ECT 305
ANALOG AND DIGITAL COMMUNICATION

Module-5

Digital Modulation Schemes

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Syllabus

- Digital modulation schemes.
- Baseband BPSK system and the signal constellation.
- BPSK transmitter and receiver.
- Base band QPSK system and Signal constellations.
- Plots of BER Vs SNR with analysis.
- QPSK transmitter and receiver.
- Quadrature amplitude modulation and signal constellation.

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Introduction

In *baseband pulse transmission*, which we studied in Chapter 4, a data stream represented in the form of a discrete pulse-amplitude modulated (PAM) signal is transmitted directly over a low-pass channel. In *digital passband transmission*, on the other hand, the incoming data stream is modulated onto a carrier (usually sinusoidal) with fixed frequency limits imposed by a band-pass channel of interest; passband data transmission is studied in this chapter.

The communication channel used for passband data transmission may be a microwave radio link, a satellite channel, or the like. Yet other applications of passband data transmission are in the design of passband line codes for use on digital subscriber loops and orthogonal frequency-division multiplexing techniques for broadcasting. In any event, the modulation process making the transmission possible involves switching (keying) the amplitude, frequency, or phase of a sinusoidal carrier in some fashion in accordance with the incoming data. Thus there are three basic signaling schemes, and they are known as

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Introduction

amplitude-shift keying (ASK), *frequency-shift keying* (FSK), and *phase-shift keying* (PSK). They may be viewed as special cases of amplitude modulation, frequency modulation, and phase modulation, respectively.

Figure 6.1 illustrates these three methods of modulation for the case of a source supplying binary data. The following points are noteworthy from Figure 6.1:

- Although in continuous-wave modulation it is usually difficult to distinguish between phase-modulated and frequency-modulated signals by merely looking at their waveforms, this is not true for PSK and FSK signals.
- Unlike ASK signals, both PSK and FSK signals have a constant envelope.

This latter property makes PSK and FSK signals impervious to amplitude nonlinearities, commonly encountered in microwave radio and satellite channels. It is for this reason, in practice, we find that PSK and FSK signals are preferred to ASK signals for passband data transmission over nonlinear channels.

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Waveforms

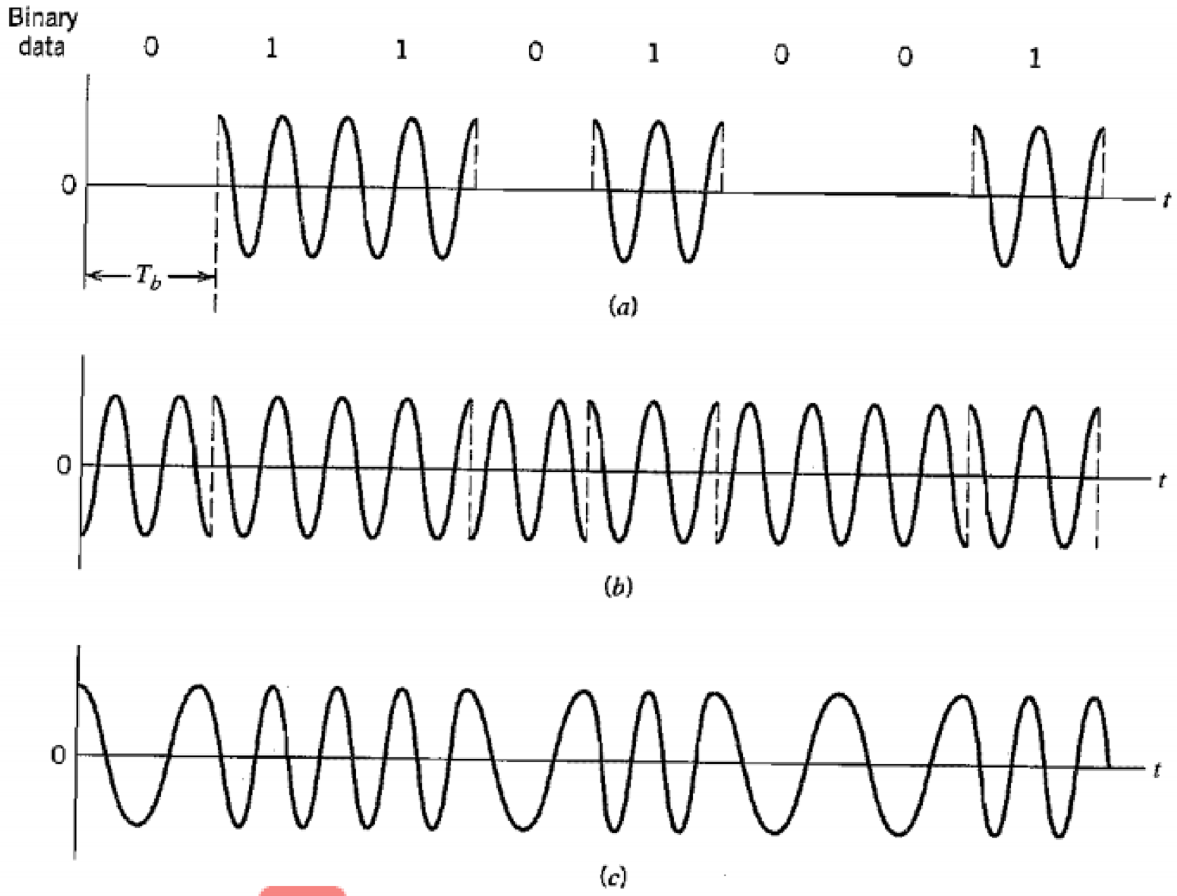

FIGURE 6.1 Illustrative waveforms for the three basic forms of signaling binary information. (*a*) Amplitude-shift keying. (*b*) Phase-shift keying. (*c*) Frequency-shift keying with continuous phase.

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Passband Transmission Model

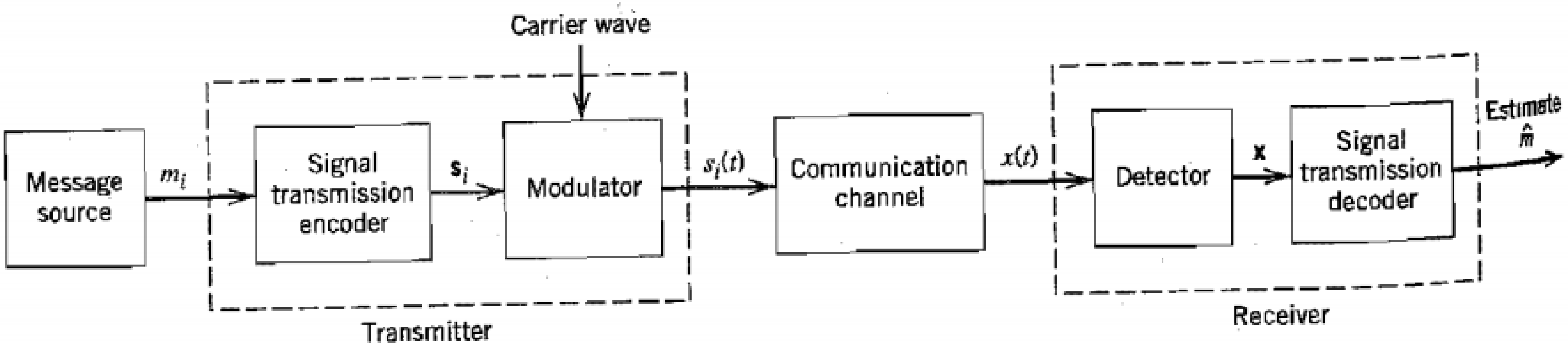

FIGURE 6.2 Functional model of passband data transmission system.

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Coherent Phase Shift Keying-BPSK

Binary Phase-Shift Keying

In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \tag{6.8}$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \tag{6.9}$$

where $0 \leq t \leq T_b$, and E_b is the *transmitted signal energy per bit*. To ensure that each transmitted bit contains an integral number of cycles of the carrier wave, the carrier frequency f_c is chosen equal to n_c/T_b for some fixed integer n_c. A pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees, as defined in Eqautions (6.8) and (6.9), are referred to as *antipodal signals*.

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Binary Phase Shift Keying

From this pair of equations it is clear that, in the case of binary PSK, there is only one basis function of unit energy, namely,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \qquad 0 \le t < T_b \tag{6.10}$$

Then we may express the transmitted signals $s_1(t)$ and $s_2(t)$ in terms of $\phi_1(t)$ as follows:

$$s_1(t) = \sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b \tag{6.11}$$

and

$$s_2(t) = -\sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b \tag{6.12}$$

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Binary Phase Shift Keying

A coherent binary PSK system is therefore characterized by having a signal space that is one-dimensional (i.e., $N = 1$), with a signal constellation consisting of two message points (i.e., $M = 2$). The coordinates of the message points are

$$\begin{aligned} s_{11} &= \int_0^{T_b} s_1(t)\phi_1(t)\,dt \\ &= +\sqrt{E_b} \end{aligned} \tag{6.13}$$

and

$$\begin{aligned} s_{21} &= \int_0^{T_b} s_2(t)\phi_1(t)\,dt \\ &= -\sqrt{E_b} \end{aligned} \tag{6.14}$$

The message point corresponding to $s_1(t)$ is located at $s_{11} = +\sqrt{E_b}$, and the message point corresponding to $s_2(t)$ is located at $s_{21} = -\sqrt{E_b}$. Figure 6.3 displays the signal-space diagram for binary PSK. This figure also includes two inserts, showing example waveforms of antipodal signals representing $s_1(t)$ and $s_2(t)$. Note that the constellation of Figure 6.3 has minimum average energy.

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Binary Phase Shift Keying

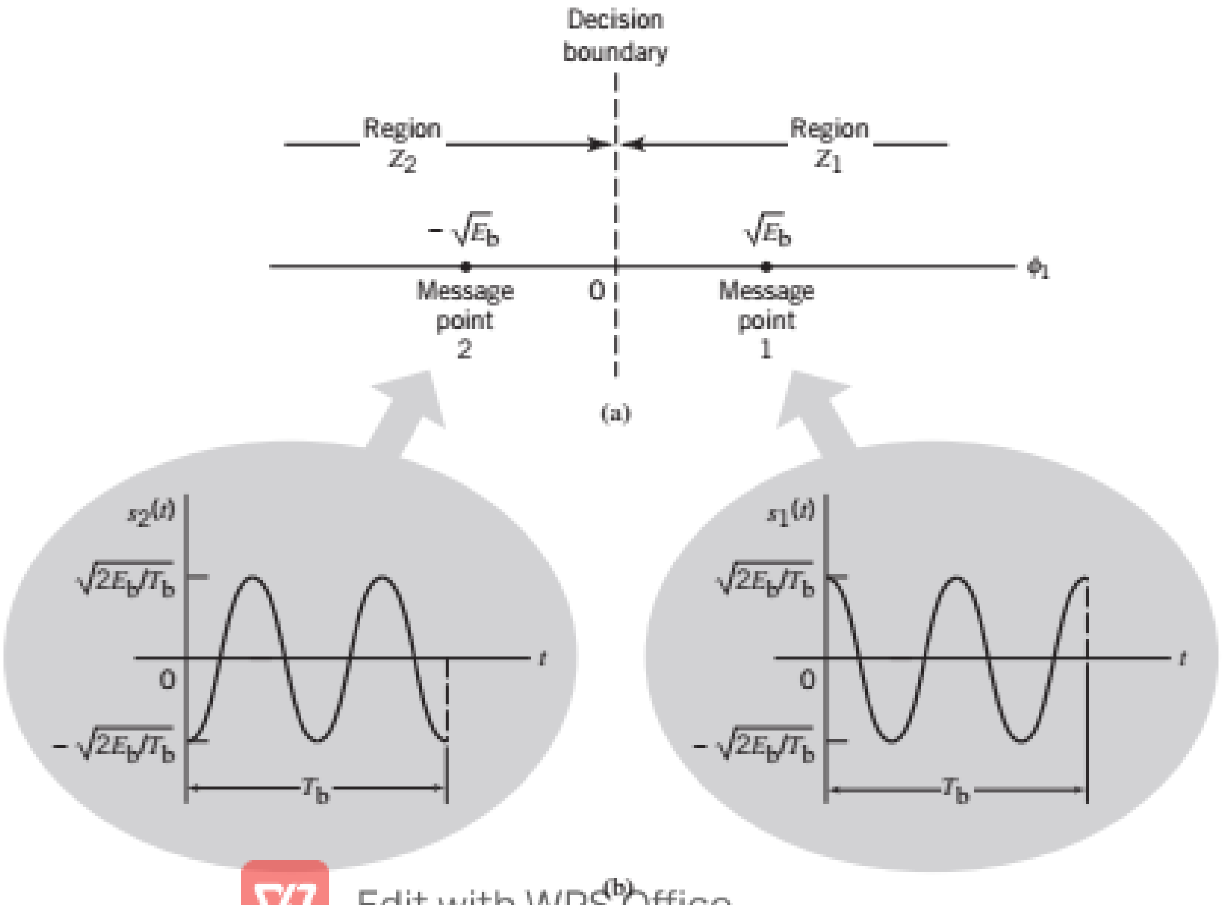

Figure 7.13
(a) Signal-space diagram for coherent binary PSK system. (b) The waveforms depicting the transmitted signals $s_1(t)$ and $s_2(t)$, assuming $n_c = 2$.

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Error Probability Binary Phase Shift Keying

Error Probability of Binary PSK

To realize a *rule for making a decision* in favor of symbol 1 or symbol 0, we apply Equation (5.59) of Chapter 5. Specifically, we partition the signal space of Figure 6.3 into two regions:

- The set of points closest to message point 1 at $+\sqrt{E_b}$.
- The set of points closest to message point 2 at $-\sqrt{E_b}$.

This is accomplished by constructing the midpoint of the line joining these two message points, and then marking off the appropriate decision regions. In Figure 6.3 these decision regions are marked Z_1 and Z_2, according to the message point around which they are constructed.

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Error Probability Binary Phase Shift Keying

The decision rule is now simply to decide that signal $s_1(t)$ (i.e., binary symbol 1) was transmitted if the received signal point falls in region Z_1, and decide that signal $s_2(t)$ (i.e., binary symbol 0) was transmitted if the received signal point falls in region Z_2. Two kinds of erroneous decisions may, however, be made. Signal $s_2(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_1 and so the receiver decides in favor of signal $s_1(t)$. Alternatively, signal $s_1(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_2 and so the receiver decides in favor of signal $s_2(t)$.

To calculate the probability of making an error of the first kind, we note from Figure 6.3 that the decision region associated with symbol 1 or signal $s_1(t)$ is described by

$$Z_1\colon 0 < x_1 < \infty$$

where the observable element x_1 is related to the received signal $x(t)$ by

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\, dt \qquad (6.15)$$

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Error Probability Binary Phase Shift Keying

The conditional probability density function of random variable X_1, given that symbol 0 [i.e., signal $s_2(t)$] was transmitted, is defined by

$$\begin{aligned} f_{X_1}(x_1|0) &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right] \\ &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] \end{aligned} \tag{6.16}$$

The conditional probability of the receiver deciding in favor of symbol 1, given that symbol 0 was transmitted, is therefore

$$\begin{aligned} p_{10} &= \int_0^\infty f_{X_1}(x_1|0)\, dx_1 \\ &= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1 \end{aligned} \tag{6.17}$$

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Error Probability Binary Phase Shift Keying

Putting

$$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b}) \tag{6.18}$$

and changing the variable of integration from x_1 to z, we may rewrite Equation (6.17) in the compact form

$$\begin{aligned} p_{10} &= \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\, dz \\ &= \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \end{aligned} \tag{6.19}$$

$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

where erfc(·) is the complementary error function.

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Error Probability Binary Phase Shift Keying

Consider next an error of the second kind. We note that the signal space of Figure 6.3 is symmetric with respect to the origin. It follows therefore that p_{01}, the conditional probability of the receiver deciding in favor of symbol 0, given that symbol 1 was transmitted, also has the same value as in Equation (6.19).

Thus, averaging the conditional error probabilities p_{10} and p_{01}, we find that the *average probability of symbol error* or, equivalently, the *bit error rate for coherent binary PSK* is (assuming equiprobable symbols)

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{6.20}$$

As we increase the transmitted signal energy per bit, E_b, for a specified noise spectral density N_0, the message points corresponding to symbols 1 and 0 move further apart, and the average probability of error P_e is correspondingly reduced in accordance with Equation (6.20), which is intuitively satisfying.

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Generation Binary Phase Shift Keying

To generate a binary PSK signal, we see from Equations (6.8)–(6.10) that we have to represent the input binary sequence in polar form with symbols 1 and 0 represented by constant amplitude levels of $+\sqrt{E_b}$ and $-\sqrt{E_b}$, respectively. This signal transmission encoding is performed by a polar nonreturn-to-zero (NRZ) level encoder. The resulting binary wave and a sinusoidal carrier $\phi_1(t)$, whose frequency $f_c = (n_c/T_b)$ for some fixed integer n_c, are applied to a product modulator, as in Figure 6.4*a*. The carrier and the timing pulses used to generate the binary wave are usually extracted from a common master clock. The desired PSK wave is obtained at the modulator output.

To detect the original binary sequence of 1s and 0s, we apply the noisy PSK signal $x(t)$ (at the channel output) to a correlator, which is also supplied with a locally generated coherent reference signal $\phi_1(t)$, as in Figure 6.4*b*. The correlator output, x_1, is compared with a threshold of zero volts. If $x_1 > 0$, the receiver decides in favor of symbol 1. On the other hand, if $x_1 < 0$, it decides in favor of symbol 0. If x_1 is exactly zero, the receiver makes a random guess in favor of 0 or 1.

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Generation Binary Phase Shift Keying

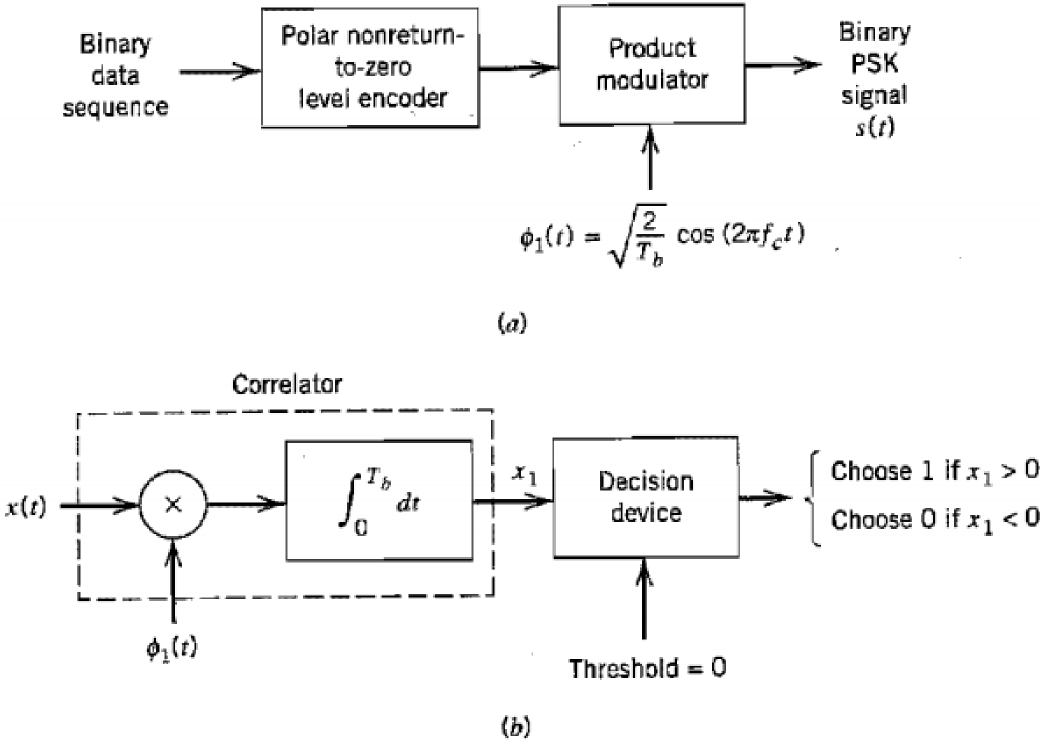

FIGURE 6.4 Block diagrams for (*a*) binary PSK transmitter and (*b*) coherent binary PSK receiver.

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Power Spectra of Binary PSK Signals

From the modulator of Figure 6.4*a*, we see that the complex envelope of a binary PSK wave consists of an in-phase component only. Furthermore, depending on whether we have symbol 1 or symbol 0 at the modulator input during the signaling interval $0 \le t \le T_b$, we find that this in-phase component equals $+g(t)$ or $-g(t)$, respectively, where $g(t)$ is the *symbol shaping function* defined by

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}, & 0 \le t \le T_b \\ 0, & \text{otherwise} \end{cases} \tag{6.21}$$

We assume that the input binary wave is random, with symbols 1 and 0 equally likely and the symbols transmitted during the different time slots being statistically independent. In Example 1.6 of Chapter 1 it is shown that the power spectral density of a random binary wave so described is equal to the energy spectral density of the symbol shaping function divided by the symbol duration. The energy spectral density of a Fourier transformable signal $g(t)$ is defined as the squared magnitude of the signal's Fourier transform. Hence, the baseband power spectral density of a binary PSK signal equals

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$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2} = 2E_b \operatorname{sinc}^2(T_b f) \tag{6.22}$$

This power spectrum falls off as the inverse square of frequency, as shown in Figure 6.5.

Figure 6.5 also includes a plot of the baseband power spectral density of a binary FSK signal, details of which are presented in Section 6.5. Comparison of these two spectra is deferred to that section.

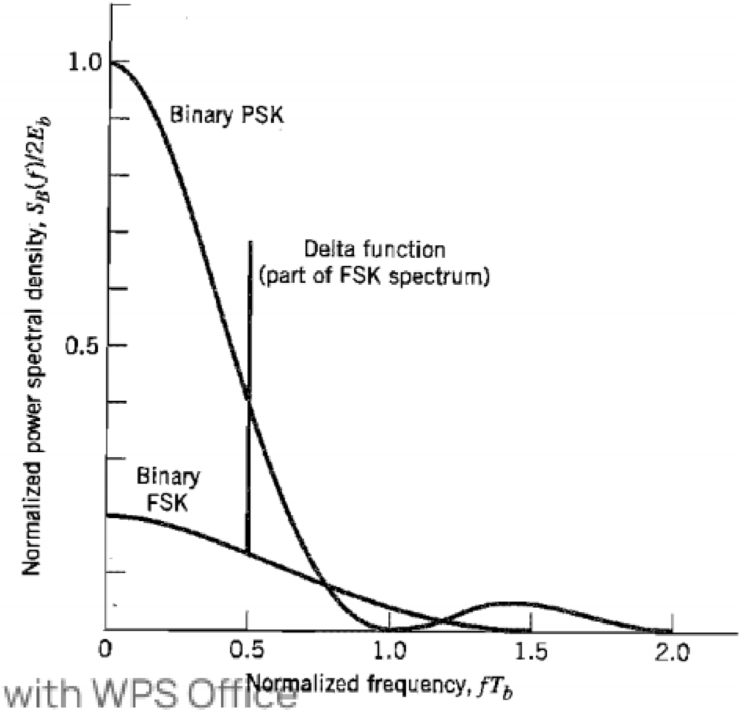

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FIGURE 6.5 Power spectra of binary PSK and FSK signals.

		Complementary Error Function Table											
x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)
0	1.000000	0.5	0.479500	1	0.157299	1.5	0.033895	2	0.004678	2.5	0.000407	3	0.00002209
0.01	0.988717	0.51	0.470756	1.01	0.153190	1.51	0.032723	2.01	0.004475	2.51	0.000386	3.01	0.00002074
0.02	0.977435	0.52	0.462101	1.02	0.149162	1.52	0.031587	2.02	0.004281	2.52	0.000365	3.02	0.00001947
0.03	0.966159	0.53	0.453536	1.03	0.145216	1.53	0.030484	2.03	0.004094	2.53	0.000346	3.03	0.00001827
0.04	0.954889	0.54	0.445061	1.04	0.141350	1.54	0.029414	2.04	0.003914	2.54	0.000328	3.04	0.00001714
0.05	0.943628	0.55	0.436677	1.05	0.137564	1.55	0.028377	2.05	0.003742	2.55	0.000311	3.05	0.00001608
0.06	0.932378	0.56	0.428384	1.06	0.133856	1.56	0.027372	2.06	0.003577	2.56	0.000294	3.06	0.00001508
0.07	0.921142	0.57	0.420184	1.07	0.130227	1.57	0.026397	2.07	0.003418	2.57	0.000278	3.07	0.00001414
0.08	0.909922	0.58	0.412077	1.08	0.126674	1.58	0.025453	2.08	0.003266	2.58	0.000264	3.08	0.00001326
0.09	0.898719	0.59	0.404064	1.09	0.123197	1.59	0.024538	2.09	0.003120	2.59	0.000249	3.09	0.00001243
0.1	0.887537	0.6	0.396144	1.1	0.119795	1.6	0.023652	2.1	0.002979	2.6	0.000236	3.1	0.00001165
0.11	0.876377	0.61	0.388319	1.11	0.116467	1.61	0.022793	2.11	0.002845	2.61	0.000223	3.11	0.00001092
0.12	0.865242	0.62	0.380589	1.12	0.113212	1.62	0.021962	2.12	0.002716	2.62	0.000211	3.12	0.00001023
0.13	0.854133	0.63	0.372954	1.13	0.110029	1.63	0.021157	2.13	0.002593	2.63	0.000200	3.13	0.00000958
0.14	0.843053	0.64	0.365414	1.14	0.106918	1.64	0.020378	2.14	0.002475	2.64	0.000189	3.14	0.00000897
0.15	0.832004	0.65	0.357971	1.15	0.103876	1.65	0.019624	2.15	0.002361	2.65	0.000178	3.15	0.00000840
0.16	0.820988	0.66	0.350623	1.16	0.100904	1.66	0.018895	2.16	0.002253	2.66	0.000169	3.16	0.00000786
0.17	0.810008	0.67	0.343372	1.17	0.098000	1.67	0.018190	2.17	0.002149	2.67	0.000159	3.17	0.00000736
0.18	0.799064	0.68	0.336218	1.18	0.095163	1.68	0.017507	2.18	0.002049	2.68	0.000151	3.18	0.00000689
0.19	0.788160	0.69	0.329160	1.19	0.092392	1.69	0.016847	2.19	0.001954	2.69	0.000142	3.19	0.00000644

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Quadrature Phase Shift Keying

The provision of reliable performance, exemplified by a very low probability of error, is one important goal in the design of a digital communication system. Another important goal is the efficient utilization of channel bandwidth. In this subsection, we study a bandwidth-conserving modulation scheme known as coherent quadriphase-shift keying, which is an example of *quadrature-carrier multiplexing*.

In *quadriphase-shift keying* (QPSK), as with binary PSK, information carried by the transmitted signal is contained in the phase. In particular, the phase of the carrier takes on one of four equally spaced values, such as $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$. For this set of values we may define the transmitted signal as

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Quadrature Phase Shift Keying

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}} \cos\left[2\pi f_c t + (2i - 1)\dfrac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \tag{6.23}$$

where $i = 1, 2, 3, 4$; E is the transmitted signal energy per symbol, and T is the symbol duration. The carrier frequency f_c equals n_c/T for some fixed integer n_c. Each possible value of the phase corresponds to a unique dibit. Thus, for example, we may choose the foregoing set of phase values to represent the *Gray-encoded* set of dibits: 10, 00, 01, and 11, where only a single bit is changed from one dibit to the next.

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Quadrature Phase Shift Keying

Signal-Space Diagram of QPSK

Using a well-known trigonometric identity, we may use Equation (6.23) to redefine the transmitted signal $s_i(t)$ for the interval $0 \le t \le T$ in the equivalent form:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t) \quad (6.24)$$

where $i = 1, 2, 3, 4$. Based on this representation, we can make the following observations:

- There are two orthonormal basis functions, $\phi_1(t)$ and $\phi_2(t)$, contained in the expansion of $s_i(t)$. Specifically, $\phi_1(t)$ and $\phi_2(t)$ are defined by a pair of *quadrature carriers*:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \qquad 0 \le t \le T \quad (6.25)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \le t \le T \quad (6.26)$$

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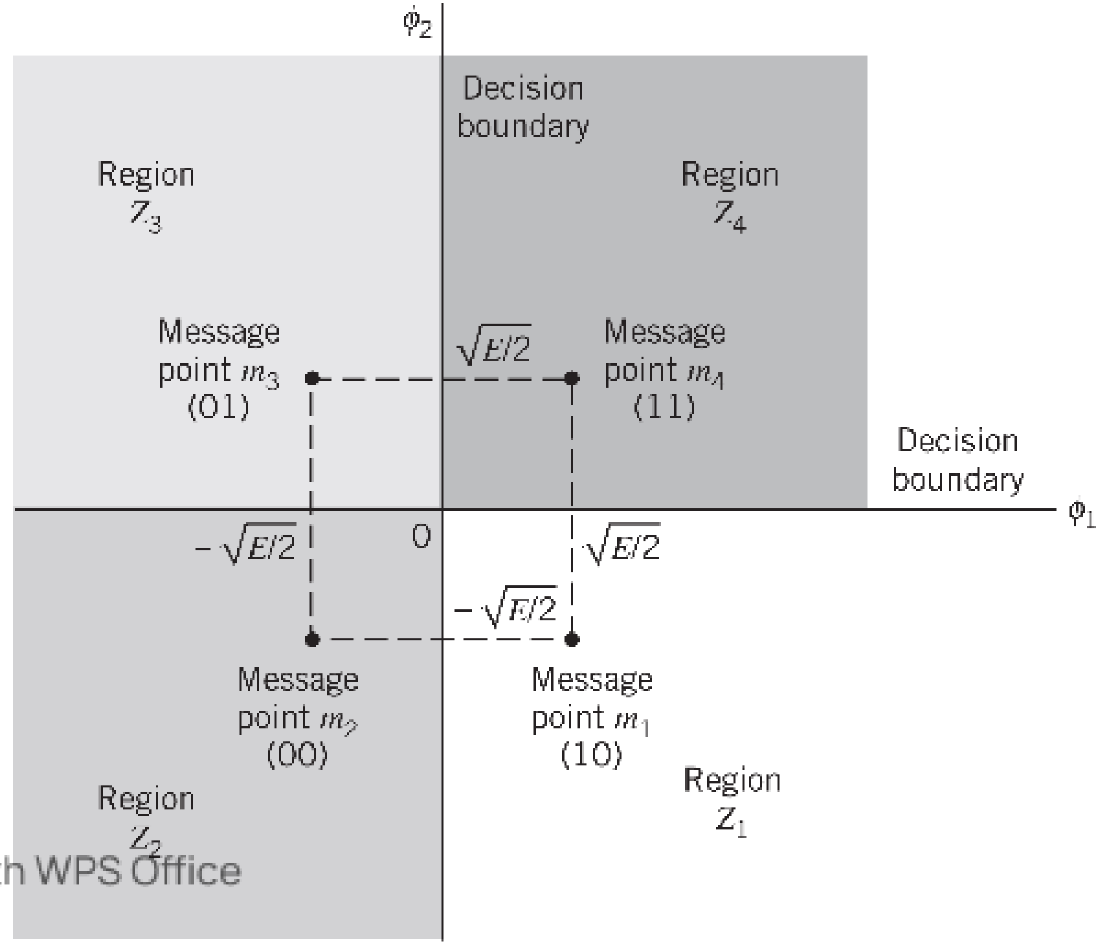
Quadrature Phase Shift Keying

TABLE 6.1 *Signal-space characterization of QPSK*

Gray-encoded Input Dibit	*Phase of QPSK Signal (radians)*	*Coordinates of Message Points*	
		s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$



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Quadrature Phase Shift Keying

- There are four message points, and the associated signal vectors are defined by

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E}\cos\left((2i-1)\dfrac{\pi}{4}\right) \\ -\sqrt{E}\sin\left((2i-1)\dfrac{\pi}{4}\right) \end{bmatrix}, \qquad i = 1, 2, 3, 4 \qquad (6.27)$$

The elements of the signal vectors, namely, s_{i1} and s_{i2}, have their values summarized in Table 6.1. The first two columns of this table give the associated dibit and phase of the QPSK signal.

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Quadrature Phase Shift Keying

Figure 6.7 illustrates the sequences and waveforms involved in the generation of a QPSK signal. The input binary sequence 01101000 is shown in Figure 6.7*a*. This sequence is divided into two other sequences, consisting of odd- and even-numbered bits of the input sequence. These two sequences are shown in the top lines of Figures 6.7*b* and 6.7*c*. The waveforms representing the two components of the QPSK signal, namely, $s_{i1}\phi_1(t)$ and $s_{i2}\phi_2(t)$, are also shown in Figures 6.7*b* and 6.7*c*, respectively. These two waveforms may individually be viewed as examples of a binary PSK signal. Adding them, we get the QPSK waveform shown in Figure 6.7*d*.

To define the decision rule for the detection of the transmitted data sequence, we partition the signal space into four regions, in accordance with Equation (5.59) of Chapter 5. The individual regions are defined by the set of points closest to the message point represented by signal vectors $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$, and $\mathbf{s}_4$. This is readily accomplished by constructing the perpendicular bisectors of the square formed by joining the four message points and then marking off the appropriate regions. We thus find that the decision regions are quadrants whose vertices coincide with the origin. These regions are marked Z_1, Z_2, Z_3, and Z_4, in Figure 6.6, according to the message point around which they are constructed.

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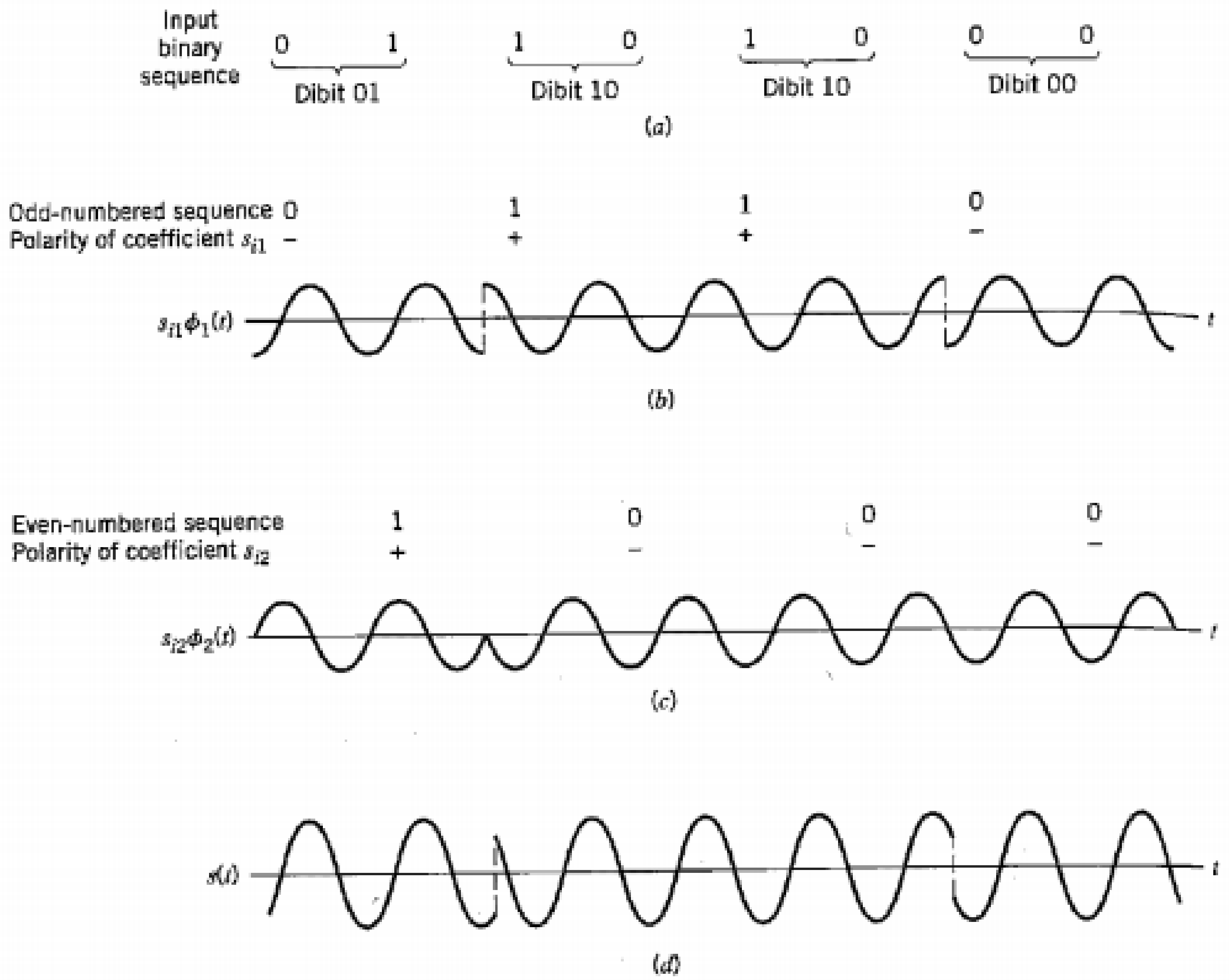

FIGURE 6.7 (*a*) Input binary sequence. (*b*) Odd-numbered bits of input sequence and associated binary PSK wave. (*c*) Even-numbered bits of input sequence and associated binary PSK wave. (*d*) QPSK waveform defined as $s(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$.

BER-Quadrature Phase Shift Keying

Error Probability of QPSK

In a coherent QPSK system, the received signal $x(t)$ is defined by

$$x(t) = s_i(t) + w(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, 3, 4 \end{cases} \tag{6.28}$$

where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$. Correspondingly, the observation vector $\mathbf{x}$ has two elements, x_1 and x_2, defined by

$$\begin{aligned} x_1 &= \int_0^T x(t)\phi_1(t)\, dt \\ &= \sqrt{E} \cos\left[(2i - 1)\frac{\pi}{4}\right] + w_1 \\ &= \pm\sqrt{\frac{E}{2}} + w_1 \end{aligned} \tag{6.29}$$

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BER-Quadrature Phase Shift Keying

and

$$
\begin{aligned}
x_2 &= \int_0^T x(t)\phi_2(t)\, dt \\
&= -\sqrt{E}\,\sin\left[(2i-1)\frac{\pi}{4}\right] + w_2 \qquad (6.30) \\
&= \mp\sqrt{\frac{E}{2}} + w_2
\end{aligned}
$$

Thus the observable elements x_1 and x_2 are sample values of independent Gaussian random variables with mean values equal to $\pm\sqrt{E/2}$ and $\mp\sqrt{E/2}$, respectively, and with a common variance equal to $N_0/2$.

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BER-Quadrature Phase Shift Keying

The decision rule is now simply to decide that $s_1(t)$ was transmitted if the received signal point associated with the observation vector $\mathbf{x}$ falls inside region Z_1, decide that $s_2(t)$ was transmitted if the received signal point falls inside region Z_2, and so on. An erroneous decision will be made if, for example, signal $s_4(t)$ is transmitted but the noise $w(t)$ is such that the received signal point falls outside region Z_4.

To calculate the average probability of symbol error, we note from Equation (6.24) that a coherent QPSK system is in fact equivalent to two coherent binary PSK systems working in parallel and using two carriers that are in phase quadrature; this is merely a statement of the quadrature-carrier multiplexing property of coherent QPSK. The in-phase channel output x_1 and the quadrature channel output x_2 (i.e., the two elements of the observation vector $\mathbf{x}$) may be viewed as the individual outputs of the two coherent binary PSK systems. Thus, according to Equations (6.29) and (6.30), these two binary PSK systems may be characterized as follows:

- The signal energy per bit is $E/2$.
- The noise spectral density is $N_0/2$.

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BER-Quadrature Phase Shift Keying

Hence, using Equation (6.20) for the average probability of bit error of a coherent binary PSK system, we may now state that the average probability of bit error in *each* channel of the coherent QPSK system is

$$P' = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E/2}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) \tag{6.31}$$

Another important point to note is that the bit errors in the in-phase and quadrature channels of the coherent QPSK system are statistically independent. The in-phase channel makes a decision on one of the two bits constituting a symbol (dibit) of the QPSK signal, and the quadrature channel takes care of the other bit. Accordingly, the *average probability of a correct decision* resulting from the combined action of the two channels working together is

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BER-Quadrature Phase Shift Keying

$$P_c = (1 - P')^2$$
$$= \left[1 - \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)\right]^2$$
$$= 1 - \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) + \frac{1}{4}\operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)$$

The average probability of symbol error for coherent QPSK is therefore

$$P_e = 1 - P_c$$
$$= \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}\operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)$$

In the region where $(E/2N_0) \gg 1$, we may ignore the quadratic term on the right-hand side of Equation (6.33), so we approximate the formula for the average probability of symbol error for coherent QPSK as

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$$P_e \simeq \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) \tag{6.34}$$

BER-Quadrature Phase Shift Keying

In a QPSK system, we note that since there are two bits per symbol, the transmitted signal energy per symbol is twice the signal energy per bit, as shown by

$$E = 2E_b \tag{6.36}$$

Thus expressing the average probability of symbol error in terms of the ratio E_b/N_0, we may write

$$P_e \simeq \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{6.37}$$

With Gray encoding used for the incoming symbols, we find from Equations (6.31) and (6.36) that the *bit error rate* of QPSK is exactly

$$\text{BER} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad \text{BER} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{6.38}$$

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BER-Quadrature Phase Shift Keying

We may therefore state that a coherent QPSK system achieves the same average probability of bit error as a coherent binary PSK system for the same bit rate and the same E_b/N_0, but uses only half the channel bandwidth. Stated in a different way, for the same E_b/N_0 and therefore the same average probability of bit error, a coherent QPSK system transmits

bandwidth. For a prescribed performance, QPSK uses channel bandwidth better than binary PSK, which explains the preferred use of QPSK over binary PSK in practice.

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Quadrature Phase Shift Keying

Generation and Detection of Coherent QPSK Signals

Consider next the generation and detection of QPSK signals. Figure 6.8*a* shows a block diagram of a typical QPSK transmitter. The incoming binary data sequence is first transformed into polar form by a *nonreturn-to-zero level* encoder. Thus, symbols 1 and 0 are represented by $+\sqrt{E_b}$ and $-\sqrt{E_b}$, respectively. This binary wave is next divided by means of a *demultiplexer* into two separate binary waves consisting of the odd- and even-numbered input bits. These two binary waves are denoted by $a_1(t)$ and $a_2(t)$. We note that in any signaling interval, the amplitudes of $a_1(t)$ and $a_2(t)$ equal s_{i1} and s_{i2}, respectively, depending on the particular dibit that is being transmitted. The two binary waves $a_1(t)$ and $a_2(t)$ are used to modulate a pair of quadrature carriers or orthonormal basis functions: $\phi_1(t)$ equal to $\sqrt{2/T}\cos(2\pi f_c t)$ and $\phi_2(t)$ equal to $\sqrt{2/T}\sin(2\pi f_c t)$. The result is a pair of

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Quadrature Phase Shift Keying

binary PSK signals, which may be detected independently due to the orthogonality of $\phi_1(t)$ and $\phi_2(t)$. Finally, the two binary PSK signals are added to produce the desired QPSK signal.

The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals $\phi_1(t)$ and $\phi_2(t)$, as in Figure 6.8*b*. The correlator outputs x_1 and x_2, produced in response to the received signal $x(t)$, are each compared with a threshold of zero. If $x_1 > 0$, a decision is made in favor of symbol 1 for the in-phase channel output, but if $x_1 < 0$, a decision is made in favor of symbol 0. Similarly, if $x_2 > 0$, a decision is made in favor of symbol 1 for the quadrature channel output, but if $x_2 < 0$, a decision is made in favor of symbol 0. Finally, these two binary sequences at the in-phase and quadrature channel outputs are combined in a *multiplexer* to reproduce the original binary sequence at the transmitter input with the minimum probability of symbol error in an AWGN channel.

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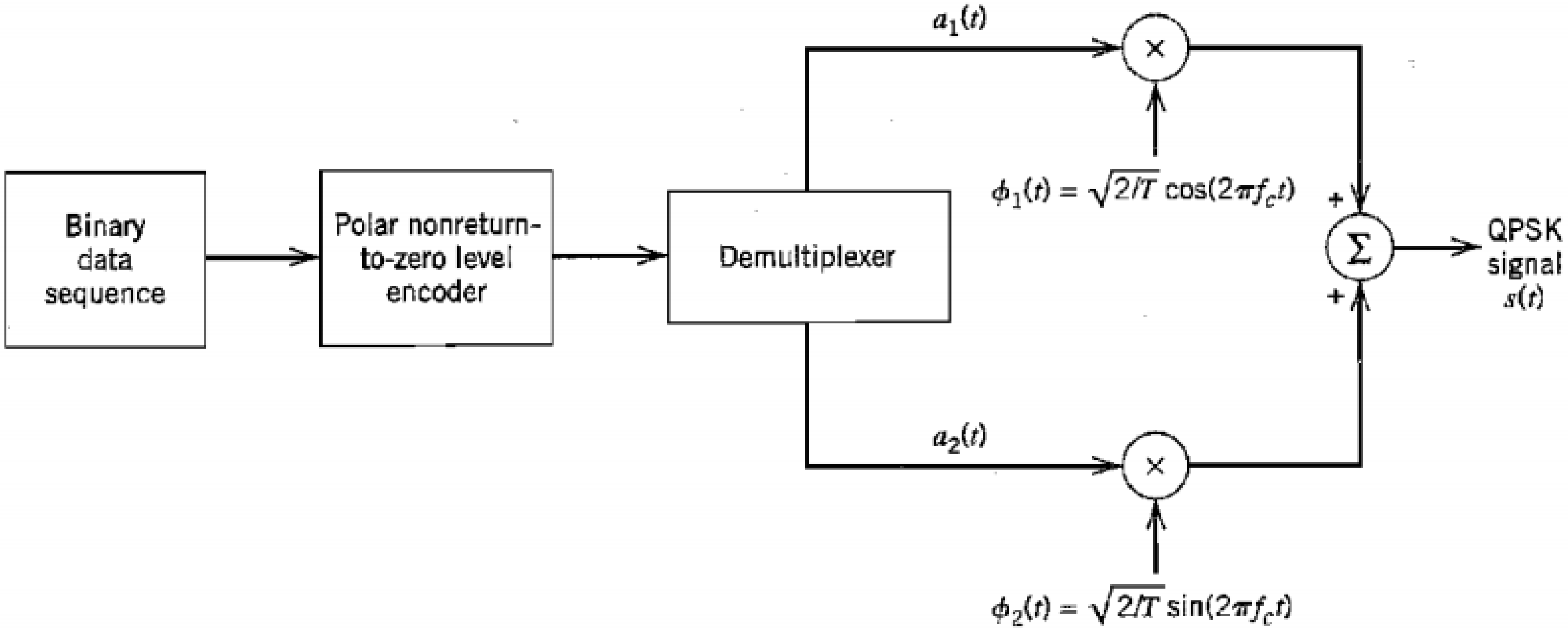
Quadrature Phase Shift Keying



(a)

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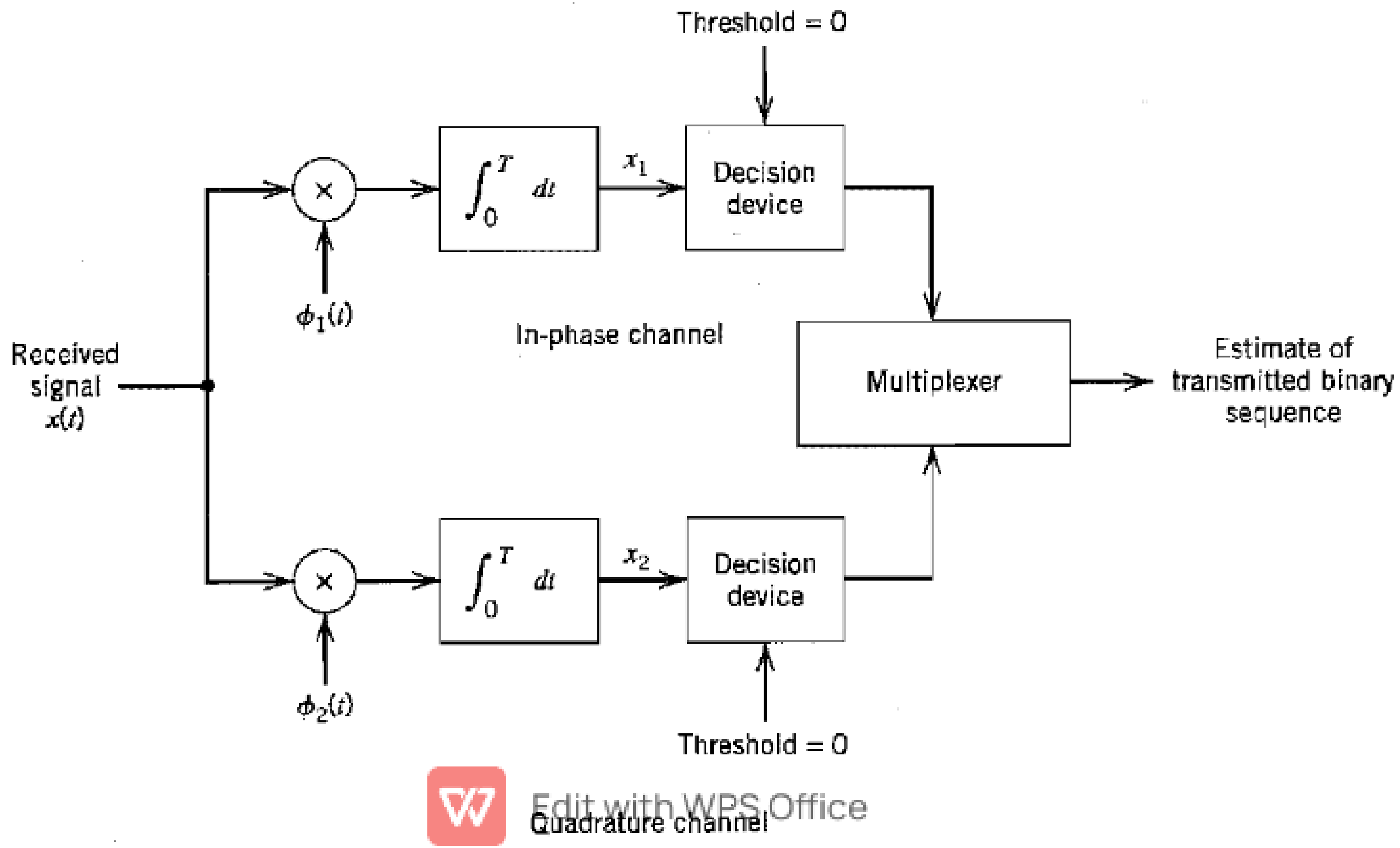
Quadrature Phase Shift Keying



(b)

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Power Spectra of QPSK Signals

Assume that the binary wave at the modulator input is random, with symbols 1 and 0 being equally likely, and with the symbols transmitted during adjacent time slots being statistically independent. We make the following observations pertaining to the in-phase and quadrature components of a QPSK signal:

1. Depending on the dibit sent during the signaling interval $-T_b \le t \le T_b$, the in-phase component equals $+g(t)$ or $-g(t)$, and similarly for the quadrature component. The $g(t)$ denotes the symbol shaping function, defined by

$$g(t) = \begin{cases} \sqrt{\dfrac{E}{T}}, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases} \tag{6.39}$$

Hence, the in-phase and quadrature components have a common power spectral density, namely, $E \operatorname{sinc}^2(Tf)$.

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Power Spectra of QPSK Signals

2. The in-phase and quadrature components are statistically independent. Accordingly, the baseband power spectral density of the QPSK signal equals the sum of the individual power spectral densities of the in-phase and quadrature components, so we may write

$$\begin{aligned} S_B(f) &= 2E\ \mathrm{sinc}^2(Tf) \\ &= 4E_b\ \mathrm{sinc}^2(2T_b f) \end{aligned} \tag{6.40}$$

Figure 6.9 plots $S_B(f)$, normalized with respect to $4E_b$, versus the normalized frequency fT_b. This figure also includes a plot of the baseband power spectral density of a certain form of binary FSK called minimum shift keying, the evaluation of which is presented in Section 6.5. Comparison of these two spectra is deferred to that section.

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Power Spectra of QPSK Signals

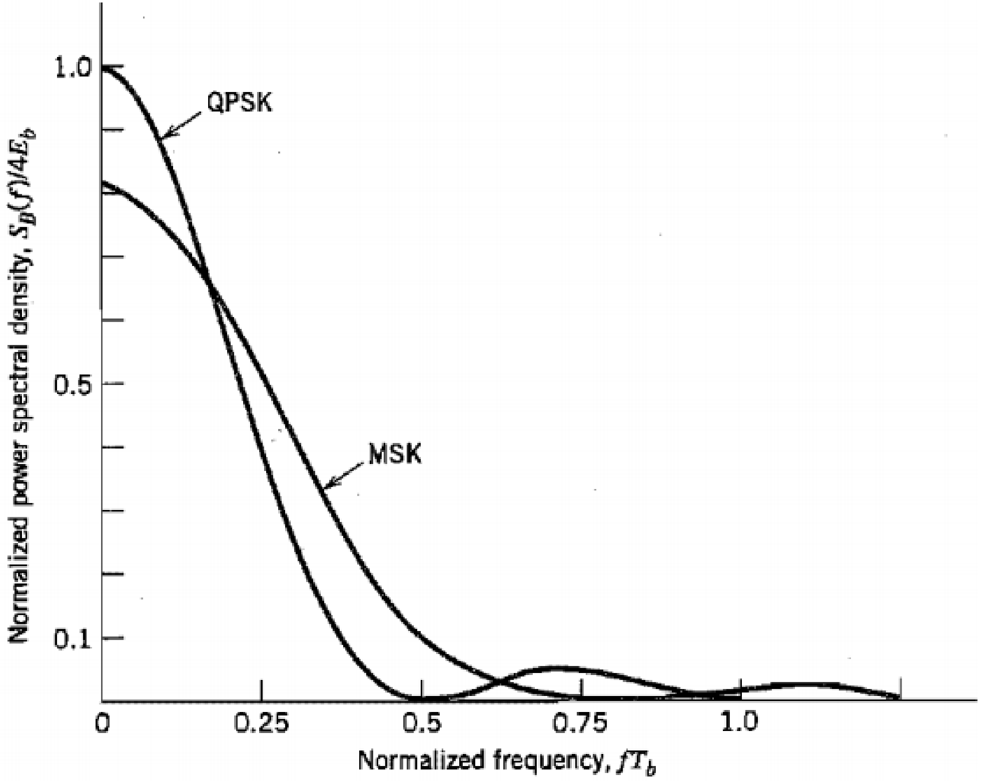

FIGURE 6.9 Power spectra of QPSK and MSK signals.

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Hybrid Amplitude/Phase Modulation Scheme

In an *M*-ary PSK system, the in-phase and quadrature components of the modulated signal are interrelated in such a way that the envelope is constrained to remain constant. This constraint manifests itself in a circular constellation for the message points. However, if this constraint is removed, and the in-phase and quadrature components are thereby permitted to be independent, we get a new modulation scheme called *M-ary quadrature amplitude modulation* (QAM). This latter modulation scheme is *hybrid* in nature in that the carrier experiences amplitude as well as phase modulation.

The passband basis functions in *M*-ary QAM may not be periodic for an arbitrary choice of the carrier frequency f_c with respect to the symbol rate $1/T$. Ordinarily, this aperiodicity is of no real concern. By reformulating the expression for the transmitted signal in a certain way, it is possible to eliminate the time variation of the basis functions on successive symbol transmissions, and thereby simplify implementation of the transmitter. In particular, the transmitter is made to appear "carrierless," while fully retaining the essence of the hybridized amplitude and phase modulation process. This is indeed the idea behind the carrierless amplitude/phase modulation (CAP).

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M-ARY Quadrature Amplitude Modulation

In Chapters 4 and 5, we studied M-ary pulse amplitude modulation (PAM), which is one-dimensional. M-ary QAM is a two-dimensional generalization of M-ary PAM in that its formulation involves two orthogonal passband basis functions, as shown by

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \qquad 0 \leq t \leq T \tag{6.53}$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \leq t \leq T \tag{6.54}$$

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M-ARY Quadrature Amplitude Modulation

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$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \leq t \leq T \tag{6.54}$$

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M-ARY Quadrature Amplitude Modulation

Let the ith message point s_i in the (ϕ_1, ϕ_2) plane be denoted by $(a_i d_{\min}/2, b_i d_{\min}/2)$, where $d_{\min}$ is the minimum distance between any two message points in the constellation, a_i and b_i are integers, and $i = 1, 2, \ldots, M$. Let $(d_{\min}/2) = \sqrt{E_0}$, where E_0 is the energy of the signal with the lowest amplitude. The transmitted M-ary QAM signal for symbol k, say, is then defined by

$$s_k(t) = \sqrt{\frac{2E_0}{T}}\, a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}\, b_k \sin(2\pi f_c t), \qquad \begin{array}{l} 0 \le t \le T \\ k = 0, \pm 1, \pm 2, \ldots \end{array} \tag{6.55}$$

The signal $s_k(t)$ consists of two phase-quadrature carriers with each one being modulated by a set of discrete amplitudes, hence the name *quadrature amplitude modulation*.

Depending on the number of possible symbols M, we may distinguish two distinct QAM constellations: square constellations for which the number of bits per symbol is even, and cross constellations for which the number of bits per symbol is odd. These two cases are considered in the sequel in that order.

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M-ARY Quadrature Amplitude Modulation

QAM Square Constellations

With an *even* number of bits per symbol, we may write

$$L = \sqrt{M} \tag{6.56}$$

where L is a positive integer. Under this condition, an M-ary QAM square constellation can always be viewed as the *Cartesian product* of a one-dimensional L-ary PAM constellation with itself. By definition, the Cartesian product of two sets of coordinates (representing a pair of one-dimensional constellations) is made up of the set of all possible ordered pairs of coordinates with the first coordinate in each such pair taken from the first set involved in the product and the second coordinate taken from the second set in the product.

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M-ARY Quadrature Amplitude Modulation

In the case of a QAM square constellation, the ordered pairs of coordinates naturally form a square matrix, as shown by

$$\{a_i, b_i\} = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\ \vdots & \vdots & & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1) \end{bmatrix} \tag{6.57}$$

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M-ARY Quadrature Amplitude Modulation

EXAMPLE 6.3

Consider a 16-QAM whose signal constellation is depicted in Figure 6.17*a*. The encoding of the message points shown in this figure is as follows:

- Two of the four bits, namely, the left-most two bits, specify the quadrant in the (ϕ_1, ϕ_2)-plane in which a message point lies. Thus, starting from the first quadrant and proceeding counterclockwise, the four quadrants are represented by the dibits 11, 10, 00, and 01.
- The remaining two hits are used to represent one of the four possihle symbols lying within each quadrant of the (ϕ_1, ϕ_2)-plane.

Note that the encoding of the four quadrants and also the encoding of the symbols in each quadrant follow the Gray coding rule.

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M-ARY Quadrature Amplitude Modulation

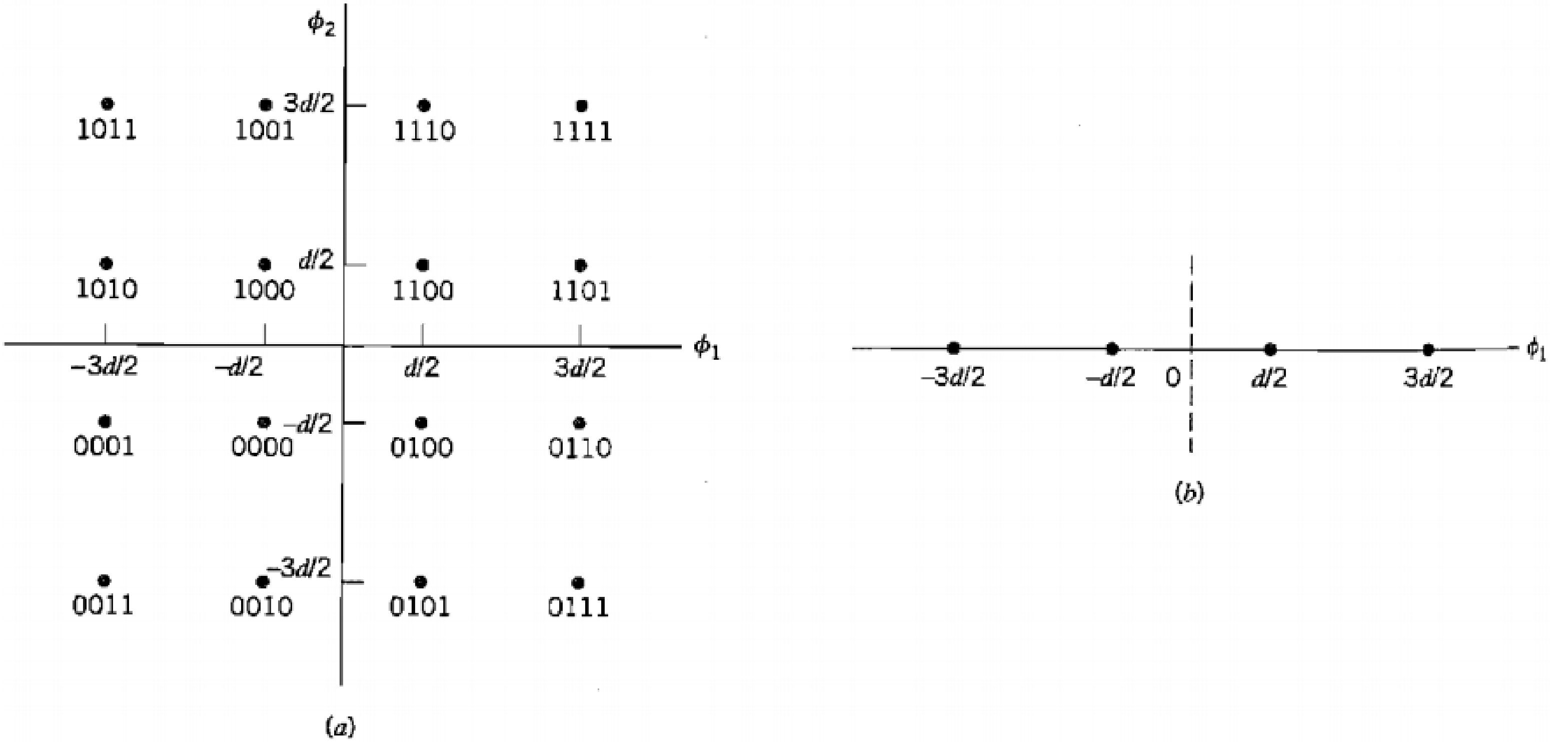

FIGURE 6.17 (*a*) Signal-space diagram of *M*-ary QAM for $M = 16$; the message points in each quadrant are identified with Gray-encoded quadbits. (*b*) Signal-space diagram of the corresponding 4-PAM signal.

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M-ARY Quadrature Amplitude Modulation

For the example at hand, we have $L = 4$. Thus the square constellation of Figure 6.17*a* is the Cartesian product of the 4-PAM constellation shown in Figure 6.17*b* with itself. Moreover, the matrix of Equation (6.57) has the value

$$\{a_i, b_i\} = \begin{bmatrix} (-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\ (-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\ (-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\ (-3, -3) & (-1, -3) & (1, -3) & (3, -3) \end{bmatrix}$$

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M-ARY Quadrature Amplitude Modulation

To calculate the probability of symbol error for M-ary QAM, we exploit the property that a QAM square constellation can be factored into the product of the corresponding PAM constellation with itself. We may thus proceed as follows:

1. The probability of correct detection for M-ary QAM may be written as

$$P_c = (1 - P'_e)^2 \tag{6.58}$$

   where P'_e is the probability of symbol error for the corresponding L-ary PAM with $L = \sqrt{M}$.

2. The probability of symbol error P'_e is defined by

$$P'_e = \left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right) \tag{6.59}$$

   (Note that $L = \sqrt{M}$ in the M-ary QAM corresponds to M in the M-ary PAM considered in Problem 4.27.)

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M-ARY Quadrature Amplitude Modulation

3. The probability of symbol error for M-ary QAM is given by

$$\begin{aligned} P_e &= 1 - P_c \\ &= 1 - (1 - P_e')^2 \\ &\simeq 2P_e' \end{aligned} \tag{6.60}$$

where it is assumed that P_e' is small enough compared to unity to justify ignoring the quadratic term.

Hence, using Equations (6.58) and (6.59) in Equation (6.60), we find that the probability of symbol error for M-ary QAM is approximately given by

$$P_e \simeq 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right) \tag{6.61}$$

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M-ARY Quadrature Amplitude Modulation

The transmitted energy in M-ary QAM is variable in that its instantaneous value depends on the particular symbol transmitted. It is therefore more logical to express P_e in terms of the *average* value of the transmitted energy rather than E_0. Assuming that the L amplitude levels of the in-phase or quadrature component are equally likely, we have

$$E_{av} = 2\left[\frac{2E_0}{L}\sum_{i=1}^{L/2}(2i-1)^2\right] \tag{6.62}$$

where the multiplying factor of 2 outside the square brackets accounts for the equal contributions made by the in-phase and quadrature components. The limits of the summation and the multiplying factor of 2 inside the square brackets take account of the symmetric nature of the pertinent amplitude levels around zero. Summing the series in Equation (6.62), we get

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M-ARY Quadrature Amplitude Modulation

$$E_{av} = \frac{2(L^2 - 1)E_0}{3} = \frac{2(M - 1)E_0}{3} \tag{6.63}$$

Accordingly, we may rewrite Equation (6.61) in terms of E_{av} as

$$P_e \simeq 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{3E_{av}}{2(M - 1)N_0}}\right) \tag{6.64}$$

which is the desired result.

The case of $M = 4$ is of special interest. The signal constellation for this value of M is the same as that for QPSK. Indeed, putting $M = 4$ in Equation (6.64) and noting that for this special case E_{av} equals E, where E is the energy per symbol, we find that the resulting formula for the probability of symbol error becomes identical to that in Equation (6.34), and so it should.

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Comparison of Digital Modulation Schemes Using a Single Carrier

Probability of Error

In Table 6.8 we have summarized the expressions for the bit error rate (BER) for coherent binary PSK, conventional coherent binary FSK with one-bit decoding, DPSK, noncoherent binary FSK, coherent QPSK, and coherent MSK, when operating over an AWGN channel. In Figure 6.45 we have used the expressions summarized in Table 6.8 to plot the BER as a function of the signal energy per bit-to-noise spectral density ratio, E_b/N_0.

Table 6.8 *Summary of formulas for the bit error rate of different digital modulation schemes*

Signaling Scheme	*Bit Error Rate*
(a) Coherent binary PSK Coherent QPSK Coherent MSK	$\frac{1}{2}\,\mathrm{erfc}(\sqrt{E_b/N_0})$
(b) Coherent binary FSK	$\frac{1}{2}\,\mathrm{erfc}(\sqrt{E_b/2N_0})$
(c) DPSK	$\frac{1}{2}\exp(-E_b/N_0)$
(d) Noncoherent binary FSK	$\frac{1}{2}\exp(-E_b/2N_0)$

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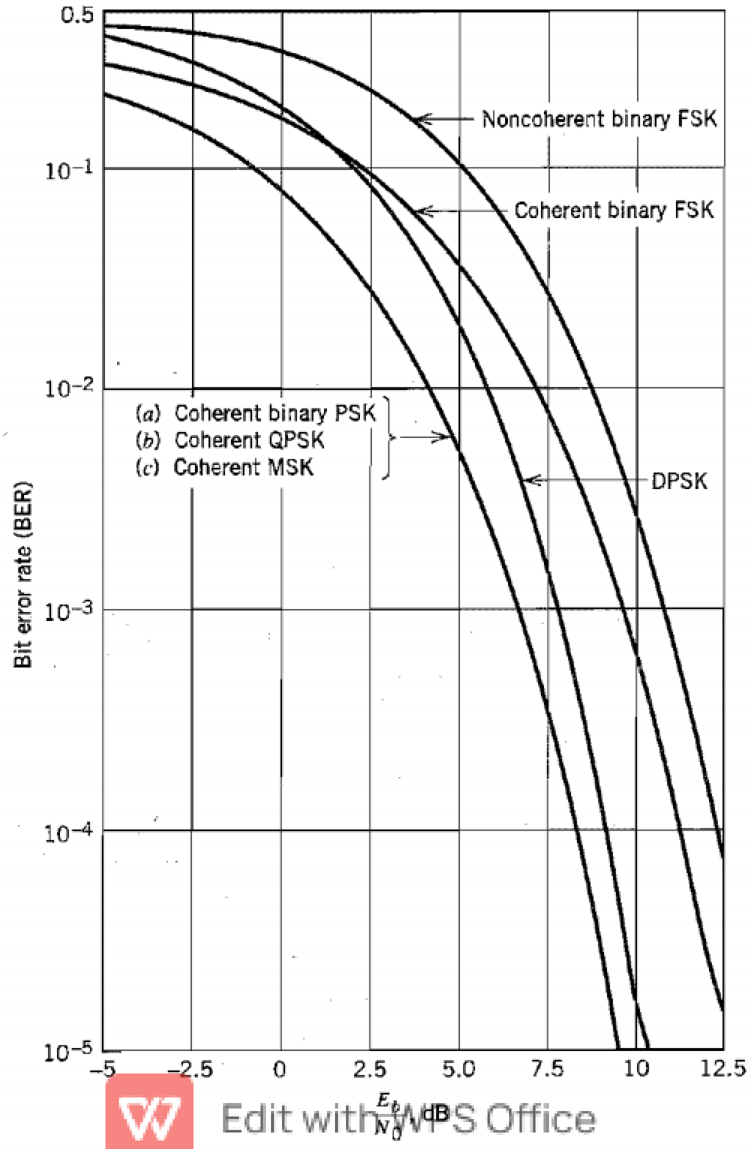

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FIGURE 6.45 Comparison of the noise performance of different PSK and FSK schemes.

Based on the performance curves shown in Figure 6.45, the summary of formulas given in Table 6.8, and the defining equations for the pertinent modulation formats, we can make the following statements:

1. The bit error rates for all the systems decrease monotonically with increasing values of E_b/N_0; the defining curves have a similar shape in the form of a *waterfall*.
2. For any value of E_b/N_0, coherent binary PSK, QPSK, and MSK produce a smaller bit error rate than any of the other modulation schemes.
3. Coherent binary PSK and DPSK require an E_b/N_0 that is 3 dB less than the corresponding values for conventional coherent binary FSK and noncoherent binary FSK, respectively, to realize the same bit error rate.
4. At high values of E_b/N_0, DPSK and noncoherent binary FSK perform almost as well (to within about 1 dB) as coherent binary PSK and conventional coherent binary FSK, respectively, for the same bit rate and signal energy per bit.
5. In coherent QPSK, two orthogonal carriers $\sqrt{2/T}\cos(2\pi f_c t)$ and $\sqrt{2/T}\sin(2\pi f_c t)$ are used, where the carrier frequency f_c is an integer multiple of the symbol rate $1/T$, with the result that two independent bit streams can be transmitted simultaneously and subsequently detected in the receiver.

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Bandwidth Efficiency of M-ary Digital Modulation Techniques

In Table 6.9, we have summarized typical values of power-bandwidth requirements for coherent binary and M-ary PSK schemes, assuming an average probability of symbol error equal to 10^{-4} and the systems operating in identical noise environments. This table shows that, among the family of M-ary PSK signals, QPSK (corresponding to $M = 4$) offers the best trade-off between power and bandwidth requirements. For this reason, we find that QPSK is widely used in practice. For $M > 8$, power requirements become excessive; accordingly, M-ary PSK schemes with $M > 8$ are not as widely used in practice. Also, coherent M-ary PSK schemes require considerably more complex equipment than coherent binary PSK schemes for signal generation or detection, especially when $M > 8$. (Coherent 8-PSK is used in digital satellite communications.)

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Basically, M-ary PSK and M-ary QAM have similar spectral and bandwidth characteristics. For $M > 4$, however, the two schemes have different signal constellations. For M-ary PSK the signal constellation is circular, whereas for M-ary QAM it is rectangular. Moreover, a comparison of these two constellations reveals that the distance between the message points of M-ary PSK is smaller than the distance between the message points of M-ary QAM, for a fixed peak transmitted power. This basic difference between the two schemes is illustrated in Figure 6.46 for $M = 16$. Accordingly, in an AWGN channel, M-ary QAM outperforms the corresponding M-ary PSK in error performance for $M > 4$.

TABLE 6.9 ***Comparison of power-bandwidth requirements for M-ary PSK with binary PSK. Probability of symbol error = 10^{-4}***

Value of M	$\dfrac{(\text{Bandwidth})_{M\text{-ary}}}{(\text{Bandwidth})_{\text{Binary}}}$	$\dfrac{(\text{Average power})_{M\text{-ary}}}{(\text{Average power})_{\text{Binary}}}$
4	0.5	0.34 dB
8	0.333	3.91 dB
16	0.25	8.52 dB
32	0.2	13.52 dB

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However, the superior performance of M-ary QAM can be realized only if the channel is free of nonlinearities.

As for M-ary FSK, we find that for a fixed probability of error, increasing M results in a reduced power requirement. However, this reduction in transmitted power is achieved at the cost of increased channel bandwidth. In other words, M-ary FSK behaves in an opposite manner to that of M-ary PSK. We will revisit this issue in an information-theoretical context in Chapter 9, and thereby develop further insight into the contrasting behaviors of M-ary PSK and M-ary FSK.

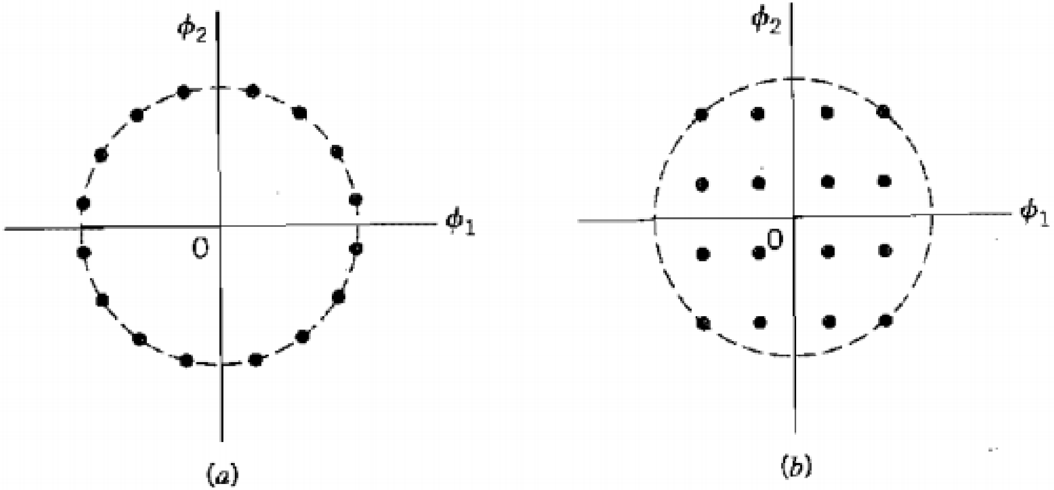

FIGURE 6.46 Signal constellations for (a) M-ary PSK and (b) corresponding M-ary QAM, for $M = 16$.

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