A signal can be analysed in 2 domains

1) Time domain analysis → Here we are analysing the variation of parameters w.r. to time.

It is known as Temporal characteristics

eg: Autocorrelation, crosscorrelation.

2) Frequency domain analysis: Here we are analysing the variation of signal parameters w.r. to frequency. which is known as finding out spacial characteristics.

eg: Energy spectrum, power spectrum

Power spectral Density

Simple definition of power spectral density is power per unit bandwidth. It's unit is W/Hz.

OR

It can be defined as the distribution of average power of a s/l in the frequency domain

the power spectral density is denoted by S(ω) or S(f)

if X(t) is the random process X(f) is it's power spectral density.

Also we can calculate the power spectral density from auto correlation function.

we know that the autocorrelation gives the measure of similarity b/w a signal and its time delayed version

i.e. $R_x(\tau) = E\left[x_1(t)\, x(t+\tau)\right]$

$\tau = t_2 - t_1$

→ Fourier transform of the autocorrelation function of a random process $x(t)$ gives its power spectral density

Fourier transform of a signal is given by

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

inverse fourier transform

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

So for getting power spectral density we have to take the fourier transform of $R_x(\tau)$

$$\boxed{S_x(f) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau} \quad \text{—— (1)}$$

Thus for getting autocorrelation function from power spectral density we have to take inverse fourier transform

$$\boxed{R_x(\tau) = \int_{-\infty}^{\infty} S_x(f)\, e^{j2\pi f\tau}\, df} \quad \text{—— (2)}$$

This relation between autocorrelation and power spectral density is known as weiner khintchine relation.

$$R_x(\tau) \xleftrightarrow{F.T} S_x(f)$$

Properties of Power Spectral density

1. Power spectral density is always non-negative i.e; it is always greater than or equal to zero.

2. Power spectral density is always a real valued ~~quantit~~ function because power is a real quantity.

3. $S_X(f)$ is an even function or symmetric.

$$S_x(f) = S_x(-f)$$

4. Putting $f=0$ in the eqn of power spectral density

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) \cdot e^{-j2\pi f \tau} \, d\tau$$

$$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau) \, e^{j2\pi \cdot 0} \, d\tau = \int_{-\infty}^{\infty} R_X(\tau) \, d\tau$$

$$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau) \, d\tau$$ ⇒ value of power spectral density at zero frequency gives the total area under the autocorrelation function graph

5. Putting $\tau = 0$ in the equation of autocorrelation fn.

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) \, e^{j2\pi f \tau} \, df$$

$$R_X(0) = \int_{-\infty}^{\infty} S_X(f) \, e^{j2\pi \cdot 0} \, df$$

$$R_X(0) = \int_{-\infty}^{\infty} S_X(f) \, df$$ ⇒ Auto correlation function at zero gives the area under the power spectral density.

we can see that what is meant by $R_x(0)$

w.k. that $R_x(\tau) = E[x(t)\, x(t+\tau)]$

put $\tau=0$ $R_x(0) = E[x(t)\, x(t)]$

$= E[x^2(t)] \Rightarrow$ Mean square value of Random process $x(t)$

Mean square value of Random Process = power of the Random process

So Auto correlation function at zero $R_x(0) \Rightarrow$ power of the Random process

from the property

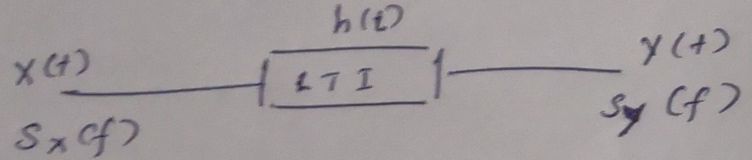
$$R_x(0) = \int_{-\infty}^{\infty} S_x(f)\, df$$

Power of the Random process is equal to the area under the power spectral density.

or

Mean square value is equal to the area under the power spectral density curve.

6. If we pass a R.P $x(t)$ through a linear time invariant system then the output is a random process denoted by $y(t)$



The relation between input and output power spectral density will be equal to

$$S_y(f) = |H(f)|^2 S_x(f)$$

$h(t) \rightarrow$ impulse response

$H(f) \rightarrow$ F.T. transform of $h(t)$

$|H(f)|$ = magnitude response. $H(f)$ - frequency response.

$\therefore$ The power spectral density of the output random process $Y(t)$ is equals to the powerspectral density of the input random process multiplied by the squared magnitude response of the LTI system.

7. If we multiply the R.P by $\cos 2\pi f_c t$, then the power spectral density is given by

$$X(t) \cos 2\pi f_0 t \xrightarrow{PSD} \frac{1}{4}\left[S_X(f-f_0) + S_X(f+f_0)\right)$$

$\therefore$ its power spectral density is shifted in frequency and amplitude is reduced by $\frac{1}{4}$

$$X(t) \sin 2\pi f_0 t \xrightarrow{PSD} \frac{1}{4}\left[S_X(f-f_0) - S_X(f+f_0)\right]$$

So we can conclude that when $X(t)$ modulates a carrier signal; the power spectral density is shifted by f_0 and amplitude will be reduced to $\frac{1}{4}$.