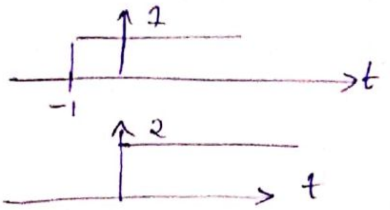
Signals & Systems (1)

Part A

1. $x(t) = u(t+1) + 2u(t) - u(t-4) - 2u(t-6)$

$u(t+1)$

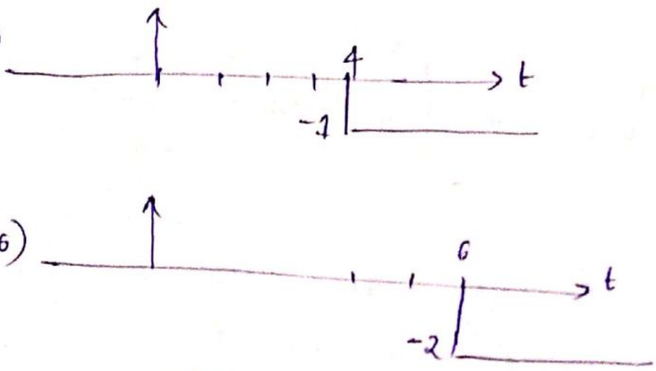
$2u(t)$



1 mark

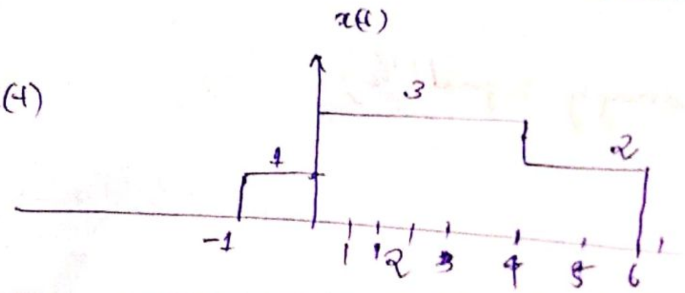
$-u(t-4)$

$-2u(t-6)$



1 mark

$x(t)$



1 Mark

2. $x_1[n] = [1, -2, 3, -1]$ & $x_2[n] = [-1, 2, 1, 3]$ are orthogonal.

$$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k] \quad (½)$$

	1	-2	3	-1
-1	-1	2	-3	1
2	2	-4	6	-2
1	1	-2	3	-1
3	3	-6	9	-3

$[-1, 4, -6, 8, -5, 8, -3]$

not orthogonal. (1)

Simplification & convolution sum calculation (1½)

(2)

3. $x(t) = e^{2t} u(-t)$. energy or power signal?

$$E = \int_{-\infty}^{\infty} (x(t))^2 dt = \int_{-\infty}^{\infty} e^{4t} u(-t)$$

eqn (1)

put $-t = u$

as $t \to 0$ $u \to 0$

$t \to \infty$ $u \to -\infty$

$dt = -du$

$$= \int_{0}^{\infty} e^{4t} u(-t).dt = \int$$

$$= \int_{-\infty}^{-\infty} e^{-4u} du = \int_{0}^{\infty} e^{-4u} du = \frac{-1}{-4}$$

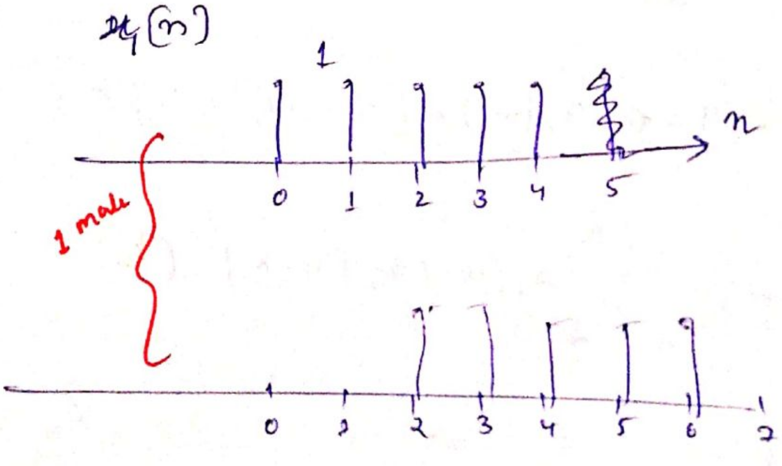
$$= \frac{1}{4}$$ Energy signal with $E = 1/4$ (1½)

& power $= 0$

Stating as energy signal (½)

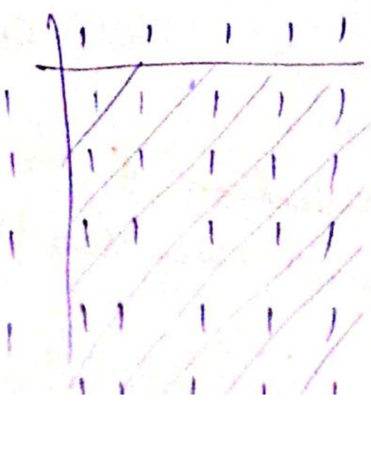
4. Convolution sum of $x_1(n) = u(n) - u[n-5]$

$x_2[n] = u(n-2] - u(n-7]$



$x_1(n) = [1, 1, 1, 1, 1]$

$x_2(n) = [0, 0, 1, 1, 1, 1, 1]$



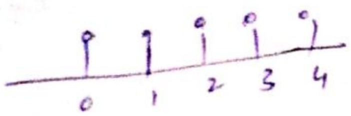
~~1, 2, 3,~~ ~~4,~~ 5, 4, 3, 1.

3

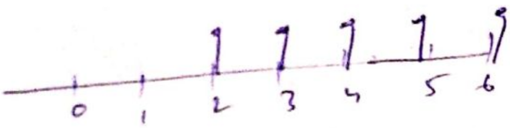
	1	1	1	1	1
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

0, 0, 1, 2, 3, 4, 5, 4, 3, 2, 1

$x_1(n) =$



$x_2(n) =$

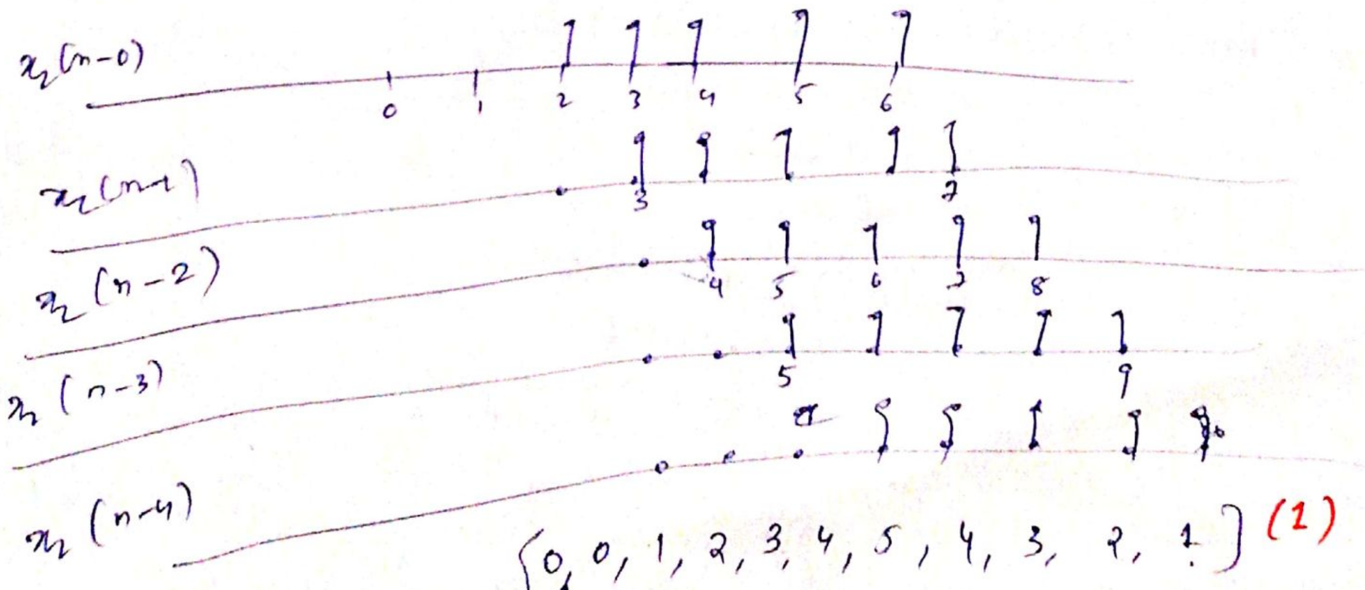


$y(n) = x_1(n) * x_2(n)$ 1/2

$$= \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$$= \sum_{k=0}^{4} x_1(k)\, x_2(n-k)$$

$= x_2(n-0) + x_2(n-1) + x_2(n-2) + x_2(n-3) + x_2(n-4)$ — 1/2



[0, 0, 1, 2, 3, 4, 5, 4, 3, 2, 1] (1)

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5. $x(t) = A \cos 2\pi f_c t$. Fourier Series expansion

$$x[k] = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt.$$

$$x(t) = \sum_{k=-\infty}^{\infty} x[k]\, e^{jk\omega_0 t}$$

we have $x(t) = A \cos 2\pi f_c t$.

$\omega_0 = 2\pi f_c = \frac{2\pi}{T}$

$T = \frac{1}{f_c}$

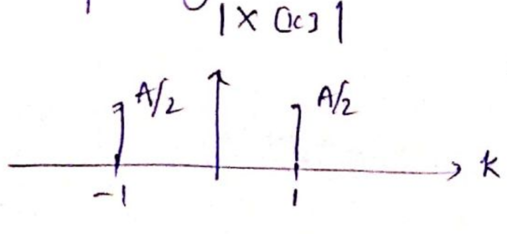
F.S expansion $x(t) = \sum_{k=-\infty}^{\infty} x[k]\, e^{jk 2\pi f_c t}$ — (1)

$$x(t) = A \cdot \tfrac{1}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right]$$

$$= A/2 \cdot e^{j2\pi f_c t} + A/2\, e^{-j2\pi f_c t}$$

comparing with eqtn (1), $x[-1] = A/2$. $x[+1] = A/2$.

have only magnitude spectra.



6. (a) (i) $h(t) = u(t+1) - u(t-1)$

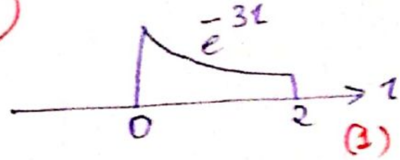
- not memoryless
- not causal.
- stable.

(ii) $e^{2n} u(n-1)$ → not stable.

→ causal.

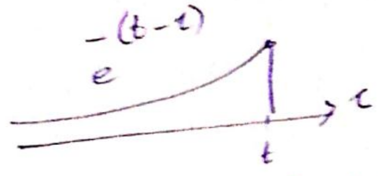
→ not memoryless.

6 (b) $x(\tau) = \begin{cases} e^{-3\tau} & 0 < \tau < 2 \\ 0 & \text{otherwise} \end{cases}$ (5)



(1) $x(\tau)$

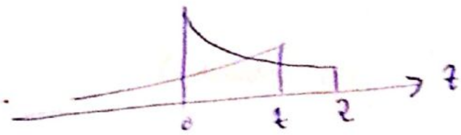
$h(t-\tau) = \begin{cases} e^{-(t-\tau)} & \tau < t \\ 0 & \text{otherwise} \end{cases}$



(1) $h(t-\tau)$

$y(t) = x(t) * h(t)$

$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} w_t(\tau)\, d\tau$ (1 Mark)



when $t<0$, $w_t(\tau) = 0$ 1 Mark.

when $0 < t \leq 2$

$y(t) = \int_0^t e^{-3\tau} \cdot e^{-(t-\tau)}\, d\tau \qquad 0 \leq t \leq 2$

$= e^{-t} \int_0^t e^{-2\tau}\, d\tau$

$= -\frac{e^{-t}}{2} \left[e^{-2\tau} \right]_0^t = -\frac{e^{-t}}{2} \left[e^{-2t} - 1 \right] = \frac{-e^{-3t} + e^{-t}}{2}$ 1 Mark

$= \frac{e^{-t}}{2} [\cdot$

when $t > 2$

$y(t) = \int_0^2 e^{-3\tau} e^{-(t-\tau)}\, d\tau = -\frac{e^{-t}}{2} \left[e^{-2\tau} \right]_0^2 = -\frac{e^{-t}}{2} \left[e^{-4} - 1 \right]$

$= -\frac{\left[e^{-(4+t)} - e^{-t} \right]}{2} \qquad t > 2$ (1 Mark)

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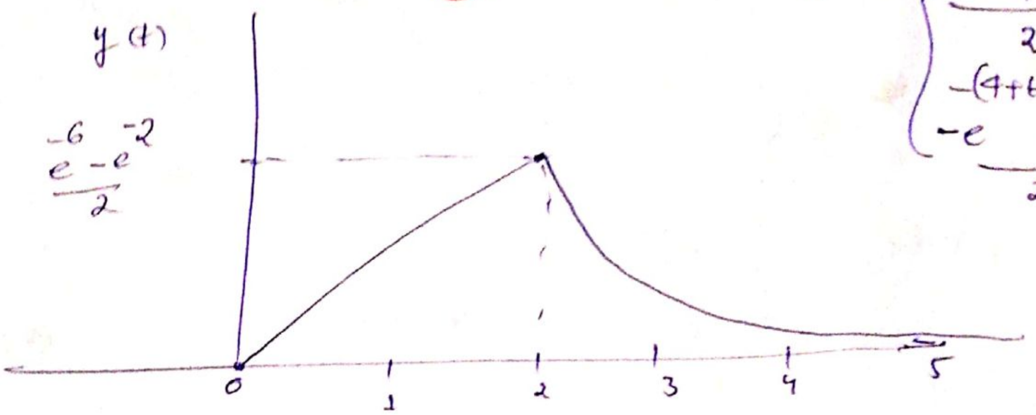
$$y(t) = \begin{cases} \dfrac{-e^{\ldots} + e^{\ldots}}{2} & \\ \dfrac{-e^{-(4+t)} + 1}{2} & t > 2 \end{cases}$$



at t = 2.

$$\frac{e^{-2} - e^{-6}}{2}$$

$w_t(\tau)$ values (3)

$$y(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2} \cdot e^{-t}\left[1 - e^{-2t}\right] & 0 \le t < 2 \\ e^{-t}/2 \left[1 - e^{-4}\right] & t \ge 2 \end{cases}$$

(2 Mark)

7. (a) (i) $y(t) = \cos(x(t))$ — time invariant. 1

— ~~non~~ stable. 1

— non linear 1

(ii) $y(n) = x(n) + \dfrac{1}{x(n-1)}$

— ~~linear~~ non linear 1

— time invariant. 1

— stable. 1

(b) convolution integral. $x_1(t) = 2u(t-1) - 2u(t-3)$

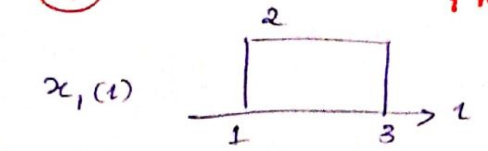
$x_2(t) = u(t+1) - 2u(t-1) + u(t-3)$

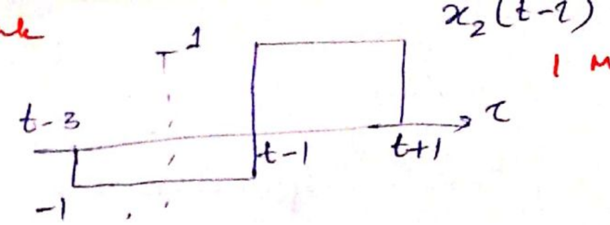
$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau.$$ 1 Mark

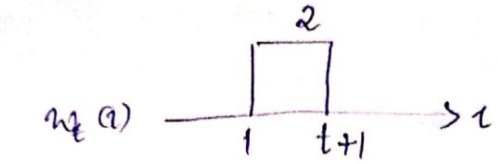
(7)

1 Mark

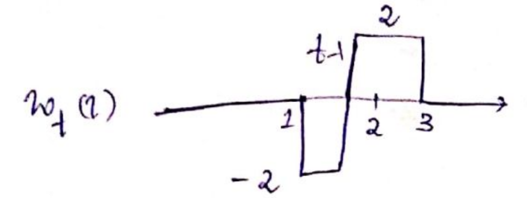




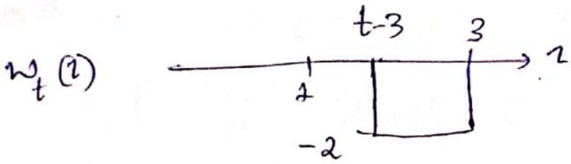
1 Mark



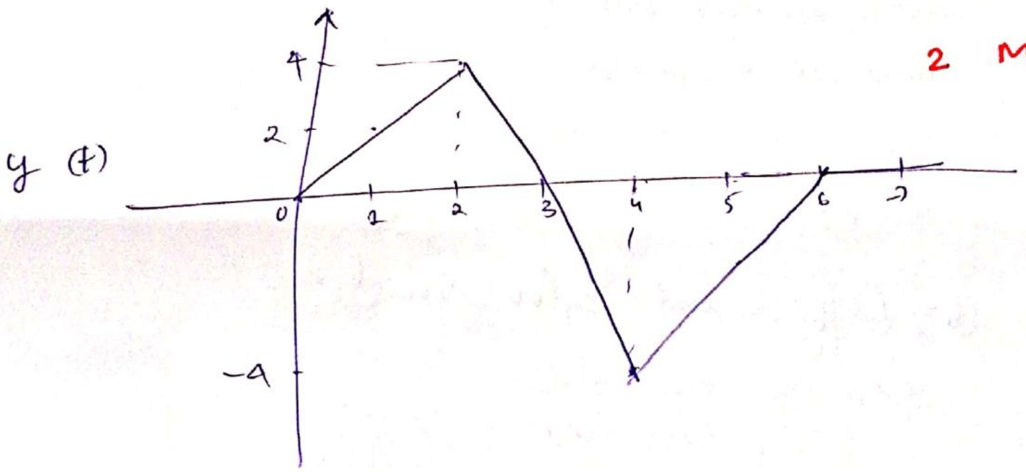
$0 \le t < 2$ 1 Mark



$2 \le t < 4$ 1 Mark



$4 \le t < 6.$ 1 Mark



2 Mark

8. Difference equation solution:

$y[n] - \frac{1}{4}y[n-1] = x[n]$; $x(n) = (1/2)^n$

$y[-1] = 8.$

homogeneous equation $\rightarrow$ 1 Mark. 2 Mark.

natural response. $y(n) = 2(1/2)^n \cdot \cancel{u(n) +} \; C_1(1/4)^n$

particular solution. $y^p(n) = 2\cdot(1/2)^n u(n).$ 1 Mark

$C_2 = 2$ — 1 Mark

Complete solution : $y(n) = (1/4)^n + 2(1/2)^n$ $\quad C_1 = 1$

~~1 Mark.~~

2 ~~2~~ Mark

8 (b) differential eqtn.

$$\frac{d}{dt} y(t) + 10\, y(t) = 2\, x(t) \; ; \; x(t) = u(t) \; ; \; y(0) = 1$$

homogenous eqtn. 1
natural response. 2
particular solution 2
complete solution 2

9 (a). Difference eqtn $y[n] - \frac{1}{2} y[n-1] = 2\, x[n]$;

$$x[n] = 2\left(-\tfrac{1}{2}\right)^n u[n] \; ; \; y[-1] = 3.$$

homogenuous eqtn.
natural response.
particular solution.
complete solution

(b) Differential eqtn solution.
homogeneous eqtn.
natural response.
particular solution.
complete solution.

10. → F.S representation

$$x(t) = 1 + \sin(\omega_0 t) + 2\cos(\omega_0 t) + \cos\left(2\omega_0 t + \pi/4\right)$$

$$\left.\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} x[k]\, e^{jk\omega_0 t} \\ X[k] &= \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt \end{aligned}\right\} \text{1 Mark}$$

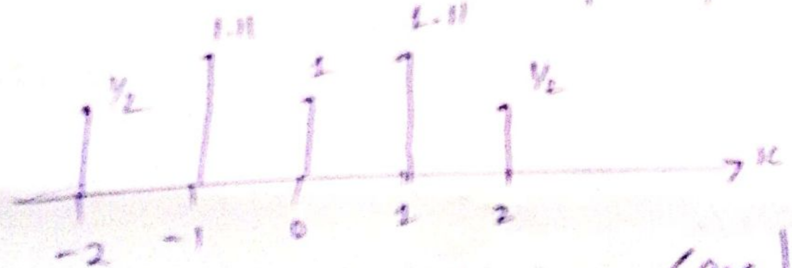
(9)

$$x(t) = 1 + \left(1+\tfrac{1}{2j}\right) e^{j\omega_0 t} + \left(1-\tfrac{1}{2j}\right) e^{-j\omega_0 t} + \frac{e^{j\pi/4}}{2} e^{j2\omega_0 t} + \frac{e^{-j\pi/4}}{2} e^{-j2\omega_0 t}$$

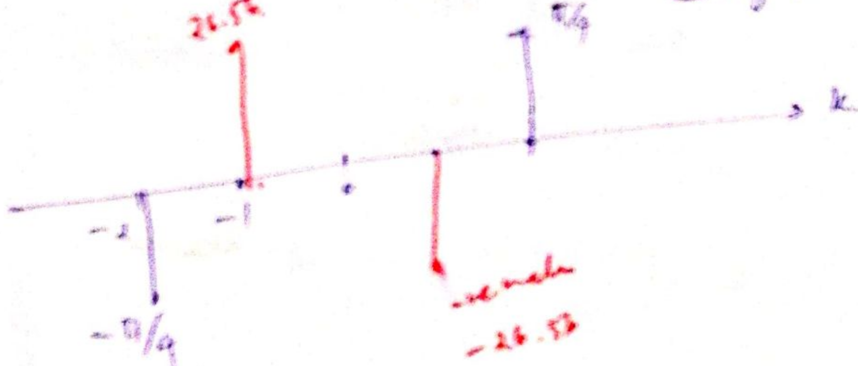
2 Marks

$$\left.\begin{aligned}
x[0] &= 1 \\
x[1] &= 1 + \frac{1}{2j} = 1 - \frac{j}{2} = \sqrt{1^2+(1/2)^2} \angle \tan^{-1}\left(\frac{-1/2}{1}\right) \\
x[-1] &= 1 - \frac{1}{2j} = 1 + j/2 = \sqrt{1^2+(1/2)^2} \angle \tan^{-1}\left(\frac{1/2}{1}\right) \\
x[2] &= \frac{e^{j\pi/4}}{2} \\
x[-2] &= \frac{e^{-j\pi/4}}{2}
\end{aligned}\right\} \text{2 Marks}$$

$|x(k)|$ 1 Marks



$\angle ang\, |X(k)|$. 1 Mark



11. F.S representation of $x(t)$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$$

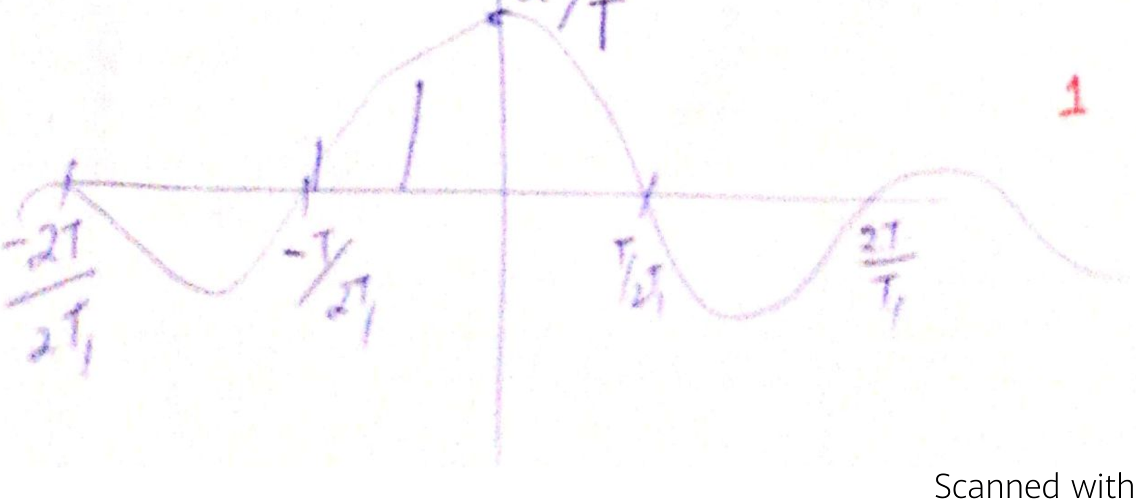
$$\left.\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t} \\ X[k] &= \frac{1}{T}\int_{\langle T \rangle} x(t) e^{-jk\omega_0 t}\, dt \end{aligned}\right\} \quad 1 \text{ Mark}$$

$$= \frac{1}{T} \cdot \frac{-1}{j\omega_0 k}\left[e^{-j\omega_0 t k}\right]_{-T_1}^{T_1} \quad - 1 \text{ Mark}$$

$$= \frac{\sin\left(\frac{2\pi k T_1}{T}\right)}{\pi/k} \quad - 2 \text{ Mark.}$$

$$= \frac{2T_1}{T} \cdot \frac{\sin\left(\pi\, 2kT_1/T\right)}{\left(\pi\, 2k\frac{T_1}{T}\right)}$$

$$= \frac{2T_1}{T} \operatorname{Sinc}\left(\frac{2kT_1}{T}\right) \quad - 2 \text{ Mark}$$



1 Mark