CS561: Artificial Intelligence

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Fuzzy Logic

26th August 2019

Resources

- Main Text:
  - Artificial Intelligence: A Modern Approach by Russell & Norvik, Pearson, 2003.
- Other Main References:
  - Principles of AI - Nilsson
  - AI - Rich & Knight
  - Knowledge Based Systems – Mark Stefik
- Journals
  - AI, AI Magazine, IEEE Expert,
  - Area Specific Journals e.g, Computational Linguistics
- Conferences
  - IJCAI, AAAI

Positively attend lectures!

Modeling Human Reasoning

Fuzzy Logic

Fuzzy Logic tries to capture the human ability of reasoning with imprecise information

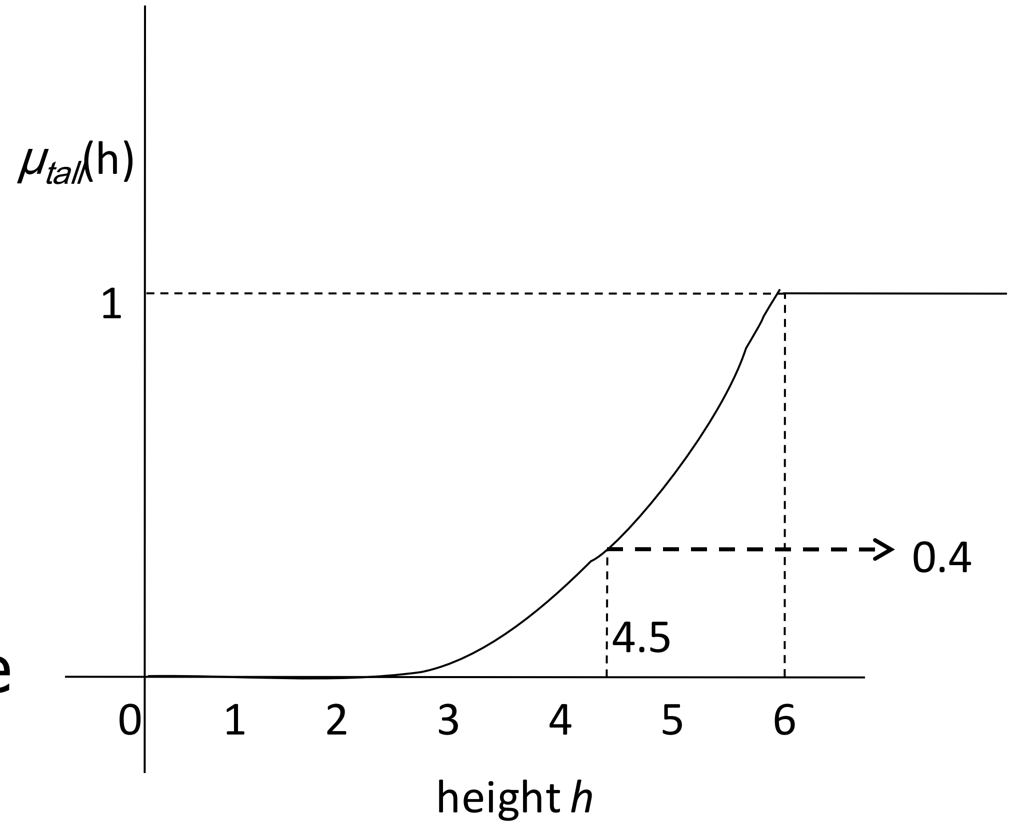
- Works with imprecise statements such as:

  In a process control situation, "*If* the temperature is <u>moderate</u> and the pressure is <u>high</u>, *then* turn the knob <u>slightly right</u>"
- The rules have "Linguistic Variables", typically adjectives qualified by adverbs (adverbs are <u>hedges</u>).

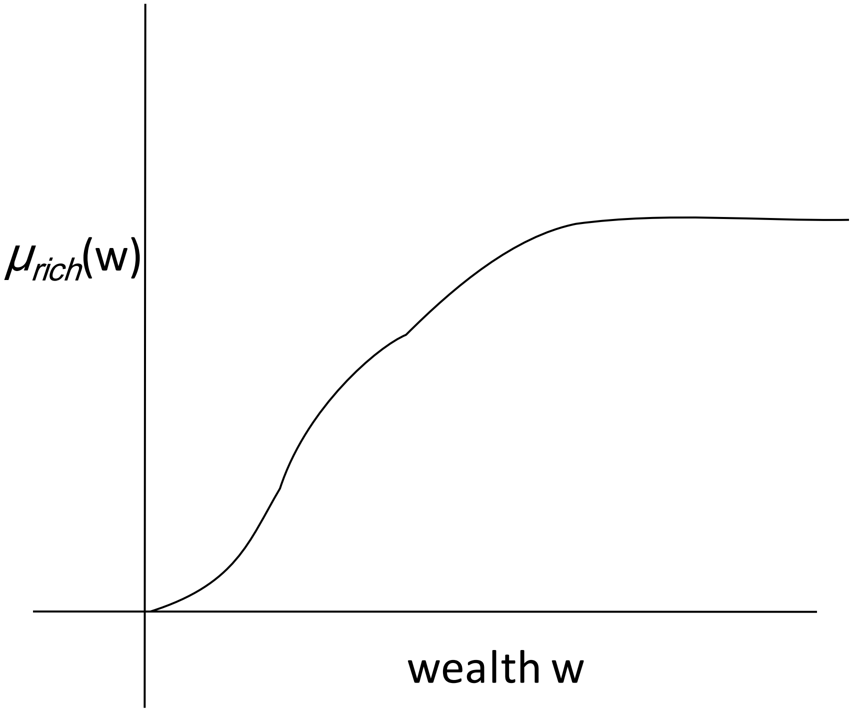
Linguistic Variables

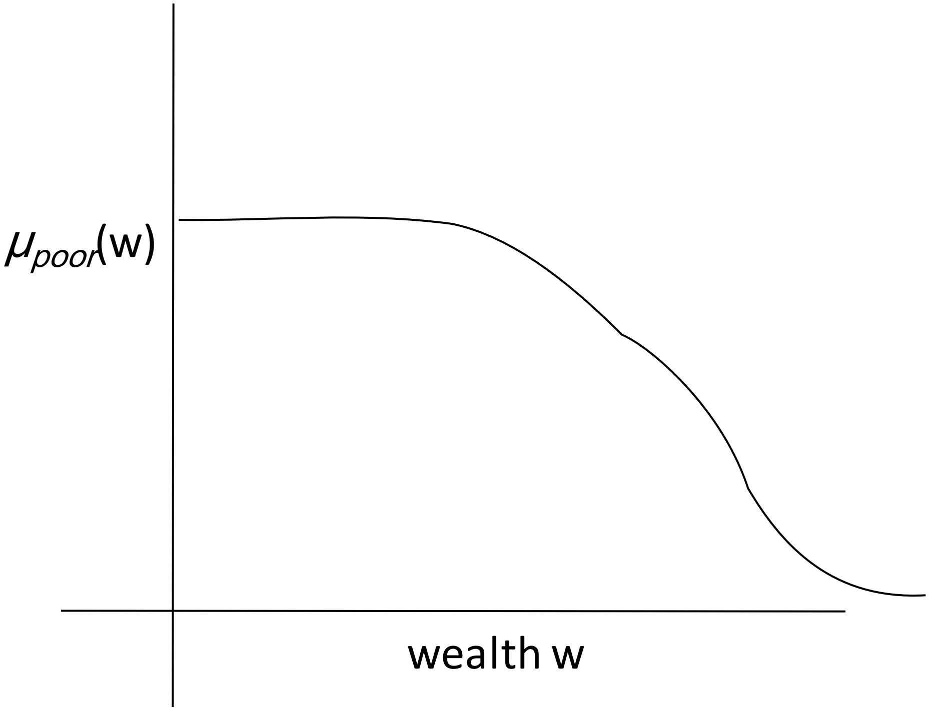
- Fuzzy sets are named by Linguistic Variables (typically adjectives).
- Underlying the LV is a numerical quantity

  E.g. For 'tall' (LV), 'height' is numerical quantity.
- Profile of a LV is the plot shown in the figure shown alongside.



Example Profiles





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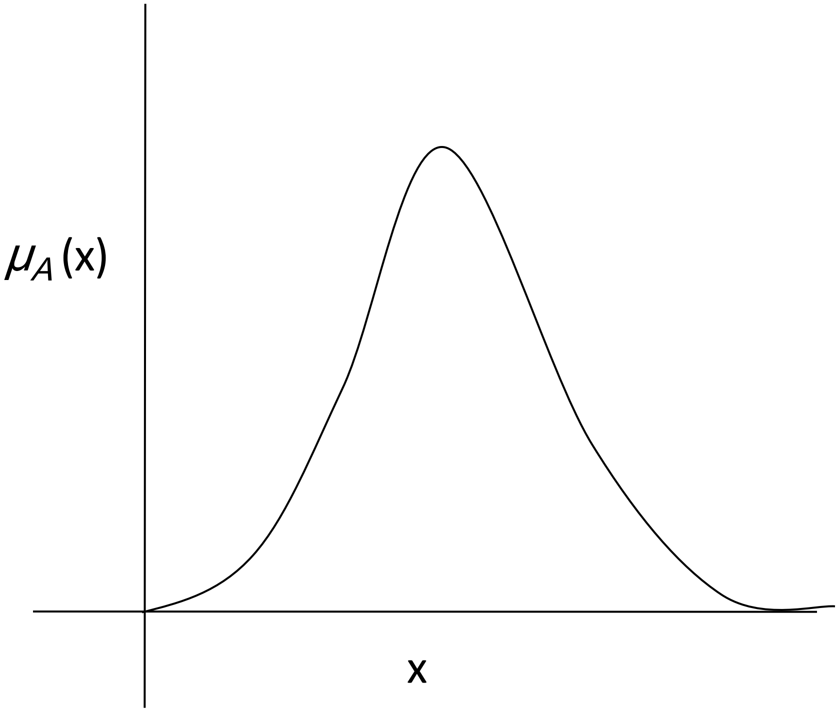
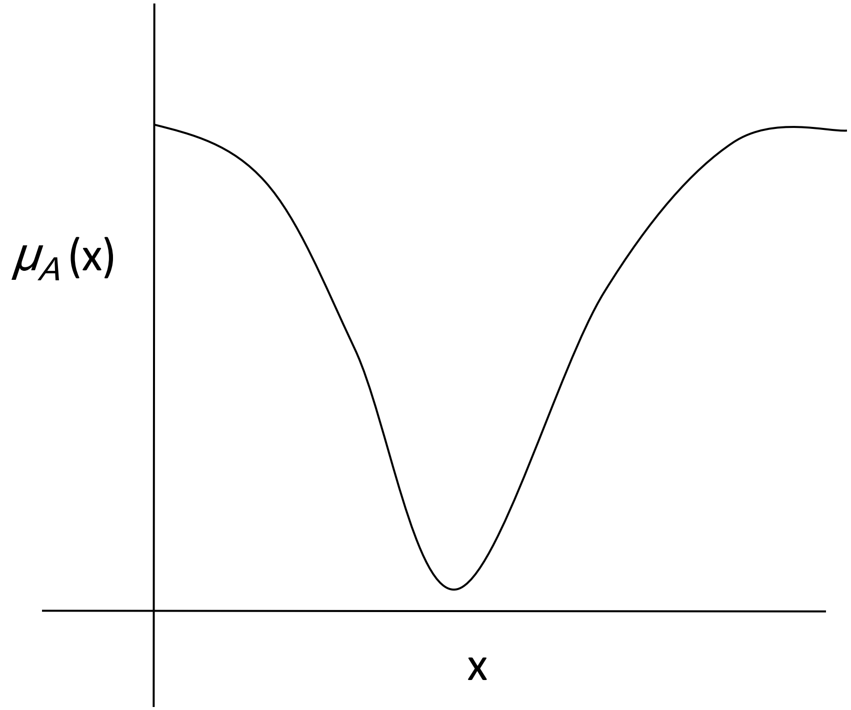


Profile representing
moderate (*e.g.* moderately rich)



Profile representing
extreme

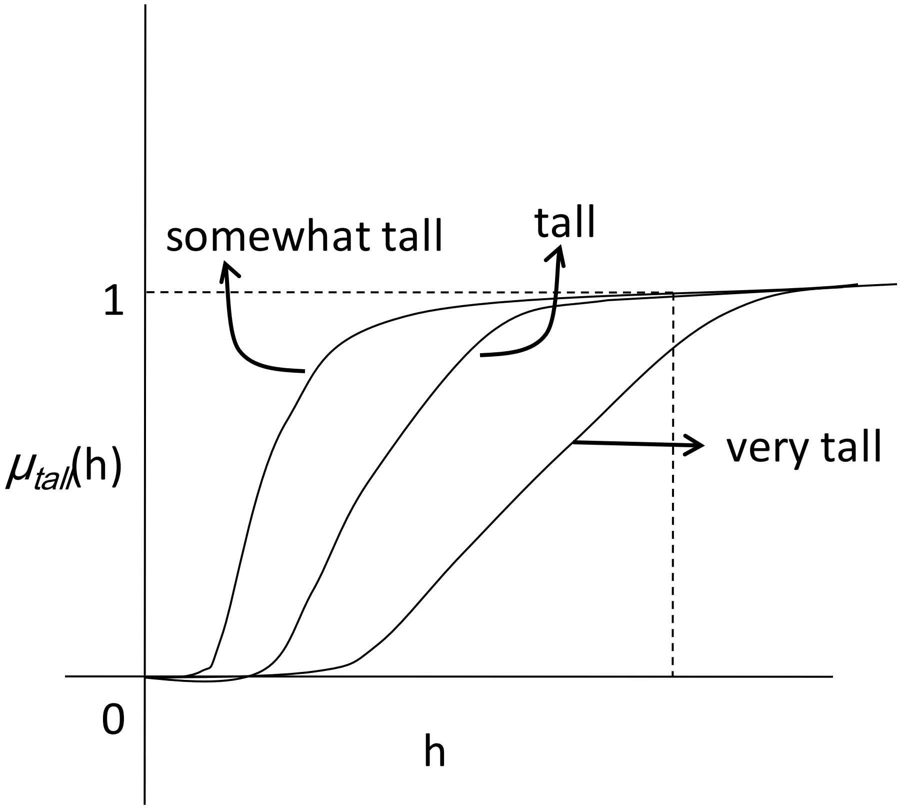
Concept of Hedge

- Hedge is an intensifier
- Example:

  LV = tall, LV_1 = very tall, LV_2 = somewhat tall
- 'very' operation:

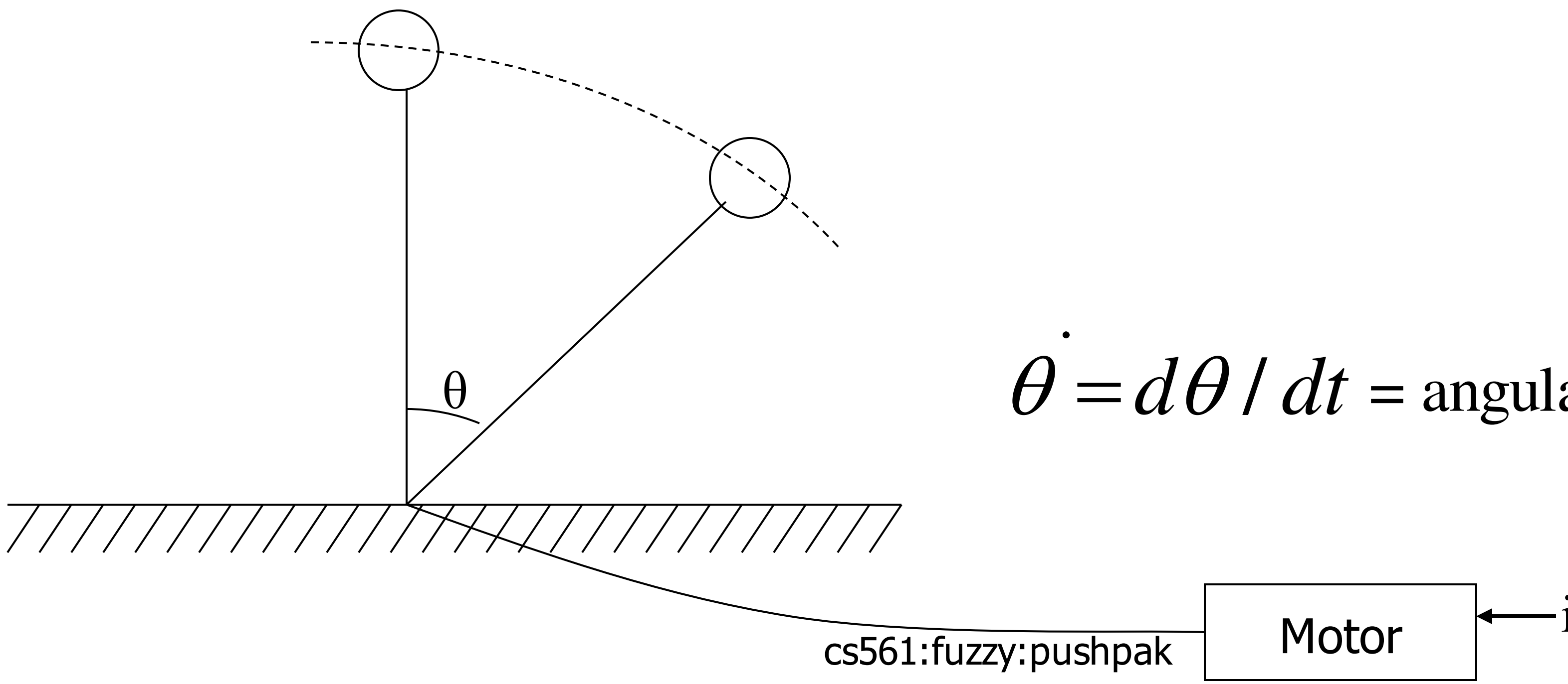
  $\mu_{very\ tall}(x) = \mu^2_{tall}(x)$
- 'somewhat' operation:

$\mu_{somewhat\ tall}(x) = \sqrt{(\mu_{tall}(x))}$



An Example

Controlling an inverted pendulum:



$\dot{\theta} = d\theta / dt$ = angular velocity

The goal: To keep the pendulum in vertical position ($\theta=0$) in dynamic equilibrium. Whenever the pendulum departs from vertical, a torque is produced by sending a current 'i'

Controlling factors for appropriate current

Angle θ, Angular velocity $\dot{\theta}$

<u>Some intuitive rules</u>

If θ is +ve small and $\dot{\theta}$ is –ve small

then current is zero

If θ is +ve small and $\dot{\theta}$ is +ve small

then current is –ve medium

Control Matrix

$\dot{\theta}$ \ θ	-ve med	-ve small	Zero	+ve small	+ve med
-ve med					
-ve small		+ve med	+ve small	Zero	
Zero		+ve small	Zero	-ve small	
+ve small		Zero	-ve small	-ve med	
+ve med					

Region of interest

Each cell is a rule of the form

If θ is <> and $\dot{\theta}$ is <>

then i is <>

<u>4 “Centre rules”</u>

1. if θ = = Zero and $\dot{\theta}$ = = Zero then i = Zero
2. if θ is +ve small and $\dot{\theta}$ = = Zero then i is –ve small
3. if θ is –ve small and $\dot{\theta}$ = = Zero then i is +ve small
4. if θ = = Zero and $\dot{\theta}$ is +ve small then i is –ve small
5. if θ = = Zero and $\dot{\theta}$ is –ve small then i is +ve small

Alternatives to fuzzy logic model human reasoning (1/2)

- Non-numerical
  - Non monotonic Logic
    - Negation by failure (*"innocent unless proven guilty"*)
    - Abduction (*P→Q AND Q gives P)*
  - Modal Logic
    - New operators beyond AND, OR, IMPLIES, Quantification etc.
  - Naïve Physics

Abduction Example

- **If**

  there is rain (P)
- **Then**

  there will be no picnic (Q)
- Abductive reasoning:

  Observation: There was no picnic(Q)

  Conclude : There was rain(P); *in absence of any other evidence*

Alternatives to fuzzy logic model human reasoning (2/2)

- Numerical
  - Fuzzy Logic
  - Probability Theory
    - Bayesian Decision Theory
  - Possibility Theory
  - Uncertainty Factor based on Dempster Shafer Evidence Theory (e.g. *yellow_eyes→jaundice; 0.3)*

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- Works with imprecise statements such as:

  In a process control situation, "*If* the temperature is moderate and the pressure is high, *then* turn the knob slightly right"
- The rules have "Linguistic Variables", typically adjectives qualified by adverbs (adverbs are hedges).

Theory of Fuzzy Sets

- Intimate connection between logic and set theory.
- Given any set 'S' and an element 'e', there is a very natural predicate, $\mu_s(e)$ called as the *belongingness predicate*.
- The predicate is such that,

$$\mu_s(e) = 1, \quad \text{iff } e \in S$$
$$= 0, \quad \text{otherwise}$$

- For example, $S = \{1, 2, 3, 4\}$, $\mu_s(1) = 1$ and $\mu_s(5) = 0$
- A predicate $P(x)$ also defines a set naturally.

  $S = \{x \mid P(x) \text{ is } true\}$

  For example, $even(x)$ defines $S = \{x \mid x \text{ is even}\}$

Fuzzy Set Theory (contd.)

- Fuzzy set theory starts by questioning the fundamental assumptions of set theory *viz.*, the belongingness predicate, μ, value is 0 or 1.
- Instead in Fuzzy theory it is assumed that,

  $$\mu_s(e) = [0, 1]$$

- Fuzzy set theory is a generalization of classical set theory *aka* called Crisp Set Theory.
- In real life, *belongingness* is a fuzzy concept.

  Example: Let, T = "tallness"

  $$\mu_T(\text{height}=6.0\text{ft}) = 1.0$$

  $$\mu_T(\text{height}=3.5\text{ft}) = 0.2$$

  An individual with height 3.5ft is "tall" with a degree *0.2*

Representation of Fuzzy sets

Let $U = \{x_1, x_2, \ldots, x_n\}$

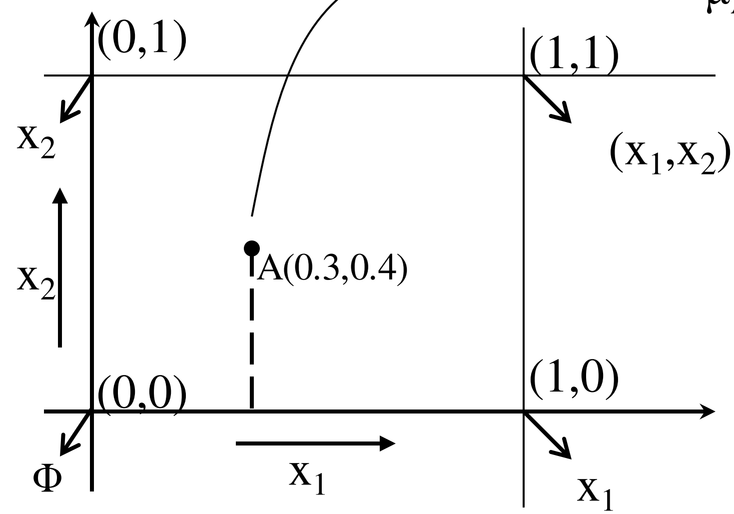
$|U| = n$

The various sets composed of elements from U are presented as points on and inside the n-dimensional hypercube. The crisp sets are the corners of the hypercube.

$\mu_A(x_1)=0.3$

$\mu_A(x_2)=0.4$

$U=\{x_1,x_2\}$



A fuzzy set A is represented by a point in the n-dimensional space as the point $\{\mu_A(x_1), \mu_A(x_2), \ldots, \mu_A(x_n)\}$

Degree of fuzziness

The centre of the hypercube is the *most fuzzy* set. Fuzziness decreases as one nears the corners

Measure of fuzziness

Called the entropy of a fuzzy set

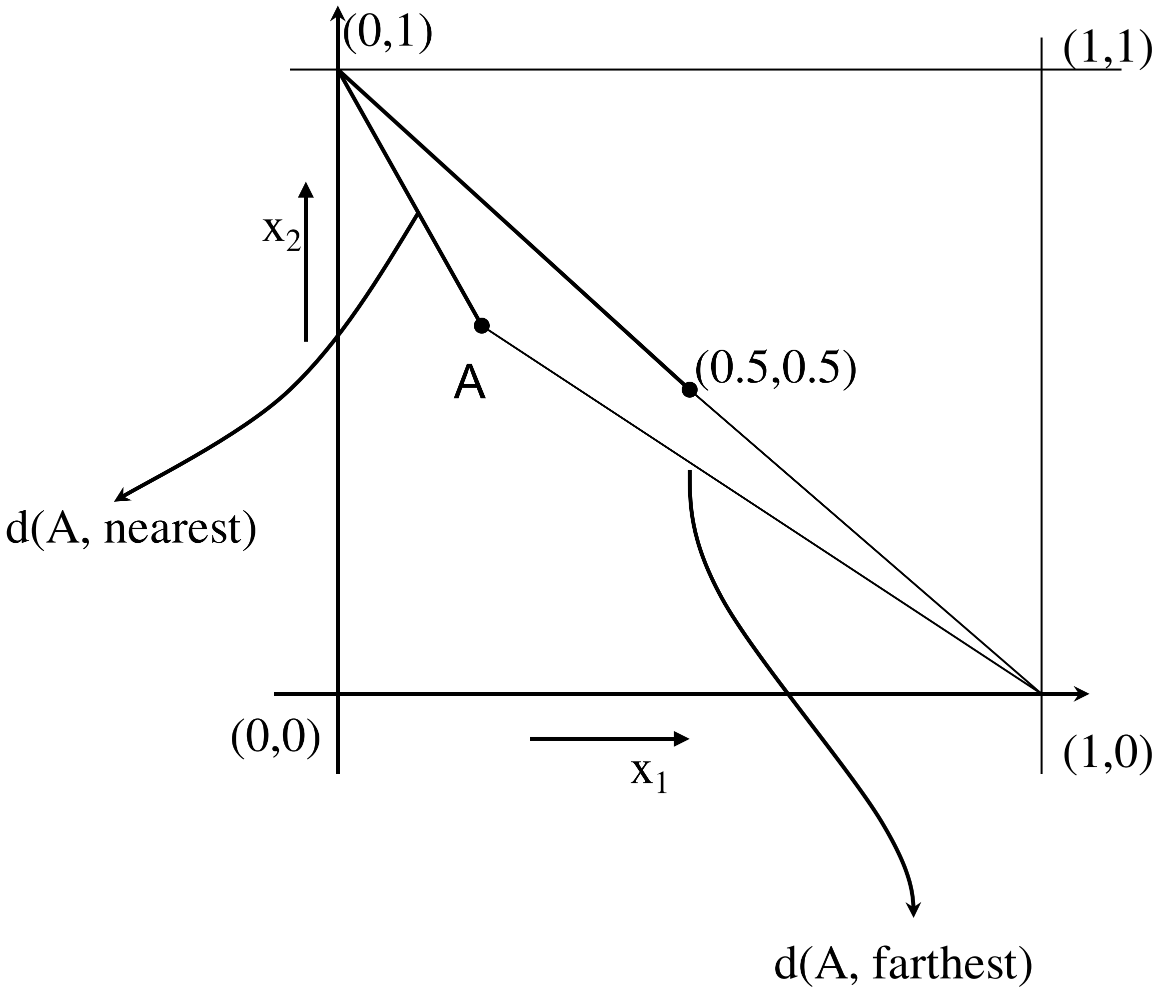
Fuzzy set

Farthest corner

$$E(S) = d(S, nearest) / d(S, farthest)$$

Entropy

Nearest corner


(0,1)
(1,1)
x_2
A
(0.5,0.5)
d(A, nearest)
(0,0)
x_1
(1,0)
d(A, farthest)

<u>Definition</u>

Distance between two fuzzy sets

$$d(S_1, S_2) = \sum_{i=1}^{n} \underbrace{| \mu_{s_1}(x_i) - \mu_{s_2}(x_i) |}_{L_1 \text{ - norm}}$$

Let C = fuzzy set represented by the centre point

d(c,nearest) = |0.5-1.0| + |0.5 – 0.0|

= 1

= d(C,farthest)

=> E(C) = 1

Definition

Cardinality of a fuzzy set

$$m(s) = \sum_{i=1}^{n} \mu_s(x_i)$$ (generalization of cardinality of classical sets)

Union, Intersection, complementation, subset hood

$$\mu_{s_1 \cup s_2}(x) = \max(\mu_{s_1}(x), \mu_{s_2}(x)), \forall x \in U$$

$$\mu_{s_1 \cap s_2}(x) = \min(\mu_{s_1}(x), \mu_{s_2}(x)), \forall x \in U$$

$$\mu_{s^c}(x) = 1 - \mu_s(x)$$

Example of Operations on Fuzzy Set

- Let us define the following:
  - Universe $U=\{X_1, X_2, X_3\}$
  - Fuzzy sets
    - $A=\{0.2/X_1 , 0.7/X_2 , 0.6/X_3\}$ and
    - $B=\{0.7/X_1 ,0.3/X_2 ,0.5/X_3\}$

  Then Cardinality of A and B are computed as follows:

  Cardinality of $A=|A|=0.2+0.7+0.6=1.5$

  Cardinality of $B=|B|=0.7+0.3+0.5=1.5$

  While distance between A and B

  $d(A,B)=|0.2-0.7)+|0.7-0.3|+|0.6-0.5|=1.0$

  What does the cardinality of a fuzzy set mean? In crisp sets it means the number of elements in the set.

Example of Operations on Fuzzy Set (cntd.)

Universe $U=\{X_1, X_2, X_3\}$

Fuzzy sets $A=\{0.2/X_1, 0.7/X_2, 0.6/X_3\}$ and $B=\{0.7/X_1, 0.3/X_2, 0.5/X_3\}$

$A \cup B= \{0.7/X_1, 0.7/X_2, 0.6/X_3\}$

$A \cap B= \{0.2/X1, 0.3/X2, 0.5/X3\}$

$A^c = \{0.8/X_1, 0.3/X_2, 0.4/X_3\}$

Laws of Set Theory

- The laws of Crisp set theory also holds for fuzzy set theory (verify them)
- These laws are listed below:
  - Commutativity: $A \cup B = B \cup A$
  - Associativity: $A \cup (B \cup C)=(A \cup B) \cup C$
  - Distributivity: $A \cup (B \cap C)=(A \cap C) \cup (B \cap C)$

    $A \cap (B \cup C)=(A \cup C) \cap(B \cup C)$
  - De Morgan's Law: $(A \cup B)^C= A^C \cap B^C$

    $(A \cap B)^C= A^C \cup B^C$

Distributivity Property Proof

- Let Universe $U=\{x_1,x_2,...x_n\}$

  $p_i = \mu_{AU(B\cap C)}(x_i)$

  $= \max[\mu_A(x_i), \mu_{(B\cap C)}(x_i)]$

  $= \max[\mu_A(x_i), \min(\mu_B(x_i),\mu_C(x_i))]$

  $q_i = \mu_{(AUB)\cap(AUC)}(x_i)$

  $= \min[\max(\mu_A(x_i), \mu_B(x_i)), \max(\mu_A(x_i), \mu_C(x_i))]$

Distributivity Property Proof

- Case I: $0<\mu_C<\mu_B<\mu_A<1$

  $p_i = \max[\mu_A(x_i), \min(\mu_B(x_i),\mu_C(x_i))]$

  $= \max[\mu_A(x_i), \mu_C(x_i)]=\mu_A(x_i)$

  $q_i = \min[\max(\mu_A(x_i), \mu_B(x_i)), \max(\mu_A(x_i), \mu_C(x_i))]$

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- Case II: $0<\mu_C<\mu_A<\mu_B<1$

  $p_i = \max[\mu_A(x_i), \min(\mu_B(x_i),\mu_C(x_i))]$

  $= \max[\mu_A(x_i), \mu_C(x_i)]=\mu_A(x_i)$

  $q_i = \min[\max(\mu_A(x_i), \mu_B(x_i)), \max(\mu_A(x_i), \mu_C(x_i))]$

  $= \min[\mu_B(x_i), \mu_A(x_i)]=\mu_A(x_i)$

  Prove it for rest of the 4 cases.

Note on definition by extension and intension

$S_1 = \{ x_i | x_i \bmod 2 = 0 \}$ – Intension

$S_2 = \{0,2,4,6,8,10,\ldots\ldots\ldots\ldots\}$ – extension

How to define subset hood?

Meaning of fuzzy subset

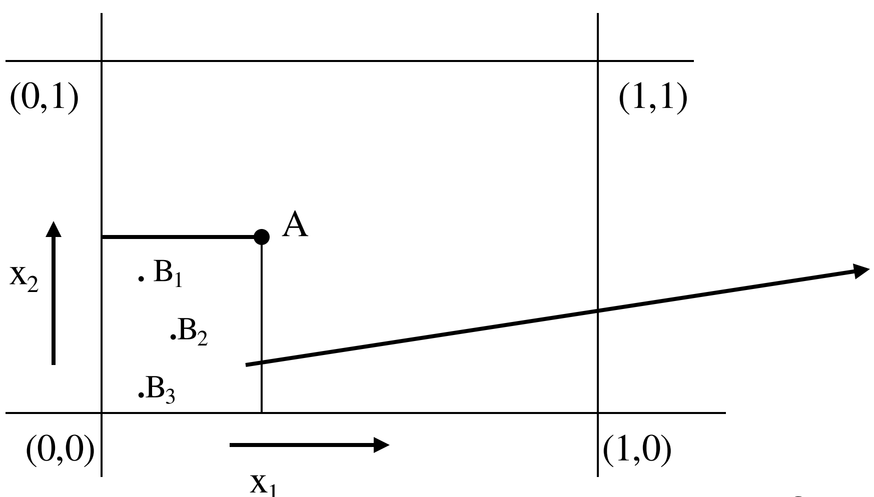
Suppose, following classical set theory we say

$$B \subset A$$

if

$$\mu_B(x) \le \mu_A(x) \forall x$$

Consider the n-hyperspace representation of A and B



Region where $\mu_B(x) \le \mu_A(x)$

This effectively means

$B \in P(A)$ CRISPLY

$P(A)$ = Power set of A

Eg: Suppose

A = {0,1,0,1,0,1,……………….,0,1} – 10^4 elements

B = {0,0,0,1,0,1,……………………,0,1} – 10^4 elements

Isn't $B \subset A$ with a degree? (only differs in the 2^{nd} element)

Subset operator is the “odd man” out

- AUB, $A \cap B$, A^c are all “Set Constructors” while $A \subseteq B$ is a Boolean Expression or predicate.
- According to classical logic
  - In Crisp Set theory $A \subseteq B$ is defined as

    $$\forall x \quad x \in A \Rightarrow x \in B$$

  - So, in fuzzy set theory $A \subseteq B$ can be defined as

    $$\forall x \quad \mu_A(x) \Rightarrow \mu_B(x)$$

Zadeh's definition of subsethood goes against the grain of fuzziness theory

- Another way of defining $A \subseteq B$ is as follows:

$$\forall x \quad \mu_A(x) \le \mu_B(x)$$

But, these two definitions imply that $\mu_{P(B)}(A)=1$
where P(B) is the power set of B

Thus, these two definitions violate the fuzzy principle that every belongingness except Universe is fuzzy

Fuzzy definition of subset

Measured in terms of "fit violation", i.e. violating the condition $\mu_B(x) \le \mu_A(x)$

Degree of subset hood S(A,B)= 1- degree of superset

$$= 1 - \frac{\sum_x \max(0, \mu_B(x) - \mu_A(x))}{m(B)}$$

$$m(\mathrm{B}) = \text{cardinality of B}$$

$$= \sum_x \mu_B(x)$$

We can show that $E(A) = S(A \cup A^c, A \cap A^c)$

Exercise 1:

Show the relationship between entropy and subset hood

Exercise 2:

Prove that

$$S(B, A) = m(A \cap B) / m(B)$$

Subset hood of B in A

Fuzzy sets to fuzzy logic

Forms the foundation of fuzzy rule based system or fuzzy expert system

Expert System

Rules are of the form

If

$C_1 \wedge C_2 \wedge \ldots\ldots\ldots\ldots C_n$

then

A_i

Where C_is are conditions

Eg: C_1=Colour of the eye yellow

C_2= has fever

C_3=high bilurubin

A = hepatitis

In fuzzy logic we have fuzzy predicates

Classical logic

$P(x_1, x_2, x_3 \ldots\ldots x_n) = 0/1$

Fuzzy Logic

$P(x_1, x_2, x_3 \ldots\ldots x_n) = [0,1]$

Fuzzy OR

$$P(x) \vee Q(y) = \max(P(x), Q(y))$$

Fuzzy AND

$$P(x) \wedge Q(y) = \min(P(x), Q(y))$$

Fuzzy NOT

$$\sim P(x) = 1 - P(x)$$

Fuzzy Implication

- Many theories have been advanced and many expressions exist
- The most used is Lukasiewitz formula
- t(P) = truth value of a proposition/predicate. In fuzzy logic t(P) = [0,1]
- $\underbrace{t(P \rightarrow Q) = \min[1, 1 - t(P) + t(Q)]}_{\text{Lukasiewitz definition of implication}}$

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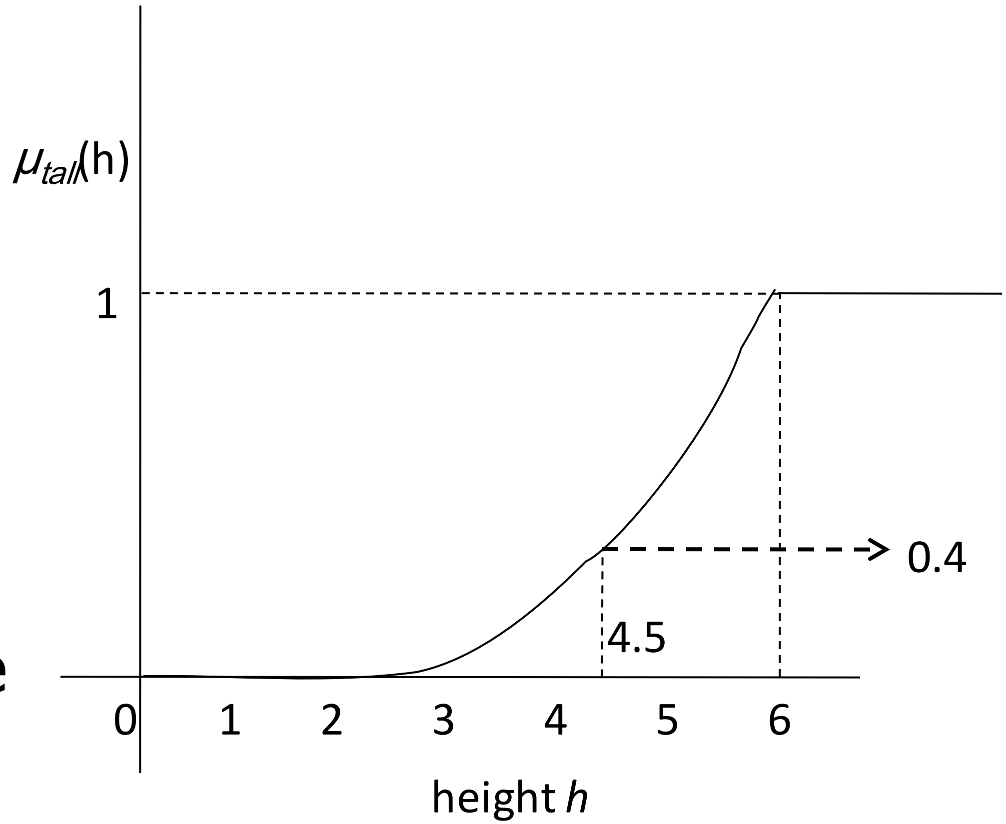

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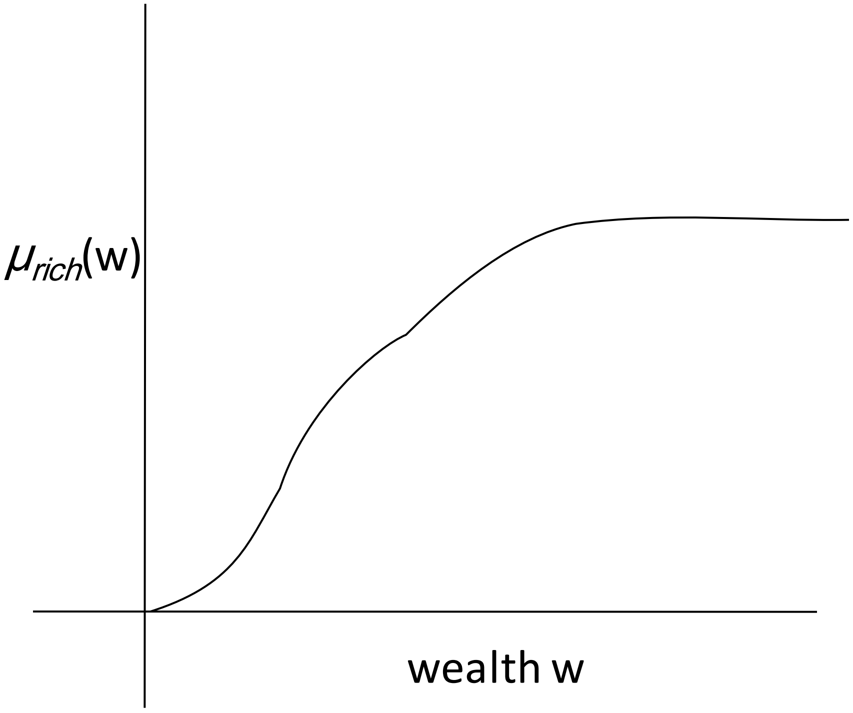

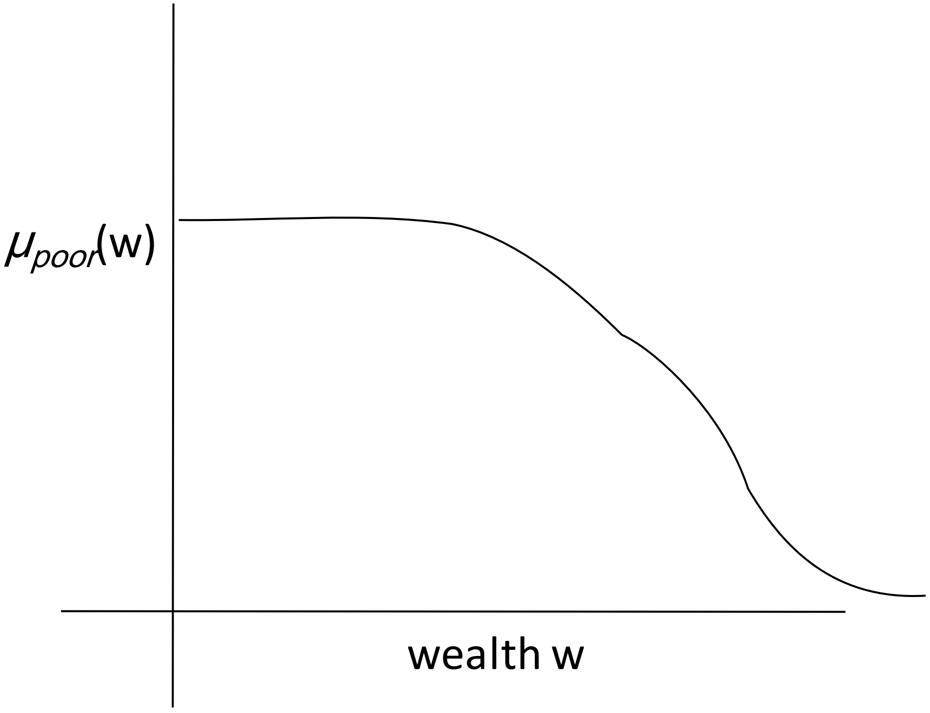

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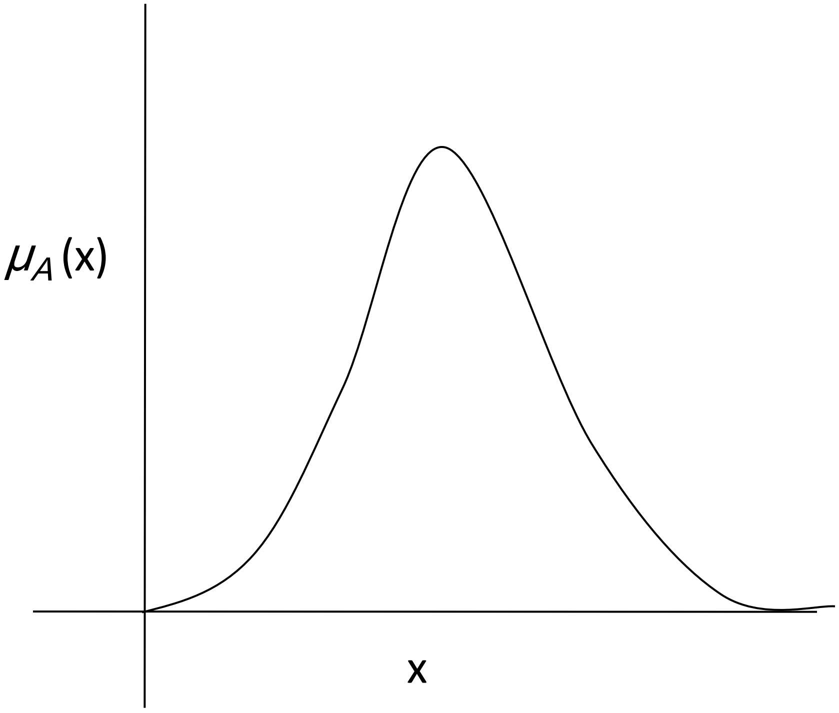

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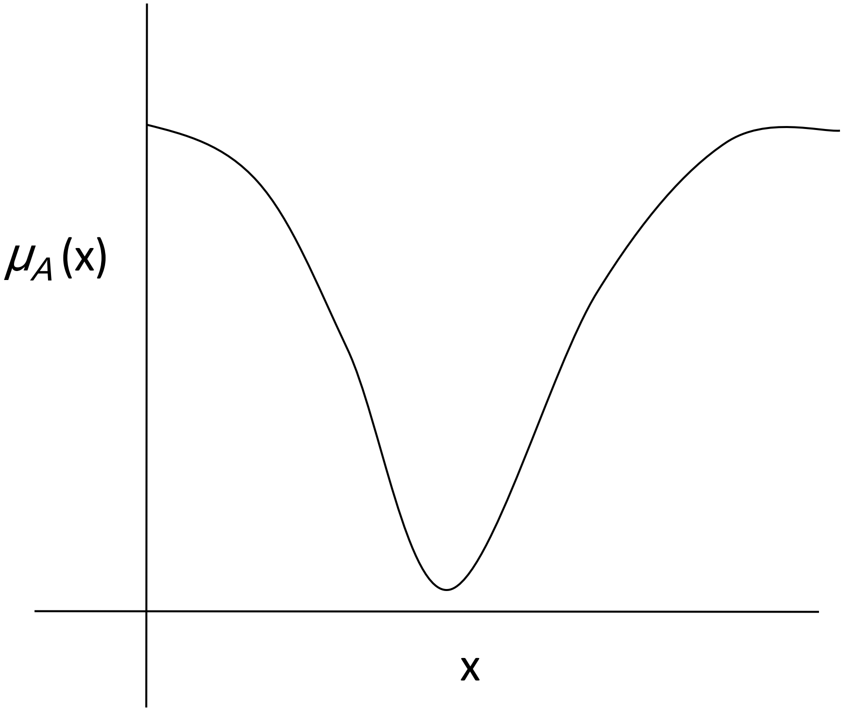

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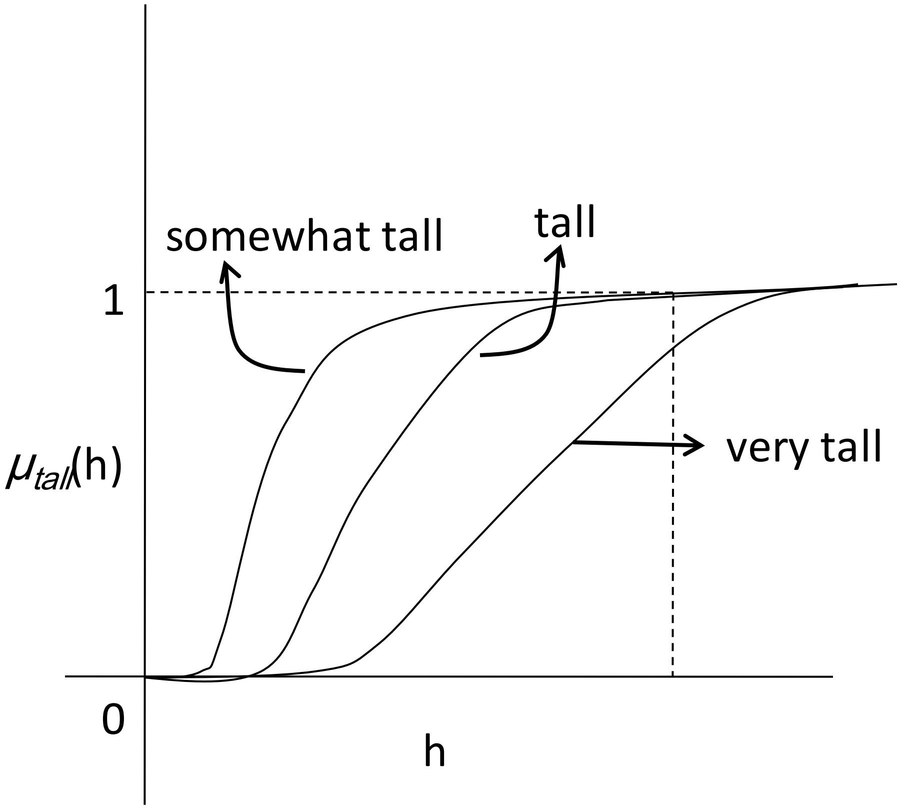


Fuzzy Inferencing

- Two methods of inferencing in classical logic
  - Modus Ponens
    - Given p and $p \rightarrow q$, infer q
  - Modus Tolens
    - Given $\sim q$ and $p \rightarrow q$, infer $\sim p$
- How is fuzzy inferencing done?

A look at reasoning

- Deduction: $p, p \rightarrow q$ |- q
- Induction: $p_1, p_2, p_3, \ldots$ |- *for_all p*
- Abduction: $q, p \rightarrow q$ |- p
- Default reasoning: Non-monotonic reasoning: Negation by failure
  - If something cannot be proven, its negation is asserted to be true
  - E.g., in Prolog

Fuzzy Modus Ponens in terms of truth values

- Given $t(p)=1$ and $t(p \rightarrow q)=1$, infer $t(q)=1$
- In fuzzy logic,
  - given $t(p)>=a$, $0<=a<=1$
  - *and* $t(p \rightarrow >q)=c$, $0<=c<=1$
  - What is $t(q)$
- How much of truth is transferred over the channel

$$p \Rightarrow q$$

Lukasiewitz formula for Fuzzy Implication

- t(P) = truth value of a proposition/predicate. In fuzzy logic t(P) = [0,1]
- $\underbrace{t(P \to Q) = \min[1, 1 - t(P) + t(Q)]}_{\text{Lukasiewitz definition of implication}}$

Use Lukasiewitz definition

- $t(p \rightarrow q) = min[1, 1 - t(p) + t(q)]$
- We have $t(p\text{-}{>}q)=c$, i.e., $min[1, 1 - t(p) + t(q)] = c$
- Case 1:
- $c=1$ gives $1 - t(p) + t(q) >= 1$, i.e., $t(q) >= a$
- Otherwise, $1 - t(p) + t(q) = c$, i.e., $t(q) >= c + a - 1$
- Combining, $t(q) = max(0, a + c - 1)$
- This is the amount of truth transferred over the channel $p \rightarrow q$

Two equations consistent

$$Sub(B, A) = 1 - Sup(B, A)$$

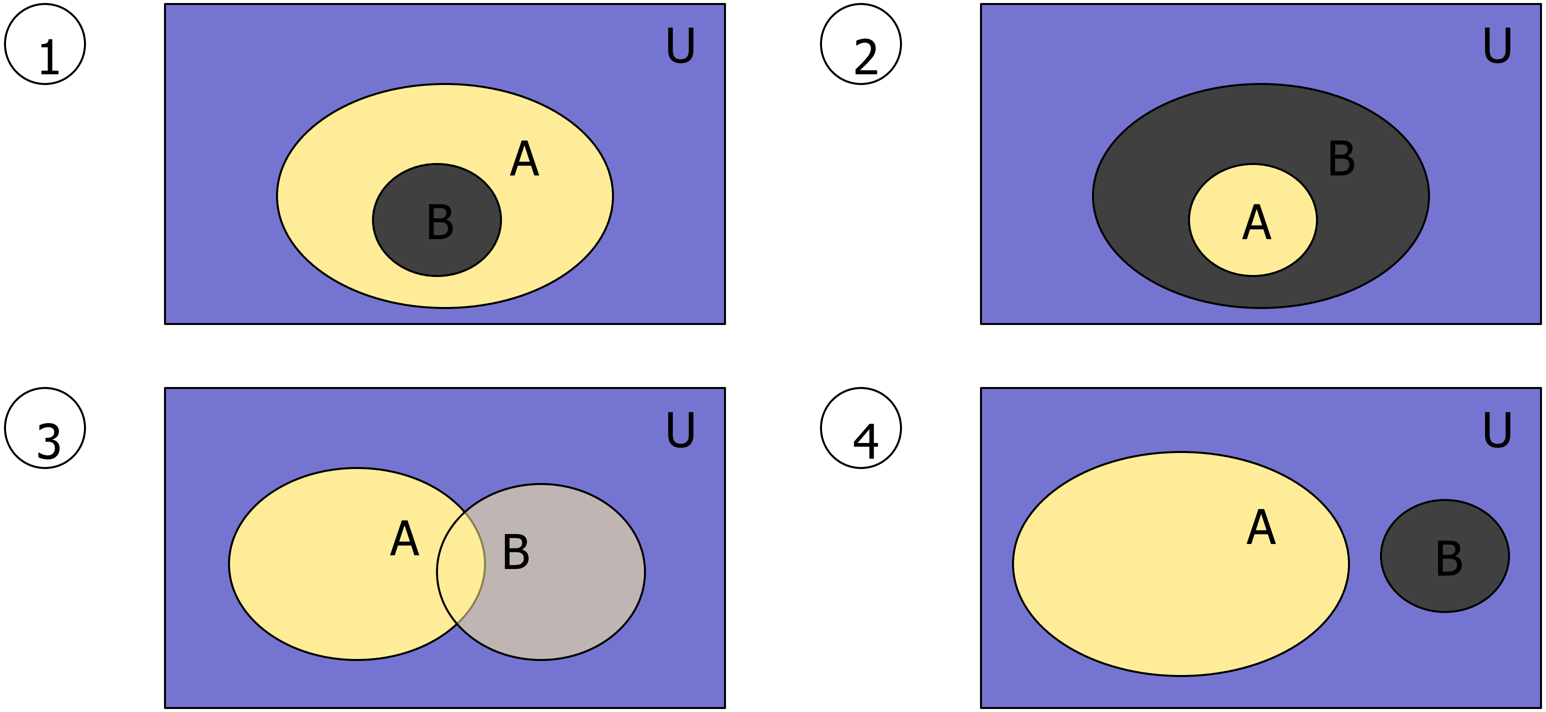
$$= 1 - \frac{\sum_{x_i \in U} \max(0, \mu_B(x_i) - \mu_A(x_i))}{\sum_{x_i \in U} \mu_B(x_i)} \qquad \text{where } U = \{x_1, x_2, ..., x_n\}$$

$$t(\mu_B(x_i) \rightarrow \mu_A(x_i)) = \min(1, 1 - t(\mu_B(x_i)) + t(\mu_A(x_i)))$$

- These two equations are consistent with each other

Proof

- Let us consider two crisp sets A and B



Proof (contd...)

- Case I:

  $\mu_A(x_i) = 1 \quad \text{only when } \mu_B(x_i) = 1 \quad So, \mu_B(x_i) - \mu_A(x_i) <= 0$

  - So,

$$Sub(B, A) = 1 - \frac{\sum_{x_i \in U} \max(0, \mu_B(x_i) - \mu_A(x_i))}{\sum_{x_i \in U} \mu_B(x_i)}$$

$$= 1 - \frac{0}{\sum_{x_i \in U} \mu_B(x_i)} = 1$$

Proof (contd…)

$$Since \qquad \mu_B(x_i) \rightarrow \mu_A(x_i) <= 0$$

$$L = t(\mu_B(x_i) \rightarrow \mu_A(x_i)) = \min(1, 1-(t(\mu_B(x_i)) - t(\mu_A(x_i))))$$

$$= \min(1, 1-(-ve)) = 1$$

- Thus, in case I these two equations are consistent with each other
- Prove them for other three cases

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Lukasiewitz formula for Fuzzy Implication

- t(P) = truth value of a proposition/predicate. In fuzzy logic t(P) = [0,1]
- $\underbrace{t(P \rightarrow Q) = \min[1, 1 - t(P) + t(Q)]}_{\text{Lukasiewitz definition of implication}}$

Use Lukasiewitz definition

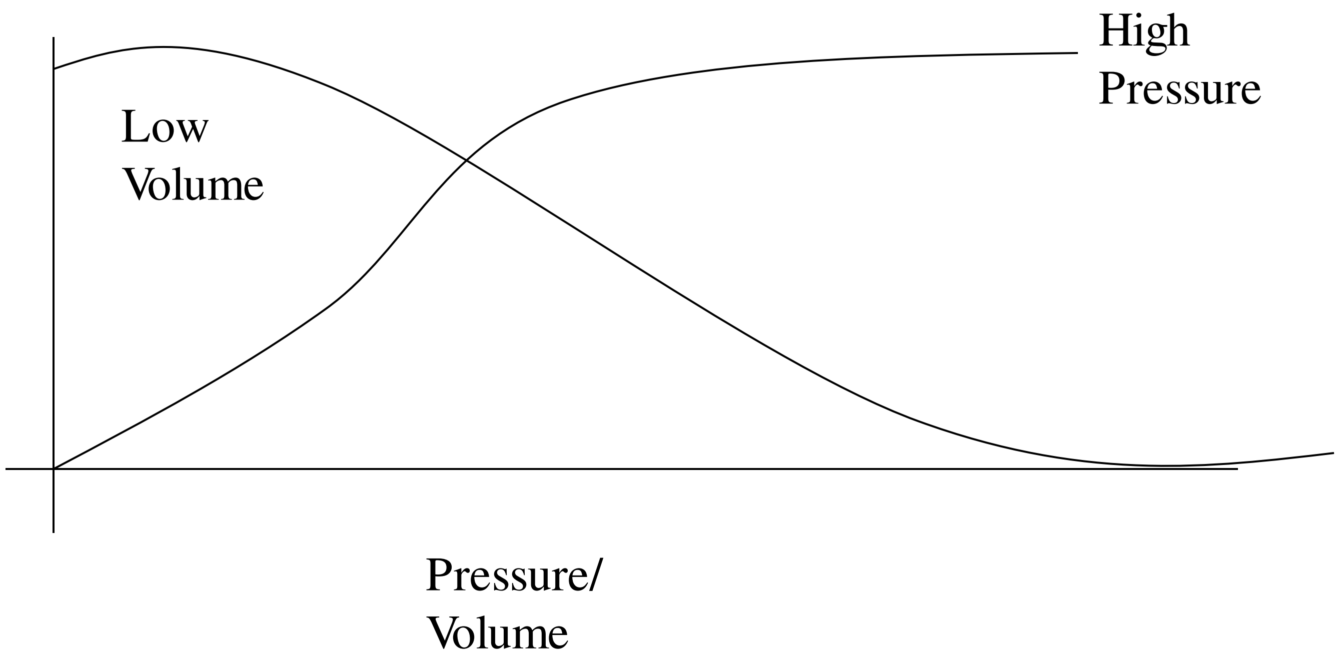
- $t(p \rightarrow q) = min[1, 1 - t(p) + t(q)]$
- We have $t(p\text{->}q)=c$, *i.e.*, $min[1, 1 - t(p)+t(q)]=c$
- Case 1:
- $c=1$ gives $1 - t(p)+t(q)>=1$, *i.e.*, $t(q)>=a$
- Otherwise, $1 - t(p)+t(q)=c$, *i.e.*, $t(q)>=c+a-1$
- Combining, $t(q)=max(0,a+c-1)$
- This is the amount of truth transferred over the channel $p \rightarrow q$

ANDING of Clauses on the LHS of implication

$$t(P \wedge Q) = \min(\ t(P), t(Q))$$

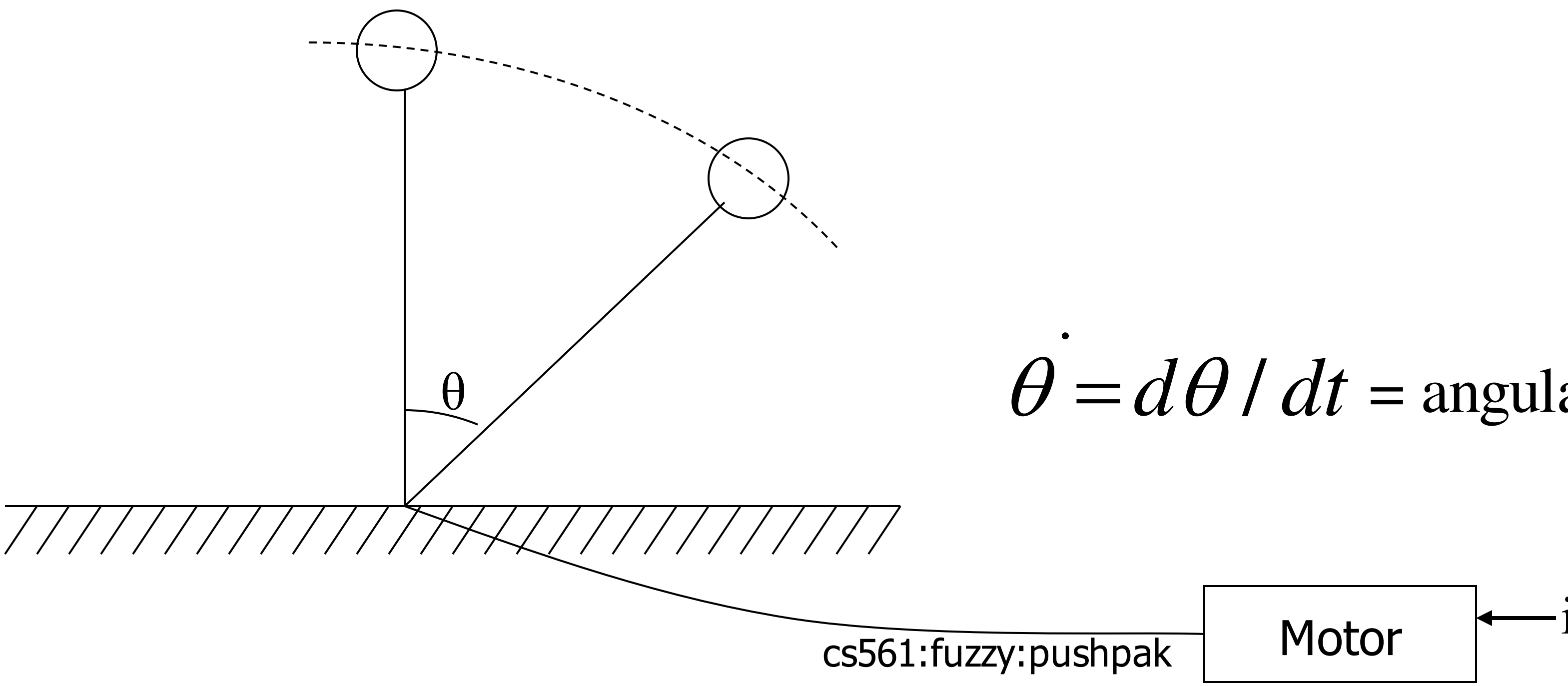
Eg: If pressure is high then Volume is low

$$t(high(\ pressure) \rightarrow low(volume))$$



An Example

Controlling an inverted pendulum:



$\dot{\theta} = d\theta / dt$ = angular velocity

The goal: To keep the pendulum in vertical position ($\theta=0$) in dynamic equilibrium. Whenever the pendulum departs from vertical, a torque is produced by sending a current 'i'

Controlling factors for appropriate current

Angle θ, Angular velocity $\dot{\theta}$

<u>Some intuitive rules</u>

If θ is +ve small and $\dot{\theta}$ is –ve small

then current is zero

If θ is +ve small and $\dot{\theta}$ is +ve small

then current is –ve medium

Control Matrix

$\dot{\theta}$ \ θ	-ve med	-ve small	Zero	+ve small	+ve med	
-ve med						
-ve small		+ve med	+ve small	Zero		
Zero		+ve small	Zero	-ve small		
+ve small		Zero	-ve small	-ve med		
+ve med						

Region of interest

Each cell is a rule of the form

If θ is <> and $\dot{\theta}$ is <>

then i is <>

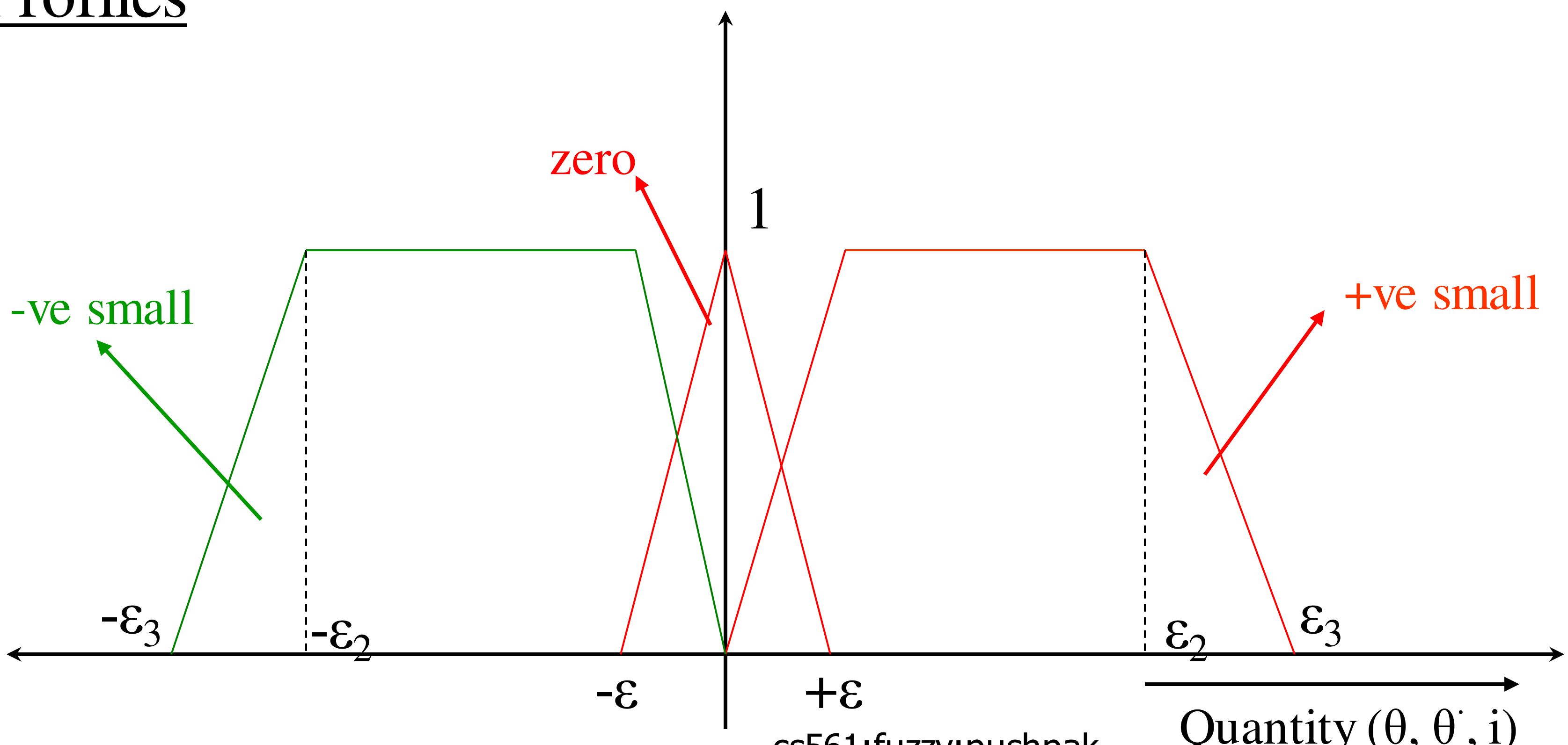
<u>4 "Centre rules"</u>

1. if θ = = Zero and $\dot{\theta}$ = = Zero then i = Zero
2. if θ is +ve small and $\dot{\theta}$ = = Zero then i is –ve small
3. if θ is –ve small and $\dot{\theta}$ = = Zero then i is +ve small
4. if θ = = Zero and $\dot{\theta}$ is +ve small then i is –ve small
5. if θ = = Zero and $\dot{\theta}$ is –ve small then i is +ve small

Linguistic variables

1. Zero
2. +ve small
3. -ve small

Profiles

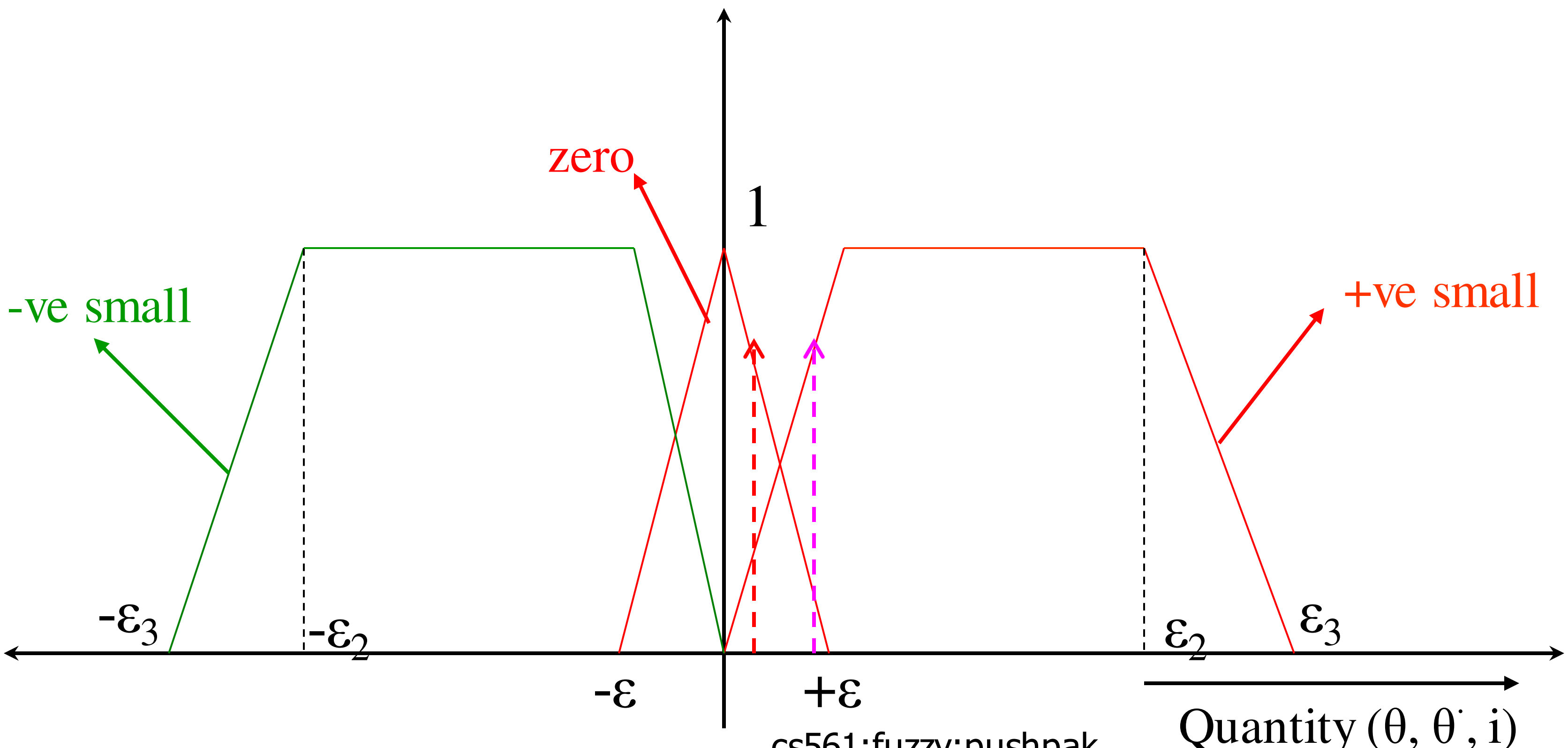


Inference procedure

1. Read actual numerical values of θ and $\dot{\theta}$
2. Get the corresponding μ values μ_{Zero}, $\mu_{(+ve\ small)}$, $\mu_{(-ve\ small)}$. This is called FUZZIFICATION
3. For different rules, get the fuzzy I-values from the R.H.S of the rules.
4. "Collate" by some method and get ONE current value. This is called DEFUZZIFICATION
5. Result is one numerical value of 'i'.

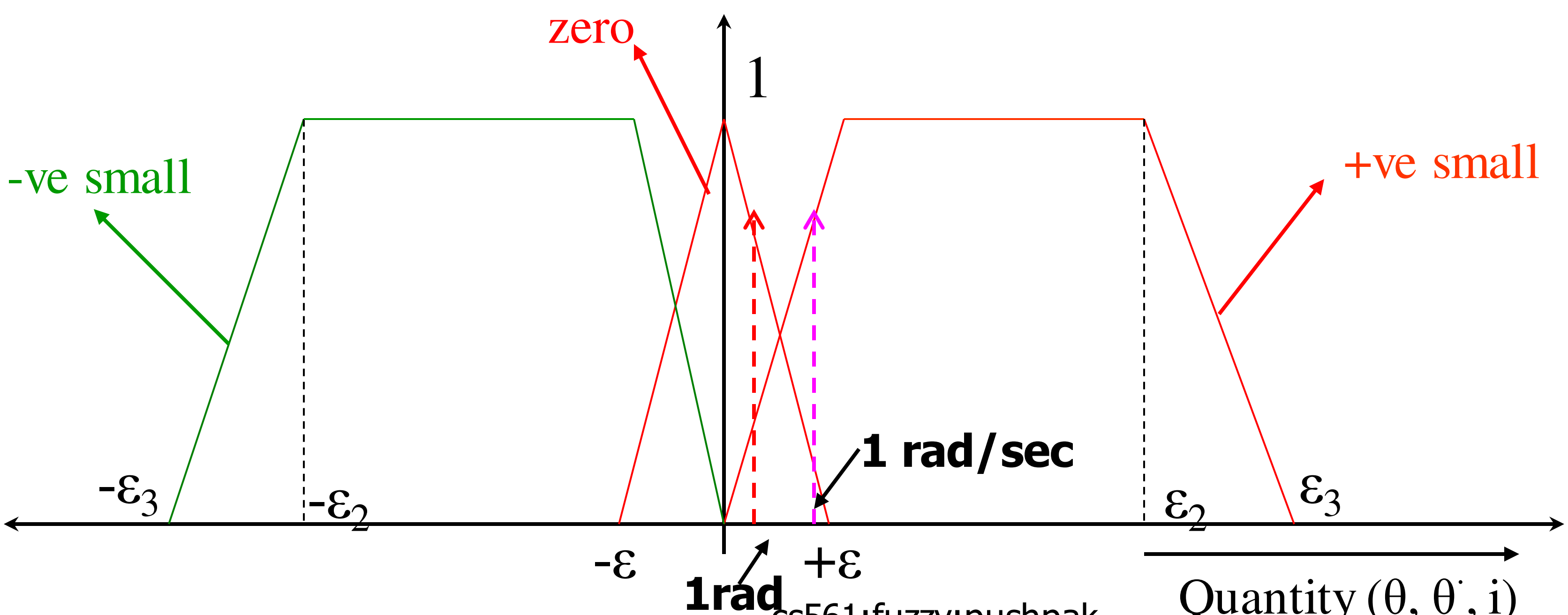
Rules Involved

if θ is Zero and dθ/dt is Zero then i is Zero
if θ is Zero and dθ/dt is +ve small then i is –ve small
if θ is +ve small and dθ/dt is Zero then i is –ve small
if θ +ve small and dθ/dt is +ve small then i is -ve medium



Fuzzification

Suppose θ is 1 radian and dθ/dt is 1 rad/sec
$\mu_{zero}(\theta = 1) = 0.8$ (say)
$M_{+ve\text{-}small}(\theta = 1) = 0.4$ (say)
$\mu_{zero}(d\theta/dt = 1) = 0.3$ (say)
$\mu_{+ve\text{-}small}(d\theta/dt = 1) = 0.7$ (say)

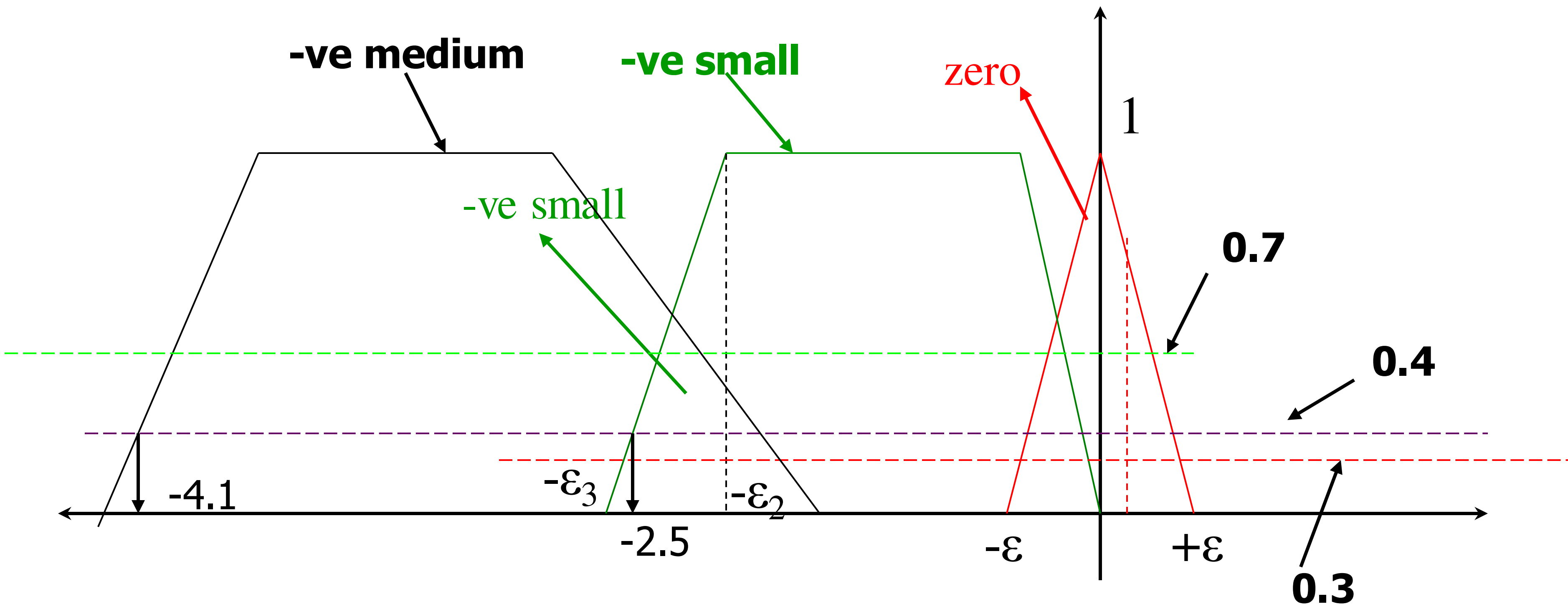


Fuzzification

Suppose θ is 1 radian and $d\theta/dt$ is 1 rad/sec
$\mu_{zero}(\theta = 1) = 0.8$ (say)
$\mu_{+ve\text{-}small}(\theta = 1) = 0.4$ (say)
$\mu_{zero}(d\theta/dt = 1) = 0.3$ (say)
$\mu_{+ve\text{-}small}(d\theta/dt = 1) = 0.7$ (say)

if θ is Zero and $d\theta/dt$ is Zero then i is Zero
min(0.8, 0.3)=0.3
hence $\mu_{zero}(i) = 0.3$
if θ is Zero and $d\theta/dt$ is +ve small then i is –ve small
min(0.8, 0.7)=0.7
hence $\mu_{-ve\text{-}small}(i) = 0.7$
if θ is +ve small and $d\theta/dt$ is Zero then i is –ve small
min(0.4, 0.3)=0.3
hence μ-ve-small(i)=0.3
if θ +ve small and $d\theta/dt$ is +ve small then i is -ve medium
min(0.4, 0.7)=0.4
hence $\mu_{-ve\text{-}medium}(i) = 0.4$

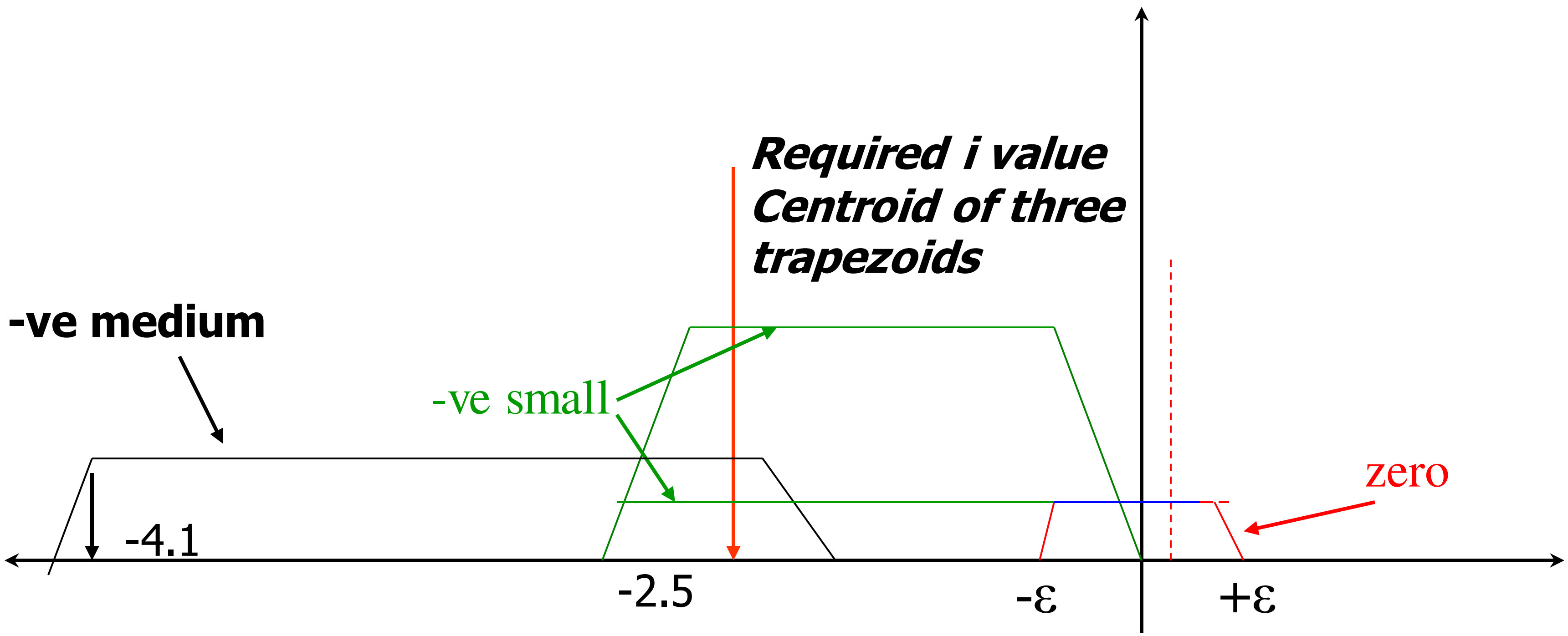
Finding *i*



Possible candidates:

i=0.5 and -0.5 from the "zero" profile and μ=0.3
i=-0.1 and -2.5 from the "-ve-small" profile and μ=0.3
i=-1.7 and -4.1 from the "-ve-small" profile and μ=0.3

Defuzzification: Finding *i* by the *centroid* method



Possible candidates:

i is the x-coord of the centroid of the areas given by the blue trapezium, the green trapeziums and the black trapezium