MAT202, MAT204 Module. 4

Numerical Methods - 1

Text: Erwin Kreyszig, Advanced Engineering Mathematics, 10^{th} Edition. John Wiley & Sons. 2016.

Section: 19.1, 19.2, 19.3, 19.5.

Errors in numerical computation - round off, truncation and relative error, Solution of equations - Newton-Raphson method and Regula-falsi method. Interpolation - finite differences, Newton's forward and backward difference method, Newton's divided difference method and Lagrange's method. Numerical integration - Trapezoidal rule and Simpsons $1/3^{rd}$ rule.

(Proof or derivation of the formulae not required for any of the methods in this module)

$x^2+2x+1=0$

$\underbrace{x=-1,-1}$ Root

$(-1)^2+2\times-1+1=0$

$x^3-2x+5=0$

$x^5-2x+3=0$

$x\cos x+e^x=10$

$x\sin x-x^2=0$

Newton Raphson Method

$$\boxed{x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}}$$

Note:

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

Using Newton Raphson method, Compute the real root of $f(x) = x^3 - 2x - 5$ correct to 5 decimal places.

$2.094, -1.0472 \pm i\,1.1359$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x) = x^3 - 2x - 5$ $\quad$ $f'(x) = 3x^2 - 2$

$f(0) = -5$ (-ve)

$f(1) = 1 - 2 - 5 = -6$ (-ve)

$f(2) = 8 - 4 - 5 = -1$ (-ve)

$f(3) = 27 - 6 - 5 = 16$ (+ve)

Root lies between 2 and 3

$|f(2)| = |-1| = 1$

$|f(3)| = |16| = 16$

$$\boxed{|f(3)| > |f(2)|}$$

$x_0 = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \frac{[8 - 4 - 5]}{[12 - 2]}$$

$$\boxed{x_1 = 2.1}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.1 - \frac{f(2.1)}{f'(2.1)}$$

$$= 2.1 - \frac{[(2.1)^3 - 2(2.1) - 5]}{[3(2.1)^2 - 2]}$$

$$= 2.0945681$$

$$\boxed{x_2 = 2.09457}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.09457 - \frac{f(2.09457)}{f'(2.09457)}$$

$$= 2.09457 - \frac{[(2.09457)^3 - 2(2.09457) - 5]}{[3(2.09457)^2 - 2]}$$

$$= 2.094551$$

$$= \underline{\underline{2.09455}}$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.09455 - \frac{f(2.09455)}{f'(2.09455)}$$

$$= 2.09455 - \frac{[(2.09455)^3 - 2(2.09455) - 5]}{[3(2.09455)^2 - 2]}$$

$$= 2.094551$$

$$= \underline{\underline{2.09455}}$$

$\therefore$ App root is 2.09455 Correct to 5 decimal place.

Using Newton Raphson method, Compute a real root of $f(x) = e^{2x} - x - 6$ lying between 0 and 1.

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$f(x) = e^{2x} - x - 6$ $\quad f'(x) = 2e^{2x} - 1$

$f(0) = e^0 - 0 - 6 = -5$ (-ve)

$f(1) = e^2 - 1 - 6 = +ve = 0.389$

$|f(0)| = |-5| = 5$

$|f(1)| = |0.389| = 0.389$

$|f(0)| > |f(1)|$

$$\boxed{x_0 = 1}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 1 - \frac{[e^2 - 1 - 6]}{[2e^2 - 1]}$$

$$= 0.97176$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.972 - \frac{f(0.972)}{f'(0.972)}$$

$$= 0.972 - \frac{[e^{2(0.972)} - (0.972) - 6]}{[2e^{2(0.972)} - 1]}$$

$$= 0.97086$$

Using Newton Raphson method, Compute a real root of $f(x) = e^{2x} - x - 6$ lying between 0 and 1.

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$f(x) = e^{2x} - x - 6$ $\quad$ $f'(x) = 2e^{2x} - 1$

$f(0) = e^0 - 0 - 6 = -5$ (-ve)

$f(1) = e^2 - 1 - 6 = +ve = 0.389$

$|f(0)| = |-5| = 5$

$|f(1)|$ ... $9| = 0.389$

$f(0$ ... $(1))$

$$\boxed{x_0 = 1}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 1 - \frac{[e^2 - 1 - 6]}{[2e^2 - 1]}$$

$= 0.97176$

$= \underline{0.972}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.972 - \frac{f(0.972)}{f'(0.972)}$$

$$= 0.972 - \frac{[e^{2(0.972)} - (0.972) - 6]}{[2e^{2(0.972)} - 1]}$$

$= 0.97086$

$= \underline{0.971}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.971 - \frac{f(0.971)}{f'(0.971)}$$

$$= 0.971 - \frac{\left[e^{2(0.971)} - 0.971 - 6\right]}{\left[2e^{2(0.971)} - 1\right]}$$

$$= 0.97087$$

$$= \underline{0.971}$$

$\therefore$ App root is 0.971 correct to three decimal place.

Find the value of $(24)^{1/3}$ using Newton Raphson method

$\sqrt{5}$ $\sqrt{12}$ $(12)^{1/3}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$(a^m)^n = a^{mn}$

$(24^{1/3})^3 = 24$

$x = (24)^{1/3}$

$x^3 = (24^{1/3})^3$

$x^3 = 24$

$x^3 - 24 = 0$

$f(x) = x^3 - 24$ $\quad f'(x) = 3x^2$

$f(0) = -24$ (-ve)

$f(1) = 1 - 24 = -23$ (-ve)

$f(2) = 8 - 24 = -16$ (-ve)

$f(3) = 27 - 24 = 3$ (+ve)

$x=2$ $\quad$ $x=3$

-ve $\quad$ 0 $\quad$ +ve

Root lies between 2 and 3

$|f(2)| = |-16| = 16$

$|f(3)| = |3| = 3$

$|f(2)| > |f(3)|$

$\boxed{x_0 = 3}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{f(3)}{f'(3)}$$

$$= 3 - \frac{[3^3 - 24]}{[3(3)^2]}$$

$\boxed{x_1 = 2.889}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.889 - \frac{f(2.889)}{f'(2.889)}$$

$$= 2.889 - \frac{[(2.889)^3 - 24]}{[3(2.889)^2]}$$

$= 2.8845$

$\boxed{x_2 = 2.885}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.885 - \frac{f(2.885)}{f'(2.885)}$$

$$= 2.885 - \frac{[(2.885)^3 - 24]}{[3(2.885)^2]}$$

$$= 2.88449$$

$$\boxed{x_3 = \underline{2.884}}$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.884 - \frac{f(2.884)}{f'(2.884)}$$

$$= 2.884 - \frac{[(2.884)^3 - 24]}{[3(2.884)^2]}$$

$$= 2.88449$$

$$\boxed{x_4 = \underline{2.884}}$$

$\therefore$ App root is 2.884 Correct to 3 decimal place.

~~x_3~~

$x = 2\sin x$.

Find the real root of $3x - \cos x - 1 = 0$ by Newton Raphson method. Correct to 6 decimal.

$x = \sqrt{5}$

$x^2 = 5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x) = 3x - \cos x - 1 \qquad f'(x) = 3 + \sin x.$

$f(0) = 0 - 1 - 1 = -2$ (-ve)

$f(1) = 3 - \cos 1 - 1 = 1.4596$ (+ve)

$\therefore$ Root lies bet: 0 and 1

$|f(0)| = |-2| = 2$

$|f(1)| = |1.4596| = 1.4596$

$|f(0)| > |f(1)|$

$\boxed{x_0 = 1}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 1 - \left[\frac{3 - \cos 1 - 1}{3 + \sin 1}\right]$$

$= 0.6200159$

$= \underline{0.620016}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.620016 - \frac{f(0.620016)}{f'(0.620016)}$$

$$= 0.620016 - \frac{[3(0.620016) - \cos(0.620016) - 1]}{[3 + \sin(0.620016)]}$$

$= 0.60712065$

$= \underline{0.607121}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.607121 - \frac{f(0.607121)}{f'(0.607121)}$$

$$= 0.607121 - \frac{[3(0.607121) - \cos(0.607121) - 1]}{[3 + \sin(0.607121)]}$$

$$= 0.607101648$$

$$= 0.607102$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.607102 - \frac{f(0.607102)}{f'(0.607102)}$$

$$= 0.6071016$$

$$= \underline{\underline{0.607102}}$$

$\therefore$ App root is 0.607102

Correct to 3 decimal plac

Find the root of $x = 2\sin x$

Starting with $x = 1$ using N.R.

$\underline{\underline{1.896}}$

RVS Maths Academy

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.607121 - \frac{f(0.607121)}{f'(0.607121)}$$

$$= 0.607121 - \frac{[3(0.607121) - \cos(0.607121) - 1]}{[3 + \sin(0.607121)]}$$

$$= 0.607101648$$

$$= 0.607102$$

$f(x) = x - 2\sin x$

$f'(x) = 1 - 2\cos x$

(Radian)

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.607102 - \frac{f(0.607102)}{f'(0.607102)}$$

$$= 0.6071016$$

$$= \underline{\underline{0.607102}}$$

$\therefore$ App root is 0.607102

Correct to 3 decimal places

Find the root of $x = 2\sin x$

Starting with $\underline{x=1}$ using N.R.

$\underline{1.896}$

Find a root of $x\log_{10}x - 1.2 = 0$ by N.R method correct to 3 decimal places.

$x = 2$, $x = 2.5$, 2.7, 0, $x = 3$

$-ve$, $-ve$, $-ve$, 0, $+ve$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\frac{d}{dx}\log_e x = \frac{1}{x}$$

$$\frac{d}{dx}\log_{10}x = \frac{\log_{10}e}{x} = \frac{0.4343}{x}$$

$$f(x) = x\log_{10}x - 1.20$$

$$\cancel{f'(x)=}$$

$$f'(x) = \cancel{x}\left[\frac{0.4343}{\cancel{x}}\right] + \log_{10}x\,(1) - 0$$

$$= 0.4343 + \log_{10}x$$

$f(1) = \log_{10}1 - 1.2 = -1.2$ (-ve)

$f(2) = 2\log_{10}(2) - 1.2 = -0.5979$ (-ve)

$f(3) = 3\log_{10}(3) - 1.2 = 0.231$ (+ve)

$f(2.5) = 2.5\log_{10}(2.5) - 1.2 = -0.205$ (-ve)

$f(2.7) = 2.7\log_{10}(2.7) - 1.2 = -0.0353$ (-ve)

Root lies 2.7 and 3

$|f(2.7)| = |-0.0353| = 0.0353$

$|f(3)| = |0.231| = 0.231$

$|f(3)| > |f(2.7)|$

$x_0 = 2.7$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2.7 - \frac{f(2.7)}{f'(2.7)}$$

$$= 2.7 - \frac{[2.7\log_{10}(2.7) - 1.2]}{0.4343 + \log_{10}(2.7)}$$

$$= \underline{\underline{2.740}}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.740 - \frac{f(2.740)}{f'(2.740)}$$

$$= 2.740 - \frac{[2.740\log_{10}(2.740) - 1.2]}{\ldots}$$

$$= \underline{\underline{2.741}}$$

Trig: → Radian.

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.741 - \frac{f(2.741)}{f'(2.741)}$$

$$= 2.741 - \frac{[2.741\log_{10}(2.741) - 1.2]}{[0.4343 + \log_{10}(2.741)]}$$

$$= \underline{\underline{2.741}}$$

∴ App Root is 2.741 Correct to 3 decimal place.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2.7 - \frac{f(2.7)}{f'(2.7)}$$

$$= 2.7 - \frac{[2.7\log_{10}(2.7) - 1.2]}{0.4343 + \log_{10}(2.7)}$$

$$= \underline{\underline{2.740}}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.740 - \frac{f(2.740)}{f'(2.740)}$$

$$= 2.740 - \frac{[2.740\log_{10}(2.740) - 1.2]}{[0.4343 + \log_{10}(2.740)]}$$

The method of false position (Regula Falsi Method)

$$\boxed{x = \frac{a f(b) - b f(a)}{f(b) - f(a)}}$$

Root lies bet 2 & 3

where $f(a)$ and $f(b)$ are of opposite sign.

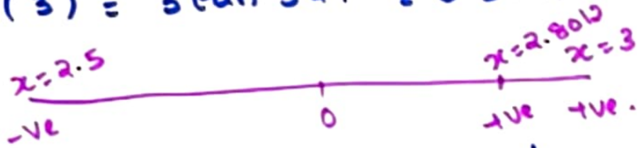
Solve the equation $x \tan x = -1$ by Regula Falsi method Starting with $x_0 = 2.5$ and $x_1 = 3$ Correct to 4 decimal places.

$$x = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$f(x) = x \tan x + 1$$

$$f(2.5) = 2.5 \tan 2.5 + 1 = -0.8675 \text{ (-ve)}$$

$$f(3) = 3 \tan 3 + 1 = 0.5724 \text{ (+ve)}$$



Root lies bet $\overset{a}{2.5}$ and $\overset{b}{3}$

$$x_1 = \frac{2.5 f(3) - 3 f(2.5)}{f(2.5) - f(3)}$$

$$= \frac{2.5[0.5724] - 3[-0.8675]}{[0.5724] - [-0.8675]}$$

$$= \underline{\underline{2.8012}}$$

$$f(2.8012) = 2.8012 \tan(2.8012) + 1$$

$$= 0.00787$$

$$= \underline{\underline{0.0079}} \text{ (+ve)}$$

Root lies between $\overset{a}{2.5}$ and $\overset{b}{2.8012}$.

$$x_2 = \frac{2.5\,f(2.8012) - 2.8012\,f(2.5)}{f(2.8012) - f(2.5)}$$

$$= \frac{2.5\,[0.0079] - 2.8012\,[-0.8675]}{0.0079 - (-0.8675)}$$

$$= \underline{2.7984}$$

$$f(2.7984) = 2.7984 \tan 2.7984 + 1$$

$$= 0.0000390 \quad (+ve)$$

Root lies bet $\overset{a}{2.5}$ and $\overset{b}{2.7984}$

$$x_3 = \frac{2.5\,f(2.7984) - 2.7984\,f(2.5)}{f(2.7984) - f(2.5)}$$

$$= \frac{2.5\,(0.00039) - 2.7984\,(-0.8675)}{0.00039 - (-0.8675)}$$

$$= \underline{2.7984}$$

App root is 2.7984 correct to 4 decimal places.

<u>Hw</u> $3x + \sin x - e^x = 0$ between 0 and 1.

Find approximate value of the real root of $x\log_{10}x = 1.2$ by **method of false position.**

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$f(x) = x\log_{10}x - 1.2$.

$f(1) = -1.2$ (-ve)

$f(\overset{a}{2}) = 2\log_{10}2 - 1.2 = -0.59794$

$f(\overset{b}{3}) = 3\log_{10}3 - 1.2 = 0.23136$

Root lies between 2 and 3

$$x = \frac{2f(3) - 3f(2)}{f(3) - f(2)}$$

$$= \frac{2(0.23136) - 3(-0.59794)}{(0.23136) - (-0.59794)}$$

$$= \underline{\underline{2.72}}$$

$f(2.72) = 2.72\log_{10}(2.72) - 1.2$

$= -0.01797$ (-ve)

Root lies bet $\overset{a}{2.72}$ and $\overset{b}{3}$

$$x_2 = \frac{2.72 f(3) - 3 f(2.72)}{f(3) - f(2.72)}$$

$$= \frac{2.72[0.23136] - 3[-0.01797]}{[0.23136] - [-0.01797]}$$

$$= 2.74$$

$$f(2.74) = 2.74 \log_{10}(2.74) - 1.2$$

$$= -0.00056 \quad (-ve)$$

Root lies bet $\overset{a}{2.74}$ and $\overset{b}{3}$

$$x_3 = \frac{2.74 f(3) - 3 f(2.74)}{f(3) - f(2.74)}$$

$$= \frac{2.74[0.23136] - 3[-0.00056]}{[0.23136] - [-0.00056]}$$

$$= \underline{2.740}$$

App root is $\underline{2.740}$ Correct to 3 decimal place.

1) $xe^x = 3 \rightarrow$ 3 decimal place. $\underline{1.048}$

2) $x^3 - 2x - 5 = 0 \rightarrow$ 3 decimal place.

Lagrange's Interpolation.

x	x_0	x_1	x_2	$\cdots$	x_n
y	y_0	y_1	y_2	$\cdots$	y_n

(x, y)

x	1	4	5	6	
y	12	14	20	30	$f(3)$

$$P_n(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} \times y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots(x-x_n)} \cdot y_2 + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

Find unique polynomial $P_3(x)$ of degree 3 or less, graph of which passes through the points $(-1,3), (0,-4), (1,5)$ and $(2,-6)$ (x,y)

	x_0	x_1	x_2	x_3
x	-1	0	1	2
y	3	-4	5	-6
	y_0	y_1	y_2	y_3

$$P_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\times y_3$$

$$= \frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)}\times 3 + \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)}\times -4 +$$

$$\frac{(x+1)(x-0)(x-2)}{(1+1)(1-0)(1-2)}\times 5 + \frac{(x+1)(x-0)(x-1)}{(2+1)(2-0)(2-1)}\times -6$$

$$= \frac{\overset{(x^2-x)}{x(x-1)}(x-2)}{\cancel{-6}\;2}\times \cancel{3}\,1 + \frac{\overset{(x^2-1)}{(x+1)(x-1)}(x-2)}{\cancel{2}}\times \cancel{-4}\,2 +$$

$$\frac{\overset{(x^2+x)}{(x+1)x}(x-2)}{-2}\times 5 + \frac{\overset{(x^2+x)}{(x+1)x}\overset{1\,(x^2-1)}{(x-1)}}{6}\times -6$$

$$= -\frac{1}{2}\left[x^3-x^2-2x^2+2x\right]-2\left[x^3-x-2x^2+1\right]+$$

$$-\frac{5}{2}\left[x^3+x^2-2x^2-2x\right]-\left[x^3-x\right]$$

$$= -\frac{1}{2}\left[x^3-3x^2+2x\right]-2\left[x^3-2x^2-x+1\right]-\frac{5}{2}\left[x^3-x^2-2x\right]$$

$$-x^3+x$$

$= x^3\left[-\frac{1}{2}-2-\frac{5}{2}-1\right]+x^2\left[\frac{3}{2}+4+\frac{5}{2}\right]+x\left[-1+2+5+1\right]-4$

$= -6x^3+8x^2+7x-4$

$P(1.5) = -6(1.5)^3+8(1.5)^2+7(1.5)-4$

$=$ ______

HW

Using Lagranges formula fit a polynomial to the data.

x :	0	1	3	4
y :	-12	0	6	12

Find the value of y when $x=2$

Ans: 4

of degree 3 or less, graph of which
(0,-4), (1,5) and (2,-6) (x,y)

$= \frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} \times 3 + \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)} \times -4 +$

$\frac{(x+1)(x-0)(x-2)}{(1+1)(1-0)(1-2)} \times 5 + \frac{(x+1)(x-0)(x-1)}{(2+1)(2-0)(2-1)} \times -6$

$= \frac{\overbrace{x(x-1)}^{(x^2-x)}(x-2)}{-6\ 2} \times 3^{1} + \frac{\overbrace{(x+1)(x-1)}^{(x^2-1)}(x-2)}{2} \times -4^{2} +$

$\frac{\overbrace{(x+1)x}^{(x^2+x)}(x-2)}{-2} \times 5 + \frac{\overbrace{(x+1)x}^{(x^2+x)}(x-1)}{6} \times -6^{1}\ (x^2-1)$

$= -\frac{1}{2}\left[x^3-x^2-2x^2+2x\right]-2\left[x^3-x-2x^2+2\right]+$

$-\frac{5}{2}\left[x^3+x^2-2x^2-2x\right]-\left[x^3-x\right]$

$= -\frac{1}{2}\left[x^3-3x^2+2x\right]-2\left[x^3-2x^2-x+2\right]-\frac{5}{2}\left[x^3-x^2-2x\right.$

U.Q Apply Lagrange's interpolation method to find the value of y, when $x=10$ from the following table.

	x_0	x_1	x_2	x_3
x	5	6	9	11
y	12	13	14	16
	y_0	y_1	y_2	y_3

$$P_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$P(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13 + \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= \underline{14.667}$$

U.Q Using Lagranges interpolation fit a polynomial to given data and hence find $y(2)$

	x_0	x_1	x_2
x	1	3	4
y	1	27	64
	y_0	y_1	y_2

$$\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \times y_0 +$$

Apply Lagrange's interpolation formula to find $f(6)$ given that $f(1)=2$, $f(2)=4$, $f(3)=8$, $f(4)=16$ and $f(7)=128$.

	x_0	x_1	x_2	x_3	x_4
x:	1	2	3	4	7
y:	2	4	8	16	128
	y_0	y_1	y_2	y_3	y_4

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\, y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4.$$

$$f(6) = \frac{(6-2)(6-3)(6-4)(6-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 + \frac{(6-1)(6-3)(6-4)(6-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4 + \frac{(6-1)(6-2)(6-4)(6-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 + \frac{(6-1)(6-2)(6-3)(6-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16 + \frac{(6-1)(6-2)(6-3)(6-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$= \underline{66.67}$$

1Q. Compute a 4D value of ln 9.2 from ln(9.0) = 2.1972, ln(9.5) = 2.2513 by linear Lagrange interpolation, determine the error using ln 9.2 = 2.2192 (4D)

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	x_0	x_1
x:	9.0	9.5
y:	2.1972	2.2513
	y_0	y_1

$$P(x) = \frac{(x-x_1)}{(x_0-x_1)} \times y_0 + \frac{(x-x_0)}{(x_1-x_0)} \times y_1$$

$$P(9.2) = \frac{(9.2-9.5)}{(9-9.5)} \times 2.1972 + \frac{(9.2-9)}{(9.5-9)} \times 2.2513$$

$$\bar{a} = \underline{\underline{2.2188}}$$

$a = 2.2192$

$$\text{Error} = a - \bar{a}$$

$$= 2.2192 - 2.2188$$

$$= \underline{\underline{0.0004}}$$

Q. $\ln(9.0) = 2.1972$
$\ln(9.5) = 2.2513$
$\ln(11) = 2.3979$

find $\ln(9.2)$ 4D

Ans $\ln(9.2) = \underline{\underline{2.2192}}$

By means of Lagranges formula, prove that $y_3 = 0.05(y_0+y_6) - 0.3(y_1+y_5) + 0.75(y_2+y_4)$

$y_0, y_1, y_2, y_3, y_4, y_5, y_6$

	x_0	x_1	x_2	x_3	x_4	x_5
$x:$	0	1	2	4	5	6
$y:$	y_0	y_1	y_2	y_4	y_5	y_6
	y_0	y_1	y_2	y_3	y_4	y_5

$$y(3) = \frac{(3-1)(3-2)(3-4)(3-5)(3-6)}{(0-1)(0-2)(0-4)(0-5)(0-6)} \times y_0 + \frac{(3-0)(3-2)(3-4)(3-5)(3-6)}{(1-0)(1-2)(1-4)(1-5)(1-6)} \times y_1 + \frac{(3-0)(3-1)(3-4)(3-5)(3-6)}{(2-0)(2-1)(2-4)(2-5)(2-6)} \times y_2 + \frac{(3-6)(3-1)(3-2)(3-5)(3-6)}{(4-6)(4-1)(4-2)(4-5)(4-6)} \times y_4$$

$$+ \frac{(3-0)(3-1)(3-2)(3-4)(3-6)}{(5-0)(5-1)(5-2)(5-4)(5-6)} \times y_5 + \frac{(3-0)(3-1)(3-2)(3-4)(3-5)}{(6-0)(6-1)(6-2)(6-4)(6-5)} \times y_6.$$

$$= \frac{1}{20} y_0 - \frac{3}{10} y_1 + \frac{3}{4} y_2 + \frac{3}{4} y_4 - \frac{3}{10} y_5 + \frac{1}{20} y_6$$

$$= \frac{1}{20}(y_0+y_6) - \frac{3}{10}(y_1+y_5) + \frac{3}{4}(y_2+y_4)$$

$$= 0.05(y_0+y_6) - 0.3(y_1+y_5) + 0.75(y_2+y_4)$$

Apply **Lagranges formula inversely** to obtain the root of the equation $f(x)=0$, given that $f(30)=-30$, $f(34)=-13$, $f(38)=3$ and $f(42)=18$

	x_0	x_1	x_2	x_3
x:	30	34	38	42.
y:	-30	-13	3	18
	y_0	y_1	y_2	y_3.

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} \times x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} \times x_1 + \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} \times x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} \times x_3$$

$$f(x) = \cancel{(0+30)(0-3)(0-18)}$$

$$= \frac{(0+13)(0-3)(0-18)}{(-30+13)(-30-3)(-30-18)} \times 30 + \frac{(0+30)(0-3)(0-18)}{(-13+30)(-13-3)(-13-18)} \times 34 + \frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)} \times 38 + \frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)} \times 42.$$

$$= \underline{\underline{37.23}}$$

x: 1998

y: 2005

Newtons Divided Difference interpolation formula.

$$P_n(x) = y_0 + y(x_0,x_1)(x-x_0) + y(x_0,x_1,x_2)(x-x_0)(x-x_1) + y(x_0,x_1,x_2,x_3)(x-x_0)(x-x_1)(x-x_2) + \cdots + y(x_0,x_1,\ldots,x_n)(x-x_0)(x-x_1)\cdots(x-x_{n-1})$$

Using Newtons divided difference interpolating polynomial estimate $f(7)$ from the following data.

	x_0	x_1	x_2	x_3
x:	5	6	9	11
y:	12	13	14	16
	y_0	y_1	y_2	y_3

$$P(x) = y_0 + f(x_0, x_1)(x - x_0) + f(x_0, x_1, x_2)(x - x_0)(x - x_1) + \dots$$

x	y	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$
5	12	$\frac{y_1 - y_0}{x_1 - x_0} = \frac{13-12}{6-5} = 1$	$\frac{\frac{1}{3} - 1}{9-5} = \frac{1}{6}$	
6	13			$\frac{\frac{2}{15} - \frac{1}{6}}{11-5} = \frac{1}{20}$
9	14	$\frac{14-13}{9-6} = \frac{1}{3}$	$\frac{1 - \frac{1}{3}}{11-6} = \frac{2}{15}$	
11	16	$\frac{16-14}{11-9} = 1$		

$$f(x) = 12 + (1)(x-5) + \frac{1}{6}(x-5)(x-6) + \frac{1}{20}(x-5)(x-6)(x-9)$$

$$f(7) = 12 + (7-5) + \frac{1}{6}(7-5)(7-6) + \frac{1}{20}(7-5)(7-6)(7-9)$$

$$= \underline{\underline{13.4667}}$$

Using Newtons divided difference interpolating polynomial estimate $f(7)$ from following data.

	x_0	x_1	x_2	x_3
x:	5	6	9	11
y:	12	13	14	16
	y_0	y_1	y_2	y_3

iii) Compute $f(9.2)$ from the following data using Newtons divided difference formula

x	8 (x_0)	9 (x_1)	9.5 (x_2)	11 (x_3)
y	2.079442 (y_0)	2.197225 (y_1)	2.251292 (y_2)	2.397895 (y_3)

x	y	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$
8	2.079442			
		$\frac{2.197225-2.079442}{9-8} = 0.117763$		
9	2.197225		$\frac{0.108134-0.117763}{9.5-8} = -0.06433$	
		$\frac{2.251292-2.197225}{9.5-9} = 0.108134$		$\frac{-0.005200+0.0064}{11-8} = 0.0041$
9.5	2.251292		$\frac{0.097735-0.108134}{11-9} = -0.005200$	
		$\frac{2.397895-2.251292}{11-9.5} = 0.097735$		
11	2.397895			

Newtons Divided Difference interpolation formula.

$$P_n(x) = y_0 + f(x_0,x_1)(x-x_0) + f(x_0,x_1,x_2)(x-x_0)(x-x_1) + f(x_0,x_1,x_2,x_3)(x-x_0)(x-x_1)(x-x_2) + \dots + f(x_0,x_1,\dots,x_n)(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

$$-2.079442 + 0.117783(x-8) + (-0.06433)(x-8)(x-9) + (0.00411)(x-8)(x-9)(x-9.5)$$

$$f(9.2) = -2.079442 + 0.117783(9.2-8) + (-0.06433)(9.2-8)(9.2-9) + (0.00411)(9.2-8)(9.2-9)(9.2-9.5)$$

$$= \underline{2.219208}$$

The following table gives the speed $v(t)$ of a particle (in cm/s) as a function of time t (in Seconds)

time t	0	1	4	5
Speed $v(t)$	0	1	40	65

Find a polynomial to model the above data using Newton's divided difference method and hence find Speed of the particle at time $t=3s$.

x	y	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$
0	0	$\frac{1-0}{1-0}=1$	$\frac{13-1}{4-0}=3$	$\frac{3-3}{5-0}=0$
1	1	$\frac{40-1}{4-1}=13$	$\frac{25-13}{5-1}=3$	
4	40	$\frac{65-40}{5-4}=25$		
5	65			

$$f(x) = y_0 + f(x_0, x_1)(x - x_0) + f(x_0, x_1, x_2)(x - x_0)(x - x_1) + f(x_0, x_1, x_2, x_3)(x - x_0)(x - x_1)(x - x_2) + \cdots$$

$$= 0 + (1)(x - 0) + 3(x - 0)(x - 1) + 0$$

$$= x + (3x^2 - 3x)$$

$$= 3x^2 - 2x$$

$$V(t) = 3t^2 - 2t.$$

Speed at $t = 3s$ $\quad V(3) = 3 \times 9 - 2 \times 3$

$$= 21 \text{ cm/s}$$

Newtons forward difference interpolation formula. [Equal interval]

$$P_n(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

where $u = \frac{x - x_0}{h}$ and $h = x_1 - x_0$

Newtons Backward difference interpolation formula. [Equal interval]

$$P_n(x) = y_n + u\,\nabla y_n + \frac{u(u+1)}{2!}\Delta^2 y_n + \frac{u(u+1)(u+2)}{3!}\Delta^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\Delta^4 y_n + \cdots$$

where $u = \frac{x - x_n}{h}$ and $h = x_1 - x_0$

The following table gives the values of the function $f(x) = e^{0.1x}$ at certain values of x. Compute the approximate value of $f(0.75)$ and $f(1.8)$ and compare with exact value:

N.F.I N.B.I

x	0	0.5	1	1.5	2
$f(x)$	1.0000	1.0513	1.1052	1.1618	1.2214

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	
0	1.0000				
		$1.0513 - 1 = 0.0513$			
0.5	1.0513		$0.0539 - 0.0513 = 0.0026$		
		$1.1052 - 1.0513 = 0.0539$		$0.0027 - 0.0026 = 0.0001$	
1	1.1052		$0.0566 - 0.0539 = 0.0027$		$0.0003 - 0.001 = 0.0002$
		$1.1618 - 1.1052 = 0.0566$		$0.0030 - 0.0027 = 0.0003$	
1.5	1.1618		$0.0596 - 0.0566 = 0.0030$		
		$1.2214 - 1.1618 = 0.0596$			
2	1.2214				

To find $f(0.75)$

By Newton foward into: formula:

$$P(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$h = x_1 - x_0 = 0.5 - 0 = 0.5 \qquad u = \frac{x - x_0}{h} = \frac{0.75 - 0}{0.5} = 1.5$$

$$P(x) = 1.0000 + 1.5(0.0513) + \frac{1.5(1.5-1)}{2!}(0.0026) + \frac{1.5(1.5-1)(1.5-2)}{3!}(0.0001)$$
$$+ \frac{1.5(1.5-1)(1.5-2)(1.5-3)}{4!}(0.0002)$$

$$= \underline{\underline{1.0779}} \ (4D)$$

$$e^{0.1x} = e^{0.1(0.75)} = \underline{\underline{1.0779}}$$

To find $f(1.8)$

By Newtons Backward Interpolation formula.

$$p(x) = y_n + u\,\Delta y_n + \frac{u(u+1)}{2!}\Delta^2 y_n + \frac{u(u+1)(u+2)}{3!}\Delta^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\Delta^4 y_n.$$

$h = x_1 - x_0 = 0.5$ $\qquad u = \frac{x - x_n}{h} = \frac{1.8 - 2}{0.5} = \underline{-0.4}.$

$$p(x) = 1.2214 + (-0.4)(0.0596) + \frac{(-0.4)(-0.4+1)}{2!}(0.0030) + \frac{(-0.4)(-0.4+1)(-0.4+2)}{3!}(0.0003) + \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{4!}(0.0002)$$

$= \underline{1.1972}$ $\qquad e^{0.1x} = \underline{0.1972}$ $\qquad (4.D)$

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Compute a 7.D value for $x = 1.72$ from the four values in the following data. → using i) Newton forward formula (ii) Newtons backward formula

x	1.7	1.8	1.9	2.0
y	0.3979849	0.3399864	0.281816	0.2238908

$x = 1.72$

Newtons forward inte.

$$P(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)\ldots}{4!}$$

$h = 1.8 - 1.7 = 0.1$

$$u = \frac{x - x_0}{h} = \frac{1.72 - 1.7}{0.1} = 0.2$$

$$= 0.3979849 + (0.2)(-0.0579985) + \frac{(0.2)(0.2-1)}{2!}(0.0001693)$$

$$+ \frac{(0.2)(0.2-1)(0.2-2)}{3!}(0.0004093)$$

$$= \underline{\underline{0.386413}}$$

$x = 1.72$

Newtons forward inte:

$$p(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$u = \frac{x - x_0}{h} = \frac{1.72 - 1.7}{0.1} = 0.2$$

$h = 1.8 - 1.7 = 0.1$

$= 0.3979849 + (0.2)(-0.0579985) + (0.2$

Newtons Backward interpolation

$$P(x) = y_n + u\Delta y_n + \frac{u(u+1)}{2!}\Delta^2 y_n + \frac{u(u+1)(u+2)}{3!}\Delta^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\Delta^4 y_n + \cdots$$

$$h = x_1 - x_0 = 1.8 - 1.7 = 0.1$$

$$u = \frac{x - x_n}{h} = \frac{1.72 - 2}{0.1} = -2.8$$

$$= 0.2238908 + (-2.8)(-0.0579278) + \frac{(-2.8)(-2.8+1)}{2!}(0.0002400)$$

$$+ \frac{(-2.8)(-2.8+1)(-2.8+2)}{3!}(0.0004093)$$

$$= 0.3864184$$

U.Q The following table gives the values of $\sin\theta$ where θ is in degrees. using Newtons interpolation formula estimate the value of

a) $\sin(8°)$ and b) $\sin(27°)$.

$h=5$

θ	5	10	15	20	25	30
$\sin\theta$	0.0871	0.1736	0.2588	0.342	0.4226	0.5

a) 0.1392

b) 0.4531

U.Q

Year:	1961	1971	1981	1991	2001	2011
No: of people	16	19	23	28	34	41

$h=10$

2005

N.B.I

36.68.

Trapezoidal Rule:

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

OR.

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[(\text{Sum of } 1^{st} \text{ and last } \cancel{\text{term}}\ \text{ordinates}) + 2(\text{Sum of remaining ordinates})\right]$$

No: of intervals = Any

Error bound

k is the maximum value of $|f''(x)|$ on the interval $[a,b]$ so $|f''(x)| \le k$ on $[a,b]$.

Actual error for Trapezoidal rule

$$|E_T| \le \frac{k(b-a)^3}{12n^2}$$

Simpsons one third rule:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4(\cdots) + 2(y_2+y_4+\cdots)\right]$$

OR.

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(\text{Sum of } 1^{st} \text{ and last ordinates}) + 4(\text{Sum of odd ordinates}) + 2(\text{Sum of even ordinates})\right]$$

No: of intervals = Even.

Error bound for Simpsons one third rule

$$|E_S| \le \frac{k(b-a)^5}{180n^4}$$

Evaluate $\int_0^1 e^{-\frac{x^2}{2}}\,dx$ using i) Trapezoidal rule ii) Simsons one third rule. iii) Also obtain an error bound for trapezoidal rule.

$f(x) = e^{-\frac{x^2}{2}}$ no: of interval $n = 5$

$\int_a^b$

$$\boxed{h = \frac{b-a}{n}}$$

$$h = \frac{1-0}{5} = 0.2$$

x	0	0.2	0.4	0.6	0.8	1
$y = f(x) = e^{-x^2/2}$	1	0.9801987	0.9231163	0.8352702	0.7261490	0.6065307
	y_0	y_1	y_2	y_3	y_4	y_5

Trapezoidal rule.

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$= \frac{0.2}{2}\left[(1+0.6065307)+2[0.9801987+0.9231163+0.8352702+0.7261490]\right]$$

$$= 0.8535999$$

... using i) Trapezoidal ...

... bound for trapezoidal rule

$\int e^{-\frac{x^2}{2}} dx$ ... 5

i) Trapezoidal rule

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[(y_0+y_5) + 2[\ldots]\right]$$

$$= \frac{0.2}{2}\left[(1+0.6065307) + 2[0.9801987 + 0.9231163 + 0.8352702 + 0.7261490]\right]$$

$$= 0.6535999$$

0.4	0.6	0.8	1	
0.9231163	0.8352702	0.7261490	0.6065307	
y_2	y_3	y_4	y_5	

Error bound.

$$|E_T| \le \frac{K(b-a)^3}{12n^2}$$

$$\int_0^1$$

$$f(x) = e^{-x^2/2}$$

$$f'(x) = e^{-x^2/2}\left(-\frac{2x}{2}\right)$$

$$= -x e^{-x^2/2}$$

$$f''(x) = -\left[x e^{-x^2/2}\left(-\frac{2x}{2}\right) + e^{-x^2/2}(1) \right]$$

$$= -e^{-x^2/2}\left[-x^2+1\right]$$

$$= e^{-x^2/2}(1-x^2)$$

$$f''(0) = 1 \qquad f''(1) = e^{-1}(1-1) = 0$$

$$k = 1$$

$$|E_T| \le \frac{k(b-a)^3}{12n^2}$$

$$\le \frac{1\,(1-0)^3}{12\times 25}$$

$$= 0.003333$$

Simsons one third rule.

$\boxed{n=8}$

$h = \dfrac{b-a}{n} = \dfrac{1-0}{8}$

$\boxed{h = 0.125}$

x	$y = e^{-x^2/2}$	
0	1	y_0
0.125	0.9922179	y_1
0.250	0.9692332	y_2
0.325	0.9321025	y_3
0.5	0.8824969	y_4
0.625	0.8225776	y_5
0.750	0.7548396	y_6
0.875	0.6819408	y_7
1	0.6065307	y_8

$$\int_0^1 e^{-x^2/2}\,dx = \frac{h}{3}\Big[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\Big]$$

$$= \frac{0.125}{3}\Big[(1+0.6065307)+4(0.9922179+0.9321025+0.8225776+0.6819408)+2(0.9692332+0.8824969+0.7548396)\Big]$$

$$= 0.855626$$

4.6 Compute $\int_0^1 \frac{dx}{1+x^2}$ using (i) trapezoidal method and (ii) Simson's method with step size $h=0.25$. Compare the results with exact value obtained by actual integration.

$n+1=5$

$n=4$

x	$f(x)=\frac{1}{1+x^2}$
0	1 y_0
0.25	0.9412 y_1
0.5	0.8 y_2
0.75	0.64 y_3
1	0.5 y_4

Trapezoidal rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[(1+0.5)+2(0.9412+0.8+0.64)\right]$$

$$= 0.7828$$

Simsons rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{0.25}{3}\left[(1+0.5)+4(0.9412+0.64)+2(0.8)\right]$$

$$= 0.7854$$

$$\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1$$

$$= \tan^{-1}1 - \tan^{-1}0$$

$$= \frac{\pi}{4}$$

$$= \underline{\underline{0.7854}} \quad (4.D)$$

Simpson's method is more accurate. up to 4 decimal places.

Q. The speed of a moving particle was measured at different points of time. The time t when the first measurement was recorded is taken as t = 0. Subsequent speeds at different times are as shown in the following table.

Time (t) in Seconds	0	10	20	30	40	50	60
Velocity (v) in m/s	35	39	44	50	56	43	40
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$n+1=7$

$\boxed{n=6}$

Evaluate the distance travelled by the particle in 60 Seconds.

$V = \frac{x}{t}$

$V = \frac{dx}{dt}$

$dx = V\,dt$

$x = \int_0^{60} V\,dt$

$h = \frac{b-a}{n} = \frac{60-0}{6} = 10$

Simson 1/3 rule:

$\int_0^6 V\,dt = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$

$= \frac{10}{3}\left[(35+40)+4(39+50+43)+2(44+56)\right]$

$= \underline{2676.67}$

U.Q A river is 80 m wide. The depth y in meters at a distance x meters from one bank is given by the following table:

$h = 10.$

x	0	10	20	30	40	50	60	70	80
y	0	5	8	10	15	12	7	3	1

Find approximately the area of cross section using Simson's $\frac{1}{3}$rd rule.

$$\int_0^{80} f(x)\,dx = \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$= 603$ sq. meters.