Discrete Random Variable

- Probability mass function (Pmf)

x:	0	1	2	...	n
$p(x)$	P_1	P_2	P_3	.....	P_n

* $\sum p(x) = 1$

* Mean (μ) Expectation of x.

$$E[x] = \sum x\, p(x)$$

* Varience (σ^2) Variance of x

$$Var(x) = E[x^2] - E[x]^2$$

$$\boxed{\begin{aligned} E[x^2] &= \sum x^2 p(x) \\ E[x^3] &= \sum x^3 p(x) \end{aligned}}$$

* Standard deviation (σ)

$$S.D = \sqrt{Var(x)}$$

* Distribution function [Cummulative distr: fun].

$$f(x) = P[x \le a]$$

* Properties.

$$E[ax] = aE[x]$$

$$E[ax+b] = aE[x] + b$$

$E[ax] + E[b]$
$aE[x] + b$

$$E[\text{constant}] = \text{Constant}.$$

$$Var(\text{constant}) = 0, \quad Var(ax) = a^2 Var(x)$$

$$V(ax+b) = a^2 Var(x) + 0$$

① If x is a discrete random Variable with Pmf $f(x) = \frac{x}{10}$, $x = 1, 2, 3, 4$.

find:

i) $P(x \le 2)$ ii) $P\left(\frac{1}{2} < x < 5/2 \,/\, x > 1\right)$ iii) the cumulative distribution function (CDF) of x.

iv) The Smallest value of λ for which $P(x \le \lambda) > \frac{1}{2}$

Conditional Prob: form:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Pmf:

x	1	2	3	4
$f(x) = \frac{x}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

$$P(x \le 2) = P(x = 1) + P(x = 2)$$

$$= \frac{1}{10} + \frac{2}{10}$$

$$= \frac{3}{10}$$

$$P\left[\frac{1}{2} < x < 5/2 \,/\, x > 1\right]$$

$$= \frac{P\left[\frac{1}{2} < x < \frac{5}{2}\right] \cap P[x > 1]}{P[x > 1]}$$

$$= \frac{P\left(1 < x < \frac{5}{2}\right)}{P[x > 1]}$$

$$= \frac{P[x = 2]}{P[x = 2] + P(x = 3) + P(x = 4)}$$

$$= \frac{P[x = 2]}{1 - P[x \le 1]} = \frac{\frac{2}{10}}{1 - \frac{1}{10}}$$

$$= \frac{\frac{2}{10}}{\frac{9}{10}} = \frac{2}{9}$$

iii) cdf

x	1	2	3	4
$P(x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$
$f(x) = P[x \le a]$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10}$	1

$$f(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{10} & 1 \le x < 2 \\ \frac{3}{10} & 2 \le x < 3 \\ \frac{6}{10} & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

$P[x \le \lambda] > \frac{1}{2}$

$\Rightarrow \lambda = 3$

$f(x) = P[x \le a]$

$\frac{1}{10} < \frac{1}{2}$

$\frac{3}{10} < \frac{1}{2}$

$\frac{6}{10} > \frac{1}{2}$

$P[x \le 3] = \frac{6}{10}$

$P[x \le \lambda] > \frac{1}{2}$

$\Rightarrow P[x \le \lambda] = f(\lambda) > \frac{1}{2}$

when $\lambda = 3$ is smallest value of λ with $P(x \le \lambda) > \frac{1}{2}$.

$P(x \le \lambda) > \frac{1}{2}$

$\Rightarrow \lambda = 3$

$f(x) = P[x \le a]$

$P[x \le 1] = \frac{1}{10} < \frac{1}{2}$

$P(x \le 2] = \frac{3}{10} < \frac{1}{2}$

$P[x \le 3] = \frac{6}{10} > \frac{1}{2}$

$P[x \le 4] = 1 > \frac{1}{2}$

... has the following probability distributi...

x	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{10}$	$15k^2$	$\frac{1}{5}$	$2k$	$\frac{3}{10}$	$3k$

$0 \le p(x) \le 1$

Compute (i) k (ii) $p(x<2)$ (iii) $p(-2<x<2)$ (iv) $p(x\le 2 \mid x>0)$ (v) mean and variance of x.

1) $\sum f(x) = 1$

$$\frac{1}{10} + 15k^2 + \frac{1}{5} + 2k + \frac{3}{10} + 3k = 1$$

$$\frac{4}{10}\,\frac{2}{5} + \frac{1}{5} + 15k^2 + 5k = 1$$

$$15k^2 + 5k + \frac{3}{5} = 1$$

$$15k^2 + 5k + \frac{3}{5} - 1 = 0$$

$$15k^2 + 5k - \frac{2}{5} = 0$$

$$75k^2 + 25k - 2 = 0$$

$$k = \frac{1}{15},\ -\frac{2}{5}$$

$\frac{1}{15}$ $\quad \frac{2}{15}$

Prob

x	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{3}{15}$

$$P(x<2) = p(x=-2) + p(x=-1) + p(x=0) + p(x=1)$$

$$= \frac{1}{10} + \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{15}{30} = \frac{1}{2}$$

OR

$$P(x<2) = 1 - p[x \ge 2]$$

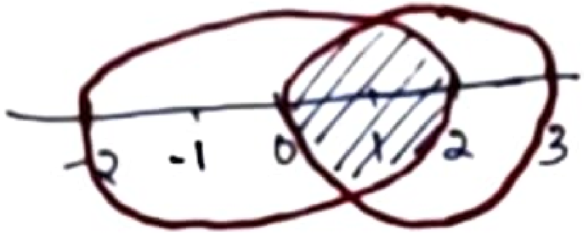
$$= 1 - [p(x=2) + p(x=3)]$$

$$= 1 - \left[\frac{3}{10} + \frac{3}{15}\right] = \frac{1}{2}$$

$P(-2<x$

$$= \frac{1}{15} + \frac{1}{5} + \frac{2}{15}$$

$$= \underline{\frac{2}{5}}$$



iv) $P(x \le 2 \mid x > 0)$ $\qquad P[A|B] = \dfrac{P[A \cap B]}{P[B]}$

$$= \frac{P[x \le 2] \cap P[x > 0]}{P[x > 0]}$$

$$= \frac{P[0 < x \le 2]}{P[x > 0]}$$

$$= \frac{P[x=1] + P[x=2]}{P[x=1] + P[x=2] + P[x=3]} = \frac{\frac{2}{15} + \frac{3}{10}}{\frac{2}{15} + \frac{3}{10} + \frac{3}{15}}$$

$$= \frac{\frac{13}{30}}{\frac{19}{30}} = \underline{\frac{13}{19}}$$

...llowing probability distrib...

3

3k

$0 \le p(A) \le 1$

... $x<2)$ (iv) $p(x \le 2 \mid x>0)$ (v) mean and varience of x.

Prof.

x	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{3}{15}$

$$p(x<2) = p(x=-2)+p(x=-1)+p(x=0)+p(x=1)$$

$$= \frac{1}{10}+\frac{1}{15}+\frac{1}{5}+\frac{2}{15} = \frac{15}{30} = \frac{1}{2}$$

OR.

$$p(x<2) = 1-p[x\ge 2]$$

$$= 1-[p(x=2)+p(x=3)]$$

$$= 1-\left[\frac{3}{10}+\frac{3}{15}\right] = \frac{1}{2}$$

$=0$

0

0

$-\frac{2}{5}$

$= \frac{13}{19}$

Mean

$$E(x) = \sum x f(x)$$

$$= -2\times\frac{1}{10}+(-1)\times\frac{1}{15}+0+1\times\frac{2}{15}+2\times\frac{3}{10}+3\times\frac{3}{15}$$

$$= \frac{16}{15}$$

$$Var(x) = E[x^2]-E[x]^2$$

$$E[x^2] = \sum x^2 f(x)$$

$$= 4\times\frac{1}{10}+1\times\frac{1}{15}+0+1\times\frac{2}{15}+4\times\frac{3}{10}+9\times\frac{3}{15}$$

$$= \frac{18}{5}$$

$$\therefore Var(x) = \frac{18}{5}-\left(\frac{16}{15}\right)^2 = \frac{554}{225}$$

$$S.D = \sqrt{Var} = \sqrt{\frac{554}{225}}$$

The possible values of a random variable x are 0, 1, 2 and 3. if $p(x=2)=2p(x=0)=4p(x=3)$ and $4p(x=1)=3p(x=2)$. find Pmf and cdf of x.

Also calculate $p(x\neq 0)$

$$p(x=2)=2p(x=0)=4p(x=3)=a$$

$p(x=2)=a$ $\quad$ $2p(x=0)=a$ $\quad$ $4p(x=3)=a$

$p(x=0)=\frac{a}{2}$ $\quad$ $p(x=3)=\frac{a}{4}$

$$4p(x=1)=3p(x=2)=b$$

$4p(x=1)=b$ $\quad$ $3p(x=2)=b$

$p(x=1)=\frac{b}{4}$ $\quad$ $p(x=2)=\frac{b}{3}$

$$a=\frac{b}{3}$$

$$\boxed{b=3a}$$

x:	0	1	2	3
$p(x)$	$\frac{a}{2}$	$\frac{b}{4}$	a	$\frac{a}{4}$

$\therefore \sum p(x)=1$

$$\frac{a}{2}+\frac{b}{4}+a+\frac{a}{4}=1$$

$$2a+b+4a+a=4$$

$$7a+b=4 \quad [\text{Since } b=3a]$$

$$\frac{b}{4}=\frac{3a}{4}$$

$$7a+3a=4$$

$$10a=4$$

$$a=0.4$$

Pmf

x	0	1	2	3
$p(x)$	0.2	0.3	0.4	0.1

x	0	1	2	3
$p(x)$	0.2	0.3	0.4	0.1
$F(x)$ $=p(x \leq 0$	0.2	0.5	0.9	1

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \leq x < 1 \\ 0.5 & 1 \leq x < 2 \\ 0.9 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

x	0	1	2	3
$p(x)$	0.2	0.3	0.4	0.1
$F(x)$ $=p(X\le 0)$	0.2	0.5	0.9	1

$p(x \neq 0) = 1 - p(x=0)$

$= 1 - 0.2$

$= 0.8$

$E[ax+b] =$

The possible values of a random Variable x are 0,1,2 and 3. if $p(x=2) = 2p(x=0) = 4p(x=3)$ and $4p(x=1) = 3p(x=2)$. find Pmf and cdf of x.

Also calculate $p(x \neq 0)$

$p(x=2) = 2p($ [illegible] $3) = a.$

$p(x=2) = a$

x:	0	1	2	3
$p(x)$	$\frac{a}{2}$	$\frac{b}{4}$	a.	$\frac{a}{4}$

$\therefore \sum p(x) = 1$

$\frac{a}{2} + \frac{b}{4} + a + \frac{a}{4} = 1$

The Cal Poly Department of statistics has a lab with six Computers reserved for statistics majors. Let x denote the number of these Computers that are in use at a particular time of day. Suppose that probability distribution of x is as given in the following table: The first row of table lists the possible x values and the Second row gives probability of each such value:

x	0	1	2	3	4	5	6
$p(x)$	0.05	0.10	0.15	0.25	0.20	0.15	0.10

i) find probability that atmost 2 Computers are in use.

ii) Probability that atleast 3 Computers are in use.

iii) Probability that between 2 and 5 Computers inclusive are in use.

iv) Probability that number of Computers in use is strictly between 2 and 5.

v) find c.d.f

i) P(almost 2 computers are in use) $= P(x \le 2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= 0.05 + 0.10 + 0.15$

$= \underline{0.30}$

almost $\le$

atleast

ii) P(atleast 3 Computers are in use) $= P(x \ge 3)$

$= P(x=3) + P(x=4) + P(x=5) + P(x=6)$

$= 0.25 + 0.20 + 0.15 + 0.10$

$= \underline{0.70}$

OR $P(x \ge 3) = 1 - P(x < 3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - 0.30$

$= \underline{0.70}$

iii) P(between 2 and 5 Computers <u>inclusive</u> are in use) $= P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= 0.15 + 0.25 + 0.20 + 0.15$

$= \underline{0.75}$

iv) P (strictly between 2 and 5) $= P(x=3) + P(x=4)$

$= 0.25 + 0.20 = \underline{0.45}$

x	0	1	2	3	4	5	6
$p(x)$	0.05	0.10	0.15	0.25	0.20	0.15	0.10
$F(x) = P[X \le x]$	0.05	0.15	0.30	0.55	0.75	0.9	1

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.05 & 0 \le x < 1 \\ 0.15 & 1 \le x < 2 \\ 0.30 & 2 \le x < 3 \\ 0.55 & 3 \le x < 4 \\ 0.75 & 4 \le x < 5 \\ 0.9 & 5 \le x < 6 \\ 1 & x > 6 \end{cases}$$

2) Six lots of Components are ready to be shipped by a Certain Supplier. The number of defective Components in each lot is as follows.

Lot	1	2	3	4	5	6
No: of defectives	0	2	0	1	2	0

RVS

$\frac{\text{No. of fa}}{\text{Total no. of}}$

$\frac{3}{6}$

one of these lots is to be randomly Selected for shipment to a particular customer. find the probability mass function, Also find $p(x \le 0)$, $p(x \le 1)$, $p(x \le 1.5)$

Let x be no: of defectives in the selected lot.

Possible Values of x are $0, 1, 2$.

$P(x=0) = \frac{3}{6}$

$P(x=1) = \frac{1}{6}$

$P(x=2) = \frac{2}{6}$

Pmf

x:	0	1	2
$p(x)$	$\frac{3}{6}$	$\frac{1}{6}$	$\frac{2}{6}$

1.5

$P(x \le 0) = p(x=0) = \frac{3}{6}$

$p(x \le 1) = p(x=0) + p(x=1) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$p(x \le 1.5) = p(x \le 0) + p(x=1) = \frac{2}{3}$

3) Consider a group of five potential blood donors - 'a, b, c, d and e' of whom only a and b have type O^{+ve} blood. five blood Samples, one from each individual, will be typed in random order until an O^{+ve} individual is identified. if Y be the number of typings necessary to identify an O^{+ve} individual. find pmf of Y. Also draw the line graphs and histogram graph of pmf.

2) Six lots of Components are ready to be shipped by a Certain Supplier. The number of defective Components in each lot is as follows.

Lot	1	2	3	4	5	6
No: of defectives	0	2	0	1	2	0

$\frac{\text{fav. no. of a}}{\text{Total no. of}}$

$\frac{3}{6}$

... to be randomly Selected for shipment to a particular

3) Consider a group of five potential blood donors - 'a, b, c, d and e' of whom only a and b have type O^{+ve} blood. Five blood samples, one from each individual, will be typed in random order until an O^{+ve} individual is identified. If Y be the number of typings necessary to identify an O^{+ve} individual. Find pmf of Y. Also draw the line graphs and histogram graph of pmf.

$$\underbrace{a, b}_{O^{+ve}}, \underbrace{c, d, e}_{\text{diff.}}$$

$P(Y=1) =$ (first selection is O^{+ve})

$= P(a \text{ or } b) = \frac{2}{5} = 0.4$

$P(Y=2) =$ (Second Selection is O^{+ve})

$= P(c,d,e) \times P(a,b) = \frac{3}{5} \times \frac{2}{4} = 0.3$

$P(Y=3) =$ (3^{rd} Selection is O^{+ve})

$= P(c,d,e) \times P(c,d,e) \times P(a,b) = \frac{3}{5} \frac{2}{4} \frac{2}{3} = 0.2$

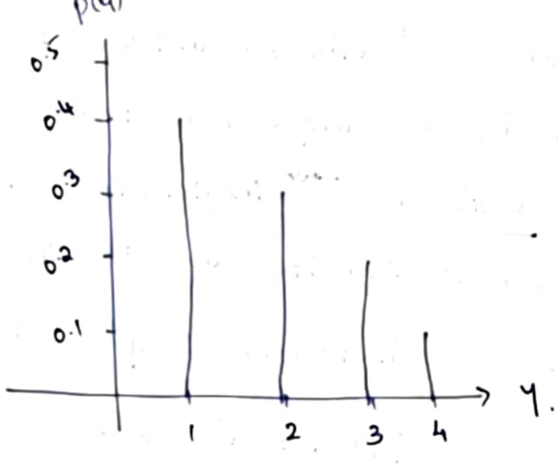
$P(Y=4) =$ (4^{th} Selection is O^{+ve})

$= P(c,d,e) \times P(c,d,e) \times P(c,d,e) \times P(a,b)$

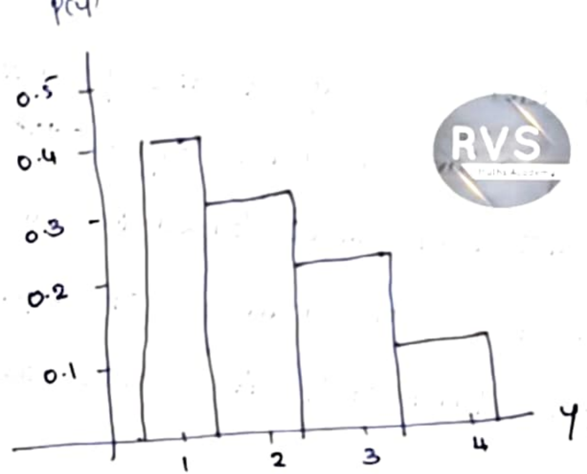
$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times 1 = 0.1$

Pmf

Y	1	2	3	4
$P(y)$	0.4	0.3	0.2	0.1



Line graph.



Histogram graph.

4) Starting at a fixed time, we observe the gender of each newborn child at a certain hospital until a boy (B) is born. Let $P(B) = p$, assume that successive births are independent and define the random variable x by number of births observed.

Find the P.m.f.

$P(B) = p$

$P(G) = 1 - p(B)$

$= 1 - p$

$P(x=1) = (\text{1}^{st} \text{ birth boy})$

$= P(B) = p$

$P(x=2) = (2^{nd} \text{ birth boy})$

$= P(G)\,p(B) = (1-p)^1 p$

$P(A) = 1 - P(\bar{A})$

$P(x=3) = (3^{rd} \text{ birth boy})$

$= P(G)\,p(G)\,p(B)$

$= (1-p)^2 p$

$p(x=4) = (4^{th} \text{ birth boy})$

$= P(G)\,P(G)\,P(G)\,P(B)$

$= (1-p)^3 p$

⋮

x	1	2	3	4	5	6	...	x
$p(x)$	p	$(1-p)p$	$(1-p)^2 p$	$(1-p)^3 p$	$(1-p)^4 p$	$(1-p)^5 p$		$(1-p)^{x-1} p$

P.m.f.

$$P(x) = \begin{cases} (1-p)^{x-1} p & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

Cumulative distribution function.

if $f(x)$ is (cdf) is given. with out finding pmf. we can find. probabilities.

$$\boxed{\begin{aligned} P(a \le x \le b) &= f(b) - f(a-1) \\ P(x=a) &= f(a) - f(a-1) \end{aligned}}$$

$P(2 \le x \le 5) = f(5) - f(1)$

$P(x=4) = f(4) - f(3)$

(1) Let x be the no: of days sick leave taken by a randomly selected employee of a large company during a particular year. if the maximum number of allowable sick days per year is 14, Possible values of x are $0, 1, 2, \ldots, 14$. with $f(0) = 0.58$, $f(1) = 0.72$, $f(2) = 0.76$, $f(3) = 0.81$, $f(4) = 0.88$ and $f(5) = 0.94$. find $P(2 \le x \le 5)$, $P(x=3)$. ~~with~~ using cdf.

cdf

x:	0	1	2	3	4	5
$f(x) = P(x \le x)$	0.58	0.72	0.76	0.81	0.88	0.94

i) $P(\overset{a}{2} \le x \le \overset{b}{5}) = f(5) - f(1) = 0.94 - 0.72$

$= 0.22$

ii) $P(x=3) = f(3) - f(2) = 0.81 - 0.76 = 0.05$

An insurance Company offers its policy holders a number of different premium payment options. for a randomly selected policy holder. Let x be the number of months between successive payments. The cdf of x is as follows.

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.30 & 1 \le x < 3 \\ 0.40 & 3 \le x < 4 \\ 0.45 & 4 \le x < 6 \\ 0.60 & 6 \le x < 12 \\ 1 & 12 \le x. \end{cases}$$

a) what is the pmf of x ?

b) Using just the cdf just Compute $p(3 \le x \le 6)$ and $p(4 \le x)$

$p(3)+p(4)+p(6)$
$+p(6)$
$= \underline{\underline{0.30}}$

$P(x=4) = F(4) - F(3)$

$F(2) = P(x \le 2)$

Pmf

x:	1	3	4	6	12
$F(x) = P[x \le x]$	0.30	0.40	0.45	0.60	1
$p(x)$	0.30	0.1	0.05	0.15	0.4

$$\Sigma p(x) = 0.3 + 0.1 + 0.05 + 0.15 + 0.4 = \underline{\underline{1}}$$

$$p(\overset{a}{3} \le x \le \overset{b}{6}) = F(b) - F(a-1)$$
$$= F(6) - F(2)$$
$$= 0.60 - 0.30$$
$$= \underline{\underline{0.30}}$$

$F(2) = P(x \le 2)$
$F(3) = P(x \le 3)$
$F(x) = P(x \le x)$

$$p(4 \le x) = P(x \ge 4) = 1 - P[x < 4]$$
$$= 1 - P[x \le 3]$$
$$= 1 - F(3)$$
$$= 1 - 0.40 = \underline{\underline{0.6}}$$

A Consumer organization that evaluates new automobiles customarily reports the number of major defects in each car examined. Let x denote the number of major defects in a randomly selected car of a certain type. The cdf of x is as follows.

$$f(x) = \begin{cases} 0 & x < 0 \\ 0.06 & 0 \le x < 1 \\ 0.19 & 1 \le x < 2 \\ 0.39 & 2 \le x < 3 \\ 0.67 & 3 \le x < 4 \\ 0.92 & 4 \le x < 5 \\ 0.97 & 5 \le x < 6 \\ 1 & 6 \le x. \end{cases}$$

Calculate following probabilites directly from cdf:

a) $p(x=2)$ b) $p(x>3)$ c) $p(2 \le x \le 5)$

d) $p(2 < x < 5)$

x: 0 1 2 3 4 5 6

a) $p(x=2) = f(2) - f(1)$
$= 0.39 - 0.19$
$= \underline{\underline{0.2}}$

$p(x=a) = f(a) - f(a-1)$

$p(a \le x \le b) = f(b) - f(a-1)$

$f(x) = p[x \le x]$

b) $p(x>3) = 1 - p[x \le 3]$
$= 1 - f(3)$
$= 1 - 0.67$
$= \underline{\underline{0.33}}$

c) $p(\overset{a}{2} \le x \le \overset{b}{5}) = f(5) - f(1)$
$= 0.97 - 0.19$
$= \underline{\underline{0.78}}$

d) $p(2 < x < 5) = p(\overset{a}{3} \le x \le \overset{b}{4})$
$= f(4) - f(2)$
$= 0.92 - 0.39$ $= \underline{\underline{0.53}}$

A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB or 16 GB of memory. The table gives the distribution of Y = the amount of memory in a purchased drive.

Y:	1	2	4	8	16
P(y)	0.05	0.10	0.35	0.40	0.10

find c.d.f and $F(2.7)$, $F(7.999)$

Y	1	2	4	8	16
P(y)	0.05	0.10	0.35	0.40	0.10.
$F(y) = P[Y \leq y]$	0.05	0.15	0.50	0.9	1

$$F(y) = \begin{cases} 0 & y \leq 1 \\ 0.05 & 1 \leq y < 2. \\ 0.15 & 2 \leq y < 4 \\ 0.50 & 4 \leq y < 8 \\ 0.9. & 8 \leq y < 16 \\ & y \geq 16 \end{cases}$$

... 8 GB or 16 GB of memory.

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$P(y)$	0.05	0.10	0.35	0.40	0.10

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Y	1	2	4	8	16
$P(y)$	0.05	0.10	0.35	0.40	0.10
$F(y) = P[Y \le y]$	0.05	0.15	0.50	0.9	1

$$F(y) = \begin{cases} 0 & y \le 1 \\ 0.05 & 1 \le y < 2 \\ 0.15 & 2 \le y < 4 \\ 0.50 & 4 \le y < 8 \\ 0.9 & 8 \le y < 16 \\ 1 & y \ge 16 \end{cases}$$

$$F(2.7) = P[y \le 2.7]$$
$$= \underline{P(y=1) + P(y=2)}$$
$$= P(y \le 2)$$
$$= F(2) = \underline{\underline{0.15}}$$

$$P(7.999) = P[y \le 7.999)$$
$$= P(y \le 4)$$
$$= F(4)$$
$$= \underline{\underline{0.50}}$$

4Q In a Certain city during the month of July there will be 0, 1, 2, 3 ... power failures with probabilities 0.4, 0.3, 0.2, 0.1. Find the mean and Variance of probability distribution.

x:	0	1	2	3.
$p(x)$:	0.4	0.3	0.2	0.1

Mean $E[x] = \sum x\, p(x)$

$= 0 + 0.3 + 0.4 + 0.3$

$E[x] = 1$

$\overset{\sigma^2}{Var(x)} = E[x^2] - E[x]^2$

$E[x^2] = \sum x^2\, p(x)$

$= 0 + 0.3 + 0.8 + 0.9.$

$= 2$

$\therefore Var(x) = E[x^2] - E[x]^2$

$= 2 - 1$

$= 1$

S.D $\sigma = \sqrt{Var(x)}$

$= 1$

A library has an upper limit of 6 on the number of videos that can be checked out to an individual at one time. Consider only those who check out videos and let x denote the number of videos checked out to a randomly Selected individual. The Pmf of x is as follows. find mean ~~and~~, Varience and standard deviation.

x:	1	2	3	4	5	6
$p(x)$	0.30	0.25	0.15	0.05	0.10	0.15.

Mean $E(x) = \sum x\, p(x)$

$= 0.30 + 2\times 0.25 + 3\times 0.15 + 4\times 0.05 + 5\times 0.10 + 6\times 0.15$

$= \underline{\underline{2.85}}$

$Var(x) = E[x^2] - E[x]^2$

$E[x^2] = \sum x^2 p(x)$

$= 0.30 + 4\times 0.25 + 9\times 0.15 + 16\times 0.05 + 25\times 0.10 + 36\times 0.15$

$= 11.35.$

$\therefore \sigma^2 = Var(x) = E[x^2] - E[x]^2$

$= 11.35 - (2.85)^2 = \underline{\underline{3.2275}}$

S.D $\sigma = \sqrt{Var(x)} = \sqrt{3.2275}$

$= \underline{\underline{1.8}}$

$E[\text{constant}] = \text{Constant}$

$V(\text{Constant}) = 0$

$E[ax+b] = E[ax] + E[b]$

$\boxed{E[ax+b] = aE(x) + b.}$

$V(ax+b) = V(ax) + V(b)$

$= a^2V(x) + 0$

$\boxed{V(ax+b) = a^2V(x)}$

$E[ax] = aE[x]$

$V(ax) = a^2V(x)$

The cost of Certain Vehicle diagnostic test depends on the number of cylinders X in the Vehicles engine. Suppose the cost function is given by $h(x) = 20 + 3x + 0.5x^2$. Find $E[Y]$ if $Y = h(x)$.

x	:	4	6	8
$p(x)$	:	0.5	0.3	0.2

Y:	
P(Y)	

$E[Y] = E[h(x)]$ $\quad Y = h(x)$

$= E[20 + 3x^2 + 0.5x^2]$

$= E[20] + E[3x^2] + E[0.5x^2]$

$= 20 + 3E[x] + 0.5E[x^2]$ —— ①

$E[x] = \sum x\,p(x)$

$= 4 \times 0.5 + 6 \times 0.3 + 8 \times 0.2$

$= \underline{5.4}$

$E[x^2] = \sum x^2 p(x)$

$= 16 \times 0.5 + 36 \times 0.3 + 64 \times 0.2$

$= 31.6$

① $\Rightarrow E[Y] = 20 + 3 \times (5.4) + 0.5(31.6)$

$= \underline{\underline{52}}$

A Computer Store has purchased three Computers of a Certain type at $500 apiece. It will Sell them for $1000 apiece. The manufacturer has agreed to repurchase any Computers still unsold after a specified period at $200 apiece. Let x denote the number of Computers and Suppose that $p(0)=0.1$, $p(1)=0.2$, $p(2)=0.3$ and $p(3)=0.4$ with $h(x)$ denote the profit associated with Selling x-units, the given Information implies $h(x)=$ revenue $-$ Cost $= 800x-900$

find expected profit and Varience.

x:	0	1	2	3
$p(x)$:	0.1	0.2	0.3	0.4

$$E[h(x)] = E[800x - 900]$$

$$= 800\,E[x] - 900$$

$V(ax) = a^2 V(x)$

$V(a) = 0$

$$E[x] = \sum x p(x)$$

$$= 0 + 0.2 + 0.6 + 1.2 = \underline{\underline{2}}$$

$$E[h(x)] = 800 \times 2 - 900$$

$$= \underline{\underline{700}}$$

Expected profit $= \underline{\underline{\$700}}$

$$Var(h(x)) = V(800x - 900)$$

$$= (800)^2 V(x) + 0$$

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \sum x^2 p(x)$$

$$= 0 + 0.2 + 1.2 + 3.6 = 5$$

$$Var(x) = 5 - 4$$

$$= \underline{\underline{1}}$$

$$Var\,h(x) = (800)^2 \times 1$$

$$= 640000$$

Variance $= \underline{\underline{\$640000}}$

$$S.D = \sqrt{Var}$$

$$= \sqrt{640000}$$

$$= \underline{\underline{800}}$$

$$E[ax+b] = aE[x]+b$$

$$E[b] = b$$

$$Var(c) = 0$$

$$Var(ax+b) = a^2 Var(x) + Var(b)$$

$$= a^2 Var(x)$$

$$\boxed{Var(ax) = a^2 Var(x)}$$

Let x be a random Variable taking Values 1, 2, 3 and 4 with probabilities $\frac{1}{6}, \frac{2}{6}, \frac{2}{6}, \frac{1}{6}$ respectively. find mean and Varience of i) $Y = 2x+3$ ii) $Z = \sin x$

x	1	2	3	4
$p(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

$$\boxed{E[ax+b] = aE[x]+b}$$

$$E[y] = E[2x+3]$$
$$= 2E[x]+3$$
$$= 2 \times \frac{5}{2} + 3 = 8$$

$$\boxed{E[y] = 8}$$

$$\boxed{V(ax+b) = a^2 Var(x) + 0}$$

$$Var[y] = Var[2x+3]$$
$$= 2^2 Var(x)$$
$$= 4 \times \frac{11}{12} = \frac{11}{3}$$

$$\boxed{Var(y) = \frac{11}{3}}$$

Mean

$$E[x] = \sum x\, p(x)$$
$$= \frac{1}{6} + \frac{4}{6} + \frac{6}{6} + \frac{4}{6}$$
$$= \frac{15}{6} = \frac{5}{2}$$

$$Var(x) = E[x^2] - E[x]^2$$
$$E[x^2] = \frac{1}{6} + \frac{8}{6} + \frac{18}{6} + \frac{16}{6}$$
$$= \frac{43}{6}$$
$$Var(x) = \frac{43}{6} - \left(\frac{5}{2}\right)^2 = \frac{11}{12}$$

$$E[Z] = E[\sin x]$$

$$= \sum (\sin x)\, p(x)$$

$$= \frac{1}{6}\sin(1) + \frac{2}{6}\sin(2) + \frac{2}{6}\sin(3) + \frac{1}{6}\sin(4)$$

$$= 0.\underline{\underline{3643}}$$

$$\mathrm{Var}(Z) = [$$

Q) Let X be a random Variable with $E[x]=1$ and $E[x(x-1)]=4$. find $Var(x)$ and $Var(2-3x)$, $Var(\frac{x}{2})$

$E[x]=1$

$E[x(x-1)]=4$

$E[x^2-x]=4$

$E[x^2]-E[x]=4$

$E[x^2]-1=4$

$E[x^2]=4+1$

$=5$

$Var(x)=E[x^2]-E[x]^2$

$=5-1$

$=\underline{4}$

$Var(c)=0$

$Var(ax)=a^2Var(x)$

$Var(2-3x)=Var(2)+(-3)^2Var(x)$

$=0+9Var(x)$

$=9\times 4$

$=\underline{36}$

$Var(\frac{x}{2})=(\frac{1}{2})^2Var(x)$

$=\frac{1}{4}\times 4$

$=\underline{1}$

Find a and b if $\underline{y = ax+b}$ has mean 4 and Varience 16. where x is a R.V with mean 8 and Varience 4.

$E[y] = 4$ $\qquad$ $Var(y) = 16$

$E[x] = 8$ $\qquad$ $Var(x) = 4$

$E[y] = 4$

$\Rightarrow E[ax+b] = 4$

$a\,E[x] + b = 4$

$8a + b = 4$ —— (1)

$Var(y) = 16$

$\Rightarrow Var(ax+b) = 16$

$a^2 \, Var(x) + 0 = 16$

$4a^2 = 16$

$a^2 = 4$

$a = \pm 2$

when $a = 2$

(1) $\Rightarrow 16 + b = 4$

$b = 4 - 16$

$= \underline{\underline{-12}}$

when $a = -2$

(1) $\Rightarrow -16 + b = 4$

$b = 4 + 16$

$= \underline{\underline{20}}$

The following is the PMF of a random Variable

x:	0	1	3	7	13
$p(x)$:	$\frac{1}{8}$	a	$\frac{1}{6}$	$\frac{1}{4}$	b

find a and b if $P(x^2 = 4x - 3) = \frac{1}{2}$

$$\boxed{\Sigma p(x) = 1}$$

$$\frac{1}{8} + a + \frac{1}{6} + \frac{1}{4} + b = 1$$

$$a + b = \frac{11}{24} \quad \text{--- (1)}$$

$$x^2 = 4x - 3$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1, 3$$

$$P[x^2 = 4x - 3] = \frac{1}{2}$$

$$P[x = 1 \text{ or } x = 3] = \frac{1}{2}$$

$$P[x=1] + P[x=3] = \frac{1}{2}$$

$$a + \frac{1}{6} = \frac{1}{2}$$

$$a = \frac{1}{2} - \frac{1}{6}$$

$$= \frac{1}{3}$$

$$(1) \Rightarrow \frac{1}{3} + b = \frac{11}{24}$$

$$b = \frac{11}{24} - \frac{1}{3} = \frac{1}{8}$$

A random Variable x takes Values $-1, 1, 3$ with equal probabilites and 5 with probability $\frac{1}{2}$ find

1. Probability distribution of x (pmf)
2. $P[|x-3|>1]$

x:	-1	1	3	5
$p(x)$:	a	a	a	$\frac{1}{2}$

$$\Sigma p(x) = 1$$

$$a+a+a+\frac{1}{2} = 1$$

$$3a = 1-\frac{1}{2}$$

$$= \frac{1}{2}$$

$$\boxed{a = \frac{1}{6}}$$

x:	-1	1	3	5
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

$|x| \le a$

$\Rightarrow -a \le x \le a$

$$P[|x-3|>1] = 1 - P[|x-3| \le 1]$$

$$= 1 - P[-1 \le x-3 \le 1]$$

$$= 1 - P[-1+3 \le x \le 1+3]$$

$$= 1 - P[2 \le x \le 4]$$

$$= 1 - P(x=3)$$

$$= 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$

An experiment consist of 4 tossess of a fair Coin. find the probability distribution for the total no: of heads. Also find the distribution function.

Let x denotes the no: of Heads

HHHH   HHHT   HHTH   HTHH   THHH

TTHH   HHTT   HTTH   HTHT   THTH   THHT

HTTT   THTT   TTHT   TTTH

TTTT

Total outcome : 16

Prob = $\frac{\text{fav: case.}}{\text{Total no: cas}}$

Pmf

cdf

x:	0	1	2	3	4
$p(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$
$F(x)$	$\frac{1}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{15}{16}$	1

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{16} & 0 \le x < 1 \\ \frac{5}{16} & 1 \le x < 2 \\ \frac{11}{16} & 2 \le x < 3 \\ \frac{15}{16} & 3 \le x < 4 \\ & x \ge 4 \end{cases}$$

Consider a lot of 10 items Containing 3 defectives from a Sample of 4 items is drawn at random. Let the random Variable x denote the no: of defective items in the Sample. find

i) Pmf of x

ii) $P(x<1)$

iii) Distribution function of x.

$\boxed{3 \quad 10 \quad 7}$ 3 - defective
7 - good.

$$\frac{3C_0 \; 7C_4}{10C_4}$$

$nC_0 = 1$
$nC_1 = n.$
$nC_n = 1$

Let x denote. no: of defective items

x takes Values. $0, 1, 2, 3, \cancel{4}$

$P(x=0) =$ P(no: defec: items. in Sample of 4 chosen)

$$= \frac{3C_0 \; 7C_4}{10C_4} = \frac{1 \times \frac{7\times \cancel{6}^{\,3} \times \cancel{5} \times \cancel{4}}{\cancel{1\cdot2\cdot3\cdot4}}}{\frac{\cancel{10}^{\,3}\times\cancel{9}\times\cancel{8}^{\,2}\times\cancel{7}}{\cancel{1\cdot2\cdot3\cdot4}}} = \frac{1}{6}$$

$P(x=1) = P(\text{1 defect item})$

$$= \frac{3C_1 \; 7C_3}{10C_4} = \frac{1}{2}.$$

$P(x=2) = P(\text{2 defective item})$

$$= \frac{3C_2 \; 7C_2}{10C_4.} = \frac{3}{10}.$$

$P(x=3) = P(\text{3 def: item})$

$$= \frac{3C_3 \; 7C_1}{10C_4} = \frac{1}{30}$$

Pmf

x:	0	1	2	3
$p(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$
E.D.F $F(x)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{29}{30}$	1

$$F(x) = \begin{cases} 0 & x<0 \\ \frac{1}{6} & 0\le x<1 \\ \frac{2}{3} & 1\le x<2 \\ \frac{29}{30} & 2\le x<3 \\ 1 & x\ge 3 \end{cases}$$

$P(x<1) = P(x=0)$

$$= \frac{1}{6}$$

Find the expected no. of throws of an unbiased die till the face marked 3 turns up.

Let x denote the no. of throws x takes value. 1, 2, 3, 4, ...

x:	1	2	3	4	....
$p(x)$:	$\frac{1}{6}$	$\frac{5}{6}\cdot\frac{1}{6}$	$\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$	$\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$	....

$1-\frac{1}{6}=\frac{5}{6}$.

$$E[x] = \sum x\,p(x)$$

$$= \frac{1}{6} + 2\times\frac{5}{6}\cdot\frac{1}{6} + 3\times\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + 4\times\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \cdots$$

$$= \frac{1}{6}\left[1 + 2\left(\frac{5}{6}\right) + 3\left(\frac{5}{6}\right)^2 + 4\left(\frac{5}{6}\right)^3 + \cdots\right]$$

$(1-x)^{-2} = 1+2x+3x^2+4x^3 + \cdots$

$$= \frac{1}{6}\left[1-\frac{5}{6}\right]^{-2}$$

$$= \frac{1}{6}\left[\frac{1}{6}\right]^{-2} = \frac{1}{6}\times 36 = \underline{\underline{6}}$$

A Coin is tossed until head appears. find the expected no: of toss:

x: be no: of toss until head appears.

$x: 1, 2, 3, 4 \dots$

x:	1	2	3	4	$\dots$
$p(x)$:	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^4$	$\dots$

$$E[x] = \sum x p(x)$$

$$= \frac{1}{2} + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4 + \dots$$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

$$= \frac{1}{2}\left[1 + 2\frac{1}{2} + 3\cdot\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \dots\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{2}\right]^{-2} = \frac{1}{2}\left[\frac{1}{2}\right]^{-2} = \frac{1}{2}\cdot 4 = \underline{\underline{2}}$$

Binomial Distribution:

Let x be a random variable with probability density function

$$p(x) = nC_x \, p^x q^{n-x} \quad , \quad x = 0, 1, 2, \ldots, n$$

where p is the probability of success and q is the probability of failure with

$$\boxed{p+q=1}$$

$n \rightarrow$ no. of trials.

Success | failure.

$p = P(s)$ | $q = P(f)$

Note

$$p(x) = \sum_{x=0}^{n} nC_x \, p^x q^{n-x} = q^n + nC_1 \, p \, q^{n-1} + nC_2 \, p^2 q^{n-2} + \cdots + p^n \qquad p+q=1$$

$$= (p+q)^n$$

$$= (1)^n = 1$$

ii/ Mean and Varience of Binomial Distribution.

$$p(x) = nC_x \, p^x \, q^{n-x} \qquad x = 0, 1, 2, \ldots, n.$$

$$\boxed{nC_x = \frac{n!}{x!\,(n-x)!}}$$

$$E[x] = \sum x \, p(x)$$

$$= \sum_{x=0}^{n} x \, nC_x \, p^x \, q^{n-x}$$

$$x! = \underline{1.2.\cdots}\,\underline{(x-1)}x = (x-1)!\,x$$

$$= \sum_{x=0}^{n} x \, \frac{n!}{x!\,(n-x)!} \, p^x \, q^{n-x}$$

$$n! = 1.2\cdots(n-1)n = (n-1)!\,n.$$

$$= \sum_{x=1}^{n} x \, \frac{n(n-1)!}{x(x-1)!\,(n-x)!} \, p^x \, q^{n-x}.$$

$$p^x p^{-1} = p^{x-1}$$

$$n-1-(x-1) = n-1-x+1 = n-x$$

$$= \sum_{x=1}^{n} np \, \frac{(n-1)!}{(x-1)!\,(n-x)!} \, \frac{p^x}{p} \, q^{n-x}.$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!} \, p^{x-1} \, q^{n-x}$$

$$= np \sum_{x=1}^{n} (n-1)C_{(x-1)} \, p^{x-1} \, q^{n-x}$$

$$= np \, (p+q)^{n-1}$$

$$\sum nC_x \, p^x q^{n-x} = (p+q)^n = 1$$

$$p+q = 1$$

$$\underline{\underline{= np}}$$

$$\boxed{\therefore \text{ Mean} = np}$$

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \sum x^2 p(x)$$

$$x! = 1.2 \cdots (x-2)(x-1)x$$
$$= (x-2)!\,(x-1)x$$

$$= \sum_{x=0}^{n} x^2\, nC_x\, p^x q^{n-x}.$$

$$nC_x = \frac{n!}{x!\,(n-x)!}.$$

$$= \sum_{x=0}^{n} [x^2 - x + x]\, nC_x\, p^x q^{n-x}.$$

$$n-2-(x-2)$$
$$n-x$$

$$= \sum_{x=0}^{n} x(x-1)\, nC_x\, p^x q^{n-x} + \sum_{x=0}^{n} x\, nC_x\, p^x q^{n-x}.$$

$$p^x p^{-2} = p^{x-2}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} p^x q^{n-x} + np.$$

$$= \sum_{x=2}^{n} \cancel{x(x-1)} \frac{n(n-1)(n-2)!}{\cancel{x(x-1)}(x-2)!\,(n-x)!} p^x q^{n-x} + np.$$

$$= \sum_{x=2}^{n} n(n-1)p^2 \frac{(n-2)!}{(n-2)!\,(n-x)!} \frac{p^x}{p^2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2} (n-2)C_{(x-2)}\, p \cdot q + np.$$

$$= n(n-1)p^2 (p+q)^{n-2} + np \qquad (p+q)=1$$

$$= n(n-1)p^2 + np.$$

$$= n^2p^2 - np^2 + np$$

$$Var(x) = E[x^2] - E[x]^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np(1-p) \qquad \begin{aligned} p+q&=1 \\ q&=1-p \end{aligned}$$

$$\boxed{Var(x) = npq.}$$

$$\boxed{S.D = \sqrt{npq}}$$

The mean and Varience of a binomial random Variable x are 16 and 8 respectively. find $p(x=0)$ and $p(x=1)$.

given mean = 16 and Varience = 8

$np = 16$ —(1)

$npq = 8$ —(2)

$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{8}{16}$

$\boxed{q = \frac{1}{2}}$

$p+q=1$

$p = 1-q$

$= 1-\frac{1}{2}$

$\boxed{p = \frac{1}{2}}$

$(1) \Rightarrow n \times \frac{1}{2} = 16$

$\boxed{n = 32}$

$\therefore$ Prof of B.D is

$p(x) = nC_x\, p^x q^{n-x}$ $x = 0,1,2,\dots,n$.

$= 32C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{32-x}$ $x = 0,1,2,\dots,32$

$\boxed{p(x) = 32C_x \left(\frac{1}{2}\right)^{32}}$

$p(x=0) = 32C_0 \left(\frac{1}{2}\right)^{32}$

$= \left(\frac{1}{2}\right)^{32}$

$p(x=1)$

$= 32C_1 \left(\frac{1}{2}\right)^{32}$

$= 32 \left(\frac{1}{2}\right)^{32}$

$= 2^5 \frac{1}{2^{32}}$

$= \left(\frac{1}{2}\right)^{27}$

$P(x) = nC_x\, p^x q^{n-x}$ $x = 0,1,2,\dots,n$.

$p+q=1$

mean $= np$ Varience $= npq$.

$S.D = \sqrt{npq}$.

$x^m x^n = x^{m+n}$

$B(n,p,q)$

$x+32-x$

$nC_0 = 1$ $nC_1 = n$ $nC_n = 1$

In a Binomial Distribution consisting of 6 independent trials, probabilites of 1 and 2 Success are 0.28336 and 0.0506 respectively. Find the parameter of the distribution.

Pmf of B.D is

$p(x) = nC_x\, p^x q^{n-x} \quad x = 0,1,2,\ldots,n.$

$\boxed{n=6}$

$p(x) = 6C_x\, p^x q^{6-x} \quad x = 0,1,2,\ldots,6$

$p(x=1) = 0.28336$

$6C_1\, p\, q^5 = 0.28336$

$6 p q^5 = 0.28336 \quad \text{--- (1)}$

$p(x=2) = 0.0506$

$6C_2\, p^2 q^4 = 0.0506$

$15 p^2 q^4 = 0.0506 \quad \text{--- (2)}$

$nC_1 = n$

$\frac{(1)}{(2)} \quad \frac{\cancel{6}^{2} \cancel{p} q^{\cancel{5}}}{\cancel{15}^{5}\, p^{\cancel{2}} \cancel{q^4}} = \frac{0.28336}{0.0506} \qquad \frac{\cancel{6}.5}{1.\cancel{0}}$

$\frac{2q}{5p} = 5.6 \qquad p+q=1$

$q = 1-p$

$2q = 28p$

$2(1-p) = \cancel{28}^{14} p$

$1-p = 14p.$

$1 = 15p$

$\underline{p = \frac{1}{15}}$

$q = 1-p$

$= 1 - \frac{1}{15}$

$\underline{= \frac{14}{15}}$

Pmf

$p(x) = 6C_x \left(\frac{1}{15}\right)^x \left(\frac{14}{15}\right)^{6-x} \quad x = 0,1,2,\ldots 6$

If on the average rain falls on 10 days in every 30 days. obtain the probability that rain will fall atleast 3 days of a given week.

$p = P(\text{rain will fall}) = \frac{10}{30} = \frac{1}{3}$

$q = 1 - p$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

$n = 7$ $\qquad B(n, p)$

Pmf of B.D is

$P(x) = nC_x\, p^x q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$= 7C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{7-x} \quad x = 0, 1, 2, \ldots, 7$

P(atleast 3 days rain fall)

$= P(x \geq 3)$

$= P(3) + P(4) + P(5) + P(6) + P(7)$

or

$= 1 - P[x < 3]$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - \left[7C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^7 + 7C_1 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^6 + 7C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^5\right]$

$= \underline{\underline{0.4294}}$

Q. Ten Coins are thrown Simultaneously. Find the probability of getting atleast 7 heads.

$n = 10$

$p = P(\text{getting head}) = \frac{1}{2}$

$q = 1 - p$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

$x^m x^n = x^{m+n}$

$x + 10 - x = 10$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$= 10C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} \quad x = 0, 1, 2, \ldots, 10$

$= 10C_x \left(\frac{1}{2}\right)^{10} \quad x = 0, 1, 2, \ldots, 10$

P(getting atleast 7 heads)

$\frac{1}{2}$

$10.$

$= p(x \geq 7)$

$= p(x=7) + p(x=8) + p(x=9) + p(x=10)$

$= 10C_7 \left(\frac{1}{2}\right)^{10} + 10C_8 \left(\frac{1}{2}\right)^{10} + 10C_9 \left(\frac{1}{2}\right)^{10}$

$+ 10C_{10} \left(\frac{1}{2}\right)^{10}$

$= \left(\frac{1}{2}\right)^{10} [120 + 45 + 10 + 1]$

$= 0.171875$

probability that a building inspector, who randomly selects 4 of the new buildings, will catch (i) none (ii) exactly one (iii) atleast two of the new buildings that violate the building code.

$n = 4$

$p = p$(new building that violate building code)

$= \frac{6}{18} = \frac{1}{3}$

$q = 1 - p$

$= 1 - \frac{1}{3} = \frac{2}{3}$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$= 4C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{4-x} \quad x = 0, 1, 2, 3, 4$

i) P(none of the building that violate building code)

$= p(x=0)$

$= 4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^4$

ii) p(exactly one building violate building code)

$nC_1 = n.$

$= p(x=1)$

$= 4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 = 4 \, \frac{8}{81} = \frac{32}{81}$

iii) p(atleast two of the new building violate building code) $= p(x \geq 2)$

$= p(2) + p(3) + p(4)$

$= 1 - [p(x < 2)] = 1 - [p(x=0) + p(x=1)]$

$= 1 - \left[\left(\frac{2}{3}\right)^4 + \frac{32}{81}\right] = 1 - \frac{48}{81} = \frac{33}{81}$

Q) The probability that a component is acceptable is 0.93. Then Ten components are picked at random. What is the probability that (i) atleast nine are acceptable (ii) Atmost three are acceptable.

$n = 10$

$P = p(\text{Component is acceptable})$

$= 0.93$

$q = 1 - P$

$= 1 - 0.93$

$= 0.07$

Pmf of B.D

$$P(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots, 10$$

$$= 10C_x \, (0.93)^x (0.07)^{10-x} \quad x = 0, 1, 2, \ldots, 10$$

P(atleast nine are acceptable)

$$= p(x \geq 9)$$

$$= p(x = 9) + p(x = 10)$$

$$= 10C_9 \, (0.93)^9 (0.07)^1 + 10C_{10} \, (0.93)^{10}$$

P(atmost three are acceptable).

$$= p(x \leq 3)$$

$$= p(x=0) + p(x=1) + p(x=2) + p(x=3)$$

$$= 10C_0 (0.93)^0 (0.07)^{10} + 10C_1 (0.93)^1 (0.07)^9 + 10C_2 (0.93)^2 (0.07)^8 + 10C_3 (0.93)^3 (0.07)^7$$

$$=$$

If on an average one vessel in every 10 is wrecked, find the probability that out of 5 vessels expected to arrive, atleast 4 will arrive safely.

$n = 5$

$p = p(\text{safe arrival of ship}) = 1 - q$

$= 1 - \frac{1}{10} = \frac{9}{10}$

$q = \frac{1}{10}$

$\therefore$ Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \dots, n.$

$= 5C_x \left(\frac{9}{10}\right)^x \left(\frac{1}{10}\right)^{5-x} \quad x = 0, 1, 2, 3, 4, 5$

$p(\text{atleast } 4 \text{ ship will arrive safely})$

$= p(x \geq 4)$

$= p(x = 4) + p(x = 5).$

$= 5C_4 \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right) + 5C_5 \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^0$

$= 0.918$

U.Q An expert shot hits a target 95% of the time. If the expert shoots 15 times in succession. Find the probability that.

i) he misses the target only once.

ii) he misses the target only at last shot.

$n = 15$

$q = P(\text{hit the target}) = \frac{95}{100} = 0.95.$

$p = P(\text{miss the target}) = 1 - q.$

$= 1 - 0.95$

$= 0.05$

Pmf of B.D is

$P(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \dots, n.$

$= 15C_x \, (0.05)^x \, (0.95)^{15-x} \quad x = 0, 1, 2, \dots, 15$

i) P(miss the target only once)

$= P(x=1)$

$= 15C_1 \, (0.05)^1 \, (0.95)^{14}$

$= 0.365$

ii) P(miss the target only at last shot).

$= P(x = 15)$

$= 15C_{15} \, (0.05)^{15} \, (0.95)^0$

$= (0.05)^{15}$

During one Stage in the manufacture of integrated circuit chips, a coating must be applied. if 70% of the chips receive a thick enough coating. find the probability among 15 chips.

a) atleast 12 will have thick enough coatings.

b) atmost 6 will have thick enough coatings.

c) exactly 10 will have thick enough coatings.

$n = 15$

$p = P(\text{chip receive thick coating}) = \frac{70}{100} = 0.7$

$q = 1 - p$

$= 1 - 0.7 = 0.3$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$= 15C_x \, (0.7)^x \, (0.3)^{15-x} \quad x = 0, 1, 2, \ldots, 15$

i) P(atleast 12 will have thick coating).

$= P(x \geq 12)$

$= P(12) + P(x=13) + P(x=14) + P(x=15).$

$= 15C_{12}(0.7)^{12}(0.3)^3 + 15C_{13}(0.7)^{13}(0.3)^2 +$
$15C_{14}(0.7)^{14}(0.3) + 15C_{15}(0.7)^{15}(0.3)^0$

$= 0.2969.$

ii) P (atmost 6 will have thick coating)

$= P(x \leq 6)$

$= P(x=0) + P(x=1) + P(x=2) + P(x=3) +$
$P(x=4) + P(x=5) + P(x=6)$

$= 0.01$

applied. if 70% of the chips receive a thick enough ...
15 chips.

a) atleast 12 will have thick enough coatings.
b) atmost 6 will have thick enough coatings.
c) exactly 10 will have thick enough coatings.

$n = 15$

$P = p(\text{chip receive thick coating}) = \frac{70}{100} = 0.7$

$q = 1 - P$
$= 1 - 0.7 = 0.3$

Pmf of B.D is

$p(x) = nC_x \, p^x \, q^{n-x} \quad x = 0, 1, 2, \ldots, n.$

$= 15C_x (0.7)^x (0.3)^{15-x} \quad x = 0, 1, 2, \ldots, 15$

i) P(atleast 12 will have thick coating).

$= p(x \geq 12)$

$= p(12) + p(x=13) + p(x=14) + p(x=15).$

$= 15C_{12}(0.7)^{12}(0.3)^3 + 15C_{13}(0.7)^{13}(0.3)^2 + 15C_{14}(0.7)^{14}(0.3) + 15C_{15}(0.7)^{15}(0.3)^0$

$= \underline{0.2969}.$

ii) P (atmost 6 will have thick coating)

$= p(x \leq 6)$

$= p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4) + p(x=5) + p(x=6)$

$= \underline{0.0152}$

iii) P(exactly 10 will have coat)

$= p(x=10)$

$= 15C_{10}(0.7)^{10}(0.3)^5$

1) The probability that the noise level of a wide-band amplifier will exceed 2dB is 0.05

u.q Find the probabilities that among 12 such amplifiers the noise level of.

a) one will exceed 2dB $P(x=1) = 0.3412$

b) atmost two will exceed 2 dB $P(x \le 2) = 0.979$

c) two or more will exceed 2dB

$P(x \ge 2)$
$= 1 - P(x < 2)$
$= 0.1185$

$n = 12$

$p = p(\text{noise exceed}) = 0.05$

$q = 1 - p = 1 - 0.05 = 0.95$

2) An agricultural co-operative claims that 90% of the watermelons shipped out are ripe and ready to eat. Find the probability that among 18 watermelons shipped out.

a) all 18 are ripe and ready to eat $P(x=18) = 0.15009$

b) atmost 16 are ripe and ready to eat. $P(x \ge 16) = 0.7338$

c) atmost 14 are ripe and ready to eat.

$P(x \le 14)$
$= 1 - P(x > 14)$
$= 0.0982$

$n = 18$

$p = p(\text{ripe and ready eat}) = \frac{90}{100} = 0.9.$

$q = 1 - p = 1 - 0.9 = 0.1$

3)

Four coins are tossed and no. of heads noted. The experiment is repeated 40 times and the following distribution is obtained. Fit a binomial distribution to the data.

Sol.

No. of Heads: (x) :	0	1	2	3	4
frequencies (f) :	4	10	15	9	2

$n = 4$

$$\text{mean} = \bar{x} = \frac{\sum xf}{\sum f} = \frac{0+10+30+27+8}{40} = 1.875$$

$np = 1.875$

$4p = 1.875$

$p =$

and the following distribution is ...

Sol. No. of Heads: (x) : 0 1 2 3 4

frequencies (f) : 4 10 15 9 2

$n=4$

$$\text{mean} = \bar{x} = \frac{\sum xf}{\sum f} = \frac{0+10+30+27+8}{40} = \underline{\underline{1.875}}$$

$N = \sum f$
$= 40$

$np = 1.875$

$4p = 1.875$

$p = 0.46875$

$q = 1 - p$

$= 0.53125$

Pmf of BD is $P(x) = nC_x\, p^x q^{n-x} \quad x = 0,1,2,\dots,n$

$= 4C_x (0.46875)^x (0.53125)^{4-x} \quad x = 0,1,2,3,4$

x	$p(x)$	$N \times p(x)$
0	0.0795	$3.18 \simeq 3$
1	0.2809	$11.236 \simeq 11$
2	0.3721	$14.884 \simeq 15$
3	0.2191	$8.704 \simeq 9$
4	~~0.46875~~ 0.0484	$1.936 \simeq 2$
		40

$\sum p(x) = 1$

HW A garden sows 4 seeds in each of 100 plant pots. The no. of pots in which 0,1,2,3 and 4 of seeds germinated is given in the following table. fit a binomial distribution to the data.

No. of Seeds germinated	0	1	2	3	4
No. of pots	13	35	34	15	3

$p = \frac{2}{5}$

than Rs 7000. Suppose 10 undergraduate students are selected randomly to be interviewed about credit card usage.

a) what is the probability that two of the students will have a credit card balance greater than Rs. 7000?

b) what is the Probability that none will have a credit card balance greater than Rs 7000?

(c) what is the Probability that atleast three will have a credit card balance greater than Rs 7000?

$p = p$(credit balance is greater than Rs 7000)

$= \frac{9}{100}$

$= 0.09$

$q = 1 - p = 1 - 0.09$

$= 0.91$

$n = 10$

Pmf of B.D is $p(x) = nC_x \, p^x q^{n-x} \quad x = 0, 1, \ldots, n$

$p(x) = 10C_x (0.09)^x (0.91)^{10-x} \quad x = 0, 1, 2, \ldots, 10$

i) P(two of the students will have credit card balance greater than 7000)

$$= p(x=2)$$

$$= 10C_2 (0.09)^2 (0.91)^8$$

$$= \underline{\underline{0.1714}}$$

ii) P(<u>none</u> will have credit card balance greater than 7000).

$$= p(x=0)$$

$$= 10C_0 (0.09)^0 (0.91)^{10}$$

$$= \underline{\underline{0.3894}}$$

iii) P(atleast 3 will have credit card balance greater than 7000)

$$= p(x \geq 3) = 1 - p(x<3) = [1 - [p(x=0) + p(x=1) + p(x=2)]]$$

$$= 1 - [0.3894 + 10C_1 (0.09)^1 (0.91)^9 + 0.1714]$$

$$= \underline{\underline{0.0540}}$$

Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let x denote the number among 15 randomly selected copies that fail the test. Find the probability that i) atmost 8 fail the test. ii) exactly 8 fail the test iii) atleast 8 fail the test iv) between 4 and 7 fail the test.