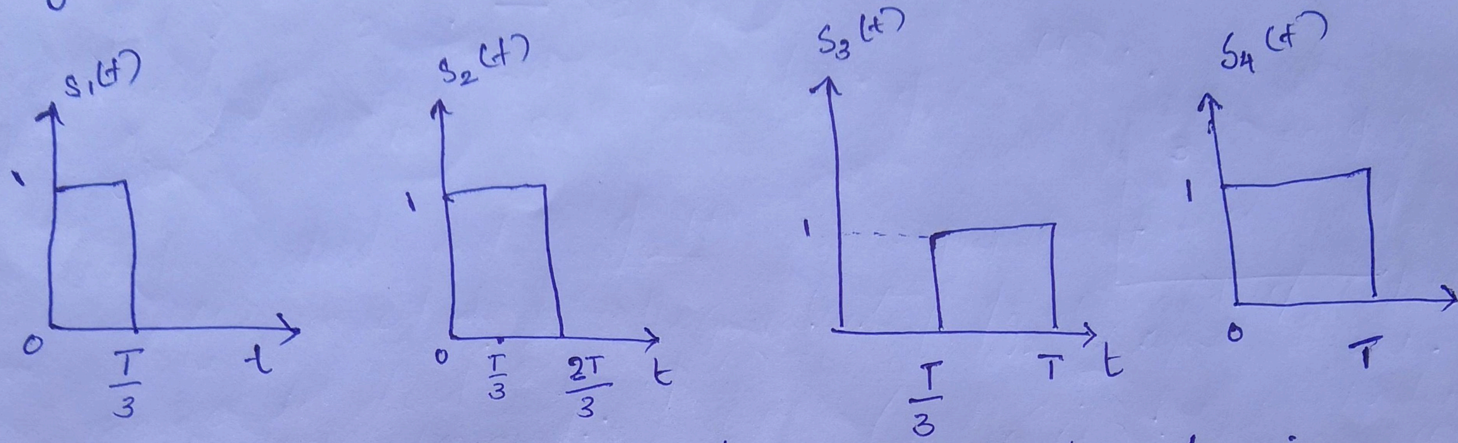
Use Gram Schmidt orthogonalisation procedure to find orthonormal basis functions for the set of signals



In the above given signals period of s/l is different but amplitude is same.

<u>Ans:</u> first we have to calculate $\phi_1(t)$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

$$E_1 = \int_0^T s_1^2(t)\, dt = \int_0^{T/3} (1)^2 dt = [t]_0^{T/3} = \underline{\underline{\frac{T}{3}}}.$$

$$\therefore \phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \frac{s_1(t)}{\sqrt{E_1}} = \begin{cases} \sqrt{\frac{3}{T}} & 0 \le t \le \frac{T}{3} \\ 0 & \text{otherwise} \end{cases}$$

<u>step: 2</u>

$$g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t) = \int_0^{T/3} 1\sqrt{\frac{3}{T}}\, dt = \sqrt{\frac{3}{T}}\,[t]_0^{T/3} = \sqrt{\frac{3}{T}} \times \frac{T}{3} = \sqrt{\frac{T}{3}}$$

$s_2(t)$ signal range is given as 0 to $\frac{2T}{3}$. But when we multiply $s_2(t)$ with $\phi_1(t)$ we need to consider the range from 0 to $T/3$ Because after $T/3$; $\phi_1(t) = 0$:.

$$\phi_2(t) = \frac{s_2(t) - s_{21}\,\phi_1(t)}{\sqrt{E_2 - s_{21}^2}}$$

$$E_2 = \int_0^T s_1^2(t)\,dt = \int_0^{2T/3} (1)^2\,dt = \left[t\right]_0^{\frac{2T}{3}} = \frac{2T}{3}$$

$$\phi_2(t) = \underbrace{\frac{1 - \sqrt{\frac{T}{3}} \times \sqrt{\frac{3}{T}}}{\sqrt{\frac{2T}{3} - \frac{T}{3}}}}_{\text{in range } 0 \text{ to } T/3} + \underbrace{\frac{1 - \sqrt{\frac{T}{3}} \times 0}{\sqrt{\frac{2T}{3} - \frac{T}{3}}}}_{\text{in range } \frac{T}{3} \text{ to } \frac{2T}{3}} + \underbrace{\frac{0 - \sqrt{\frac{T}{3}} \times 0}{\sqrt{\frac{2T}{3} - T_3}}}_{\frac{2T}{3} \text{ to } T}$$

[when doing the above step we must consider the range of $s_i(t)$ and $\phi_i(t)$]

$$= \cancel{\frac{1-1}{\sqrt{\frac{2T}{3} - \frac{T}{3}}}} + \frac{1 - 0}{\sqrt{\frac{2T}{3} - \frac{T}{3}}} + \cancel{0}$$

$$\phi_2(t) = \begin{cases} \dfrac{1}{\sqrt{\frac{T}{3}}} & \frac{T}{3} \le t \le \frac{2T}{3} \\ 0 & \text{elsewhere} \end{cases}$$

Step 3 : Next we have to calculate $\phi_3(t)$ from $S_3(t)$
This $\phi_3(t)$ should be orthogonal to $\phi_1(t)$ and $\phi_2(t)$ and energy should be normalised to one.

For this we have to find out projection of $S_3(t)$ on $\phi_1(t) \rightarrow S_{31}$
similarly find out projection of $S_3(t)$ on $\phi_2(t) \rightarrow S_{32}$.
Then S_{31} and S_{32} should subtract from $S_3(t)$. Then we will get an orthogonal s/g $g_3(t)$. From this we will find out $\phi_3(t)$

$$S_{31} = \int_0^T S_3(t)\,\phi_1(t)\,dt = 0.$$

$S_3(t)$ is given in the range of $T/3$ to T. From 0 to $T/3$ it is zero. and $\phi_1(t)$ exist from 0 to $T/3$ after that it is zero. so when we multiply $S_3(t)$ and $\phi_1(t)$ in the entire duration it will get 0.

$$S_{32} = \int_0^T S_3(t)\,.\,\phi_2(t)\,dt = \int_{T/3}^{2T/3} 1 . \sqrt{\frac{3}{T}}\,dt = \sqrt{\frac{3}{T}}\,[t]_{T/3}^{2T/3} = \sqrt{\frac{3}{T}}\left[\frac{2T}{3} - \frac{T}{3}\right]$$

$$= \sqrt{\frac{3}{T}} \times \frac{T}{3} = \frac{\sqrt{3}}{\sqrt{T}} \times \frac{\sqrt{T} \times \sqrt{T}}{\sqrt{3} \times \sqrt{3}}$$

$S_3(t) = 1$ from $T/3$ to T

$$= \sqrt{\frac{T}{3}}$$

$\phi_2(t) = \sqrt{\frac{3}{T}}$ from $\frac{T}{3}$ to $\frac{2T}{3}$ after that it is zero. So multiplication within the integral exist b/w $T/3$ to $2T/3$ only.

$$g_3(t) = S_3(t) - S_{31}\,\phi_1(t) - S_{32}\,\phi_2(t)$$

$$\underbrace{\left(0 - \sqrt{\frac{T}{3}} \times 0\right)}_{\text{in } 0 \text{ to } T/3} + \underbrace{\left(1 - \sqrt{\frac{T}{3}} \times \sqrt{\frac{3}{T}}\right)}_{T/3 \text{ to } \frac{2T}{3}} + \underbrace{\left(1 - \sqrt{\frac{T}{3}} \times 0\right)}_{\text{in } \frac{2T}{3} \text{ to } T}$$

$$0 \quad + \quad 1 - 1 \quad + \quad 1 - 0$$

$$g_3(t) = \begin{cases} 1 & \frac{2T}{3} \le t \le T \\ 0 & \text{elsewhere.} \end{cases}$$

Next 3^{rd} basis function is finding out by normalising $g_3(t)$

$$\phi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t)\,dt}} = \frac{1}{\sqrt{\frac{T}{3}}}$$

$$\int_0^T g_3^2(t)\,dt = \int_{2T/3}^{T} 1^2\,dt = [t]_{2T/3}^{T} = T - \frac{2T}{3} = \underline{\underline{\frac{T}{3}}}$$

$$\phi_3(t) = \begin{cases} \sqrt{\frac{3}{T}} & \frac{2T}{3} \le t \le T \\ 0 & \text{else where} \end{cases}$$

<u>step 4</u>. calculate $\phi_4(t)$; it should be orthogonal to $\phi_1(t)$, $\phi_2(t)$ and $\phi_3(t)$. For that we have to find out

S_{41} - projection of $S_4(t)$ on $\phi_1(t)$

S_{42} - projection of $S_4(t)$ on $\phi_2(t)$

S_{43} - projection of $S_4(t)$ on $\phi_3(t)$

Then subtract all these from $S_4(t)$ and hence we will get $g_4(t)$

$$g_4(t) = S_4(t) - S_{41}\,\phi_1(t) - S_{42}\,\phi_2(t) - S_{43}\,\phi_3(t)$$

$$S_{41} = \int_0^T \underbrace{S_4(t)}_{0 \text{ to } T}\,\underbrace{\phi_1(t)}_{0 \text{ to } T/3}\,dt = \int_0^{T/3} (1)\cdot\sqrt{\frac{3}{T}}\,dt = \sqrt{\frac{3}{T}}\cdot\frac{T}{3} = \sqrt{\frac{T}{3}}$$

$$S_{42} = \int_0^T S_4(t)\,\underbrace{\phi_2(t)}_{T/3 \text{ to } 2T/3}\,dt = \int_{T/3}^{2T/3} (1)\sqrt{\frac{3}{T}}\,dt = \sqrt{\frac{3}{T}}\cdot\left[\frac{2T}{3} - \frac{T}{3}\right] = \sqrt{\frac{3}{T}}\times\frac{T}{3} = \sqrt{\frac{T}{3}}$$

$$S_{43} = \int_0^T S_4(t)\,\underbrace{\phi_3(t)}_{\frac{2T}{3} \text{ to } T} = \int_{2T/3}^{T} (1)\cdot\sqrt{\frac{3}{T}}\,dt = \sqrt{\frac{3}{T}}\cdot\left[T - \frac{2T}{3}\right] = \sqrt{\frac{3}{T}}\times\frac{T}{3} = \sqrt{\frac{T}{3}}$$

$g_4(t) = s_4(t) - s_{41}\phi_1(t) - s_{42}\phi_2(t) - s_{43}\phi_3(t)$

$$g_4(t) = \underbrace{1 - \sqrt{\frac{T}{3}} \cdot \sqrt{\frac{3}{T}} - \sqrt{\frac{T}{3}} \times 0 - \sqrt{\frac{T}{3}} \times 0}_{0 \text{ to } T/3} \longrightarrow 0$$

$$+ 1 - \underbrace{\sqrt{\frac{T}{3}} \times 0 - \sqrt{\frac{T}{3}} \times \sqrt{\frac{3}{T}} - \sqrt{\frac{3}{T}} \times 0}_{T/3 \text{ to } 2T/3} \Rightarrow 0$$

$$+ 1 - \underbrace{\sqrt{\frac{T}{3}} \times 0 - \sqrt{\frac{T}{3}} \times 0 - \sqrt{\frac{T}{3}} \cdot \sqrt{\frac{3}{T}}}_{2T/3 \text{ to } T} \Rightarrow 0.$$

$g_4(t) = 0 \quad \therefore \phi_4(t) = 0.$

$\phi_1(t) = \sqrt{3/T} \quad 0 \le t \le T/3$, $\quad \phi_2(t) = \sqrt{3/T} \quad T/3 \le t < 2T/3$

$\qquad 0 \quad$ elsewhere $\qquad\qquad\qquad 0 \quad$ elsewhere.

$\phi_3(t) = \sqrt{3/T} \quad 2T/3 \le t \le T$, $\quad \phi_4(t) = 0.$

$\qquad 0 \quad$ elsewhere

The three basis functions $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$ form an orthonormal set.

Here we have $M = 4$ and $N = 3$ which means that the 4 signals $s_1(t)$, $s_2(t)$, $s_3(t)$ & $s_4(t)$ do not form a linearly independant set. This is confirmed by noting $s_4(t) = s_1(t) + s_3(t)$

Any of these 4 signals can be expressed as a linear combination of three basis functions.

we can plot the basis function as follows.