Coordinate Geometry

10^{th} Maths - Chapter 7

This is Problem-4 from Exercise 7.3

1. Find the area of quadrilateral whose vertices, taken in order, are $\begin{pmatrix}-4\\-2\end{pmatrix}, \begin{pmatrix}-3\\-5\end{pmatrix}, \begin{pmatrix}3\\-2\end{pmatrix}$ and $\begin{pmatrix}2\\3\end{pmatrix}$.
   Solution:
   The input parameters for this problem are available in Table (1) By

Symbol	**Value**	**Description**
$\mathbf{A}$	$\begin{pmatrix}-4\\-2\end{pmatrix}$	First point
$\mathbf{B}$	$\begin{pmatrix}-3\\-5\end{pmatrix}$	Second point
$\mathbf{C}$	$\begin{pmatrix}3\\-2\end{pmatrix}$	Third point
$\mathbf{D}$	$\begin{pmatrix}2\\3\end{pmatrix}$	Fourth point

Table 1

joining $\mathbf{B}$ to $\mathbf{D}$, you will get two triangles $\mathbf{ABD}$ and $\mathbf{BCD}$.
In general, the area of $\triangle\mathbf{ABD}$ can be expressed as

$$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{D})\right\| \tag{1}$$

$$\mathbf{A}-\mathbf{B}=\begin{pmatrix}-4\\-2\end{pmatrix}-\begin{pmatrix}-3\\-5\end{pmatrix}=\begin{pmatrix}-1\\3\end{pmatrix} \tag{2}$$

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix}-4\\-2\end{pmatrix}-\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-6\\-5\end{pmatrix} \tag{3}$$

Substituting the values of (2) and (3) in (1),

$$\frac{1}{2}\begin{vmatrix}-1 & 3\\-6 & -5\end{vmatrix}=\frac{23}{2} \tag{4}$$

Also, the area of $\triangle\mathbf{BCD}$ can be expressed as

$$\frac{1}{2}\left\|(\mathbf{B}-\mathbf{C})\times(\mathbf{B}-\mathbf{D})\right\| \tag{5}$$

$$\mathbf{B}-\mathbf{C}=\begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}-6\\-5\end{pmatrix} \tag{6}$$

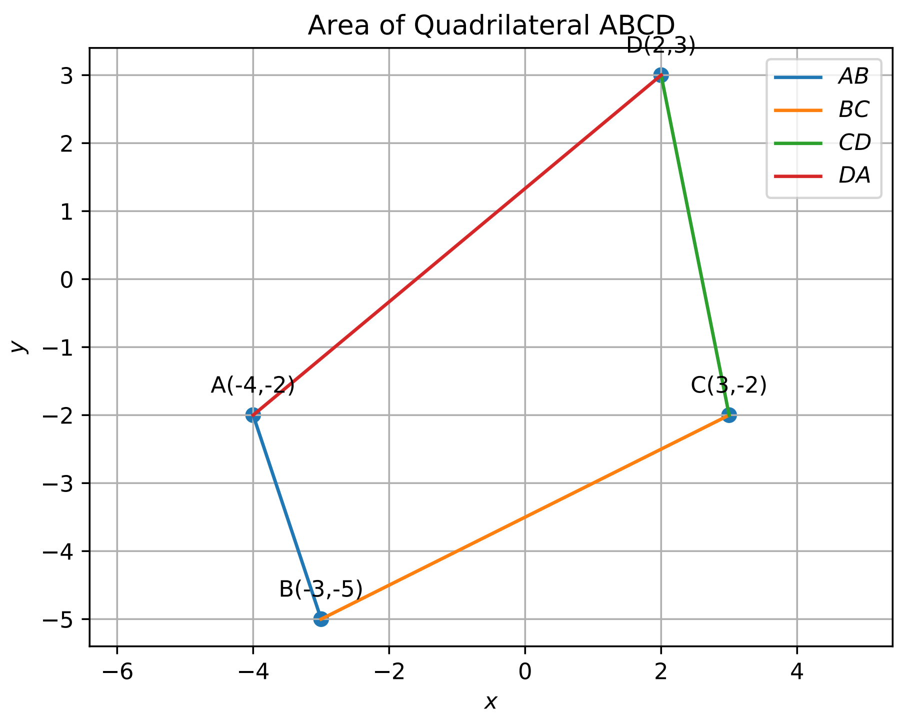
$$\mathbf{B}-\mathbf{D}=\begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-3\\-8\end{pmatrix} \tag{7}$$

Substituting the values of (6) and (7) in (5),

$$\frac{1}{2}\begin{vmatrix}-6 & -3\\-5 & -8\end{vmatrix}=\frac{33}{2} \tag{8}$$

Area of Quadrilateral $\mathbf{ABCD}$ = area of $\triangle\mathbf{ABD}$+ area of $\triangle\mathbf{BCD}$,

$$\frac{23}{2}+\frac{33}{2}=28 \tag{9}$$



Area of Quadrilateral ABCD
D(2,3)
A(-4,-2)
C(3,-2)
B(-3,-5)
AB
BC
CD
DA
x
y
−6
−4
−2
0
2
4
3
2
1
0
−1
−2
−3
−4
−5

Figure 1