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MODULE - I

Antenna Parameters

1. Gain
2. Directivity
3. Beam solid angle
4. Beam width
5. Effective apterature
6. Effective height
7. Polarisation
8. Antena temperature
9. Radiation resistance & efficiency
10. Principle of reciprocity

Antenna is a metallic device used for radiating radio waves. It is a transitional structure b/w free space wave & guiding device.

1. Gain

The ratio of power gain in a given direction to the power gain of reference antenna in it's referenced direction. Which is given by, Gain, G $= \frac{4\pi \times U(\theta,\phi)}{P_{in}(\text{reference antenna})}$

where, U represent power of antenna in given direction.

2. Directivity

The directivity of the antenna is given by, the ratio of maximum radiation intensity to average radiation intensity. This mathematically represented as,

$$D = \frac{U(\theta,\phi)_{max}}{U_{average}}$$

Average pointing vector for a sphere given by,

$$U_{average} = S(\theta,\phi)_{ave}$$

$$= \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S(\theta,\phi)\,d\Omega$$

∴ directivity for a sphere

$$D = \frac{S(\theta,\phi)_{max}}{S_{ave}}$$

$$= \frac{1}{\frac{S_{ave}}{S(\theta,\phi)_{max}}} = \frac{1}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}\,d\Omega}$$

$$= \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi}\frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}\,d\Omega}$$

The denominator can be taken as beam area, denoted by Ω_A.

Gain, $G = \frac{4\pi}{\Omega_A}$

If beam solid angle is small, beam area also small, providing high directivity.

2 The normalized field pattern is given by $E_n = \sin\theta\sin\phi$. For $0 \le \theta \le \phi$ where θ zenith angle, ϕ is azenith angle. Find exact directivity & approximate directivity.

Exact directivity, $D = \frac{4\pi}{\Omega_A}$

Beam area $= \int_0^{2\pi}\int_0^{\pi}\frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}\,d\Omega$

The above formula become the radiated power $\frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}\,d\Omega = P_n(\theta,\phi)$

= square of field pattern of antenna

$d\Omega = d\theta\,d\phi\sin\theta$

$$= \int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} E_n^2(\theta,\phi)\,d\Omega$$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$4\sin^3\theta = 3\sin\theta - \sin 3\theta$

$\sin^3\theta = \frac{1}{4}[-\sin 3\theta + 3\sin\theta]$

$$= \int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi}\sin^2\theta\sin^2\phi\,d\theta\,d\phi\,\sin\theta$$

$$= \int_0^{\pi}\int_0^{\pi}\sin^3\theta\sin^2\phi\,d\theta\,d\phi$$

$$= \int_0^{\pi}\sin^3\theta\,d\theta\int_0^{\pi}\sin^2\phi\,d\phi \quad —①$$

$$\int_0^{\pi}\sin^3\theta\,d\theta = \frac{1}{4}\int_0^{\pi}-\sin 3\theta + 3\sin\theta\,d\theta$$

$$= \frac{1}{4}\int_0^{\pi}(3\sin\theta - \sin 3\theta)\,d\theta$$

$$= \frac{1}{4}\left[-3\cos\theta + \frac{\cos 3\theta}{3}\right]_0^{\pi}$$

$$= \left[\frac{-3}{4}\times -1 + \frac{1}{12}\times -1\right] - \left[-\frac{3}{4} + \frac{1}{12}\right]$$

$$= 4/3 \quad —②$$

$$\int_0^{\pi} \sin^2\phi \, d\phi = \int_0^{\pi} \frac{1-\cos 2\phi}{2} \, d\phi$$

$$= \frac{1}{2}\int_0^{\pi} 1-\cos 2\phi \, d\phi$$

$$= \frac{1}{2}\left[\phi - \frac{\sin 2\phi}{2}\right]_0^{\pi}$$

$$= \frac{1}{2}\left[\pi\right]$$

$$= \pi/2 \quad \text{——— (3)}$$

sub (2) & (3) in (1) $\Rightarrow$

$$\int_0^{\pi} \sin^3\theta \, d\theta \int_0^{\pi} \sin^2\phi \, d\phi = 4/3 \times \pi/2$$

$$= 4\pi/6 = \underline{\underline{2\pi/3}}$$

$$\text{Gain} = \frac{4\pi}{2\pi/3} = 4\pi \times 3/2\pi = \underline{6}$$

which is the exact directivity.

Approximate directivity, directivity in terms of half power beam width. For this condition, directivity $D = \frac{41253}{\theta^o_{HP} \times \phi^o_{HP}}$

Approximate directivity expressed in degree and for half power beam width $\theta = \phi = 90^o$.

ie, $D = \frac{41253}{90 \times 90} = \underline{\underline{5.1}}$

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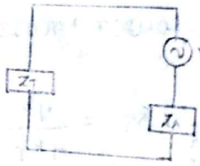
Effective aperature

Consider a dipole receiving antenna having equivalent circuit



$Z_T \rightarrow$ terminating load impedence

$Z_A \rightarrow$ internal antenna impedence

$V \rightarrow$ Equivalent voltage.

From the figure current through the terminating impedence,

$$I = \frac{V}{Z_T + Z_A}$$

where, $Z_T = R_T + jX_T$

$Z_A = R_A + jX_A$

$$\therefore I = \frac{V}{R_T + R_A + j(X_A + X_T)}$$

Power delivered by antenna to the terminating impedence P obtained by,

$$P = |I|^2 R_T$$

$$|I| = \frac{V}{\sqrt{(R_T+R_A)^2 + (X_A+X_T)^2}}$$

$|a+jb| = \sqrt{a^2+b^2}$

$$P = \frac{V^2}{(R_T+R_A)^2 + (X_A+X_T)^2} \times R_T$$

And also we know total power P absorbed from the wave $P = SA$.

where $S \rightarrow$ pointing vector

$A \rightarrow$ Area of the antenna.

Both of the power to be equivated we get,

$$SA = \frac{V^2}{(R_T+R_A)^2+(X_A+X_T)^2} \times R_T$$

For maximum power transfer, $R_T = R_A$ & $X_A = -X_T$

$$SA = \frac{V^2}{(2R_T)^2} \times R_T = \frac{V^2}{4R_T}$$

$$A = \frac{V^2}{4R_TS}$$

This A can be represented as maximum effective aperature. And also antenna impedence R_A related with maximum effective aperature can be written as,

$$A = \frac{V^2}{4R_AS}$$

And also antenna impedence contain radiation resistance and loss resistance. ie,

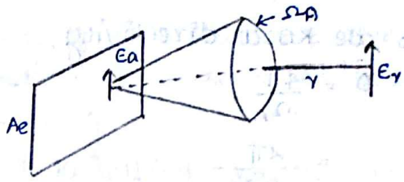
$$R_A = R_r + R_L$$

∴ above equation becomes,

$$\boxed{A = \frac{V^2}{4(R_r+R_L)S}}$$

⇒ Derive a relation b/w directivity and effective aperature.

consider an antenna with effective aperature A_e which radiate all of its power in a conical pattern of beam area Ω_A as shown in figure.



Assuming a uniform field E_a over the aperature of the power radiated is,

$$P = SA = \frac{E_a^2}{Z_0} A_e$$

$Z_0 \rightarrow$ intrinsic impedence of the medium

Assuming uniform field E_r in the far field at a distance r, the power radiated is given by,

$$P = \frac{E_r^2}{Z_0} r^2 \Omega_A$$

To equivating both of the equation,

$$\frac{E_a^2}{Z_0} A_e = \frac{E_r^2}{Z_0} r^2 \Omega_A$$

$$E_r^2 = \frac{E_a^2 A_e}{r^2 \Omega_A}$$

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We know, beam area Ω_A is given by, the integral of normalized power pattern. And also we know,

$$|E_r| = \frac{|E_a| A_e}{r\lambda}$$

$$\frac{E_a^2 A_e}{r^2 \Omega_A} = \frac{E_a^2 A_e^2}{r^2 \lambda^2}$$

$$\Omega_A = \frac{\lambda^2}{A_e}$$

where $\lambda \rightarrow$ wavelength in meter

$A_e \rightarrow$ Antenna aperature

$\Omega_A \rightarrow$ Beam area in square meter (m^2)

And also we know directivity.

$$D = \frac{4\pi}{\Omega_A}$$

$$= \frac{4\pi}{\lambda^2/A_e}$$

$$\boxed{D = \frac{4\pi A_e}{\lambda^2}}$$

Assuming no losses, A_e = maximum effective Aperature

$$\boxed{\begin{aligned} A_e &= A_{em} \\ D &= \frac{4\pi A_{em}}{\lambda^2} \end{aligned}}$$

This is the relation b/w maximum effective aperature & directivity.

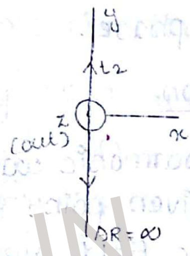
Polarization

The polarization of a uniform plane wave refer to time varying behaviour of electric field strength vector at fi some fixed point in space. Consider for an example, a uniform plane wave travelling the z-direction E & H vector line in x-y plane. If $\vec{E_y} = 0$ and E_x present, the wave said to be polarised in x-direction. If $\vec{E_x} = 0$ and only E_y present, the wave is said to be polarized in y-direction. If both E_x & E_y are present, are in the phase, the resultant electric field pass a direction depend on the relative magnitude of E_x & E_y.

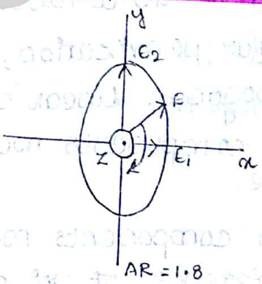
Polarisation may be classified as;

1. linear
2. circular
3. Eliptical

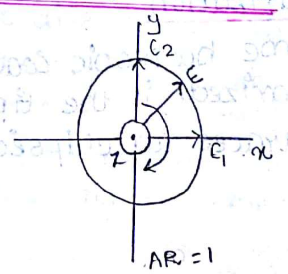
1. Linear polarization



2. Elliptical polarization



3. Circular polarization



Linear polarization

→ A time harmonic wave is linearly polarized at a given point in space, if the electric field vector at that point is always oriented along the same straight line at every instand of time.

→ It contain electric or magnetic vector component.

→ Two orthogonal linear component are in 180° out of phase.

Circular polarization.

→ A time harmonic wave is circularly polarized at a given point in space, if electric or magnetic field vector at the point rices a circle as a function of time.

→ In circular polarization, the field must have 2 orthogonal linear components.

→ The two components must have the same magnitude.

→ The two components must have a time phase difference of 90°, 270° etc.

Elliptical polarization

→ A time harmonic wave is elliptically polarized, if the tip of the field vector traces an ellipsecal focus on the space.

→ The elliptical polarization contain the field must have 2 orthogonal linear components.

→ The 2 components of same or different mangnitude.

→ If 2 components are not same magnitude, phase difference is 0, 180° etc.

→ If 2 components are same magnitude it become circular polarization.

Poynting vector for elliptically & circularly polarized wave.

In complex notation the poynting vector is $S = \frac{1}{2} E \times H^*$

$$= \frac{1}{2} Re\left[E \times H^*\right]$$

$$= \frac{1}{2}\left[\frac{E^2}{Z_0}\right]\hat{z}$$

where, $E \rightarrow$ amplitude of EF

$$E = \sqrt{E_1^2 + E_2^2}$$

$$S = \frac{1}{2}\left[\frac{E_1^2 + E_2^2}{Z_0}\right]\hat{z}$$

2 An elliptically polarized wave travelling in +ve z-direction in air has x & y component

$E_x = 3\sin(\omega t - \beta z)$ V/m

$E_y = 6\sin(\omega t - \beta z + 75°)$ V/m

Find the average power per unit area produced by the wave.

$E_1 = 3$, $E_2 = 6$, $Z_0 = 377$

$$S = \frac{1}{2}\left[\frac{E_1^2 + E_2^2}{Z_0}\right]\hat{z}$$

$$= \frac{1}{2}\left[\frac{9+36}{377}\right]\hat{z}$$

$$= 0.059\ W/m^2$$

Antenna Temperature

For a lossless antenna, if T absolute temperature in kelvin, k - boltzmann constant 1.38×10^{-23} J/k, P - power per unit. Then, power = the product of boltzmann constant & absolute temp.

$$P = kT_A$$

⇒ Antenna temperature in terms of flux density S & effective aperature Ae.

The power receiver from the source, $P = SA_eB$

where, S → power density per unit band-width

A_e → Effective aperature

B → band width.

since, $P = kT_AB$

$$SA_eB = kT_AB$$

$$\boxed{T_A = \frac{SA_e}{k}}$$ kelvin

⇒ Antenna temp. in terms of beam area

Antenna temp, $\boxed{T_A = \frac{\Omega_S}{\Omega_A} T_S}$

where T_A → Antenna temp

Ω_S → solid Angle

T_S → source temperature

Ω_A → beam area

2 What is the effective noise temp. Noise temp is given by 1.3 dB.

Effective noise temp $T_e = (F-1)T_0$

T_0, absolute temp = 290 k

$10 \log F = 1.3$

$F = 1.35$

$T_e = 101.5$ kelvin

Antenna field zones

The field around an antenna divided into two region;

1. Near field [Fresnel zone)

Reactive near field: $R \leq 0.62\sqrt{L^3/\lambda}$

Radiating near field: $R \leq 2L^2/\lambda$

2. Far field

$$R > 2L^2/\lambda$$

$E_1 = 3$, $E_2 = 6$, $Z_0 = 377$

$$S = \frac{1}{2}\left[\frac{E_1^2 + E_2^2}{Z_0}\right]\hat{z}$$

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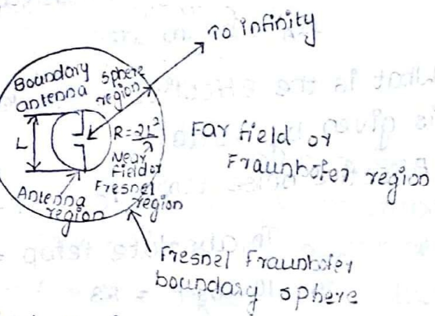
The boundary b/w these two zone at a radius $R = 2L^2/\lambda$

where $L \rightarrow$ max. dimensions of antenna in meter

$\lambda \rightarrow$ wavelength in meter



1. Near field Region

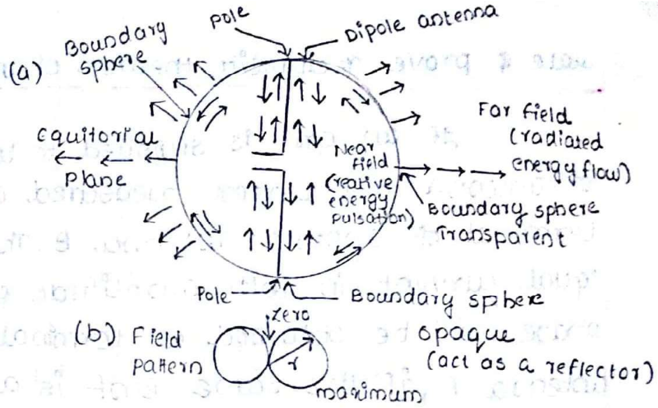
* The longitudinal component of EF may be significant & power flow not entirly radial.

* The shape of the field pattern depend on the distance.

2. Far field region

* Power flow radially directed & shape of field pattern independent of the distance.

⇒ Energy flow in near field & far field can be drawn as;



Duality of Antennas

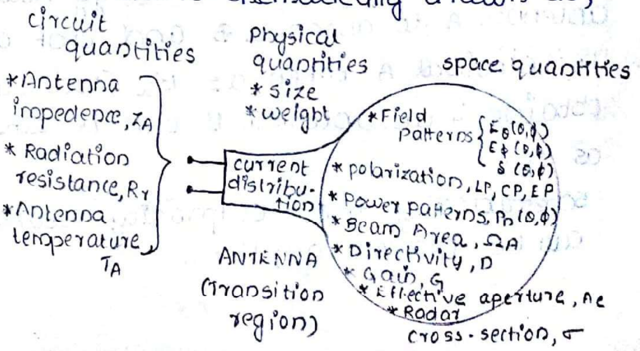
The antenna parameters can be classified as; circuit quantities, physical quantities & space quantities.

Duality as circuit device contain resistance & temperature.

Space quantities contain field pattern, polarization, beam area, directivity, gain, aperature & radar crosssection.

Physical quantities contain size & weight.

This is schematically drawn as;



state & prove reciprocity theorem of antenna

If an Emf is supplied to terminal of antenna A & current measured at terminal of another antenna B. Then equal current in both amplitude & phase will be obtained at terminal of antenna A, if the same emf is applied to terminal of antenna B.

Proof :

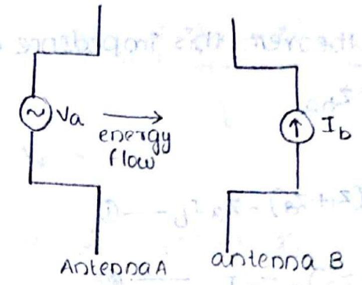
Assumptions;

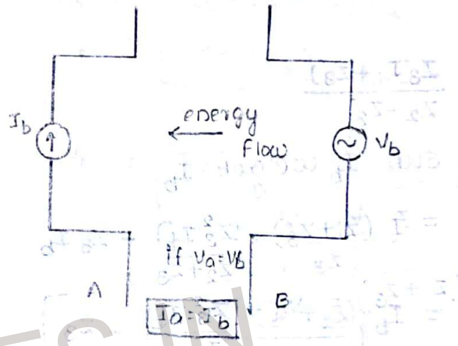
1. EMF are of same frequency
2. Media are linear & passive
3. Transmitting & receiving pattern of antenna are same.
4. Power flow same for both antennas.

Case I :

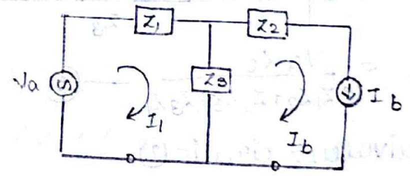
Transmitting antenna energy flow from antenna A to antenna B and emf applied to terminal A taken as Va and current obtained at antenna B can be taken as I_b.

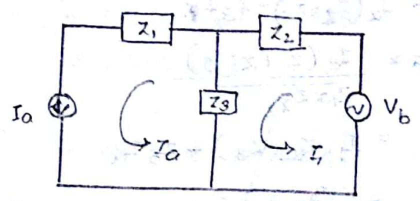
schematically the reciprocity theorem can be proved as;





Equivalent circuit

① 

② 

Transfer impedence,

$Z_{ab} = V_a / I_b$

$Z_{ba} = V_b / I_a$

By reciprocity theorem, this impedence are equal.

$Z_{ab} = Z_{ba}$

fig (1),

$V_a = I_1(Z_1+Z_3) - Z_3 I_b$ —— ①

$0 = I_b(Z_2+Z_3) - Z_3 I_1$ —— ②

② ⇒

$I_1 = \frac{I_b(Z_2+Z_3)}{Z_3}$

in eqn ① sub I_1 we get I_b

$V_a = \frac{I_b(Z_2+Z_3)}{Z_3}(Z_1+Z_3) - Z_3 I_b$

$= I_b\left[\frac{(Z_2+Z_3)(Z_1+Z_3)}{Z_3} - Z_3\right]$

$= I_b\left[\frac{Z_1Z_2+Z_2Z_3+Z_3Z_1+Z_3^2-Z_3^2}{Z_3}\right]$

$I_b = \frac{V_a Z_3}{Z_1Z_2+Z_2Z_3+Z_3Z_1}$ —— ⑤

From equivalent circuit ②

$0 = I_a(Z_1+Z_3) - Z_3 I_a$

$I_1 = \frac{Z_3 I_a(Z_1+Z_3)}{Z_2+Z_3}$

$V_b = I_1(Z_2+Z_3) - Z_3 I_a$

$= \left(\frac{Z_3 I_a}{Z_2+Z_3}\right)(Z_1+Z_3) - Z_3 I_a$

$= I_a\left[\frac{Z_3(Z_1+Z_3)}{Z_2+Z_3} - Z_3\right]$

$= I_a\left[\frac{Z_3 Z_1 + Z_3^2 - Z_3Z_2 - Z_3^2}{Z_2+Z_3}\right]$

~~$V_b =$~~

$0 = I_a(Z_1+Z_3) - Z_3 I_1$ —— ③

$V_b = I_1(Z_2+Z_3) - Z_3 I_a$ —— ④

$I_1 = \frac{I_a(Z_1+Z_3)}{Z_3}$

$V_b = \left(\frac{I_a(Z_1+Z_3)}{Z_3}\right)(Z_2+Z_3) - Z_3 I_a$

$= I_a\left[\frac{(Z_1+Z_3)(Z_2+Z_3)}{Z_3} - Z_3\right]$

$V_b = I_a\left[\frac{Z_1Z_2+Z_1Z_3+Z_3Z_2+Z_3^2-Z_3^2}{Z_3}\right]$

$I_a = \frac{V_b Z_3}{Z_1Z_2+Z_1Z_3+Z_3Z_2}$ —— ⑥

If $V_a = V_b$ from eqn ⑤ & ⑥

we can write

$I_a = I_b$

This prove the reciprocity theorem.

1. Derive radiation resistance of a short electric dipole (R_r)
2. Derive radiation resistance of a half wave dipole antenna.
3. Derive directivity of a short dipole antenna (D).
4. Derive directivity of the half-wave dipole antenna.

PROOF - 2

Derive radiation resistance of a short electric dipole (R_r)

The radiation resistance of short dipole is determined by following steps.

(1) Total power radiated is obtained by integrating point vector of far field over a large sphere.

(2) The power is equated to I^2R.

$I \rightarrow$ rms current on the dipole

Total power radiated

The 3 components of poynting vector is

$S_\theta = -E_r H_\phi$

$S_\phi = E_\theta H_r$

$S_r = E_\theta H_\phi$

For a short dipole antenna $\therefore H_r = E_r = 0$

ie, $S_\theta = S_\phi = 0$

ie, S_r only exist and $S_r = \frac{1}{2} Re[E_\theta \times H_\phi^*]$

This is the formula for poynting vector also.

$$\frac{E_\theta}{H_\phi} = \eta$$

$$E_\theta = \eta H_\phi$$

$$S_r = \frac{1}{2} Re[\eta H_\phi H_\phi^*]$$

$$= \frac{1}{2} |H_\phi|^2 \eta$$

The total power P is radiated then $P = \iint S_r ds$

$$S_r = \frac{1}{2} |H_\phi|^2 \times \eta$$

& for spherical coordinates $ds = r^2 \sin\theta \, d\theta \, d\phi$

$$P = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{1}{2} |H_\phi|^2 \times \eta \times r^2 \sin\theta \, d\theta \, d\phi$$

$$|H_\phi| = \frac{I_0 L}{4\pi} \sin\theta \left(\frac{\omega}{cr}\right)$$

$$P = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{1}{2} \left(\frac{I_0 L}{4\pi}\right)^2 \sin^2\theta \left(\frac{\omega}{cr}\right)^2 \eta \times r^2 \sin\theta \, d\theta \, d\phi$$

$$P = \frac{1}{2} \left(\frac{I_0 L}{4\pi}\right)^2 \left(\frac{\omega^2}{c^2}\right) \cdot \eta \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta \, d\phi$$

$$\int_{\theta=0}^{\pi} \sin^3\theta \, d\theta = \int_{\theta=0}^{\pi} \frac{1}{4} (3\sin\theta - \sin 3\theta)$$

$$= \frac{1}{4} \left[(-3\cos\theta)_0^\pi + \left(\frac{\cos 3\theta}{3}\right)_0^\pi \right]$$

$$= \frac{1}{4} \left[(-3\cos\pi + 3\cos 0) + \left(\frac{\cos 3\pi}{3} - \cos 0\right) \right]$$

$$= \frac{1}{4} \left[(3+3) + \left(-\frac{1}{3} - 1\right) \right]$$

$$= \frac{4}{3}$$

$$\int_{\phi=0}^{2\pi} 4/3\, d\phi = (4/3\,\phi)_0^{2\pi} = 4/3 \times 2\pi \qquad 377 = 120\pi$$

$$P = 1/2\left(\frac{I_0 L}{4\pi}\right)^2 \frac{\omega^2}{c^2} \times 377 \times 4/3 \times 2\pi$$

$$= 1/2\left(\frac{I_0 L}{4\pi}\right)^2 \frac{\omega^2}{c^2} \times 120\pi \times 4/3 \times 2\pi$$

$$= (I_0 L)^2 \frac{\omega^2}{c^2}(2/3)\left(\frac{1}{4^2\pi^2}\right)(120\pi)(2\pi)$$

Also substituting $\omega = 2\pi f$ & $c = f\lambda$

$\omega^2 = 4\pi^2 f^2 \qquad c^2 = f^2\lambda^2$

$$P = (I_0 L)^2\left(\frac{4\pi^2 f^2}{f^2\lambda^2}\right) \times 10$$

$$P = \frac{(I_0 L)^2\, 40\pi^2}{\lambda^2}$$

$$P = \frac{40\pi^2 I_0^2 L^2}{\lambda^2} \text{ watt/m}^2$$

Equating this value to $I^2 R_r$

But $I = I_0/\sqrt{2}$

$\therefore I^2 R_r = (I_0/\sqrt{2})^2 R_r$

$$\frac{40\pi^2 I_0^2 L^2}{\lambda^2} = \frac{I_0^2 \times R_r}{2}$$

$$\boxed{R_r = 80\pi^2 (L/\lambda)^2}$$

Radiation resistance of a short dipole antenna.

PROOF - 3

Derive radiation resistance of a half wave dipole antenna

To find the radiation resistance the poynting vector is integrated over a large sphere and computing the power then equating to I^2R then getting radiation resistance.

Power $P = \iint S_r\, ds$

$$S_r = 1/2\, \mathrm{Re}\left[E_\theta \times H_\phi^*\right]$$

$E_\theta / H_\phi = \eta$

$E_\theta = \eta H_\phi \qquad S_r = 1/2\, \mathrm{Re}\left[\eta H_\phi H_\phi^*\right]$

$$= 1/2\, |H_\phi|^2 \eta$$

$$ds = r^2 \sin\theta\, d\theta\, d\phi$$

$$\therefore P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} 1/2\, |H_\phi|^2 \eta\, r^2 \sin\theta\, d\theta\, d\phi$$

$$|H_\phi| = \frac{I_m}{2\pi r} e^{-j\beta r}\left(\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right)$$

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} 1/2\, \frac{I_m^2}{4\pi^2 r^2} e^{-2j\beta r}\left(\frac{\cos^2(\pi/2\cos\theta)}{\sin^2\theta}\right) 377\, r^2 \sin\theta\, d\theta\, d\phi$$

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{I_m^2}{8\pi^2} e^{-2j\beta r} \times 120\pi\left[\frac{\cos^2(\pi/2\cos\theta)}{\sin^2\theta}\right]\sin\theta\, d\theta\, d\phi$$

$$P = 30\, I_m^2 \int_0^{\pi} \frac{\cos^2(\pi/2\cos\theta)}{\sin\theta}\, d\theta$$

$\lambda/2$ radiation resistance

$$P = 60 I_m^2 \int_0^{\pi/2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} d\theta$$

$\int_0^{\pi/2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} d\theta$ is integrated by using trapezoidal rule we get the given integral value is 0.6096.

Total power: $P = 60 I_m^2 \times 0.6096$

$= 36.5 I_m^2$

This is the power radiated from Half wave dipole antenna.

Inorder to computing radiation resistance, this power is equivated to I^2R.

ie, $P = I^2 R$

$= (I_m/\sqrt{2})^2 R$

$= \frac{I_m^2}{2} R$

$P = 36.5 I_m^2$

$I_m^2/2 \, R = 36.5 I_m^2$

$R = 36.5 \times 2$

$\therefore R_r (\lambda/2 \text{ Antenna}) = 73\,\Omega$

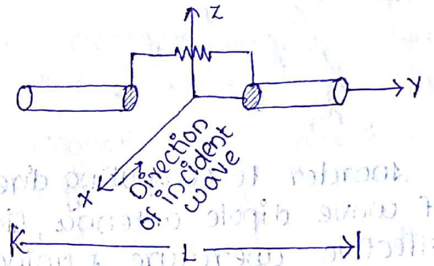
PROOF - 4

Derive directivity of short dipole antenna

Inorder to computing directivity first computing effective apperature then using the formula

$D = \frac{4\pi A_{em}}{\lambda^2}$ $A_{em} \rightarrow$ maxi. effective apperature

consider a short dipole with uniform current introduced by incident wave have length 'L' current 'I'. This are shown in fig as.



The max effective aperature of antenna is,

$A_{em} = \frac{V^2}{4 S R_r}$ $V \rightarrow$ induced voltage, $S \rightarrow$ Poynting vector, $R_r \rightarrow$ Radiation resistance

$V = E L$

$S = E \cdot H$

$= E \cdot E/\eta$

$= E^2/\eta$

$R_r = 80\pi (L/\lambda)^2$

$A_{em} = \frac{E^2 L^2 \eta}{4 \times E^2 \cdot 80\pi (L/\lambda)^2}$

$= \frac{120\pi \lambda^2}{320\pi^2}$

$= \frac{3\lambda^2}{8\pi}$

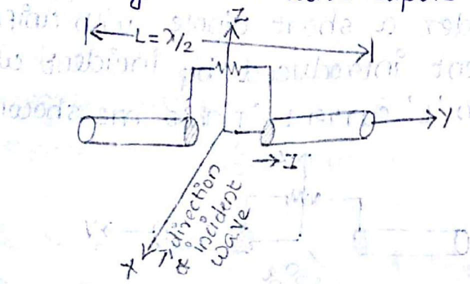
$D = \frac{4\pi}{\lambda^2} \times \frac{3\lambda^2}{8\pi}$

$= 1.5$

Directivity of short dipole antenna is 1.5.

PROOF - 5

Directivity of half wave dipole antenna



Inorder to computing directivity of a half wave dipole antenna, first computing effective aperature & using the formula, $D = \frac{4\pi}{\lambda^2} A_{em}$

Also we know, $A_{em} = \frac{V^2}{4S R_r}$

where S = Poynting vector $= E \cdot H = E^2/\eta$

$R_r \rightarrow$ radiation resistance of half wave dipole antenna

$R_r = 73\Omega$

$V \rightarrow$ induced voltage

dv for half wave dipole antenna is obtained by,

$dv = E\,dy \cos\left(\frac{2\pi}{\lambda}\right)y$

$V = 2\int_0^{\lambda/4} E \cos\left(\frac{2\pi}{\lambda}y\right)dy$

$= \left[E \sin\left(\frac{2\pi y}{\lambda}\right) \times \lambda/2\pi\right]_0^{\lambda/4}$

$= 2E \sin\pi/2 \times \lambda/2\pi$

$= E\lambda/\pi$

$A_{em} = \frac{\frac{E^2\lambda^2}{\pi^2}}{4\,E^2/\eta \times 73\Omega}$

$= \frac{E^2\lambda^2}{\pi^2} \times \frac{\eta}{4E^2 \times 73\Omega}$

$= \frac{\lambda^2\eta}{4\times\pi^2\times 73\Omega}$

$= \frac{\lambda^2 120\pi}{73\pi^2 \times 4}$

$= \frac{\lambda^2}{\pi} 0.41 = 0.13\lambda^2$

$D = \frac{4\pi}{\lambda^2} \times 0.13\lambda^2$

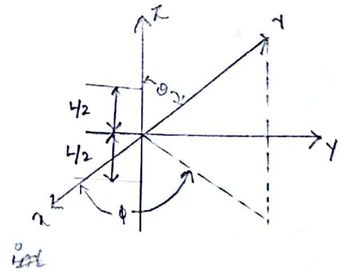
$= 1.63$

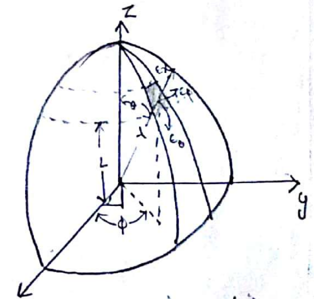
Derive field components H_r, H_θ, H_ϕ (magnetic field components) & E_r, E_θ, E_ϕ (electric field components) of a short dipole antenna

Derive electro magnetic field components of a half wave dipole antenna.

* Magnetic field component of short dipole antenna (ie, H_r, H_θ, H_ϕ)

consider a short dipole antenna with length L & is associated electric field component on spherical surface can be drawn as;





KT orientation

From maxwell eqn $\nabla \cdot B = 0$

The magnetic flux B is solenoid. It can be represented as curl of another vector A.

ie, $\nabla \cdot \nabla \times A = 0$

And also we know $B = \mu H = \nabla \times A$ —①

The vector potential expressed in spherical coordinate $(\nabla \times A)_r$, $(\nabla \times A)_\theta$, $(\nabla \times A)_\phi$

$$(\nabla \times A)_r = \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}(A_\phi \sin\theta) - \frac{\delta}{\delta\phi}(A_\theta)\right] \quad —②$$

$$(\nabla \times A)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\phi}A_r - \frac{\delta}{\delta r}(A_\phi r)\right] \quad —③$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\delta}{\delta r}(A_\theta r) - \frac{\delta}{\delta\theta}A_r\right] \quad —④$$

By using rectangular to spherical coordinate transformation

$$\left.\begin{aligned} A_r &= A_z\cos\theta \\ A_\theta &= -A_z\sin\theta \\ A_\phi &= 0 \end{aligned}\right\} —⑤$$

sub. eqn ⑤ in ②

$$(\nabla \times A)_r = \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}(0) - \frac{\delta}{\delta\phi}(A_z\sin\theta)\right]$$

$$= \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\phi}(A_z\sin\theta)\right]$$

$$= 0$$

Compare eqn ① we get,

$(\nabla \times A)_r = 0$

$\mu H_r = 0$

$$(\nabla \times A)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\phi}A_z\cos\theta - \frac{\delta}{\delta r}(0)\right]$$

$$\boxed{H_\theta = 0}$$

The vector potential of z-direction is obtained by,

$$A_z = \frac{\mu I_0 L}{4\pi r} e^{j\omega(t - r/c)}$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\delta}{\delta r}(-A_z\sin\theta r) - \frac{\delta}{\delta\theta}(A_z\cos\theta)\right]$$

$$= \frac{1}{r}\left[-\frac{\delta}{\delta\theta}\left[\frac{\mu I_0 L}{4\pi r} e^{j\omega(t-r/c)}\cos\theta\right] + \frac{\delta}{\delta r}\left(-\frac{\mu I_0 L}{4\pi r} e^{j\omega(t-r/c)} \sin\theta r\right)\right]$$

$$= \frac{1}{r}\left[-\frac{\mu I_0 L}{4\pi} e^{j\omega(t-r/c)} \times -\frac{j\omega}{c}\sin\theta + \frac{\mu I_0 L}{4\pi r} e^{j\omega(t-r/c)}\sin\theta\right]$$

$$= \frac{1}{r}\frac{\mu I_0 L}{4\pi} e^{j\omega(t-r/c)} \sin\theta\left[\frac{j\omega}{c} + \frac{1}{r}\right]$$

compare eqn ① we get

$$\mu H_\phi = \frac{1}{r}\frac{\mu I_0 L}{4\pi}\sin\theta\, e^{j\omega(t-r/c)}\left[\frac{j\omega}{c} + \frac{1}{r}\right]$$

$$\boxed{H_\phi = \frac{I_0 L}{4\pi}\sin\theta\, e^{j\omega(t-r/c)}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]}$$

H_r, H_θ, H_ϕ are the magnetic field components of short dipole antenna.

From maxwell eqn $\nabla \cdot B = 0$

The magnetic flux B is solenoid. It can be represented as curl of another vector A.

ie, $\nabla \cdot \nabla \times A = 0$

And also we know $B = \mu H = \nabla \times A$ —①

The vector potential expressed in spherical coordinate $(\nabla \times A)_r$, $(\nabla \times A)_\theta$, $(\nabla \times A)_\phi$

$$(\nabla \times A)_r = \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}(A_\phi \sin\theta) - \frac{\delta}{\delta\phi}(A_\theta)\right] \quad —②$$

$$(\nabla \times A)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\phi}A_r - \frac{\delta}{\delta r}(A_\phi r)\right] \quad —③$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\delta}{\delta r}(A_\theta r) - \frac{\delta}{\delta\theta}A_r\right] \quad —④$$

By using rectangular to spherical coordinate transformation

$$\left.\begin{aligned} A_r &= A_z\cos\theta \\ A_\theta &= -A_z\sin\theta \\ A_\phi &= 0 \end{aligned}\right\} —⑤$$

sub. eqn ⑤ in ②

$$(\nabla \times A)_r = \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}(0) - \frac{\delta}{\delta\phi}(A_z\sin\theta)\right]$$

$$= \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\phi}(A_z\sin\theta)\right]$$

$$= 0$$

compare eqn ① we get,

$$(\nabla \times A)_r = 0$$

$$\mu H_r = 0$$

$$\boxed{H_r = 0}$$

$$(\nabla \times A)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\phi}A_z\cos\theta - \frac{\delta}{\delta r}(0)\right]$$

$$\boxed{H_\theta = 0}$$

The vector potential of z-direction is obtained by,

$$A_z = \frac{\mu I_0 L}{4\pi r} e^{j\omega(t - r/c)}$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\delta}{\delta r}(-A_z\sin\theta r) - \frac{\delta}{\delta\theta}(A_z\cos\theta)\right]$$

$$= \frac{1}{r}\left[-\frac{\delta}{\delta\theta}\left[\frac{\mu I_0 L}{4\pi r} e^{j\omega(t-r/c)}\cos\theta\right] + \frac{\delta}{\delta r}\left[-\frac{\mu I_0 L}{4\pi r} e^{j\omega(t-r/c)}\sin\theta r\right]\right]$$

$$= \frac{1}{r}\left[-\frac{\mu I_0 L}{4\pi} e^{j\omega(t-r/c)} \times -j\omega/c \sin\theta + \frac{\mu I_0 L}{4\pi r} e^{j\omega(t-r/c)}\sin\theta\right]$$

$$= \frac{1}{r}\frac{\mu I_0 L}{4\pi} e^{j\omega(t-r/c)}\sin\theta\left[\frac{j\omega}{c} + \frac{1}{r}\right]$$

compare eqn ① we get

$$\mu H_\phi = \frac{1}{r}\frac{\mu I_0 L}{4\pi}\sin\theta e^{j\omega(t-r/c)}\left[\frac{j\omega}{c} + \frac{1}{r}\right]$$

$$\boxed{H_\phi = \frac{I_0 L}{4\pi}\sin\theta e^{j\omega(t-r/c)}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]}$$

H_r, H_θ, H_ϕ are the magnetic field components of short dipole antenna.

⇒ Electric field components E_r, E_θ, E_ϕ short dipole antenna

From maxwell equation

$$\nabla \times H = \frac{\delta D}{\delta t} = \varepsilon \frac{\delta E}{\delta t}$$

$$(\nabla \times H)_r = \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}(H_\phi \sin\theta) - \frac{\delta}{\delta\phi}(H_\theta)\right]$$

$$(\nabla \times H)_r = \varepsilon \delta E_r/\delta t$$

$$(\nabla \times H)_r = \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}(H_\phi \sin\theta)\right]$$

$$\varepsilon\frac{\delta E_r}{\delta t} = \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}\frac{I_0 L}{4\pi}\sin\theta\, e^{j\omega(t-r/c)}\left(\frac{j\omega}{cr}+1/r^2\right)\sin\theta\right]$$

$$= \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}\frac{I_0 L}{4\pi}\sin^2\theta\, e^{j\omega(t-r/c)}\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right)\right]$$

$$= \frac{1}{r\sin\theta}\left[\frac{\delta}{\delta\theta}\sin^2\theta\right] e^{j\omega(t-r/c)}\frac{I_0 L}{4\pi}\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right)$$

$$= \frac{1}{r\sin\theta}\frac{I_0 L}{4\pi}e^{j\omega(t-r/c)}\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right)\frac{\delta}{\delta\theta}\left[\frac{1-\cos 2\theta}{2}\right]$$

$$= \frac{1}{r\sin\theta}\frac{I_0 L}{8\pi}e^{j\omega(t-r/c)}\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right) \times +\sin 2\theta \times 2$$

$$= \frac{1}{r\sin\theta}\frac{I_0 L}{4\pi}e^{j\omega(t-r/c)}\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right) \times 2\sin\theta\cos\theta$$

$$= \frac{1}{r}\frac{I_0 L}{2\pi}e^{j\omega(t-r/c)}\cos\theta\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right)$$

integrating w.r.to t

$$E_r = \frac{I_0 L}{2\pi\varepsilon}e^{j\omega(t-r/c)} \times \frac{1}{j\omega}\cos\theta\left(\frac{j\omega}{cr^2}+\frac{1}{r^3}\right)$$

$$\boxed{E_r = \frac{I_0 L}{2\pi\varepsilon}e^{j\omega(t-r/c)}\cos\theta\left(\frac{1}{cr^2}+\frac{1}{j\omega r^3}\right)}$$

$$(\nabla \times H)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\phi}H_r - \frac{\delta}{\delta r}(H_\phi r)\right]$$

$$\frac{\varepsilon\delta E_\theta}{\delta t} = \frac{1}{r}\left[-\frac{\delta}{\delta r}\left[\frac{I_0 L}{4\pi}r\sin\theta\, e^{j\omega(t-r/c)}\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right)\right]\right]$$

$$= \frac{1}{r}\left[\frac{I_0 L}{4\pi}\sin\theta\frac{-\delta}{\delta r}e^{j\omega(t-r/c)}\left(\frac{j\omega}{c}+\frac{1}{r}\right)\right]$$

$$= \frac{-1}{r}\frac{I_0 L}{4\pi}\sin\theta\left[e^{j\omega(t-r/c)} \times -1/r^2 - \left(\frac{j\omega}{c}+\frac{1}{r}\right)e^{j\omega(t-r/c)} \times \frac{j\omega}{c}\right]$$

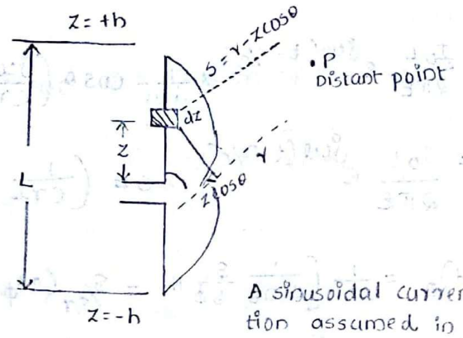
$$= \frac{I_0 L}{4\pi}\sin\theta \times e^{j\omega(t-r/c)}\left[\frac{1}{r^3}+\frac{j^2\omega^2}{c^2 r}+\frac{1}{r^2}\frac{j\omega}{c}\right]$$

$$\boxed{E_\theta = \frac{I_0 L}{4\pi\varepsilon}\sin\theta\, e^{j\omega(t-r/c)}\left[\frac{1}{j\omega r^3}+\frac{j\omega}{c^2 r}+\frac{1}{cr^2}\right]}$$

$$\boxed{E_\phi = 0}$$

⇒ Derive field radiated by half wave dipole antenna

Half wave dipole antenna is a straight radiator usually fed in the centre with a physical length of half wave length producing a maximum of radiation in the plane normal to the axis.



A sinusoidal current distribution assumed in centre fed dipole antenna

Total length $L = 2h$

where, $h = L/2$ in free space

since the current is assumed sinusoidally distributed as,

$$I = I_m \sin\beta(h-z),\ z>0$$
$$= I_m \sin\beta[h+z],\ z<0$$

where $\beta = 2\pi/\lambda$

$I_m \rightarrow$ Peak value of current

The vector potential at a distant point P due to this current element Idz is given by,

$$dA_z = \frac{\mu I e^{j\omega(t-r/c)}}{4\pi s}$$

$$A_z = \int_{-h}^{+h} \frac{\mu I e^{j\omega(t-r/c)}}{4\pi s} dz$$

where $r = r - z\cos\theta$

In the above term the exponential term become $e^{j\beta z\cos\theta}$ and the denominator contain s approximately equal to r.

$$\therefore A_z = \int_{-h}^{h} \frac{\mu I e^{j\beta z\cos\theta}}{4\pi r} dz$$

$$A_z = \frac{\mu}{4\pi r}\int_{-h}^{h} +I e^{j\beta z\cos\theta} dz$$

substitute $\beta = 2\pi/\lambda$ and take the conditions for current,

$$A_z = \frac{\mu}{4\pi r}\int_{0}^{h} e^{j2\pi/\lambda\, z\cos\theta}\, I_m \sin 2\pi/\lambda(h-z)\, dz$$

$$A_z = \frac{\mu}{4\pi r}\int_{-h}^{0} I_m \sin 2\pi/\lambda(h+z)\, e^{j2\pi/\lambda\, z\cos\theta} dz$$

Integrating above identity by using trignometric eqn, we get

$$\boxed{A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]}$$

magnetic field component H_ϕ,

$$\mu H_\phi = \frac{1}{r}\frac{\delta}{\delta r}\left[-A_z \sin\theta . r\right]$$

$$\mu H_\phi = \frac{1}{r}\frac{\delta}{\delta r}\left[-\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]\sin\theta . r\right]$$

$$= \frac{1}{r}\frac{\mu I_m}{2\pi\beta}\frac{\cos(\pi/2\cos\theta)}{\sin\theta} e^{-j\beta r} \cdot -j\beta$$

$$\boxed{H_\phi = \frac{1}{r}\cdot\frac{I_m}{2\pi} e^{-j\beta r}\frac{\cos(\pi/2\cos\theta)}{\sin\theta}}$$

Electric field component E_θ,

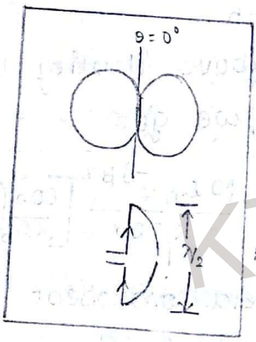
$$E_\theta = \eta H_\phi$$

$$= 377 \times H_\phi$$

$$= 377 \times \frac{I_m}{2\pi r} e^{-j\beta r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]$$

$$\boxed{E_\theta = \frac{60 I_m}{r} e^{-j\beta r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]}$$

The radiation pattern of $\lambda/2$ antenna can be drawn;



Radiation pattern of $\lambda/2$ antenna

The pattern more directional than pattern of short dipole antenna.

The beam width b/w half power point of $\lambda/2$ antenna is 78° which is less than that of short dipole antenna, which is 90°.

MODULE-3

1. Array of 2 point sources
2. Linear array of N isotropic point sources
3. Principle of pattern Multiplication

4 1. Broad side array
5 2. END fire array
6 3. Binomial arrays
7 4. Dolph chebyshev arrays

1. Explain different conditions of array of 2 point sources

case I
Two isotropic point sources of same amplitude and phase

case II
equal amplitude & opposite phase (180°)

Case III
equal amplitude & 90° phase

case IV
equal amplitude & any phase difference

Case I

Two isotropic point sources of same amplitude and phase (we have to compute total field pattern, amplitude field pattern, direction of maximum & minimum,