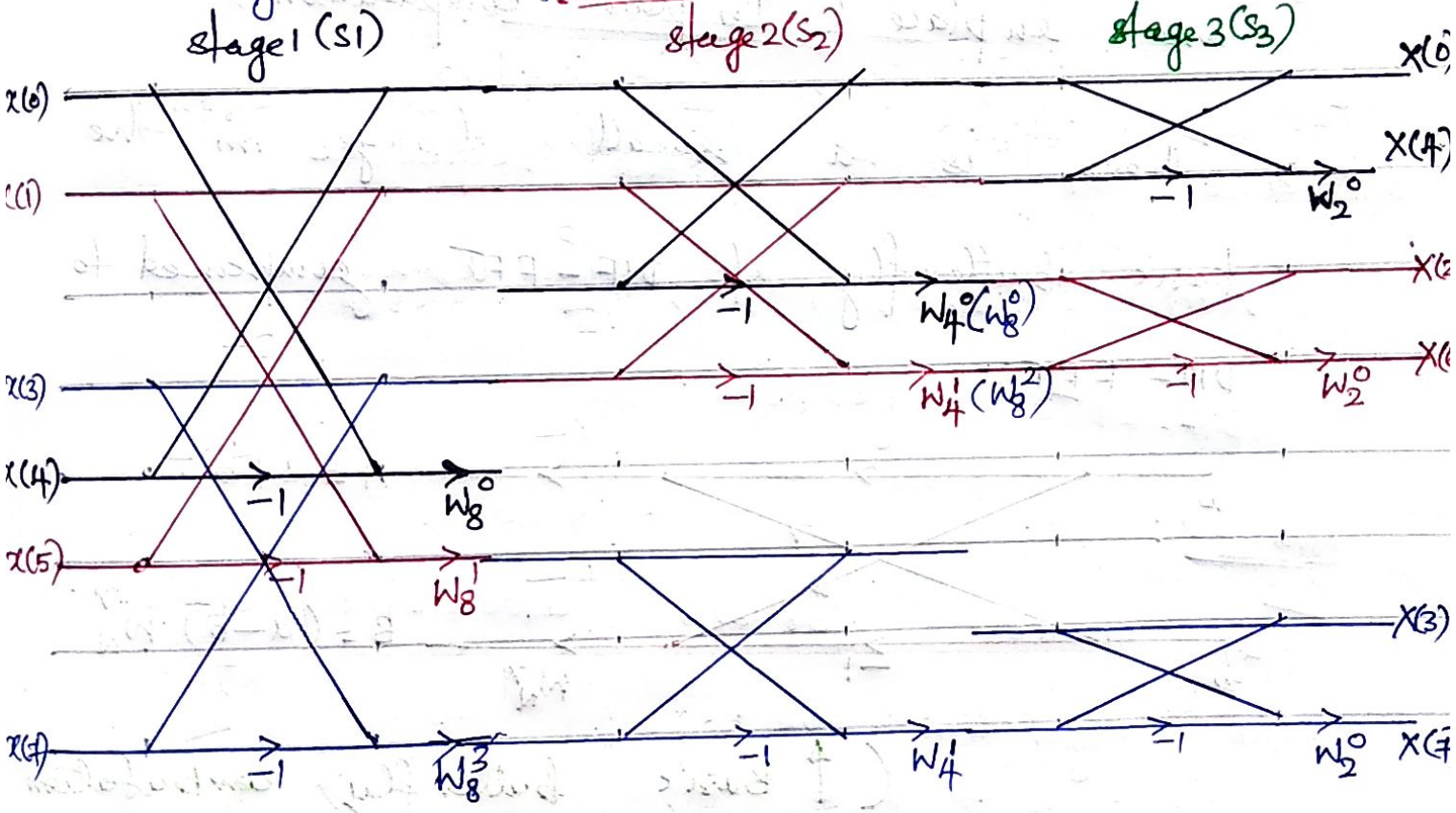
Radix-2 Decimation in Frequency - FFT Algorithm (DIF-FFT):

In DIF-FFT Algorithm the output sequence X(k) is divided into smaller and smaller ~~subgroups~~ subsequences.

→ The flow graph of radix-2 DIF-FFT Algorithm for N = 8 ⇓



- In DIF-FFT Algorithm input(x(n)) is applied in natural order but its DFT X(k) is obtained at output side in ~~but~~ bit reversed order.
- As in the case of DIT-FFT here also

for evaluating N-point DFT, where $N = 2^v$.

then no. of stages required.

$$v = \log_2 N$$

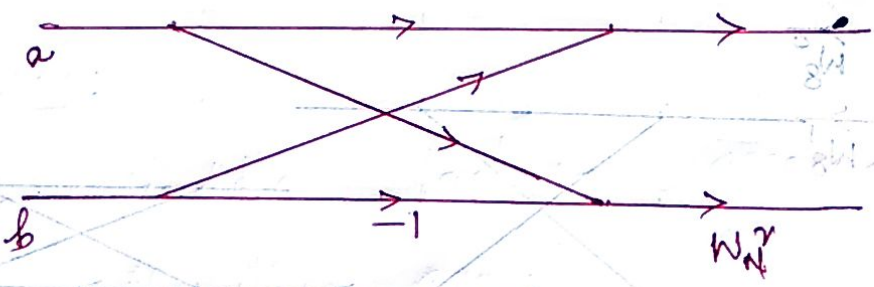
eg: if $N = 8 \longrightarrow$ no. of stages $v = \log_2(8)$

$= 3$ stages

(Or) $N = 4 \Longrightarrow v = \log_2(4) = 2$ stages.

→ ~~Here~~ In DIF-FFT also computations are in place (In place computation)

→ There is a small change in the basic butterfly of DIF-FFT compared to DIT-FFT.



(↑ Basic butterfly computation in radix-2 DIF-FFT Algorithm).

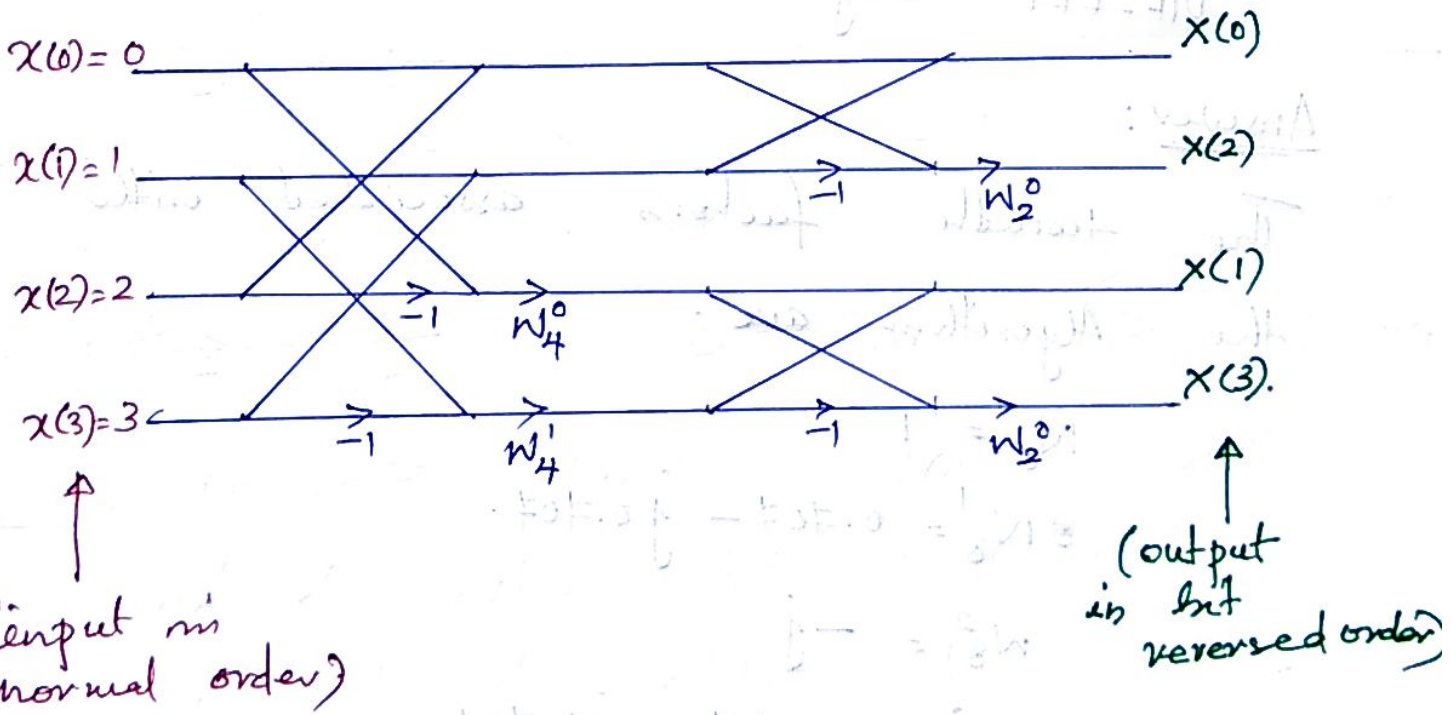
Q) Compute DFT of the sequence $x(n) = \{0, 1, 2, 3\}$ using radix-2 DIF-FFT Algorithm.

Answer:

Bit reversal in the case of N=4.

i/p index	binary	bit reversed	o/p index
x(0) → 0	00	00	0 → X(0)
x(1) → 1	01	10	2 → X(2)
x(2) → 2	10	01	1 → X(1)
x(3) → 3	11	11	3 → X(3)

DIF - FFT Algorithm.



Input	S_1	S_2 (stage 2)
0	$0+2. = 2$	$2+4 = 6$
1	$1+3 = 4$	$2-4 = -2$
2	$(0-2)W_4^0 = -2.$	$-2+2j$
3	$(1-3)W_4^1 = (1-3)(-j)$ $= 2j$	$-2-2j$

$$x(k) = \{6, -2, -2+2j, -2-2j\}$$

Q). Find the DFT of the sequence

$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIF-FFT Algorithm.

Answer:

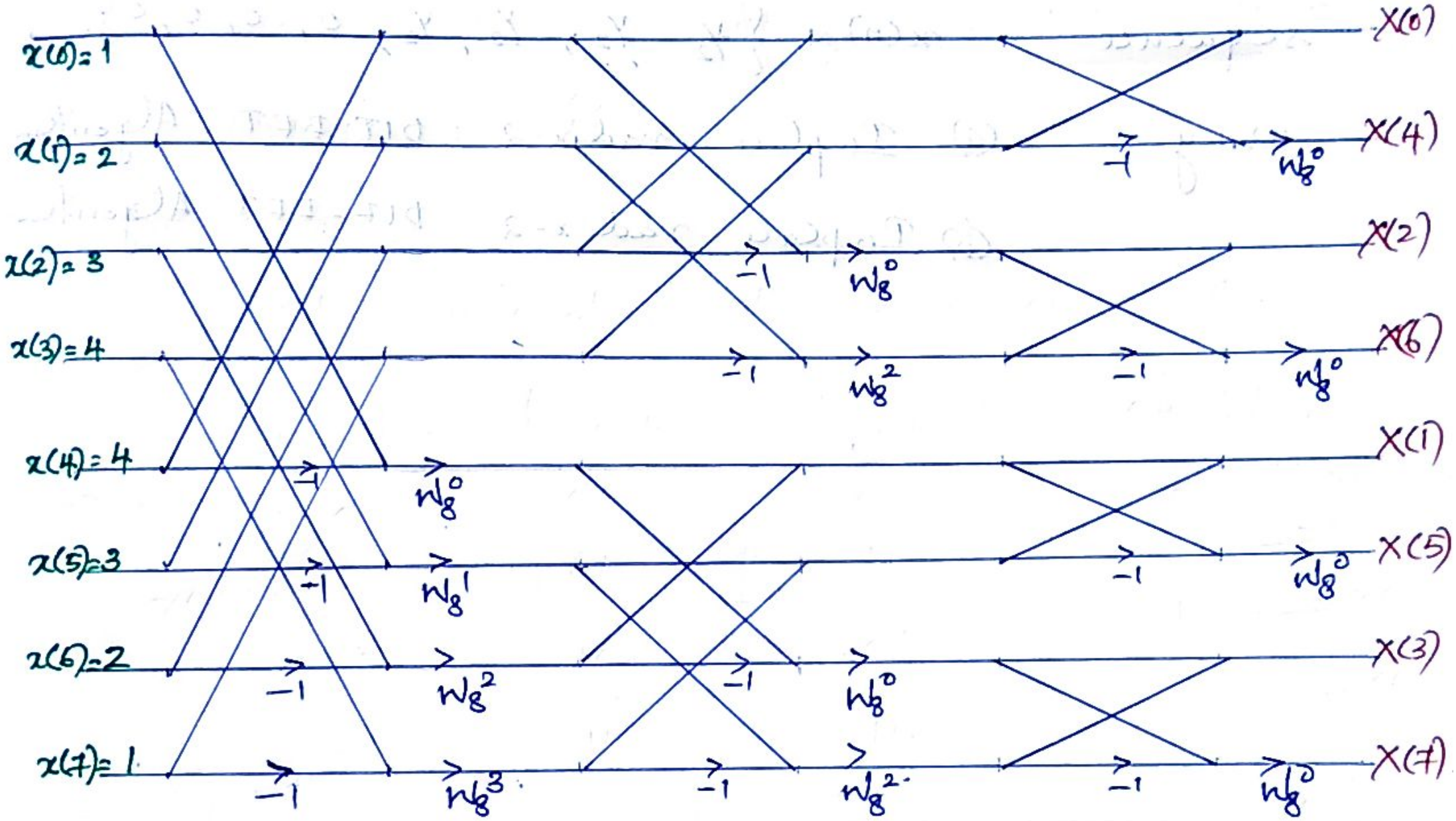
The twiddle factors associated with the Algorithm are.

$$W_8^0 = 1$$

$$W_8^1 = 0.707 - j0.707.$$

$$W_8^2 = -j$$

$$W_8^3 = -0.707 - j0.707.$$


x(0) = 1
x(1) = 2
x(2) = 3
x(3) = 4
x(4) = 4
x(5) = 3
x(6) = 2
x(7) = 1
X(0)
X(4)
X(2)
X(6)
X(1)
X(5)
X(3)
X(7)
-1
W_8^0
W_8^1
W_8^2
W_8^3

Input	S_1	S_2	Output
1	$1+4=5$	$5+5=10$	$10+10=20$
2	$2+3=5$	$5+5=10$	$10-10=0$
3	$3+2=5$	$5-5=0$	0
4	$4+1=5$	$(5-5)W_8^2=0$	0
4	$(1-4)W_8^0=-3$	$[-3+(-j)]=-3-j$	$-3-j-2.828-j1.414$ $=-5.828-j2.414$
3	$(2-3)(0.707-j0.707)$ $=-0.707+j0.707$	$-0.707+j0.707+$ $(-2.121-j2.121)$ $=-2.828-j1.414$	$-3-j+2.828+j1.414$ $=-0.172+j0.414$
2	$(3-2)(-j)=-j$	$-3-(-j)=-3+j$	$-3+j+2.828-j1.414$ $=-0.172-j0.414$
1	$(4-1)(-0.707-j0.707)$ $=-2.121-j2.121$	$(-0.707+j0.707+2.121$ $+j2.121)(-j)$ $=2.828-j1.414$	$-3+j-2.828+j1.414$ $=-5.828+2.414j$

$$X(k)=\{20,-5.828-j2.414,0,-0.172-j0.414,0,-0.172+j0.414,0,$$
$$-5.828+j2.414\}$$