Response Letter

Dear Prof. Ouyang Qi:

Thank you very much for the referee report on our manuscript (Ref. No.:

#CHA23-AR-00405) entitled "Epidemic spreading on multi-layer networks with

active nodes". We are encouraged by the positive comments given by referees. We

have revised the manuscript to fully address all referee comments. Please find

enclosed a detailed, point-to-point response to all referee comments with all changes

clearly specified. All changes are italicized and bolded in the manuscript.

We wish to take this opportunity to thank the referees for their time and

insightful comments that have resulted in an improved manuscript. We would also

like to thank you for handling our manuscript. We hope our revised manuscript can be

judged to have met the high standards of Chaos.

With kind regards,

Yachun Gao

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(On behalf of all co-authors)

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Point-to-point response to referee comments

To Reviewer #1

Reviewer statements: The study proposes a new model to learn about the transmission dynamic of infectious diseases and it's very interesting. In this work, a SEIS transmission model with active nodes is constructed based on a multi-layer network, in order to deeply discover the impacts of the latency time as well as the mobile crowds. And while the introduction of the active nodes can lead to the multi-space propagation of the disease, it isn't the determinant factor of widespread outbreak. Besides, the latency time can contribute to the short-period cumulation of infected individuals. These results are quite attractive, and they're also perspective for relative policy formulation. I believe this topic is of great interest to our readers, but I also think that some improvements are needed before it is ready for publication. And my suggestions are as follows:

Response: Thank you for your positive evaluation and comments.

Comment 1. In this manuscript, the practical significances of the model should be explained more detail.

Response 1: Thank you for your valuable suggestion, helping us present our work in a clearer way. In the revised manuscript, we have improved the practical significances showed in the first paragraph of Section IV, page 10. We synthesize the practical implications of our findings based on the cumulative effect of the latent time and the phase transition at the zero point of the relative proportion of active nodes(The conclusions of Section III), and provide corresponding suggestions for policy formulation in infectious disease prevention and control. The changes are shown in page 10.

Comment 2. The literature research section could be a little more in-depth, while latest literature could be cited.

Response 2: Thank you for your comment and it's quite helpful. In order to make the

literature research more in-depth, we have added several references, corresponding to Refs. 13-15, 16, 19, and 20 in the references. The literature research section has also been optimized based on these references and will be discussed in Response 3 in detail.

Comment 3. The introduction part of this manuscript is not so impressive. Improve it with concise and clear descriptions.

Response 3: Thank you for your comment, in order to improve this, we have not only revised the introduction, and accordingly, appropriate adjustments have been made to the abstract in the hope of leaving a deeper impression on the reader.

- (1) In the Introduction (Section I), we have expanded the original third paragraph into two paragraphs based on the two categories of biological and social influences on disease transmission. At the end of the third paragraph of Section 1 in the revised manuscript, an elaboration of the SEIR model has been added to complement the relevant literature; and before the fourth paragraph of Section 1, a more detailed description of how social factors influence the disease transmission process has been added. Besides, some wordings in the original text has been also revised or deleted to make the narrative more coherent. The final version has been bolded and italicized in the manuscript, shown in the left column of Page 2.
- (2) We also modify the Abstract (in page 1) according to the changes made in the introduction section. We first concisely and directly point out two important influences in the disease transmission process that are of interest to this paper: the latent time and the mobile crowds, and then further introduce the model and show the conclusions.

Comment 4. It would be better to discuss the results in the context of the COVID-19. **Response 4**: Thank you for your comment. The applications of our model is indeed reflected in the context of the COVID-19, and for this reason, we have added some application aspects to each part of Section III (Results and Discussions) to deeply discuss the results, which is shown in the revised manuscript as follows:

- (1) At the end of the last paragraph of Part 1. *The Relative Proportion of Active Nodes*, we have modified the original description by adding recommendations for epidemic prevention and control policies when epidemic transmission enters a steady state, and by simply describing and evaluating the policies under the current development of COVID-19, shown in page 7.
- (2) At the end of the penultimate paragraph of Part 2. The Combined Effects of The Expose Rate and Recovery Rate, we have followed up on what was said above and have discussed measures that can be taken at the societal level and at individual level to prevent epidemics with a long latent period like COVID-19 becoming normalized (i.e., reach a steady state), shown in page 8.
- (3) At the end of Part 3. *The Effects of the Transmission Rate*, we have added a policy development recommendation based on the results, namely, that priority could be given to regulating Expose Rate and Recovery Rate for epidemics with long latent time and the ability to reinfect individuals, such as COVID-19. This is to highlight the research implications of the results, shown in the first paragraph in the right column, page 9.

Comment 5. The code should be shared with the reviewer and/or editor for verification.

Response 5: Thank you for your comment. Here we provide the code shown in https://github.com/TigerWoodss/codes_seis/blob/main/seis.md, which is the main Monte Carlo simulation for our model.

Comment 6. The descriptions of each picture should be more detailed, so that the readers can understand more clearly. For example, FIG. 1 just simply describes the meaning of different color nodes in the diagram, but not the dynamic process of the model, which is not conducive to the reader to control the model according to the picture.

Response 6: Thank you for your comment. We have added a description of how the active nodes jump randomly between layers of FIG. 1 (in page 3). And we have added

a description of the three states in the phase diagram to FIG. 5 (in page 8) and FIG. 7 (in page 9). Both of these changes are shown in italics and bold in the corresponding figure annotations in the revised manuscript.

Comment 7. At the theoretical level, due to the large number of formulas in this manuscript, the descriptions of each formula should also be a little more organized and more detailed, such as Eq. (4a) (4b) and Eq(6a)-(6c), so that the reader can understand the meaning of the formulas more quickly.

Response 7: Thank you for your valuable suggestion. For Eq.(4a)(4b), we have added an explanation of the meaning of the second term on the right side of the equations, which are $(1 - \beta^E)^{g_i^E(t)}$ and $(1 - \beta)^{g_i^I(t)}$, shown in the first graph of Section II. *THE MODEL*, Subsection B. *The Horizontal Disease Transmission*, Part 1. *Stochastic Representation*, in page 4, just above Eq.(4a)(4b). And for Eq.(6a)-(6c), we have explained in more detail the meaning of each term in the equations, also shown in Section II. *THE MODEL*, Subsection B. *The Horizontal Disease Transmission*, Part 1. *Stochastic Representation*, page 4, just below Eq.(6a)-(6c).

To Reviewer #2

Reviewer statements: The authors simulate progression of an epidemic with multi-layered networks. Building on a susceptible-Infected-Susceptible (SIS) model, the authors have included Exposed (E) state of nodes to simulate incubation time. To simulate mobility, the authors have incorporated active nodes which perform random walk within a layer and between layers. Their study concludes that it is not advisable to institute a lockdown because it can impact people's lives. The authors have done a good job in underlaying the theoretical basis of their proposed model. However, improving the language of the paper can make it more comprehensible.

Response: Thank you for your positive evaluation and comments. According to your suggestions, we have read the manuscript for several times and have polished the language overall.

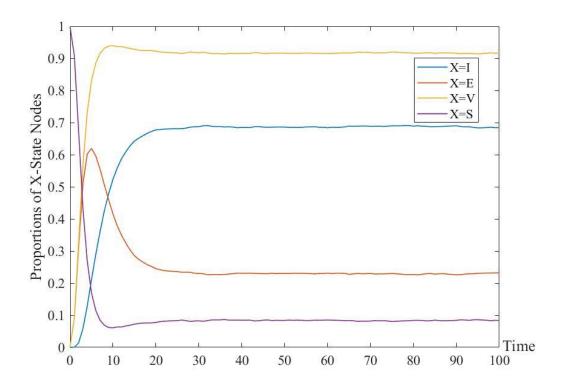
Major comments:

Comment 1. Is there any way the mobility (active nodes) can be decided based on real world data? If possible, it would help to see some comparison with real data.

Response 1: Thank you for your valuable suggestion, helping us present our work in a clearer way. Deciding the mobility from real data can further demonstrate the relevance of our model, and the movement of individuals across geographic spaces is an extremely complex matter. Through extensive literature research, we finally found a way to obtain the proportion of mobile crowds that is applicable to our model in the article named *Population flow drives spatio-temporal distribution of COVID-19 in China* published in *Nature* (see Ref. 36 in the manuscript), which is the mobile phone geolocation data. The data can reliably quantify human movement-provide precise, verifiable and real-time information. And in this way, the paper counts the population outflow from Wuhan (a city in China) during the 24 days prior to the COVID-19 outbreak, from which we obtain an average daily population outflow from Wuhan during that period of 4.3%. This data is higher than usual because it's during the Chinese Spring Festival. And we've added this description to the first paragraph of Section III. *RESULTS AND DISCUSSIONS*, Part 1. *The Relative Proportion of Active Nodes*, shown in page 6.

Comment 2. One example is to select the value of δ a from literature to make the model capture a more realistic outbreak scenario.

Response 2: Thank you for your valuable suggestion. Here we use the data 4.3% gained in Response 1 with other parameters unchanged to simulate a more realistic scenario and the results are as follows:



As we can see from the figure, due to the "cumulative effect" of the latent time, the E-State curve still shows a "peak" while the V-State curve shows a "shoulder". Also we can see that under this scale of ground simulation experiments, the outbreak is very rapid and the final scale of infection is large and goes into the Full-Domain Outbreak State. It indicates that the risk of disease transmission is also high at times like holidays when the rate of migration of individuals is high, so rapid measures are needed to interrupt spreading before the peak is brought out.

And according to this result, we have given our simulations for different δ some practical meanings in the manuscript. We have used $\delta=0$ to simulate the lockdown policy, $\delta=0.0001$ to simulate the light-lockdown policy(i.e, very few individuals can move), $\delta=0.001$ to simulate the normal-state life of individuals, $\delta=0.01$ to simulate the holidays, and $\delta=0.1$ to simulate the extreme cases. All of these changes are shown in the first paragraph of Section III. *RESULTS AND DISCUSSIONS*, Part 1. *The Relative Proportion of Active Nodes*, in page 6. And we added some descriptions of the result in the third paragraph of the revised manuscript, which will be discussed in detail in the next Response.

Comment 3. For the case with $\delta = 0$, there is barely any peak in infections. Does that mean in order to have a full blown outbreak there needs to be at least 1 active node? This aspect needs explanation.

Response 3: Yes. Thank you for your valuable suggestion. As we have given practical meanings for different δ in Response 2, therefore, we have added the descriptions of the curves of the proportions of nodes in each state and have explained the reasons why there isn't a full outbreak when $\delta = 0$: the active nodes cannot bring the virus from their own layer to another layer. The specific additions are shown in the third paragraph of Section III. *RESULTS AND DISCUSSIONS*, Part 1. *The Relative Proportion of Active Nodes*, in page 6 and page 7.

Other minor comments:

Comment 4. In section 2, subsection A, do the authors mean $W_{i-Act}(t+1) = W_{i-Act}(t)$

Response 4: Thank you for your comment. And this formula is true in the steady state because all physical quantities do not vary with time in the steady state, so we have $W_{i-Act}(t+1) = W_{i-Act}(t)$.

Comment 5. In section 2, subsection B, the authors may want to rephrase ".... the asymptomatic infected individuals who are suffering from the latent time of a disease..."

Response 5: Thank you for your comment. And we have changed the description into "The state E (Exposed), is to simulate the asymptomatic infected individuals who can infect others."

Comment 6. It would be nice to have a table with the descriptions all the symbols in one place, as such it becomes hard to keep track of all the symbols.

Response 6: Thank you for your comment. And we have added the Table of Main Symbol in Appendix A, in page 12.

Comment 7. The font on some of the plots are too small to be legible.

Response 7: Thank you for your comment. We have increased the font size of all plots.