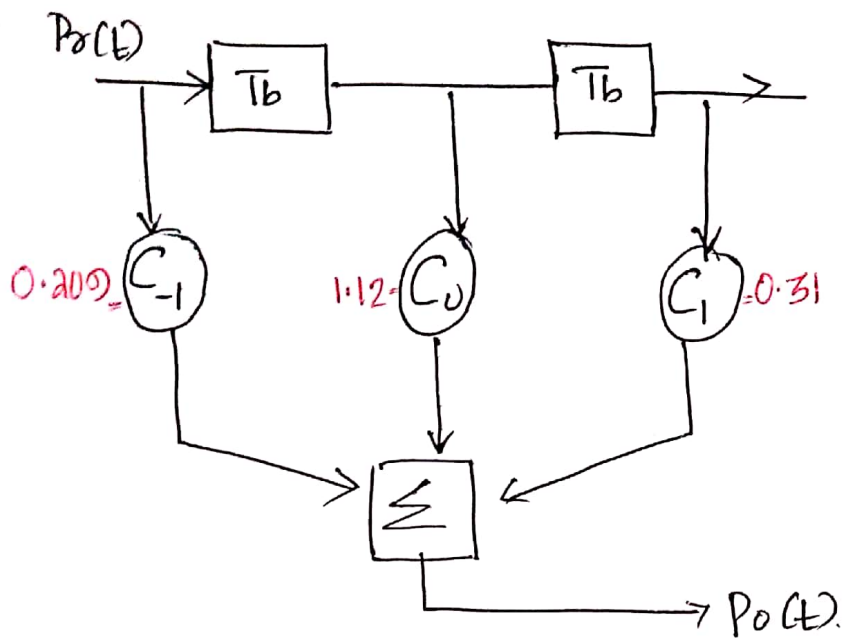
$$1 = -0.3C_{-1} + 1C_0 + -0.2C_1$$

$$0 = 0.1C_{-1} + -0.3C_0 + 1C_1$$

$$\Rightarrow C_{-1} = 0.209$$

$$C_0 = 1.12$$

$$C_1 = 0.31$$



→ The name Zero forcing corresponds to bringing down the intersymbol interference (ISI) to zero.

→ Disadvantages of ZF Equalizers

- \* The zero-forcing equalizer removes all ISI, & is ideal when the channel is noiseless.
- \* When the channel is noisy, the zero-forcing equalizer will amplify the noise greatly at frequencies  $f$  where the channel response  $H(j2\pi f)$  has a small magnitude in the attempt to invert the channel completely.