A discrete random Variable X is Said to be Poisson random Variable with parameter $\lambda > 0$, if its Pmf is

$$\boxed{p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, \ldots}$$

$$\boxed{e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots}$$

Mean and Varience of P.D

Mean

$x! = \underbrace{1.2\cdots(x-1)}\,x$

$= (x-1)!\,x$

$$E[x] = \sum x\, p(x)$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=1}^{\infty} \cancel{x} \frac{e^{-\lambda}\lambda^{x}}{(x-1)!\,\cancel{x}}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x}}{(x-1)!}$$

$$= e^{-\lambda}\left[\frac{\lambda}{1} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \frac{\lambda^4}{3!} + \cdots\right]$$

$$= \lambda e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right]$$

$$\boxed{x^m x^n = x^{m+n}}$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$e^{0} = 1$

$$\boxed{\text{Mean} = \lambda}$$

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \sum x^2 p(x)$$

$$= \sum [x^2 - x + x]\, p(x)$$

$$= \sum x(x-1)\, p(x) + \underbrace{\sum x p(x)}_{\lambda}$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^x}{x!} + \lambda$$

$$= \sum_{x=2}^{\infty} \cancel{x(x-1)} \frac{e^{-\lambda}\lambda^x}{(x-2)!\,\cancel{(x-1)x}} + \lambda$$

$$= e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!} + \lambda$$

$x! = 1.2. \cdots (x-2)(x-1)x$

$= (x-2)!\,(x-1)x$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

$x^m x^n = x^{m+n}$

$e^0 = 1$

$$= e^{-\lambda}\left[\frac{\cdots}{1} \cdots \frac{\cdots}{1!} \cdots \frac{\cdots}{2!} \cdots \frac{\cdots}{3!}\right]$$

$$= \lambda^2 e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right] + \lambda$$

$$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda$$

$$= \lambda^2 + \lambda$$

$$\therefore Var(x) = E[x^2] - E[x]^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$\boxed{Var(x) = \lambda}$$

$$S.D = \sqrt{\lambda}.$$

v.v.imp

Poisson Approximation to Binomial:

The Probability Distribution of a Binomial random Variable $X = B(n,p)$ approaches that of a Poisson random Variable as $n \to \infty$ and $p \to 0$ in such a way that $np = \lambda$ stays constant.

[n is large and p is small]

$$np = \lambda$$

$$\boxed{p = \frac{\lambda}{n}} \qquad q = 1-p \qquad \frac{1}{\infty} = 0$$

$$n \cdot n^{x-1} = n^{1+x-1} = n^x$$

Pmf of B.D is

$$P(x) = nC_x \, p^x q^{n-x} \qquad x = 0,1,2,\dots,n$$

$$= \frac{n!}{x!(n-x)!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$\boxed{\lim_{n\to\infty}\left[1-\frac{\lambda}{n}\right]^n = e^{-\lambda}}$$

$$= \frac{\cancel{1.2.3\ldots(n-(x+1))(n-x)}(n-(x-1))\ldots(n-2)(n-1)n}{x!\,\cancel{[1.2.3\ldots(n-x)]}}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n(n-1)(n-2)\ldots(n-(x-1))}{x!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\cancel{n^x}\left[1-\frac{1}{n}\right]\left[1-\frac{2}{n}\right]\cdots\left[1-\frac{x-1}{n}\right]}{x!}\frac{\lambda^x}{\cancel{n^x}}\left(1-\frac{\lambda}{n}\right)^{n-x}$$

As $n \to \infty$

$$= \lim_{n\to\infty}\frac{\lambda^x\left[1-\frac{\lambda}{n}\right]^{n-x}}{x!}$$

$$= \frac{\lambda^x}{x!}\lim_{n\to\infty}\frac{\left[1-\frac{\lambda}{n}\right]^n}{\left[1-\frac{\lambda}{n}\right]^x} = \frac{\lambda^x}{x!}\lim_{n\to\infty}\left[1-\frac{\lambda}{n}\right]^n$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0,1,2,\dots$$

If x is a poisson Variate Such that $p(x=2)=p(x=3)$. find

Pmf of P.D is $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x=0,1,2,\dots$

$$p(x=2) = p(x=3)$$

$$\frac{\cancel{e^{-\lambda}}\,\lambda^2}{\underset{1\cdot\cancel{2}}{2!}} = \frac{\cancel{e^{-\lambda}}\,\lambda^3}{\underset{1\cdot\cancel{2}\cdot3}{3!}}$$

$$1 = \frac{\lambda}{3}$$

$$\boxed{\lambda=3}$$

$\therefore$ Pmf of PD is $p(x) = \frac{e^{-3}3^x}{x!}$ $x=0,1,2,\dots$

$p(x=4) = \frac{e^{-3}3}{4!}$

$= 0.168$

If x is a poisson variate such that $p(x=2) = 9p(x=4) + 90p(x=6)$. Find the standard deviation.

Prob of PD is $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0,1,2,\ldots$

$$p(x=2) = 9p(x=4) + 90p(x=6)$$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$$\cancel{e^{-\lambda}}\left[\frac{\lambda^2}{\cancel{2}}\right] = \cancel{e^{-\lambda}}\left[\frac{\overset{3}{\cancel{9}}\lambda^4}{\underset{\cancel{12}\,4}{\cancel{24}}} + \frac{\cancel{90}\lambda^6}{\underset{\underset{4}{\cancel{360}}}{\cancel{720}}}\right]$$

$$\lambda^2 = \frac{3}{4}\lambda^4 + \frac{\lambda^6}{4}$$

$$\cancel{\lambda^2}^{\,1} = \cancel{\lambda^2}\left[\frac{3}{4}\lambda^2 + \frac{\lambda^4}{4}\right]$$

$$\frac{\lambda^4}{4} + \frac{3}{4}\lambda^2 = 1$$

$$\lambda^4 + 3\lambda^2 = 4$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

$$(\lambda^2+4)(\lambda^2-1) = 0$$

$\lambda^2+4=0$ $\quad$ $\lambda^2-1=0$

$\lambda^2=-4$ $\quad$ $\lambda^2=1$

$\lambda=\pm 2i$ $\quad$ $\lambda=\pm 1$

$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $\lambda > 0$

Since $\lambda > 0$ $\quad \underline{\underline{\lambda = 1}}$

$\therefore$ S.D $= \sqrt{\lambda}$

$= \underline{\underline{1}}$

if x is a poisson Variate such that $p(x=0)=p(x=1)=k$. S.T $k=\frac{1}{e}$.

Pmf of P.D is

$$P(x)=\frac{e^{-\lambda}\lambda^{x}}{x!} \quad x=0,1,2,\ldots$$

$0! = 1$

$(a)^0 = 1$

$$p(x=0)=p(x=1)=k$$

$$\frac{e^{-\lambda}\lambda^{0}}{0!}=\frac{e^{-\lambda}\lambda}{1!}=k$$

$$p(x=0)=p(x=1) \Rightarrow e^{-\lambda}=e^{-\lambda}\lambda$$

$$\boxed{\lambda=1}$$

$$p(x=0)=k.$$

$$\frac{e^{-\lambda}\lambda^{0}}{0!}=k.$$

$$e^{-\lambda}=k.$$

Since $\lambda=1$

$$k=e^{-1}$$

$$=\frac{1}{e}.$$

A car hire firm has two cars which it hires out day by day. The no. of demands (λ) for a car on each day is distributed as Poisson Distribution with mean 1.5. Calculate the proportion of days on which (i) there is no demand (ii) Some demand is refused.

Pdf of P.D

$$p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0, 1, 2, \dots$$

given $\lambda = 1.5$

$$p(x) = \frac{e^{-1.5} (1.5)^x}{x!} \quad x = 0, 1, 2, \dots$$

$p(\text{there is no demand}) = p(x=0)$

$$= \frac{e^{-1.5} (1.5)^0}{0!} = e^{-1.5} = 0.2231$$

(ii) $p(\text{demand is refused})$

$$= p(x > 2)$$

$$= 1 - [p(x \le 2)]$$

$$= 1 - (p(x=0) + p(x=1) + p(x=2))$$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!} \right]$$

$$= 0.1913$$

U.Q There are 250 typographical errors in a book of 1000 pages. The no. of errors per page is supposed to follow a poisson distribution. What is the probability that a randomly selected page will have more than 2 errors.

Let λ = mean no. of errors per page.

$$\lambda = \frac{250}{1000} = 0.25$$

Pmf of P.D is

$$p(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad x = 0, 1, 2, \ldots$$

$$= \frac{e^{-0.25} (0.25)^{x}}{x!} \quad x = 0, 1, 2, \ldots$$

P(more than 2 errors)

$$= P(x > 2)$$

$$= 1 - P(x \leq 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-0.25} (0.25)^{0}}{0!} + \frac{e^{-0.25} (0.25)^{1}}{1!} + \frac{e^{-0.25} (0.25)^{2}}{2!} \right]$$

$$= 0.0021$$

It is known that 2% of the bolts produced by a Company are defective. The bolts are supplied in boxes of 200 bolts. What is the probability that a randomly choosen box contains not more than 5 defective bolts.

$n = 200$

$p = p(\text{defective}) = \frac{2}{100} = 0.02$

$\therefore \lambda = np$

$= 200 \times 0.02 = \underline{\underline{4}}$

$\therefore$ Pmf of P.D is

$p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0, 1, 2, \ldots$

$= \frac{e^{-4} 4^x}{x!} \quad x = 0, 1, 2, \ldots$

p(not more than 5 defective bolts)

$= p(x \leq 5)$

$= p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4) + p(x=5)$

$= e^{-4}\left[\frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!}\right]$

$= \underline{\underline{0.785}}$

A manufacturer who produces medicine bottles find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. using P.D. find how many boxes will contain i) no defective ii) atleast 2 defective.

$n = 500 \qquad N = 100$

$p = p(\text{defective}) = \frac{0.1}{100} = 0.001$

$\lambda = np$

$= 500 \times 0.001$

$= 0.5$

Pmf of PD is $p(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad x = 0, 1, 2, \ldots$

$= \frac{e^{-0.5} (0.5)^{x}}{x!} \quad x = 0, 1, 2, \ldots$

i) $P(\text{no defective}) = p(x=0)$

$= \frac{e^{-0.5} (0.5)^{0}}{0!}$

$= 0.6065$

Req: Number $= N \times p(x=0)$

$= 100 \times 0.6065 = 60.6$

$\simeq 61$

ii) $P(\text{atleast 2 defective}) = p(x \geq 2)$

A manufacturer who produces medicine bottles ...

defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using P.D find how many boxes will contain i) no defective ii) atleast 2 defective.

$n = 500 \qquad N = 100$

$p = p(\text{defective}) = \frac{0.1}{100} = 0.001$

$\lambda = np$

$= 500 \times 0.001$

$= 0.5$

Pmf of PD is $p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0, 1, 2, \dots$

$= \frac{e^{-0.5} (0.5)^x}{x!} \quad x = 0, 1, 2, \dots$

i) $P(\text{no defective}) = p(x=0)$

$= \frac{e^{-0.5} (0.5)^0}{0!}$

$= \underline{0.6065}$

Req: Number $= N \times p(x=0)$

$= 100 \times 0.6065 = 60.6$

$\simeq \underline{61}$

ii) $P(\text{atleast 2 defective}) = p(x \geq 2)$

$= 1 - p(x < 2)$

$= 1 - [p(x=0) + p(x=1)]$

$= 1 - \left[\frac{e^{-0.5}(0.5)^0}{0!} + \frac{e^{-0.5}(0.5)}{1!} \right]$

Req: Num $= N \times p(x \geq 2)$

$= 100 \times 0.009$

$= 9$

The no: of accidents in a year to taxi drivers in a city follow P.D with mean equal to 3. out of 2000 taxi drivers find no: of drivers with

i) no accident in a year.

ii) more than 3 accident in a year.

$\lambda = 3$

$N = 2000$

i) $p(x=0)$

Req: $N_0 = N \times p(x=0)$

$= 2000 \times p(x=0)$

$= 100$

ii) $p(x \geq 3) = 1 - p(x < 3)$

$= 1 - [p(0) + p(1) + p(2)]$

$=$

Req: $N_0 = N \times p(x \geq 3)$

$= 2000 \times p(x \geq 3)$

$= 705$

The no. of accidents per day was recorded in a district for a period of 1500 days and the following results were obtained. Fit a poisson distribution and compute the theoretical frequencies.

No. of accidents per day:	0	1	2	3	4	5
observed frequency:	342	483	388	176	111	0

$N = \Sigma f = 342 + 483 + 388 + 176 + 111 = 1500$

$\Sigma x f = 0 + 483 + 2 \times 388 + 3 \times 176 + 4 \times 111 = 2231$

$\bar{x} = \frac{\Sigma x f}{\Sigma f} = \frac{2231}{1500} = \underline{\underline{1.49}}$

Mean $\lambda = \underline{\underline{1.49}}$

Pmf of P.D is $p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0, 1, 2 \cdots$

$= \frac{e^{-1.49} (1.49)^x}{x!} \quad x = 0, 1, 2, \cdots$

x	$p(x)$	$N \times p(x)$
0	0.23	345
1	0.34	510
2	0.25	375
3	0.12	180
4	0.05	75
5	0.01	15
	$\Sigma p(x) = 1$	1500

It is known that on a production line the probability that an item is faulty is 0.1. 50 items are chosen at random and checked for faults. Find the probability that there will be no faulty items and also the probability that there will be 3 faulty items using

i) binomial distribution   ii) Poisson distribution.

i) $P = P(\text{faulty item}) = 0.1$

$n = 50$

$q = 1 - P = 0.9$

Pmf of B.D is $p(x) = n C_x \, p^x q^{n-x} \quad x = 0,1,2,\dots,n$

$p(x) = 50 C_x (0.1)^x (0.9)^{50-x} \quad x = 0,1,2,\dots,50$

• P(no faulty items)

$= P(x=0) = 50C_0 (0.1)^0 (0.9)^{50}$

$= 0.005153775$

$P(3 \text{ faulty items}) = p(x=3)$

$= 50 C_3 (0.1)^3 (0.9)^{47}$

$= 0.1385651$

$\lambda = np = 50 \times 0.1 = 5$

Pmf of PD is $p(x) = \dfrac{e^{-\lambda} \lambda^x}{x!} \quad x = 0,1,2,\dots$

$= \dfrac{e^{-5} 5^x}{x!} \quad x = 0,1,2,\dots$

i) $P(x=0) = \dfrac{e^{-5} 5^0}{0!} = 0.0067394$

ii) $P(x=3) = \dfrac{e^{-5} 5^3}{3!} = 0.1403739$

Let x denote the number of creatures of a particular type Captured in a trap during a given time period. Suppose that x has a poisson distribution with μ (λ) $= 4.5$, So on average traps will Contain 4.5 Creatures. Find the probability that a

i) trap Contains exactly five creatures.

ii) atmost five Creatures.

$\mu = 4.5$ $[\lambda = 4.5]$

Pmf of P.D is

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad x = 0,1,2,\dots$$

$$= \frac{e^{-4.5}(4.5)^{x}}{x!} \quad x = 0,1,2,\dots$$

i) p(trap contains exactly five creature).

$$= p(x=5)$$

$$= \frac{e^{-4.5}(4.5)^{5}}{5!}$$

$$= \underline{0.1708}$$

ii) P(atmost five creatures) $= p(x \le 5)$

$$= p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4) + p(x=5)$$

$$= e^{-4.5}\left[\frac{(4.5)^0}{0!} + \frac{(4.5)^1}{1!} + \frac{(4.5)^2}{2!} + \frac{(4.5)^3}{3!} + \frac{(4.5)^4}{4!} + \frac{(4.5)^5}{5!}\right]$$

$$= \underline{0.7029}$$

If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing atleast one such error is 0.005 and errors are independent from page to page. What is the probability that one of its 400 page novels will contain exactly one page with errors? Atmost three pages with errors?

$n = 400$

$p = P(\text{error}) = 0.005$

$\lambda = np$

$= 400 \times 0.005$

$= 2$

Pmf of PD is $p(x) = \dfrac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2, \ldots$

$= \dfrac{e^{-2} 2^x}{x!} \quad x = 0, 1, 2, \ldots$

i) P(exactly one page with error)

$= p(x=1)$

$= \dfrac{e^{-2} 2}{1!} = \underline{0.270671}$

ii) P(atmost three pages with error)

$= p(x \le 3)$

$= p(0) + p(1) + p(2) + p(3)$

$= e^{-2}\left[\dfrac{2^0}{0!} + \dfrac{2^1}{1!} + \dfrac{2^2}{2!} + \dfrac{2^3}{3!}\right]$

$= \underline{0.8576}$

V.V.Imp

Accidents occur at an intersection at a poisson rate of 2 per day. what is the probability that in january there would be atleast 3 days (not necessarily consecutive) without any accidents?

Let X : no: of accidents per day.

$\lambda = 2$

Pmf of P.D is

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad x = 0,1,2,\ldots$$

$$= \frac{e^{-2}2^{x}}{x!} \quad x = 0,1,2,\ldots$$

P(there will be without any accidents on a given day)

$$= P(X=0) = \frac{e^{-2}2^{0}}{0!} = 0.1353$$

Let $p = P$(without any accidents)

$= 0.1353$

$n = 31$ [31 days in January]

$\mu = np$

$= 31 \times 0.1353 = 4.1943$

Let Y denotes no: of days in Jan: without any accidents

Pmf $$P(y) = \frac{e^{-4.1943}(4.1943)^{y}}{y!} \quad y = 0,1,2,3\ldots$$

P(atleast 3 days without any accidents)

$= P(Y \geq 3) = 1 - P(Y < 3)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 -$

probability that in January there would be atleast 3 days without any accidents?

Let x : no: of accidents per day.

$\lambda = 2$

Pmf of P.D is

$$p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0,1,2,\dots$$

$$= \frac{e^{-2} 2^x}{x!} \quad x = 0,1,2,\dots$$

P(there will be without any accident on a given day)

$$= p(x=0) = \frac{e^{-2} 2^0}{0!} = 0.1353$$

Let $p = p$(without any accidents).

$= 0.1353$.

$n = 31$ [31 days in January]

$\mu = np$

$= 31 \times 0.1353 = 4.1943$

Let y denotes no: of days in Jan: without any accident

Pmf $\quad p(y) = \frac{e^{-4.1943} (4.1943)^y}{y!} \quad y = 0,1,2,3\dots$

P(atleast 3 days without any accidents)

$= P(y \geq 3) = 1 - P(y < 3)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - e^{-4.1943} \left[\frac{(4.1943)^0}{0!} + \frac{(4.1943)^1}{1!} + \frac{(4.1943)^2}{2!} \right]$

$= 0.7890$

Traffic accidents at a particular intersection follow poisson distribution with an average rate of 1.4 per week. what is the probability

a) that the next week is accident free?

c) there will be atleast 2 accidents during the next two weeks?

e) ~~there~~ will be exactly 2 accidents tomorrow.

f) that the next accident will not occur for three days.

b) That there will be exactly 3 accidents next week.

d) there will be exactly 5 accidents during the next four weeks?

g) there will be exactly three accident-free weeks during the next eight weeks?

h) there will be exactly five accident-free days during the next week?

$\lambda = 1.4$ (1 week)

pmf of P.D is $p(x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$ $x = 0,1,2,\ldots$

$= \dfrac{e^{-1.4}(1.4)^{x}}{x!}$ $x = 0,1,2,\ldots$

~~accident~~

a) p(next week is accident-free) $= p(x=0)$

$= \dfrac{(1.4)^{0}(e^{-1.4})}{0!}$

$= \underline{\underline{0.2466}}$

b) p(there will be exactly ...

$$= p(x=3)$$

$$= \frac{e^{-1.4}(1.4)^3}{3!} = \underline{\underline{0.1128}}$$

c) <u>next two weeks</u>. $\lambda = 1.4 + 1.4$

$$= 2.8$$

$$p(x) = \frac{e^{-2.8}(2.8)^x}{x!} \quad x = 0, 1, 2, \ldots$$

$$p(\text{atleast 2 accidents}) = p(x \geq 2)$$

$$= 1 - p[x < 2]$$

$$= 1 - [p(0) + p(1)]$$

$$= 1 - e^{-2.8}\left[1 + \frac{(2.8)^1}{1}\right] = \underline{\underline{0.7689}}$$

d) Next four ...

$$\lambda = 1.4 + 1.4 + 1.4 + 1.4$$

$$= 5.6$$

$$p(x) = \frac{e^{-5.6}(5.6)^x}{x!} \quad x = 0, 1, 2, \ldots$$

b) p(there will be exactly 3 accidents in next week)

$= p(x=3)$

$= \frac{e^{-1.4}(1.4)^3}{3!} = \underline{0.1128}$

c) next two weeks. $\lambda = 1.4 + 1.4$
$= 2.8$

$p(x) = \frac{e^{-2.8}(2.8)^x}{x!}$ $x = 0,1,2,\ldots$

p(atleast 2 accidents) $= p(x \geq 2)$

$= 1 - p[x < 2]$

$= 1 - [p(0) + p(1)]$

$= 1 - e^{-2.8}\left[1 + \frac{(2.8)^1}{1}\right] = \underline{0.7689}$

d) Next

$\lambda = 1.4 + 1.4 + 1.4 + 1.4$
$= 5.6$

$p(x) = \frac{e^{-5.6}(5.6)^x}{x!}$ $x = 0,1,2,\ldots$

p(exactly 5 accidents) $= p(x=5)$

$= \frac{e^{-5.6}(5.6)^5}{5!}$

$= \underline{0.1697}$

e) (one day) $= \frac{1.4}{7} = \underline{0.2} = \lambda.$

$p(x) = \frac{e^{-0.2}(0.2)^x}{x!}$ $x = 0,1,2,\ldots$

p(exactly 2 accidents tomorrow) $= p(x=2)$

$= \frac{e^{-0.2}(0.2)^2}{2!}$

f) next 3 days $= 0.2 + 0.2 + 0.2$

$\lambda = 0.6$

Prob $p(x) = \dfrac{e^{-0.6} (0.6)^x}{x!}$ $\quad x = 0, 1, 2, \dots$

$p(\text{no accident}) = p(x=0)$

$= \dfrac{e^{-0.6} (0.6)^0}{0!}$

$= \underline{\underline{0.5488}}$

g) 1 week $\quad \lambda = 1.4$

$P = p(\text{no accident}) = 0.2466$

$n = 8$

$q = 1 - p = 0.7534$

Prob of B.D $\Rightarrow$

$p(x) = nC_x\, p^x q^{n-x}$ $\quad x = 0, 1, 2, \dots, n.$

$= 8C_x (0.2466)^x (0.7534)^{8-x}$ $\quad x = 0, 1, 2, \dots, 8$

P(exactly 3 accident free weeks in 8 weeks)

$= p(x=3) = 8C_3 (0.2466)^3 (0.7534)^5$

$= \underline{\underline{0.20384}}$

h) $\lambda = 0.2$ $\quad p(x=0) = \dfrac{e^{-0.2} (0.2)^0}{0!} = 0.81873$

1 week $\quad n = 7$ $\quad q = 1 - p = 0.18127$

$p(x) = 7C_x (0.81873)^x (0.18127)^{7-x}$ $\quad x = 0, 1, 2, \dots, 7.$

P(exactly 5 accident free days in a week (7 days))

$= p(x=5) = 7C_5 (0.81873)^5 (0.18127)^3$

$= \underline{\underline{0.25365}}$

U.Q

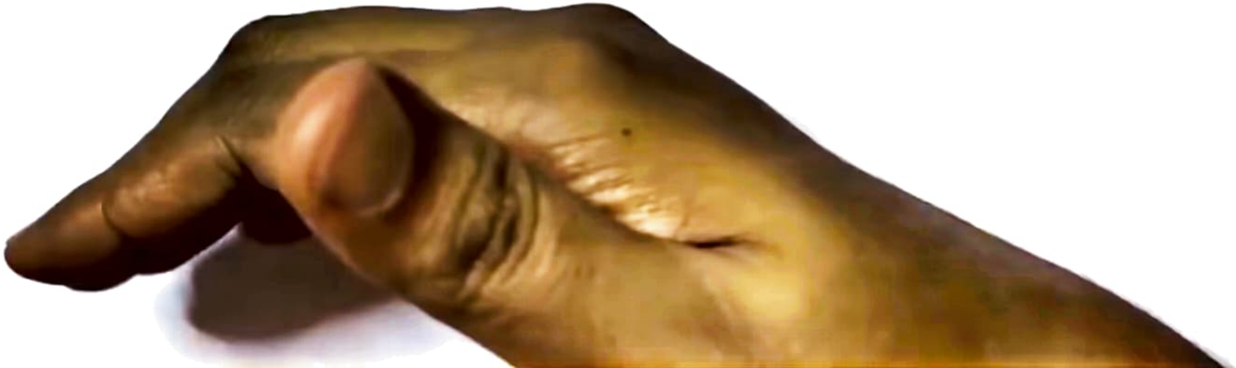
Earthquakes occur in a region at an average rate of 5 per year according to a Poisson distribution. What is the probability that i) no earthquake would occur next year?

ii) no earthquakes would occur in exactly two of the next five years.

$\lambda = 5$

$p(x=2)$

$n = 5$



Discrete two-dimensional random Variables:

p.m.f

x	0, 1, 2, ... n.
$p(x)$	

$$P(x) = \frac{1}{2^x} \qquad x = 0, 1, 2, \dots, 10$$

1) Joint Probability mass fun: [Joint Pmf]

Suppose x and y are discrete random Variables. defined by.

$$P(x,y) = P[X=x, Y=y]$$

i) $\sum p(x) = 1$

ii) $p(x) \geq 0$

$0 \leq p(x) \leq 1$

Note:

$$\sum p(x,y) = 1$$

$$0 \leq p(x,y) \leq 1$$

y \ x	0	1	2
0	(.)	.	
1			
2			
3			

Marginal Distributions

Let x and y be discrete random Variables with joint Pmf $P(x,y)$. Then the Probability mass fun: of x and y are given respectively.

marginal Pmf of x is $P(X=x) = \sum_y P(x,y)$

Marginal Pmf of y is $P(Y=y) = \sum_x P(x,y)$

Independent Random Variables

Two discrete random Variables x and y are said to be independent if for all x, y.

$$p(x,y) = P_x(x)\; P_y(y)$$

Expectation of two random Variables :

Let $f(x,y)$ be a function of a discrete random Variables x and y.

$$E[f(x,y)] = \sum f(x,y)\, p(x,y)$$

Note:

$$E[xy] = \sum x\,y\, p(x,y)$$

if x and y are independent $E[xy] = E[x]\,E[$

Two discrete random Variables x and y are said to be independent if for all x, y.

$$P(x,y) = P_x(x)\ P_y(y)$$

Expectation of two random Variables :

Let $f(x,y)$ be a function of a discrete random Variables x and y.

$$E[f(x,y)] = \sum f(x,y)\ P(x,y)$$

Note:

$$E[XY] = \sum x y\ p(x,y)$$

if x and y are independent $E[XY] = E[X]\,E[Y]$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Prob: Distribution

$$P(X=x \mid Y=y) = \frac{P[X=x, Y=y]}{P[Y=y]}$$

From the following table for Bivariate distribution of (X, Y). find

i) $P(X \le 1)$ ii) $P(Y \le 3)$ iii) $P(X \le 1, Y \le 3)$ iv) $P(X \le 1 \mid Y \le 3)$ v) $P(Y \le 3 \mid X \le 1)$

vi) $P(X + Y \le 4)$ vii) The marginal distribution of X or Marginal PMF of X

viii) The marginal distribution of Y or marginal PMF of Y.

ix) The Conditional distribution of X given $Y = 2$. $(X/_{Y=2})$

x) Examine X and Y are independent xi) $E[Y - 2X]$

J.P.m.f

$X \backslash Y$	1	2	3	4	5	6
0	$P(0,1)$ 0	$P(0,2)$ 0	$P(0,3)$ $\frac{1}{32}$	$P(0,4)$ $\frac{2}{32}$	$P(0,5)$ $\frac{2}{32}$	$P(0,6)$ $\frac{3}{32}$.
1.	$P(1,1)$ $\frac{1}{16}$	$P(1,2)$ $\frac{1}{16}$	$P(1,3)$ $\frac{1}{8}$	$P(1,4)$ $\frac{1}{8}$	$P(1,5)$ $\frac{1}{8}$	$P(1,6)$ $\frac{1}{8}$
2.	$P(2,1)$ $\frac{1}{32}$	$P(2,2)$ $\frac{1}{32}$	$P(2,3)$ $\frac{1}{64}$	$P(2,4)$ $\frac{1}{64}$	$P(2,5)$ 0	$P(2,6)$ $\frac{2}{64}$

$P(x, y)$

$x = 0, 1, 2.$

$y = 1, 2, 3, 4, 5, 6$

(x, y)

$P(2,4) = \frac{1}{64}$

$P(1,3) = \frac{1}{8}.$

From the following table for Bivariate distribution of (X,Y). find

i) $P(X \le 1)$ ii) $P(Y \le 3)$ iii) $P(X \le 1, Y \le 3)$ iv) $P(X \le 1 \mid Y \le 3)$ v) $P(Y \le 3 \mid X \le 1)$

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x) Examine X and Y are independent xi) $E[Y-2X]$

J.P.m.f

X \ Y	1	2	3	4	5	6	$P_X(x)$
0	$P(0,1)$ 0	$P(0,2)$ 0	$P(0,3)$ $\frac{1}{32}$	$P(0,4)$ $\frac{2}{32}$	$P(0,5)$ $\frac{2}{32}$	$P(0,6)$ $\frac{3}{32}$.	$\frac{8}{32}$
1.	$P(1,1)$ $\frac{1}{16}$	$P(1,2)$ $\frac{1}{16}$	$P(1,3)$ $\frac{1}{8}$	$P(1,4)$ $\frac{1}{8}$	$P(1,5)$ $\frac{1}{8}$	$P(1,6)$ $\frac{1}{8}$	$\frac{20}{32}$
2.	$P(2,1)$ $\frac{1}{32}$	$P(2,2)$ $\frac{1}{32}$	$P(2,3)$ $\frac{1}{64}$	$P(2,4)$ $\frac{1}{64}$	$P(2,5)$ 0	$P(2,6)$ $\frac{2}{64}$	$\frac{4}{32}$.
$P_Y(y)$	$P(Y=1)$ $\frac{3}{32}$	$P(Y=2)$ $\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$	1

$P(x,y)$

$x = 0,1,2.$

$y = 1,2,3,4,5,6$

(x,y)

$P(2,4) =$

$(1,3)$

i) $P(x \le 1) = P(x=0) + P(x=1)$

$$= \frac{8}{32} + \frac{20}{32} = \frac{28}{32}$$

$$= \frac{7}{8}$$

ii) $P(y \le 3) = P(y=1) + P(y=2) + P(y=3)$

$$= \frac{3}{32} + \frac{3}{32} + \frac{11}{64}$$

$$= \frac{23}{64}$$

x : 0 1 2

y : 1 2 3 4 5 6

iii) $P(x \le 1, y \le 3) = P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)$

$$= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8}$$

$$= \frac{9}{32}$$

iv) $P(x \le 1 \mid y \le 3) = \dfrac{P[x \le 1, y \le 3]}{P[y \le 3]}$

$$= \frac{\frac{9}{32}}{\frac{23}{64}}$$

$$= \cancel{\frac{9}{28}} \quad \frac{18}{23}$$

v) $P(y \le 3 \mid x \le 1) = \dfrac{P[x \le 1, y \le 3]}{P[x \le 1]}$

$$= \frac{\frac{9}{32}}{\frac{7}{8}}$$

$$= \frac{9}{28}$$

vi) $P(x+y \leq 4)$

y: 1 2 3 4 5 6

0+5, 1+4 = 5

$= p(0,1) + p(0,2) + p(0,3) + p(0,4) +$
$p(1,1) + p(1,2) + p(1,3) +$
$p(2,1) + p(2,2)$

$$= 0 + 0 + \frac{1}{32} + \frac{2}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32}$$

$$= \frac{13}{32}$$

vii) Marginal PMF of x.

x	0	1	2
$P_x(x)$	$\frac{8}{32}$	$\frac{20}{32}$	$\frac{4}{32}$

viii) Marginal PMF of Y.

y	1	2	3	4	5	6
$P_y(y)$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$

ix) Conditional distribution of x given $Y=2$ is

$P[X = x_i \mid Y = 2]$ $\quad x = 0, 1, 2$

$$P[X=0 \mid Y=2] = \frac{P[x=0, Y=2]}{P[Y=2]}$$

$$= \frac{P(0,2)}{P(Y=2)} = \frac{0}{\left(\frac{3}{32}\right)} = 0$$

$$P[X=1 \mid Y=2] = \frac{P[x=1, Y=2]}{P[Y=2]}$$

$$= \frac{P(1,2)}{P(Y=2)} = \frac{\frac{1}{16}}{\frac{3}{32}} = \frac{2}{3}$$

$$P[X=2 \mid Y=2] = \frac{P[x=2, Y=2]}{P[Y=2]}$$

$$= \frac{\frac{1}{32}}{\frac{3}{32}} = \frac{1}{3}$$

x) $P(x,y) = P_x(x)\, P_y(y)$ if x and y are independent.

$p(x=0) \times p(x=1) = p(0,1)$

But $\frac{8}{32} \times \frac{3}{32} \neq 0$

$\therefore$ x and y are not independent.

xi) $E[y-2x] = E[y] - 2E[x]$

$E[x] = \sum x\, p(x)$

$= 0 \times \frac{8}{32} + 1 \times \frac{20}{32} + 2 \times \frac{4}{32} = \frac{28}{32}$

x) $P(x,y) = P_x(x)\, P_y(y)$ if x and y are independent.

$$p(x=0) \times p(x=1) = p(0,1)$$

But $\frac{8}{32} \times \frac{3}{32} \neq 0$

$\therefore$ x and y are not independent.

xi) $E[Y-2X] = E[Y] - 2E[X]$

$$E[x] = \sum x\, p(x)$$

$$= 0 \times \frac{8}{32} + 1 \times \frac{20}{32} + 2 \times \frac{4}{32} = \underline{\underline{\frac{28}{32}}}$$

$$E[4] = \sum y\, p(y)$$

$$= 1 \times \frac{3}{32} + 2 \times \frac{3}{32} + 3 \times \frac{11}{64} + 4 \times \frac{13}{64} + 5 \times \frac{6}{32} + 6 \times \frac{16}{64}$$

$$= \underline{\underline{\frac{259}{64}}}$$

$\therefore$ $E[4-2x] = E[4] - 2E[x]$

$$= \frac{259}{64} - 2 \times \frac{28}{32}$$

$$= \underline{\underline{\frac{147}{64}}}$$

Discrete two-dimensional random Variables:

p.m.f

x	0, 1, 2, ... n
$p(x)$	

1) Joint Probability mass fun: [Joint Pmf]

Suppose x and y are discrete random Variables defined by

$p(x) = \frac{1}{2^x}$ $x = 0, 1, 2, \ldots, 10$

$$P(x,y) = P[X = x, Y = y]$$

i) $\sum p(x) = 1$

ii) $p(x) \geq 0$

$0 \leq p(x) \leq 1$

Note:

$$\sum p(x,y) = 1$$

$$0 \leq p(x,y) \leq 1$$

$y \backslash x$	0	1	2
0	(.,)	.	
1			
2			
3			

Marginal Distributions

Let x and y be discrete random Variables with joint Pmf $P(x,y)$. Then the Probability mass fun. of x and y are given respectively.

marginal Pmf of x is $P(X = x) = \sum_{y} P(x,y)$

Let X and Y have the following joint Probability distribution.

Y \ X	2	4
1	0.10	0.15
3	0.20	0.30
5	0.10	0.15

Show that X and Y are Independent.

Y \ X	2	4	$P_y(y)$
Y = 1	0.10 P(2,1)	0.15	P(Y=1) = 0.25
Y = 3	0.20	0.30	P(Y=3) = 0.5
Y = 5	0.10	0.15 P(4,5)	P(Y=5) = 0.25
$P_x(x)$	P(X=2) = 0.40	0.60	1

If x and y are independent then

$$p(x,y) = P_x(x)\, P_y(y)$$

$$0.25 \times 0.40 = 0.10$$

$$P_y(y=1) \times P_x(x=2) = P(2,1)$$

$$0.25 \times 0.60 = 0.15$$

$$P_y(y=5) \times P_x(x=4) = P(4,5)$$

∴ X and Y are Independent Random Variable.

The joint Probability mass function of random Variables x and Y is given by.

$$p(x,y) = \begin{cases} \dfrac{x(x+y)}{70} & x = 1,2,3 \quad y = 3,4 \\ 0 & \text{otherwise.} \end{cases}$$

find $E[x]$ and $E[Y]$

$y \backslash x$	1	2	3	$P_y(y)$
$y=3$	$P(1,3)$ $\frac{4}{70}$	$P(2,3)$ $\frac{10}{70}$	$P(3,3)$ $\frac{18}{70}$	$P(y=3)$ $\frac{32}{70}$
$y=4$	$(1,4)$ $\frac{5}{70}$	$P(2,4)$ $\frac{12}{70}$	$P(3,4)$ $\frac{21}{70}$	$P(y=4)$ $\frac{38}{70}$
$P_x(x)$	$P(x=1)$ $\frac{9}{70}$	$P(x=2)$ $\frac{22}{70}$	$P(x=3)$ $\frac{39}{70}$	1

$x=1 \quad y=3$

$p(1,3) = \frac{1(1+3)}{70} = \frac{4}{70}$

$x=1 \quad y=4$

$p(1,4) = \frac{1(1+4)}{70} = \frac{5}{70}$

$\frac{2(2+3)}{70} =$ 5

$\frac{2(2+4)}{70}$

$\frac{3(3+3)}{70}$

$\frac{3(3+4)}{70}$

marginal Pmf of x.

$x:$	1	2	3
$P_x(x)$	$\frac{9}{70}$	$\frac{22}{70}$	$\frac{39}{70}$

$Var(x) = E(x^2) - E(x)^2$

$E(x^2) =$

$$E[x] = \sum x \, P_x(x)$$

$$= 1 \times \frac{9}{70} + 2 \times \frac{22}{70} + 3 \times \frac{39}{70}$$

$$= \underline{\underline{\frac{170}{70}}}$$

marginal Pmf of Y is

$y:$	3	4
$P_y(y)$	$\frac{32}{70}$	$\frac{38}{70}$

$$E[Y] = \sum y \, P_y(y)$$

$$= 3 \times \frac{32}{70} + 4 \times \frac{38}{70}$$

$$= \underline{\underline{\frac{248}{70}}}$$

$P(x \leq 1)$

The joint Probability mass function of random Variables x and y is given by.

$x \backslash y$	$y=0$	$y=1$	$y=2$
$x=0$	$p(0,0)$ $\frac{1}{6}$	$p(0,1)$ $\frac{1}{4}$	$p(0,2)$ $\frac{1}{8}$
$x=1$	$p(1,0)$ $\frac{1}{8}$	$p(1,1)$ $\frac{1}{6}$	$p(1,2)$ $\frac{1}{6}$

find the following i) $Var(x)$ ii) $Var(y)$ and iii) $E[XY]$

$x \backslash y$	$y=0$	$y=1$	$y=2$	$P_x(x)$
$x=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$	$P(x=0)$ $\frac{13}{24}$
$x=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$P(x=1)$ $\frac{11}{24}$
$P_y(y)$	$P(y=0)$ $\frac{7}{24}$	$P(y=1)$ $\frac{10}{24}$	$P(y=2)$ $\frac{7}{24}$	1

marginal Pmf of x

x:	0	1
$P_x(x)$	$\frac{13}{24}$	$\frac{11}{24}$

$$E[x] = \sum x \, P_x(x)$$
$$= 0 + \frac{11}{24} = \frac{11}{24}$$

$$E[x^2] = \sum x^2 \, P_x(x)$$
$$= 0 + 1 \times \frac{11}{24} = \frac{11}{24}.$$

$$\therefore Var(x) = E[x^2] - E[x]^2$$
$$= \frac{11}{24} - \left(\frac{11}{24}\right)^2$$
$$= \frac{143}{576}$$

marginal Pmf of y

y:	0	1	2
$P_y(y)$	$\frac{7}{24}$	$\frac{10}{24}$	$\frac{7}{24}$

$$E[y] = \sum y \, P_y(y)$$
$$= 0 + \frac{10}{24} + \frac{14}{24} = 1$$

$$E[y^2] = \sum y^2 \, P_y(y)$$
$$= 0 + \frac{10}{24} + \frac{28}{24} = \frac{38}{24}.$$

$$Var(y) = E[y^2] - E[y]^2$$
$$= \frac{38}{24} - 1 = \frac{14}{24}$$

$$E[xy] = \sum xy \, p(x,y)$$

$$E[XY] = \sum_{x=0}^{1} \sum_{y=0}^{2} xy\, p(x,y)$$

$x: 0 \quad 1$

$y: 0 \quad 1 \quad 2$

$p(1,2)$

$$= 0 \times 0 \times \frac{1}{6} + 0 \times 1 \times \frac{1}{4} + 0 \times 2 \times \frac{1}{8} +$$

$$+ 1 \times 0 \times \frac{1}{8} + 1 \times 1 \times \frac{1}{6} + 1 \times 2 \times \frac{1}{6}$$

$$= \frac{1}{2}$$

$E[XY]$

$E[X] = ?$

$E[Y] = ?$

$E[XY] =$

if X and Y independent

$$E[XY] = E[X]\,E[Y]$$

$$= \frac{11}{24} \cdot 1$$

$$= \frac{11}{24}$$