Generator Matrix

$\boxed{\text{Generator Matrix of Order } k \times n}$.

Polynomials $g(x)$, $xg(x)$, $x^2g(x)$ and $x^3g(x)$ represen
code vector polynomial of the same cyclic code.

Eg:

$$g(x) = 1 + x + x^3 \qquad (n, k) = (7, 4).$$

$$= 1\cdot 1 + 1\cdot x + 0\cdot x^2 + 1\cdot x^3 + 0\cdot x^4 + 0\cdot x^5 + 0\cdot x^6$$

The codevector corresponding to $g(x)$ is $\underline{\underline{1101000}}$.

$$x\,g(x) = x(1+x+x^3)$$
$$= x + x^2 + x^4.$$

$\therefore$ code vector corresponding to $x g(x)$ is ~~00~~ 0110100

$$x^2 g(x) = x^2(1+x+x^3)$$
$$= x^2 + x^3 + x^5$$

$\therefore$ code vector corresponding to $x^2 g(x)$ is 0011010.

$$x^3 g(x) = x^3(1+x+x^3)$$
$$= x^3 + x^4 + x^6.$$

$\therefore$ code vector corresponding to $x^3 g(x)$ is 0001101

$$[G]_{k\times n} = \begin{array}{c} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \to g(x) \\ \to x\,g(x) \\ \to x^2 g(x) \\ \to x^3 g(x) \end{array} \qquad \begin{array}{c} g(x) \\ \vdots \\ \vdots \\ x^{k-1} g(x) \end{array}$$

The generator matrix G is not in systematic form, ie cannot be visualised in $[P \mid I_K] = [P \mid I_4]$ form.

⇒ The last 4 elements of 1st and 2nd row of [G] coincides with 1st. 2 rows of I_K. But not the last rows.

⇒ it can be transformed into a systematic form by adding first row to 3rd row and placing the result in third row.

$$[G]_{k\times n} = \begin{array}{c} \left[\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} \to g(x) \\ \to x\,g(x) \\ \to x^2 g(x) \\ \to x^3 g(x) \end{array} \qquad \begin{array}{c} g(x) \\ \vdots \\ \vdots \\ x^{k-1} g(x) \end{array}$$

R3<- R1+R3

$$[G] \Rightarrow \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

Tx.

1 + 2 + 1 → 4.

$$G = \begin{bmatrix} 110 & 1000 \\ 011 & 0100 \\ 111 & 0010 \\ 101 & 0001 \end{bmatrix}.$$

R3 ← R1+R3

R4 ← R1+R2+R4

$n, k = 7, 4$

now $G = [P \mid I_4]$.

Parity Check matrix

The rows of H matrix are

$$H = x^k h(x^{-1}) \quad x^{k+1} h(x^{-1}) \quad \dots \quad x^{n-1} h(x^{-1}$$

we know that

$$x^n + 1 = g(x)\, h(x).$$

for (7,4) cyclic code, we have $n = 7$.

$$x^7 + 1 = g(x)\, h(x).$$

parity check polynomial

$$h(x) = \frac{x^7+1}{g(x)}.$$

$$x^3+x+1 \overline{\big)\, x^7+1 \,}\quad \text{quotient: } x^4+x^2+x+1$$

$$x^7+1$$

$$\underline{x^7+x^5+x^4}$$

$$x^5+x^4+1$$

$$\underline{x^5+x^3+x^2}.$$

$$x^4+x^3+x^2+1.$$

$$\underline{x^4+\quad x^2+x}.$$

$$x^3+x+1$$

$$\underline{x^3+x+1}$$

$$\underline{\underline{0}}.$$

$\therefore \quad h(x) = x^4 + x^3 + x + 1.$

Reciprocal of $h(x)$ is defined as $x^k h(x^{-1})$.

This polynomial is also a factor of $(1+x^n)$.

Let us consider $x^4 h(x^{-1})$ for a $(7,4)$ cyclic code.

$$h(x) = 1 + x^3 + x + x^4.$$

$$h(x^{-1}) = 1 + \frac{1}{x} + \frac{1}{x^3} + \frac{1}{x^4}.$$

$$x^4 \cdot h(x^{-1}) = x^4\left(1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^4}\right).$$

$$= x^4+x^3+x^2+1.$$

Code is $[1\ 0\ 1\ 1\ 1\ 0\ 0]$.

$$x^5\, h(x^{-1}) = x^5\left(1+1/x+1/x^2+1/x^4\right).$$

$$= x^5+x^3+x^4+x.$$

Code is $[0\ 1\ 0\ 1\ 1\ 1\ 0]$.

$$x^5 h(x^{-1}) = x^5 (1 + 1/x + 1/x^2 + 1/x^4).$$

$$= x^5 + x^3 + x^4 + x.$$

code is [0 1 0 1 1 1 0].

$$x^6 h(x^{-1}) = x^6 (1 + 1/x + 1/x^2 + 1/x^4).$$

$$= x^6 + x^5 + x^4 + x^2.$$

code is [0 0 1 0 1 1 1].

H is a $(n-k) \times n$ matrix.

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

not of form $[I_{n-k} \,\vdots\, P^T]$

Add 1st row to 3rd row. & result placed in 1st row.

R1 ← R1+R3

$$H = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$H^T = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right]$$