ECT - 204

SIGNALS AND SYSTEMS

ASSIGNMENT 2

Bimal . B
ECE - B
RollNo: 16

1) Determine the fourier series representation of the signal $x(t) = \sin 4t + \cos 6t$ plot magnitude & phase spectrum

ans $x(t) = \sin 4t + \cos 6t$

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$$= \frac{e^{j4t} - e^{-4tj}}{2j} + \frac{e^{6tj} + e^{-6tj}}{2} \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$x(t) = \frac{1}{2j} e^{4tj} - \frac{1}{2j} e^{-4tj} + \frac{1}{2} e^{6tj} + \frac{1}{2} e^{-6tj} \quad \text{--- (1)}$$

FSR is given as : $\sum_{k=-\infty}^{\infty} x[k] e^{jk\omega_0 t}$ --- (2)

fundamental frequency

$x(t) = x_1(t) + x_2(t)$

$\omega_1 = 4 \Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{\pi}{2}$

$\omega_2 = 6 \Rightarrow T_2 = \frac{2\pi}{\omega_2} = \pi/3$

$\therefore$ fundamental time period $T = LCM(T_1, T_2)$

$T = \lambda_1 T_1 = \lambda_2 T_2$

i.e. $\lambda_1 T_1 = \lambda_2 T_2$

$$\frac{T_2}{T_1} = \frac{\lambda_1}{\lambda_2} = \frac{2}{3} = \frac{\lambda_1}{\lambda_2}$$

$\lambda_1 = 2, \quad \lambda_2 = 3$

Hence $T = \lambda_1 T_1 = 2 \times \frac{\pi}{2} = \pi$

$\Rightarrow$ fundamental frequency $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

Subs ω_0 in (2)

on comparing with (2) we get

$x[2] = \frac{1}{2j} = -\frac{j}{2}$, $x[-2] = \frac{-1}{2j} = \frac{j}{2}$ $x[3] = x[-3] = \frac{1}{2}$

magnitude - $|x[k]| = \sqrt{\left(\frac{1}{2}\right)^2} = \frac{1}{2}$ for $k = 2, -2$

$|x[k]| = \sqrt{(1/2)^2} = 1/2$ for $k = 3, -3$

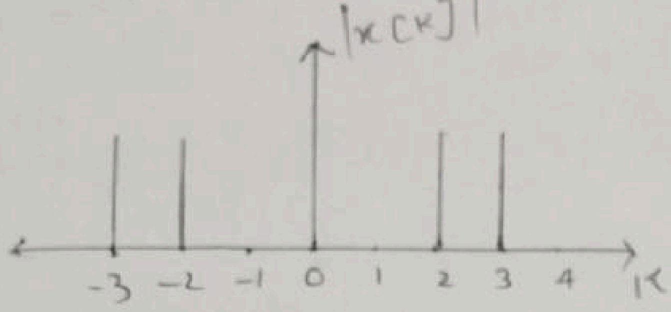
phase angle :-

$$\angle x[K] = -\frac{j}{2} = \tan^{-1}\left(\frac{-1}{0}\right) = -\pi/2 \text{ at } K = 2$$

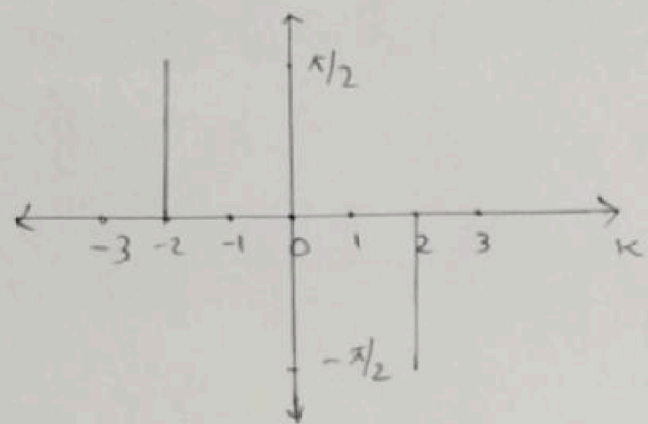
$$j/2 = \tan^{-1}\left(\frac{1/2}{0}\right) = \pi/2 \text{ at } K = -2$$

$$1/2 = \tan^{-1}(0) = 0 \text{ for } K = 3, -3$$

magnitude spectrum



phase spectrum



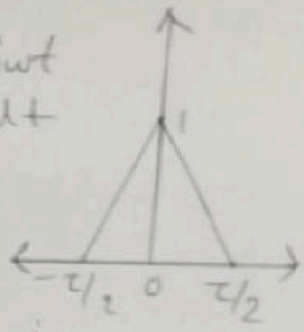
2) find the fourier transform of the signal defined by

$$x(t) = \begin{cases} 1 - \frac{2|t|}{\tau} & \text{for } |t| < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

Ans: fourier transform of a signal $x(t)$ is given by

$$X(t) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

The given signal is of a triangular function



$$x(\omega) = \int_{-\tau/2}^{0}\left(1+\frac{2t}{\tau}\right)e^{-j\omega t}\,dt + \int_{0}^{\tau/2}\left(1-\frac{2t}{\tau}\right)e^{-j\omega t}\,dt$$

$$= \int_{0}^{\tau/2}\left(1-\frac{2t}{\tau}\right)e^{j\omega t}\,dt + \int_{0}^{\tau/2}\left(1-\frac{2t}{\tau}\right)e^{-j\omega t}\,dt$$

$$= \int_{0}^{\tau/2} e^{j\omega t}\,dt - \int_{0}^{\tau/2}\frac{2t}{\tau}e^{j\omega t}\,dt + \int_{0}^{\tau/2} e^{-j\omega t}\,dt - \int_{0}^{\tau/2}\frac{2t}{\tau}e^{-j\omega t}\,dt$$

$$= \int_{0}^{\tau/2}\left[e^{j\omega t}+e^{-j\omega t}\right]dt - \frac{2}{\tau}\int_{0}^{\tau/2} t\left[e^{j\omega t}+e^{-j\omega t}\right]dt$$

$$= \int_{0}^{\tau/2} 2\cos\omega t\,dt - \frac{2}{\tau}\int_{0}^{\tau/2} 2t\cos\omega t\,dt$$

$$= 2\left[\frac{\sin\omega t}{\omega}\right]_{0}^{\tau/2} - \frac{4}{\tau}\left(\left[\frac{t\sin\omega t}{\omega}\right]_{0}^{\tau/2} - \int_{0}^{\tau/2}\frac{\sin\omega t}{\omega}\,dt\right)$$

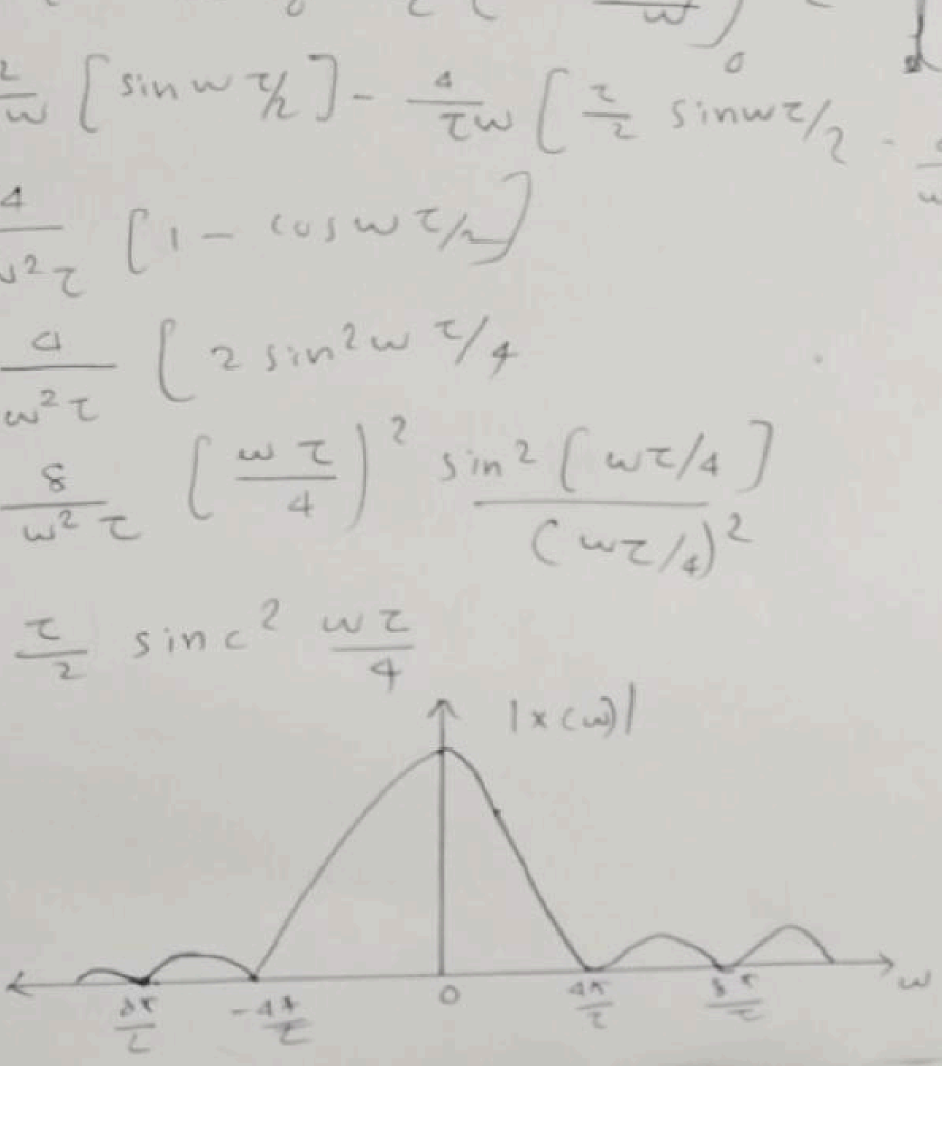
$$= \frac{2}{\omega}\left[\sin\omega\tau/2\right] - \frac{4}{\tau\omega}\left[\frac{\tau}{2}\sin\omega\tau/2\right] - \frac{4}{\omega^2\tau}\left[\cos\frac{\omega\tau}{2} - 1\right]$$

$$= \frac{4}{\omega^2\tau}\left[1-\cos\omega\tau/2\right]$$

$$= \frac{4}{\omega^2\tau}\left[2\sin^2\omega\tau/4\right]$$

$$= \frac{8}{\omega^2\tau}\left(\frac{\omega\tau}{4}\right)^2\frac{\sin^2\left[\omega\tau/4\right]}{(\omega\tau/4)^2}$$

$$= \frac{\tau}{2}\,\mathrm{sinc}^2\,\frac{\omega\tau}{4}$$



3) find the inverse laplace transform of $X(s) = \frac{8-(s-2)(4s+10)}{(s+1)(s^2+4s+4)}$

∴ ROC $Re(s) < -1$

Ans: given $X(s) = \frac{8-(s-2)(4s+10)}{(s+1)(s^2+4s+4)}$

$$\rightarrow \frac{8-(s-2)(4s+10)}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$8-(s-2)(4s+10) = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

when $s = -2$

$$8-(-4)(2) = -C \Rightarrow C = -16$$

when $s = -1$

$$8-(-3)(6) = A \Rightarrow A = 26$$

when $s = 0$

$$28 = 4A + 2B + C$$

$$2B = -60 \quad B = -30$$

poles $= -1, -2$

ROC of $X(s)$ is $Re(s) > -1$, Hence signal is right sided. so the individual ROC's must be $Re(s) > -1$ $Re(s) > -2$

$$\Rightarrow X(s) = \frac{26}{s+1} - \frac{30}{s+2} - \frac{16}{(s+2)^2}$$

$$x(t) = L^{-1}\{X(s)\} = L^{-1}\left\{\frac{26}{s+1}\right\} - L^{-1}\left\{\frac{30}{s+2}\right\} - L^{-1}\left\{\frac{16}{(s+2)^2}\right\}$$

$$= 26e^{-t}u(t) - 30e^{-2t}u(t) - 16te^{-2t}u(t)$$

Hence

$$x(t) = 26e^{-t}u(t) - 30e^{-2t}u(t) - 16te^{-2t}u(t)$$

4) The transfer function of an LTI System is given by $H(s) = \frac{2s^2+9s-11}{(s+1)(s^2+s-6)}$ find the Impulse response of the System if i) stable ii) is causal. will the System be both stable & causal

$$H(s) = \frac{2s^2+9s-11}{(s+1)(s^2+s-6)}$$

$$= \frac{2s^2+9s-11}{(s+1)(s-2)(s+3)}$$

$$\frac{2s^2+9s-11}{(s+1)(s-2)(s+1)} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{s+3}$$

$$2s^2+9s-11 = A(s-2)(s+3) + B(s+1)(s+3) + C.(s+1)(s-2)$$

at $s=-1$
$-18 = -6A$
$\underline{\underline{A = 3}}$

at $s = 2$
$15B = 15$
$\underline{\underline{B = 1}}$

at $s = -3$
$-20 = 10C$
$\underline{\underline{C = -2}}$

$$H(s) = \frac{3}{s+1} + \frac{1}{s-2} + \frac{-2}{s+3}$$

poles are $\underline{\underline{s = -1, 2, -3}}$

i) $h(t) = 3e^{-t}u(t) + -e^{2t}u(-t) - 2e^{-3t}u(t)$

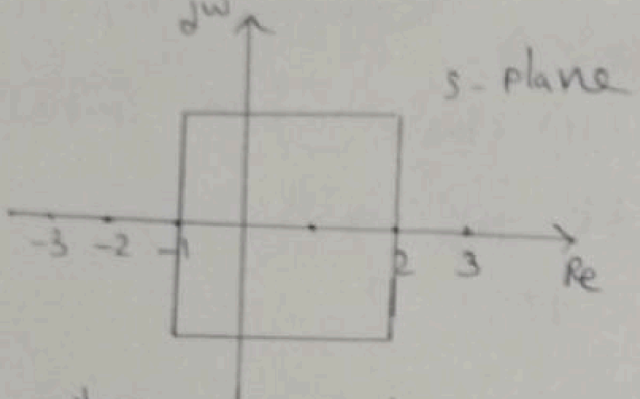
i.e for $h(t)$ to be stable the ROC must contain $j\omega$ axis and ROC should not contain any pole

$\therefore$ Individual ROC must be

$Re(s) > -3$, $Re(s) > -1$ & $Re(s) < 2$

ie, $-1 < Re(s) < 2$

$\therefore h(t) = 3e^{-t}u(t) + -e^{2t}u(-t) - 2e^{-3t}u(t)$



Hence the given syste in stasble

ii) $\because$ the ROC is not at the right of right most pole. Hence it is not causal

$\therefore$ One pole lies in the right half of the s-plane the system is not both stable & causal.