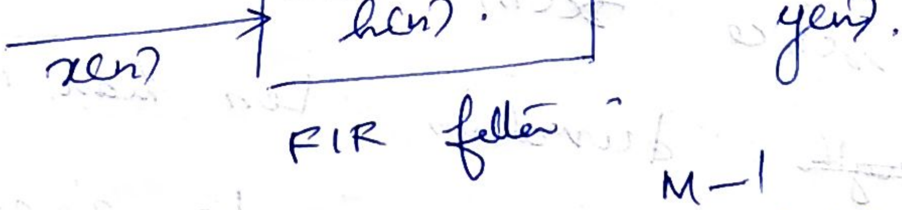
Linear filtering methods based on DFT
(Linear Convolution)

— Linear convolution is used if we have to find output of a linear filter to a given input sequence.

Linear convolution using circular convolution

$x(n)$ — of length L — input of linear s/m.

$h(n) \rightarrow$ impulse response of s/m of length M.



FIR filter

$$y(n) = x(n) * h(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

— Since $x(n)$ and $h(n)$ are finite duration $y(n)$ is also finite duration of length $L+M-1$.

— In frequency domain

$$Y(\omega) = X(\omega) H(\omega).$$

— By using DFT, DFT of size $N \geq L+M-1$ is required to represent $y(n)$.

$$Y(k) = Y(\omega) \Big|_{\omega = \frac{2\pi k}{N}}$$

$$Y(k) = X(\omega) H(\omega) \Big|_{\omega = \frac{2\pi k}{N}}$$

$$\therefore \quad Y(k) = X(k) H(k) \quad k = 0, \ldots N-1$$

— Since $x(n)$ and $h(n)$ has ~~length~~ duration less than N, we simply pad zeros to increase the length to N.

Q) Determine the response of the FIR filter with impulse response

$$h(n) = \{\underset{\uparrow}{1}, 2, 3\}$$

to the input sequence

$$x(n) = \{\underset{\uparrow}{1}, 2, 2, 1\}$$

using circular convolution method.

Ans:

Here $L = 4$ [length of $x(n)$]

$M = 3$ [length of $h(n)$].

ie o/p of an FIR filter is given by linear convolution of $x(n)$ and $h(n)$.

ie $y(n) = x(n) * h(n)$.

But here we have to find linear convolution using circular convolution.

In order to find linear convolution using circular convolution.

① make the two input of same length which is as required o/p length i.e $L+M-1=6$

i.e $x(n) = \{\underset{\uparrow}{1}, 2, 2, 1, 0, 0\}$

$h(n) = \{\underset{\uparrow}{1}, 2, 3, 0, 0, 0\}$

② then find their circular convolution by any of three methods already studied

ⓐ concentric circle method

ⓑ matrix method.

ⓒ DFT - IDFT method.

(a) Matrix method.

$$y(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 & 2 \\ 2 & 1 & 0 & 0 & 1 & 2 \\ 2 & 2 & 1 & 0 & 0 & 1 \\ 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 9 \\ 11 \\ 8 \\ 3 \end{bmatrix}$$

$$y(n) = \underline{\{1, 4, 9, 11, 8, 3\}}.$$

HW
→ In the same way we can find output using concentric circle method. and DFT - IDFT method.

Time Domain aliasing

If length of input N is less than L+M-1 results in time domain aliasing.

eg: if $N=4$

then

										1 + 8	4 + 3
						9	11	1 + 8	4 + 3		
		9	11	1 + 8	4 + 3						
1 + 8	4 + 3			9	7						
9	7	9	11	9	7	9	11	9	7	9	11

then $y(n) = \{9, 7, 9, 11\}$.

We can check the answer of Linear convolution by a method we already studied

$$y(n) = x(n) * h(n)$$

	1	2	3
1	1	2	3
2	2	4	6
2	2	4	6
1	1	2	3

$y(n) = \{1, 4, 9, 11, 8, 3\}$