INFORMATION THEORY & CODING

LECTURE 1

Course Outcomes

- State Shannon's Hartley Theorem and obtain the relation between capacity, Bandwidth and SNR.
- Determine the bandwidth efficiency of channels using Shannon's Limit.

Communication system

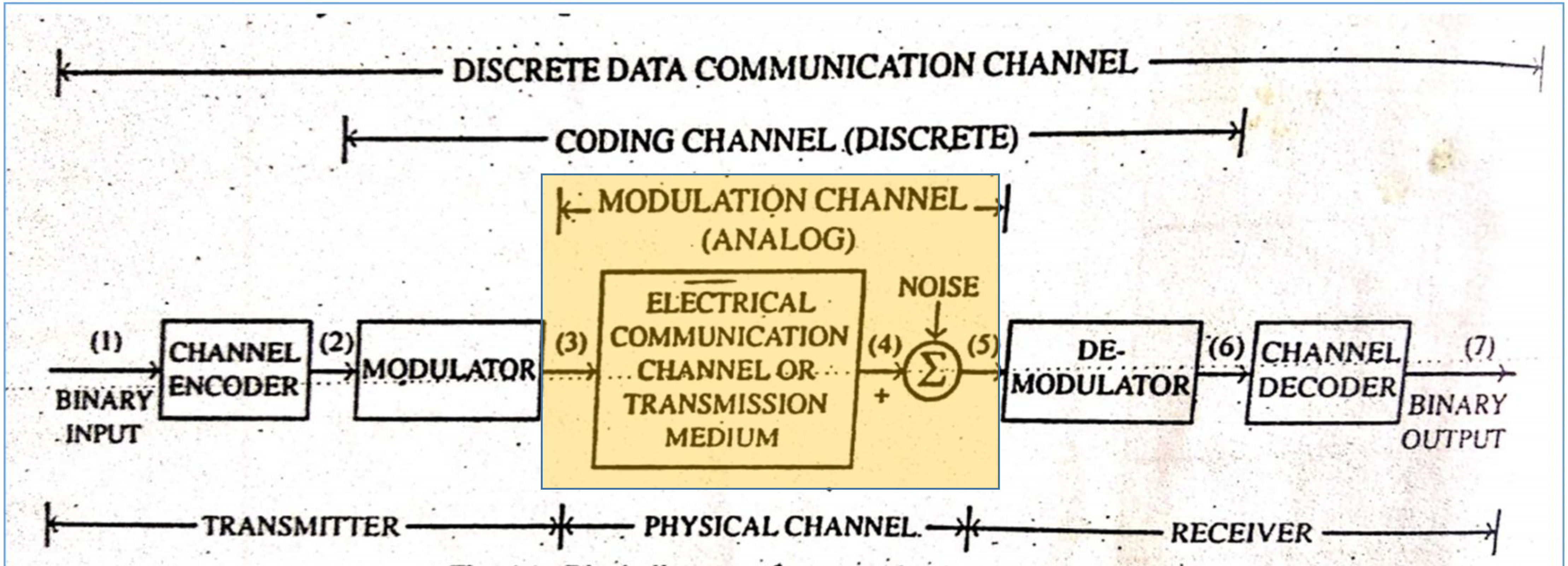


Fig. 4.1 : Block diagram of communication system

The communication channel b/w points (3) & (5) is analog and continous in nature.

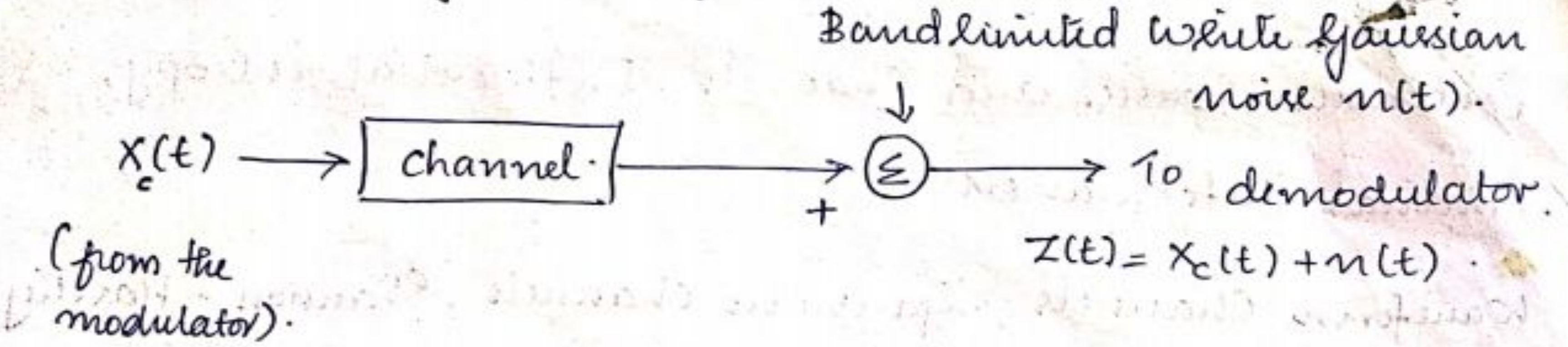
The input signals are continous function of time and the fn of the channel is to produce output of electrical waveform presented at its input.

The channel modifies the input waveform in a random fashion due to additive noise usually pure Gaussian noise. This is because shot noise, radiation noise etc tend to have Gaussian distribution

Analog part of channel

Modulation & demodulation are designed to get rid of Gaussian noise.

Analog portion of Communication Channel.



→ The input to channel is a random process $X_c(t)$ which consists of collection of all waveforms generated by the modulator.

→ Bandwidth of X_c and channel is assumed to be B Hz.

→ The additive noise at the channel output is zero mean.

→ The capacity of this portion of the channel is got by maximising the rate of information transmission wrt distribution of $X_c(t)$.

C= max(Rt)
Rt = I(x,y) * rs

Basics of Random variables

- A random variable is a numerical description of the outcome of a statistical experiment.
- A random variable that may assume only a finite number or an infinite sequence of values is said to be discrete; one that may assume any value in some interval -is said to be continuous.
- 2 types of random variables
- Discrete random variable(DRV)
- Continuous random variable(CRV) :

Random variable

- If the random variable assumes only discrete or integer values, then it is called Discrete Random variable
- eg: no of telephone calls received by an office during an hour in a day
- If the random variable assuming all values (both integer and fractional) then it is called Continuous Random Variable
- Eg: temperature measured at different instants in a day measured in deg Celsius

Probability Distribution function-DRV & CRV

Probability Distribution Function : If X is a DRV, then a function $f(x) = P(X = x)$ [where x represents the values which the random variable X takes] is defined as Probability Distribution Function (PDF) or Probability Mass Function (PMF) or Frequency Function (FF), satisfying the following two conditions:

(a) $f(x) \geq 0$ for all values of x

(b) $\sum_{\forall x} f(x) = \sum_{\forall x} P(X = x) = 1$

Probability Density Function (PDF) : If X is a CRV, then a function $f(x) = P(X = x)$ is defined as PDF satisfying the following two conditions:

(a) $f(x) \geq 0$ for all values of x

(b) $\int_{-\infty}^{+\infty} f(x).dx = 1$

Note : The meaning of condition (b) above is the total area under the curve f(x) from $-\infty$ to $+\infty$ is equal to unity. If we consider the area under the curve f(x) between any two values $x = a$ and $x = b$ as shown in figure 1.8, then this area represents the probability that the random variable X takes all values between 'a' and 'b' given by

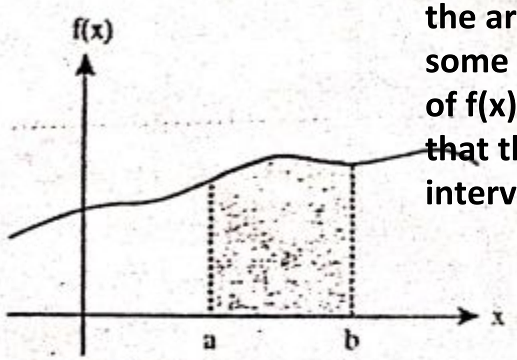


Fig. 1.8 : Illustrating area under curve f(x)

the area under the graph of f(x) corresponding to some interval, obtained by computing the integral of f(x) over that interval, provides the probability that the variable will take on a value within that interval.

$$P(a \leq X \leq b) = \int^{a} f(x).dx \quad \text{..... (1.29)}$$

Differential Entropy of Continuous Ensembles

Entropy of Continuous Signals.

Entropy of discrete message symbol is given by

$$H(S) = \sum_{i=1}^{q} P(S_i) \log_2 \frac{1}{P(S_i)}.$$

Consider coutinous random variable 'X' to be a limiting form of a discrete random variable which assumes a value $X_i = \Delta x$ where $i = 0 \pm 1, \pm 2, \pm 3 \ldots$ and Δx approches zero.

By definition a coutinous random variable (CRV) 'X' assume a value in the interval $(x_i, x_i + \Delta x)$ with probability $f_X(x_i)\Delta x$.

Hence permitting Δx to approach zero, the ordinary entropy of CRV 'X' may be written in the limit as follows.

$$H(X) = \lim_{\Delta x \to 0} \sum_{i} f_X(x_i)\Delta x \log \frac{1}{f_X(x_i)\Delta x}.$$

$$= \lim_{\Delta x \to 0} \left[\sum_{\forall i} f_x(x_i)\, \Delta x \log \frac{1}{f_x(x_i)} - \sum_{\forall i} f_x(x_i)\, \Delta x \log \Delta x \right.$$

As $\Delta x \to 0$.

$f_x(x_i)\, \Delta x \to f(x)\, dx$ and summation becomes integration

and thus we have above equation rewritten as

$$H(x) = \int_{-\infty}^{\infty} f(x) \log \frac{1}{f(x)}\, dx - \lim_{\Delta x \to 0} \log \Delta x \int_{-\infty}^{\infty} f(x)\, dx .$$

we know $\int_{-\infty}^{\infty} f(x)\, dx = 1$ as $f(x)$ is prob. DENSITY(PDF) fn.

then $H(x) = \int_{-\infty}^{\infty} f(x) \log \frac{1}{f(x)}\, dx - \lim_{\Delta x \to 0} \log \Delta x .$

from the previous equation as Δx approaches zero, $\log \Delta x$ approaches infinity.

ie the entropy of a continuous RV is infinitely large.

CRV can have any large value b/w ($-\infty$ and ∞).

$\Rightarrow$ We delete $\log \Delta x$ by adopting $H(x)$ as "differential entropy"

Since the information transmitted over a channel is actually the difference b/w 2 entropy terms that have common reference, the info will be the same as difference b/w corresponding differential entropy terms:

$$\boxed{H(x) = \int_{-\infty}^{\infty} f(x) \log \frac{1}{f(x)} \, dx \quad \text{bits/sample}}$$

Mutual information of a continuous noisy channel

$$H(x,y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \log f(x,y)\, dx\, dy.$$ Joint Entropy

and

Equivocation

$$H(x|y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \log f(x|y)\, dx\, dy.$$

where $f(x,y)$ is the joint probability density fn.

$f(x|y)$ is the conditional probability of X given $Y=y$.

Mutual information $I(X,Y)$

$$I(X,Y) = H(X) - H(X|Y)$$

also $I(X,Y) = H(Y) - H(Y/X)$ H(Y/X) = H(N)

and $I(X,Y) = H(X) + H(Y) - H(X,Y)$

Channel capacity $C = \max\left[I(X,Y)\right]$.

Rate of transmission

Rate of transmission

The rate of transmission of continuous signals is given by

$$R_t = \text{Mutual Information } I(X,Y) \text{ bits/sample.}$$

$$R_t = I(X,Y) = H(Y) - H(y/x).$$

$$R_t = H(y) - H(N).$$

The channel capacity is given by.

$$C = R_{t\,max} = [H(x) - H(N)]_{max}.$$

CONCLUSION

- Expression for Differential entropy
- Mutual Information
- Rate of Transmission

THANK YOU