Stochastic Optimization Algorithms – Home Problem 1

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Problem 1.1

1. Initially the function is defined as follows:

$$f(x_1\,,x_2;\mu)=\begin{cases}(x_1-1)^2+2(x_2-2)^2; \quad x_1^2+x_2^2-1\ \le 0\\ (x_1-1)^2+2(x_2-2)^2+\ \mu(x_1^2+x_2^2-1)^2 \quad otherwise\end{cases}$$

2. Now the gradient of the above function is calculated for the given conditions.

1) when the constraints are fulfilled:

$$\nabla f(X;\mu)=\left(\frac{\partial f}{\partial x_1},\frac{\partial f}{\partial x_2}\right)=(2(x_1-1)\,,4(x_2-2))$$

2) when the constraints are not fulfilled:

$$\nabla f(X;\mu)=\left(\frac{\partial f}{\partial x_1},\frac{\partial f}{\partial x_2}\right)=((2(x_1-1)+4\mu x_1(x_1^2+x_2^2-1)\ ),(4(x_2-2)+4\mu x_2(x_1^2+x_2^2-1)))$$

3. For unconstrained condition ($\mu=0$), the minimum is found as

$$\frac{\partial f}{\partial x_1}=0\ \ \rightarrow\ \ 2(x_1-1)\ =0\ \ \rightarrow\ \ x_1=1$$

$$\frac{\partial f}{\partial x_2}=0\ \ \rightarrow\ \ 4(x_2-2)\ =0\ \ \rightarrow\ \ x_2=2$$

The minimum points are $(x_1,x_2)=(1{,}2)$

5. The Penalty method is carried out to find the minimum points with the following parameters:

Step size, $\eta=0.0001\ and\ \mu=[1,10,100,1000]$ and Tolerance T = 10^-6

Table 1: Minimum points for various penalty values

$\boldsymbol{\mu}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$
1	0.433777	1.210166
10	0.331354	0.995540
100	0.313738	0.955252
1000	0.311790	0.950732

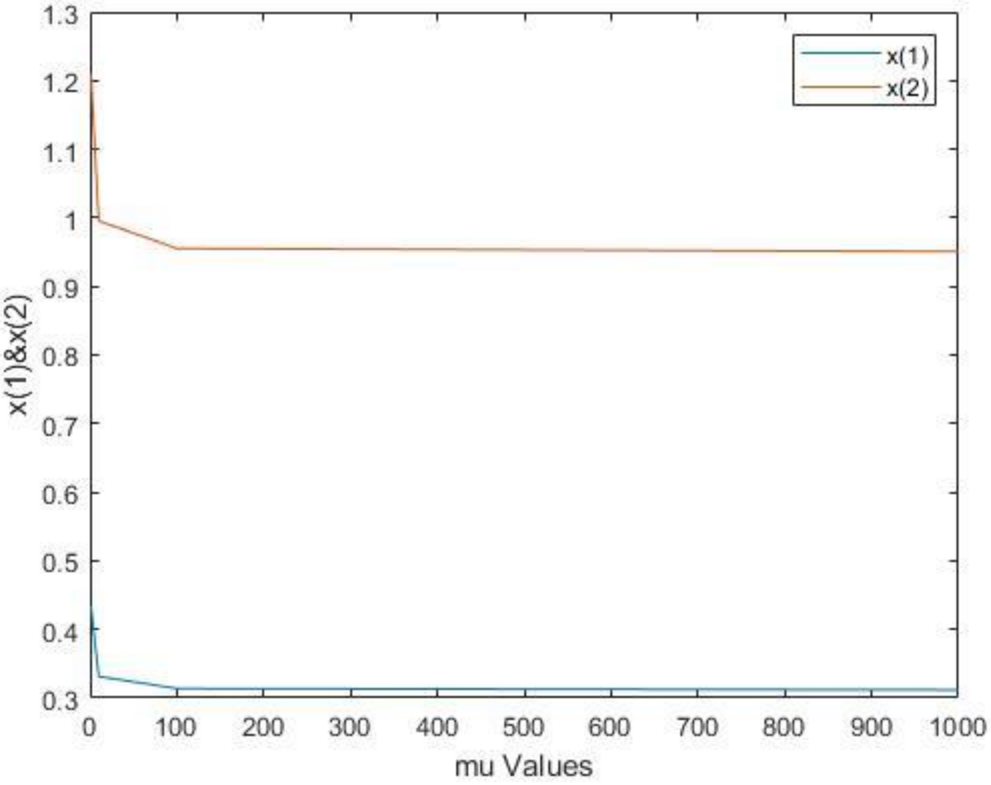


Figure 1: x_1 and x_2 for various μ values

From the above figure, it is noted that both x_1 and x_2 values converge to steady state as mu value increases.

Problem 1.3

Here we implement the Genetic algorithm to find the minimum of the function,

$$g(x_1 , x_2) = (1.5 - x_1 + x_1x_2)^2 + (2.25 - x_1 + x_1x_2^2)^2 + (2.625 - x_1 + x_1x_2^3)^2.$$

a) In this part, the RunSingle.m file is modified with different parameter values and tested for successful implementation of the algorithm. Finally, a set of parameters were fixed and the process was repeated 10 times to calculate the following:

Parameters:

Table 1: Parameters for run of 10 epochs

populationSize = 100	numberOfVariables = 2	crossoverProbability = 0.5
maximumVariableValue = 5	tournamentSize = 3	mutationProbability = 0.3
numberOfGenes = 50	tournamentProbability = 0.75	numberOfGenerations = 1000

With the above parameters, the following values were obtained for 10 runs.

When the simulation was run with more number of generations (2000), it had some room to converge to the minimum point and the points were mostly the same, as opposed to what we see below for a lesser number of generations (1000) where there are variations for (x_1, x_2)

Table 2:Function value and Variable value

Iterations	Function Value	x_1	x_2
1	4.39113e-05	2.98403	0.49569
2	0.000217	3.02673	0.50444
3	0.000457	3.02946	0.51087
4	0.000681	2.93817	0.48316
5	0.000977	3.03248	0.50210
6	2.7152e-05	2.99219	0.49893
7	0.000757	2.94614	0.48984
8	0.00018	2.96717	0.49150
9	0.000234	3.03842	0.50880
10	0.003294	2.97090	0.49561

b) Here, the RunBatch.m file was used to run the algorithm for fixed parameter values, except varying the mutation probability ranging from 0 to 1, including 0.02. The process was repeated 100 times for each mutation probability and median and average fitness values were calculated.

Table 3: Fitness values at different Mutation probability

P_{mut}	Median Fitness Values
0	132.464
0.01	6178.126
0.02	75464792.5192
0.03	4983220.0943
0.04	691877.30626
0.6	778.7764
0.7	982.7209
0.8	906.9743
0.9	3440.5597
1	118.262

Below is the plot for the mutation probability Vs median fitness values.

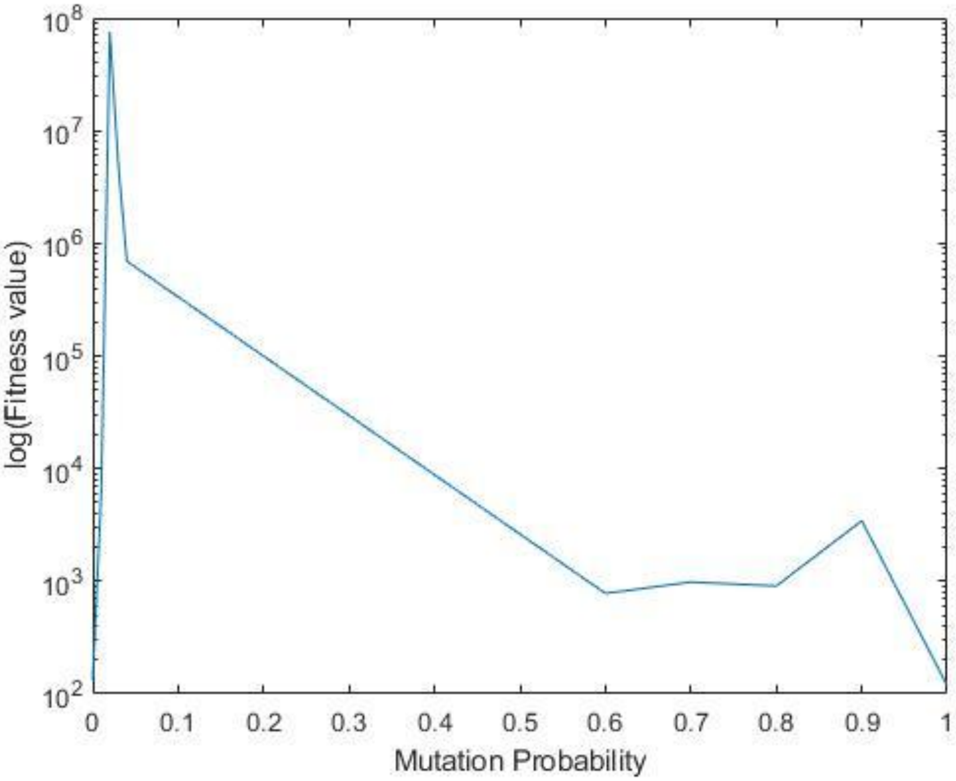
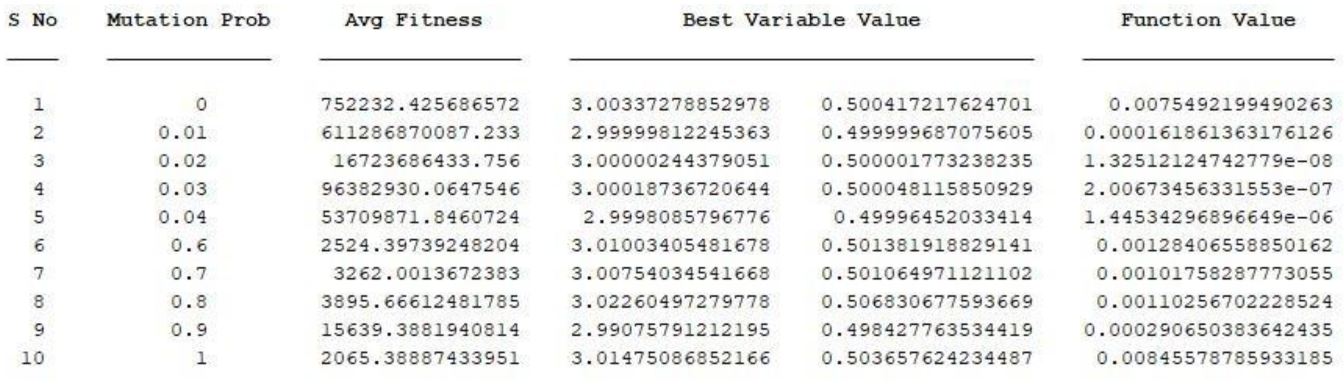


Figure 2: Graph of P_{mut} Vs Fitness Value

As can be seen from *Table 4*, the algorithm returns the maximum fitness value for mutation probability = 0.02 which corresponds to (1/m), where m is the number of genes (50). This is in accordance to the Onemax function, which finds the binary string for a given length that maximizes the sum of its digits.

c) From the many iterations, we make an initial guess of the actual minimum of (x_1, x_2) at (3, 0.5), as seen below.



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S No   Mutation Prob     Avg Fitness                  Best Variable Value                   Function Value
____   _____________   ________________   ______________________________________   ____________________

  1          0          752232.425686572   3.00337278852978     0.500417217624701     0.0075492199490263
  2       0.01         611286870087.233   2.99999812245363     0.499999687075605   0.000161861363176126
  3       0.02          16723686433.756   3.00000244379051     0.500001773238235   1.32512124742779e-08
  4       0.03         96382930.0647546   3.00018736720644     0.500048115850929   2.00673456331553e-07
  5       0.04         53709871.8460724    2.9998085796776      0.49996452033414   1.44534296896649e-06
  6        0.6         2524.39739248204   3.01003405481678     0.501381918829141    0.00128406558850162
  7        0.7          3262.0013672383   3.00754034541668     0.501064971121102    0.00101758287773055
  8        0.8         3895.66612481785   3.02260497279778     0.506830677593669    0.00110256702228524
  9        0.9         15639.3881940814   2.99075791212195     0.498427763534419   0.000290650383642435
 10          1         2065.38887433951   3.01475086852166     0.503657624234487    0.00845578785933185
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Figure 3: Values after 100 runs of algorithm for each P_{mut}

Analytically solving, we should get $\nabla g(x_1, x_2) = 0$ at (3 , 0.5)

$$\frac{\partial g}{\partial x_1} = 2(1.5 - \ x_1 + x_1 x_2)(x_2 - 1) + 2(2.25 - x_1 + x_1 x_2^2)(x_2^2 - 1) + 2(2.625 - x_1 + x_1 x_2^3)(3x_2^2 - 1)$$

$$\frac{\partial g}{\partial x_2} = 2(1.5 - \ x_1 + x_1 x_2)(x_1) + 2(2.25 - x_1 + x_1 x_2^2)(2x_2) + 2(2.625 - x_1 + x_1 x_2^3)(3x_1 x_2^2)$$

substituting (3 , 0.5), we get $\left(\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}\right) = (0,0)$. From this, it is evident that $(x_1, x_2) = (3,0.5)$ is a stationary point.