CONTENTS

- Quick recap
- Problems based on Shannon Hartley Law

Example 1

- A gaussian channel has a 10 MHz Bandwidth. If S/N ratio is 100.Calculate the channel capacity and maximum rate of information?

Given $B = 10 \times 10^6$ Hz

$$\frac{S}{N} = 100$$

$$\therefore \quad C = B \log\left(1+\frac{S}{N}\right) = 10 \times 10^6 \log(1 + 100)$$

$$\therefore \quad C = 66.59 \times 10^6 \text{ bits/sec}$$

N = B

=N/B

Maximum Information rate $R_{max} = C_{\infty} = \left(\frac{S}{N}\right) B \log_2 e$

$$\therefore \quad R_{max} = (100)(10 \times 10^6) \log_2 e$$

$$\therefore \quad R_{max} = 1.44 \times 10^9 \text{ bits/sec}$$

Example 2

). A Gaussian channel has 1MHz BW. Calculate the channel capacity if the signal power to the noise power spectral density ratio $(S/\eta) = 10^5$ Hz. Also find max. information rate.

$$C = B\log_2\left(1 + \frac{S}{NB}\right) \qquad N = \eta B$$

$$= B\log_2\left(1 + \frac{S}{\eta B}\right)$$

$$= 1\,\text{MHz} \cdot \log_2\left(1 + \frac{10^5}{10^6}\right)$$

$$= 137.5 \text{ Kbits/sec}$$

Max Information Rate $R_{max} = \frac{S}{\eta} 1.44$

$= 1.44 \times 10^5 = \underline{\underline{144 \text{ kbits/sec}}}.$

Example 3

Example 4.27 : A Gaussian channel has a bandwidth of 4 KHz and a two sided noise power spectral density $\eta/2$ of 10^{-14} watts/Hz. Signal power at the receiver has to be maintained at a level less than or equal to 0.1 milli watt. Calculate the capacity of the channel.

Given $B = 4000$ Hz

$$\frac{\eta}{2} = 10^{-14} \text{ watts/Hz}$$

$\therefore$ Noise power $= N = \eta B = 2 \times 10^{-14} \times 4000$

$= 8 \times 10^{-11}$ watts.

Signal power $S \leq 0.1 \times 10^{-3}$ watts

$$\left(\frac{S}{N}\right)_{max} = \frac{0.1 \times 10^{-3}}{8 \times 10^{-11}} = 1.25 \times 10^{6}$$

$\therefore$ Channel capacity $= B \log\left(1+\frac{S}{N}\right)$

$= 4000 \log(1 + 1.25 \times 10^{6})$

$\therefore$ $C = 81014$ bits/sec

Example4

3) Alphanumeric data are entered into a computer from a remote terminal through a voice grade telephone channel. The channel has a BW of 3.4KHz and output SNR of 20dB. The terminal has a total of 128 symbols. Assume that the symbols are equiprobable and the successive transmission are statistically independent.

a) calculate channel capacity

b) Find the average information content per character

c) Calculate the max. symbol rate for which error-free transmission over the channel is possible.

Given:

$B = 3.4 KHz = 3400 Hz$.

$10 \log_{10} \frac{S}{N} = 20 dB$

$\frac{S}{N} = 100$.

No: of characters $= q = 128$ equiprobable characters.

a) Channel capacity C

$$C = B\log_2\left(1+\frac{S}{N}\right).$$

b) Avg information ~~av~~ content per character

(it is max. since all the characters are equiprobable).

$$H_{max} = \log_2 q = \log_2 128 = 7 \text{ bits/character}.$$

c) avg information rate $R_s = r_s H$

For error free transmission $R_s < C$.

$r_s H < C$

$r_s < \frac{C}{H}$

$r_s < \frac{22638}{7} \simeq 3234$ ~~b~~symbols/sec.

∴ maximum symbol rate for which error-free transmission

over the channel is possible ~~if $r_s <$~~ $= 3234$ symbols/sec

Example 5

1) A voice grade channel of the telephone n/w has a BW 3.4KHz

a) find C for SNR = 30dB

b) find minimum S/N ratio required to support informat transmission thru telephone channel at rate of 4800bits/sec.

Given:

$$B = 3.4\,KHz = \underline{\underline{3400}}\ Hz$$

$$10 \log_{10} \frac{S}{N} = 30\,dB \Rightarrow$$

$$\Rightarrow \log_{10} S/N = 3$$

$$\therefore \underline{\underline{S/N = 10^3}} \quad \underline{\underline{10^3}}$$

a) $C = B \log_2 (1 + S/N)$.

$= 3400 \log_2 (1 + 1000)$.

$= \underline{\underline{33889 \text{ bits/sec}}}$.

b) $C = 4800$ bits/sec $\qquad S/N = ?$

$C = B \log_2 (1 + S/N)$

$4800 = 3400 \log_2 \left(1 + \frac{S}{N}\right)$.

$\log_2 (1 + S/N) = \frac{4800}{3400}$

$$(1 + S/N) = 2^{48/34}$$

$$\frac{S}{N} = 2^{24/17} - 1 = \underline{\underline{1.66}}.$$

or $\quad \frac{S}{N}$ in dB $= 10\log_{10} 1.66 = \underline{\underline{2.2\,dB}}$

$$\underline{\underline{\frac{S}{N} = 2.2\,dB}}.$$

CONCLUSION

- Problems on channel capacity, S/N