INFORMATION THEORY & CODING:

LECTURE 4

CONTENTS

- Quick recap
- Implications of Shannon’s Hartley Law
- Shannon’s limit
- Bandwidth-efficiency diagram

Shannon's Limit

- We define an ideal system where the data is transmitted at the rate of Rt = channel capacity C
- We may express the average transmitted signal power as S,

$$S = E_b C$$

Where E_b = transmitted energy per bit in joules.

Using $N = \eta B$ and $S = E_b C$ in equation (4.216), we get for an ideal system

$$C = B \log_2 \left(1 + \frac{E_b}{\eta} \frac{C}{B}\right)$$

C = B log(1+S/N)

$$\text{or} \quad \frac{C}{B} = \log_2 \left(1 + \frac{E_b}{\eta} \frac{C}{B}\right)$$

Bandwidth- Efficiency

The quantity $\left(\frac{C}{B}\right)$ is called ***"Bandwidth-efficiency"*** and the quantity (E_b/η) i

$$\frac{E_b}{\eta} = \frac{2^{C/B} - 1}{(C/B)}$$

$$\frac{C}{B} = \log_2\left(1 + \frac{E_b}{\eta}\frac{C}{B}\right)$$

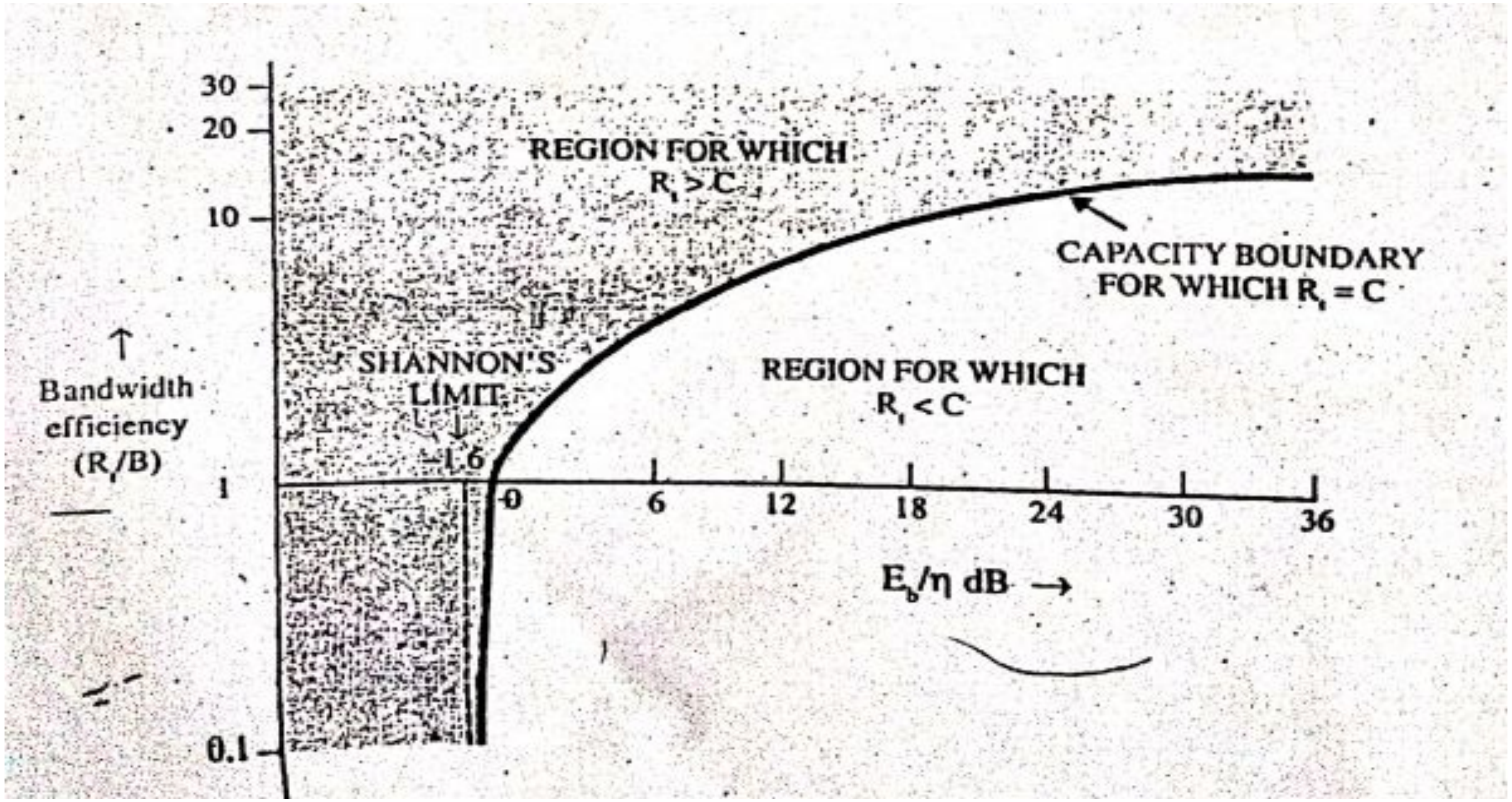
$x = \log_z y$

$z^x = y$

Bandwidth – Efficiency diagram

- We plot Rt/B as a function of Eb/

This diagram represents the capacity boundary for which Rt= C



Observations from the diagram

1. For infinite bandwidth, the signal energy-to-noise ratio E_b/η approaches the limiting value.

$$\left(\frac{E_b}{\eta}\right)_\infty = \lim_{B\to\infty}\left(\frac{E_b}{\eta}\right) = \lim_{B\to\infty}\left[\frac{2^{C/B}-1}{(C/B)}\right]$$

Let $\frac{C}{B} = x$. As $B \to \infty$, $x \to 0$

$$\therefore \quad \left(\frac{E_b}{\eta}\right)_\infty = \lim_{x\to 0}\left[\frac{2^x-1}{x}\right] \qquad \text{...... (4.221)}$$

Using L'Hospital Rule, the above limit can be evaluated as below:

$$\text{Let} \quad y = 2^x$$

Taking ln on both sides

$$ln\, y = x\, ln2$$

Differentiating, $\frac{1}{y}\, dy = (ln2)\, dx$

$$\therefore \quad \frac{dy}{dx} = y\,(ln2) = 2^x\,(ln2) \qquad \text{...... (4.222}$$

Differentiating both numerator and denominator of the RHS of equation (4.221) with respect to 'x', we get

$$\left(\frac{E_b}{\eta}\right)_{\infty} = \lim_{x\to 0}\left[\frac{\frac{d}{dx}(2^x-1)}{\frac{d}{dx}(x)}\right]$$

$$= \lim_{x\to 0}\left[\frac{2^x(ln2)}{1}\right] \text{ by using equation (4.222)}$$

$$= 2^0\, ln2$$

$$\therefore \left(\frac{E_b}{\eta}\right)_{\infty} = ln2 = 0.693$$

Shannon' s Limit

$$\text{or} \quad \left(\frac{E_b}{\eta}\right)_\infty \text{ in dB} = 10 \log_{10}(0.693)$$

$$\therefore \quad \left(\frac{E_b}{\eta}\right)_\infty \text{ in dB} \cong -1.6 \text{ dB} \qquad \text{......} (4.223)$$

This value of – 1.6 dB is called the **"Shannon's Limit"**. The corresponding value of channel capacity is given by equation (4.217) as

$$C_\infty = B \frac{S}{N} \log_2 e \text{ bits/sec}$$

$$= \frac{S}{\eta} \log_2 e$$

N.B

$$\boxed{C_\infty = 1.44 \frac{S}{\eta}}$$

2. The capacity boundary, defined as the curve for critical bit rate Rt= C separates the error free transmission(Rt<C) from those of with error free transmission is not possible(Rt>C)

3. The diagram highlights the trade off between Eb/ and Rt/B

CONCLUSION

- Shannon's limit : $\left(\frac{E_b}{\eta}\right)_{\infty}$ in dB $\cong -1.6$ dB
- Bandwidth Efficiency Diagram