Coordinate Geometry

10^{th} Maths - Chapter 7

This is Problem-7 from Exercise 7.1

1. The point on the x-axis which is equidistant from $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

Solution:
The input parameters for this problem are available in Table 1

Symbol	Value	Description
A	$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$	First point
B	$\begin{pmatrix} -2 \\ 9 \end{pmatrix}$	Second point
O	?	Desired point

Table 1

If **O** lies on the x-axis and is equidistant from the points **A** and **B**,

$$\|\mathbf{O}-\mathbf{A}\| = \|\mathbf{A}-\mathbf{B}\| \tag{1}$$

$$\implies \|\mathbf{O}-\mathbf{A}\|^2 = \|\mathbf{O}-\mathbf{B}\|^2 \tag{2}$$

which can be expressed as

$$(\mathbf{O}-\mathbf{A})^\top (\mathbf{O}-\mathbf{A}) = (\mathbf{O}-\mathbf{B})^\top (\mathbf{O}-\mathbf{B})$$

$$\implies \|\mathbf{O}\|^2 - 2\mathbf{O}^\top\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{O}\|^2 - 2\mathbf{O}^\top\mathbf{B} + \|\mathbf{B}\|^2 \tag{3}$$

which can be simplified to obtain

$$\mathbf{O} = o\mathbf{e}_1 \tag{4}$$

where

$$o = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2\left(\mathbf{A}-\mathbf{B}\right)^\top \mathbf{e}_1} \tag{5}$$

now substituting the A and B values in eq.5

$$\left(\mathbf{A}-\mathbf{B}\right)^\top = \left(\begin{pmatrix}2\\-5\end{pmatrix} - \begin{pmatrix}-2\\9\end{pmatrix}\right)^\top = \begin{pmatrix}4 & -14\end{pmatrix} \tag{6}$$

$$\|\mathbf{A}\|^2 = 21 \tag{7}$$

$$\|\mathbf{B}\|^2 = 85 \tag{8}$$

upon substituting the values in eq 5. the value of o $= -7$
Hence, the desired point is O is $\begin{pmatrix}-7\\0\end{pmatrix}$.

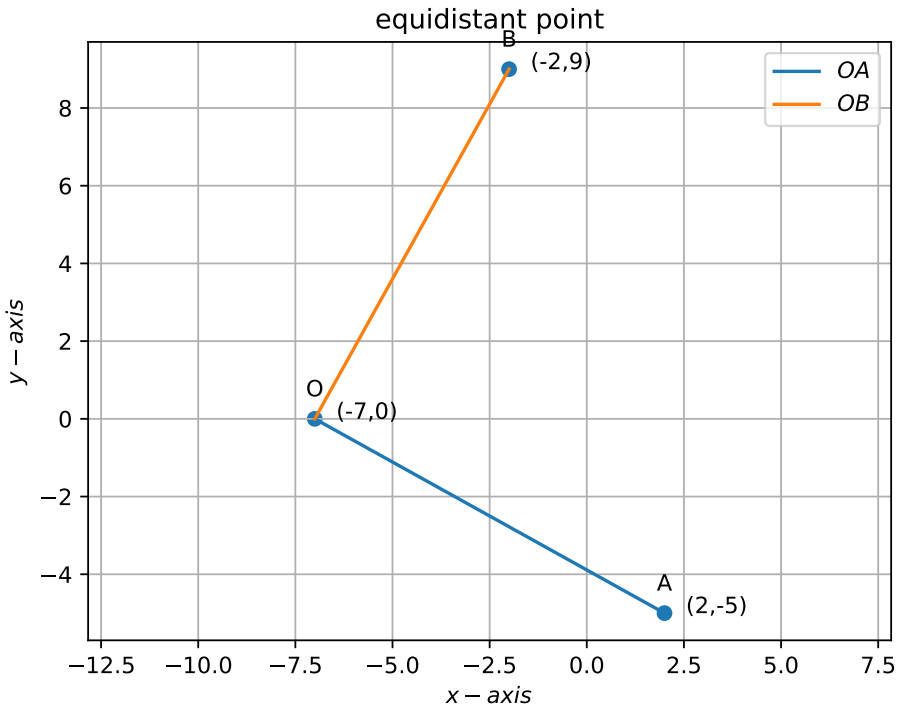
equidistant point
B
(-2,9)
OA
OB
O
(-7,0)
A
(2,-5)
8
6
4
2
0
−2
−4
−12.5
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
y − axis
x − axis

Figure 1