Arun Hari Anand

① a) Wiley cheats $\Rightarrow$ Wiley gets caught

b) p is true $\Rightarrow$ q is true

~~c) You pay the fee $\Rightarrow$ You can access the website~~

c) You can access the website $\Rightarrow$ You pay the subscription fee

② a) It rains $\Rightarrow$ I open my umbrella

If it doesn't rain, I don't open my umbrella

b) I miss class $\Rightarrow$ I am unwell

& If I am not unwell, then I do not miss class

c) ~~If you~~ You are not curious and knowledgeable $\Rightarrow$ You can't invent

If you can't invent, then You are not curious and knowledgeable

③ a) P

b) T

c) P

d) P

e) T

f) P

g) P

h) F

i) P

j) T

k) T

l) $P \Rightarrow q$

④ a)

p	q	$p \Rightarrow q$	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$	$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

QED: From the above, ~~When~~ $p \Rightarrow q \equiv \neg q \Rightarrow \neg p$ under all circumstances b)

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

QED: From the above, $\neg(p \wedge q) \equiv \neg p \vee \neg q$ under all circumstances

c)

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

QED: From the above, $\neg(p \vee q) \equiv \neg p \wedge \neg q$ under all circumstances

d) proposition $x \equiv [p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)]$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$	x
T	T	T	T	T	T	T	T	T
F	T	T	T	F	F	F	F	T
T	F	T	T	T	F	T	T	T
T	T	F	T	T	T	F	T	T
F	T	F	F	F	F	F	F	T
F	F	T	F	F	F	F	F	T
T	F	F	F	F	F	F	F	T
F	F	F	F	F	F	F	F	T

QED: From the above, proposition x (that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$) is always true under all circumstances.

5) a) $\{4, 9, 16, 25\}$

b) $\{\phi, \{a\}, \{c\}, \{e\}, \{a, c, e\}\}$

c) $\{0, 1\}$

d) ϕ

e) $\{1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 40, 41, 42,$ [illegible] $44, 45, 46, 47, 48, 49\}$

f) $\{a, c, o, m, d, t, e\}$

g) $\{\{2,4\}, \{2,4,6,8\}, \{2,6\}, \{2,8\}, \{4,6\}, \{4,8\}\}$

6) a) $\{4,6,7,9,2,3,5\}$

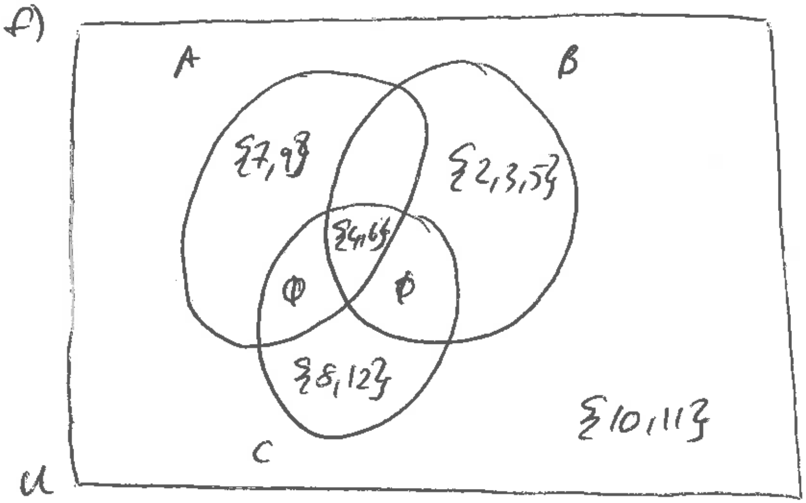
b) $\{4,6\}$

c) $\{7,8,9,12\}$

d) $\{2,3,4,5,6,8,10,11,12\}$

e) ϕ

f)



7) a) $(A \cup B) - (A \cap B)$

b) $(A \cap B) \cup (A \cap C) \cup (B \cap C)$

c) $A \times (B \cup C)$

8) ~~$\{\{a,b\} \mid a \neq b \neq 0 \wedge ab<0 \wedge (|a|=|b|^2 \vee |b|=|a|^2)\}$~~ ~~$a\in\mathbb{Z} \wedge b\in\mathbb{Z}$~~

$\{\{a,b\} \mid a \in \mathbb{Z} \wedge b \in \mathbb{Z} \wedge ab<0 \wedge (|a|=|b|^2 \vee |b| = |a|^2)\}$

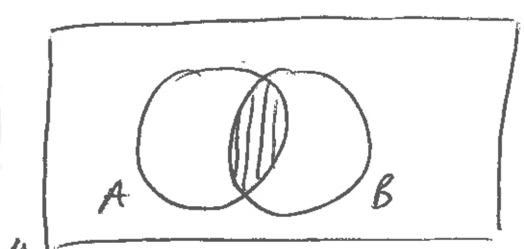
9) a) ϕ

b) A

c) A

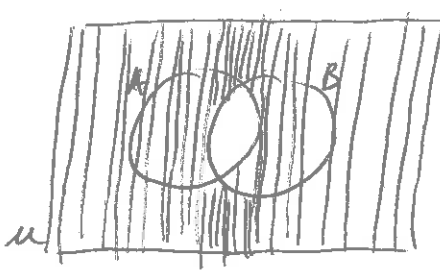
~~d)~~

10) a) True



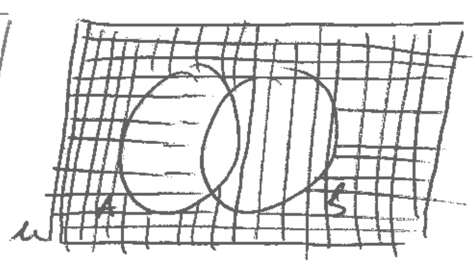
$A \cap B$

Fig1



$\overline{A \cap B}$

Fig 2



$\bar{A} \cup \bar{B}$

Fig3 $|||$ = $\bar{A}$ $\equiv$ = $\bar{B}$

Fig2 = Fig3 ; QED

b) $B \cap C$ True

$B \cap C$

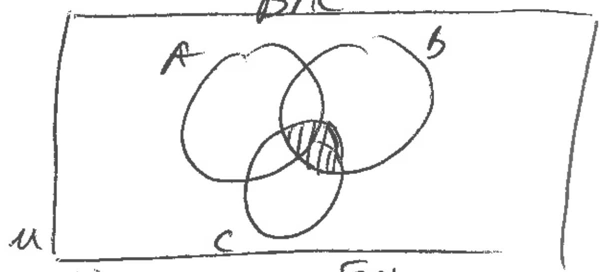


Fig 1

$A \cup (B \cap C)$

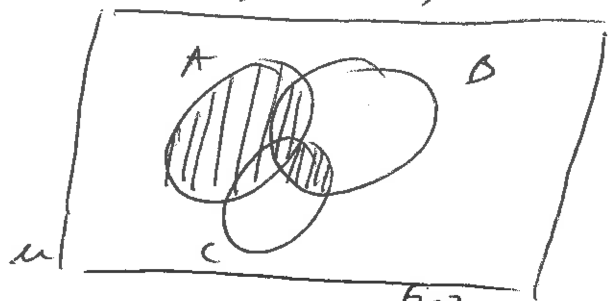


Fig 2

$A \cup B$

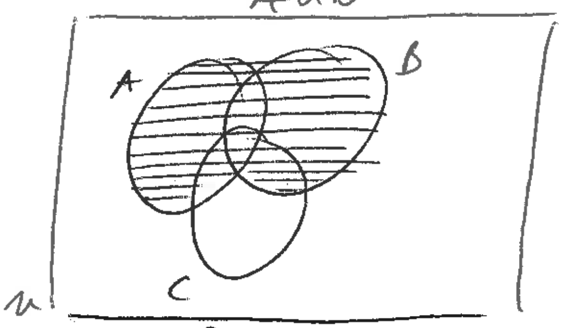


Fig 3

$A \cup C$

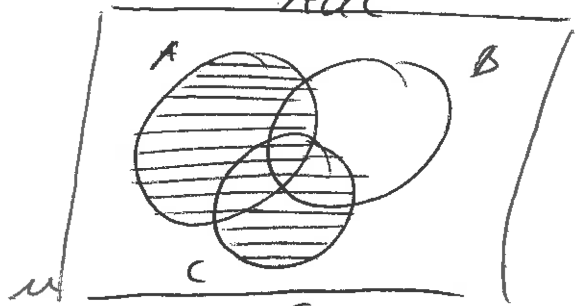


Fig 4

$(A \cup B) \cap (A \cup C)$

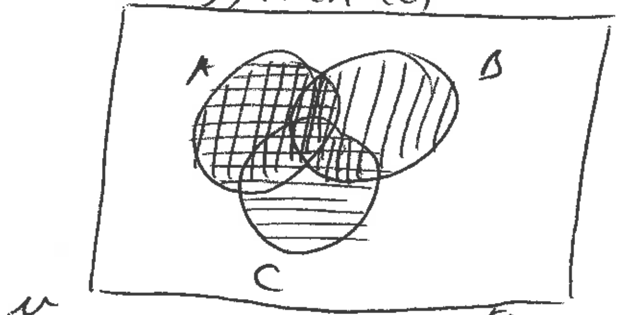


Fig 5

~~$A \cap$~~ $A \cup B$ = |||

$A \cup C$ = ≡

More clearly

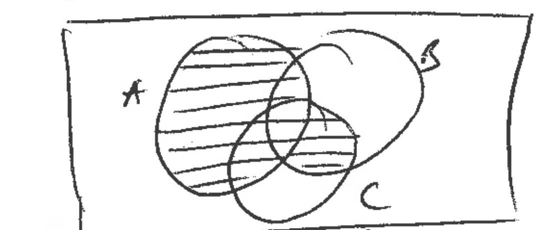


Fig 6

QED: Fig 2 and Fig 6 are the same

c) True

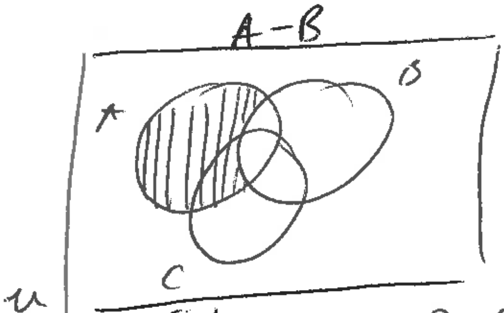


Fig1

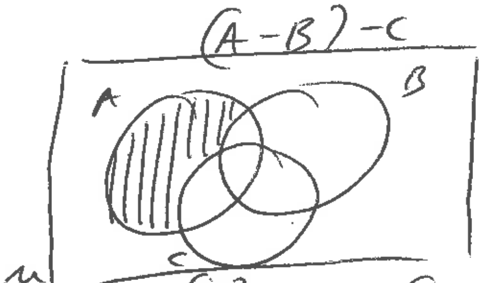


Fig2

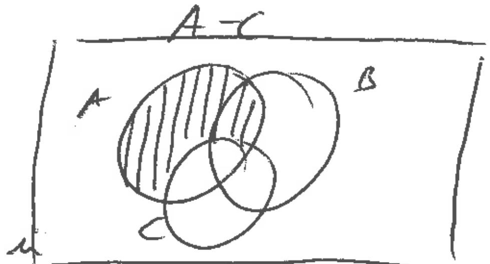


Fig3

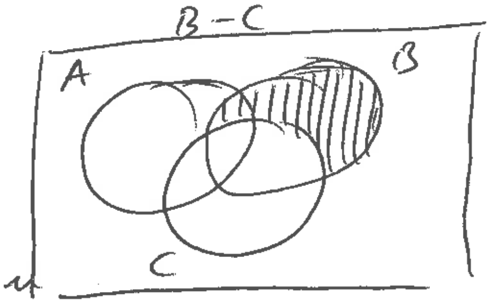


Fig4

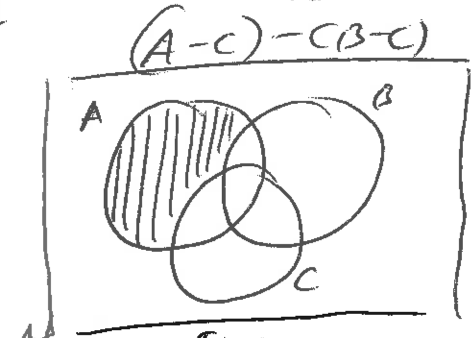
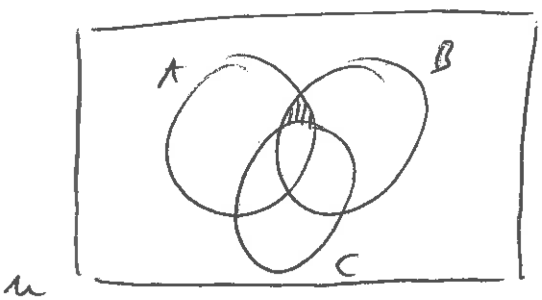


Fig5

Fig 2 = Fig 5, so this is true. ~~th~~ In $(A-C)-(B-C)$, the only piece that must be removed from $A-C$ is $(A-C)\cap(B-C)$. This intersection is represented as follows:



So, since this is the only part removed, this is how we were able to represent $(A-C)-(B-C)$

11 a) $(A\cap B)\cup(A\cap\overline{B}) = A \longrightarrow$ True

Assume $x \in (A\cap B)\cup(A\cap\overline{B})$

$\Leftrightarrow (x\in A \wedge x\in B) \vee (x\in A \wedge x\notin B)$ - By definitions of union and intersection of sets

$\Leftrightarrow x\in A \wedge (x\in A \vee x\notin B)$ - By DeMorgans Law, which states that $p\wedge(q\vee r) \equiv (p\wedge q)\vee(p\wedge r)$; or alternatively $(p\wedge q)\vee(p\wedge r) \equiv p\wedge(q\vee r)$

$\Leftrightarrow x\in A \wedge T$; Because $x\in B \vee x\notin B$ always evaluates to true (tautology) because x is either an element of B or it is not, by the definition of sets.

$\Leftrightarrow x\in A$ by the definition of $\wedge$

~~from~~ $\Leftrightarrow$ X So, ~~iff~~ iff $x\in(A\cap B)\cup(A\cap\overline{B})$; it means $x\in A$.

From the above, $(A\cap B)\cup(A\cap\overline{B}) = A$

QED

Going the other way

Assume that $x \in A$

It is a given that ~~$A \cup \bar{A}$~~ $B \cup \bar{B} = U$ (by complement law)

Thus $x \in U$, and so $x \in (B \cup \bar{B})$

$\Leftrightarrow x \in B \vee x \in \bar{B}$ by definition of $\cup$

$\Leftrightarrow x \in A \wedge (x \in B \vee x \in \bar{B})$; grouping all of the givens together.

$\Leftrightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in \bar{B})$ by distributive law

~~$\Leftrightarrow x \in A \cup$~~

$\Leftrightarrow (x \in A \cap B) \vee (x \in A \cap \bar{B})$ by definition of $\wedge$

$\Leftrightarrow x \in (A \cap B) \cup (A \cap \bar{B})$ by definition of $\vee$

As we have have shown that both the left side and the right side are subsets of each other, they must be the same set. So the left side equals the right side.

Thus $(A \cap B) \cup (A \cap \bar{B}) = A$

QED

b) $(A-B) \cup (B-A) = A \cup B$

False

$A-B$

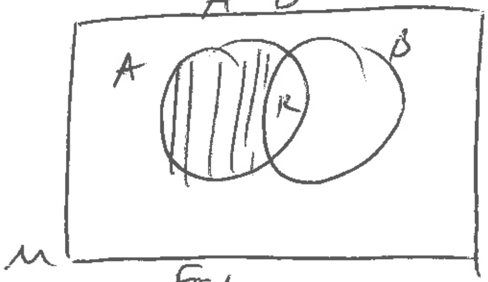


Fig1

$B-A$

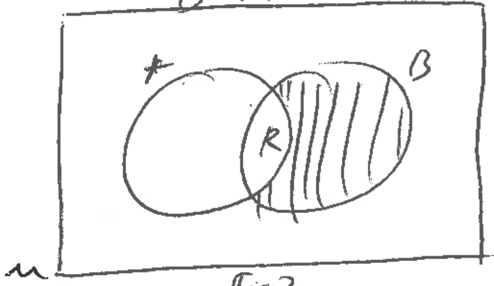


Fig2

$(A-B) \cup (B-A)$

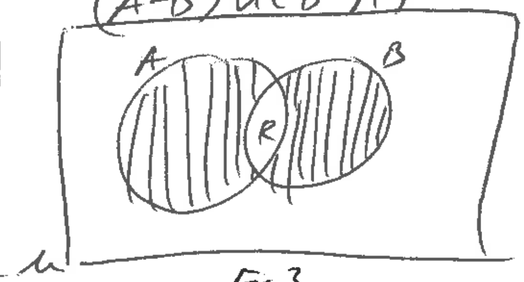


Fig3

$A \cup B$

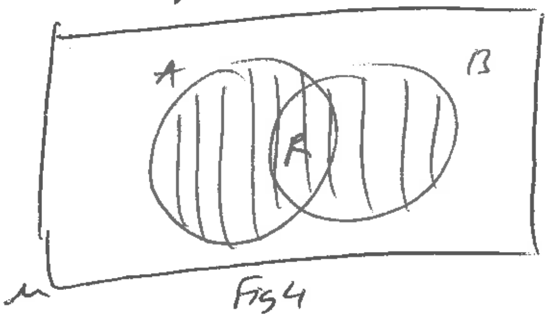


Fig4

The region R, consisting of $A \cap B$, is not shaded in for fig3, but it is for fig4. So ~~$A\cap B$ elements +~~ any element $a \in A \cap B$ will be a counterexample

for ex: $A = \{1,2,3,4\}$ $B = \{3,4,5,6\}$

$A - B = \{1,2\}$ $B - A = \{5,6\}$ $(A-B) \cup (B-A) = \{1,2,5,6\}$

$A \cup B = \{1,2,3,4,5,6\}$

So $3 \in (A-B) \cup (B-A)$, but $3 \notin A \cup B$

$4 \in (A-B) \cup (B-A)$, but $\cancel{3} \notin A \cup B$

So this equality is false.

c) $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

Assume $X \in \overline{A \cap B \cap C}$

$\Leftrightarrow X \notin A \cap B \cap C$ – By definition of complement

$\Leftrightarrow \neg(X \in A \cap B \cap C)$ – By definition of $\notin$ and $\neg$

$\Leftrightarrow \neg(X \in A \wedge X \in B \wedge X \in C)$ – By definition of $\cap$ and $\wedge$

$\Leftrightarrow X \notin A \vee X \notin B \vee X \notin C$ – By definition of $\neg$ and DeMorgan's Law

$\Leftrightarrow$ ~~So if $X \in \overline{A \cap B \cap C}$, then $X \notin A$~~ $X \in \overline{A \cap B \cap C} \Leftrightarrow X \notin A \vee X \notin B \vee X \notin C$

~~$\Leftrightarrow \overline{A \cap B \cap C} \Leftrightarrow X \notin \bar{A} \cup \bar{B} \cup \bar{C}$~~

$\Leftrightarrow \overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$ – By definition of $\notin$ and $\vee$

~~QED~~

Going the other way,

Assume $x \in \bar{A} \cup \bar{B} \cup \bar{C}$

$\iff x \in \bar{A} \vee x \in \bar{B} \vee x \in \bar{C}$

$\iff x \notin A \vee x \notin B \vee x \notin C$

$\iff \neg(x \in A \wedge x \in B \wedge x \in C)$

$\iff \neg(x \in A \cap B \cap C)$

$\iff x \notin A \cap B \cap C$

$\iff x \in \overline{A \cap B \cap C}$

~~QED~~ From the above, it can be ~~the~~ shown that the left side and right side of the equation are subsets of one another. Thus, the left and right side must be equivalent.

Thus, $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

QED