ASSIGNMENT

MATHS

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ECE - B
S4

Q. Solve the system of equation using

a) Gauss Jacobi Method.

b) Gauss Seidal Method

$x - 2y + 5z = 12$, $5x + 2y - z = 6$, $2x + 6y - 3z = 5$

a) Gauss Jacobi Method.

$x - 2y + 5z = 12$

$5x + 2y - z = 6$

$2x + 6y - 3z = 5$

after re-arranging.

$5x + 2y - z = 6$

$2x + 6y - 3z = 5$

$x - 2y + 5z = 12$

here

$|5| > |2| + |-1|$

$|6| > |2| + |-3|$

$|5| > |1| + |-2|$

then

$$\left.\begin{aligned} x &= \frac{6 - 2y + z}{5} \\ y &= \frac{5 - 2x + 3z}{6} \end{aligned}\right\} \quad (1).$$

$$z = \frac{12 - x + 2y}{5} \Bigg\} \in (1)$$

Taking $x^{(0)} = 0,\ y^{(0)} = 0,\ z^{(0)}$ as initial approximation.

First Iteration

$$x^{(1)} = \frac{6 - 2y^{(0)} + z^{(0)}}{5} = 1.2$$

$$y^{(1)} = \frac{5 - 2x^{(0)} + 3z^{(0)}}{6} = \frac{5}{6} = 0.83$$

$$z^{(1)} = \frac{12 - x^{(0)} + 2y^{(0)}}{5} = \frac{12}{5} = 2.4$$

Second Iteration

$$x^{(2)} = \frac{6 - 2y^{(1)} + z^{(1)}}{5} = \frac{6 - 2 \times 0.83 + 2.4}{5} = 1.348$$

$$y^{(2)} = \frac{5 - 2x^{(1)} + 3z^{(1)}}{6} = \frac{5 - 2 \times 1.2 + 3 \times 2.4}{6} = 1.63$$

$$z^{(2)} = \frac{12 - x^{(1)} + 2y^{(1)}}{5} = \frac{12 - 1.2 + 2 \times 0.83}{5} = 2.492$$

Third Iteration

$$x^{(3)} = \frac{6 - 2y^{(2)} + z^{(2)}}{5} = \frac{6 - 2 \times 1.63 + 2.492}{5} = 1.046$$

$$y^{(3)} = \frac{5 - 2x^{(2)} + 3z^{(2)}}{6} = \frac{5 - 2 \times 1.348 + 3 \times 2.492}{6} = 1.63$$

$$z^{(2)} = \frac{12 - x^{(2)} + 2y^{(2)}}{5} = \frac{12 - (1.348) + 2.(1.63)}{5} = 2.78$$

Fourth Iteration

$$x^{(4)} = \frac{6-2y^{(3)}+z^{(3)}}{5} = \frac{6-2\times 1.63+2.732}{5} = 1.104$$

$$y^{(4)} = \frac{5-2x^{(3)}+3z^{(3)}}{6} = \frac{5-2\times 1.046+3\times 2.782}{6} = 1.875$$

$$z^{(4)} = \frac{12-x^{(3)}+2y^{(3)}}{5} = \frac{12-1.046+2\times 1.63}{5} = 2.842$$

Fifth Iteration

$$x^{(5)} = \frac{6-2y^{(4)}+z^{(4)}}{5} = \frac{6-2(1.875)+2.842}{5} = 1.0184$$

$$y^{(5)} = \frac{5-2x^{(4)}+3z^{(4)}}{6} = \frac{5-2\times(1.104)+3\times(2.842)}{6} = 1.88$$

$$z^{(5)} = \frac{12-x^{(4)}+2y^{(4)}}{5} = \frac{12-1.104+2\times 1.875}{5} = 2.92$$

b) Gauss Seidel method.

After re arranging the given equation.

$$5x-2y-z=6$$
$$2x+6y-3z=5$$
$$x-2y+5z=12$$

ie, now

$$|a_1| > |b_1| + |c_1|$$

$$|b_2| > |a_2| + |c_2|$$

$$|c_3| > |b_3| + |a_3|$$

Here:

$$|5| > |2| + |-1|$$

$$|6| > |2| + |-3|$$

$$|5| > |1| + |-2|$$

then

$$x = \frac{6 - 2y + z}{5}$$

$$y = \frac{5 - 2x + 3z}{6}$$

$$z = \frac{12 - x + 2y}{5}$$

taking initial aproximation as $x^{(0)} = 0,\ y^{(0)} = 0,\ z^{(0)}$

then.

First Iteration

$$x^{(1)} = \frac{6 - 2y^{(0)} + z^{(0)}}{5} = \frac{6 - 2(0) + 0}{5} = 1.2$$

$$y^{(1)} = \frac{5 - 2x^{(1)} + 3z^{(0)}}{6} = \frac{5 - 2 \times 1.2 + 3 \times 0}{6} = 0.43$$

$$z^{(1)} = \frac{12 - x^{(1)} + 2y^{(1)}}{5} = \frac{12 - 1.2 + 2 \times 0.43}{5} = 2.332$$

Second Iteration

$$x^{(2)} = \frac{6-2y^{(1)}+z^{(1)}}{5} = \frac{6-2\times0.43+2.332}{5} = 1.494$$

$$y^{(2)} = \frac{5-2x^{(2)}+3z^{(1)}}{6} = \frac{5-2\times1.494+3\times2.332}{6}$$

$$= 1.501$$

$$z^{(2)} = \frac{12-x^{(2)}+2y^{(2)}}{5} = \frac{12-1.494+2\times1.501}{5} = 2.701$$

Third Iteration

$$x^{(3)} = \frac{6-2y^{(2)}+z^{(2)}}{5} = \frac{6-2\times1.501+2.701}{5} = 1.13$$

$$y^{(3)} = \frac{5-2x^{(3)}+3z^{(2)}}{6} = \frac{5-2\times1.139+3\times2.701}{6} = 1.304$$

$$z^{(3)} = \frac{12+2y^{(3)}-x^{(3)}}{5} = \frac{12-1.139+3\times1.304}{5} = 2.893$$

Fourth Iteration

$$x^{(4)} = \frac{6-2y^{(3)}+z^{(3)}}{5} = \frac{6-2\times1.804+2.893}{5} = 1.057$$

$$y^{(4)} = \frac{5-2x^{(4)}+3z^{(3)}}{6} = \frac{5-2\times1.057+3\times2.893}{6} = 1.927$$

$$z^{(4)} = \frac{12-x^{(4)}+2y^{(4)}}{5} = \frac{12-1.057+2(1.927)}{5} = 2.95$$

Fifth Iteration

$$x^{(5)} = \frac{6 - 2y^{(4)} + z^{(4)}}{5} = \frac{6 - 2 \times 1.927 + 2.959}{5} = 1.021$$

$$y^{(5)} = \frac{5 - 2x^{(5)} + 3z^{(4)}}{6} = \frac{5 - 2 \times 1.021 + 3 \times 2.959}{6} = 1.972$$

$$z^{(5)} = \frac{12 - x^{(5)} + 2y^{(5)}}{5} = \frac{12 - 1.021 + 2 \times 1.972}{5} = 2.98$$