Arun Hari Anand

CS 30 HW#2

① a) $A \cap A = A$
b) $A \cup A = A$
c) $A \cap \bar{A} = \phi$
d) $A \cap U = A$
e) $A \cap \phi = \phi$
f) $\bar{\phi} = U$
g) $\overline{A \cap B} = \bar{A} \cup \bar{B}$
h) $\overline{A \cup B} = \bar{A} \cap \bar{B}$
i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
j) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

② a) It is 1-to-1, onto and a bijection
b) It is not 1-to-1, not onto and ∴ not a bijection
c) It is 1-to-1, not onto, and ∴ not a bijection
d) It is not 1-to-1, it is onto and ∴ not a bijection

③ a) $f(n) = x$
b) $f(x) = \begin{cases} 2^x, & x \geq 0 \\ 2^{-x}+1, & x < 0 \end{cases}$
c) For a rational number x, such that $x = \frac{p}{q}$ for some $p, q \in \mathbb{Z}$, such that p and q have no common factors, $f(x) = 2^p \cdot 3^q$ or $(2^p)(3^q)$
d) $f(a) = \begin{cases} \frac{1}{a+2}, & a \geq 0 \\ \frac{1-a}{2-a}, & a < 0 \end{cases}$

④ a) $\{13, 14, 15, 16, 55\}$
b) ϕ
c) $\{z+6 \mid x \in \mathbb{N}\}$
d) $\mathbb{Z}$

⑤ A number p is prime iff $p > 1 \wedge \forall_{n \in \mathbb{Z}} [(n > 0 \wedge F(n,p)) \Rightarrow (n = 1 \vee n = p)]$

⑥ The GCF of 2 integers m and n is d iff
$\exists_{z,y \in \mathbb{Z}} [S(m,z,d) \wedge S(n,y,d) \wedge \{\forall_{p,q,r \in \mathbb{Z}} (\neg S(m,p,q) \vee \neg S(n,p,r) \vee E(p,d))\}]$

⑦ a) True, because when $y = x+1$, for all values of $z \in \mathbb{Z}$, z cannot be greater than x, not equal to y, and still be less than y. This is because there does not exist an integer between x and $x+1$.

b) False, because no matter what value of y we pick, there will always be a z such that $x < z < y$. In other words, there always exists a real number between any 2 real numbers x and y.

8 a) $H(n) \equiv \forall_{s \in S} (score(s) \leq n)$

b) $B(s) \equiv H(score(s))$

c) $P \equiv \exists_{x,y \in S} (x \neq y \wedge B(x) \wedge B(y))$

d) $Q \equiv P \wedge \forall_{x,y,z \in S} [(x \neq y \neq z \wedge B(y) \wedge B(z)) \Rightarrow \neg B(x)]$

e) $R(s) \equiv H(score(s) + 10)$

f) $t \equiv \exists_{x \in S} [R(x) \wedge \forall_{y \in S} (score(y) > score(x) \Rightarrow H(score(y)))]$

9 a) $P(x): x = a$

b) $P(x): (x = a) \vee (x = b)$

10 a) $\forall_{x \in S} \exists_{y \in S} [\{\forall_{t_1 \in A} (P(x, t_1) \Rightarrow P(y, t_1))\} \wedge \{\exists_{t_2 \in A} (P(y, t_2) \wedge \neg P(x, t_2))\}]$

b) $\exists_{x \in S} \forall_{y \in S} [\{\exists_{t_1 \in A} (P(x, t_1) \wedge \neg P(y, t_1))\} \vee \{\forall_{t_2 \in A} (\neg P(y, t_2) \vee P(x, t_2))\}]$