Steps:

① N

② $H_N(s) \big|_{\Omega_c = 1 rad/sec}$ → Normalised Butterworth table. (TF)

③ Analog frequency transformation.

$H_N(s) = \frac{1}{D(s)}$

For N=1

~~H(s)~~ $D(s) = s+1$

N=2

$D(s) = s^2 + \sqrt{2}s + 1$

Case I : Lp → Lp

$$s \Rightarrow \frac{s}{\Omega_c}$$

$$H(s) = H_N(s) \Big|_{s \to \frac{s}{\Omega_c}}$$

Case II: Lp → HP.

$$s \Rightarrow \frac{\Omega_c}{s}$$

$$H(s) = H_N(s) \Big|_{s \to \frac{\Omega_c}{s}}$$

Case III: LP → BP.

$$s \Rightarrow \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$$

Ω_u → uppercut off freq

Ω_l → lowercut off freq

Case IV: LP → BR.

$$s \Rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$$

Another way for analog freq transformatⁿ:

(N → order N → poles)

$$H_N(s) = \frac{1}{(s-s_1)(s-s_2)\cdots(s-s_N)}$$

Normalised poles;

$$s_k = e^{j\phi_k} \quad \text{—} ① , \; k = 1, 2, \ldots N.$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}, \quad k = 1, 2, \ldots N$$

Poles

Normalised poles

Unnormalised poles

Unnormalised poles;

$$s_k' = \Omega_c s_k \quad \text{—} ②$$

Q. An analog Butterworth filter has a PB attenuation of -2dB at a frequency of 20rad/sec and a minimum stop-band attenuation of -10dB at a frequency of 30rad/sec. Design the analog LPF.

= $\Omega_p = 20$ rad/sec $\Omega_s = 30$ rad/sec.

$\alpha_p = -2$dB $\alpha_s = -10$dB.

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\log\left(\sqrt{15.387}\right)}{\log(3/2)}$$

$$= \frac{0.5935}{0.176}$$

$$N \geq \underline{\underline{3.37}}.$$

$$\underline{\underline{N = 4}}.$$

$$H_N(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.865s + 1)}$$

$$\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}}$$

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{20}{(10^{0.1\times2} - 1)^{1/8}}$$

$$\Omega_c = \underline{\underline{21.38 \text{ rad/sec}}}.$$

$$H(s) = H_N(s)\Big|_{s = \frac{s}{\Omega_c}} \quad \left\{ \frac{s}{\Omega_c} = \left(\frac{s}{21.38}\right) \right\}.$$

$$H(s) = \frac{1}{\left(\frac{s^2}{\Omega_c^2} + 0.765\frac{s}{\Omega_c} + 1\right)\left(\frac{s^2}{\Omega_c^2} + 1.865\frac{s}{\Omega_c} + 1\right)}$$

$$= \frac{1}{\cancel{(10^{-3} \times 2.18 s^2 + 0.0465 + 1)}}$$

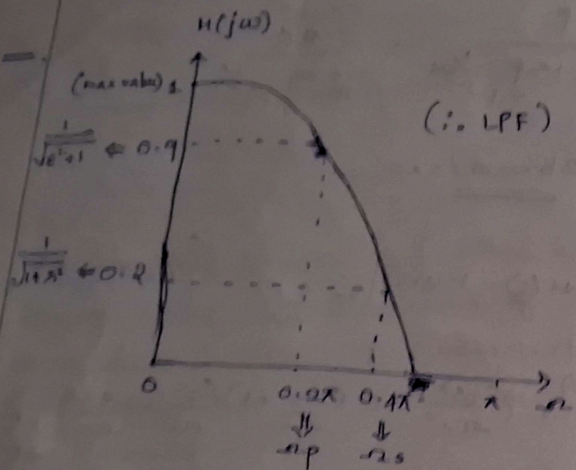
$$= \frac{1}{(2.18\times10^{-3}s^2 + 0.0355 + 1)(2.18\times10^{-3}s^2 + 0.0865 + 1)}$$

=

Q. For the given specification, design analog BWF

$0.9 \le |H(j\omega)| \le 1$ for $0 \le \Omega \le 0.2\pi$

$|H(j\omega)| \le 0.2$ for $0.4\pi \le \Omega \le \pi$



(∴ LPF)

$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$ $\qquad$ $\frac{1}{\sqrt{1+\varepsilon^2}} = 0.9$

$0.04 = \frac{1}{1+\lambda^2}$ $\qquad$ $\sqrt{1+\varepsilon^2} = 1.11$

$1+\lambda^2 = 25$ $\qquad$ $\varepsilon^2 = 0.234$

$\lambda^2 = 24$ $\qquad$ $\varepsilon = 0.48$

$\lambda = \underline{4.89}$.

$A = \frac{\lambda}{\varepsilon} = \frac{4.89}{0.48} = \underline{10.18}$.

$\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = \underline{2}$

$N \ge \frac{\log(A)}{\log(1/k)} = \frac{\log(10.18)}{\log(2)} =$

$N \ge 3.34 \approx 4$ $\qquad$ $\underline{N = 4}$.

$H_N(s) = \frac{1}{(s^2+0.765s+1)(s^2+1.865s+1)}$

$\cancel{s \to \frac{s}{\Omega_s} = \frac{s}{0.4\pi} = \frac{s}{1.25}}$

$\cancel{H(s) = \frac{1}{s^4+}}$

$\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}} = \frac{0.2\pi}{(0.48)^{1/4}} = \frac{0.2\pi}{0.83} = \underline{0.75}$.

$s \to \frac{s}{\Omega_c} = \frac{s}{\underline{0.75}}$.

$$H(s) = \frac{1}{\left(\left(\frac{s}{0.75}\right)^2 + 0.765\frac{s}{0.75} + 1\right)\left(\left(\frac{s}{0.75}\right)^2 + \frac{1.865}{0.75} + 1\right)}$$

$$H(s) = \frac{1}{\left(\frac{s^2}{0.562} + 1.015s + 1\right)\left(\frac{s^2}{0.562} + 2.48s + 1\right)}.$$

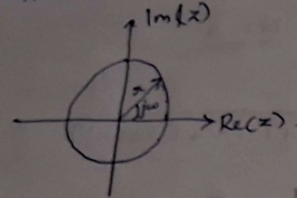
$$= \frac{1}{(1.77s^2+1.01s+1)(1.77s^2+2.48s+1)}$$

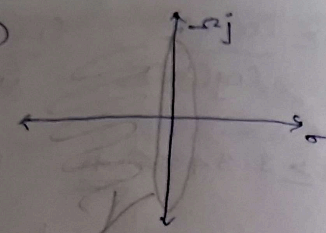
$$= \frac{1/1.77}{(s^2+0.575s+0.56)(s^2+1.401s+0.56)}$$

$$= \frac{0.564}{(s^2+0.575s+0.56)(s^2+1.401s+0.56)}$$

$s_k = \sigma_k + j\Omega_k$ (poles)

$z_k = re^{j\omega_k}$





→ Whatever poles marked on the img axis should be marked on the unit circle.

→ Whatever poles marked on the LHB should be marked ~~on~~ inside the unit circle.

This a condition should be satisfied to do the transformation.

2 Method for transformation for analog to digital TF.

① Impulse Invarience.

② Bilinear transformation.

Type of Transformat^n	Transformat^n	Band edge freq of new filter
LP	$s \to \frac{\Omega_p}{\Omega_p'} s$	Ω_p'
HP	$s \to \frac{\Omega_p \Omega_p'}{s}$	Ω_p'
BP	$s \to \Omega_p \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$	$\Omega_l \, \Omega_u$
Band stop	$s \to \Omega_p \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$	$\Omega_u \, \Omega_l$.

This taple shows the freq transformations for analog filter. (prototype LPF has band edge freq Ω_p).

① IMPULSE INVARIENCE METHOD

$z_k = e^{s_k T}$ — ① ($T \to$ sampling period)

$z_k = e^{(\sigma_k + j\Omega_k)T}$ — ② {analog → digital by sampling}

$z_k = e^{\sigma_k T + j\Omega_k T}$ — ② ($s_k \to$ pole)

$z_k = re^{j\omega_k}$ — ③

Comparing ② and ③

① $e^{j\Omega_k T} = re^{j\omega_k}$ ($\sigma = 0$, real part 0).

Here $r = 1$. Also $\Omega_k T = \omega_k$ that means digital ~~poles~~ freq is related to T times the analog pole freq.

② $\sigma < 0$. when $r < 1$.

If analog pole is $s_1 = \sigma_1 + j\Omega_1$
then digital pole $z_1 = e^{s_1 T} = e^{(\sigma_1 + j\Omega_1)T}$.

If we have an analog pole having the same real part. but diff freq.

$s_2 = \sigma_1 + j(\Omega_1 + \frac{2\pi}{T})$.

then: $z_2 = e^{s_2 T} = e^{(\sigma_1 + j(\Omega_1 + \frac{2\pi}{T}))T}$

$= e^{\sigma_1 T} e^{j\Omega_1 T} \underbrace{e^{2\pi}}_{1}$

$= e^{\sigma_1 T} e^{j\Omega_1 T}$

$\therefore z_1 = z_2$

- If analog pole is having same real and freq multiple diff with multiple of 2π, it will be mapped to the same place (one place) in the z-plane (digital).

- So it is not much efficient for HPF because we will be losing high freq component in the analog domain. (disadvantage of impulse.) i.e, Many to one mapping.

$$\left\{ X[z] = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \right\}.$$

Design Procedure:

$H(s) \longrightarrow H(z)$

$H(z)$ for IIR filter;

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}.$$

For an IIR filter the impulse is ∞ but it should be maintained as causal;

So for IIR filter;

$$H[z] = \sum_{n=0}^{\infty} h[n] z^{-n} \quad \text{— (1)}$$

We know;

$$H(s) = \frac{1}{(s-s_1)(s-s_2)\cdots(s-s_N)}.$$

Applying partial fraction;

$$H(s) = \frac{c_1}{(s-s_1)} + \frac{c_2}{(s-s_2)} + \cdots + \frac{c_N}{(s-s_N)}$$

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{s-s_k} \quad \text{— (2)}$$

By taking inverse we will get $h[n]$.

Taking inverse of (2).

$$L^{-1}\{H(s)\} = h(t)$$

$$h(t) = \sum_{k=1}^{N} c_k e^{s_k t}$$

$t = \frac{n\pi}{T}$

Do sampling to convert t to n;

where $t = nT$.

$$h(nT) = \sum_{k=1}^{N} c_k e^{s_k nT}.$$

$$h(n) = \sum_{k=1}^{N} c_k e^{s_k nT}. \quad \text{— (3)}$$

(3) in (1)

$$H[z] = \sum_{n=0}^{\infty} \left[\sum_{k=1}^{N} c_k e^{s_k nT} \right] z^{-n}.$$

$$= \sum_{k=1}^{N} c_k \sum_{n=0}^{\infty} e^{s_k nT} z^{-n}$$

$$= \sum_{k=1}^{N} c_k \sum_{n=0}^{\infty} \left(e^{s_k T} z^{-1}\right)^{n}.$$

$$H[z] = \sum_{k=1}^{N} C_k \left(\frac{1}{1 - e^{S_k T} z^{-1}} \right) \quad - (4)$$

Comparing (3) and (4)

~~For the analog~~ Each analog pole is mapped

$$\frac{1}{s - s_k} \xrightarrow{\text{mapped}} \frac{1}{1 - e^{S_k T} z^{-1}}$$

For each analog pole can be mapped as given above in digital.

2 ways: $H(s) \rightarrow$ partial fraction $\rightarrow$ inverse $\rightarrow h(t) \rightarrow h(n)$ then $H[z]$

$H(s) \rightarrow$ partial fraction $\rightarrow$ inverse $\rightarrow$ poles mapp then $H[z]$.

Q. Design an IIR filter having analog butterworth. If $H(s) = \dfrac{2}{(s+1)(s+2)}$ using impulse invariance method. Assume $T = 1$ sec.

$s = -1, -2$

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$$\frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}.$$

$$2 = A(s+2) + B(s+1)$$

$s = -2$; $\quad s = -1$;

$2 = -B \quad 2 = A$

$B = -2$.

$$= \frac{2}{s+1} + \frac{-2}{s+2}.$$

$$H(s) = \frac{2}{s+1} - \frac{2}{s+2}.$$

$$h(t) = 2e^{-t} - 2e^{-2t}.$$

$$h(n) = 2e^{-nT} - 2e^{-2nT}.$$

$T = 1$ sec;

$$h(n) = 2e^{-n} - 2e^{-2n}$$

$$H[z] = \frac{2}{1 - e^{-1} z^{-1}} - \frac{2}{1 - e^{-2} z^{-1}}$$

$$= \frac{2}{1 - 0.36 z^{-1}} - \frac{2}{1 - 0.135 z^{-1}}$$

$$= \frac{2(1 - 0.135 z^{-1}) - 2(1 + 0.36 z^{-1})}{(1 - 0.36 z^{-1})(1 - 0.135 z^{-1})}$$

$$= \frac{\cancel{2} - 0.27 z^{-1} - \cancel{2} + 0.72 z^{-1}}{1 - 0.135 z^{-1} - 0.36 z^{-1} + 0.0486 z^{-2}}$$

$$= \frac{0.45 z^{-1}}{1 - 0.495 z^{-1} + 0.0486 z^{-2}}.$$

② Bilinear Transformation Method.

- $H[z] = H[s]\Big|_{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$

$s = \sigma + j\Omega$; $z = re^{j\omega}$

After solving we get a relation connecting analog freq and digital freq:

$$\boxed{\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)}$$

when $\theta = \frac{\omega}{2}$

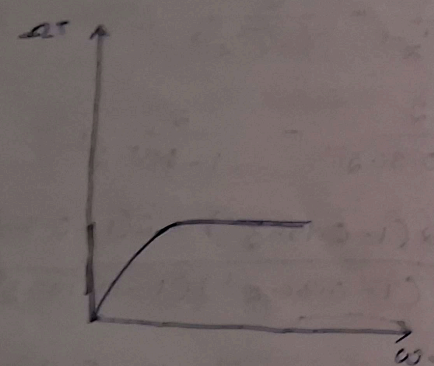
for small value of θ

$\tan\theta \approx \theta$.

$\therefore \Omega = \frac{2}{T}\theta = \frac{2}{T}\frac{\omega}{2}$

$= \frac{\omega}{T}$

$\underline{\omega = \Omega T}$



When freq ↑, it will become non-linear so it will have same value (freq).

- Warping effect is the effect where high freq component in analog domain is lased ~~in the digital~~ when mapping in the digital domain in Bilinear transformation.

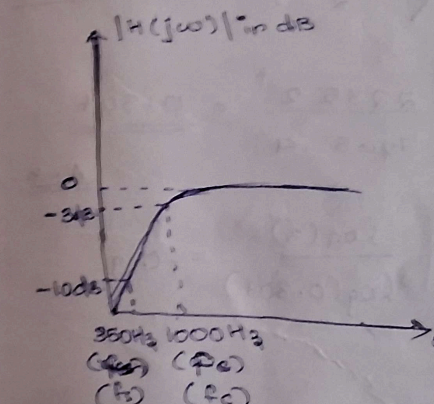
=

To eliminate warping effect we use Pre-warpping method.

* $\Omega p = \frac{2}{T}\tan\left(\frac{\omega p}{2}\right)$. ($f \to x$) (o kitya T) analog; $\omega p \to$ PB edge freq in digital domain

* $\Omega s = \frac{2}{T}\tan\left(\frac{\omega s}{2}\right)$. analog; $\omega s \to$ SB edge freq in digital domain.

? Using bilinear transformation design a HPF, monotonic in PB with cut off freq of 1000Hz and down ~~count~~ 10dB at 350Hz. The sampling freq is 3000Hz.



monotonic → following the response

As it is monotonic; ~~ωc~~ ωp. $fc = fp$.

$\omega s_{an} = 2\pi f s_{an}$

$5200\,Hz = 2\pi f s_{an}$.

$f s_{an} = 795.7$.

Step ①: Map digital specificat^n to analog

$$N \geq \frac{\log(A)}{\log(1/K)}$$

$A = \frac{\lambda}{\varepsilon}$ $\quad 1/K = \frac{\omega s}{\omega p}.\ \frac{\Omega s}{\Omega p}.$

$$\frac{\sqrt[2]{10^{0.1\alpha s}-1}}{\sqrt{10^{0.1\alpha p}-1}}.$$

$\Omega p = \frac{2}{T}\tan\left(\frac{\omega p}{2}\right) = \frac{12}{T}$ ~~tan~~ $\left(\frac{1020}{2}\right)$

$f_{smp} = 5000\,Hz$

$T = 1/5000 = \underline{2\times10^{-4}}\,s$

$\omega_p = 2\pi f_p$ $\quad\quad$ $\omega_s = 2\pi f_s$

$\omega_p = \underline{\underline{2000\pi}}$. $\quad\quad$ $\omega_s = 2\pi 350$

$= \underline{\underline{700\pi}}$.

$\Omega_p = \frac{2}{2\times10^{-4}} \tan\left(\frac{2000\pi T}{2}\right) = \underline{\underline{7265.4\ rad/s}}$

$\Omega_s = \frac{2}{2\times10^{-4}} \tan\left(\frac{700\pi T}{2}\right) = \underline{\underline{2235.2}}\ rad/s$.

$1/k = \frac{\Omega_s}{\Omega_p} = \frac{2235.2}{7265.4} = \underline{\underline{0.307}}$.

$\underline{\underline{A=3}}$.

$N = \frac{\log(A)}{\log(1/k)} = \frac{\log(3)}{\log(0.307)} = -0.9$

$\underline{\underline{N=1}}$.

$A = \frac{\sqrt{10^{0.1(10)} - 1}}{\sqrt{10^{(0.1)(3)} - 1}} = \sqrt{\frac{10-1}{10^{0.3}-1}} = \sqrt{\frac{9}{0.995}} = \underline{\underline{3}}$.

$H_N(s) = \frac{1}{s+1}$ $\quad\quad$ $\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}}$ $\quad\quad$ $\varepsilon = \underline{\underline{1}}$.

$H(s) = H_N(s)\Big|_{s = \frac{\Omega_c}{s}}$ $\quad\quad$ $\Omega_c = \underline{\underline{7265.4}}$

$(\Omega_p = \Omega_c)$

$H(s) = \frac{1}{\frac{\Omega_c}{s} + 1}$.

$= \frac{1}{\frac{7265.4}{s} + 1} = \frac{1}{\frac{7265.4 + s}{s}}$

$= \underline{\underline{\frac{s}{s + 7265.4}}}$.

? Design a digital butterworth filter satisfying the constraint $0.707 \leq |H(e^{j\omega})| \leq 1$. for $0 \leq \omega \leq \pi/2$, $|H(e^{j\omega})| \leq 0.2$ for $\frac{3\pi}{4} \leq \omega \leq \pi$ with $T = 1\,sec$ using (a) bilinear transformation
(b) impulse invarience.

=

(a) $\frac{1}{\sqrt{1+\varepsilon^2}} = 0.707$.

$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$

$\omega_p = \pi/2$.

$\omega_s = \frac{3\pi}{4}$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p T}{2}\right) = 2\tan\left(\frac{\pi/2}{2}\right) = \underline{\underline{2}}\ \text{rad/s}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s T}{2}\right) = 2\tan\left(\frac{3\pi/4}{2}\right) = \underline{\underline{4.8}}\ \text{rad/s.}$$

$$\frac{1}{\sqrt{1+\epsilon^2}} = 0.707 \qquad \frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\sqrt{1+\epsilon^2} = 1.414 \qquad 1+\lambda^2 = 25$$

$$1+\epsilon^2 = 2.00 \qquad \lambda^2 = 24$$

$$\epsilon^2 = 1 \qquad \underline{\underline{\lambda = 4.8}}.$$

$$\underline{\underline{\epsilon = 1}}.$$

$$A = \lambda/\epsilon = \underline{\underline{4.8}}.$$

$$N \geq \frac{\log A}{\log(1/k)} \qquad 1/k = \frac{\Omega_s}{\Omega_p} = \underline{\underline{2.4}}.$$

$$N \geq \frac{\log(4.8)}{\log(2.4)} = \underline{\underline{1.79 \approx 2}}.$$

$$H_N(s) = \frac{1}{s^2+\sqrt{2}s+1}\Bigg|_{s = \frac{s}{\Omega c}} \qquad \Omega c = \frac{\Omega p}{\epsilon^{1/N}} = \frac{2}{1^{1/2}} = \underline{\underline{2}}.$$

$$H(s) = \frac{1}{\frac{s^2}{4}+\frac{\sqrt{2}s}{2}+1}$$

$$= \frac{1}{\frac{s^2}{4}+\frac{s\sqrt{2}}{\cancel{2}2}+1}$$

$$= \frac{1}{\frac{s^2}{4}+\frac{2s\sqrt{2}}{4}+\frac{4}{4}} \qquad 2\sqrt{2} = 2.82$$

$$= \underline{\underline{\frac{4}{s^2+2s\sqrt{2}+4}}}.$$

$$H[z] = H(s)\Big|_{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H[z] = \frac{4}{\left(2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]\right)^2 + 2.82\times 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 4}$$

$$= \frac{4}{4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2 + 5.65\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 4}.$$

$$= \frac{4(1+z^{-1})^2}{4(1-z^{-1})^2 + 5.65(1-z^{-1})(1+z^{-1}) + 4(1+z^{-1})^2}$$

$$= \quad . \quad \cancel{4+4z^{-1}}$$

$$= \frac{4(1+z^{-1})^2}{4(1-z^{-1})^2 + \cancel{5.65 - 5.65z^{-1}}\ 5.65(1-z^{-1})(1+z^{-1}) + 4(1+z^{-1})^2}$$

$$= \frac{4(1+2z^{-1}+z^{-2})}{4(1-2z^{-1}+z^{-2}) + 5.65(1+\cancel{z^{-1}}-\cancel{z^{-1}}-z^{-2}) + 4(1+2z^{-1}+z^{-2})}$$

$$= \frac{4+8z^{-1}+4z^{-2}}{4-\cancel{8z^{-1}}+4z^{-2}+5.65+5.65z^{-2}+4+\cancel{8z^{-1}}+8z^{-2}}$$

$$= \underline{\underline{\frac{4+8z^{-1}+4z^{-2}}{17.65z^{-2}+13.65}}}.$$

(b) $\Omega_p = \omega_p T$ $\quad \Omega_s = \omega_s T$

$\Omega_s = \omega_s$

$\Omega_p = \omega_p$

$\Omega_p = \pi/2 \quad \Omega_s = 3\pi/4$

$\epsilon = 1 \quad \lambda = 4.8$

$A = \ldots \; 4.8$

$$N \geq \frac{\log(A)}{\log(1/k)}$$

$$1/k = \frac{\Omega_s}{\Omega_p} = \frac{3\pi/4}{\pi/2} = \frac{3\times 2}{4} = \frac{3}{2}$$

$$N \geq \frac{\log(4.8)}{\log(3/2)}$$

$N \geq 3.86$

$N \approx 4$.

$$H_N(s) = \frac{1}{(s^2+0.76s+1)(s^2+1.86s+1)}$$

$$H(s) = H_N(s)\Big|_{\frac{s}{\Omega_c}}$$

$$\Omega_c = \frac{\Omega_p}{\epsilon^{1/N}} = \pi/2 \text{ rad/sec}.$$

$$H(s) = \frac{1}{\left(\left(\frac{s}{\pi/2}\right)^2 + 0.76\frac{s}{\pi/2} + 1\right)\left(\left(\frac{s}{\pi/2}\right)^2 + 1.86\frac{s}{\pi/2} + 1\right)}$$

$$= \frac{1}{\left(\left(\frac{s}{1.57}\right)^2 + \frac{0.76}{1.57}s + 1\right)\left(\left(\frac{s}{1.57}\right)^2 + \frac{1.865}{1.57} + 1\right)}$$

$$= \frac{1}{\left(\frac{0.405s^2}{0.405} + 0.48s + 1\right)(0.405s^2 + 1.18s + 1)}$$

$0.405(s^2 + 0.48s$

$$= \frac{1}{(0.405)(s^2+1.18s+2.46)(s^2+2.91s+2.46)}$$

$$= \frac{1/0.405}{(s^2+1.18s+2.46)(s^2+2.91s+2.46)}$$

$$= \frac{2.46}{(s^2+1.18s+2.46)(s^2+2.91s+2.46)}$$

$$\frac{A}{s+1.456+j0.6} + \frac{A^*}{s+1.456-j0.6} + \frac{B}{s+0.6+j1.45} + \frac{B^*}{s+0.6-j1.45}$$

$A = 0.727 + j1.757$

$A^* = 0.727 - j1.757$.

$B = -0.727 - j0.3$

$B^* = -0.727 + j0.3$

$$\frac{0.727+j1.757}{s+1.456+j0.6} + \frac{0.727-j1.757}{s+1.456-j0.6} + \frac{-0.727-j0.3}{s+0.6+j1.45} + \frac{-0.727+j0.3}{s+0.6+j1.45}$$

$$= (0.727+j1.757)e^{-(1.456+j0.6)t} \ldots$$

FIR filter design by frequency sampling method

For type-1 method, frequency samples are taken at intervals of:

$$K F_s/N, \quad k = 0, 1, \ldots, N-1.$$

For type-2 method, frequency samples are taken at intervals of:

$$f_k = (k + 1/2) F_s/N, \quad k = 0, 1, \ldots N-1$$

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j(2\pi/N)nk}.$$

$$x[k] = x[e^{j\omega}]\Big|_{\omega = \frac{2\pi k}{N}}.$$

$$h(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} H(k)$$

$$H(k) = H(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}}$$

$$H\left(\frac{2\pi k}{N}\right)(k+\alpha)$$

Type 1: $\alpha = 0$

Type 2: $\alpha = 1/2$.

imp

Type 1:

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} 2|H(k)| \cos[2\pi k(n-\alpha)/N] + H(0)\right] \text{ For N even.}$$

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{(N-1)/2} 2|H(k)| \cos[2\pi k(n-\alpha)/N] + H(0)\right]$$

for N odd.

Type 2:

$$h(n) = \frac{2}{N}\left[\sum_{k=0}^{\frac{N-3}{2}} |H(k)| \cos\pi\left(\frac{(2k+1)(n-\alpha)}{N}\right)\right], \text{ N is odd.}$$

$$h(n) = \frac{2}{N}\left[\sum_{k=0}^{\frac{N}{2}-1} |H(k)| \cos\pi\left\{\frac{(2k+1)(n-\alpha)}{N}\right\}\right], \text{ N is even.}$$

? Determine the coefficient of a linear phase FIR filter of length M=15, that has a symmetric unit sample response and a frequency response that satisfies condition:

$$H\left[(2\pi k)/\underbrace{15}_{M}\right] = 1, \quad k = 0, 1, 2, 3$$
$$= 0, \quad k = 4, 5, 6, 7.$$

M=N

= Here only 7 is given but N=15. Requires 14 values.

$$|H[k]| = |H^*(N-k)|$$

$$|H(8)| = |H^*(15-8)| = H(7) = \underline{\underline{0}}.$$

$$|H(9)| = |H^*(6)| = 0 = \underline{\underline{0}}$$

$$|H(10)| = |H^*(5)| = \underline{\underline{0}}$$

$$|H(11)| = |H^*(4)| = \underline{\underline{0}}.$$

$$|H(12)| = |H^*(3)| = \underline{\underline{1}}$$

$$|H(13)| = |H^*(2)| = \underline{\underline{1}}.$$

$$|H(14)| = |H^*(1)| = \underline{\underline{1}}.$$

N=15 (odd)

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{7} 2|H(k)| \cos\left(\frac{2\pi k(n-\alpha)}{N}\right) + H(0)\right]$$

$$\alpha = \frac{N-1}{2} = \frac{14}{2} = \underline{\underline{7}}.$$

$$h(n) = \frac{1}{15}\left[\sum_{k=1}^{7} 2|H(k)|\cos\left(\frac{2\pi k(n-7)}{15}\right) + \underbrace{H(0)}_{1}\right]$$

$$h(n) = \frac{1}{15}\left[\sum_{k=1}^{3} 2|H(k)|\cos\frac{2\pi k(n-7)}{15} + 1\right]$$

$$h(n) = \frac{1}{15}\left[1 + \sum_{k=1}^{3} 2|H(k)|\cos\frac{2\pi k(n-7)}{15}\right]$$

$$h(1) = \frac{1}{15}\left[1 + \left|\frac{2\cos 2\pi(-6)}{15} + \frac{2\cos 4\pi(-6)}{15} + \frac{2\cos 6\pi(-6)}{15}\right|\right]$$

$$= \frac{1}{15}\left[1 + \left|2\cos\frac{12\pi}{15} + 2\cos\frac{24\pi}{15} + 2\cos\frac{36\pi}{15}\right|\right]$$

$$= \frac{1}{15}\left[1 + \{-1.61 + 0.61 + 0.61\}\right]$$

$$= 0.041$$

$$h(0) = \frac{1}{15}\left[1 + \left\{2\cos\frac{2\pi(-7)}{15} + 2\cos\frac{4\pi(-7)}{15} + 2\cos\frac{6\pi(-7)}{15}\right\}\right]$$

$$= -0.05$$

(3)

$$h(2) = \frac{1}{15}\left[1 + 2\cos\frac{10\pi}{15} + 2\cos\frac{20\pi}{15} + 2\cos\frac{30\pi}{15}\right]$$

$$= 0.066$$

(4)

$$h(3) = \frac{1}{15}\left[1 + 2\cos\frac{8\pi}{15} + 2\cos\frac{16\pi}{15} + 2\cos\frac{24\pi}{15}\right]$$

$$= -0.0364$$

(3)

$$h(4) = \frac{1}{15}\left[1 + 2\cos\frac{6\pi}{15} + 2\cos\frac{12\pi}{15} + 2\cos\frac{18\pi}{15}\right]$$

$$= -0.1078$$

(2)

$$h(5) = \frac{1}{15}\left[1 + 2\cos\frac{4\pi}{15} + 2\cos\frac{8\pi}{15} + 2\cos\frac{12\pi}{15}\right]$$

$$= 0.034$$

(1)

$$h(6) = \frac{1}{15}\left[1 + 2\cos\frac{2\pi}{15} + 2\cos\frac{4\pi}{15} + 2\cos\frac{6\pi}{15}\right]$$

$$= 0.318$$

$$h(7) = \frac{1}{15}\left[1 + 2\cos 0 + 2\cos 0 + 2\cos 0\right]$$

$$= \frac{1}{15}\left[1 + 2 + 2 + 2\right]$$

$$= 0.466$$

{filter coefficient h(n) values}

$h(0) = h(14)$ $\quad h(4) = h(10)$

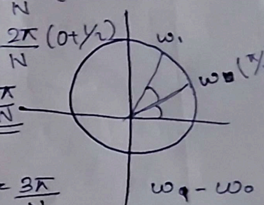
$h(1) = h(13)$ $\quad h(5) = h(9)$

$h(2) = h(12)$ $\quad h(6) = h(8)$

$h(3) = h(11)$

$\omega_k = \frac{2\pi}{N}(k + 1/2)$

$\omega_0 = \frac{2\pi}{N}(0 + 1/2) = \frac{\pi}{N}$



$\omega_1 = \frac{3\pi}{N}$

$\frac{3\pi}{N} - \frac{\pi}{N} = \frac{2\pi}{N}$

$\omega_1 - \omega_0$ always $\frac{2\pi}{N}$

$\omega_2 - \omega_1$ ....

? A requirement exist for a LPF satisfying the following specifications.

passband 0-5 KHz

sampling freq 18 KHz.

filter length 9

Obtain filter coefficient.