Module-4

- The factors affecting the choice of realization methods:
  - → Computational complexity.
  - → Memory requirement.
  - → Finite-word length effect.

- **STRUCTURES FOR FIR SYSTEMS**

In general, an FIR system is described by the difference equation;

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k).$$

or equivalently by the system function

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}.$$

$$h(n) = \begin{cases} b_n & , 0 \le n \le M-1 \\ 0 & , \text{otherwise.} \end{cases}$$

Delay element;

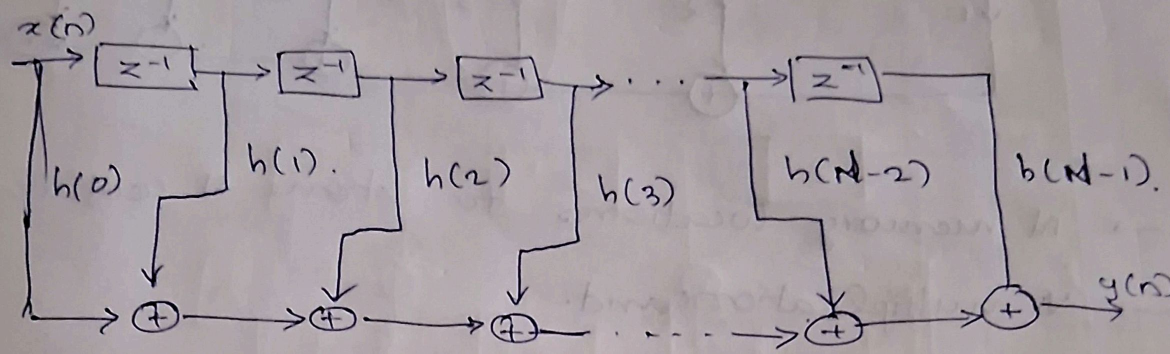
$$x[n] \overset{z}{\longleftrightarrow} x[z]$$

$$x[n-k] \overset{ZT}{\longleftrightarrow} x[z]z^{-k}$$

$$x[n-1] \overset{z}{\longleftrightarrow} x[z]z^{-1}$$

* Direct form Structure

$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$



Transversal or tapped-delay-line filter.

$$H[z] = \sum_{k=0}^{N-1} h(n) z^{-k}.$$

$$= h(0) z^{-0} + h(1) z^{-1} + h(2) z^{-2} + \dots + h(N-1) z^{-(N-1)}$$

Difference equation; $\quad x[n] z^{-1} \quad x[n] z^{-2}.$

$$y(n) = h(0) x(n) + h(1) x(n-1) + h(2) x(n-2) + \dots + h(N-1)\, x(n-(N-1)) \quad \text{— ①}$$

→ N-1 memory locatⁿ to store the N-1 i/p

? Realize an FIR filter having the TF

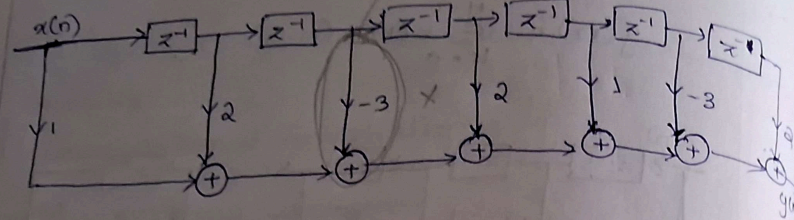
$H[z] = 1 + 2z^{-1} - 3z^{-2} + 2z^{-3} + z^{-4} - 3z^{-5} + 2z^{-6}$ in direct form

Ans: N = 7.

Comparing ①;

$h(0) = 1 \quad h(1) = 2 \quad h(2) = -3 \quad h(3) = 2 \quad h(4) = 1$

$h(5) = -3 \quad h(6) = 2. \quad$ if z^{-3} que X



→ N memory locations to store N coefficients.

→ N multiplications and.

→ N-1 additions.

* <u>LINEAR Phase Structure</u>

For type 1 and 2 linear phase filters,

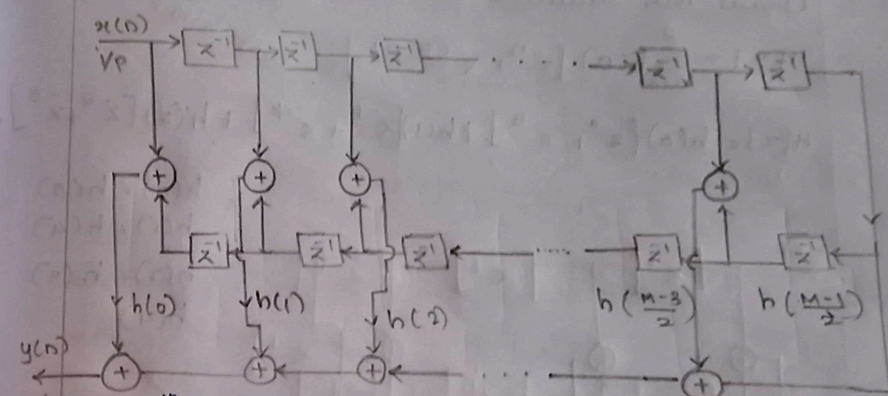
$h(n) = \pm h(M-1-n)$ ↓ adv Structure having min no. of multiplicat^n.

$$H(z) = \sum_{n=0}^{\frac{(N-1)}{2}-1} h(n)\left[z^{-n} \pm z^{-(N-1-n)}\right] + h\left(\frac{N-1}{2}\right) z^{-\frac{(N-1)}{2}}$$

N → odd.

$$H(z) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\left[z^{-n} + z^{-(N-1-n)}\right], \quad N \to \text{even}.$$

N = odd;



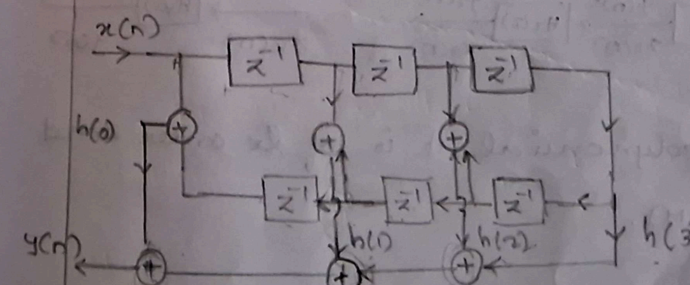
multiplicat^n reduced → $M \to M/2$ M → even $\quad M \to \frac{M-1}{2}$ for odd

? Draw the FIR filter structure with the min number of multipliers for M = 7 / N = 7.

Ans: $H[z] = \sum_{n=0}^{\frac{N-1}{2}-1} h(n)\left[z^{-n} + z^{-(N-1-n)}\right] + h\left(\frac{N-1}{2}\right) z^{-(N-1)/2}.$

$$H[z] = \sum_{n=0}^{2} \left[h(n)\left(z^{-n} + z^{-(6-n)}\right)\right] + h(3) z^{-3}$$

$$H[z] = h(0)\left(z^{-0} + z^{-6}\right) + h(1)\left(z^{-1} + z^{-5}\right) + h(2)\left(z^{-2} + z^{-4}\right) + h(3) z^{-3}.$$

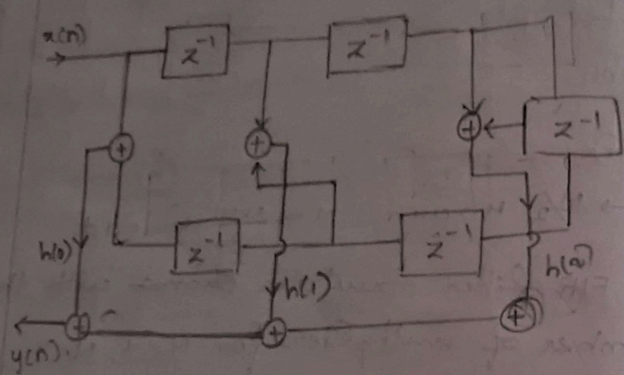


? Draw the structure for even case. Take $M = 6$.

$= H[z] = \sum_{n=0}^{2} h(n)\left[z^{-n} + z^{-(5-n)}\right]$

$H[z] = h(0)\left[z^{-0} + z^{-5}\right] + h(1)\left[z^{-1} + z^{-4}\right] + h(2)\left[z^{-2} + z^{-3}\right]$

$h(0) = h(5)$
$h(1) = h(4)$
$h(2) = h(3)$
$h(4)$



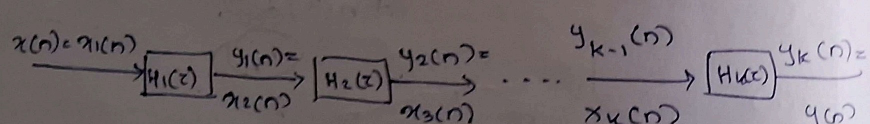
$\cancel{\frac{6-1}{2}}$

$6/2 = 3$ Mult

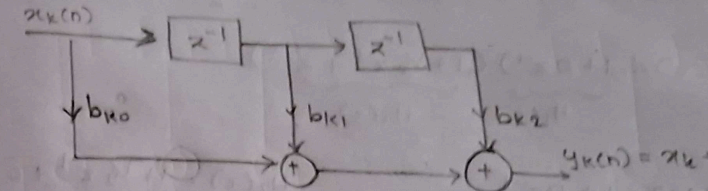
* CASCADE FORM.

$$H(z) = \prod_{k=1}^{K} H_k(z)$$

$$H_k(z) = b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2} \quad k = 1, 2, \dots K.$$



As it is
In terms of polynomial it is of the order and not length(M).



? Obtain the cascade realization of a sys function

$$H(z) = \underbrace{(1 + 2z^{-1} - z^{-2})}_{H_1(z)} \underbrace{(1 + z^{-1} + z^{-2})}_{H_2(z).}$$

(4th order)

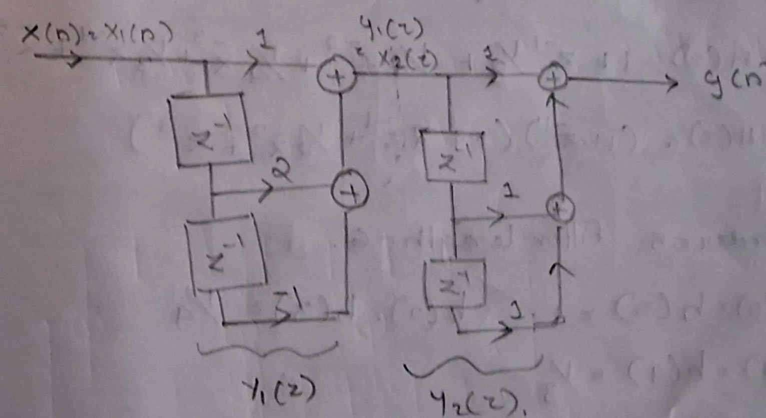
$H_1(z) = (1 + 2z^{-1} - z^{-2})$

$Y_1(z) = X_1[z] + 2z^{-1}X_1[z] - z^{-2}X_1[z].$

$H_2(z) = 1 + z^{-1} + z^{-2}.$

$Y_2[z] = X_2[z] \times H_2[z]$

$= X_2[z] + z^{-1}X_2[z] + X_2[z]z^{-2}.$



? Obtain the structure $H(z) = 1 + \frac{5}{2} z^{-1} + 2z^{-2} + 2z^{-3}$.

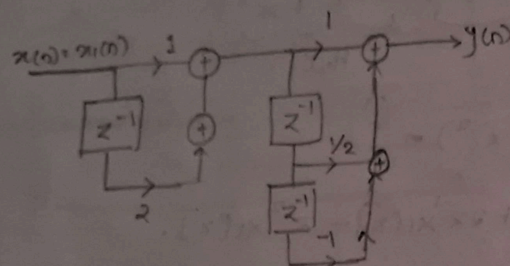
$H(z) = \underbrace{(1+2z^{-1})}_{H_1(z)} \underbrace{(1 + \frac{1}{2}z^{-1} + z^{-2})}_{H_2(z)}$

$H_2(z) = 1 + \frac{1}{2} z^{-1} + z^{-2}$

$H_1(z) = 1 + 2z^{-1}$

$Y_1[z] = X_1[z] + 2X_1[z]z^{-1}$

$Y_2(z) = X_2[z] + X_2[z]\frac{1}{2}z^{-1} + X_2[z]z^{-2}$



? Realise the following system function using min no. of multiplication.

(i) $H(z) = 1 + z^{-1}\frac{1}{3} + \frac{1}{4}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{3}z^{-4} + z^{-5}$.

(ii) $H(z) = (1+z^{-1})(1 + 2z^{-1} + \frac{1}{2}z^{-2} + z^{-3})$

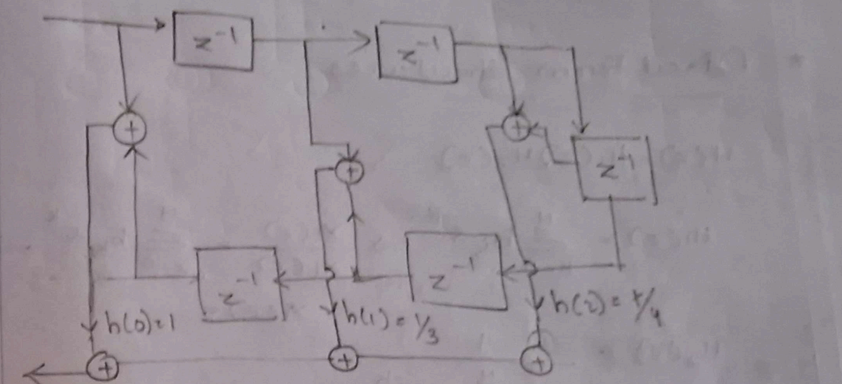
(i) Order = 5 Filter length = 6,

$h(0) = h(5) = 1 \quad h(2) = h(3) = \frac{1}{4}$

$h(1) = h(4) = \frac{1}{3}$

Hence it can be realized by min no. of multiplication. (linear phase)

(i) $H(z) = 1[z^{0} + z^{-5}] + \frac{1}{3}[z^{-1} + z^{-4}] + \frac{1}{4}[z^{-2} + z^{-3}]$



(ii) $H[z] = 1 + 2z^{-1} + \frac{1}{2}z^{-2} + z^{-3} + z^{-1} + 2 + \frac{1}{2}z^{-1} + z^{-2}$

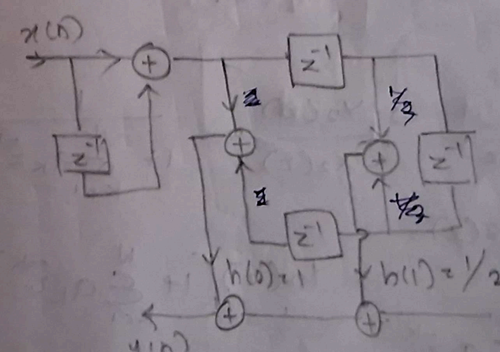
$= 3 + \frac{7}{2}z^{-1} + \frac{3}{2}z^{-2} + z^{-3}$

$H(z) = (1+z^{-1})(1 + 2z^{-1} + \frac{1}{2}z^{-2} + z^{-3})$.

Here Order = 3 Length = 4

$h(0) = h(3) = 1$

$h(1) = h(2) = \frac{1}{2}$



$1 + \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} + z^{-3}$

$+ z^{-1} + \frac{3}{2}z^{-1} + \frac{3}{2}z^{-2}$

$+ 2z^{-2} + 3z^{-2} + 5z^{-1}$

$z^{-3} + \frac{5}{2}z^{-2}$

IIR Structures

* Direct Form Structures

$H(z) = H_1(z) H_2(z)$

$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \Rightarrow \frac{Y_1(z)}{X_1(z)} = \sum_{k=0}^{M} b_k z^{-k} \quad \text{—(1)}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

(1) $\Rightarrow Y_1(z) = \sum_{k=0}^{M} b_k z^{-k} X_1(z)$

Inverse z transform;

$$y_1(n) = \sum_{k=0}^{M} b_k \, x_1(n-k).$$

$$y_1(n) = b_0 x_1(n) + b_1 x_1(n-1) + \dots + b_M x_1(n-M).$$

Illy:

$$H_2(z) = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}} \Rightarrow \frac{Y_2(z)}{X_2(z)} = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

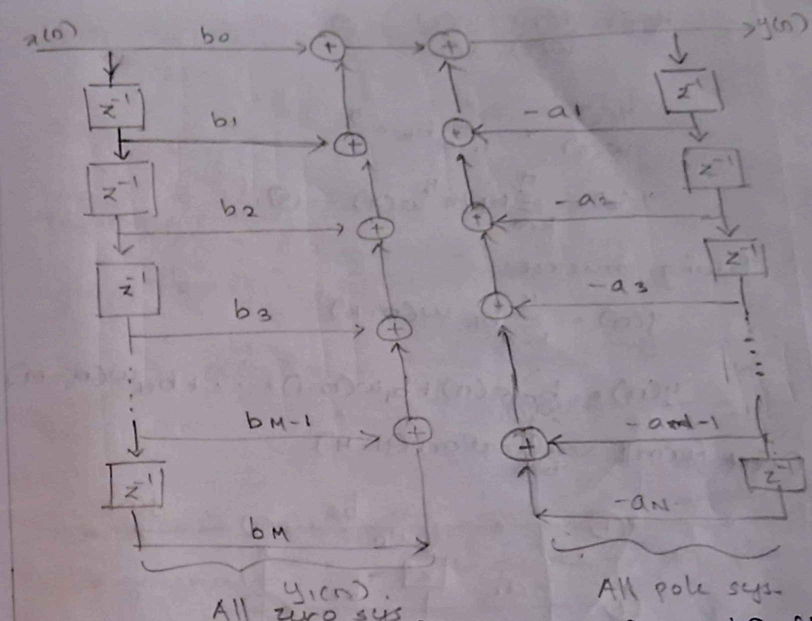
$$Y_2(z) = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}} X_2(z). \Rightarrow Y_2(z)\left(1 + \sum_{k=1}^{N} a_k z^{-k}\right) = X_2(z)$$

$$Y_2(z) = X_2(z) - \sum_{k=1}^{N} a_k z^{-k} Y_2(z).$$

Inverse;

$x_2(n) = y_1(n)$

$$y(n) = y_2(n) = x_2(n) - \sum_{k=1}^{N} a_k y(n-k).$$



$$y(n) = y_1(n) - a_1 y(n-1) - a_2 y(n-2) + \dots + a_N y(n-N).$$

disad.

This realization requires $M+N+1 \rightarrow$ multiplication

$M+N \rightarrow$ addition and $M+N+1$ memory location.

So to avoid this disadvantage we have.

→ Direct Form II structure:

It has $M+N+1 \rightarrow$ multiplication

$M+N \rightarrow$ addition.

and the max of (M,N) memory locatn

$$H[z] = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=0}^{N} a_k z^{-k}} = \frac{Y(z)}{X(z)} \Rightarrow \text{TF of IIR filter. (1)}$$

$H(z) = \frac{Y(z)}{W(z)} \frac{W(z)}{X(z)}$ — ②

$\frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k}$

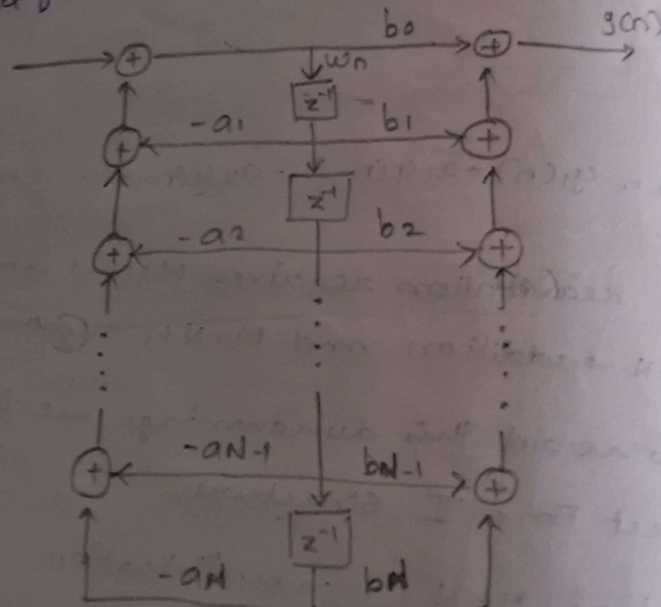
$Y(z) = \sum_{k=0}^{M} b_k z^{-k} W(z)$ — ⑤.

Taking inverse;

$y(n) = \sum_{k=0}^{M} b_k \, w(n-k)$

$y(n) = b_0 w(n) + b_1 w(n-1) + \dots + b_M w(n-M)$.

Direct form II realization (N = M)



Taking ③

$\frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$

$X(z) = W(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right]$

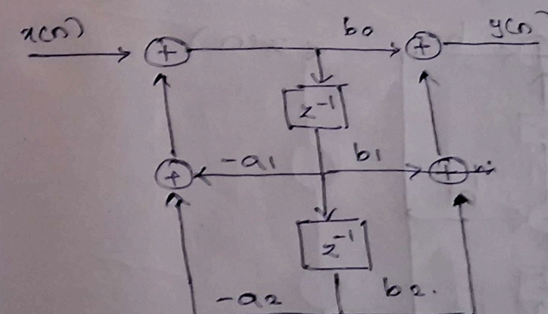
$X(z) - \sum_{k=1}^{N} a_k z^{-k} W(z) = W(z)$ — ④

Taking inverse;

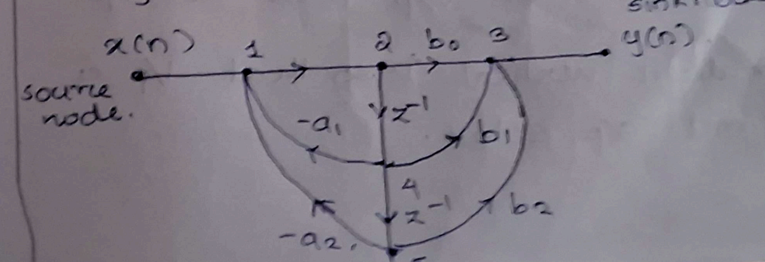
$w(n) = x(n) - \sum_{k=1}^{N} a_k w(n-k)$.

$w(n) = x(n) - a_1 w(n) - a_2 w(n-2) + \dots$
$a_N w(n-N)$.

Graphical rep of s/g from the ~~sink~~ source node (i/p generate) to sink node → s/g flow graph.



s/g flow graph

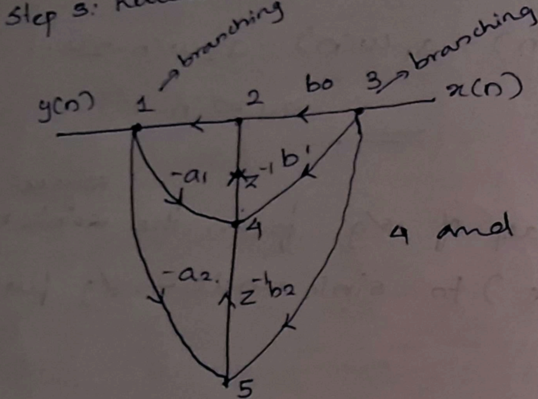


* Direct form II Transpose Structure.

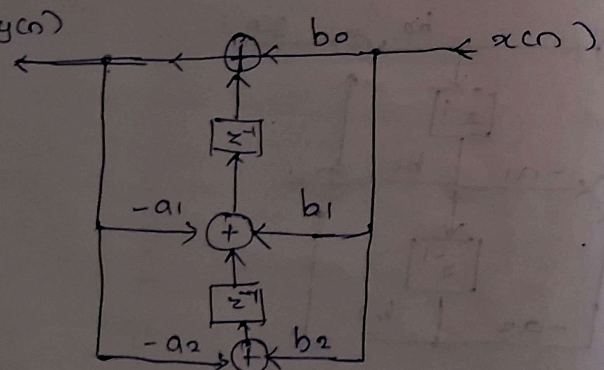
Step 1: Replace the o/p and i/p and i/p as o/p

Step 2: Replace all the summing point as branching point and all branching point as summing point.

Step 3: Reverse the direction of all the values.

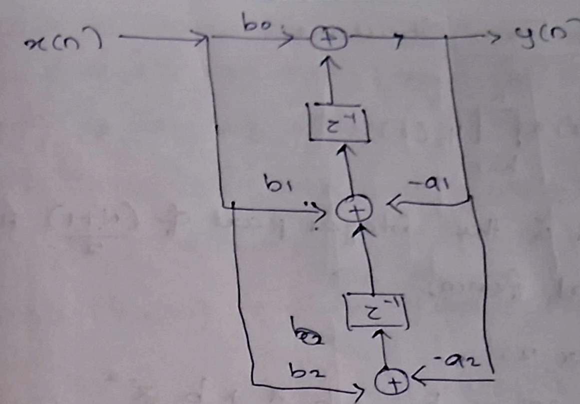


4 and 5 now summing node

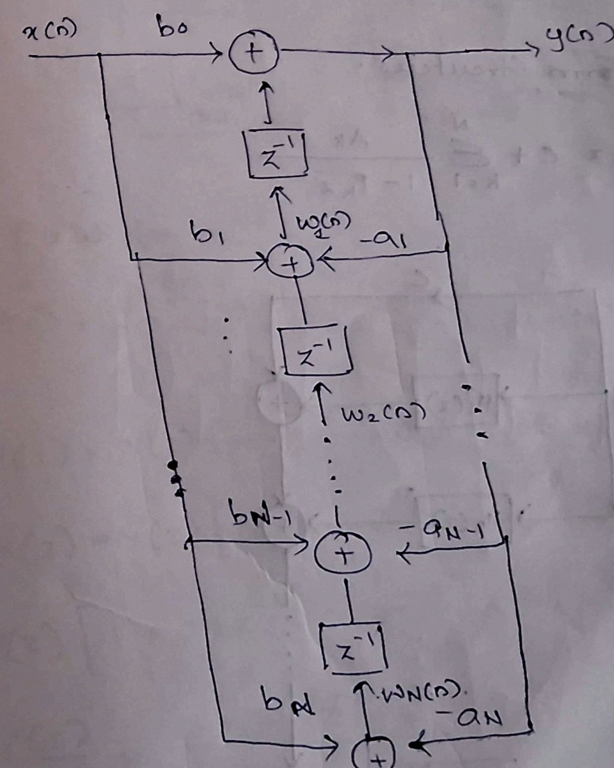


Now first we have o/p and last i/p

Now make it vertical so that i/p is first and o/p is last.



General case;



* Cascade Form Structure.

$$H(z) = \prod_{k=1}^{K} H_k(z).$$

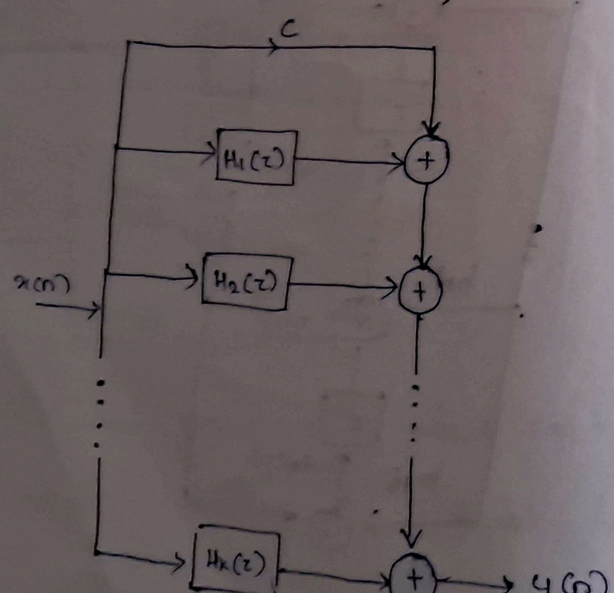
where K is the integer part of $\frac{(N+1)}{2}$ $H_k(z)$ is the general form;

If order 2;

$$H_k(z) = \frac{b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}}$$

* Parallel Form Structure.

$$H(z) = C + \sum_{k=1}^{N} \frac{A_k}{1 - P_k z^{-1}}$$

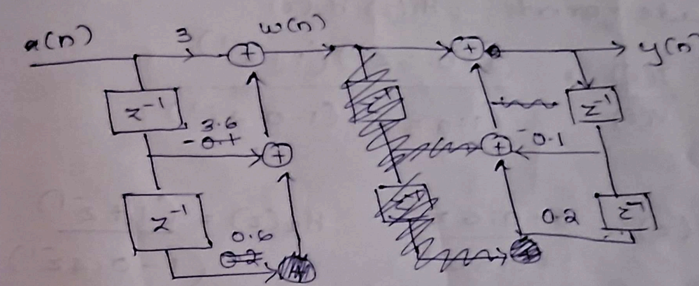


Q. Obtain the direct form I and direct form II cascade and parallel form for the following system given by difference equation

$y(n) = -0.1 y(n-1) + 0.2 y(n-2) + 3x(n) + 3.6 x(n-1) + 0.6 x(n-2)$

Ans:

$w_1(n) = 3x(n) - 3.6x(n-1) + 0.6x(n-2)$ $[y_1(n)]$



Direct form I.

Direct form II :

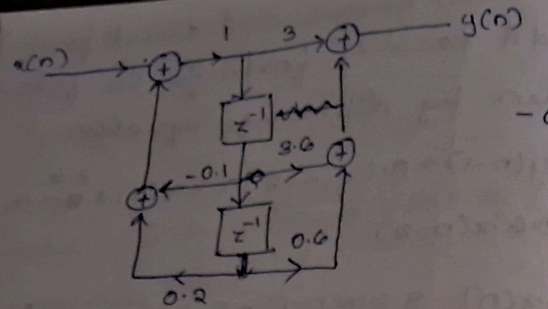
$$Y(z) = -0.1 z^{-1} y(z) + 0.2 z^{-2} y(z) + 3x(z) + 3.6 x(z) z^{-1} + 0.6 x(z) z^{-2}.$$

$$Y(z) = Y(z)[-0.1 z^{-1} + 0.2 z^{-2}] + X[z][3 + 3.6 z^{-1} + 0.6 z^{-2}]$$

$$Y(z) - Y(z)[-0.1 z^{-1} + 0.2 z^{-2}] = X[z][\;"\;]$$

$$Y(z)[1 + 0.1 z^{-1} - 0.2 z^{-2}] = X[z][3 + 3.6 z^{-1} + 0.6 z^{-2}]$$

$$\frac{Y(z)}{X(z)} = \frac{3 + 3.6 z^{-1} + 0.6 z^{-2}}{1 + 0.1 z^{-1} - 0.2 z^{-2}}$$

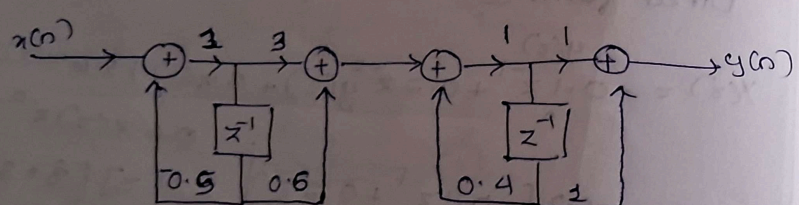


$-(-0.1)$

Cascade form: $H_1(z)\,H_2(z)$

$$\frac{Y(z)}{X(z)} = \frac{(3+0.6z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

$$H_1(z) = \frac{3+0.6z^{-1}}{1+0.5z^{-1}} \qquad H_2(z) = \frac{(1+z^{-1})}{(1-0.4z^{-1})}$$



Parallel form:

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

the form; $\times z^{2}$

$$H(z) = \frac{3z^2+3.6z+0.6}{z^2+0.1z-0.2}$$

$\div z$;

$$\frac{H(z)}{z} = \frac{3z^2+3.6z+0.6}{z(z^2+0.1z-0.2)}$$

$$= \frac{3z^2+3.6z+0.6}{z(z-0.4)(z+0.5)}$$

$$\frac{H(z)}{z} = \frac{A}{z} + \frac{B}{z-0.4} + \frac{C}{z+0.5}$$

$$\frac{H(z)}{z} = A(z-0.4)(z+0.5) + Bz(z+0.5) + Cz(z-0.4)$$

$3z^2+3.6z+0.6 = A(z-0.4)(z+0.5) + Bz(z+0.5) + Cz(z-0.4)$.

$z=0$;

$0.6 = A(-0.4)(0.5)$

$A = \frac{0.6}{-0.2} = \underline{\underline{-3}}$

$z=0.4$;

$2.52 = B\,0.36$

$\underline{\underline{B=7}}$

$z=-0.5$;

$0.45 = C\,0.45$

$\underline{\underline{C=-1}}$

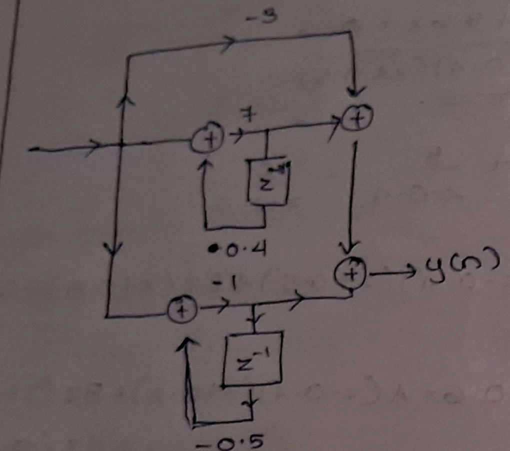
$$\frac{H(z)}{z} = \frac{-3}{z} + \frac{7}{z-0.4} + \frac{-1}{z+0.5}$$

$$\frac{H(z)}{z} = \frac{-3}{z} + \frac{7}{z-0.4} + \frac{-1}{z+0.5}$$

$\frac{-3}{z^{-2}} + \frac{7}{z^{-2}-0.4z^{-1}} + \frac{-1}{}$

$\times z;$

$$H(z) = -3 + \frac{7}{1-0.4z^{-1}} + \frac{1}{1+0.5z^{-1}}$$



Transpose direct II form:

S/g from direct II form.