13.2. TESTING OF HYPOTHESIS

A hypothesis is some statement or assertion about a population which we want to verify on the basis of information available from a sample.

There are two types of hypothesis

1. Null hypothesis
2. Alternative Hypothesis

13.2.1. Null and Alternative Hypothesis

Null and Alternate Hypothesis

According to R.A. Fisher, "Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true".

In testing of hypothesis we always begin with the assumption or hypothesis which is the assumed value of a parameter. This is called Null hypothesis. The null hypothesis asserts that there is no significant difference between the sample statistic and the population parameter. If there is any difference between the sample statistic and the population parameter then it may be due to fluctuations in sampling from the same population.

Null hypothesis is the hypothesis which is to be verified with the help of given sample. That is null hypothesis is the hypothesis which is under test. In hypothesis testing, we must state the assumed or hypothesized value of the population parameter before we begin sampling. The assumption we wish to test is called the null hypothesis and is symbolized by 'H_0'.

Example: We want to test the hypothesis that the population mean is equal to 500. We would symbolize it as follows and read it as,

The null hypothesis is that the population mean = 500 written as,

$H_0 : \mu = 500$

Alternative Hypothesis

A hypothesis which is different from Null hypothesis is called Alternative hypothesis. It is denoted by H_1. The two hypothesis H_0 and H_1 are opposite of each other. That is if one of the hypothesis is accepted then the other is rejected and vice versa.

Example: If we want to test success rate of a particular treatment, we make null hypothesis for success rate 'p' (for the test value of 0.99) as

$H_0 : p = 0.99$ and alternative hypothesis is among

$H_1 : p \neq 0.99$

$H_1 : p < 0.99$

$H_1 : p > 0.99$

Example: If we want to test if the attribute of educational qualification has any influence on income of the individual, we make null hypothesis as

H_0 : Educational qualification has no influence on income of an individual and alternative hypothesis is

H_1 : Educational qualification has an influence on income of the individual

13.2.2. Interpreting the Level of Significance

The purpose of hypothesis testing is not to question the computed value of the sample statistic but to make a judgment about the difference between that sample statistic and a hypothesized value for population parameter.

The next step after stating the null and alternative hypotheses is to decide what criterion to be used for deciding whether to accept or reject the null hypothesis. If we assume the hypothesis is correct, then the significance level will indicate the percentage of sample statistic that is outside certain limits (in estimation, the confidence level indicates the percentage of sample statistic that falls within the defined confidence limits).

13.2.3. Hypotheses are Accepted and Not Proved

Even if our sample statistic does fall in the non - shaded region (the region shown in figure 12.1 that makes up 95 percent of the area under the curve), this does not prove that our null hypothesis (H_0) is true; it simply does not provide statistical evidence to reject it.

Therefore, whenever we say that we accept the null hypothesis, we actually mean that there is no sufficient statistical evidence to reject it. Use of the term accept, instead of do not reject, has become standard practice. It means that when sample data do not suggest us to reject a null hypothesis, we believe as if that hypothesis is true.

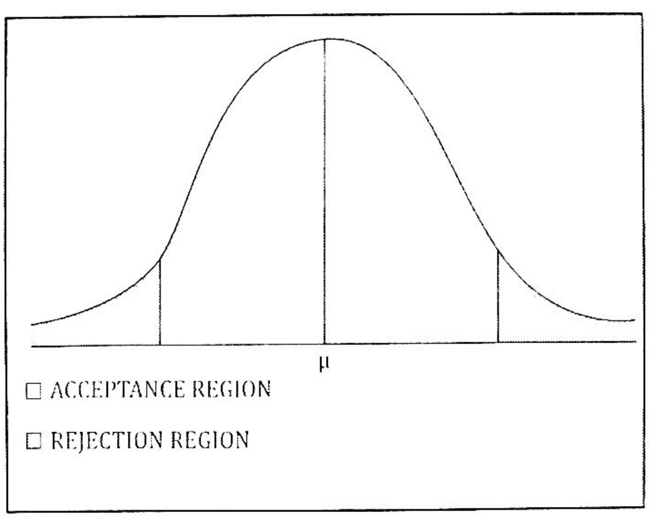

Fig 1: Acceptance and rejection region of sample

13.3. SELECTING A SIGNIFICANCE LEVEL

There is no single standard or universal level of significance for testing hypotheses. In some instances, a 5% and 1% level of significance is used which means that our decision is correct to the extent of 95% or 99%. Hence, it is possible to test a hypothesis at any level of significance. But remember that our choice of the minimum standard for an acceptable probability, or the significance level, is also the risk we assume of rejecting a null hypothesis when it is true.

The higher the significance level we use for testing a hypothesis, the higher the probability of rejecting a null hypothesis when it is true. The 5% level of significance implies we are ready to reject a true hypothesis in 5% of cases.

If the significance level is high then we would rarely accept the null hypothesis when it is not true but, at the same time, often reject it when it is true.

When testing a hypothesis we come across four possible situations.

Possible situations when testing a hypothesis

		Decision from Sample	
True State		Reject H_0	Accept H_0
	H_0 True	Wrong (Type–I Error)	Correct
	H_0 False (H_1 True)	Correct	Wrong (Type II Error)

The combinations are:

1. If the null hypothesis is true, and the test result make up to accept it, then we have made a right decision.
2. If null hypothesis is true, and the test result make us to reject it, then we have made a wrong decision (**Type I error**). It is also known as **Consumer's Risk, denoted by α.**
   α is also known as level of significance.
3. If hypothesis is false, and the test result make us to accept it, then we have made a wrong decision (**Type II error**). It is known as **producer's risk, denoted by β, where, 1 –β is called power of the Test.**
4. If hypothesis is false, test result make us to reject it – we have made a right decision.

13.4. ONE – TAILED TEST AND TWO TAILED TEST

There are two types of problems of tests of hypothesis

1. Two tailed Test
2. One tailed Test

One tailed test is again classified into two types

a) Right Tailed Test
b) Left Tailed Test

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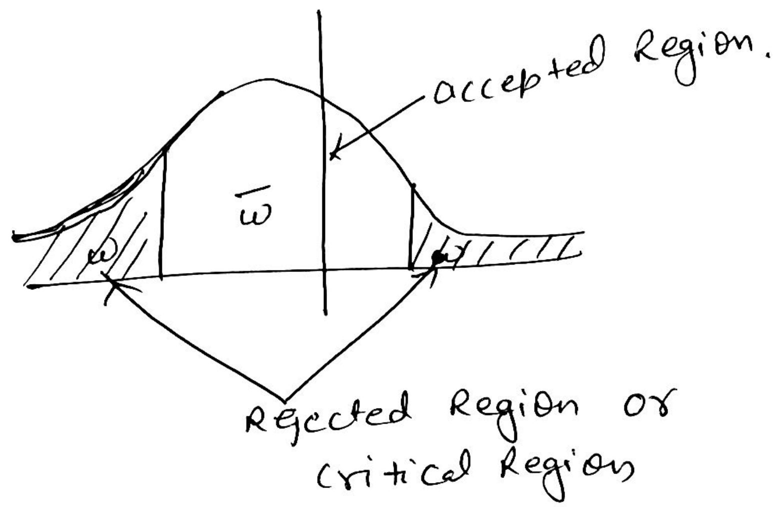
Critical Region: The value of the test Statistic on which the Null Hypothesis is Rejected is called critical Region. Usually it is denoted by 'ω'.

The Accepted Region is denoted by $\bar{\omega}$.



Type I and Type II Errors.

α = Probability of Type I Error

= Probability of rejecting H_0 when H_0 is true.

$\Rightarrow \boxed{P(x \in \omega \mid H_0) \equiv \alpha}$

where $x = x_1, x_2, \ldots x_n$.

β = Probability of type II Error

= Probability of accepting H_0 when H_0 is false.

$\Rightarrow \boxed{P(x \in \bar{\omega} \mid H_1) \equiv \beta}$

and we know that

$$\alpha + \beta = 1 . \Rightarrow \quad \alpha = 1 - \beta$$

$$\cancel{P(x \in \omega \mid H_0) + P(x \in \bar{\omega}_0 \mid H_1) = 1} .$$

$$\boxed{P(x \in \omega \mid H_0) = 1 - \beta .}$$

Q1 Given the frequency function: $f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{elsewhere} \end{cases}$

and that you are testing the null hypothesis $H_0: \theta = 1$ against $H_1: \theta = 2$, by means of a single observed value of 'x'. What would be the size of the type I and type II errors, if you chose the interval (i) $0.5 \le x$, (ii) $1 \le x \le 1.5$ as the critical region. Also obtain the power function of test.

Soln $H_0: \theta = 1$, $H_1: \theta = 2$

(i) $\omega = \{x : 0.5 \le x\} = \{x : x \ge 0.5\}$

$\bar{\omega} = \{x : x < 0.5\}$

$\alpha = P(x \in \omega \mid H_0)$

$= P(x \ge 0.5 \mid \theta = 1)$

$= P(0.5 \le x \le \theta \mid \theta = 1)$

$= P(0.5 \le x \le 1 \mid \theta = 1)$

$= \int_{.5}^{1} [f(x,\theta)]_{\theta=1} \, dx = \int_{.5}^{1} 1 \, dx = .5$

Similarly

$\beta = P(x \in \bar{\omega} \mid H_1)$

$= P(x \le 0.5 \mid \theta = 2)$

$= \int_{0}^{.5} [f(x,\theta)]_{\theta=2} \, dx = \int_{0}^{.5} \frac{1}{2} \, dx = .25$

(ii) $\omega = \{x : 1 \le x \le 1.5\}$

$$\alpha = P\{x \in \omega \mid \theta = 1\}$$

$$= \int_1^{1.5} [f(x,\theta)]_{\theta=1}\, dx. = 0.$$

Since under $H_0 : \theta = 1$, $f(x,\theta) = 0$, for $1 \le x \le 1.5$.

$$\beta = P\{x \in \bar{\omega} \mid \theta = 2\}$$

$$= 1 - P\{x \in \omega \mid \theta = 2\}.$$

$$= 1 - \int_1^{1.5} [f(x,\theta)]_{\theta=2}\, dx$$

$$= 1 - \left(\frac{x}{2}\right)_1^{1.5} = .75.$$

Power function $= 1 - \beta = 1 - .75 = \underline{\underline{.25}}.$

If $x \geq 1$ is the critical region for testing $H_0: \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population

$$f(x,\theta) = \theta \exp(-\theta x), \quad 0 \leq x \leq \infty$$

Find the value of type I and type II error.

Soln $\omega = \{x : x \geq 1\}$ and $\bar{\omega} = \{x : x < 1\}$.

$H_0 : \theta = 2$, $H_1 : \theta = 1$.

$$\alpha = P\{x \in \omega \mid H_0\}$$

$$= P\{x \geq 1 \mid \theta = 2\}.$$

$$= \int_1^{\infty} [f(x,\theta)]_{\theta=2}\, dx$$

$$= \int_1^{\infty} 2e^{-2x}\, dx$$

$$= 2\left(\frac{e^{-2x}}{-2}\right)_1^{\infty} = \frac{1}{e^2}$$

$$\beta = P\{x \in \bar{\omega} \mid H_1\}$$

$$= P\{x < 1 \mid \theta = 1\}$$

$$= \int_0^1 e^{-x}\, dx.$$

$$= \frac{e-1}{e}.$$

Q3 Let p is the probability that a coin will head in a single toss in order to test $H_0: p$ against $H_1: p = 3/4$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I and power of test

Sol $H_0: p = 1/2$, $H_1: p = \frac{3}{4}$, $\omega = \{x : x \geq 4\}$, $\bar{\omega}\{x : x \leq 3\}$

$$P(X=x) = {}^nC_x p^x (1-p)^{n-x}$$

$$= {}^5C_x p^x (1-p)^{5-x}.$$

$$\alpha = P\{X \geq 4 | H_0\}.$$

$$= P\{X=4 | p=1/2\} + P\{X=5 | p=1/2\}$$

$$= {}^5C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^{5-4} + {}^5C_5 \left(\tfrac{1}{2}\right)^5. \quad = \frac{3}{16}.$$

$$\beta = P(x \in \bar{\omega} | H_1)$$

$$= \cancel{P(x \leq 3 | 3/4)}$$

$$= 1 - P\left(X=4 | p=\tfrac{3}{4}\right) + P\left(X=5 | p=\tfrac{3}{4}\right)$$

$$= 1 - \left[{}^5C_4 \left(\tfrac{3}{4}\right)^4 \left(\tfrac{1}{4}\right) + {}^5C_5 \left(\tfrac{3}{4}\right)^5\right].$$

$$= 47/128.$$

Power of test $= 1 - \beta = 1 - \frac{47}{128}.$

$$= \frac{81}{128}.$$

13.4. ONE – TAILED TEST AND TWO TAILED TEST

There are two types of problems of tests of hypothesis

1. Two tailed Test
2. One tailed Test

One tailed test is again classified into two types

a) Right Tailed Test
b) Left Tailed Test

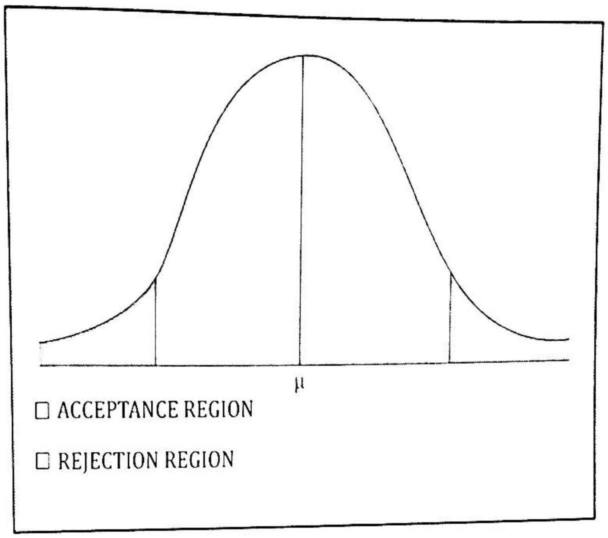
Two - Tailed Test: A two tailed test is the test of any statistical hypothesis where the Alternative hypothesis is written with the symbol "≠".

That is, a two - tailed test of a hypothesis will reject the null hypothesis if the sample mean is significantly higher than or lower than the hypothesized population mean. Thus, in a two - tailed test, rejection region is split in two parts under the distribution curve.

A two-tailed test is appropriate when:

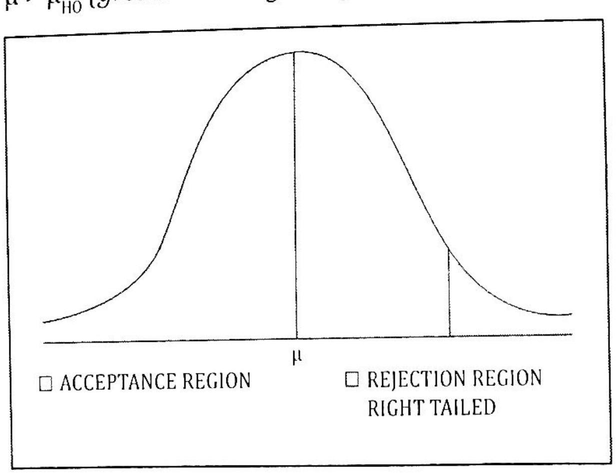
the null hypothesis is $\mu = \mu_{H0}$ (where $\mu = \mu_{H0}$ is some specified value)

the alternative hypothesis is $\mu \neq \mu_{H0.}$



One Tailed Test: When the hypothesis about the population mean is rejected only for the value of falling into one of the tails of the sampling distribution, then it is called One tailed test

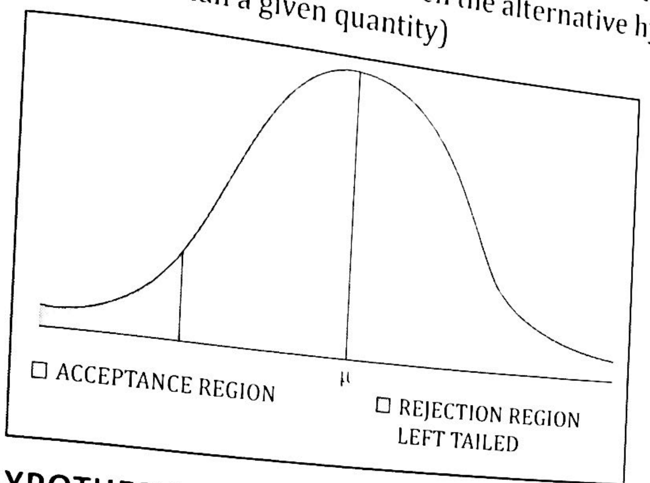
Right Tailed Test: A *Hypothesis Test where the rejection region is located to the extreme right of the distribution. A right - tailed test is conducted when the alternative hypothesis* (H_1) *contains the condition* H_1: $\mu > \mu_{H0}$ *(greater than a given quantity).*



Right-tailed Test

next

Left Tailed Test: A Hypothesis Test where the rejection region is located to the extreme left of the distribution. A left-tailed test is conducted when the alternative hypothesis (H_1) contains the condition H_1: $\mu < \mu_{H0}$ (less than a given quantity)



13.5. TESTS OF HYPOTHESIS CONCERNING LARGE SAMPLES

When the size of sample exceeds 30, it is called as large sample otherwise it is considered as small sample. Following are the assumptions for the tests of hypothesis for large samples:

(i) The sampling distribution of a sample statistics is approximately normal.

(ii) Values given by the samples are sufficiently close to the population value and can be used in its place for the standard error of the estimate.

13.5.1. Testing of Hypothesis About Population Mean

(i) We shall first take the hypothesis testing concerning the population parameter μ by considering the two – tailed test: $H_0: \mu = \mu_0$ (μ_0 *is the hypothesised value of* μ)
Since the best unbiased estimator of μ is the sample mean $\bar{x}$, $\bar{x} \sim N(\mu, \sigma_{\bar{x}})$,

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}, \text{where } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}}$$

If the calculated value of $z < -z_{\frac{\alpha}{2}}$ or $> z_{\frac{\alpha}{2}}$, the null hypothesis is rejected.

(ii) If the hypothesis involves a right – tailed test. For example,

$H_0: \mu \leq \mu_0$ *and* $H_1: \mu > \mu_0$

For the calculated value $z > z_\alpha$, the null hypothesis is rejected.

(iii) If the hypothesis involves a left – tailed test, i.e.,

$H_0: \mu \geq \mu_0$ *and* $H_1: \mu < \mu_0$

For the calculated value $z > -z_\alpha$, the null hypothesis is rejected.

Example: The mean life time of a sample of 100 electrical bulbs produced by a company is found to be 1,580 hours with standard deviation of 90 hours. Test the hypothesis that the mean life time of the bulbs produced by the company is 1,600 hrs.

Solution: The null hypothesis is that there is no significant difference between the sample mean and hypothetical poulation mean, i.e.

$H_0: \mu = \mu_0$ and $H_1: \mu \neq \mu_0$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}, \text{ where } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}}$$

$$z = \frac{1580 - 1600}{90/\sqrt{100}} = -2.22$$

13.5.2. Testing Hypothesis for the Difference Between Two Means

The test statistics for the difference between two normally distributed population mean is based on the general form of standard normal statistic as given below:

$$z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}}$$

Where $\theta = \mu_1 - \mu_2$. Since the best unbiased estimator of $\theta = \mu_1 - \mu_2$ is $\bar{x}_1 - \bar{x}_2$, therefore $\hat{\theta} = \bar{x}_1 - \bar{x}_2$.

The standard deviation $\sigma_{\hat{\theta}}$ of the sampling distribution of $(\bar{x}_1 - \bar{x}_2)$ is given by

$$\sigma^2_{\hat{\theta}} = \sigma^2_{\bar{x}_1 - \bar{x}_2} = Var(\bar{x}_1 - \bar{x}_2) = Var(\bar{x}_1) + Var(\bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

The test statistic z is given by $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

The null Hypothesis is $H_0: \mu_1 - \mu_2 = 0$

Hence, the z statistic $= \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

At 5% level of significance, the critical value of z for two tailed test is ±1.96. If the computed value of z is greater than 1.96 or less than -1.96, then reject H_0, otherwise accept H_0.

Note: If σ_1^2 and σ_2^2 are not known then for large samples then s_1^2 and s_2^2 can be used..

Example: Details of two companies are

	Company A	Company B
Mean life (in hours)	1,300	1,288
Standard Deviation (in hrs)	82	93
Sample size	100	100

Which brand of test tubes are better if the desired risk is 5%.

next