Q) Given $x(n) = \{1, -2, 3, -4, 5, -6\}$ without calculating DFT find the following quantities: (KTU Dec 2018)

(a) $X(0)$ (b) $\sum_{k=0}^{5} X(k)$ (c) $X(3)$

(d) $\sum_{k=0}^{5} |X(k)|^2$ (e) $\sum_{k=0}^{5} (-1)^k X(k)$.

Answer: Here $N=6$.

(a). we have

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}.$$

$N=6 \Rightarrow$

$$X(k) = \sum_{n=0}^{5} x(n)\, e^{-j\frac{2\pi}{6}kn} \quad \text{——— (1)}$$

to find $X(0)$ put $k=0$ in eqn (1)

$$X(0) = \sum_{n=0}^{5} x(n)\, \underbrace{e^{-j\frac{2\pi}{6}\times 0\times n}}_{e^0=1}$$

$$= \sum_{n=0}^{5} x(n)$$

$$= x(0) + x(1) + x(2) + x(3) + x(4) + x(5)$$

$$= 1 + -2 + 3 + -4 + 5 + -6 = -3$$

(b). we have IDFT equ.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{6} \sum_{k=0}^{5} X(k)\, e^{j\frac{2\pi}{6}kn} \quad \text{—— (2)}$$

to find $\sum_{k=0}^{5} X(k)$ put $n=0$ in eqn (2)

$$6 \times x(0) = \sum_{k=0}^{5} X(k)\, \underbrace{e^{j\cdot\frac{2\pi}{N}\cdot 0 \cdot n}}_{1}$$

$$\therefore \sum_{k=0}^{5} X(k) = 6 \times x(0) = 6 \times 1 = \underline{\underline{6}}$$

(c) to find $X(3)$ put $k=3$ in eqn (1).

$$X(3) = \sum_{n=0}^{5} x(n)\, e^{-j\frac{2\pi\, 3n}{6}}$$

$$= \sum_{n=0}^{5} x(n)\, e^{-j\pi n} \qquad (e^{-j\pi} = -1)$$

$$= \sum_{n=0}^{5} x(n)\, (-1)^n.$$

$$= x(0)(-1)^0 + x(1)(-1)^1 + x(2)(-1)^2 + x(3)(-1)^3 + x(4)(-1)^4 + x(5)(-1)^5$$

$$= 1 + 2 + 3 + 4 + 5 + 6 = 21$$

(d) from Parseval's theorm

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2 \quad \text{—— (3)}$$

$\therefore$ Here $N=6$.

$$\sum_{n=0}^{5} |x(n)|^2 = \frac{1}{6} \sum_{k=0}^{5} |X(k)|^2$$

$$\therefore \sum_{k=0}^{5} |X(k)|^2 = 6 \times \sum_{n=0}^{5} |x(n)|^2$$

$$= 6\left[|x(0)|^2 + |x(1)|^2 + \cdots |x(5)|^2\right]$$

$$= 6\left[1^2 + |-2|^2 + |3|^2 + |-4|^2 + |5|^2 + |-6|^2\right]$$

$$= 6\left[1 + 4 + 9 + 16 + 25 + 36\right]$$

ie $\sum_{k=0}^{5} |X(k)|^2 = 6 \times 91 = 546$

(e) to find $\sum_{k=0}^{5} (-1)^k X(k)$

Consider IDFT equation

$$x(n) = \frac{1}{6} \sum_{k=0}^{5} x(k)\, e^{j\frac{2\pi}{6}kn}$$

put $n=3$ in above equation

$$x(3) = \frac{1}{6} \sum_{k=0}^{5} x(k)\, e^{j\frac{2\pi}{6}k\cdot 3}$$

$$\therefore \sum_{k=0}^{5} x(k)\, e^{j\pi k} = 6 \times x(3)$$

$$(e^{j\pi} = -1)$$

$$\sum_{k=0}^{5} (-1)^k x(k) = 6 \times -4$$

$$= \underline{\underline{-24}}$$

Q. State circular frequency shift property of DFT. 4 point DFT of the signal $x(n) = \{a, b, c, d\}$ is $X(k)$. Find the IDFT of $X(k-2)$? (KTU- Dec 2018)

Ans: Circular frequency shift property of DFT

if $x(n) \xrightarrow[N]{DFT} X(k)$

$$e^{j\frac{2\pi}{N}mn}\, x(n) \xrightarrow[N]{DFT} X((k-m))_N$$

Here $x(n) = \{a, b, c, d\} \xrightarrow{DFT} X(k)$.

$$X(k-2) \xrightarrow{IDFT} e^{j\frac{2\pi}{4}2n}\, x(n)$$

$$= e^{j\pi n}\, x(n) = (-1)^n x(n).$$

$$= \{(-1)^0 \cdot a, (-1)^1 \cdot b, (-1)^2 \cdot c, (-1)^3 d\}.$$

$$= \{a, -b, c, -d\}$$

2b) How will you obtain linear convolution from circular convolution? For $x(n) = \{1, 2, 3\}$ and $h(n) = \{1, -2\}$ obtain linear convolution $x(n) * h(n)$ using circular convolution (KTU - Dec 2018)

$L = 3, \quad M = 2$

linear convln o/p length $= L + M - 1$

$= 3 + 2 - 1 = 4$

we write

$x(n) = \{1, 2, 3, 0\}$ $\quad h(n) = \{1, -2, 0, 0\}$

$$x(n) \; (N)_4 \; h(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -1 \\ -4 \\ -7 \\ -6 \end{bmatrix}$$

$$y(n) = \{-1, -4, -7,$$