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Properties of cyclic codes

1. For a (n, k) cyclic codes there exists a generator polynomial of degree $(n-k)$ given by

$$g(x) = g_0 + g_1 x + g_2 x^2 + \dots g_{n-k} x^{n-k}$$

The generator polynomial is unique ie H is the only code vector polynomial of minimum degree $(n-k)$.

Properties of cyclic codes….

2) The generator polynomial $g(x)$ of a (n,k) cyclic code is a factor of x^n+1

$$x^n+1 = g(x)\,h(x).$$

where $h(x)$ is another polynomial of degree k called "parity check polynomial".

3) If $g(x)$ is a polynomial of degree $(n-k)$ and is a factor of x^n+1, then it generates (n,k) cyclic code.

Properties of cyclic codes….

1) The code vector polynomial $c[x]$ can be found by using $c[x] = d[x]\,g[x]$. (non-systematic).

where $d(x)$ is message vector polynomial of degree k | $D(x)$.

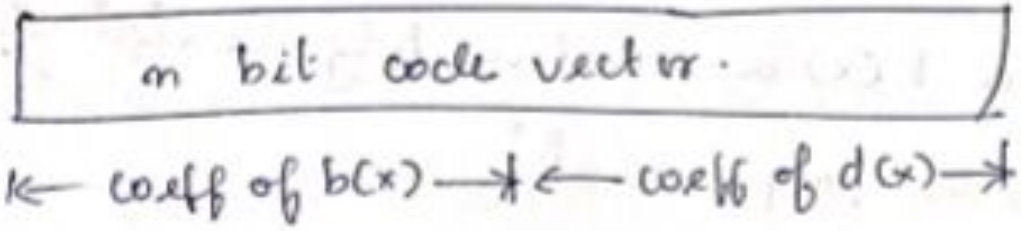
$\therefore d(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3 + \ldots + d_{k-1} x^{k-1}$.

This method generalis non-systematic cyclic codes.

Properties of cyclic codes….

b) To generate a systematic cyclic code the remainder polynomial $b(x)$ is got from division of $x^{n-k}\, d(x)$ by $g(x)$.

The coefficient of $b(x)$ are placed in the beginning of code vectors followed by coefficient of message polynomial $d(x)$ to get code vectors.



Example

Eg.

For a (7,4) single error correcting cyclic code

$D(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3$ and $x^n + 1 = x^7 + 1 = (1+x+x^3)(1+x+x^2+x^4)$

using generator polynomial $g(x) = 1 + x + x^3$. Find all the 16 code vectors of the cyclic code both in systematic & non-systematic form.

Non-systematic form:

$$C(x) = D(x)\,g(x)$$

Given msg vector $D = d_0\,d_1\,d_2\,d_3 = 0001$.

msg polynomial $D(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3$.

$D = 0001 = x^3$.

$D(x) = x^3$.

$g(x)$ is given $= 1 + x + x^3$.

$$C(x) = D(x) \cdot g(x)$$

$$= x^3(1 + x + x^3)$$

$$= x^3 + x^4 + x^6 \Rightarrow$$ $C_0=0\ C_1=0\ C_2=0\ C_3=1\ C_4=1\ C_5=0\ C_6=1$

code word for $D = 0001$ $C = 0001101$.

For $D = 0011$

$D(x) = x^2 + x^3$

$g(x) = 1 + x + x^3$

$C(x) = d(x) \cdot g(x)$

$= (x^2 + x^3)(1 + x + x^3)$

$= x^2 + x^3 + x^3 + x^4 + x^5 + x^6$

$= x^2 + \cancel{2x^3} + x^4 + x^5 + x^6$

$= 0\ 0\ 1\ 0\ 1\ 1\ 1$

Find codewords for all the msg vectors.

msg D	code vector C
0000	0000000
0001	0001101
0010	0011010
0011	0010111
0101	0111001
0110	0101110
0111	0100011
1000	1101000
1001	1100101
1010	1110010
1011	1111111
1100	1011100
1101	1010001
1110	1000110
1111	1001011
10100	0110100

Systematic cyclic codes

In systematic form, the first 3 bits are check bits & last 4 are msg bits

Check bits are got from remainder polynomial b

$$b(x) = \frac{x^{n-k}\, D(x)}{g(x)} + Q(x) \text{ (not needed)}$$

$Q(x)$: quotient term.

Steps for encoding cyclic systematic codes

Summary.

Steps involved in encoding procedure for an (n,k) cyclic codes are systematic in structure.

1) Multiply message polynomial $m(x)$ by X^{n-k}.

2) Divide $x^{n-k}\, m(x)$ by $g(x)$ obtaining remainder $b(x)$

3) Add $b(x)$ to $x^{n-k}\, m(x)$ obtaining code polynomial $C[x]$.

let $D = [0\ 0\ 0\ 1]$.

$D(x) = x^3$.

$x^{n-k} D(x) = x^{7-4} \cdot x^3 = x^6$

$x^3 + x + 1 \overline{)\ x^6}$

$x^3 + x + 1$

$x^6 + x^4 + x^3$

$x^4 + x^3$

$x^4 + x^2 + x$

$x^3 + x^2 + x$

$x^3 + x + 1$

$x^2 + 1$ (Remainder)

$b(x) = x^2 + 1$

$= 1 + 0 \cdot x + 1 \cdot x^2$

$= \underline{101}$.

Let $U = [0\ 0\ 1\ 0]$.

$D(x) = x^2$.

$x^{n-k} D(x) : x^{7-4} D(x) = x^3 \cdot x^2 = x^5$

$$x^3+x+1 \overline{)\ x^5}\ \ x^2+1$$

$$x^5 + x^3 + x^2$$

$$x^3 + x^2$$

$$x^2 + x + 1$$

$$x^2 + x + 1$$

$b(x) = x^2 + x + 1$

$= 111$.

message	codevectors
0000	000 0000
0001	101 0001
0010	111 0010
0011	010 0011
0100	011 0100
0101	110 0101
0110	100 0110
0111	001 0111
1000	110 1000
1001	011 1001
1010	001 1010
1011	100 1011
1100	000 101 1100
1101	000 1101
1110	010 1110
1111	111 1111

In systematic code, the first 3 bits are check bits and last 4 bits are message bits