Computational Lab(P 342) Project

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1 Question 1: Random Walk

In this question, we were asked to perform random walks with varying number of steps, ranging from 250 steps to 3000 steps, with the step size to be a constant. For each number of steps, 100 walks were performed in Python code, and of them were plotted from each set of steps.

The 'randomness' in a two dimensional random walk has been reflected in the randomness of the angle chosen between 0 and 360 degrees, as the walker can go in any direction on the 2D plane that he wants. Thus, the x coordinates and y coordinates are expressed as

$$x = x_0 + \cos(\theta)$$

and

$$y = y_0 + \sin(\theta)$$

where θ is the random angle that has been generated by the code.

The radial distance and the average distance travelled in x and y direction was calculated for each random walk. RMS value of the radial distances was calculated for the 100 walks from each of the step sizes, and a plot of RMS distance vs $N^{1/2}$ was plotted. The plot appeared to be proportional to the value of $\sqrt{N}$ with some minor deviations. The deviations may be a result of the apparently 'small' number of steps taken, N=250,500,100,2000,3000 as Statistical analysis is based on the Law of Large numbers.

2 Question 2: Numerical Volume of an Ellipsoid using Monte Carlo method

In this question, we have been asked to numerically calculate the volume of an Ellipsoid i.e a 3D ellipse, with the axes length of 1 unit, 1.5 units and 2 unit. Monte Carlo method is based on generation of a huge number of random numbers in a certain range, that fill up the whole space that we want to find the volume of. Assuming the the number of Random numbers is large enough,

$$\frac{V_{ellipsoid}}{V_{Cuboid}} = \frac{N_e}{N_c}$$

where N_e is the number of points lying inside the ellipsoid and N_c is the number of points lying inside the cuboid.

As a,b and c are the semi axes of the ellipsoid, we have

$$V_{ellipsoid} = \frac{N_e}{N_c} \cdot 2a \cdot 2b \cdot 2c = \frac{N_e}{N_c} \cdot 8abc$$

Here, plot of Volume obtained for each value of N(no. of random points) ranging from N=500 to N=50000 was plotted, and it was observed that the graph obtained hovered about the analytical volume of the ellipsoid i,e 12.57 cubic units. As the value of N increased, the deviation tended to decrease.

A plot of fractional error in the volume foreach value of N was plotted against value of N. The graph was seen to hover around 0 error, and it became close to 0 as we increases the value of N.

Lastly, a 3 dimensional plot of the ellipsoid was generated in Python using the random numbers generated along z, y and z axis.