Efficiency of CSMA/CD

Assume that the access medium is partitioned into timeslots and that a packet takes k timeslots to be transmitted. Remember that the efficiency is the *ratio of successfully used timeslots*, that is, the efficiency is

$$\text{efficiency} = \frac{k}{k+x}$$

where x is the expected number of unsuccessful timeslots (collision or empty).

1. Let Y be a random variable denoting the number of slots until a success: $P(Y=m) = \beta(1-\beta)^{m-1}$, where β is the probability of success for a single timeslot. If every of the N nodes transmits in a given timeslot with probability p: $\beta = Np(1-p)^{N-1}$.

   Y has a geometric distribution with mean

   $$E[Y] = \sum_{m=0}^{\infty} m\beta(1-\beta)^{m-1} = \frac{\beta}{1-\beta}\sum_{m=0}^{\infty} m(1-\beta)^m = \frac{\beta}{1-\beta}\frac{1-\beta}{\beta^2} = \frac{1}{\beta}$$

   Let X be a random variable denoting the number of consecutive wasted slots: $X = Y - 1$, and x its mean value:

   $$\begin{aligned} x &= E[X] = E[Y] - 1 = \frac{1-\beta}{\beta} \\ &= \frac{1-Np(1-p)^{N-1}}{Np(1-p)^{N-1}} \\ \text{efficiency} = \frac{k}{k+x} &= \frac{k}{k + \frac{1-Np(1-p)^{N-1}}{Np(1-p)^{N-1}}} \end{aligned}$$

2. Maximising the efficiency is equivalent to minimising x, which is equivalent to maximising β. p^* maximising β can be calculated as follows:

   $$\frac{\partial}{\partial p}\beta = \frac{\partial}{\partial p}Np(1-p)^{N-1} = N(1-p)^{N-1} - \frac{Np(1-p)^{N-1}(N-1)}{1-p} = 0 \longrightarrow p^* = \frac{1}{N}$$

3. By knowing that $(1 - \frac{1}{N-1})^{N-1}$ approaches $1/e$ for $N \to \infty$ we can calculate the maximum efficiency as

   $$\text{efficiency} = \frac{k}{k + \frac{1-(1-1/N)^{N-1}}{(1-1/N)^{N-1}}} \quad \xrightarrow{N\to\infty} \quad \frac{k}{k+e-1}$$

4. $\frac{k}{k+e-1}$ approaches 1 as $k \to \infty$.