Module 4

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Contents

Signal-Space Analysis

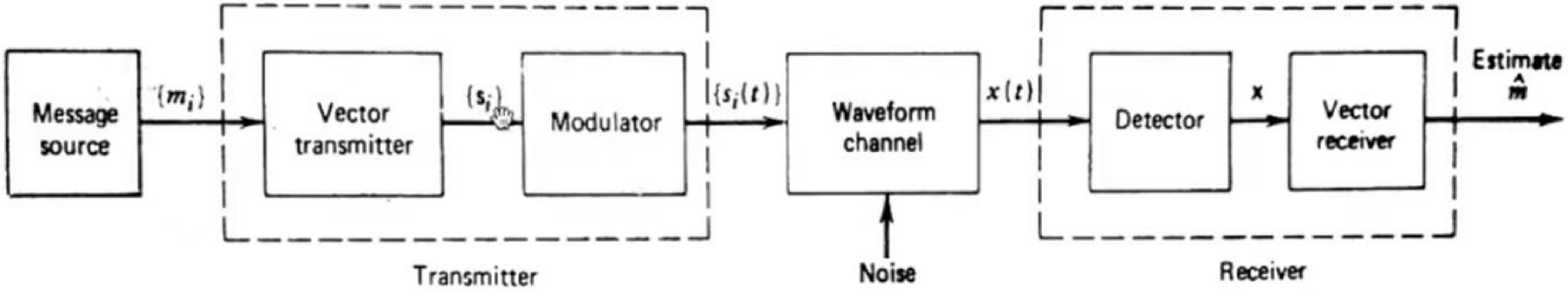


Figure 1: Conceptual model of digital communication system

Message source:

One message symbol m_i each T_m seconds, there are m different symbols and all of them occur equally likely,

$$p_i = P\{m_i \text{ emitted}\} = \frac{1}{M} \quad \text{for all } i$$

Vector encoder:

Mapping a symbol to a real-valued vector of dimension $N \leq M$,

$$m_i \mapsto \mathbf{s}_i = \left(s_{i1}, s_{i2}, \ldots, s_{iN}\right)$$

Modulator:

Mapping a real-valued vector to a real-valued waveform in an interval $0 \leq t < T_s$ with finite energy

$$\mathbf{s}_i \mapsto s_i(t)$$

$$E_i = \int_0^T s_i^2(t)dt < \infty$$

Waveform channel:

LTI system, bandwidth accomodates $s_i(t)$ without distortion, and noise is added.

$$r_i(t) = s_i(t) + n(t), \quad 0 \le t < T_s$$

where $n(t)$ is an additive white Gaussian noise.

Demodulator:

Mapping the received signal to a real valued vector of dimension N,

$$r_i(t) \mapsto \mathbf{r}_i = (r_{i1}, r_{i2}, \ldots, r_{iN})$$

Vector detector:

Mapping $\mathbf{r}_i$ to one of m messages,

$$\mathbf{r}_i \mapsto \hat{m}_i$$

Decision is made according to statistically optimal to minimize probability of symbol error

$$P_e = \sum_{i=1}^{M} P\{\hat{m}_i \neq m_i / m_i\} P\{m_i\}$$

Geometric Representation of Signals

- **Geometric representation** for any set of M energy signals $\{s_i(t)\}$ is a linear combination of N orthonormal basis functions where $N \leq M$
- Given a set of real-valued energy signals $s_1(t), s_2(t), \ldots, s_M(t)$

Basis function

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \begin{cases} 0 \leq t \leq T \\ i = 1,2, \ldots, M \end{cases}$$

- where the coefficients of the expansion are

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \begin{cases} i = 1,2, \ldots, M \\ j = 1,2, \ldots, N \end{cases}$$

- The real-valued basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are **orthonormal**

Orthonormal basis functions satisfy

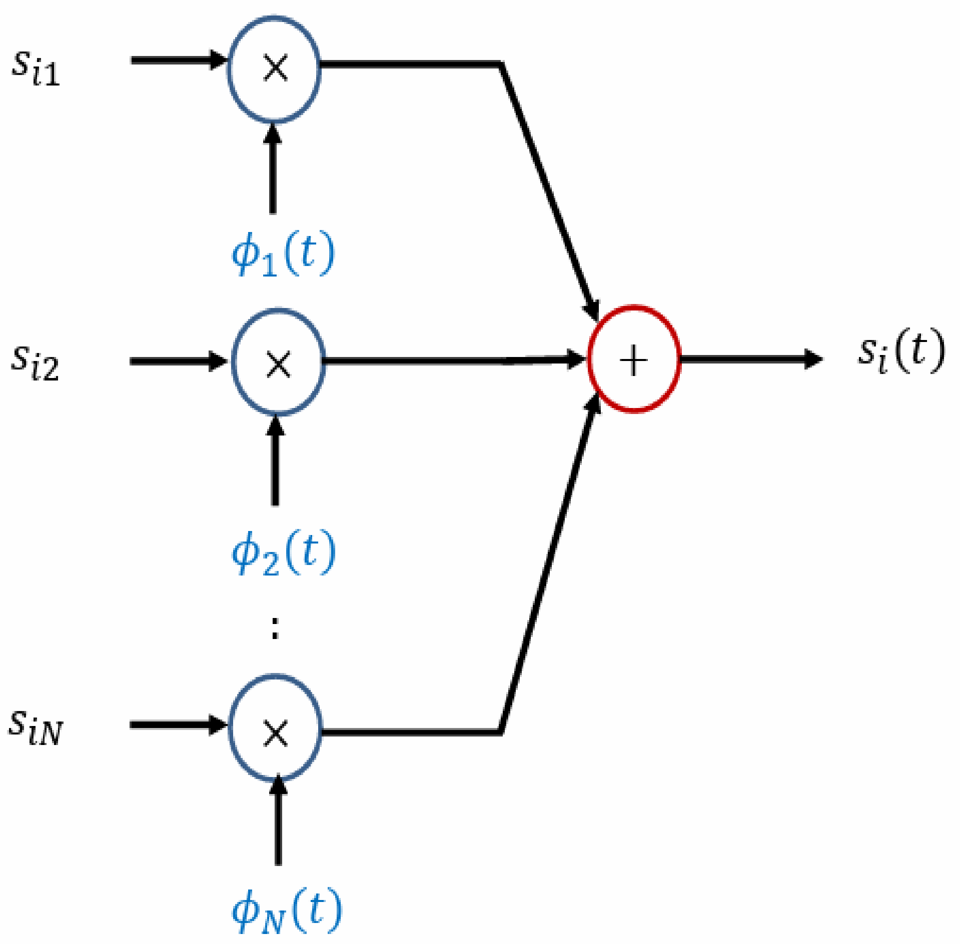
$$\int_0^T \phi_i(t)\phi_j(t)dt = \delta_{ij} = \begin{cases} 1, \text{if } i = j \\ 0, \text{if } i \neq j \end{cases}$$

where δ_{ij} is the *Kronecker delta*

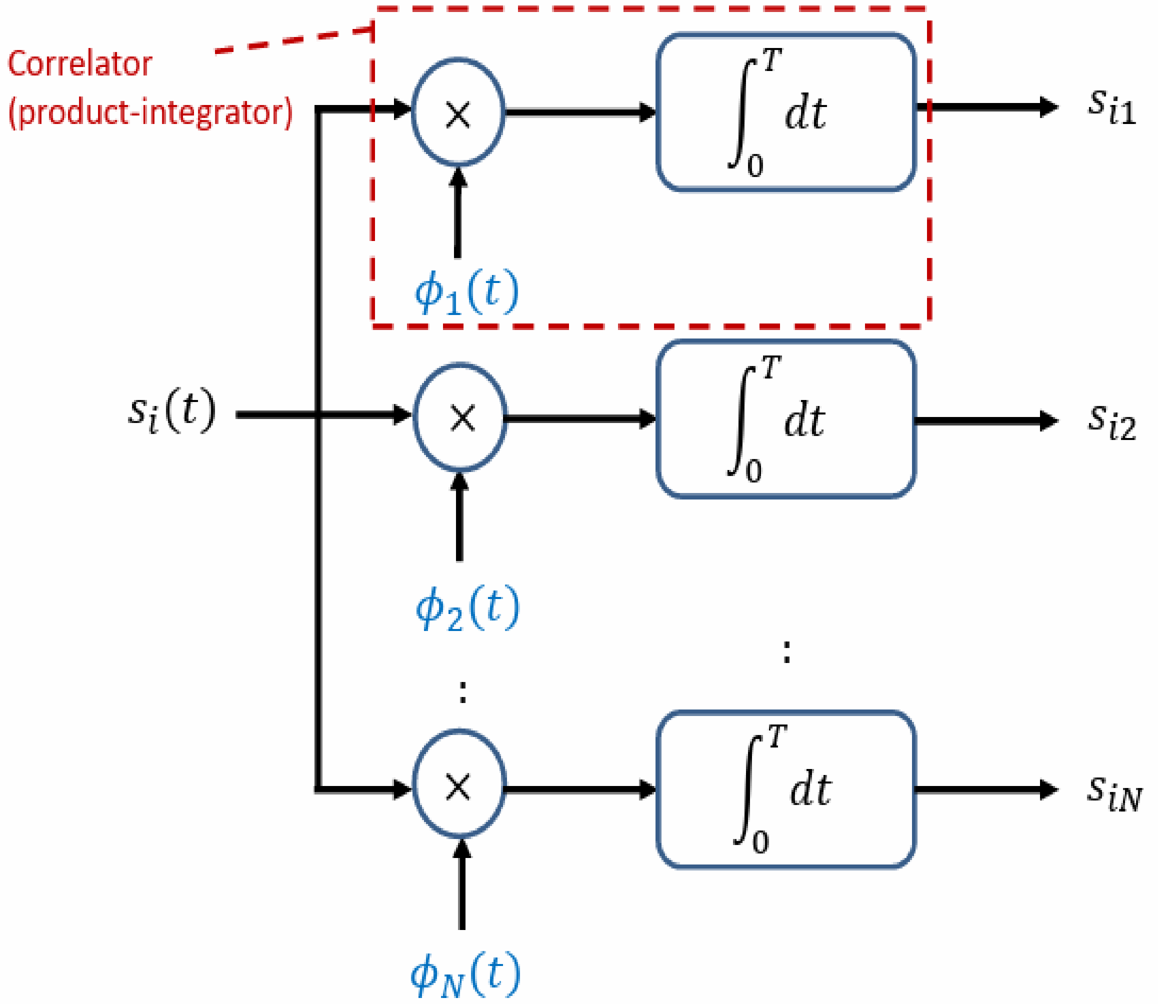
- Orthonormal basis functions imply two conditions:
    1. Each basis function is **normalized** to have unit energy
    2. Basis functions $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$ are **orthogonal** with respect to each other over the interval $0 \leq t \leq T$

- The set of coefficients $\{s_{ij}\}$, $j = 1, \dots, N$ can be visualized as an N**-dimensional vector**
- $(s_{i1}, s_{i2}, \dots, s_{iN})$ denoted by $\boldsymbol{s_i}$ which has one-to-one relation with the transmitted signal $s_i(t)$

Synthesizer for generating the signal $s_i(t)$



Analyzer for generating the elements of $\boldsymbol{s}_i$



- Each signal in the set $\{s_i(t)\}$ is determined by **signal vector**, $\boldsymbol{s_i}$ given by

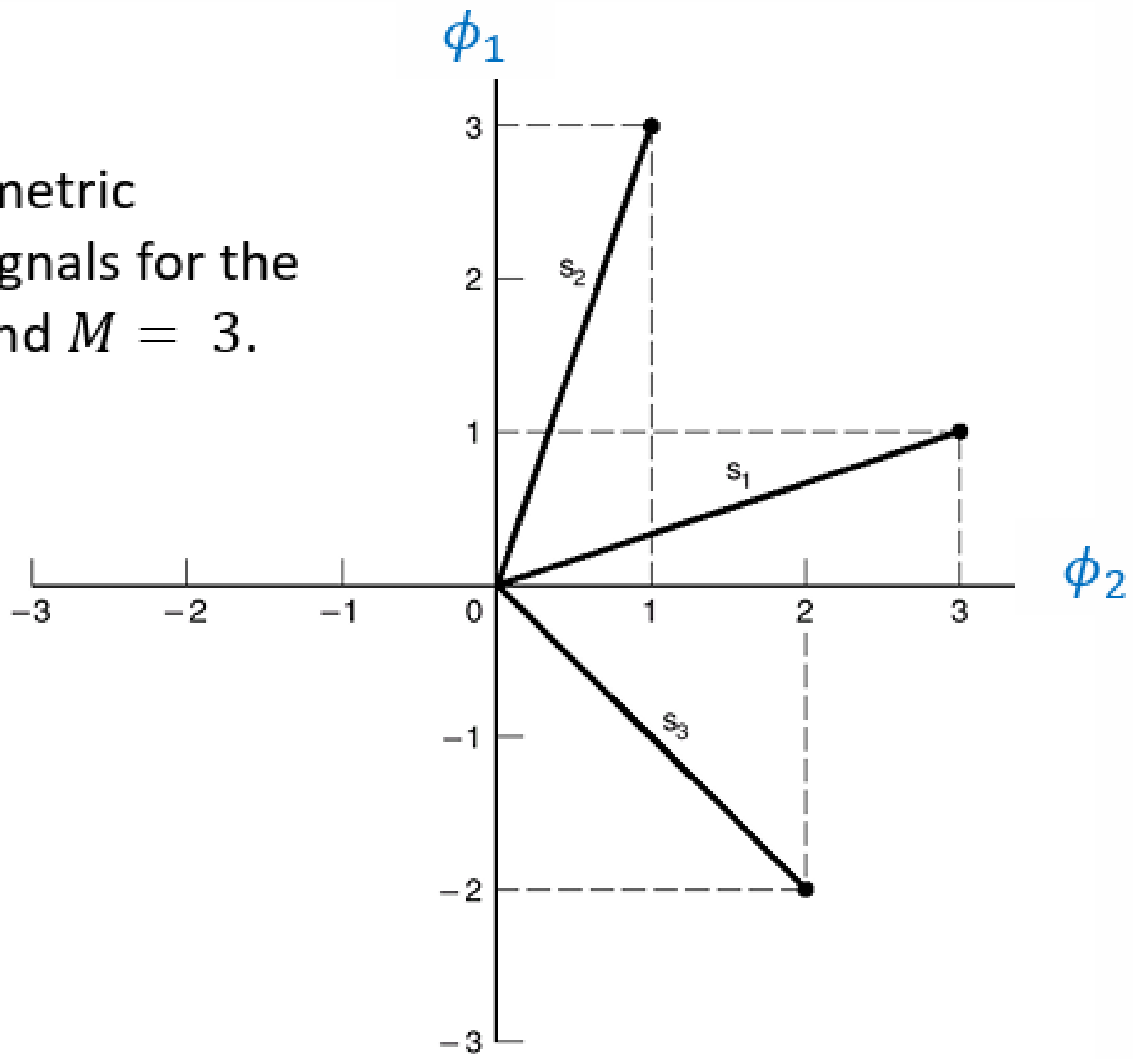
Column vector
$$\boldsymbol{s_i} = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, i = 1,2,\dots,M$$

- Extend the conventional notion of 2D and 3D (Euclidean space) to an N-dimensional Euclidean space.

→ Set of signal vectors $\{\boldsymbol{s_i} | i = 1,2,..,M\}$ represents set of M points in the N-dimensional Euclidean space (**signal space**)

- **What is the importance of representing energy signals geometrically?**
- For mathematical tractable analysis and providing the basis to noise consideration in digital communication system.

Illustrating the geometric representation of signals for the case when $N = 2$ and $M = 3$.



- $\|\boldsymbol{s_i}\|$ represents the norm (length) of signal vector $\boldsymbol{s_i}$
- Inner product (dot product) of $\boldsymbol{s_i}$

$$\|\boldsymbol{s_i}\|^2 = \boldsymbol{s_i}^T\boldsymbol{s_i} = \sum_{j=1}^{N} s_{ij}^2$$

where s_{ij} is the j^{th} element of $\boldsymbol{s_i}$, $(.)^T$ denotes matrix transposition

- **Show that energy of $\boldsymbol{s_i(t)} = \|\boldsymbol{s_i}\|^2$**

$$E_i = \int_0^T s_i^2(t)dt = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\,\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}\phi_k(t)\right]dt$$

$$= \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}\,s_{ik}\int_0^T \phi_j(t)\phi_k(t)dt = \sum_{j=1}^{N} s_{ij}^2 = \|\boldsymbol{s_i}\|^2$$

- Energy of a signal $s_i(t)$ equals to the squared length of signal vector $\boldsymbol{s_i}$

Euclidean distance

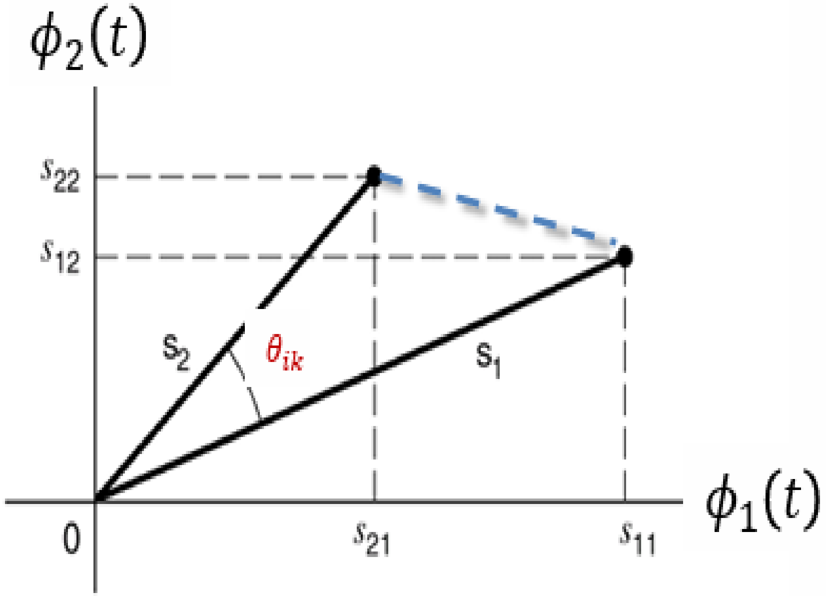
- Euclidean distance (d_{ik}) between two points represented by the signal vectors $\boldsymbol{s_i}$ and $\boldsymbol{s_k}$ is

$$d_{ik}^2 = \|\boldsymbol{s_i} - \boldsymbol{s_k}\|^2 = \sum_{j=1}^{N} \left(s_{ij} - s_{kj}\right)^2 = \int_0^T \left(s_i(t) - s_k(t)\right)^2 dt$$

- The angle, θ_{ik}, between two signal vectors $\boldsymbol{s_i}$ and $\boldsymbol{s_k}$ is given by

$$\cos\theta_{ik} = \frac{\boldsymbol{s_i^T s_k}}{\|\boldsymbol{s_i}\|\|\boldsymbol{s_k}\|}$$

- Signal vectors $\boldsymbol{s_i}$ and $\boldsymbol{s_k}$ are **orthogonal** (perpendicular) if $\boldsymbol{s_i^T s_k} = 0 \Rightarrow \theta_{ik} = 90°$



Schwarz Inequality

- For any pairs of energy signals $s_1(t)$ and $s_2(t)$,

$$\left(\int_{-\infty}^{\infty} s_1(t)s_2(t)dt\right)^2 \leq \int_{-\infty}^{\infty} s_1^2(t)dt \int_{-\infty}^{\infty} s_2^2(t)dt$$

Triangular Inequality

$$|\mathbf{s}_1(t) + \mathbf{s}_2(t)| \leq |\mathbf{s}_1(t)| + |\mathbf{s}_2(t)|$$

Formulas for Two Signals

- Assume we have a pair of signals: $s_i(t)$ and $s_j(t)$, each represented by its vector,
- Then:

$$s_{ij} = \int_0^T s_i(t) s_k(t) dt = s_i^T \mathrm{s}_\mathrm{k} \qquad (5.13)$$

Inner product of the signals is equal to the inner product of their vector representations [0,T]

Inner product is invariant to the selection of basis functions

Gram-Schmidt Orthogonalization Procedure

- The Gram–Schmidt orthogonalization procedure permits the representation of any set of M energy signals, as linear combinations of N orthonormal basis Auctions, where $N < M$.

- That is to say, we may represent the given set of real-valued energy signals $s_1(t), s_2(1), ..., s_m(t)$, each of duration T seconds, in the form

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t) \qquad \begin{array}{l} 0 \leqslant t \leqslant T \\ i = 1, 2, \ldots, M \end{array} \tag{1}$$

- where, the coefficients of the expansion are defined by,

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt \qquad \begin{array}{l} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{array} \tag{2}$$

- The real-valued basis functions $\phi_1 t, \phi_2(t), \ldots, \phi_N(t)$ are orthonormal, that is;

$$\int_0^T \phi_i(t)\phi_j(t)dt = \begin{cases} 1 & \text{if i = j} \\ 0 & \text{if i} \neq \text{j} \end{cases} \tag{3}$$

Stage-1

- Establish whether or not the given set of signals $s_1(t), s_2(t), \ldots, s_M(t)$ is linearly independent.
- If they are not linearly independent, then there exists a set of coefficients $a_1, a_2, \ldots, a_M$, not all equal to zero, such that we may write

$$a_1 s_1(t) + a_2 s_2(t) + \cdots + a_M s_M(t) = 0 \qquad 0 \leq t \leq T \tag{4}$$

- Suppose, in particular, that $a_M(t) \neq 0$. Then we may express the corresponding signal $s_M(t)$ as

$$s_M(t) = -\left[\frac{a_1}{a_M} s_1(t) + \frac{a_2}{a_M} s_2(t) + \cdots + \frac{a_{M-1}}{a_M} s_{M-1}(t)\right] \tag{5}$$

which implies that the signal $s_M(t)$ may be expressed in terms of the remaining $(M-1)$ signals.

- If they are not linearly independent, then there exists a set of coefficients $b_1, b_2, \ldots, b_{M-1}$, not all equal to zero, such that we may write

$$b_1 s_1(t) + b_2 s_2(t) + \cdots + b_{M-1} s_{M-1}(t) = 0 \qquad 0 \leq t \leq T \quad (6)$$

- Suppose, in particular, that $b_{M-1}(t) \neq 0$. Then we may express the corresponding signal $s_{M-1}(t)$ as linear combination of the remaining $M-2$ signals as

$$s_{M-1}(t) = -\left[\frac{b_1}{b_{M-1}} s_1(t) + \frac{b_2}{b_{M-1}} s_2(t) + \cdots + \frac{b_{M-2}}{b_{M-1}} s_{M-2}(t)\right] \quad (7)$$

- Let $s_1(t), s_2(t), \ldots, s_N(t)$ denote this subset of linearly independent signals, where $N \leq M$.

Stage-2

As a starting point, define the first basis function as

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \tag{8}$$

where, E_1 is the energy of the signal $s_1(t)$.

Rearraging Eq. (8) we get,

$$\begin{aligned} s_1(t) &= \sqrt{E_1}\phi_1(t) \\ &= s_{11}\phi_1(t) \end{aligned} \tag{9}$$

where, the coefficient $s_{11} = \sqrt{E_1}$ and $\phi_1(t)$ has unit energy.

To define the second basis function, we define a new intermediate function as

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \tag{10}$$

which is orthogonal to $\phi_1(t)$ over the interval $0 \leq t \leq T$.
The second basis function is then given by

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)dt}} \tag{11}$$

Substituting Eq. (11) in Eq. (10) and simplifying we get

$$\begin{aligned}
\phi_2(t) &= \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{\int_0^T (s_2(t) - s_{21}\phi_1(t))^2}} \\
&= \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{\int_0^T [s_2(t)]^2 dt + \int_0^T s_{21}^2\phi_1^2(t)dt - \int_0^T 2\ s_2(t)\ s_{21}\phi_1(t)dt}}
\end{aligned}$$

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{\int_0^T s_2^2(t)dt + s_{21}^2 \int_0^T \phi_1^2(t)dt - 2\ s_{21} \int_0^T\ s_2(t)\phi_1(t)dt}}$$

$$= \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 + s_{21}^2 - 2\ s_{21} \times s_{21}}}$$

$$= \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 + s_{21}^2 - 2\ s_{21}^2}}$$

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \tag{12}$$

where, E_2 is the energy of signal $s_2(t)$ given as

$$E_2 = \int_0^T s_2^2(t)dt$$

Continuing in this fashion, we may define

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\phi_j(t) \tag{13}$$

where the coefficients $s_{ij},\ j = 1, 2, \ldots, i-1$, are themselves defined by

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt \tag{14}$$

Then it follows that the set of functions

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)dt}} \tag{15}$$

forms an orthonormal set.

GS Procedure in Brief

Procedure

Step 1.

$$g_1(t) = s_1(t) \qquad \text{(direction)}$$

$$\phi_1(t) = \frac{g_1(t)}{\|g_1(t)\|} \qquad \text{(unit length)}$$

$$= \frac{s_1(t)}{\sqrt{E_1}} \qquad E_1 = \int_0^T s_1^2(t)\,dt$$

$$\Rightarrow s_1(t) = \sqrt{E_1}\,\phi_1(t) = s_{11}\phi_1(t)$$

where $s_{11} = \sqrt{E_1}$

and $\phi_1(t)$ has unit energy.

Step 2. Compute:

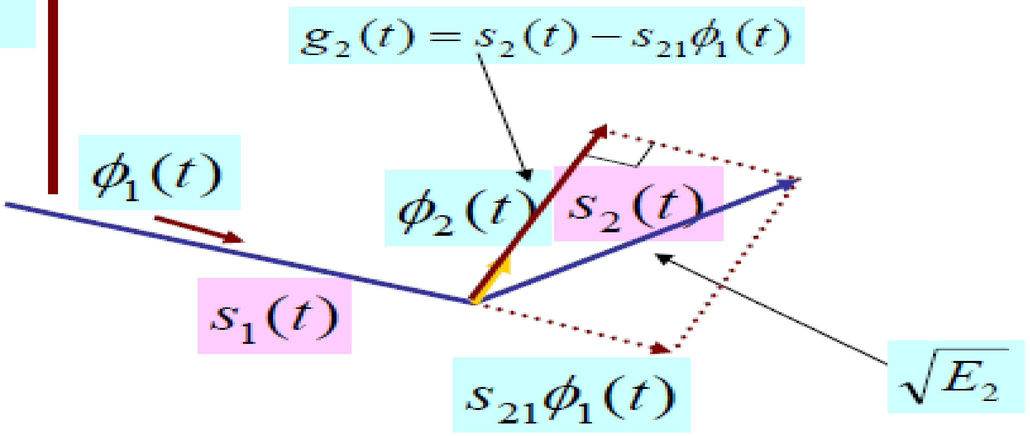
$$s_{21} = \int_0^T s_2(t)\phi_1(t)\,dt$$

Set

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \qquad (g_2(t) \perp \phi_1(t))$$

$$\Rightarrow \langle g_2(t), \phi_1(t) \rangle = 0 \qquad \text{(direction)}$$

($\phi_2(t)$ is the normalized version of $g_2(t)$)



Step 2.

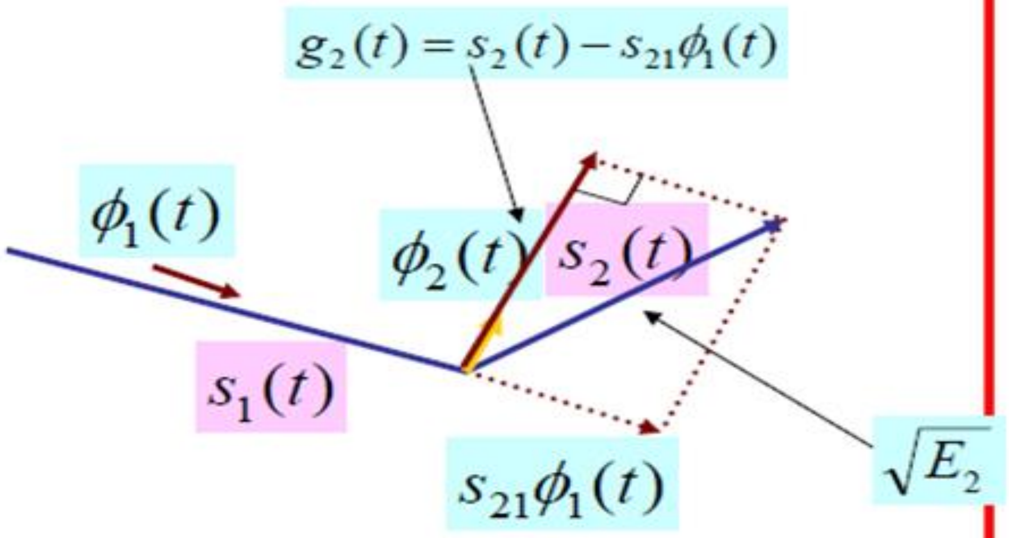
Compute: $s_{21} = \int_0^T s_2(t)\phi_1(t)dt$

Set

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \quad (g_2(t) \perp \phi_1(t))$$

$$\Rightarrow < g_2(t), \phi_1(t) > = 0 \quad \text{(direction)}$$

$\phi_2(t)$ is the normalized version of $g_2(t)$



Compute the norm of $g_2(t)$:

$$E_2 = \int_0^T s_2^2(t)dt$$

$$\|g_2(t)\| = \sqrt{\int_0^T g_2^2(t)dt} = \sqrt{E_2 - 2s_{21}^2 + s_{21}^2} = \sqrt{E_2 - s_{21}^2}$$

Set $\phi_2(t) = \dfrac{g_2(t)}{\|g_2(t)\|} = \dfrac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)dt}}$

$$\phi_2(t) = \frac{g_2(t)}{\|g_2(t)\|} = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}}$$

We have

$$\| \phi_2(t) \| = \int_0^T \phi_2^2(t)dt = 1$$

and $\int_0^T \phi_1(t)\phi_2(t)dt = 0$

Step *n*. Compute: $s_{nj} =< s_n(t), \phi_j(t) > , \ j = 1,\ldots,n\text{-}1$

$$g_n(t) = s_n(t) - \sum_{j=1}^{n-1} s_{nj}\phi_j(t) \qquad \text{(direction)}$$

$$\| g_n(t) \| = \sqrt{E_n - \sum_{j=1}^{n-1} s_{nj}^2}$$

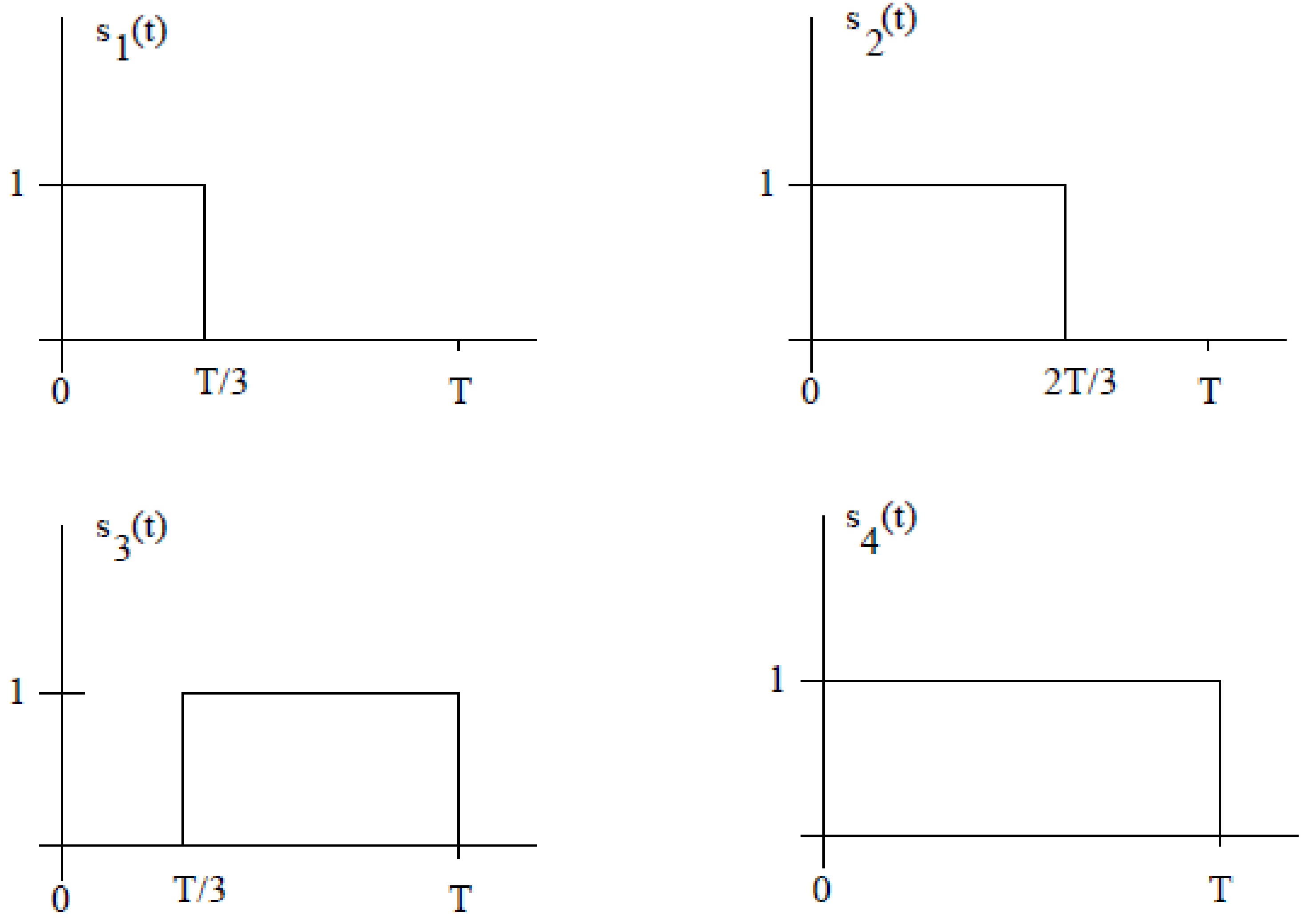
$$\left(E_n = \left\| s_n(t) \right\|^2 = \int_0^T s_n^2(t)\,dt \right)$$

$$\phi_n(t) = \frac{g_n(t)}{\left\| g_n(t) \right\|} \qquad \text{(unit length)}$$

$$= \frac{s_n(t) - \sum_{j=1}^{n-1} s_{nj}\phi_j(t)}{\sqrt{E_n - \sum_{j=1}^{n-1} s_{nj}^2}}$$

components of $s_n(t)$ already accounted for by $\phi_1(t),\ldots,\phi_{n-1}(t)$

Example. A set of four waveform is illustrated as below. Find an orthonormal set for this set of signals by applying the Gram-Schmidt procedure.



Step 1: This signal set is not linearly independent because

$$s_4(t) = s_1(t) + s_3(t)$$

Therefore, we will use $s_1(t)$, $s_2(t)$, and $s_3(t)$ to obtain the complete set of basis functions.

Step 2:

a)

$$E_1 = \int_0^T s_1^2(t)dt = T/3$$

$$f_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \begin{cases} \sqrt{3/T} \ , & 0 \leq t \leq T/3 \\ 0 \ , & \text{else} \end{cases}$$

b)

$$\begin{aligned} s_{21} &= \int_0^T s_2(t) f_1(t) dt \\ &= \int_0^{T/3} \sqrt{3/T} dt = \sqrt{T/3} \end{aligned}$$

$$E_2 = \int_0^T s_2^2(t) dt = 2T/3$$

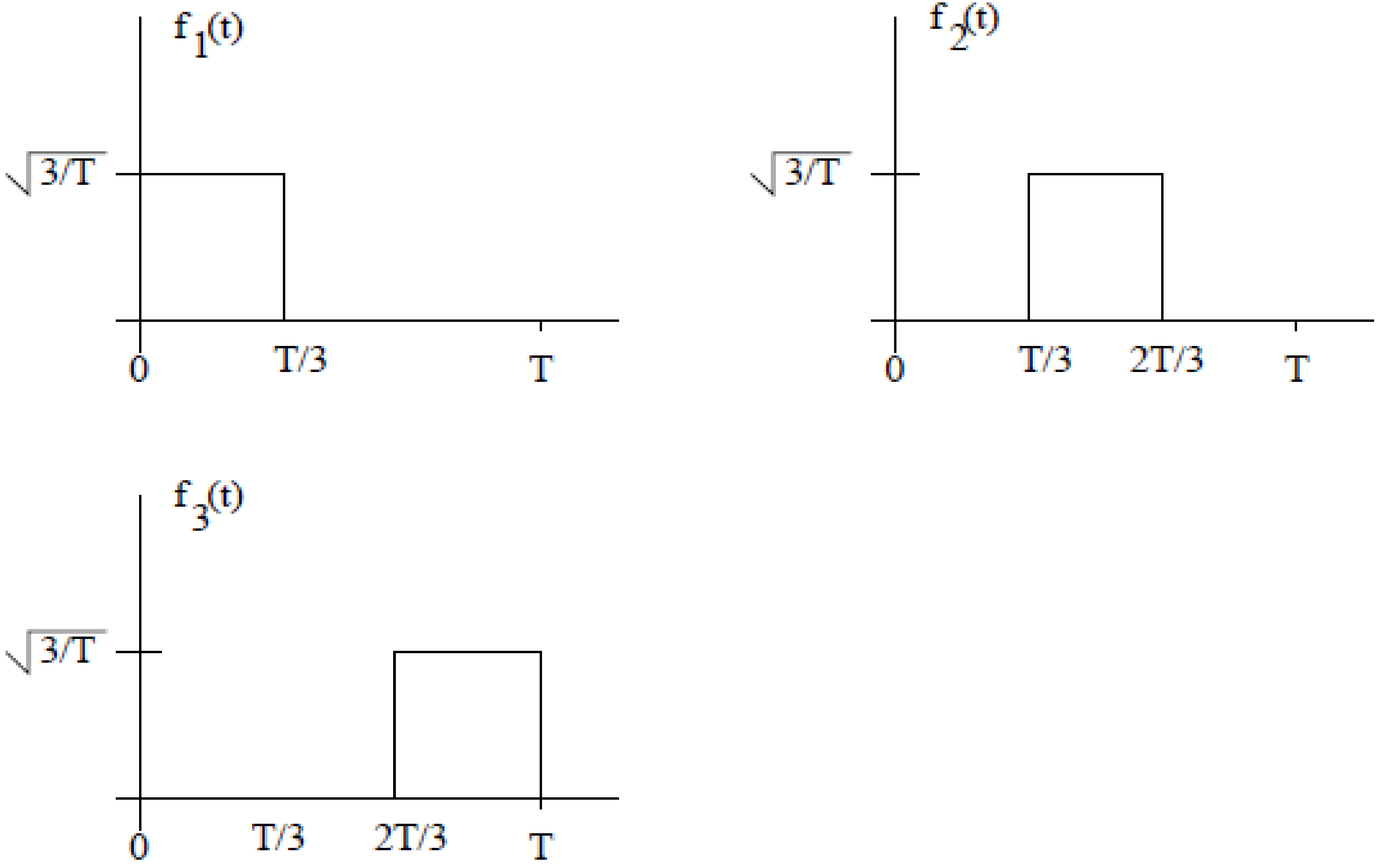
$$\begin{aligned} f_2(t) &= \frac{s_2(t) - s_{21} f_1(t)}{\sqrt{E_2 - s_{21}^2}} \\ &= \begin{cases} \sqrt{3/T} \ , & T/3 \le t \le 2T/3 \\ 0 \ , & \text{else} \end{cases} \end{aligned}$$

c)

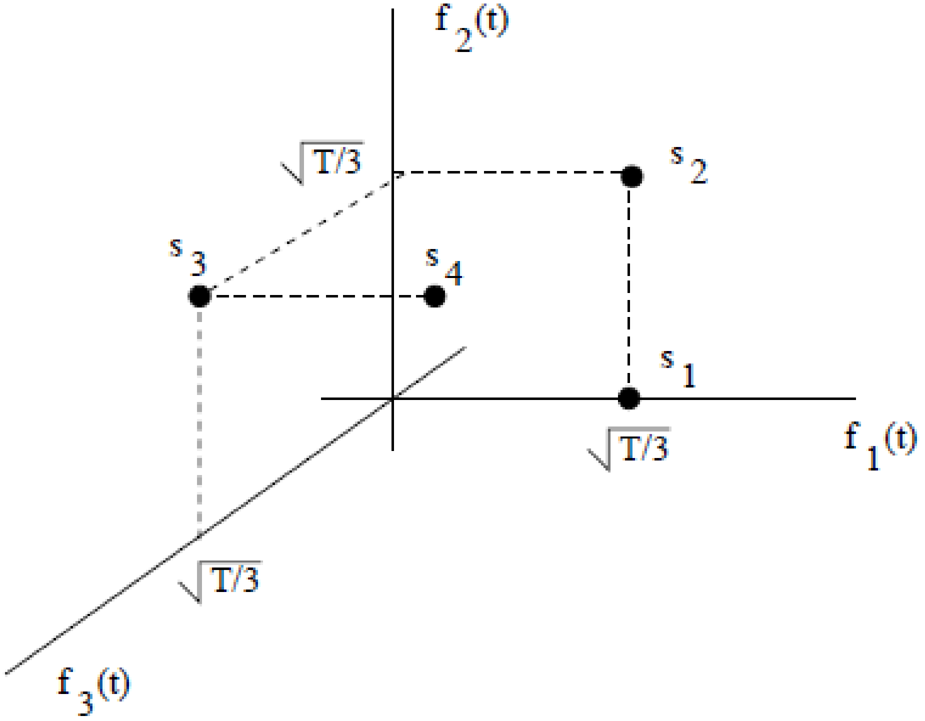
$$\begin{aligned} s_{31} &= \int_0^T s_3(t) f_1(t) dt = 0 \\ s_{32} &= \int_0^T s_3(t) f_2(t) dt \\ &= \int_{T/3}^{2T/3} \sqrt{3/T} dt = \sqrt{T/3} \end{aligned}$$

$$\begin{aligned} g_3(t) &= s_3(t) - s_{31} f_1(t) - s_{32} f_2(t) \\ &= \begin{cases} 1 \ , & 2T/3 \le t \le T \\ 0 \ , & \text{else} \end{cases} \end{aligned}$$

$$\begin{aligned} f_3(t) &= \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t) dt}} \\ &= \begin{cases} \sqrt{3/T} \ , & 2T/3 \le t \le T \\ 0 \ , & \text{else} \end{cases} \end{aligned}$$


$f_1(t)$
$\sqrt{3/T}$
0
T/3
T
$f_2(t)$
$\sqrt{3/T}$
0
T/3
2T/3
T
$f_3(t)$
$\sqrt{3/T}$
0
T/3
2T/3
T

$$
\begin{aligned}
s_1(t) &\leftrightarrow \mathbf{s}_1 = (\sqrt{T/3}, 0, 0) \\
s_2(t) &\leftrightarrow \mathbf{s}_2 = (\sqrt{T/3}, \sqrt{T/3}, 0) \\
s_3(t) &\leftrightarrow \mathbf{s}_3 = (0, \sqrt{T/3}, \sqrt{T/3}) \\
s_4(t) &\leftrightarrow \mathbf{s}_4 = (\sqrt{T/3}, \sqrt{T/3}, \sqrt{T/3})
\end{aligned}
$$

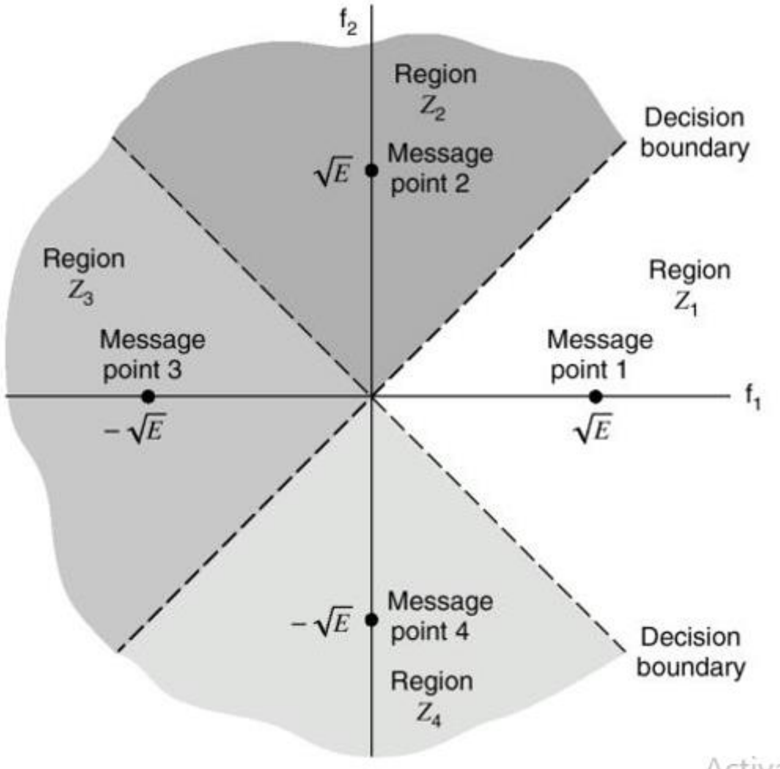


Constellation Diagram

- Geometrical representation of set of energy signals by assuming orthonormal basis function as geometrical axis is known as **Constellation Diagram.**
- Representing Waveform as points.

Properties

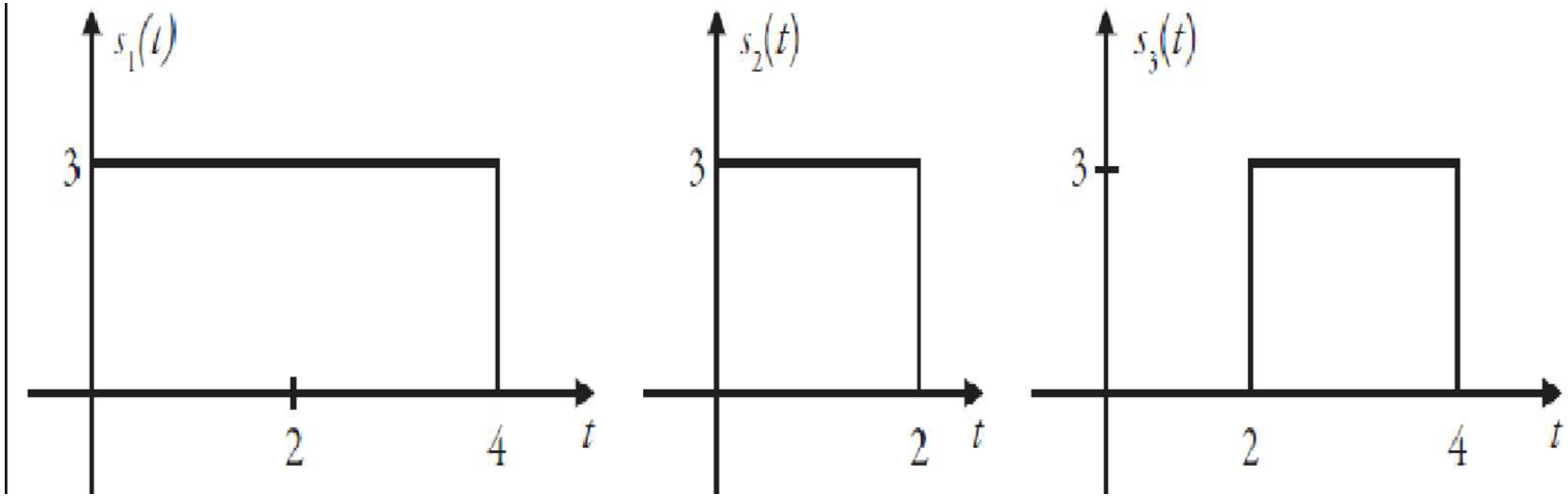
- Square of distance to any point from origin represents the energy of the signal by that point.
- Square of distance between two points represents the difference in energy between the signals represented by those points.
- As distance between points increases, the probability of error decreases.



Illustrating the partitioning of the observation space into decision regions for the case when N = 2 and M = 4; it is assumed that the M transmitted symbols are equally likely.

Question

Apply Gram Schmidt orthogonalisation to obtain orthonormal basis functions for the signals shown below. Express the signals in terms of orthonormal basis functions.



The signals are not linearly independent since $s_1(t)=s_2(t)+s_3(t)$

<u>Step I</u>

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{\int_0^T s_1^2(t)\,dt}} = \frac{s_1(t)}{s_{11}}$$

$$\int_0^T s_1^2(t)\,dt = \int_0^4 3^2\,dt = 9(t)_0^4 = 36$$

$$s_{11} = \sqrt{36} = 6$$

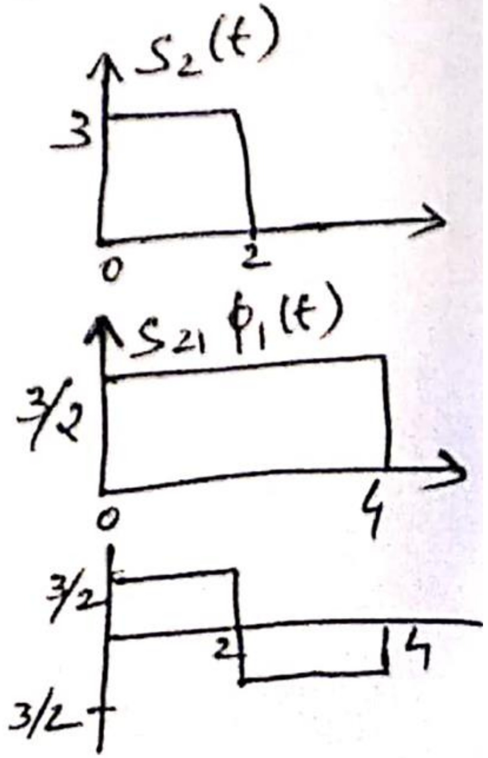
$$\phi_1(t) = \frac{s_1(t)}{\sqrt{36}} = \frac{s_1(t)}{6}$$

$$\phi_1(t) = \begin{cases} 1/2 & 0 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Step 2

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt$$

$$= \int_0^2 3\ \tfrac{1}{2}\ dt = \tfrac{3}{2}\,(t)_0^2 = 3$$



$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$

$$g_2(t) = \begin{cases} 3/2 & 0 \le t \le 2 \\ -3/2 & 2 \le t \le 4 \end{cases}$$

$$\phi_2(t) = \frac{g_2(t)}{\| g_2(t) \|}$$

$$\int_0^T g_2^2(t)\,dt = \int_0^2 (3/2)^2\,dt + \int_2^4 (-3/2)^2\,dt$$

$$= 9/4\,(2-0) + 9/4\,(4-2)$$

$$= 9/2 + 9/2 = 9$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}} = \frac{g_2(t)}{3}$$

$$\phi_2(t) = \begin{cases} 1/2 & 0 \le t \le 2 \\ -1/2 & 2 \le t \le 4 \end{cases}$$

$$S_{22} = \sqrt{\int_0^T g_2^2(t)\,dt} = 3$$

Step 3

$$s_{31} = \int_0^T s_3(t)\,\phi_1(t)\,dt = \int_2^4 3 \times \tfrac{1}{2}\,dt .$$

$$= \tfrac{3}{2}(4-2) = 3$$

$$s_{32} = \int_0^T s_3(t)\,\phi_2(t)\,dt$$

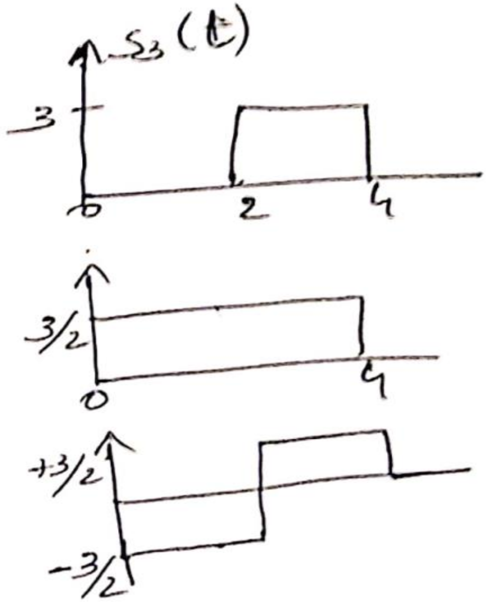
$$= \int_2^4 3 \times -\tfrac{1}{2}\,dt = -\tfrac{3}{2}(4-2) = -3$$

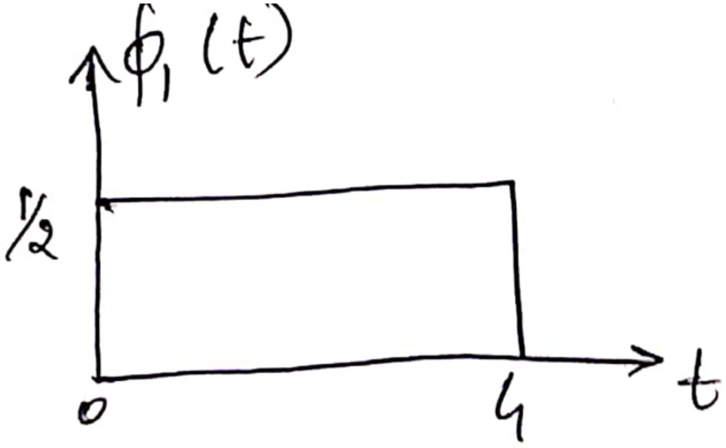
$$g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$$

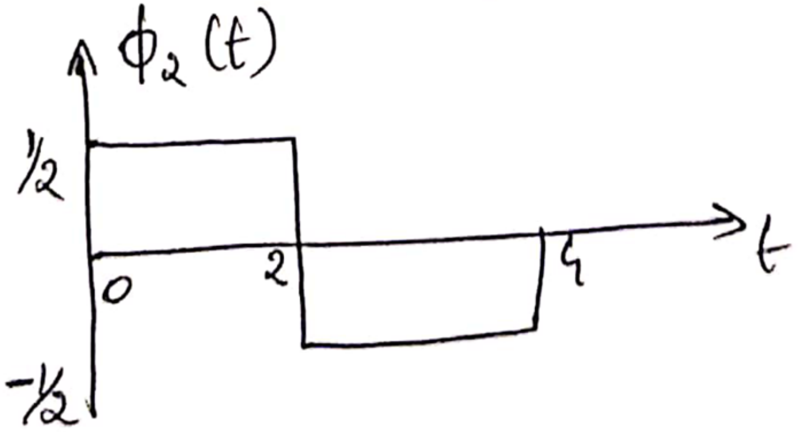
$$= 0$$

$\therefore$ $\phi_3(t)$ does not exit

$$s_{33} = 0$$





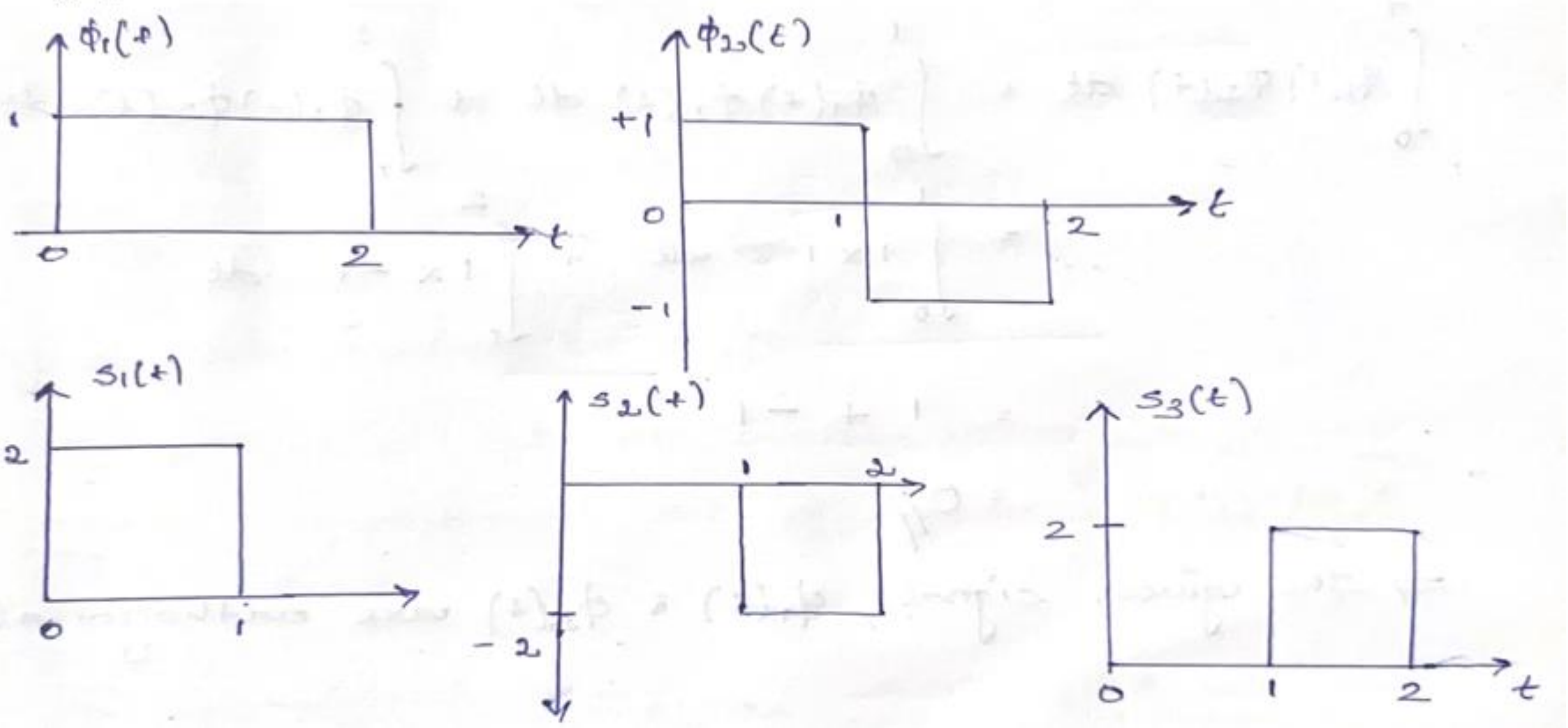


$$s_1(t) = s_{11}\,\phi_1(t) = 6\,\phi_1(t)$$
$$s_2(t) = s_{21}\,\phi_1(t) + s_{22}\,\phi_2(t) = 3\,\phi_1(t) + 3\,\phi_2(t)$$
$$s_3(t) = s_{31}\,\phi_1(t) + s_{32}\,\phi_2(t) = 3\,\phi_1(t) - 3\,\phi_2(t)$$

Q) i. Check whether the given signals ϕ_1 and ϕ_2 are orthogonal.

ii. Obtain corresponding orthonormal functions.

iii. Express the given signals S_1(t), $S_2(t)$ and $S_3(t)$ shown below in terms of ϕ_1 and ϕ_2.



Question:

Two functions $s_1(t)$ and $s_2(t)$ are given in Fig. 1

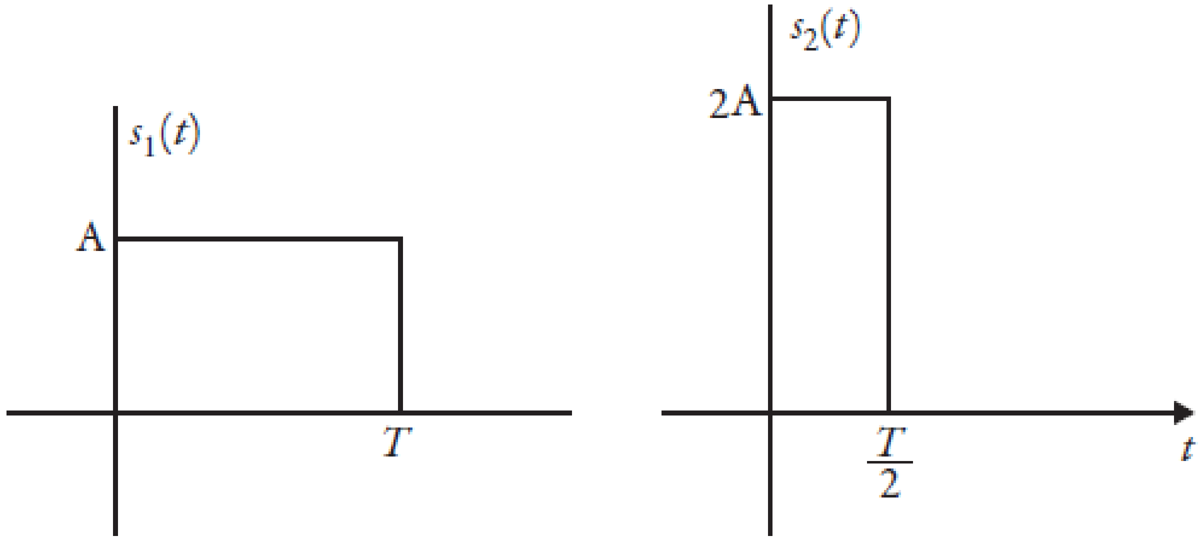
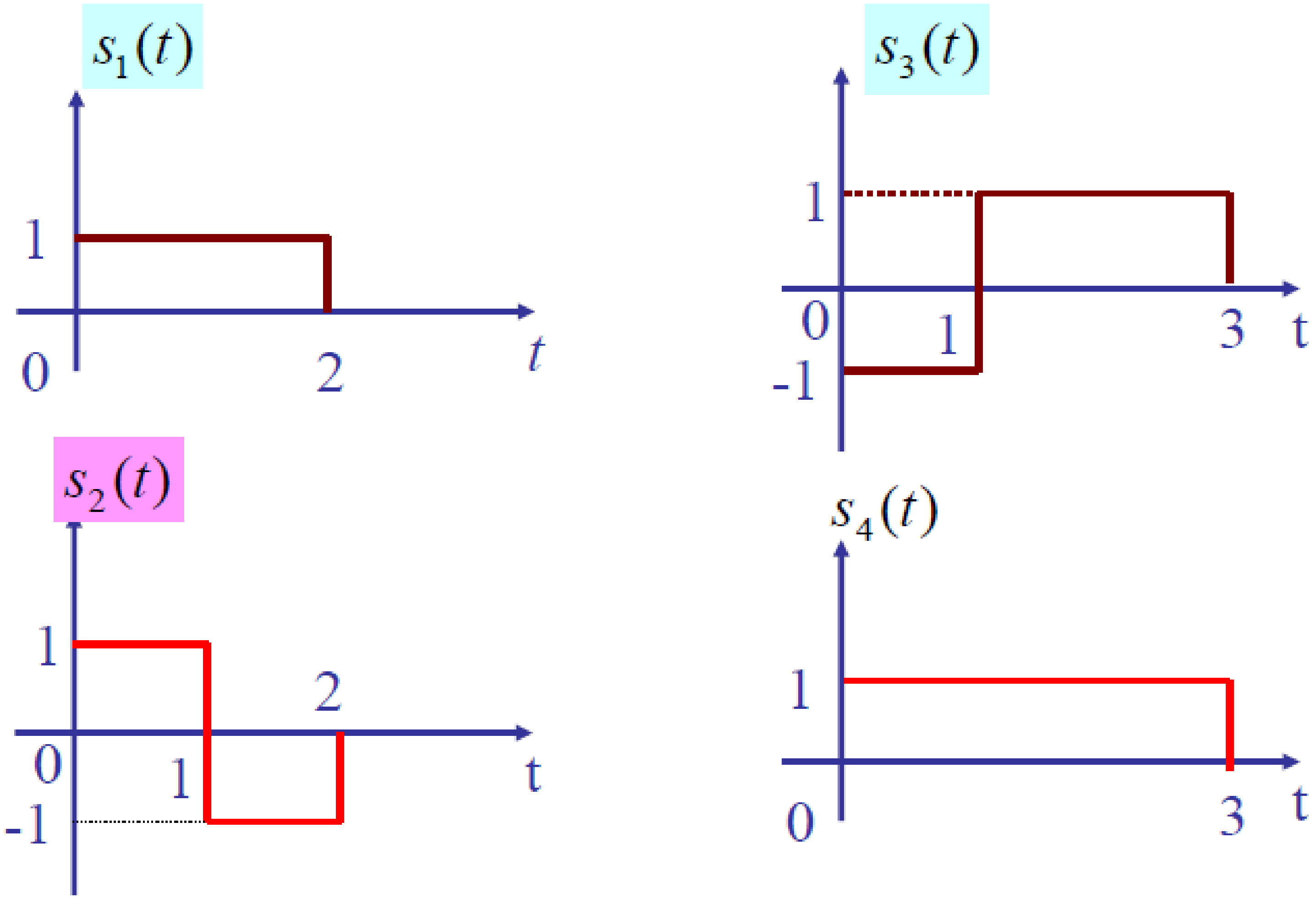


Fig. 1: $s_1(t)$ and $s_2(t)$

(1) Using the Gram–Schmidt orthogonalization procedure, express these functions in terms of orthonormal functions.

(2) Sketch $\phi_1(t)$ and $\phi_2(t)$.

Example. A set of four waveform is illustrated as below. Find an orthonormal set for this set of signals by applying the Gram-Schmidt procedure.



This signal set is not linearly independent because

$$s_4(t) = s_1(t) + s_2(t) + s_3(t)$$

Step 1. $E_1 = \int_0^T s_1^2(t)dt = \int_0^3 s_1^2(t)dt = \int_0^2 1\,dt = 2\,, \quad T = 3$

$$\phi_1(t) = s_1(t)/\sqrt{2}\,, \qquad 0 \le t \le 3$$

Step 2.

$$s_{21} = \int_0^T s_2(t)\phi_1(t)dt = \int_0^3 s_2(t)\phi_1(t)dt$$
$$= \frac{1}{\sqrt{2}}\int_0^2 1\,dt + \int_0^2 (-1\cdot 1)\,dt = 0$$

$$\Rightarrow \quad s_2(t) \perp \phi_1(t)$$

Set

$$\phi_2(t) = s_2(t)/\sqrt{E_2} = s_2(t)/\sqrt{2}$$
$$(\,E_2 = \int_0^T s_2^2(t)dt = 1 + 1 = 2)$$

Step 3.

$$s_{31} = \int_0^T s_3(t)\phi_1(t)dt = 0$$

$$s_{32} = \int_0^T s_3(t)\phi_2(t)dt = -\sqrt{2}$$

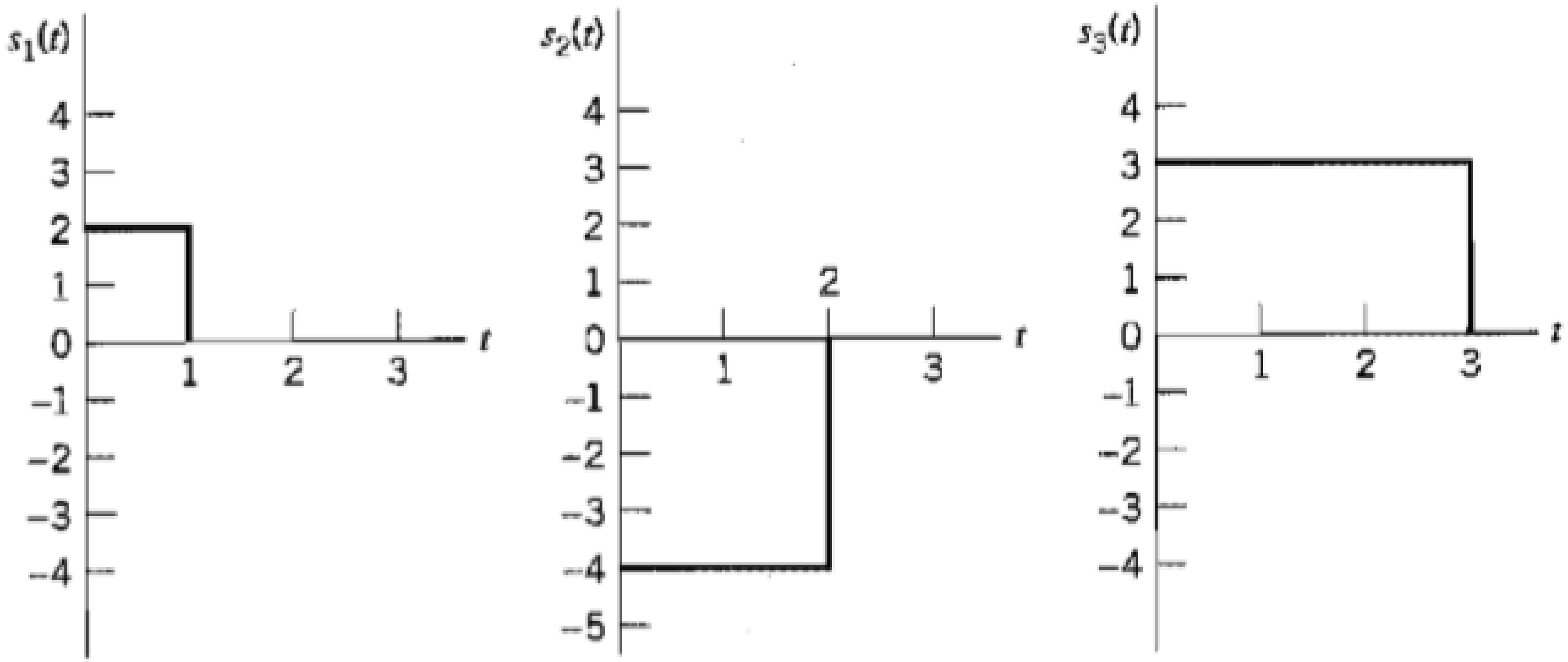
$$\Rightarrow \; g_3(t) = s_3(t) + \sqrt{2}\phi_2(t)$$

$$\|g_3(t)\| = \left\{\int_0^T g_3^2(t)dt\right\}^{1/2} = 1$$

$$\phi_3(t) = \frac{g_3(t)}{\| g_3(t) \|} = s_3(t) + \sqrt{2}\phi_2(t)$$

Question:

Using Gram Schmidt orthogonalization procedure, find the orthonormal basis functions for the signals $s_1(t), s_2(t), s_3(t)$ shown in the figure below



Question:

Apply Gram Schmidt orthogonalisation to obtain orthonormal basis functions for the signals shown below. Express the signals in terms of orthonormal basis functions.