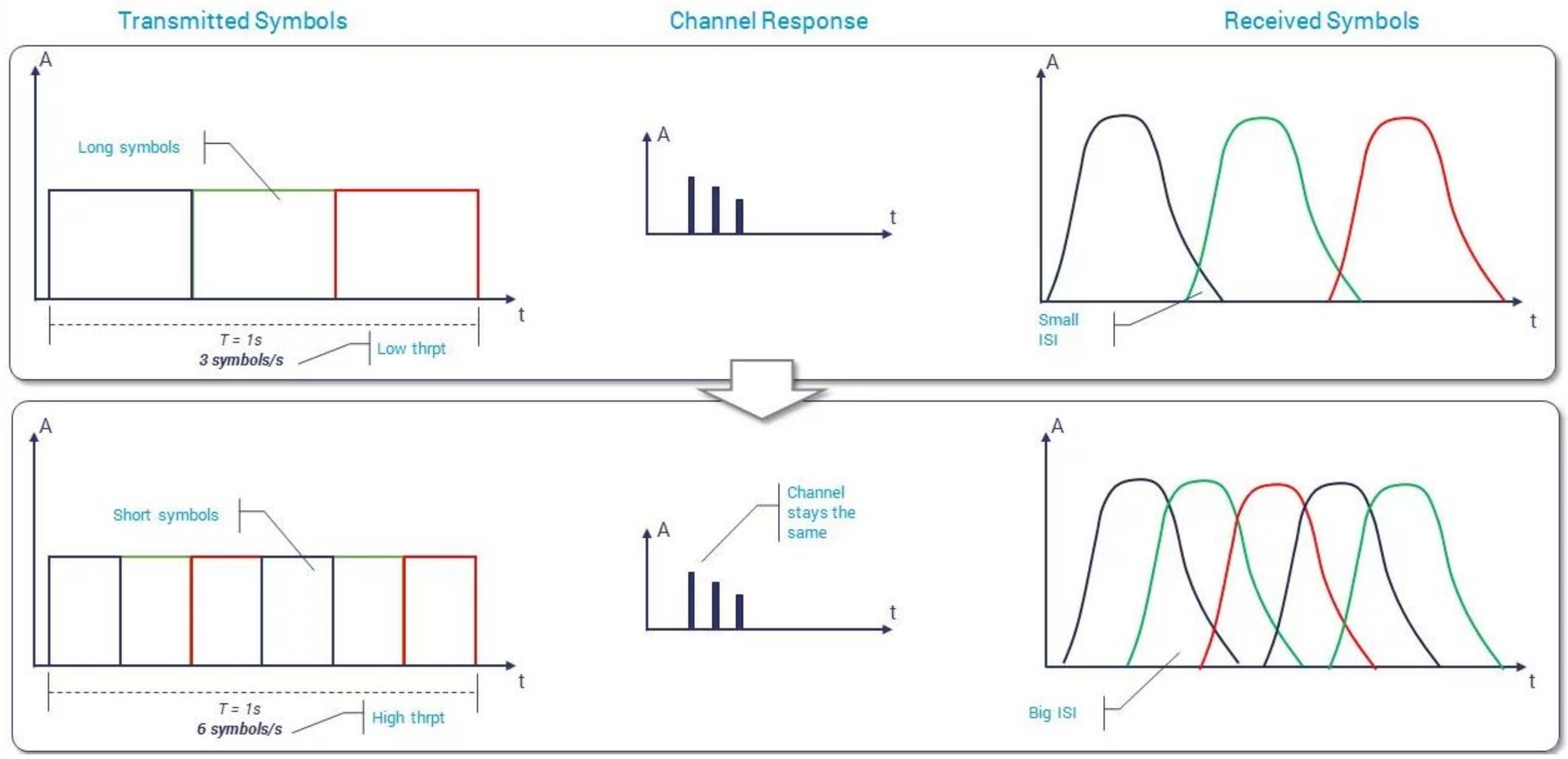
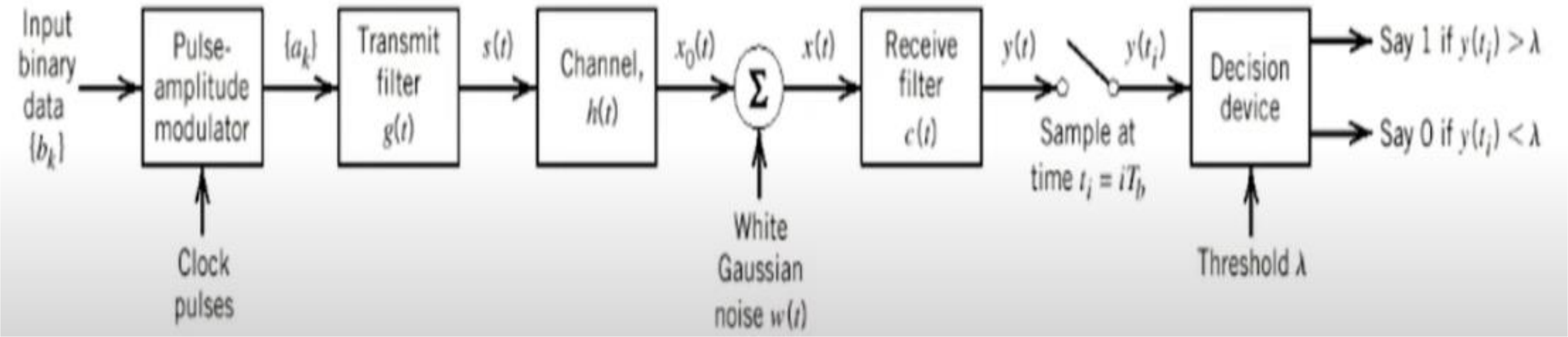
# Module 4 - Part 2

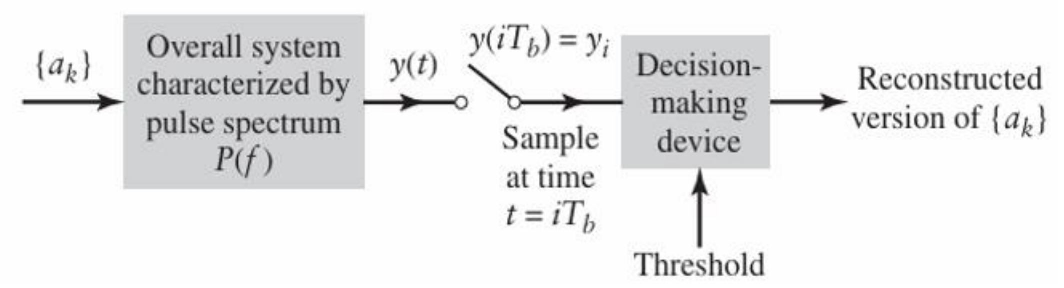
**Prepared by :**
**Asst. Prof. POOJA P P**
**ECE Department**
**GCEK**

# Inter Symbol Interference

- Arises due to imperfections in the frequency response of the channel.
- Occurs when a pulse spreads out in such a way that it interfere with adjacent pulses at the sample instants.
- Example:






**Baseband binary data transmission. (a) Block diagram of the system, depicting its constituent components all the way from the source to destination. (b) Simplified representation of the system.**

- $\boldsymbol{b_k}$ - input binary data stream.
- At time $t = kT_b$ , $T_b$ is the bit duration , k = 0, ±1, ±2, ±3……
- The binary data stream $b_k$ is applied to a line encoder, the purpose of which is to produce a level-encoded signal denoted by $\boldsymbol{a_k}$

$$a_k = \begin{cases} +1 \text{ if symbol } b_k \text{ is } 1 \\ -1 \text{ if symbol } b_k \text{ is } 0 \end{cases} \dashrightarrow (1)$$

- The level-encoded signal $\boldsymbol{a_k}$ is applied to a transmit filter to produce a sequence of pulses, whose basic shape is denoted in the time and frequency domains by **g(t)** and **G(f)** respectively.
- Transmitted signal **s(t)**

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b) \longrightarrow (2)$$

- The signal s(t) is modified and transmitted over the channel whose impulse response is **h(t).**
- The channel adds the white gaussian noise (random noise) **w(t)** to the signal at the receiver input.
- The noisy signal **x(t)** is then passed through a receiver filter of impulse response **c(t)**.
- The resulting filter output **y(t)** sampled and given to the decision device.

➢ If sampled value is more than the threshold we will say it as binary ‘1’.

➢ If the sampled value is lower than threshold we will say it as binary ‘0’.

- The channel output is

$$x_0(t) = s(t) * h(t) \qquad\qquad * \text{ represents convolution}$$

- The receiver filter input

$$x(t) = x_0(t) + w(t) = [s(t)*h(t)] + w(t)$$

$$x(t) = \sum_k a_k [g(t - kT_b) * h(t)] + w(t)$$

- The receiving filter output

$$y(t) = x(t)*c(t)$$

$$y(t) = x(t) * c(t) = \left[\sum_k a_k [g(t - kT_b) * h(t)] + w(t)\right] * c(t)$$

$$y(t) = \sum_k a_k [g(t - kT_b) * h(t) * c(t)] + w(t) * c(t) \longrightarrow (3)$$

- The scaled pulse **p(t)** is obtained by double convolution of impulse response of transmitter g(t), the impulse response of channel h(t) and impulse response of receiving filter c(t), as

$$p(t) = g(t)*h(t)*c(t)$$

- We assume that p(t) is normalized by setting **p(0) =1**.

- Convolution in time domain is transformed into multiplication in frequency domain.

$$P(f) = G(f)\, H(f)\, C(f)$$

Where P(f), G(f), H(f) and C(f) are frequency response of p(t), g(t), h(t) and c(t) respectively.

- Using Inverse Fourier Transform

$$g(t - kT_b) * h(t) * c(t) = F^{-1}\left[e^{-j2\pi fkT_b} G(f) H(f)\, C(f)\right]$$

$$\Rightarrow g(t - kT_b) * h(t) * c(t) = \int_{-\infty}^{\infty} e^{-j2\pi fkT_b} G(f)\ H(f)\, C(f)\, e^{j2\pi ft}\, df$$

$$\Rightarrow g(t - kT_b) * h(t) * c(t) = \int_{-\infty}^{\infty} G(f)\ H(f)\, C(f)\, e^{j2\pi f(t-kT_b)}\, df$$

$$\Longrightarrow g(t - kT_b) * h(t) * c(t) = \int_{-\infty}^{\infty} P(f)\, e^{j2\pi f(t-kT_b)}\, df$$

$$\Longrightarrow g(t - kT_b) * h(t) * c(t) = p(t - kT_b)$$

Where, $p(t) = \int_{-\infty}^{\infty} P(f)\, e^{j2\pi ft}\, df$

From eq (3), $y(t) = \sum_k a_k\, p(t - kT_b) + n(t)$

Where, $n(t) = w(t) * c(t)$

- The received filter output y(t) is sampled at $\boldsymbol{t_i = kT_b}$ , $T_b$ is the bit duration , k = 0, ±1, ±2, ±3……

$$y(t_i) = \sum_k a_k\, p(t_i - kT_b) + n(t_i)$$

$$\Longrightarrow y(t_i) = \sum_k a_k\, p(iT_b - kT_b) + n(iT_b)$$

$$y(t_i) = \sum_k a_k\, p[(i - k)T_b] + n(t_i)$$

$$\underbrace{y(t_i) = a_i}_{\text{When } k = i} + \underbrace{\sum_{k=-\infty}^{\infty} a_k\, p[(i-k)T_b]}_{\text{ISI term}} + n(t_i)$$

**When k= i**

**p(0) = $\sqrt{E}$**

**= 1 (normalised)**

**ISI term**

The above equation has two terms (if noise ignored):

(i) First term is produced by the $i^{th}$ transmitted bit. Theoretically, only this term should be present.

(i) Second term represents the residual effect of all the transmitted bits, obtained at the time of sampling the $i^{th}$ bit. This residual effect is known as the inter symbol interference(ISI).

# Nyquist Criterion for Distortion less Baseband Binary Transmission

- We know, $$y(t_i) = a_i + \sum_{k=-\infty}^{\infty} a_k \, p[(i-k)T_b] + n(t_i)$$

- TIME DOMAIN CRITERION:
- From the above equation, the second term must be zero to eliminate the effect of ISI. This is possible if the received pulse p(t) is controlled such that,

$$p[(i-k)T_b] = \begin{cases} 1 \text{ for } i = k \\ 0 \text{ for } i \neq k \end{cases} \dashrightarrow (1)$$

- **Use a pulse shape that has a nonzero amplitude at its center and zero amplitude at $t = \pm nT\ b$ ($n$ = 1,2,3,……).**

- If p(t) satisfies the above condition, then we get a signal which is free from ISI. ie $y(t_i) = a_i$.

- FREQUENCY DOMAIN CRITERION:

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = \frac{1}{f_b} = T_b$$

- **The pulse that have zero ISI, should have a spectrum if shifted to the multiple value of the rate should result in a constant.**
- Above equation is called **Nyquist pulse shaping criterion for baseband transmission**

# Ideal Nyquist Channel

- The simplest way of satisfying

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = T_b \; (or) \; \sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b \longrightarrow (1)$$

- For n =0, the LHS corresponds to P(f) and it represents a frequency function with the narrowest band which satisfies the above equation.
- The range of frequencies for P(f) will extend from –W to +W ( -$B_0$ to + $B_0$ ) where W or $B_0$ corresponds to half the bit rate.
- This equation is to specify the frequency function **P(f) to be in the form of a rectangular function**

$$P(f) = \begin{cases} \frac{1}{2W}; & -W < f < W \\ 0; & |f| > W \end{cases}$$

$R_b$ - Nyquist Rate
W – Nyquist Bandwidth

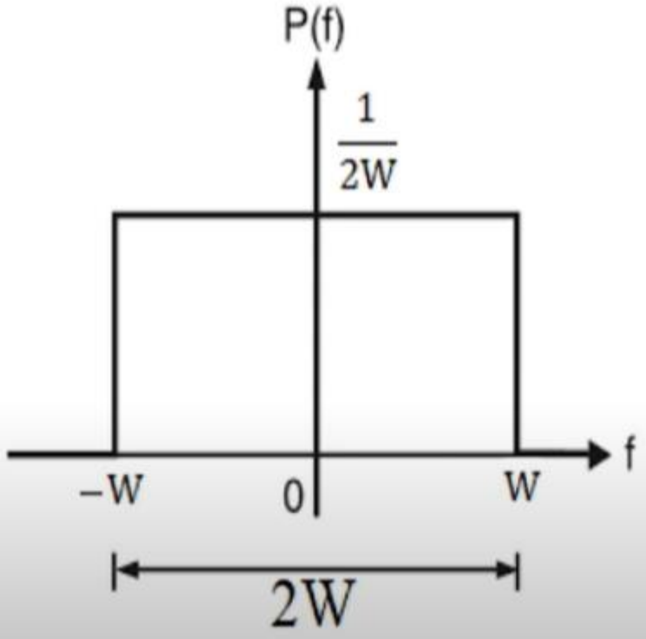
$$P(f) = \frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right)$$

$$W = \frac{R_b}{2} = \frac{1}{2T_b}$$

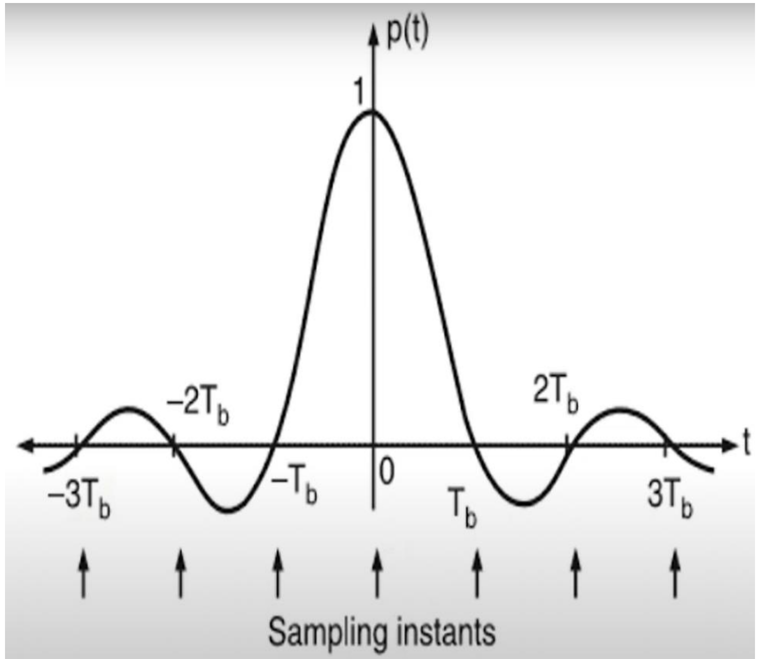
- The signal that produces zero ISI can be obtained by taking the IFT of P(f)

$$p(t) = F^{-1}[P(f)] ==> p(t) = F^{-1}\left[\frac{1}{2W}\,\mathrm{rect}\left(\frac{f}{2W}\right)\right]$$

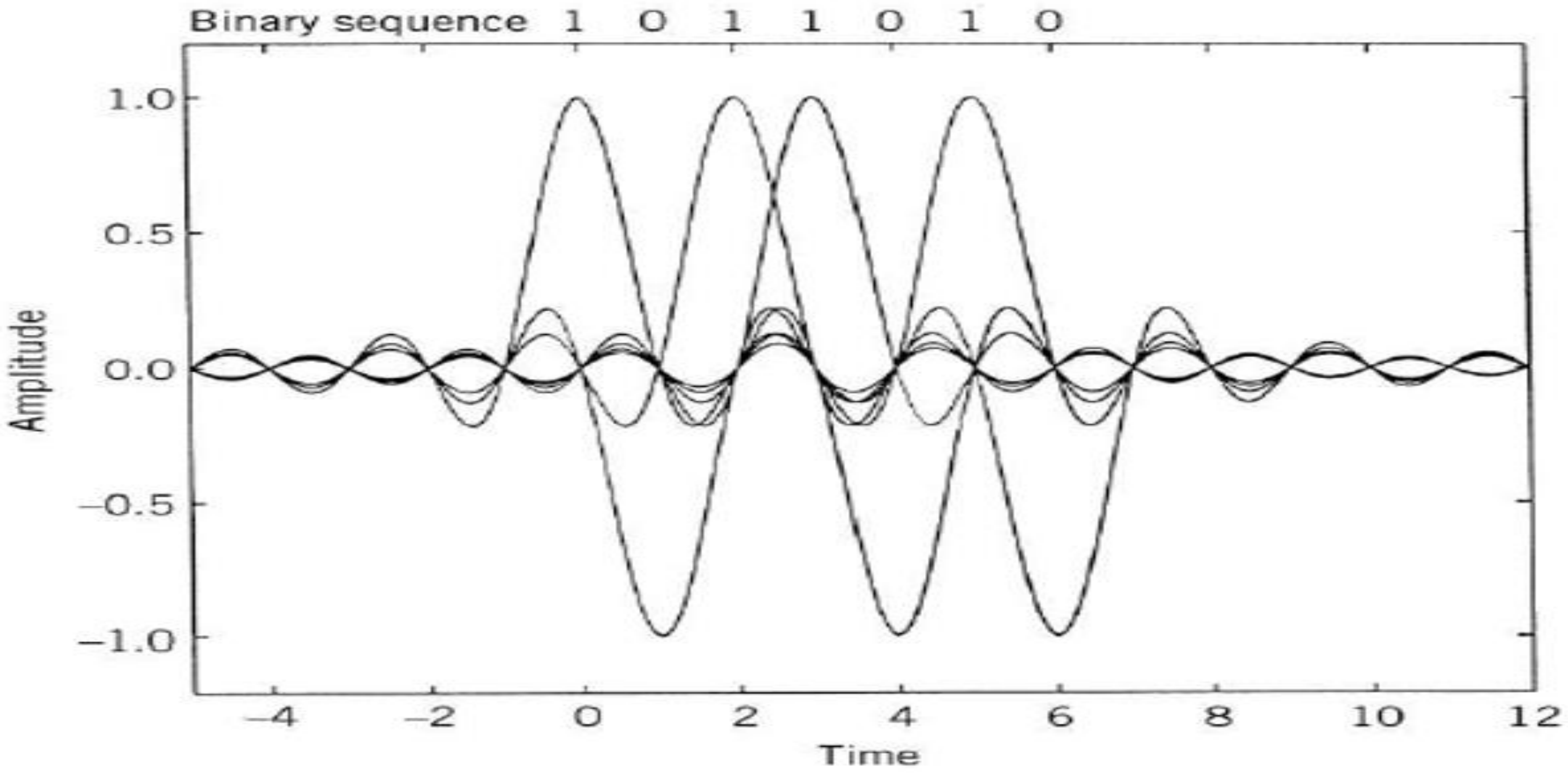
$$==> p(t) = \mathrm{sinc}(2Wt)\ (\text{or})\ p(t) = \frac{\sin(2\pi Wt)}{2\pi Wt}$$



(a) Graphical representation of P(f)



(b) Time-domain representation



Binary sequence 1 0 1 1 0 1 0
1.0
0.5
0.0
−0.5
−1.0
Amplitude
−4
−2
0
2
4
6
8
10
12
Time

## Advantages of using Sinc Pulse

- Bandwidth requirement (of the channel) is reduced.
- ISI is reduced.

## Possible difficulties;

- The function P(f) varies from –W to +W and zero elsewhere. This is physically unrealizable because of abrupt transitions at the edges ±W.
- Sinc pulse is not fast decaying (**decays very slowly 1/*t* rate**)

- Practical solution is the Raised Cosine Channel.

# Raised Cosine Channels or Raised Cosine Spectrum

- We extend the minimum value of **W** = $\boldsymbol{\frac{R_b}{2}}$ to an adjustable value between **W and 2W.**
- We know

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = T_b \text{ (or) } \sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b = \frac{1}{R_b} = \frac{1}{f_b}$$

$$\ldots + P(f + 3R_b) + P(f + 2R_b) + P(f + R_b) + P(f) + P(f - R_b) + P(f - 2R_b) + P(f - 3R_b) \ldots = \frac{1}{R_b} = T_b$$

- We will consider only three terms (three harmonics) and restrict the bandwidth to (–W,+W)

$$P(f + 2W) + P(f) + P(f - 2W) = \frac{1}{2W}; \ -W \le f \le W$$

$W = \frac{R_b}{2}$

$R_b = 2W$

- There are several possible band-limited functions to satisfy the above equation.
- Of great practical interest is the raised cosine spectrum.
- Raised Cosine spectrum consists of

➢ **Flat portion**, which occupies the frequency band $\boldsymbol{0 \leq |f| \leq f_1}$ for some $f_1$ parameter to be defined.

➢ **Roll-off portion**, which occupies the frequency band $\boldsymbol{f_1 < |f| < 2W - f_1}$

- The mathematical representation of raised cosine pulse is

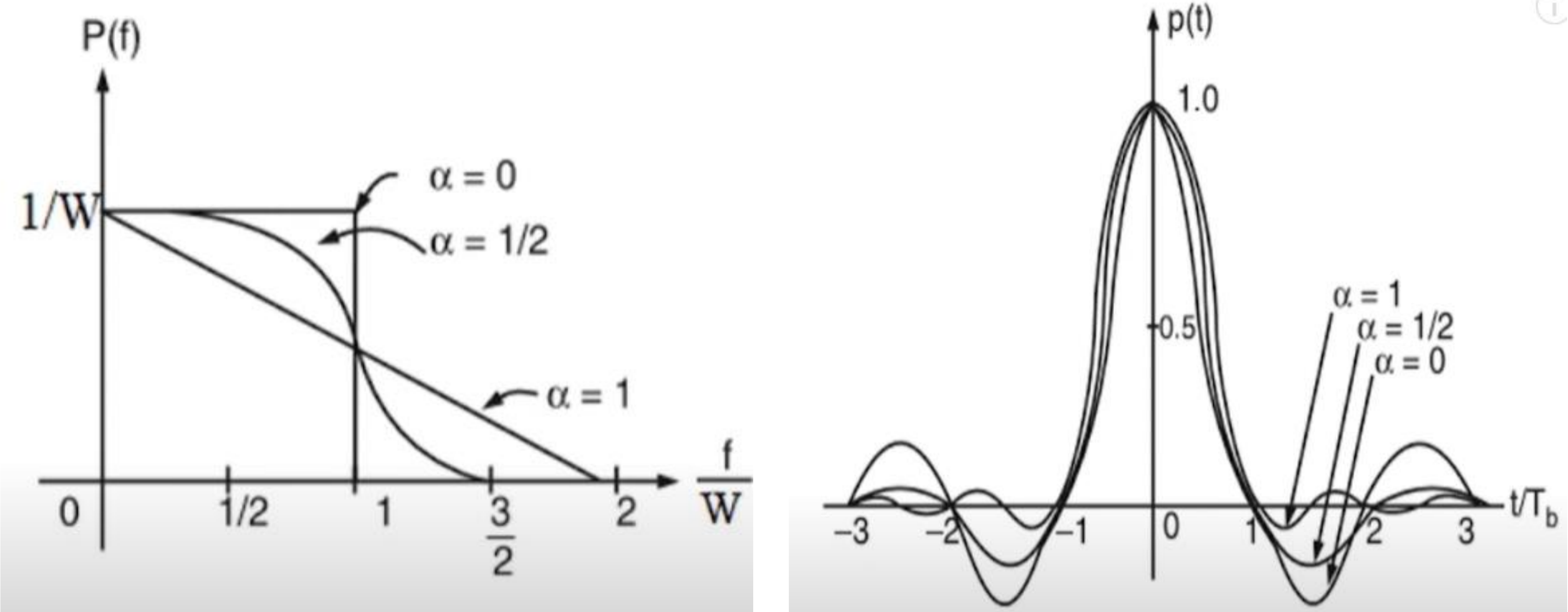
$$P(f) = \begin{cases} \dfrac{1}{2W}; \quad \text{(flat portion)} & 0 \leq |f| \leq f_1 \\ \dfrac{1}{4W}\left\{1 - \sin\left[\dfrac{\pi(|f| - W)}{2W - 2f_1}\right]\right\}; & f_1 \leq |f| < 2W - f_1 \\ 0; & |f| \geq 2W - f_1 \end{cases}$$

- The frequency parameter $f_1$ & bandwidth **W** are related by

$$\alpha = 1 - \frac{f_1}{W}$$

- $\alpha$ - Roll off factor, indicates the excess bandwidth over the ideal solution W.
- The transmission Bandwidth $B_T$

$$B_T = 2W - f_1 \implies B_T = W(1+\alpha)$$




(a) Frequency response

(b) Time response

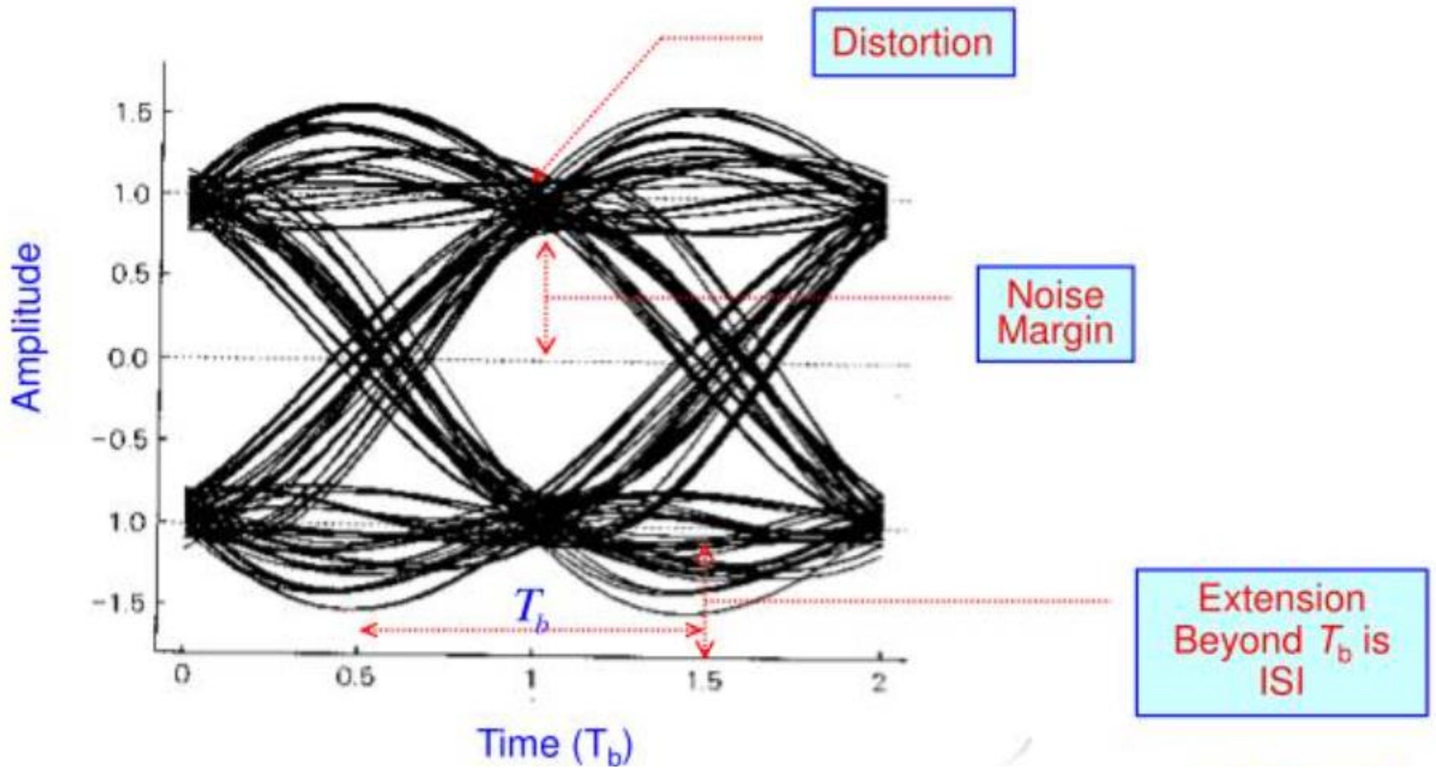
**Fig.** Responses for different roll-off factors, $\alpha$

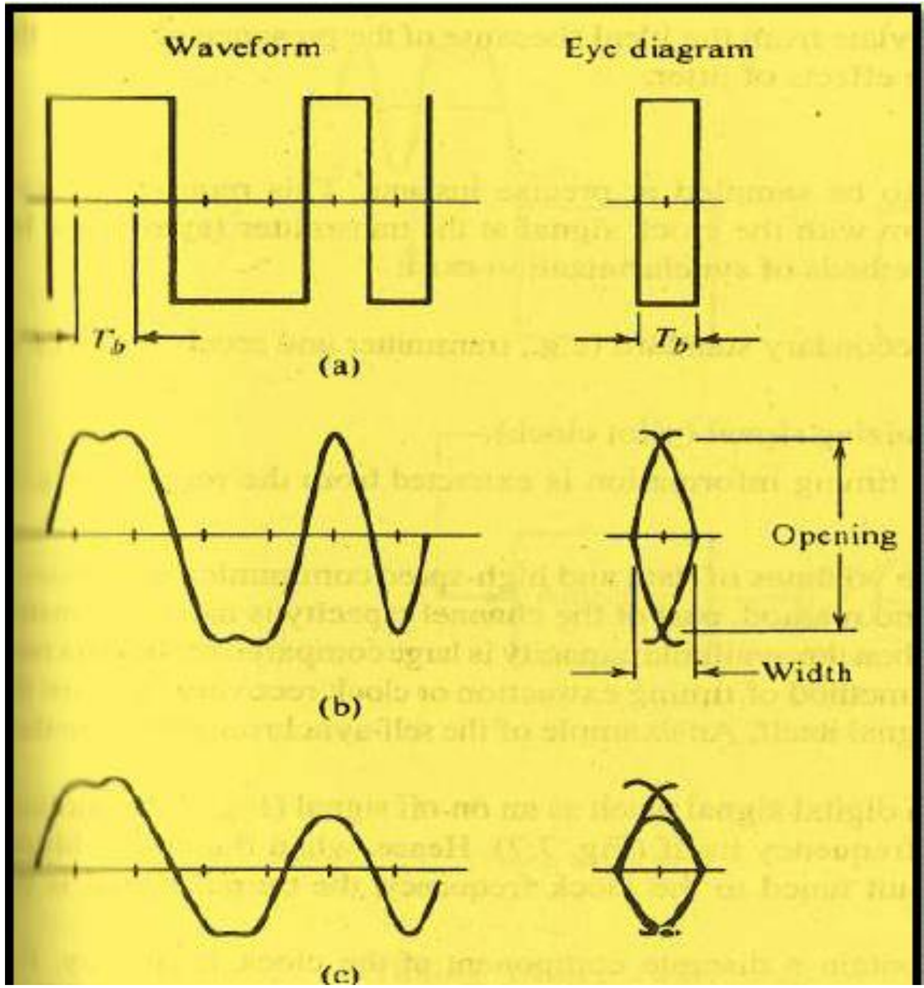
# Eye Pattern or Eye Diagram

How ISI is measured?

- In CRO, it is measured as eye diagram.
- At x terminal received signal is connected and at y terminal sawtooth wave is connected.

- More the opening of eye --- Less ISI






a. **Ideal channel**, infinite BW, pulse received without distortion.

b. **Distortion channel,** finite BW, received signal will rounded and spread out.
***- full opening at mid-pt***

c. **Noise channel,** ISI is not zero, the eye close partially at the mid-pt