Module IV - Design of IIR filters
(Infinite Impulse Response filter)

~~Basi~~

- Basically digital filter is a linear time invariant (LTI) discrete time system.

Digital filters two types.

① FIR filters (Finite Impulse Response filter)

② IIR filters (Infinite Impulse Response filter)

FIR filters:

- Present output sample depends on present input sample and previous input samples

- ie FIR filters are non-recursive type (non feed back filters).

IIR filters:

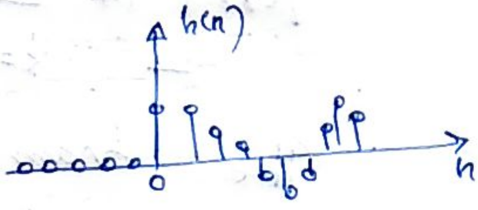
- present output sample depends on present input, past input samples and past output samples.

- ie IIR filters are recursive type filters (has feedback connection).

Design of IIR filters

The impulse response $h(n)$ for a realizable filter is (or causal).

$h(n) = 0$ for $n \leq 0$ ie



and for stability $h(n)$ must satisfy the condition $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

- IIR digital filters are described by the difference equation:

$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k) - \sum_{k=1}^{N} a_k\, y(n-k). \quad \text{①}$$

- The transfer function $H(z)$ is the z-transform of impulse response $h(n)$.

$$h(n) \xrightarrow{z} H(z) = \sum_{n=0}^{\infty} h(n)\, z^{-n}.$$

To get $H(z)$ apply z-transform to eqn ①.

$$Y(z) = \sum_{k=0}^{M} b_k\, z^{-k} X(z) - \sum_{k=1}^{N} a_k\, z^{-k} Y(z). \quad \text{②}$$

if $x(n) \rightarrow X(z)$.

then $x(n-m) \xrightarrow{z} z^{-m} X(z)$.

if $y(n) \xrightarrow{z} Y(z)$

$y(n-m) \xrightarrow{z} z^{-m} Y(z)$.

re arranging ②

$$Y(z)\left[1+\sum_{k=1}^{N} a_k z^{-k}\right] = X(z)\sum_{k=0}^{M} b_k z^{-k} \quad \text{③}$$

The transfer function $H(z)$ is also defined as:

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{z \text{ transform of output, } y(n)}{z \text{ transform of input, } x(n)}$$

∴ from equation ③

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}}$$

— The design of IIR filter for a given specification is finding filter coefficients $\{a_k\}$ and $\{b_k\}$ of above equation.

5. Infinite Impulse Response Filters

5.1 Introduction

Basically a digital filter is a linear time-invariant discrete time system. The terms Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) are used to distinguish filter types. The FIR filters are of non-recursive type, whereby the present output sample depends on the present input sample and previous input samples, whereas the IIR filters are of recursive type, whereby the present output sample depends on the present input, past input samples and output samples. The properties and the design of FIR filters are discussed in detail in chapter 6. In this chapter the design of IIR filters that are realizable and stable are discussed in detail.

The impulse response $h(n)$ for a realizable filter is

$$h(n) = 0 \quad \text{for} \quad n \leq 0 \tag{5.1a}$$

and for stability it must satisfy the condition

$$\sum_{n=0}^{\infty} |h(n)| < \infty. \tag{5.1b}$$

IIR digital filters have the transfer function of the form

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{5.2}$$

The design of an IIR filter for the given specifications is to find filter coefficients $a_k s$ and $b_k s$ of Eq.(5.2).

5.2 Frequency Selective Filters

A filter is one, which rejects unwanted frequencies from the input signal and allow the desired frequencies. The range of frequencies of signal that are passed through the filter is called passband and those frequencies that are blocked is called stopband.

The filters are of different types.

1. Lowpass filter, 2. Highpass filter, 3.Bandpas filter, 4. Bandreject filter.

1. *Lowpass filter*

The magnitude response of an ideal lowpass filter allows low frequencies in the passband $0 < \Omega < \Omega_c$ to pass, whereas the higher frequencies in the stopband $\Omega > \Omega_c$ are blocked. The frequency Ω_c between the two bands is the cutoff frequency, where the magnitude $|H(j\Omega)| = 1/\sqrt{2}$.

In practice it is impossible to obtain the ideal response. The practical response of a lowpass filter is shown in solid line in Fig. 5.1a.

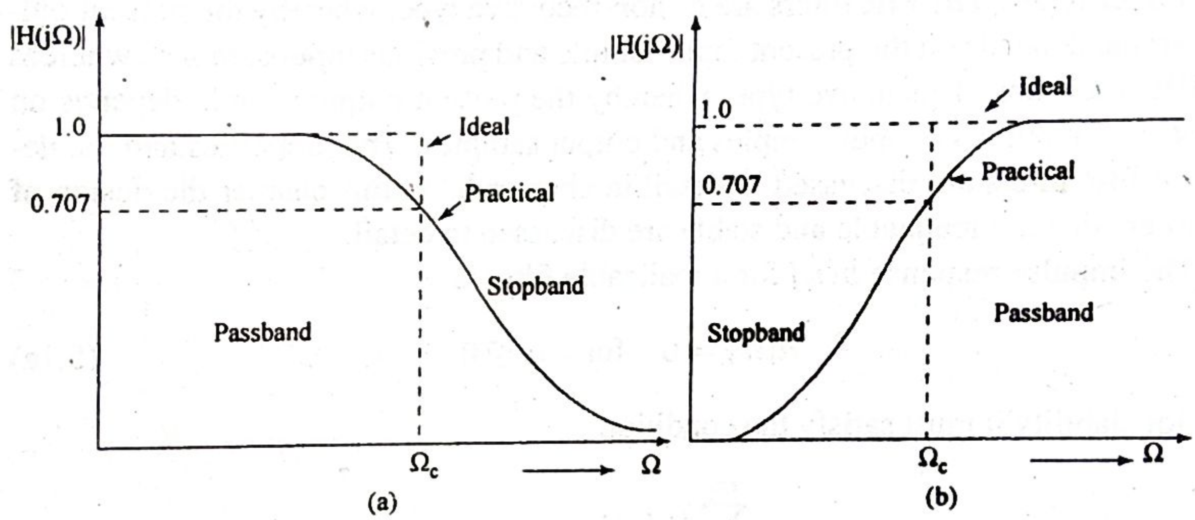


Fig. 5.1 Magnitude response of filters (a) Lowpass (b) Highpass

2. *Highpass filter*

The highpass filter allows high frequencies above $\Omega > \Omega_c$ and rejects the frequencies between $\Omega = 0$ and $\Omega = \Omega_c$. The magnitude response of an ideal and practical highpass filter is shown in Fig. 5.1b.

3. *Bandpass filter*

It allows only a band of frequencies Ω_1 to Ω_2 to pass and stops all other frequencies. The ideal and practical response of bandpass filter are shown in Fig. 5.2.

4. *Bandreject filter*

It rejects all the frequencies between Ω_1 and Ω_2 and allows remaining frequencies. The magnitude response of an ideal and practical filters is shown in Fig. 5.3.

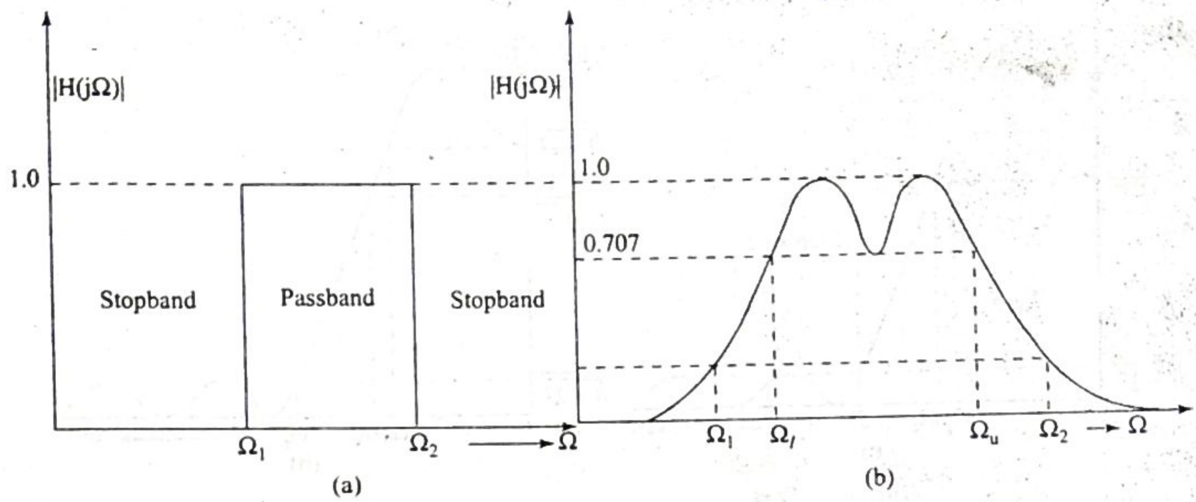


Fig. 5.2 Magnitude response of Bandpass filter (a)Ideal (b) Practical

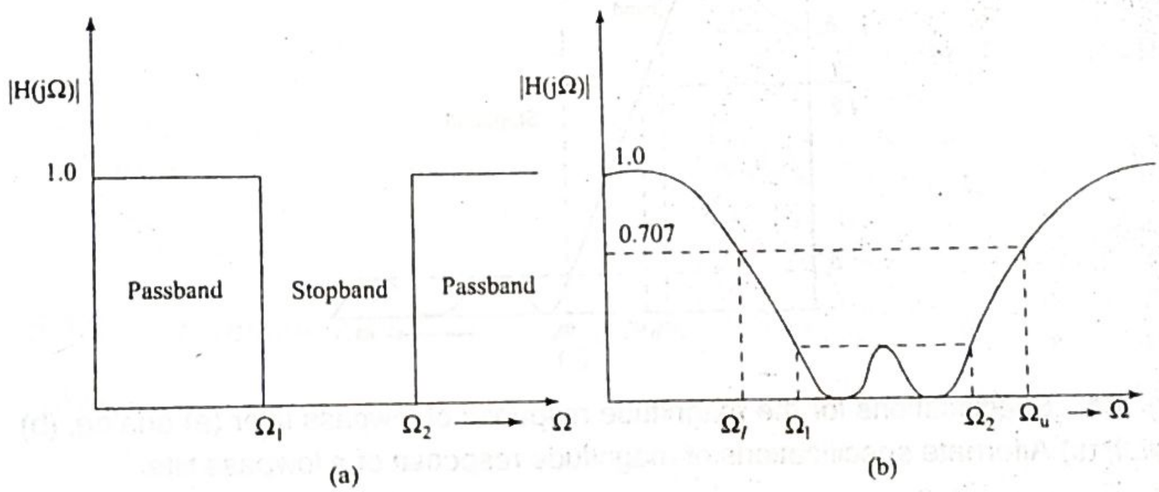


Fig. 5.3 Magnitude response of Bandreject filter (a) Ideal (b) Practical

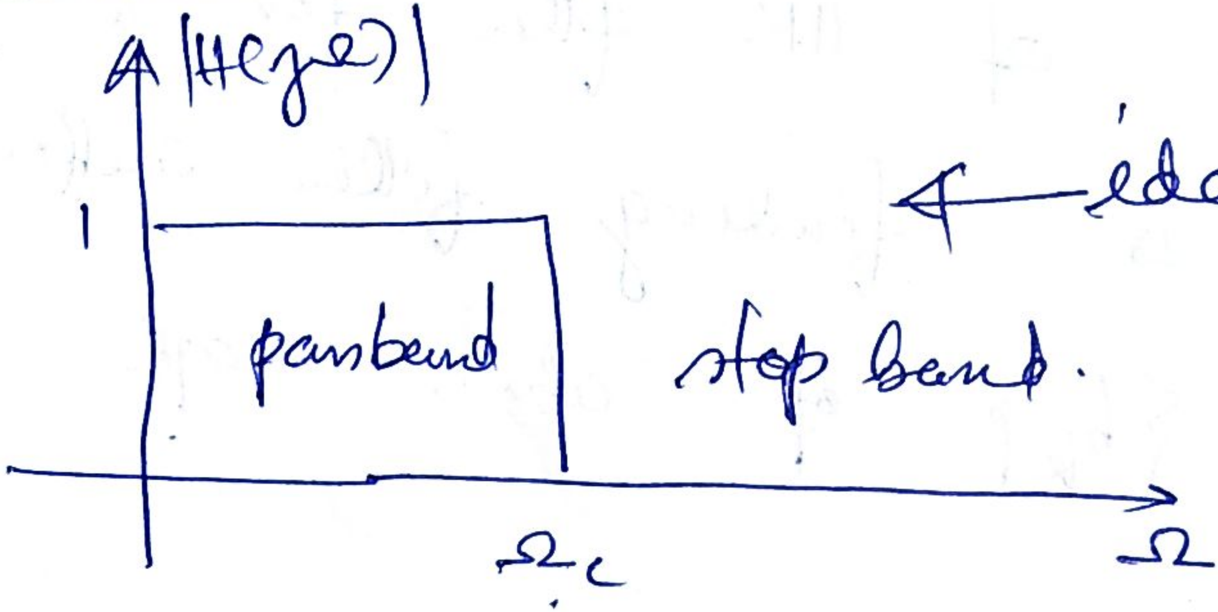
5.3 Design of Digital filters from Analog filters

The most common technique used for designing IIR digital filters known as indirect method, involves first designing an analog prototype filter and then transforming the prototype to a digital filter. For the given specifications of a digital filter, the derivation of the digital filter transfer function requires three steps.

1. Map the desired digital filter specifications into those for an equivalent analog filter.
2. Derive the analog transfer function for the analog prototype.
3. Transform the transfer function of the analog prototype into an equivalent digital filter transfer function.

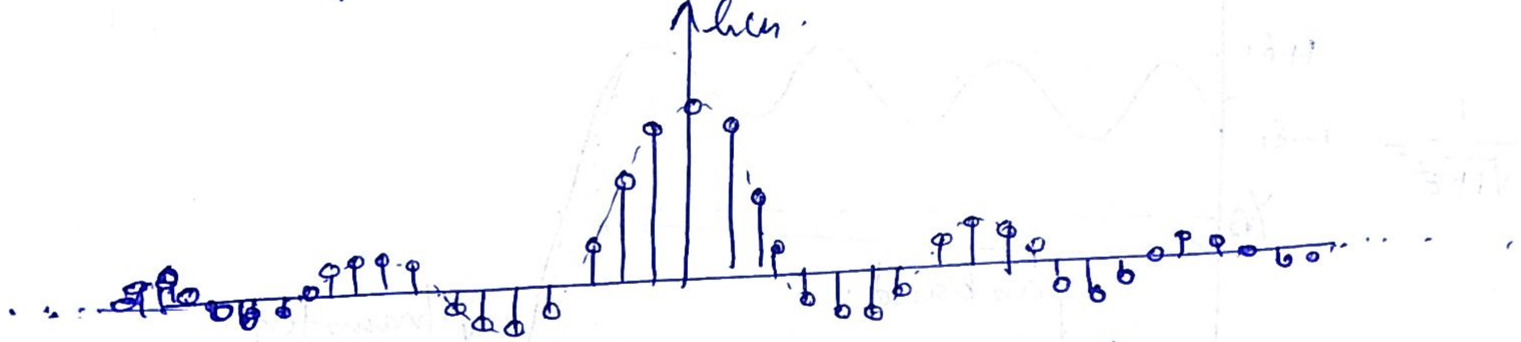
Fig. 5.4b shows the magnitude response of a digital lowpass filter. The various parameters in the figure are

Magnitude response of low pass filter



But ideal filters are not stable and causal and therefore they are not physically realizable.

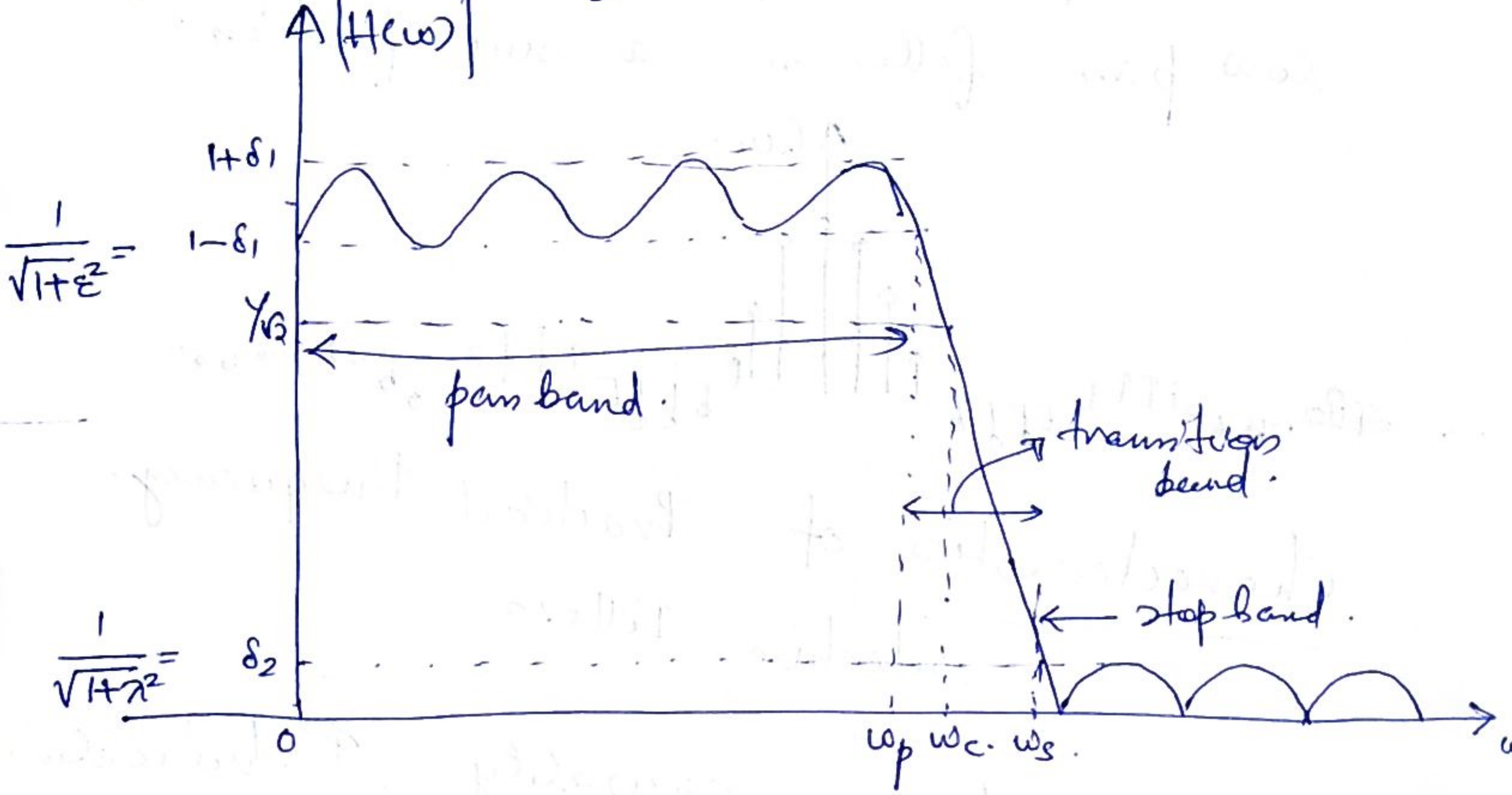
- The impulse response of an ideal low pass filter is a sinc function.



Characteristics of Practical Frequency-Selective Filters

- To achieve causality, ~~B~~ truncation of h(n) results in Gibb's phenomenon.
- i.e small amount of ripple in pass band (which is tolerable)., and a ~~a~~ small amount of ripple in stop ~~band~~ band (which is tolerable) and transition from passband to stop band

is not sharp and the transition of frequency response from passband to stopband define transition band or transition region.



$\omega_c \rightarrow$ cutoff frequency.

$\omega_p \rightarrow$ edge of pass band.

$\omega_s \rightarrow$ edge of stop band (begining of stop band).

$\omega_s - \omega_p \rightarrow$ width of transition band.

band width $\rightarrow$ width of passband is usually called bandwidth of the filter.

$\delta_p \longrightarrow$ ripples in the pass band.

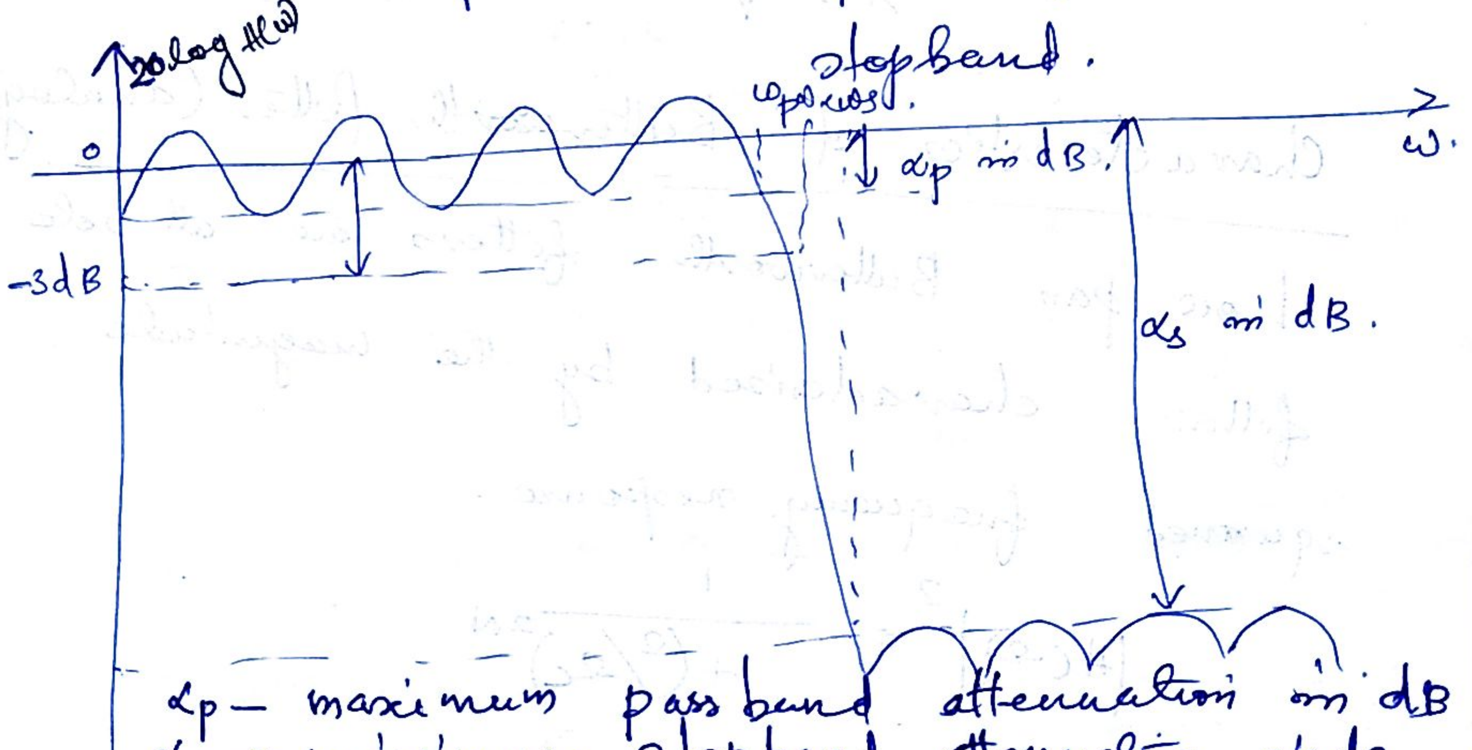
$\delta_s \longrightarrow$ ripples in the stopband.

To accomodate large dynamic range in graph of the frequency response of the filter it is common practice to use a logarithmic scale for $|H(\omega)|$

— In any filter design problem a filter is specified by. δ_p, δ_s, ω_s and ω_p.

$\varepsilon \longrightarrow$ parameter specifying allowable passband

$\lambda \longrightarrow$ parameter specifying allowable stopband.

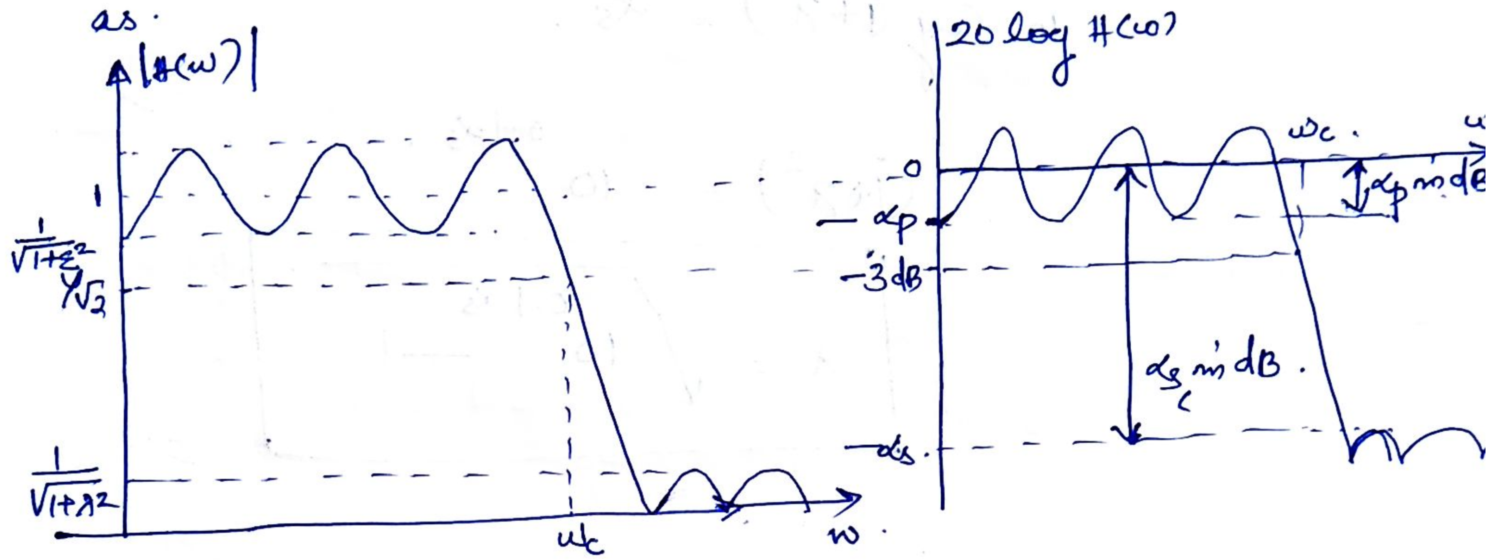


α_p — maximum pass band attenuation in dB

$\alpha_s \rightarrow$ minimum stopband attenuation in dB

Relation between (ε and α_p) and (λ and α_s)

Frequency response of a digital filter in normal scale and dB scale is as.



From these two plots.

$$20 \log \frac{1}{\sqrt{1+\varepsilon^2}} = -\alpha_p$$

$$-20 \log\left(\sqrt{1+\varepsilon^2}\right) = -\alpha_p$$

$$20 \log (1+\varepsilon^2)^{1/2} = \alpha_p$$

$$10 \log (1+\varepsilon^2) = \alpha_p$$

$$\log (1+\varepsilon^2) = 0.1\,\alpha_p$$

$$(1+\varepsilon^2) = 10^{0.1\alpha_p}$$

$$\boxed{\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}}$$

Similarly.

$$20 \log \frac{1}{\sqrt{1+\lambda^2}} = -\alpha_s.$$

$$-20 \log \sqrt{1+\lambda^2} = -\alpha_s.$$

$$10 \log (1+\lambda^2) = \alpha_s.$$

$$(1+\lambda^2) = 10^{0.1\alpha_s}.$$

$$\boxed{\lambda = \sqrt{10^{0.1\alpha_s} - 1}}$$