Properties of Regular Languages

For regular languages L_1 and L_2 we will prove that:

Union: $L_1 \cup L_2$

Concatenation: $L_1 L_2$

Star: $L_1{}^*$

Reversal: $L_1{}^R$

Complement: $\overline{L_1}$

Intersection: $L_1 \cap L_2$

Are regular Languages

We say: Regular languages are **closed under**

Union: $L_1 \cup L_2$

Concatenation: $L_1 L_2$

Star: $L_1 *$

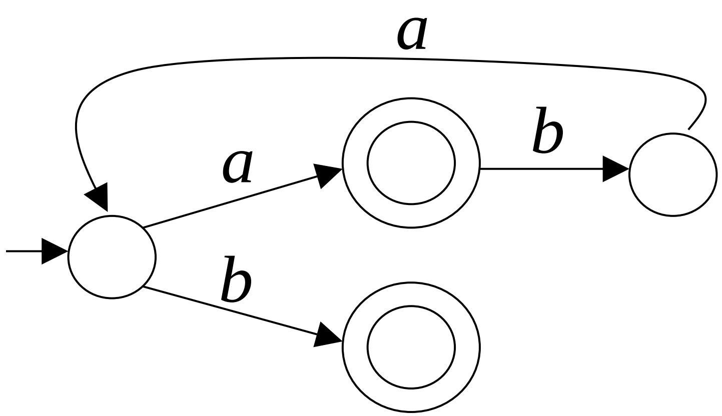
Reversal: L_1^R

Complement: $\overline{L_1}$

Intersection: $L_1 \cap L_2$

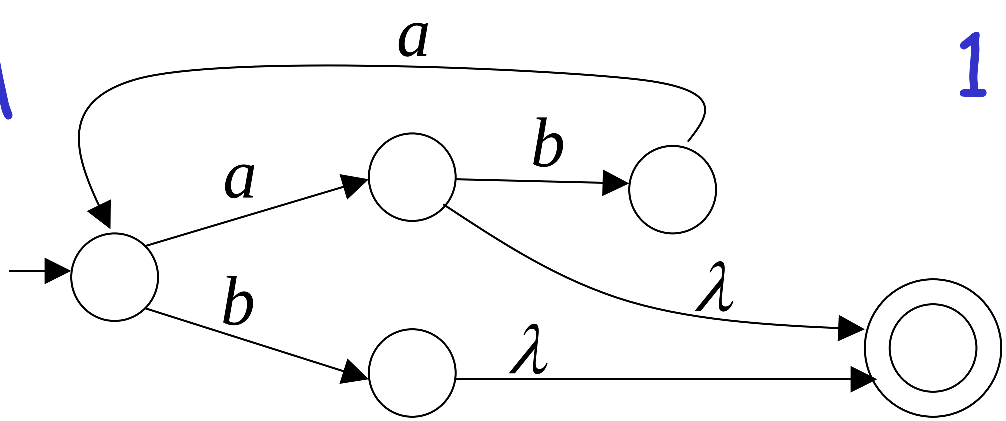
A useful transformation: use one accept state

NFA



2 accept states

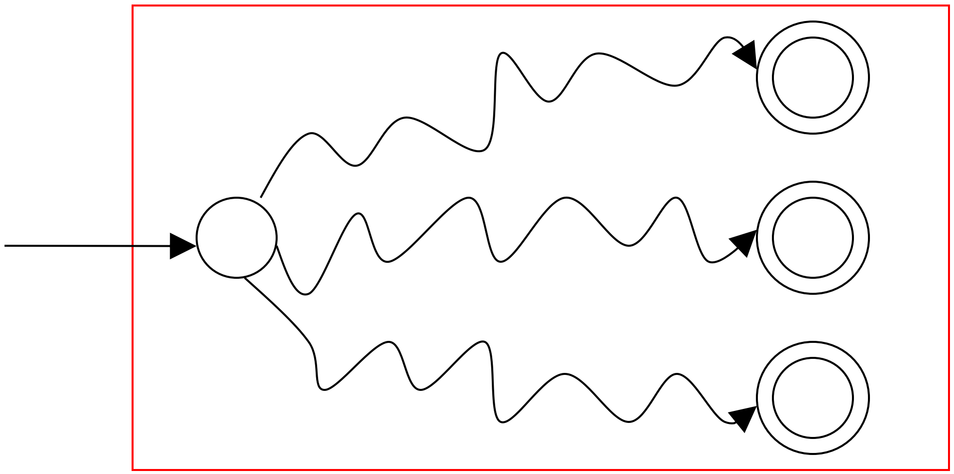
Equivalent
NFA



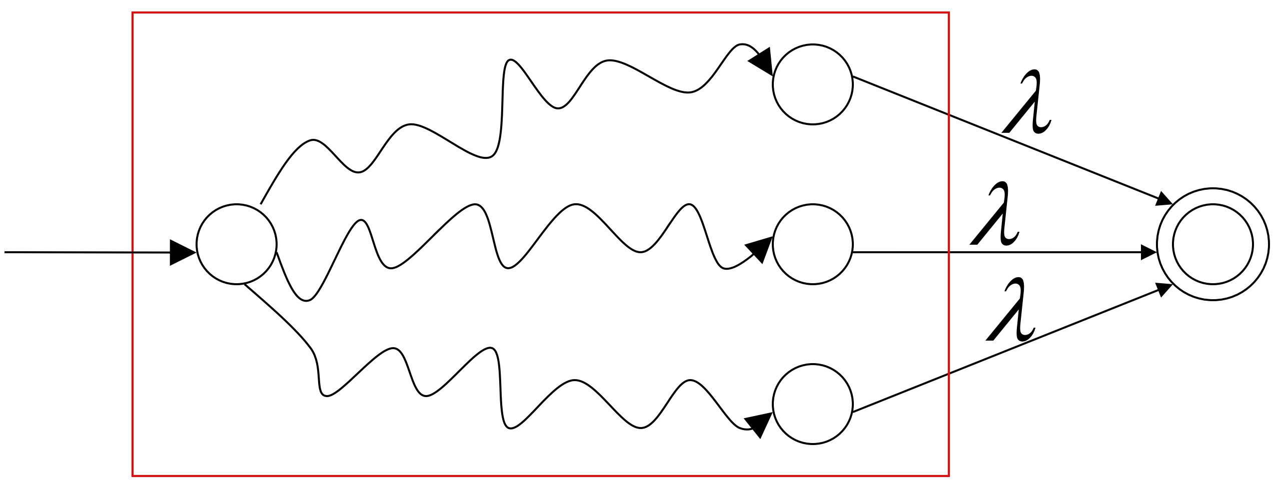
1 accept state

In General

NFA



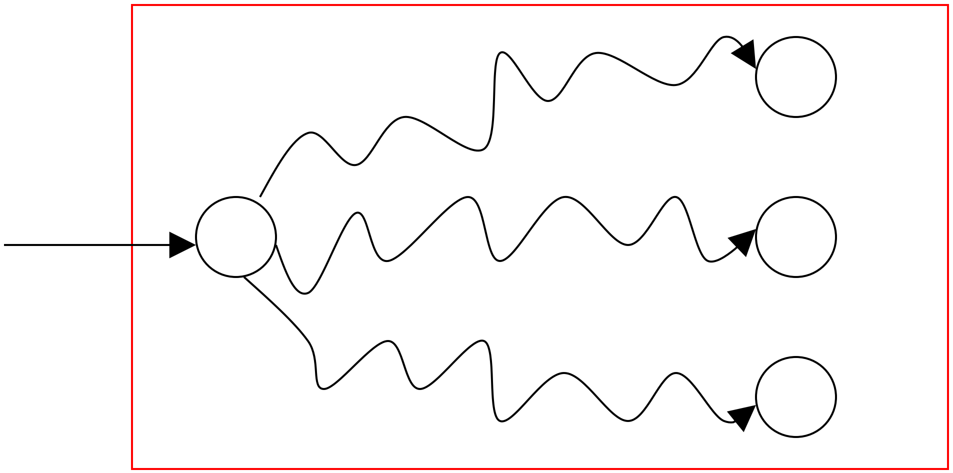
Equivalent NFA

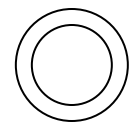


Single accepting state

Extreme case

NFA without accepting state





Add an accepting state
without transitions

Take two languages

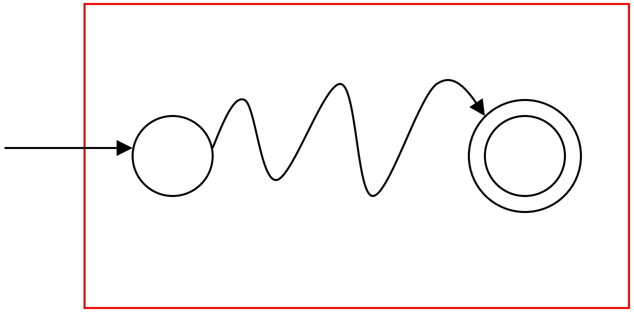
Regular language L_1

Regular language L_2

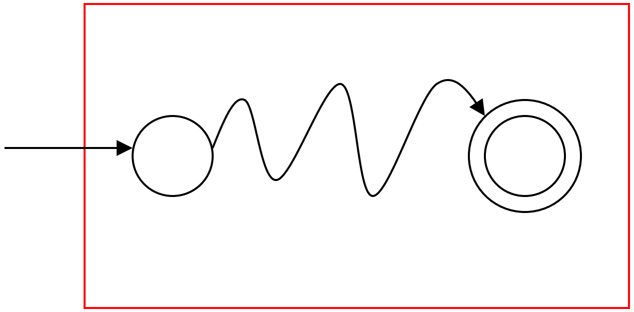
$$L(M_1) = L_1$$

$$L(M_2) = L_2$$

NFA M_1



NFA M_2



Single accepting state

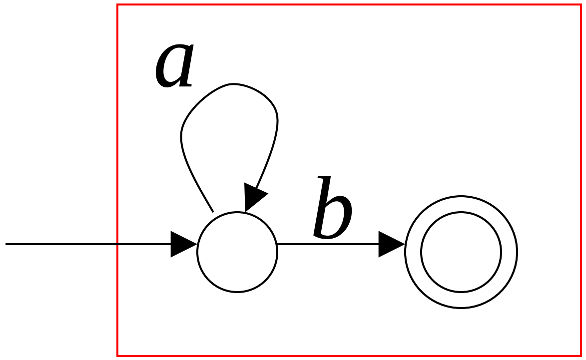
Single accepting state

Example

$n \geq 0$

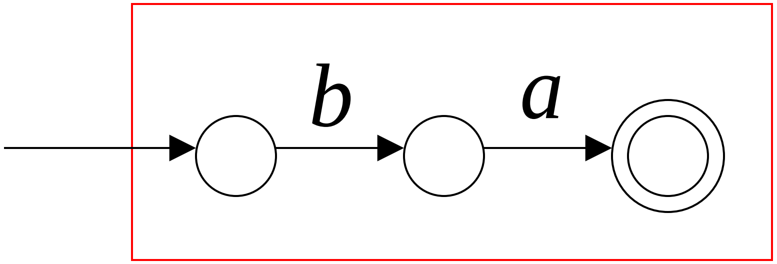
$L_1 = \{a^n b\}$

M_1



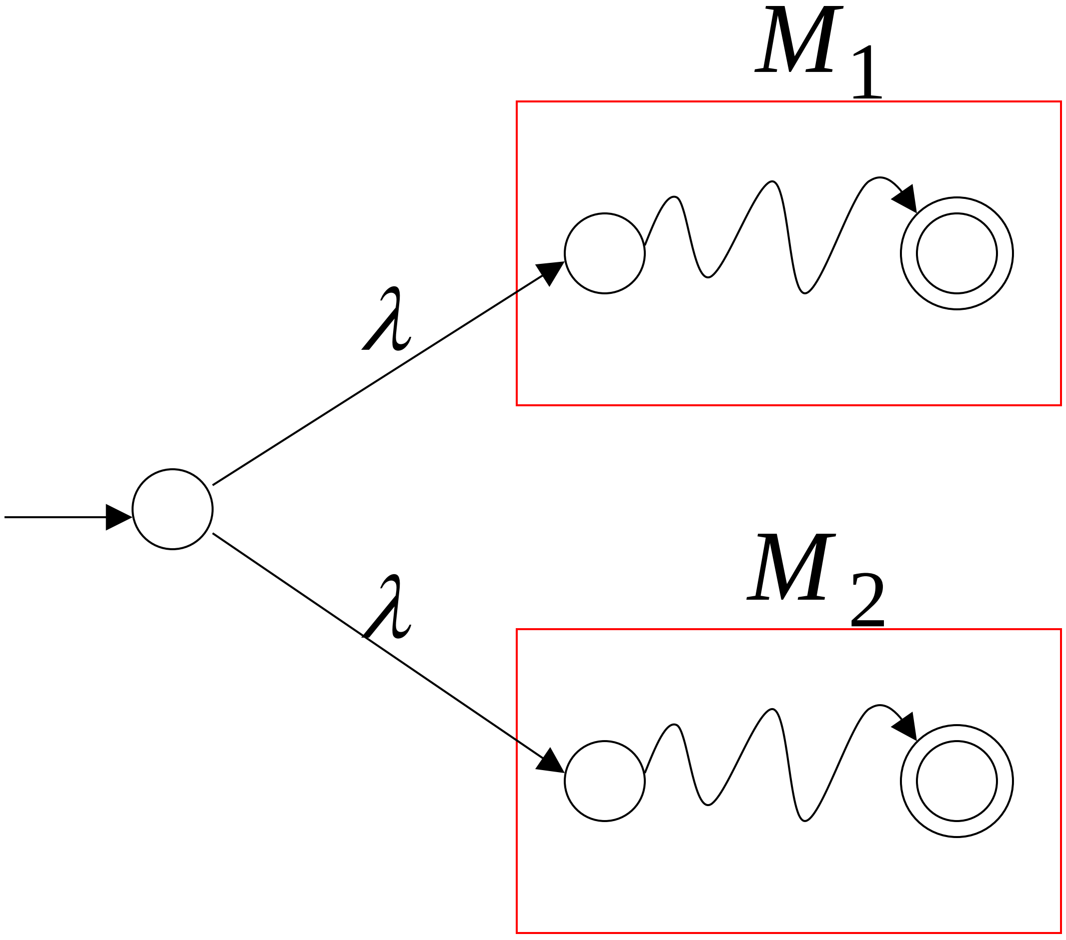
$L_2 = \{ba\}$

M_2



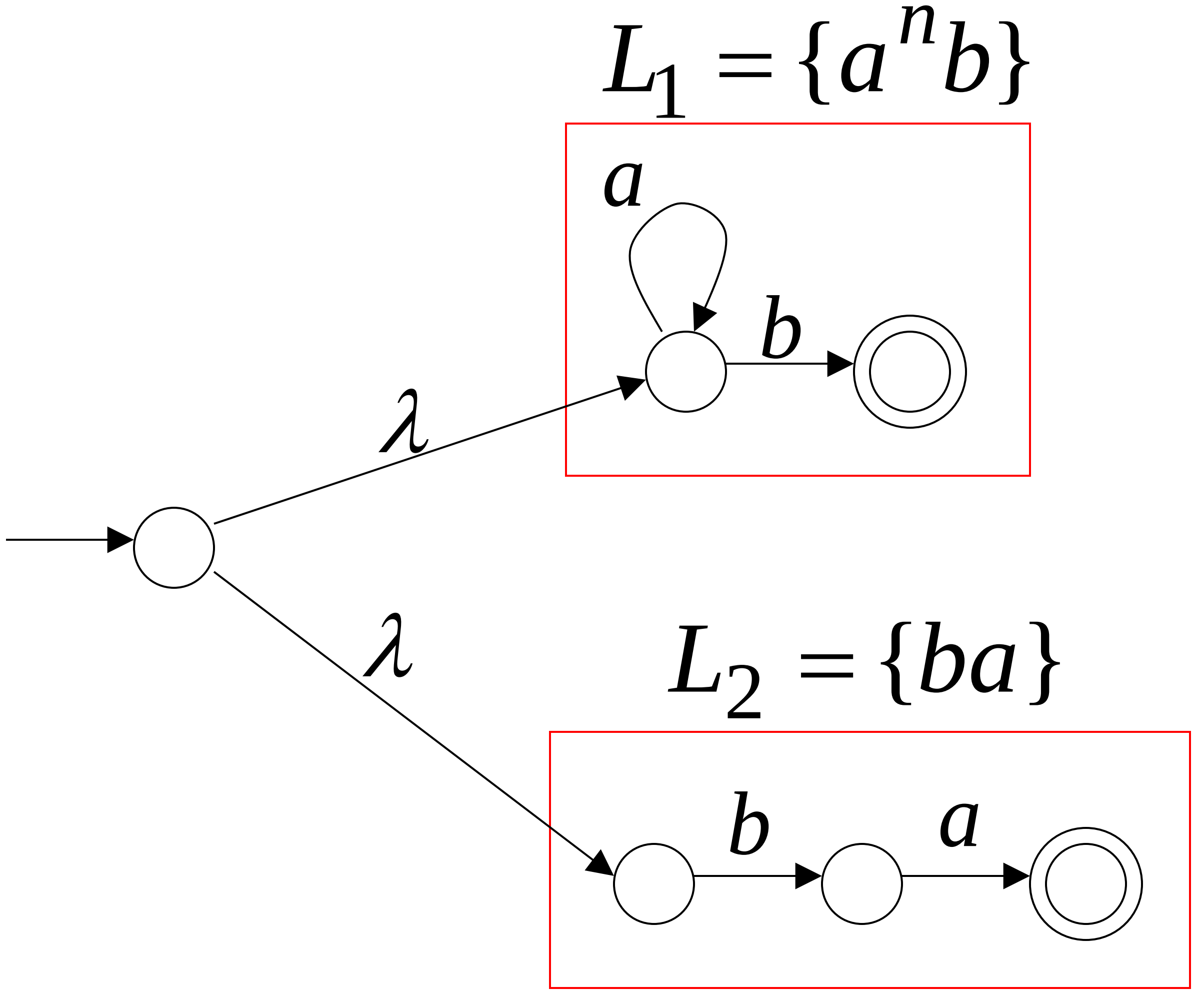
Union

NFA for $L_1 \cup L_2$



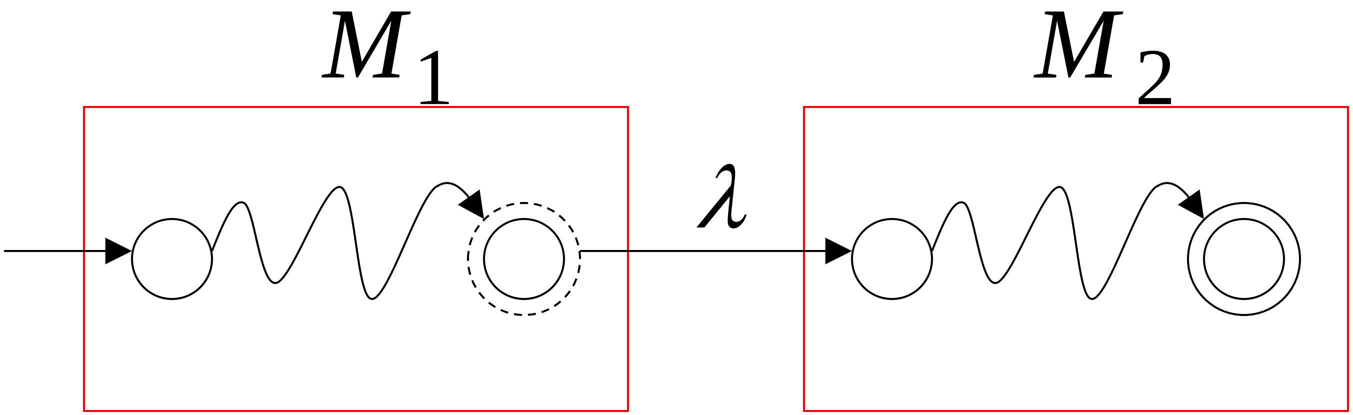
Example

NFA for $L_1 \cup L_2 = \{a^n b\} \cup \{ba\}$



Concatenation

NFA for $L_1 L_2$

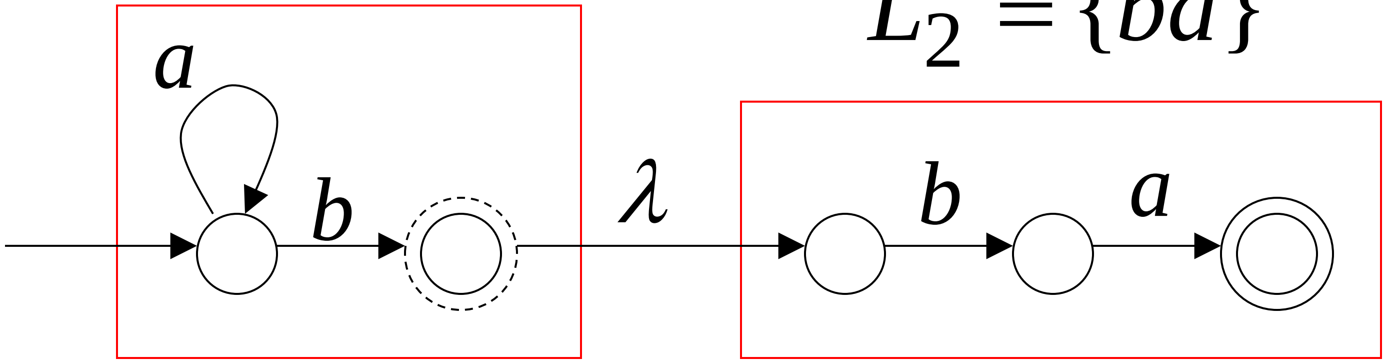


Example

NFA for $L_1 L_2 = \{a^n b\}\{ba\} = \{a^n bba\}$

$L_1 = \{a^n b\}$

$L_2 = \{ba\}$

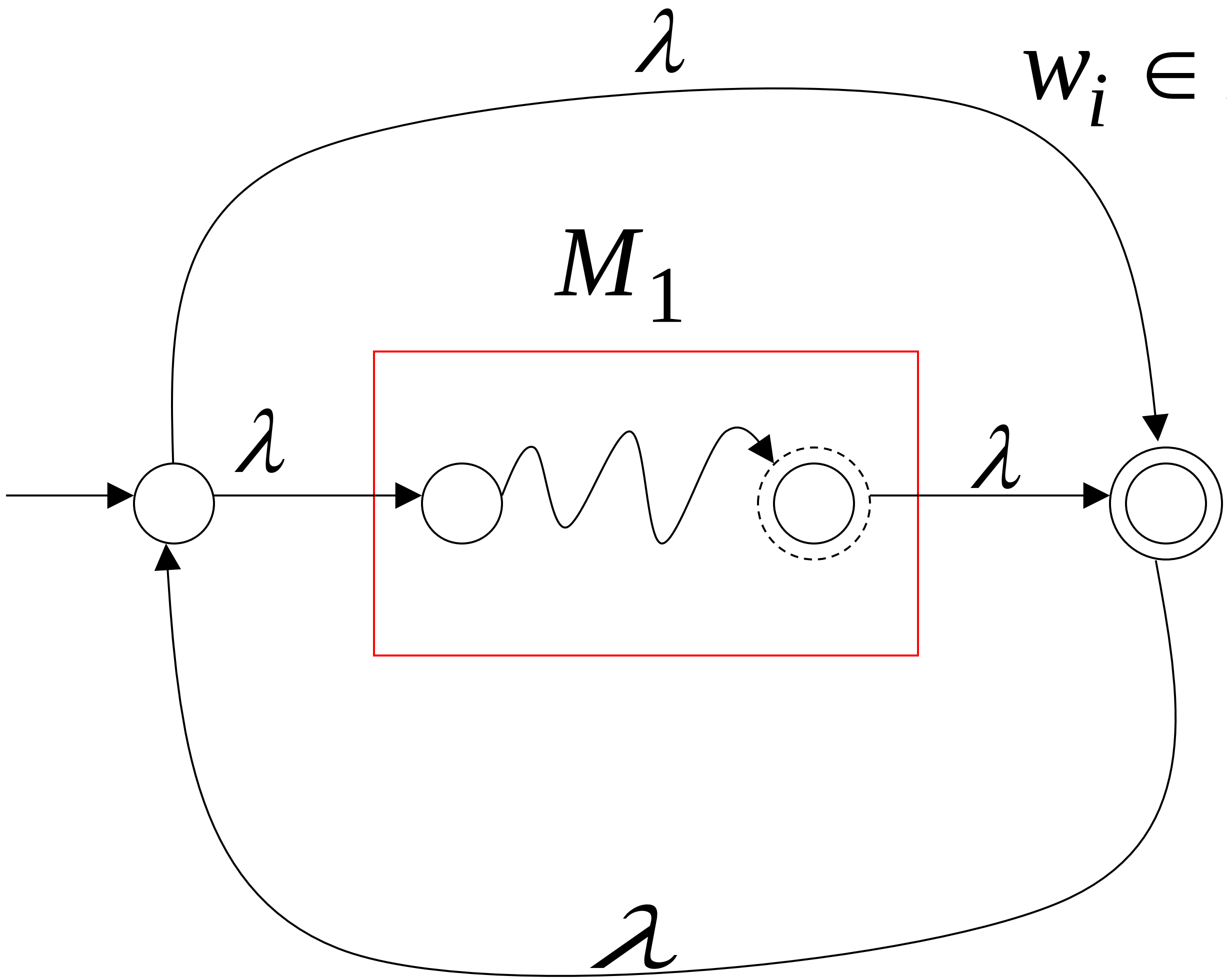


Star Operation

NFA for $L_1{}^*$

$w = w_1 w_2 \cdots w_k$

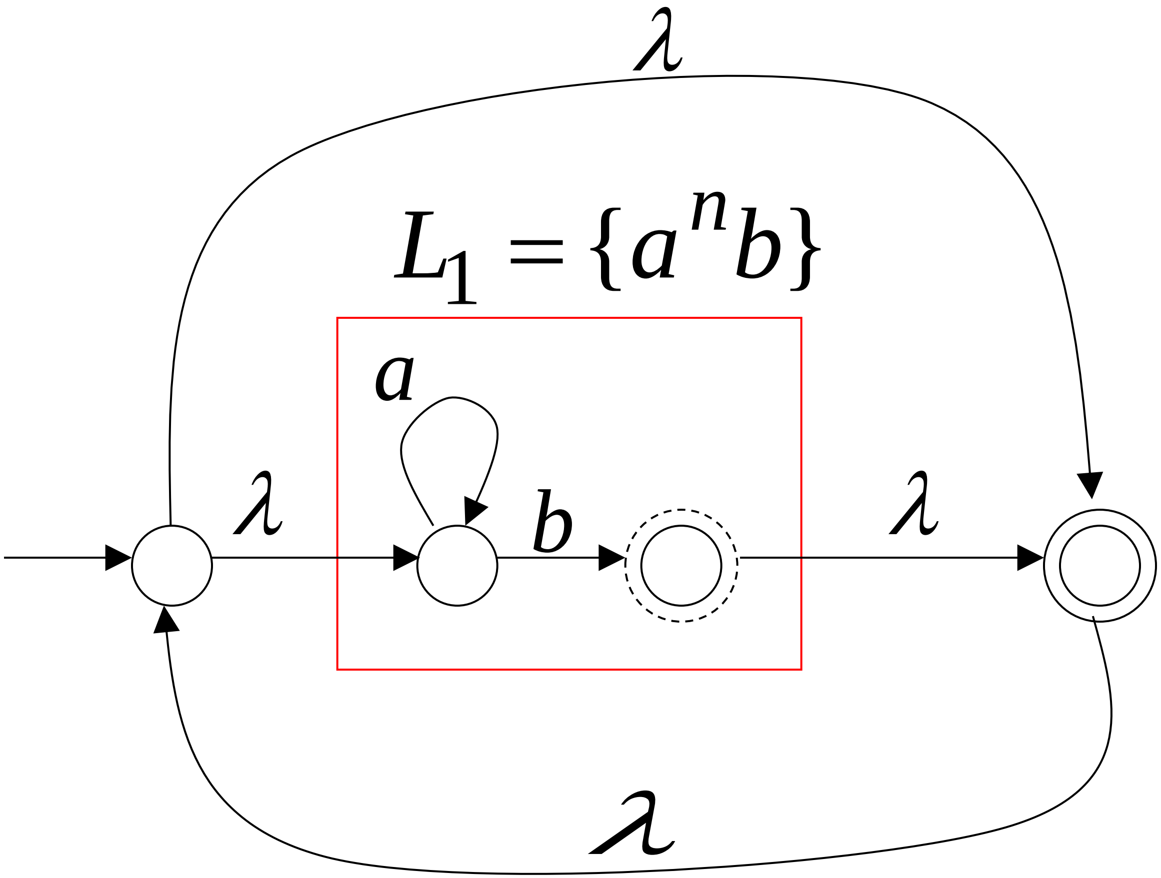
$w_i \in L_1$



$\lambda \in L_1{}^*$

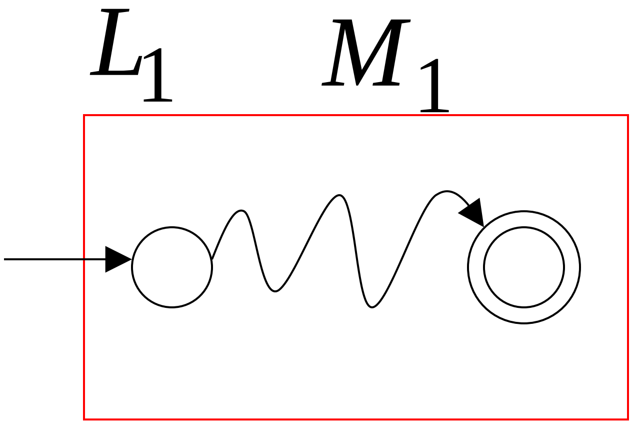
Example

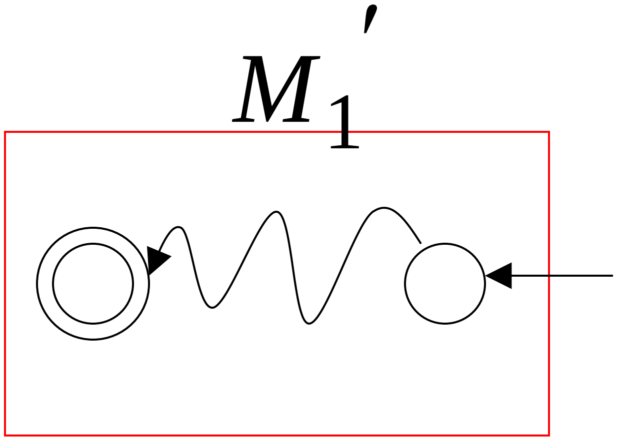
NFA for $L_1{}^* = \{a^n b\}^*$



Reverse

NFA for L_1^R





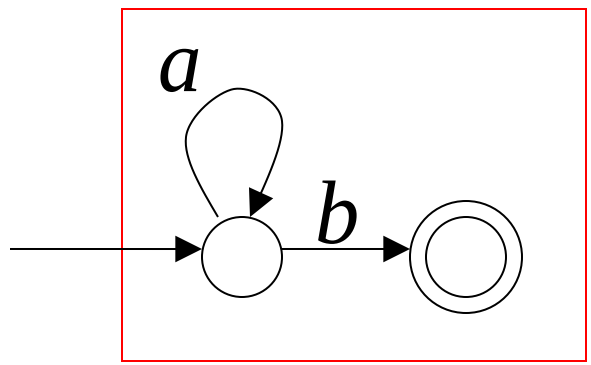
1. Reverse all transitions

2. Make initial state accepting state and vice versa

Example

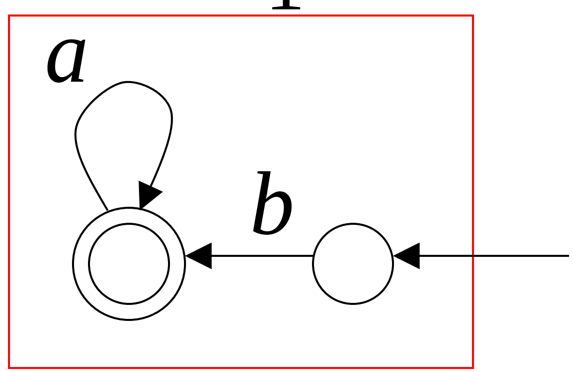
M_1

$L_1 = \{a^n b\}$

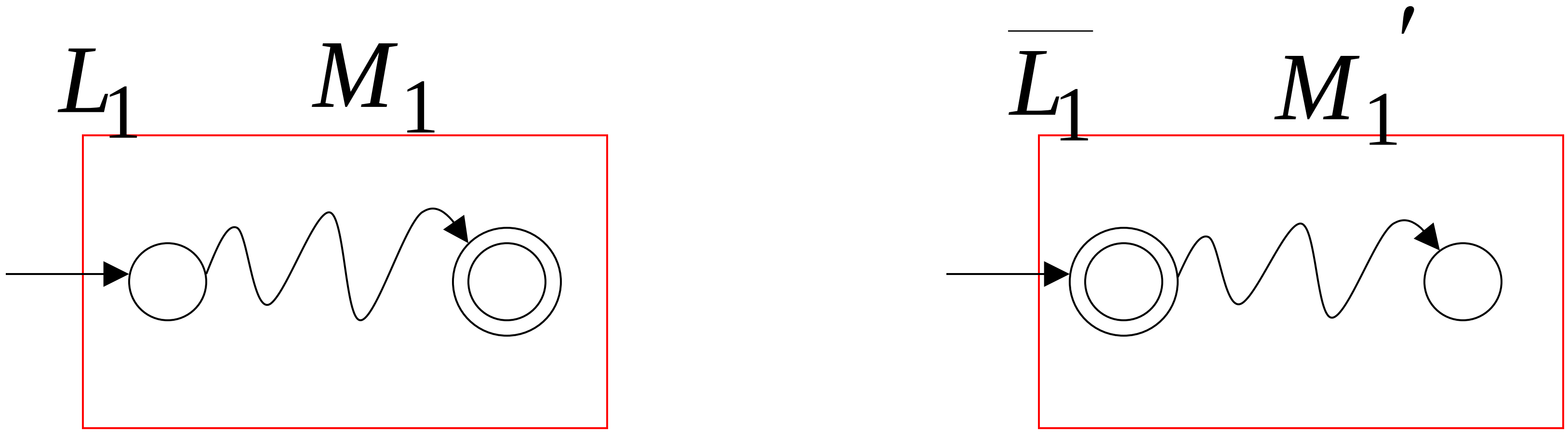


M_1'

$L_1^R = \{ba^n\}$



Complement



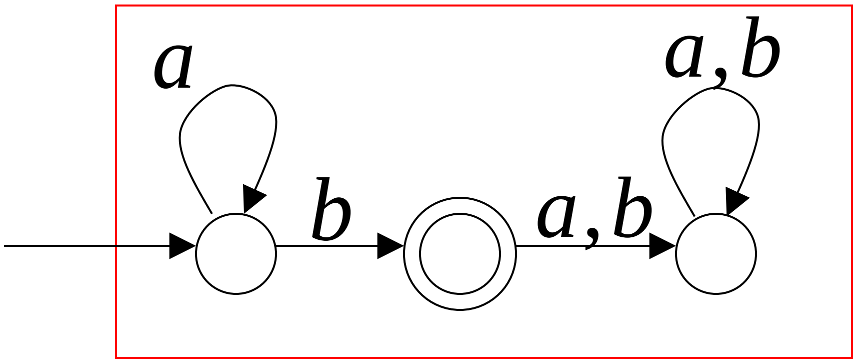
1. Take the DFA that accepts L_1

2. Make accepting states non-final, and vice-versa

Example

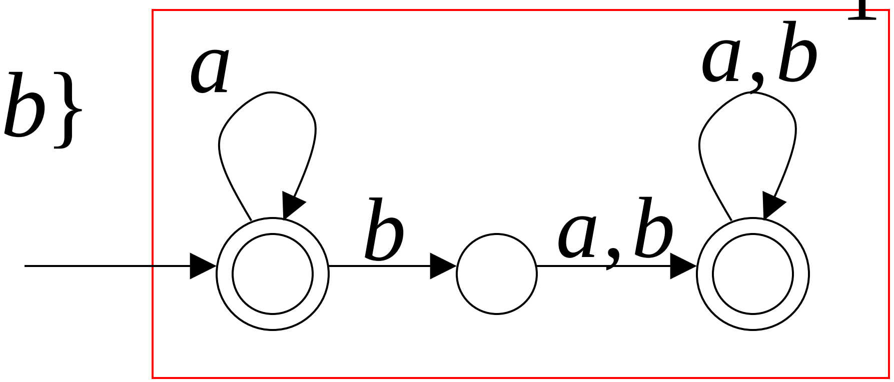
$$L_1 = \{a^n b\}$$

M_1



$$\overline{L_1} = \{a,b\}^* - \{a^n b\}$$

M_1'



Intersection

L_1 regular

We show $\Rightarrow$ $L_1 \cap L_2$ regular

L_2 regular

DeMorgan's Law: $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

$L_1 \; , \; L_2$ regular

$\Rightarrow$ $\overline{L_1} \; , \; \overline{L_2}$ regular

$\Rightarrow$ $\overline{L_1} \cup \overline{L_2}$ regular

$\Rightarrow$ $\overline{\overline{L_1} \cup \overline{L_2}}$ regular

$\Rightarrow$ $L_1 \cap L_2$ regular

Example

$L_1 = \{a^n b\}$ regular

$L_2 = \{ab, ba\}$ regular

$\Rightarrow$ $L_1 \cap L_2 = \{ab\}$

regular

Another Proof for Intersection Closure

Machine M_1

DFA for L_1

Machine M_2

DFA for L_2

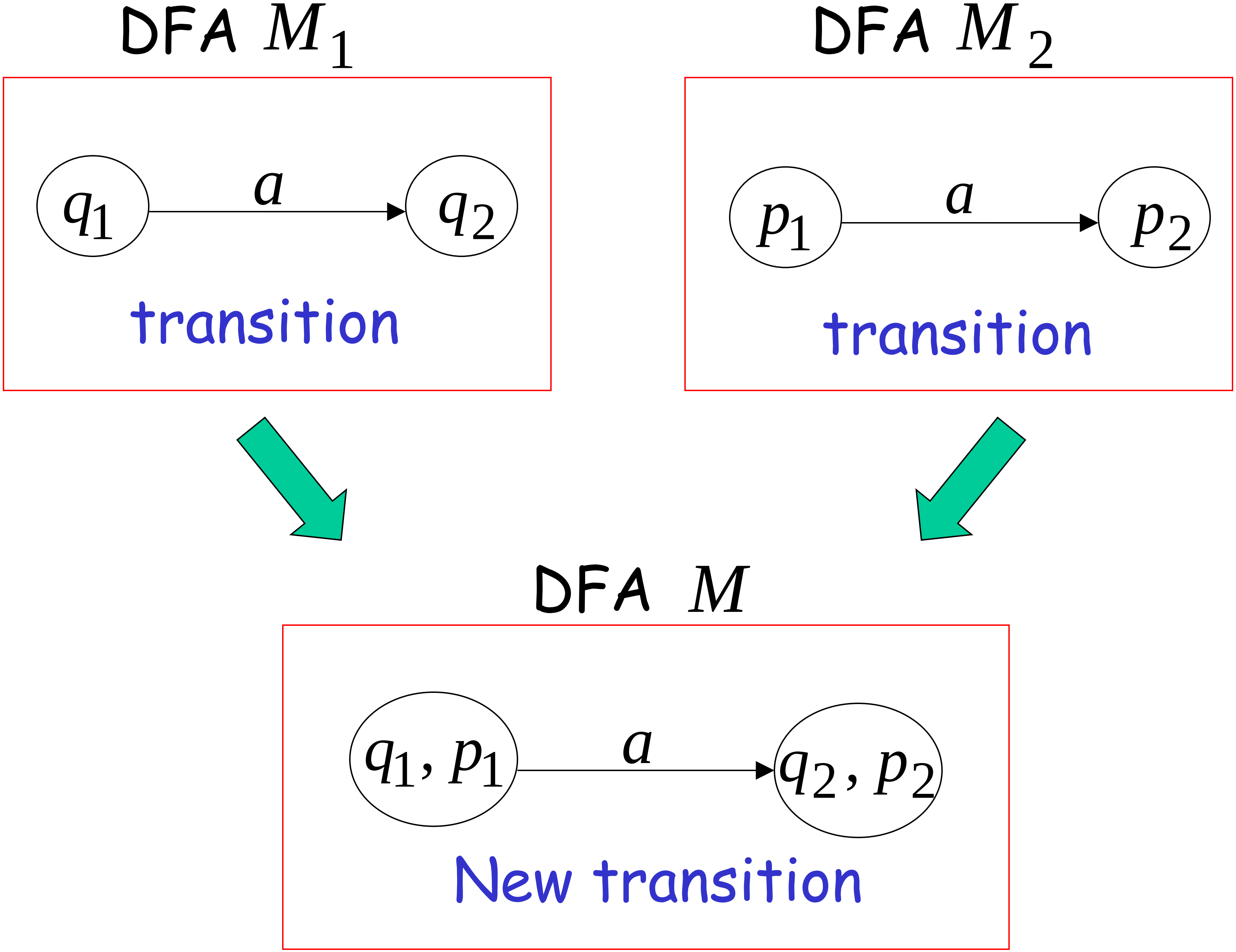
Construct a new DFA M that accepts $L_1 \cap L_2$

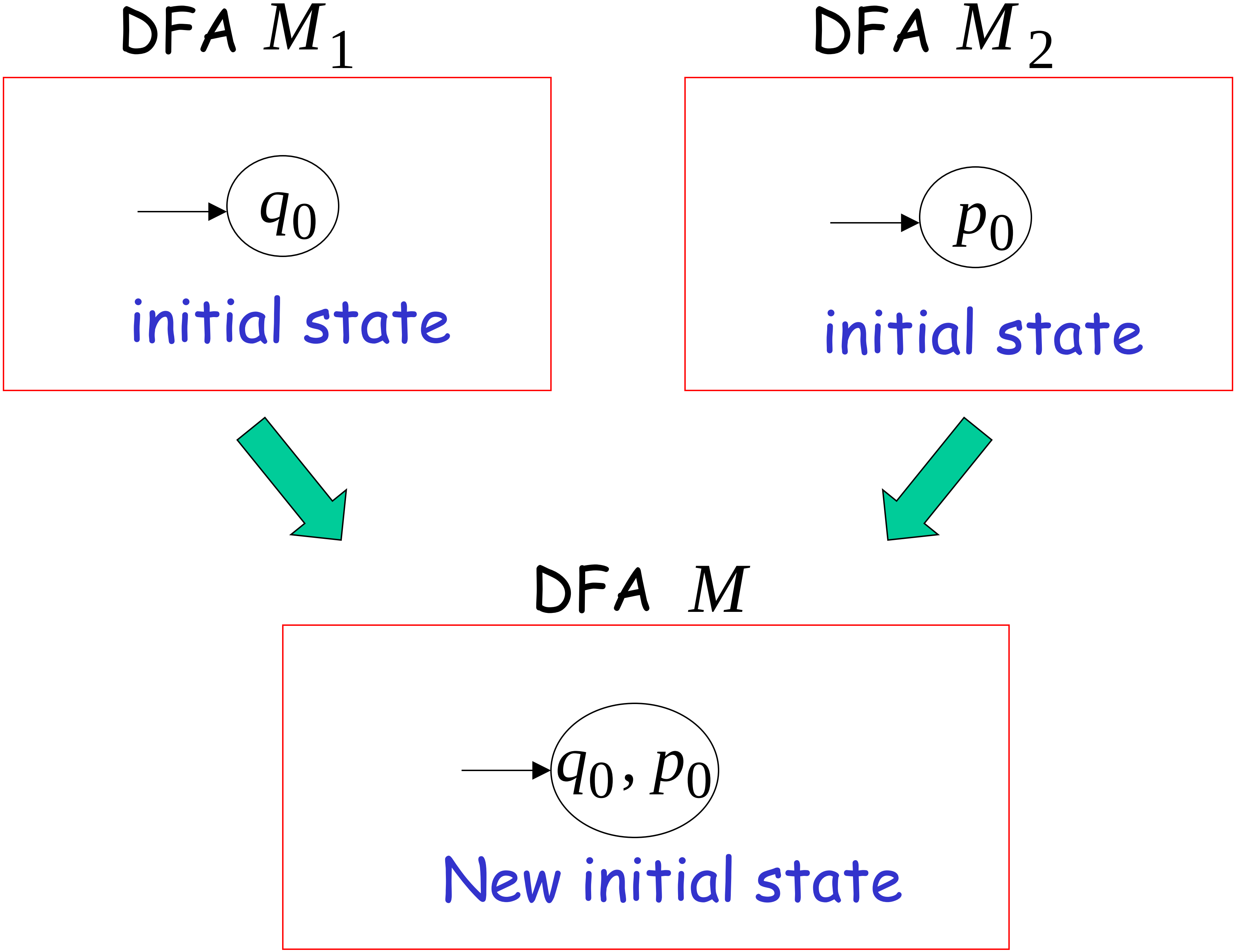
M simulates in parallel M_1 and M_2

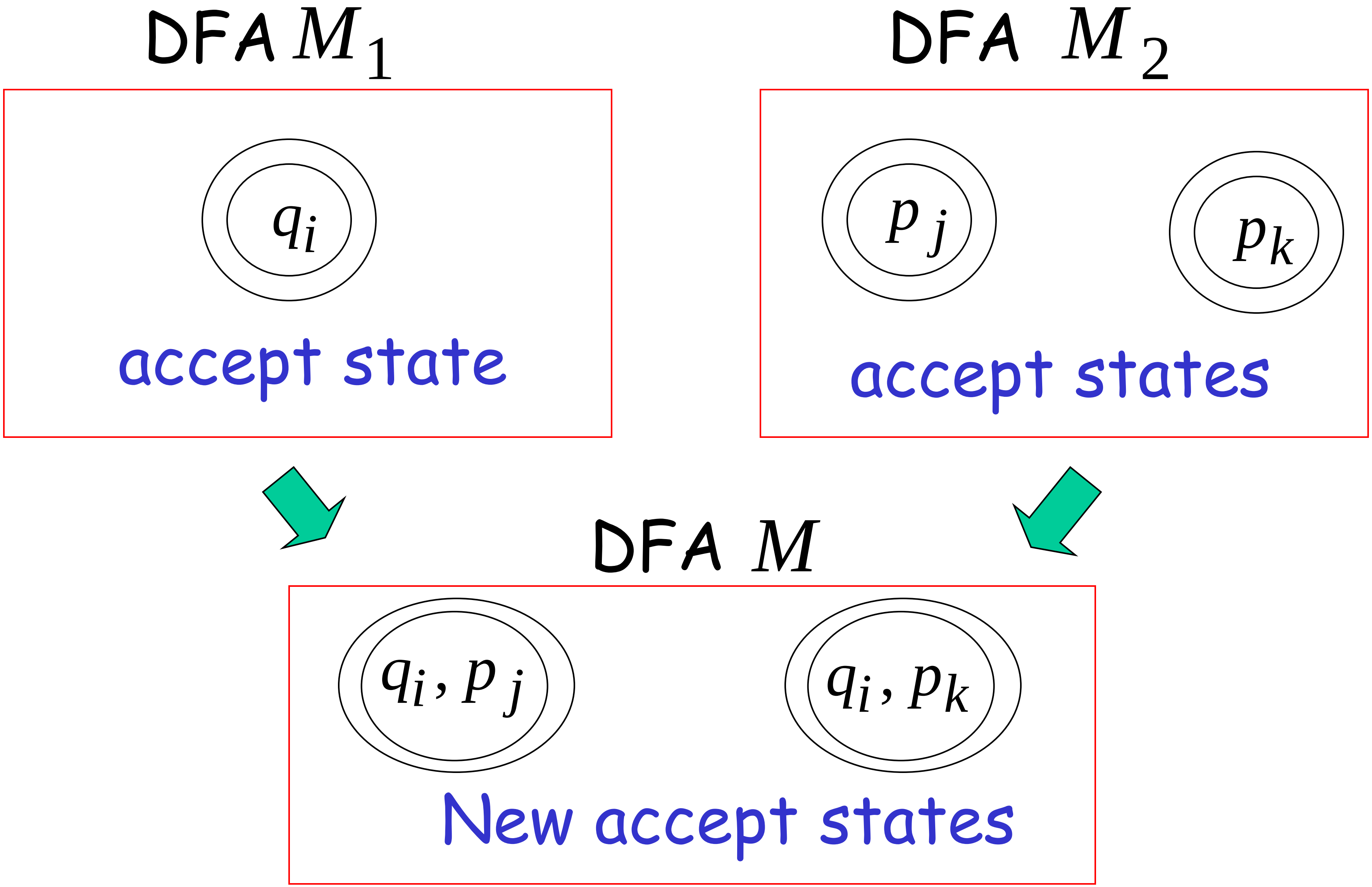
States in M

q_i, p_j

State in M_1 State in M_2


DFA M_1
q_1
a
q_2
transition
DFA M_2
p_1
a
p_2
transition
DFA M
q_1, p_1
a
q_2, p_2
New transition


DFA M_1
q_0
initial state
DFA M_2
p_0
initial state
DFA M
q_0, p_0
New initial state



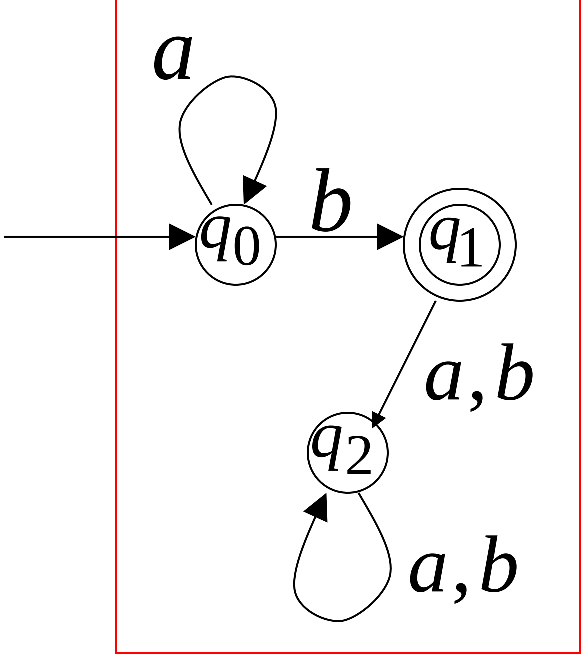
Both constituents must be accepting states

Example:

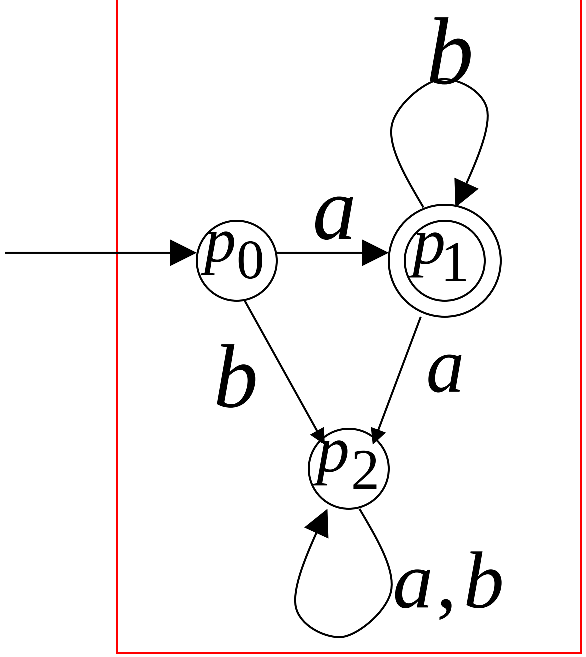
$$L_1 = \{a^n b\} \quad n \geq 0$$

$$L_2 = \{ab^m\} \quad m \geq 0$$

M_1

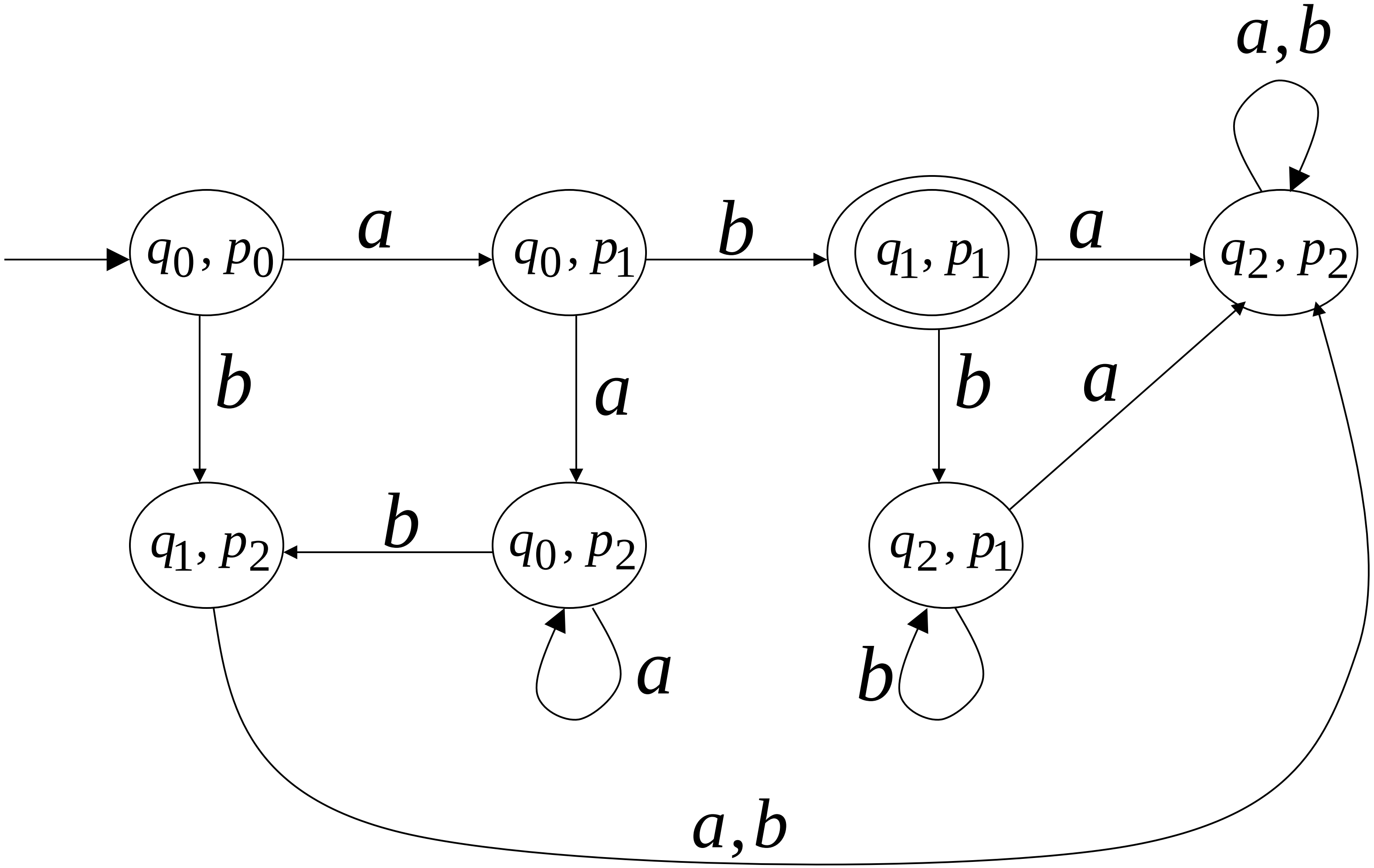


M_2



Automaton for intersection

$$L = \{a^n b\} \cap \{ab^n\} = \{ab\}$$



M simulates in parallel M_1 and M_2

M accepts string w if and only if:

M_1 accepts string w

and M_2 accepts string w

$$L(M) = L(M_1) \cap L(M_2)$$