TRANSISTOR EQUIVALENT CIRCUIT MODEL

- TOTAL RESPONSE OF AN AMPLIFIER:
  - DC RESPONSE + AC RESPONSE
- AC ANALYSIS
  - SMALL SIGNAL ANALYSIS
  - LARGE SIGNAL ANALYSIS
- What is an Equivalent Model??
- An equivalent model is the combination of circuit elements properly chosen to represent the actual behavior of the device under specific operating point
- Why Equivalent model?
- Cannot apply network theorems directly on practical devices. So need equivalent model to use this theorems to find out different network parameters.
- TRANSISTOR MODELS
  - Hybrid Model
  - re Model
  - Hybrid Pi Model

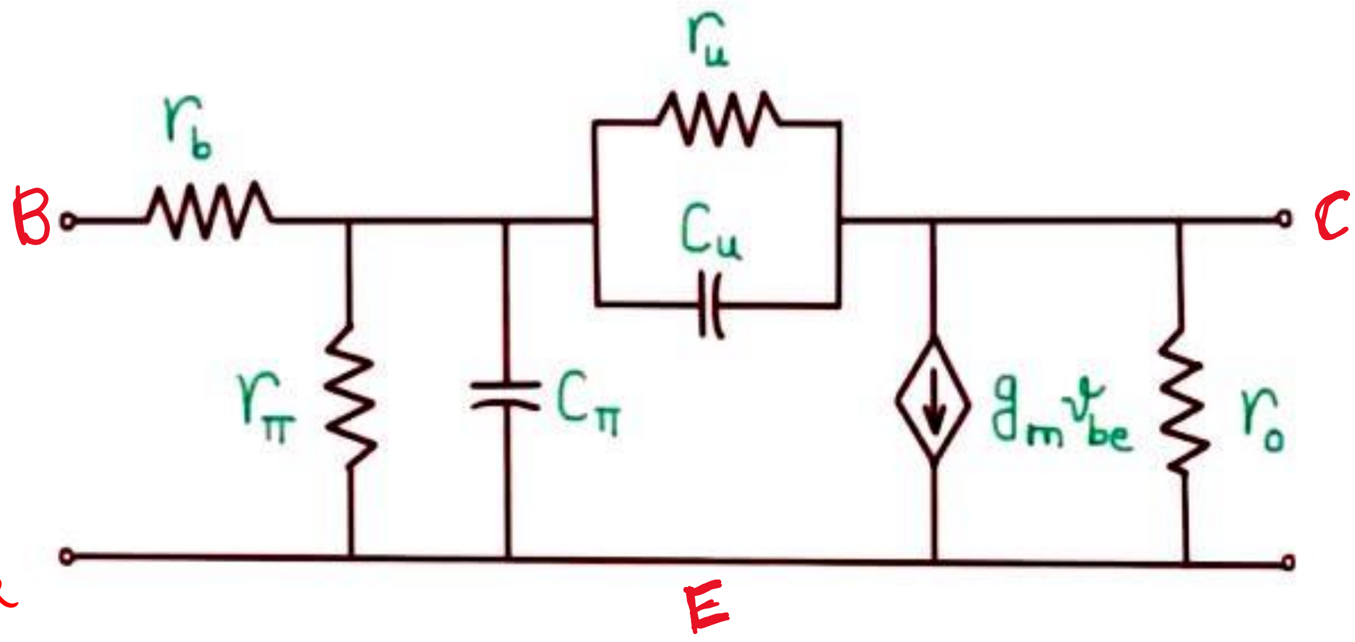
HYBRID π MODEL

- **C_μ -** Transition capacitance between base and collector. Early effect representation
- **C_π –** Diffusion Capacitance. Minority carriers stored in Base region
- **r_π –** small input resistance between base and emitter seen from base
- **r_μ -** collector base reverse resistance (large value)
- **r_b -** base terminal resistance (small value)
- **r_o –** output resistance seen from output, collector emitter resistance
- **$g_m\ v_{be}$ –** current source

$g_m \rightarrow$ Trans conductance

$i_c = g_m v_{be}$

$i_c = h_{21} i_b$



VOLTAGE CONTROLLED CURRENT SOURCE

CURRENT CONTROLLED CURRENT SOURCE

HYBRID π MODEL

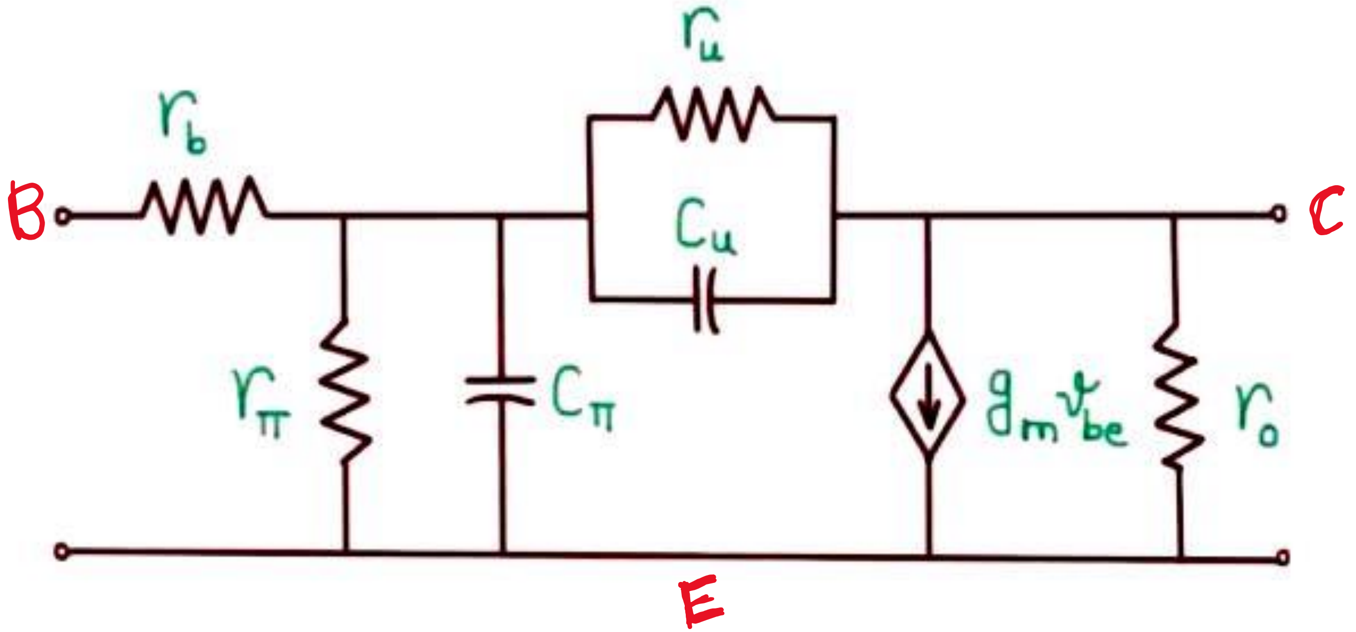
DC conditions of a transistor

$$I_C = I_0 \, e^{(V_{BE}/V_T)}$$

If an input signal v_{be} also applied to the base

$$i_C = I_0 \, e^{(V_{BE} + v_{be})/V_T}$$

$$= I_0 \, e^{V_{BE}/V_T} \cdot e^{v_{be}/V_T}$$



$$i_C = I_C \cdot e^{v_{be}/V_T} = I_C \left(1 + \frac{v_{be}}{V_T}\right)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!}$$

$$i_C = I_C + I_C \cdot \frac{v_{be}}{V_T} \rightarrow i_c$$

$$e^{v_{be}/V_T} = 1 + \frac{v_{be}}{V_T} + \dots$$

$$v_{be} << V_T$$

DC component

Signal Component

HYBRID π MODEL

$$i_c = I_c \cdot \frac{v_{be}}{V_T}$$

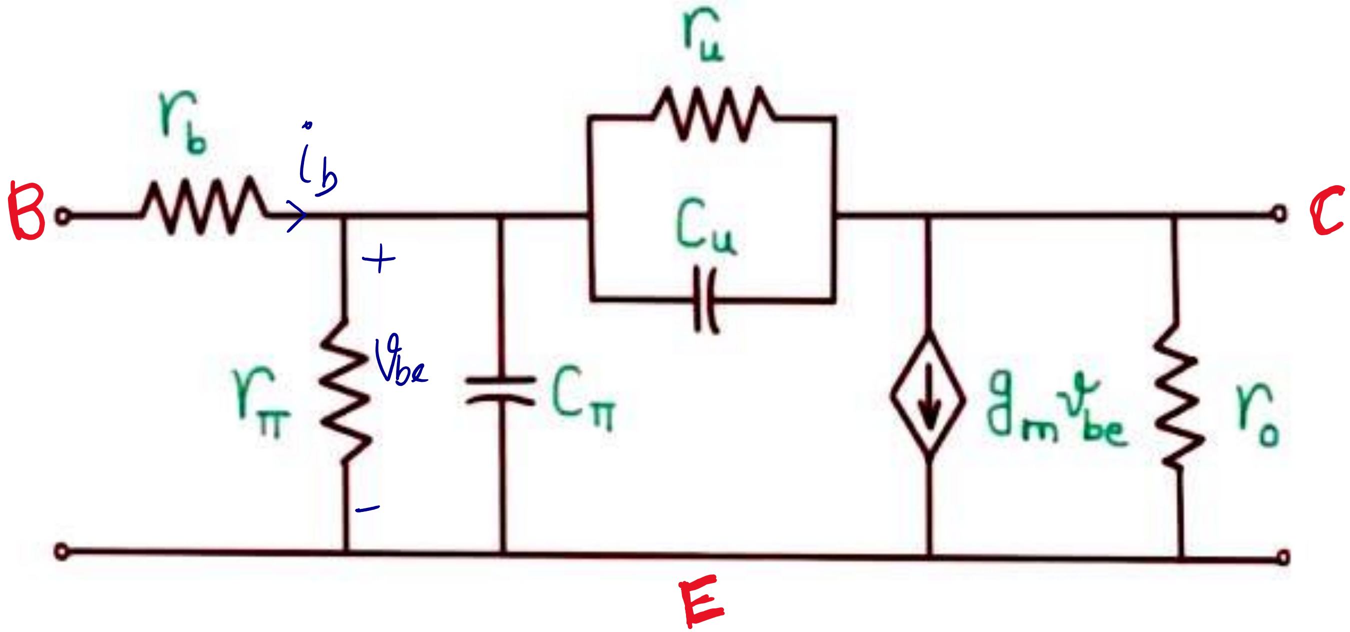
TRANSCONDUCTANCE, $g_m = \frac{I_c}{V_T}$

$$i_c = g_m v_{be}$$

$$i_b = \frac{i_c}{\beta} \Rightarrow \quad i_b = \frac{g_m v_{be}}{\beta}$$

$$r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$$

$$g_m = \frac{I_c}{V_T} \Rightarrow \quad r_\pi = \frac{\beta V_T}{I_c} = \frac{V_T}{I_B}$$



$$\boxed{i_c = g_m v_{be}}$$

$$\boxed{i_b = \frac{g_m v_{be}}{\beta}}$$

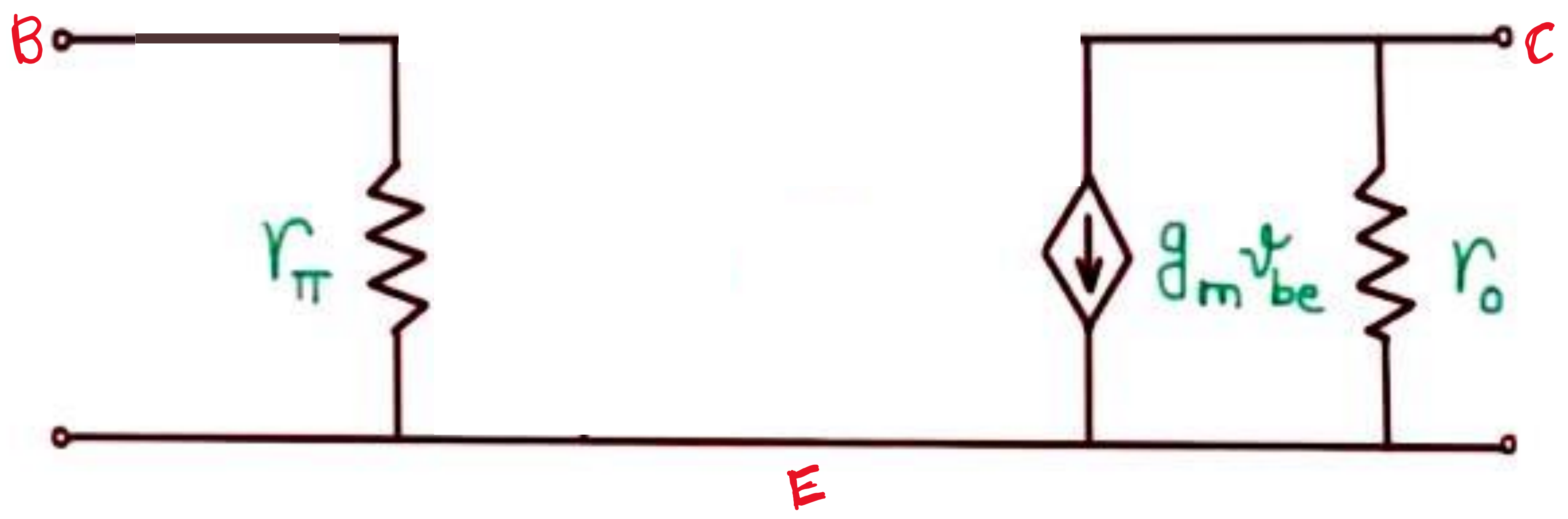
$$\boxed{r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}}$$

HYBRID π MODEL

r_b - very small

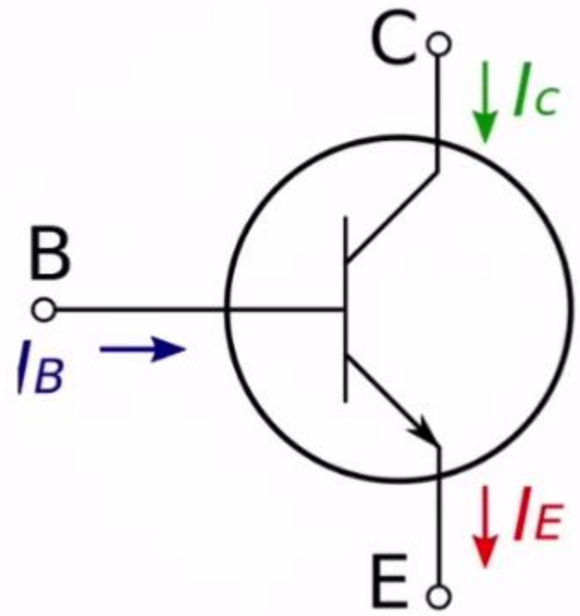
r_μ - very large

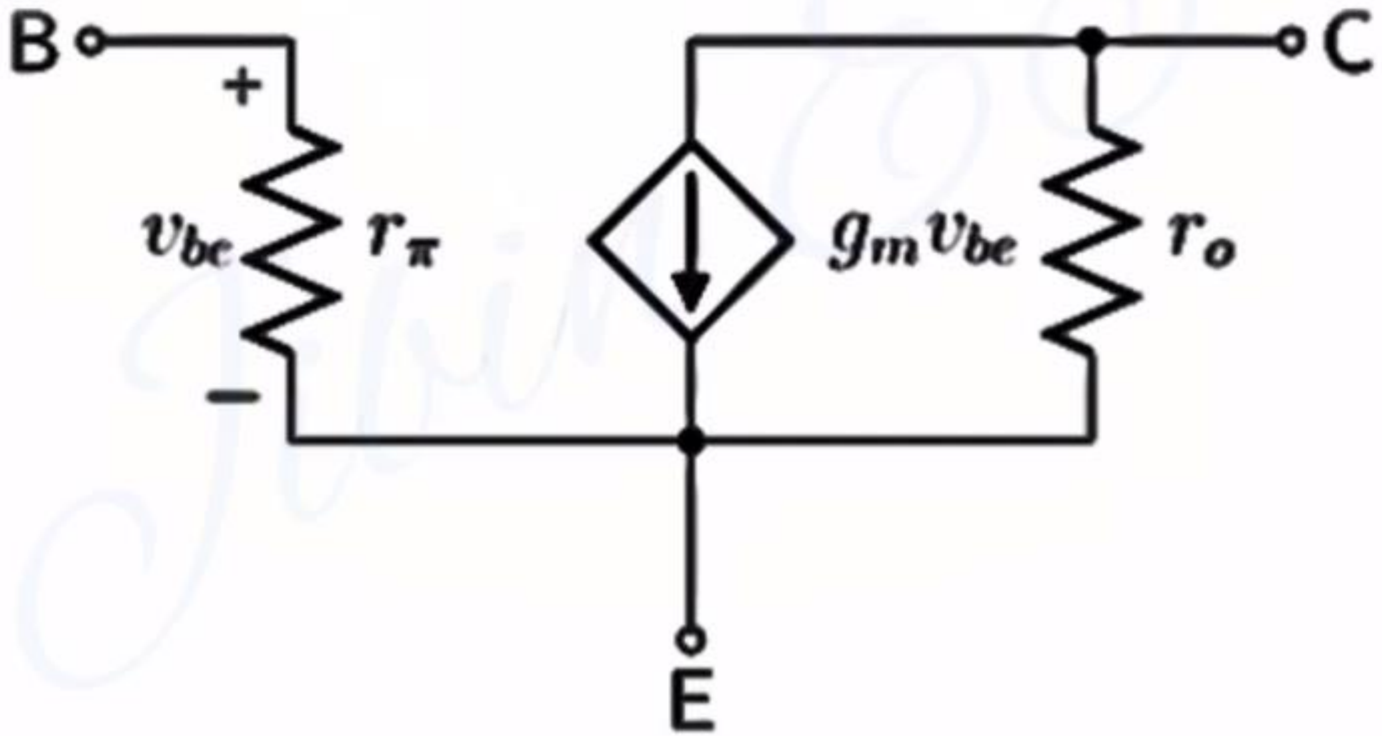
$X_c = \frac{1}{2\pi f C}$



$f\downarrow$ $X_c\uparrow$ (OPEN CIRCUIT)

HYBRID π MODEL

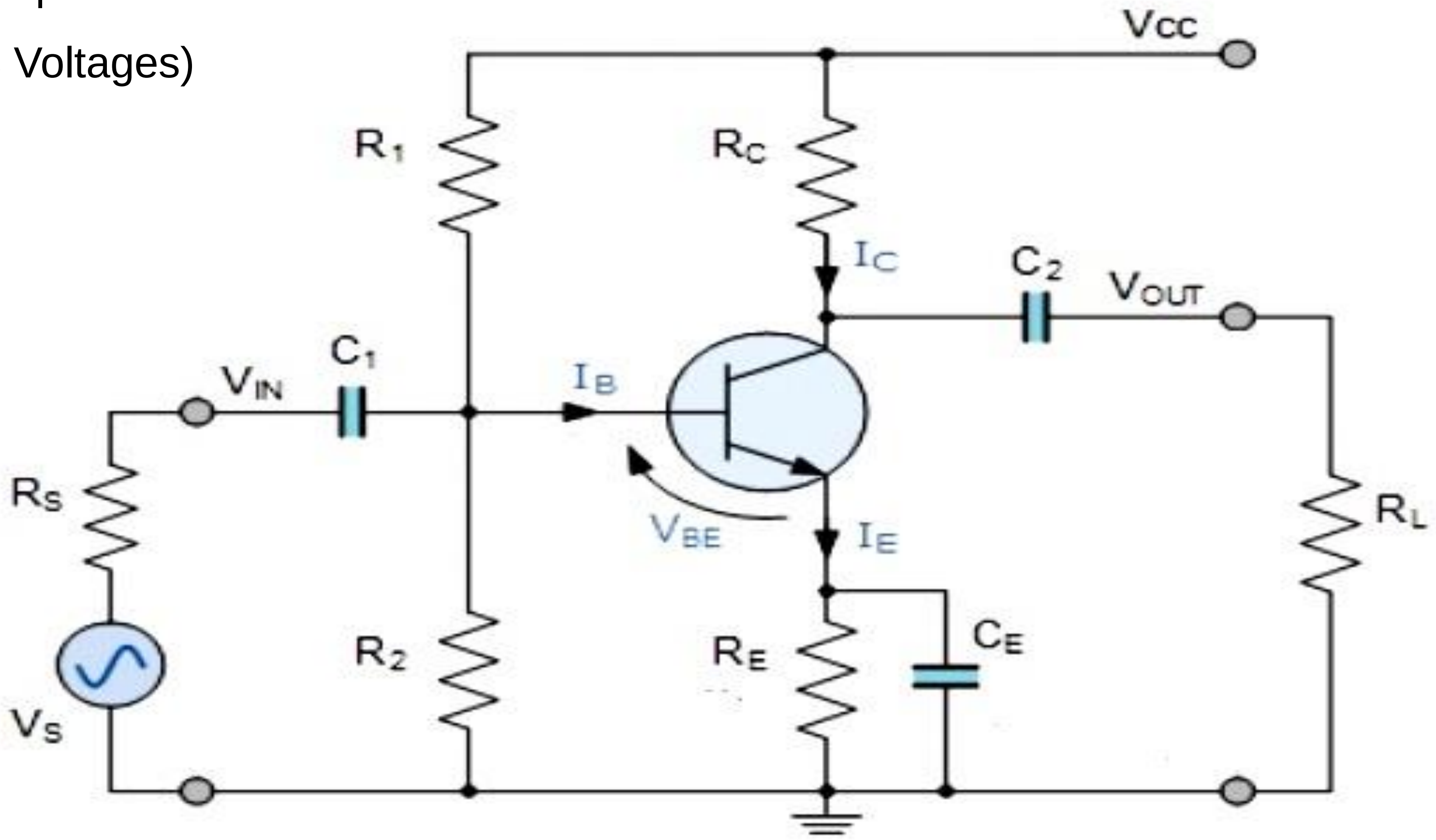




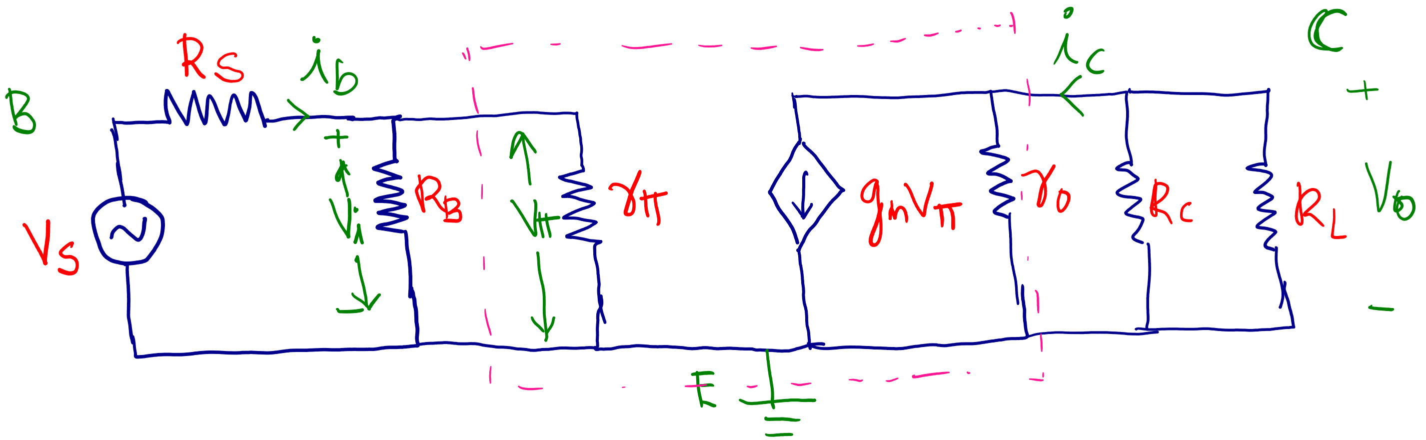
CE AMPLIFIER WITH BYPASS CAPACITOR

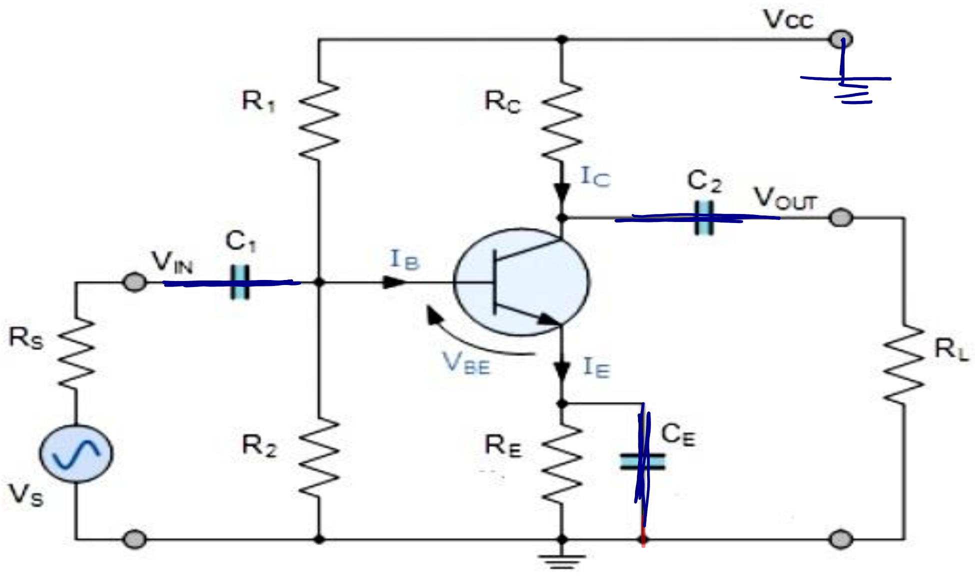
STEPS TO CONVERT TO HYBRID π MODEL

- Short circuit all Bypass and Coupling Capacitors
- Remove all external power supply (DC Voltages) and connect that node to ground

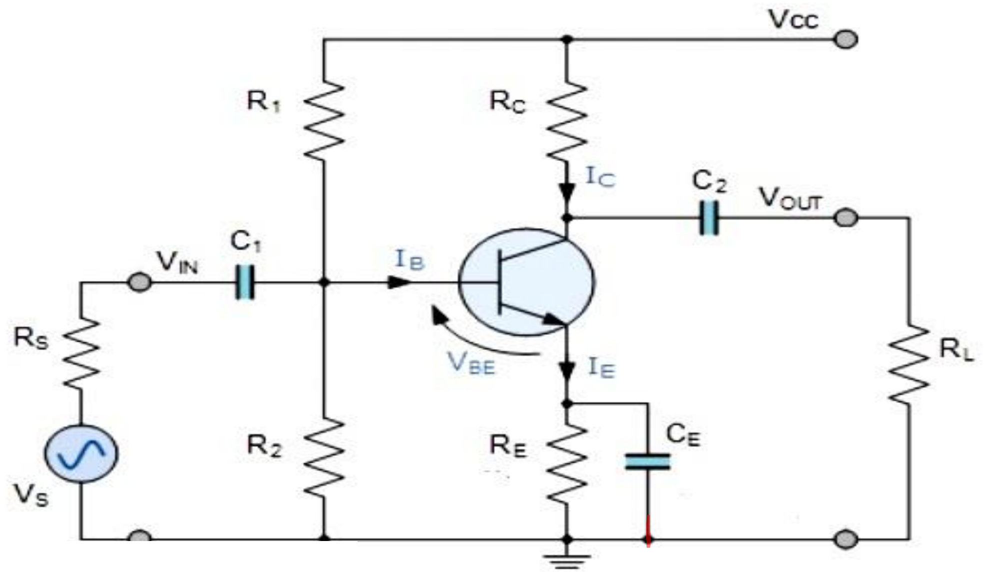


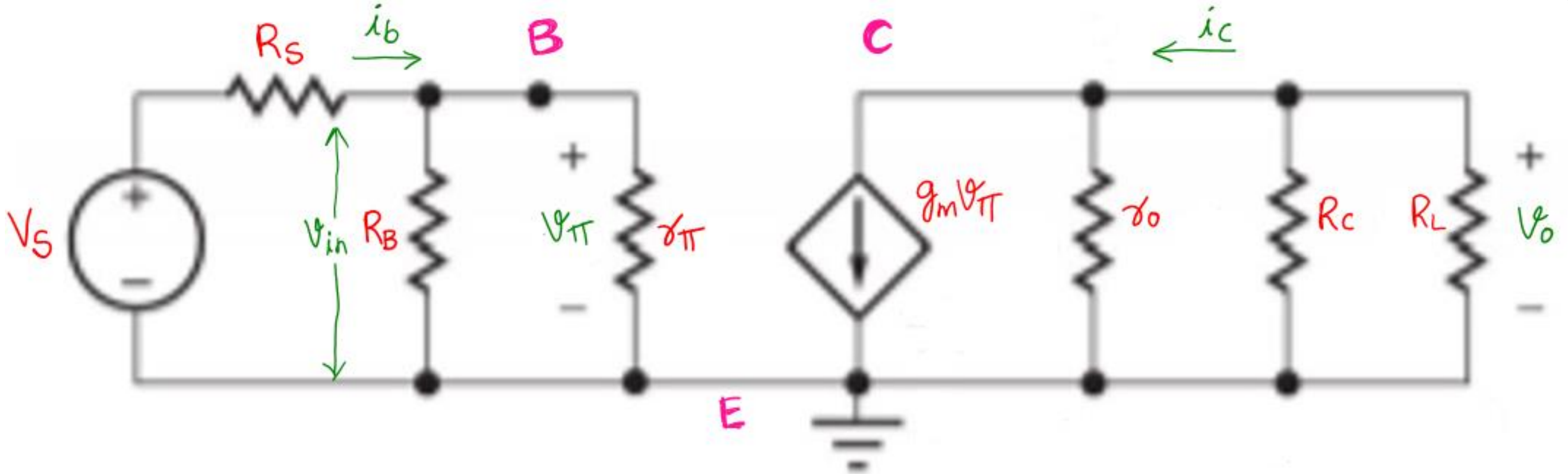
EQUIVALENT CIRCUIT (CE amplifier with bypass capacitor)



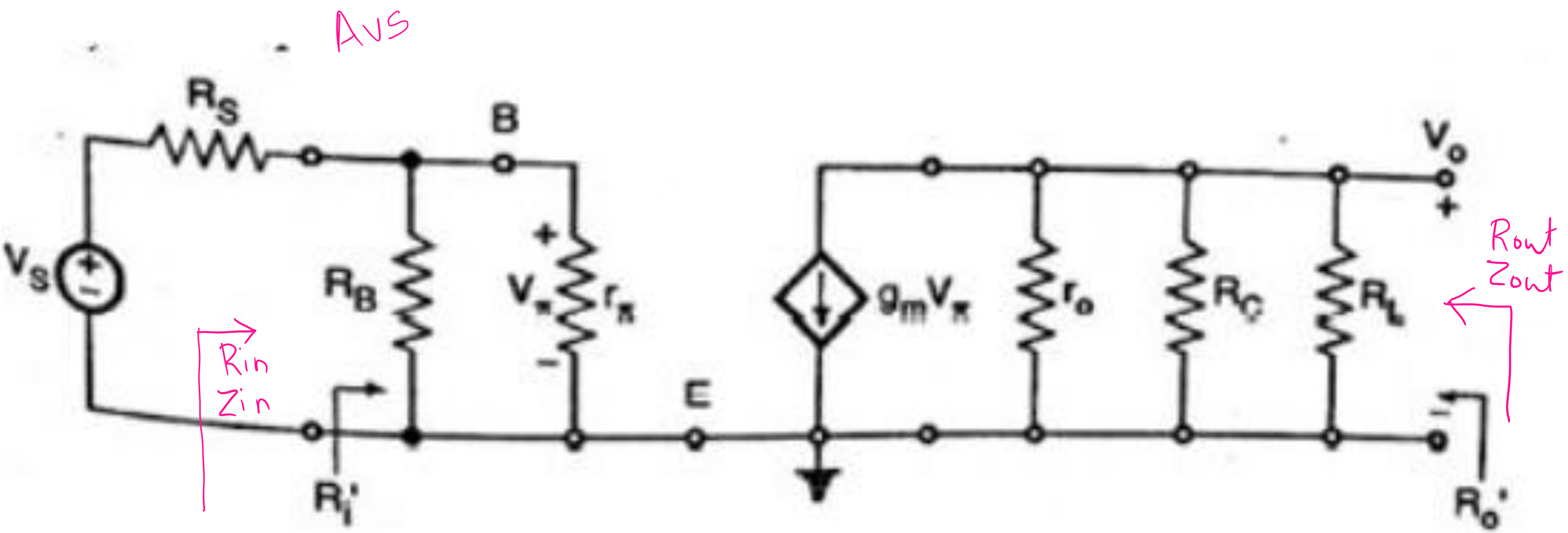


EQUIVALENT CIRCUIT (CE amplifier with bypass capacitor)





EQUIVALENT CIRCUIT (CE amplifier with bypass capacitor)

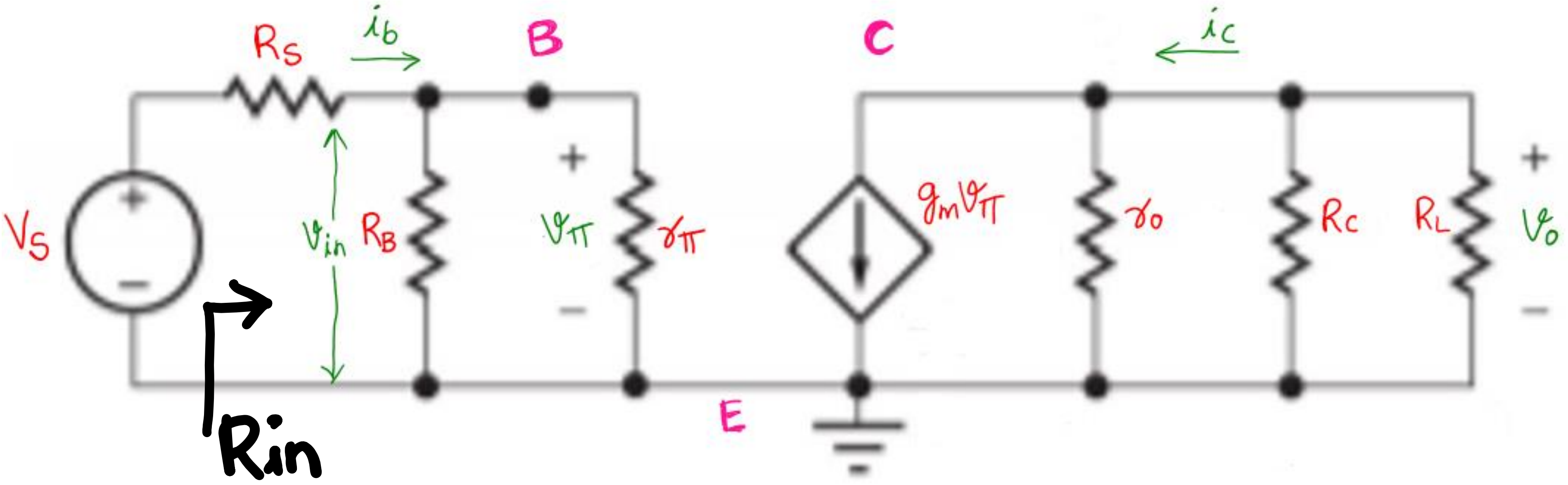


EQUIVALENT CIRCUIT (CE amplifier with bypass capacitor)

PARAMETERS TO DETERMINE

- INPUT RESISTANCE (R_{in})
- OUTPUT RESISTANCE (R_{OUT})
- VOLTAGE GAIN WITHOUT V_S (A_V)
- VOLTAGE GAIN WITH V_S (A_{VS})
- CURRENT GAIN (A_i)

Input Resistance (Rin) or Input Impedance (Zin)



$$\boxed{R_{in} = Z_{in} = R_B \parallel r_\pi}$$

Voltage Gain (without V_s) (A_v)

$$A_v = \frac{v_{out}}{v_{in}} = \frac{i_{out} \times R_{out}}{v_{in}}$$

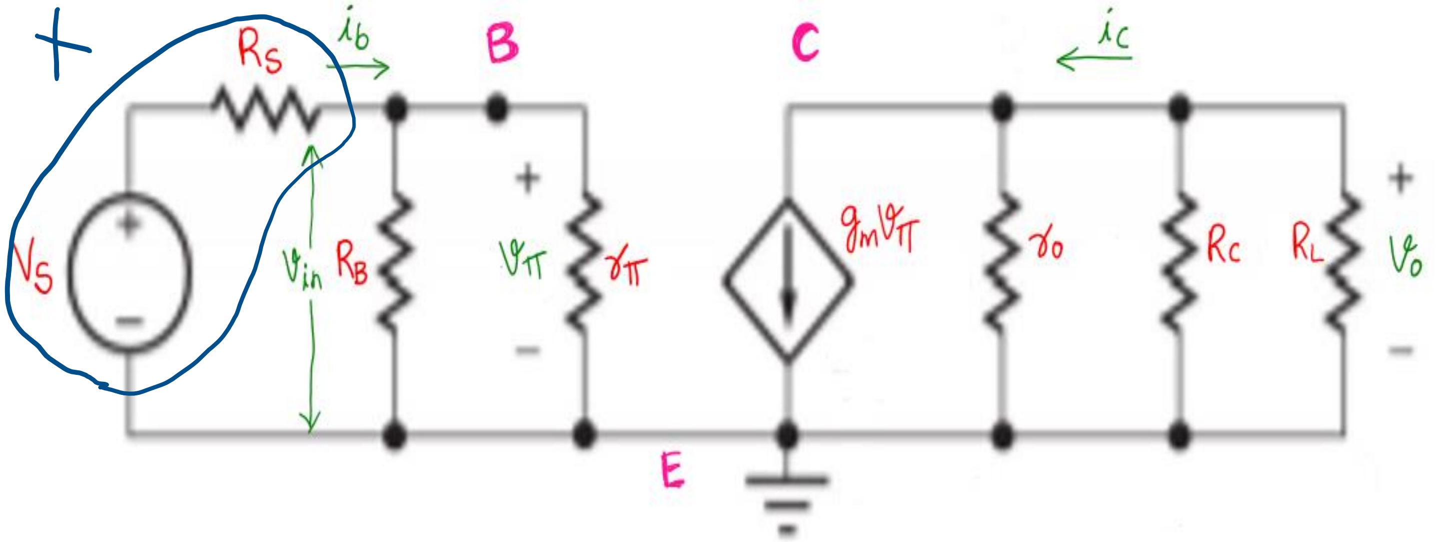
$$i_{out} = -g_m v_\pi$$

$$R_{out} = (R_L \| R_C \| r_o)$$

$$V_\pi \cong v_{in}$$

$$i_{out} = -g_m v_{in}$$

$$A_v = \frac{-g_m v_{in} \times (R_L \| R_C \| r_o)}{v_{in}}$$

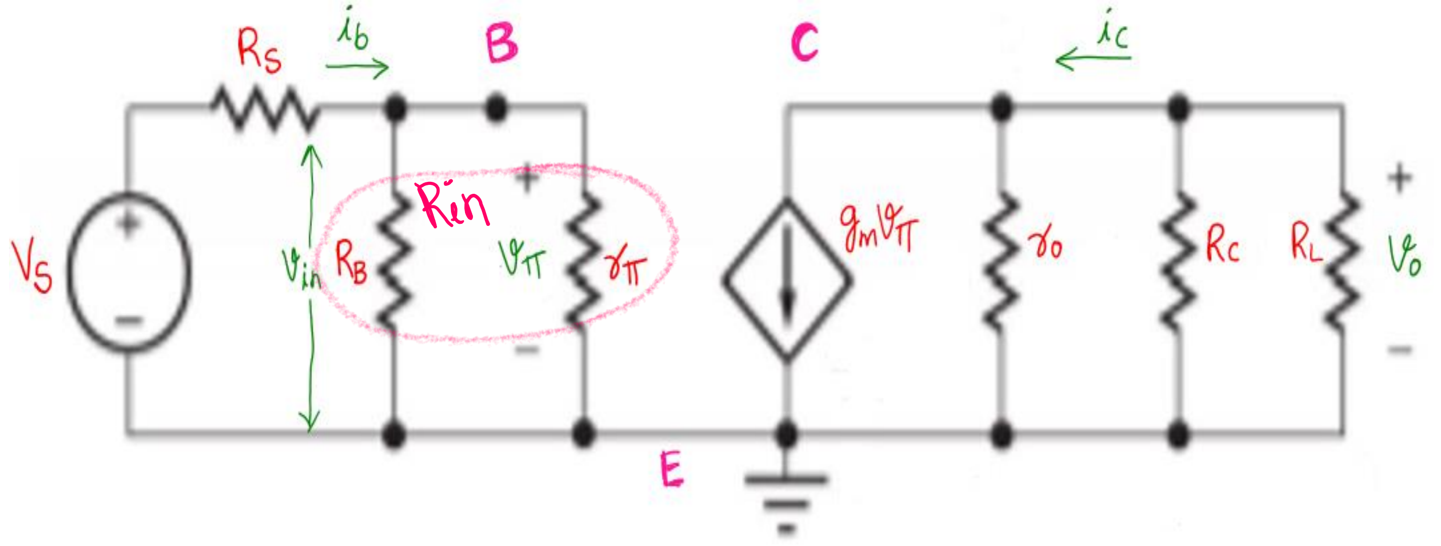


$$\boxed{A_v = -g_m (R_L \| R_C \| r_o)}$$

Voltage Gain (with V_s) (A_{Vs})

$$A_{Vs} = \frac{v_{out}}{v_s}$$

$$R_B \| r_\pi \cong R_{in}$$



APPLYING VOLTAGE DIVIDER RULE AT INPUT SIDE

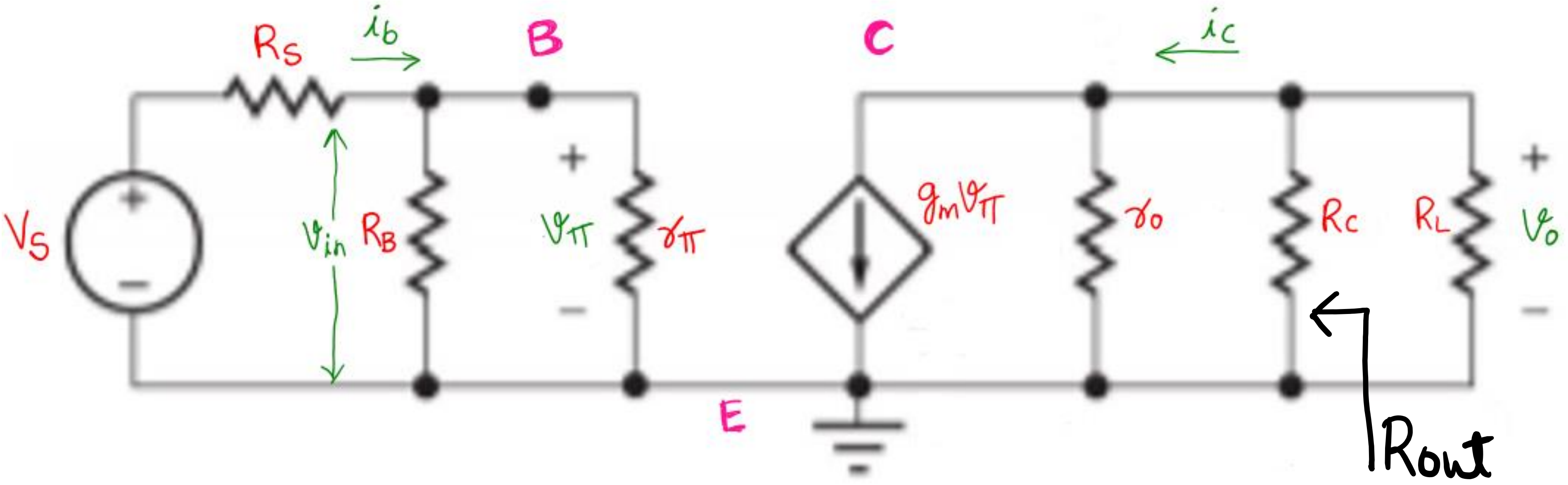
$$v_{in} = \frac{V_s \cdot R_{in}}{R_{in} + R_s} \qquad \frac{v_{in}}{V_s} = \frac{R_{in}}{R_{in} + R_s}$$

$$A_{Vs} = \frac{v_{out}}{v_s} = \frac{v_{out}}{v_s} \times \frac{v_{in}}{v_{in}} = \frac{v_{out}}{v_{in}} \times \frac{v_{in}}{v_s}$$

$$\boxed{A_{Vs} = A_V \left(\frac{R_{in}}{R_{in} + R_s} \right)}$$

$$\boxed{A_{Vs} = -g_m (R_C \| R_L \| r_o) \left(\frac{R_{in}}{R_{in} + R_s} \right)}$$

Output Resistance (Rout) or Output Impedance (Zout)



$$R_{out} = Z_{out} = (R_C \parallel r_o)$$

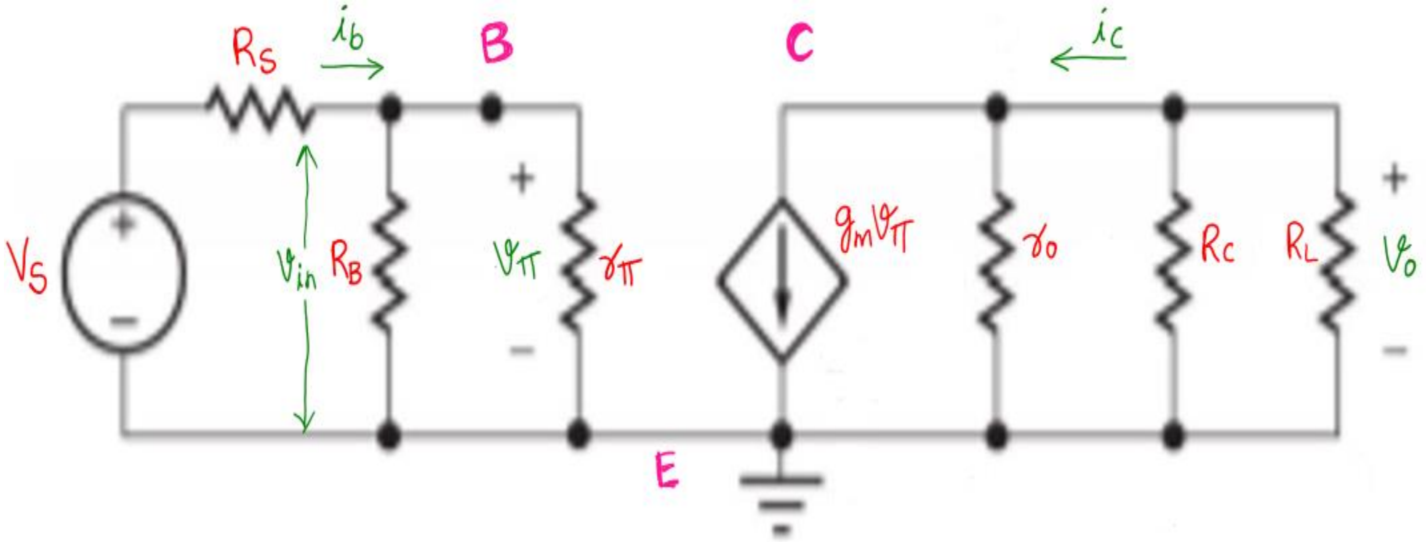
Current Gain (A_i)

$$A_i = \frac{i_{out}}{i_{in}}$$

$$i_{out} = v_{out}/R_L$$

$$R_{in} \cong R_B \| r_\pi$$

$$i_{in} = v_{in}/R_{in}$$

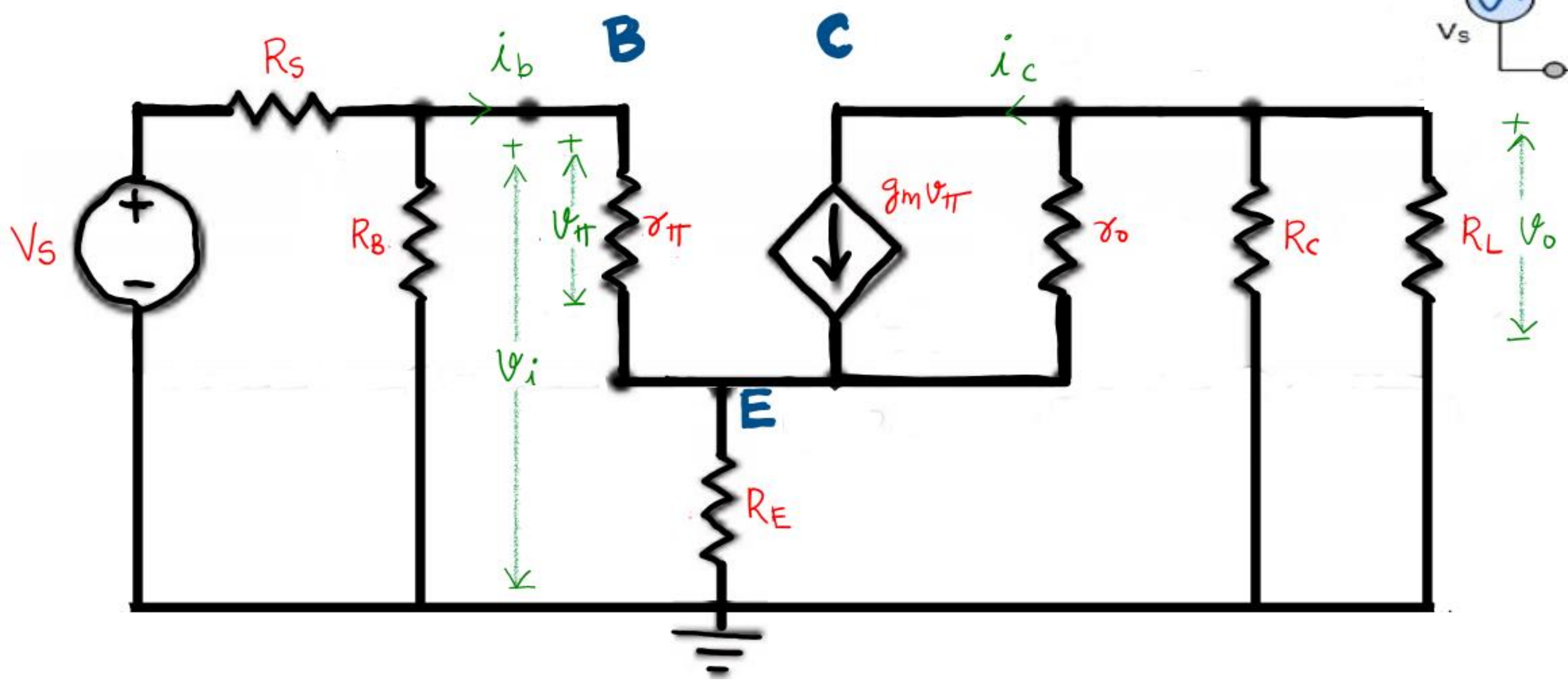


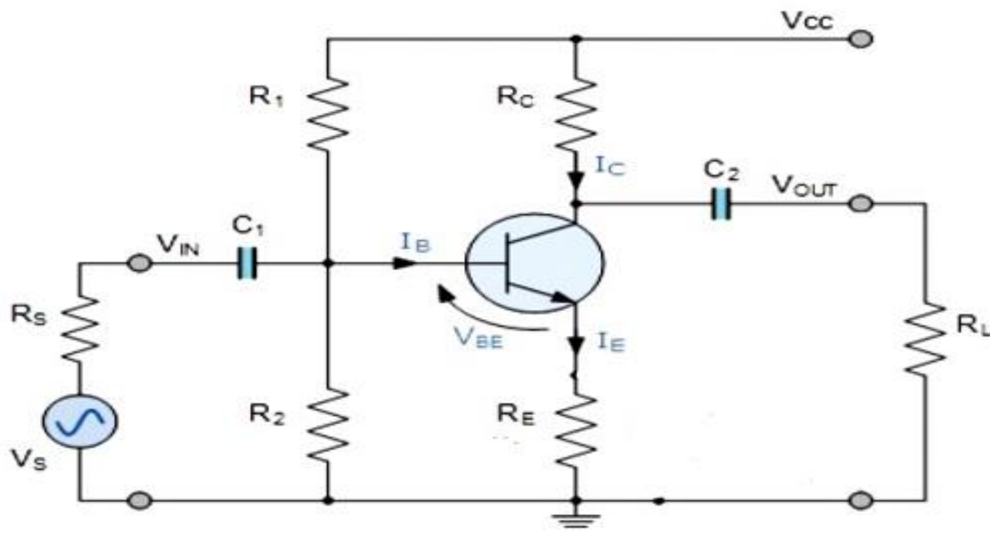
$$A_i = \frac{i_{out}}{i_{in}} = \frac{v_{out}/R_L}{v_{in}/R_{in}} = \frac{v_{out}}{v_{in}} \times \frac{R_{in}}{R_L}$$

$$\boxed{A_i = A_v \cdot \frac{R_{in}}{R_L}}$$

$$\boxed{A_i = -g_m (R_C \| R_L \| r_o) \cdot \frac{(R_B \| r_\pi)}{R_L}}$$

CE amplifier without bypass capacitor





CE amplifier without bypass capacitor

PARAMETERS TO DETERMINE

- INPUT RESISTANCE (R_{in})
- VOLTAGE GAIN WITHOUT V_S (A_V)
- VOLTAGE GAIN WITH V_S (A_{VS})
- CURRENT GAIN (A_i)

Input Resistance (Rin) or Input Impedance (Zin)

$$R_{in} = R_B \| R_i' \qquad R_i' = \frac{v_{in}}{i_b}$$

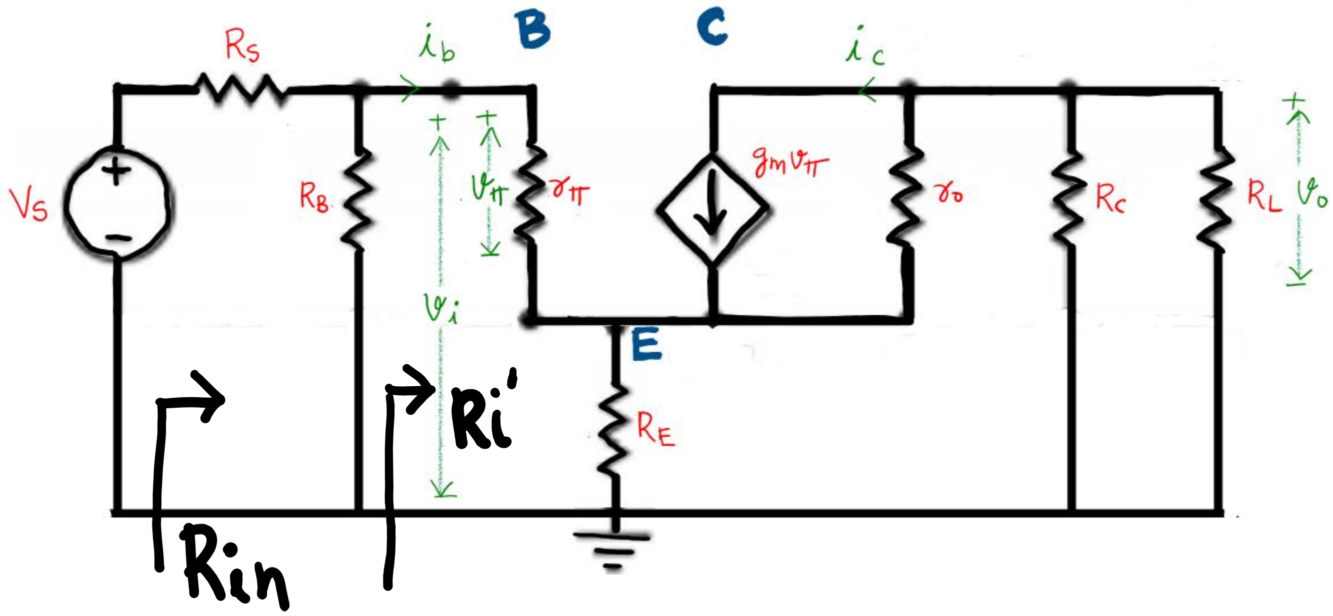
$$v_{in} = i_b r_\pi + (i_b + g_m v_\pi) R_E$$

$$v_{in} = i_b \left(r_\pi + \left(1 + \frac{g_m v_\pi}{i_b}\right) R_E \right)$$

$$v_{in} = i_b \left(r_\pi + (1 + g_m r_\pi) R_E \right)$$

$$R_i' = \frac{v_{in}}{i_b} = \frac{i_b \left(r_\pi + (1 + g_m r_\pi) R_E \right)}{i_b}$$

$$\boxed{R_{in} = R_B \| \left(r_\pi + (1 + g_m r_\pi) R_E \right)}$$



$$g_m r_\pi = \beta \qquad R_i' = R_E(1+\beta) + r_\pi$$

$$\boxed{R_{in} = R_B \| \left(r_\pi + (1+\beta) R_E \right)}$$

Voltage Gain without Vs (Av)

$$A_V = \frac{V_{out}}{V_{in}} \qquad V_{out} = i_{out} \times R_{out}$$

NEGLECTING r_o term

$$V_{out} = (-g_m v_\pi) \cdot (R_C \| R_L)$$

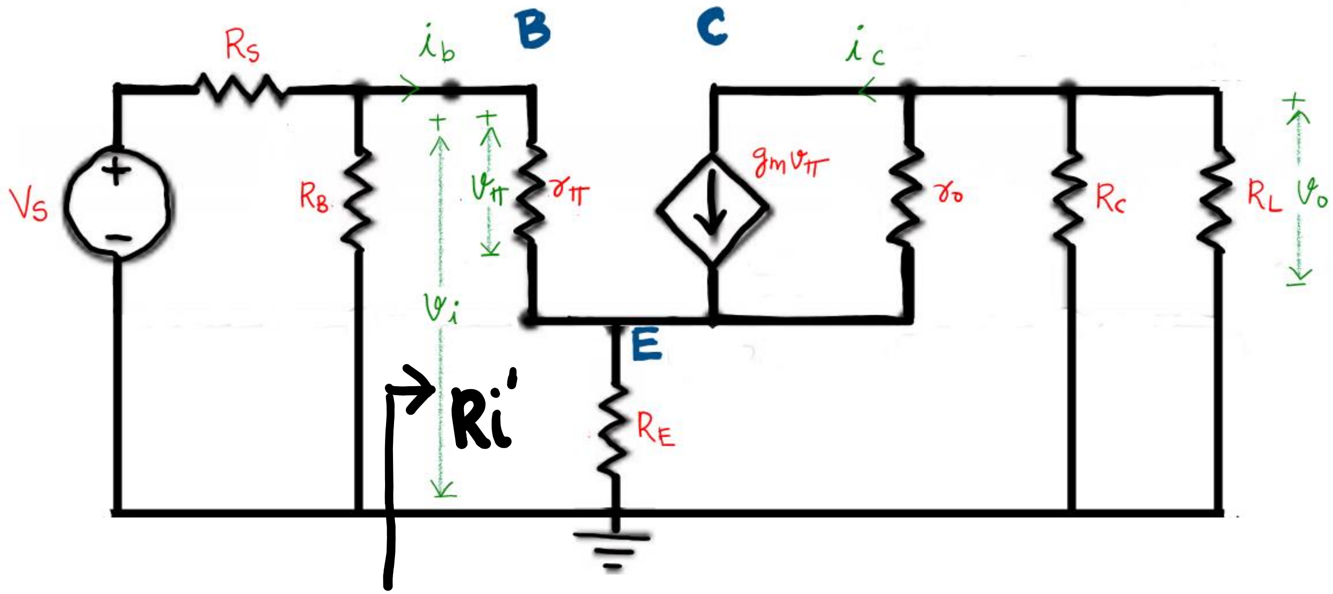
$$v_\pi = i_b r_\pi \qquad V_{in} = R_i' \cdot i_b$$

$$A_V = \frac{(-g_m \cancel{i_b} r_\pi) \cdot (R_C \| R_L)}{R_i' \cdot \cancel{i_b}}$$

$$R_i' = R_E(1+\beta) + r_\pi = R_E + \beta R_E + r_\pi$$

$$R_i' \simeq \beta R_E \qquad \beta = g_m r_\pi$$

$$R_i' = g_m r_\pi R_E$$



$$A_V = \frac{-\cancel{g_m r_\pi}(R_C \| R_L)}{\cancel{g_m r_\pi} R_E}$$

$$\boxed{A_V = \frac{-(R_C \| R_L)}{R_E}}$$

Voltage Gain with Vs (Avs)

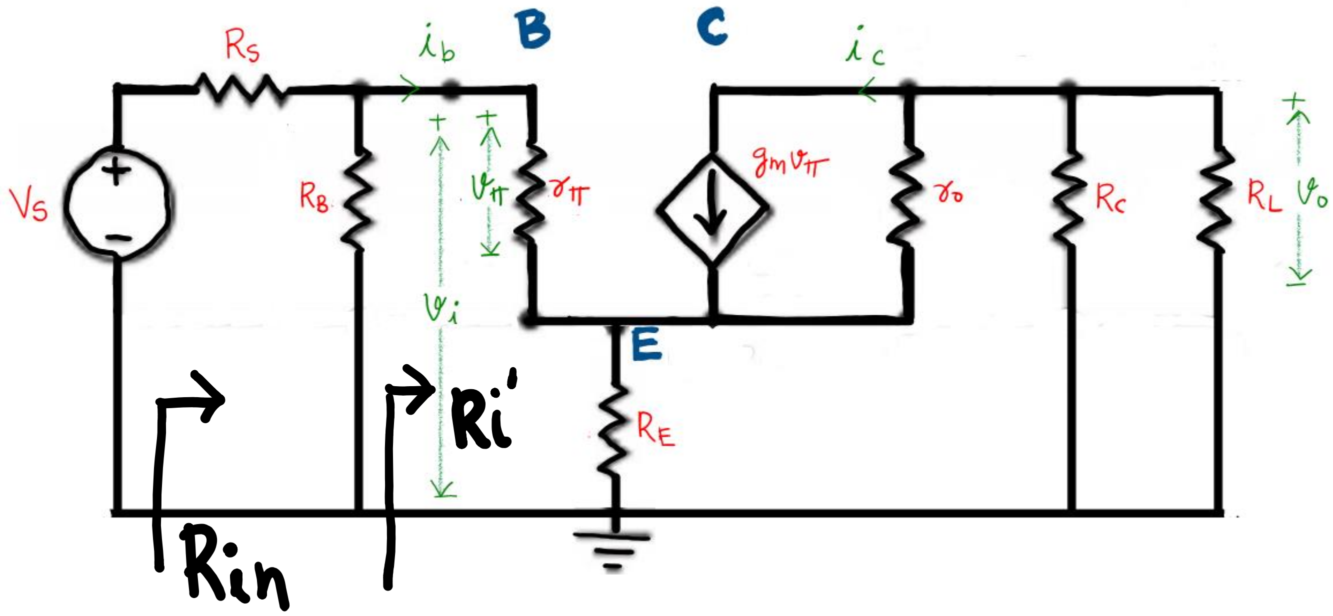
$$A_{vs} = \frac{v_{out}}{v_s}$$

APPLYING VOLTAGE DIVIDER RULE AT INPUT SIDE

$$v_{in} = \frac{v_s \cdot R_{in}}{R_s + R_{in}} \qquad \frac{v_{in}}{v_s} = \frac{R_{in}}{R_s + R_{in}}$$

$$A_{vs} = \frac{v_{out}}{v_s} = \frac{v_{out}}{v_s} \times \frac{v_{in}}{v_{in}} = \frac{v_{out}}{v_{in}} \times \frac{v_{in}}{v_s}$$

$$A_{vs} = A_v \times \left(\frac{R_{in}}{R_{in} + R_s}\right)$$



$$\boxed{A_{vs} = \frac{-(R_c \| R_L)}{R_E}\left(\frac{R_{in}}{R_{in} + R_s}\right)}$$

Current Gain (Ai)

$$A_i = \frac{i_{out}}{i_{in}}$$

$$i_{out} = V_{out}/R_L$$

$$i_{in} = V_{in}/R_{in}$$

$$A_i = \frac{i_{out}}{i_{in}} = \frac{V_{out}/R_L}{V_{in}/R_{in}} = \frac{V_{out}}{V_{in}} \times \left(\frac{R_{in}}{R_L}\right)$$

$$A_i = A_v\left(\frac{R_{in}}{R_L}\right)$$

$$\boxed{A_i = \frac{-(R_c \| R_L)}{R_E}\left(\frac{R_{in}}{R_L}\right)}$$