MAT 202, MAT 204



Numerical Methods. I

Text: Excom Kreyszig, Advanced Engineering Mathematics 110th Edi: John Wily

[Sections 20.3,20.5,21.1]

Solution of linear systems - Grauss Siedal and Jacobi iteration

methods. Curve fitting-method of least squares, fitting straight lines.
and parabolas. Solution of ordinary differential equations- Eules and

classical Runge-kutta method of second and fourth order, Adams-Moulton predictor - correction method (Boot or desivation of the. formulae not required for any of the methods in this module)

Jacobi Iteration method

Corronge the given system of equation in diagonaly dominant

3714 674 4 273 =0

(3 x1) - x2 + x3 = 1 371 - 672 1273 = 0

3+3=b<7

Solve the following System of equations by Gauss-Jacobi method.

$$x+y+5ux=110$$
, $6x+15y+2x=72$, $27x+6y-7=85$

Let initial Values be $x_{0}=0$, $Z_{0}=0$, $Z_{0}=0$
 $x+15y+22=72$.

 $x+4y+5ux=110$
 $x_{1}=\frac{1}{21}[85-640+70]=\frac{1}{21}[85-0+0]=3.1681=3.168$
 $x+4y+5ux=110$
 $x_{1}=\frac{1}{21}[85-640+70]=\frac{1}{21}[85-0+0]=3.1681=3.168$
 $x_{2}=\frac{1}{21}[85-640+70]=\frac{1}{21}[10-0-0]=3.0370=337$
 $x_{3}=\frac{1}{21}[10-x-4]=\frac{1}{21}[85-640+70]=\frac{1}{21}[10-0-0]=3.0370=337$
 $x_{3}=\frac{1}{21}[85-640+7]=\frac{1}{21}[85-640+70]=3.1517$

 $x_1 = \frac{1}{27} [85 - 641 + 71] = \frac{1}{27} [85 - 6(44) + (2.087)] = 2.157$

40 = 15 [72 - 67(1-22)] = 15 [72-6(3-146)-2(2-037)]= 3.269 73 = 54 [110-21-41] = 1 [110-3.148-4.8] = 1.8698

$$x_3 = \frac{1}{27} \left[85 - 642 + 73 \right] = \frac{1}{27} \left[85 - 6(3.267) + 3(1.896) \right] = 3.6852 = 3.685$$

$$43 = \frac{1}{27} \left[17 - 6x_2 - 322 \right] = \frac{1}{15} \left[17 - 6(3.157) - 3(1.896) \right] = 3.6852 = 3.685$$

$$\frac{43}{15} = \frac{1}{15} \left[\frac{12 - 6x_2 - 2x_3}{15} \right] = \frac{1}{15} \left[\frac{12 - 6(3.151) - 4 - 6x_3}{15} \right] = \frac{1}{15} \left[\frac{110 - 2x_2 - 4x_3}{15} \right] = \frac{1}{15} \left[\frac{110 - 2x_3 - 4x_3}{15} \right] = \frac{1}{15} \left[\frac{1}{15} \right] = \frac{1}{15} \left[$$

$$\frac{h^{th} \text{ Iteration}}{24 = \frac{1}{27} \left[85 - 643 + 73 \right] = \frac{1}{27} \left[85 - 6(3.686) + 1.937 \right] = 2.461}{1 - 27 - 6(3.686) - 2(1.937)} = \frac{1}{27} \left[12 - 6(3.686) + 1.937 \right] = \frac{1}{27} \left[12 - 6(3.686) + 1.937$$

$$x_{4} = \frac{1}{27} \left[85 - 643 + 73 \right] = \frac{1}{27} \left[85 - 6(3.685) - 2(1.937) \right] = 3.545$$

$$y_{3} = \frac{1}{15} \left[72 - 6x_{3} - 2x_{3} \right] = \frac{1}{15} \left[72 - 6(2.492) - 2(1.937) \right] = 3.545$$

$$7_{4} = \frac{1}{5} \left[110 - 2x_{3} - 43 \right] = \frac{1}{5} \left[110 - 2.492 - 3.685 \right] = 1.923$$

$$\frac{3}{25} = \frac{1}{27} \left[85 - 640 + 20 \right] = \frac{1}{27} \left[85 - 6(3505) + \frac{1.923}{2} \right] = 2.032$$



$$y_{5} = \frac{1}{15} \left[12 - 6 \times 4 - 2 \times 4 \right] = \frac{1}{15} \left[12 - 6 \left(2 \cdot 4 + 6 \right) - 2 \left(1 \cdot 4 \cdot 2 \right) \right] = 3.583$$

$$75 = \frac{1}{5u} \left[110 - 2u - 4u \right] = \frac{1}{5u} \left[10 - (2.401) - (3.545) \right] = 1.927$$

$$\frac{6^{hh} \text{ Iteration}}{26 = \frac{1}{27} \left[85 - 695 + 25\right] = \frac{1}{27} \left[85 - 6(3.583) + 1.927\right] = 2.423}$$

$$\frac{1}{26} \left[72 - 625 - 225\right] = \frac{1}{15} \left[72 - 6(2.432) - 2(1.927)\right] = 3.570$$

$$\frac{1}{26} \left[72 - 625 - 225\right] = \frac{1}{15} \left[72 - 6(2.432) - 2(1.927)\right] = 3.563$$

$$y_{6} = \frac{1}{15} \left[72 - 6x_{5} - 32_{5} \right] = \frac{1}{15} \left[72 - 6(342 - 3.563) = 1.926.$$

$$Z_{6} = \frac{1}{54} \left[110 - 2x_{5} - 3x_{5} \right] = \frac{1}{54} \left[110 - 3.432 - 3.563 \right] = 1.926.$$

$$\frac{\text{The Iteration}}{x_7 = \frac{1}{27} \left[85 - 696 - 76 \right] = \frac{1}{27} \left[85 - 6(3.570) + 1.926 \right] = 2.426}$$

$$\chi_{7} = \frac{1}{27} \left[88 - 696 - 626 \right] = \frac{1}{27} \left[12 - 6(2423) - 2(1926) \right] = 3.574$$

$$\chi_{7} = \frac{1}{15} \left[17 - 6x_{6} - 2x_{6} \right] = \frac{1}{15} \left[12 - 6(2423) - 2(1926) \right] = 3.574$$

$$\chi_{7} = \frac{1}{15} \left[100 - 2x_{6} - 46 \right] = \frac{1}{15} \left[100 - 2.423 - 3.576 \right] = 1.926$$

$$\chi_{7} = \frac{1}{15} \left[100 - 2x_{6} - 46 \right] = \frac{1}{15} \left[100 - 2.423 - 3.576 \right] = 1.926$$

$$\frac{8^{4h} \cdot 1 + e^{2} + e^{2} + e^{2}}{78 = \frac{1}{27} \left[85 - 6 \cdot (3.574) - 1.976 \right] = 2.475}$$

$$\frac{78 = \frac{1}{27} \left[85 - 6 \cdot (3.574) - 1.976 \right] = 2.475}{15}$$

$$\frac{78 = \frac{1}{15} \left[72 - 6 \cdot (3.574) - 2 \cdot (1.976) \right] = 3.573}{15}$$

$$\frac{78 = \frac{1}{54} \left[110 - 27 - 27 \right] = \frac{1}{54} \left[110 - 2.476 - 3.574 \right] = 1.976}{110}$$

10th Iteration 710= 1 [85-649129]= 1 [85-6(3.573)+1.926]= 2.426

Grauss - Siedel Method

First arrange the system of equation as diagonally dominant mission

Initial Values are. 40=0, Zo=0

Diagonally dominant from: X1= 1 [85-646+ 20] = 3.148 2724 69-2=85 -(1)

$$y = \frac{1}{15} [72 - 62 - 22]$$
 $z = \frac{1}{54} [110 - 24 - 4]$

which value (40=01 70=0)

$$Z_1 = \frac{1}{54} \left[110 - 21 - 21 \right] = \frac{1}{54} \left[110 - 3 \cdot 144 - 3 \cdot 541 \right] = 1.913$$

Zz= = = [110-xz-4z] = = = [110-2.432-3.572]= 1.926

$$y_1 = \frac{1}{15} [72 - 6x_1 - 220] = \frac{1}{15} [72 - 6(3.144) - 0] = 3.54$$

 $z_1 = \frac{1}{15} [72 - 6x_1 - 220] = \frac{1}{15} [72 - 6(3.144) - 0] = 3.54$

$$x_3 = \frac{1}{27} \left[85 - 64 + 72 \right] = \frac{1}{27} \left[85 - 6(3.572) + (1.926) \right] = 3.6$$

$$y_{3} = \frac{1}{27} \left[85 - 6 \frac{1}{3} + \frac{1}{23} \right] = \frac{1}{27} \left[85 - 6 \frac{1}{3} + \frac{1}{23} \right] = \frac{1}{15} \left[72 - 6 \left(2 + \frac{1}{3} \frac{1}{3} \right) - 2 \left(1 + \frac{1}{3} \frac{1}{3} \right) \right] = 3.573$$

$$y_{3} = \frac{1}{15} \left[72 - 6 \frac{1}{3} - \frac{1}{3} \frac{1}{3} \right] = \frac{1}{15} \left[72 - 6 \left(2 + \frac{1}{3} \frac{1}{3} \right) - 2 \left(1 + \frac{1}{3} \frac{1}{3} \right) \right] = 3.573$$

$$73 = \frac{1}{15} \left[110 - 23 - 43 \right] = \frac{1}{54} \left[110 - 2426 - 3.573 \right] = 1926$$
 $73 = \frac{1}{54} \left[110 - 23 - 43 \right] = \frac{1}{54} \left[110 - 2426 - 3.573 \right] = 1926$

$$\frac{L^{4h} \ \text{Iteration}}{Z_{4z} = \frac{1}{27} \left[85 - 6(3.573) + (1.926) \right] = 2.426}$$

$$Z_{4z} = \frac{1}{27} \left[85 - 6(3.573) + (1.926) \right] = 3.573$$

$$Y_{4z} = \frac{1}{15} \left[12 - 6(2.426) - 2(1.926) \right] = 3.573$$

$$Y_{4z} = \frac{1}{15} \left[12 - 6(2.426) - 2(1.926) \right] = 1.926$$

$$y_{u} = \frac{1}{15} \left[\frac{12 - 624 - 223}{54} \right] = \frac{1}{15} \left[\frac{110 - 2.426 - 3.573}{54} \right] = 1.926$$

$$Z_{h} = \frac{1}{54} \left[\frac{110 - 24 - 426}{54} \right] = \frac{1}{54} \left[\frac{110 - 2.426 - 3.573}{54} \right] = 1.926$$

$$\therefore Solution 2 = 2.426, \quad y = 3.573, \quad z = 1.926$$

$$\therefore Solution 2 = 2.426, \quad y = 3.573, \quad z = 1.926$$



$$\frac{3^{**d}}{7} \frac{1}{(85-64)} + \frac{7}{2} = \frac{1}{27} \frac{1}{(85-6(9.572)+(1.926)]} = 3.426$$

$$\frac{7}{27} \frac{1}{(85-64)} + \frac{7}{2} = \frac{1}{27} \frac{1}{(85-6(2.426))} - \frac{1}{2} (1.926) = 3.573$$

$$\frac{1}{3} = \frac{1}{15} \frac{1}{(10-2)} - \frac{1}{27} \frac{1}{(10-2)} + \frac{1}{27} \frac{1}{(10-2)} = 3.573$$

$$\frac{1}{3} = \frac{1}{15} \frac{1}{(10-2)} - \frac{1}{27} \frac{1}{(10-2)} + \frac{1}{27} \frac{1}{(10-2)} = 3.573$$

$$\frac{1}{27} \frac{1}{(85-64)} + \frac{7}{27} = \frac{1}{27} \frac{1}{(85-6(2.426))} + \frac{1}{2} \frac{1}{(1.926)} = 3.573$$

$$\frac{1}{27} \frac{1}{(1.926)} = \frac{1}{27} \frac{1}{(1.926)} = 3.573$$

74= 2.432 , 42 = 3.572, Z.=1.926

 $y_{u-} = \frac{1}{15} \left[12 - 6 \times_4 - 2 \times_3 \right] = \frac{1}{15} \left[12 - 6 \left(2 \cdot 4 \times 6 \right) - 2 \left(1 \cdot 9 \times 6 \right) \right] = 3.573$ $Z_{4} = \frac{1}{5u} \left[110 - 2u - y_{4} \right] = \frac{1}{5u} \left[110 - 2 426 - 3 573 \right] = 1926.$:: Salu in 7: 2.476, 4:3.573, 7:1.976 Correct to 3.D.

$$x_3 = \frac{1}{a_0} \left[17 - \frac{1}{4} + \frac{1}{4} z_1 \right] = \frac{1}{a_0} \left[17 - \frac{1}{4} - \frac{1}{4} - \frac{1}{4} \left[17 - \frac{1}{4} -$$

$$\chi_3 = \frac{1}{20} \left[17 - \frac{1}{12} + \frac{1}{2} Z_z \right] = \frac{1}{20} \left[17 - \left(-0.9998 \right) + 2 \left(0.9998 \right) \right] = (0.0000)$$

$$7_{3} = \frac{1}{20} \left[(1 - \frac{1}{3}) + \frac{1}{2} \left[-\frac{1}{2} \left[-\frac{1}$$

$$33z \frac{1}{20} \left[-18 - 3x3 + 72 \right] = \frac{1}{20} \left[-18 - 3(10000) + (0.9998) \right] = 10000$$

$$73 = \frac{1}{20} \left[25 - 2x5 + 3y3 \right] = \frac{1}{20} \left[25 - 2(10000) + 3(-10000) \right] = 10000$$

 $24 = \frac{1}{20} \left[17 - \frac{1}{3} + 273 \right] = \frac{1}{20} \left[17 - (-1.0000) + 2(1.0000) \right] = 1.0000$

$$\chi_{4} = \frac{1}{20} \left[(17 - 93 + 273) \right] = \frac{1}{20} \left[(18 - 374 + 73) \right] = \frac{1}{20} \left[(18 - 3(10000) + 10000) \right] = (10000)$$

$$\chi_{4} = \frac{1}{20} \left[(18 - 374 + 73) \right] = \frac{1}{20} \left[(25 - 2(10000) + 3(-10000)) \right] = (10000)$$

$$\chi_{4} = \frac{1}{20} \left[(25 - 274 + 394) \right] = \frac{1}{20} \left[(25 - 2(10000) + 3(-10000)) \right] = (10000)$$

$$\chi_{4} = \frac{1}{20} \left[(25 - 274 + 394) \right] = \frac{1}{20} \left[(25 - 2(10000) + 3(-10000)) \right] = (10000)$$

$$\chi_{4} = \frac{1}{20} \left[(25 - 274 + 394) \right] = \frac{1}{20} \left[(25 - 2(10000) + 3(-10000)) \right] = (10000)$$

$$\chi_{4} = \frac{1}{20} \left[(25 - 274 + 394) \right] = \frac{1}{20} \left[(25 - 2(10000) + 3(-10000)) \right] = (10000)$$

$$\chi_{4} = \frac{1}{20} \left[(25 - 274 + 394) \right] = \frac{1}{20} \left[(25 - 2(10000) + 3(-10000)) \right] = (10000)$$

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$$\chi_{4} = \frac{1}{20} \left[(25 - 274 + 394) \right] = \frac{1}{20} \left[(25 - 2(10000) + 3(-10000)) \right] = (10000)$$

Principle of Least Squares:

The principle of least squares states that the sum in squares of difference between the actual value and the approximated value should be minimum to E = \(\frac{7}{2} \) (yi - f(xi)) is minimum.

Normal equ Ey = a Ex+nb

and of basapala. A= 0+Px+cxs Normal equation. mad equation Zyenasb Zx+c Zx2 Zzy= a Zx + b Zz2, (Zx8 Zxy= a Z x 3 b Zx3 1 c Zx4 Note: A= 025+P2+(. Parabolo 9= 0x2+p

Zy. na+bZzi

4: 0+px3

5x4=05x+P5x4

Fit a Straight line by method of least squares of the following data.

2 5 10 15 20 25

9 15 19 23 26 36

equi.os	2:pre	y=016x
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Nes	roal equation.
(Zy=ma+bZx
	Exy = 9 Ex + 6 Ex

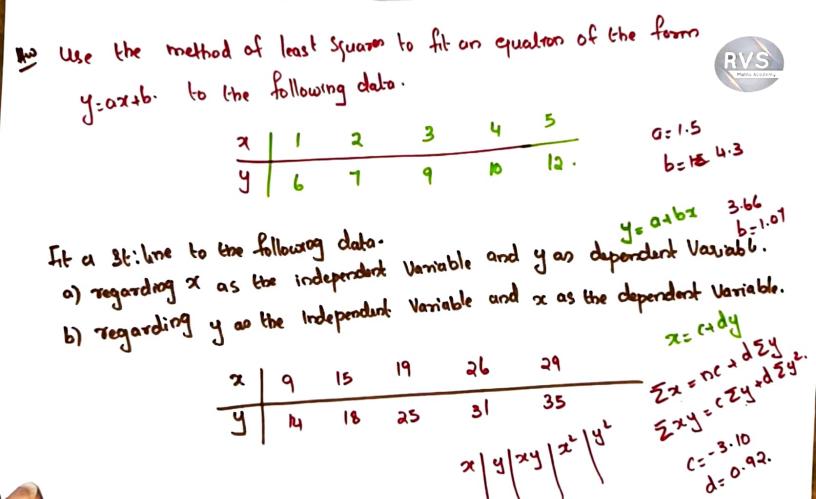
٦	7	24	×r
5	15	75	25
	19	196	100
10	23	345	225
15	13	520	400
20	3 F		625
25	30	750	645
Zx= 75	5 Σy= 113	22y = 168	5 توري 1375



16:0:14

-185 = 0 -250b.

b= -185 = 0.74



The following table gives the tensile force x (in thousands of pounds) applied be steel specimen and the resulting elongation (in thousands of prish)

11	2 1 2 3 4 5 20 10 63 76 85	adjak Hha
	9 14 33 40 Hence Pr	Palet Cia
find th	y 14 33 40 63 76 85 equation of the least square line that fits the above data . Hence pre- equation of the least square line that fits the above data. Hence pre- equation of the least square line that fits the above data. Hence	

find the equation of the	15 3.5	thousand	pounds.	
that the equation of the tensile for elongation when the tensile for	2	4	24	7
eniof stips dialpy.	۱ ٦	38	76	9
normal quatron is	3 4	40 63	565	16
Ey=na+b Ex	5	76 85	380 510	36
Zzy = a Zz 4 b Zzł	22 = 21	zy: 311	Exy: 135	2 23.91
	-	_		

$$311 = 6a + 21 b - (1)$$

$$1342 = 214 + 91b - (2)$$

$$0 \times 21 \Rightarrow 6531 = 126a + 441b$$

$$20 \times 6 \Rightarrow 8052 = 126a + 546b$$

$$-1521 = 0 - 105b$$

$$b = -1521 = 14.4857$$

-105

b=14.4857

(1) => 311 = 6 a + 21 x 14.4857 Q= 311 - 21x 14.4857 9 = 1.1333 equiof stitu

14=1.1333 + 14.4557 x when x=3.5 9=1.1333+14.4857 (3.5) = 51.8333

Use method of least squares to fit a curve of the torm.

The following data by converting the equation
$$y=ax+bx^2$$
 to linear form.

 $x \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 5$
 $y \mid 2.1 \mid 4.4 \mid 1.2 \mid 10.4 \mid 15.7 \mid 18.3$
 $y=ax+bx^2$

1 2.1 2.1 2.1 1

2 4.4 2.2 4.4 4

2 4.4 2.2 4.4 4

Y=a+bx

Y=a+bx

Y=a+bx

Towned equ.

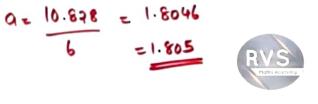
 $y=ax+bx^2$

5 15.7 3.14 15.7 25

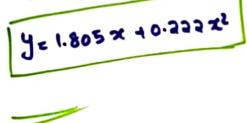
 $y=ax+bx$
 $y=ax+bx$

$$-23.31 = 0 - 105b$$

$$6 = -\frac{23.31}{-105} = 0.222$$



.. y : ax +bx

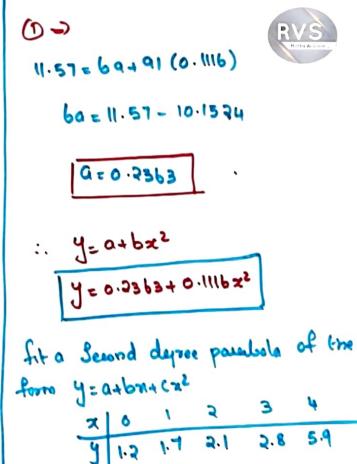


Fit a curve of the form y=a+bx to the following data.

2		2	3	ч	5	6
4	0.56	0.89	1.04	1.63	2.95	4.5

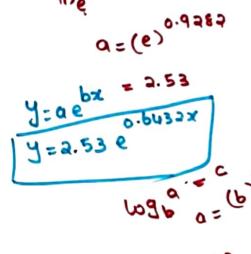


	×	y. 1	مد	x²y	24
4=01pz3	-	0.56	1,	0.56	1
poswal form;	ą	0.89	4	3.56	81
Zy=na+bZz2	3	1.04	9 16	9.36	256
Zzq=aZz2+bZz4	4	1.63	25	36.08	625
•	5	2.95 4.5	36	73.75	१२१८
	-	Zy : \$ 9.51	Zz2:91	Z×4=27	16.81 2275



Fit a curve of the form y=aebx to the following data. 71.89 29.23 10.11 6.03 ZY y = a ebx by = lo (aeba) log(ab) = loga + logb 1505.0 2.03 1.7967 logen x 17967 6.03 elna + Ineba 4.6270 2.3135 10-11 Put log = Y 18-6526 5 8845 =lna+bx 13.5005 100 EA 3-3759 29.23 Y= A+b= 30 11.0727 28.5771 ZY= nA+bZx 11. 0727 = 5A 4 10b 28.5771=10A+30b _ () 5 -4- A Zz +6222

A= 100 = 0 9282



Numerical Solution of ordinary differential equation (ODE) dy 44= ex 1) Eulers Method y=cfap1

yn+1 = yn+ bf (xn140)

dy = e7-4 f(n/y) = e7-4

y, = yo + h f(xo, yo) (26,40) y== 41+bf(21141)

h=21-20

Solut numerically the initial value problem
$$y'-y=e^2\cos x$$
 1900=0 using

Enter method on the interval $0 \le x \le 1$ taking Step Size $b=0.3$.

You = $y_0 + b f(x_0, y_0)$
 $y'-y=e^2\cos x$
 $y'=e^2\cos x + y$
 $y'=e^2\cos$

1 = 0.8 Ju: 1.3209

Use Euler's method to Solve dy = x+xy+y, y(0)=1, compare y at 200. by lating troops. y, = yo + h f (20, yo). = 14015 f(0,1) 1904 = 904 pf(xn, 90) = 140.15 [04041] fixiy)= x+xy+4 given yeor=1 use Euler's method to evaluate y(0.4) with head from the quatron dy = 42 27 y0=1 20 = 0 x1=0.15 41= ? 20=0 40=1 fmy)= "

given you)=1 9" = A3 4 pol(431 A3) 24=8-1 41=3 h= 21-20 = 0.15-0 X3:03 43:1 74 = 0.4 Yuz

Second order Runge-kutta Method

$$y_{n+1} = y_{n+1} + \frac{1}{2} (k_1 + k_2)$$
 $k_1 = b + (x_{n+1}y_n)$
 $k_2 = b + (x_{n+h}, x_{n+k_1})$

taking Step Size b=0.2 A1= 201 (K1+ K2) Jun = 20+ 7 (K1+ K) Ya = Y1+ 1 (k1+ k2) K1= bf(x0140) k1: bf(20140) ki= bf(21141) =02 (100,05) =0.2 1 (0.2,0.826] Kz p f(xo+b, york) =02[05-04] =0.5 [0.838-(0.5)31] = 0 3572 f(x14) = 4-x3+1 Kz= bf(20+6, y0+k1) kz=bf(x14b1414 k1) =021(02,08) =02f(04,11832) p:0.5 = 0.2 [1.1832 - (04)21] =0.5 [0.8- (0.5),41] given 4(0)=0.8 4=0.5 = 0.4046 42 = 0.826+ } [0.3572+0.408 = 0 352 X5 0 41=05+= (03+0352) 21=5 X120-2 - 1.2069 - 0 826 42 -7 24: 6.4

Using Modified Euler's method (Rk roethod of order 2) Solve dy -y-x2+1,

Use Runge-kutta Second order find the Value of y at x = 0.25 and x=0.5 given dy = 2xy, y(0)=1 21 =0.98 AI = 11083 a1=20+7 (K1+K2) A== A++ (K++ P) Yna = Yn+ + (k1+ 16) kiz bf (20140) kis bf(xiigi) kie bf(znign) 20.25 \$ (011) =0.25 f(0.25, 1.0626) kae hf (xnah, ynak) =0.25[0] =0.25 [240.25,1.0625] f(x14) = 2x4 ks = hf(x0+h, y0+ k1) given yeo) = 1 K2= bf (xqab, 41ak) yo=1 =0.25 4 (0.25,1) =0.25 f(0.5,1.1953) y,= ". 2,00.25 = 0.25 [2x0-5x1.1958] =0.25 [2x0.25x1] x, = 0.5 4,=? =0.125 . o . 2988 42-1.0625+7 [0.133840.2988] A1= 1+ = (0+0.132) h= 21-20 = 0.25-0 = 1.0625 4251.2783

2150.5

1) Solve by improved Euler's method dy = y+e7, y(0)=0 for 7:0-210-4 fromy) 4=0 x=0 4 4= 3 41 = 0 20214 h= 21.20 0 6.2474 6.54116 (3) Solve by Rk method of order 2. y'=x2-y.14(0)=1 find correct

(3) Solve by KK muthod at some in y = x - y - 1 y (0) = 1 tind correctly

20 = 0 y = 1

20 = 0 y = 1

21 = 0.9055

21 = 0.9055

Fourth order Runge-kutta method [Classical Runge-kutta Method]

$$\begin{array}{lll}
Y_{n+1} = Y_n + \frac{1}{6} \left[k_1 + a k_2 + 2 k_3 + k_4 \right] \\
k_1 = b f(x_{n_1} y_n) \\
k_2 = b f(x_n + \frac{b}{2} + y_n + \frac{k_1}{2}) \\
k_3 = b f(x_n + \frac{b}{2} + y_n + \frac{k_2}{2}) \\
k_4 = b f(x_n + b + y_n + k_3)
\end{array}$$

$$\begin{array}{lll}
K_1 = y_0 + \frac{1}{6} \left[k_1 + a k_2 + 2 k_3 + k_4 \right] \\
k_1 = b f(x_0 + y_0) \\
k_2 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_1}{2}) \\
k_3 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_2}{2}) \\
k_4 = b f(x_0 + b + y_0 + k_3)
\end{array}$$

$$\begin{array}{lll}
K_1 = y_0 + \frac{1}{6} \left[k_1 + a k_2 + 2 k_3 + k_4 \right] \\
k_5 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_1}{2}) \\
k_6 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_2}{2})
\end{array}$$

$$\begin{array}{lll}
k_1 = y_0 + \frac{1}{6} \left[k_1 + a k_2 + 2 k_3 + k_4 \right] \\
k_2 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_1}{2}) \\
k_3 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_2}{2})
\end{array}$$

$$\begin{array}{lll}
k_1 = y_0 + \frac{1}{6} \left[k_1 + a k_2 + 2 k_3 + k_4 \right] \\
k_2 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_1}{2}) \\
k_3 = b f(x_0 + \frac{b}{2} + y_0 + \frac{k_2}{2})
\end{array}$$

$$\begin{array}{lll}
k_4 = b f(x_0 + b + y_0 + k_2) \\
k_4 = b f(x_0 + b + y_0 + k_2)
\end{array}$$

Criven the initial value problem
$$\frac{dy}{dx} = [x+y], y(0) = 1, \text{ use Runge Rights}$$

method of bourth order to find $y(0.2)$ with $b=0.1$
 $y_{0.11} = y_{0.11} + \frac{1}{6} [k_{11} + 2k_{21} + 2k_{31} + k_{11}]$
 $y_{0.12} = y_{0.11} + \frac{1}{6} [k_{11} + 2k_{21} + 2k_{31} + k_{11}]$
 $y_{0.13} = y_{0.11} + \frac{1}{6} [k_{11} + 2k_{21} + 2k_{31} + k_{11}]$
 $y_{0.14} = y_{0.11} + \frac{1}{6} [k_{11} + 2k_{21} + 2k_{31} + k_{11}]$
 $y_{0.15} = y_{0.15} + y_{$

2 =0.1

41=1.104921

Ku= hf (26+ h, yo+ ks)

$$k_{3} = hf(x_{1} + \frac{h}{h}, y_{1} + \frac{k_{2}}{h})$$

$$= o \cdot 1 f(o \cdot 15 + 1 \cdot 164931 + o \cdot 11447)$$

$$= o \cdot 1 f(o \cdot 15 + 1 \cdot 163156)$$

$$= o \cdot 1 f(o \cdot 15 + 1 \cdot 163156)$$

$$= o \cdot 1 f(x_{1} + h) \cdot y_{1} + k_{3})$$

$$= o \cdot 1 f(x_{1} + h) \cdot y_{1} + k_{3})$$

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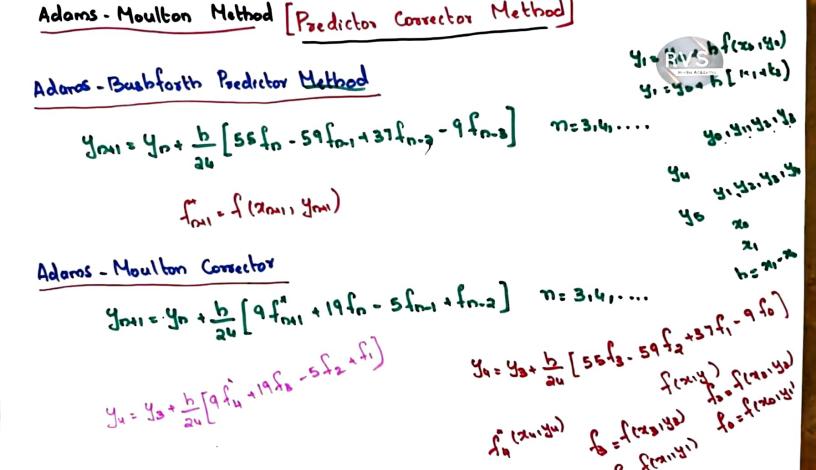
$$= o \cdot 1 f(x_{3} + h) \cdot y_{3} + k_{3}$$

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$$=$$



Using Adams method find
$$y(0.4)$$
 given $y' = \frac{xy}{a}$, $y(0) = 1$, $y(0.1) = 1.01$,

 $y(0.2) = 1.022$, $y(0.3) = 1.023$.

Adams Bookforth predictor formula.

 $y' = \frac{xy}{a}$
 $y' = \frac{xy}{a}$

$$x_{3} = 0.3$$

$$x_{3} = 0.3$$

$$x_{3} = 0.3$$

$$x_{4} = 0.4$$

$$y_{4} = 7$$

$$y_{5} = f(x_{5}, y_{5}) = f(0.3, 1.023) = 0.3 \times 1.023 = 0.1023$$

$$y_{5} = f(x_{5}, y_{5}) = f(0.3, 1.023) = 0.3 \times 1.023 = 0.1534$$

$$y_{6} = f(x_{5}, y_{5}) = f(0.3, 1.023) = 0.3 \times 1.023 = 0.1534$$

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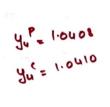
44=1.0408

b . 0 1

Adams moulton Corrector Servedor

You =
$$\frac{1}{3}$$
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=1.0410



Solve the introl value problem dy = x-y , y(0)=1 to find y (0.4) by Adams method. Starting Solution required are to be obtained using Runge kutter reathod of order 4 using Step Size hoo.1 To find yiryziya By R.k method. fixid) = x-d3 y = you + [k + a k + a k + ku] Kiepfarido) = 0.1 {(011) = 0.1 [0-(0)] = 0.1 P=0.1 40=1 Kz= bfexny) = of flow bf(xo+ b, yo+ k) 74 = 0 X1=0.1 = 0.1 {(0.05 - 0.90 = 23] N3 = 0.3 A3 : 5) = -0.08525 24: 5 Jogans. 1 13:0.3 k3= hf(26+ = 1 y0+ k2)= 0.1 f(0.05,0.9165) 24 = 0.h Ku: H(x0+h, y0+k3) = 0.1 fp.1, 0.8341) = -0.07341

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-0.5089 y" = y3+ b [9fu + 19f3 - 5f2 + f1]

_{= 0} 7797