8.1. INTRODUCTION

To transmit digital data over a band-pass channel, it is necessary to modulate the incoming data onto a carrier wave with fixed frequency limits imposed by channel. The data may represent digital computer outputs or PCM waves generated by digitizing voice or video signals. The channel may be telephone channel, microwave radio link, satellite channel or an optical fiber.

Modulation

Modulation is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave. In digital communications, the modulating wave consists of binary data or an M-ary encoded version of it and the carrier is sinusoidal wave.

Different Shift Keying methods that are used in digital modulation techniques are

- **Amplitude shift keying [ASK]**
- **Frequency shift keying [FSK]**
- **Phase shift keying [PSK]**

This may be viewed as special cases of amplified modulation, frequency modulation and phase modulation respectively.

Digital Communication

When digital data is transmitted directly or modulates some carrier, it is called digital communication. In digital communications, the modulating wave consists of binary data or an M-ary encoded version of it. For the carrier, it is necessary to use a sinusoidal wave with a sinusoidal carrier the modulator can distinguish one signal from another is a step change in the amplitude, frequency or phase of the carrier. The result of this modulation process is amplitude shift keying (ASK), frequency shift keying (FSK) and phase shift keying (PSK).

Amplitude Shift Keying (ASK)

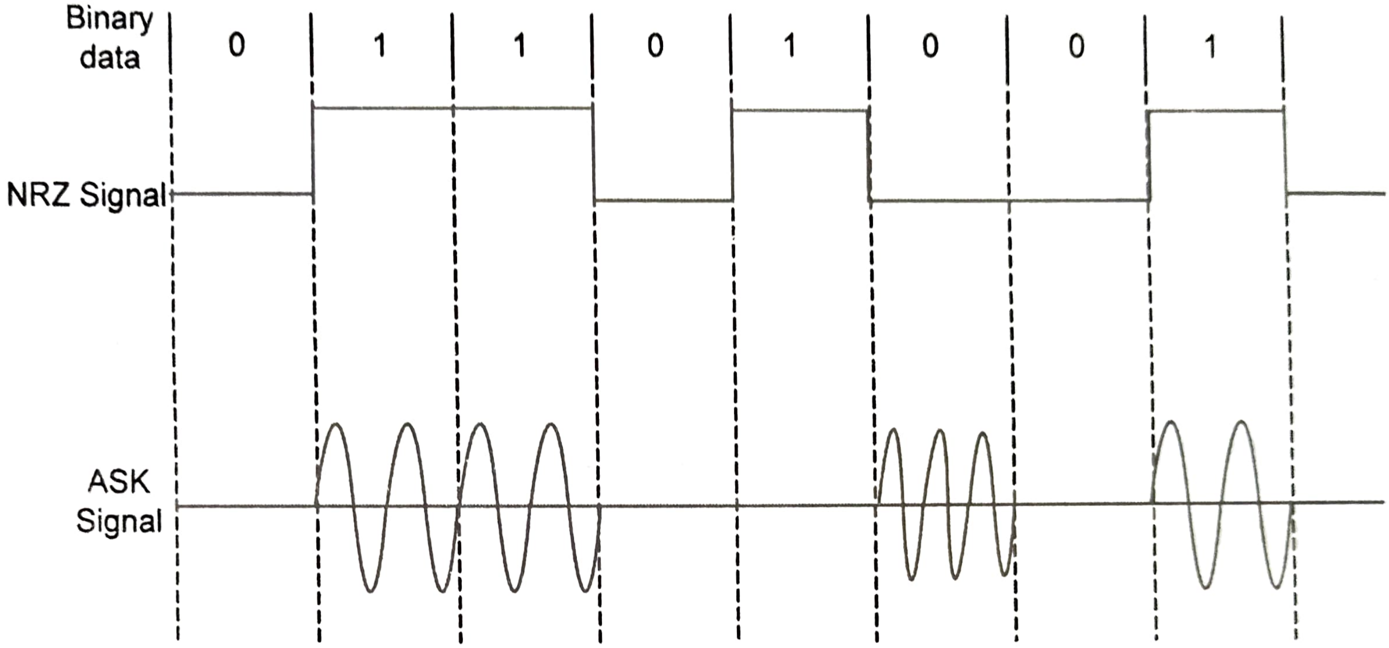


Fig. 8.1. Amplitude Shift Keying

The amplitude shift keying is also called on-off keying (OOK). This is the simplest digital modulation technique changes in both amplitude and phase of the carrier are combined to produce amplitude phase keying.

Frequency Shift Keying

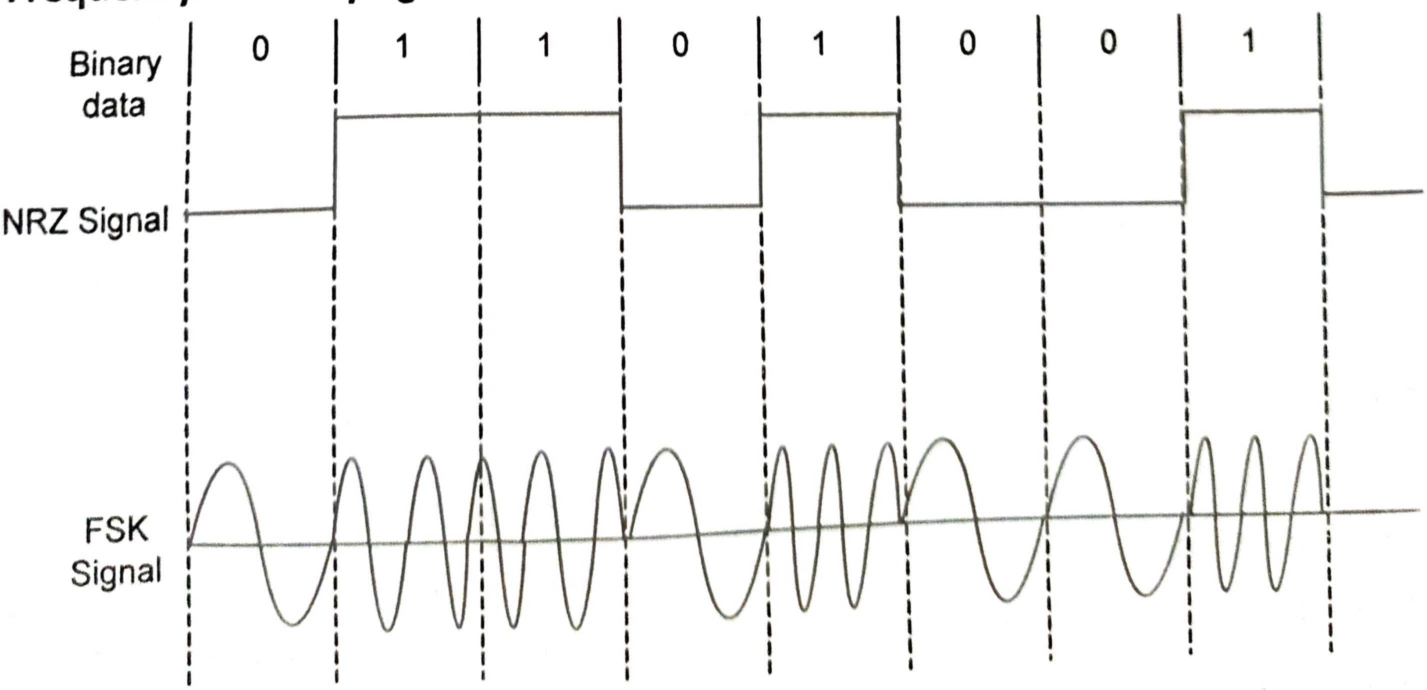


Fig. 8.2. Frequency Shift Keying

In frequency shift keying, the frequency of the carrier is shifted according to the binary symbols. The phase of the carrier is unaffected, (i.e) we have two different frequency signals according to binary symbols.

Phase Shift Keying

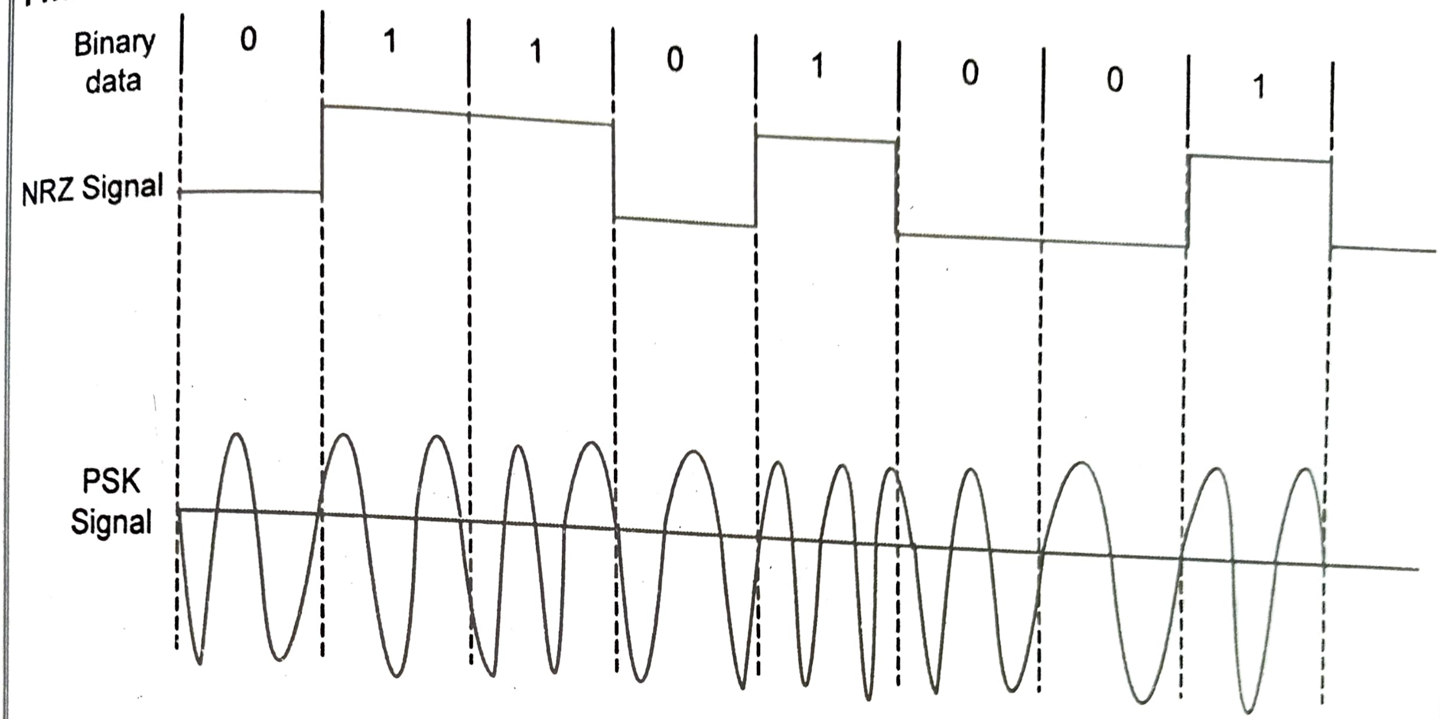


Fig. 8.3. Phase Shift Keying

In phase shift keying, the phase of the carrier signal is varied according to the binary input signal. In this technique, the digital data modulates phase of the carrier.

8.1.1. CLASSIFICATION OF DIGITAL MODULATION DETECTION TECHNIQUES

Digital modulation Detection techniques may be classified into coherent and non coherent techniques. Each of these two classes may be subdivided into binary and M-ary techniques.

1. Coherent Detection

Coherent detection is performed by cross correlating the received signal with each one of the replicas and then making a decision based on comparisons with preselected thresholds. It is also called synchronous detection.

2. Non-Coherent Detection

In this detection, knowledge of the carrier wave's phase is not required. Hence it is also called envelope detection. It is simple but higher probability of error.

There are a multiple of modulation/detection schemes available to the designer of a digital communication system required for data transmission over a band-pass channel.

The choice is made in favor of the scheme that attains as many of the following design goals as possible

1. Maximum data rate
2. Minimum probability of symbol error
3. Minimum transmitted power
4. Minimum channel bandwidth
5. Maximum resistance to interfering signals
6. Minimum circuit complexity

8.1.2. COMPARISON BETWEEN COHERENT & NON-COHERENT DETECTION

S. No.	Coherent Detection	Non-Coherent Detection
1.	Receiver has exact knowledge of the carrier wave is phase reference	Knowledge of the carrier wave's phase is not required
2.	It is also called as synchronous detection	It is also called as envelope detection
3.	The receiver is phase – locked to the transmitter	No phase synchronization between local oscillator used in the receiver
4.	Error probability is less	Error probability is high
5.	Receiver is more complicated	Receiver is less complicated

8.2. COHERENT BINARY MODULATION TECHNIQUES

As mentioned previously, binary modulation has three basic forms:

1. Amplitude-Shift Keying (ASK)
2. Phase-Shift Keying (PSK) and

3. Frequency-Shift Keying (FSK).

In this section, we present the noise analysis for the coherent detection of PSK and FSK signals, assuming an additive white Gaussian noise (AWGN) model.

8.3. COHERENT BINARY PHASE SHIFT KEYING (BPSK)

8.3.1. PRINCIPLE OF BPSK

In phase shift keying, the phase of the carrier signal is varied according to the binary input signal. In this technique, the digital data modulates phase of the carrier.

The carrier phase is changed between 0° and 180° based on binary input signal. So it is also called as binary phase shift keying (BPSK).

In a coherent Binary PSK system, the pair of signals, $S_1(t)$ and $S_2(t)$ used to represent binary symbol 1 and 0.

$$\text{For symbol } 1 \Rightarrow S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad \ldots (1)$$

$$\text{For symbol } 0 \Rightarrow S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad \ldots (2)$$

where

E_b is the transmitted signal Energy per bit

f_c – carrier frequency

Energy:

There is only one basic function of unit energy, namely

$$\phi_1(t) = +\sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \; 0 \le t < T_b \qquad \ldots (3)$$

Then we may expand the transmitted signals $S_1(t)$ and $S_2(t)$ in terms of $\phi_1(t)$ as follows.

$$S_1(t) = \sqrt{E_b}\, \phi_1(t) \qquad 0 \le t < T_b \qquad \ldots (4)$$

$$S_2(t) = -\sqrt{E_b}\, \phi_1(t) \qquad 0 \le t < T_b \qquad \ldots (5)$$

8.3.2. GEOMETRICAL REPRESENTATION

A Binary PSK system is characterized by having a signal space i.e. one dimensional (N = 1) and with two message points (i.e. M = 2) as shown below.

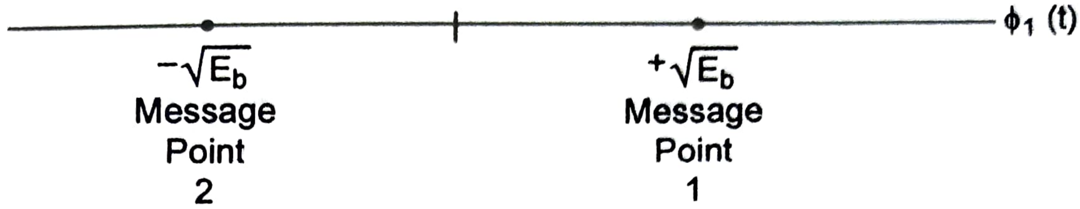


Fig. 8.4. Geometrical representation of BPSK Signal

The coordinates of the message points equal

$$S_{11} = \int_0^{T_b} S_1(t)\ \phi_1(t)\, dt$$

$$S_{11} = +\sqrt{E_b}$$

and
$$S_{21} = \int_0^{T_b} S_2(t)\ \phi_1(t)\, dt$$

$$= -\sqrt{E_b}$$

The message point corresponding to S_1 (t) is located at $S_{11} = +\sqrt{E_b}$ and the message point corresponding to S_2 (t) is located at $S_{21} = -\sqrt{E_b}$

At the receiver the point at $+\sqrt{E_b}$ on ϕ_1 (t) represents symbol 1 and point at $-\sqrt{E_b}$ represents symbol 0. The separation between these two points represent the isolation in symbols '1' and '0' in BPSK signal. This separation is normally called distance d, from above figure it is clear that the distance between the two points is

$$d = +\sqrt{E_b} - (-\sqrt{E_b})$$

$$d = +\sqrt{E_b} + \sqrt{E_b}$$

$$d = 2\sqrt{E_b}$$

As this distance '*d*' increases, the isolution between the symbols in BPSK signal is more. Therefore probability of error reduces.

8.3.3. GENERATION

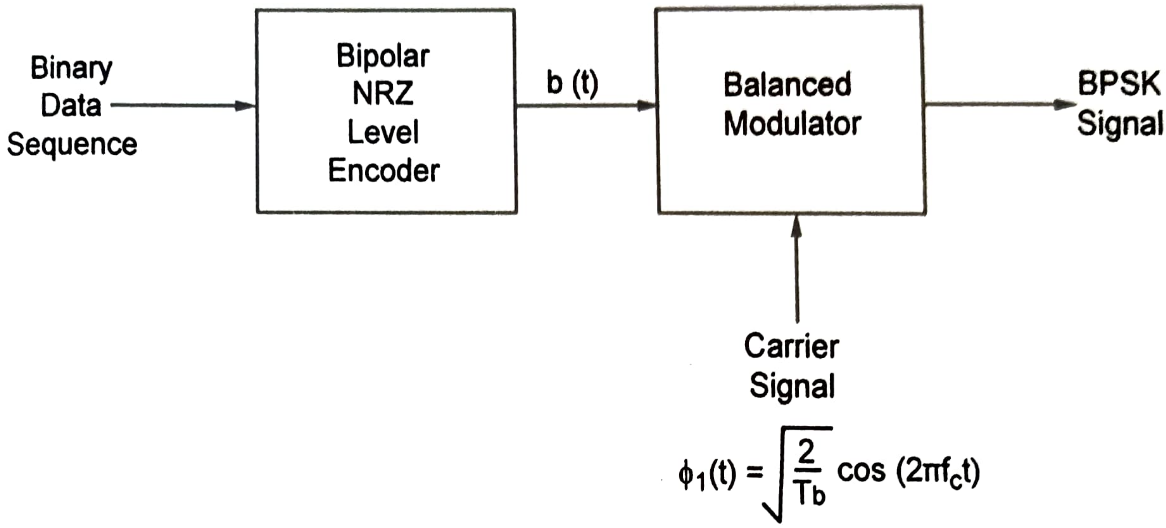


Fig. 8.5. BPSK generation scheme

The input binary sequence symbol 1 can be represented as $+\sqrt{E_b}$ and symbol 0 represented as $-\sqrt{E_b}$. This binary wave $b(t)$ and carrier signal are applied to a balanced modulator. The carrier and the liming pulses used to generate the binary wave are usually extracted from a common master clock. The desired BPSK wave is obtained at the modulator output.

8.3.4. DETECTION

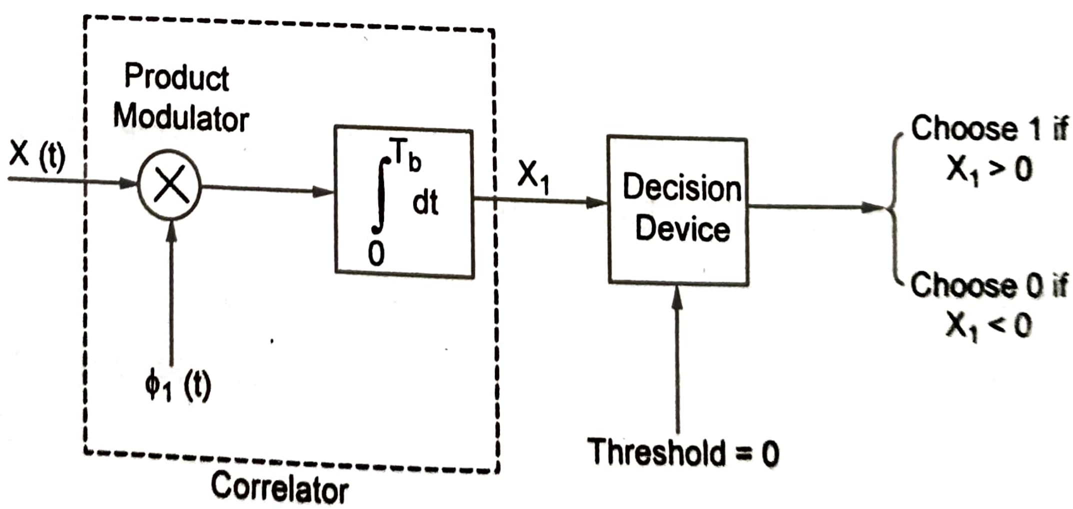


Fig. 8.6. Binary PSK Receiver

- To detect the original binary sequence from noisy BPSK signal $x(t)$, the received signal from the channel is applied to a correlator.
- A locally generated coherent reference signal $\phi_1(t)$, is also applied to the correlator. The output of the multiplier is integrated over one bit period (T_b). The correlator output x_1 is compared with a threshold of zero volts.

1. If the correlator output is exceeded the threshold, the receiver decides in favour of symbol "1"

$$x_1 > 0 \approx 1$$

2. If the correlator output is not exceeded the threshold, the receiver decides in favour of symbol "0".

$$x_1 < 0 \approx 0$$

3. If x_1 is exactly zero, the receiver makes a random guess in favour of 0 or 1.

$$x_1 = 0 \approx 0 \text{ or } 1$$

8.3.5. BANDWIDTH

The Bandwidth of BPSK signal is

$$\text{BW} = \text{higher frequency} - \text{lowest frequency in the main lobe.}$$

$$\text{BW} = (f_0 + f_b) - (f_0 - f_b)$$

$$\boxed{\text{BW} = 2f_b}$$

where $f_b = \dfrac{1}{\text{T}_b}$ = maximum frequency in the baseband signal

f_0 = carrier frequency

∴ The maximum bandwidth of PSK signal is equal to twice of the highest frequency contained in baseband signal.

8.3.6. BAUD RATE

For BPSK, the modulated waveform changes at the rate equal to bit rate. Hence

$$\boxed{\text{Baud rate} = f_b}$$

8.3.7. POWER SPECTRAL CHARACTERISTICS

In BPSK Generation, from the modulator we see that the complex envelope of a binary PSK wave consists of an in-phase component only. Depending on whether we have a symbol 1 or symbol 0 at the modulator input during the signaling interval $0 \leq t \leq \text{T}_b$, we find that this in-phase component equals $+\,g(t)$ or $-\,g(t)$, where $g(t)$ is the symbol shaping function denoted by

$$g(t) = \begin{cases} \sqrt{\dfrac{2\,E_b}{T_b}} & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases}$$

We assume that the input binary wave is random, with symbol 1 and 0 equally likely and the symbols transmitted during the different time slots being statistically independent.

The power spectral density of a random binary wave is equal to energy spectral density of the symbol shaping function divided by the symbol duration.

The energy spectral density of a Fourier transformable signal $g(t)$ is defined as the squared magnitude of the signals Fourier transform. Hence the baseband power spectral density of a BPSK wave equals

$$S_B(f) = \frac{2\,E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2}$$

$$= 2\,E_b\,\text{sinc}^2(T_b f)$$

8.3.8. PROBABILITY OF ERROR

The decision region associated with symbol 1 or signal $S_1(t)$ is described by

$$Z_1 \;:\; 0 < x_1 < 1$$

where x_1 is the observation scalar

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

where $x(t)$ is the received signal

when symbol 0 or signal $S_2(t)$ is transmitted, is defined by

$$f x_1(x_1 | 0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - S_{21})^2\right]$$

$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b}\,)^2\right]$$

The conditional probability of the receiver deciding in favor of symbol 1, given that symbol 0 was transmitted, is therefore

$$P_e(0) = \int_0^{\infty} f x_1 (x_1 | 0)\, d x_1$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left[-\frac{1}{N_0} (x_1 + \sqrt{E_b}\,)^2\right] d x_1$$

Putting $Z = \frac{1}{\sqrt{N_0}} (x_1 + \sqrt{E_b}\,)$ and changing the variable of integration from x_1 to Z, we may rewrite above equation in the form of

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-Z^2)\, dz$$

$$= \frac{1}{2}\, erfc\left[\sqrt{\frac{E_b}{N_0}}\right]$$

where

$erfc\left[\sqrt{\frac{E_b}{N_0}}\right]$ is the complement error function

Similarly

$$P_e(1) = \frac{1}{2}\, erfc\left[\sqrt{\frac{E_b}{N_0}}\right]$$

By averaging probability of symbol error for BPSK equals

$$\boxed{P_e = \frac{1}{2}\, erfc\left[\sqrt{\frac{E_b}{N_0}}\right]}$$

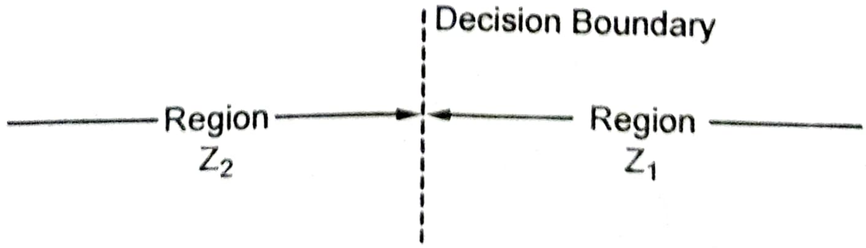


Fig. 8.7.

8.3.9. ADVANTAGES

- It gives minimum possibility of error
- It has a very good noise immunity
- It has lower bandwidth compared to BFSK signal
- It has the best performance of all the systems in presence of noise

8.3.10. DISADVANTAGES

- Generation and detection is not easy
- Ambiguity in output signal

8.5. COHERENT QUADRATURE-MODULATION TECHNIQUES

There are two goals are important in the design of a digital communication system such as

1. Very low probability of error
2. Efficient utilization of channel bandwidth

QPSK is an example of bandwidth conserving modulation schemes for the transmission of binary data. QPSK is an extension of binary PSK.

8.5.1. QUADRIPHASE-SHIFT KEYING (QPSK)

In quadriphase-shift keying (QPSK), the phase of the carrier takes on one of four equally spaced values, such as $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$ as shown by

$$S_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\dfrac{\pi}{4}\right] & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases} \quad \ldots(1)$$

where,

i = 1, 2, 3, 4

E - is the transmitted signal energy per symbol

T - is the symbol duration

f_c - carrier frequency equals to $\dfrac{n_c}{T}$ for some fixed integer n_c.

Each possible value of the phase corresponds to a unique pair of bits called a dibit. For example we may choose the foregoing set of phase values to represent the gray encoded set of dibits 10, 00, 01 and 11.

Using trigonometric identify, we may rewrite equation (1) in the equivalent form of

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t), & \text{for } 0 \le t \le T, \\ 0, & \text{elsewhere} \end{cases} \quad \ldots(2)$$

1. There are only two orthonormal basis functions, $\phi_1(t)$ and $\phi_2(t)$. The appropriate forms for $\phi_1(t)$ and $\phi_2(t)$ are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \le t \le T$$

and

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \le t \le T$$

2. There are four message points and the associated signal vectors are defined by

$$S_i = \begin{bmatrix} \sqrt{E} \cos\left[(2i-1)\frac{\pi}{4}\right] \\ -\sqrt{E} \sin\left[(2i-1)\frac{\pi}{4}\right] \end{bmatrix} \qquad i = 1, 2, 3, 4$$

S_{i1} and S_{i2} are called elements of the signal vectors and their values are shown in below table. The first two columns of this table give the associated dibits and phase of the QPSK signal.

Signal space characterization of QPSK

Input dibit	Phase of QPSK signal	Coordinates of message points	
		S_{i1}	S_{i2}
1 0	$\frac{\pi}{4}$	$+\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
0 0	$\frac{3\pi}{4}$	$-\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
0 1	$\frac{5\pi}{4}$	$-\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$
1 1	$\frac{7\pi}{4}$	$+\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$

8.5.2. GEOMETRICAL REPRESENTATION

A QPSK signal is characterized by having a two dimensional signal constellation (N = 2) and four message points (M = 4) as shown below.

The decision regions are quadrants whose vertices coincide with the origin. These regions are marked as Z_1, Z_2, Z_3 and Z_4.

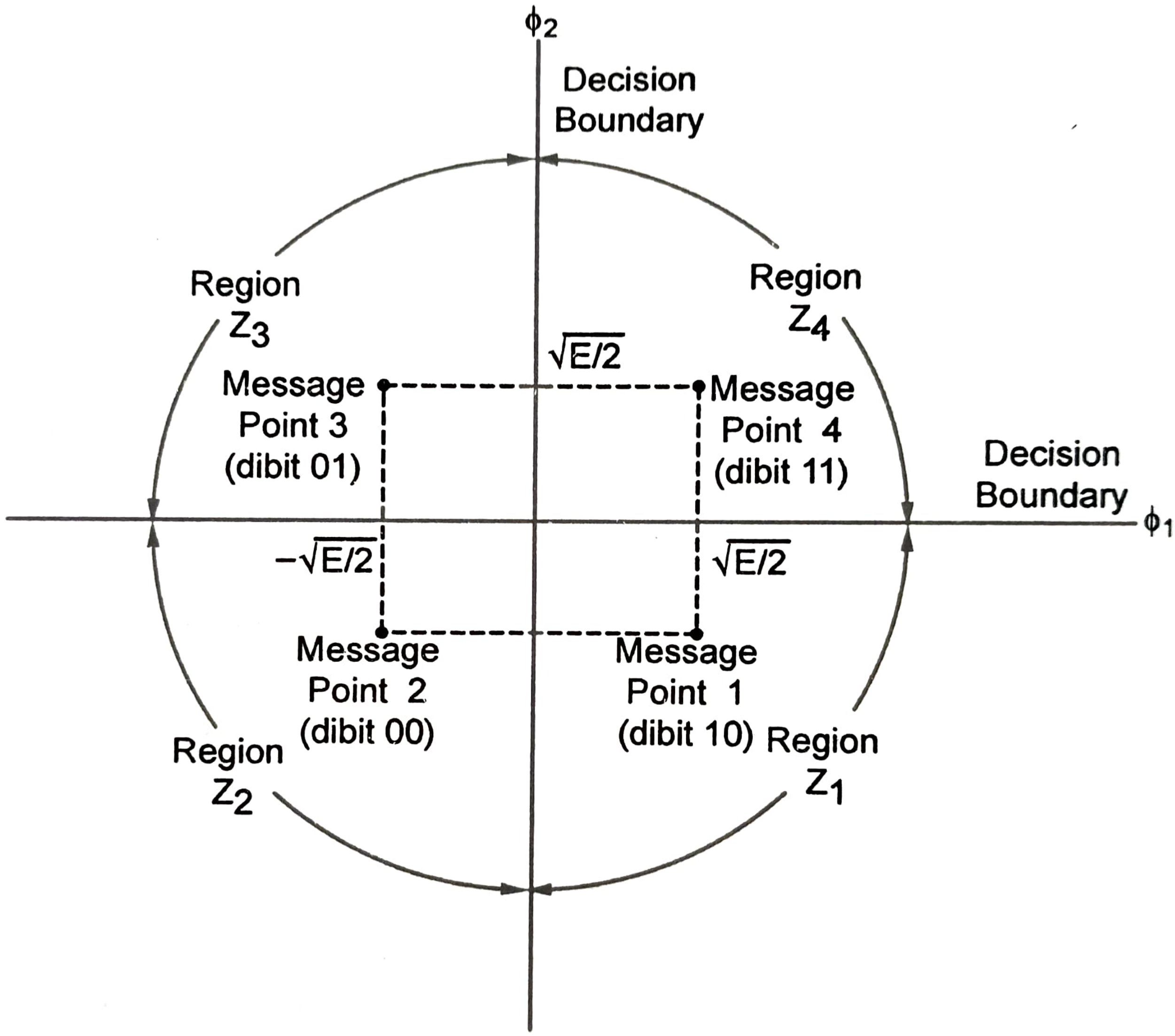


Fig. 8.12. Signal space diagram for coherent QPSK system

The received signal $x(t)$ is defined by

$$x(t) = S_i(t) + \omega(t) \qquad 0 \le t \le T$$

$$i = 1, 2, 3, 4$$

Where

$\omega(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $\frac{N_0}{2}$.

The observation vector x has two elements x_1 and x_2 that are defined by

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] + \omega_1$$

and

$$x_2 = \int_0^T x(t)\,\phi_2(t)\,dt = -\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] + \omega_2$$

where $i = 1, 2, 3, 4$

8.5.3. GENERATION

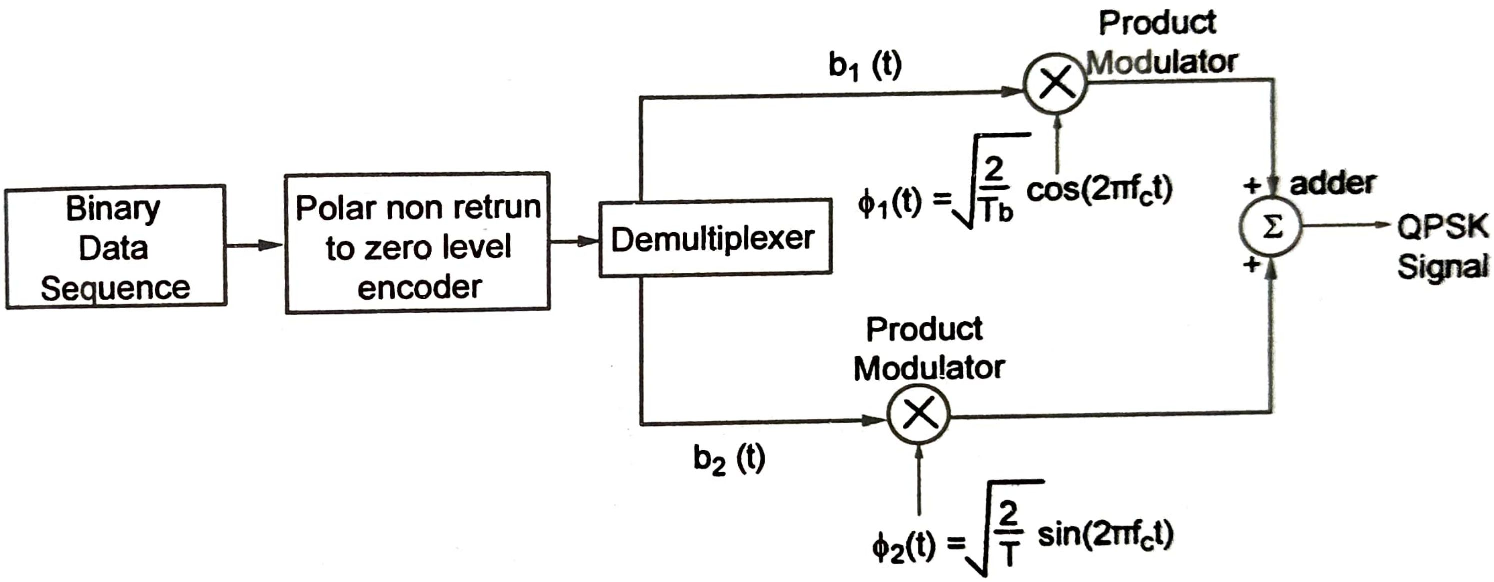


Fig. 8.13. QPSK transmitter

NRZ Encoder

The input binary sequence is first transformed into polar form by a NRZ encoder. Thus symbols 1 and 0 are represented by $+\sqrt{E_b}$ and $-\sqrt{E_b}$.

Demultiplexer

The NRZ encoder output is given to demultiplexer to divide the binary wave into two separate binary waves consisting of the odd and even numbered input bits. These two binary waves are denoted by $b_1(t)$ and $b_2(t)$.

The amplitudes of $b_1(t)$ and $b_2(t)$ equal S_{i1} and S_{i2}, depending on the particular dibit that is being transmitted. The two binary waves $b_1(t)$ and $b_2(t)$ are used to modulate a pair of quadrature carriers or orthonormal basis functions $\phi_1(t)$

equal to $\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $\phi_2(t)$ equal to $\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$. The two binary waves are added to produce the desired QPSK wave.

Symbol Duration

For a QPSK symbol duration is twice as long as the bit duration T_b of the input binary wave (i.e) for a given rate $\frac{1}{T_b}$, a QPSK wave requires half the transmission bandwidth of the corresponding binary PSK wave.

8.5.4. DETECTION

The QPSK receiver consists of a pair of correlators with a common input and supplies with a locally generated pair of coherent reference signals $\phi_1(t)$ and $\phi_2(t)$.

The correlator outputs x_1 and x_2 are each compared with a threshold of zero volts.

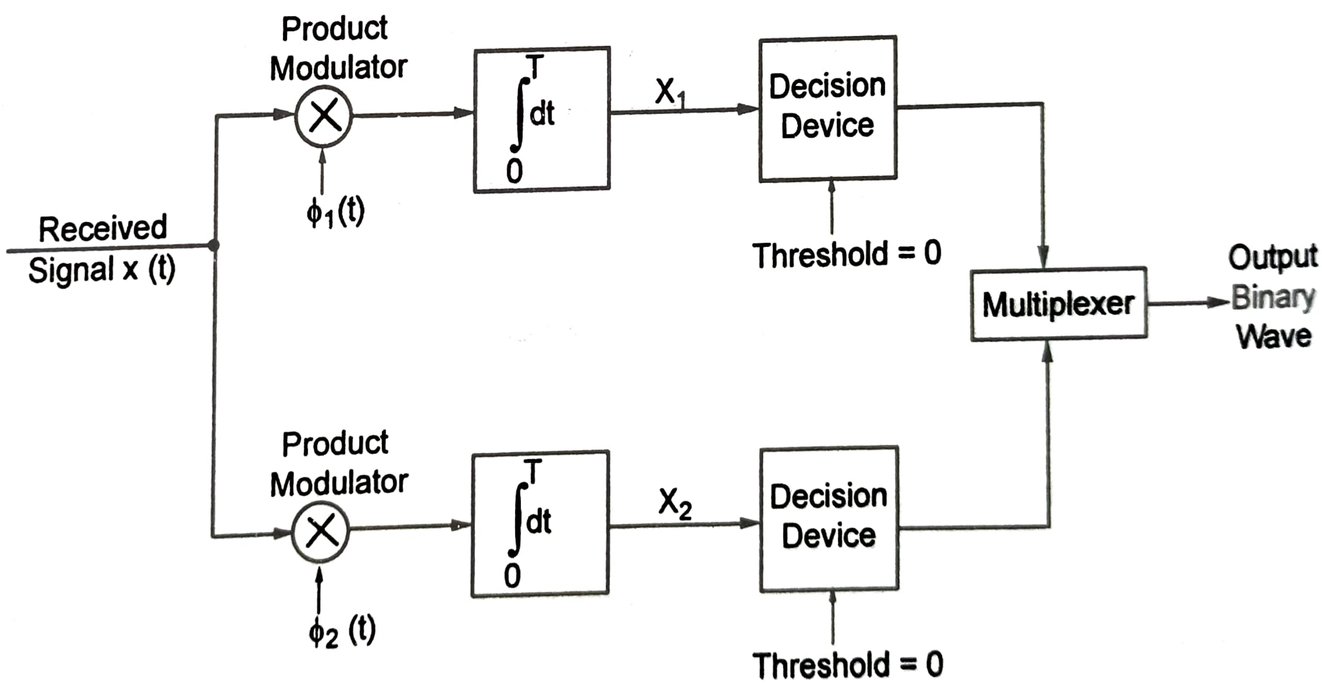


Fig. 8.14. QPSK Receiver

1. If $x_1 > 0$, a decision is made in favor of symbol 1 for the upper or in-phase channel output, but if $x_1 < 0$ a decision is made in favor of symbol 0.
2. If $x_2 > 0$, a decision is made in favor of symbol 1 for the lower or quadrature channel output, but if $x_2 < 0$, a decision is made in favor of symbol 0.

Multiplexer

The two binary sequence x_1 and x_2 are combined in a multiplexer to reproduce the original binary sequence at the transmitter input with the minimum probability of symbol error.

8.5.5. BANDWIDTH

In QPSK the two waveforms $\phi_1(t)$ and $\phi_2(t)$ form the baseband signals. One bit period for both these signals is equal to $2\,T_b$. Therefore bandwidth of QPSK is

$$\text{BW} = 2 \times \frac{1}{2\,T_b} \text{ or BW} = f_b$$

The bandwidth of QPSK signal is half of the bandwidth of PSK signal.

8.5.6. POWER SPECTRAL CHARACTERISTICS

We know that QPSK has two components such as In-phase and quadrature components. Based on these two components we have following observations.

1. Depending on the dibit sent during the signaling interval $-T_b \le t \le T_b$, the inphase component equals $+g(t)$ or $-g(t)$ and similarly for the quadrature component.

   The symbol shaping function $g(t)$ can be denoted as

$$g(t) = \begin{cases} \sqrt{\dfrac{E}{T}} & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

Hence, in-phase and quadrature components have a common power spectral density namely E sinC² (T_f)

2. The in-phase and quadrature components are statistically independent. The baseband power spectral density of the QPSK signal equals the sum of the individual power spectral densities of the in-phase and quadrature components.

$$S_B(f) = 2E\,\text{sinc}^2(T_f)$$
$$= 4\,E_b\,\text{sinc}^2(2\,T_b f)$$

8.5.7. PROBABILITY OF ERROR

The transmitted signal can be partition into four regions.

1. The set of points closest to the message point associated with signal vector S_1.
2. The set of points closest to the message point associated with signal vector S_2.
3. The set of points closest to the message point associated with signal vector S_3.
4. The set of points closest to the message point associated with signal vector S_4.

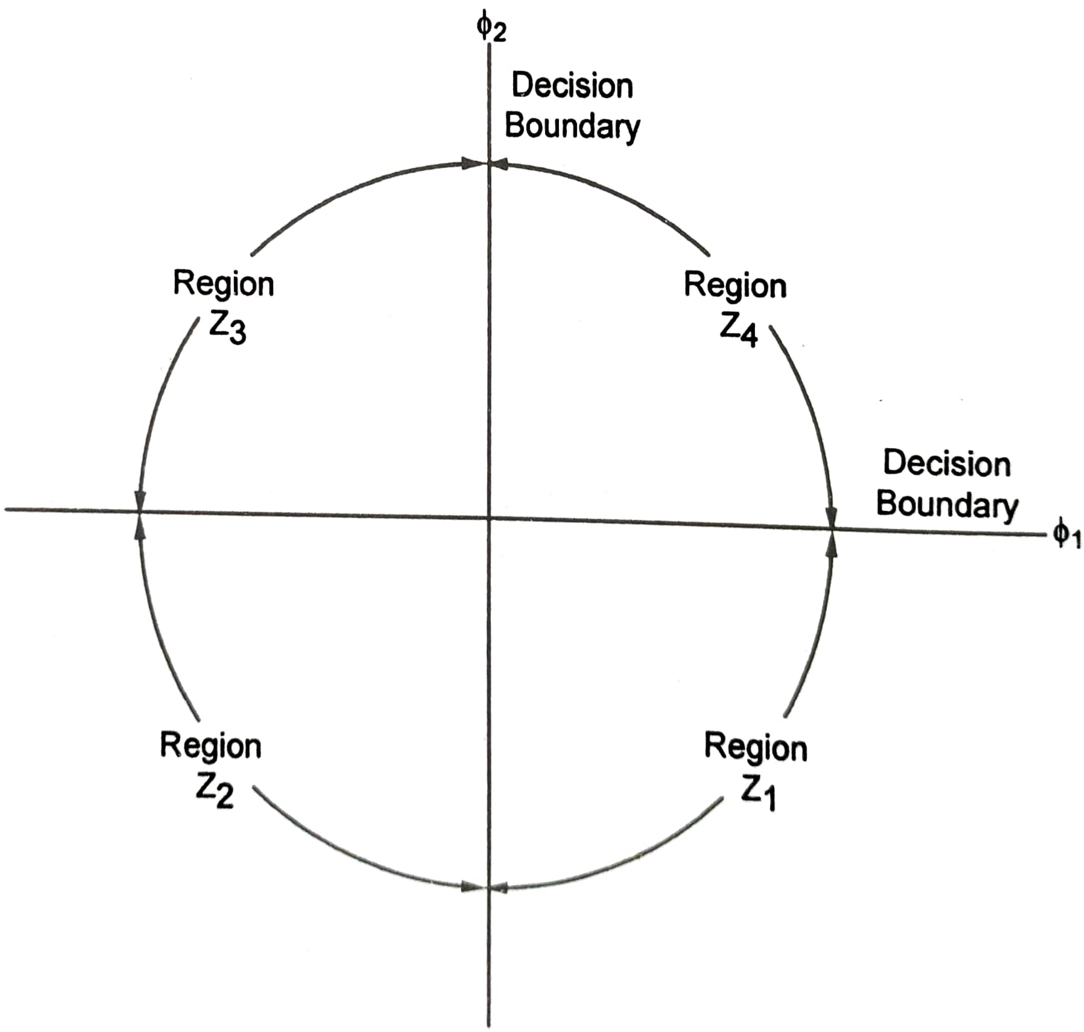


Fig. 8.15. Space diagram

$S_1(t)$ was transmitted if the received signal point associated with the observation vector X falls inside region Z_1. $S_2(t)$ was transmitted if the received signal point falls inside region Z_2 and so on.

The receiver will make a correct decision to provided proper symbol transmission. For a correct decision when signal $S_4(t)$ is transmitted then the elements x_1 and x_2 of the observation vector X must be both positive as shown in Fig.8.16.

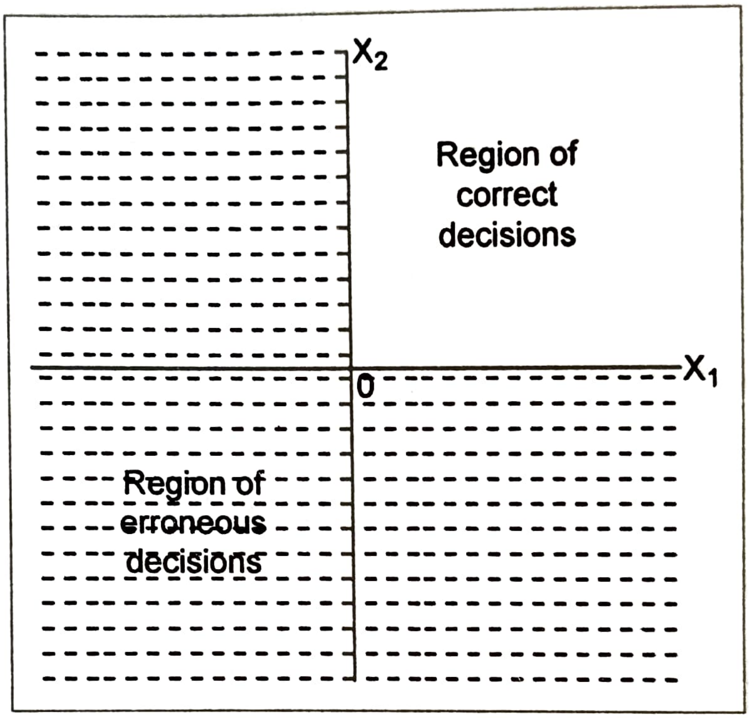


Fig. 8.16. Illustrating the region of correct decision and the region of erroneous decision, given that the signal $S_4(t)$ was transmitted.

The probability of a correct decision P_c equals the conditional probability of the joint event $x_1 > 0$ and $x_2 > 0$, given that signal $S_4(t)$ was transmitted.

Both X_1 and X_2 are Gaussian random variables with a conditional mean equal to $\sqrt{\frac{E}{2}}$ and a variance equal to $\frac{N_0}{2}$.

$$P_c = \int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(x_1 - \sqrt{E/2}\,)^2}{N_0}\right] dx_1 \cdot \int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(x_2 - \sqrt{E/2}\,)^2}{N_0}\right] dx_2 \quad ...(1)$$

In above equation, the first integral on the right side is the conditional probability of the event $x_1 > 0$ and the second integral is the conditional probability of the event $x_2 > 0$, both given that signal $S_4(t)$ was transmitted.

Let, $$\frac{x_1 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}} = \frac{x_2 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}$$

$$= Z$$

Then, changing the variables of integration from x_1 and x_2 to Z, then we can rewrite equation (1) in the form

$$P_c = \left[\frac{1}{\sqrt{\pi}} \int_{-\sqrt{E/2\,N_0}}^{\infty} \exp(-Z^2)\, dz\right]^2 \quad \ldots(2)$$

However, from the definition of the complementary error function we find that

$$\frac{1}{\sqrt{\pi}} \int_{\sqrt{E/2\,N_0}}^{\infty} \exp(-Z^2)\, dz = 1 - \frac{1}{2}\, erfc\left[\frac{\sqrt{E}}{2\,N_0}\right] \quad \ldots(3)$$

Accordingly, we have

$$P_c = \left[1 - \frac{1}{2}\, erfc\sqrt{\frac{E}{2\,N_0}}\right]^2$$

$$P_c = 1 - erfc\sqrt{\frac{E}{2\,N_0}} + \frac{1}{4}\, erfc^2\left(\sqrt{\frac{E}{2\,N_0}}\right) \quad \ldots(4)$$

The average probability of symbol error for coherent QPSK is

$$P_e = 1 - P_c$$

where P_c = probability of a correct decision

$$P_e = 1 - \left[1 - erfc\left[\sqrt{\frac{E}{2\,N_0}}\right] + \frac{1}{4}\, erfc^2\left[\sqrt{\frac{E}{2\,N_0}}\right]\right]$$

$$P_e = erfc\left[\sqrt{\frac{E}{2\,N_0}}\right] - \frac{1}{4}\, erfc^2\left[\sqrt{\frac{E}{2\,N_0}}\right] \quad \ldots(5)$$

In the region where $\frac{E}{2\,N_0} >> 1$

then we may ignore the second term on the right side of equation (5) and approximate the formula for the average probability of symbol error for coherent QPSK as

$$P_e \approx erfc\left[\sqrt{\frac{E}{2N_0}}\right] \qquad ...(6)$$

In a QPSK system, we know that there are two bits per symbol. So the transmitted signal per symbol is twice the signal energy per bit that is

$$E = 2E_b \qquad ...(7)$$

Expressing the average probability of symbol error interms of the ratio $\frac{E_b}{N_0}$, we may write

$$P_e = erfc\left[\sqrt{\frac{E_b}{N_0}}\right] \qquad ...(8)$$

8.5.8. ADVANTAGES

1. For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK
2. Carrier power remains constant
3. Effective utilization of available bandwidth is possible.
4. Low error probability.
5. Because of reduced bandwidth, the information transmission rate of QPSK is higher.

8.5.9. DISADVANTAGES

1. Interchannel interference is large due to side lobes.
2. Complex circuits are needed for generation and detection.

8.9. QUADRATURE AMPLITUDE MODULATION (QAM)

Quadrature Amplitude Modulation or QAM is a form of modulation which is widely used for modulating data signals onto a carrier used for radio communications. It is widely used because it offers advantages over other forms of data modulation such as PSK, although many forms of data modulation operate along each other.

Quadrature Amplitude Modulation, QAM is a signal in which two carriers shifted in phase by 90 degrees are modulated and the resultant output consists of both amplitude and phase variations. In view of the fact that both amplitude and phase variations are present it may also be considered as a mixture of amplitude and phase modulation.

i.e $\boxed{\text{QAM} = \text{ASK} + \text{PSK}}$

A motivation for the use of quadrature amplitude modulation comes from the fact that a straight amplitude modulated signal, i.e. double sideband even with a suppressed carrier occupies twice the bandwidth of the modulating signal. This is very wasteful of the available frequency spectrum. QAM restores the balance by placing two independent double sideband suppressed carrier signals in the same spectrum as one ordinary double sideband suppressed carrier signal.

8.9.1. ANALOG AND DIGITAL QAM

Quadrature amplitude modulation, QAM may exist in what may be termed either analog or digital formats. The analog versions of QAM are typically used to allow multiple analog signals to be carried on a single carrier. For example it is used in PAL and NTSC television systems, where the different channels provided by QAM enable it to carry the components of chroma or colour information. In radio applications a system known as C-QUAM is used for AM stereo radio. Here the different channels enable the two channels required for stereo to be carried on the single carrier.

Digital formats of QAM are often referred to as "Quantised QAM" and they are being increasingly used for data communications often within radio communications systems. Radio communications systems ranging from cellular technology as in the case of LTE through wireless systems including WiMAX, and Wi-Fi 802.11 use a variety of forms of QAM, and the use of QAM will only increase within the field of radio communications.

8.9.2. DIGITAL / QUANTISED QAM BASICS

Quadrature amplitude modulation, QAM, when used for digital transmission for radio communications applications is able to carry higher data rates than ordinary amplitude modulated schemes and phase modulated schemes. As with phase shift keying, etc, the number of points at which the signal can rest, i.e. the number of points on the constellation is indicated in the modulation format description, e.g. 16QAM uses a 16 point constellation.

When using QAM, the constellation points are normally arranged in a square grid with equal vertical and horizontal spacing and as a result the most common forms of QAM use a constellation with the number of points equal to a power of 2 i.e. 4, 16, 64 . . . .

By using higher order modulation formats, i.e. more points on the constellation, it is possible to transmit more bits per symbol. However the points are closer together and they are therefore more susceptible to noise and data errors.

Normally a QAM constellation is square and therefore the most common forms of QAM16QAM, 64QAM and 256QAM.

The advantage of moving to the higher order formats is that there are more points within the constellation and therefore it is possible to transmit more bits per symbol. The downside is that the constellation points are closer together and therefore the link is more susceptible to noise. As a result, higher order versions of QAM are only used when there is a sufficiently high signal to noise ratio.

To provide an example of how QAM operates, the constellation diagram below table shows the values associated with the different states for a 16QAM signal. From this it can be seen that a continuous bit stream may be grouped into fours and represented as a sequence.

8.9.3. TYPES OF QAM

Name	Bits per symbol (N)	Number of symbols (M) ($M = 2^N$)
4 QAM	2	4
8 QAM	3	8
16 QAM	4	16
32 QAM	5	32
64 QAM	6	64

8.9.4. PRINCIPLE OF QAM

The general form of QAM is defined by the transmitted signal such as

$$S_i(t) = \sqrt{\frac{2E_0}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}}\, b_i \sin(2\pi f_c t) \qquad 0 \le t \le T \quad \ldots(1)$$

Where, E_0 is the energy of the signal with the lowest amplitude a_i & b_i are a pair of independent intergers chosen in accordance with the location of the pertinent message point.

The signal $S_i(t)$ consists of two phase quadrature carriers, each of which is modulated by a set of discrete amplitudes, hence it called quadrature amplitude modulation.

The signal $S_i(t)$ can be expanded in terms of a pair of basis functions

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \le t \le T \qquad \ldots(2)$$

and

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \le t \le T \qquad \ldots(3)$$

The coordinates of the i^{th} message point are $a_i\sqrt{E}$ and $b_i\sqrt{E_0}$, where (a_i, b_i) is an element of the L-by-L matrix

$$\{a_i, b_i\} = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\ \vdots & \vdots & & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1) \end{bmatrix}$$

Where, $L = \sqrt{M}$

For, example, For the 16 point QAM L = 4 then we have the matrix

$$\{a_i, b_i\} = \begin{bmatrix} (-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\ (-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\ (-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\ (-3, -3) & (-1, -3) & (1, -3) & (3, -3) \end{bmatrix}$$

8.9.5. GEOMETRICAL REPRESENTATION

Case 1: M = 16

The signal constellation for QAM consists of a square lattice of message points. Below Figure shows signal constellation for M = 16.

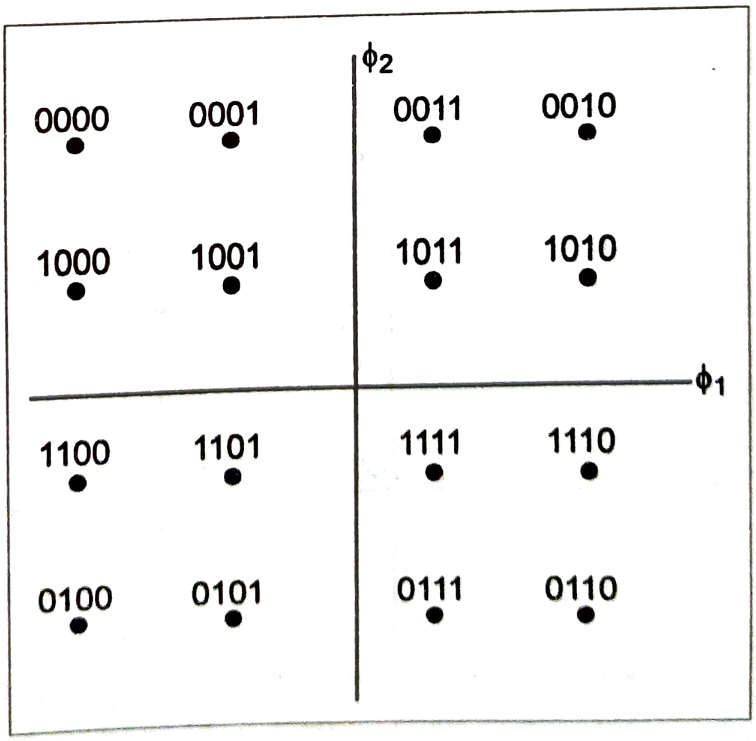
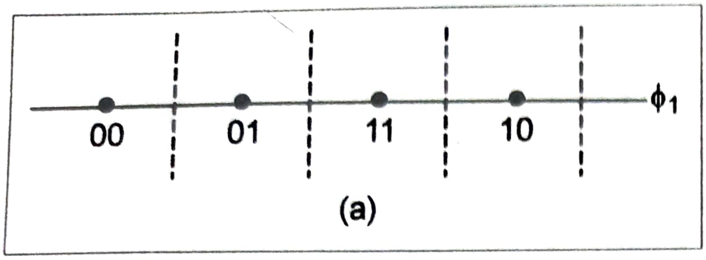


Fig. 8.20. Signal constellation of QAM for M=16

For M = 16, the corresponding signal constellations for the in-phase and quadrature components of the amplitude phase modulated wave are shown below.



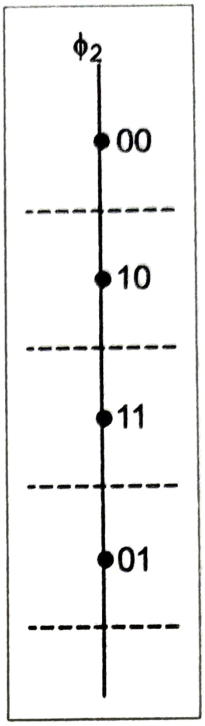


Fig. 8.21. Decomposition of signal constellation of QAM (for M=16) into two signal – space diagrams for (a) in-phase comment $\phi_1(t)$ and (b) quadrature comment $\phi_2(t)$

Case 2: M = 4

The signal constellation for M = 4 is shown in below Figure, which is recognized to be the same as QPSK.

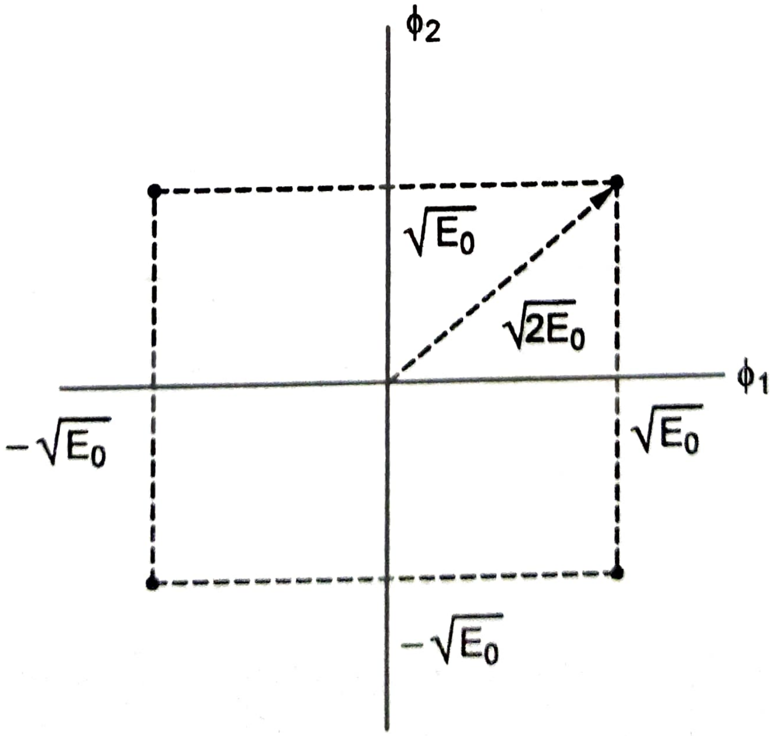


Fig. 8.22. Signal constellation for the QAM for M = 4

8.9.6. GENERATION

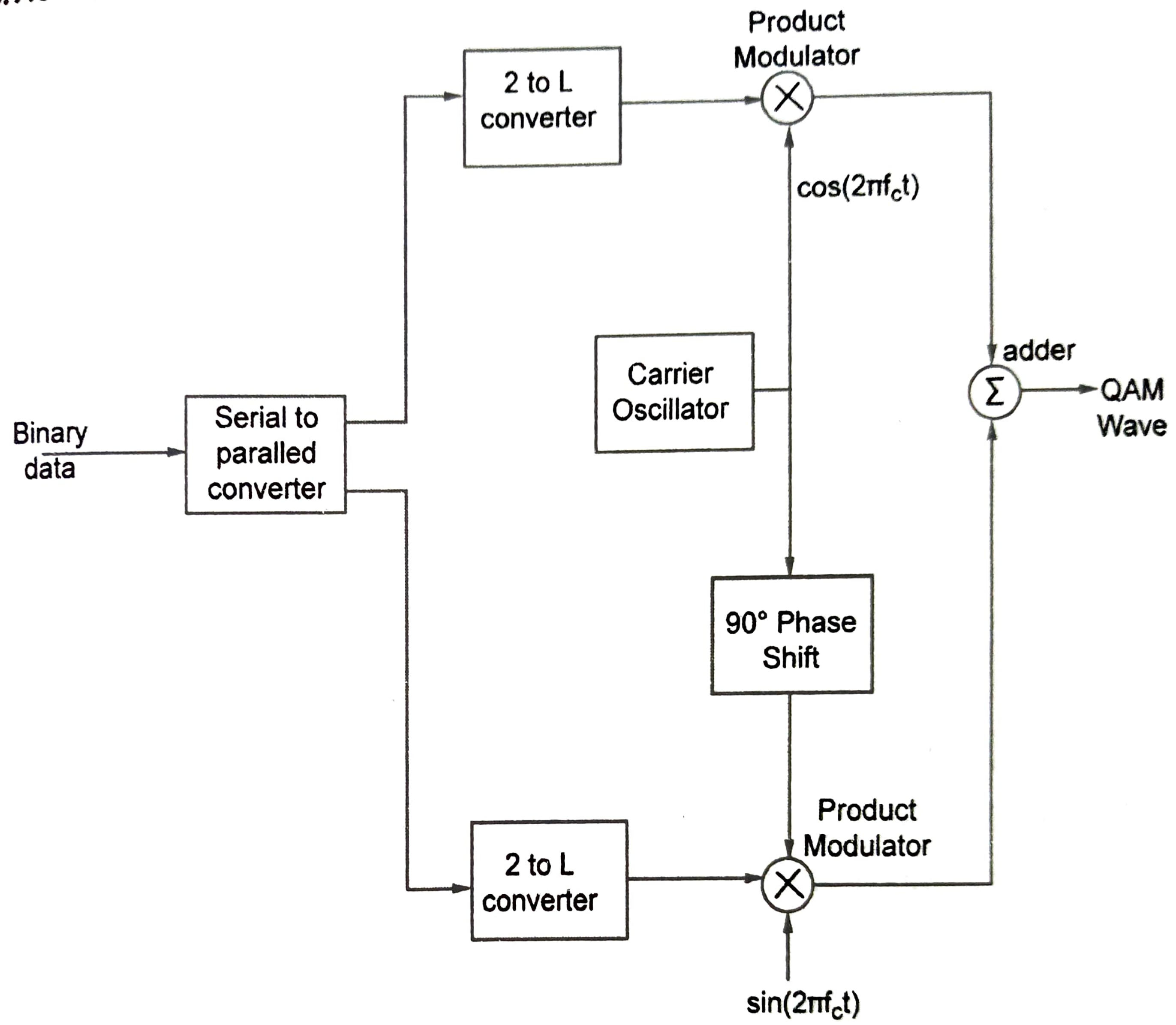


Fig. 8.23. QAM System Transmitter

Serial To paralled Converter

Serial to paralled converter accepts a binary sequence at a bit rate $R_b = 1/T_b$ and produces two paralled binary sequences whose bit rates are $R_b/2$ each,

2-to-L Converters

In 2 to L converters, where $L = \sqrt{M}$, generate polar L-Level signals in response to the respective in-phase and quadrature channel inputs.

Quadrature carrier multiplexing of the two polar L-Level signals to generate desired QAM signal.

8.9.7. DETECTION

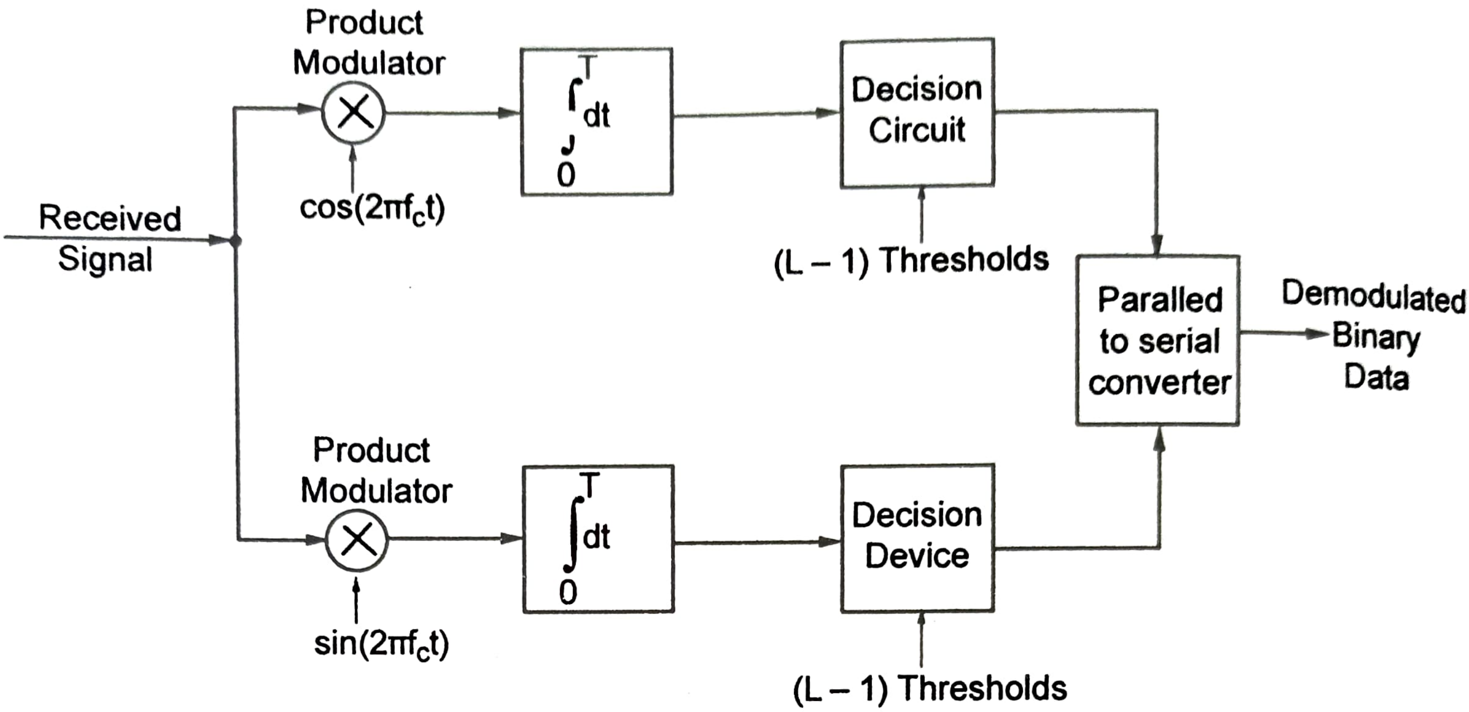


Fig. 8.24. QAM System Receiver

Decision Circuit

Decoding of each baseband channel is accomplished at the output of the decision circuit, which is designed to compare the L-Level signals against L-1 decision thresholds.

Paralled-to-Serial Converter

The two binary sequences are combined in the parallel to serial converter to Reproduce the original binary sequence.

8.9.8. BANDWIDTH

Bandwidth of QAM signal will be

$$\boxed{\mathrm{BW} = \frac{2 f_b}{\mathrm{W}}}$$

8.9.9. POWER SPECTRAL CHARACTERISTICS

Power spectral density of baseband QAM signal will be

$$S(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s} \right]^2$$

We know that $E_s = P_s T_s$

The above equation gives power spectral density of QAM when they modulate the carrier, the main lobe given by above equation is shifted at carrier frequency f_c.

$$S(f) = \frac{P_s T_s}{2}\left[\frac{\sin\pi(f-f_c)T_s}{\pi(f-f_c)T_s}\right]^2 + \frac{P_s T_s}{2}\left[\frac{\sin\pi(f+f_c)T_s}{\pi(f+f_c)T_s}\right]^2$$

This equation gives power spectral density of QAM signal.

8.9.10. PROBABILITY OF ERROR

To calculate the probability of symbol error for QAM, we proceed as follows.

1. The in-phase and quadrature components of QAM are independent so the probability of correct detection for such a scheme may be written as

$$P_c = (1 - P_e')^2 \qquad \ldots(1)$$

where P_e' is the probability of symbol error for either component.

2. The signal constellation for the in-phase or quadrature component has a geometry similar to PAM with a corresponding number of amplitude levels.

$$P_e' = \left[1 - \frac{1}{L}\right] erfc\left[\sqrt{\frac{E_0}{N_0}}\right] \qquad \ldots(2)$$

where L is the square root of M

3. The probility of symbol error for QAM is given by

$$\begin{aligned} P_e &= 1 - P_c \\ &= 1 - (1 - P_e')^2 \qquad \ldots(3) \\ &\simeq 2P_e' \end{aligned}$$

where it is assumed that P_e' is small compared to unity. The probability of symbol error for QAM is given by

$$P_e \simeq 2\left[1 - \frac{1}{\sqrt{M}}\right] erfc\left[\sqrt{\frac{E_0}{N_0}}\right] \qquad \ldots(4)$$

8.9.11. TRANSMITTED ENERGY

The trasmitted energy in QAM is variable in its instantaneous value depends on the particular symbol transmitted. So we can express P_e interms of the average value of the transmitted energy rather than E_0.

Assuming that the L amplitude levels of the in-phase or quadrature component are equally likely, we have

$$E_{av} = 2\left[\frac{2\,E_0}{L}\sum_{i=1}^{L/2}(2\,i-1)^2\right] \qquad ...(5)$$

Summing the series in equation (5) we get

$$E_{av} = \frac{2\,(L^2-1)\,E_0}{3}$$

$$= \frac{2\,(M-1)\,E_0}{3} \qquad ...(6)$$

According, we may rewrite equation (4) interms of E_{av} as

$$P_e \simeq 2\left[1-\frac{1}{\sqrt{M}}\right] erfc\left[\sqrt{\frac{3\,E_{av}}{2\,(M-1)\,N_0}}\right] \qquad ...(7)$$

which is the desired result

8.9.12. ADVANTAGES

- It has better noise immunity than PSK
- It is easy to design than PSK

8.9.13. DISADVANTAGES

- It has higher error probability than QPSK
- Relatively complex

QAM vs other modulation formats

As there are advantages and disadvantages of using QAM it is necessary to compare QAM with other modes before making a decision about the optimum mode. Some radio communications systems dynamically change the modulation scheme dependent upon the link conditions and requirements - signal level, noise, data rate required, etc.

The table below compares various forms of modulation.

Summary of types of modulation with data capacities				
Modulation	**Bits per symbol**	**Error margin**		**Complexity**
OOK	1	1/2	0.5	Low
BPSK	1	1	1	Medium
QPSK	2	$1 / \sqrt{2}$	0.71	Medium
16 QAM	4	$\sqrt{2} / 6$	0.23	High
64QAM	6	$\sqrt{2} / 14$	0.1	High

Typically it is found that if data rates above those that can be achieved using 8-PSK are required, it is more usual to use quadrature amplitude modulation. This is because it has a greater distance between adjacent points in the I - Q plane and this improves its noise immunity. As a result it can achieve the same data rate at a lower signal level.

However the points no longer the same amplitude. This means that the demodulator must detect both phase and amplitude. Also the fact that the amplitude varies means that a linear amplifier is required to amplify the signal.

QAM Applications

QAM is in many radio communications and data delivery applications. However some specific variants of QAM are used in some specific applications and standards.

For domestic broadcast applications for example, 64 QAM and 256 QAM are often used in digital cable television and cable modem applications. In the UK, 16 QAM and 64 QAM are currently used for digital terrestrial television using DVB - Digital Video Broadcasting. In the US, 64 QAM and 256 QAM are the mandated modulation schemes for digital cable as standardized by the SCTE in the standard ANSI/SCTE 07 2000.

In addition to this, variants of QAM are also used for many wireless and cellular technology applications.