ECT401	MICROWAVES AND ANTENNAS	CATEGORY	L	T	P	CREDIT
		PCC	2	1	0	3

Preamble: This course aims to impart knowledge on the basic parameters of antenna, design and workingofvariousbroad band antennas,arraysand itsradiationpatterns.It also introduces various microwave sources, their principle of operation and study of various microwave hybrid circuits and microwave semiconductor devices.

Prerequisite: ECT 302 ELECTROMAGNETICS

Course Out Comes: After the completion of the course the student will be able to:

CO1-K2	Understand the basic conceptofantennas anditsparameters.
CO2-K3	Analyzethe far filed pattern of Short dipole and Half wave dipole antenna.
CO3-K3	Design of various broad band antennas, arrays and its radiation patterns.
CO4-K2	Illustratethe principle of operation of cavity resonators and various microwave sources.
CO5-K2	Explain various microwave hybrid circuits and microwave semiconductor devices.

Mapping of course outcomes with program outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	3		1								2
CO2	3	3	3	1	2							2
CO3	3	3	3	1	3							2
CO4	3	3	2	1								2
CO5	3	3	2	1								2

Assessment Pattern:

Bloom's Category	**Continuous Assessment Tests**		**End Semester Examination**
	1	**2**	
Remember			
Understand K2	20	20	40
Apply K3	30	30	60
Analyse			
Evaluate			
Create			

Mark distribution:

Total Marks	CIE	ESE	ESE Duration
150	50	100	3 Hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks
Continuous Assessment Test (2 numbers) : 25 marks
Assignment/Quiz/Course project : 15 marks

End Semester Examination Pattern

Maximum Marks: 100 **Time: 3 hours**

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 subdivisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Define isotropic radiator and derive the expression for its electric field strength.
2. Explain the terms
   i) Antenna temperature ii) Antenna efficiency iii) Beam efficiency
   iv) Radiation pattern v) Antenna Polarization
3. Show that the directivity of a half wave dipole is 4 (from the expression for average power).
4. Find the radiation intensity of a current element with corresponding field strength in the direction of maximum radiation of $E_m = \frac{60}{r\sqrt{80}} V/m$

Course Outcome 2 (CO2):

1. Show that the directivity of a half wave dipole is 4 (from the expression for average power).
2. Derive expressions for the Far Field components and Radiation Resistance and Directivity of a short dipole antenna.
3. State and Prove Reciprocity Theorem.

Course Outcome 3 (CO3):

1. Derive the relation for normalized electrical field in the case of ‘n’ isotropic array sources $En= (AF)_n$.
2. Explain the working of a horn antenna. Write down the expression for gain, HPBW and BWFN.
3. Design an Endfire Array and plot its radiation pattern.
4. Design a LPDA with $\tau = 0.85$, $\sigma = 0.03$ for the frequency range 15-45 MHz.

Course Outcome 4 (CO4):

1. Determine the resonant frequency of an air filled rectangular cavity operating in the dominant mode with dimensions as,a=4cm, b=5cm and d=6cm.
2. Derive power output and efficiency of a reflex klystron.
3. What is the significance of slow wave structures used in microwave circuits?Explain different slow wave structures with neat sketches.
4. With neat diagram explain the operation of a travelling wave tube.
5. With the help of figures explain the bunching process of an 8-cavity cylindrical magnetron.

Course Outcome 5 (CO5):

1. Explain S-parameters and its properties.
2. With a schematic describe the operation of a four port circulator. Obtain the simplified S matrix of a perfectly matched, lossless four port circulator.
3. Explain RWH theory of Gunn Oscillation.
4. Define Gunn Effect and with the help of figures explain different modes of operation of Gunn diode.

Syllabus

Module	Course contents	Hours
I	**Basic antenna parameters:** gain, directivity, beam width and effective aperture calculations, effective height, wave polarization, radiation resistance, radiation efficiency, antenna field zones. Duality and Principles of reciprocity, Helmholtz theorem (derivation required), Field, directivity and radiation resistance of a short dipole and half wave dipole (far field derivation).	7
II	**Broad band antenna:** Principle of Log periodic antenna array and design, Helical antenna: types and design. Designof Microstrip RectangularPatchantennas and feeding methods. Principles of Horn, Parabolic dish antenna (expression for E, H and Gain without derivation), Mobile phone antenna – Inverted F antenna.	6
III	**Arrays of point sources**, field of two isotropic point sources, principle of pattern multiplication, linear arrays of 'n' isotropic point sources. Array factor, Grating lobes.DesignofBroadside,End fire andDolph Chebyshevarrays. Concept of Phase array.	8
IV	**Microwaves:**Introduction, advantages, Cavity Resonators- Derivation of resonance frequency of Rectangular cavity. **Single cavity klystron- Reflex Klystron** Oscillators: Derivation of Power output, efficiency and admittance. **Magnetron oscillators:** Cylindrical magnetron, Cyclotron angular frequency, Power output and efficiency.**Travelling Wave Tube:** Slow wave structures, Helix TWT,Amplification process, Derivation of convection current, axialelectric field, wave modes and gain.	8

V	**Microwave Hybrid circuits**: Scattering parameters, Waveguide Tees- Magic tees, Hybrid rings.Formulation of S-matrix.Directional couplers: Two hole directional couplers, S-matrix. Circulators and Isolators. Phase Shifter. **Microwave Semiconductor Devices**: Amplifiers using MESFET. Principle of Gunn diodes: Different modes, Principle of operation Gunn Diode Oscillators.	6

Text Books:

1. Balanis, Antenna Theory and Design, 3/e, Wiley Publications.
2. John D. Krauss, Antennas for all Applications, 3/e, TMH.
3. K D Prasad, Antenna and Wave Propagation, Satyaprakash Publications
4. Samuel Y. Liao, Microwave Devices and Circuits, 3/e, Pearson Education, 2003.
5. Robert E. Collin, Foundation of Microwave Engineering, 2/e, Wiley India, 2012.

References:

1. Collin R.E, Antennas & Radio Wave Propagation, McGraw Hill. 1985.
2. Jordan E.C. & K. G. Balmain, Electromagnetic Waves & Radiating Systems, 2/e, PHI.
3. Raju G.S.N., Antenna and Wave Propagation, Pearson, 2013.
4. Sisir K.Das& Annapurna Das, Antenna and Wave Propagation, McGraw Hill,2012
5. Thomas A.Milligan, Modern Antenna Design, IEEE PRESS, 2/e, Wiley Inter science.
6. Das, Microwave Engineering, 3/e, McGraw Hill Education India Education , 2014
7. David M. Pozar, Microwave Engineering,4/e, Wiley India, 2012.

Course Contents and Lecture Schedule.

No	Topic	No.of. Lectures
	Module I	
1.1	Basicantennaparameters (all parameters and related simple problems), Relation between parameters (derivation required)	**2**
1.2	Principlesofreciprocity (proof required), Duality. Conceptofretardedpotential	**1**
1.3	Helmholtz theorem (derivation required)	
1.4	Derivation of Field, directivity and radiation resistance of a short dipole	**2**
1.5	Derivation of Field, directivity and radiation resistance of a half wave dipole.	**2**
	Module II	
2.1	Principle of Log periodic antenna array and design, Helical antenna: types and design	**2**
2.2	Design of Rectangular Patch antennas and feeding techniques	**2**
2.3	Principles of Horn,Parabolic dish antenna, (expression for E, H, G without derivation).	**1**
2.4	Mobile phone antenna – Inverted F antenna.	**1**
	Module III	

3.1	Arrays of point sources, field of two isotropic point sources, principle of pattern multiplication	**2**
3.2	Linear arrays of 'n' isotropic point sources. Grating lobes. Array factor (derivation)	**2**
3.3	Design of Broadside, End fire and Dolph Chebyshev arrays.	**3**
3.4	Concept of Phase array.	**1**
	Module IV	
4.1	**Microwaves:**Introduction, advantages, Cavity Resonators-Types, Derivation of resonance frequency of Rectangular cavity (problems required)	**1**
4.2	**Single cavity klystron- Reflex Klystron** Oscillators: Derivation of Power output, efficiency and admittance.(problems required)	**2**
4.3	**Magnetron oscillators:** Cylindrical magnetron, Cyclotron angular frequency, Power output and efficiency.(problems required)	**2**
4.4	**Travelling Wave Tube:** Slow wave structures, Helix TWT,Amplification process, Derivation of convection current, axialelectric field, wave modes and gain. (problems required)	**3**
	Module V	
5.1	**Microwave Hybrid circuits**: Scattering parameters, Waveguide Tees- Magic tees, Hybrid rings.Formulation of S-matrix.	**1**
5.2	Directional couplers: Two hole directional couplers, S-matrix. Circulators and Isolators. Phase Shifter.	**2**
5.3	**Microwave Semiconductor Devices**: Amplifiers using MESFET.	**1**
5.4	Principle of Gunn diodes: Different modes, Principle of operation Gunn Diode Oscillators.	**2**

Simulation Assignments (ECT 401)

The following simulation assignments can be done with MATLAB/HFSS/CST Microwave Studioor any Open software.

- Simulation of radiation pattern of
    a) Microstrip patch antenna
    b) Arrays
    c) Helical antenna

Model Question paper

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SEVENTH SEMESTER B. TECH DEGREE EXAMINATION

Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

Max. Marks:100 Duration: 3 Hours

PART A

(*Answer All Questions*)

1	Derive an expression for aperture area of an antenna.	(3)
2	(i) Obtain the radiation resistance of a thin dipole antenna of length λ/15.	(3)
	(ii) Find HPBW of an antenna which has a field given by: $E(\theta) = Cos^2\theta, for\ \ 0 \le \theta \le 90^0$.	
3	Why Log Periodic antenna is called as Frequency Independent antenna, explain?	(3)
4	Briefly explain about Inverted F antenna.	(3)
5	Explain (i) Pattern Multiplication (ii) Grating lobes	(3)
6	Demonstrate the working principle of Phase Arrays.	(3)
7	Derive the resonant frequency of a rectangular cavity resonator.	(3)
8	What are re-entrant cavities? Show that they support infinite number of resonant frequencies.	(3)
9	Explain with figure a ferrite isolator can support only forward direction waves.	(3)
10	Write a short note on Phase shifter.	(3)

PART B

(*Answer one question from each module. Each question carries 14 marks*)

MODULE I

11 a)	Define the terms (i) Retarded potential (ii) Antenna field zones	(4)
b)	Derive expressions for the Far Field components and Radiation Resistance and Directivity of a short dipole antenna.	(10)

OR

12a)	State and prove Helmholtz theorem	(7)
b)	(i) Compute the radiation resistance, power radiated and efficiency of an antenna having total resistance of 50Ω and effective height of 69.96m and a current of 50A (rms) at 0.480MHz. (ii) Calculate the effective aperture of a short dipole antenna operating at 100 MHz.	(7)

MODULE II

13 a) Explain the working of a parabolic dish antenna. Write down the expression for gain, HPBW and BWFN. (6)

b) Design a rectangular microstrip antenna using a dielectric substrate with dielectric constant of 2.2, h = 0.1588 cm so as to resonate at 10 GHz. (8)

OR

14 a) Explain the working of a Log periodic dipole array and explain its design steps. (7)

b) Explain axial mode helical antenna. Write down the expression for gain, HPBW,BWFN and radiation resistance of axial mode helical antenna. (7)

MODULE III

15 Derive expression for array factor of N isotropic sources for end-fire array and also the expression for major lobe, minor lobes and Nulls of the array. (14)

OR

16 a) Explain Chebyshev array and write down the expression for array factor. (7)

b) Design a Broadside Array and plot its radiation pattern. (7)

MODULE IV

17a) A reflex klystron operates under the following conditions: Vo=500V, Rsh =10KΩ, fr= 8 GHz, L=1 mm, e/m = l.759 x 1011 (MKS system) The tube is oscillating at fr at the peak of the n = 2 or mode. Assume that the transit time through the gap and beam loading to be neglected. Determine: - (7)

(a) The value of the repeller voltage Vr.
(b) The direct current necessary to give a microwave gap voltage of 200V.
(c) The electronic efficiency under this condition.

b) Assuming pi mode of oscillations explain how a magnetron can sustain its oscillations using the cross field. (7)

OR

18 a) Show that the axial electric field of TWT varies with convection current. (7)

b) Explain the electronic admittance of the gap in the case of reflex klystron. With admittance diagram explain the condition required for oscillation in a reflex Klystron. (7)

MODULE V

19 a) Explain the working of a microwave amplifiers using MESFET (8)

b) Explain the constructional features of two-hole directional coupler and derive the S Matrix. (6)

OR

20 a) Draw the J-E characteristics of Gunn diode and explain its operation. (10)

b) Discuss the constructional features of magic tees and derive its S Matrix. Why are they called so? (4)

ECL411	ELECTROMAGNETICS LAB	CATEGORY	L	T	P	CREDIT
		PCC	0	0	3	2

Preamble: This course aims to

(i) Provide practical experience in design and analysis of few electronic devices and circuits used for Microwave and Optical communication engineering.

(ii) Familiarize students with simulation of basic Antenna experiments with simulation tools.

Prerequisite: Nil

Course Outcomes: After the completion of the course the student will be able to

CO1	Familiarize the basic Microwave components and to analyse few microwave measurements and its parameters.
CO2	Understand the principles of fiber-optic communications and the different kind of losses, signal distortion and other signal degradation factors.
CO3	Design and simulate basic antenna experiments with simulation tools.

Mapping of course outcomes with program outcomes

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	3	3						3			3
CO2	3	3	3						3			3
CO3	3	3	3	2	3				3			3

Assessment Mark distribution

Total Marks	CIE	ESE	ESE Duration
150	75	75	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 15 marks
Continuous Assessment : 30 marks
Internal Test (Immediately before the second series test): 30 marks

Microwave & Antenna

Module 1

- Basic antenna parameters (all parameters & related simple problems). Relation between parameters (derivation required)
- Principles of reciprocity (proof required). Duality. Concept of retarded potential.
- Helmholtz theorem (derivation required)
- Derivation of Field, directivity and radiation resistance of a short dipole.
- Derivation of Field, directivity and radiation resistance of a half wave dipole.

Antenna

An antenna is a device that radiates or receives electromagnetic wave. It acts as a transitional structure between free space & guiding structure. The guiding structure could be a transmission line like a coaxial cable or waveguide. The guiding structure transports energy to antenna or from the antenna. Antenna not only transmits or receives energy, it optimizes or accelerates the radiated energy in some direction & suppress it in others. Antenna is a reciprocal device that is same antenna can be used for transmission & reception.

Different types of antenna

1. wire antenna eg dipole antenna
   loop antenna
   helix antenna.
2. Aperture antenna
   eg horn antenna
3. microstrip antenna
4. reflector antenna
   eg parabolic reflector
   corner reflector
5. lens antenna

How does an antenna radiate? (Radiation Mechanism)

- For radiation, there must be a time varying current or acceleration (or deceleration) of charge. For acceleration or deceleration of charge, the wire must be curved, bent, discontinous, terminated or truncated.

- For example, when an open ended conducting wire, is initially energized, the charges in the wire, are accelerated at the source end of the wire and decelerated due to reflection from its end. The accelerated charges in motion produces time varying magnetic field, which in turn produces a time varying electric field.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$(2)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$(1)$$

① Time varying current produces a magnetic field

② Time varying magnetic field produces an electric field

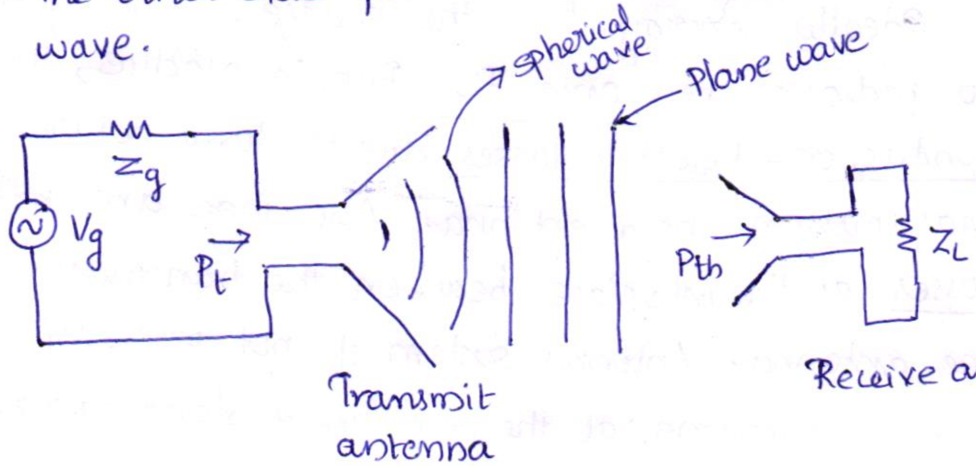
③ Time varying electric field produces magnetic field.

Equivalent circuit of antenna

An antenna is inherently a bidirectional device (or reciprocal) ie they can be used as a transmitter or receiver.

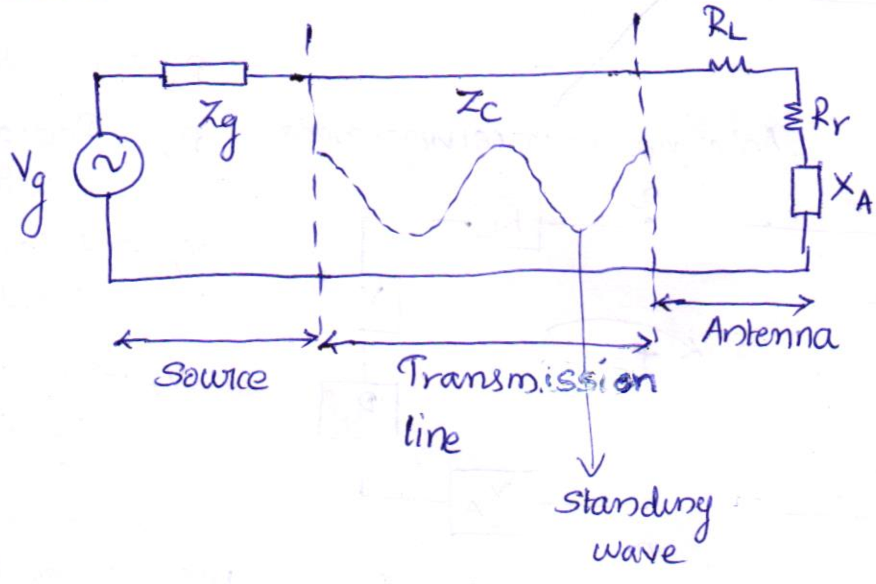
Basic operation of transmitting & receiving antenna.

A transmitting antenna can be viewed as a device that converts a guided electromagnetic wave on a transmission line into a plane wave propagating in free space. Thus one side of an antenna appears as an electrical circuit element, while the other side provides an interface with a propagating plane wave.



A receiving antenna intercepts a portion of an incident plane wave & delivers a receive power P_r to the receiver load impedance.

Thevenin equivalent of antenna system in transmitting mode



V_g → Voltage generator

Z_g → source impedance

Z_c → characteristic impedance of the transmission line

The transmitting antenna is represented by a load $Z_a = (R_L + R_r) + jX_a$ connected to the transmission line.

$R_L \rightarrow$ is resistance that accounts for conduction & dielectric losses. (loss resistance of antenna)

$R_r \rightarrow$ Radiation resistance of antenna

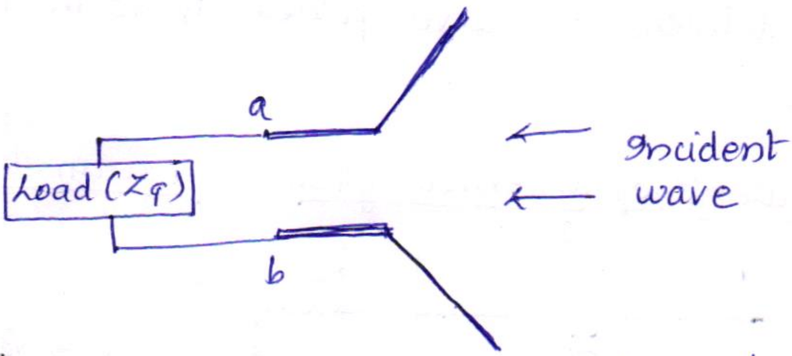
$X_A \rightarrow$ represents imaginary part of the impedance associated with radiation by the antenna. (antenna reactance)

Ideally, energy from the source is entirely transmitted to radiation resistance R_r. But in practice, there will be conduction-dielectric losses due to lossy nature of transmission line & antenna. Also there are reflection losses at the interface between the transmission line & the antenna. Antenna system if not properly designed, causes reflections at the interface & standing waves result.

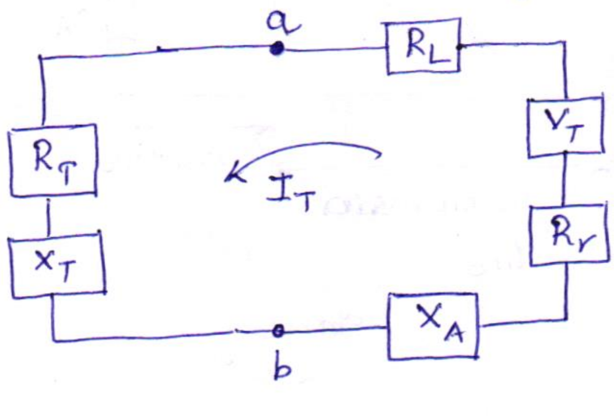
Draw the equivalent circuit of a receiver antenna (6 marks) (KTU May 2019)

Thevenin equivalent circuit of receiving antenna

(Balanis pg 95)



Antenna in receiving mode



$R_L \rightarrow$ loss resistance

$R_r \rightarrow$ Radiation resistance

$X_A \rightarrow$ Antenna reactance

$V_T \rightarrow$ Thevenin voltage due to radiation power received by the antenna.

Antenna when used in receiving mode, the incident wave impinges upon the antenna & it induces a voltage V_T which is analogous to V_g of the transmitting mode.

Fundamental properties of antenna

(i) Input impedance

Input impedance (Z_A) is the impedance presented by an antenna at its terminals or the ratio of voltage to current at a pair of terminals or the ratio of electric & magnetic fields at a point

$$Z_A = R_A + jX_A$$

$X_A \rightarrow$ antenna reactance

$$R_A = R_r + R_L$$

where R_r is radiation resistance

R_L is loss resistance

- Radiation resistance (R_r) is the fictitious resistance which when substituted in series with an antenna will consume the same power as it actually radiated by the antenna.
- Loss resistance (R_L) of the antenna accounts for conduction & dielectric losses.
- Input impedance of an antenna is generally a function of frequency. Input impedance of the antenna depends on many factors including its geometry, its method of excitation & its proximity to surrounding objects.
- Since input impedance is a function of frequency, the antenna will be matched to the interconnecting transmission line and other associated equipment only within a bandwidth.

② Radiation Pattern

Radiation pattern of an antenna is a mathematical or graphical representation of radiation properties of the antenna like power flux density, radiation intensity, field strength & polarization

Various parts of radiation pattern is called radiation pattern lobes. A radiation lobe is a portion of radiation pattern bounded by region of relatively weak radiation intensity.

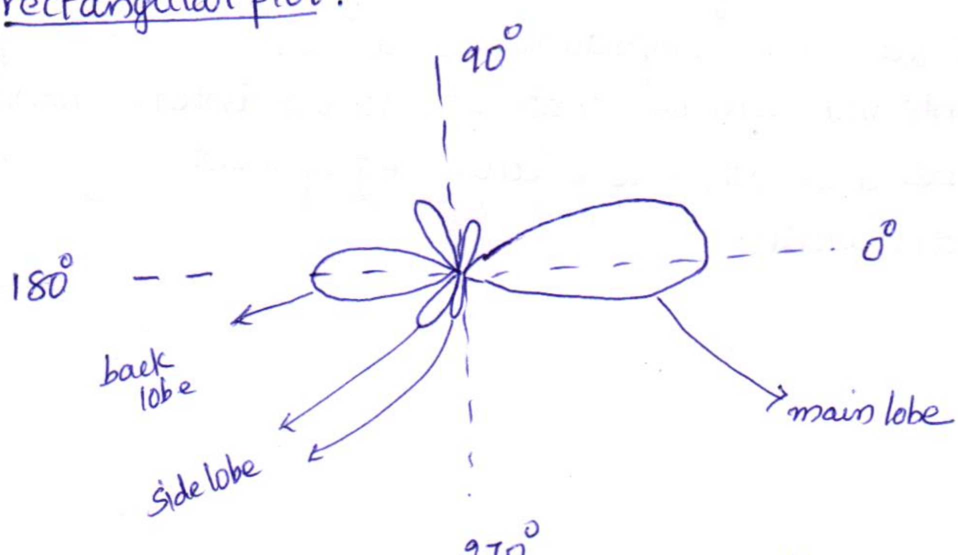
Major lobe or (main lobe) is the radiation lobe containing the direction of maximum radiation.

Minor lobes represent radiation in undesirable direction & they have to be minimized.

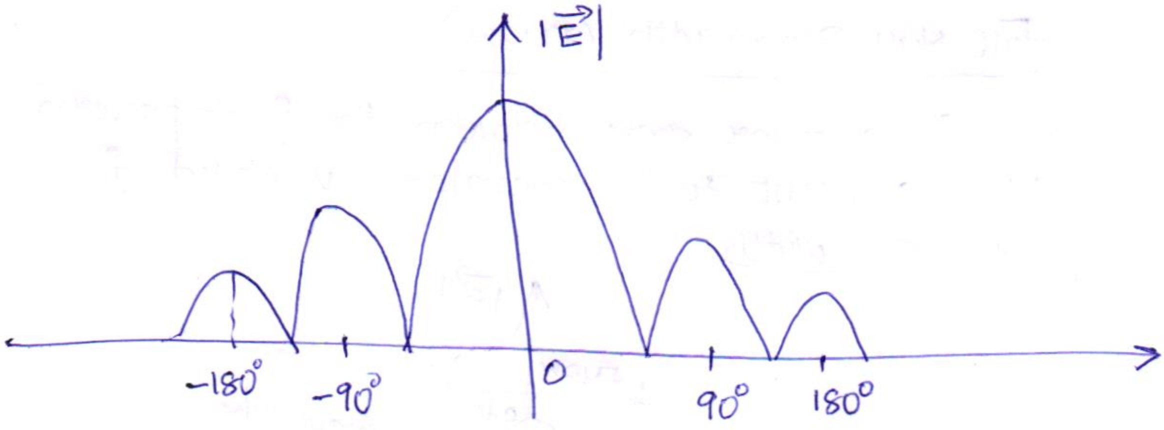
Side lobes are normally largest of the minor lobes. Attainment of a sidelobe level smaller than -30dB usually requires very careful design & construction Low sidelobe ratios are very important to minimize false target indications through sidelobes.

Back lobe is the radiation lobe whose axis makes an angle approximately 180° with respect to the main beam of the antenna.

Radiation pattern can be plotted as polar plot or rectangular plot.



Polar plot of radiation pattern.



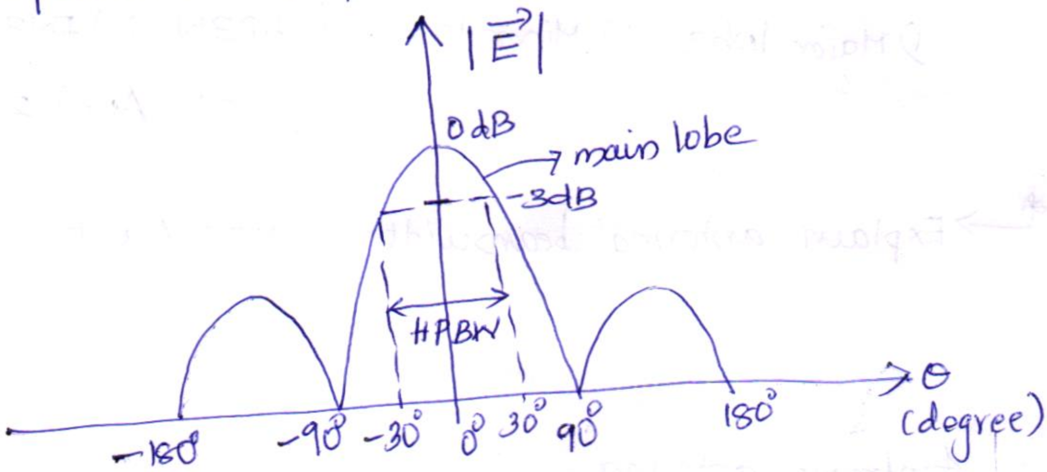
Rectangular plot of radiation pattern

Beam width

Beamwidth is the aperture angle from which where most of the power is radiated. The 2 main considerations of this beamwidth are half power beamwidth (HPBW) & Full Null Beamwidth (FNBW)

Half power beamwidth (HPBW)

Half power beamwidth is the angle between the half-power (-3dB) points of the main lobe

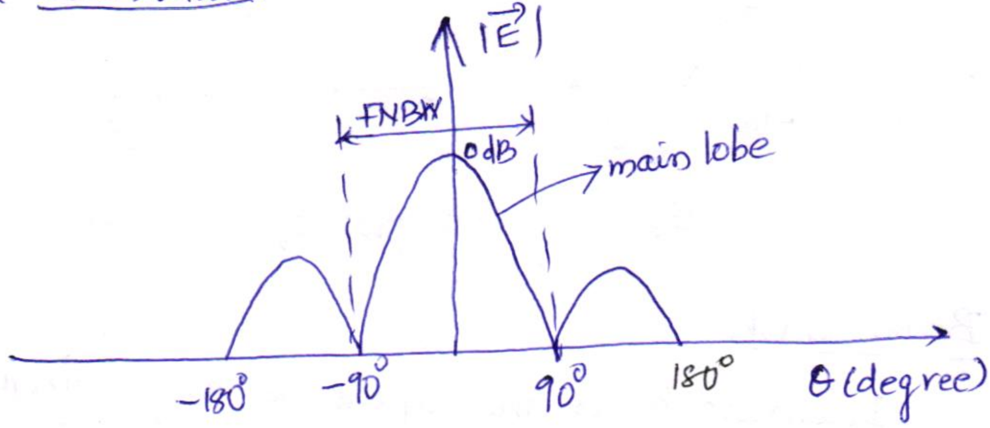


For example, in the above radiation pattern the HPBW

$= 30-(-30) = 60^{\circ}$

Full Null Beamwidth (FNBW)

The angular span between the first pattern nulls adjacent to the mainlobe is called full null beamwidth.



For example, in the above radiation pattern the FNBW

$= 90 - (-90)$

$= 180^{\circ}$

*→ Explain the radiation pattern of antenna. Sketch the radiation pattern & mark the following

i) Major lobe ii) Minor lobe iii) HPBW iv) FNBW

(KTU, April 2018)

*→ Explain antenna beamwidth (KTU March 2018)

Isotropic antenna

Isotropic antenna is a hypothetical lossless antenna having equal radiation in all directions.

Directional antenna

A directional antenna is one having the property of radiating or receiving EM waves more effectively in some direction than in others.

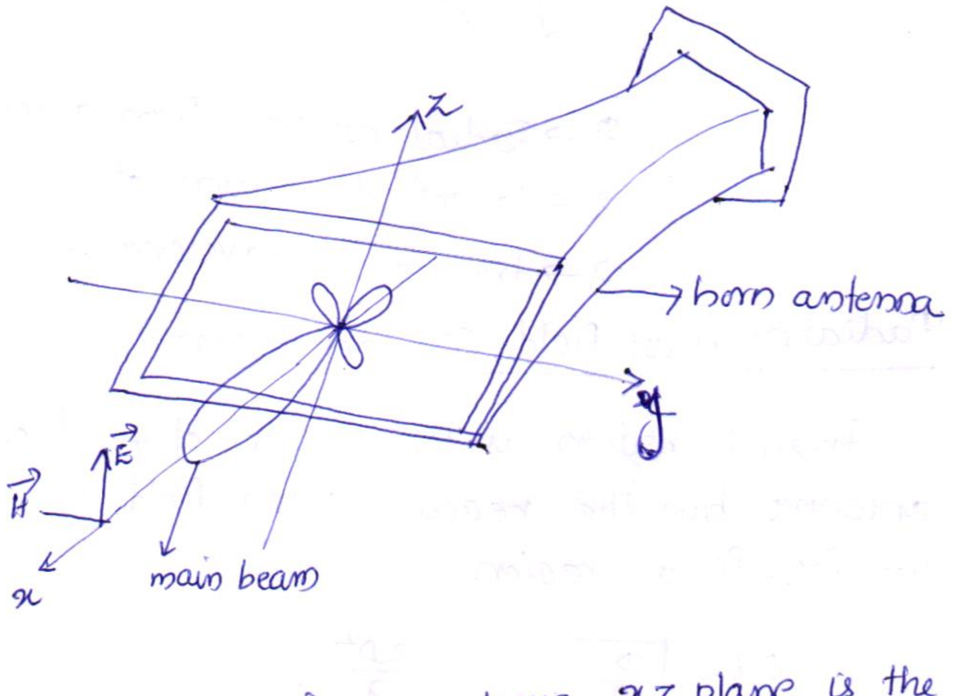
Omnidirectional antenna

Omnidirectional antenna is a type of directional antenna. Example the antenna may have directional pattern in elevation plane + non-directional pattern in azimuth plane.

Principal planes

Radiation pattern is 3-dimensional. For ease of understanding + better visualization cuts are taken in principal planes.

For a linear polarized antenna, there are 2 principal planes viz E plane + H-plane. E-plane is defined as the plane containing the electric field vector + the direction of maximum direction. H-plane is that plane containing magnetic field vector + the direction of maximum radiation



For example, in figure above, xz plane is the E-plane or elevation plane + xy plane is the H-plane or azimuthal plane.

With the help of a neat figure explain about the antenna field zones (6 marks) (KTU - Dec 2022) $\swarrow$*

Antenna field zones

The space surrounding an antenna is usually subdivided into 3 regions

1) reactive near field
2) radiating near field (Fresnel zone)
3) far field (Franhofer region)

The boundaries representing these regions are not unique & no abrupt changes in field configurations are noted.

Reactive near field region

Reactive near field region is that portion of field wherein the reactive field predominates

$$R < 0.62\sqrt{\frac{D^3}{\lambda}}$$

$R \rightarrow$ Radial distance from antenna
$D \rightarrow$ largest dimension of antenna
$\lambda \rightarrow$ free space wavelength.

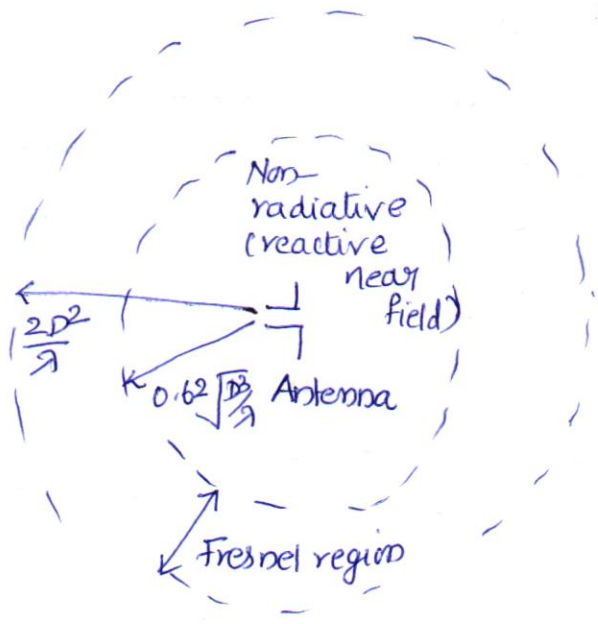
Radiating near field (Fresnel region)

Fresnel region is the region of field of an antenna b/w the reactive near field region & the far-field region

$$0.62\sqrt{\frac{D^3}{\lambda}} < R < \frac{2D^2}{\lambda}$$

Far-field (Fraunhofer region)

Fraunhofer region is the region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna $\frac{2D^2}{\lambda} < R < \infty$



Radiation Power Density ($\vec{W}_{rad}$)

Radiation power density (or time average Poynting vector is given by:

$$\vec{W}_{rad} = \frac{1}{2} Re(\vec{E} \times \vec{H}^*)$$

Average power radiated by an antenna (or radiated power) is given by:-

$$P_{rad} = \oiint \vec{W}_{rad} \cdot \vec{dA}$$

where $|\vec{dA}| = r^2 \sin\theta \, d\theta \, d\phi$

Radiation Intensity ($\vec{U}$)

Radiation intensity is the power radiated from the antenna per unit solid angle. It is a far field parameter

$$\vec{U} = r^2 \vec{W}_{rad}$$

$\therefore$ Radiated Power

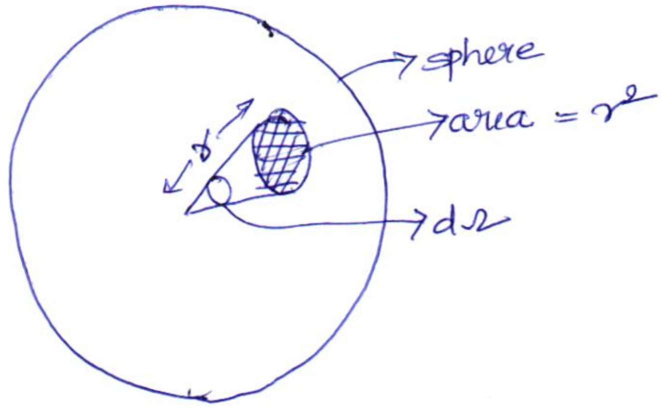
$$P_{rad} = \oiint \vec{U} \cdot d\Omega$$

$$= \int_0^{2\pi} \int_0^{\pi} |\vec{U}| \sin\theta \, d\theta \, d\phi$$

Solid angle (Ω)

Solid angle is that fraction of the surface of a sphere that a particular object covers, as seen by an observer at the sphere's center. The unit of solid angle is steradian

Steradian is the solid angle with its vertex at the center of a sphere of radius r, that is subtended by a spherical surface area equivalent to that of a square with each side of length 'r'

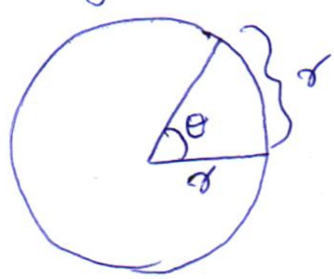


$$d\Omega = \frac{ds}{r^2}$$

$ds \rightarrow$ area
$r \rightarrow$ radius of sphere

$$ds = r^2 \sin\theta \, d\theta \, d\phi$$

Radian is the plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is 'r'



Power radiated by an isotropic antenna

Isotropic antenna is a hypothetical lossless antenna having equal radiation in all directions. Thus radiation intensity ($\vec{U}$) & radiation power density ($\vec{W}$) are independent of the angle θ & ϕ

$\therefore U = U_0$ a constant

Power radiated by isotropic antenna

$$P_{rad} = \oiint U_0 \, d\Omega$$

$$= U_0 \oiint d\Omega$$

$$= U_0 \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$= U_0 \left[-\cos\theta\right]_0^{\pi} \left[2\pi\right]$$

$$= U_0 [4\pi]$$

$$\boxed{P_{rad} = 4\pi U_0}$$

Directivity

Directivity of an antenna is defined as the ratio of the radiation intensity in a given direction $U(\theta, \phi)$ from the antenna to the radiation intensity averaged over all directions (U_0)

$$D = \frac{U(\theta, \phi)}{U_0} = \frac{4\pi U(\theta, \phi)}{P_{rad}}$$

If the direction is not mentioned, the direction of maximum radiation intensity is implied

Maximum Directivity (D_0)

$$D_{max} = D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}$$

Directivity of an isotropic antenna is

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi U_0}{4\pi U_0} = 1 = 0\ dB$$

The directivity of an isotropic antenna is '1'. Since its power is radiated equally well in all directions. For all other sources, the maximum directivity is greater than unity & Thus directivity is a relative "figure of merit" which gives an indication of the directional properties of the antenna as compared with those of an isotropic source. The values of directivity will be equal to or greater than zero dB & equal to or less than the maximum directivity $(0 \leq D \leq D_0)$

Directivity is often expressed in dB

$$D_0\,(dB) = 10 \log_{10}(D_0)$$

Qn Define directivity & gain of an antenna (KTU)

Antenna Efficiency

Antenna efficiency takes into account the losses at the input terminals & within the structure of the antenna. Losses are due to

1. reflections – If the antenna is not matched to the transmission line, then there will be reflection.
2. I^2R losses – conductor + dielectric losses.

1. Reflection efficiency (mismatch) (e_r)

Reflection efficiency takes into account the losses at the input terminal when the antenna is not matched to the transmission line.

The voltage reflection coefficient at input terminals of the antenna (Γ)

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

where

Z_{in} is the antenna input impedance

Z_0 is characteristic impedance of the transmission line.

Reflection efficiency

$$\boxed{e_r = 1 - |\Gamma|^2}$$

This loss is external to the antenna and can be eliminated by use of proper matching circuit.

2. Antenna radiation efficiency (e_{cd})

Antenna radiation efficiency (e_{cd}) takes into account the conductor + dielectric losses within the structure of antenna

$$e_{cd} = e_c e_d$$ where e_c = conduction efficiency, e_d = dielectric efficiency

Resistive losses, due to imperfect metals & dielectric materials result in a difference between the power delivered to the input of an antenna & power radiated by that antenna.

$$e_{cd} = e_c e_d = \frac{P_{rad}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}}$$

where P_{rad} is the power radiated by the antenna, P_{in} is the power supplied to the input of the antenna & P_{loss} is power lost in the antenna.

The radiation efficiency of antenna (e_{cd}) relates gain to directivity

Gain (G) = $e_{cD} D$

$D \rightarrow$ Directivity

$e_{cd} \rightarrow$ radiation efficiency

Overall efficiency (e_0) of the antenna is

$$\boxed{e_0 = e_r e_{cd}}$$

Gain

Gain is a measure that takes into account the efficiency of the antenna as well as its directional capabilities (NB: Directivity is a measure of the directional properties only)

Absolute Gain

Absolute Gain is the ratio of the intensity in a given direction to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically

$$G(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{in}} \rightarrow (1)$$

$$P_{rad} = e_{cd} P_{in} \longrightarrow (2)$$

$$(1) + (2) \Rightarrow G(\theta,\phi) = e_{cd} \frac{U(\theta,\phi)}{P_{in}}$$

$$= \frac{e_{cd}\, U(\theta,\phi)\, 4\pi}{P_{rad}}$$

$$= e_{cd} D(\theta,\phi)$$

Maximum value of gain (G_0)

$$G_0 = e_{cd} D_0$$

Gain is often expressed in dB

$$G_0 (dB) = 10 \log_{10} (e_{cd} D_0)$$

The antenna directivity is a function only of the shape of radiation pattern of an antenna & is not affected by losses in the antenna itself. But antenna gain is the product of directivity & radiation efficiency. Sometimes the effect of impedance mismatch loss is included in the gain of an antenna, this is referred to as the realized gain

Qn Differentiate between Gain & Directivity of an antenna (3 marks)
[KTU 2022]

Polarization

Polarization of an antenna in a given direction is defined as the polarization of electromagnetic wave transmitted by the antenna. When the direction is not stated, the polarization is taken to be the polarization in the direction of maximum gain. In practice, polarization of radiated energy varies with the direction from the center of the antenna, so that different parts of the pattern may have different polarizations.

Polarization describes how position of the tip of the electric field vector $\vec{E}$ is varying with time in the plane perpendicular to the direction of propagation

Polarization may be classified as

1) Linear
2) Circular
3) Elliptical

Linear polarization

A time harmonic wave is linearly polarized at a given point in space if the electric field (or magnetic field) vector at that point is always oriented along the same straight line at every instant of time. This is accomplished if the field vector possesses

1. only one component
2. Two orthogonal linear components that are in time phase or 180° (or multiples of 180°) out of phase.

Example

① $\vec{E} = \cos(\omega t - \beta z + \pi/2)\,\hat{a}_x$

② $\vec{E} = 3\cos(\omega t - \beta z)\,\hat{a}_y + 2\cos(\omega t - \beta z)\,\hat{a}_x$ → The time harmonic wave has 2 orthogonal components (oriented along $\hat{a}_x$ & $\hat{a}_y$), which are in time phase ⇒ linear polarization.

Circular polarization

A time harmonic wave is circularly polarized at a given point in space if the electric (or magnetic) field vector at that point traces a circle as a function of time.

The necessary & sufficient conditions to accomplish this are the following:

① The field must have 2 orthogonal linear components

② The two components must have the same magnitude

③ The two components must have a time phase difference of odd multiples of 90°

If the field rotation is clockwise, the wave is right-hand circularly polarized (RHCP). If the field rotation is anticlockwise, the wave is left-hand circularly polarized (LHCP)

eg $\vec{E} = \sin(\omega t - \beta z)\hat{a}_x + \cos(\omega t - \beta z)\hat{a}_y \longrightarrow$ RHCP

$\vec{E} = \sin(\omega t - \beta z)\hat{a}_y + \cos(\omega t - \beta z)\hat{a}_x \longrightarrow$ LHCP.

Elliptical polarization

A time harmonic wave is elliptically polarized if the tip of the field vector (electric or magnetic) traces an elliptical locus in space.

A wave is elliptically polarized if it is not linearly or circularly polarized

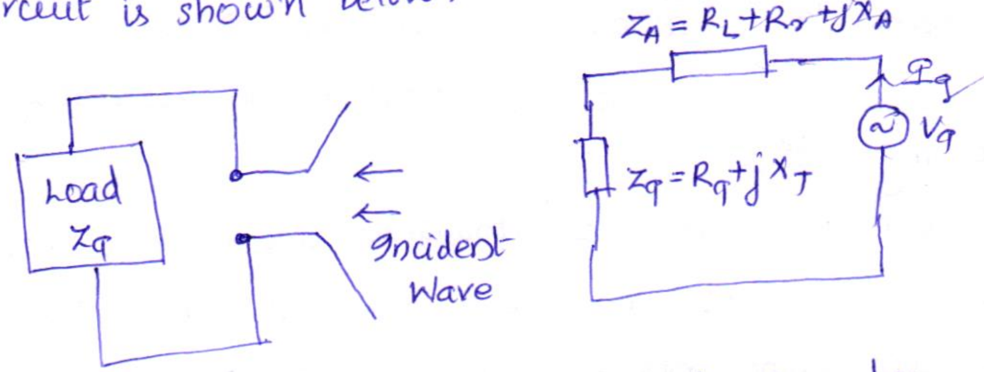
$$\vec{E} = \cos(\omega t - \beta z)\hat{a}_x + 2\sin(\omega t - \beta z)\hat{a}_y$$

Polarization efficiency (polarization mismatch) or loss factor

Polarization efficiency / polarization mismatch is defined as the ratio of power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density & direction of propagation, whose state of polarization has been adjusted for a maximum received power.

Effective Aperture Area (A_{em})

Consider a receiving antenna shown below. When an electromagnetic wave falls on the antenna, it induces a voltage V_T. The equivalent circuit is shown below:



Power delivered to the load is given by

$$P_q = \frac{1}{2}|I_g|^2 R_q = \frac{1}{2}\frac{|V_q|^2 R_q}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2} \rightarrow ①$$

Assuming conjugate matching,

$$R_L + R_r = R_q \quad \& \quad X_T = -X_A \rightarrow ②$$

From ① & ②, maximum power delivered to load,

$$P_q = \frac{1}{2}\frac{|V_q|^2 R_q}{(2)^2 (R_L + R_r)^2} = \frac{1}{8}\frac{|V_q|^2 (R_L + R_r)}{(R_L + R_r)^2}$$

$$P_q = \frac{1}{8}\frac{|V_q|^2}{(R_L + R_r)} \rightarrow ③$$

P_q can be expressed as a fraction of incident power.

$$P_q = A_{em} W_i$$

where A_{em} = maximum effective aperture area

W_i = incident power density

$$\boxed{A_{em} = \frac{|V_q|^2}{8 W_i (R_L + R_r)}} \rightarrow ⑤$$

Maximum effective area is defined as the equivalent area, which when multiplied by the incident power density leads to the maximum power delivered to the load.

Power dissipated as heat (P_l) is given by:

$$P_l = \frac{1}{2} |I_g|^2 R_L$$

$$= \frac{1}{2} \frac{|V_T|^2 R_L}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}$$

Substituiting ②

$$P_l = \frac{1}{2} \frac{|V_T|^2 R_L}{(2)^2 (R_r + R_L)^2} = \frac{|V_T|^2 R_L}{8(R_r + R_L)^2} \longrightarrow ⑥$$

P_l can be expressed as a fraction of incident power

$$P_l = A_L W_i \longrightarrow ⑦$$

From ⑥ & ⑦

$$A_L = \frac{|V_T|^2 R_L}{8 W_i (R_L + R_r)^2} \longrightarrow ⑧$$

Loss area is thus defined as the equivalent area, which when multiplied by the incident power density leads to power dissipated as heat through R_L

Scattered Power (or radiated power)

$$P_r = \frac{1}{2} |I_g|^2 R_r$$

$$= \frac{1}{2} \frac{|V_T|^2 R_r}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}$$

Substituting ②

$$P_r = \frac{1}{2} \frac{|V_T|^2 R_r}{(2)^2 (R_L + R_r)^2} = \frac{1}{8} \frac{|V_T|^2 R_r}{(R_L + R_r)} \longrightarrow ⑨$$

P_r can be expressed as a function of incident power.

$$P_r = A_s W_i \longrightarrow ⑩$$

where A_s is the scattering area
W_i is the incident power density.

From (9) & (10)

$$\boxed{A_s = \frac{|V_q|^2 R_r}{8W_i(R_L+R_r)^2}} \longrightarrow (11)$$

Scattering area is defined as the equivalent area, which when multiplied by incident power density gives the scattering power.

Total power captured (P_c) is given by:

$$P_c = \frac{1}{2}|I_g|^2 (R_L+R_r+R_T)$$

$$= \frac{1}{2}\frac{|V_q|^2 (R_L+R_r+R_q)}{(R_r+R_L+R_q)^2+(X_A+X_q)^2}$$

Assuming conjugate matching (From (1))

$$P_c = \frac{1}{2}\frac{|V_q|^2}{2}\frac{(R_L+R_r+R_q)}{(R_r+R_L)^2}$$

$$= \frac{1}{8}|V_q|^2\frac{(R_L+R_r+R_q)}{(R_L+R_r)^2} \longrightarrow (12)$$

P_c can be expressed as a fraction of incident power

$$\boxed{P_c = A_c W_i} \longrightarrow (13)$$

From (12) & (13)

$$A_c = \frac{|V_q|^2(R_L+R_r+R_q)}{8W_i(R_L+R_r)^2} \longrightarrow (14)$$

<u>Capture area</u> is defined as the equivalent area, which when multiplied by the incident power density leads to the total power captured, collected, or intercepted by the antenna.

In general, capture area is the sum of effective area, scattering area and loss area.

Capture area = Effective area + scattering area + loss area.

Qn Discuss in detail about about effective aperture of antenna (8 marks)
KTU April 2018

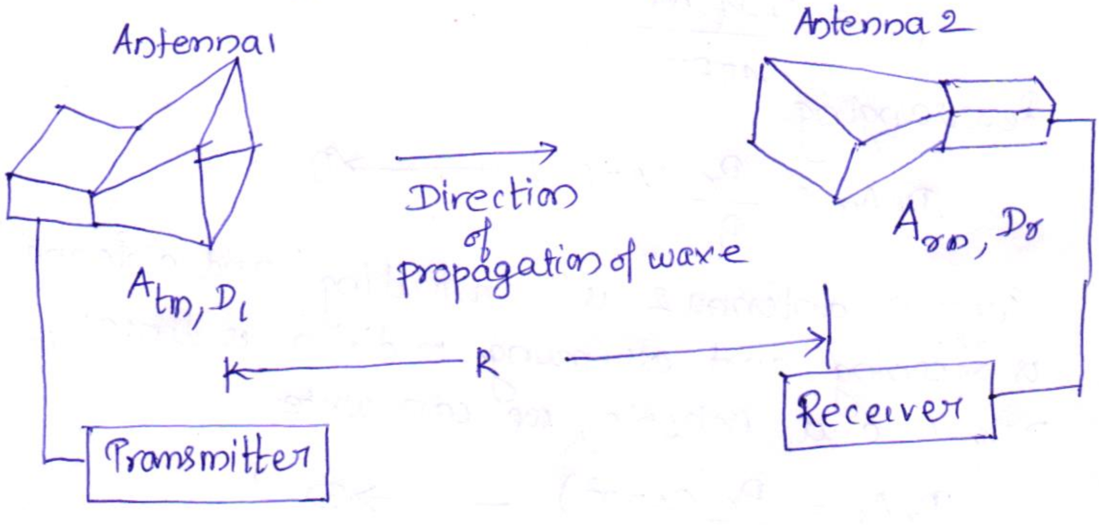
Aperture efficiency (η_p)

Aperture efficiency of an antenna is defined as the ratio of maximum effective area (A_{em}) to the physical area (A_p) of the antenna.

$$\eta_p = \frac{A_{em}}{A_p} = \frac{\text{Maximum effective area}}{\text{Physical area}}$$

Relation between directivity and maximum effective area.

Consider the geometric arrangement shown below:



Antenna 1 is used as a transmitter and antenna 2 is used as a receiver.

$A_t \rightarrow$ Effective Area of transmitting antenna

$A_r \rightarrow$ Effective Area of receiving antenna

$D_t \rightarrow$ Directivity of transmitting antenna

$D_r \rightarrow$ Directivity of receiving antenna.

- If antenna 1 was isotropic antenna, the radiated power density (W_o) at a distance R is given by:

$$W_0 = \frac{P_t}{4\pi R^2} \longrightarrow ①$$

where P_t is the total radiated power.

- We know antenna 1 has a directivity D_t. So the transmitted power density will be

$$W_t = W_o D_t = \frac{P_t D_t}{4\pi R^2} \longrightarrow ②$$

Power collected (received) by antenna 2 & transferred to load is

$$P_t = W_t A_r$$

$$= \frac{P_t D_t A_r}{4\pi R^2}$$

Rearranging

$$D_t A_r = \frac{P_r (4\pi R^2)}{P_t} \longrightarrow ①$$

Suppose antenna 2 is transmitting and antenna 1 is receiving and assuming medium is linear, passive and isotropic, we can write

$$D_r A_t = \frac{P_r (4\pi R^2)}{P_t} \longrightarrow ②$$

① & ② $\Rightarrow D_t A_r = D_r A_t$

$$\frac{D_t}{D_r} = \frac{A_t}{A_r} \longrightarrow ③$$

Directivity of antenna is directly proportional to effective area. This equation ③ can be written as:

$$\frac{D_{ot}}{D_{or}} = \frac{A_{tm}}{A_{rm}} \longrightarrow ④$$

where $D_{0t} \rightarrow$ maximum directivity of tx antenna
$D_{0r} \rightarrow$ " " of rx antenna
$A_{tm} \rightarrow$ " effective area of tx antenna
$A_{rm} \rightarrow$ " " " rx antenna

If antenna 1 is isotropic, then $D_{0t} = 1$

$$A_{tm} = \frac{A_{rm}}{D_{0r}} \longrightarrow (5)$$

Maximum effective area of an isotropic antenna

$$A_{em} = \frac{\lambda^2}{4\pi} \longrightarrow (6)$$

$$(5) \ \& \ (6) \Rightarrow \frac{\lambda^2}{4\pi} = \frac{A_{rm}}{D_{0r}}$$

$$\Rightarrow A_{rm} = D_{0r} \frac{\lambda^2}{4\pi}$$

Thus in general, maximum effective area (A_{em}) of any antenna is related to the maximum directivity D_0 as

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 \longrightarrow (7)$$

For aperture antennas, the maximum directivity (D_{max}) can be written as:

$$D_{max} = \frac{4\pi A}{\lambda^2} \longrightarrow (8)$$

where A = physical area of aperture.

$$(7) \ \& \ (8) \Rightarrow \frac{1}{D_{max}} = \frac{A_{em}}{D_0 A}$$

$$\frac{D_0}{D_{max}} = \frac{A_{em}}{A} = \eta_{ap}$$

Thus the aperture efficiency (η_{ap}) can also be defined as the ratio of actual directivity (D) of aperture antenna to the maximum directivity (D_{max})

$$\eta_{ap} = \frac{D_0}{D_{max}}$$

In practice, there are several factors that can serve to reduce the directivity of an antenna from the maximum possible value.

For example

1) nonideal amplitude or phase characteristic of the aperture field.

2) spill over of feed pattern in the case of reflector antenna

3) aperture blockage.

Effective length

The effective length of an antenna is a quantity used to determine the voltage induced on an open circuit terminal of the antenna when a wave impinges on it.

$$V_{oc} = \vec{E_i} \cdot \vec{l_e}$$

where $\vec{E_i}$ = incident electric field

$\vec{l_e}$ = vector effective length.

Effective length of a linearly polarized antenna receiving a plane wave in a given direction is defined as "the ratio of the magnitude of the open circuit voltage developed at the terminals of antenna (V_{oc}) to the magnitude of electric field strength in the direction of antenna polarization

$$|\vec{l_e}| = \frac{V_{oc}}{|E_i|}$$

For example, consider a vertical dipole of length $l = \lambda/2$, immersed in an incident field E_i.

- Assuming the antenna is oriented for maximum response,
- If the current distribution is uniform

$$l_e = l \quad (l \text{ is the physical length})$$

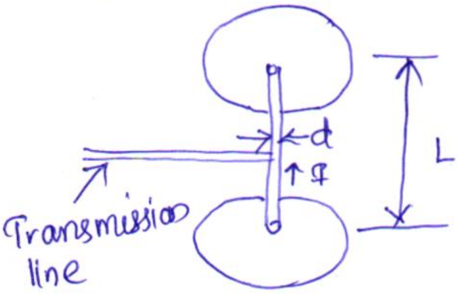
- If the current distribution is sinusoidal

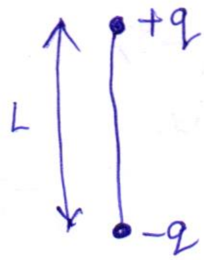
$$l_e = \frac{2}{\pi} = 0.64\, l$$

Short Electric dipole

Any linear antenna may be considered as consisting of a large number of very short conductors connected in series, it is of interest to examine first the radiation properties of short conductors.

A short linear conductor is often called a short dipole ($\lambda/50 < l < \lambda/10$). Plates at the ends of the dipole provide capacitive loading





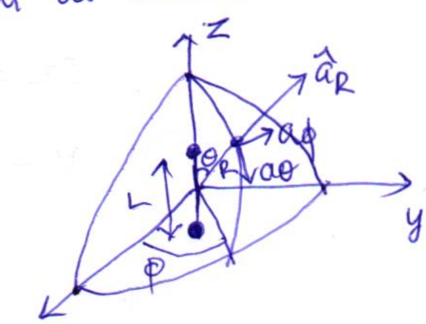
The short length & the presence of these plates result in a uniform current I along the length L of the dipole. The diameter 'd' is small compared to its length ($d << \lambda$)

$$\frac{dq}{dt} = I$$

Fields of short dipole

When current is flowing in a short dipole, the effect of current is not felt instantaneously at point P, but only after an interval equal to the time required for the disturbance to propagate over the distance R (propagation time). It is called retardation effect

Consider a short dipole placed as shown below

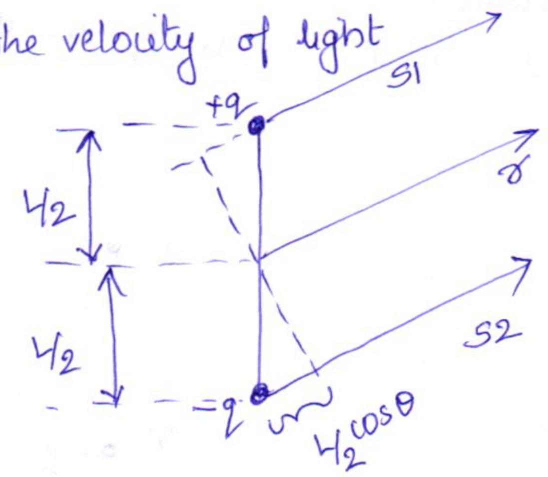


Retarded current

$$[I] = I_0 e^{j\omega(t - R/c)}$$

where $\frac{R}{c}$ is the phase retardation

Phase retardation is the interval required for the disturbance to travel the distance 'R' where c is the velocity of light



The retarded scalar potential V of a charge distribution is

$$V = \frac{1}{4\pi \varepsilon_0} \int \frac{[\rho]}{R} \, dv$$

Since the region of charge in the case of the dipole being considered is confined to the points at the ends

$$V = \frac{1}{4\pi \varepsilon_0} \left\{ \frac{[q]}{S1} - \frac{[q]}{S2} \right\}$$

where $[q]$ is the retarded charge

$$S1 = R - \frac{L}{2}\cos\theta$$

$$S2 = R + \frac{L}{2}\cos\theta$$

V = Retarded potential.

$$[q] = \int [I]\, dt$$

$$= I_0 \int e^{j\omega(t - \frac{s}{c})} \cdot dt$$

$$= \frac{I_0}{j\omega} e^{j\omega(t-\frac{s}{c})}$$

$$= \frac{[I]}{j\omega}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{[I]}{j\omega s_1} - \frac{[I]}{j\omega s_2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{I_0}{j\omega}\right]\left[\frac{e^{j\omega(t - s_1/c)}}{s_1} - \frac{e^{j\omega(t-\frac{s_2}{c})}}{s_2}\right]$$

$$= \frac{I_0}{4\pi\varepsilon_0 j\omega}\left[\frac{e^{j\omega(t-\frac{R}{c}+\frac{L}{2c}\cos\theta)}}{R-\frac{L}{2}\cos\theta} - \frac{e^{j\omega(t-\frac{R}{c}-\frac{L}{2c}\cos\theta)}}{R+\frac{L}{2}\cos\theta}\right]$$

This is the retarded potential.

<u>Magnetic Vector Potential</u>

$$\vec{A} = \hat{a}_z \frac{\mu_0 [I] L}{4\pi R}$$

$$= \hat{a}_z \frac{\mu_0 I_0 L}{4\pi R}\left(e^{j\omega(t-R/c)}\right)$$

Converting to spherical coordinates

$$A_R = A_z\cos\theta = \frac{\mu_0 I_0 L\, e^{j\omega(t-R/c)}\cos\theta}{4\pi R}$$

$$A_\theta = -A_z\sin\theta$$

$$= -\frac{\mu_0 I_0 L}{4\pi R} e^{j\omega(t-\frac{R}{c})}\sin\theta$$

$$\vec{H} = \frac{1}{\mu_0}(\nabla\times\vec{A}) = \frac{1}{\mu_0 R^2\sin\theta}\begin{vmatrix} \hat{a}_R & R\hat{a}_\theta & R\sin\theta\,\hat{a}_\phi \\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi \\ A_R & RA_\theta & 0 \end{vmatrix}$$

$$= \frac{1}{\mu_0 R^2\sin\theta}\left[\hat{a}_R\left(-\underbrace{\cancel{\frac{\partial RA_\theta}{\partial\phi}}}_{0}\right) - R\hat{a}_\theta \underbrace{\cancel{\frac{\partial A_R}{\partial\phi}}}_{0}\right] + \frac{R\sin\theta\,\hat{a}_\phi}{\mu_0 R^2\sin\theta}\left(\frac{\partial(RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial\theta}\right)$$

$$\vec{H} = \frac{\hat{a}_\phi}{\mu_0 R}\left(\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right)$$

$$= \frac{\hat{a}_\phi}{\mu_0 R}\left[\frac{\partial}{\partial R}\left\{\frac{-\mu_0 L I_0 e^{j\omega(t-R/c)}}{4\pi R}(R\sin\theta)\right\} - \frac{\partial}{\partial \theta}\left\{\frac{\mu_0 L I_0 e^{j\omega(t-R/c)}}{4\pi R}\cos\theta\right\}\right]$$

$$= \frac{\hat{a}_\phi}{\mu_0 R}\left[\frac{I_0 \sin\theta \mu_0 L e^{j\omega(t-R/c)}}{4\pi\left(\frac{c}{j\omega}\right)} + \frac{\mu_0 L I_0 e^{j\omega(t-R/c)}\sin\theta}{4\pi R}\right]$$

$$\vec{H} = \frac{\hat{a}_\phi I_0 L \sin\theta}{4\pi} e^{j\omega(t-R/c)}\left[\frac{j\omega}{cR} + \frac{1}{R^2}\right]$$

$$H_R = H_\theta = 0$$

$$H_\phi = \frac{I_0 L \sin\theta}{4\pi} e^{j\omega(t-\frac{R}{c})}\left[\frac{j\omega}{cR} + \frac{1}{R^2}\right]$$

$$\nabla \times \vec{H} = j\omega\varepsilon_0 \vec{E}$$

$$\vec{E} = \frac{1}{j\omega\varepsilon_0}(\nabla \times \vec{H})$$

$$= \frac{1}{j\omega\varepsilon_0}\frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{a}_R & R\hat{a}_\theta & R\sin\theta\,\hat{a}_\phi \\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & R\sin\theta H_\phi \end{vmatrix}$$

$$= \frac{1}{j\omega\varepsilon_0 R^2\sin\theta}\left[\hat{a}_R\left(\frac{\partial}{\partial\theta}(R\sin\theta H_\phi)\right) - R\hat{a}_\theta\left(\frac{\partial}{\partial R}(R\sin\theta H_\phi)\right)\right]$$

$$E_R = \frac{1}{j\omega\varepsilon_0 R^2\sin\theta}\frac{\partial}{\partial\theta}(R\sin\theta H_\phi)$$

$$E_R = \frac{1}{j\omega\varepsilon_0 R^2 \sin\theta}\left[\frac{\partial}{\partial\theta}\left\{\frac{R\sin^2\theta\, I_0 L}{4\pi}\, e^{j\omega(t-R/c)}\left[\frac{j\omega}{CR}+\frac{1}{R^2}\right]\right\}\right]$$

$$= \frac{\cancel{R}}{j\omega\varepsilon_0}\,\frac{I_0 L}{4\pi}\,\frac{e^{j\omega(t-R/c)}}{R^{\cancel{2}}\,\cancel{\sin\theta}}\left[\frac{j\omega}{CR}+\frac{1}{R^2}\right]\left[2\sin\theta\cos\theta\right]$$

$$E_R = \frac{I_0 L\cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{cR^2}+\frac{1}{j\omega R^3}\right]e^{j\omega(t-R/c)}$$

$$E_\theta = \frac{1}{j\omega\varepsilon_0 R^2\sin\theta}(-R)\frac{\partial}{\partial R}(R\sin\theta\, H_\phi)$$

$$= \frac{-1}{j\omega\varepsilon_0 R\sin\theta}\frac{\partial}{\partial R}\left[\frac{R I_0 L\sin^2\theta}{4\pi}\, e^{j\omega(t-R/c)}\left[\frac{j\omega}{CR}+\frac{1}{R^2}\right]\right]$$

$$= -\frac{I_0 L\sin\theta}{4\pi\varepsilon_0 R\, j\omega}\left[\frac{-j\omega}{c}\, e^{j\omega(t-\frac{R}{c})}\left(\frac{j\omega}{c}\right)+\frac{\partial}{\partial R}\left\{\frac{e^{j\omega(t-R/c)}}{R}\right\}\right]$$

$$= -\frac{I_0 L\sin\theta}{4\pi\varepsilon_0 R\, j\omega}\left[-\left(\frac{j\omega}{c}\right)^2 e^{j\omega(t-R/c)}+\frac{1}{R}\left(\frac{-j\omega}{c}\right)e^{j\omega(t-R/c)} - \frac{1}{R^2}e^{j\omega(t-R/c)}\right]$$

$$= \frac{I_0 L\sin\theta}{4\pi\varepsilon_0}\, e^{j\omega(t-R/c)}\left[\frac{j\omega}{c^2R}+\frac{1}{R^2C}+\frac{1}{j\omega R^3}\right]$$

So the fields are

Electric Field

$$\begin{cases} E_R = \dfrac{I_0 L\cos\theta}{2\pi\varepsilon_0}\left[\dfrac{1}{CR^2}+\dfrac{1}{j\omega R^3}\right]e^{j\omega(t-R/c)} \\ E_\theta = \dfrac{I_0 L\sin\theta}{4\pi\varepsilon_0}\, e^{j\omega(t-R/c)}\left[\dfrac{j\omega}{c^2R}+\dfrac{1}{R^2C}+\dfrac{1}{j\omega R^3}\right] \\ E_\phi = 0 \end{cases}$$

Magnetic Field

$$\begin{cases} H_R = 0 \\ H_\theta = 0 \\ H_\phi = \dfrac{I_0 L\sin\theta}{4\pi}\, e^{j\omega(t-R/c)}\left[\dfrac{j\omega}{CR}+\dfrac{1}{R^2}\right] \end{cases}$$

Far Field approximation

As R becomes large $\frac{1}{R^2}$ & $\frac{1}{R^3}$ terms become negligibly small.

$$H_\phi = \frac{j\omega}{CR}\left(\frac{I_0 L \sin\theta}{4\pi} e^{j\omega(t-R/c)}\right)$$

$$= \frac{j\beta}{R}\left(\frac{I_0 L \sin\theta}{4\pi} e^{j\omega(t-R/c)}\right)$$

$$\boxed{H_\phi = \frac{j I_0 \beta L}{4\pi R} \sin\theta \, e^{j\omega(t-R/c)}} \longrightarrow (1)$$

$$E_\theta = \frac{I_0 L \sin\theta \, e^{j\omega(t-\frac{R}{c})}}{4\pi \varepsilon_0} \left(\frac{j\omega}{c^2 R}\right)$$

$$\boxed{E_\theta = \frac{j I_0 \beta L \sin\theta \, e^{j\omega(t-\frac{R}{c})}}{4\pi \varepsilon_0 c R}} \longrightarrow (2)$$

$$\boxed{E_R = 0} \longrightarrow (3)$$

Radiation Resistance

$$\frac{E_\theta}{H_\phi} = \eta_0 \longrightarrow (4)$$

Average Power, $P_{avg} = \frac{1}{2} Re(E \times H^*)$

$$= \frac{1}{2}\eta_0 \left(\frac{I_0 \beta L}{4\pi R} \sin\theta\right)^2 \leftarrow \left\{\text{From } (1), (2) \& 4\right\}$$

$$= \frac{\eta_0}{2} \frac{I_0^2 \beta^2 L^2 \sin^2\theta}{16\pi^2 R^2}$$

Total power radiated

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_{avg}\, R^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{I_0^2 \beta^2 L^2 \eta_0}{2\times 16\pi^2}(2\pi)\int_{\theta=0}^{\pi} \sin^3\theta\, d\theta$$

$$= \frac{I_0^2 \beta^2 L^2 \eta_0}{16\pi}\left(\frac{-4}{3}\right)$$

$$= \frac{I_0^2 \beta^2 L^2 \eta_0}{12\pi}$$

$$= \frac{I_0^2 (2\pi)^2 L^2 \eta_0}{\lambda^2 (12\pi)}$$

$$= \frac{\eta_0 \pi I_0^2 L^2}{3\lambda^2}$$

$$\int \sin^3\theta \cdot d\theta$$
$$= \int \sin\theta\,(1-\cos^2\theta)\,d\theta$$
$$= \int \sin\theta - \int \sin\theta\cos^2\theta\, d\theta$$
$$= \frac{1}{3}\cos^3\theta - \cos\theta$$

$$\left[\tfrac{1}{3}\cos^3\theta - \cos\theta\right]_{\theta=0}^{\pi}$$
$$= \tfrac{1}{3} - 1 - (-\tfrac{1}{3} + 1)$$
$$= \tfrac{1}{3} - 1 + \tfrac{1}{3} - 1$$
$$= \tfrac{2}{3} - 2$$
$$= -\tfrac{4}{3}$$

$$\left(\frac{I_0}{\sqrt{2}}\right)^2 R_r = \frac{\eta_0 \pi I_0^2 L^2}{3\lambda^2}$$

$$R_r = \frac{\eta_0 \pi I_0^2 L^2}{3\lambda^2} \times \frac{2}{I_0^2}$$

$$= \frac{120\pi^2 L^2 \times 2}{3\lambda^2}$$

$$= 80\pi^2 \left(\frac{L}{\lambda}\right)^2$$

$$\boxed{R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2}$$

Radiation resistance of a short dipole is $80\pi^2\left(\frac{L}{\lambda}\right)^2$ where L is the length of the dipole & λ is the wavelength.

Directivity of short dipole

We know,

$$P_{avg} = \frac{1}{2} \operatorname{Re}(E \times H^*)$$

$$= \frac{\eta_0}{2} \frac{I_0^2 \beta^2 L^2 \sin\theta}{16\pi^2 R^2}$$

$$= \frac{\eta_0 I_0^2 \beta^2 L^2 \sin\theta}{32\pi^2 R^2}$$

Maximum radiation intensity

$$U_{max} = R^2 [P_{avg}]_{max}$$

$$= \frac{R^2 \eta_0}{2} \frac{I_0^2 \beta^2 L^2 \sin(90^\circ)}{16\pi^2 R^2}$$

$$U_{max} = \frac{\eta_0 I_0^2 \beta^2 L^2}{32\pi^2} \longrightarrow (1)$$

Total power radiated is given by:

$$P_r = \frac{\eta_0 \beta^2 I_0^2 L^2}{12\pi} \longrightarrow (2)$$

Directivity

$$D = \frac{4\pi U_{max}}{P_r}$$

$$= \frac{4\pi \eta_0 I_0^2 \beta^2 L^2}{32\pi^2} \frac{12\pi}{\eta_0 \beta^2 I_0^2 L^2}$$

$$= \frac{12}{8}$$

$$= 1.5$$

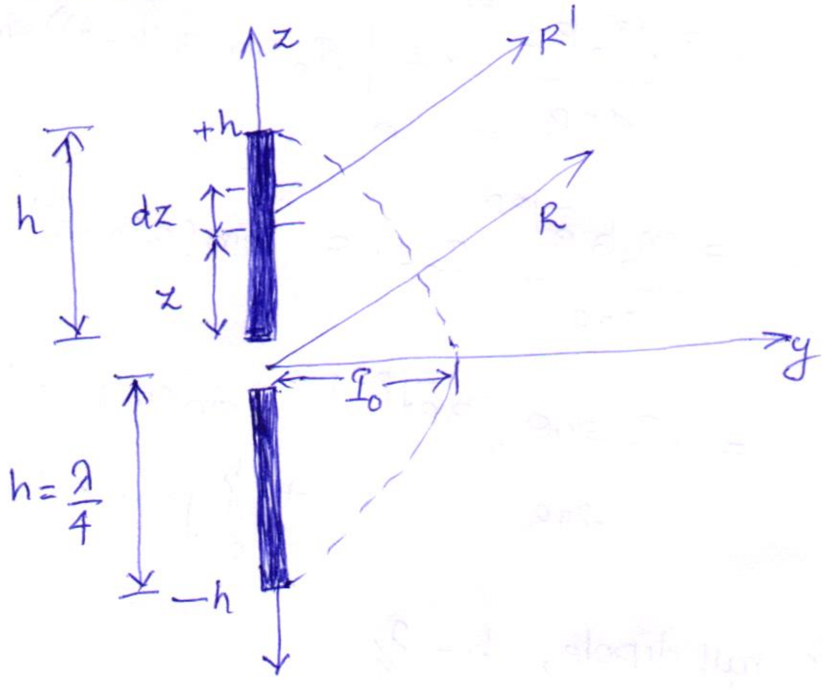
$$(D)_{dB} = 10 \log(1.5) = 1.76 \text{ dB}$$

Half dipole

Half dipole is a linear wire antenna with physical length ($L = \lambda/2$). The antenna is symmetrically fed at the center by a balanced two wire transmission line.

Consider a half wave dipole antenna of length ($L = 2h$) oriented along z-direction with center at origin. Since the length is now comparable with wavelength, current distribution cannot be uniform as we assumed for short dipole.

Analysis shows that the current distribution on a linear dipole antenna is sinusoidal with zero current at the ends of the antenna.



current distribution is given by

$$I(z) = \begin{cases} I_0 \sin(\beta(h+z)) & \text{for } z<0 \\ I_0 \sin(\beta(h-z)) & \text{for } z<0 \end{cases}$$

The radiated electric field due to the dipole can be obtained by dividing the dipole into small short dipoles with appropriate currents & superimposing the radiation field with proper phase.

The radiated far field $\vec{E}$ contribution from differential current element $I dz$ is given by:

$$dE_\theta = \eta_0 \, dH_\phi = \frac{j(I dz)}{4\pi}\left(\frac{e^{-j\beta R'}}{R'}\right)\eta_0 \beta \sin\theta$$

$$R' = R - z\cos\theta$$

For $R >> h$, $R' \approx R$

The far field $\vec{E}$ component is given by

$$E_\theta = \eta_0 H_\phi$$

$$= \frac{j\eta_0 \beta}{4\pi}\int_{-h}^{h} \frac{I e^{-j\beta R'}}{R'} \sin\theta \, dz$$

$$= \frac{j\eta_0 \beta e^{-j\beta R}}{4\pi R} \times 2 \int_0^h I_0 \sin(\beta(h-z)) \sin\theta \, e^{j\beta z\cos\theta} dz$$

$$= \frac{j\eta_0 \beta \, e^{-j\beta R}}{2\pi R} I_0 \sin\theta \int_0^h \sin(\beta(h-z)) \begin{bmatrix} \cos(\beta z\cos\theta) \\ + j\sin(\beta z\cos\theta) \end{bmatrix} dz$$

$$= \frac{j I_0 \sin\theta \, \eta_0 \beta e^{-j\beta R}}{2\pi R} \begin{bmatrix} \int_0^h \sin(\beta(h-z)) \cos(\beta z\cos\theta) dz \\ + \int_0^h j\sin(\beta(h-z)) \sin(\beta z\cos\theta) dz \end{bmatrix}$$

For half dipole, $h = \lambda/4$

So $\beta h = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \pi/2$

Thus

$$E_\theta = \frac{j I_0 \sin\theta \, \eta_0 \beta e^{-j\beta R}}{2\pi R} \begin{bmatrix} \int_0^h \sin(\pi/2 - \beta z) \cos(\beta z\cos\theta) dz \\ + \int_0^h \sin(\pi/2 - \beta z) \sin(\beta z\cos\theta) dz \end{bmatrix}$$

$$E_\theta = \frac{j I_0 \sin\theta\, \eta_0\, \beta\, e^{-j\beta R} \cos(\frac{\pi}{2}\cos\theta)}{2\pi R(\beta)\sin^2\theta}$$

$$E_\theta = \frac{j I_0 \eta_0\, e^{-j\beta R} \cos(\pi/2 \cos\theta)}{2\pi R \sin\theta}$$

Normalizing E_θ

$$F(\theta) = \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$$

The factor $|F(\theta)|$ is the E-plane pattern function for half dipole antenna

$$|F(\theta)| = \left| \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right|$$

<u>Direction of nulls</u>

$|F(\theta)| = 0$

$\Rightarrow \cos(\pi/2 \cos\theta) = 0$

$\Rightarrow \frac{\pi}{2}\cos\theta = \pi/2$

$\Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 0^\circ, 180^\circ \ldots$

<u>Direction of maximum</u>

$|F(\theta)| = 1$

when $\theta = \pi/2,\ 3\pi/2$

$$\int_0^h \sin(\pi/2 - \beta z)\cos(\beta z \cos\theta)\,dz$$

$$= \int_0^h \cos(\beta z)\cos(\beta z\cos\theta)\,dz$$

$$= \frac{1}{2}\int_0^h \cos(\beta z + \beta z\cos\theta)\,dz + \frac{1}{2}\int_0^h \cos(\beta z - \beta z\cos\theta)\,dz$$

$$= \frac{1}{2}\left[\frac{\sin(\beta z + \beta z\cos\theta)}{\beta(1+\cos\theta)}\right]_0^h + \frac{1}{2}\left[\frac{\sin(\beta z - \beta z\cos\theta)}{\beta(1-\cos\theta)}\right]_0^h$$

$$= \frac{1}{2}\left[\frac{\sin(\pi/2 + \pi/2\cos\theta)}{\beta(1+\cos\theta)}\right] + \frac{1}{2}\left[\frac{\sin(\pi/2 - \pi/2\cos\theta)}{\beta(1-\cos\theta)}\right]$$

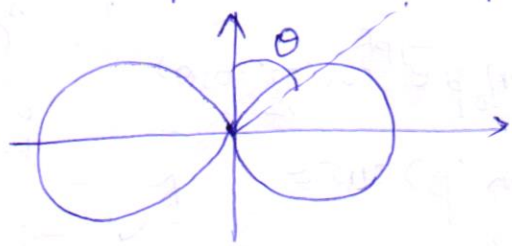
$$= \frac{1}{2}\frac{\cos(\pi/2\cos\theta)}{\beta(1+\cos\theta)} + \frac{1}{2}\frac{\cos(\pi/2\cos\theta)}{\beta(1-\cos\theta)}$$

$$= \frac{1}{2\beta}\left[\frac{\cos(\pi/2\cos\theta)(1-\cos\theta) + (1+\cos\theta)\cos(\pi/2\cos\theta)}{\beta^2(1-\cos^2\theta)}\right]$$

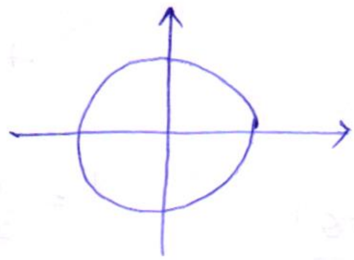
$$= \frac{1}{2\beta}\left[\frac{2\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\cos(\pi/2\cos\theta)}{\beta\sin^2\theta} \quad //$$

So the $|\vec{E}|$ field pattern in E-plane is



The E-field pattern in H-plane is



The 3-D radiation pattern of half dipole is of the shape of a doughnut

<u>Half power beamwidth</u>

$$F(\theta) = \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} = \frac{1}{\sqrt{2}}$$

$$\frac{\sin\theta}{\sqrt{2}} = \cos(\pi/2 \cos\theta)$$

$\theta = 51°, 129°$

Half power beamwidth of half-wave dipole

$= 129 - 51 = 78°$

<u>Full Null Beamwidth</u>

$FNBW = 180 - 0 = 180°$ //

<u>Radiation Resistance</u>

$$P_{avg} = \frac{1}{2} Re\{E \times H^*\} \longrightarrow ①$$

$$E_\theta = \frac{j I_0 \eta_0 e^{-j\beta R} \cos(\pi/2 \cos\theta)}{2\pi R \sin\theta} \longrightarrow ②$$

$$\frac{|E_\theta|}{|H_\phi|} = \eta_0 \longrightarrow ③$$

①, ② & ③ ⇒

$$P_{avg} = \frac{1}{2}\left[\frac{I_0^2 \eta_0 \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{4\pi^2 R^2 \sin^2\theta}\right]$$

$$= \frac{120\pi I_0^2 \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{2\times 4\pi^2 R^2 \sin^2\theta}$$

$$= \frac{15 I_0^2 \cos^2\left(\pi/2 \cos\theta\right)}{\pi R^2 \sin^2\theta}$$

Total power radiated

$$P_r = \oint\!\!\oint \vec{P}\cdot\vec{ds} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{15 I_0^2 \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} R^2 \sin\theta\, d\theta\, d\phi$$

$$= 30 I_0^2 \int_{\theta=0}^{\pi} \frac{\cos^2\left(\pi/2 \cos\theta\right) d\theta}{\sin\theta}$$

$$= 36.54\, I_0^2$$

$$\left(\frac{I_m}{\sqrt{2}}\right)^2 R_r = 36.54\, I_m^2$$

$$R_r = 2\times 36.54$$

$$= 73.1\Omega$$

Radiation resistance $= 73.1\Omega$

Directivity

Maximum radiation intensity

$$U_{max} = R^2 P_{avg}(90^\circ)$$

$$= \frac{15 I_0^2 R^2}{\pi R^2}$$

$$= \frac{15 I_0^2}{\pi}$$

$$\text{Directivity } (D) = \frac{4\pi U_{max}}{P_r} = \frac{4\pi\,(15 I_0^2)}{\pi\times 36.54 I_0^2} = \frac{60}{36.54} = 1.64$$

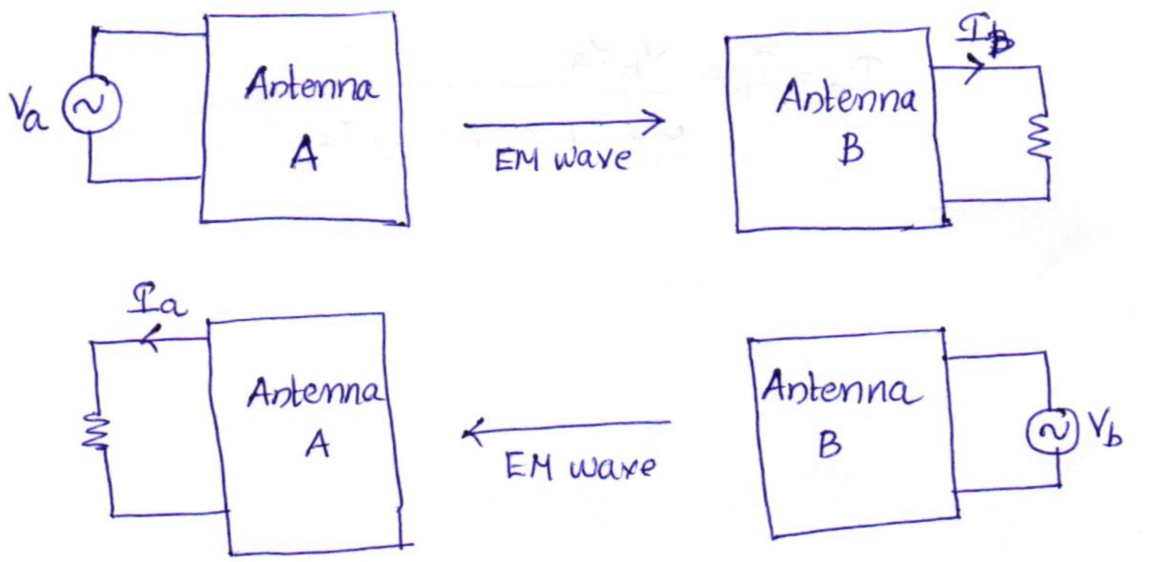
Directivity in dB

$$(D)_{dB} = 10 \log (1.64)$$

$$= 2.153 \text{ dB}.$$

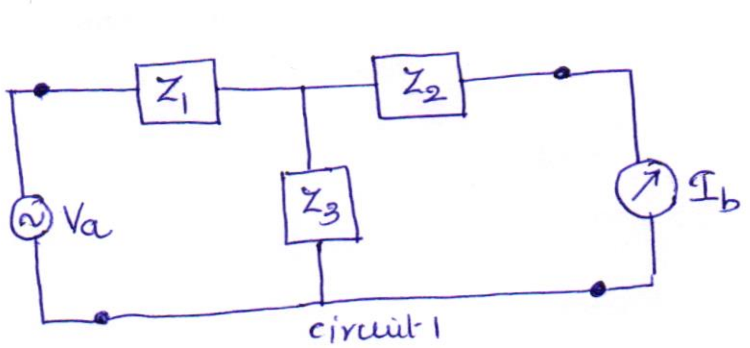
Reciprocity Theorem

- Principle of reciprocity states that the receive and transmit properties of an antenna are identical.
- If an emf is applied to the terminals of an antenna A then a current will be obtained at the terminals of another antenna B. Similarly if an emf is applied to the terminals of antenna B then an equal current (in both amplitude and phase) will be obtained at the terminals of antenna A.
- If $V_a = V_b$, then by reciprocity theorem $I_a = I_b$

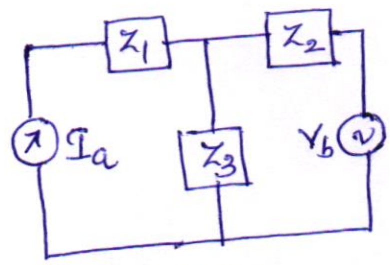


Proof

- Let the antenna & the space between them be replaced by a network of linear, passive, bilateral impedances.



circuit 1



circuit 2

- The current I_b is

(from circuit 1) $$I_b = I_1 \frac{Z_3}{Z_2+Z_3} \longrightarrow (1)$$

$$I_1 = \frac{V_a}{Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}} = \frac{V_a (Z_2 + Z_3)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \longrightarrow (2)$$

From (1) & (2)

$$I_b = \frac{V_a Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

By interchanging the current & emf position (circuit 2),

$$I_a = \frac{V_b Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

If $V_a = V_b$, then $I_a = I_b$ thus proved

$$I_a = I_b = \frac{V_b Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

Module II

- Principle of Log periodic antenna array and design, Helical antenna : types and design.
- Design of Rectangular Patch antenna & feeding techniques.
- Principle of Horn, Parabolic dish antenna, (expression for E, H, G without derivation.
- Mobile phone antenna - Inverted F antenna.

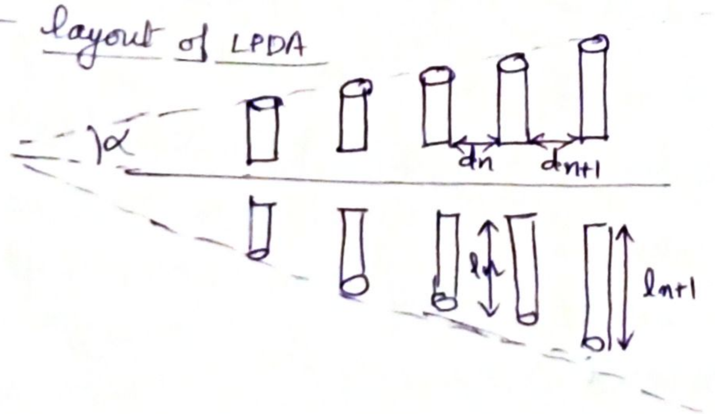
Log Periodic Dipole Antenna (LPDA)

- Log Periodic Dipole Antenna (LPDA) was invented by Isbell & DuHamel in 1960. Carrel developed the mathematical model and design equations for LPDA.
- It is a multielement, directional, wideband antenna.
- It consists of a number of non-identical half wave dipole driven elements of gradually increasing length.
- Elements are spaced at intervals following a logarithmic function of the frequency, decided by spacing factor (σ)
- Relationship between length of adjacent dipole elements vary logarithmically - decided by the scaling factor (τ).

$$\ln(f_2) - \ln(f_1) = \ln(1/\tau)$$

- The impedance of the antenna is periodic with the logarithm of the frequency.
- The gain of a typical LPDA ranges from 7.5 dBi to 12 dBi
- They are used in VHF and UHF bands.
- An LPDA can be designed to over a large bandwidth, sometimes as large as $f_U : f_L = 10:1$

- Layout of LPDA



$\alpha \rightarrow$ half angle

$l_n, l_{n+1} \rightarrow$ length of dipoles

$d_n, d_{n+1} \rightarrow$ spacing b/w dipoles.

- A log periodic dipole antenna has 3 regions
  1. active region
  2. transmission region
  3. unexcited region.

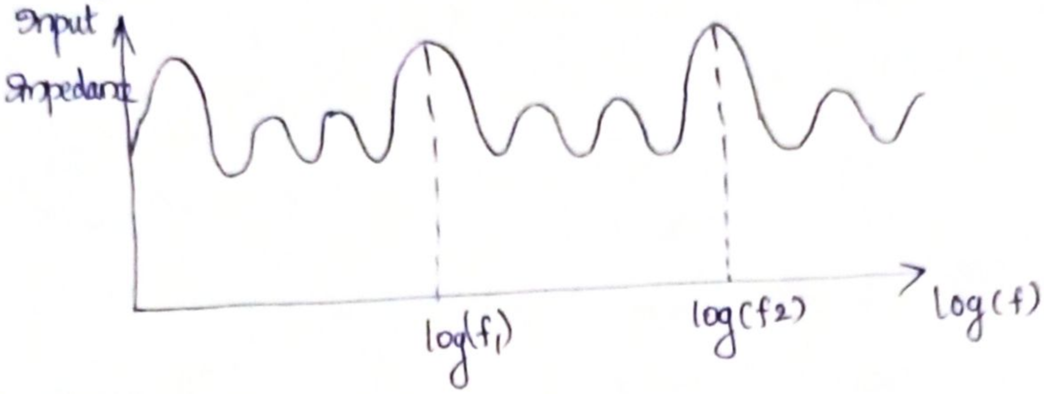
- The transmission region consists of all dipole elements which are reasonably less than a half wavelength long at a given frequency and the portion of feeder to which these elements are attached. One field originates at the feed point and propagates along the feeder in the direction of larger elements. This is called the transmission field. The other field originates in the vicinity of half-wavelength dipole and propagates in the direction of smaller elements, manifesting itself as radiated field.

- At any frequency within the design band there are several elements of nearly half-wavelength dimensions. The current in these elements is large compared to the current on the remaining elements. These elements contribute most of the radiation and form the so-called "active region". As frequency decreases from f_n to τf_n, the active region shifts from the group of elements to next.

- The unexited region consists of dipole elements with length greater than half wavelength long at a given frequency. These dipoles act as directors, reflecting electromagnetic radiation towards the feed direction.

Thus the radiated field travel towards the feed forming a unidirectional end-fire pattern. towards the vertex. The radiated wave of a LPDA is linearly polarized & it has horizontal polarization when the plane of the antenna is parallel to the ground.

If the input impedance of an LPDA is plotted as a function of frequency, it will be repetitive. If it is plotted as a function of logarithm of frequency it will be periodic with each cycle being exactly identical to the preceding one. Hence the name log-periodic.



Design Procedure

- To start with design parameters τ & σ are fixed for a given directivity. As τ increases, the number of elements (& thus the size of the antenna) increases. A larger τ gives larger gain. Thus choosing τ & σ involves a tradeoff.

- The desired bandwidth

$$B = \frac{f_U}{f_L}$$ where f_U = highest frequency of operation

f_L = lowest frequency of operation

Once τ & σ are fixed, half angle α can be determined from the below equation

$$\sigma = \frac{1-\tau}{4\tan\alpha}$$

For a LPDA, the operational bandwidth depends on the distance of the active region can move before it is distorted by the smallest or largest dipole element. Thus the width of the active region decides the operational bandwidth of the antenna. The bandwidth of the active region (B_{ar}) can be determined by the equation

$$B_{ar} = 1.1 + 7.7(1-\tau)^2 \cot\alpha$$

The structure bandwidth (B_s) is given by:

$$B_s = B \cdot B_{ar}$$

The number of half wave dipole elements (N) can be calculated as:

$$N = 1 + \frac{\log(B_s)}{\log(1/\tau)}$$

The length of the largest dipole is given by

$$l_{max} = \frac{\lambda_{max}}{4} = \frac{c}{4 f_{min}}$$

$c \rightarrow$ velocity of light

$f_{min} \rightarrow$ lowest frequency of operation

The spacing between the dipoles can be found using the relation:

$$\sigma = \frac{d_n}{4 l_n}$$

$l_n \rightarrow$ length of n^{th} dipole

$d_n \rightarrow$ spacing between n^{th} & $(n+1)^{th}$ element.

If the average characteristic impedance of the dipole (Z_a) is known, the thickness of every dipole can be calculated as:

$$Z_a = 120\left(\log_e(l/a) - 2.25\right)$$

Qn Design a LPDA operating in the frequency band 54 MHz to 216 MHz. Given $\tau = 0.865$ & $\sigma = 0.157$. The input impedance is 50 ohm. The elements should be made of aluminium tubing with 1.9 cm outside diameter for the largest element

$\tau = 0.865$

$\sigma = 0.157$

$$\sigma = \frac{1-\tau}{4\tan\alpha} \Rightarrow 0.157 = \frac{1-0.865}{4\tan\alpha}$$

$$\tan\alpha = \frac{1-0.865}{4(0.157)}$$

$\Rightarrow$ ~~α~~ $\alpha = 120^{\circ}$

$$B = \frac{f_0}{f_L} = \frac{216}{54} = 4$$

$$B_{av} = 1.1 + 7.7(1-\tau^2)\cot\alpha$$

$$= 1.1 + 7.7(1-0.865^2)\cot \alpha$$

$$= \underline{1.753}$$

$$B_s = B_{ar} B$$

$$= 1.753 \times 4 = 7.012$$

$$l_{max} = \frac{c}{f_{min} \times 4} = \frac{3\times10^8}{54\times10^6\times4} = 1.38\,m$$

$$N = 1 + \frac{\log(B_s)}{\log(1/\tau)} = 1 + \frac{\log(7.012)}{\log\left(\frac{1}{0.865}\right)} = 1 + \frac{0.8468}{0.0629}$$

$$= 14.44 \approx 14$$

Length of the dipole element (l_n)	Spacing between dipole elements
$l_1 = 1.38\,m$	$d_1 = 4l_1\sigma = 0.866\,cm$
$l_2 = 1.38\times0.865 = 1.1937\,m$	$d_2 = 4l_2\sigma = 0.75\,m$
$l_3 = 1.1937\times0.865 = 1.0325\,m$	$d_3 = 4l_3\sigma = 0.65\,m$
$l_4 = 1.0325\times0.865 = 0.89\,m$	$d_4 = 4l_4\sigma = 0.56\,m$
$d_5 = 0.89\times0.865 = 0.77\,m$	$d_5 = 4l_5\sigma = 0.47\,m$
$l_6 = 0.77\times0.865 = 0.67\,m$	$d_6 = 4l_6\sigma = 0.42\,m$
$l_7 = 0.67\times0.865 = 0.58\,m$	$d_7 = 4l_7\sigma = 0.36\,m$
$l_8 = 0.58\times0.865 = 0.50\,m$	$d_8 = 4l_8\sigma = 0.31\,m$
$l_9 = 0.50\times0.865 = 0.43\,m$	$d_9 = 4l_9\sigma = 0.27\,m$
$l_{10} = 0.43\times0.865 = 0.37\,m$	$d_{10} = 4l_{10}\sigma = 0.23\,m$
$l_{11} = 0.37\times0.865 = 0.32\,m$	$d_{11} = 4l_{11}\sigma = 0.20\,m$
$l_{12} = 0.32\times0.865 = 0.28\,m$	$d_{12} = 4l_{12}\sigma = 0.18\,m$
$l_{13} = 0.28\times0.865 = 0.24\,m$	$d_{13} = 4l_{13}\sigma = 0.15\,m$
$l_{14} = 0.24\times0.865 = 0.21\,m$	

Length $(l) = 1.38\,m$

Radius $(a) = \frac{0.019}{2} = 0.095\,m$

$Z_0 = 120(\log_e(l/a) - 2.25)$

$= 120\left(\log_e\left(\frac{1.38}{0.095}\right) - 2.25\right)$

$= 120(\log_e(145.816) - 2.25)$

$= 327.88\,\Omega$ $\longrightarrow$ average characteric impedance

Radius of each element can be calculated as follows:

$Z_0 = 120(\log_e(l/a) - 2.25)$

$\frac{327.88}{120} + 2.25 = \log_e(l/a)$

$\log_e(l/a) = 4.982$

$(l/a) = 148.81$

$a = \frac{l}{148.81}$

Radius of each dipole element

$a_1 = 9.5\,mm$

$a_2 = \frac{l_2}{148.81} = 8.2\,mm$

$a_3 = \frac{l_3}{148.81} = 7.1\,mm$

$a_4 = \frac{l_4}{148.81} = 6.1\,mm$

$a_5 = \frac{l_5}{148.81} = 5.3\,mm$

$a_6 = \frac{l_6}{148.81} = 4.6\,mm$

$a_7 = \frac{l_7}{148.81} = 4.0\,mm$

$a_8 = \frac{l_8}{148.81} = 3.4\,mm$

$a_9 = \frac{l_9}{148.81} = 3\,mm$

$a_{10} = \frac{l_{10}}{148.81} = 2.6\,mm$

$a_{11} = \frac{l_{11}}{148.81} = 2.2\,mm$

$a_{12} = \frac{l_{12}}{148.81} = 1.9\,mm$

$a_{13} = \frac{l_{13}}{148.81} = 1.7\,mm$

$a_{14} = \frac{l_{14}}{148.81} = 1.4\,mm$

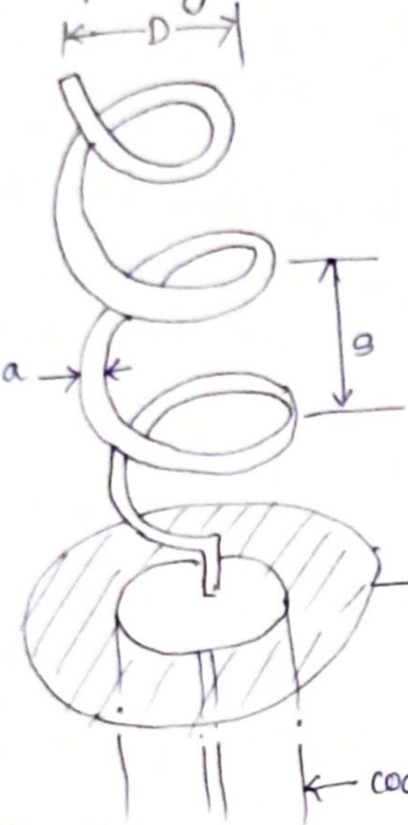
Helical Antenna

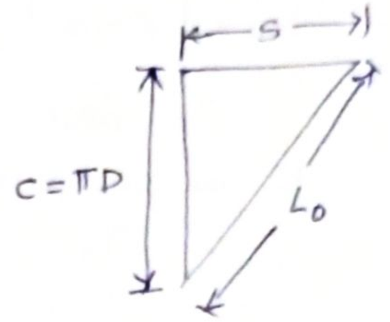
- A conducting wire is wound in the form of a screw thread forming a helix.
- Usually helical antenna is used with a ground plane.
- The ground plane can be flat or cupped.
- The diameter of ground plane should be $\geq 3\lambda/4$.
- The geometrical configuration of helix is shown below:

$N \rightarrow$ No of helix

$D \rightarrow$ Diameter

$S \rightarrow$ spacing between each turn.





Total length of the antenna $(L) = NS$

circumference of the helix $(C) = \pi D$

Length of the wire between each turn $(L_0) = \sqrt{S^2 + C^2}$

Total length of the wire $(L_n) = NL_0 = N\sqrt{S^2+C^2}$

The pitch angle (α) is given by

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) = \tan^{-1}(S/C)$$

When $\alpha = 0^\circ$ – ~~helical~~ helix reduces to a loop antenna.

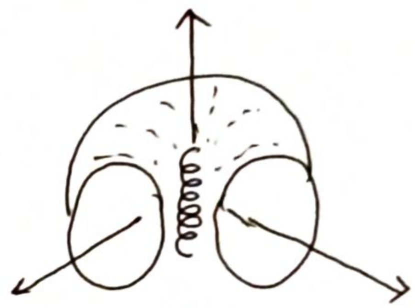
$\alpha = 90^\circ$ – " " " " linear wire

For helical antenna $0^\circ < \alpha < 90^\circ$

- The radiation characteristic of antenna can be controlled by controlling the geometric parameters like pitch angle & size of the conducting wire.
- The general polarization of the antenna is elliptical.
- The helical antenna can operate in 2 principal mode
  (1) normal (broadside) mode
  (2) axial (endfire) mode.

<u>Normal (broadside) mode</u>

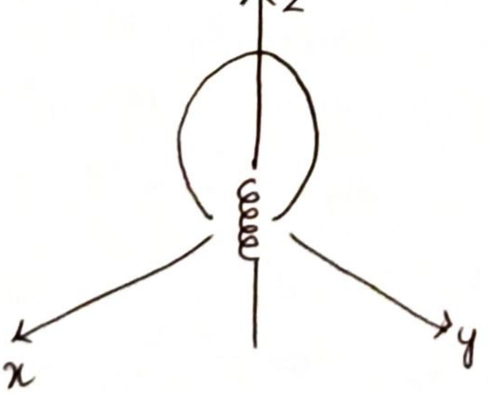
The radiation has its maximum in a plane normal to the axis & is nearly null along axis



Broadside array.

<u>Axial (end fire) mode</u>

The radiation has its maximum along the axis of the helix



Normal Mode

- To achieve the normal mode of operation, the dimension of the helix is usually kept small compared to wavelength

$$(N L_0 << \lambda_0)$$

- In the normal mode, the helix can be considered as N small loops and N short dipoles connected together in series

Equivalent

- The fields are obtained by superposition of the fields of small loops of diameter D + fields of short dipoles of length S.

- Field due to short dipole is

$$E_\theta = j\eta \frac{\beta I_0 S e^{-j\beta r}}{4\pi r} \sin\theta$$

- Field due to loop of diameter D is

$$E_\phi = \eta \frac{\beta^2 (D/2)^2 I_0 e^{-j\beta r}}{4r} \sin\theta$$

- The ratio of magnitudes of E_θ + E_ϕ gives the axial ratio (AR)

$$AR = \frac{|E_\theta|}{|E_\phi|} = \frac{4S}{\pi \beta D^2} = \frac{2\lambda S}{(\pi D)^2}$$

Axial Ratio

$$AR = \frac{2N+1}{2N}$$

Normalized far field pattern is given by

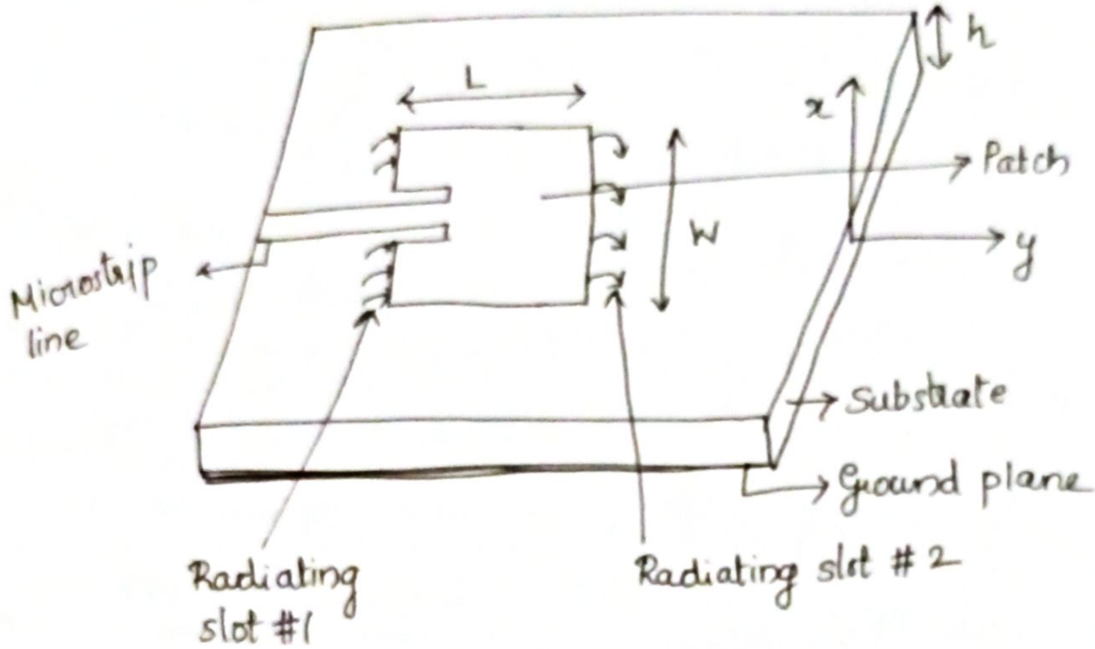
$$E = \sin\left(\frac{\pi}{2N}\right) \cos\theta \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin \psi/2} \quad \text{where } \psi = k_0\left(S\cos\theta - \frac{L_0}{P}\right)$$

$$P = L_0/\lambda_0 \text{ for ordinary}$$

$$\frac{S/\lambda_0 + 1}{} \text{ end fire direction}$$

Microstrip Patch Antenna

— Microstrip patch antenna consists of a radiating patch on a dielectric substrate of certain thickness, with ground plane at the bottom of the substrate. The radiating patch can be of different shapes like circle, rectangle, triangle, square etc. In microstrip patch antenna, the ground plane and the radiating patch is made up of conductor like copper, aluminium, gold etc.



Layout of rectangular patch antenna

Applications

— Microstrip patch antennas are widely used in aircraft, space craft & missile applications where size, weight etc are constrains

— They are used in mobile, radio and wireless communication systems

Advantages

— low profile
— conformable to planar & nonplanar surfaces
— compact
— can be easily formed into arrays.

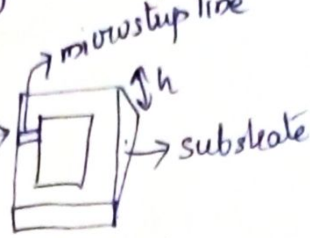
- They are cheaper & easy to install
- can be easily integrated to microstrip circuits

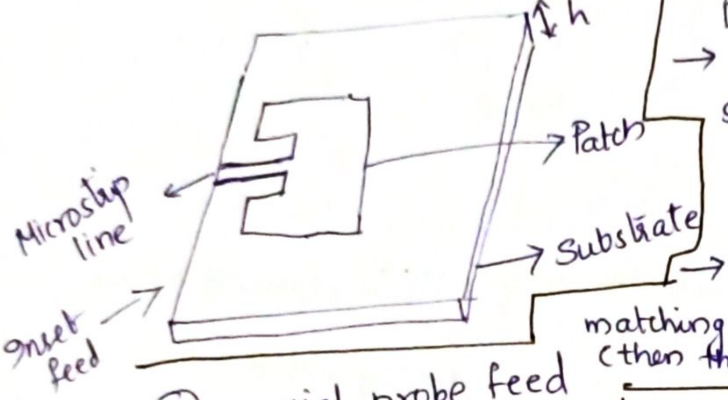
<u>Disadvantages</u>

- low efficiency
- low power handling capability
- high Q
- poor polarization purity
- spurious feed radiation
- very narrow frequency bandwidth

<u>Feeding Methods</u>

① Microstrip line feed

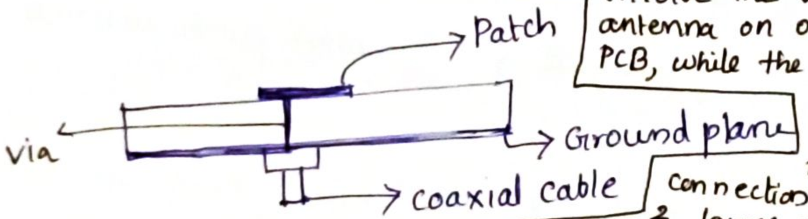




→ In this type, a conducting strip is connected directly to the edge of the microstrip patch. The patch is wider than the conducting strip.

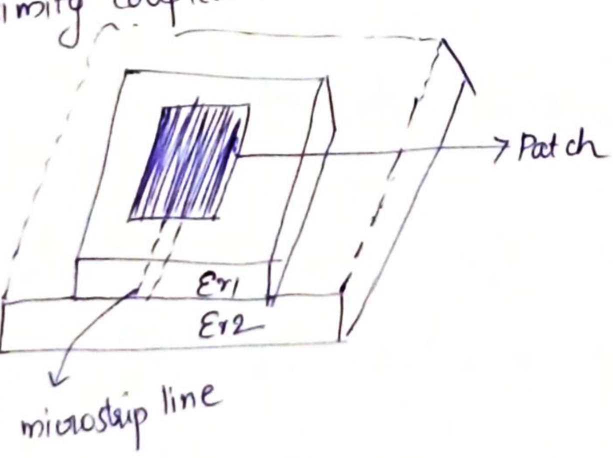
→ For the purpose of impedance matching an inset is cut in the patch (then the feed is called inset feed)

② coaxial probe feed



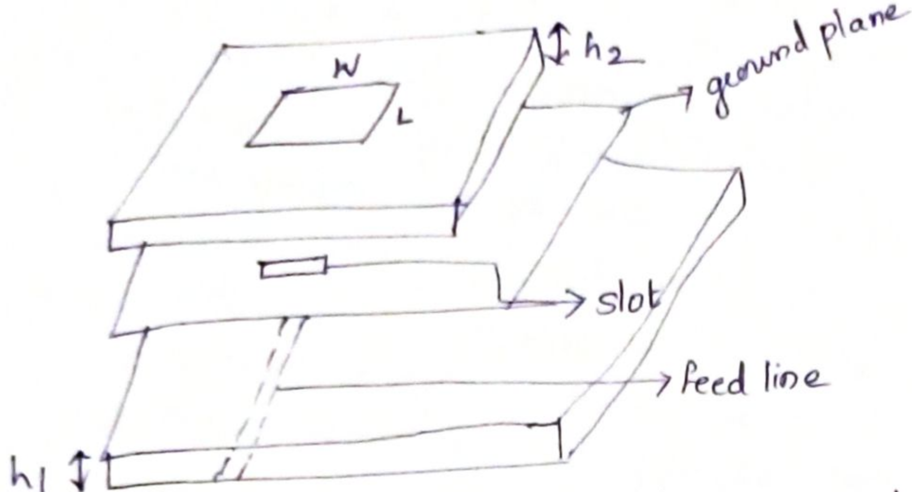
→ coaxial-fed patch antenna involve the use of a patch antenna on one side of the PCB, while the feedline is routed on the back side of the PCB. The connection between the 2 layers is implemented using a via.

③ Proximity coupled feed



- In proximity coupled feed, two dielectric substrates are used
- The microstrip patch is there on the upper dielectric substrate & the feedline is there between two substrates

④ Aperture coupled feed



This feed is having two substrates, which are different from each other & are seperated by a ground plane. In this method, the microstrip patch and feed line are coupled through a slot in the ground plane.

Substrate

The dielectric constant of substrates used for microstrip patch antennas are typically in the range $2.2 < \varepsilon_r < 12$. Lower the permittivity of the substrate, wider is the fringing field and better is the radiation. With the decrease of permittivity antenna bandwidth and efficiency increases. But lowering the permittivity decreases the input impedance & increases the size of the antenna.

Thickness of substrate (h) is chosen such that

$h > 0.06\lambda_g$

where λ_g is the guided wavelength given by

$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon}}$$

A tradeoff has to be made while selecting the substrate thickness. As thickness of substrate increases, surface wave results in undesired radiation, decrease antenna efficiency and introduces spurious coupling between different circuits or antenna elements. Also surface waves reaching the outer boundaries of an open microstrip structure are reflected & diffracted by the edges. These diffracted wave provide an additional contribution to radiation, degrading the antenna pattern by increasing the side lobe & cross polarization levels. Thus thickness should be chosen such that surface waves are suppressed. A thick substrate with low dielectric constant yields better efficiency, larger bandwidth & better radiation, where as a thin substrate with higher dielectric constant yields compact antenna, with less efficiency & narrower bandwidth. Thus a tradeoff has to be made between the antenna dimensions & antenna performance.

Methods of Analysis

The main methods for analysing microstrip patch antenna are

(1) transmission line model
(2) cavity model
(3) full wave model

Transmission line Model

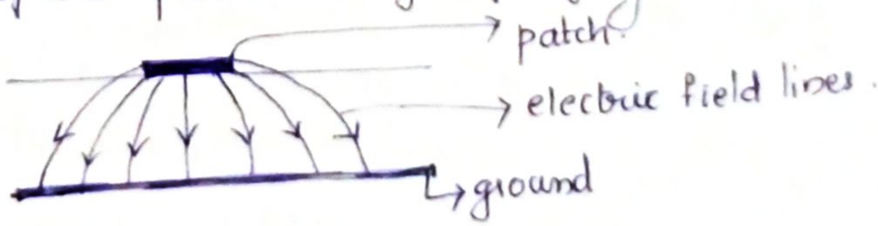
- easiest of all models
- but yields less accurate results.
- However it does shed some physical insight
- proposed by Munson in 1974

- The interior region of the patch is modelled as a transmission line. The characteristic impedance Z_0 & propagation constant β for the line are determined by the patch size and substrate parameters.

- Consider a rectangular patch of dimension $L \times W$. The periphery of this patch is described by 4 walls/edges at $x = 0, L$ and $y = 0, W$, The 4 edges of the patch are classified as radiating type or non-radiating type depending on the field variation along their length. This classification is based on the observation that a radiating edge is associated with slow field variation along its length. The nonradiating edge, on the other hand should have an integral multiple of half wavelength variation along the edge, such that there is an almost complex cancellation of the radiated power from the edge. For the TM_{10} mode in the patch, the edges at $x = 0, L$ are the radiating types because the electric field is uniform along these edges. The walls $y = 0, W$ are nonradiating types because of half-wave variation of the fields along these edges. The edges at $x = 0, L$ radiate most of the power.

Thus in transmission line model, the microstrip antenna is represented by two radiating slots seperated by a low-impedance transmission line of length L.

Fringing effects – Because the dimensions of the patch are finite along length & width, the fields at the edges of the patch undergo fringing



As $\frac{W}{h} >> 1$ and $\varepsilon_r >> 1$, the electric field line concentrate mostly in the substrate. Fringing in this case makes the microstrip line look wider electrically compared to its physical dimensions. Since some of the waves travel in the substrate and some in air, an effective dielectric constant (ε_{eff}) is introduced to account for fringing and wave propagation in the line

$\frac{W}{h} > 1$

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + \frac{12h}{W}\right]^{-1/2}$$

$\varepsilon_r \rightarrow$ dielectric constant

Design Procedure

① For an efficient radiator, a practical width that leads to good radiation efficiencies is

$$W = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r + 1}}$$

$\varepsilon_r \rightarrow$ dielectric constant
$f_r \rightarrow$ resonant frequency
$c \rightarrow$ speed of light

② Find the effective dielectric constant

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + \frac{12h}{W}\right]^{-1/2}$$

③ Find the effective length (L_{eff}) of the patch

- Because of fringing effects, electrically the patch looks greater than the physical dimension

- The extended length (ΔL) is given by

$$\Delta L = 0.412h \frac{(\varepsilon_{eff} + 0.3)\ (W/h + 0.264)}{(\varepsilon_{eff} - 0.258)\ (\frac{W}{h} + 0.8)}$$

- The effective length is given by:

$$L_{eff} = \frac{c}{2f_r\sqrt{\epsilon_{eff}}}$$

④ The physical length (L) is given by

$$L = L_{eff} - 2\Delta L$$

Qn. Design a rectangular microstrip antenna using a substrate (RT/duroid 5880) with dielectric constant of 2.2, h = 0.1588 cm so as to resonate at 10 GHz

$f_r = 10\,GHz$

$h = 0.01588\,cm$

$\epsilon_r = 2.2$

$$W = \frac{c}{2f_r}\sqrt{\frac{2}{\epsilon_r+1}} = \frac{3\times10^8}{2\times10\times10^9}\sqrt{\frac{2}{2.2+1}}$$

$$= \frac{3\times10^8}{20\times10^9}\sqrt{0.625}$$

$$= \frac{3}{200}\times0.79$$

$$= 0.01185\,m$$

$$= 1.185\,cm.$$

$$\epsilon_{eff} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+\frac{12h}{W}\right]^{1/2}$$

$$= \frac{3.2}{2} + \frac{1.2}{2}\left[1+\frac{12\times0.1588\times10^{-2}}{1.185\times10^{-2}}\right]^{1/2}$$

$$= \underline{\underline{1.972}}$$

$$\Delta L = 0.412h\,\frac{[\epsilon_{eff}+0.3]\left[\frac{W}{h}+0.268\right]}{[\epsilon_{eff}-0.258]\left[\frac{W}{h}+0.8\right]}$$

$$= 0.412\times0.1588\,\frac{[1.972+0.3]\left(\frac{1.186}{0.1588}+0.264\right)}{[1.972-0.258]\left(\frac{1.186}{0.1588}+0.8\right)}$$

$$= 0.081\,cm //$$

$$L_{eff} = \frac{c}{2 f_r \sqrt{\varepsilon_{eff}}}$$

$$= \frac{3 \times 10^8}{2 \times 10 \times 10^9 \sqrt{1.972}}$$

$$= \frac{1.5 \times 10^{-2}}{\sqrt{1.972}} = \frac{1.5 \times 10^{-2}}{1.4} = \underline{\underline{1.071 \times 10^{-2}\ m}}$$

$$= \underline{1.071\ cm}$$

$$L = L_{eff} - 2\Delta L$$

$$= 1.071 - 2 \times 0.081$$

$$= 1.071 - 0.162$$

$$= 0.909\ cm$$

Length of the patch = 0.909 cm

Width of the patch = 1.185 cm

Antenna for mobile handset

- The evolution of mobile phone antennas was driven by 2 major forces

① user demand

Mobile phones incorporating internal antennas has better appearance, are smaller in size and thus are easier to carry so that the customer demand ~~was~~ is immediately adhered to regardless of considering any disadvantages in antenna performance.

② new frequency bands were added with the introduction of new standard generations.

Requirements

The main design challenges of mobile phone antenna are the requirement of small size and should accomodate multisystem in multibands including all cellular 2G, 3G, 4G and other noncellular radiofrequency (RF) bands.

Moreover the need for a nice appearance and should meet all standards and requirements such as specific absorption rate (SAR) and hearing aid compatibilty (HAC) and over the air (OTA).

The antenna for mobile phone must be compact. The size should be reduced without compromising radiation efficiency

Specific Absorption Rate (SAR)

An antenna in a mobile handset is in close proximity to the user. So while designing such an antenna, both communication aspect and potential health risk has to be taken into account. Human tissue close to antenna absorb electromagnetic radiation. Specific absorption rate (SAR) in a 1g or 10g tissue mass is a determining parameter in the health discussion. As per FCC 1987, the limit of SAR is 1.6 W/g in 1g of tissue.

- SAR of electromagnetic energy can be calculated from the electric field within the tissue as

$$SAR = \frac{1}{V} \int_{sample} \frac{\sigma(r)|E(r)|^2 dr}{\rho(r)}$$

where $\sigma(r) \rightarrow$ sample electrical conductivity

$E(r) \rightarrow$ rms electric field

$\rho \rightarrow$ sample density.

Types of antennas used in cellphone

(1) External antennas

- The mobile phones that were manufactured for the 1st generation and also in the begining of the second generation had an external antenna placed on the top corner of the handset.

eg ① whip (monopole) antenna
② helical antenna.

Whip antenna

① has a length close to quarter wavelength in the frequency of operation.

② They are fed similar to dipole.

③ They are very efficient radiators

④ Most whip antenna can be retracted into the phone when not in use.

Helical antenna

- Helical antenna can acheive the coverage of the same frequency band with a shorter antenna element compared to whip antenna.
- In the normal mode, helical antenna has a radiation pattern similar to monopole element.

- A combination of helical & whip antennas was also used.

Advantage of external antennas

- high efficiency
- broadband matching

Disadvantages of external antenna

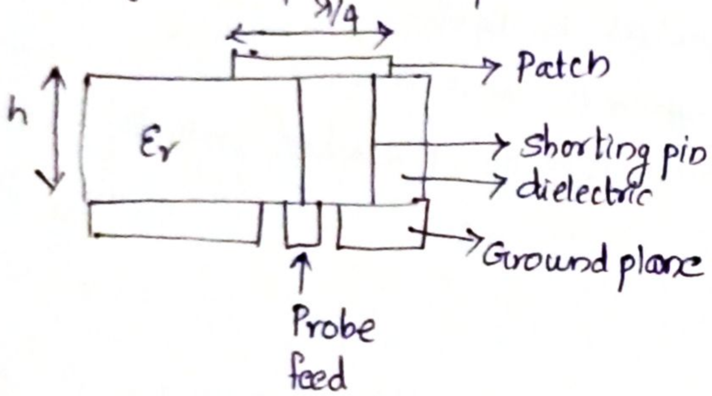
→ large size
→ high SAR.

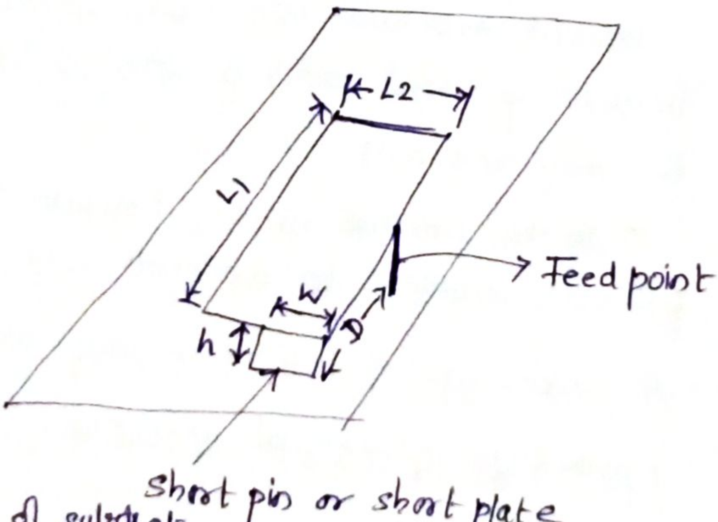
Internal Antennas

- Antenna is embedded inside the phone & not visible to user.
- eg planar inverted F- Antenna (PIFA) is one of the most commonly used internal antenna.

PIFA (Planar Inverted F Antenna)

- PIFA is a quarter wavelength patch antenna over a ground plane, having one or more ground connections generally close to the feed. This ground connection acts as a shunt inductor and cancels out the high capacitance between the PIFA element and the ground plane at particular frequencies.





ε_r → permittivity of substrate

L1 → Length of PIFA

L2 → Width of PIFA

W → width of shorting pin / shorting plate.

D → distance of feed from the shorting pin.

h → height of PIFA from ground plane.

- The PIFA is resonant when:

$$L1 + L2 - W = \lambda/4$$

Suppose $W=0$ (ie short is just a pin), PIFA is resonant when

$$L1 + L2 = \lambda/4$$

Suppose $W = L2$, ie the shorting pin runs the entire width of the patch, PIFA is resonant when:

$$L_1 = \lambda/4$$

- The input impedance of PIFA can be controlled by adjusting the distance 'D' between the short pin & feed. The closer the feed is to the shorting pin, the impedance will decrease; the impedance can be increased by moving it farther from the shorting edge. Thus impedance can be tuned by varying 'D'.

- The length of PIFA antenna can be further reduced by capacitive loading.

<u>Advantages</u>

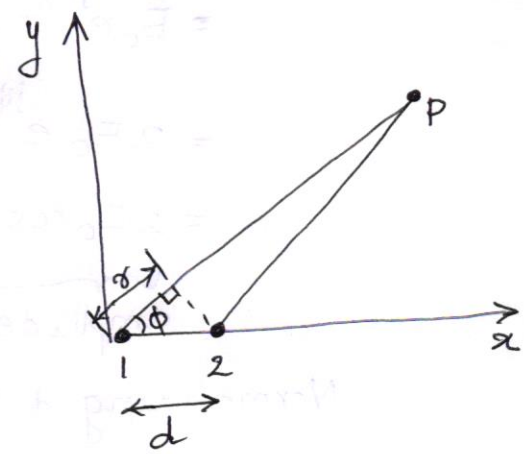
① easy to manufacture
② low profile
③ low cost
④ omnidirectional radiation pattern
⑤ impedance matching is easy by tuning the shorting strip, & the feeding distance & thickness of the shorting pin.
⑥ low SAR
⑦ PIFA antenna can be made multiband. For example triband PIFA antenna with resonance at 900 MHz, 1800 MHz & 1900 MHz are available

Module II

- Array of point sources, field of two isotropic point source, principle of pattern multiplication
- Linear arrays of 'n' isotropic point sources. Grating lobes, Array factor (derivation)
- Design of broadside, end fire and Dolph-Chebyshev arrays.
- Concept of phase array.

Two element array of point sources

Consider an array of two point sources (isotropic). All the sources are excited with equal amplitude currents. Let the spacing between any 2 adjacent elements of the array (inter element spacing) be 'd'. The phase shift between currents on any adjacent antenna elements of the array (progressive phase shift) be δ.



The field due to the array at a far away point P is proportional to the current and is the superposition of fields due to individual elements (having equal amplitude but different phases). The phase of the field has 2 components

1) phase due to phase of excitation current (δ)
2) " " " propagation.

Phase difference $\psi = 2\pi$ (path difference) $+ \delta$

$$\cos\phi = \frac{r}{d}$$

$$r = d\cos\phi \text{ meters}$$

$$= \frac{d}{\lambda}\cos\phi \text{ wavelengths}$$

Phase difference

$$\psi = \frac{2\pi}{\lambda} d\cos\phi + \delta$$

$$\boxed{\psi = \beta d\cos\phi + \delta}$$

Case I : Two isotropic point sources with seperation $d=\frac{\lambda}{2}$, fed with current of same amplitude & phase

The electric field at a point P is the superposition of $\vec{E}$ due to each source

$$E = E_0 + E_0 e^{j\psi}$$
$$= E_0 e^{j\psi/2}\left[e^{-j\psi/2} + e^{j\psi/2}\right]$$
$$= 2E_0 e^{j\psi/2} \cos\psi/2$$
$$= \underbrace{2E_0 \cos\psi/2}_{\text{amplitude}} \; \underbrace{e^{j\psi/2}}_{\text{phase}}$$

Normalixing & taking the amplitude term

$$|E| = \left|\cos\frac{\psi}{2}\right|$$

$\delta = 0$ & $d = \lambda/2$

$$\Rightarrow \frac{\psi}{2} = \frac{\beta d\cos\phi + \delta}{2}$$
$$= \frac{\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi}{2}$$
$$= \frac{\pi}{2}\cos\phi$$

$$|E| = \left|\cos\frac{\psi}{2}\right| = \left|\cos\left(\pi/2\cos\phi\right)\right|$$

Direction of maxima

$$\frac{\pi}{2}\cos\phi = 0 \Rightarrow \cos\phi = 0$$
$$\Rightarrow \phi = 90^\circ, 270^\circ \text{ etc.}$$

Direction of null

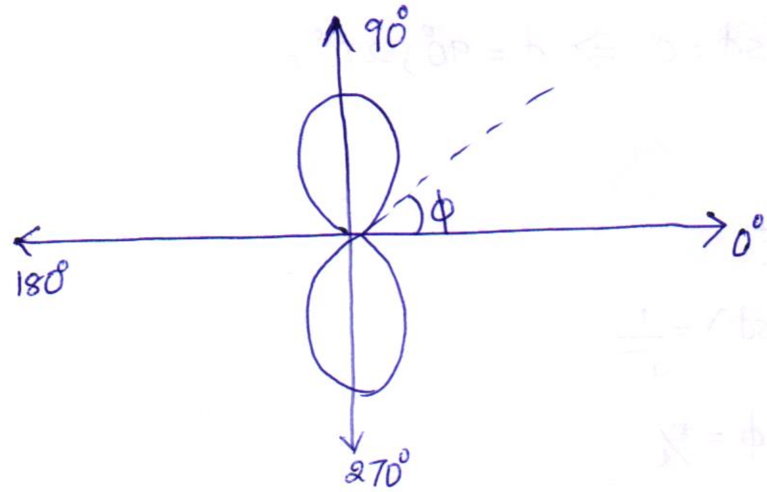
$$|E| = \left|\cos\left(\pi/2\cos\phi\right)\right|$$
$$\pi/2\cos\phi = \pi/2 \Rightarrow \cos\phi = 1$$
$$\Rightarrow \phi = 0^\circ, 180^\circ$$

HPBW

$$|E| = \frac{1}{\sqrt{2}} \Rightarrow \left| \cos\left(\frac{\pi}{2}\cos\phi\right)\right| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\pi}{2}\cos\phi = \pi/4$$

$$\cos\phi = \frac{1}{2} \Rightarrow \phi = 60^\circ, 120^\circ$$



Case 2: Two isotropic point sources with seperation $d = \lambda/2$, fed with current of same amplitude & phase quadrature

$$\delta = \pi$$

$$\psi = \beta d\cos\phi + \delta$$

$$= \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi + \pi$$

$$= \pi\cos\phi + \pi$$

$$E = E_0 + E_0 e^{j\psi}$$

$$= E_0 + E_0 e^{j(\pi\cos\phi + \pi)}$$

$$= E_0 + E_0 e^{j\pi\cos\phi} \cdot e^{j\pi}$$

$$= E_0 - E_0 e^{j\pi\cos\phi}$$

$$= E_0 e^{\frac{-j\pi\cos\phi}{2}} \left[e^{\frac{j\pi\cos\phi}{2}} - e^{\frac{-j\pi\cos\phi}{2}} \right]$$

$$= 2j\sin\left(\frac{\pi}{2}\cos\phi\right) E_0 e^{\frac{-j\pi\cos\phi}{2}}$$

$$= \underbrace{2jE_0\sin(\pi/2\cos\phi)}_{\text{Amplitude}} \underbrace{e^{\frac{-j\pi\cos\phi}{2}}}_{\text{Phase}}$$

Normalizing the E field. The field pattern is given by

$$|E| = |\sin(\pi/2 \cos\phi)|$$

Direction of Null

$$\sin(\pi/2 \cos\phi) = 0$$

$$\pi/2 \cos\phi = 0 \Rightarrow \phi = 90^\circ, 270^\circ$$

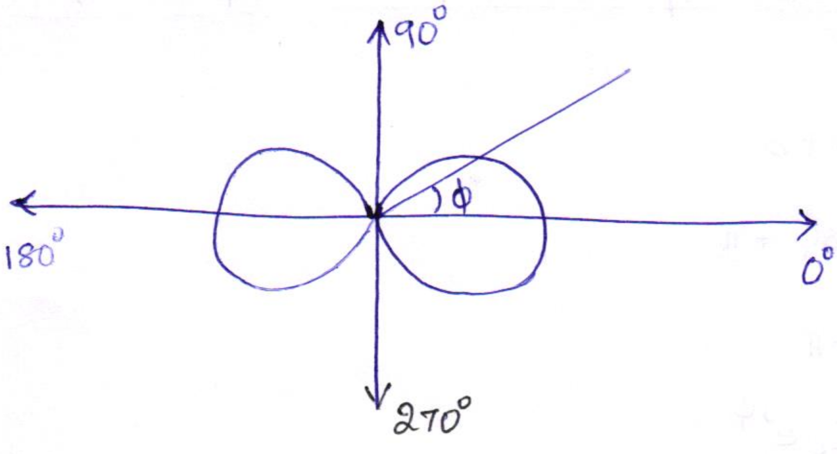
HPBW

$$|E| = \frac{1}{\sqrt{2}}$$

$$\sin(\pi/2 \cos\phi) = \frac{1}{\sqrt{2}}$$

$$\pi/2 \cos\phi = \pi/4$$

$$\cos\phi = 1/2 \Rightarrow \phi = 60^\circ, 120^\circ$$



Case 3: Two isotropic point sources with seperation $d = \frac{\lambda}{2}$, fed with current of same amplitude but out of phase by 90°

$$E = E_0 + E_0 e^{j\psi}$$

$$d = \frac{\lambda}{2}, \quad \delta = \frac{\pi}{2}$$

$$\psi = \beta d \cos\phi + \delta$$

$$= \frac{2\pi}{\lambda} \frac{\lambda}{2} \cos\phi + \pi/2 = \pi\cos\phi + \pi/2$$

$$E = E_0 + E_0 e^{j(\pi/2 + \pi\cos\phi)}$$
$$= E_0 \left[e^{j(\pi/4 + \pi/2\cos\phi)}\right]\left[e^{-j(\pi/4 + \pi/2\cos\phi)} + e^{j(\pi/4 + \pi/2\cos\phi)}\right.$$
$$= 2E_0 e^{j\psi/2} \cos(\pi/2\cos\phi + \pi/4)$$
$$= \underbrace{2E_0 \cos(\pi/2\cos\phi + \pi/4)}_{\text{Amplitude}} \underbrace{e^{j\psi/2}}_{\text{Phase}}$$

Normalizing, Let $2E_0 = 1$

The field pattern is given by

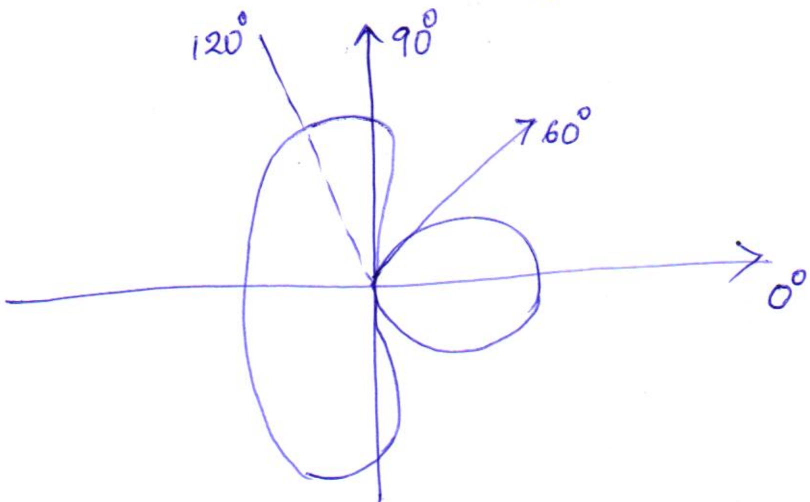
$$|E| = |\cos(\pi/2\cos\phi + \pi/4)$$

Direction of maxima

$$\pi/2\cos\phi + \pi/4 = 0 \Rightarrow \frac{\pi}{2}\cos\phi = \pi/4 \Rightarrow \cos\phi = -1/2$$
$$\Rightarrow \phi = 120^\circ, 240^\circ$$

Direction of minima (null)

$$\frac{\pi}{2}\cos\phi + \pi/4 = \pi/2 \Rightarrow \pi/2\cos\phi = \pi/4$$
$$\Rightarrow \cos\phi = 1/2 \Rightarrow \phi = 60^\circ$$



The field pattern is cardiod-shaped, unidirectional pattern.

Case 4: General case of 2 isotropic point sources of equal amplitude and any phase.

$$\psi = \beta d \cos\phi + \delta$$

$$E = E_0 + E_0 e^{j\psi}$$

$$= E_0 e^{j\psi/2}\left[e^{j\psi/2} + e^{-j\psi/2}\right]$$

$$= 2E_0 e^{j\psi/2} \cos\psi/2$$

$$E = e^{j\psi/2}\, 2E_0 \cos\psi/2$$

Normalized Field pattern is

$$\boxed{|E| = \left|\cos\frac{\psi}{2}\right|}$$

<u>Antenna Array</u> are groups of similar antennas arranged in various configurations with proper amplitude and phase relations to give certain desired radiation characteristics.

<u>Linear array</u> : If the array elements are placed along a line (called the axis of the array) it is said to be a linear array.

<u>Uniform linear array</u> :

It is a linear array in which the array elements are equispaced and are excited with uniform current with constant progressive phase shift (phase shift between adjacent antenna elements)

<u>Array factor of a uniform linear array</u>

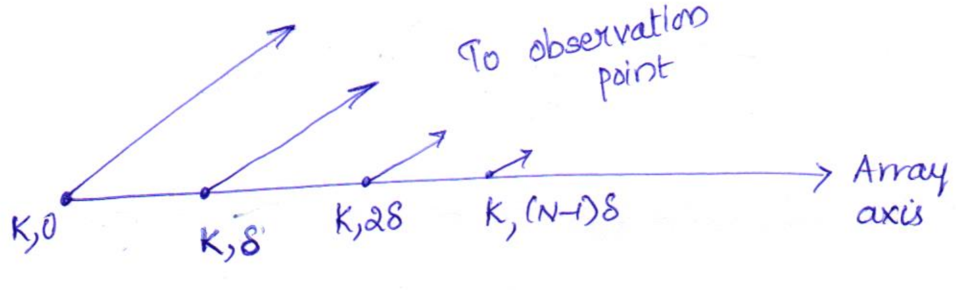
Consider an array having N elements and let the antennas be isotropic. All elements are excited with equal amplitude currents. The spacing between any 2 adjacent elements of the array (inter element spacing) is 'd'. The phase shift between currents on ~~array~~ any 2 adjacent antenna elements of the array (progressive phase shift is δ).

The field due to antenna at a far away point is proportional to the current & is superposition of fields due to individual elements (have equal amplitude but different phases). The phase of the field has 2 components.

1) Phase due to phase of excitation current is δ
2) phase due to propagation $= \beta d \cos\phi$

Total phase difference between fields due to adjacent elements

$$\psi = \beta d \cos\phi + \delta$$



Let the $\vec{E}$ due to individual antennas have unit amplitude at $\vec{P}$

The total electric field at P is

$$E = 1 + e^{j\psi} + e^{j2\psi} + e^{3j\psi} + \cdots\cdots e^{j(N-1)\psi} \longrightarrow ①$$

$① \times e^{j\psi} \Rightarrow$

$$E e^{j\psi} = e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \cdots\cdots e^{jN\psi} \longrightarrow ②$$

$$② - ① \Rightarrow E(e^{j\psi} - 1) = e^{jN\psi} - 1$$

$$E = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \longrightarrow ③$$

$$= \frac{e^{\frac{jN\psi}{2}}\left[e^{\frac{-jN\psi}{2}} - e^{\frac{jN\psi}{2}}\right]}{e^{\frac{j\psi}{2}}\left[e^{-j\psi/2} - e^{j\psi/2}\right]}$$

$$= e^{\frac{j(N-1)\psi}{2}}\left[\frac{2j\sin\left(\frac{N\psi}{2}\right)}{2j\sin(\psi/2)}\right]$$

$$= \underbrace{e^{j(N-1)\psi/2}}_{\text{phase}}\ \underbrace{\left[\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right]}_{\text{amplitude}}$$

From ③ $\Rightarrow E = \dfrac{1 - e^{jN\psi}}{1 - e^{j\psi}}$

The maximum electric field is obtained when all the terms in series add in phase ie $\psi = 0$

$$\underset{\psi \to 0}{Lt}\ \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} = \underset{\psi \to 0}{Lt}\ \frac{e^{jN\psi} \times jN}{e^{j\psi} \times j} = N$$

So the maximum value of E is N. The radiation pattern is generally normalized with respect to the maximum value N

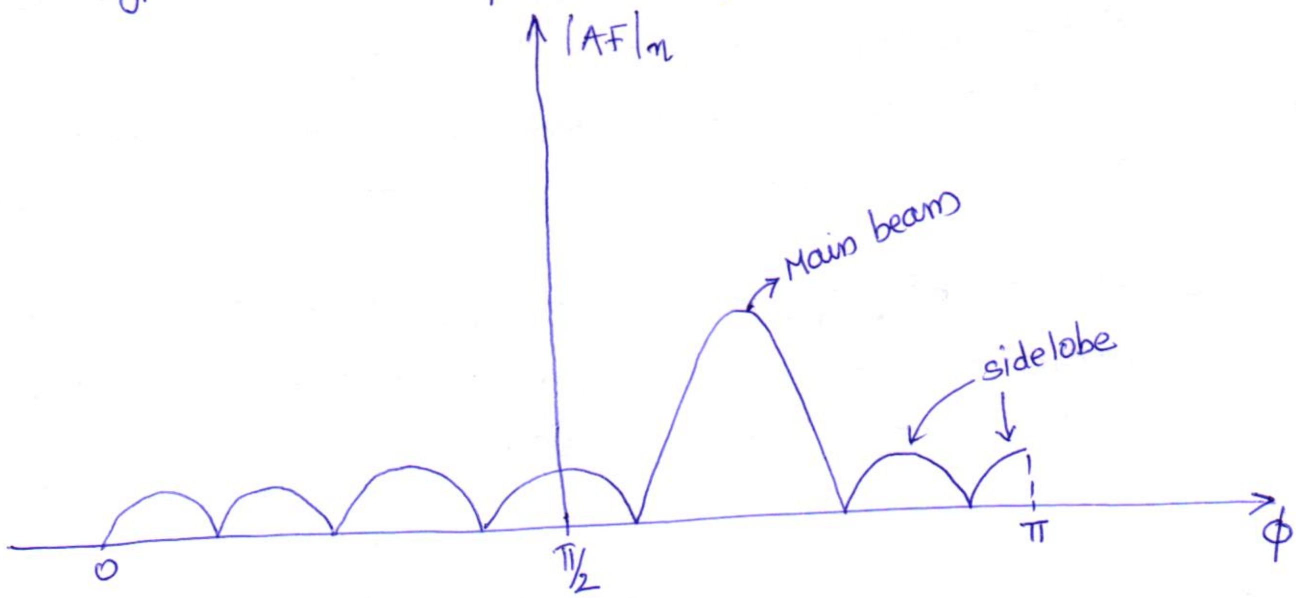
$$|E| = \frac{1}{N} \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

This is called the array factor (AF) of N-element linear array.

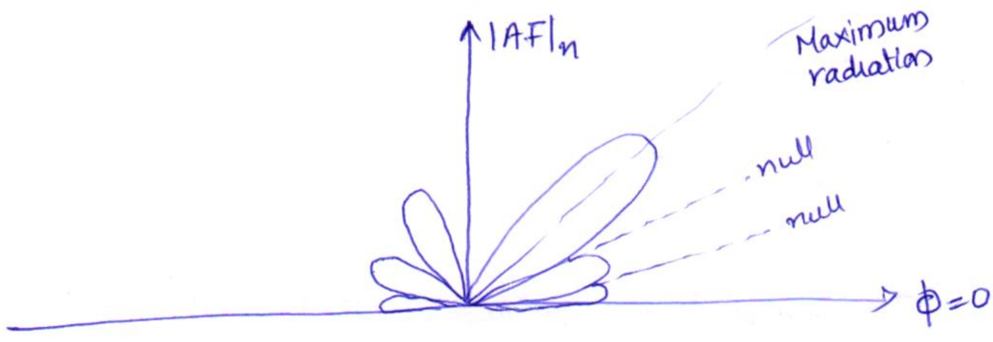
$$AF_1 = \frac{1}{N} \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

The array factor is a function of geometry of the array and the excitation phase. By varying the seperation 'd' and/or the phase 'δ' between the elements, the seperation characteristics of the array factor and of the total field of the array can be controlled. Each array has its own array factor. The array factor, in general, is a function of the number of elements, their geometrical arrangement, their relative magnitudes, their relative phases & their spacings.

A typical radiation pattern is given below:-



Radiation pattern as cartesian plot.



Direction of maximum radiation

Direction of maximum radiation (also called the direction of main beam) is one of the important feature of the array.

Direction of maximum radiation is obtained when $\psi = 0$. If the direction of maximum radiation is denoted by ϕ_{max}, then

$$\psi = \beta d \cos\phi_{max} + \delta = 0$$

$$\Rightarrow \cos\phi_{max} = \frac{-\delta}{\beta d}$$

$$\phi_{max} = \cos^{-1}\left(\frac{-\delta}{\beta d}\right) = \cos^{-1}\left(\frac{\delta\lambda}{2\pi d}\right)$$

$$\boxed{\phi_{max} = \cos^{-1}\left(\frac{\delta\lambda}{2\pi d}\right)}$$

Two things can be noted from the equation.

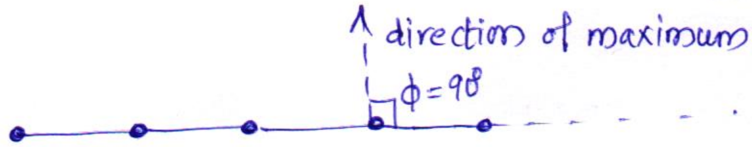
1. The direction of the maximum radiation is independent of the number of elements in the array.
2. The direction of the main beam can be changed from 0 to π by changing the progressive phase shift δ from $-\beta d$ to $+\beta d$

— When $\delta = -\beta d$, $\phi_{max} = \cos^{-1}(-1) \Rightarrow \phi_{max} = 0°$ or $180°$. The maximum radiation direction is along the axis of the array. Such an array is called <u>end fire array</u>.

— Suppose $\delta = 0$, then

$$\phi_{max} = \cos^{-1}(0)$$
$$= \pi/2$$

i.e. The main beam direction is perpendicular to the array axis. Such an array is called <u>broadside array</u>.



Direction of Null

The nulls of radiation pattern can be obtained by equating the array factor to zero

$$\sin\left(\frac{N\psi}{2}\right) = 0$$

$$\frac{N\psi}{2} = \pm m\pi \qquad m = 1, 2, 3 \ldots$$

$$\psi = \pm \frac{2m\pi}{N} \qquad m = 1, 2 \ldots$$

$$\beta d \cos\phi_{null} + \delta = \pm \frac{2m\pi}{N}$$

$$\cos\phi_{null} = \frac{\pm 2m\pi}{N\beta d} - \frac{\delta}{\beta d}$$

$$= \frac{\pm 2m\pi}{N \frac{2\pi}{\lambda} d} - \frac{\delta}{\beta d}$$

$$\cos\phi_{null} = \frac{\pm m\lambda}{Nd} - \frac{\delta}{\beta d}$$

$$\phi_{null} = \cos^{-1}\left(\frac{\pm m\lambda}{Nd} - \frac{\delta}{\beta d}\right) \qquad m = 1, 2 \ldots$$

Case 1 : End Fire array ($\delta = -\beta d$)

$$\phi_{null} = \cos^{-1}\left(1 \pm \frac{m\lambda}{Nd}\right)$$

$$\boxed{\phi_{null} = \cos^{-1}\left(1 - \frac{m\lambda}{Nd}\right)}$$ ($\because \cos\theta$ is never greater than 1)

$m = 1, 2 \ldots$

Case 2 : Broadside array ($\delta = 0$)

$$\cos(\phi_{null}) = \pm \frac{m\lambda}{Nd}$$

$$\phi_{null} = \cos^{-1}\left(\pm \frac{m\lambda}{Nd}\right)$$

where $m = 1, 2 \ldots$

Direction of sidelobes

Local maximum in the radiation pattern is called sidelobe. There is one sidelobe between 2 adjacent nulls except the main beam. Whenever the numerator of the array factor is maximum, there is a sidelobe in the radiation pattern

$$\sin\left(\frac{N\psi}{2}\right) = \pm 1$$

$$\frac{N\psi}{2} = \pm(2m+1)\frac{\pi}{2}$$

$$\psi = \pm\frac{(2m+1)\pi}{N}$$

$$\beta d \cos\phi_{SL} + \delta = \pm\frac{(2m+1)\pi}{N}$$

$$\cos\phi_{SL} = \pm\frac{(2m+1)\pi}{N\beta d} - \frac{\delta}{\beta d}$$

$$\phi_{SL} = \cos^{-1}\left(\pm\frac{(2m+1)\pi}{\beta d N} - \frac{\delta}{\beta d}\right)$$

where $m = 1, 2 \dots$

Case 1: Broadside array ($\delta = 0$)

$$\phi_{SL} = \cos^{-1}\left\{\pm\frac{(2m+1)\pi}{\beta d N}\right\}$$

where $m = 1, 2 \dots$

Case 2: End fire array

$$\phi_{SL} = \cos^{-1}\left(\pm\frac{(2m+1)}{\beta d N} + 1\right)$$

$$\boxed{\phi_{SL} = \cos^{-1}\left(1 - \frac{(2m+1)}{\beta d N}\right)}$$

where $m = 1, 2 \dots$

Amplitudes of mth sidelobe

We know,

$$(AF)_n = \frac{1}{N} \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

For sidelobes

$$\sin\left(\frac{N\psi}{2}\right) = 1$$

$$\psi = \pm \frac{(2m+1)\pi}{N}$$

So amplitude of mth sidelobe is

$$SL_m = \frac{1}{N}\left[\frac{1}{\sin\left(\pm\frac{(2m+1)\pi}{N}\right)}\right]$$

Amplitude of main beam $= \frac{1}{N}$

For a large array $N \gg 1$ and the side lobe amplitude is approximately

$$\approx \frac{1}{N} \frac{1}{(m+\frac{1}{2})\frac{\pi}{N}} = \frac{2}{(2m+1)N}$$

The sidelobe amplitudes are independent of array size & the direction of main beam.

Principle of pattern multiplication

The total field pattern of an array of nonisotropic but similar point sources is the product of

1) pattern of individual sources and

2) pattern of an array of isotropic sources having the same location, relative amplitudes and phase as the nonisotropic point sources

The total field E is

$$E = \underbrace{f(\theta,\phi)\,F(\theta,\phi)}_{\text{field pattern}} \quad \angle\underbrace{(f_p(\theta,\phi) + F_p(\theta,\phi))}_{\text{phase pattern}}$$

where

$f(\theta,\phi)$ = Field pattern of individual source

$f_p(\theta,\phi)$ = phase pattern of individual source

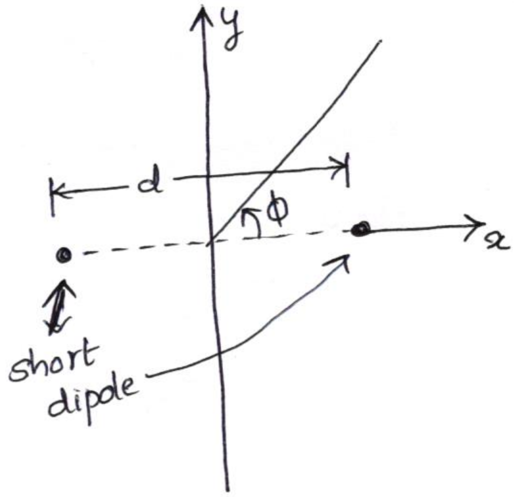
$F(\theta,\phi)$ = Field pattern of array of isotropic sources

$F_p(\theta,\phi)$ = phase pattern of array of " "

Qn Assume 2 identical point sources seperated by a distance 'd' each source having the field pattern given by

$$E = E_0 \sin\phi$$

as might be obtained by 2 short dipoles arranged as shown below. Let $d = \lambda/2$ and phase angle $\delta = 0$. Find the total field pattern.



$$AF_n = \frac{1}{N} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin(\psi/2)}$$

$$\psi = \beta d \cos\phi + \delta$$

$$\delta = 0,\ d = \lambda/2 \Rightarrow \psi = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi + 0$$

$$\psi = \pi \cos\phi$$

$N = 2$

$$(AF)_n = \frac{1}{2} \frac{\sin\left(\frac{2\pi\cos\phi}{2}\right)}{\sin(\pi/2 \cos\phi)}$$

$$= \frac{1}{\cancel{2}} \frac{\cancel{2}\,\cancel{\sin(\pi/2\cos\phi)}\cos(\pi/2\cos\phi)}{\cancel{\sin(\pi/2\cos\phi)}}$$

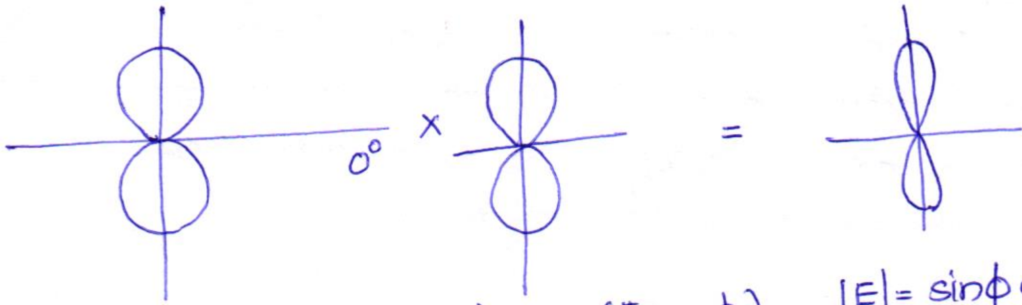
$$(AF)_n = \cos(\pi/2 \cos\theta)$$

Field pattern of individual source $F(\theta,\phi) = \sin\phi$

Field pattern of array of isotropic source

$$F(\theta,\phi) = \cos(\pi/2 \cos\phi)$$

$$\boxed{|E| = \sin\phi \cos(\pi/2 \cos\phi)}$$



$|E| = \sin\phi$ $\quad$ $(AF)_n = \cos(\frac{\pi}{2}\cos\phi)$ $\quad$ $|E| = \sin\phi\cos(\pi/2\cos\phi)$

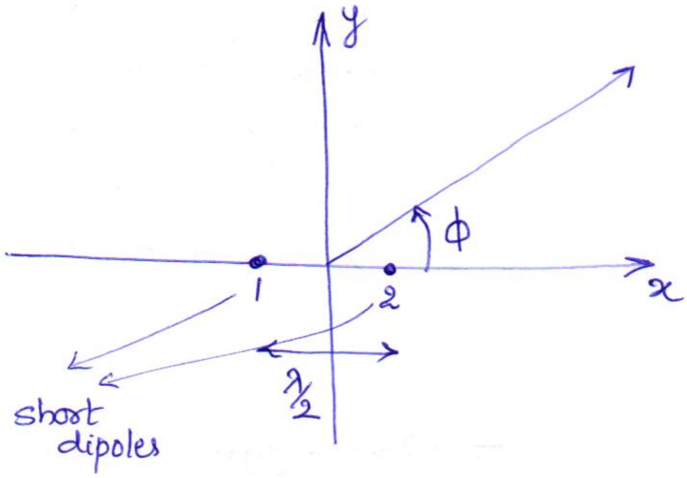
Qn. Consider 2 identical point sources seperated by a distance d, each source having a field pattern given by

$$E = E_0 \cos\phi$$

as might be obtained by two short dipoles arranged as in figure below

$d = \lambda/2$ and $\delta = 0$

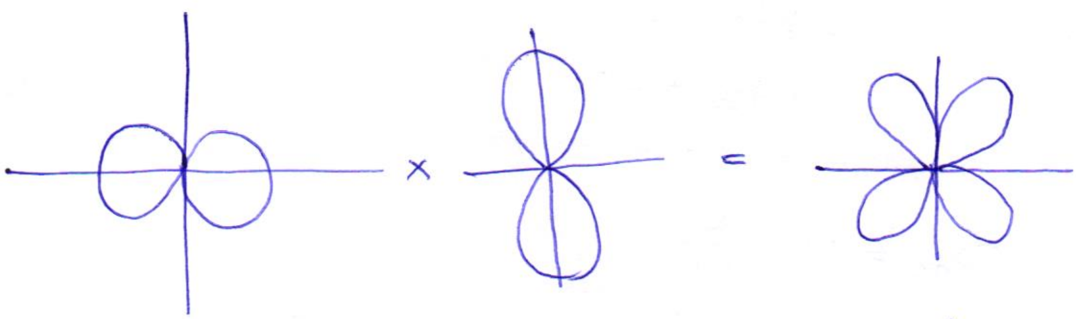
Find the total field pattern



$(AF)_n = \cos(\pi/2\cos\phi)$

$|E|_n = \cos\phi$

Total field $|E| = \cos\phi\cos(\pi/2\cos\phi)$



$E = E_0\cos\phi$ $\quad$ $\cos(\pi/2\cos\phi)$ $\quad$ $\cos\phi\cos(\pi/2\cos\phi)$

Broadside array

Broadside array is a uniform linear array in which the direction of maximum radiation is perpendicular to the axis of array. The progressive phase shift $\delta = 0$ ie the elements are fed in phase.

$$\psi = \beta d \cos\phi$$

Direction of maximum radiation is obtained when $\psi = 0$.

$$\phi_{max} = \cos^{-1}(0) = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

Direction of null for a broadside array

$$\phi_{null} = \cos^{-1}\left(\pm \frac{m\lambda}{Nd}\right) \quad m = 1, 2 \ldots$$

HPBW for broadside array

$$HPBW = \frac{\lambda}{dN}$$

Directivity of broadside array

$$D_{BS} = \frac{2dN}{\lambda}$$

End fire array

End fire array is ~~of~~ a uniform linear array in which the direction of maximum radiation is along the axis of the array. The progressive phase shift $\delta = -\beta d$

$$\psi = \beta d \cos\phi + \delta$$

$$= \beta d \cos\phi - \beta d$$

Direction of maximum radiation is obtained when $\psi = 0$

$$\phi_{max} = \cos^{-1}\left(\frac{\delta}{\beta d}\right)$$

$$= \cos^{-1}\left(\frac{-\beta d}{\beta d}\right) = \cos^{-1}(-1) = 0^{\circ} \text{ or } 180^{\circ}$$

Direction of null is given by:

$$\phi_{null} = \cos^{-1}\left(1 - \frac{m\lambda}{Nd}\right)$$

HPBW of end fire array

$$HPBW = \sqrt{\frac{2\lambda}{dN}}$$

Directivity of end fire array

$$D_{ES} = \frac{8dN}{\lambda}$$

Qn. Determine and plot the field pattern of a broadside array with 4 elements spaced $\lambda/2$ apart.

$N = 4$
$\delta = 0$
$d = \lambda/2$

<u>Direction of maximum radiation</u>

$\phi_{max} = 90^\circ$ ($\because$ broadside array)

<u>Direction of Null</u>

$$\phi_{null} = \cos^{-1}\left(\pm \frac{m\lambda}{Nd}\right) = \cos^{-1}\left(\pm \frac{m\lambda}{4\lambda/2}\right) = \cos^{-1}(\pm m/2)$$

where $m = 1, 2 \ldots\ldots$

$m=1 \Rightarrow \cos^{-1}(\pm 1/2) \Rightarrow \phi_{null1} = 60^\circ$

$m=2 \Rightarrow \cos^{-1}(\pm 1) \Rightarrow \phi_{null2} = 0^\circ$

$m=3 \Rightarrow \cos^{-1}(\pm 3/2) \Rightarrow$ invalid.

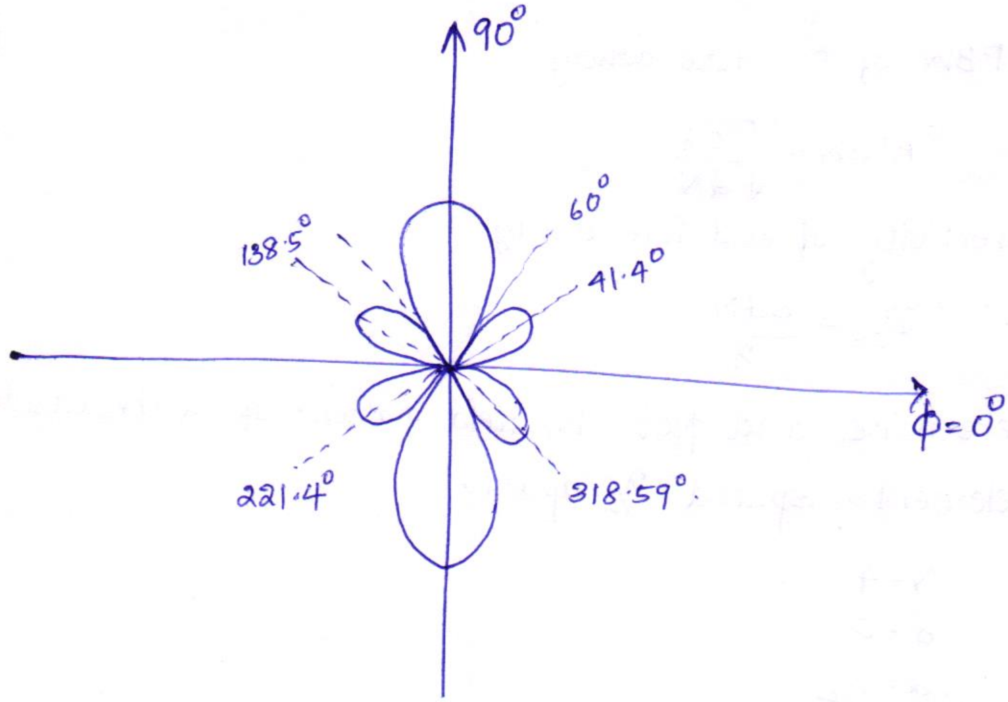
<u>Direction of sidelobes</u>

$$\phi_{SL} = \cos^{-1}\left(\pm \frac{(2m+1)\pi}{\beta d N} - \frac{\delta}{\beta d}\right) \qquad m = 1, 2 \ldots\ldots$$

$$\delta = 0 \Rightarrow \phi_{SL} = \cos^{-1}\left(\pm \frac{(2m+1)\pi}{\frac{2\pi}{\lambda}\frac{\lambda}{2}4}\right) = \cos^{-1}\left(\pm \frac{(2m+1)}{4}\right)$$

$m=1 \Rightarrow \cos^{-1}(\pm 3/4) = 41.4^\circ, 138.5^\circ, 318.5^\circ$

$m=2 \Rightarrow \cos^{-1}(\pm 5/4) \Rightarrow$ invalid



Qn Determine and plot the field pattern of an end fire array with 4 elements spaced $\lambda/2$ apart.

$N=4$

$\delta = -\beta d$

$d = \lambda/2$

Direction of maximum radiation.

$\phi_{max} = 0^\circ, 180^\circ$ ($\because$ end fire array).

Direction of Null

$$\phi_{null} = \cos^{-1}\left(1 - \frac{m\lambda}{Nd}\right)$$

$$= \cos^{-1}\left(1 - \frac{m\lambda}{4\times\lambda/2}\right)$$

$$= \cos^{-1}\left(1 - \frac{m}{2}\right) \quad ; m = 1, 2 \ldots$$

$m=1 \Rightarrow \cos^{-1}(1-1/2) \Rightarrow \phi_{null1} = 60^\circ$

$m=2 \Rightarrow \cos^{-1}(1-1) \Rightarrow \phi_{null2} = 90^\circ$

$m=3 \Rightarrow \cos^{-1}(1-3/2) \Rightarrow \phi_{null3} = 120^\circ$

Direction of sidelobes

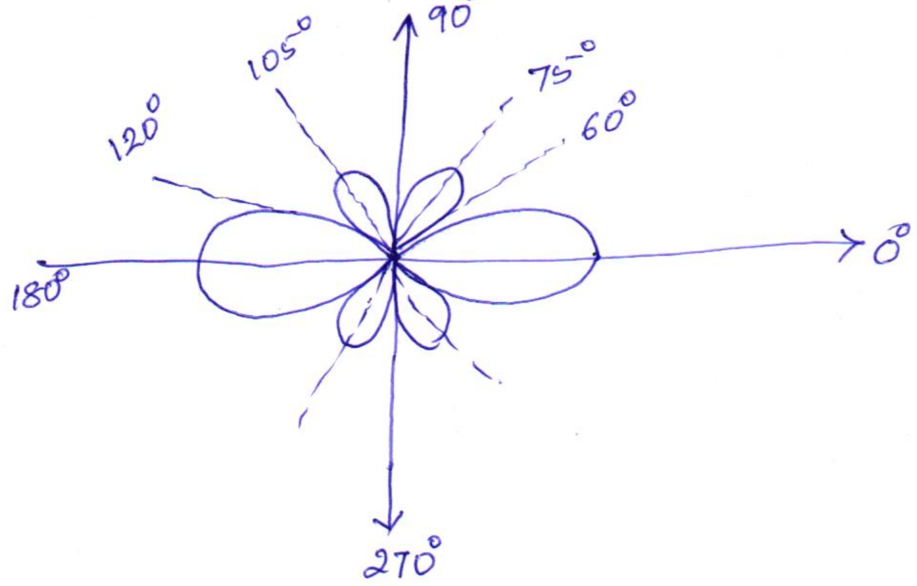
$$\phi_{SL} = \cos^{-1}\left(\pm\frac{(2m+1)\pi}{\beta d N} + 1\right) \quad m = 1, 2, \ldots$$

$$= \cos^{-1}\left(\pm\frac{(2m+1)\pi}{\frac{2\pi}{\lambda}\frac{\lambda}{2}\times 4} + 1\right) = \cos^{-1}\left(1 - \frac{(2m+1)}{4}\right) \quad m = 1, 2 \ldots$$

$m=1 \Rightarrow \phi_{SL1} = \cos^{-1}(1-3/4) = \cos^{-1}(1/4) = 75^\circ$

$m=3 \Rightarrow \phi_{SL3} = \cos^{-1}(1-7/4) = \cos^{-1}(-3/4) = 105^\circ$

$m=4 \Rightarrow \phi_{SL4} = \cos^{-1}(1-9/4) = \cos^{-1}(5/4) \Rightarrow$ invalid.



Dolph Tchebyshev Array (DCA)

Dolph proposed (in 1946) a method for designing arrays with any desired side-lobe level for a given HPBW. This method is based on the approximation of the pattern of the array by a Tchebyshev polynomial of order m, high enough to meet the requirement of the sidelobe levels. A DCA with no sidelobe (side lobe level of $-\infty$ dB) reduces to the binomial design.

Tchebyshev Polynomial

The Tchebyshev polynomial of order m is defined by

$$T_m(x) = \begin{cases} (-1)^m \cosh(m \cosh^{-1}(x)) & x \leq -1 \\ \cos^{-1}(m \cos^{-1}(x)) & -1 \leq x \leq 1 \\ \cosh(m \cosh^{-1}(x)) & x \geq 1 \end{cases} \longrightarrow (1)$$

A Tchebyshev polynomial $T_m(x)$ of any order 'm' can be derived via a recursion formula, provided $T_{m-1}(x)$ and $T_{m-2}(x)$ are known.

$$T_m(x) = 2x\, T_{m-1}(x) - T_{m-2}(x) \longrightarrow (2)$$

Put m=0 in equation (1)

$$T_0(x) = 1$$

Put m=1 in equation (1)

$$T_1(x) = x$$

From (2)

$$T_2(x) = 2x\, T_1(x) - T_0(x)$$
$$= 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$
$$T_4(x) = 8x^4 - 8x^2 + 1$$
$$T_5(x) = 16x^5 - 20x^3 + 5x \quad \text{etc.}$$

If $|x| \leq 1$, then Tchebyshev polynomials are related to cosine functions through $x = \cos\frac{\psi}{2}$ so that

$$T_m(x) = \cos\left(\frac{m\psi}{2}\right)$$

$$e^{\frac{jm\psi}{2}} = \cos\left(\frac{m\psi}{2}\right) + j\sin\left(\frac{m\psi}{2}\right)$$

$$= \left(\cos\left(\frac{\psi}{2}\right) + j\sin(\psi/2)\right)^m$$

$$\cos\left(\frac{m\psi}{2}\right) = \mathrm{Re}\left(\cos(\psi/2) + j\sin(\psi/2)\right)^m$$

$m=0 \Rightarrow T_0(x) = \cos\left(\frac{m\phi}{2}\right) = 1$

$m=1 \Rightarrow T_1(x) = \cos\left(\frac{m\phi}{2}\right) = \cos(\psi/2)$

$m=2 \Rightarrow T_2(x) = \cos\left(\frac{m\phi}{2}\right) = 2\cos^2(\psi/2) - 1$

$m=3 \Rightarrow T_3(x) = \cos(3\phi/2) = 4\cos^3(\psi/2) - 3\cos(\psi/2)$

Linear array with non-uniform distribution

If the linear array has even number of elements, then the total field

$$E_n^2 = 2\sum_{k=0}^{N-1} A_k \cos\left(\frac{2k+1}{2}\psi\right)$$

If the linear array has odd number of elements, then the total field

$$E_n^2 = 2\sum_{k=0}^{N} A_k \cos\left(\frac{2k\psi}{2}\right)$$

Dolph Tchebyschelf Array Design

The main goal is to approximate the desired array factor with a Tchebyshelf polynomial such that

1) the side lobe level meets the requirement

2) the main beamwidth is as smal as possible.

Step 1

An array of N elements has an array factor approximated with a Tchebyscheff polynomial of order $m = N-1$

$$T_m = AF_n$$

Step 2

Let the side lobe level be R, then solve for x_0

$$T_m(x_0) = R$$

Step 3

Since $R > 1$, $x_0 > 1$. We know $x = \cos(\psi/2)$

Since $-1 \leq \cos\psi/2 < 1$, the relationship between x & $\cos\psi/2$ must be normalized as

$$\cos\frac{\psi}{2} = \frac{x}{x_0} = w.$$

Now $-1 \leqslant w \leqslant 1$

Step 4

The pattern polynomial may be expressed as a polynomial in 'w'. Equating the Tchebyscheff polynomial & array polynomial

$$T_m(x) = E_n.$$

The coefficient of the array polynomial is then obtained yielding Dolph-Tchebyscheff amplitude distribution

Qn. Design a broadside Dolph-Tschebycheff array of 8 elements with spacing $d = \lambda/2$ between the elements. The sidelobe levels are 26 dB below the main lobe maximum. Find the excitation coefficients of the array.

$SSL(dB) = 26\ dB$

$20\log_{10}(R) = 26 \Rightarrow R = 10^{26/20} = 10^{1.3} \approx 20$

$m = N-1 = 8-1 = 7$

$T_7(x_0) = 20$

$\cosh(7\cosh^{-1}(x_0)) = 20$

$7\cosh^{-1}(x_0) = \cosh^{-1}(20) = 3.69$

$\cosh^{-1}(x_0) = 0.526$

$x_0 = \cosh(0.526) = 1.15$

$T_0(x) = 1$

$T_1(x) = x$

$T_2(x) = 2x^2 - 1$

$T_3(x) = 4x^3 - 3x$

$T_4(x) = 8x^4 - 8x^2 + 1$

$T_5(x) = 16x^5 - 20x^3 + 5x$

$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$

$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$

$$E_n^c = 2\sum_{K=0}^{7} A_K \cos\left(\frac{2K+1}{2}\psi\right)$$

$$E_8 = A_0 w + A_1(4w^3 - 8w) + A_2(16w^5 - 20w^3 + 5w) + A_3(64w^7 - 112w^5 + 56w^3 - 7w)$$

where $w = \frac{x}{x_0} = \cos\psi/2$

$$E_8 = A_0\frac{x}{x_0} + A_1\left(4\frac{x^3}{x_0^3} - \frac{3x}{x_0}\right) + A_2\left(\frac{16x^5}{x_0^5} - \frac{20x^3}{x_0^3} + \frac{5x}{x_0}\right) + A_3\left(64\frac{x^7}{x_0^7} - 112\frac{x^5}{x_0^5} + 56\frac{x^3}{x_0^3} - \frac{7x}{x_0}\right)$$

$$E_8 = 64 A_3 \frac{x^7}{x_0^7} + (16 A_2 - 112 A_3)\frac{x^5}{x_0^5} +$$

$$(4A_1 - 20A_2 + 56 A_3)\frac{x^3}{x_0^3} + (5A_2 - 3A_1 - 7A_3 + A_0)\frac{x}{x_0} \rightarrow (1)$$

Tchebyscheff polynomial of like degree is

$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x \longrightarrow (2)$$

Comparing (1) & (2)

$$\frac{64A_3}{x_0^7} = 64 \Rightarrow A_3 = x_0^7 = 1.15^7 = 2.66$$

IIIrly $A_2 = 4.56$

$A_1 = 6.82$

$A_0 = 8.25$

Normalizing

$$A_{3n} = \frac{2.66}{2.66} = 1$$

$$A_{2n} = \frac{4.56}{2.66} = 1.7$$

$$A_{1n} = \frac{6.82}{2.66} = 2.6$$

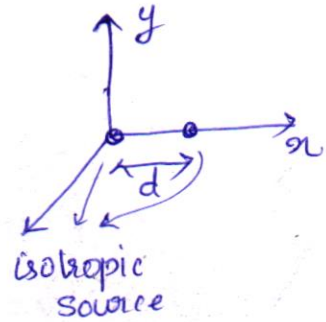
$$A_{0n} = \frac{8.25}{2.66} = 3.1$$

The exitation coefficients are 1, 1.7, 2.6, 3.1, 3.1, 2.6, 1.7, 1

Binomial array

The binomial broadside array was proposed by JS Stone to synthesize patterns without sidelobes.

First, consider a two element array. The elements are identical isotropic sources. They are fed in phase.



$$\psi = \beta d \cos\phi$$

Array factor is given by:

$$AF = 1 + e^{j\psi}$$

Let $e^{j\psi} = z$, then $AF = 1 + z$

$$|AF|^2 = |1 + e^{j\psi}|^2$$
$$= (1+\cos\psi)^2 + \sin^2\psi$$
$$= 1 + \cos^2\psi + 2\cos\psi + \sin^2\psi$$
$$= 2 + 2\cos\psi$$
$$= 2(1+\cos\psi)$$
$$= 4\cos^2\psi/2$$

$|AF| = 2\cos\psi/2$ where $\psi = \beta d\cos\phi$

Direction of Null

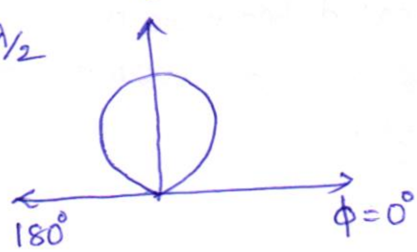
$$\phi_{null} = \cos^{-1}\left(\pm\frac{m\lambda}{Nd}\right) = \cos^{-1}\left(\pm\frac{m\lambda}{2d}\right)$$

First Null

$$m=1 \Rightarrow \phi_{null_1} = \cos^{-1}\left(\pm\frac{\lambda}{2d}\right)$$

As long as $d < \frac{\lambda}{2}$, null doesnot exist. When $d = \lambda/2$ null occurs at $0°$ & $180°$

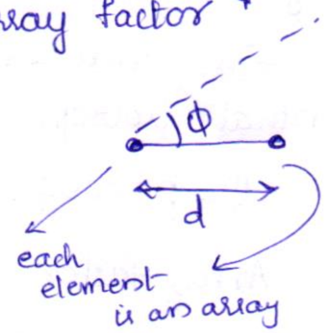
Thus in the "visible" range of ϕ, all secondary lobes are eliminated when $d \leq \lambda/2$



Now consider a two element array whose elements are identical and same as array givenabove. The distance between the two arrays is 'd'

The new array will have an array factor

$$(AF) = (1+z)(1+z)$$
$$= 1 + 2z + z^2$$



The new array will also have no sidelobes. Continuing the process for an N-element array.

$$(AF) = (1+z)^{N-1}$$

If $d \le \frac{\lambda}{2}$, the above AF doesnot have sidelobes regardless of the number of elements N.

The excitation amplitude can be obtained easily by binomial expansion. Making use of Pascal's triangle we write down the excitation amplitude coefficients as:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

An array with a binomial distribution of excitation amplitudes is called a binomial array.

<u>Advantage</u>

The pattern has no minor lobes

<u>Disadvantage</u>

1. increased beamwidth
2. large ratio of current amplitudes required in large arrays.

Grating lobes

Array factor (AF) of N element linear array is given by:

$$AF = \frac{1}{N}\frac{\sin(N\psi/2)}{\sin(\psi/2)} \quad \text{where } \psi = \beta d\cos\phi + \delta$$

$d \rightarrow$ seperation b/w elements
$\delta \rightarrow$ phase b/w elements
$\beta = \frac{2\pi}{\lambda}$

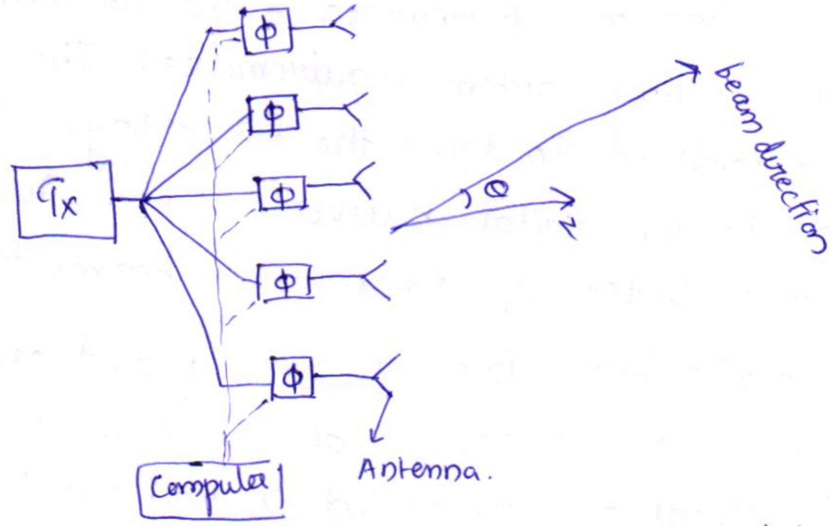
If d/λ & δ are properly chosen, only one main beam exist in the "visible" space, $(-90° < \phi < 90°)$. Large spacing $(d > \lambda/2)$ will produce additional main beams which are called grating lobes (GL)

Phased arrays

Phased arrays are collections of antennas arranged in a pattern. Phased arrays has the following advantages over individual elements

1. improved spatial resolution : view more detailed resolution when localizing and imaging a target.
2. Electronic steering : The antenna beam can be steered.
3. Interference suppression :

A phased array consists of an array of antenna elements powered by a transmitter (Tx). The feed current for each element passes through a phase shifter (ϕ) controlled by a computer (C)



By changing the phase shifts, the computer can instantly change angle θ of the beam.

There are 4 most common phase arrays, which are:

① passive electronically scanned array (PESA)

② active electronically scanned array (AESA)

③ Hybrid beam forming phased array

④ digital beam forming (DBF) array.

- A passive phased array or passive electronically scanned array (PESA) is a phased array in which the antenna elements are connected to a single transmitter/receiver.

- An active phased array or active electronically scanned array (AESA) is a phased array in which each antenna element has an analog transmitter/receiver (T/R) module, which creates the phase shifting required to electronically steer the antenna beam. Unlike PESAs they can radiate several beams of radio waves at multiple frequencies in different directions simultaneously. Each beam former has a receiver/exciter connected to it.

- A hybrid beam forming phased array can be thought of as a combination of an AESA and a digital beam forming phased array. It uses subarrays that are active phased arrays (for instance, a subarray may be 64, 128 or 256 elements & the number of elements depends upon system requirements). The subarrays are combined to form the full array. Each subarray has its own digital receiver/exciter. This approach allows clusters of simultaneous beams to be created.

- A digital beam forming (DBF) phased array has a digital receiver/exciter at each element in the array. The signal at each element is digitized by the receiver/transmitter. This means that antenna beams can be formed digitally in a field programmable gate array (FPGA) or the ~~array~~ computer. This approach allows for multiple simultaneous antenna beams to be formed.

Module IV

Microwaves

Introduction, advantages, Cavity Resonators – Types, Derivation of resonant frequency of rectangular cavity (problems)

Single cavity klystron

Reflex Klystron Oscillators : Derivation of power output, efficiency and admittance (problem required)

Magnetron oscillators

Cylindrical magnetron, Cyclotron angular frequency, Power output and efficiency (problems required)

Travelling wave tube

Slow wave structure, Helix, TWT, Amplification process, Derivation of convection current, axial electric field, wave modes + gain (problems required)

Microwaves

— Microwaves are a type of electromagnetic wave with wavelength ranging from one meter to one millimeter. with frequencies between 300MHz and 300 GHz.

$f_1 = 300\ MHz$
$= 300 \times 10^6\ Hz$
$= 3 \times 10^8\ Hz$

$\lambda_1 = \frac{c}{f_1}$
$= \frac{3 \times 10^8}{3 \times 10^8}$
$= 1m$

$f_2 = 300\ GHz$
$= 300 \times 10^9\ Hz$
$= 3 \times 10^{11}\ Hz$

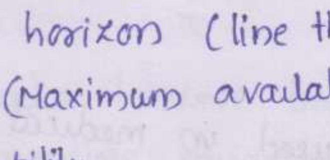
$\lambda_2 = \frac{c}{f_2}$
$= \frac{3 \times 10^8}{3 \times 10^{11}}$
$= 10^{-3} m$
$= 1mm$

— The term microwaves indicates the wavelength is in micron ranges.

microwave: micro → micron range; wave → wavelength

Features of microwaves

① Electromagnetic wave with small wavelength (1m to 1mm)
② Microwaves travel by line of sight (LOS). ie they travel in a direct path from source to receiver.



These waves may be diffracted, refracted, reflected or absorbed by the atmosphere & obstructions with materials and generally cannot travel over horizon (line that seperates earth & sky) or behind obstacle. (Maximum available visual horizon – 64 km).

Advantages
③ High bandwidth capability
④ High antenna gain

Applications of microwaves

① Point to point Communication

i) TV programs are transmitted by communication satellites using LOS microwave propagation

ii) Telephone and data signals are transmitted by microwave relay stations.

② Radars

③ Microwave heating

Microwave heating is used in industrial processes for drying and curing products. It can also be used to treat cancer patients.

④ Microwave oven

Microwave oven is a device with a high power magnetron that emits microwave radiation at 2.4 GHz. Water (moisture) in the food absorb energy and causes dielectric heating.

⑤ Wireless Data Networks

Microwaves are used in wireless LAN (eg Bluetooth) WiMax and also in broadband internet links

⑥ Remote Sensing

Satellites are used to monitor on or above the earth's surface by using microwave signals to detect weather conditions, ozone, soil moisture, forest & exploration of natural resources.

⑦ Radio Astronomy

⑧ Medical applications

— Microwaves are used in medical fields like heart stimulation, haemorrhage control, sterilization etc.

— MRI scanning

History of microwaves.

- 1873 – James clerk Maxwell predicted the existence of electromagnetic waves (Maxwell's equation)
- 1888 – Heinrich Hertz (german Physicist) demonstrated the existence of radio waves using spark radio transmitter.
- 1894 – Jagadish chandra Bose generated millimeter waves (60 GHz) using spark oscillator
- Later researches was done on similarities between radio waves and light waves to test Maxwell's Theory. Most of the researchers ~~who~~ were interested in producing short wavelength radio waves in UHF and microwave ranges, using prism, lenses etc.
- By 1930, first low power vaccum tubes were developed. Using this a few watts of power at frequencies of few GHz were ~~was~~ used in communication.
- 1931 – Microwave relay link for English channel was developed. (Bidirectional communication)
- RADAR (Radio detection & Ranging) was developed during World War II. Many technological developments happened during World War II. Radar Antennas was fitted on aircraft to detect or localize enemy aircraft (in cm range)
  eg Klystron tube, Magnetron tube.
- After World War II, microwaves were rapidly exploited commercially. Due to high frequencies, they had a very large bandwidth (information carrying capacity)
  eg A single microwave beam could carry tens of thousands of phone calls.
- Later microwave radar became the central technology used in Air traffic control, maritime navigation, Anti-aircraft defence etc. Later many microwave antennas were developed eg Parabolic Antenna.

- Microwave heating – Microwaves were generated using vacuum tubes / magnetron to cook food in microwave oven.
- Later developments was done on solid state microwave devices & microwave ICs used to reduce the size of microwave devices.

Advantages

The unique advantages of microwaves over low frequency radio signals are:-

1) Large bandwidth

Due to high bandwidth capability, more information can be transmitted over the channel. Thus microwaves are used for point-to-point communication

2) Better directivity/gain

As frequency increases, wavelength decreases ($\because \lambda = c/f$). Thus for a given antenna size, more antenna gain is possible at higher frequencies because of a shorter wavelength (λ)

$$G = \frac{4\pi A_e}{\lambda^2}$$

where G = Gain

A_e = Antenna aperture

λ = wavelength.

Shorter wavelength allows microwave energy to be concentrated to a small area (Example applications are microwave oven, industrial heating etc).

3) Small size antennas

Antenna size is inversely proportional to the transmission frequency. Microwaves being high frequency, antenna size can be reduced.

4) Low power consumption

- The power required to transmit a high frequency signal is less than the power required for transmitting a low frequency signal.
- As microwaves are high frequency signals, the power consumption is low.

5) Effect of Fading

- Variation in the received signal strength due to atmospheric changes and/or ground reflections in the signal propagation path are called fading. Fading effect is observed to be severe at low frequency than at high frequency. ~~This~~ Due to LOS propagation, microwave communication is reliable because of less fading effects.

6) Transparency property of microwaves

Study of atmospheric layers, ionosphere, sun, other planet characteristics, and remote sensing is possible with microwaves. At microwave frequencies, the electromagnetic properties of many materials change with frequency. This is due to molecular, atomic, and nuclear resonances of conducting materials and substances when they are exposed to microwave fields.

7) Better resolution for radars due to smaller wavelengths.

8) Low power requirements

The transmitter/receiver power requirements at microwave frequencies are less compared with low frequencies due to narrow beamwidths.

The main disadvantages of microwaves are
1) Expensive components
2) Line of sight will be disrupted if any obstacles, such as new buildings are in any way.

Cavity Resonator

- A cavity resonator is a waveguide that is shorted at both ends.
- A cavity resonator can hold electromagnetic energy. The constructive and destuctive interference of multiple reflected waves cause resonances.
- Cavity resonators are used in microwave circuits for high frequencies similar to standard resonant LC circuits at low frequencies.
- When the waveguide's one end is terminated in the shorting plane, there will be reflections, resulting in standing waves. The general mode of propagation in the cavity resonator is TE_{mnp} or TM_{mnp}.
- Applications of cavity resonator

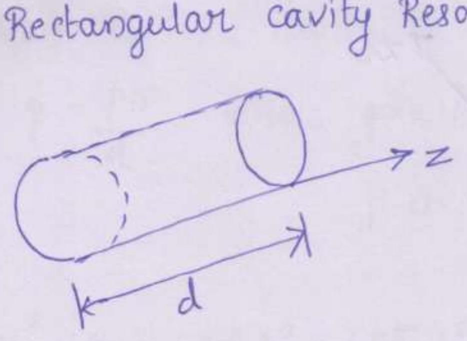
  Cavity resonators are used as

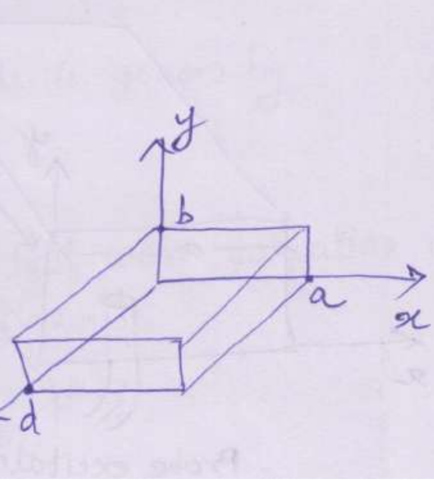
  1) Tuned circuits
  2) cavity in klystron amplifier / oscillator
  3) cavity magnetron
  4) cavity resonators are used in duplexers of radar.
  5) In cavity wavemeters for frequency measurement.

- Types of cavity resonators

  In microwave applications, the commonly used cavity resonators are

  1. Circular Cavity Resonator
  2. Rectangular Cavity Resonator.





Rectangular Cavity Resonator

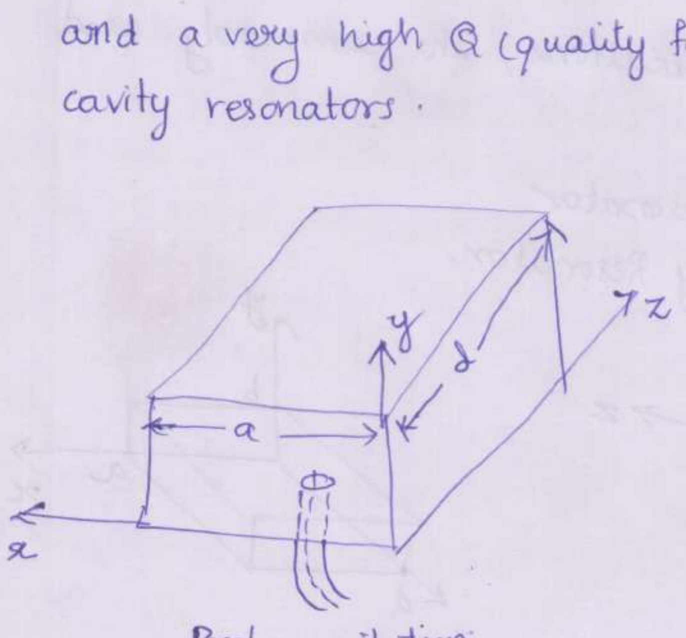
Why cavity resonator?

① At UHF (300MHz to 3GHz) and higher frequencies, ordinary lumped-circuit elements like R, L and C are difficult to make and stray fields become important

② Circuits with dimensions comparable to the operating wavelength become efficient radiators and will interfere with other circuits and systems.

③ Furthermore, conventional wire circuits tend to have a high effective resistance both because of energy loss through radiation and as a result of skin effect.

To provide a resonant circuit at UHF and higher frequencies, we look to an enclosure (a cavity) completely surrounded by conducting walls. Such a shielded enclosure confines electromagnetic fields inside and furnishes large areas for current flow, thus eliminating radiation and high-resistance effects. These enclosures have natural resonant frequencies and a very high Q (quality factor) and are called cavity resonators.



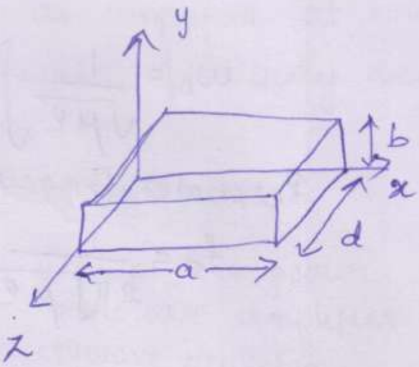
Probe excitation

Derivation of resonant frequency of rectangular cavity resonator

For a rectangular waveguide,

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon \longrightarrow (1)$$

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (2)$$



γ is the propagation constant

$$\gamma = \alpha + j\beta$$

$\alpha \rightarrow$ attenuation constant

$\beta \rightarrow$ phase constant $\quad \beta = \frac{2\pi}{\lambda_g}$

$\lambda_g \rightarrow$ guided waveguide

$m \rightarrow$ no of half waves ~~periodicity~~ variation in x direction

$n \rightarrow$ no of half wave periodicity in y direction

a & b are length and breadth of cavity resonator.

$$(1) \Rightarrow \omega^2 \mu \varepsilon = h^2 - \gamma^2 \longrightarrow (3)$$

Substituting (2) & (3)

$$\omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2 \longrightarrow (4)$$

For wave propagation $\alpha = 0 \Rightarrow \gamma = j\beta$

$$\Rightarrow \gamma^2 = (+j\beta)^2 = -\beta^2 \longrightarrow (5)$$

$$(4) \,\&\, (5) \Rightarrow \omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2 \longrightarrow (6)$$

Condition for the cavity to resonate is given by

$$\beta = \frac{p\pi}{d} \quad \text{where } p = 1, 2, 3 \ldots$$

p is no of half wave variation in z direction.

$$\omega_0^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\omega_0^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]$$

$$\omega_0 = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

Resonator frequency

$$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

The general wave mode through the cavity resonator is denoted by TE_{mnp} for transverse electric (TE) wave and TM_{mnp} for transverse magnetic (TM) wave. The mode with lowest resonant frequency is TE_{101} (dominant mode).

Qn. Calculate the lowest resonant frequency of a rectangular cavity resonator of dimension $a = 2$ cm, $b = 1$ cm & $d = 3$ cm.

TE_{101} has lowest resonant frequency.

$$f_0 = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$= \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{2\times10^{-2}}\right)^2 + \left(\frac{0}{1\times10^{-2}}\right)^2 + \left(\frac{1}{3\times10^{-2}}\right)^2}$$

$$= 9.0138\times10^9 \text{ Hz}$$

$$= 9.0138 \text{ GHz}$$

Qn A rectangular cavity resonator excited by TE_{101} mode at 20 GHz, have the dimensions $a = 2$ cm & $b = 1$ cm, calculate the length of the cavity

$$f_0 = \frac{1}{2\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$20\times10^9 = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{2\times10^{-2}}\right)^2 + \left(\frac{1}{d}\right)^2} \Rightarrow \underline{\underline{d = 0.809 \text{ cm}}}$$

Microwave Sources

— High power microwave sources uses specialized vaccum tubes to generate microwaves. In microwave tubes the electron transit time is utilized for microwave oscillation or amplification. The principle uses an electron beam on the which space-charge waves interact with electromagnetic fields in the microwave cavities to transfer energy to the output circuit of the cavity.

— There are basically two types of microwave tubes:

i) O-Type microwave tube

- Tubes in the O-type category are sometimes called linear or rectilinear beam tubes in recognition of the straight path taken by the electron beam.
- In this class of devices, both velocity and density modulation take place, creating the bunching effect
- The electron bundles thus created have a period in the microwave region.
- Examples of O-type tubes are klystrons + travelling wave tubes (TWT)

ii) M-type microwave tube

- Principle feature of such tubes is that electrons travel in a curved path. Hence the name M-type.
- These are crossed field devices where static magnetic field is perpendicular to the electric field eg Magnetrons.
- The O-type tubes differ from M-type in that electrons travel in a straight line under the influence of parallel electric + magnetic fields.

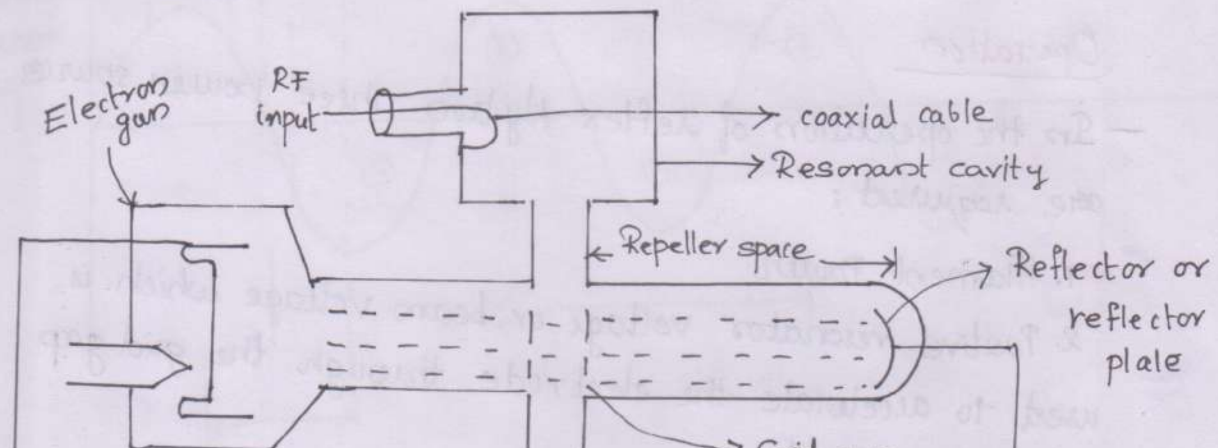
Klystron

- Klystron is most widely used tube as amplifier at microwave frequencies. Klystron works on the principle of velocity and current modulation.
- There are 2 basic configurations of Klystron tubes
  1) Two cavity or multicavity klystron – used as low power microwave amplifier
  2) Reflex klystron – used as low power microwave oscillator.

Reflex Klystron

Reflex klystron is a single cavity velocity modulated tube in which single cavity does the functions of both the buncher and cavity resonators.

Construction



- It consists of
  ① an anode cavity
  ② resonant cavity
  ③ electron gun
  ④ a filament surrounded by cathode
  & ⑤ a focussing electrode. as shown in the above diagram.

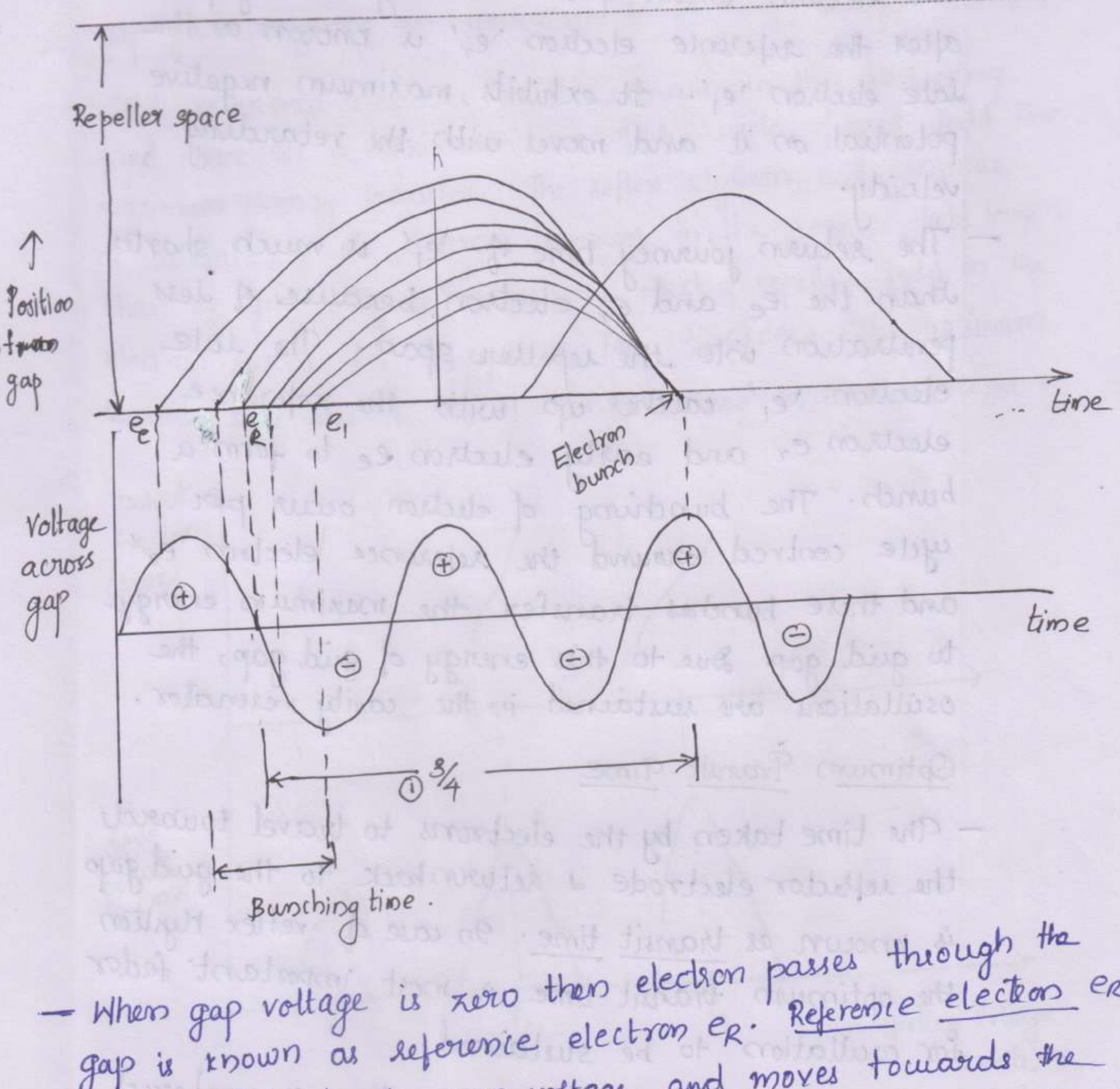
- Reflector electrode is placed at a short distance from resonator grid and is at negative potential with respect to the cathode. The electron beam is modulated when passed through anode resonant cavity, (which is at positive potential).
- The electrons travel towards ~~the~~ repeller electrode after passing through gap in the cavity. Because of the high negative field, the electrons never reach the reflector electrode + are returned back towards the gap on their return journey, the electrons give more energy to gap and oscillations are sustained. Because of the reflex action of electron beam, it is called reflex klystron.

Operation

- In the operation of reflex klystron three power sources are required:

1. Filament Power
2. Positive resonator voltage or beam voltage which is used to accelerate the electrode through the grid gap of resonant cavity.
3. Negative repeller voltage used to push the electrons back to resonator grids.

- The electrons are focussed into a beam by electrostatic fields set up by the resonator potential V_A. The resonator potential also causes the resonant cavity to begin ~~osillations~~ oscillations at ~~the~~ its natural frequency when the tube is energized. These oscillations causes the RF voltage across the grid gap of the cavity that changes the direction of electrostatic field affects the electrons in the beam as they pass through the grid gap. This can be easily explained by the applegate diagram shown below:



- When gap voltage is zero then electron passes through the gap is known as reference electron e_R. Reference electron e_R is unaffected by the gap voltage and moves towards the reflector electrode. This electron gets reflected by the negative potential on the reflector electrode. It returns back to the gap.
- The electron which passes through the gap before reference electron (e_r) is known as early electron (e_e). This electron exhibits the maximum positive potential + accelerated e_e moves with great velocity + it penetrates deep into repeller space.

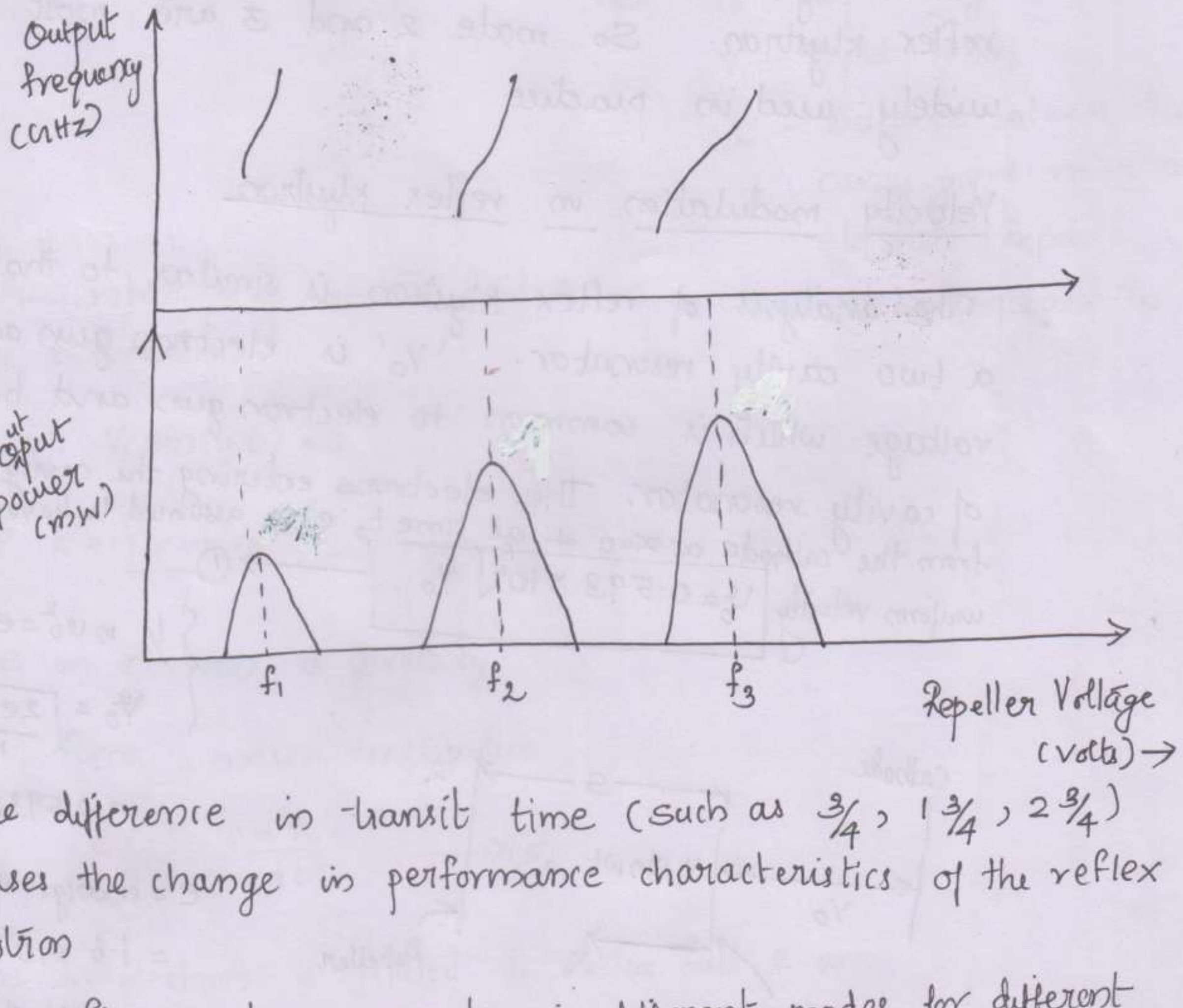
- In return journey from reflector electrode to grid gap, the 'e_e' takes greater time than e_r because of more penetration into repeller space.
- The electron which passes through the gap after the reference electron 'e_r' is known as the late electron 'e_l'. It exhibits maximum negative potential on it and moves with the retarding velocity.
- The return journey time of 'e_l' is much shorter than the e_e and e_r electron because of less penetration into the repeller space. The late electron 'e_l' catches up with the reference electron e_r and early electron e_e to form a bunch. The bunching of election occur per cycle centred around the reference electron 'e_r' and these bunches transfer the maximum energy to grid gap. Due to this energy of grid gap, the oscillations are sustained in the cavity resonator.

Optimum Transit Time

- The time taken by the electrons to travel towards the reflector electrode & return back to the grid gap is known as transit time. In case of reflex klystron the optimum transit time is most important factor for oscillation to be sustained.
- The most optimum departure time is centered around the reference electron which is 180° phase difference from the sine wave voltage across resonator gap. The cavity resonator gives up energy thereby accelerating the electrons & gains energy thereby retarding electrons.

- When the positive gap voltage is applied it provides the maximum retardation to the electron, the e^- returns towards the gap. This causes electrons to fall through negative voltage between gap grids & giving up maximum amount of energy to the gap.

- The reference electrons must remain in the reflecting field space for a maximum time of 3/4 cycle of grid field for maximum energy transfer. The reflex klystron will continue to oscillate if the electrons remain in the repeller field longer than 3/4 cycle. Figure below shows the effect of repeller field on the electron bunch for 3/4 cycle & for $1\frac{3}{4}$ cycles. The optimum transit time should be $T = n + \frac{3}{4}$ where n is any integer.



- The difference in transit time (such as $\frac{3}{4}$, $1\frac{3}{4}$, $2\frac{3}{4}$) causes the change in performance characteristics of the reflex klystron.
- The reflex klystron operates in different modes for different characteristics caused by transit time. The modes are described below. Mode curves are shown above.

Mode 1 : Reflex klystron operates in this mode when the repeller voltage produces an electron transit time of 3/4 cycle.

Mode 2 : Reflex klystron operates in this mode when repeller voltage produces an electron transit time of 1 3/4 cycle

The choice of mode is determined by the difference of power available from each mode. Mode 1 gives the larger output power but voltage requirement is also higher which affects the efficiency of the reflex klystron. So mode 2 and 3 are most widely used in practice

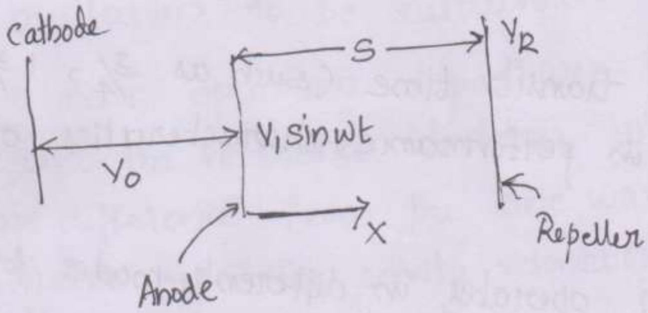
Velocity modulation in reflex klystron

The analysis of reflex klystron is similar to that of a two cavity resonator. 'V_0' is electron gun anode voltage which is common to electron gun and body of cavity resonator. The electrons entering the cavity gap from the cathode at $z=0$ & at time t_0 e^- is assumed to have a uniform velocity $\boxed{v_0 = 0.593 \times 10^6 \sqrt{V_0}}$ → ①

$$\frac{1}{2} m v_0^2 = eV_0$$

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

$$= 0.593 \times 10^6 \sqrt{V_0}$$



e = charge of e^-
$= 1.6 \times 10^{-19}$ colomb

m = mass of electron
$= 9.1 \times 10^{-31}$ kg

After velocity modulation, electrons leave the cavity gap at $z=d$ at time t_1 with a velocity.

$$v(t_1) = v_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right] \longrightarrow ②$$

where β_i = beam coupling factor of input cavity

θ_g = gap transit angle.

- Due to the effect of retarding electric field E, the electrons are forced back to the cavity at time t_2 & is given by:

$$E = \frac{V_r - V_0 - V_1\sin(\omega t)}{L} \longrightarrow ③$$

V_r = Repeller voltage

V_0 - Gap voltage

V_1 - Input voltage

L - Distance between the cavity gap & reflector electrode (Repeller

The retarding electric field E is assumed to be constant in the z-direction

$\therefore V_1\sin(\omega t) = 0$

$$\Rightarrow E = \frac{V_r - V_0}{L} \longrightarrow ④$$

Force on electron is given by:

Force = mass x acceleration

$$= m\frac{d^2x}{dt^2} \longrightarrow ⑤$$

Force on electron is related to electric field E as

$$\frac{m\,d^2x}{dt^2} = -eE \longrightarrow ⑥$$

⑤ & ⑥ $\Rightarrow$

$$m\frac{d^2x}{dt^2} = -eE = -e\frac{(V_r - V_0)}{L} \longrightarrow ⑦$$

$$\frac{d^2x}{dt^2} = \frac{-e}{mL}(V_r - V_0)$$

Integrating

$$\frac{dx}{dt} = \frac{-e}{mL}(V_r - V_0)t + C_1 \longrightarrow (8)$$

At time $t = t_1$, $\frac{dx}{dt} = v_1$

$$v_1 = \frac{-e}{mL}(V_r - V_0)t_1 + C_1$$

$$C_1 = v_1 + \frac{e}{mL}(V_r - V_0)t_1 \longrightarrow (9)$$

(8) & (9) $\Rightarrow$ $\frac{dx}{dt} = \frac{-e}{mL}(V_r - V_0)t + v_1 + \frac{e}{mL}(V_r - V_0)t_1$

$$= \frac{-e}{mL}V_r t + \frac{e}{mL}V_0 t + v_1 + \frac{e}{mL}V_r t_1 - \frac{e}{mL}V_0 t_1$$

$$= \frac{e}{m}(V_0 - V_r)(t - t_1) + v_1$$

$$\frac{dx}{dt} = \frac{-e}{mL}(V_r - V_0)(t - t_1) + v_1$$

Again integrating wrt time

$$x = \frac{-e}{mL}(V_r - V_0)\int_{t_1}^{t}(t - t_1)\,dt + v_1\int_{t_1}^{t}dt$$

$$= \frac{-e}{mL}(V_r - V_0)(t - t_1)^2 + v_1(t - t_1) + C_2$$

At $t = t_1$ distance travelled by electron $x = d = C_2$

Then

$$x = \frac{-e}{2mL}(V_r - V_0)(t - t_1)^2 + v_1(t - t_1) + d$$

The electron leaves the cavity gap at $x=d$ at time t_1 with velocity v_1 & it returns to the gap at $x=d$ at time t_2 then at $t=t_2$, $x=d$

$$0 = \frac{-e(V_r - V_0)}{2mL}(t_2-t_1)^2 + v_1(t_2-t_1)$$

$$(t_2-t_1) = \frac{2mLv_1}{e(V_r-V_0)}$$

Round-Trip transit time (t_2-t_1) in repeller region is

$$(t_2-t_1) = \frac{2mLv_1}{e(V_r-V_0)} \longrightarrow (10)$$

From (1) & (10)

$$(t_2-t_1) = \frac{2mLV_0}{e(V_r-V_0)}\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

By multiplying with ω

$$\omega(t_2-t_1) = \frac{\omega \cdot 2mLV_0}{e(V_r-V_0)}\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

If $\frac{2mLV_0}{e(V_r-V_0)} = T_0$ where T_0 is the round trip transit angle of center of bunch electron.

$$\omega t_2 - \omega t_1 = \omega T_0 + \frac{\omega T_0 \beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

Let $\omega T_0 = \theta_g'$

$$\frac{\omega T_0 \beta_i V_1}{2V_0} = x'$$

Then $$\boxed{\omega t_2 - \omega t_1 = \theta_g' + x'\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)} \longrightarrow (11)$$

~~When~~ $\boxed{\theta_g' = \omega T_0}$ is the dc transit angle

& $\boxed{X' = \theta_g' \dfrac{\beta_i V_1}{2 V_0}}$ is the bunching parameter

For maximum energy transfer,

$$\sin(\omega t_1 - \frac{\theta_g}{2}) = 0$$

$$⑪ \Rightarrow \omega t_2 - \omega t_1 = \theta_g' \qquad (\because \theta_g' = \omega T_0)$$

$$\omega t_2 - \omega t_1 = \omega T_0$$

$$\omega(t_2 - t_1) = \omega T_0$$

$$\omega(t_2 - t_1) = \omega T_0 = 2\pi N = 2\pi(n - 1/4)$$

where $N = n - 1/4$ is the number of modes for any integer n

$n \rightarrow$ cycle number

$$\omega(t_2 - t_1) = 2\pi n - \frac{2\pi}{4}$$

$$\omega(t_2 - t_1) = 2\pi n - \pi/2 = \theta_g' \longrightarrow ⑫$$

Efficiency

The fundamental component of the current induced in the cavity by the modulated electron beam is given by

$$i_{2ind} = -\beta_i I_2 = 2 I_0 \beta_i J_1(X') \cos(\omega t_2 - \theta_0')$$

The magnitude of the fundamental component is

$$I_2 = 2 I_0 \beta_i J_1(X') \longrightarrow ⑬$$

$J_1(X')$ is Bessel function

The dc power supplied by the beam voltage V_0 is

$$P_{dc} = V_0 I_0 \longrightarrow (14)$$

where V_0 = Beam voltage
I_0 = Beam current

The ac power delivered to the load is given by

$$P_{ac} = \frac{V_1}{\sqrt{2}} \cdot \frac{I_2}{\sqrt{2}} = \frac{V_1 I_2}{2} \longrightarrow (15)$$

From (13) & (15)

$$\therefore P_{ac} = \frac{V_1 \times 2 I_0 \beta_i J_1(X')}{2}$$

$$P_{ac} = V_1 I_0 \beta_i J_1(X') \longrightarrow (16)$$

We know

$$X' = \frac{\theta_g' \beta_i V_1}{2 V_0}$$

where θ_g' is the dc transit angle
β_i is the beam coupling coefficient.

$$V_1 = \frac{2 X' V_0}{\beta_i \theta_g'} \longrightarrow (17)$$

$$(12) \Rightarrow \omega(t_2 - t_1) = 2\pi n - \pi/2 = \theta_g' \longrightarrow (18)$$

$$(18) \,\&\, (17) \Rightarrow V_1 = \frac{2 X' V_0}{\beta_i (2\pi n - \pi/2)} \longrightarrow (19)$$

$$(19) + (16) \Rightarrow P_{ac} = \frac{2 X' V_0}{\cancel{\beta_i}(2\pi n - \pi/2)} I_0 \cancel{\beta_i} J_1(X')$$

$$P_{ac} = \frac{2 X' V_0 I_0 J_1(X')}{2\pi n - \pi/2} \longrightarrow (20)$$

Efficiency is the ratio between the maximum power transferred to the output of klystron to the power input to the klystron.

$$\text{Efficiency } (\eta) = \frac{P_{ac}}{P_{dc}}$$

$$= \frac{\dfrac{2X' V_0 I_0 J_1(X')}{2n\pi - \pi/2}}{V_0 I_0} \quad \text{From } 14, 20$$

$$\boxed{\eta = \frac{2X' J_1(X')}{2\pi n - \pi/2}} \longrightarrow 21$$

- For a reflex klystron, the factor $X' J_1(X')$ reaches a maximum value of 1.25 at $X' = 2.408$ & $J_1(X') = 0.52$
- Maximum power is transferred in mode 2

$$\eta_{max} = \frac{2 \times 1.25}{2\pi \times 2 - \pi/2}$$

$$= 22.8\%$$

Electronic admittance of reflex klystron

Electronic admittance of a reflex klystron is defined as the ratio of current induced in the cavity by the modulation of electron beam (i_2) to voltage across the cavity gap (v_2)

$$Y_e = \frac{i_2}{v_2}$$

The induced current can be expressed as

$$i_{2ind} = 2 I_0 \beta_i J_1(X') \cos(\omega(t_2 - \theta_0'))$$

$\cos(\omega t_2 - \omega\theta_g')$
$= \cos\omega t_2 \cos\theta_g' - \sin\omega t_2 \sin\theta_g'$
$= \cos\theta_g' - \sin\theta_g'$

Induced current expressed in phasor form:

$$i_{2ind} = 2 I_0 \beta_i J_1(X') e^{-j\theta_0'} \longrightarrow (1)$$

The voltage across the gap can be expressed in phasor form as

$$V_2 = V_1 e^{-j\pi/2} \longrightarrow (2)$$

Electronic admittance is given by

$$Y_e = \frac{i_{2ind}}{V_2} = \frac{2 I_0 \beta_i J_1(X') e^{-j\theta_0'}}{V_1 e^{-j\pi/2}} \longrightarrow (3)$$

From bunching parameter

$$X' = \frac{\beta_i V_1 \theta_0'}{2 V_0} \longrightarrow (4)$$

$$\Rightarrow V_1 = \frac{2 V_0 X'}{\beta_i \theta_0'} \longrightarrow (5)$$

Substitute (5) in (3)

$$Y_e = \frac{2 I_0 \beta_i J_1(X') e^{-j\theta_0'}}{\frac{2 V_0 X'}{\beta_i \theta_0'} e^{-j\pi/2}}$$

$$\boxed{Y_e = \frac{I_0}{V_0} \frac{\beta_i^2 \theta_0'}{2} \frac{2 J_1(X')}{X'} e^{j(\pi/2 - \theta_0')}}$$

where I_0 = Beam current

V_0 = Beam voltage

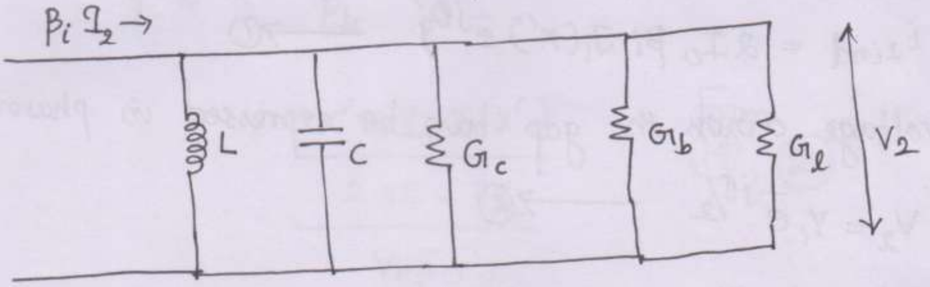
β_i = Beam coupling coefficient

θ_0' = Transit angle of bunch electron

J_1 = current density

X' = Bunching parameter

Equivalent circuit of Reflex Klystron



The above represents the equivalent circuit of Reflex klystron. In the circuit L and C represent energy storage elements of the cavity, G_c represents the copper losses of the cavity.

$G_b \rightarrow$ beam loading conductance.

$G_l \rightarrow$ load conductance

The necessary condition for oscillations is that the magnitude of the negative real part of the electronic admittance should not be less than the total conductance of the cavity circuit

Electronic admittance Y_e in rectangular form is

$$Y_e = G_e + jB_e$$

Condition for oscillation is

$$|-G_e| \geq G$$

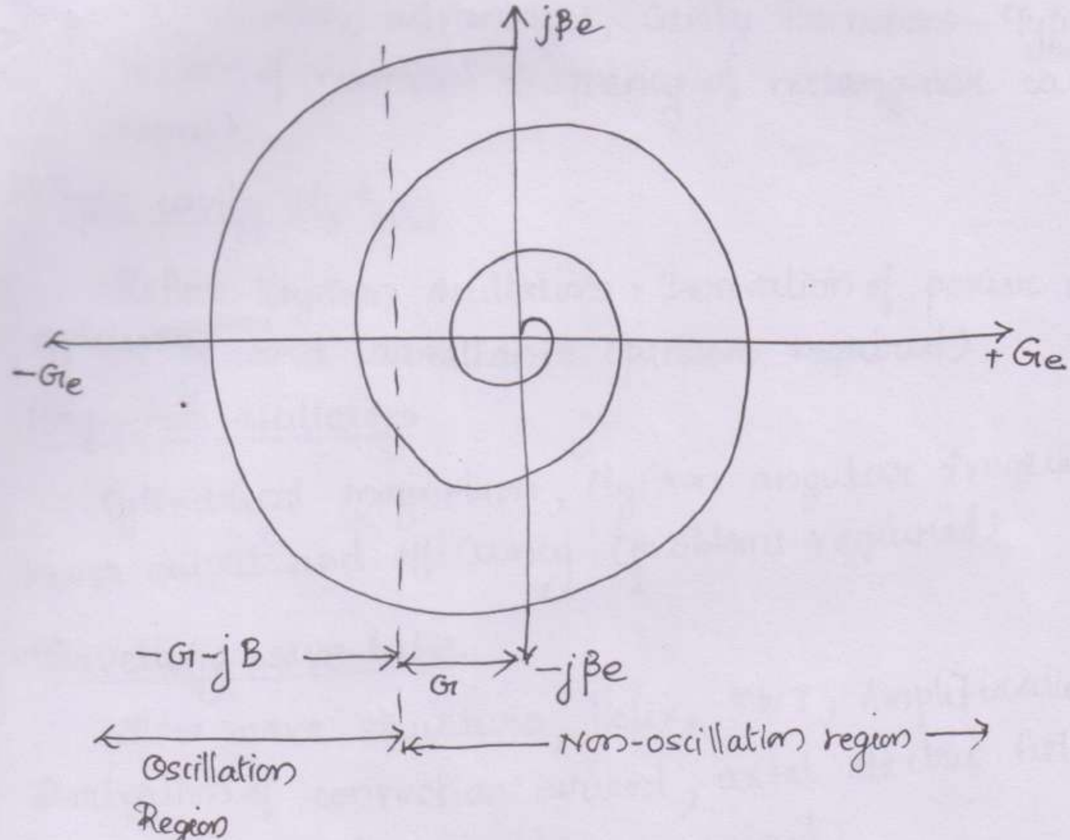
where $G = G_c + G_b + G_l = \frac{1}{R_{sh}}$

$R_{sh} \rightarrow$ effective shunt resistance.

The rectangular plot of electronic admittance Y_e is a spiral & any value of θ_0' for which the spiral lies in the area to the left of line $(-G - jB)$ will yield oscillation ie

$$\theta_0' = (n - 1/4)\,2\pi = N \cdot 2\pi$$

where N is the mode number.



Qn A reflex Klystron operates at the peak mode of $n=2$ with beam voltage $V_0 = 300V$. Beam current $I_0 = 20mA$, signal voltage $V_1 = 40V$. Determine

a) input power in watts

b) output power in watts.

c) Efficiency.

peak mode $\Rightarrow X' J_1(X') = 1.25$; $n=2$

$$P_{dc} \text{ (output power)} = V_0 I_0 = 300 \times 20 \times 10^{-3}$$

$$= 6 \text{ Watts.}$$

$$P_{ac} \text{ (output power)} = \frac{2 V_0 I_0 X' J_1(X')}{2n\pi - \pi/2}$$

$$= \frac{2 \times 300 \times 20 \times 10^{-3} \times 1.25}{2 \times 2 \times \pi - \pi/2} = \frac{15}{3.5\pi} = \frac{10}{\pi} = \cancel{3.18}\ 1.36 \text{ Watts.}$$

$$\text{Efficiency } (\eta) = \frac{P_{ac}}{P_{dc}} \times 100 = \frac{1.36}{6} \times 100 = 22.7\%$$

Microwave Sources

- High power microwave sources uses specialized vaccum tubes to generate microwaves. In microwave tubes the electron transit time is utilized for microwave oscillation or amplification. The principle uses an electron beam on the which space-charge waves interact with electromagnetic fields in the microwave cavities to transfer energy to the output circuit of the cavity.
- There are basically two types of microwave tubes :

i) O-Type microwave tube

- Tubes in the O-type category are sometimes called linear or rectilinear beam tubes in recognition of the straight path taken by the electron beam.
- In this class of devices, both velocity and density modulation take place, creating the bunching effect
- The electron bundles thus created have a period in the microwave region.
- Examples of O-type tubes are klystrons + travelling wave tubes (TWT)

ii) M-type microwave tube

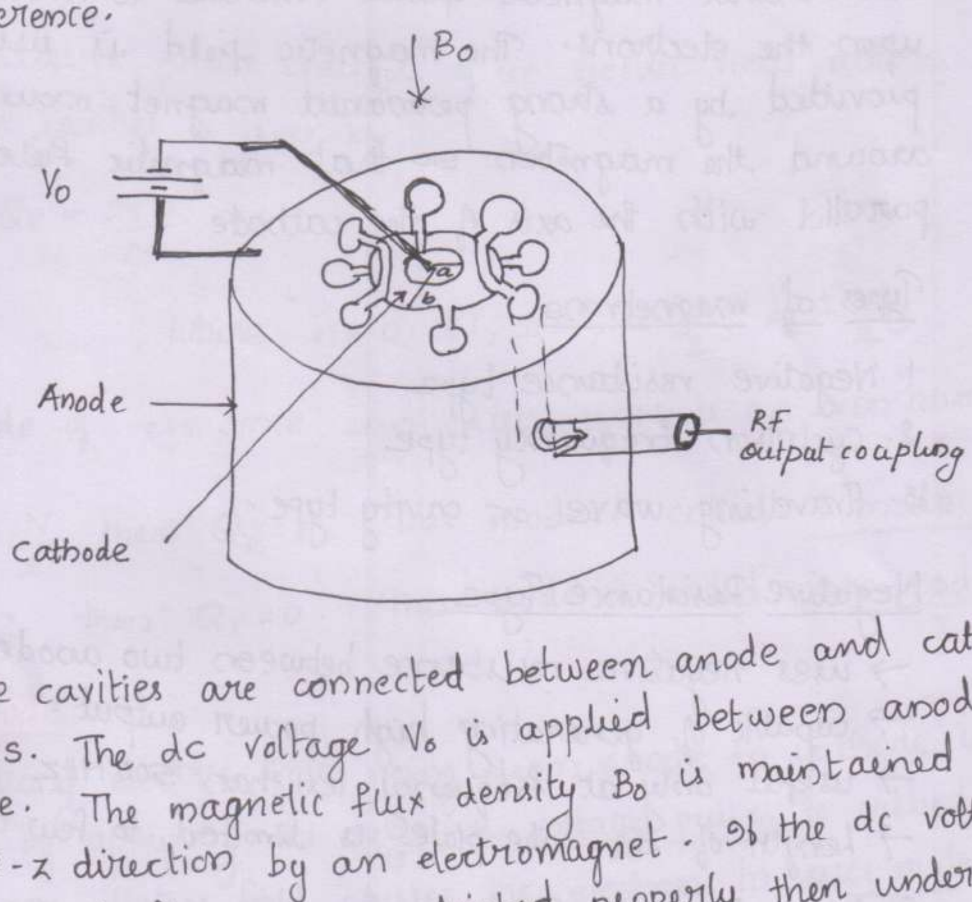
- Principle feature of such tubes is that electrons travel in a curved path. Hence the name M-type.
- These are crossed field devices where static magnetic field is perpendicular to the electric field eg Magnetrons.
- The O-type tubes differ from M-type in that electrons travel in a straight line under the influence of parallel electric + magnetic fields.

Magnetrons

- Magnetrons provide microwave oscillation of very high peak power. The magnetron was invented by Hull in 1921 + in 1939 improved high power magnetron was developed by Randall and Boot.
- The magnetrons are cross field tubes in which electric + magnetic fields are perpendicular to each other. So these tubes are called M-type microwave tubes.

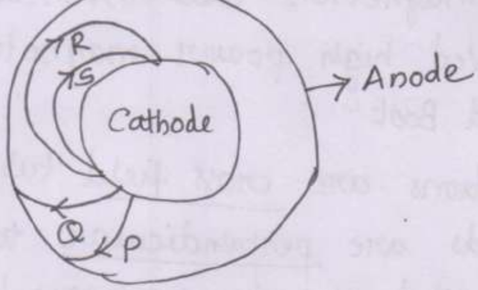
Construction

Cylindrical magnetron consists of a cylindrical cathode of finite length and radius 'a' at the centre surrounded by a cylindrical anode of radius 'b'. The anode has several re-entrant cavities which are equi-spaced around the circumference.



These cavities are connected between anode and cathode by slots. The dc voltage V_0 is applied between anode & cathode. The magnetic flux density B_0 is maintained in positive-z direction by an electromagnet. If the dc voltage (V_0) and magnetic flux (B_0) are adjusted properly then under the combined forces the electrons follow the cycloidal path between anode-cathode space.

Figure below shows the cycloidal path of electrons under balanced electric and magnetic field strength.



Electron path P $\rightarrow \vec{B} = 0$

" Q $\rightarrow \vec{B}$ is small

" R $\rightarrow \vec{B}$ is critical value (cutoff)

" S $\rightarrow \vec{B}$ is greater than critical value.

The open space between cathode and anode is called the interaction space. In this space the electric and magnetic fields interact to exert force upon the electrons. The magnetic field is usually provided by a strong permanent magnet mounted around the magnetron so that magnetic field is parallel with the axis of the cathode

Types of magnetrons

1. Negative resistance type
2. Cyclotron frequency type
3. Travelling wave or cavity type.

Negative Resistance Type

→ uses negative resistance between two anode segments
→ capable of generating high power output.
→ useful only at frequency less than 500 MHZ
→ Length of the tube plate is limited to few centimetres.

Cyclotron Frequency Type

→ The working of these magnetrons depends upon the synchronisation between an alternating component of electric field and periodic oscillation of electrons in the direction parallel to this field.

→ useful only for frequencies greater than 100 MHz.

Travelling wave or Cavity Type

→ These magnetrons provide the oscillations of very high peak power. These are very useful in radar applications. The working of these magnetrons depend upon the interaction of electrons with a rotating electromagnetic field of constant angular velocity.

Modes of operation

– A N-cavity tightly coupled system will have N-modes of operation. Each anode is characterized by a combination of frequency and phase of oscillation relative to the adjacent cavity. These modes are self consistent so that the total phase shift around the ring of cavity resonators is $2n\pi$ where n is an integer. For an 8-cavity magnetron, minimum phase shift should be 45° $(45 \times 8 = 360^\circ)$

– The relative phase change of ac electric field across adjacent cavities is given by

$$Q_v = \frac{2\pi n}{N}$$

$$\text{where } n = 0, \pm 1, \pm 2, \pm\left(\frac{N}{2}-1\right), \pm\frac{N}{2}$$

$\frac{N}{2}$ mode of resonance can exist if N is an even number.

– If $n = \frac{N}{2}$ then $Q_v = \pi$. This mode is called π mode

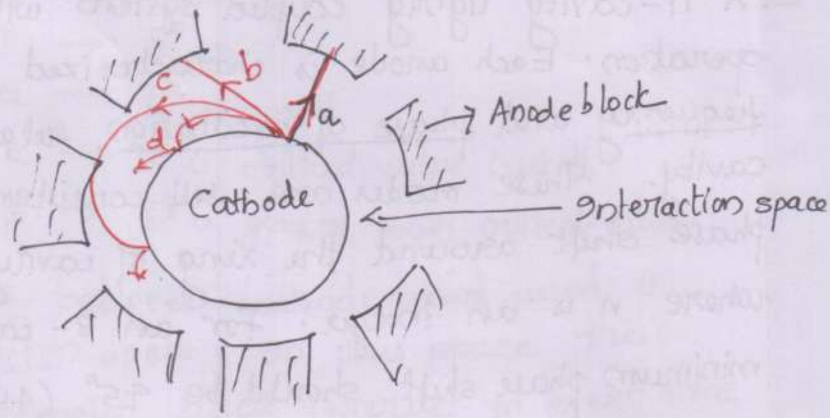
If $n = 0$ then $Q_v = 0$. This mode is called zero-mode

zero-mode

– The strong electric field going from anode to cathode is created by applying the negative voltage pulse to cathode. The strong electric field causes the electrons to accelerate towards the anode after they have been accelerated by the cathode.

- The electrons get accelerated when moving against electric field, where as electrons are decelerated when moving in the same direction as the electric field.

- When $\vec{B} = 0$, the electrons travel in a straight line from cathode to anode due to radial electric field force acting on it. (path 'a') 17



- When magnetic field is slightly increased, it will exert lateral force on electron & e^- travels from cathode to anode in a curved path. The radius (R) of curved path is given by

$$R = \frac{mv}{eB}$$

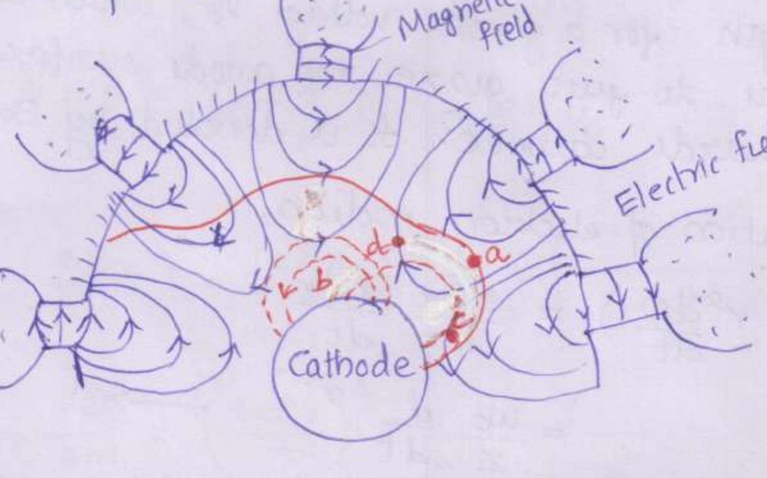
$m \rightarrow$ mass of electron
$v \rightarrow$ velocity of electron
$e \rightarrow$ charge of electron
$m \rightarrow$ mass of electron.

- When magnetic field is increased then the electron doesnot reach to anode (path c), the anode current becomes zero. The magnetic field required to return electrons back to the cathode is called critical magnetic field (B_{oc}) is also known as critical magnetic field.

- If the magnetic field is made larger than the critical field ($B > B_{oc}$), the ~~electrons exerts~~ a greater force is exerted on the electron and it returns back to cathode faster than electron 'c'. This is represented by path 'd'. All such electrons may cause back heating of the cathode. This can be avoided by switching off the heater supply after commencement of oscillation.

π-mode

Assume that the RF oscillations are initiated due to some noise transient within the magnetron and oscillations are sustained by the device operation. When $n = \frac{N}{2}$ then there is π-mode of operation. The anode poles are π radians apart in phase.



The electron 'a' slows down in the presence of oscillations thus transferring energy from cathode to anode. The electrons which participate in transferring the energy to the RF field are called forward electrons and they are responsible for the bunching effect.

The electron 'b' is accelerated by the RF field and it takes energy from the oscillations resulting in increased velocity. It bends more sharply spends very little time in interaction space & returns back to cathode. These electrons are called unfounded electrons and donot participate in bunching process.

The electron 'd' slows down and falls back in step with electron 'a'. This results in forward electron like a, c, d to confined to spokes or electron clouds. The spokes so formed in π-mode rotate with an angular velocity corresponding to 2 poles per cycle. This process is called phase focussing effect corresponding to a bunch of forward electrons around the reference electrons 'a'. The phase focussing effect of these forward electrons imparts enough energy to the RF oscillations so that they are sustained.

Cut-off magnetic field Density (B_{oc})

Cut-off magnetic field strength is the magnetic field strength for a given voltage V_0, which causes the electrons to just graze the anode surface & return towards cathode. It is denoted by B_{oc}.

From equation of electron motion,

$$\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{eB_z}{m}\frac{r\,dr}{dt}$$

$$= \frac{\omega_c}{2}\frac{d}{dt}(r^2) \longrightarrow ①$$

where $\omega_c = \frac{eB_z}{m}$ and is called cyclotron angular frequency.

integrating wrt t.

$$r^2\frac{d\phi}{dt} = \frac{\omega_c r^2}{2} + K \quad \text{where } K \text{ is a constant}$$

For $r = a$ (radius of cathode)

$$\frac{d\phi}{dt} = 0 \Rightarrow K = \frac{-\omega_c a^2}{2}$$

$$r^2\frac{d\phi}{dt} = \frac{\omega_c r^2}{2} - \frac{\omega_c a^2}{2}$$

$$r^2 \frac{d\phi}{dt} = \frac{\omega_c}{2}(r^2 - a^2)$$

$$\frac{d\phi}{dt} = \frac{\omega_c}{2}\left(1 - \frac{a^2}{r^2}\right)$$

When $r = b$

$$\frac{d\phi}{dt} = \frac{\omega_c}{2}\left(1 - \frac{a^2}{b^2}\right) \longrightarrow (A)$$

Electron velocity is given as

$$b^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m}V_0 \longrightarrow (B)$$

$$(A) + (B) \Rightarrow b^2\left(\frac{\omega_c}{2}\left(1 - \frac{a^2}{b^2}\right)\right)^2 = \frac{2e}{m}V_0$$

Substituting $\omega_c = \frac{eB_{oc}}{m}$ at grazing

$$b^2\left[\frac{eB_{oc}}{2m}\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2e}{m}V_0$$

$$\left(\frac{eB_{oc}}{2m}\right)^2 = \left(\frac{2eV_0}{m}\right)\left(\frac{1}{b^2}\right)\left[\frac{1}{1 - \frac{a^2}{b^2}}\right]^2$$

$$\frac{eB_{oc}}{2m} = \left(\frac{2eV_0}{m}\right)^{1/2} \cdot \frac{1}{b} \cdot \frac{1}{1 - \frac{a^2}{b^2}}$$

$$B_{oc} = \frac{2m}{e}\left(\frac{2eV_0}{m}\right)^{1/2} \frac{1}{b\left(1 - \frac{a^2}{b^2}\right)}$$

$$\boxed{B_{oc} = \frac{\left(8V_0 \frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}}$$

The above equation is called Hull cut-off magnetic equation.

For a given magnetic field B_0, the cut off voltage is given by:

$$\boxed{V_{OC} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2}$$

The above equation is called Hull cut-off voltage equation.

Cyclotron Angular Frequency

The magnetic field is normal to the motion of electron that travel in the cycloidal path. The outward centrifugal force is equal to the magnetic force on the electrons

$$\frac{mv^2}{r} = eVB$$

$e \rightarrow$ charge of electron
$v \rightarrow$ velocity of electron
$r \rightarrow$ radius of cycloidal path
$B \rightarrow$ magnetic field intensity
$V \rightarrow$ voltage on electrons

The cyclotron angular frequency of the circular motion of electron is

$$\omega = \frac{V}{r}$$

$$= \frac{eB}{m}$$

The period of complete revolution is

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi m}{eB}$$

For oscillation to occur, the feedback should be in phase as it is integral multiples of 2π radians. If there are N-cavities, the phase should be

$$\phi = \frac{2\pi n}{N}$$ where n is any integer indicating mode of oscillation.

Magnetron oscillators are generally operated in π-mode. The condition for maximum transfer of energy from the electrons to the RF field takes place when cyclotron frequency of electron is equal to the angular velocity of RF wave

- Angular velocity $= \frac{d\phi}{dt}$
- Cyclotron frequency is the ratio of angular frequency to phase constant of π-mode field

$$w_c = \frac{w}{\beta}$$

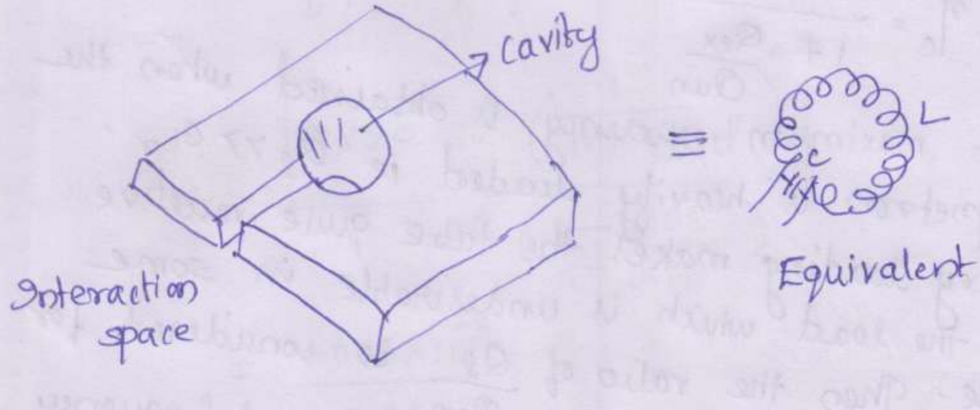
where $\beta = \frac{2\pi n}{NL}$ where L is separation b/w resonant cavities.

- For maximum energy transfer from electron to RF field

$$\frac{d\phi}{dt} = \frac{\omega}{\beta} \Rightarrow \omega = \beta \frac{d\phi}{dt}$$

Efficiency of magnetron

Equivalent circuit of magnetron is shown below. The resonant circuit and dc power supply determines the efficiency of the magnetron. In this circuit Ye is admittance and 'L' is inductance of the resonator.



Equivalent circuit.

The unloaded quality factor of the resonator is given by:

$$Q_{in} = \frac{w_0 C}{G_r}$$

where $w_0 = 2\pi f_0$ is angular resonant frequency of magnetron

C = capacitance

G_r = Conductance of the resonator

The external quality factor of the load circuit is

$$Q_{ex} = \frac{w_0 C}{G_l} \qquad G_l = \text{conductance of loaded circuit}$$

Loaded quality factor Q_l of the resonant circuit is expressed by,

$$Q_l = \frac{w_0 C}{G_r + G_l}$$

The circuit efficiency is defined as

$$\eta_c = \frac{G_l}{G_l + G_r}$$

$$= \frac{1}{1 + \frac{G_r}{G_l}}$$

$$\eta_c = \frac{1}{1 + \frac{Q_{ex}}{Q_{un}}}$$

The maximum efficiency is obtained when the magnetron is heavily loaded ie $G_l >> G_r$. Heavy loading makes the tube quite resistive to the load which is undesirable in some case. Then the ratio of $\frac{Q_l}{Q_{en}}$ is considered for high circuit efficiency of magnetron of frequency stability.

Qn. A 250 kW pulsed cylindrical magnetron has the following parameters

Anode voltage = 25 kV

Peak anode current = 25 A

Magnetic field $B = 0.35\ Wb/m^2$

Radius of cathode = 4 cm.

Radius of cylinder = 8 cm.

calculate efficiency of the magnetron, cyclotron frequency, cut off magnetic field. Assume power loss = 18.5 kW

Electronic efficiency

$$\eta_e = \frac{V_0 I_0 - P_{loss}}{P_{dc}}$$

$$= \frac{25\times10^3 \times 25 - 18.5\times10^3}{25\times10^3\times 25} = 0.97 = 97\%$$

cyclotron frequency

$$f_c = \frac{eB}{2\pi m} = \frac{1.759\times10^{11}\times0.35}{2\pi} \quad \left(\frac{e}{m} = 1.759\times10^{11}\right)$$

$$= 9.79\ GHz$$

cut off magnetic field

$$B_c = \frac{\left(8V_0\frac{m}{e}\right)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)}$$

$$= \frac{\left[8\times25\times10^3\left(\frac{1}{1.759\times10^{11}}\right)\right]^{1/2}}{0.08\left(1-\frac{0.04^2}{0.08^2}\right)}$$

$$= \frac{1.066\times10^{-3}}{0.06}$$

$$= 17.76\ mWb/m^2$$ //

Qn An x-band pulsed cylindrical magnetron has the following operating parameters

Anode voltage $V_0 = 26\ kV$

Beam current $I_0 = 27\ A$

Magnetic flux density $B_0 = 0.336\ Wb/m^2$

Radius of cathode cylinder $a = 5\ cm$

Radius of vane edge to center $b = 10\ cm$.

calculate

i) cyclotron angular frequency

ii) cut off voltage for given B_0

iii) cut off magnetic flux density for given V_0.

i) cyclotron angular frequency

$$w_c = \frac{e}{m} B_0$$

$$= 1.759 \times 10^{11} \times 0.336$$

$$= 5.91 \times 10^{10}\ rad$$

ii) cut off voltage V_{oc} is given by

$$V_{oc} = \frac{e}{m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

$$= \frac{1.759 \times 10^{11}}{8} (0.336)^2 (10 \times 10^{-2})^2 \left(1 - \frac{5^{-2}}{10^2}\right)^2$$

$$= 139.5\ KV$$

iii) cut off magnetic flux density.

$$B_{oc} = \frac{\left(8 V_0 \frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)} = \frac{8 \times 26 \times 10^3 \times \frac{1}{1.759 \times 10^{11}}}{\left(10 \times 10^{-2}\left(1 - \frac{5^2}{10^2}\right)\right)}$$

$$= 14.495\ mWb/m^2$$

Qn An x-band pulsed conventional magnetron has the following operating parameters

Anode voltage = 5.5 KV
Beam current = 4.5 A
Operating frequency = 9 GHz
Resonator conductance = 2×10^{-4} mho
Loaded conductance = 2.5×10^{-5} mho
Vane capacitor = 2.5×10^{-12} F
Duty cycle = 0.002
Power loss = 18.5 KW

Compute
i) resonant frequency
ii) unloaded Q
iii) The loaded Q
iv) The electronic efficiency

$V_0 = 5.5$ KV
$I_0 = 4.5$ A
$f = 9 \times 10^9$ Hz
$G_r = 2 \times 10^{-4}$ mho
$G_\ell = 2.5 \times 10^{-5}$ mho
$C = 2.5$ pF
$P_{loss} = 18.5$ KW

i) resonant frequency $\omega_r = 2\pi f$

$$= 2\pi \times 9 \times 10^9$$

$$= 56.5 \times 10^9 \text{ rad/s}$$

ii) unloaded $Q = \frac{\omega_r C}{G_r}$

$$= \frac{56.5 \times 10^9 \times 2.5 \times 10^{-12}}{2 \times 10^{-4}}$$

$$= 706.2$$

Loaded Q

$$Q_l = \frac{\omega_r C}{G_r + G_l}$$

$$= \frac{56.5 \times 10^9 \times 2.5 \times 10^{-12}}{(200+25) \times 10^{-6}}$$

$$= 628$$

The electronic efficiency

$$\eta_e = \frac{P_{gen}}{V_0 I_0} = \frac{V_0 I_0 - P_{loss}}{P_{dc}}$$

$$= \frac{(5.5 \times 10^3)(4.5) - (18.5 \times 10^3)}{5.5 \times 10^3 \times 4.5}$$

$$= 25\%$$

Qn. A normal circular magnetron has the following parameters

Inner radius = 0.15 m

Outer radius = 0.45 m

Magnetic flux density = 1.2×10^{-3} Wb/m²

a) Determine Hull cutoff voltage

b) Determine Hull cutoff magnetic flux density if beam voltage < 6000 V

$a = 0.15$ m

$b = 0.45$ m

$B_0 = 1.2$ mWb/m²

a) Hull cutoff voltage

$$V_{0c} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

$$= \frac{1.759 \times 10^{11}}{8} (1.2 \times 10^{-3})^2 (0.45)^2 \left(1 - \frac{0.15^2}{0.45^2}\right)^2$$

$$V_{0c} = 5065.92 \text{ V}$$

$$= 5.07 \text{ kV}$$

Hull cut off magnetic flux density

$$B_{oc} = \frac{\left(8 V_0 \frac{m}{e}\right)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)}$$

$$= \frac{\left(8 \times 6000 \times \frac{1}{1.759\times10^{11}}\right)^{1/2}}{0.45\left(1-\frac{0.15^2}{0.45^2}\right)}$$

$$=$$

$$= 10.43 \text{ mWb/m}^2$$

Q. A normal circular magnetron has the following parameter Cathode radius = 2mm and anode radius = 4mm. Determine the Hull cut off voltage if the magnetic flux density is 0.3 Wb/m² & the cut off magnetic flux density if $V_0 = 15$ KV

$$a = 2\times10^{-3} \text{ m}$$
$$b = 4\times10^{-3} \text{ m}$$
$$B_0 = 0.3 \text{ Wb/m}^2$$

Hull cut off voltage

$$V_{oc} = \frac{e}{8m} B_0^2 b^2 \left(1-\frac{a^2}{b^2}\right)^2$$

$$= \frac{1.759\times10^{11}}{8} (0.3)^2 (4\times10^{-3})^2 \left(1-\frac{2^2}{4^2}\right)^2$$

$$= 23.746 \text{ KV}.$$

Cut off magnetic flux density

$$B_{oc} = \frac{\left(8 V_0 \frac{m}{e}\right)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)} = \frac{\left(8\times5\times10^{3}\times\frac{1}{1.759\times10^{11}}\right)^{1/2}}{4\times10^{-3}\left(1-\frac{2^2}{4^2}\right)}$$

$$= 0.15486$$

$$= 154.86 \text{ mWb/m}^2 //$$

Qn A magnetron has a cathode radius of 2.5 mm and an anode radius of 5 mm. What is the Hull cut off potential if a 0.27 Wb/m^2 magnetic field is applied ?

$$F = 2.95 \times 10^9 \, Hz$$

$$B = 0.27 \, Wb/m^2$$

$$N = 8$$

$$a = 5 \, mm = 0.005 \, m$$

$$b = 2.5 \, mm = 0.0025 \, m$$

$$V_{Ht} = \frac{2\pi F B}{N}(b^2 - a^2)$$

$$\cancel{V_{Ht}} = 2 \times 3.14 \times 2.95 \times 10^9 \, \cancel{Hz} \, (0.27) \left[(0.005)^2 - (0.0025)^2\right]$$

$$= \frac{(5.01 \times 10^9)(1.88 \times 10^{-5})}{8}$$

$$= 11.7 \, KV$$

Qn. A pulsed cylindrical magnetron is operated with the following parameters

Anode voltage = 25 kV

Beam current = 25 A

Magnetic density = 0.34 Wb/m^2

Radius of cathode cylinder = 5 cm

Radius of anode cylinder = 10 cm.

Calculate

a) angular frequency

b) cut off voltage

c) cut off magnetic flux density

Angular frequency $= \dfrac{e\beta_0}{m} = 1.759 \times 10^{11} \times 134$

$= 0.5981 \times 10^{11}$ radians

Cut-off voltage $= \left(\dfrac{e\beta_0^2 b^2}{8m}\right)\left(1 - \dfrac{a^2}{b^2}\right)^2$

$= \dfrac{1}{8} \times 1.759 \times 10^{11} \times (0.34)^2 \times (10 \times 10^{-2})^2 \times \left(1 - \dfrac{5^2}{10^2}\right)$

$= 142.97 \text{ kV}$

Cut off magnetic flux density

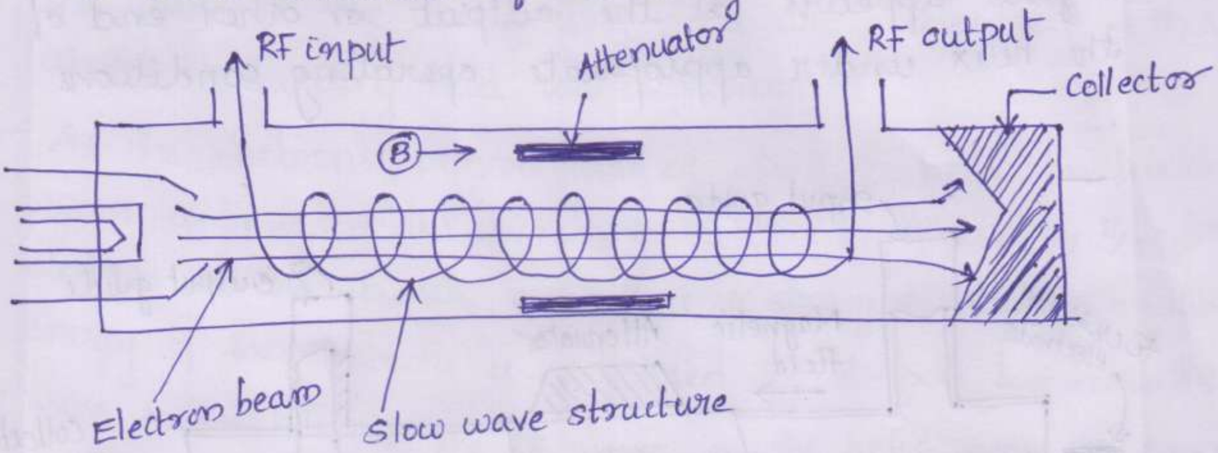
$= \dfrac{(8 V_0 m/e)^{1/2}}{b(1 - a^2/b^2)}$

$= \dfrac{(8 \times 25 \times 10^3 \times 1)^{1/2}}{(1.759 \times 10^{11})^{1/2}} \left(10 \times 10^{-2}\left(1 - \dfrac{5^{2}}{10^2}\right)\right)^{-1}$

$= 142.2 \text{ mWb/m}^2.$

Travelling Wave Tubes (TWT)

- Travelling wave tube (TWT) is an O-type, parallel field linear beam device.
- TWT is an amplifier which makes use of a distributed interaction between an electron beam and a travelling wave.
- Most TWT are non-resonant devices
- The basic structure of travelling-wave tube is shown below:



- The electron gun emits electron. TWT is basically a slow wave structure. The goal of this design is to slow down the RF wave, which propagates at the speed of light (c), to a phase velocity close to the velocity of the electron beam. Under this condition, direct interaction occurs between the RF wave and the electron beam. Synchronized velocities allow both velocity and density modulation of electron beam. The slow wave structure is also called a periodic delay line.

Helix TWT

A helix travelling wave tube consists of an electron gun and a slow wave structure. The electron beam is focussed by magnetic focussing field and guide it through the centre of the long axial helix. The slow wave structure is either the helical type or folded backlive. Helix is a loosely wound thin conducting helical wire. This is termed as O-type TWT. The signal to be amplified is applied to the end of the helix adjacent to the electron gun. The applied signal propagates around the turns of the helix & produces an electric field at the centre of the helix, directed along the helix axis. The amplified signal appears at the output or other end of the helix under appropriate operating conditions

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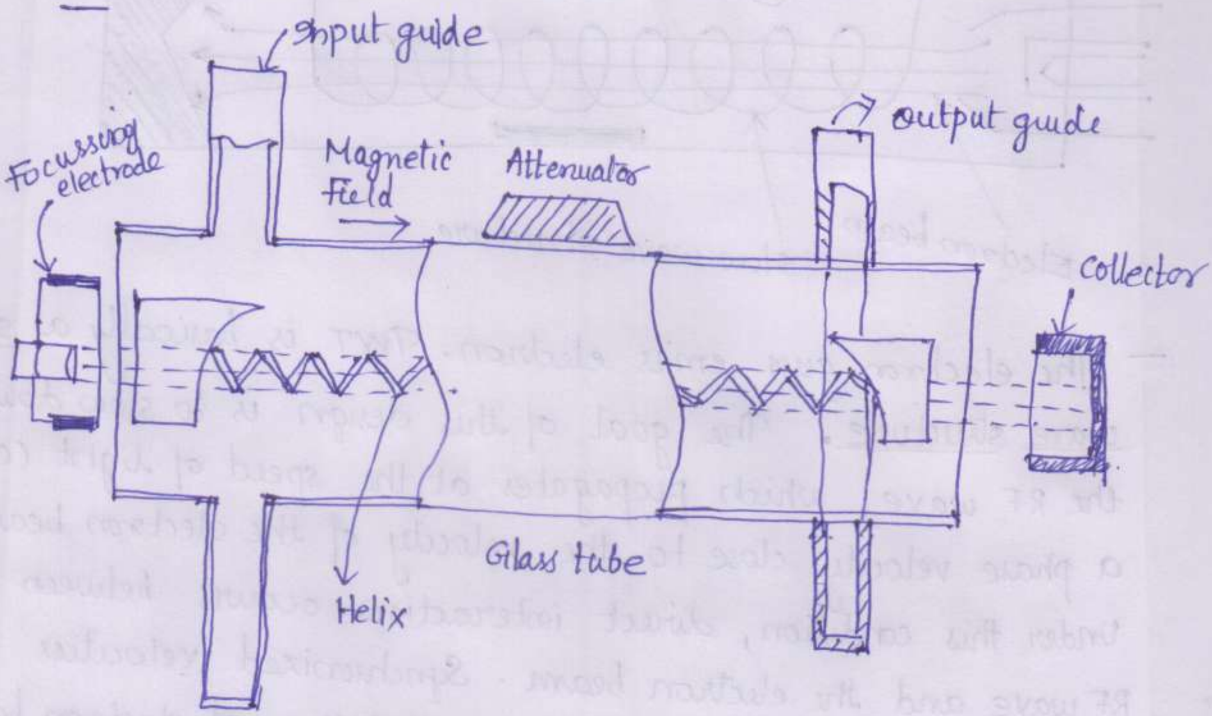


— The helix is made positive wrt cathode and collector more +ve. Thus the beam is attracted to the collector & acquires a high velocity. The speed with which the electric field advances axially is equal to the velocity of light multiplied by the ratio of helix circumference

— Electrons leaving the cathode at random quickly encounter the weak axial RF field at the input end of the helix, which is due to the input signal. As electron passes across the gap, velocity modulation & bunching takes place. When the electrons enter the helix tube, an interaction takes place between the moving axial electric field and the moving electrons. The electrons transfer energy to the wave on the helix. This interaction causes the signal wave on the helix to become larger. The electrons entering the helix at zero field are not affected by the signal wave, those electrons entering the helix at the accelerating field are accelerated & those at the retarding field are deaccelerated.

- As the electrons travel further along the helix, the bunching process continues & the bunching shifts the phase by $\pi/2$. Each electron in the bunch encounters a stronger, also the microwave energy of the electrons is delivered by electron bunch to the wave on the helix & the RF wave on the helix grows exponentially and also reaches its maximum at the output end. Thus amplification of RF wave is accomplished.

Slow wave structures

- Several different forms of slow wave structure are commonly used in TWTs single helix, folded or double helix, ring bar and coupled resonant cavity.
- The helix form of slow wave structure uses a conductor wound into a helical space. In most devices the slow wave helix is wound from flat tungsten or molybdenum, but in a few devices hollow tubing is used. The latter design hollow tubing is used. The pitch (p) of the helix is scaled to reduce the RF wave phase velocity to the electron beam velocity. The phase velocity (v_p) of an RF signal travelling along a slow wave helix is given by:

$$v_p = \frac{cP}{\sqrt{P^2 + (\pi d)^2}}$$

$v_p \rightarrow$ phase velocity in m/s
$c \rightarrow 3\times10^8$ m/s
$P \rightarrow$ Helix pitch in meters
$d \rightarrow$ Helix diameter.

Need of slow wave structure

1. Slow wave structures are used to reduce the wave velocity in a certain direction so that the signal & the electron beam can interact.
2. For producing larger gain over a wide bandwidth.

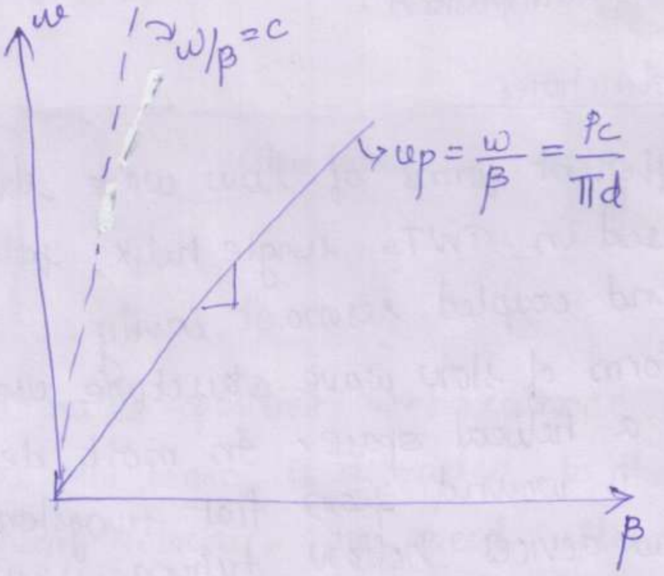
Amplification process in TWT

— The slow wave structures of helix is ~~de~~ characterized by the Brillouin diagram (ω-β diagram)

— For a very small pitch angle, the phase velocity along free space is approximately given by

$$v_p = \frac{\omega}{\beta} = \frac{pc}{\pi d}$$

ω → angular frequency.
β → wave number.



The group velocity of the wave is the slope of the graph & is given by

$$v_{gr} = \frac{\partial \omega}{\partial \beta}$$

- The phase shift per period of the fundamental wave on the structure is given by:

$$\theta_r = \beta_0 L$$

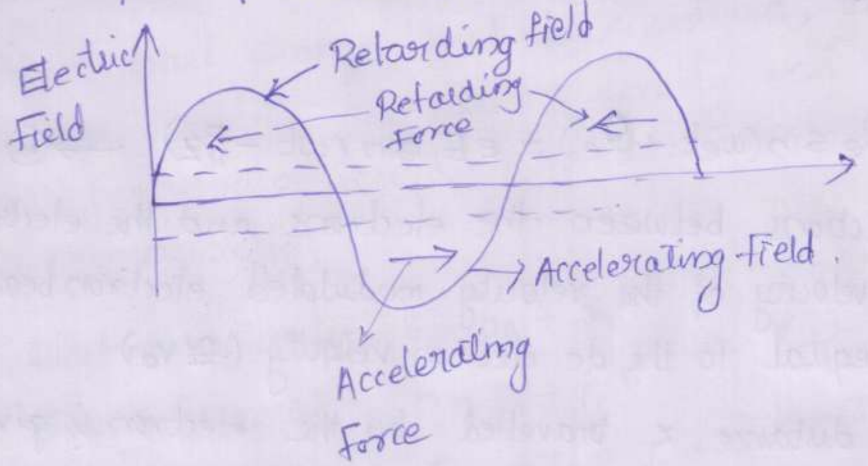
where $\beta_0 = \frac{\omega}{v_0}$ is the phase constant of the average beam velocity.

L = period of the slow-wave structure or pitch

The dc transit time is given by

$$T_0 = \frac{L}{v_0}$$

— The electrons entering the retarding field (ie during negative RF cycle) are deaccelerated & the electrons entering the accelerating field (ie during positive RF cycle) are accelerated. Both these electrons form a bunch with these electrons that enter the helix during the zero RF field. The bunched electrons give up their energy to the wave travelled on the helix and amplified signal obtained at the output of the helix.



- The motion of electrons in the helix tube TWT can be analyzed in terms of axial electric field. If the travelling wave is propagating in z-direction, the z-component of the electric field can be expressed as

$$E_z = E_1 \sin(\omega t - \beta z) \longrightarrow ①$$

where E_1 is magnitude of electric field in the z-direction

$\beta = \frac{\omega}{v_p}$ $\qquad \beta \rightarrow$ axial phase constant

Force on electron

$$F = -eE = m\frac{dv}{dt} \longrightarrow (2)$$

(1) & (2) $\Rightarrow$ $m\frac{dv}{dt} = -eE_1 \sin(\omega t - \beta_p z) \longrightarrow (3)$

Assuming that the velocity of electron is given by

$$v = v_0 + v_e \cos(\omega_e + \theta_e) \longrightarrow (4)$$

$v_0 \rightarrow$ DC electron velocity

$v_e \rightarrow$ Magnitude of velocity fluctuation in the velocity modulated electron beam.

$\omega_e \rightarrow$ Angular frequency of velocity fluctuation

θ_e = Phase angle of the fluctuation.

On differentiating equation (4)

$$\frac{dv}{dt} = -v_e \sin(\omega_e t - \theta_e) \longrightarrow (5)$$

(3) & (5) $\Rightarrow$

$$\omega_e m v_e \sin(\omega_e t + \theta_e) = eE_1 \sin(\omega t - \beta_p z) \longrightarrow (6)$$

For interactions between the electrons and the electric field, the velocity of the velocity modulated electron beam must be equal to the dc electron velocity ($\cong v_0$)

Hence the distance z travelled by the electron is given by

$$z = v_0(t - t_0) \longrightarrow (7)$$

At $t = t_0$, the electric field is assumed maximum.

(7) & (6) $\Rightarrow$

$$\omega_e m v_e \sin(\omega_e t + \theta_e) = eE_1 \sin(\omega t - \beta_p(v_0(t - t_0)))$$

Comparing LHS & RHS

$$\omega_e m v_e = eE_1 \Rightarrow \boxed{v_e = \frac{eE_1}{m\omega_e}}$$

Thus the magnitude of velocity fluctuation of the electron beam is directly proportional to the magnitude of the axial field.

comparing the sinusoidal function

$$w_e t = \omega t - \beta_p v_0 t$$

$$\Rightarrow w_e = \omega - \beta_p v_0$$

$$= \beta_p (\omega/\beta_p - v_0)$$

$$\boxed{w_e = \beta_p (v_p - v_0)} \quad (\because v_p = \omega/\beta_p)$$

Also $\boxed{\theta_e = +\beta_p v_0 t_0}$

Convection current

- convection current is the current induced by the axial electric field. It is produced by the interaction between the electron beam and the axial electric field.
- When space charge effect is considered, the electron velocity, charge density, the current density and axial electric field will perturbate according to their averages or dc values.

Mathematically,

Electron velocity $v = v_0 + v_1 e^{(j\omega t - \gamma z)} \longrightarrow (1)$

Charge density $\rho = \rho_0 + \rho_1 e^{(j\omega t - \gamma z)} \longrightarrow (2)$

current density $J = -J_0 + J_1 e^{(j\omega t - \gamma z)} \longrightarrow (3)$

(-ve sign indicate J_0 is +ve in -ve z direction)

Axial electric field $E_z = E_1 e^{(j\omega t - \gamma z)} \longrightarrow (4)$

where $\gamma = \alpha + j\beta$ is the propagation constant of the axial wave.

Force on electrons

$$F = -eE = m\frac{du}{dt} \rightarrow ⑤$$

$$⑤ + ④ \Rightarrow \frac{dv}{dt} = \frac{-eE}{m}$$

$$\frac{dv}{dt} = -\frac{e}{m}E_1 e^{(j\omega t - \gamma z)} \rightarrow ⑥$$

Differentiating ① wrt t

$$\frac{du}{dt} = v_1(j\omega - \gamma v_0)e^{(j\omega t - \gamma z)} \rightarrow ⑦$$

Chain rule for total derivatives

$$\frac{dv}{dt} = \frac{\partial v}{\partial x} + \frac{dz}{dt}\frac{\partial v}{\partial z}$$

$$\frac{dz}{dt} = u_0$$

Comparing ⑥ & ⑦

$$-\frac{eE_1}{m}e^{(j\omega t - \gamma z)} = v_1(j\omega - \gamma v_0)e^{(j\omega t - \gamma z)}$$

$$\Rightarrow \boxed{v_1 = \frac{-eE_1}{m(j\omega - \gamma v_0)}} \rightarrow ⑧$$

For a small signal, the electron beam current density can be written as:

$$J = \rho u \rightarrow ⑨$$

where J = current density
ρ = charge density
$u \rightarrow$ electron velocity.

① , ② & ⑨ $\Rightarrow$

$$J = (\rho_0 + \rho_1 e^{(j\omega t - \gamma z)})(u_0 + u_1 e^{(j\omega t - \gamma z)})$$

$$= \rho_0 u_0 + \rho_0 u_1 e^{(j\omega t - \gamma z)} + u_0\rho_1 e^{(j\omega t - \gamma z)} + \rho_1 u_1 e^{2(j\omega t - \gamma z)}$$

$$J = \rho_0 u_0 + (u_0\rho_1 + u_1\rho_0)e^{(j\omega t - \gamma z)} + \rho_1 u_1 e^{2(j\omega t - \gamma z)} \rightarrow ⑩$$

Comparing (10) & (3) $\Rightarrow$

$$\left.\begin{aligned} -J_0 &= \rho_0 v_0 \\ J_1 &= (v_0 \rho_1 + \rho_0 v_1) \\ \rho_1 v_1 &= 0 \end{aligned}\right\} \longrightarrow (11)$$

$$\therefore \quad J = -J_0 + J_1 e^{j\omega t - \gamma z} \longrightarrow (12)$$

where $J_0 = -\rho_0 v_0$

$J_1 = v_0 \rho_1 + \rho_0 v_1$

From continuity equation
ρ_1 is given by:

$$\rho_1 = \frac{-j\gamma J_1}{\omega} \longrightarrow (13)$$

(13), (8) & (11) $\Rightarrow$

$$J_1 = \rho_1 v_0 + \rho_0 v_1$$

$$= \frac{-j\gamma J_1}{\omega} v_0 + \rho_0 \left\{ \frac{-eE_1}{m(j\omega - \gamma v_0)} \right)$$

Rearranging

$$\boxed{J_1 = j \frac{\omega}{v_0} \frac{e}{m} \frac{J_0 E_1}{(j\omega - \gamma v_0)^2}} \qquad (J_0 = -\rho_0 v_0)$$

- If the magnitude of the axial electric field is uniform over the cross-sectional area of electron beam, then and $i = I_0$

I_0 – dc current

The convection current in electron beam is given by

$$\boxed{i = j \frac{\beta I_0}{2v_0 (j\beta - \gamma)^2} E_1}$$

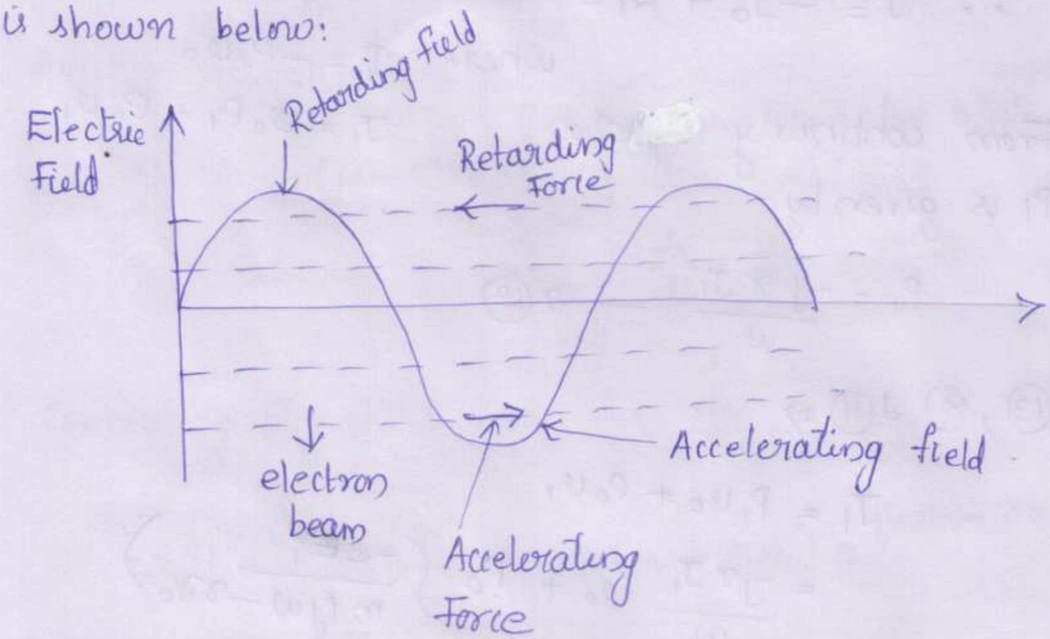
where $\beta = \frac{\omega}{v_0}$ &

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

This equation is called the electronic equation since it determines the convection current induced by the axial electric field.

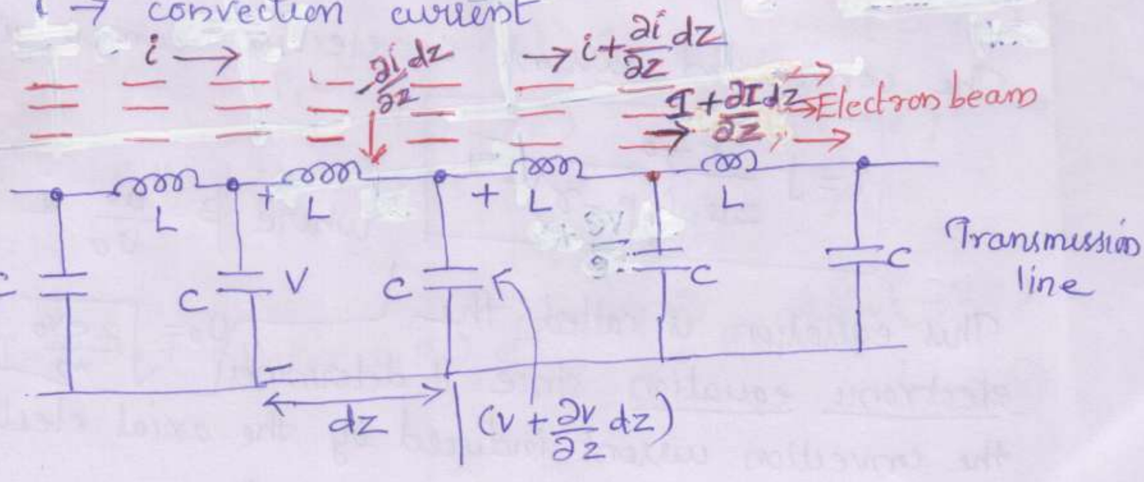
Axial electric field

The RF axial field adds to the field already present in the helix and it causes the signal power to increase with distance. The coupling relationship between the electron beam of the slow wave helix is shown below:



For simplicity, the slow wave helix can be represented by a distributed lossless transmission line. The parameter of slow wave helix are:-

- $L \rightarrow$ inductance per unit length
- $C \rightarrow$ capacitance per unit length
- $I \rightarrow$ alternating current in the transmission line
- $V \rightarrow$ alternating voltage in the transmission line
- $i \rightarrow$ convection current



Since transmission line is coupled to a convection electron beam current, a current is induced in the line. Applying KCL & transmission line theory

$$\frac{\partial I}{\partial z} = -c\frac{dv}{dt} - \frac{\partial i}{\partial z} \longrightarrow (1)$$

Substituting $\frac{\partial}{\partial z} = -\gamma$ & $\frac{\partial}{\partial t} = +j\omega$

Equation (1) becomes

$$-\gamma I = -cj\omega V + \gamma i \longrightarrow (2)$$

From KVL

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \longrightarrow (3)$$

Substituting $\frac{\partial}{\partial z} = -\gamma$ & $\frac{\partial}{\partial t} = j\omega$

$$-\gamma V = -j\omega L I \longrightarrow (4)$$

Multiplying (4) with $-\gamma$

$$+\gamma^2 V = j\omega L \gamma I$$

Substituting γI from (2)

$$\gamma^2 V = j\omega L(cj\omega V - \gamma i)$$

$$= -c\omega^2 LV - j\omega\gamma L i$$

$$\boxed{\gamma^2 V = -V\omega^2 LC - \gamma i j\omega L} \longrightarrow (5)$$

If convection electron beam is absent equation (5) becomes

$$\gamma_o^2 V = -V\omega^2 LC$$

$$\boxed{\gamma_o = j\omega\sqrt{LC}} \longrightarrow (6)$$

The characteristic impedance of the line is :-

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} \longrightarrow (7)$$

Substituting ⑥ & ⑦ in ⑤

$$\gamma^2 V = -V\omega^2 LC - \gamma i j\omega L$$

$$V = \frac{-\gamma i j\omega L}{\gamma^2+\omega^2 LC} \longrightarrow (8)$$

Since $\gamma_0 Z_0 = j\omega L$ & $-\omega^2 LC = \gamma_0^2$

$$\boxed{V = \frac{-\gamma\gamma_0 Z_0 i}{\gamma^2-\gamma_0^2}}$$

The axial electric field $E = -\nabla V$

$$E = -\nabla V$$

$$= -\frac{\partial}{\partial z} V$$

$$= \gamma V$$

$$\boxed{E_1 = \frac{-\gamma^2\gamma_0 Z_0 i}{\gamma^2-\gamma_0^2}}$$

This equation is called <u>circuit equation</u> because it determines how the axial electric field of the slow wave helix is affected by the spatial ac electron beam current.

<u>Waves modes / Propagation Constant</u>

The wave modes of a helix type travelling wave tube can be determined by solving the electronic & circuit equations simultaneously for the propagation constants. Each solution for the propagation constant represents a mode of travelling wave in the tube.

Axial electric field

$$E_1 = \frac{-\gamma^2 \gamma_0 Z_0 i}{\gamma^2 - \gamma_0^2} \longrightarrow ①$$

Convection current

$$i = j \frac{\beta_e I_0}{2v_0 (j\beta_e - \gamma)^2} E_1 \longrightarrow ②$$

① & ②

$$\Rightarrow E_1 = \frac{-\gamma^2 \gamma_0 Z_0}{\gamma^2 - \gamma_0^2} \frac{j\beta I_0}{2v_0 (j\beta - \gamma)^2} E_1$$

$$(\gamma^2 - \gamma_0^2)(j\beta - \gamma)^2 = -j \frac{\gamma^2 \gamma_0 Z_0 \beta I_0}{2V_0} \longrightarrow ③$$

Equation ③ is of 4th order in γ and it has 4 roots. The exact solution can be obtained with numerical methods. However the approximate solutions may be found by equating the dc electron beam velocity to the axial phase velocity of the travelling wave,

$$\gamma_0 = j\beta \longrightarrow ④$$

$$④ \& ③ \Rightarrow (\gamma^2 - (j\beta)^2)(j\beta - \gamma)^2 = -j \frac{\gamma^2 (j\beta) Z_0 \beta I_0}{2V_0}$$

$$(\gamma - j\beta)^3 (\gamma + j\beta) = -\frac{j\gamma^2 (j\beta) Z_0 \beta I_0}{2V_0} \longrightarrow ⑤$$

Let $C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$

Then $\boxed{(\gamma - j\beta)^3 (\gamma + j\beta) = 2C^3 \beta \gamma^2} \longrightarrow ⑥$

C is called travelling wave tube gain parameter

From equation ⑥, it is clear from the term $(\gamma - j\beta)^3$ that there are 3 forward waves travelling in TWT. The term $(\gamma + j\beta)$ indicates that there is one backward wave travelling in opposite direction to electron beam in the TWT.

Qn. A helical slow wave structure has a pitch 'p' 2mm and a diameter of 4cm. Calculate the wave velocity in axial direction of helix

$$p = 2\,mm = 2 \times 10^{-3}\,m$$

$$d = 4\,cm = 4 \times 10^{-3}\,m$$

Axial phase velocity

$$v_p = \frac{cp}{\sqrt{p^2 + (\pi d)^2}} = \frac{3 \times 10^8 \times 2 \times 10^{-3}}{\sqrt{(2 \times 10^{-3}) + (\pi \times 4 \times 10^{-3})}}$$

$$= 4.971 \times 10^6\,m/s.$$

Module V

Microwave Hybrid Circuits – Scattering parameters, Waveguide Tees – Magic tees, Hybrid rings, Formulation of S-matrix, Directional couplers – Two hole directional couplers, S-matrix, Circulators and Isolators, Phase shifter

Microwave Semiconductor Devices – Amplifiers using MESFET. Principle of Gunn diode – Different modes, Principle of operation Gunn Diode Oscillators

The S-parameters

Need for S-parameter

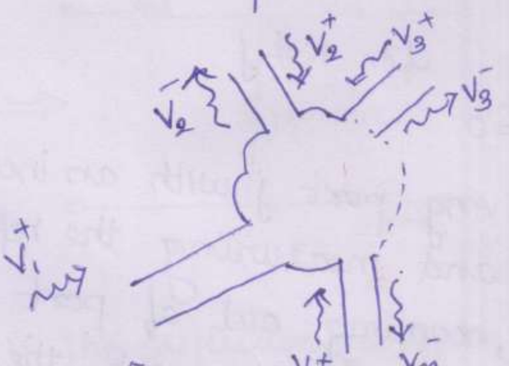
- Defining voltages and currents at microwave frequencies is difficult
- Measuring voltages and currents at microwave frequencies is difficult because direct measurement usually involve the magnitude (inferred from power) and phase of a wave travelling in a given direction or of a standing wave.

Thus equivalent voltages and currents & the related impedance and admittance matrices, become somewhat of an abstraction when dealing with high frequencies.

- Like impedance or admittance matrix for an N-port network, the scattering matrix provides a complete description of the network as seen at its N-ports. Scattering matrix can be directly measured with a vector network analyser.

Formulation of S-matrix

- consider a N-port network shown below:



Let V_n^+ be the amplitude of voltage wave incident on port N and V_n^- be the amplitude of voltage wave reflected from port N

Then S-matrix is defined as

$$\begin{bmatrix} V_1^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & & & \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

amplitude of voltage wave incident = S parameter × amplitude of reflected voltage wave

* $S_{11}, S_{12} \cdots S_{NN}$ are called <u>scattering</u> <u>parameters</u> or <u>scattering</u> <u>coefficients</u> or <u>S-parameter</u>
* S-parameters are <u>complex</u> numbers.
* Let $[V^-] = \begin{bmatrix} V_1^- \\ \vdots \\ V_N^- \end{bmatrix}$ & $[V^+] = \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & & & \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix}$$

then we can write

$$[V^-] = [S][V^+]$$

— A specific element of [S] matrix can be determined as

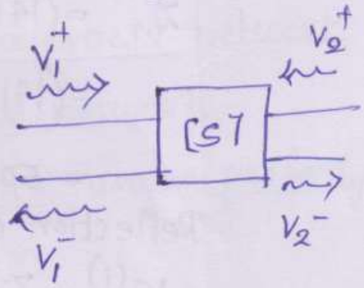
$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0} \quad \text{for } k \neq j$$

S_{ij} is found by driving port j with an incident wave of voltage V_j^+ and measuring the reflected wave amplitude, V_i^-, coming out of port i. The incident waves on all ports except the j^{th} port are set to zero which means that all ports should be <u>terminated</u> <u>in</u> <u>matched</u> <u>load</u> to <u>avoid</u> <u>reflections</u>.

- S_{ii} is the reflection coefficient seen looking into port i when all other ports are terminated in matched load.
- S_{ij} is the transmission coefficient from port j to port i when all other ports are terminated in matched load.

Qn. eg consider S-matrix $[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, here

S_{11} is reflection coefficient seen looking into port 1 when port 2 is grounded

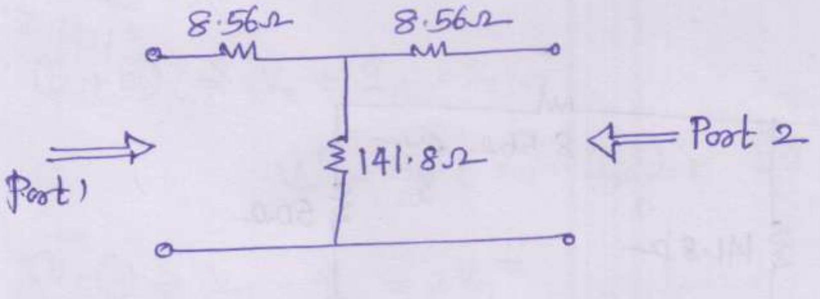


S_{12} is transmission coefficient from port 2 to port 1

S_{21} is transmission coefficient from port 1 to port 2

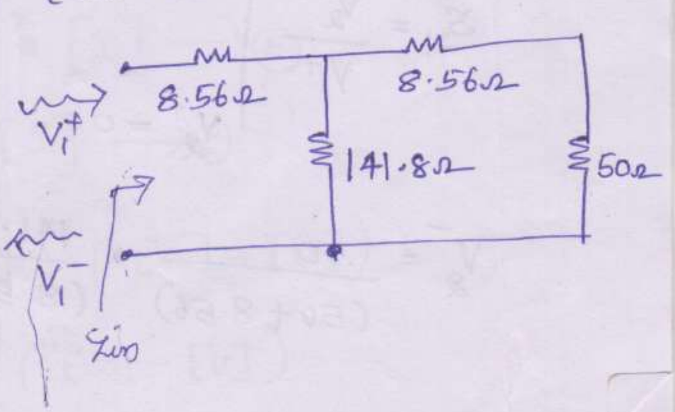
S_{22} is reflection coefficient seen looking into port 2 when port 1 is grounded.

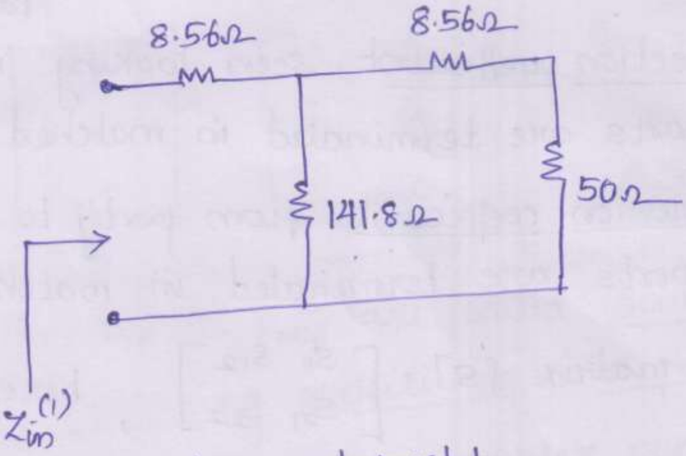
Qn. Find the S-parameters of the 3-dB attenuator circuit shown in figure.



Ans S_{11} is the reflection coefficient seen at port 1 when port 2 is terminated in matched load ($Z_0 = 50\Omega$)

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+ = 0}$$





Input impedance at port 1

$$Z_{in}^{(1)} = \left(141.8 \parallel (50 + 8.56)\right) + 8.56$$

$$= \left(141.8 \parallel 58.56\right) + 8.56 = \frac{141.8 \times 58.56}{141.8 + 58.56} + 8.56$$

$$= 50\,\Omega$$

Reflection coefficient at port 1

$$\Gamma^{(1)} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0}$$

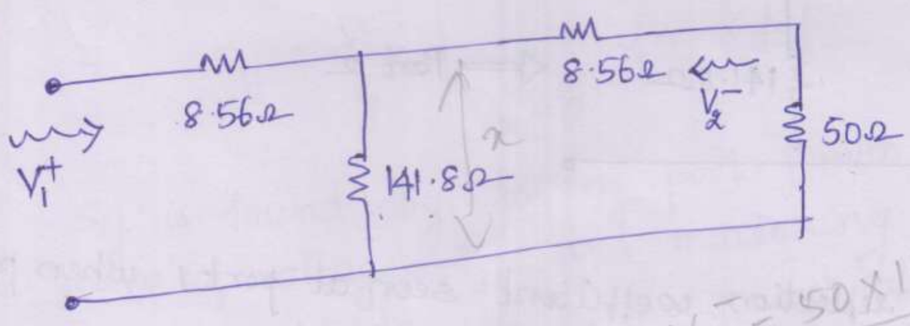
$$= \frac{50-50}{50+50}$$

$$= 0$$

$$S_{11} = \Gamma^{(1)} = 0$$

$$V_2 = \frac{50 \times .94}{(8.56 + 50)}$$

$$x = \frac{141.8}{(141.8 + 8.56)} V_1^+$$



$$V_2 = \frac{50 \times 141.8}{(141.8 + 8.56)} V_1^+$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+ = 0}$$

$$V_2^- = \frac{50}{(50 + 8.56)} \times \frac{141.8}{(8.56 + 141.8)} V_1^+ = 0.8538 \times 0.9430\, V_1^- = 0.805\, V_1^-$$

By symmetry

$S_{11} = S_{22} = 0$

$S_{21} = S_{12} = \underline{0.805}$

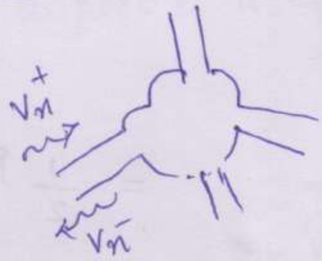
$$[S] = \begin{bmatrix} 0 & 0.805 \\ 0.805 & 0 \end{bmatrix}$$

Properties of S-matrix

- S-matrix is essentially a square matrix or order $N \times N$ for N-port network.
- S-matrix is a symmetric matrix for a reciprocal network
- S-matrix is a unitary matrix for a lossless network.
- The diagonal element of S-matrix are zero under perfectly matched conditions

S-matrix of a reciprocal network is symmetric : Proof

$$V_n = V_n^+ + V_n^- \longrightarrow (1)$$



$$I_n = I_n^+ - I_n^-$$

Assuming impedance is unity,

$$I_n = V_n^+ - V_n^- \longrightarrow (2)$$

$$(1) + (2) \Rightarrow V_n + I_n = 2V_n^+$$

$$V_n^+ = \frac{1}{2}(V_n + I_n) = \frac{I_n}{2}\left(\frac{V_n}{I_n} + 1\right) \longrightarrow (3)$$

$$(1) - (2) \Rightarrow V_n - I_n = 2V_n^-$$

$$V_n^- = \frac{1}{2}(V_n - I_n) = \frac{I_n}{2}\left(\frac{V_n}{I_n} - 1\right) \longrightarrow (4)$$

$$(3) \Rightarrow [V^+] = \frac{1}{2}([Z] + [U]) \times [I] \longrightarrow (5)$$

$$(4) \Rightarrow [V^-] = \frac{1}{2}([Z] - [U])[I] \longrightarrow (6)$$

$$[V^+]^{-1}[V^-] = ([Z] + [U])^{-1}([Z] - [U])$$

$$[S] = ([Z] + [U])^{-1}([Z] - [U])$$

$$[S]^t = \left(\left([Z]+[U]\right)^{-1}\right)^t \left([Z]-[U]\right)^t$$

If the network is reciprocal, $[Z]$ is symmetric

ie $[Z]^t = [Z] \Rightarrow [S] = [S]^t \Rightarrow$ symmetric

<u>S-matrix of a lossless junction is a unitary matrix : Proof</u>

$$P_{avg} = \frac{1}{2}\left\{[V]^t [I]^*\right\}$$

$$= \frac{1}{2} Re\left\{\left[[V^+]+[V^-]\right]^t \left[[V^+]-[V^-]\right]^*\right\}$$

$$= \frac{1}{2} Re\left\{[V^+]^t [V^+]^* - [V^+]^t [V^-]^* + [V^-]^t [V^+]^* - [V^-]^t [V^-]^*\right\}$$

$$= \frac{1}{2} Re\left\{\underbrace{[V^+]^t [V^+]^*}_{\text{Total incident Power}} - \underbrace{[V^-]^t [V^-]^*}_{\text{Total reflected Power.}}\right\} \qquad \boxed{A^t B^* = B^t A^*}$$

If the network is lossless, there is no real power delivered to the network

$\Rightarrow P_{avg} = 0$

$$\Rightarrow [V^+][V^+]^* = [V^-]^t [V^-]^* \longrightarrow (1)$$

We know

$$\frac{[V^-]}{[V^+]} = [S]$$

$$[V^-] = [S][V^+] \longrightarrow (2)$$

$$[V^+]^t [V^+]^* = \left[[S][V^+]\right]^t [S]^* [V^+]^*$$

For non-zero $[V^+]$

$[U]$ is identity matrix

$$\boxed{[S]^t [S]^* = [U]}$$

$$\Rightarrow \left[[S]^t\right]^{-1} = [S]^*$$

or when S matrix is a unitary matrix, the network is lossless

$$\sum_{K=0}^{N} S_{Kl} S_{Kj}^* = 0 \quad \text{for } i \neq j$$

<u>Return Loss</u> & <u>Insertion loss</u>

Reflection coefficient (Γ_N) looking into port N is S_{NN} if all other ports are matched.

Return loss $= -20 \log |\Gamma_N|$

~~Insert~~

Transmission coefficient from port m to port n is equal to S_{NM} if all other ports are matched.

Insertion loss $= -20 \log |\tau|$

Waveguide Tees

- In microwave circuits a waveguide or coaxial line junction with two independent ports is commonly referred to as a tee junction.
- The characteristics of 3-port junction can be explained by three theorem of the tee junction.

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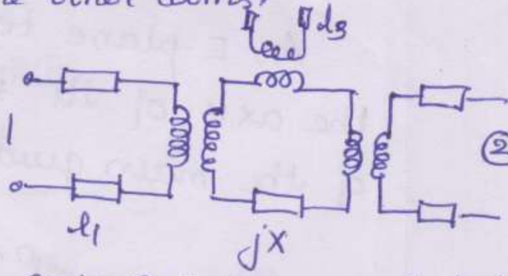
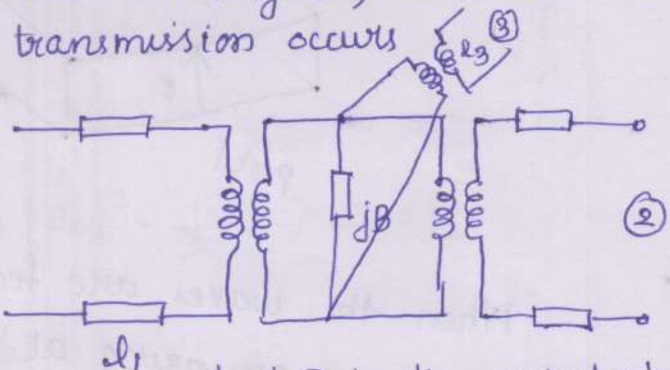
Theorem 1

- It is always possible to place a short circuit in one arm of a T junction in such a position that there is no transmission of power between the other arms.
- In the case of the series circuit, if the distance of short circuit (move a short circuiting plunger) from the terminal plane is such that the total line length from the short circuit to the transformer is an odd number of quarter wavelengths, the circuit is open at that point & no transmission occurs



Series T-junction equivalent circuit



Shunt T-junction equivalent circuit

In the shunt case, the transformer must be short circuited to prevent transmission

Theorem 2

If the Tee junction is symmetrical about an arm, a short circuit can be placed in the arm of symmetry such that the transmission b/w the other 2 arms is possible without

Theorem 3

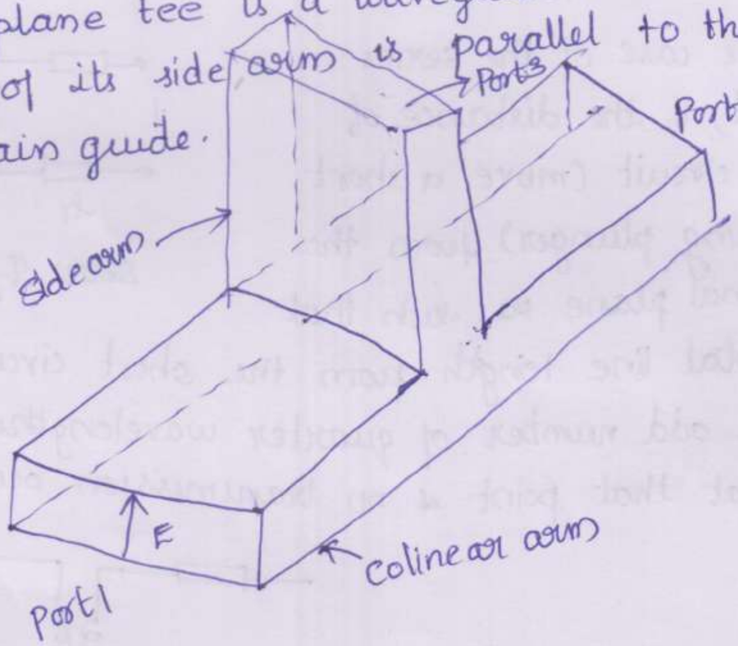
It is impossible for a general 3-port junction of arbitrary symmetry to present matched impedance at all three arms.

Waveguide Tees

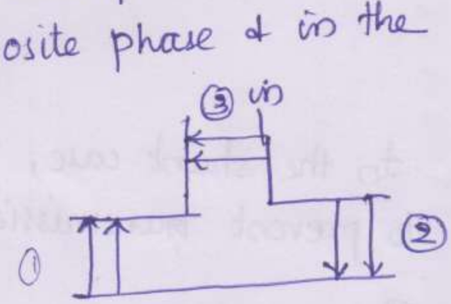
Waveguide tees may consist of E-plane tee, H-plane tee, magic tee, hybrid rings, corners, bends and twists.

E-plane tee (series tee)

An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the E-field of the main guide.



When the waves are fed into the side arm (port 3) the waves appearing at port 1 and port 2 of the the collinear arm will be in opposite phase & in the same magnitude.



$$S_{13} = -S_{23}$$

Assuming the 3rd port is matched

$$S_{33} = 0$$

The S-matrix becomes.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

lossless
$\Rightarrow [S]^t [S]^* = [U]$

Assuming lossless & symmetric,

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{11} S_{11}^* + S_{12} S_{12}^* + S_{13} S_{13}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow ①$$

Illrly $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \longrightarrow ②$

$① - ② \Rightarrow |S_{11}|^2 = |S_{22}|^2 \Rightarrow S_{11} = S_{22} \longrightarrow ③$

$|S_{13}|^2 + |S_{13}|^2 = 1 \Rightarrow S_{13} = \frac{1}{\sqrt{2}} \longrightarrow ④$

$$S_{13} S_{11}^* - S_{13} S_{12}^* = 0$$

$\Rightarrow S_{11} = S_{12} \longrightarrow ⑤$

③, ④ & ⑤ in ① $\quad |S_{11}|^2 + |S_{11}|^2 = \frac{1}{2} \Rightarrow S_{11} = 1/2$

$⑤ \Rightarrow S_{12} = 1/2$

$③ \Rightarrow S_{22} = 1/2$

$$S = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

$$V_1^- = \tfrac{1}{2} V_1^+ + \frac{1}{2} V_2^+ + \frac{V_3^+}{\sqrt{2}}$$

$$V_2^- = \frac{1}{2} V_1^+ + \frac{1}{2} V_2^+ - \frac{V_3^+}{\sqrt{2}}$$

$$V_3^- = \frac{1}{\sqrt{2}} V_1^+ - \frac{1}{\sqrt{2}} V_2^+$$

Case 1

When $V_3^- \neq 0$ + $V_2^+ = V_1^+ = 0$

$$V_1^- = \frac{V_3^+}{\sqrt{2}}$$

$$V_2^- = -\frac{V_3^+}{\sqrt{2}}$$

$$V_3^- = 0$$

Power input at port 3 gets equally divided between port 1 and port 2 with a phase difference of 180°.
E-plane Tee is a 3-dB power splitter.

Case 2

$V_1^+ = V_2^+ = V_0$, $V_3^+ = 0$

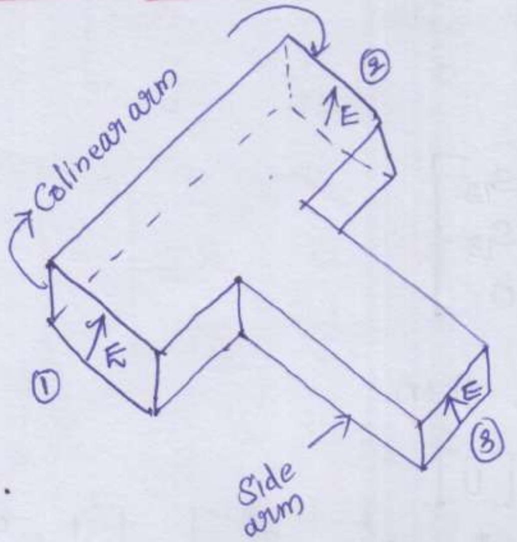
$$V_1^- = V_0$$

$$V_2^- = V_0$$

$$V_3^- = 0$$

Power output at port 3 = 0 when equal inputs are fed at port 1 and port 2

H-plane Tee (shunt Tee)



- H-plane Tee is a waveguide tee in which the axis of the side arm is parallel to the H-field of the main waveguide. Port 1 & port 2 are called collinear ports and port 3 is called H-arm or side arm

- H-plane Tee is so called because the axis of the side arm is parallel to the H-plane of the main transmission line. As all 3 arms of H-plane Tee lie in the plane of magnetic field, the field divides itself into the arms. Therefore it is also called a current junction.

- If 2 input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase & additive.

- If input is fed into port 3, the wave will split equally into port 1 and port 2 in phase & in the same magnitude

$$S_{13} = S_{23}$$

- Due to symmetry,

$$S_{12} = S_{21}$$

$$S_{32} = S_{23}$$

$$S_{13} = S_{31}$$

- Assuming perfect match at junction

$$S_{33} = 0$$

- The s-matrix is

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

If this is lossless, then

$$[S]^t [S]^* = [U]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow ①$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \longrightarrow ②$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow ③$$

$$① - ② \Rightarrow |S_{11}|^2 = |S_{22}|^2 \Rightarrow S_{11} = S_{22} \longrightarrow ④$$

$$③ \Rightarrow S_{13} = \frac{1}{\sqrt{2}} \longrightarrow ⑤$$

$$S_{13} S_{11}^* + S_{13} S_{12}^* = 0$$

$$S_{13} (S_{11}^* + S_{12}^*) = 0$$

Since $S_{13} \neq 0$ $\quad S_{11}^* = -S_{12}^* \Rightarrow \boxed{S_{11} = -S_{12}} \longrightarrow ⑥$

Using ④, ⑤ and ⑥ in ①

$$|S_{11}|^2 + |S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$\boxed{S_{11} = 1/2} \longrightarrow ⑦$$

From ⑥ $\quad S_{11} = -1/2$

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$$S = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

$$V_1^- = \frac{1}{2} V_1^+ - \frac{1}{2} V_2^+ + \frac{1}{\sqrt{2}} V_3^+$$

$$V_2^- = -\frac{1}{2} V_1^+ + \frac{1}{2} V_2^+ + \frac{1}{\sqrt{2}} V_3^+$$

$$V_3^- = \frac{1}{\sqrt{2}} V_1^+ + \frac{1}{\sqrt{2}} V_2^+$$

Case 1

$V_1^+ = 0$ & $V_2^+ = 0$ $\quad V_3^+ \neq 0$

$$\frac{V_1^-}{V_3^+} = \frac{1}{\sqrt{2}} \qquad V_2^- = \frac{1}{\sqrt{2}} V_3^+ , \; V_3^- = 0$$

$$P_3 = P_1 + P_2 = 2P_1 = 2P_2$$

The amount of power coming out of port 1 & port 2 due to input at port 3

$$= 10 \log (P_1/P_3)$$

$$= 10 \log (P_2/P_3)$$

$$= 10 \log (1/2)$$

$$= -3 \text{dB}$$

3dB power splitter.

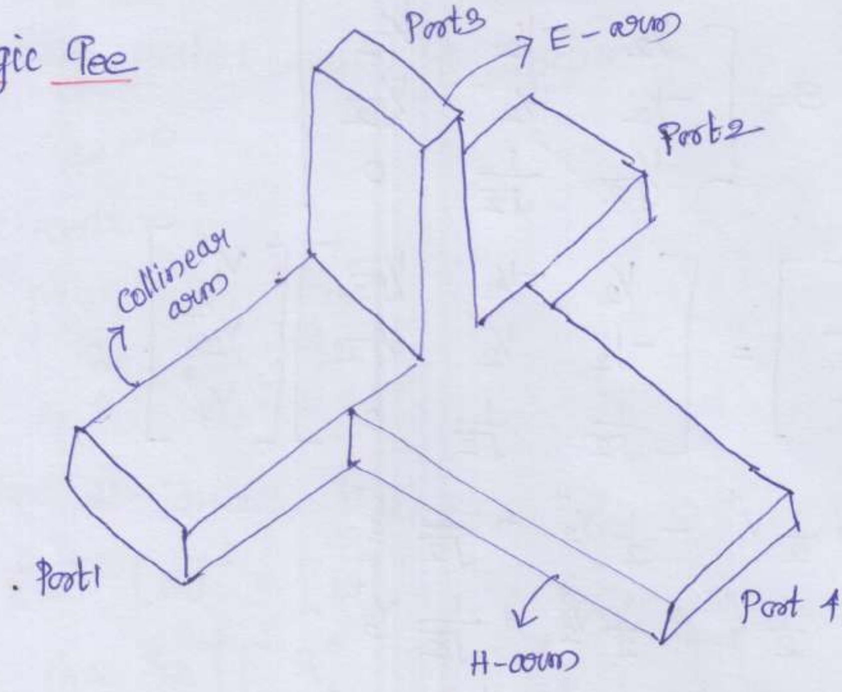
Case 2

$V_1^+ = V_2^+ = V_0 \quad V_3^+ = 0$

$$V_3^- = \frac{1}{\sqrt{2}} V_0 + \frac{1}{\sqrt{2}} V_0 = \sqrt{2} V_0$$

Power output at port 3 is the addition of power inputs from port 1 & port 2 in phase H arm is also called sum arm.

Magic Tee



- Is the combination of E-plane Tee and H-plane Tee.
- If two waves of equal magnitude and phase are fed into port 1 and port 2, output is additive at port 4, output is zero at port 3.
- If an input is given at H-arm, it divides equally into port 1 and port 2. At port 3 the output is zero.

$$S_{43} = S_{34} = 0$$

- If a wave is fed into one of the collinear arm at port 1 or port 2, it will not appear in other collinear arm at port 2 or port 1

$$S_{12} = S_{21} = 0$$

- The S-matrix of a magic Tee can be expressed as

$$S = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

Formulation of s-matrix of Magic Tee

— Using the properties of magic tee, its s-matrix can be obtained as follows:

(1) [S] is a 4x4 matrix since there are 4 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

(2) Because of H-plane Tee section:

$$S_{24} = S_{14}$$

(3) Because of E-plane Tee section:

$$S_{13} = -S_{23}$$

or $S_{23} = -S_{13}$

(4) Because of geometry of the ~~fus~~ junction an input at port ③ cannot come out of port 4 and viceversa.

$$S_{34} = S_{43} = 0$$

(5) For symmetry property $S_{ij} = S_{ji}$

ie $S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{23} = S_{32}$

$S_{14} = S_{41}$, $S_{24} = S_{42}$, $S_{34} = S_{43}$

(6) If port ③ and ④ are perfectly matched to the junction

$$S_{33} = S_{44} = 0$$

Thus [S] matrix can be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix}$$

For unitary property

$$[S][S^*] = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* & S_{14}^* \\ S_{13}^* & -S_{13}^* & 0 & 0 \\ S_{14}^* & S_{14}^* & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (1)$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (2)$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow (3)$$

$$|S_{14}|^2 + |S_{14}|^2 = 1 \longrightarrow (4)$$

Equating (1) & (2)

$$S_{11} = S_{22} \longrightarrow (5)$$

From (3) & (4)

$$2|S_{13}|^2 = 2|S_{14}|^2 \qquad 2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = |S_{14}|^2 \qquad S_{13} = \frac{1}{\sqrt{2}}$$

$$|S_{13}| = |S_{14}| = \frac{1}{\sqrt{2}} \longrightarrow (6)$$

From (1)

$$|S_{11}|^2 + |S_{12}|^2 + \tfrac{1}{2} + \tfrac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0 \longrightarrow (7)$$

$$(7) \text{ \& } (5) \Rightarrow \left.\begin{matrix} S_{11} = 0 \\ S_{12} = 0 \end{matrix}\right\} \rightarrow (8)$$

Substituting (5) & (6) in (2)

$$|S_{12}|^2 + |S_{22}|^2 + \tfrac{1}{2} + \tfrac{1}{2} = 1$$

$$|S_{12}|^2 + |S_{22}|^2 = 0$$

$$|S_{12}|^2 = -|S_{22}|^2$$

$$\Rightarrow S_{12} = S_{22} \longrightarrow (9)$$

From (5) and (7)

$$S_{22} = S_{12} = 0 \longrightarrow (10)$$

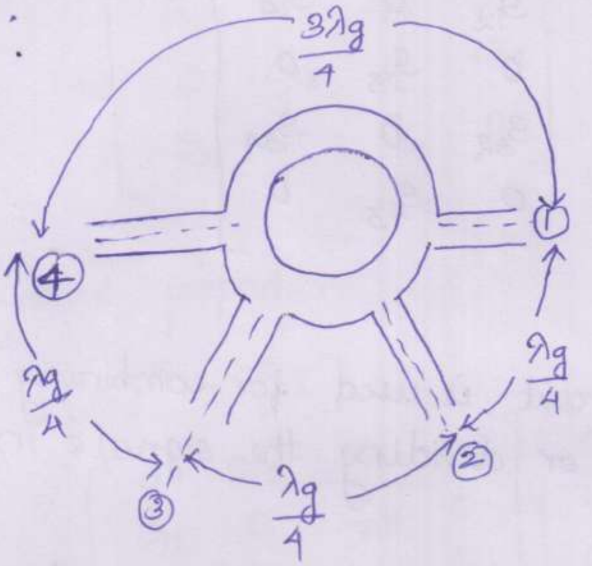
$$\therefore [S] = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

<u>Applications</u>

1. as an isolator
2. as a matching device
3. as a duplexer
4. as a balanced mixer
5. used in impedance measurement.
6. used as switch
7. used in mixer.

Hybrid ring / Rat-Race circuit

- The rat race circuit is an arrangement performing similar function as magic tee but is constructionally different from magic tee.
- The arrangement has a piece of rectangular waveguide bent in E-plane to form a complete loop. It has 4 orifice seperations from each of which a waveguide emerges.



Ports ①, ②, ③ and ④ are available with spacing of $\frac{\lambda_g}{4}$, straight spacing between port 1 and 4 is $\frac{3}{4}\lambda_g$

- If the signal is applied to port 1 it will divide equally, one half will travel in clockwise direction and other half in anticlockwise direction. A voltage null exists at port ③ since the difference in path length causes the two waves to arrive at that point 180° out-of-phase while the signal splits equally between port ③ and ④
- Similarly, the signal fed into port ② will not emerge at port ④

- Rat race (hybrid ring) and magic tee may be used interchangeably with magic tee having the advantage of smaller size but suffering from disadvantage of requiring internal matching which is not needed by rat race if thickness of ring is correctly chosen.

- S-matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Application

- The rat-race circuit is used for combining two different signals or dividing the signals into two signals.

Derivation of S-matrix of rat-race coupler

A rat-race coupler has 4 ports. Thus the S-matrix will be

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

As port 1 and port 3 are isolated ports, the S-parameters

$$S_{13} = 0$$

As port 2 and port 4 are isolated ports, the S-parameter

$$S_{24} = 0$$

From symmetry property of s-matrix, $S_{ij} = S_{ji}$

$\Rightarrow S_{12} = S_{21}, \quad S_{13} = S_{31}, \quad S_{24} = S_{42}, \quad S_{23} = S_{32}$

All ports are matched

$\Rightarrow S_{11} = S_{22} = S_{33} = S_{44} = 0$

Applying all these conditions, the s-matrix becomes

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \longrightarrow (1)$$

From unitary property,

$$[S][S]^* = [I]$$

From (1) $\Rightarrow$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} |S_{12}|^2 + |S_{14}|^2 &= 1 \\ |S_{12}|^2 + |S_{23}|^2 &= 1 \\ |S_{23}|^2 + |S_{34}|^2 &= 1 \\ |S_{14}|^2 + |S_{34}|^2 &= 1 \end{aligned}\right\} \longrightarrow (A)$$

The signal on port 1 will be split between port 2 and 4 equally in case of an ideal 3-dB rat-race coupler with a 180° phase difference

$$\left.\begin{aligned} |S_{21}| = |S_{41}| &= \frac{1}{\sqrt{2}} \\ S_{41} = \frac{1}{\sqrt{2}} e^{j\pi} &= -\frac{1}{\sqrt{2}} \end{aligned}\right\} \longrightarrow (2)$$

Substituting ② in Ⓐ ⇒

$$|S_{14}|^2 + |S_{34}|^2 = 1$$

$$\frac{1}{2} + |S_{34}|^2 = 1.$$

$$|S_{34}| = \pm \frac{1}{\sqrt{2}}$$

Similarly substituting ② in A ⇒

$$|S_{23}|^2 + \frac{1}{2} = 1$$

$$\Rightarrow |S_{23}| = \pm \frac{1}{\sqrt{2}}$$

We know, the signal that is input at port 3, is equally split between port 2 and port 4, but in opposite phase

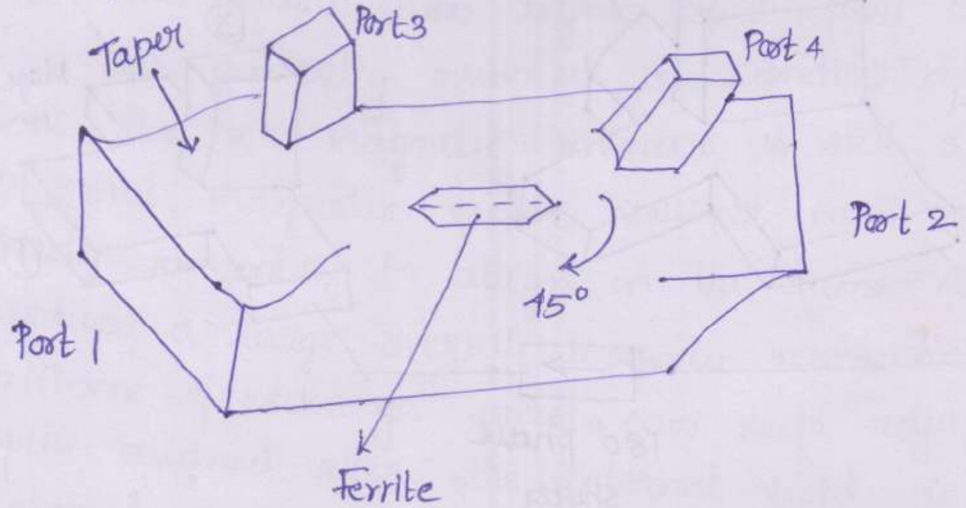
$$\Rightarrow \left. \begin{array}{l} S_{23} = -\frac{1}{\sqrt{2}} \\ S_{34} = \frac{1}{\sqrt{2}} \end{array} \right\} \longrightarrow ③$$

Thus the S-matrix becomes

$$[S] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} & 0 \\ 0 & \frac{-1}{\sqrt{2}} & 0 & 1 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

Circulator

- A microwave circulator is a multiport waveguide junction in which the wave can flow only from n^{th} port to $(n+1)^{th}$ port in one direction.
- Circulator is a non-reciprocal device
- All the four ports are matched and transmission of power takes place in cyclic order (either clockwise or anticlockwise). only.
- An ideal circulator is perfectly lossless.
- Four port circulator : principle of operation



- Working of circulator is based on principle of Faraday rotation. All the port : port 1, 2, 3 & 4 are oriented such that the E-field of transmitted signal couples to these ports successively after going through a rotation of 45° in clockwise direction.
- Power entering port 1 travels along the magnetized ferrite The direction of the E-field vector gets rotated by 45°. Therefore power entering at port 1 appears at port 2. The power cannot be coupled to port 4 because ports 2 & 4 are 90° out of phase. Similarly, port 3 is coupled to port 4 and port 4 to port 1

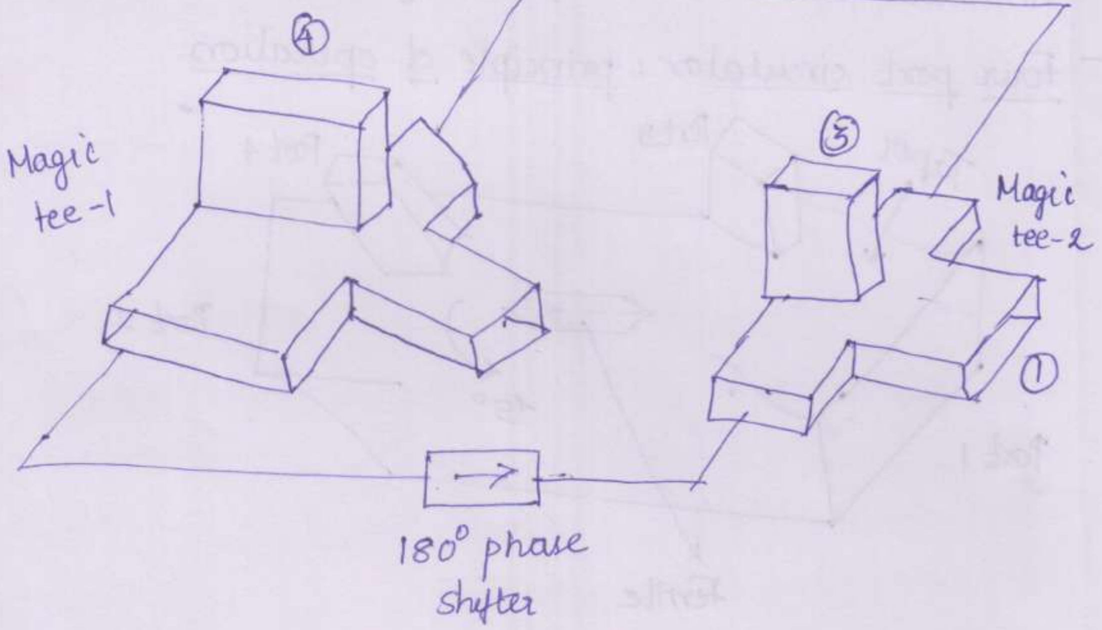
— S matrix is below

$$S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$

<u>Four port circulator using Magic Tees</u>

— A four-port circulator can be constructed by using two magic tees and a phase shifter.



Basic properties of ferrimagnetic materials {For addition reading.}

In a material magnetic moment arises due to
(1) spin (spin moment)
(2) orbiting electron (orbital moment)

- An electron orbiting around a nucleus gives rise to an effective current loop, and thus a magnetic moment.
- Spin moment is due to electron spinning ($m = \frac{qh}{2m_e}$)
  In most solids, electron spin occurs in pairs with opposite signs so that overall magnetic moment is negligible. In a magnetic material, however a large fraction of the electron spins are unpaired (more left hand spins than right hand spins or viceversa), but are generally oriented in random directions, so that the net magnetic moment is still small. An external magnetic field, however can cause the dipole moments to allign in the same direction to produce a large overall magnetic moment. The existence of exchange forces can keep adjacent electron spins aligned after the external field is removed, the material is then said to be permanently magnetized.
- The Lande g factor is a measure of relative contribution of the ~~orbi~~ ~~cobi~~ orbital moment & the spin moment to the total magnetic moment; g=1 when the moment is due to only orbital motion & g=2 when the moment is due to spin only. For ferrite material, g is in the range 1.98 to 2.01 so g=2 is a good approximation.
- Ferrite is <u>nonlinear</u> material and its permeability is an <u>assymetric</u> tensor.

$$B = \mu H$$

$$\mu = \mu_0 (1 + \bar{\chi}_m)$$

$$\tilde{\chi}_m = \begin{bmatrix} \chi_m & jK & 0 \\ jK & \chi_m & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ which is the

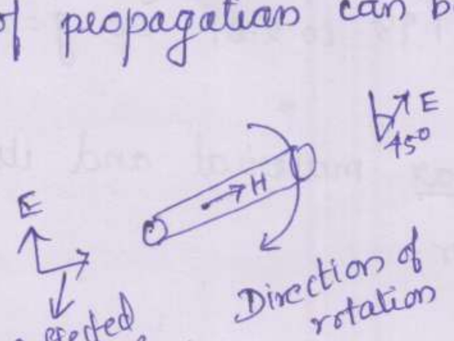
tensor magnetic susceptibility. Here χ is the diagonal susceptibity and K is the off-diagonal susceptibility.

— The elements of susceptibility or permeability tensors become infinite when the frequency ω equal to Larmor frequency ω_0. This effect is called gyromagnetic resonance.

— When a piece of ferrite is affected by a dc magnetic field, the ferrite exhibits Faraday rotation. The wave in the ferrite is rotated in clockwise direction. consequently the propagation phase constant β^+ for the forward direction differs from the propagation phase constant β^- for the backward direction. By choosing the length of ferrite slab + the dc magnetic field so that

$$w = (\beta^+ - \beta^-) l = \pi/2$$

a differential phase shift of 90^0 for the 2 direction of propagation can be obtained.



Ferrite Isolators

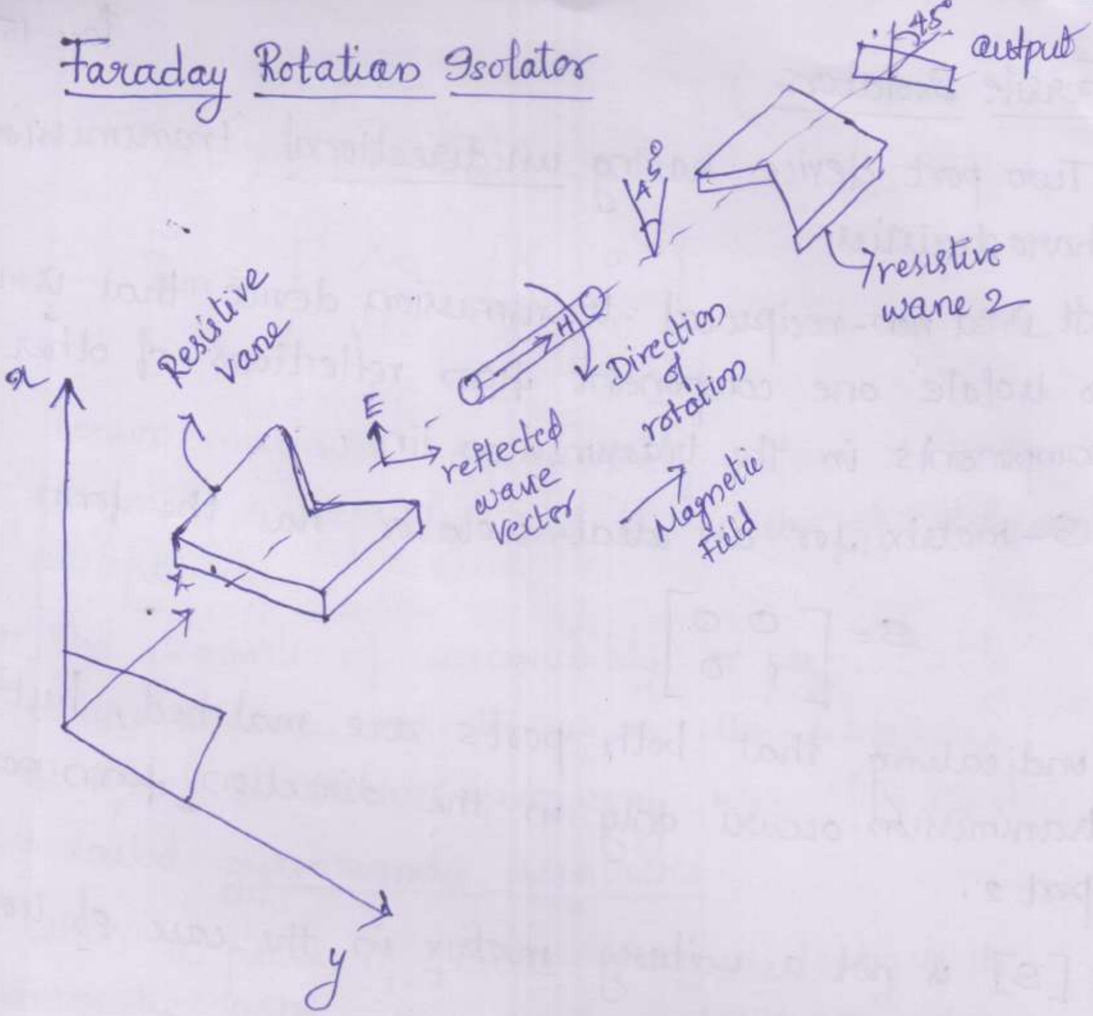
- Two port device having unidirectional transmission characteristics
- It is a non-reciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line.
- S-matrix for an ideal isolator has the form

$$S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

indicating that both ports are matched, but transmission occurs only in the direction from port 1 to port 2.

- [S] is not a unitary matrix in the case of isolator ie isolator is lossy
- A common application of an isolator, is its use between a high power source and a load to prevent possible reflections from damaging the source.
- An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Isolators are generally used to improve frequency stability of microwave generators like klystrons & magnetrons, in which the reflection from the load affects the generating frequency. As a result, the isolator maintains the frequency stability of the generator.

Faraday Rotation Isolator



The operating principle of Faraday-rotation isolator is as follows:

- The input resistive card is in the y-z plane and the output resistive card is displaced 45° with respect to the input card. The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by 45°. The degree of rotation depends on the length & diameter of the rod and on the applied dc magnetic field. An input TE_{10} dominant mode is incident to the left end of the isolator. Since TE_{10} mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation. The wave in the ferrite rod section is rotated clockwise by 45° & the is normal to the output resistive card. As a result of rotation, the wave arrives at the output end without attenuation at all. On the contrary, a reflected wave from the output end is similarly rotated clockwise 45° by the ferrite rod. However since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card. The typical performance of these isolators is about 1-dB insertion loss in the forward transmission & about 20 to 30 dB isolation in reverse attenuation.

Insertion Loss (I_L)

$$I_L(dB) = 10\log\left(\frac{P_1}{P_2}\right)$$

where P_1 is the power launched at input port

P_2 is " received at output port.

Isolation (I_s)

$$I_s(dB) = 10\log\left(\frac{P_2'}{P_1'}\right)$$

where P_1' is power at input port

P_2' is power launched at output port.

Qn A matched isolator has insertion loss of 0.5 dB & isolation 25 dB. Find the S-matrix

matched $\Rightarrow S_{11} = 0,\ S_{22} = 0$

Insertion loss = 0.5 dB = $-20\log|S_{21}|$

$$\Rightarrow S_{21} = 10^{-0.5/20} = 10^{-0.025} = 0.944061$$

Isolation $= 25dB = -20\log|S_{12}|$

$$\Rightarrow S_{12} = 10^{-25/20} = 10^{-1.2} = \underline{\underline{0.056}}$$

$$[S] = \begin{bmatrix} 0 & 0.056 \\ 0.944 & \\ & 0 \end{bmatrix}$$

Qn. A 3-port circulator has an insertion loss of 1dB, isolation 30dB & VSWR = 1.5. Find the S-matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Insertion Loss $= 1dB = -20\log|S_{21}|$

$$|S_{21}| = 10^{-1/20} = 0.89$$

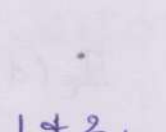
The insertion loss is same b/w port 1 & 2, 2 & 3 and 3 & 4

$$|S_{21}| = |S_{32}| = |S_{13}| = 0.89$$

The isolation between the ports is 30dB

$$30 = -20\log|S_{31}|$$

$$|S_{31}| = 10^{-30/20} = 10^{-1.5} = 0.032$$



$$|S_{31}| = |S_{23}| = |S_{12}| = 0.032$$

VSWR = 1.5

$$|\Gamma| = \frac{S-1}{S+1} = \frac{1.5-1}{1.5+1} = 0.2$$

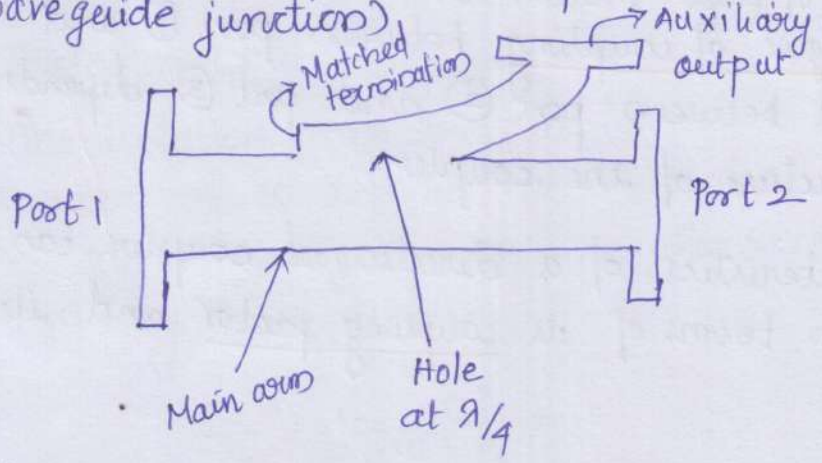
$$|S_{11}| = |\Gamma| = 0.2 \qquad |S_{11}| = |S_{22}| = |S_{33}| = 0.2$$

Thus the S-matrix of circulator is

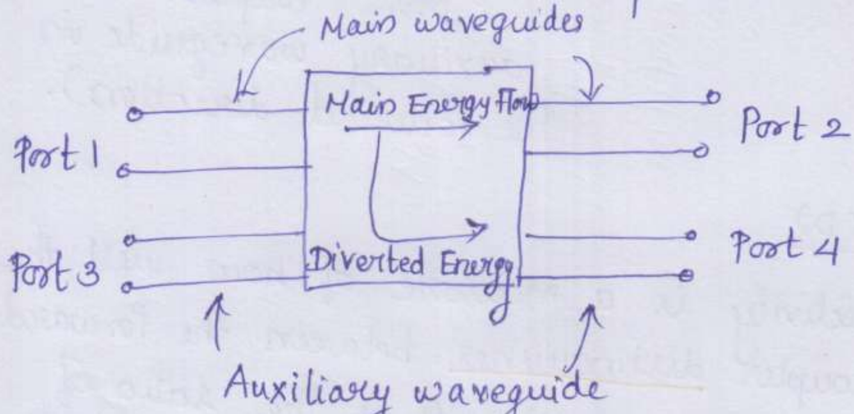
$$[S] = \begin{bmatrix} 0.2 & 0.032 & 0.89 \\ 0.89 & 0.2 & 0.032 \\ 0.032 & 0.89 & 0.2 \end{bmatrix}$$

Directional coupler

- Directional coupler is a 4-port device. (or a 4-port waveguide junction)



- It consists of a primary waveguide (1-2) + a secondary waveguide (3-4)
- Directional coupler is used to measure the unidirectional power being delivered to a load by sampling technique. In sampling technique, only a known fraction of power, in forward wave is measured. From this fractional power, the total power can be measured. There is no reflections at the junction of these 4 ports.



- With matched terminations at all its ports, the properties of an ideal directional coupler can be summarized as follows:

1) A portion of power travelling from port ① to port ② is coupled to port ④ but not to port ③

2) A portion of power travelling from port ② to port ① is coupled to port ③ but not to port ④ [bidirectional case]

③ A portion of power incident on port ③ is coupled to port ② but not to port ①

④ A portion of power incident on port ④ is coupled to port ① but not to port ③

— The degree of coupling between port ① and port ④ and between port ② and port ③ depends on the structure of the coupler.

— The characteristics of a directional coupler can be expressed in terms of its coupling factor and its directivity.

Coupling factor (C)

— If the wave is propagating from port ① to ② in the primary line, the coupling factor is defined as:

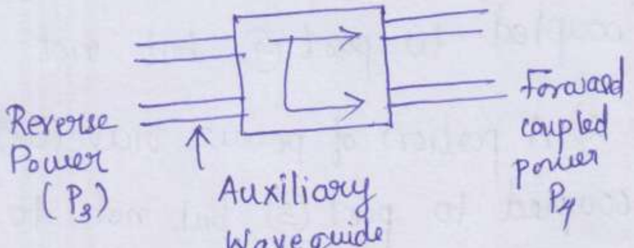
$$\text{Coupling Factor (dB)} = 10\log_{10}\left(\frac{P_1}{P_4}\right)$$

where $P_1 \rightarrow$ power input to port 1 (incident power)

$P_4 \rightarrow$ power output from port 4 (power coupled in the auxiliary waveguide in the forward direction).

Directivity (D)

The directivity is a measure of how well the directional coupler distinguishes between the forward and reverse travelling power. It is the ratio of forward coupled power at auxiliary waveguide to the reverse power at auxiliary waveguide



$$D_{(dB)} = 10\log_{10}\left(\frac{P_4}{P_3}\right)$$

- Ideally directivity should be infinity ie power output at port 3 is zero.

Isolation (I)

- The isolation measures the directive properties of directional coupler. It is defined as the ratio of incident power at main waveguide (P_1) to the reverse power at auxiliary waveguide (P_3)

$$I = 10\log_{10}\left(\frac{P_1}{P_3}\right)$$

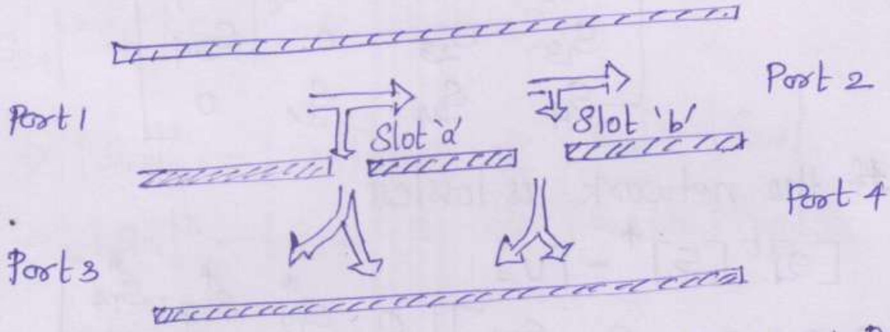
- Isolation factor represents the amount of isolation between 2 ports in a directional coupler. Ideally, it should be infinity ie power output at port 3 is zero.

$$\boxed{I = D + C}$$

I → Isolation
D → Directivity
C → Coupling

Two-hole Directional Coupler

Two hole directional coupler consists of two guides with two (holes) common between them. These two aperture holes are at a distance of $\frac{\lambda_g}{4}$.



$P_1 \rightarrow$ Input Port
$P_2 \rightarrow$ output Port
$P_3 \rightarrow$ Isolated Port
$P_4 \rightarrow$ Coupled Port.

- Energy is coupled through the slots from the main to the coupled guide. Because the slots are a quarter-wave length apart, the energy in the coupled guide will cancel in one direction & reinforce in the other direction.
- Consider a wave propagating from port 1 to port 2. When the wave passes the slot a energy is radiated into the coupled guide, where it radiates in both directions. The main guide wave continues to propagate toward slot b. Part of the wave couples through slot b into the other guide. As before the coupled wave propagates in both directions in other guide.
- The portion that propagates towards port 4 is in phase with slot a energy and thus reinforces the signal. But the portion that propagates from slot b back towards slot a is phase shifted 180°. Thus the port 3 signals from slots a and b are out of phase by 180° (because seperation ab = $\lambda g/4$) & cancel each other. We can label port 1 the input, port 2 the output, port 3 the isolated port and port 4 coupled port.

Formulation of s-matrix using directional coupler

- Consider a four port, symmetric (reciprocal) network that is matched at all ports.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

If the network is lossless

$$[S]^t [S]^* = [U]$$

$$\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & S_{24}^* \\ S_{13}^* & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & S_{24}^* & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (1)$$

$$|S_{12}|^2 + |S_{23}|^2 + |S_{24}|^2 = 1 \longrightarrow (2)$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{34}|^2 = 1 \longrightarrow (3)$$

$$|S_{14}|^2 + |S_{24}|^2 + |S_{34}|^2 = 1 \longrightarrow (4)$$

$$S_{12} S_{23}^* + S_{14} S_{34}^* = 0 \longrightarrow (5)$$

$$S_{14} S_{12}^* + S_{23}^* S_{34} = 0 \longrightarrow (6)$$

$$(5) \times S_{12}^* - (6) \times S_{34}^* \Rightarrow |S_{12}|^2 S_{23}^* + S_{12}^* S_{14} S_{34}^* - S_{14} S_{12}^* S_{34}^* - S_{23}^* |S_{34}|^2 = 0$$

$$S_{23}^* \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0$$

One way to satisfy this is $S_{23} = 0$. Similarly it can be shown that $S_{14} = 0$. The condition $S_{23} = S_{14} = 0$ results in a directional coupler.

Rewriting ① to ④

$$|S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow (7)$$

$$|S_{12}|^2 + |S_{24}|^2 = 1 \longrightarrow (8)$$

$$|S_{13}|^2 + |S_{34}|^2 = 1 \longrightarrow (9)$$

$$|S_{24}|^2 + |S_{34}|^2 = 1 \longrightarrow (10)$$

$$(7)-(8) \Rightarrow |S_{13}|^2 - |S_{24}|^2 = 0 \Rightarrow |S_{13}| = |S_{24}|$$

$$(7)-(9) \Rightarrow |S_{12}|^2 - |S_{34}|^2 = 0 \Rightarrow |S_{12}| = |S_{34}|$$

Thus the S-matrix is

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{13} \\ S_{13} & 0 & 0 & S_{12} \\ 0 & S_{13} & S_{12} & 0 \end{bmatrix}$$

Let $S_{12} = S_{34} = \alpha$

$S_{13} = \beta e^{j\theta}$

$S_{24} = \beta e^{j\phi}$

$$S_{12} S_{13}^* + S_{24} S_{34}^* = 0$$

$$\alpha\beta e^{-j\theta} + \beta e^{j\phi}\alpha = 0 \Rightarrow \alpha\beta(e^{-j\theta} + e^{j\phi}) = 0$$

$$e^{-j\theta} = -e^{j\phi} = e^{j\pi + j\phi}$$

$$-\theta = \pi + \phi \Rightarrow \theta + \phi = \pi$$

or $\boxed{\theta + \phi = \pi \pm 2n\pi}$

$e^{j\pi} = \cos\pi + j\sin\pi$
$= -1$
$e^{j\pi/2} = j$

Case 1

$\theta = \phi = \pi/2$

$S_{12} = \alpha$

$S_{13} = \beta e^{j\pi/2} = j\beta$

$S_{24} = \beta e^{j\pi/2} = j\beta$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$ ⇒ It is a symmetric coupler.

Case 2

$\theta = 0, \ \phi = \pi$

$S_{12} = \alpha$

$S_{13} = \beta e^{j0} = \beta$

$S_{24} = \beta e^{j\pi} = -\beta$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$ ⇒ It is an antisymmetric coupler.

We know

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$\alpha^2 + \beta^2 = 1$$

- Any reciprocal, lossless, matched 4 port network is a directional coupler.
- Power supplied to port 1 is coupled to port 3 with a coupling factor $|S_{13}|^2 = \beta^2$ while the remaining of the input power is delivered to port 2, the through port with the coefficient $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$. For an ideal directional coupler, no power is delivered to port 4 (isolated port.

Directivity (D)

$$D = 10\log\left(\frac{P_3}{P_4}\right) = 20\log\left(\frac{\beta}{|S_{14}|}\right)$$

Isolation (I)

$$I = 10\log\left(P_1/P_4\right) = -20\log|S_{14}|$$

is the ability of a directional coupler to seperate forward & backward travelling waves

Coupling (C)

$$C = 10\log\left(\frac{P_1}{P_3}\right) = -20\log|S_{13}| = -20\log\beta$$

$$\boxed{I = D + C}$$

An ideal directional coupler has infinite directivity and isolation

Qn A 3dB directional coupler has directivity ~~of~~ 57 dB and isolation 60 dB. Determine the S-matrix. Take port 1 as input port, port 2 as through port & port 3 as coupled port.

$$C = -20\log(\beta) = -20\log(S_{13}) = 3$$

$$\Rightarrow \beta = S_{13} = 10^{-3/20} = 0.707$$

$$I = -20\log(S_{14}) = 60$$

$$\Rightarrow S_{14} = 10^{-60/20} = 10^{-3}$$

$$|S_{12}|^2 = \alpha^2 = 1-\beta^2 = 0.5$$

$$\Rightarrow S_{12} = \sqrt{0.5} = 0.707$$

Qn Calculate the coupling factor of a directional coupler when incident power is 600 mW and power in auxiliary waveguide is 350 μWatt.

$$P_1 = 600 \text{ mWatt} = 600 \times 10^{-3} \text{ Watt}$$
$$P_4 = 350 \text{ μwatt} = 350 \times 10^{-6} \text{ Watt}$$

Coupling factor is given by

$$C = 10 \log_{10}\left(\frac{P_1}{P_4}\right)$$
$$= 10 \log_{10}\left(\frac{600 \times 10^{-3}}{350 \times 10^{-6}}\right)$$
$$= 32.34 \text{ dB}.$$

Qn For a directional coupler the incident power is 550 mwatt. Calculate the power in the main arm and auxiliary arm. The coupling ~~for~~ factor is 30 dB.

$$P_1 = 550 \text{ mwatt} = 550 \times 10^{-3} \text{ Watt}$$
$$C = 30 \text{ dB}$$

Coupling factor

$$C = 10 \log\left(\frac{P_1}{P_4}\right)$$
$$30 = 10 \log\left(\frac{550 \times 10^{-3}}{P_4}\right)$$
$$\frac{550 \times 10^{-3}}{P_4} = 1000 \Rightarrow P_4 = 550 \text{ μWatt}.$$

$\therefore$ Power in auxiliary arm = 550 μwatt

Now power in main arm = Output power &

Input power = output power + auxiliary power

Output power = Input power − Auxiliary Power

$$= P_1 - P_4$$

$$= 550 \times 10^{-3} - 550 \times 10^{-6}$$

$$= 549.45 \text{ milliwatt}$$

Qn Find the directivity in dB for a coupler if same power is applied in turn to input and output of the coupler with output terminated in each case in a matched impedance. The auxiliary output reading are 450 mW and 0.710 μW.

Auxiliary outputs $P_3 = 0.710\ \mu W$

$P_4 = 450\ mW$

$$D = 10 \log \left(\frac{P_4}{P_3} \right)$$

$$D = 10 \log \left(\frac{450 \times 10^{-3}}{0.71 \times 10^{-6}} \right)$$

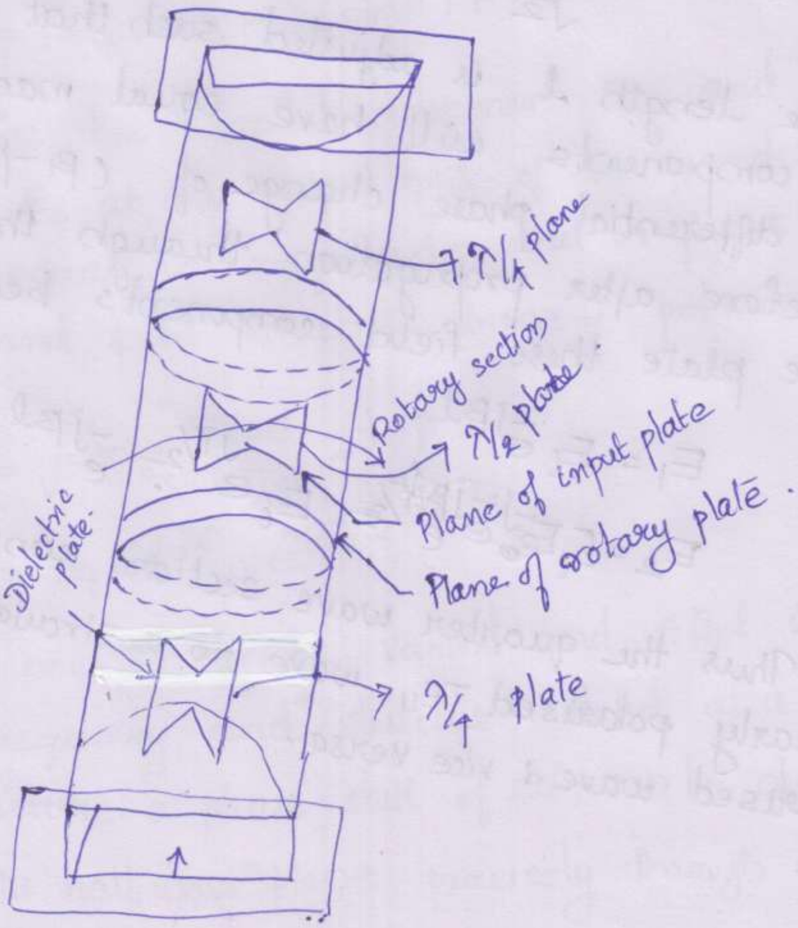
$$= 58 \text{ dB}$$

Phase shifter

A phase shifter produces a phase shift in the transmitted (input) wave which can be adjusted. Ideally it should be perfectly matched to the input + output lines + should produce zero attenuation. Examples of phase shifters include rotary phase shifter, PIN diode phase shifter etc.

Rotary phase shifter

It consists of a section of circular wave guide containing a loss less dielectric plate of length 2l called half wave section. This section can be rotated over 360° precisely between 2 sections of circular to rectangular waveguide transitions each containing loss less dielectric plates of length l called quarter wave sections oriented at an angle 45° wrt the broad wall of the rectangular wave guide ports at the input and output.



The incident TE_{10} wave in the rectangular guide becomes a TE_{11} wave in the circular guide. The half section produces a phase shift equal to twice its rotation angle θ with respect to the quarter wave section. The dielectric plates are tapered through a length of quarter wavelength at both ends for reducing reflection due to discontinuity.

The TE_{11} mode incident field E_i in the input quarter wave section can be decomposed into two transverse component perpendicular to the quarter wave plate. After propagation through the quarter wave plane, these components are:-

$$E_1 = E_i \cos 45^\circ e^{-j\beta_1 l} = E_0 e^{-j\beta_1 l}$$

$$E_2 = E_i \sin 45^\circ e^{-j\beta_2 l} = E_0 e^{-j\beta_2 l}$$

where $E_0 = \frac{E_i}{\sqrt{2}}$

The length l is adjusted such that these two components will have equal magnitude but differential phase change of $(\beta_1 - \beta_2) l = 90^\circ$ Therefore after propagation through the quarter wave plate these field components become

$$E_1 = E_i e^{-j\beta l}$$

$$E_2 = E_i e^{-j\beta l} e^{j\pi/2}$$

Thus the quarter wave sections convert a linearly polarised TE_{11} wave to a circularly polarised wave & vice versa.

After emerging from the half wave section, the field components parallel and perpendicular to the half wave plate can be represented as :

$$E_3 = (E_1 \cos\theta - E_2 \sin\theta)\, e^{-j2\beta_1 l} = E_0 e^{-j\theta} e^{-j3\beta_1 l}$$

$$E_4 = (E_2 \cos\theta - E_1 \sin\theta)\, e^{-j2\beta_2 l} = E_0 e^{-j\theta} e^{-j3\beta_1 l} e^{-j\frac{\pi}{2}}$$

Since $2(\beta_1 - \beta_2) l = \pi$

or

$$-2\beta_2 l = \pi - 2\beta_1 l.$$

After emerging from the half wave section, the field components E_3 and E_4 may again be decomposed into two TE_{11} modes, polarised parallel and perpendicular to the output end of this quarter wave plate, the field components parallel and perpendicular to the quarter wave plate can be written as .

$$E_5 = (E_3 \cos\theta - E_4 \sin\theta)\, e^{-j\beta l} = E_0 e^{-j2\theta} e^{-j4\beta_1 l}$$

$$E_6 = (E_4 \cos\theta - E_3 \sin\theta)\, e^{-j\beta}$$

Therefore the parallel component E_5 and perpendicular component E_6 at the output end of the quarter wave plate are equal in magnitude and in phase to produce a result and field which is linearly polarised TE_{11} wave

$$E_{out} = \sqrt{2}\, E_0 e^{-j2\theta} e^{-j4\beta_1 l}$$

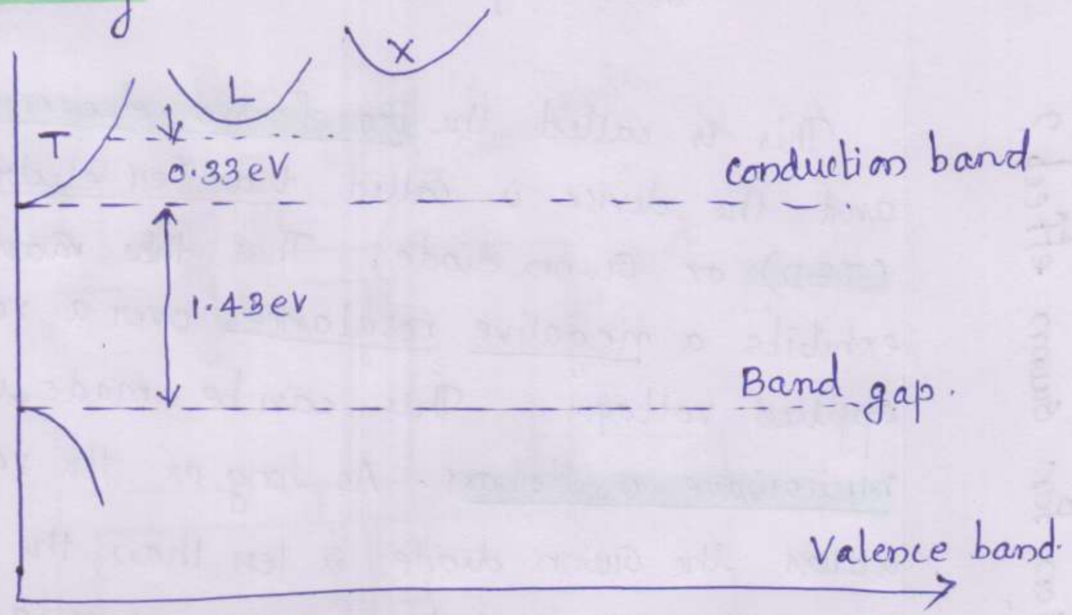
$$= E_1 e^{-j2\theta} \cdot e^{-4\beta_1 l}$$

Since now θ can be varied and $4\beta_1 l$ is fixed at a given frequency and structure fixed at a given frequency and structure a phase shift of 2θ can be obtained by rotating the half wave plane precisely through an angle θ wrt. quarter wave plates.

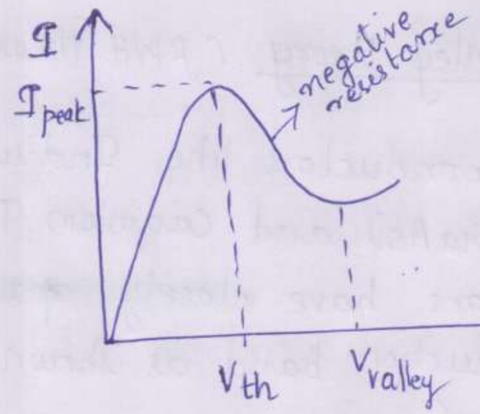
Gunn effect using two valley theory (RWH theory)

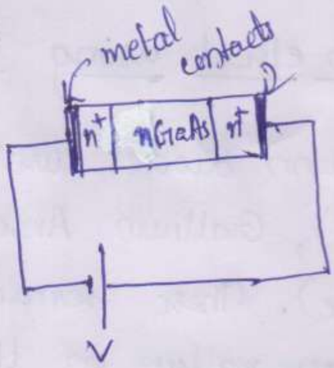
What is transferred electron effect? (3 marks)

Gunn diodes uses semiconductors like Indium Phosphide (InP), Gallium Arsenide (GaAs) and Cadmium Telluride (CdTe). These semiconductors have closely spaced energy valley in the conduction band as shown below:



When a dc voltage is applied across the material an electric field is established across it. With a low E-field in the material, most of the electrons will be located in the lower energy central valley, Γ. With a higher electric field, most of the electrons will be transferred to the high energy L and X valleys. In L and X valleys effective electron mass is larger compared to that in Γ valley. Thus the electron mobility is lower than that in the low energy Γ valley. Since the conductivity is directly proportional to mobility, the conductivity and hence the current decreases with an increase in Electric field or voltage in an intermediate range beyond a threshold voltage V_{th} as shown in figure below:





Write the necessary condition for Gunn effect?

This is called the transferred electron effect and the device is called transfer electron device (TED) or Gunn diode. Thus the Gunn diode exhibits a negative resistance over a range of applied voltages. This can be made use of in microwave oscillators. As long as the voltage across the Gunn diode is less than the critical value, V_{th} the current increases as voltage increases. This is the ohmic region.

When $V > V_{th}$, current starts decreasing with increase in voltage till voltage attains a value called valley voltage (V_{valley}). The region between peak voltage and valley voltage is called negative resistance region.

Necessary conditions for Gunn effect

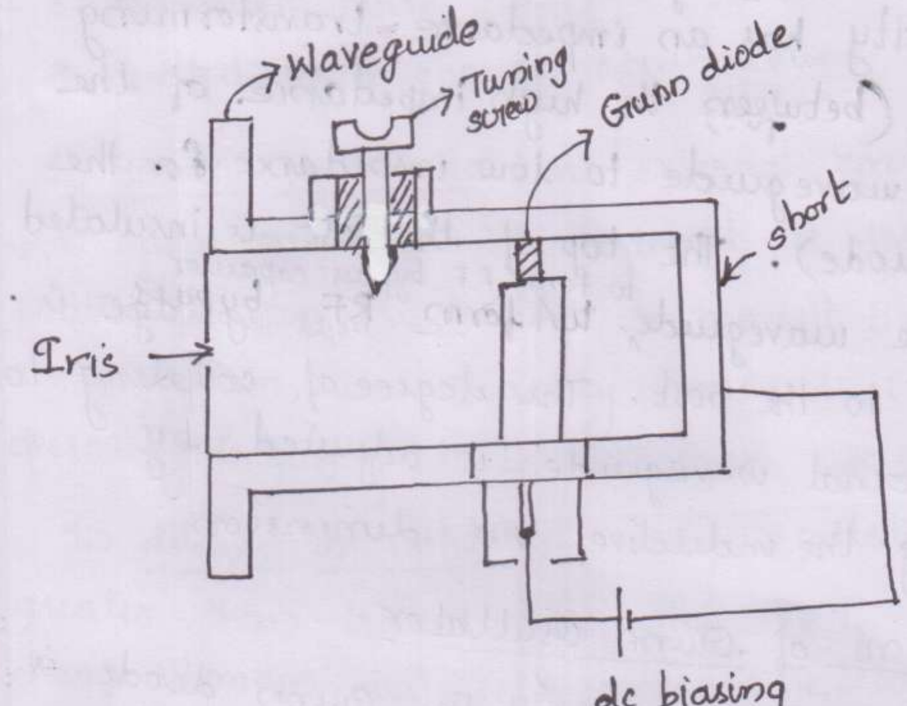
Let m_l, μ_l, n_l be the effective mass, mobility and carrier concentration for lower valley & m_u, μ_u, n_u be respective quantities for upper valley. Then condition for Gunn effect is

(1) $\mu_l > \mu_u$

(2) $\frac{dn_l}{dE}$ should be negative.

Gunn Diode Oscillator

A Gunn diode oscillator can be designed by mounting the Gunn diode inside a waveguide cavity formed by a short circuit termination at one end and by an iris at the other end as shown below



The diode is mounted at the centre perpendicular to the board wall where the electric field component is maximum under the dominant TE_{10} mode. The intrinsic frequency of oscillation depends on the electron drift velocity V_d due to high field through the effective length l

$$f_0 = \frac{V_d}{l}$$

For GaAs diode, $V_d = 10^7$ cm/s. Normally the cavity is turned to resonate at the intrinsic frequency by adjusting the short position. The tuning screw is inserted perpendicul at the center of the broad wall for frequency tuning.

The total resistive loading from the cavity and the external load should be around 20% higher than the Gunn device resistance, R_j so that the circuit resistance $\left(\frac{R_L R_j}{R_L - R_j}\right)$ will be negative.

The cavity has an impedance transforming property (from the high impedance of the output waveguide to low impedance for the Gunn diode). The top of the post is insulated from the waveguide to form RF bypass capacitor. A dc bias voltage is applied to the post. The degree of coupling to the external waveguide is adjusted by selecting the inductive iris dimension

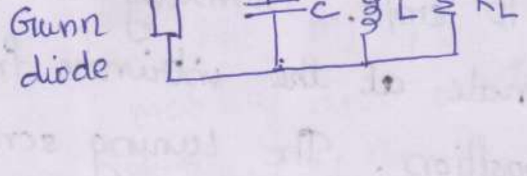
Limitations of Gunn Oscillator

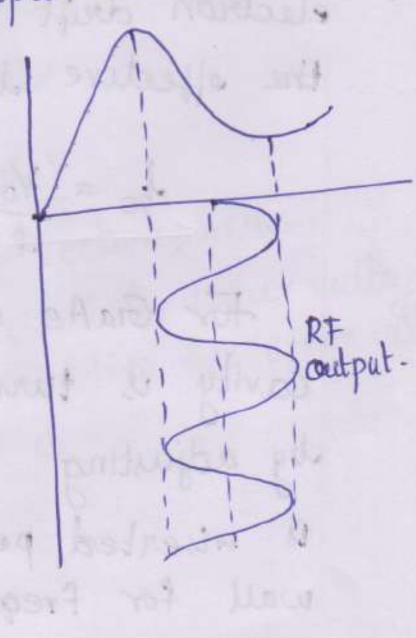
The power output of the Gunn diode is limited (due to heat dissipation)

Advantage

small size, ruggedness, low cost.

Equivalent circuit & resulting output waveform is shown below.





Write different modes of operation of a Gunn diode (3 marks)

Different modes of operation of Gunn diodes are :-

1. Transit time mode
2. Delayed or inhibited mode
3. Quenched mode
4. Limited space charge accumulation mode.

In transit time mode, no external circuit is required for operation. The device length is such that $f_L = 10^7$ cm/s Oscillation period is equal to transit time $T = T_0$. It is a low power, low efficiency mode & it require the operating frequency to be less than 30 GHz.

In delayed or inhibited mode, osullation period is greater than transit time. This $T > T_0$ and thus the charge domain will take more time to move from cathode to anode. Hence the operating frequency is less than that of transit time mode.

In quenched mode, oscillation period is less than transit time $T < T_0$ and thus the charge domain will take less time to cross the length of the device, hence operating frequency is higher than the transit time mode.

In limited space charge accumulation mode, operating frequency is 0.5 to 50 times more than that of transit time mode. This mode gives higher power & higher efficiency.

Qn In an n-type GaAs, Gunn diode has the following parameters: Applied field = 3 kV/cm; device length = 15 μm; and operating at a frequency of 10 GHz. Calculate the negative electron mobility.

The electron drift velocity is

$$V_s = 10 \times 10^9 \times 15 \times 10^{-6}$$
$$= 150 \times 10^3 \text{ m/s}$$
$$= 1.5 \times 10^7 \text{ cm/s}$$

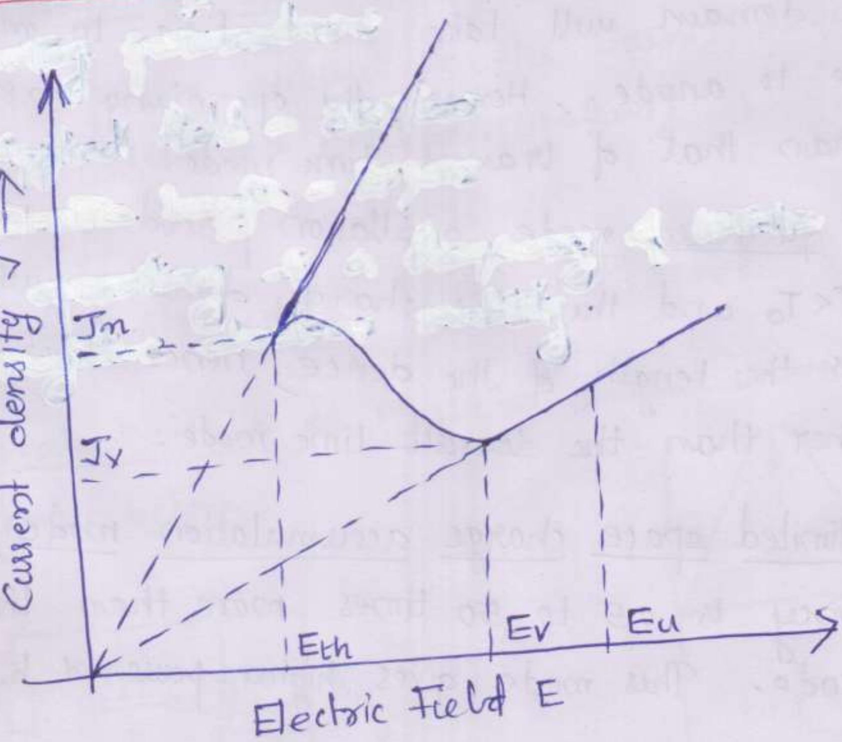
The negative electron mobility is

$$\mu = \frac{-V_s}{E} = \frac{-1.5 \times 10^7}{3 \times 10^3} = -0.5 \times 10^4$$

$$= -5000 \text{ cm}^2/\text{V-sec}$$

Draw JE characteristics of GUNN diode.

Characteristics of Gunn diode



Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Seventh Semester B.Tech Degree Examination December 2022 (2019 scheme)

Course Code: ECT401
Course Name: MICROWAVES AND ANTENNAS

Max. Marks: 100 **Duration: 3 Hours**

PART A

Answer all questions, each carries 3 marks.

		Marks
1	Differentiate between Gain and Directivity of an antenna.	(3)
2	Derive expression for effective aperture of an antenna.	(3)
3	Explain the principle of operation of a Horn Antenna.	(3)
4	Explain the working of an Inverted – F antenna.	(3)
5	Explain the principle of Pattern Multiplication.	(3)
6	Explain the concept of phased arrays.	(3)
7	Derive expressions for the resonant frequency of a rectangular cavity resonator.	(3)
8	Derive expressions for the efficiency of a Reflex Klystron	(3)
9	List the important properties of Scattering parameters.	(3)
10	What do you mean by Gunn Effect?	(3)

PART B

Answer any one full question from each module, each carries 14 marks.

Module I

11	a)	With the help of a neat figure explain about the antenna field zones.	(6)
	b)	Derive expressions for the Radiation Resistance and Directivity of a short dipole antenna.	(8)

OR

12	a)	State and prove Reciprocity Theorem.	(7)
	b)	Derive Helmholtz Equation in terms of Vector Magnetic Potential.	(7)

Module II

13	a)	Explain the axial mode and normal mode of operation of a helical antenna.	(6)
	b)	Design a rectangular patch antenna using a substrate with a dielectric constant of 10.5, h = 0.126 cm so as to resonate at 1.65 Ghz .	(8)

OR

14	a)	Explain the steps involved in the design of a Log Periodic Dipole Array.	(7)

b) With the help of neat sketches explain the working principle of parabolic dish antenna. What are the typical feed antennas used with Dish antennas ? (7)

Module III

15 a) Derive expression for the total field radiated by two isotropic point sources fed with current of same amplitude and phase. Also find the directions of maxima and minima. (7)

b) Derive expressions for the array factor of a linear array of n-isotropic point sources of equal amplitude and spacing. Derive the conditions for using this array as an end fire array. (7)

OR

16 a) Explain the difference between broadside array and end fire array (4)

b) Design a 7 element Dolph-Chebyshev array with a spacing of $d = \lambda/2$. The pattern is to be optimum with a side lobe of 22 db down the main lobe maximum. (10)

Module IV

17 a) With the help of a neat diagram explain the working of a Reflex Klystron. (7)

b) A cylindrical magnetron has the following operating parameters : $V_o = 25KV$, $I_o = 28A$, $B_o = 0.332\ Wb/m^2$, a = 5 cm , b = 10 cm. Find (7)

a) Cutoff voltage for a fixed B_o,

b) Cut of magnetic Flux Density for a fixed V_o

OR

18 a) Derive expressions for the Hull cut off Magnetic Field and Voltage of a magnetron. (7)

b) With diagram explain the amplification process in a travelling wave tube (7)

Module V

19 a) Explain the important properties of Magic Tee. Derive its Scattering parameters (7)

b) Explain the different modes of operation of Gunn Diode. (7)

OR

20 a) Explain the working of two hole directional coupler. Derive its Scattering parameters. (7)

b) With the help of neat sketches explain the working of a circulator. (7)

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Scheme of Valuation/Answer Key

(Scheme of evaluation (marks in brackets) and answers of problems/key)

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SEVENTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2022

Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

Max. Marks: 100		**Duration: 3 Hours**

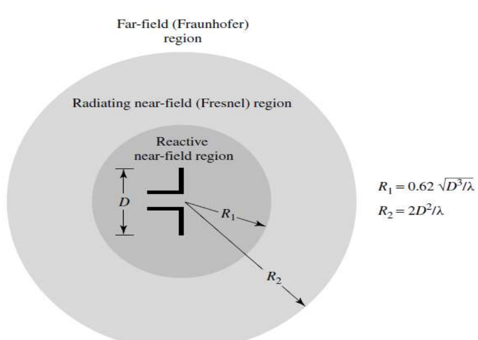
PART A

		Answer all questions, each carries 3 marks.	Marks
1		Gain – 1.5 marks, Directivity – 1.5 marks	(3)
2		Derivation of effective aperture - 3 marks	(3)
3		principle of operation of a Horn Antenna – 3 marks	(3)
4		Working of an Inverted – F antenna – 3 marks	(3)
5		Principle of Pattern Multiplication – 3 marks	(3)
6		Concept of phased arrays – 3 marks	(3)
7		Expressions for the resonant frequency of a rectangular cavity resonator – 3 marks	(3)
8		Expressions for the efficiency of a Reflex Klystron – 3 marks	(3)
9		Three properties of Scattering parameters – 3 marks	(3)
10		Gunn Effect – 3 marks	(3)

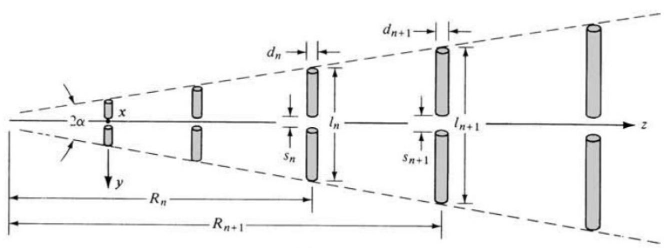
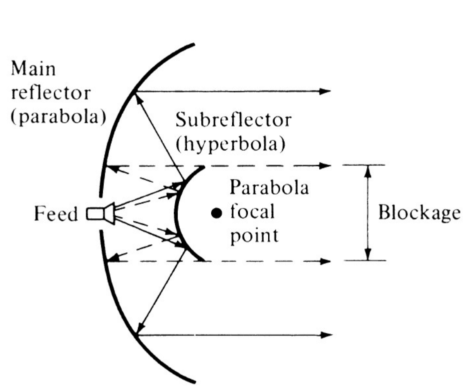
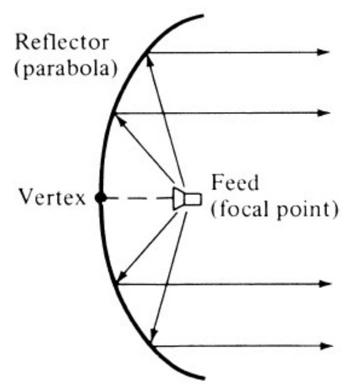
PART B

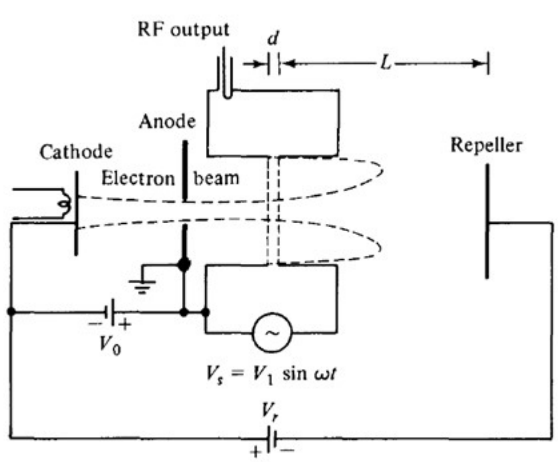
Answer any one full question from each module, each carries 14 marks.

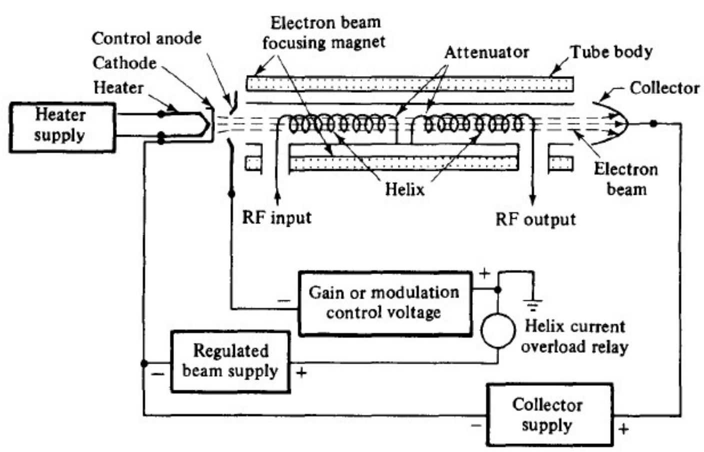
		Module I	
11	a)	Figure -3 marks, Explanation – 3 marks	(6)



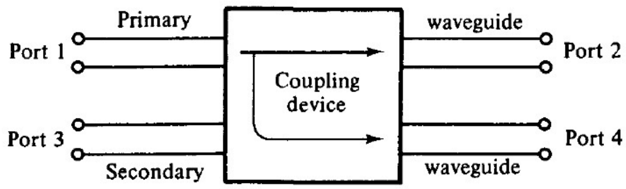
	b)	Derivation of Radiation Resistance – 4 marks, Directivity – 4 marks	(8)
		OR	
12	a)	Statement of Reciprocity Theorem – 2 marks , Derivation – 5 marks	(7)

	b)	Derivation of Helmholtz Equation (Steps + Final Answer) – 7 marks	(7)
		Module II	
13	a)	Axial mode – 3 marks, Normal mode – 3 marks	(6)
	b)	Design Formula + Steps – 6 marks. Final answer – 2 marks Maximum credit can be given to formulae/steps $W = \frac{1}{2f_r\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{2}{\epsilon_r+1}} = \frac{\upsilon_0}{2f_r}\sqrt{\frac{2}{\epsilon_r+1}}$ Width of the Patch (W) – 3.775 cm $\epsilon_{\text{reff}} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+12\frac{h}{W}\right]^{-1/2}$ Effective dielectric constant $\varepsilon_{\text{reff}}$ – 9.313 $\frac{\Delta L}{h} = 0.412\frac{(\epsilon_{\text{reff}}+0.3)\left(\frac{W}{h}+0.264\right)}{(\epsilon_{\text{reff}}-0.258)\left(\frac{W}{h}+0.8\right)}$ $\quad L = \frac{1}{2f_r\sqrt{\epsilon_{\text{reff}}}\sqrt{\mu_0\epsilon_0}} - 2\Delta L$ Extension in Length ΔL – 0.0541 cm Actual Length (L) – 2.871 cm	(8)
		OR	
14	a)	Design steps of Log Periodic Dipole Array – 7 marks  (a) Dipole array	(7)
	b)	Figure - 2 marks, Explanation – 3 marks , Two Feed Antennas – 2 marks  (d) Curved (Cassegrain feed)  (c) Curved (front-fed)	(7)

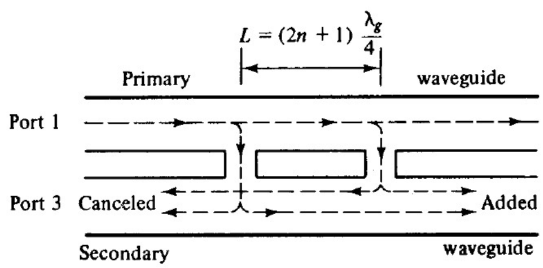
		Module III	
15	a)	Expression for the total field – 4 marks. Directions of maxima and minima – 3 marks	(7)
	b)	Expression for array factor – 5 marks. End fire array condition – 2 marks	(7)
		OR	
16	a)	broadside array + end fire array - 4 marks	(4)
	b)	Design steps of Dolph-Chebyshev array – 8 marks $AF = \sum_{n=1}^{M} w_n \cos[(2n-1)u]$ (even array) $AF = \sum_{n=0}^{M} w_n \cos(2nu)$ (odd array) $u = kd\cos\theta/2$ $(AF)_{10} = z[(a_1-3a_2+5a_3-7a_4+9a_5)/z_0] + z^3[(4a_2-20a_3+56a_4-120a_5)/z_0^3] + z^5[(16a_3-112a_4+432a_5)/z_0^5] + z^7[(64a_4-576a_5)/z_0^7] + z^9[(256a_5)/z_0^9] = 9z-120z^3+432z^5-576z^7+256z^9$ $(AF)_{10} = 2.798\cos(u)+2.496\cos(3u)+1.974\cos(5u)+1.357\cos(7u)+\cos(9u)$ where $u = [(\pi d/\lambda)\cos\theta]$ Final Answer – 2 marks	(10)
		Module IV	
17	a)	Figure of Reflex Klystron -3.5 marks, Explanation – 3.5 marks  	(7)

	b)	Formula/Steps – 5 marks, Final Answer – 2 marks Maximum credit can be given to steps/formula $V_{0c} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$ Cut Off Voltage (Voc) – 13.632 MV $B_{0c} = \frac{\left(8V_0 \frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}$ Cut off Magnetic Field (Boc) – 14.217 mWb/m^2	(7)
		OR	
18	a)	Maximum credit can be given to the derivation steps The *Hull cutoff magnetic equation* is obtained from $$B_{0c} = \frac{\left(8V_0 \frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}$$ This means that if $B_0 > B_{0c}$ for a given V_0, the electrons will not reach the anode. Conversely, the cutoff voltage is given by $$V_{0c} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$ This means that if $V_0 < V_{0c}$ for a given B_0, the electrons will not reach the anode. Equation is often called the *Hull cutoff voltage equation*. Expressions for the Hull cut off Magnetic Field – 3.5 marks Hull cut off Voltage – 3.5 marks	(7)
	b)	Figure of amplification process - 3.5 marks, Explanation – 3.5 marks  	(7)

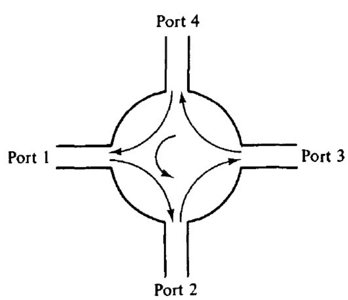
		Module V	
19	a)	Important properties of Magic Tee – 3 marks. Scattering parameters – 4 marks	(7)
	b)	Different modes of operation of Gunn Diode with figure – 7 marks	(7)
		OR	
20	a)	Working of two hole directional coupler – 3 marks. Scattering parameters – 4 marks	(7)



$$\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$



	b)	Figure of circulator - 3.5 marks, Working – 3.5 marks	(7)



$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Seventh Semester B.Tech Degree (S, FE) Examination May 2023 (2019 Scheme)

Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

Max. Marks: 100 **Duration: 3 Hours**

PART A

Answer all questions, each carries 3 marks. Marks

1		Define the term 'Beam solid angle'. Also derive the relation between beam solid angle and directivity	(3)
2		List the different types of antennas based on the radiation pattern	(3)
3		Compare different feeding method of parabolic dish antenna.	(3)
4		Why Log Periodic antenna is called as Frequency Independent antenna, explain?	(3)
5		Explain (i) Pattern Multiplication (ii) Grating lobes	(3)
6		Find the FNBW of linear array of 4 isotropic point sources with n=4, d=$\lambda/2$ and $\delta=0$?	(3)
7		Explain the working of a cavity resonator. Give a practical use of cavity resonator.	(3)
8		With the help of an example , explain the significance of slow wave structures used in microwave circuits	(3)
9		Explain with figure , the working of ferrite isolator	(3)
10		Write a short note on Phase shifter.	(3)

PART B

Answer any one full question from each module, each carries 14 marks.

Module I

11	a)	Derive the expressions for the Radiation resistance and Directivity of a short dipole antenna	(10)
	b)	The radiation resistance of a short dipole of length 0.1 times the wavelength is 8 ohms. Calculate the radiation resistance when the length of dipole is reduced by a factor of ½	(4)

OR

12	a)	A lossless half wave dipole antenna with input impedance of 73Ω is connected to a 50Ω transmission line (Let$U = B_0 sin^3(\theta)$). Find Gain and overall efficiency.	(7)
	b)	State and prove Helmholtz theorem	(7)

Module II

13	a)	Explain the design steps of a rectangular microstrip antenna	(8)
	b)	Distinguish between the normal and axial modes of radiation from a helical Antenna	(6)

OR

14	a)	Explain the working of horn antenna .Write down the expression for directivity, gain and HPBW	(7)
	b)	Explain the working principle of parabolic dish antenna. Write down the expression for directivity, gain and HPBW	(7)

Module III

15	a)	Explain Chebyshev array and write down the procedure for finding the expression array factor	(10)
	b)	Derive the expression for Array factor of an array of odd numbers of isotropic elements is positioned symmetrically along the z-axis. Assume that the non-uniform amplitude excitation is symmetrical about the origin	(4)

OR

16	a)	Design a Broadside Array and plot its radiation pattern.	(7)
	b)	Show that for an array of two isotropic point sources with identical amplitude and phase, have a broadside radiation pattern	(7)

Module IV

17	a)	With the help of neat diagrams explain the working of an magnetron.	(7)
	b)	With neat diagram describe the constructional features and working principle of a Travelling Wave Tube	(7)

OR

18	a)	Show that the axial electric field of TWT varies with convection current.	(7)
	b)	A reflex klystron operates under the following conditions: Cathode voltage , Vo=600V Rsh = l5Kohm Oscillating frequency , fr= 9 GHz , Distance between Rentrant cavity and Repeller , L =1 mm	(7)

Given J(1.832) = 0.582 The tube is oscillating at fr at the peak of the n = 2 mode or 1 3/4 mode. Assume that the transit time through the gap and beam loading can be neglected

(i) Find the value of the repeller voltage Vr.

(ii) Find the direct current necessary to give a microwave gap voltage of 200 V.

(iii) What is the electronic efficiency under this condition?

Module V

19	a)	Explain the features of magic tee and derive its s- matrix	(7)
	b)	With a schematic diagram describe the operation of a four-port circulator. Obtain the S matrix of a perfectly matched, lossless four port circulator	(7)

OR

20	a)	Explain the working of a microwave amplifiers using MESFET	(7)
	b)	Explain the working principle of a 2 hole directional coupler and derive its S matrix	(7)

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DRAFT SCHEME

1000ECT401052301

Scheme of Valuation/Answer Key

(Scheme of evaluation (marks in brackets) and answers of problems/key)

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SEVENTH SEMESTER B.TECH (S) DEGREE EXAMINATION, MAY 2023

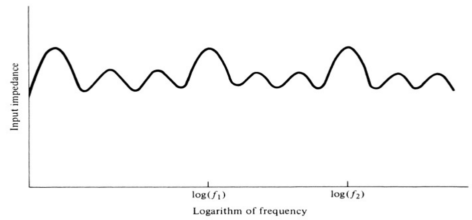
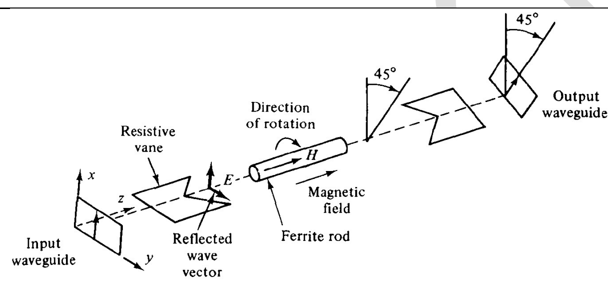
Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

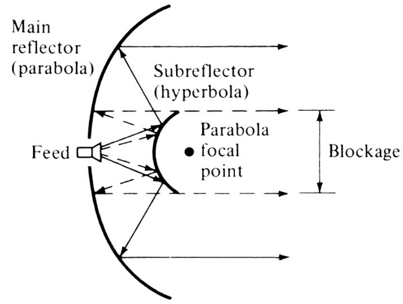
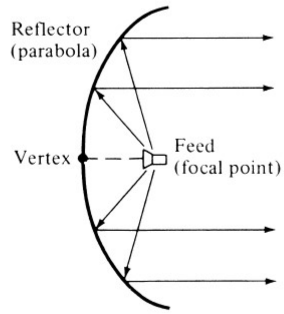
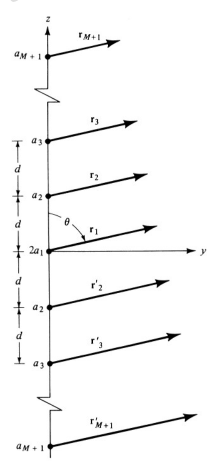
Max. Marks: 100		Duration: 3 Hours

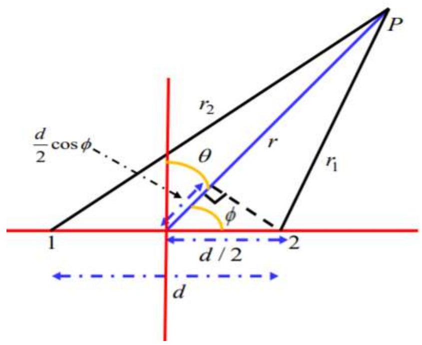
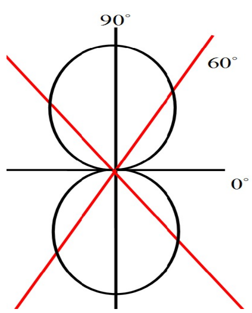
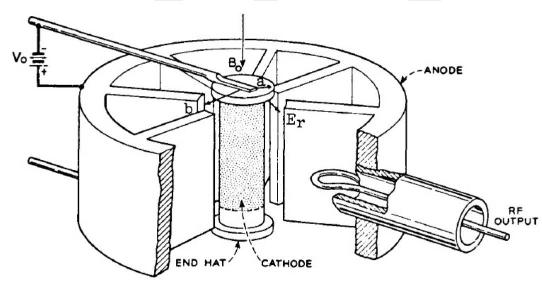
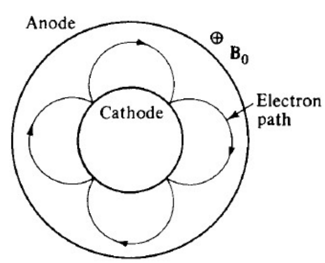
PART A

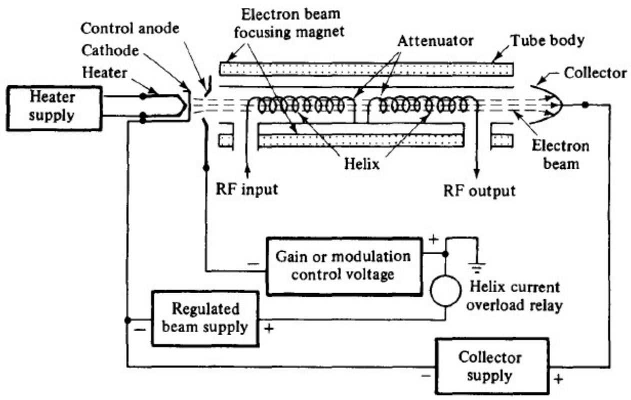
		Answer all questions, each carries 3 marks.	Marks
1		Definition of Beam solid angle-2 marks The beam solid angle Ω_A is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within Ω_A (Relation-1 mark) $\boxed{D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}}}$	(3)
2		An **Isotropic radiator** is defined as a hypothetical lossless antenna having equal radiation in all directions.---------1 mark A **directional antenna** : having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others -----------1 mark An **Omnidirectional antenna** : having an essentially nondirectional pattern in a given plane (in this case in azimuth) and a directional pattern in any orthogonal plane (in this case in elevation). An omnidirectional pattern is then a special type of a directional pattern---1 mark	(3)
3		Cassegrain feed-1.5 mark Front fed-1.5 mark	(3)
4		Log periodic antenna is called frequency independent antenna(Reason)-3 marks Repetitive variation observed in the graph of input impedance and frequency for the log periodic antenna	(3)

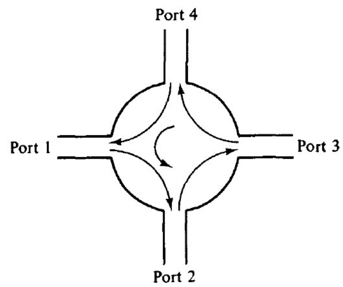
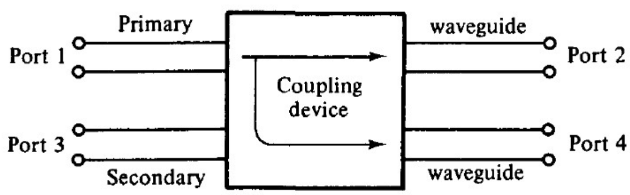
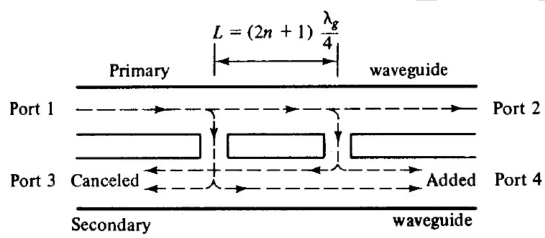
			
5		Pattern Multiplication-1.5 Marks The field pattern of an array of Non isotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources having same relative amplitude and phase (Grating lobes-1.5 marks)	(3)
6		Steps-2 marks Answer-1 mark FNBW=2*60=120^0	(3)
7		Explanation - 2 mark, Any Practical use - 1 mark Eg. Frequency meter	(3)
8		Significance of slow wave structure - 2 mark, Any Example - 1 mark Eg. Traveling wave tube	(3)
9		 Figure - 1.5 mark, Explanation - 1.5 mark	(3)
10		Any Three points=3*1= 3 marks	(3)
PART B			
Answer one full question from each module, each carries 14 marks.			
		Module I	
11	a)	Derivation the Radiation resistance-5 marks Derivation of Directivity -5 marks	(10)
	b)	Steps- 2marks, Answer-2 marks The radiation resistance of a short dipole of length 0.1 times the wavelength is 8ohms. The radiation resistance is proportional to the length normalized to the wavelength. So, radiation resistance when the length of dipole is reduced by a factor of ½ is equal to = 1⁄4 × 8 = 2 Ohms.	(4)
		OR	

12	a)	Steps-4 marks, Answers- 3 marks, Gain =16/3π, Overall efficiency=0.965	(7)
	b)	Helmholtz theorem statement-2 marks Proof- 5 marks	(7)
		Module II	
13	a)	$W = \frac{1}{2f_r\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{2}{\epsilon_r+1}} = \frac{v_0}{2f_r}\sqrt{\frac{2}{\epsilon_r+1}}$ -2 marks $\epsilon_{\text{reff}} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+12\frac{h}{W}\right]^{-1/2}$ -2 marks $\frac{\Delta L}{h} = 0.412\frac{(\epsilon_{\text{reff}}+0.3)\left(\frac{W}{h}+0.264\right)}{(\epsilon_{\text{reff}}-0.258)\left(\frac{W}{h}+0.8\right)}$ -2 marks $L = \frac{1}{2f_r\sqrt{\epsilon_{\text{reff}}}\sqrt{\mu_0\epsilon_0}} - 2\Delta L$ -2 marks	(8)
	b)	Normal mode of radiation from a helical Antenna-3 marks Axial mode of radiation from a helical Antenna-3 marks	(6)
		OR	
14	a)	working of horn antenna-5 marks Expression for directivity, gain and HPBW-2 marks	(7)
	b)	working principle of parabolic dish antenna-4 marks expression for directivity, gain and HPBW-3 marks -If a beam of parallel rays is incident upon a reflector whose geometrical shape is a parabola, the radiation will converge (focus) at a spot which is known as the *focal point* -In the same manner, if a point source is placed at the focal point, the rays reflected by a parabolic reflector will emerge as a parallel beam. -Rays that emerge in a parallel formation are usually said to be *collimated*	(7)

		 (d) Curved (Cassegrain feed)  (c) Curved (front-fed)	
		Module III	
15	a)	$$AF=\sum_{n=1}^{M} w_n \cos[(2n-1)u] \quad \text{(even array)}$$ $$AF=\sum_{n=0}^{M} w_n \cos(2nu) \quad \text{(odd array)}$$ $$u=kd\cos\theta/2$$ $$k=\beta=\frac{2\pi}{\lambda}$$ w_n are the current amplitudes M = even array factor expression- 4marks M = odd – 4marks Explanation 2 marks	(10)
	b)	Derivation and equation-4 marks 	(4)
		OR	
16	a)	Design-5 marks Plot of pattern-2 marks	(7)

	b)	Array factor derivation-4 marks  $$\mathrm{En} = \cos\left(\frac{\beta\frac{d}{2} * \cos\emptyset}{2}\right)$$ Pattern-3 marks 	(7)
		Module IV	
17	a)	  Diagram - 3 mark, Expatiation -4 mark	(7)

	b)	 Diagram - 3 mark Explanation - 4 mark	(7)
		OR	
18	a)	EXPLANATION AND PROOF-7 MARKS	(7)
	b)	$\frac{V_0}{(V_r + V_0)^2} = \left(\frac{e}{m}\right)\frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$ $= (1.759 \times 10^{11})\frac{(2\pi 2 - \pi/2)^2}{8(2\pi \times 9 \times 10^9)^2(10^{-3})^2} = 0.832 \times 10^{-3}$ $(V_r + V_0)^2 = \frac{600}{0.832 \times 10^{-3}} = 0.721 \times 10^6$ $V_r = 250$ V Assume that $\beta_0 = 1$. Since $V_2 = I_2 R_{sh} = 2I_0 J_1(X')R_{sh}$ the direct current I_0 is $I_0 = \frac{V_2}{2J_1(X')R_{sh}} = \frac{200}{2(0.582)(15 \times 10^3)} = 11.45$ mA $\text{Efficiency} = \frac{2X' J_1(X')}{2\pi n - \pi/2} = \frac{2(1.841)(0.582)}{2\pi(2) - \pi/2} = 19.49\%$ (i) Repeller voltage Vr . - 3 mark (ii) Direct current necessary to give a microwave gap voltage of 200 V - 2 mark (iii) Electronic efficiency under this condition - 2 mark	(7)
		Module V	
19	a)	Features-4 marks, S- matrix 3 marks	(7)

	b)	 $$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ Diagram - 2 mark Explanation - 3 mark, S matrix - 2 mark	(7)
		OR	
20	a)	Working of a microwave amplifiers using MESFET -7 marks	(7)
	b)	  $$\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$ Diagram - 2 mark Explantion -2mark Derivation of S matrix - 2 mark S matrx of Directional coupler - 1 mark	(7)
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Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S7 (R, S) / S7 (PT) (R) Examination December 2023 (2019 Scheme)

Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

Max. Marks: 100 **Duration: 3 Hours**

PART A

Answer all questions, each carries 3 marks. Marks

1		Define the term Beam Area of an antenna. Calculate the beam area of a field pattern given by $E(\theta)=\cos^2\theta$ for $0° \leq \theta \leq 90°$	(3)
2		Illustrate and describe the field zones of an antenna.	(3)
3		Outline the principles of a mobile phone antenna with neat diagram.	(3)
4		With suitable figures, explain any two feeding methods of Micro strip patch antenna.	(3)
5		Derive the Array factor of a two element array formed by two infinitesimal dipoles.	(3)
6		What are grating lobes? How can they be minimised?	(3)
7		Derive the equation for Cyclotron angular frequency of cylindrical magnetron.	(3)
8		Derive the equation for resonant frequency of Rectangular cavity resonator. Compute the resonant frequency of the dominant mode for an air-filled cavity of dimensions a = 5 cm, b = 2 cm, and d =15 cm.	(3)
9		What is a circulator? Write the S matrix of a perfectly matched lossless 4 port circulator.	(3)
10		Draw the J-E characteristics of a Gunn diode and explain the different regions in it.	(3)

PART B

Answer any one full question from each module, each carries 14 marks.

Module I

11	a)	State and prove Reciprocity theorem	(5)
	b)	Derive the expressions for Radiation resistance and directivity of short dipole	(9)

OR

12 a) Define the term Effective aperture. Derive the expression for the Effective aperture of antenna.
Calculate the effective aperture of an antenna which is operating at a frequency of 2GHz. (9)

b) Differentiate between Antenna efficiency and Beam efficiency. (5)

Module II

13 a) With a neat figure explain the working of Log Periodic dipole array. Write all the design equations. (7)

b) Differentiate between the Normal mode and Axial mode of a Helical antenna. Write the expressions for HPBW, BWFN and Directivity in axial mode. (7)

OR

14 a) Design a rectangular microstrip patch antenna using a substrate with dielectric constant of 2.2, h =0.1588 cm so as to resonate at 10GHz. (10)

b) Explain the principle of a Cassegrain antenna with a neat figure. (4)

Module III

15 a) What is meant by a phased array? (4)

b) Design a 4 element Dolph Chebyscheff array with $\lambda/2$ spacing between elements. The pattern is to be optimum with the side lobe level 19.1 db down the main lobe maximum. (10)

OR

16 a) Differentiate between broadside and endfire arrays. (4)

b) Derive the expression for the total field radiated by linear array of N isotropic point sources and write the expression for Array factor. (10)

Module IV

17 a) Explain the bunching process in Reflex Klystron with Applegate diagram. (7)

b) A traveling-wave tube (TWT) operates under the following parameters: (7)
Beam voltage: V_o= 3 kV Beam current: l_o = 30 mA, Characteristic impedance of helix: Zo=100Ω Circuit length: N = 50, Frequency: f = 10 GHz. Calculate (a) the gain parameter C; (b) the output power gain A_p in decibels

OR

18 a) With the help of neat sketches, explain the working of an 8 cavity Cylindrical Magnetron. (7)

	b)	Show that the magnitude of the velocity fluctuation of the electron beam is directly proportional to the magnitude of the axial electric field in a helix TWT.	(7)

Module V

19	a)	With a schematic diagram explain the constructional features of a two hole directional coupler and derive the S matrix.	(8)
	b)	With neat figure, describe the two valley model theory of semiconductors.	(6)

OR

20	a)	With neat figure, explain the features of a Hybrid ring T. Obtain the S matrix.	(8)
	b)	Describe the principle of operation of amplifiers using MESFET	(6)

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Scheme of Valuation/Answer Key

(Scheme of evaluation (marks in brackets) and answers of problems/key)

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S7 (R, S) / S7 (PT) (R) Examination December 2023 (2019 Scheme)

Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

Max. Marks: 100		Duration: 3 Hours

PART A

		Answer all questions, each carries 3 marks.	Marks
1		**The beam area or beam solid angle or Ω_A of an antenna is given by the integral of the normalized power pattern over a sphere 4 π sr. (1mark)** Ans: ***Beam area*** $\Omega_A = \theta_{HP}\Phi_{HP}$ **sr=1.26sr=4356sq.deg (2 marks)** **Note: Since Beam area is related to beamwidth, it cannot be fully considered to be out of syllabus. Hence if the question has been attempted or if beamwidth has been defined, 2 marks may be awarded.**	**(3)**
2		**Diagram – 1marks** **Explanation – 2 marks (Fresnel Zone , Fraunhoffer zone equations must be written)**	**(3)**
3		**Diagram - 1 mark** **Principle - 2 marks**	**(3)**
4		**Diagram of any two feeding methods (Microstrip line, Coaxial probe, Aperture coupling, Proximity coupling) – 1 mark** **Explanation – 2 marks**	**(3)**
5		$\mathrm{AF} = 2\cos[\frac{1}{2}(kd\cos\theta + \beta)]$ **– 1 mark** **Derivation with figure – 2 marks** **(Isotropic point source array of two elements may also be given 3 marks.)**	**(3)**
6		**Principal maxima that occur in other undesired directions,** **Explanation – 2 marks** **Minimisation by adjusting spacing between elements ($d<\lambda/2$ for endfire array and $d_{max}<\lambda$ for broadside array) - 1 mark**	**(3)**
7		$\omega_c = eB/m$ **Correct expression -1 mark** **Derivation – 2 marks**	**(3)**

8		The separation equation for both TE and TM modes is given by $$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$ For a lossless dielectric, $k^2 = \omega^2\mu\epsilon$; therefore, the resonant frequency is expressed by $$f_r = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \quad (TE_{mnp}, TM_{mnp})$$ $$f_r = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$ **Correct expression -1 mark** **Resonant frequency derivation (with above two steps sufficient) – 1 mark** **Answer: 3.16 GHz- 1 mark**	**(3)**
9		**Circulator definition-– multiport waveguide junction in which the wave can flow only from the nth port to the (n + l)th port in one direction- 1 mark** **Explanation – 1 mark** **S matrix -1 mark** $$S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$	**(3)**
10		**J-E (I-V) characteristics graph-2 marks** **Explanation - 1 mark**	**(3)**
		PART B	
		Answer one full question from each module, each carries 14 marks.	
		Module I	
11	**a)**	**Statement – 2 marks** **Proof- 3 marks**	**(5)**
	b)	**Directivity of short dipole- 1.5 Derivation - 4 marks** **Radiation resistance – 80 $\pi^2(dl/\lambda)^2$ Derivation - 4.marks** **Figure -1 mark**	**(9)**
		OR	
12	**a)**	**Definition – 3 marks** **Derivation – 4 marks** **$A_{em} = D\lambda^2 / 4\pi$ - 2 marks** **Note: If λ has been calculated and A_{em} has been written in terms of D, 2 marks may be given.**	**(9)**
	b)	**Beam efficiency is the Ratio of Main beam area to the total beam area** **Total antenna efficiency is used to take into account losses at the input terminals and within the structure of the antenna.** **Definitions-2marks**	**(5)**

		Equations and explanation-3 marks	
		Module II	
13	**a)**	**Diagram - 2.5 marks** **Explanation - 2.5 marks** **Design equations - 2 marks**	**(7)**
	b)	**Explanation of each mode- 4 marks** **Design equations – 3 marks** **HPBW = 52/C $(\lambda^3/NS)^{1/2}$** **BWFN = 115/C$(\lambda^3/NS)^{1/2}$** **Directivity = 15NSC^2/ λ^3**	**(7)**
		OR	
14	**a)**	$W = \frac{1}{2f_r\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{2}{\epsilon_r+1}} = \frac{\upsilon_0}{2f_r}\sqrt{\frac{2}{\epsilon_r+1}}$ $\epsilon_{\text{reff}} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+12\frac{h}{W}\right]^{-1/2}$ $\frac{\Delta L}{h} = 0.412\frac{(\epsilon_{\text{reff}}+0.3)\left(\frac{W}{h}+0.264\right)}{(\epsilon_{\text{reff}}-0.258)\left(\frac{W}{h}+0.8\right)}$ $L = \frac{1}{2f_r\sqrt{\epsilon_{\text{reff}}}\sqrt{\mu_0\epsilon_0}} - 2\Delta L$ **L_{eff} = L + 2 ΔL** **Length of ground plane , L_g = 6h + L** **Width of ground plane W_g= 6h + W** **Substituting ,** **W = 1.186 cm - 2 mark** **ε_{reff} = 1.97 - 1 mark** **ΔL = 0.081 cm - 1 mark** **L = 0.906 cm, L_{eff} = 1.068 cm - 2 marks** **L_g = 1.858 cm , W_g= 2.138 cm - 2 marks** **Diagram of patch antenna 2 mark**	**(10)**
	b)	**Diagram – 2 marks** **Principle- 2 marks**	**(4)**
		Module III	

15	**a)**	**Definition of phased array -2marks** **Principle- 2 marks**	**(4)**
	b)	**Design of Chebysheff array- 8 marks** **Final answer – 2 marks** **Answer:**$(AF)_4=5.625\cos\{(\pi/2)\cos\theta\}+3.375\cos\{(3\pi/2)\cos\theta\}$	**(10)**
		OR	
16	**a)**	**Any 4 points of difference -4 marks**	**(4)**
	b)	**Derivation of Equation for total field – 7 marks** **Array factor expression – 3 marks** $\boxed{(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]}$	**(10)**
		Module IV	
17	**a)**	**Reflex Klystron diagram- 2 marks** **Explanation of bunching process-3 marks** **Applegate diagram – 2 marks**	**(7)**
	b)	**a.** $C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3} = \left(\frac{30 \times 10^{-3} \times 10}{4 \times 3 \times 10^3}\right)^{1/3} = 2.92 \times 10^{-2}$ **b.** $A_p = -9.54 + 4.73\,NC = -9.54 + 47.3 \times 50 \times 2.92 \times 10^{-2} = 59.52$ dB **Correct equations -4 marks** **Final answers-3 marks**	**(7)**
		OR	
18	**a)**	**Diagram of Magnetron – 3marks** **Working –4 marks**	**(7)**
	b)	**Derivation of the relation– 7 marks**	**(7)**
		Module V	
19	**a)**	**Diagram – 4 marks** **S matrix- 4 marks**	**(8)**
	b)	**Diagram – 3 marks** **Explanation of theory – 3 marks**	**(6)**
		OR	
20	**a)**	**Hybrid ring T diagram – 2 marks** **Features- 2 marks** **S Matrix derivation- 4 marks**	**(8)**
	b)	**Diagram - 3 marks** **Principle of operation – 3 marks**	**(6)**
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Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S7 (S, FE) / S7 (PT) (S) Examination May 2024 (2019 Scheme)

Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

Max. Marks: 100 **Duration: 3 Hours**

PART A

Answer all questions, each carries 3 marks. Marks

1		Explain different types of Radiation patterns	(3)
2		Find HPBW and FNBW of an antenna which has a field given by, $E(\theta) = cos^2(\theta) \quad for\ 0 \leq \theta \leq 90°$	(3)
3		With the help of diagrams, compare different feeding method of parabolic dish antenna.	(3)
4		List major advantages and drawbacks of the Microstrip patch antennas.	(3)
5		What is the principle of pattern multiplication?	(3)
6		Find the FNBW of linear array of 4 isotropic point sources with n=4, d=λ/2 and δ= -π ?	(3)
7		Explain the working of a cavity resonator. Give a practical use of cavity resonator.	(3)
8		With the help of an example , explain the significance of slow wave structures used in microwave circuits	(3)
9		Explain with figure , the working of ferrite isolator	(3)
10		Explain Scattering parameters for an N port network.	(3)

PART B

Answer any one full question from each module, each carries 14 marks.

Module I

11	a)	The maximum radiation intensity of a 90% efficient antenna is "200" mW/unit solid angle. Find directivity and gain when (i). Input power is 125.66 mW (ii). Radiated power is 125.66 mW	(8)
	b)	Explain the terms (i) Retarded potential (ii) Antenna field Zones	(6)

OR

12 a) Derive the expressions for the Radiation resistance and Directivity of a half wave dipole antenna (10)

b) The radiation resistance of a short dipole of length 0.1 times the wavelength is 8 ohms. Calculate the radiation resistance when the length of dipole is reduced by a factor of ½. (4)

Module II

13 a) Why log periodic antenna is called frequency independent antenna? Explain the working of log periodic dipole array. (6)

b) Design a rectangular micro strip antenna using dielectric substrate with dielectric constant of 2.2, h=0.1588 c.m so as to resonate at 10 GHz (8)

OR

14 a) Explain the working of horn antenna .Write down the expression for directivity, gain and HPBW (8)

b) Briefly explain about Inverted F antenna. Also list the applications (6)

Module III

15 a) Show that for an array of two isotropic point sources with identical amplitude and phase, have a broadside radiation pattern (7)

b) Design an Endfire array and plot its radiation pattern (7)

OR

16 a) Design a broadside Dolph-Tschebyscheff array of 10 elements with spacing d between elements. The side lobes are 26 dB below the maximum of the major lobe. Find the excitation coefficients and form the array factor (14)

Module IV

17 a) With neat diagram describe the constructional features and working principle of a Travelling Wave Tube (7)

b) A reflex klystron operates under the following conditions: (7)

Cathode voltage , Vo=600V

Rsh = l5Kohm

Oscillating frequency , fr= 9 GHz ,

Distance between Rentrant cavity and Repeller , L =1 mm

Given J(1.832) = 0.582

The tube is oscillating at fr at the peak of the n = 2 mode or 1 3/4 mode

Assume that the transit time through the gap and beam loading can be neglected

(i) Find the value of the repeller voltage Vr .

(ii) Find the direct current necessary to give a microwave gap voltage of 200 V.

(iii) What is the electronic efficiency under this condition?

OR

18	a)	With neat diagram, describe the constructional features and working principle of a Reflex Klystron	(7)
	b)	With the help of neat diagrams explain the working of a magnetron.	(7)

Module V

19	a)	Explain Gunn effect with the help of Ridley–Watkins–Hilsum theory.	(7)
	b)	Explain the working principle of a 2 hole directional coupler and derive its S matrix	(7)

OR

20	a)	Discuss the constructional features of E plane tee and derive its S Matrix.	(7)
	b)	With a schematic diagram describe the operation of a four-port circulator. Obtain the S matrix of a perfectly matched, lossless four port circulator	(7)

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FINAL Scheme of Valuation/Answer Key

(Scheme of evaluation (marks in brackets) and answers of problems/key)

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

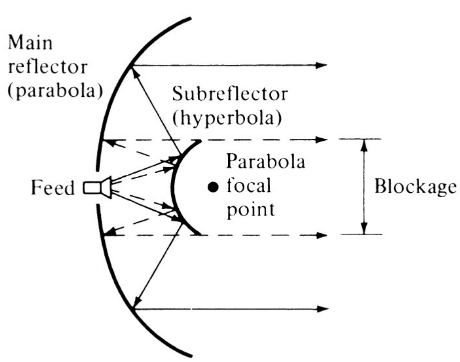
SEVENTH SEMESTER B. TECH(S,FE) DEGREE EXAMINATION, MAY 2024 (2019 Scheme)

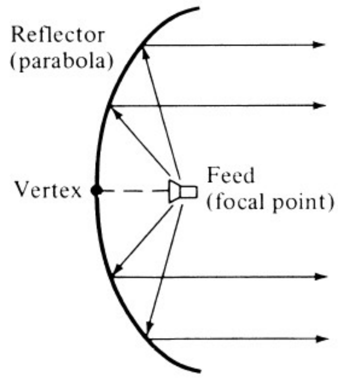
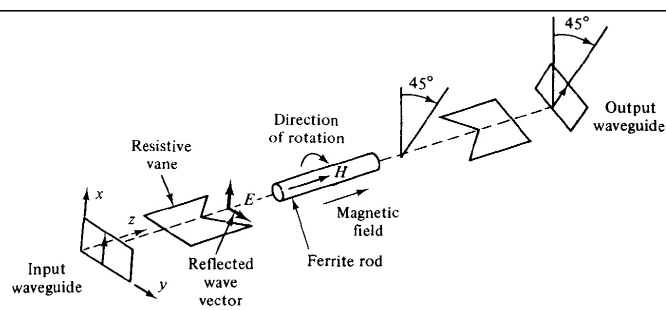
Course Code: ECT401

Course Name: MICROWAVES AND ANTENNAS

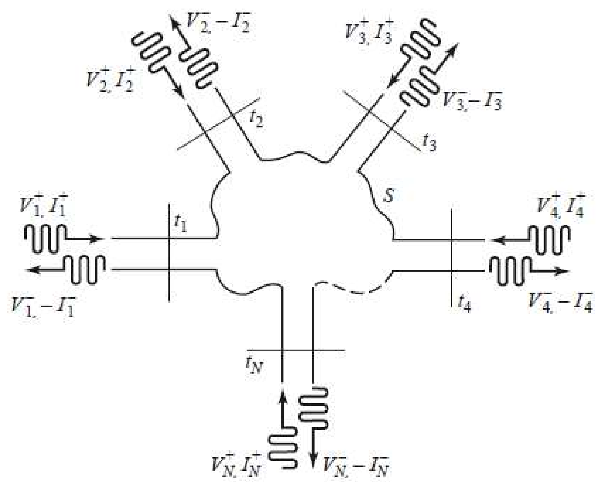
Max. Marks: 100		Duration: 3 Hours

PART A

		Answer all questions, each carries 3 marks.	Marks
1		Isotropic- 1 mark Directional-1 mark Omnidirectional-1 mark An **Isotropic radiator** is defined as a hypothetical lossless antenna having equal radiation in all directions. A **directional antenna** : having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others An **Omnidirectional antenna**: having an essentially non directional pattern in a given plane (in this case in azimuth) and a directional pattern in any orthogonal plane (in this case in elevation). An omnidirectional pattern is then a special type of a directional pattern	(3)
2		HPBW~66^0 OR 1.14 rad -1.5 mark FNBW-2*90=180^0-1.5 Mark	(3)
3		Cassegrain feed-1.5 mark  (d) Curved (Cassegrain feed) Front fed-1.5 mark	(3)

		 (c) Curved (front-fed)	
4		Advantages-2 marks Draw backs-1 mark	(3)
5		Pattern multiplication definition-3 marks The field pattern of an array of Non isotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources having same relative amplitude and phase	(3)
6		Null direction equation – 1 mark Final answer-2 marks FNBW~$2*60^0=120^0$	(3)
7		Explanation - 2 mark Any Practical use - 1 mark Eg. Frequency meter	(3)
8		Significance of slow wave structure - 2 mark Any Example - 1 mark Eg. Traveling wave tube	(3)
9		 Figure - 1.5 mark Explanation - 1.5 mark	(3)

10			(3)



$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix},$$

$$[V^-] = [S][V^+].$$

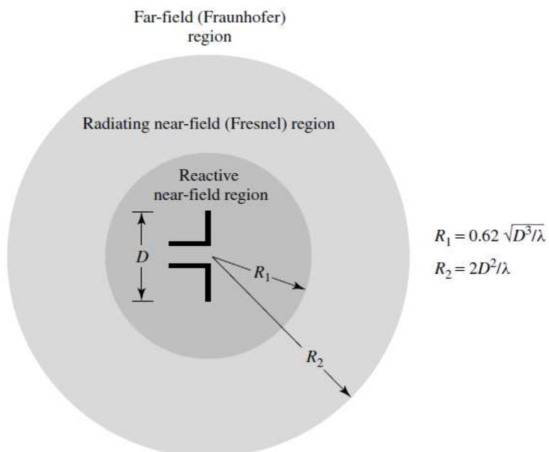
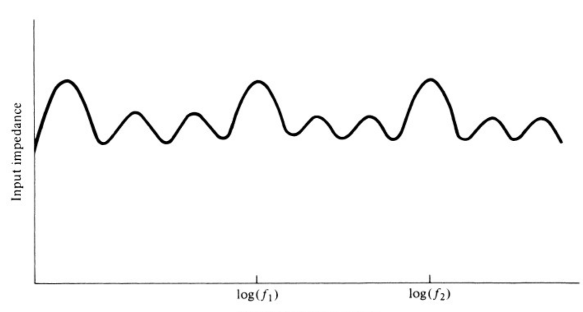
Explanation - 2 mark

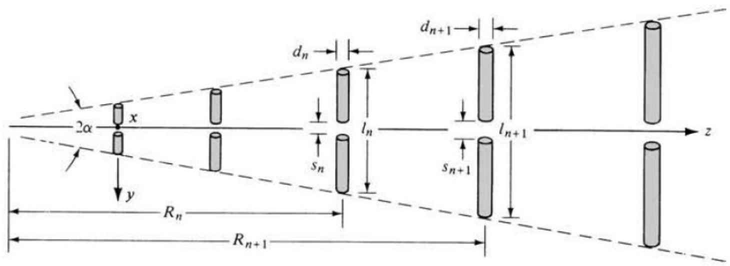
Equation - 1 mark

PART B

Answer one full question from each module, each carries 14 marks.

		Module I	
11	a)	(i) Directivity equation–(4πU/Prad,Prad=Pin*efficiency) -1 mark, Answer(D=22.22)-1mark Gain- 2 marks(G=efficiency*D, G=20.00) (ii) Directivity equation–(4πU/Prad) -1 mark, Answer(D=20)-1mark	(8)

		Gain- 2 marks(G=efficiency*D, G=18.00)	
	b)	(i) Retarded potential-equation explanation-3 marks (ii) Antenna field Zones-3 marks 	(6)
		OR	
12	a)	Radiation resistance derivation steps -4 Marks and answer -2 marks Radiation resistance=73 ohm. Directivity derivation steps -3 Marks and answer -1 mark Directivity=1.64.	(10)
	b)	Steps-2 marks, Answer=2 marks The radiation resistance of a short dipole of length 0.1 times the wavelength is 8 ohms. The radiation resistance is proportional to the length normalized to the Wave length. So, radiation resistance when the length of dipole is reduced by a factor of ½ is equal to = 1⁄4 × 8 = 2 Ohms	(4)
		Module II	
13	a)	log periodic antenna is called frequency independent antenna(Reason)-2 marks Repetitive variation observed in the graph of input impedance and frequency for the log periodic antenna  Explanation with figure -4 marks	(6)



(a) Dipole array

b) (8)

$$W = \frac{1}{2f_r\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{2}{\epsilon_r+1}} = \frac{\upsilon_0}{2f_r}\sqrt{\frac{2}{\epsilon_r+1}}$$

$$W = \frac{30}{2(10)}\sqrt{\frac{2}{2.2+1}} = 1.186 \text{ cm}$$

-2 marks

$$\epsilon_{\text{reff}} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+12\frac{h}{W}\right]^{-1/2}$$

$$\epsilon_{\text{reff}} = \frac{2.2+1}{2} + \frac{2.2-1}{2}\left(1+12\frac{0.1588}{1.186}\right)^{-1/2} = 1.972$$

-2 marks

$$\frac{\Delta L}{h} = 0.412\frac{(\epsilon_{\text{reff}}+0.3)\left(\frac{W}{h}+0.264\right)}{(\epsilon_{\text{reff}}-0.258)\left(\frac{W}{h}+0.8\right)}$$

$$\Delta L = 0.1588(0.412)\frac{(1.972+0.3)\left(\frac{1.186}{0.1588}+0.264\right)}{(1.972-0.258)\left(\frac{1.186}{0.1588}+0.8\right)}$$

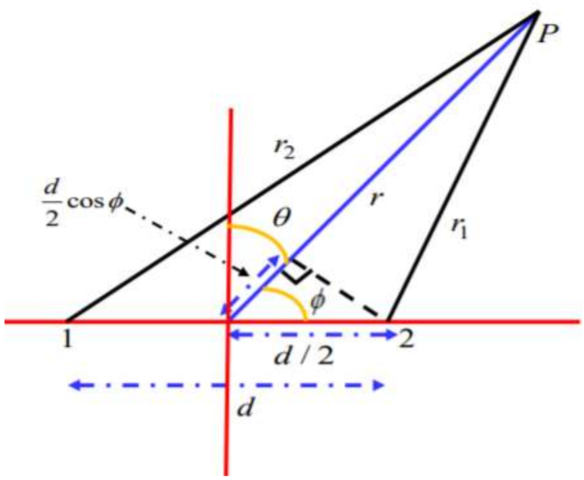
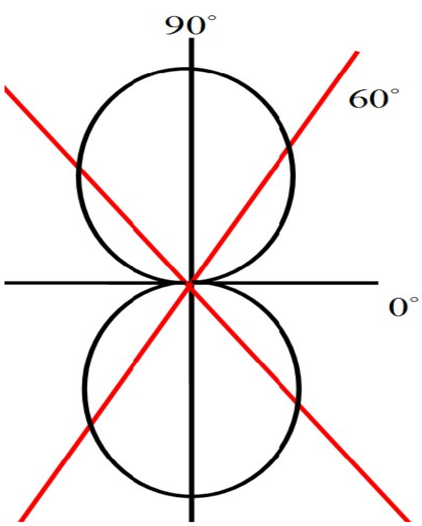
$$= 0.081 \text{ cm } (0.032 \text{ in})$$

-2 marks

$$L = \frac{1}{2f_r\sqrt{\epsilon_{\text{reff}}}\sqrt{\mu_0\epsilon_0}} - 2\Delta L$$

$$L = \frac{\lambda}{2} - 2\Delta L = \frac{30}{2(10)\sqrt{1.972}} - 2(0.081) = 0.906 \text{ cm}$$

-2 marks

		OR	
14	a)	Explanation with figure-5 marks Equations-3 marks	(8)
	b)	Explanation about Inverted F antenna with construction details-4 marks Application-2 marks	(6)
		Module III	
15	a)	Array factor derivation-4 marks  $$En = \cos\left(\frac{\beta \frac{d}{2} * cos\emptyset}{2}\right)$$ Pattern-3 marks 	(7)
	b)	Design an Endfire array and plot its radiation pattern-7 marks	(7)
		OR	
16	a)	Design steps-10 marks	(14)

Final answers-4 marks

(a) $(AF)_{2M} = \sum_{n=1}^{5} a_n \cos\left[\frac{(2n-1)}{2} d_r \cos\phi\right]$

$2M = 10, \quad M = 5$

(b) $(AF)_{2M} = a_1 \cos \frac{d_r}{2}\cos\phi + a_2 \cos \frac{3}{2} d_r \cos\phi + a_3 \cos \frac{5}{2} d_r \cos\phi + a_4 \cos \frac{7}{2} d_r \cos\phi + a_5 \cos \frac{9}{2} d_r \cos\phi$

$= a_1 \cos u + a_2 \cos 3u + a_3 \cos 5u + a_4 \cos 7u + a_5 \cos 9u$

(c) $(R_0)_{dB} = 26\,dB$

$20 \log R_0 = 26$

$R_0 = 10^{26/10} = \underline{\underline{20}}$

$$Z_0 = \frac{1}{2}\left[\left(R_0 + \sqrt{R_0^2 - 1}\right)^{1/m} + \left(R_0 - \sqrt{R_0^2 - 1}\right)^{1/m}\right]$$

$m = N - 1 = 10 - 1 = 9$

$Z_0 = \underline{\underline{1.0857}}$

(d) $(AF)_{10} = a_1\left(\frac{z}{z_0}\right) + a_2\left(4\left(\frac{z}{z_0}\right)^3 - 3\left(\frac{z}{z_0}\right)\right)$

$+ a_3\left(16\left(\frac{z}{z_0}\right)^5 - 20\left(\frac{z}{z_0}\right)^3 + 5\left(\frac{z}{z_0}\right)\right)$

$+ a_4\left(64\left(\frac{z}{z_0}\right)^7 - 112\left(\frac{z}{z_0}\right)^5 + 56\left(\frac{z}{z_0}\right)^3 - 7\left(\frac{z}{z_0}\right)\right)$

$+ a_5\left(256\left(\frac{z}{z_0}\right)^9 - 576\left(\frac{z}{z_0}\right)^7 + 432\left(\frac{z}{z_0}\right)^5 - 120\left(\frac{z}{z_0}\right)^3 + 9\left(\frac{z}{z_0}\right)\right)$

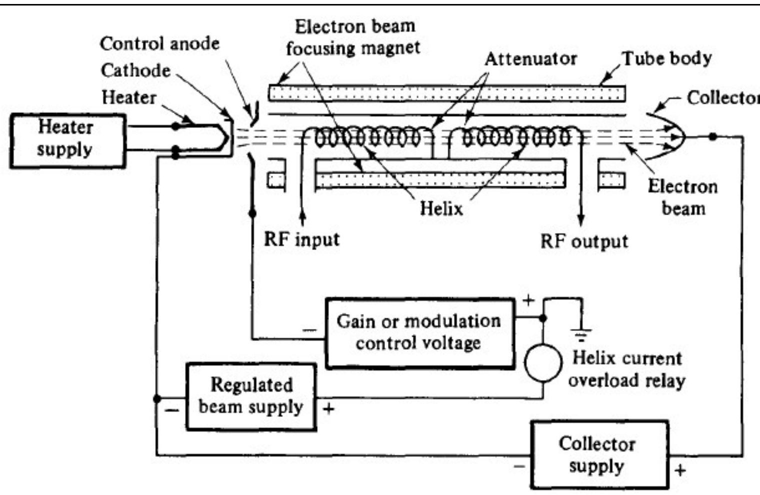
$T_m(z) = T_9(z)$

$= 256 z^9 - 576 z^7 + 432 z^5 - 120 z^3 + 9z$

Equating both eqns co-efficients of both equations

$\underline{z^9}$ $\quad 256 = a_5 \times 256 \times \left(\frac{1}{z_0}\right)^9$

$a_5 = (z_0)^9 = 2.083$

		z^7 $-576 = -a_5 \times \frac{576}{z_0^7} + a_4 \times \frac{64}{z_0^7}$ $a_4 = 2.82$ $a_3 = 4.118$ $a_2 = 5.207$ $a_1 = 5.837$ Normalised form $a_5 = 1$, $a_4 = 1.357$, $a_3 = 1.974$, $a_2 = 2.497$, $a_1 = 2.798$ $(AF)_{10} = 2.798 \cos u$ $(AF)_{10} = 2.798 \cos u + 2.496 \cos 3u + 1.974 \cos 5u + 1.357 \cos 7u + \cos 9u$	
		Module IV	
17	a)	 Diagram - 3 mark Explanation - 4 mark	(7)

b) (7)

$$\frac{V_0}{(V_r + V_0)^2} = \left(\frac{e}{m}\right)\frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$

$$= (1.759 \times 10^{11})\frac{(2\pi 2 - \pi/2)^2}{8(2\pi \times 9 \times 10^9)^2(10^{-3})^2} = 0.832 \times 10^{-3}$$

$$(V_r + V_0)^2 = \frac{600}{0.832 \times 10^{-3}} = 0.721 \times 10^6$$

$$V_r = 250 \text{ V}$$

Assume that $\beta_0 = 1$. Since

$$V_2 = I_2 R_{sh} = 2I_0 J_1(X')R_{sh}$$

the direct current I_0 is

$$I_0 = \frac{V_2}{2J_1(X')R_{sh}} = \frac{200}{2(0.582)(15 \times 10^3)} = 11.45 \text{ mA}$$

$$\text{Efficiency} = \frac{2X' J_1(X')}{2\pi n - \pi/2} = \frac{2(1.841)(0.582)}{2\pi(2) - \pi/2} = 19.49\%$$

(i) Repeller voltage Vr . - 3 mark

(ii) Direct current necessary to give a microwave gap voltage of 200 V - 2 mark

(iii) Electronic efficiency under this condition - 2 mark

OR

18 a) (7)

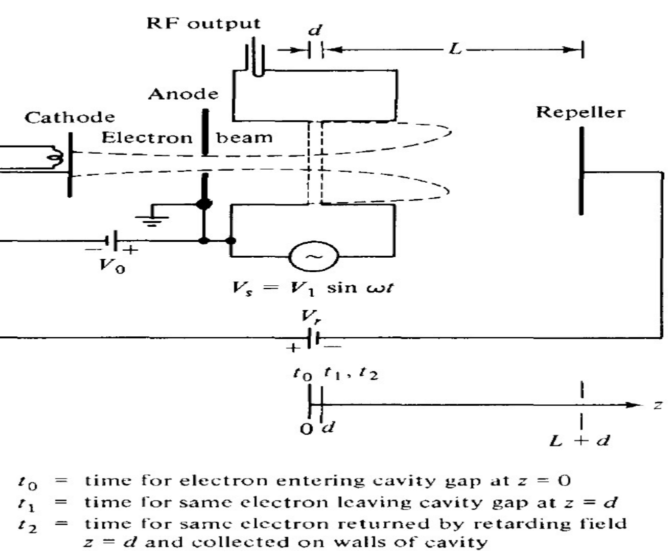
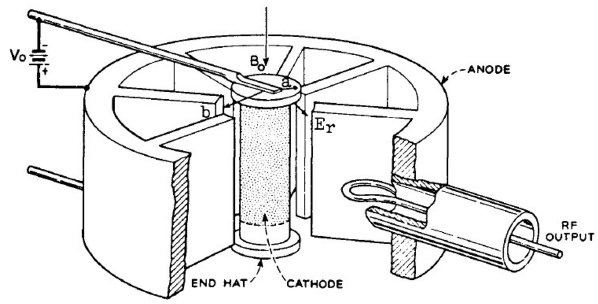
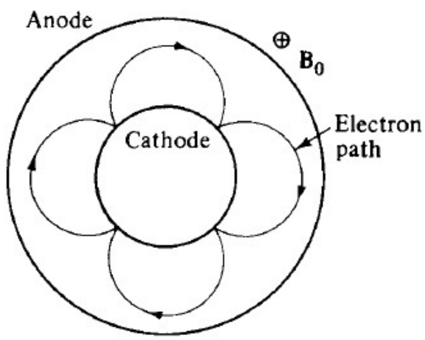
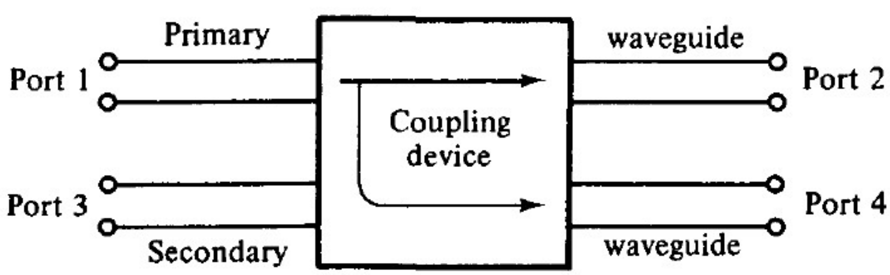
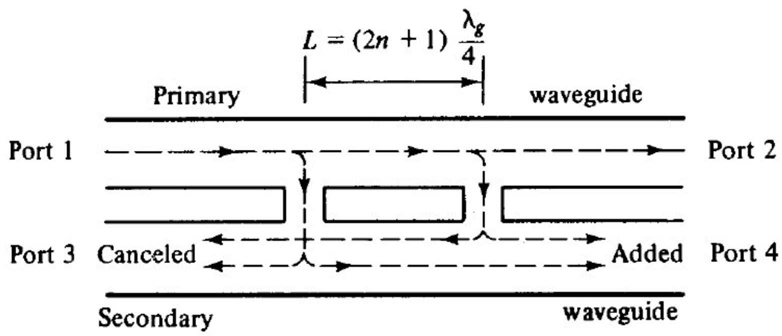
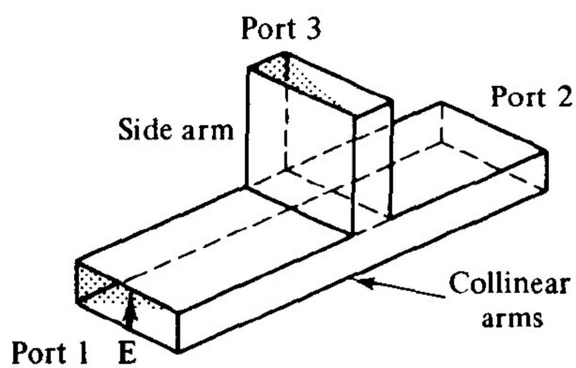
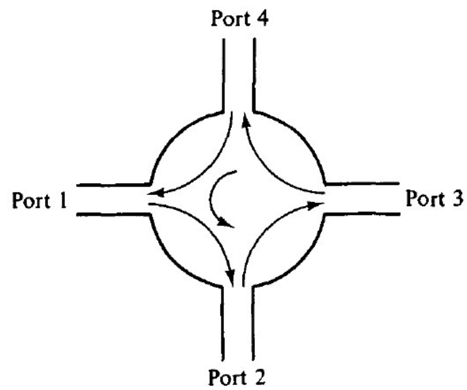


		Diagram - 3 mark Explanation - 4 mark	
	b)	Diagram - 3 mark Expatiation -4 mark	(7)
		Module V	
19	a)	Gunn Effect- 2 mark RWH theory - 5 mark	(7)
	b)		(7)

		 Diagram - 2 mark Explantion -2mark Derivation of S matrix - 2 mark $\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$ S matrx of Directional coupler - 1 mark	
		OR	
20	a)	 Diagram - 2 mark Exculpation - 3 mark Derivation of S matrix - 2 mark	(7)

		$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix}$$	
	b)	 Diagram - 2 mark Explanation - 3 mark $$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ S matrix - 2 mark	(7)

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