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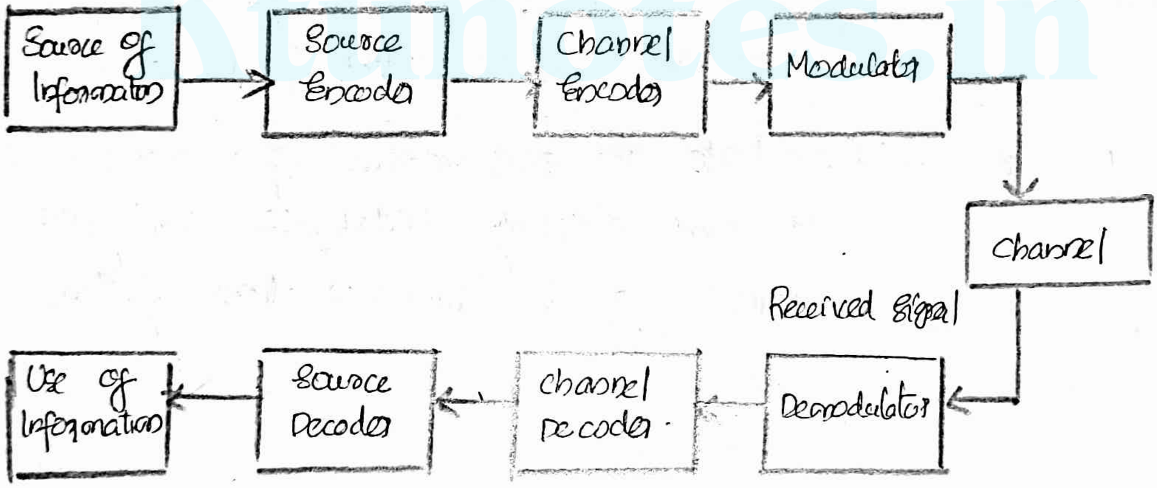
MODULE

ELEMENTS OF DIGITAL COMMUNICATION SYSTEMS



Source of Information.

There are 2 kinds of source of information are available such as

1. Analog Information Sources 2. Digital Information Sources.

Analog Information Sources.

Microphone actuated by a speech, TV camera scanning a ~~scen~~ scene, continuous amplitude signals.

Digital Information Sources.

These are teletype or numerical output of computer which consists of a sequence of discrete symbols or letters. Analog information is transformed into discrete information through the process of sampling and

Quantizing.

Source Encoder/Decoder

* The source encoder converts the input i.e, symbol sequence into a binary
sequence of '0's and '1's by assigning code words to the symbols
in the input sequence.
At the receiver, the source decoder converts the binary output of
the channel decoder into a symbol sequence.

* channel Encoder/Decoder.

Error control is accomplished by the channel coding operation that
consists of systematically adding extra bits to the output of the
source coder. These extra bits donot convey any information but
helps the receiver to detect and/or correct some of the errors in
the information bearing bits. There are 2 methods of channel coding:

1. Block Coding.

Encoder takes a block of 'k' information bits from the source encoder
and adds 'r' error control bits, where 'r' is dependent on
'k' and error control desired.

2. Convoluting coding.

The information bearing message stream is encoded in a continuous
fashion by continuously interleaving information bits and error
control bits. The channel decoder recovers the information bearing
bits from the coded binary system. Error detection and possible
correction is also performed by the channel decoder. Important
parameters of coder/decoder are:

- Method of coding
- Effeciency
- Error control capabilities
- Complexity of the circuit.

* Modulator.

It converts the input bit stream into an electrical waveform suitable
for transmission over the communication channel. Modulator can be
effectively used to minimize the effects of channel noise, to match
frequency spectrum of transmitted signal with channel characteristics,
to provide the capability to multiplex many signals.

* Demodulator.

The extraction of the message from the information bearing waveform
produced by the modulator is accomplished by the demodulator.
The output of the demodulator is bit stream. The important
parameter is the method of demodulation.

* Channel

The channel provides the electrical connection between the source and destination. The different channels are:

- Pair of wires
- Coaxial cable
- Optical fibre
- Radio channel
- Satelite channel or combination of any of these.

Advantages of Digital Communication

- The effect of distortion, noise & interference is less in a digital communication system. This is because the disturbance must be large enough to change the pulse from one state to the other.
- Regenerative repeaters can be used at fixed distance along the link, to identify and regenerate a pulse before it is degraded to an ambiguous state
- Digital circuits are more reliable & cheaper compared to analog circuits.
- The hardware implementation is more flexible than analog hardware because of the use of microprocessors, VLSI chips etc.
- Signal processing functions like encryption, compression can be employed to maintain the secrecy of the information.
- Error detecting and Error correcting codes improve the system performance by reducing the probability of error.
- Combining digital signals using TDM is simpler than combining analog signals using FDM. The different types of signals such as data, telephone, TV can be treated as identical signals in transmission and switching in a digital communication system.
- We can avoid signal jamming using spread spectrum technique.

Disadvantages of Digital Communication

- Large system Bandwidth:
  Digital transmission requires a large system bandwidth to communicate the same information in a digital format as compared to analog format
- System Synchronization:
  Digital detection requires System Synchronization whereas the analog signals generally have no such requirement.

Comparison of Analog and Digital Communication.

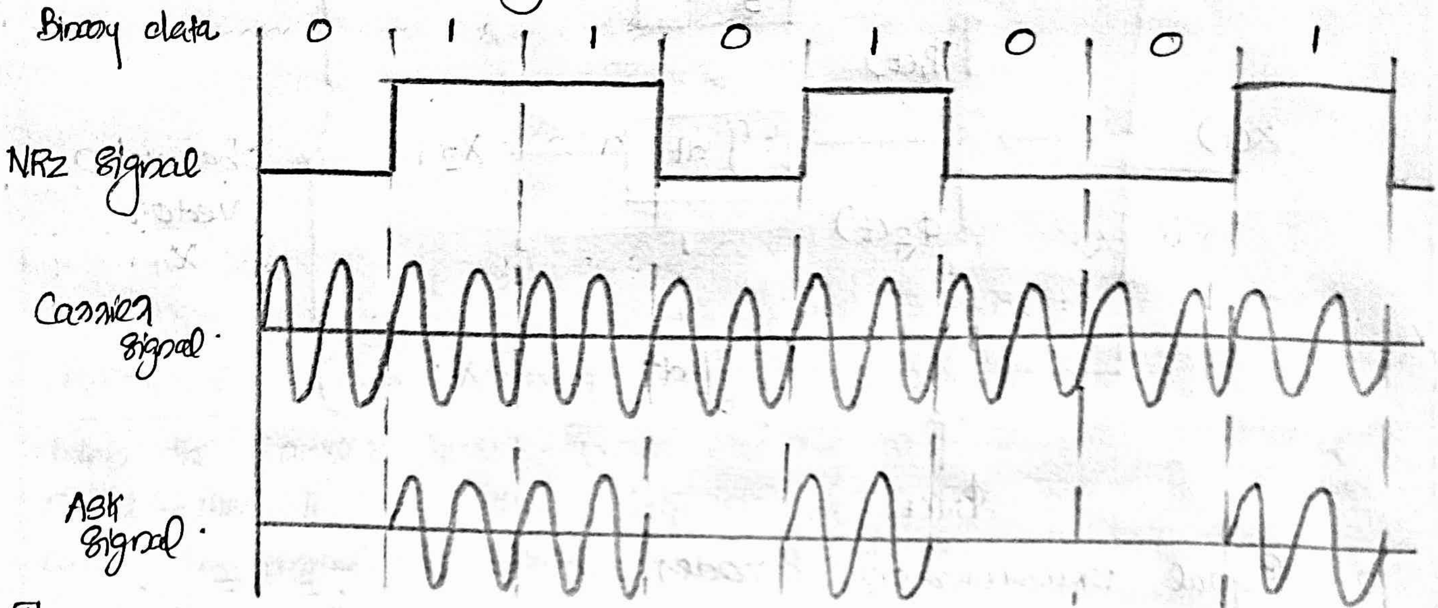
Sl No.	Analog Communication	Digital Communication
1.	The signal can take up voltage level corresponding to any real number.	The signal can take up only one among two voltage level corresponding to 1 and 0
2	One the signal (voltage) is ~~computed~~ corrupted with noise it is difficult to recover the correct value	Even if it is corrupted by channel noise it is possible to recover the original information by using a suitable threshold at the receiver.
3	Repeaters in analog communication are amplifiers which also amplifies noise thereby degrading the quality of the system.	Repeaters in digital communication can be wave regenerators which produce new waveforms after recovering the original information from the received waveform & hence noise does not accumulate through a digital communication link.
4	Since the voltages can take infinite levels, it would need infinite bits to represent data	Since the voltage can take only finite number of values, it requires finite number of bits for propogation.
5.	Transmitted in electronic pulses	Discrete transmission of data.
6	Amplitude & frequency vary together	Amplitude & frequency did not vary together, one always remains constant
7	Less tolerant to noise, make use of bandwidth	More tolerant to noise.

⇒ In science & engineering the mathematical models of signal are in 'deterministic' and 'stochastic'.

* Deterministic :- A model is said to be deterministic, if there is no uncertainity about its time - dependent behavior at any instant of time.

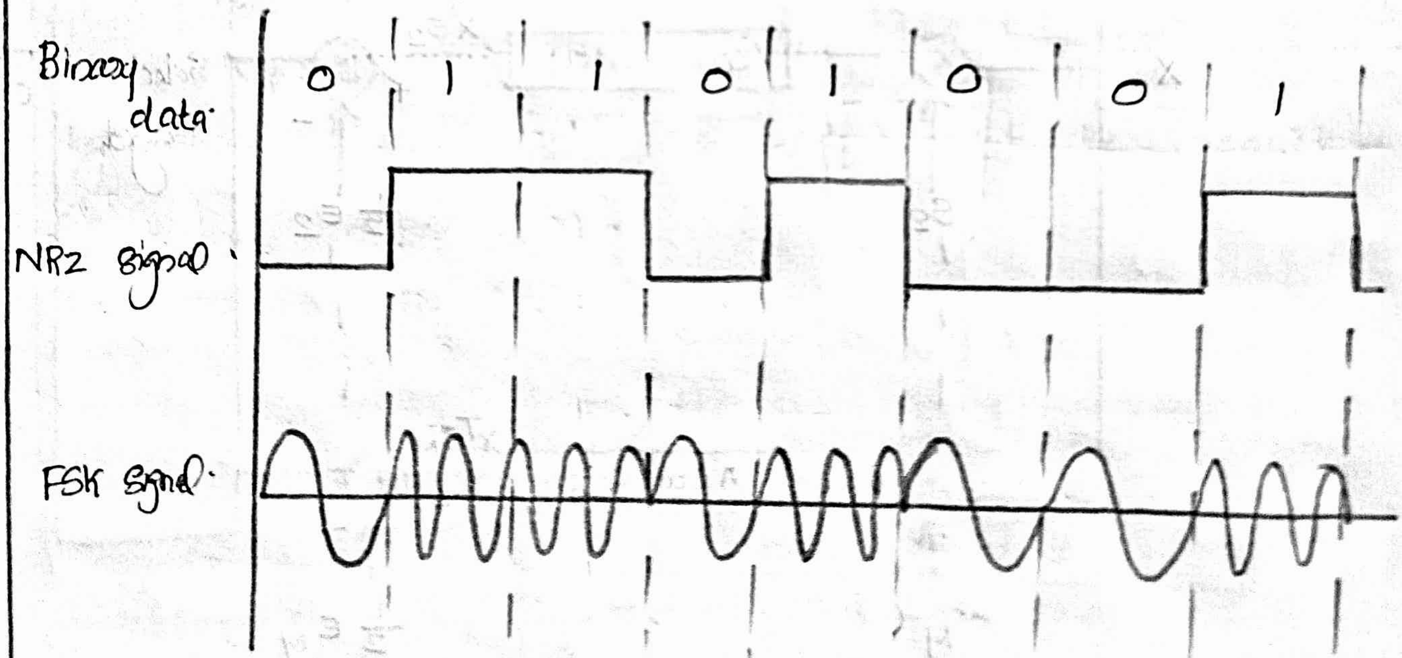
MODULE 5

Amplitude shift Keying (ASK).



The amplitude shift keying is also called On-off keying (OOK). This is simplest digital modulation technique. changes in both amplitude & phase of carrier are combined to produce amplitude phase keying.

Frequency shift keying.



Phase Shift Keying

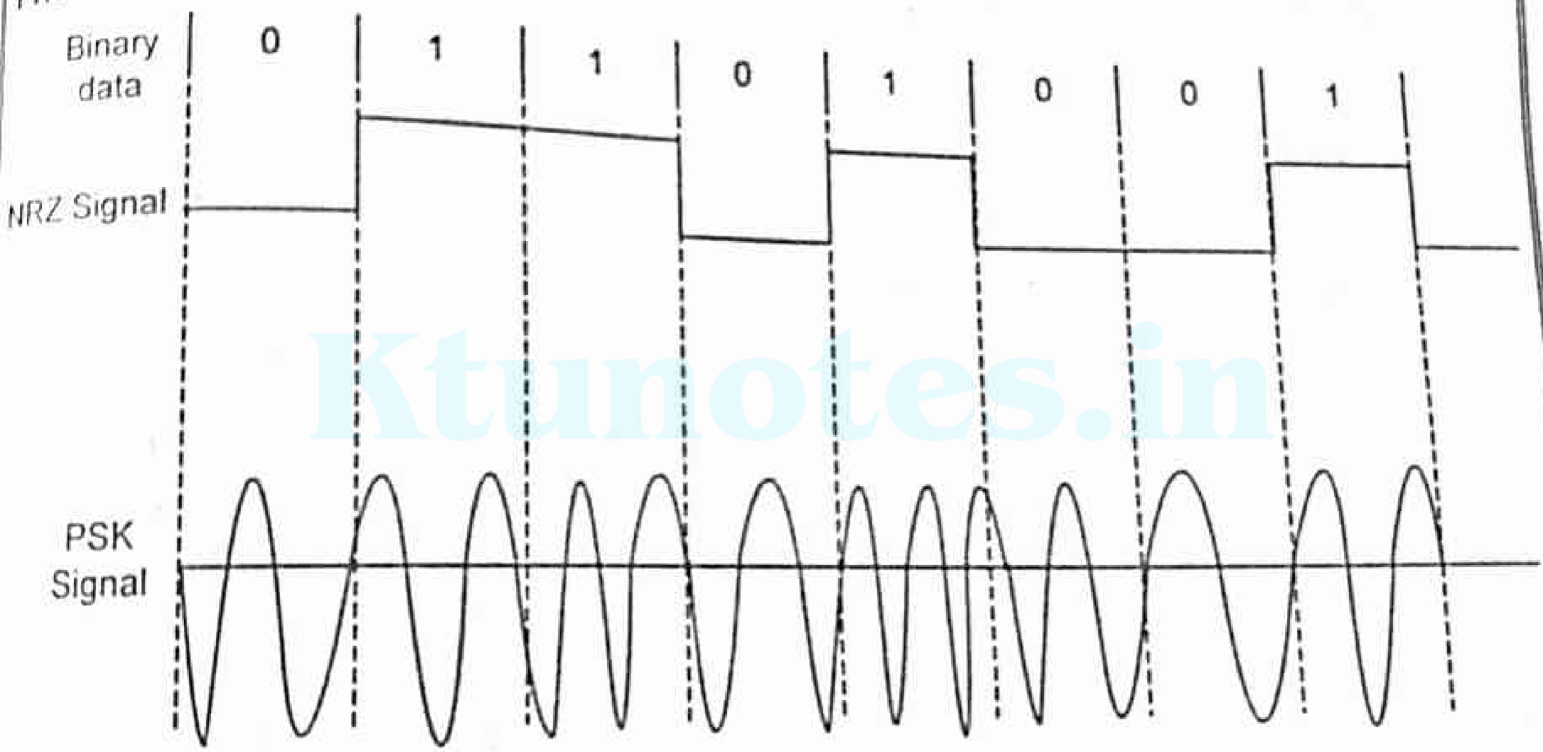


Fig. 8.3. Phase Shift Keying

The choice is made in favor of the scheme that attains as many of the following goals as possible.

1. Maximum data rate.
2. Minimum Probability of symbol error.
3. Minimum transmitted power
4. Minimum channel bandwidth
5. Maximum resistance to interfering signals.
6. Minimum circuit complexity.

COMPARISON BETWEEN COHERENT & NON-COHERENT DETECTION

Sl No	Coherent detection	Non-coherent detection
1.	Receiver has exact knowledge of the carrier wave's phase reference	Knowledge of the carrier wave's phase is not required.
2.	It is also called as synchronous detection	It is also called as envelope detection.
3.	The receiver is phase locked to the transmitter	No phase synchronization between local oscillator used in the receiver.
4.	Error probability is less	Error probability is high
5.	Receiver is more complicated	Receiver is less complicated.

Coherent Binary phase shift Keying (BPSK)

Here the phase of the carrier signal is varied according to the binary input signal.

Carrier signal, $c(t) = A\cos(2\pi f_c t + \phi)$

If the input bit is 1 $\rightarrow \phi = 0$.

bit is 0 $\rightarrow \phi = 180$.

$S_1(t) = A\cos(2\pi f_c t)$ for symbol 1

$S_2(t) = -A\cos(2\pi f_c t)$ for symbol 0.

Each signal is transmitted Tb second duration.

A $\rightarrow$ Amplitude of the signal.

$f_c \rightarrow$ Carrier frequency

The amplitude 'A' is represented bits per energy.

$E_b \rightarrow$ Energy per bit.

$$E_b = \int_0^{T_b} S_1^2(t)\,dt = \int_0^{T_b} A^2\cos^2 2\pi f_c t.$$

$$\left[\cos^2\theta = \frac{1+\cos 2\theta}{2}\right]$$

$$E_b = \frac{A^2}{2}\int_0^{T_b}(1+\cos 4\pi f_c t)\,dt.$$

$$= \frac{A^2}{2}\left[\int_0^{T_b} 1.dt + \int_0^{T_b}\cos 4\pi f_c t\,dt\right]$$

$$= \frac{A^2}{2}[T_b - 0] + 0.$$

$$E_b = \frac{A^2 T_b}{2}.$$

$$A^2 = \frac{2E_b}{T_b} \qquad A = \sqrt{\frac{2E_b}{T_b}}$$

$$\therefore S_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \quad \text{for Symbol '1'}$$

$$S_2(t) = -\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \quad \text{for Symbol '0'}$$

$$0 \le t \le T_b.$$

By Gram - Schmidt Orthogonalization (GSOP)

$N \le M$

$N \rightarrow$ No. of basic function $= 1$

$M \rightarrow$ No. of message $= 2$.

First basic functions,

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_b}} = \frac{\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t}{\sqrt{E_b}}$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

$$\phi_2(t) = 0 \qquad N = 1.$$

So BPSK is one dimensional.

$$\boxed{\begin{array}{l}\text{orthonormal basic function} \\ \int_0^{T_b} \phi_1^2(t)\,dt = 1\end{array}}$$

$$S_{ij} = \int_0^{T_b} S_i(t)\,\phi_j(t)\,dt \qquad \begin{array}{l} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{array}$$

$$M = 2, \quad N = 1$$

$$i = 1, 2, \quad J = 1$$

$$S_{11} = \int_0^{T_b} S_1(t)\,\phi_1(t)\,dt$$

We know that $\phi_1(t) = \dfrac{S_1(t)}{\sqrt{E_b}}$

$$S_1(t) = \phi_1(t)\sqrt{E_b}$$

$$S_2(t) = -\sqrt{E_b}\,\phi_1(t)$$

$$S_{11} = \int_0^{T_b} \sqrt{E_b}\,\phi_1(t)\,\phi_1(t)\,dt$$

$$= \sqrt{E_b}\int_0^{T} \phi_1^2(t)\,dt$$

$$= \underline{\sqrt{E_b}}$$

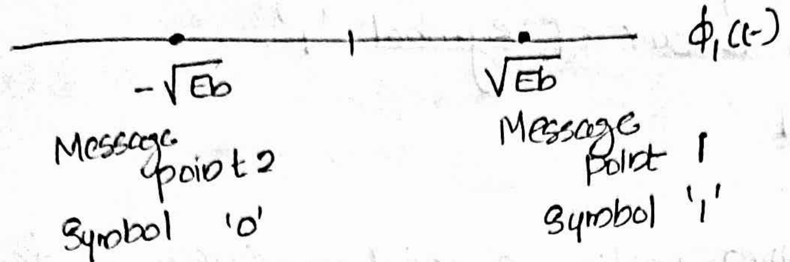
$$S_{21} = \int_0^{T_b} S_2(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} -\sqrt{E_b}\,\phi_1(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} -\sqrt{E_b}\,\phi_1^2(t)\,dt = -\sqrt{E_b}$$

The message point corresponding to $S_1(t)$ is located at $S_{11} = +\sqrt{E_b}$ and the message point corresponding to $S_2(t)$ is located at $S_{21} = -\sqrt{E_b}$.

Signal space diagram

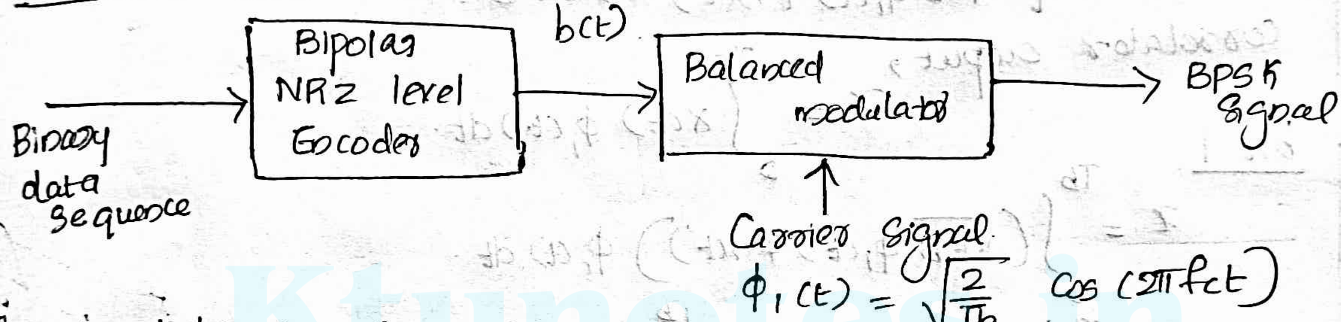


The separation between these 2 point represents isolation in symbol '1' and '0' in BPSK signal.

$$d = +\sqrt{Eb} - (-\sqrt{Eb}) = 2\sqrt{Eb}$$

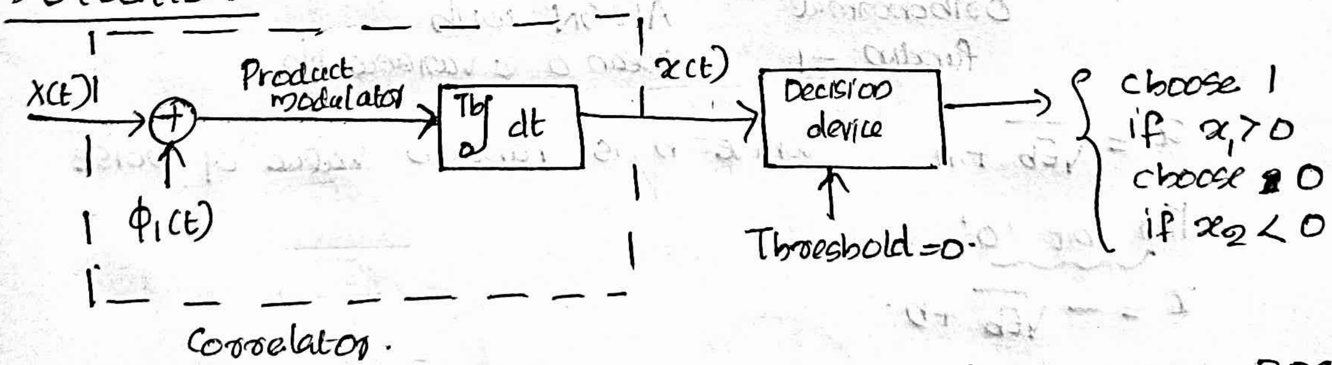
- d increases the isolation between the symbols in BPSK signal is more. So probability of error reduces.

GENERATION



- The input binary sequence sybol 1 can be represented as $+\sqrt{Eb}$ and symbol 0 represented as $-\sqrt{Eb}$. This binary wave b(t) and carrier signal are applied to balanced modulator & produce BPSK signal.

DETECTION



- To detect the original binary sequence from noisy BPSK signal x(t), the received signal from the channel is applied to a correlator
- A locally generated coherent reference signal $\phi_1(t)$, is also applied to the correlator. The output of the multiplier is integrated over one bit period (Tb). The correlator output x_1

is compared with a threshold of zero volts.

- If the correlator output is exceeded the threshold, the receiver decided in favour of symbol '1'.

$$x_1 > 0 \cong 1$$

→ If $x_1 < 0 \cong 0$

→ If $x_1 = 0$ the receiver make a random guess in favour of 0 or 1.

$$x(t) = \begin{cases} S_1(t) + n(t) \longrightarrow \text{bit } 1 \\ S_2(t) + n(t) \longrightarrow \text{bit } 0 . \end{cases}$$

$$x(t) = \begin{cases} \sqrt{Eb}\,\phi_1(t) + n(t) \longrightarrow \text{bit } 1 \\ -\sqrt{Eb}\,\phi_1(t) + n(t) \longrightarrow \text{bit } 0 . \end{cases}$$

Correlator output,

$$x = \int_0^{Tb} x(t)\,\phi_1(t)\,dt .$$

bit 1.

$$x = \int_0^{Tb} \left(\sqrt{Eb}\,\phi_1(t) + n(t)\right)\phi_1(t)\,dt .$$

$$= \int_0^{Tb} \sqrt{Eb}\,\phi_1^2(t)\,dt + \int_0^{Tb} n(t)\,\phi_1(t)\,dt .$$

$$= \sqrt{Eb}\,\underbrace{\int_0^{Tb} \phi_1^2(t)\,dt}_{\substack{\text{Orthonormal} \\ \text{function } = 1}} + \underbrace{\int_0^{Tb} n(t)\,\phi_1(t)\,dt}_{\substack{\text{AWGN with} \\ \text{mean } 0 \text{ \& variance } \frac{No}{2}}} .$$

$x = \sqrt{Eb} + n$ where n is random value of noise

for bit '0'

$$x = -\sqrt{Eb} + n .$$

$$\therefore x = \begin{cases} \sqrt{Eb} + n \longrightarrow \text{bit } 1 \\ -\sqrt{Eb} + n \longrightarrow \text{bit } 0 \end{cases}$$

x is a gaussian function and is characterised by its mean and variance.

Mean

$$E(x) = \begin{cases} E(\sqrt{E_b}) + E(n) \longrightarrow \text{bit } 1 \\ E(-\sqrt{E_b}) + E(n) \longrightarrow \text{bit } 0. \end{cases}$$

$$E(x) = \begin{cases} \sqrt{E_b} \longrightarrow \text{bit } 1 \\ -\sqrt{E_b} \longrightarrow \text{bit } 0. \end{cases}$$

Variance

$$E(x-\bar{x})^2 = E\left\{(\sqrt{E_b} + n - \sqrt{E_b})^2\right\}$$

$$= E[n^2] = \frac{N_0}{2} \rightarrow \text{bit } 1$$

Variance of noise.

bit 0

$$E(x-\bar{x})^2 = E\left\{(-\sqrt{E_b} + n - (-\sqrt{E_b}))^2\right\}$$

$$= E(n^2) = \frac{N_0}{2}$$

pdf of Gaussian random variable,

$$\cancel{f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{x}} \qquad f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(x-\mu)^2/2\sigma^2}$$

$X \rightarrow$ Random variable.

$\mu \rightarrow$ Mean

$\sigma^2 \rightarrow$ Variance.

when symbol '0' is transmitted,

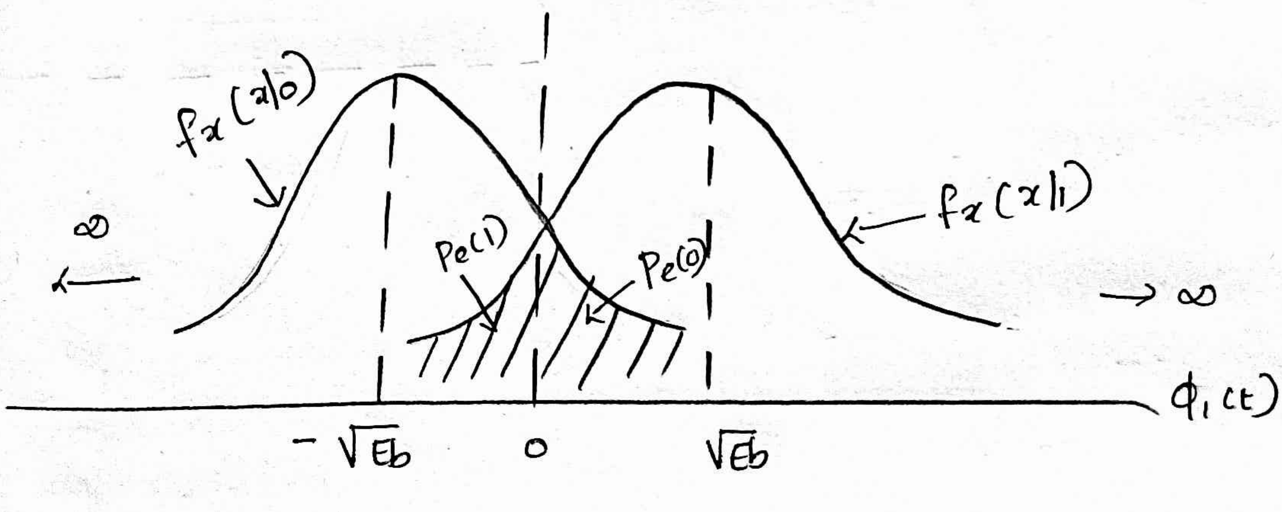
$$f_x(x/0) = \frac{1}{\sqrt{2\pi N_0/2}}\, e^{-\frac{(x+\sqrt{E_b})^2}{2N_0/2}}$$

$$= \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{(x+\sqrt{E_b})^2}{N_0}}$$

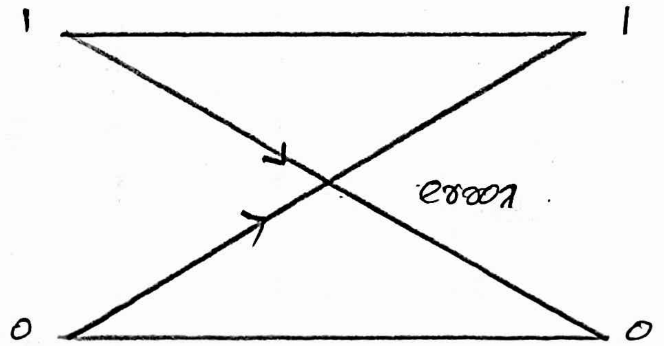
When symbol '1' is transmitted,

$$f_x(x/1) = \frac{1}{\sqrt{2\pi N_0/2}}\, e^{-\frac{(x-\sqrt{E_b})^2}{2N_0/2}}$$

$$f_x(x/1) = \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{(x-\sqrt{E_b})^2}{N_0}}$$



→ Assume binary Symmetric channel (BSC)



Assume large length of sequence $P(0) = P(1) = \frac{1}{2}$.

Maximum likelihood detection,

average probability of error, $P_e = \frac{1}{2} P_e(0) + \frac{1}{2} P_e(1)$

$$P_e = \frac{P_e(0) + P_e(1)}{2}$$

$$P_e(0) = \int_0^\infty f_x(x/_0)\,dx$$

$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x+\sqrt{E_b})^2}{N_0}} dx.$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty e^{-\frac{(x+\sqrt{E_b})^2}{N_0}} dx.$$

Usually probability of error is expressed interms of complimentary error function

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-z^2} dz$$

$$\text{Put } z = \frac{1}{\sqrt{N_0}} (x + \sqrt{E_b}) \quad \text{——— (1)}$$

Integrating x to z

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty e^{-\frac{(x+\sqrt{E_b})^2}{N_0}} dx.$$

eqn (1),

$$\sqrt{N_0}\, z = x + \sqrt{E_b}$$

$$\sqrt{N_0}\, dz = dx$$

$$\text{Put } x = 0, \quad z = \frac{\sqrt{E_b}}{\sqrt{N_0}}$$

$$\text{Put } x = \infty, \quad z = \infty.$$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{\frac{\sqrt{E_b}}{\sqrt{N_0}}}^{\infty} e^{-z^2} \sqrt{N_0}\, dz$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_o}}}^{\infty} e^{-z^2} dz$$

$$P_e(0) = \frac{1}{2}\left[\frac{2}{\sqrt{\pi}} \int_{\sqrt{E_b/N_o}}^{\infty} e^{-z^2} dz\right]$$

$$P_e(0) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{E_b/N_o}\right]$$

Similarly, $P_e(1) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_o}}\right]$

By averaging probability of symbol error for BPSK equals

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_o}}\right]}$$

Bandwidth

$BW = (f_o + f_b) - (f_o - f_b)$

$BW = 2f_b$.

$f_b = \frac{1}{T_b} \Rightarrow$ Maximum frequency in baseband signal.

f_o = carrier frequency

$\therefore$ Maximum BW of PSK = Twice the highest frequency contained ~~contained~~ in baseband signal

Baud rate = f_b

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

Coherent Quadrature Modulation Techniques

There are 2 goals are important in the design of a digital communication system such as

1. Very low probability of error.
2. Efficient utilization of channel bandwidth.

QPSK is an example of bandwidth conserving modulation schemes for the transmission of binary data. QPSK is an extension of binary PSK.

QUADRIPHASE - SHIFT KEYING (QPSK)

In quadriphase shift keying (QPSK), the phase of the carrier takes on one of four equally spaced values, such as $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ and $\frac{7\pi}{4}$ as shown by.

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right) & , 0 \le t \le T \\ 0 & , \text{elsewhere} \end{cases} \quad \text{——— (1)}$$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

where,

$i = 1, 2, 3, 4$

E = is the transmitted signal energy per symbol

T = is the symbol duration $= T_b \log_2(M)$; $M = 4$, $T = 2T_b$

f_c = carrier frequency equals to $\frac{n_c}{T}$ for some fixed integer n_c.

Each possible value of the phase corresponds to a unique pair of bits called a dibit. For eg, we may choose the foregoing set of phase values to represent the gray encoded set of dibits 10, 00, 01 and 11.

Using trigonometric identity, we may rewrite eqn (1) in the equivalent form of

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left((2i-1)\frac{\pi}{4}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left((2i-1)\frac{\pi}{4}\right) \sin(2\pi f_c t); & \text{for } 0 \le t \le T \\ 0 & ; \text{elsewhere} \end{cases}$$

——— (2)

1. These are only two ~~orthogonal~~ orthonormal basis functions, $\phi_1(t)$ and $\phi_2(t)$. The appropriate forms for $\phi_1(t)$ and $\phi_2(t)$ are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad \text{and} \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \quad 0 \le t \le T$$

$$S_i(t) = \sqrt{E} \cos\left[(2i-1)\frac{\pi}{4}\right] \phi_1(t) - \sqrt{E} \sin\left[(2i-1)\frac{\pi}{4}\right] \phi_2(t)$$

2. There are 4 message points and the associated signal vectors are defined by

$$S_i = \begin{bmatrix} \sqrt{E} \cos\left[(2i-1)\frac{\pi}{4}\right] \\ -\sqrt{E} \sin\left[(2i-1)\frac{\pi}{4}\right] \end{bmatrix}$$

$N = 2$
$M = 4$
$i = 1, 2, 3, 4$
$N \le M$

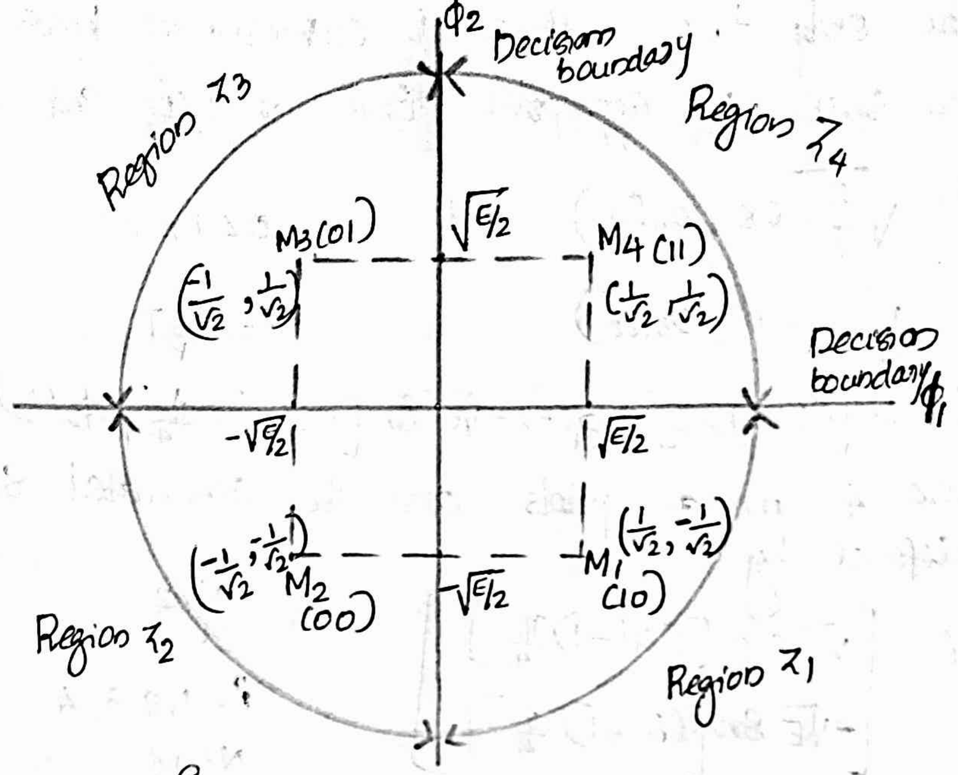
S_{i1} and S_{i2} are called elements of the signal vectors and their values are shown in below table. The first two columns of this table give the associated dibits and phase of the QPSK signal.

Signal Space characterization of QPSK			
Input dibit	Phase of QPSK signal	Coordinates of message points	
		S_{i1}	S_{i2}
10	$\pi/4$	$+\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
00	$3\pi/4$	$-\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
01	$5\pi/4$	$-\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$
11	$7\pi/4$	$+\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$

Geometrical Representation.

A QPSK signal is characterized by having a 2 dimensional signal constellation ($N = 2$) and 4 message points ($M = 4$) as shown below.

The decision regions are quadrants whose vertices coincide with the origin. These regions are marked as Z_1, Z_2, Z_3 and Z_4



Signal space diagram

The received signal $x(t)$ is defined by

$$x(t) = S_i(t) + w(t) \qquad 0 \le t \le T$$
$$i = 1, 2, 3, 4$$

where

$w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $\frac{N_0}{2}$.

The observation vector x has 2 elements x_1 and x_2 that are defined by.

$$x_1 = \int_0^T x(t)\phi_1(t)\,dt = \underbrace{\sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]}_{S_1(t)} + w_1$$

and

$$x_2 = \int_0^T x(t)\phi_2(t)\,dt = \underbrace{-\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]}_{S_2(t)} + w_2.$$

where $i = 1, 2, 3, 4$

GENERATION

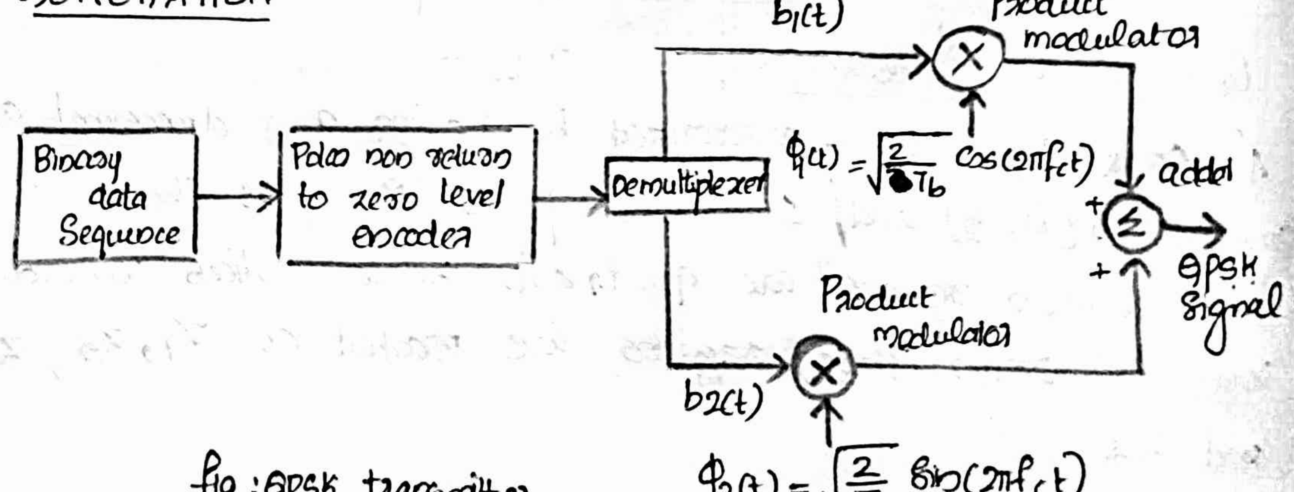


fig: QPSK transmitter.

NRZ Encoder

The input binary sequence is first transformed into polar form by a NRZ encoder. Thus symbols 1 and 0 are represented by $+\sqrt{E_b}$ and $-\sqrt{E_b}$

Demultiplexer

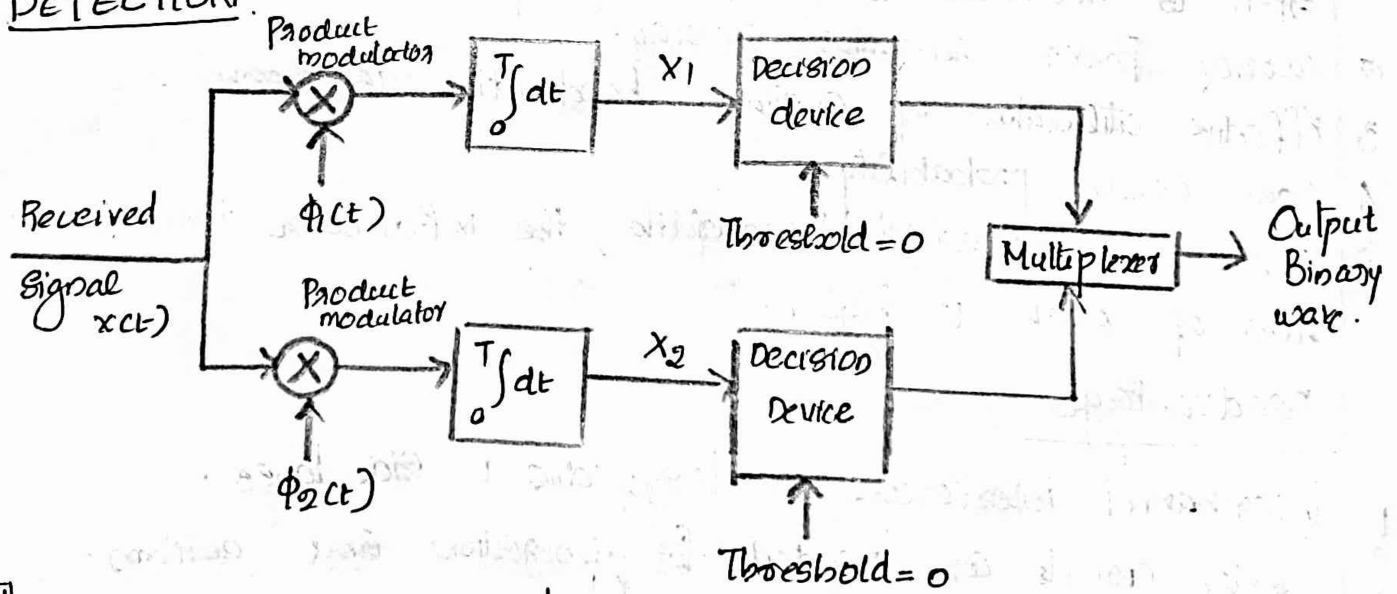
The NRZ encoder output is given to demultiplier to divide the binary wave into 2 seperate binary waves consisting of the odd and even numbered input bits. These 2 binary waves are denoted by $b_1(t)$ and $b_2(t)$

The amplitudes of $b_1(t)$ & $b_2(t)$ equal s_{i1} and s_{i2}, depending on the particular dibit that is being transmitted. The 2 binary waves $b_1(t)$ & $b_2(t)$ are used to modulate a pair of quadrature carries or orthonormal basis functions $\phi_1(t)$ equal to $\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $\phi_2(t)$ equal to $\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$. The two binary waves are added to produce the desired QPSK wave.

Symbol duration

For a QPSK symbol duration is twice as long as the bit duration T_b of the input binary wave (ie) for a given rate $\frac{1}{T_b}$, a QPSK wave requires half the transmission bandwidth of the corresponding binary PSK wave.

DETECTION



The QPSK receiver consists of a pair of correlators with a common input and supplies with a locally generated pair of coherent reference signals $\phi_1(t)$ & $\phi_2(t)$.

The correlator outputs x_1 and x_2 are each compared with a threshold of zero volts.

1. If $x_1 > 0$, a decision is made in favor of symbol 1 for the upper or in-phase channel output, but if $x_1 < 0$ a decision is made in favor of symbol 0.

2. If $x_2 > 0$, a decision is made in favor of symbol 1 for the lower or quadrature channel output, but if $x_2 < 0$, a decision is made in favor of symbol 0.

Multiplexer.

The 2 binary sequence x_1 and x_2 are combined in a multiplexer at the transmitter input to reproduce the original binary sequence with the minimum probability of symbol error.

Bandwidth

In QPSK the two waveforms $\phi_1(t)$ and $\phi_2(t)$ form the baseband signals. One bit period for both these signals is equal to $2T_b$. Therefore bandwidth of QPSK is

$$BW = 2 \times \frac{1}{2T_b} \quad \text{or} \quad BW = f_b$$

The bandwidth of QPSK signal is half of the bandwidth of PSK signal.

Advantages.

1. For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK
2. carrier Power remains constant
3. Effective utilization of available bandwidth is possible.
4. Low error probability
5. Because of reduced bandwidth, the information transmission rate of QPSK is higher.

Disadvantages

1. Interchannel interference is large due to side lobes
2. Complex circuits are needed for generation and detection.

Probability of error

$x(t) = s_i(t) + w(t) \quad ; \quad 0 \le t \le T$
$i = 1, 2, 3, 4$

$w(t) \rightarrow$ AWGN with zero mean and variance $N_0/2$.

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt$$

$$= \int_0^T (s_i(t) + w(t))\,\phi_1(t)\,dt$$

$$= \int_0^T \left(s_i(t)\phi_1(t) + w(t)\phi_1(t)\right)dt$$

$$= \int_0^T s_i(t)\,\phi_1(t)\,dt + \int_0^T w(t)\,\phi_1(t)\,dt$$

$$= \int_0^T s_i(t)\,\phi_1(t)\,dt + n$$

$$= \int_0^T \left\{\sqrt{E}\cos\left((2i-1)\frac{\pi}{4}\right)\phi_1(t) - \sqrt{E}\sin\left((2i-1)\frac{\pi}{4}\right)\phi_2(t)\right\}\phi_1(t)\,dt + n$$

$$= \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\underbrace{\int_0^T \phi_1^2(t)\,dt}_{1} - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\underbrace{\int_0^T \phi_1(t)\phi_2(t)\,dt}_{0} + n$$

$$\boxed{x_1 = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] + n} \quad i = 1, 2, 3, 4$$

$$x_2 = \int_0^T x(t)\,\phi_2(t)\,dt = \int_0^T (s_i(t) + w(t))\,\phi_2(t)\,dt$$

$$= \int_0^T s_i(t)\,\phi_2(t)\,dt + \int_0^T w(t)\,\phi_2(t)\,dt$$

$$= \int_0^T s_i(t)\,\phi_2(t)\,dt + n$$

$$= \int_0^T \left\{\sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t) - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2(t)\right\}\phi_2(t)\,dt + n$$

$$= \int_0^T \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t)\phi_2(t)\,dt + \int_0^T \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2^2(t)\,dt + n$$

$$\boxed{x_2 = \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] + n} \quad i = 1, 2, 3, 4$$

Assume $s_4(t)$ is transmitted for i/p dibit '11'

$$x_1 = \sqrt{\frac{E}{2}} + n$$

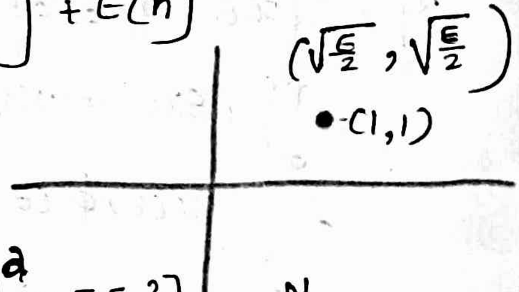
$$x_2 = \sqrt{\frac{E}{2}} + n$$

Mean $E(x_1) = E\left[\sqrt{\frac{E}{2}} + n\right]$

$$\underline{E(x_1) = \sqrt{\frac{E}{2}}}$$

$$E(x_2) = E\left[\sqrt{\frac{E}{2}} + n\right] = E\left[\sqrt{\frac{E}{2}}\right] + E(n)$$

$$\underline{E[x_2] = \sqrt{\frac{E}{2}}}$$



Variance

$$Var(x_1) = E[x - \bar{x}]^2 = E\left[\sqrt{\frac{E}{2}} + n - \sqrt{\frac{E}{2}}\right]^2 = E[n^2] = \frac{N_0}{2}$$

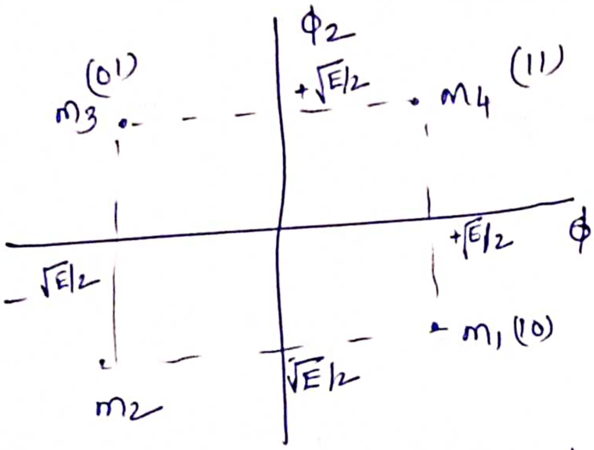
$$Var[x_2] = E[x - \bar{x}]^2 = E\left[\sqrt{\frac{E}{2}} + n - \sqrt{\frac{E}{2}}\right]^2 = E[n^2] = \frac{N_0}{2}.$$

$$\left[f_x(x) = \frac{1}{2\ldots^2} e^{-(x-\mu)^2/2\sigma^2} \right]$$

Comparison between digital modulation techniques

Sl No	Parameter	BPSK	BFSK	QPSK
1.	Variable characteristic	Phase	Frequency	phase
2.	Equation of transmitted signal s(t)	$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$	$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \frac{\pi t}{T_b}\right)$	$s(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right)$ $i = 1,2,3,4$
3	Bit per symbol	One	One	Two
4.	No. of possible symbols	Two	Two	four
5.	Detection method	Coherent	non coherent	Coherent
6	Minimum Euclidean distance	$2\sqrt{E_b}$	$\sqrt{2E_b}$	$2\sqrt{E_b}$
7	Minimum bandwidth	$2f_b$	$4f_b$	f_b
8	Symbol duration	T_b	T_b	$2T_b$
9	Probability of error	$P_e = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$	$P_e = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right]$	$P_e = \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$
10	Symbol shaping function g(t)	$g(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases}$	$g(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{T_b}\right), & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases}$	$g(t) = \begin{cases} \sqrt{\frac{E}{T}} & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$

Error probability of QPSK (BER of QPSK)



In QPSK, the received signal $x(t)$ is given by

$$x(t) = S_i(t) + w(t)$$

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt = \sqrt{E}\cos\left[(2i-1)\pi/4\right] + w_1$$

$$= \pm\sqrt{\frac{E}{2}} + w_1$$

$$x_2 = \int_0^T x(t)\,\phi_2(t)\,dt = -\sqrt{E}\cos\left[(2i-1)\pi/4\right] + w_2$$

$$= \pm\sqrt{\frac{E}{2}} + w_2.$$

decision rule

Decide in favour of m_1 if x lies in z_1

Decide in favour of m_2 if x lies in z_2.

" of m_3 if x lies in z_3.

" of m_4 if x lies in z_4

An erroneous decision will be made if, due to noise, Suppose if m_4 was transmitted but received signal point falls outside z_4.

To calculate the average probability of error, by the equation a QPSK system equivalent to 2 BPSK system.

W. know that P_e of BPSK $= \frac{1}{2} \operatorname{erfc}\left[\sqrt{E_b/N_0}\right]$

QPSK bit energy $= \frac{E}{2}$.

In QPSK P_e for the Inphase channel $= \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E}{2N_0}}\right] = p'$

P_e of Quadriphase channel $= \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E}{2N_0}}\right] = p'$

Probability for correct decision $= 1 - p'$
in phase decision (P_{CI})
or
probability for correct decision
in Quadriphase channel. (P_{CQ})

So
probability of correct decision in QPSK channel.

$$P_C = P_{CI} \cdot P_{CQ}$$

$$= (1-p')^2 = \left[1 - \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E}{2N_0}}\right]\right]^2$$

$$= 1 + \frac{1}{4} \operatorname{erfc}^2\left[\sqrt{\frac{E}{2N_0}}\right] - 2\frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$= 1 + \frac{1}{4} \operatorname{erfc}^2\left[\sqrt{\frac{E}{2N_0}}\right] - \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

Assume the signal to noise ratio.

$$SNR \ \frac{E}{2N_0} >>> 1$$

erfc of large value $\simeq$ small

So $P_C \simeq 1 - \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$

Error probability of symbol error. $\simeq 1 - P_C$.

$$P_e \simeq 1 - \left(1 - \operatorname{erfc}\sqrt{\frac{E}{2N_0}}\right)$$

$$P_e \simeq \operatorname{erfc}\sqrt{E/2N_0}$$

Bit error rate (BER) $= \frac{1}{2}$ Symbol error rate.

$\therefore$ for QPSK

$$\boxed{BER = \frac{1}{2} \, erfc \sqrt{\frac{E_b}{N_0}}}$$

$$(BER)_{QPSK} = (BER)_{BPSK}.$$

BPSK

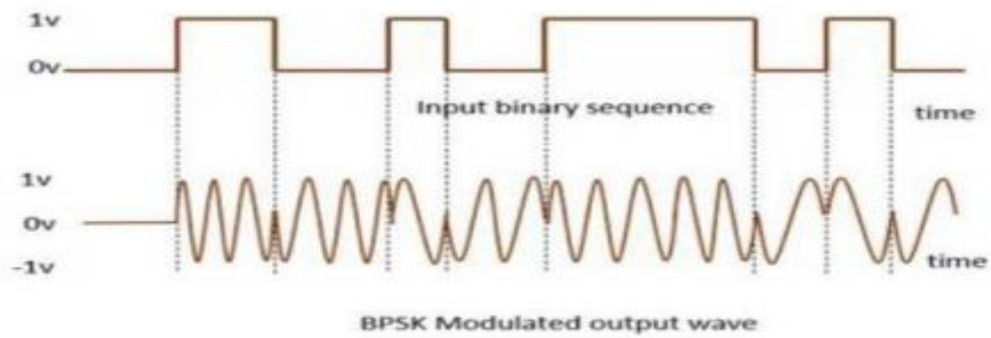


Figure 2-15 shows the output phase-versus-time relationship for a BPSK waveform. Logic 1 input produces an analog output signal with a 0° phase angle, and a logic 0 input produces an analog output signal with a 180° phase angle.

As the binary input shifts between a logic 1 and a logic 0 condition and vice versa, the phase of the BPSK waveform shifts between 0° and 180°, respectively.

BPSK signaling element *(t_s)* is equal to the time of one information bit *(t_b)*, which indicates that the bit rate equals the baud.

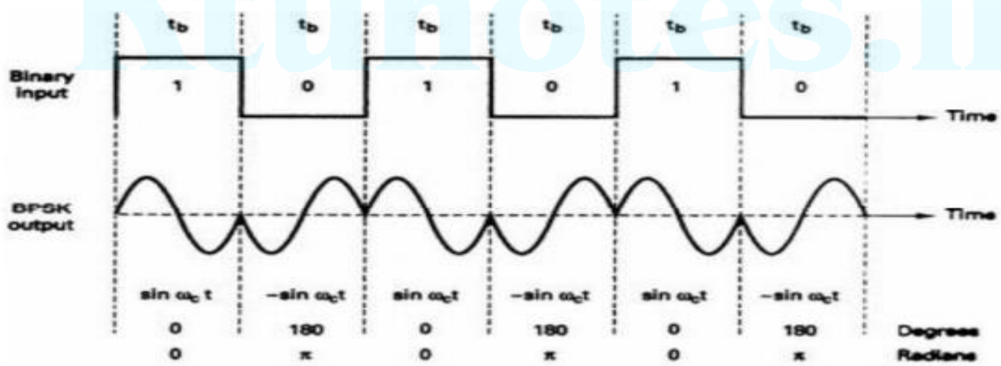
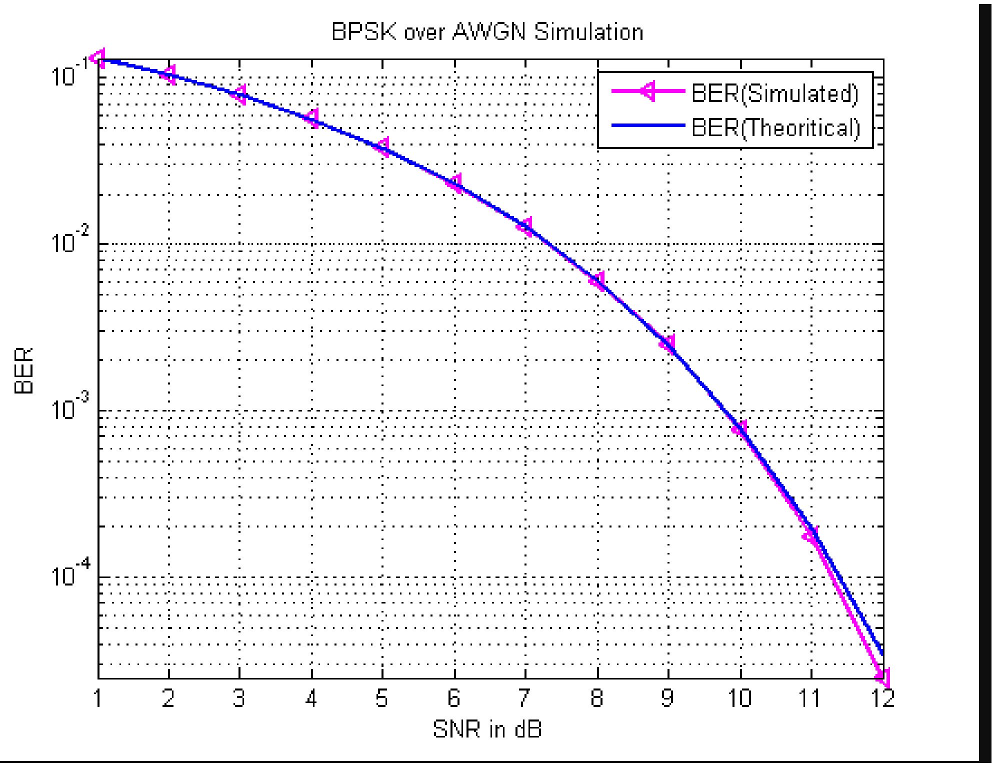
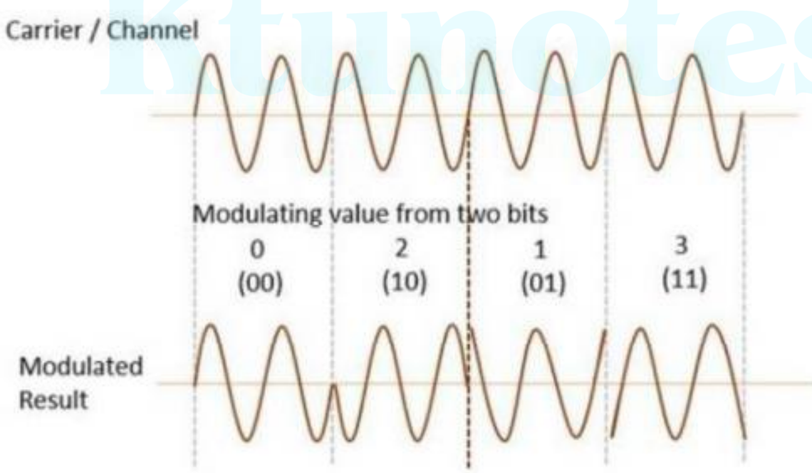


FIGURE 2-15 Output phase-versus-time relationship for a BPSK modulator

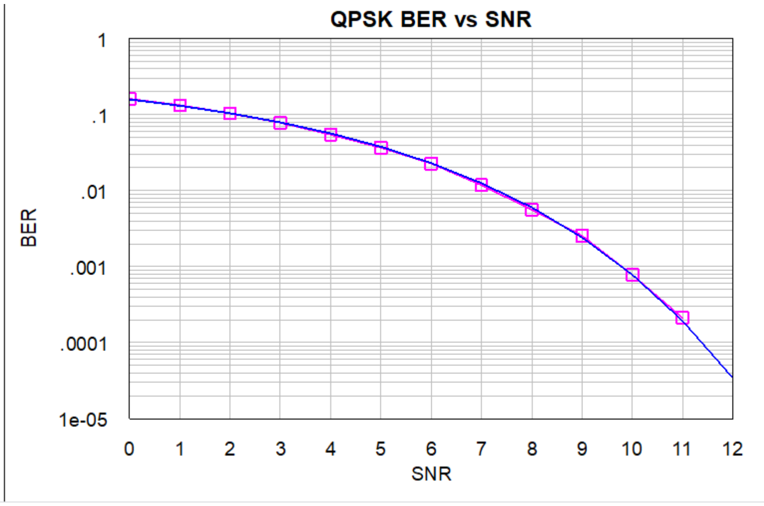


QPSK



QPSK is a variation of BPSK, and it is also a DSB-SC (Double Sideband Suppressed Carrier) modulation scheme, which sends two bits of digital information at a time, called as **bigits**.

Instead of the conversion of digital bits into a series of digital stream, it converts them into bit-pairs. This decreases the data bit rate to half, which allows space for the other users.


QPSK BER vs SNR
1
.1
.01
.001
.0001
1e-05
BER
0
1
2
3
4
5
6
7
8
9
10
11
12
SNR

QAM — Quadrature Amplitude Modulation

→ QAM is a form of modulation which is widely used for modulating data signals onto a carrier used for radio communications

→ QAM is a signal in which two carriers shifted in phase by 90° are modulated and resultant output consists of both amplitude and phase variations.

$$QAM = ASK + PSK.$$

→ QAM is able to carry higher data rates than ordinary amplitude modulated schemes and phase modulated schemes.

→ When using QAM the constellation points are normally arranged in a square grid with equal vertical and horizontal spacing and ~~too~~ as a result the most common form of QAM use a constellation with the number of points equal to a power of 2 ie 4, 16, 64 - - - -

→ The advantages of moving to the higher order formats is that there are more points within the constellation and therefore it is possible to transmit more bits per symbol

→ The ~~do~~ disadvantages is that Constellation points are closer together and therefore the link is more susceptible to noise

8.9.3. TYPES OF QAM

Name	Bits per symbol (N)	Number of symbols (M) ($M = 2^N$)
4 QAM	2	4
8 QAM	3	8
16 QAM	4	16
32 QAM	5	32
64 QAM	6	64

8.9.4. PRINCIPLE OF QAM

The general form of QAM is defined by the transmitted signal such as

$$S_i(t) = \sqrt{\frac{2E_0}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}}\, b_i \sin(2\pi f_c t) \qquad 0 \le t \le T \quad \ldots(1)$$

Where, E_0 is the energy of the signal with the lowest amplitude a_i & b_i are a pair of independent intergers chosen in accordance with the location of the pertinent message point.

The signal $S_i(t)$ consists of two phase quadrature carriers, each of which is modulated by a set of discrete amplitudes, hence it called quadrature amplitude modulation.

The signal $S_i(t)$ can be expanded in terms of a pair of basis functions

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \le t \le T \quad \ldots(2)$$

and

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \le t \le T \quad \ldots(3)$$

$(-3,-3) \quad (-1,-3) \quad (1,-3) \quad (3,-3)\rfloor$

8.9.5. GEOMETRICAL REPRESENTATION

Case 1: M = 16

The signal constellation for QAM consists of a square lattice of message points. Below Figure shows signal constellation for M = 16.

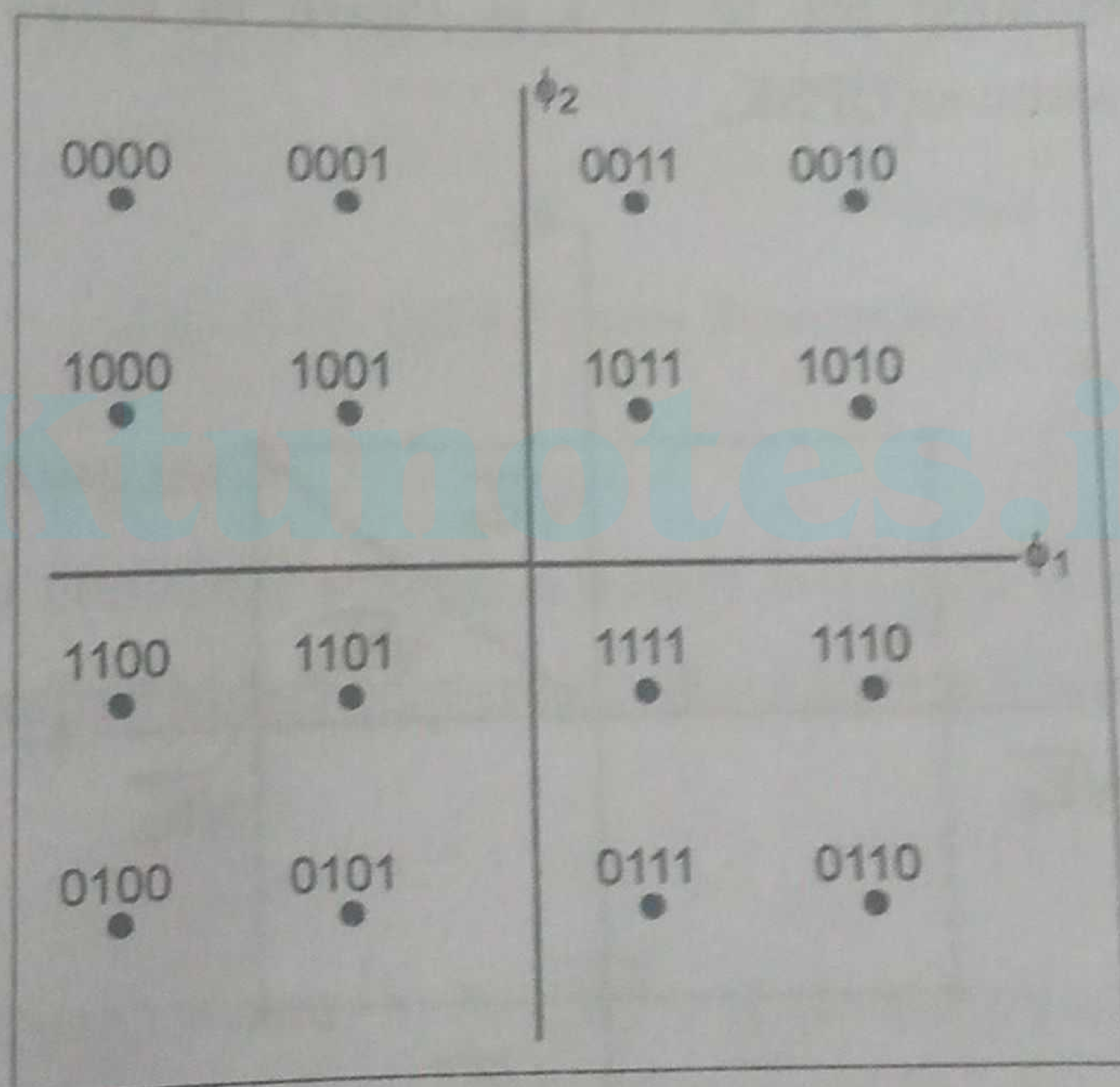


Fig. 8.20. Signal constellation of QAM for M=16

For M = 16, the corresponding signal constellations for the in-phase and quadrature components of the amplitude phase modulated wave are shown below.

Case 2: M = 4

The signal constellation for M = 4 is shown in below Figure, which is recognized to be the same as QPSK.

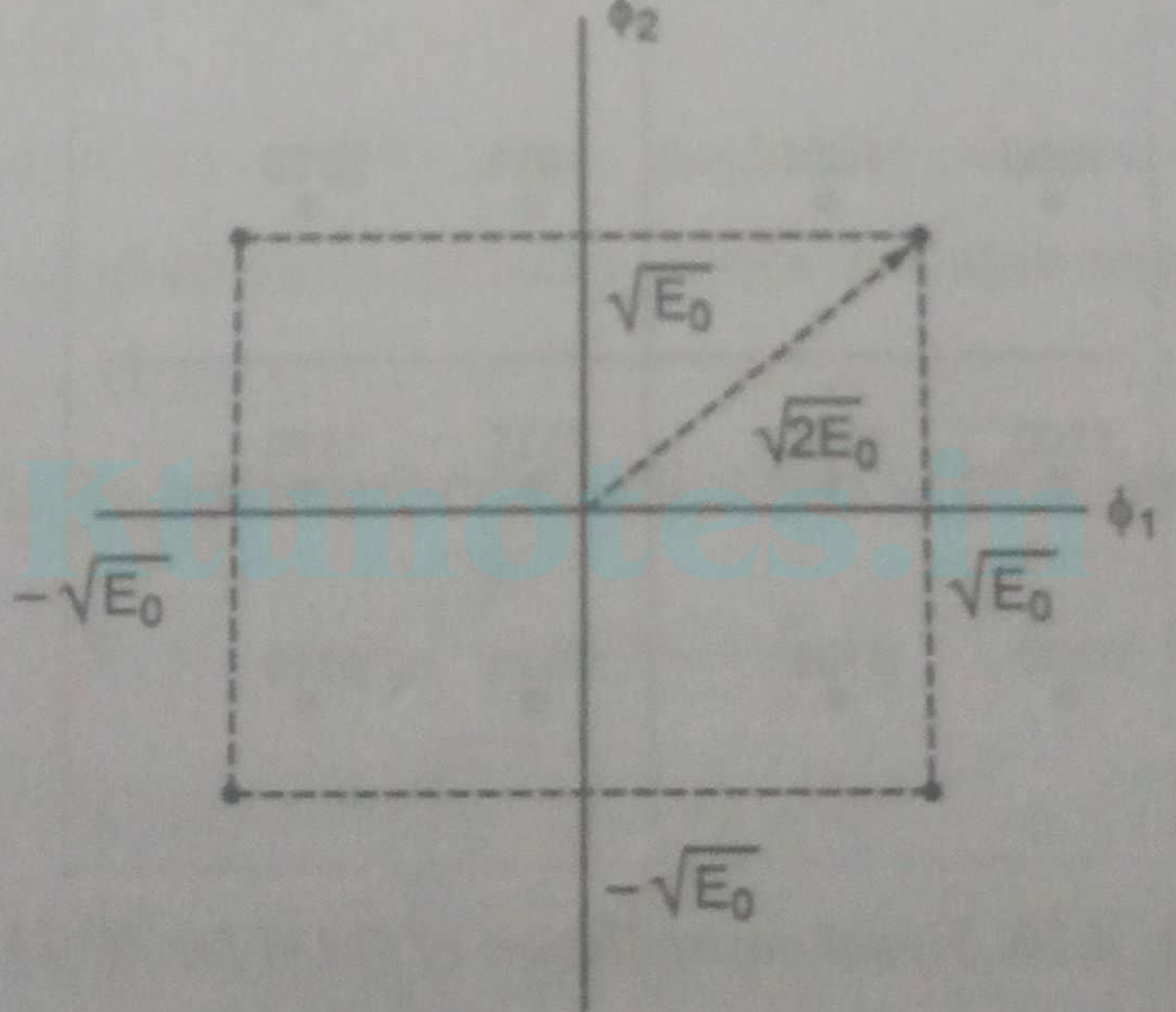


Fig. 8.22. Signal constellation for the QAM for M = 4

8.9.6. GENERATION

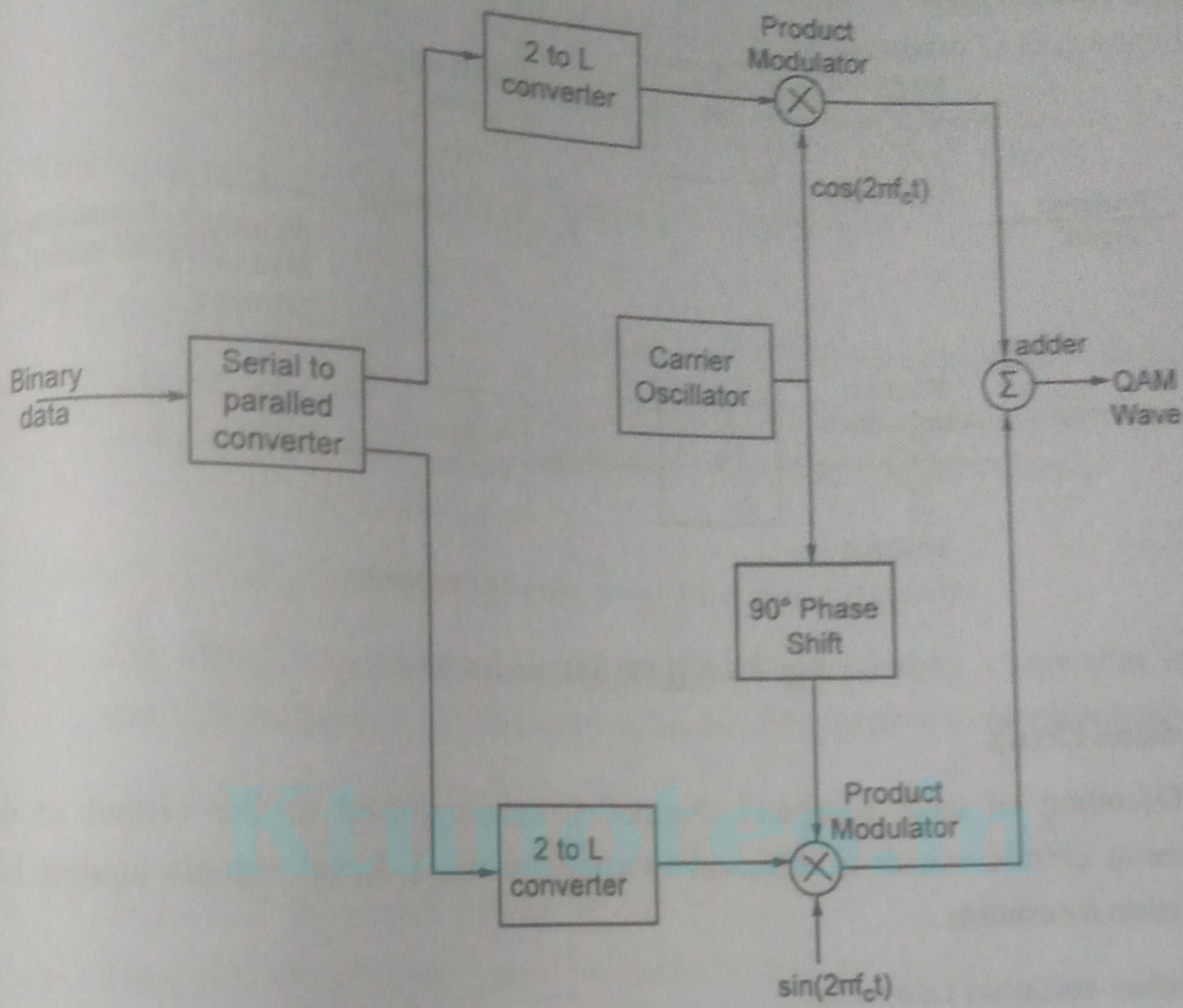


Fig. 8.23. QAM System Transmitter

Serial To paralled Converter

Serial to paralled converter accepts a binary sequence at a bit rate $R_b = 1/T_b$ and produces two paralled binary sequences whose bit rates are $R_b/2$ each,

2-to-L Converters

In 2 to L converters, where $L = \sqrt{M}$, generate polar L-Level signals in response to the respective in-phase and quadrature channel inputs.

Quadrature carrier multiplexing of the two polar L-Level signals to generate desired QAM signal.

8.9.7. DETECTION

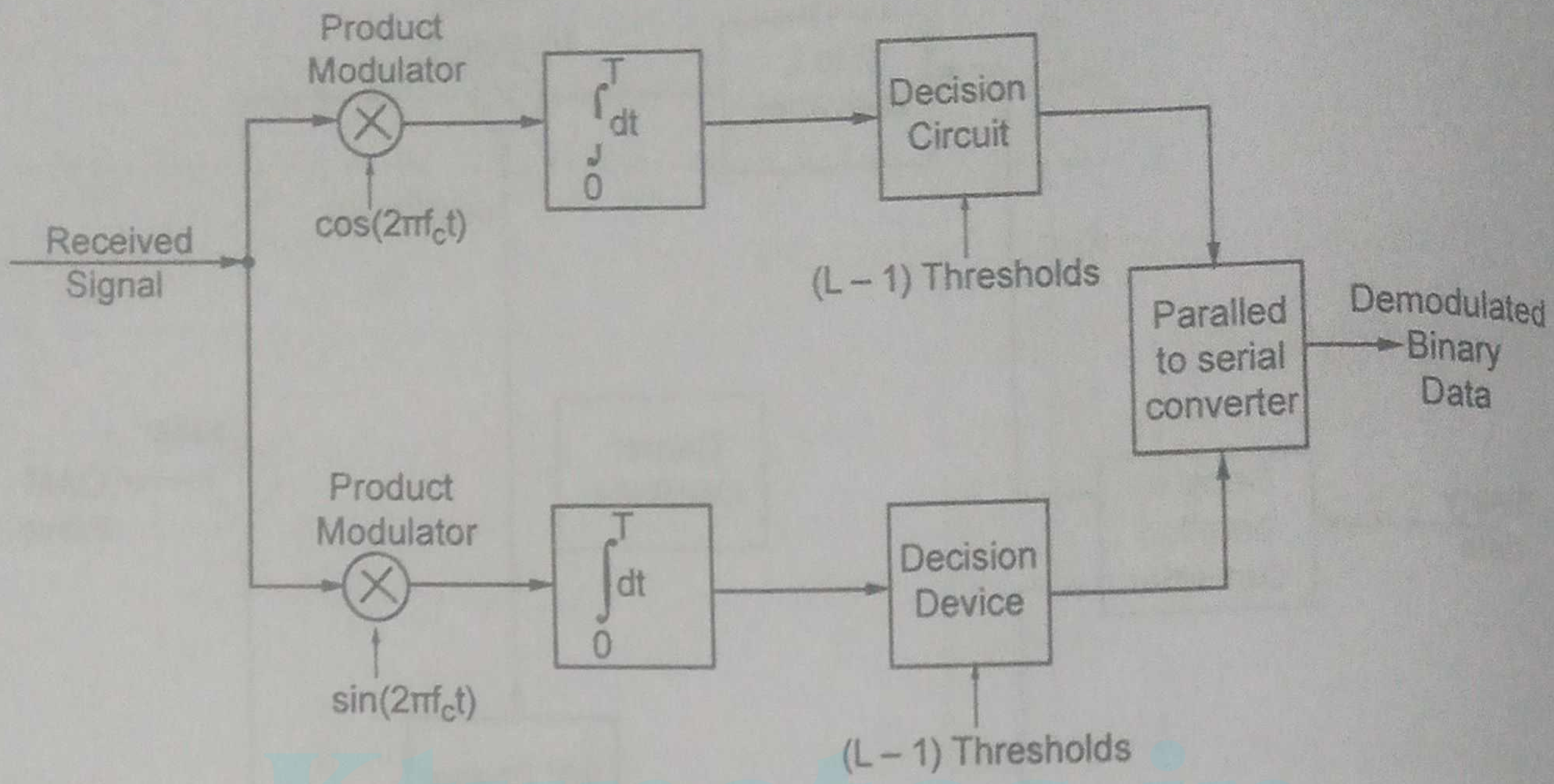


Fig. 8.24. QAM System Receiver

Decision Circuit

Decoding of each baseband channel is accomplished at the output of the decision circuit, which is designed to compare the L-Level signals against L-1 decision thresholds.

Paralled-to-Serial Converter

The two binary sequences are combined in the parallel to serial converter to Reproduce the original binary sequence.

8.9.8. BANDWIDTH

Bandwidth of QAM signal will be

$$\boxed{BW = \frac{2f_b}{W}}$$

desired result

8.9.12. ADVANTAGES

- It has better noise immunity than PSK
- It is easy to design than PSK

8.9.13. DISADVANTAGES

- It has higher error probability than QPSK
- Relatively complex

probability of error

$$P_e = 2\left[1 - \frac{1}{\sqrt{M}}\right] \operatorname{erfc}\left[\sqrt{\frac{E_0}{N_0}}\right]$$

Application

→ Data Delivery application

→ For domestic broadcast applications

For example, 64 QAM and 256 QAM are used ~~for~~ in digital cable television and cable modem application.

→ In UK for digital terrestrial television using DVB - Digital Video Broadcasting. 16 QAM and 64 QAM are used.