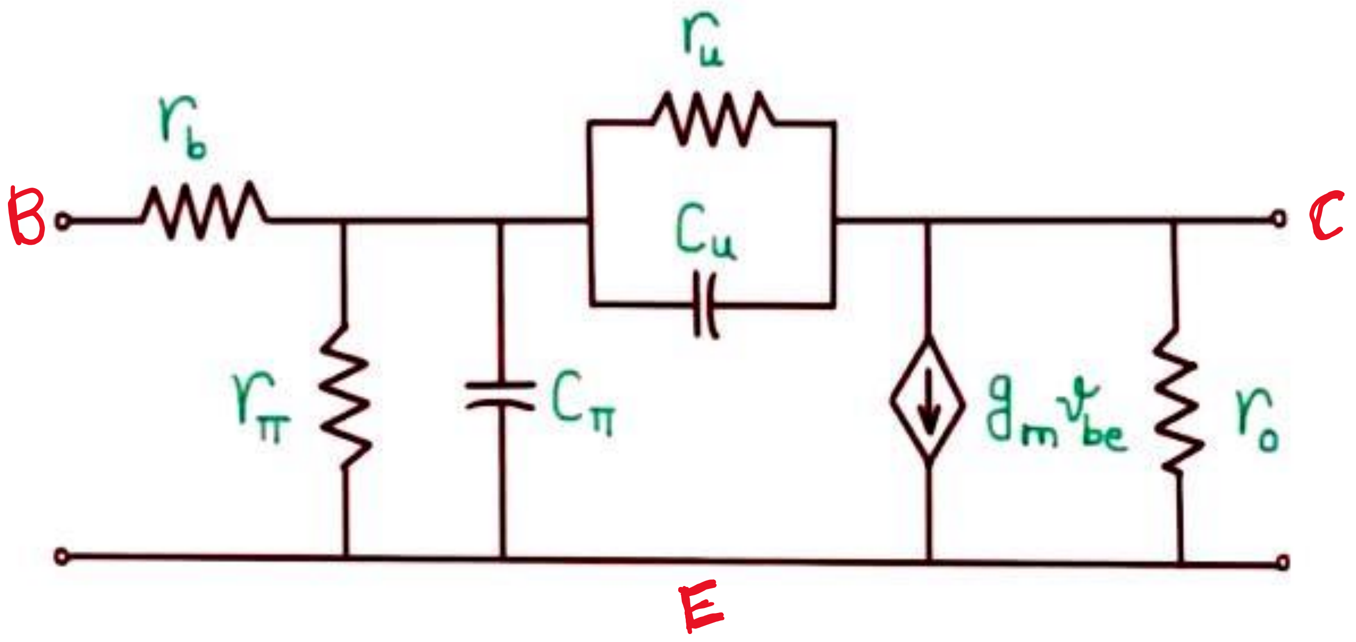
HIGH FREQUENCY EQUIVALENT CIRCUITS

- The performance of BJT is limited at higher frequencies due to the presence of junction and diffusion capacitance.
- So for analysing a transistor circuit at high frequency small signal models are not suitable.
- Equivalent circuit should be modified by including the junction capacitance of the transistor.
- At low frequencies we can analyse transistor using h-parameters, but in high frequencies it is not suitable
  - Value of h-parameters are not constant at high frequencies.
  - At high frequencies h-parameters become more complex

HYBRID π MODEL

- **C_μ -** Transition capacitance between base and collector. Early effect representation
- **C_π –** Diffusion Capacitance. Minority carriers stored in Base region
- **r_π –** small input resistance between base and emitter seen from base
- **r_μ -** collector base reverse resistance (large value)
- **r_b -** base terminal resistance (small value)
- **r_o –** output resistance seen from output, collector emitter resistance
- **g_m v_{be} –** current source



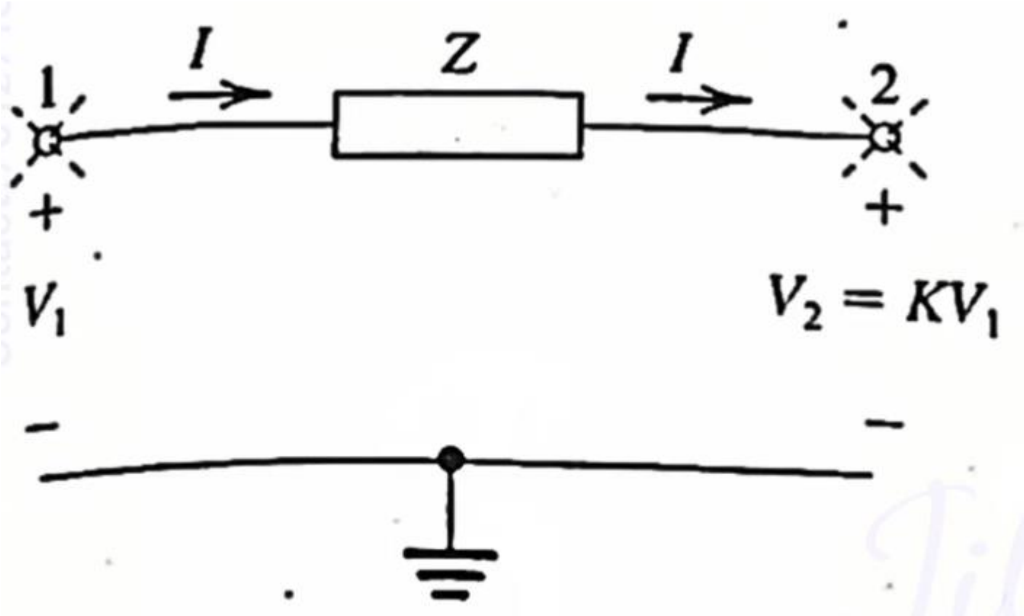
VOLTAGE CONTROLLED CURRENT SOURCE

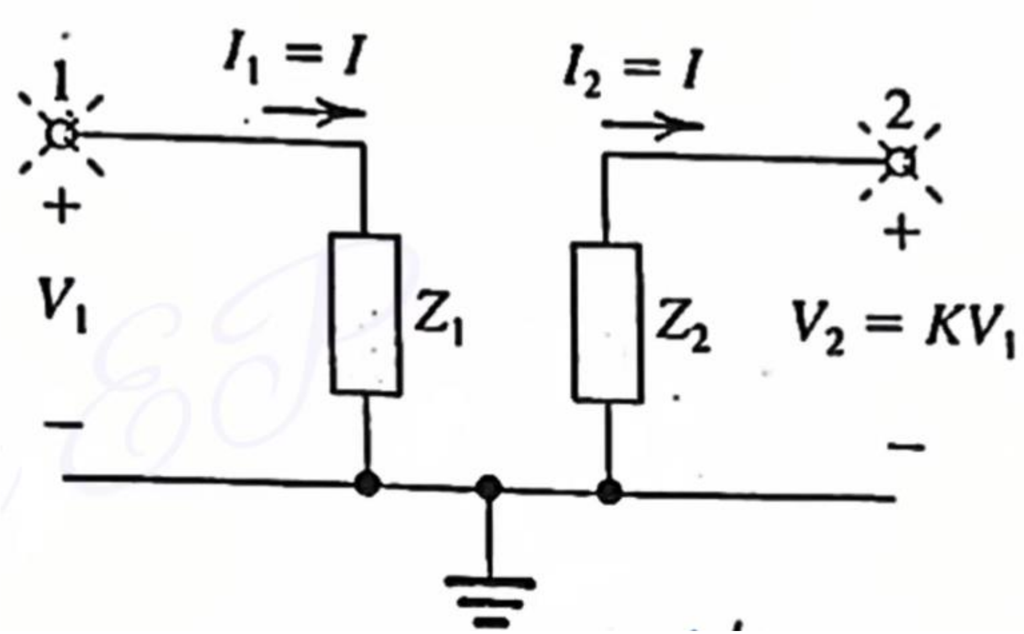
CURRENT CONTROLLED CURRENT SOURCE

MILLER'S THEOREM

- In analysis of High Frequency response of CE Amplifier, the bridging capacitances (**C_μ** and **C_π**) are replaced by an equivalent input capacitance. This effective technique is based on general theorem known as **Miller's Theorem.**

MILLER'S THEOREM

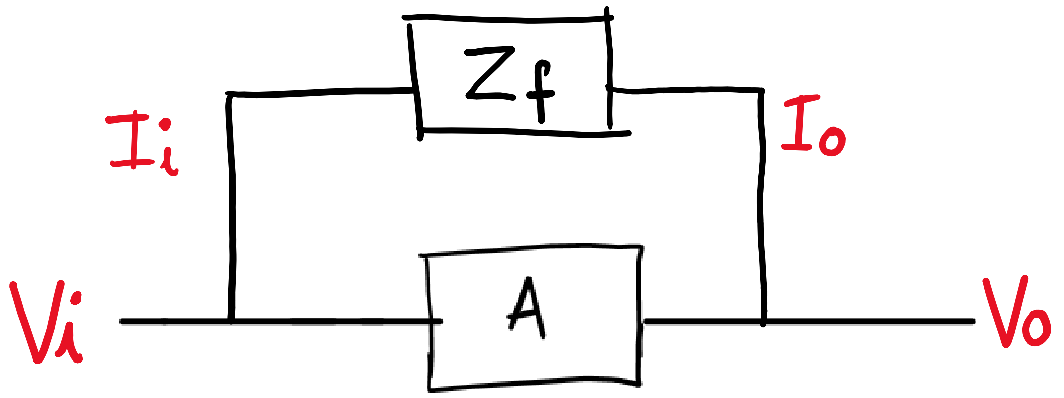




$$Z_i = \frac{Z_f}{(1-A)}$$

$$Z_o = \frac{Z_f A}{(A-1)}$$

MILLER'S THEOREM PROOF

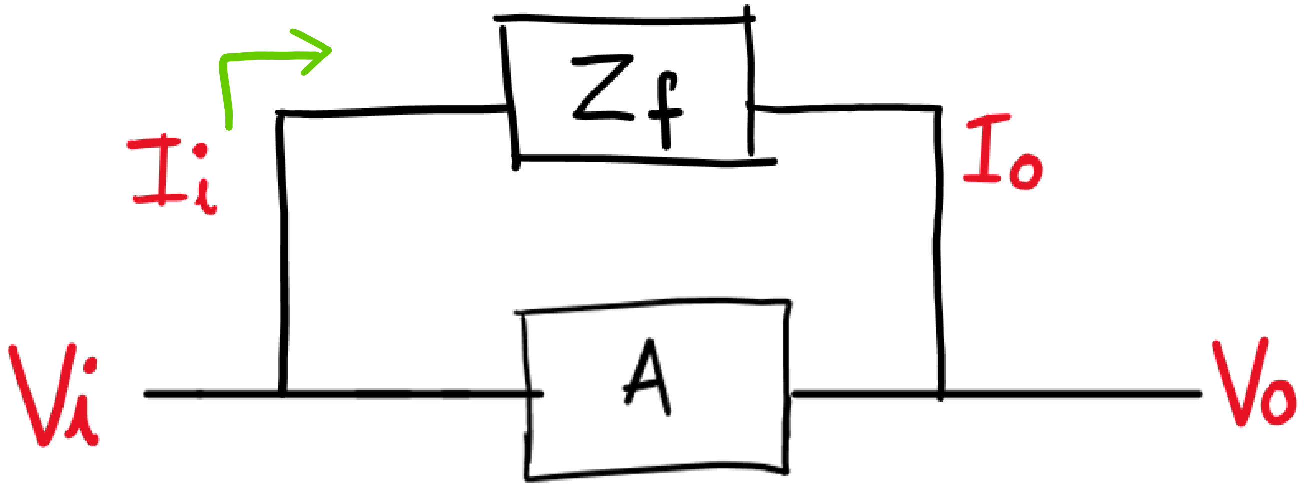


$$A = \frac{V_o}{V_i}$$

$$Z_i = \frac{V_i}{I_i}$$

$$Z_o = \frac{V_o}{I_o}$$

MILLER'S THEOREM PROOF



$$I_i = \frac{V_i - V_o}{Z_f} \longrightarrow (1)$$

$$I_i = \frac{V_i - AV_i}{Z_f} = \frac{V_i(1-A)}{Z_f}$$

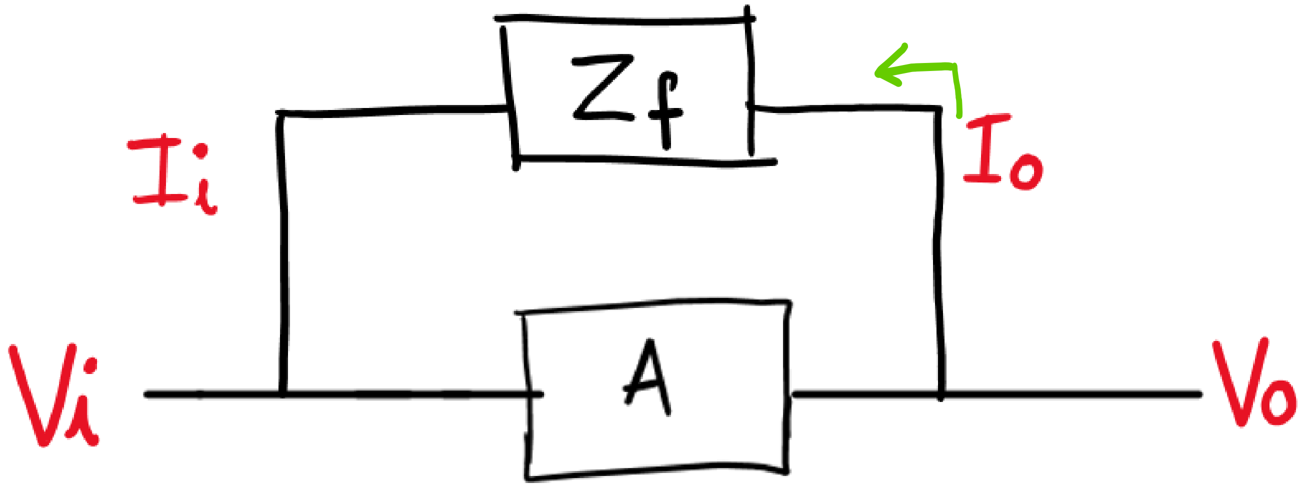
$$A = \frac{V_o}{V_i} \Rightarrow V_o = AV_i$$

we know

$$Z_i = \frac{V_i}{I_i} = \frac{Z_f}{(1-A)}$$

$$\boxed{Z_i = \frac{Z_f}{(1-A)}}$$

MILLER'S THEOREM PROOF



$$I_o = \frac{V_o - V_i}{Z_f}$$

$$A = \frac{V_o}{V_i} \Rightarrow V_i = \frac{V_o}{A}$$

$$I_o = \frac{V_o - V_o/A}{Z_f} = \frac{V_o\left(1 - 1/A\right)}{Z_f}$$

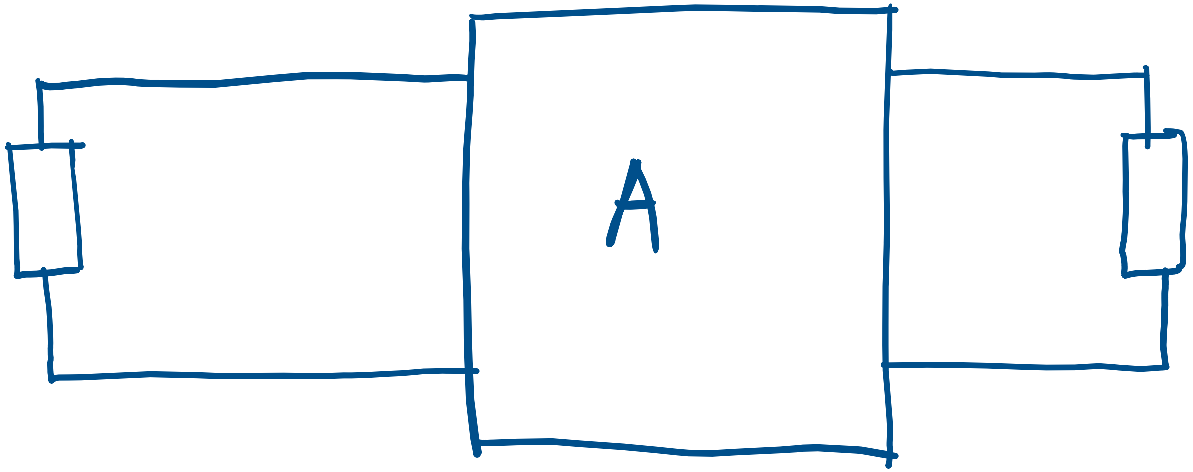
We know,

$$Z_o = \frac{V_o}{I_o} = \frac{Z_f}{\left(1 - 1/A\right)}$$

$$\boxed{Z_o = \frac{Z_f A}{(A-1)}}$$

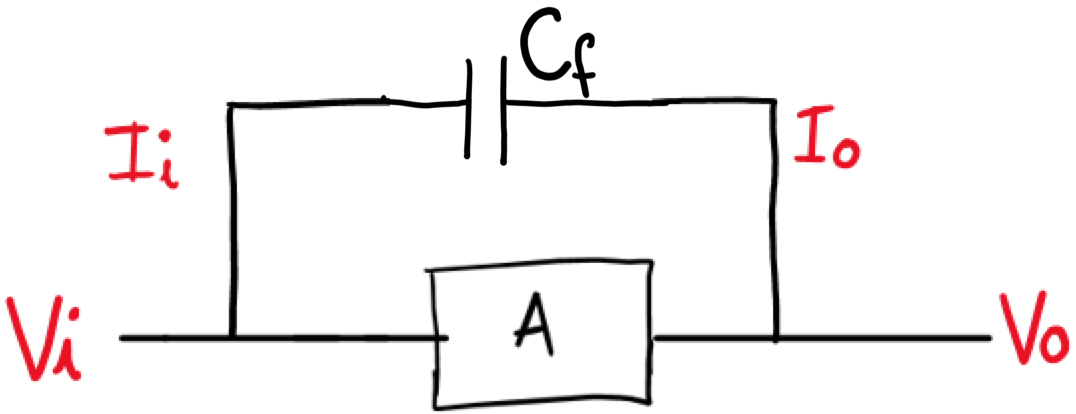
MILLER'S THEOREM PROOF

$$\frac{Z_f}{(1-A)} = Z_i$$



$$Z_o = \frac{Z_f A}{(A-1)}$$

IF CAPACITOR AS FEEDBACK COMPONENT



$$Z_i = \frac{Z_f}{(1-A)}$$

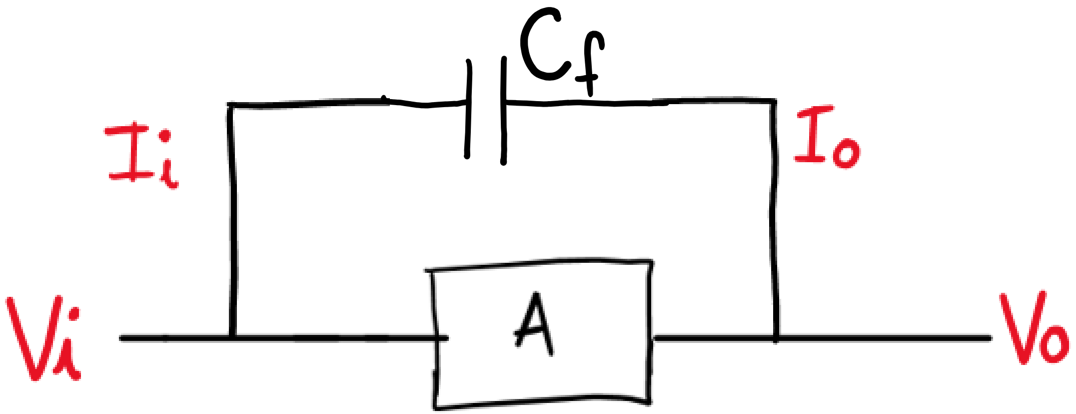
$Z_i, Z_f \rightarrow$ Capacitor

$$\frac{1}{j\omega C_i} = \frac{1/j\omega C_f}{(1-A)}$$

$$\frac{1}{C_i} = \frac{1}{C_f(1-A)}$$

$$\boxed{C_i = C_f(1-A)}$$

IF CAPACITOR AS FEEDBACK COMPONENT



$$Z_o = \frac{Z_f A}{(A-1)}$$

Z_o, Z_f - capacitor

$$\frac{1}{j\omega C_o} = \frac{A/j\omega C_f}{(A-1)}$$

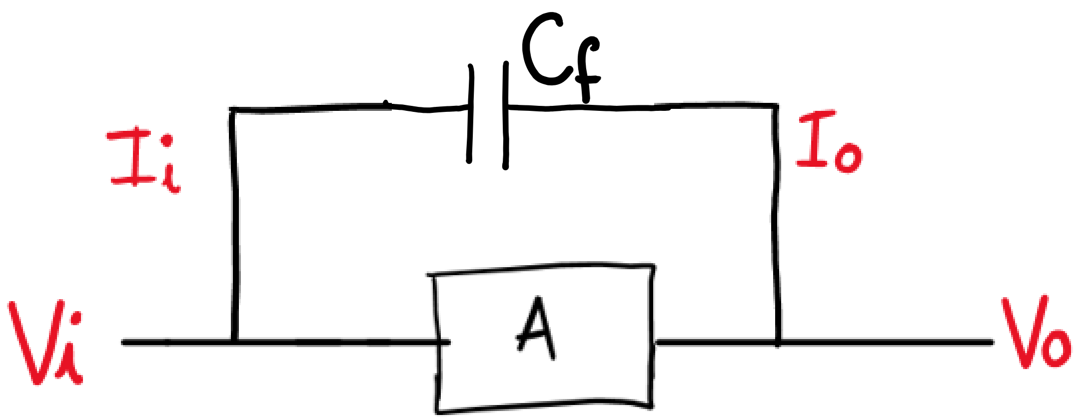
$$\frac{1}{C_o} = \frac{1}{C_f}\left(\frac{A}{A-1}\right)$$

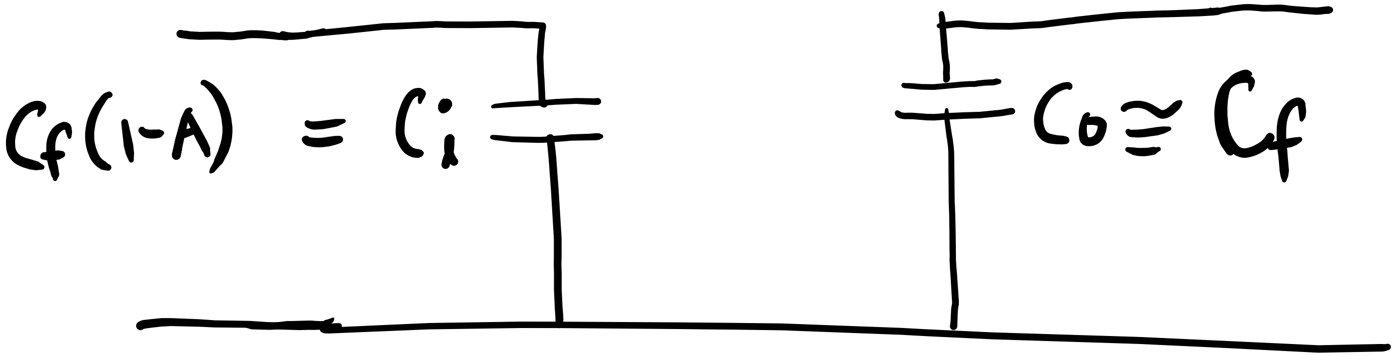
$$\left(\frac{A}{A-1}\right) \simeq 1$$

$$C_o \cong C_f$$

$$\boxed{C_o = C_f\left(\frac{A-1}{A}\right)}$$

IF CAPACITOR AS FEEDBACK COMPONENT





$$C_i = C_f(1-A)$$

$$C_o = C_f\frac{(A-1)}{A}$$

HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER

- **C_μ -** Transition capacitance between base and collector. Early effect representation
- **C_π –** Diffusion Capacitance. Minority carriers stored in Base region
- **r_π –** small input resistance between base and emitter seen from base
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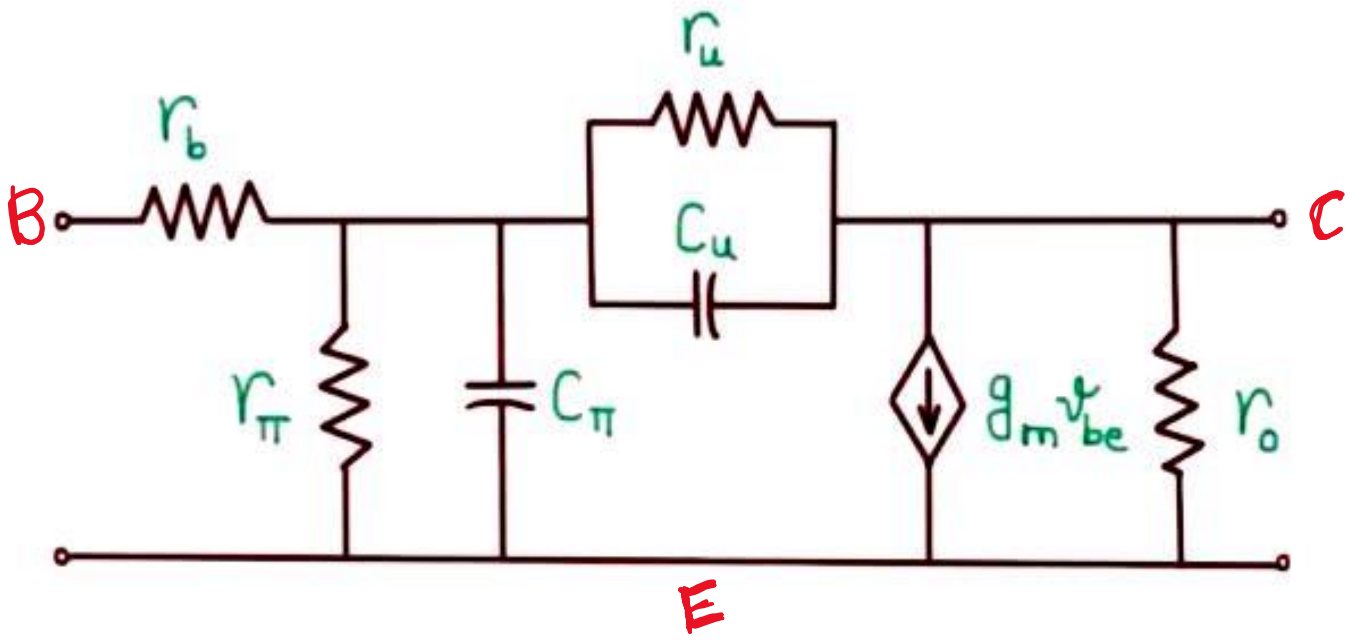
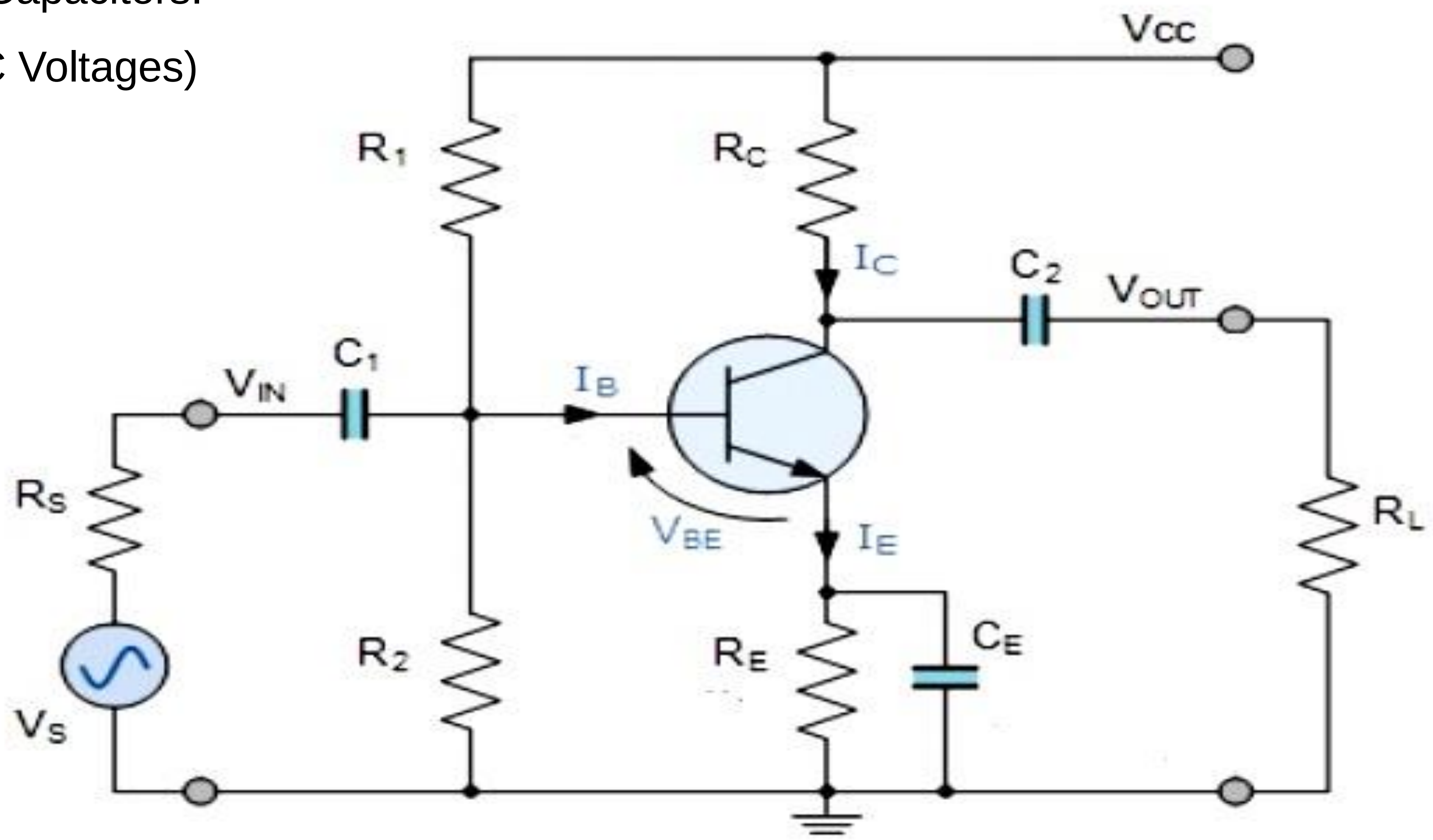


HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER

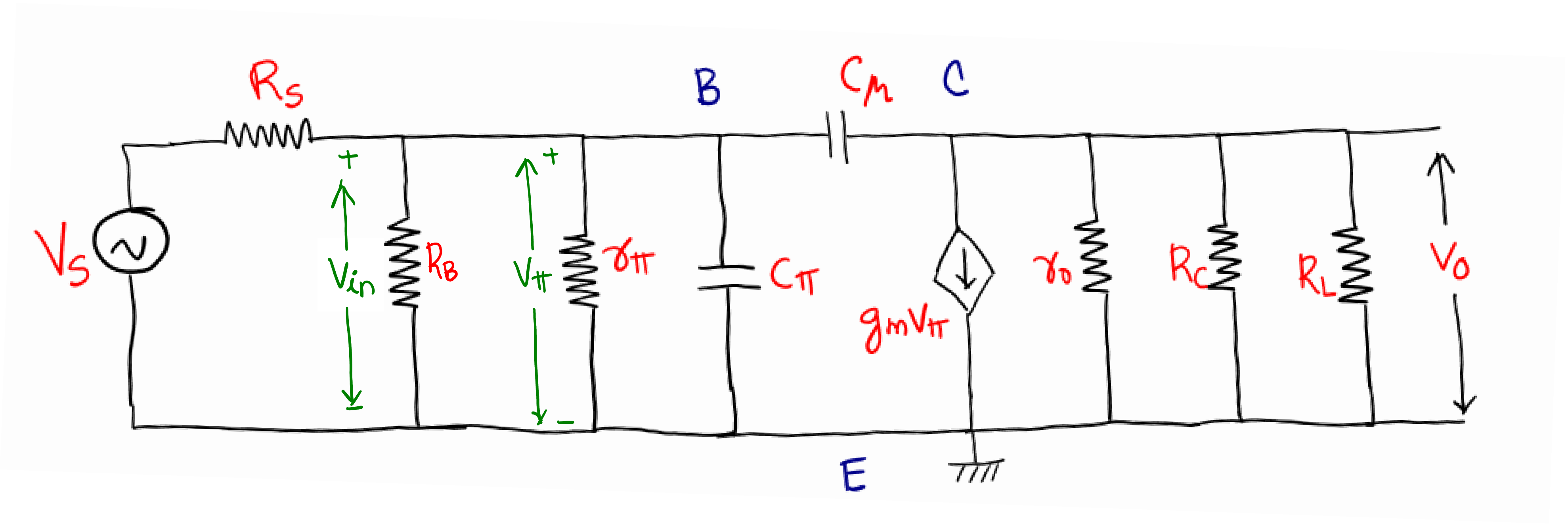
STEPS TO CONVERT TO HYBRID π MODEL

- Short circuit all Bypass and Coupling Capacitors.

  Remove all external power supply (DC Voltages) and connect that node to ground

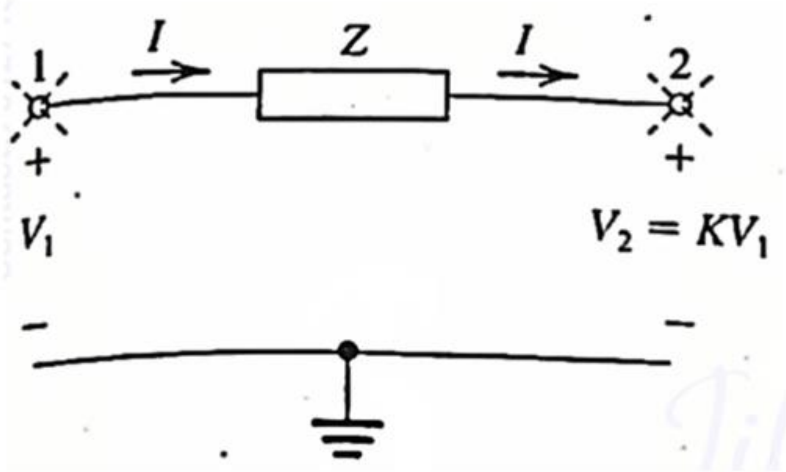


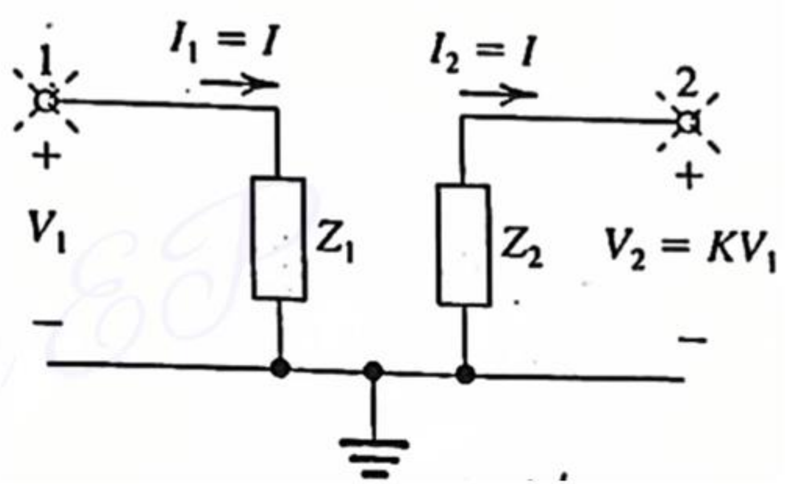
HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER



HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER

- The feedback capacitor **C_μ** can be split using **Miller's Theorem.**
- So by Miller's Theorem,

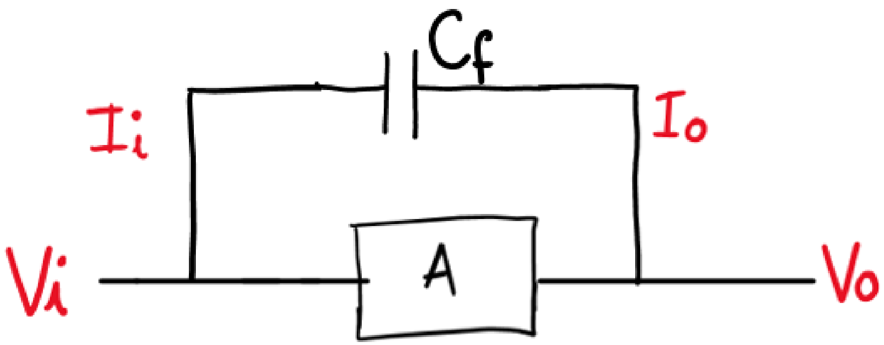




$$\boxed{Z_i = \frac{Z_f}{(1-A)}}$$

$$\boxed{Z_o = \frac{Z_f A}{(A-1)}}$$

HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER



- Applying Miller's theorem at input side

$$c_1 = c_\mu(1 - A_V)$$

HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER

- Applying Miller's theorem at input side

$$c_1 = c_\mu(1 - A_V)$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_\pi} \qquad (V_{in} \cong V_\pi)$$

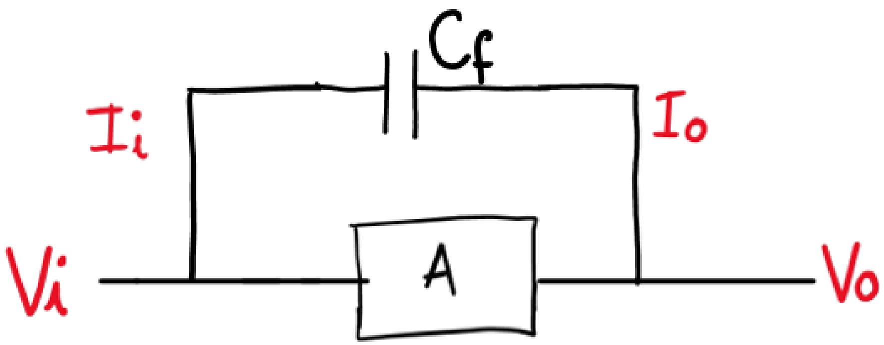
$$V_{out} = -g_m V_\pi (r_o \| R_c \| R_L)$$

$$A_v = \frac{V_{out}}{V_\pi} = \frac{-g_m \cancel{V_\pi} (r_o \| R_c \| R_L)}{\cancel{V_\pi}}$$

$$\boxed{A_v = -g_m (r_o \| R_c \| R_L)}$$

$$\boxed{C_1 = C_\mu (1 + g_m (r_o \| R_c \| R_L))}$$

HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER



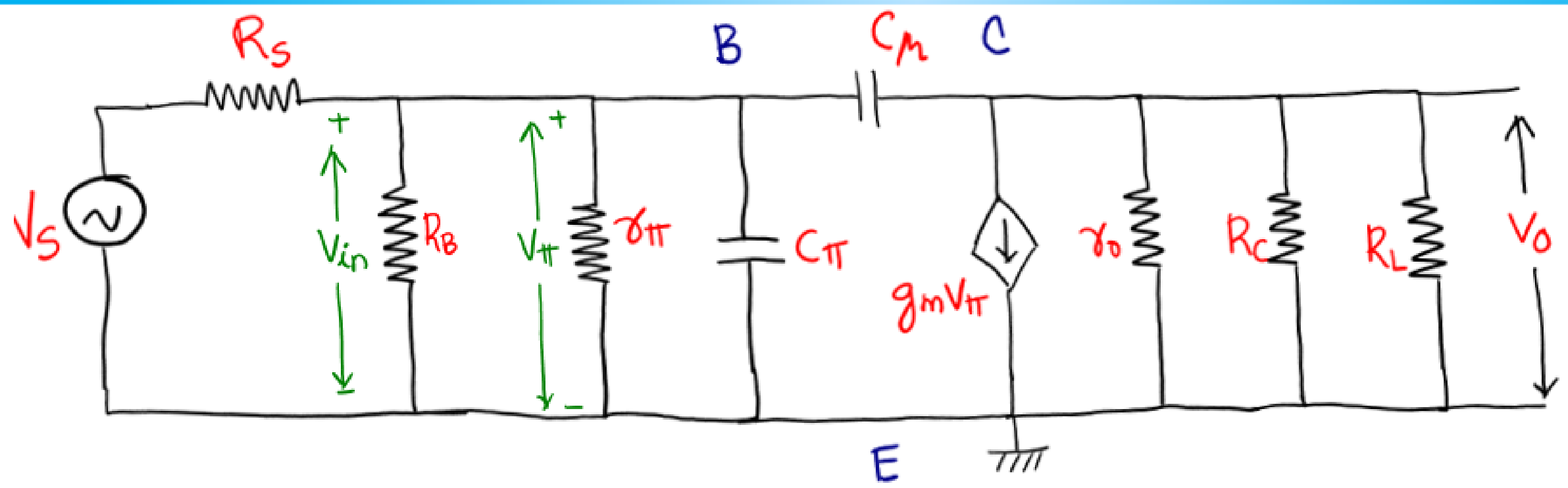
- Applying Miller's theorem at output side

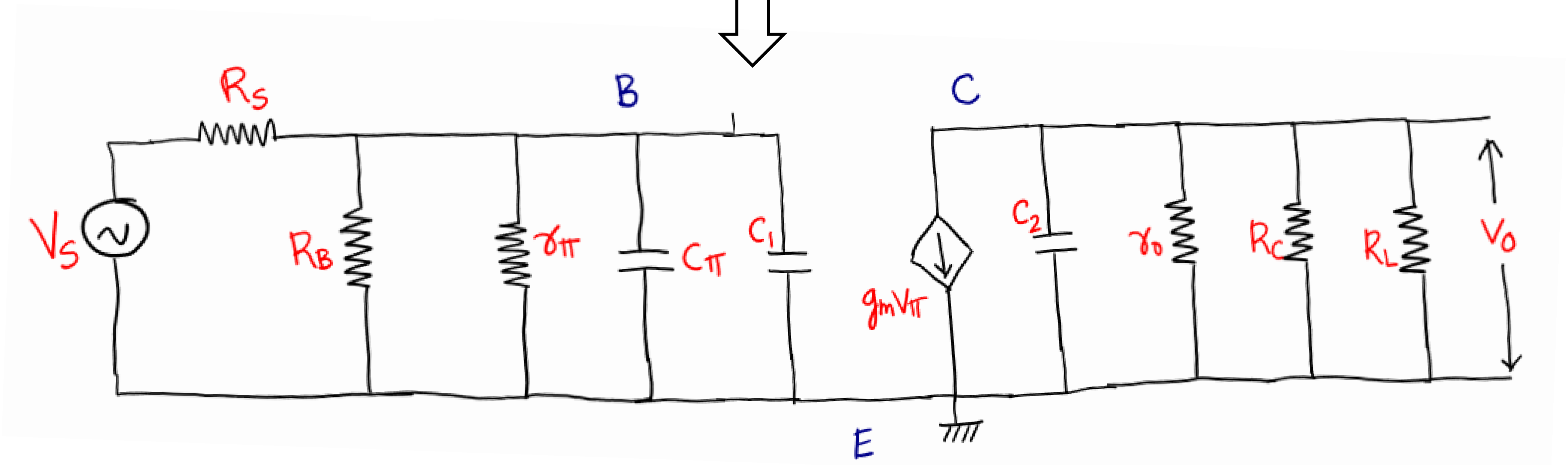
$$A_V = -g_m(r_o \| R_C \| R_L)$$

$$C_2 = C_\mu \frac{A_V - 1}{A_V}$$

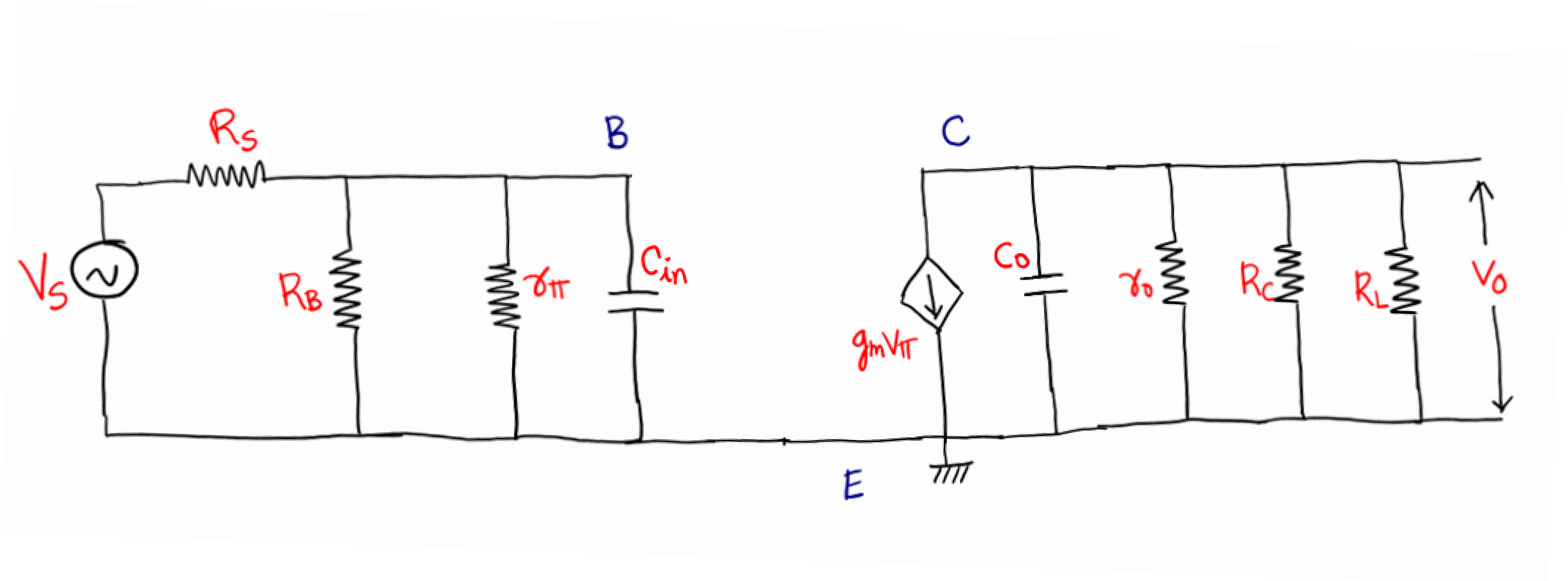
$$C_2 \cong C_\mu$$

HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER





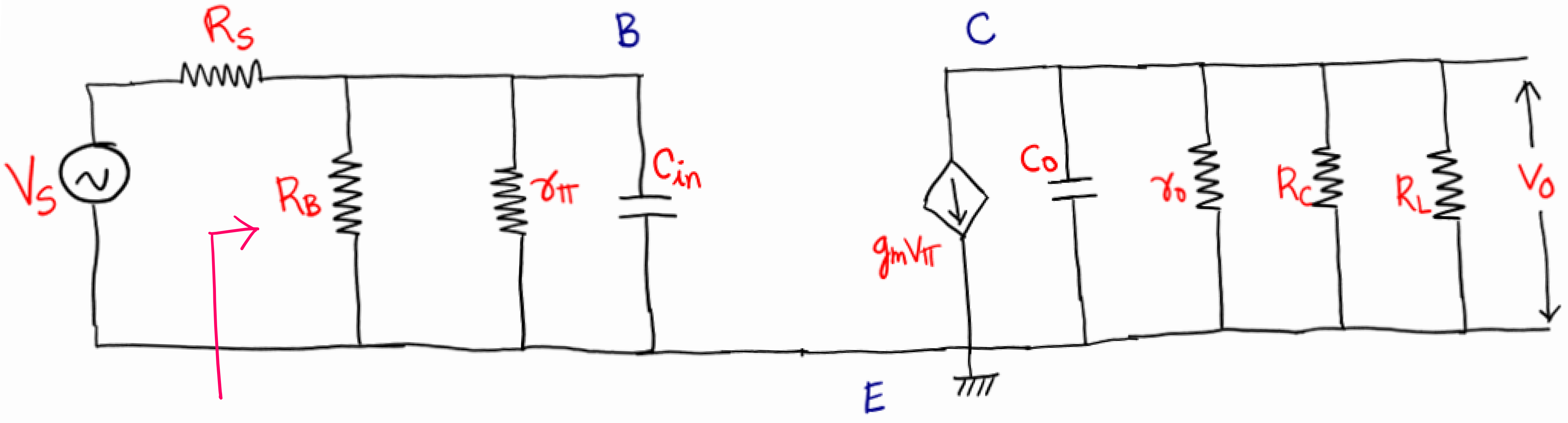
HIGH FREQUENCY HYBRID π MODEL OF CE AMPLIFIER



$$C_{in} = C_\pi + C_1$$

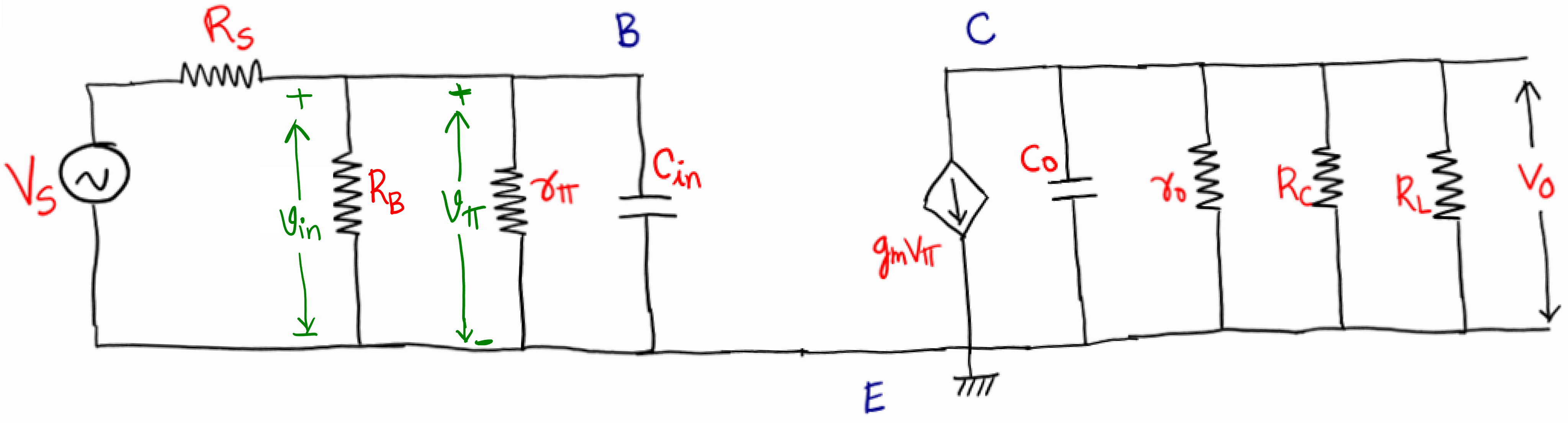
$$C_o = C_2$$

INPUT RESISTANCE



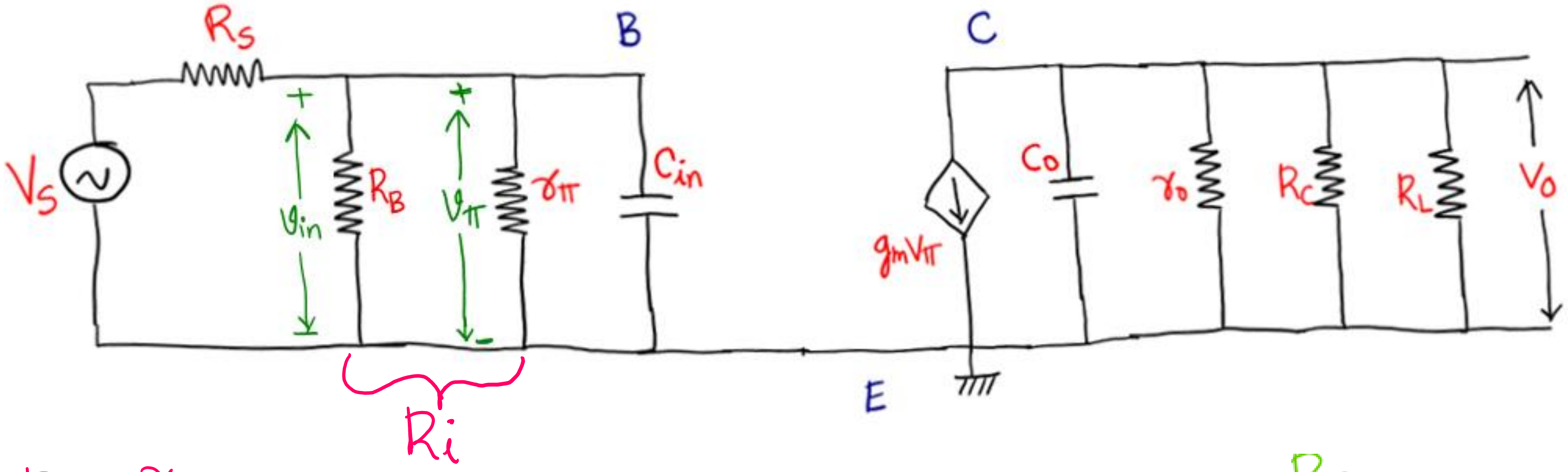
$$\boxed{R_i = R_B \,\|\, r_\pi}$$

VOLTAGE GAIN

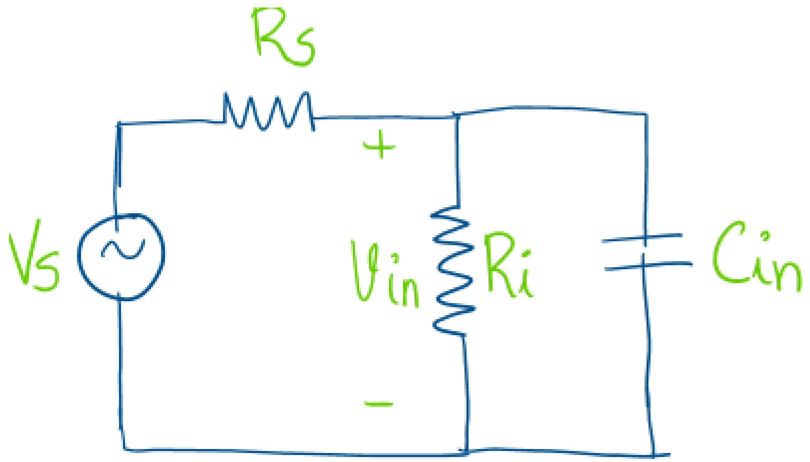


$$A_{vs} = \frac{v_{out}}{v_s} = \frac{v_{out}}{v_{in}} \times \frac{v_{in}}{v_s}$$

EFFECT OF INPUT CAPACITANCE



$R_i = R_B \| r_\pi$



By voltage Division rule

$$V_{in} = \frac{V_S \times (R_i \| 1/j\omega C_{in})}{R_S + (R_i \| 1/j\omega C_{in})}$$

EFFECT OF INPUT CAPACITANCE

$$\left(R_i \parallel 1/j\omega C_{in}\right) = \frac{R_i \times \dfrac{1}{j\omega C_{in}}}{R_i + \dfrac{1}{j\omega C_{in}}}$$

$$= \frac{R_i / \cancel{j\omega C_{in}}}{j\omega R_i C_{in} + 1 / \cancel{j\omega C_{in}}}$$

$$\left(R_i \parallel 1/j\omega C_{in}\right) = \frac{R_i}{1 + j\omega R_i C_{in}}$$

EFFECT OF INPUT CAPACITANCE

$$\left(R_i \parallel \tfrac{1}{j\omega C_{in}}\right) = \frac{R_i}{1 + j\omega R_i C_{in}} \quad ; \quad V_{in} = \frac{V_s \times \left(R_i \parallel \tfrac{1}{j\omega C_{in}}\right)}{R_s + \left(R_i \parallel \tfrac{1}{j\omega C_{in}}\right)}$$

$$V_{in} = \frac{V_s \times \left(R_i / 1 + j\omega R_i C_{in}\right)}{R_s + \left(R_i / 1 + j\omega R_i C_{in}\right)}$$

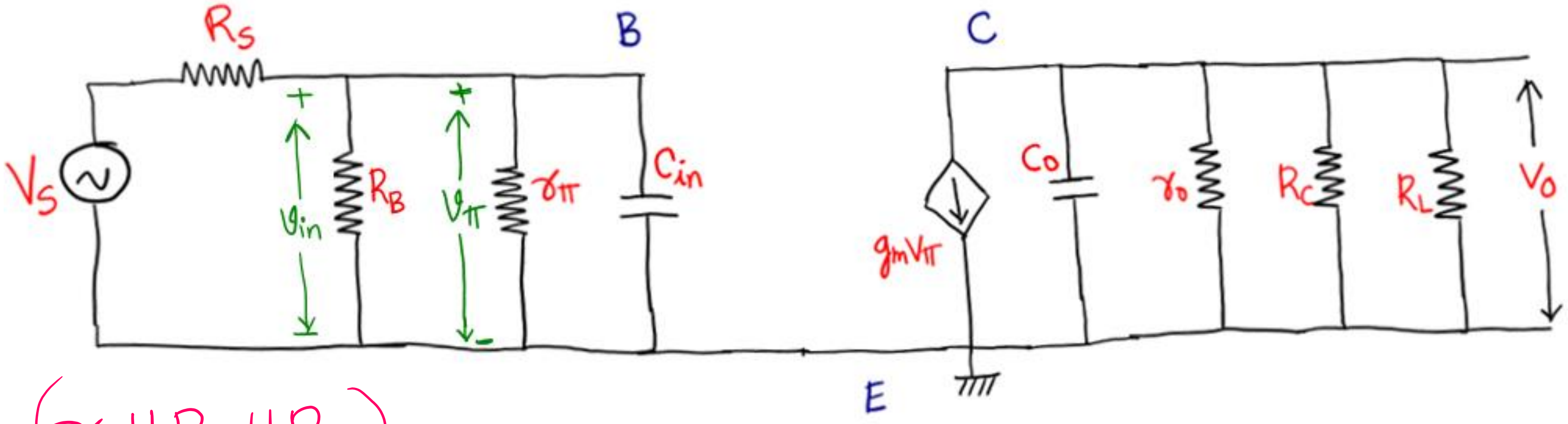
$$V_{in} = \frac{\left(V_s \cdot R_i / 1 + j\omega \cancel{R_i} C_{in}\right)}{\left(\left(R_s(1 + j\omega R_i C_{in}) + R_i\right) / 1 + j\omega \cancel{R_i} C_{in}\right)}$$

$$V_{in} = \frac{V_s \cdot R_i}{R_s + j\omega R_i R_s C_{in} + R_i}$$

$$V_{in} = \frac{V_s \cdot R_i}{(R_i + R_s)} \left(\frac{1}{1 + j\omega \left(\frac{R_i R_s}{R_i + R_s}\right) C_{in}}\right)$$

$$\frac{V_{in}}{V_s} = \left(\frac{R_i}{R_i + R_s}\right) \left(\frac{1}{1 + j\omega (R_i \parallel R_s) C_{in}}\right)$$

EFFECT OF OUTPUT CAPACITANCE



$$\frac{V_o}{V_{in}}$$

$$R_o = (r_o \| R_C \| R_L)$$

$$V_o = -g_m V_\pi \left(R_o \| 1/j\omega C_o \right)$$

$$V_{in} \cong V_\pi$$

$$\frac{V_o}{V_{in}} = \frac{-g_m \cancel{V_\pi} \left(R_o \| 1/j\omega C_o \right)}{\cancel{V_\pi}}$$

EFFECT OF OUTPUT CAPACITANCE

$$\left(R_o \parallel \frac{1}{j\omega C_o}\right) = \frac{R_o \cdot \frac{1}{j\omega C_o}}{R_o + \frac{1}{j\omega C_o}}$$

$$= \frac{R_o / \cancel{j\omega C_o}}{j\omega R_o C_o + 1 / \cancel{j\omega C_o}}$$

$$\left(R_o \parallel \frac{1}{j\omega C_o}\right) = \frac{R_o}{1 + j\omega R_o C_o}$$

$$\frac{V_o}{V_{in}} = -g_m \left(R_o \parallel \frac{1}{j\omega C_o}\right)$$

$$\frac{V_o}{V_{in}} = \frac{-g_m \cdot R_o}{1 + j\omega R_o C_o}$$

VOLTAGE GAIN

$$\frac{V_{in}}{V_s} = \left(\frac{R_i}{R_i + R_s}\right)\left(\frac{1}{1 + j\omega(R_i \parallel R_s)C_{in}}\right)$$

$$\frac{V_o}{V_{in}} = \frac{-g_m \cdot R_o}{1 + j\omega R_o C_o}$$

$$A_{Vs} = \frac{V_o}{V_s} = \frac{V_o}{V_{in}} \times \frac{V_{in}}{V_s}$$

$$A_{Vs} = \left(\frac{-g_m R_o}{1 + j\omega R_o C_o}\right)\left(\frac{R_i}{R_i + R_s}\right)\left(\frac{1}{1 + j\omega(R_i \parallel R_s)C_{in}}\right)$$

$$\boxed{A_m = \frac{-g_m R_o R_i}{R_i + R_s}}$$

$$\boxed{A_{Vs} = \frac{A_m}{(1 + j\omega R_o C_o)(1 + j\omega(R_i \parallel R_s)C_{in})}}$$

VOLTAGE GAIN IN TERMS OF FREQUENCY

$$A_{V_s} = \frac{A_m}{\underbrace{(1 + j\omega R_o C_o)}_{2}\ \underbrace{(1 + j\omega (R_i \| R_s) C_{in})}_{1}}$$

$1 \Rightarrow \quad 1 + j\omega (R_i \| R_s) C_{in}$

$\omega = 2\pi f$

$\Rightarrow \quad 1 + j2\pi f (R_i \| R_s) C_{in}$

$\Rightarrow \quad \boxed{1 + j\left(\frac{f}{f_{H1}}\right)}$

$$\boxed{f_{H1} = \frac{1}{2\pi (R_i \| R_s) C_{in}}}$$

f_{H1} – Cutoff frequency introduced by Input Capacitor

VOLTAGE GAIN IN TERMS OF FREQUENCY

$$A_{Vs} = \frac{A_m}{\underbrace{(1 + j\omega R_o C_o)}_{2}\ \underbrace{(1 + j\omega (R_i \| R_s) C_{in})}_{1}}$$

$2 \Rightarrow 1 + j\omega R_o C_o$

$\omega = 2\pi f$

$\Rightarrow 1 + j2\pi f R_o C_o$

$$\boxed{\Rightarrow 1 + j\left(\frac{f}{f_{H_2}}\right)}$$

$$\boxed{f_{H_2} = \frac{1}{2\pi R_o C_o}}$$

f_{H2} – Cutoff frequency introduced by Output Capacitor

VOLTAGE GAIN IN TERMS OF FREQUENCY

$$A_{vs} = \frac{A_m}{(1+j\omega R_o C_o)(1+j\omega (R_i \parallel R_s) C_{in})}$$

$$\boxed{A_{vs} = \frac{A_m}{\left(1+j\left(f/f_{H_2}\right)\right)\left(1+j\left(f/f_{H_1}\right)\right)}}$$

$$\boxed{f_H = \frac{1}{\sqrt{\left(1/f_{H_1}\right)^2 + \left(1/f_{H_2}\right)^2}}}$$