DFA Exercise

1. Which of the strings 0001, 01001, 0000110 are accepted by DFA given in Figure 1.

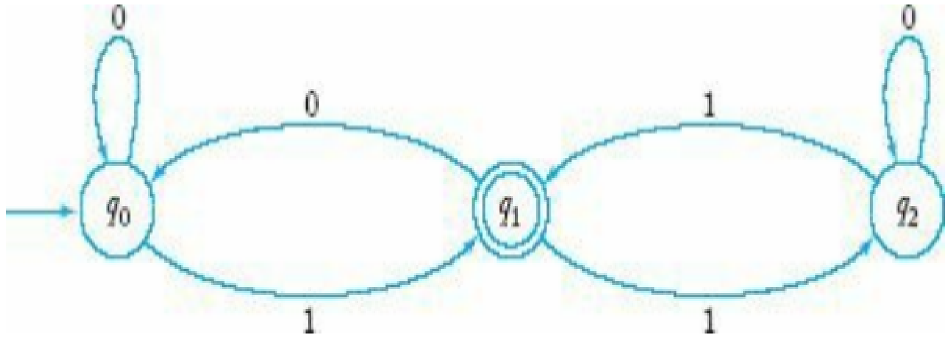

Figure 1.

2. Design DFA

   (a) accepts the string 1010 only.
   (b) accepts the strings 1100 or 1010 only.

3. Design DFA

   (a) which accepts set of all strings containing 1010 as substring.
   (b) which accepts set of all strings containing 0101 as substring.

4. Design DFA which accepts set of all binary strings.

5. Design DFA

   (a) which accepts set of all strings ending with 00.
   (b) which accepts set of all strings containing 3 consecutive zero's.
   (c) which does not accepts set of all strings containing 3 consecutive zero's.

6. Design DFA which accepts only those words that have even number of substrings a,b.

7. For $\Sigma = \{a,b\}$, construct dfa's that accept the sets consisting of

   (a) all strings with exactly one a,
   (b) all strings with at least one a,
   (c) all strings with no more than three a's,
   (d) all strings with at least one a and exactly two b's,
   (e) all the strings with exactly two a's and more than two b's.

8. Give dfa's for the languages

   (a) $L = \{ab^5 wb^2 : w \in \{a,b\}^*\}$,
   (b) $L = \{ab^n a^m : n \geq 2, m \geq 3\}$,
   (c) $L = \{w_1 abw_2 : w_1 \in \{a,b\}^*, w_2 \in \{a,b\}^*\}$,
   (d) $L = \{ba^n : n \geq 1, n \neq 5\}$.

9. Find dfa's for the following languages on $\Sigma = \{a,b\}$.

   (a) $L = \{w : |w| \bmod 3 = 0\}$.
   (b) $L = \{w : |w| \bmod 5 \neq 0\}$.
   (c) $L = \{w : n_a(w) \bmod 3 > 1\}$.
   (d) $L = \{w : n_a(w) \bmod 3 > n_b(w) \bmod 3\}$.
   (e) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 > 0\}$.
   (f) $L = \{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$.
   (g) $L = \{w : |w| \bmod 3 = 0, |w| \neq 6\}$.

10. Construct deterministic finite automata accepting each of the following languages (L1 – L4):

    (a) $L1 = \{w \in \{a, b\}^* :$ each a in w is immediately preceded and immediately followed by a b$\}$
    (b) $L2 = \{w \in \{a, b\}^* :$ w has $3k + 1$ b's for some $k \in N\}$
    (c) $L3 = \{w \in \{a, b\}^* :$ w has neither aa nor bb as a substring$\}$
    (d) $L4 = \{w \in \{a, b\}^* :$ w has even number of a's and one or two b's$\}$

11. A run in a string is a substring of length at least two, as long as possible and consisting entirely of the same symbol. For instance, the string abbbaab contains a run of b's of length three and a run of a's of length two. Find dfa's for the following languages on $\{a,b\}$.

    (a) $L = \{w :$ w contains no runs of length less than four$\}$.

(b) L= {w : every run of a's has length either two or three}.
(c) L= {w : there are at most two runs of a's of length three}.
(d) L= {w : there are exactly two runs of a's of length 3}.

12. Consider the set of strings on {0,1} defined by the requirements below. For each, construct an accepting dfa.

(a) Every 00 is followed immediately by a 1. For example, the strings 101, 0010, 0010011001 are in the language, but 0001 and 00100 are not.
(b) All strings containing 00 but not 000.
(c) The leftmost symbol differs from the rightmost one.
(d) Every substring of four symbols has at most two 0's. For example, 001110 and 011001 are in the language, but 10010 is not since one of its substrings, 0010, contains three zeros.
(e) All strings of length five or more in which the fourth symbol from the right end is different from the leftmost symbol.
(f) All strings in which the leftmost two symbols and the rightmost two symbols are identical.
(g) All strings of length four or greater in which the leftmost three symbols are the same, but different from the rightmost symbol.

13.
(a) With $\Sigma = \{a,b\}$, give a dfa for L= $w_1 a w_2 : |w_1| \geq 3, |w_2| \leq 5\}$.
(b) Design a DFA for the language $L_2 = \{ x \in \{0, 1\}^* :$ x is interpreted as a unsigned binary numeral and x mod 5 = 2}.
(c) Design a DFA that accepts the language L3 = $\{x \in \{0, 1\}^* : |x|_0$ is even and '00' is a substring of x }.

14. Find minimal dfa's for the following languages. In each case prove that the result is minimal.

(a) $L = \{a^n b^m > : n \geq 2, m \geq 1\}$.
(b) $L = \{a^n b : n \geq 0\} \cup \{b^n a : n \geq 1\}$
(c) $L = \{a^n : n \geq 0, n \neq 3\}$.
(d) $L = \{a^n : n \neq 2$ and $n \neq 4\}$.
(e) $L = \{a^n : n \bmod 3 = 0\} \cup \{a^n : n \bmod 5 = 1\}$.

15. Construct a dfa that accepts strings on {0,1} if and only if the value of the string, interpreted as a binary representation of an integer, is zero modulo five. For example, 0101 and 1111, representing the integers 5 and 15, respectively, are to be accepted.