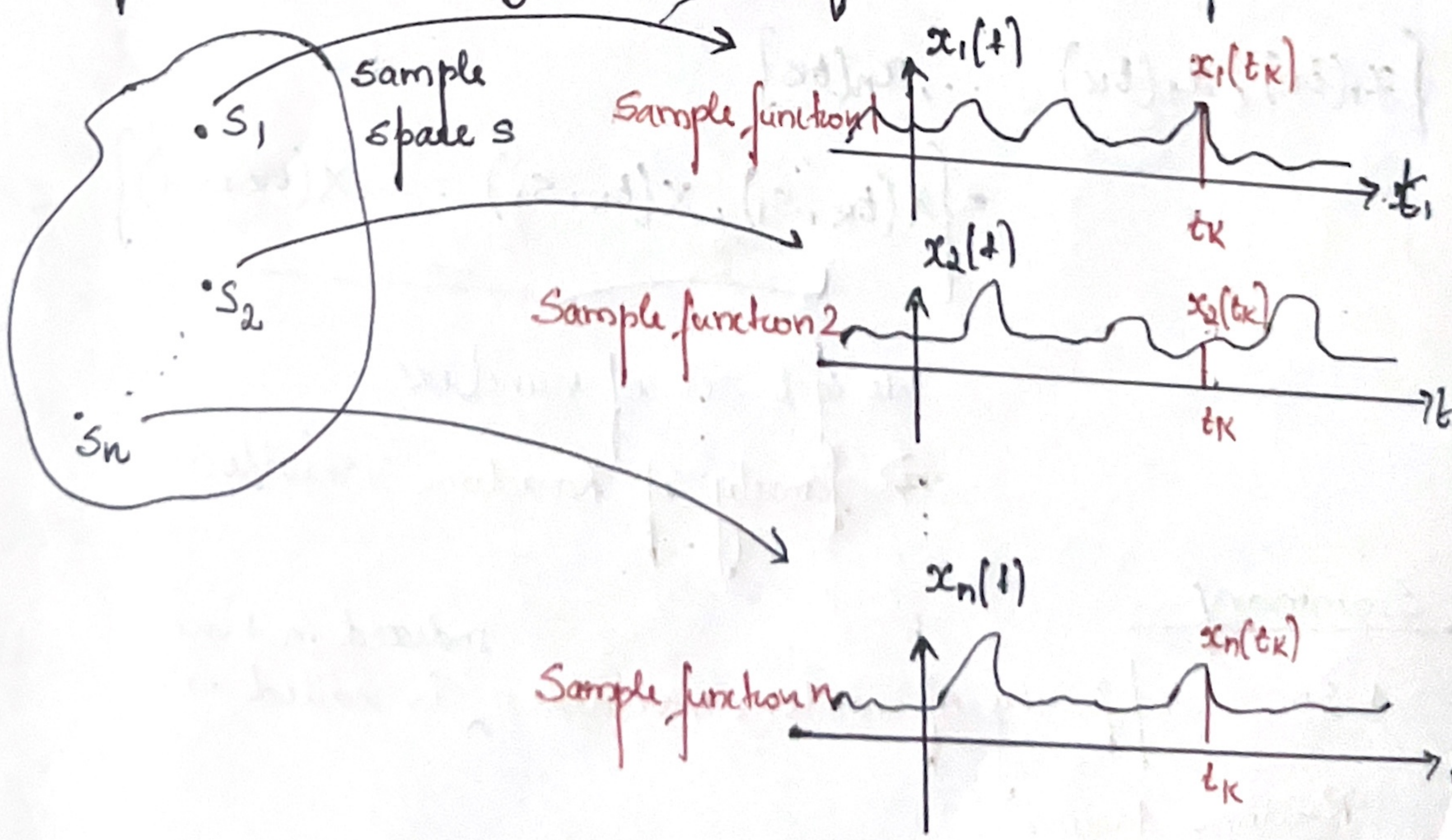
Random Process (or stochastic process)

- In real life situation, observations are made over a period of time and they are influenced by random effects, not just at a single instant but throughout the entire interval of time or sequence of times.

- A random variable is a function that assigns a real number to every outcome of a random experiment. While a random process is a function that assigns a time function to every outcome of a random experiment.



- ~~Each point in the sample space or ensemble is a function of time.~~

- Collection of all possible sample functions is called an ensemble.

• Random process is denoted by

$$x(t, s_j) \quad , \cancel{s \text{ varies from } 1 \text{ to } j}$$

• For a fixed point s_j, the function of $x(t, s)$ is called a realization or a sample function of the RV.

• We can denote the sample function as

$$x_j(t) = x(t, s_j)$$

• When waveforms are sampled at t_K instance.

$$\{x_1(t_K), x_2(t_K) \ldots, x_n(t_K)\}$$

$$= \{\underbrace{x(t_K, s_1), x(t_K, s_2) \ldots, x(t_K, s_n)}_{\text{We get set of numbers.}}\}$$

⇒ family of Random Variables.

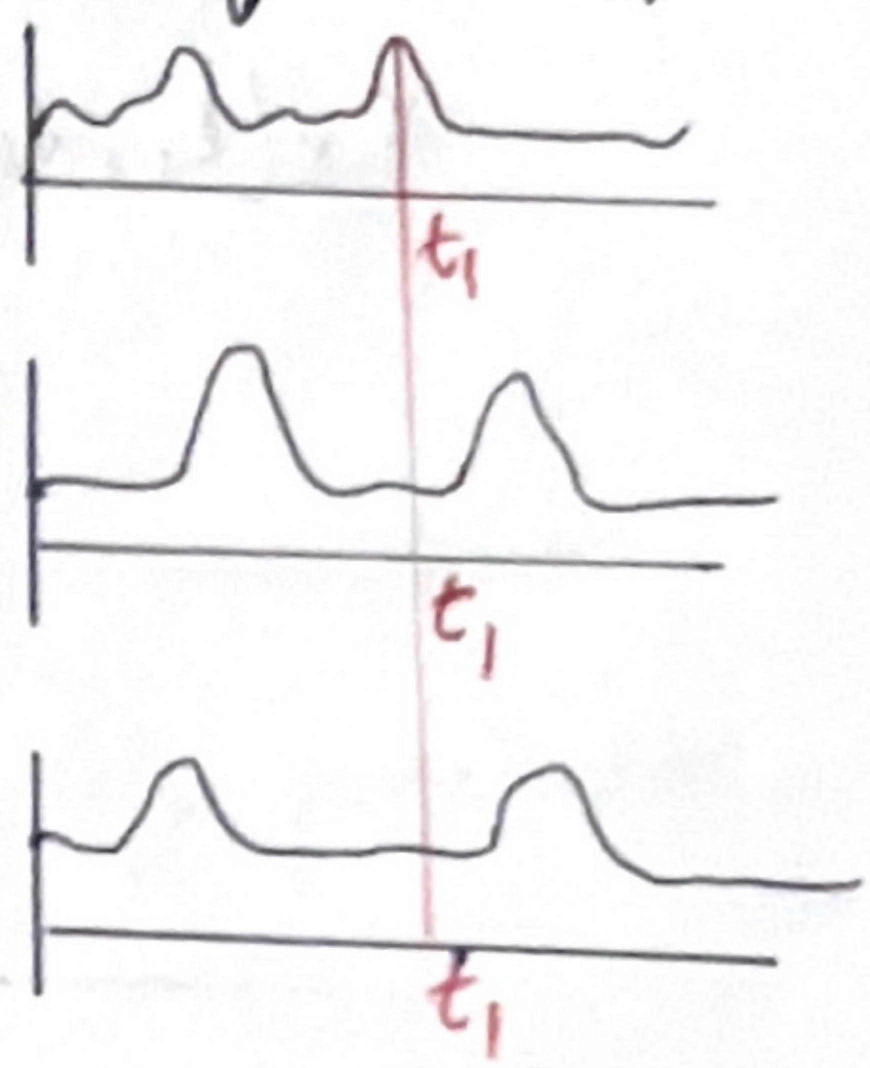
Summary

* Ensemble / family of random variables indexed in time. is called as Random Process.

* Sampling of Random process gives Random Variable

Statistical average / Ensemble mean or average.

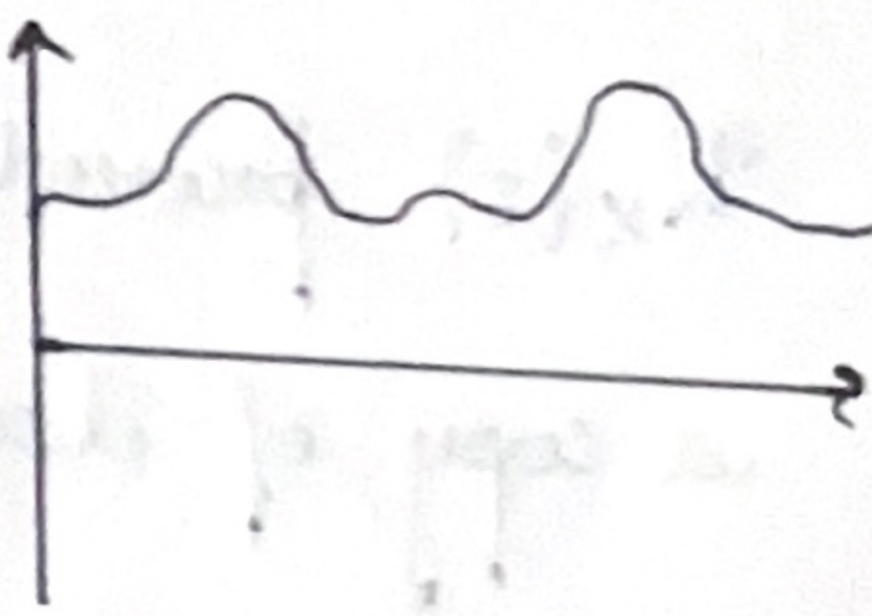
Ensemble mean is taken over the ensemble of waveforms at a fixed instant of time t



$$\mu_X = E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,.dx$$

Time average

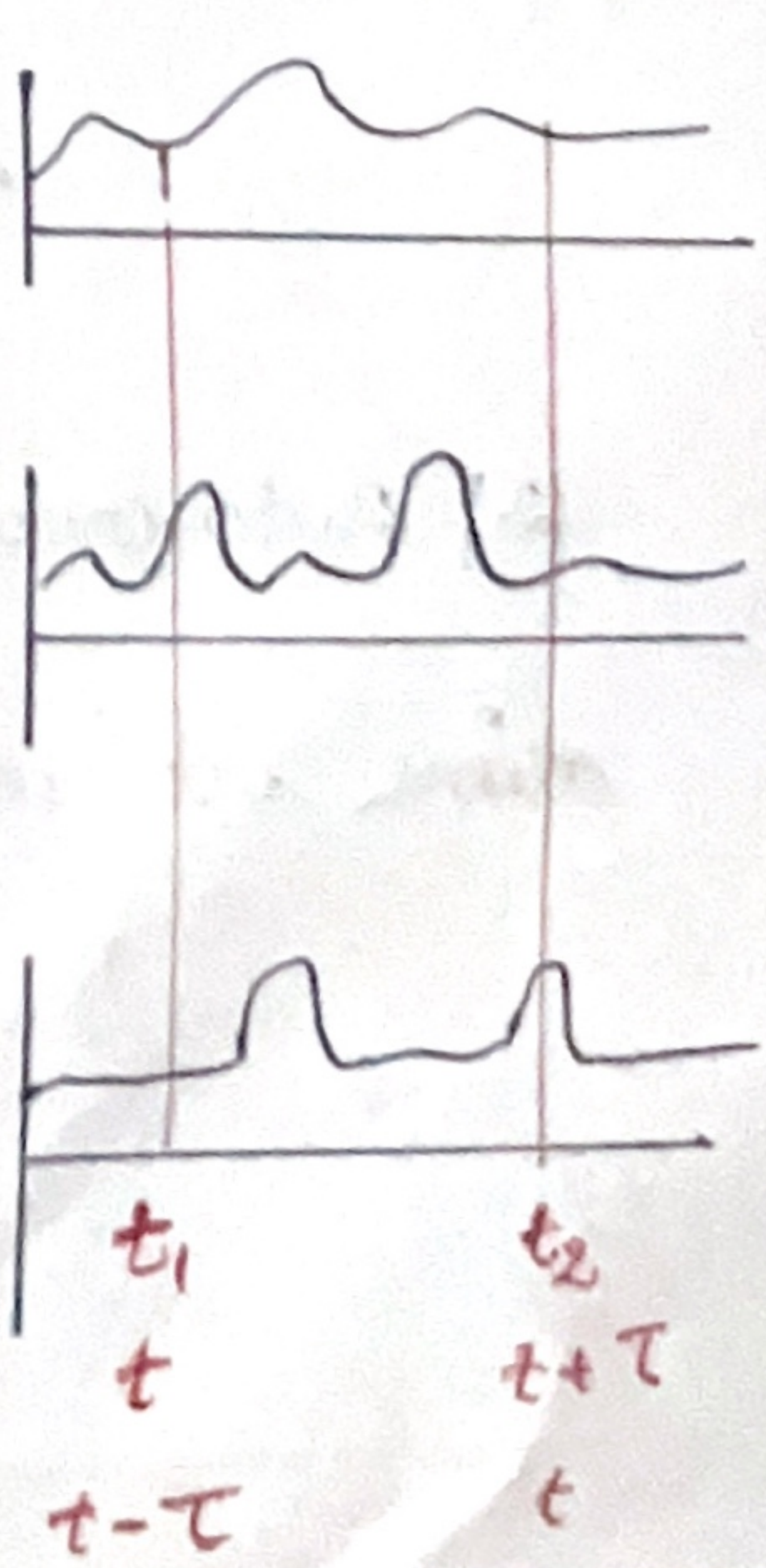
Time average of a single sample function is given by the average of all the values within the observation interval $-T$ to T



$$\mu_x(t) = \frac{1}{2T}\int_{-T}^{T} x(t)\,dt \quad , \quad -T \le t < T$$

Autocorrelation

Refers to the degree of correlation of the same variables between two successive time intervals.



$$R_X(t_1, t_2) = E[x(t_1).x(t_2)]$$
$$= E[X_1 . X_2]$$

here $X_1(t)$ & $X_2(t)$ are 2 random variables obtained at time instant t_1 & t_2. The autocorrelation function is the measure of degree to which the two random variables of the same random process are related to each other.

$$R_X(t_1, t_2) = E[x(t_1) \cdot x(t_2)]$$

$$= E[X_1 \cdot X_2]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \cdot x_2 \, f_{X_1X_2}(x_1, x_2) \cdot dx_1 \, dx_2$$

$$R_X(\tau) = E[x(t) \cdot x(t+\tau)] \underset{\tau=0}{=} m_x$$

$R_X(\tau)$ provides a measure of how closely the signal matches a copy of itself as the copy is shifted by τ units.

Properties

(1) Autocorrelation fn is always an even function.

$$R_X(\tau) = R_X(-\tau)$$

(2) Autocorrelation at 0 is greater than autocorrelation at any other point

$$R_X(0) \geq R_X(\tau)$$

(3) $R_X[0] = E[X(t) \cdot X(t)]$

$= E[X^2(t)]$ – mean square value.

– total power.

→ When $\tau = 0$, autocorrelation value is always maximum, which corresponds to the total energy of the i/p function.