- BW (no. of sidebands) = $2nf_m$
- When n=1 ------------BW = $2f_m$ (same as AM--DSB)
- These types of FM s/gs are called narrow band FM s/gs. (β<<1)
- BW in terms of β = $2(\beta+1)f_m$ ------------------------Wide band FM s/gs (β>>1)

- β=
- BW = $2(\ +1)f_m$
- **BW= $2(\ +f_m)$ -------------------------- Carson's rule**

- Carson's rule states that the bandwidth required to transmit an angle modulated wave is twice the sum of the peak frequency deviation and highest modulating signal frequency.
- Unlike AM(only 3 req components), FM has infinite no.of sidebands as well as carrier
- Bessel fn coeffs decrease in values as n increases
- Mod index determines how many side band components have significant amplitude
- Sidebands at equal distances from f_c have equal amplitudes and are symmetrical about carrier freq

FM can be divided into **Narrowband FM** and **Wideband FM** based on the values of modulation index β

Narrowband FM (β<1)

- This frequency modulation has a small BW when compared to wideband FM.
- The modulation index β is small, i.e., less than 1.
- Its spectrum consists of the carrier, the USB and LSB.
- This is used in mobile communications such as police wireless, ambulances, taxicabs, etc.

Wideband FM (β>1)

- This frequency modulation has infinite bandwidth.
- The modulation index β is large, i.e., higher than 1.
- Its spectrum consists of a carrier and infinite number of SBs, which are located symmetrically around it.
- Used in entertainment, broadcasting applications such as FM radio, TV, etc.

Narrow band FM

- consider the expression for FM wave

$$V_{FM}(t) = A_c \cos(2\prod f_c t + 2\prod k_f(t)\,dt)$$

$$V_{FM}(t) = A_c \cos(\omega_c t + k_f(t)\,dt)$$

Considering the signals as real parts of the phasor, we can express this in exponential manner as

$V_{FM}(t) = A_c \cos(t) = A_c$

$V_{FM}(t)= A_c$

Let =g(t)

Then $V_{FM}(t)= A_c$ *

$V_{FM}(t)= A_c$

- For NBFM, 1 for all values of t, then

$\approx$ 1+)

$V_{FM}(t)= A_c$ 1+)]

=cos + j

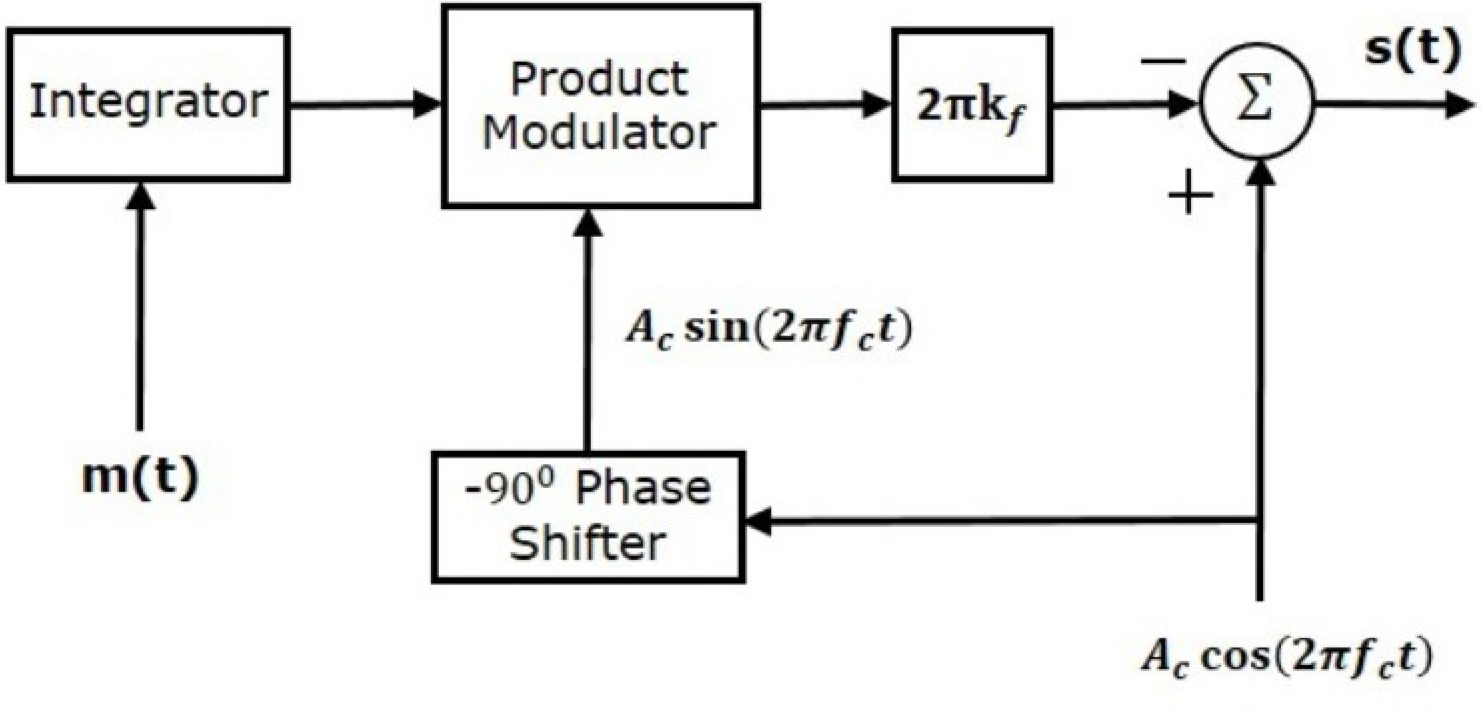
$V_{FM}(t)= A_c$ [cos + j 1+)]

$V_{FM}(t)= A_c\cos$) sin

This is the **expression for Narrow band FM**.

Or $V_{FM}(t)= A_c\cos(2\pi f_c t)-\sin(2\pi f_c t)2\pi m(t)dt$

Fig. shows the generation of narrow band FM using balanced modulator .(NBFM modulator)



Side band terms are generated by a balanced modulatr as in DSBSC s/m and then carrier term is added to it.

Mathematical Expression for Single-tone Narrow Band FM

- For a single tone NBFM, we take modulating s/g as consisting of a single freq.
- Eg : If $m(t)= A_m \cos \omega_m t$
- g(t)= (t) dt

$$= \sin \omega_m t$$

Substituting in the exp for NBFM, we get

$$V_{FM}(t)= A_c\cos \ \sin \omega_m t \sin$$

$$=A_c\cos \ \sin \omega_m t \sin$$

$$V_{FM}(t) = A_c\cos \ \sin \omega_m t \sin$$

This is the exprsn for single tone NBFM

Wide band FM

- The expression for the wideband FM is complex.
- The only way to solve this equation is by using the Bessel functions. By using the Bessel functions the equation for wideband FM wave can be expanded as follows :
- $V_{FM}(t) = A_c \{ J_0(\beta) \cos w_c t + J_1(\beta) [\cos(w_c + w_m)t - \cos(w_c - w_m)t] + J_2(\beta) [\cos(w_c + 2w_m)t + \cos(w_c - 2w_m)t] + J_3(\beta) [\cos(w_c + 3w_m)t - \cos(w_c - 3w_m)t] + \ldots\ldots \}$
- So from eqn, it is clear that

1.The FM wave consists of carrier. The first term in eqn represents the carrier.

2.The FM wave ideally consists of infinite number of sidebands. All the terms except the first one are sidebands.

3.The amplitudes of the carrier and sidebands is dependent on the J coefficients.

4.As the values of J coefficients are dependent on the modulation index β, the modulation index determines how many sideband components have significant amplitudes.

5.Some of the J coefficients can be negative. Therefore, there is a 180^o phase shift for that particular pair of sidebands.

6.The carrier component does not remain constant. As $J_0(\beta)$ is varying, the amplitude of the carrier will also vary. However, the amplitude of FM wave will remain constant.

7.For certain values of modulation index, the carrier component will disappear completely. These values are called Eigen values.

8. In FM, the total transmitted power always remains constant. It is not dependent on the modulation index. The reason for this is that the amplitude of the FM signal i.e. A_c is always constant and the power transmitted is given by,

$$P_t =$$

- Where A_c is the peak amp of FM s/g

Spectrum of NBFM & WBFM

1. Spectrum of NBFM(β<<1)

- $V_{FM}(t) = A_c \, Cos\,[2\prod f_c\, t + \beta \prod f_m\, t\,]$

$$\cos(a + b) = \cos(a)\cos(b) – \sin(a)\sin(b)$$

$V_{FM}(t) = A_c \{ \cos 2\prod f_c\, t\,.\cos(\beta \prod f_m t) - \sin 2\prod f_c\, t\,.\sin(\beta \prod f_m t)\}$

When β <<1, $\cos(\beta \prod f_m t) = 1$

$$\sin(\beta \prod f_m t) = \beta \prod f_m t$$

$V_{FM}(t) = A_c \{ \cos 2\prod f_c\, t - \beta (\prod f_m t)(\sin 2\prod f_c\, t)\}$

$$\sin a \sin b = (1/2)[\cos(a - b) - \cos(a + b)]$$

$\mathbf{V_{FM}(t) = A_c \cos 2\prod f_c\, t + f_c + f_m)t - f_c - f_m)t}$

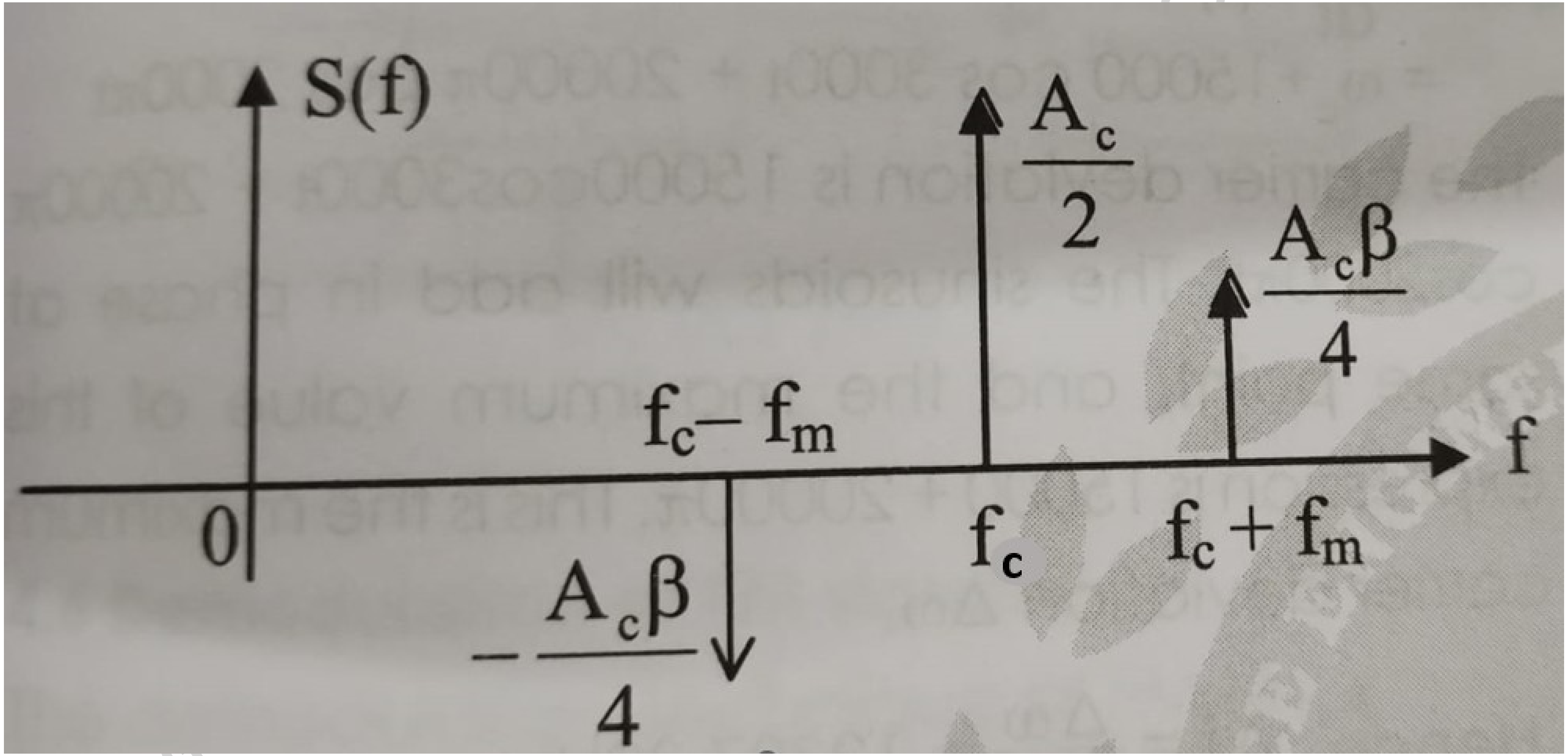
$$s(t) \;=\; A_c \cos 2\prod f_c\, t + f_c + f_m)t - f_c - f_m)t \text{ ------(i)}$$

- This is the time domain representation of NBFM s/g.
- To draw the spectrum of NBFM, we need to get its freq domain representation. So by getting the Fourier transform of this, we get

- FT(cos $2\prod f_c$ t) = ½ $(f\text{-}f_c) + (f+f_c)\}$
- So FT of eqn (i) becomes

$$s(f)= (f-f_c) + (f+f_c)\} + (f-f_c-f_m) + (f+f_c+f_m)\} - (f-f_c+f_m) + (f+f_c-f_m)\}$$

Spectrum of NBFM



- BW of NBFM = $2f_m$

Spectrum of AM and NBFM are identical except that spectral components at f_c-f_m is 180 deg out of phase.

- Power P_t=P_c(1+)

2. Spectrum of WBFM(β>>1)

- V_{FM} (t) = s(t) $= A_c \,\mathrm{Cos}\, [2\prod f_c\, t + \beta \;\prod f_m\, t\,]$
- Using Bessel Fn, Cos [A + β Sin B] =

- V_{FM} (t) =s(t) $=A_c$
- $S(t)= A_c J_0 (\beta) \mathrm{Cos}\, w_c t + A_c J_1 (\beta) \mathrm{Cos}\, (w_c + w_m)t + A_c J_{-1} (\beta) \mathrm{Cos}\, (w_c - w_m)t +$

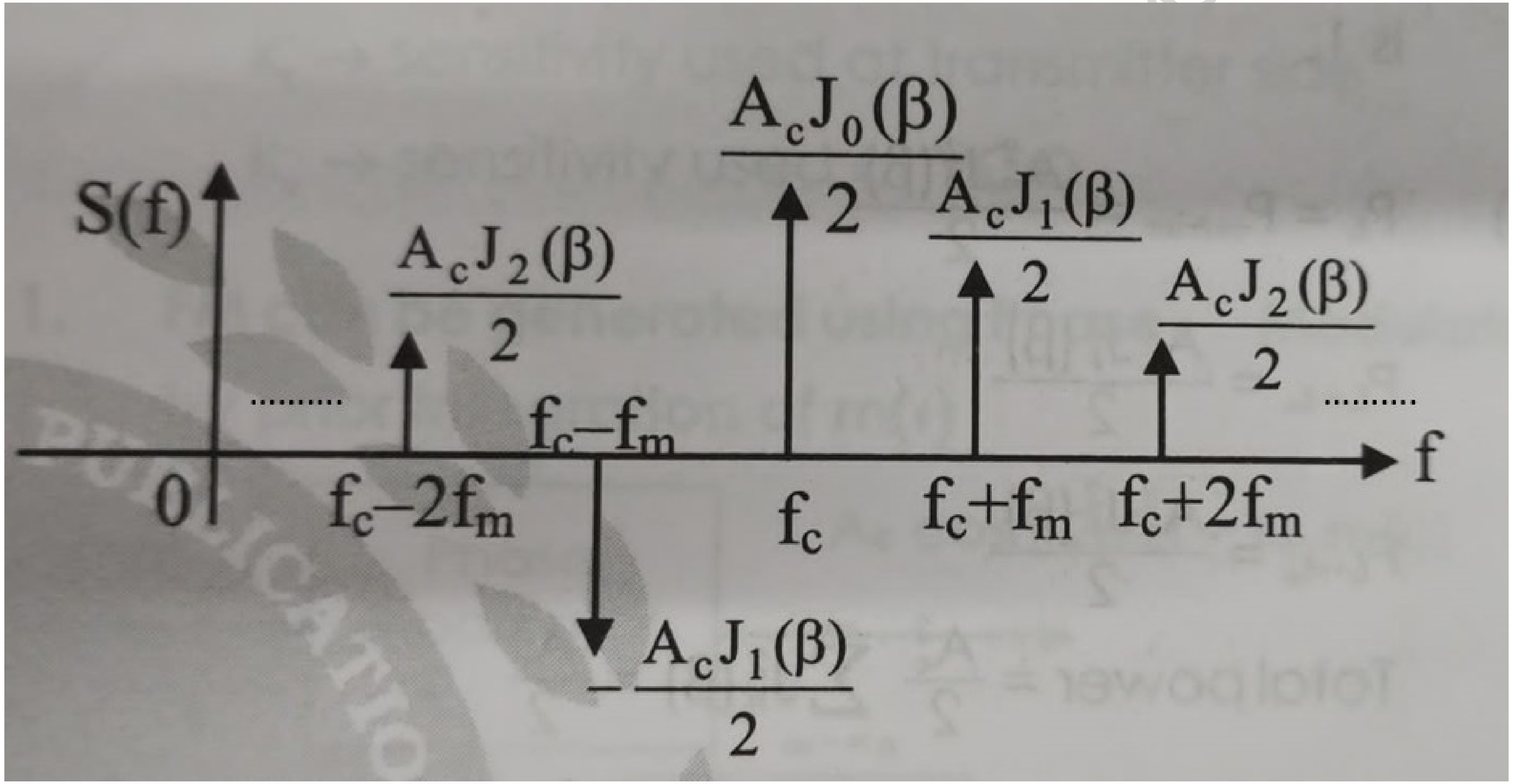
$A_c J_2 (\beta) \mathrm{Cos}\, (w_c + 2w_m)t + A_c J_{-2} (\beta) \mathrm{Cos}\, (w_c - 2w_m)t +$
…………

$= A_c J_0 (\beta) \mathrm{Cos}\, w_c t + A_c J_1 (\beta) \{\mathrm{Cos}\, (w_c + w_m)t - \mathrm{Cos}\, (w_c - w_m)t\} +$

$A_c J_2 (\beta) \{\mathrm{Cos}\, (w_c + 2w_m)t + \mathrm{Cos}\, (w_c - 2w_m)t \} +$
…………

Spectrum of WBFM

s(f)= (f-f$_c$) + (f+f$_c$)} + (f-f$_c$-f$_m$) + (f+f$_c$+f$_m$)} - (f-f$_c$+f$_m$) + (f+f$_c$-f$_m$)} + ...



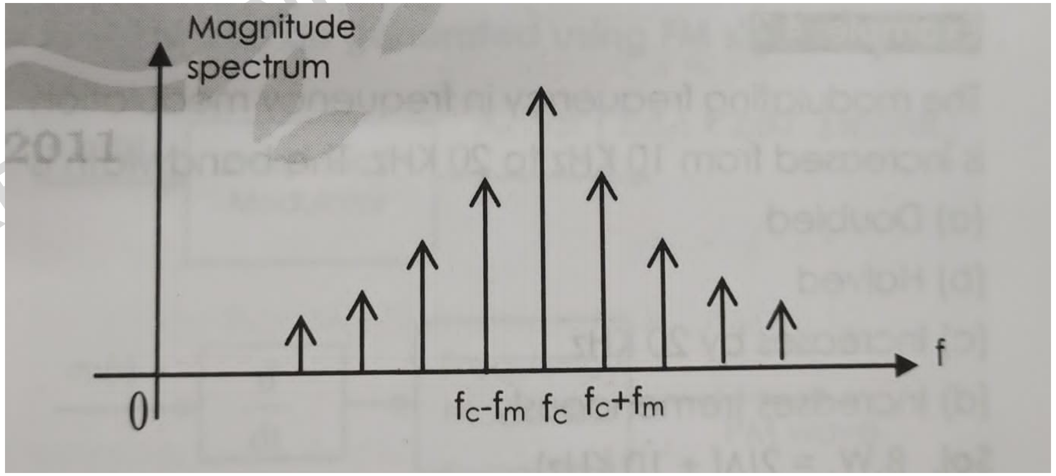
Therefore theoretical BW of WBFM is ∞

1.NBFM (Narrowband FM <1) Spectra:

1. The spectrum of NBFM consists of the carrier frequency (f_c) and two sidebands located symmetrically around the carrier frequency.
2. The sidebands' frequencies are at (f_c-f_m) and ($f_c + f_m$), where f_m is the frequency of the modulating signal.
3. The amplitude of the sidebands decreases as the distance from the carrier frequency increases.

2.WBFM (Wideband FM >1) Spectra:

1. The spectrum of WBFM consists of the carrier frequency (fc) and infinite no.of sidebands each separated by f_m.



2. The number and spacing of these sidebands depend on the modulation index ().

3. WBFM produces more sidebands compared to NBFM, and the sidebands' amplitudes and spacing are more complex.

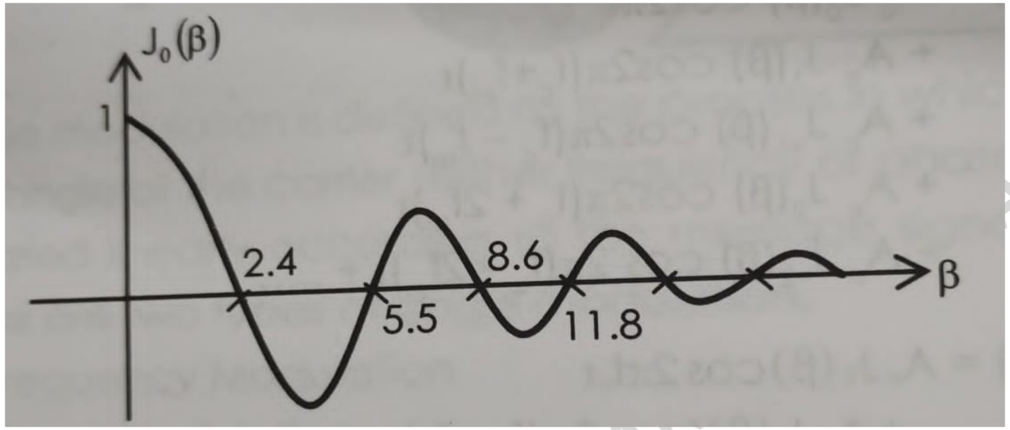
4. As the modulation index increases, the sidebands become more numerous and more closely spaced, resulting in a wider spread of frequencies in the spectrum.

5. Amplitudes of spectral components depends on Bessel fn coeff.s $J_n()$ which decrease as n increases. So amp of spectral components also decreases on both sides of the carrier.

6. In WBFM, spectrum amplitude of carrier component depends on $J_0()$ and hence on mod index .

Bessel fn coeff $J_0()$ =0 when

- For these values of amp of carrier component in the spectrum is 0.



7. Power P_{fc}=

P_{fc+fm}=

P_{fc-fm}=

Total power = =1

P_t= (assuming R=1ohm) if not, **P_t**

- Theoretical BW of WBFM = ∞
- Practical BW of WBFM using Carson's rule:

$$BW = 2(\beta + 1) f_m$$

$$\mathbf{BW = 2(+ f_m)}$$

S.No.	Parameter/Characteristics	Wideband FM	Narrowband FM
1.	Modulation index	Greater than 1	Less than or slightly greater than 1
2.	Maximum deviation	75 kHz	5 kHz
3.	Range of modulating frequency	30 Hz to 15 kHz	30 Hz to 3 kHz
4.	Maximum modulation index	5 to 2500	Slightly greater than 1
5.	Bandwidth	Large about 15 times higher than BW of narrowband FM	Small. Approximately same as that of AM
6.	Applications	Entertainment broad-casting (can be used for high quality music transmission)	FM mobile communication like police wireless, ambulance etc. (This is used for speech trans-mission)

AM

- frequency and phase remain the same.
- $V_{AM}(t) = A_c[1 + m\,\mathrm{Sin}w_m\, t][\mathrm{Sin}w_c\, t]$
- modulation index varies from 0 to 1
- It has only two sidebands.
- It has simple circuit.
- The amplitude of the carrier wave is modified in order to send the s/g
- It requires low bandwidth in the range of 10 kHz.
- received signal is of low quality.
- It works in a frequency range of 535 to 1705 Kilohertz (KHz).
- It operates in the medium and high freq.
- It has poor sound quality.

FM

- amplitude and phase remain the same.
- $V_{FM}(t) \quad = A_c \,\mathrm{Cos}\,[\omega_c t + \beta\ m(t)]$
- β can have values < 1 or > 1
- It has an infinite number of sidebands.
- It has complex circuit.
- The frequency of the carrier wave is modified in order to send the s/g.
- It requires high bandwidth in the range of 200 kHz.
- received signal is of high quality.
- It works in a frequency range of 88 to 108 Megahertz (MHz).
- It operates in the very high frequency.

AM

- Transmitted power is dependent upon mod index m. $P_t = P_c[1+\ ]$
- Amp of SB is dep on m and is always < carrier amp.
- BW = 2 times highest modulating freq.
- AM s/g is more susceptible to noise & more affected by noise than FM
- Demodulation of AM s/g is very easy and cheaper

FM

- In this carrier amp is constant, so transmitted P is constant, indep of mod index β
- Amp of carrier & SB vary with β and can be calculated using Bessel Fn.
- BW is α to mod index
- Has Noise immunity
- The ckts to produce and demodulate FM are complex and expensive than AM ckts.

Phase Modulation

- the phase of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.
- the amplitude and the frequency of the carrier signal remains constant.

- Phase modulation is used in mobile communication systems, while frequency modulation is used mainly for FM broadcasting.

Mathematical Representation of PM wave

- The equation for instantaneous phase $\varnothing_i$ in PM

$$\varnothing_i = k_p m(t)$$

- Phase sensitivity k_p unit is rad/V
- msg s/g m(t)
- Std equation of angle modulated wave is $$s(t) = A_c \cos(2\pi f_c t + \phi_i)$$
- Substituting value of $\varnothing$ $$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

- This is the eq of PM
- Now if $m(t) = A_m \cos(2\pi f_m t)$
- $S_{PM}(t) = A_c \cos(2\pi f_c t + k_p A_m \cos(2\pi f_m t))$
- $S_{PM}(t) = A_c \cos(2\pi f_c t + \Delta\varnothing \cos(2\pi f_m t))$

  Peak phase deviation $\Delta\phi = k_p A_m$
- This eqn is identical to sinusoidal FM eqn where

$$s(t) = A_c \cos(2\pi f_c t + \beta \cos(2\pi f_m t))$$

- So the trigonometric expansion will be similar to that of sinusoidal FM containing a carrier term & side freqs at $(f_c \pm nf_m)$
- Amps are given in terms of Bessel fn $J_n(\Delta\varnothing)$. Here argument is $\Delta\varnothing$ rather than freq mod index β

1. A sinusoidal modulating waveform of amplitude 5 V and a frequency of 2 KHz is applied to FM generator, which has a frequency sensitivity of 40 Hz/volt. Calculate the frequency deviation, modulation index, and bandwidth.

2. An FM wave is given by
Calculate the frequency deviation, bandwidth, and power of FM wave.

$$s\left(t\right) = 20\cos\left(8\pi \times 10^{6}t + 9\sin\left(2\pi \times 10^{3}t\right)\right)$$

- 1. $\Delta f = k_f A_m$ $\beta = \frac{\Delta f}{f_m}$ $BW = 2 f_m$ Narrow band FM

as β <1

200Hz, 0.1, 4KHz

2. $s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ $\beta = \frac{\Delta f}{f_m}$

$P_c = \frac{{A_c}^2}{2R}$

Wide band FM as β>1 $BW = 2(\beta + 1) f_m$

9KHz,20KHz, 200W(assuming R=1ohm)