INFORMATION THEORY & CODING

LECTURE 6

CONTENTS

- Quick recap
- Rate of transmission information over discrete channel
- Types of channel and find their channel capacity

Rate of transmission information over discrete channel,Rt

When we consider a discrete memoryless channel accepting symbols at the rate of 'r_s' message symbols/sec then the average rate at which information is going into the channel is given by.

$$R_{in} = H(A) \cdot r_s \text{ bits/sec}$$

H(A) = entropy of input symbols, rs = rate at which symbols accepted in channel

At the Receiver, it is not possible to reconstruct the input symbol sequence with certainty by operating on receiving sequence.

This is due to the errors in the channel. Some amount of information is lost in the channel called as equivocation $H(A/B)$. Hence net amount of information is called Mutual Information.

$$I(A,B) = H(A) - H(A/B).$$

average rate of transmission of Information,

$$R_t = [H(A) - H(A/B)] \cdot r_s \text{ bits/second.}$$

or

$$R_t = [H(B) - H(B/A)] \cdot r_s \text{ bits/second}$$

The above defination is true, when the channel is so noisy, that the o/p may become statistically independat of input

when B & A are independant $H(B/A) = H(B)$ & hence all information going into channel is lost. a no information is transmitted over the channel.

Capacity of a discrete memoryless channel

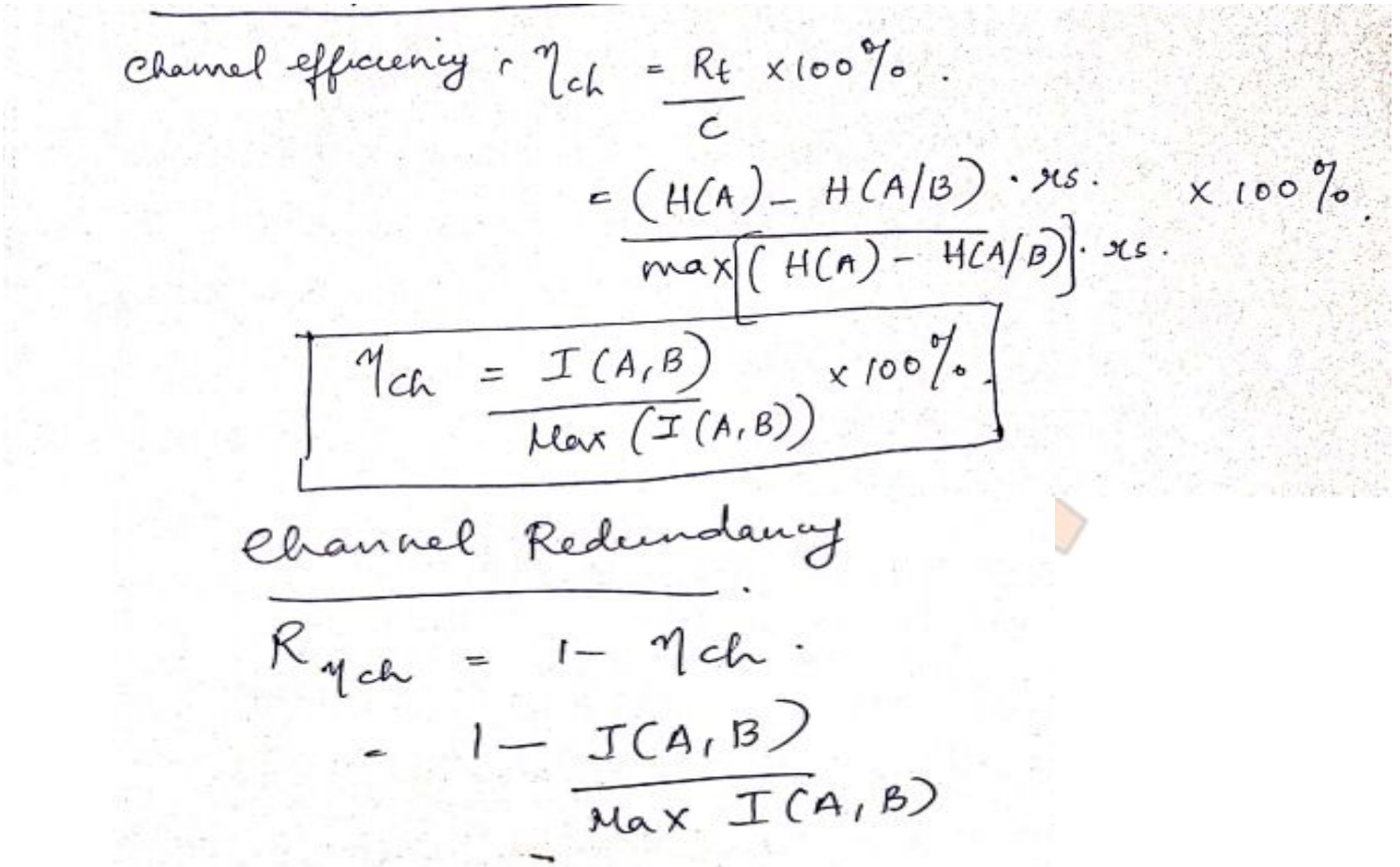
The capacity of a discrete memoryless noisy channel is defined as the maximum possible rate of information transmission over the channel. The max rate of transmission occurs when source is matched to the channel.

Channel capacity, $C = \text{Max}\{R_t\}$

$= \underline{\text{Max}\{H(A) - H(A/B)\}} \cdot r_s$.

the channel capacity can be defined as maximum of Mutual Information

Channel efficiency and redundancy



Special channels

- Symmetric/uniform channel
- **Binary symmetric channel(BSC)**
- **Binary Erasure channel(BEC)**
- Noiseless channel
- Deterministic channel
- Cascaded Channel

Noise free channel

a) Noise free channels:

For a noise-free channel, the Mutual Info. is given by $I(A,B) = H(A)$.

channel capacity $= \max [H(A)] \cdot r_s$ bits/message.

$= \log M$ bits/message. for $r = 1$

2.Symmetric channel

A channel is said to be symmetric/uniform if the 2nd and subsequent rows of the channel matrix contains same elements as that of 1st row but in a different order.

Conditions for symmetric channel

This is a channel in which

(i) $H(B/a_i)$ is independant of i, i.e. entropy corresp to each row of $P(B/A)$ is the same.

(ii) $\sum_{j=1}^{s} P(b_j/a_i)$ is independant of j, i.e. sum of all columns of $P(B/A)$ is the same.

(iii) A channel is symmetric if the rows and columns of channel matrix are seperately identical except for permutations.

eg:

a) $P(A/B) = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$

it is symmetric as rows and columns are identical except for permutations.

b) $P(B/A) = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$

not symmetric channel, as rows are identical but columns are not identical

c) $P(B/A) = \begin{bmatrix} 1/3 & 1/6 & 1/3 & 1/6 \\ 1/6 & 1/3 & 1/6 & 1/3 \end{bmatrix}$

$\Rightarrow$ it is a symmetric channel.

Channel capacity of symmetric channel

In general, channel matrix of symmetric / uniform channel can be written for 'r' input symbols and 's' output symbols.

$$P(B/A) = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_r \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & \cdots b_s \end{array} \\ \begin{bmatrix} P_1 & P_2 & P_3 \cdots & P_s \\ P_3 & P_1 & P_8 \cdots & P_{s-1} \\ \cdot & & & \\ \cdot & & & \\ P_{s-1} & P_{s-2} \cdots & P_1 & \end{bmatrix} \end{array}$$

where $P_1, P_2, P_3 \ldots P_{s-4}, P_{s-3}, P_{s-2}, P_{s-1}, P_s$ are conditional prob $P(b_j/a_i)$ whose permutation are present in other rows.