

Coordinate Geometry

10th Maths - Chapter 7

This is Problem-6 from Exercise 7.4

1. The vertices of $\triangle ABC$ are $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$. A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the $\triangle ADE$ and compare it with the area of $\triangle ABC$.

Solution: The input parameters for this problem are available in Table (1)

Symbol	Value	Description
A	$\begin{pmatrix} 4 \\ 6 \end{pmatrix}$	First point
B	$\begin{pmatrix} 1 \\ 5 \end{pmatrix}$	Second point
C	$\begin{pmatrix} 7 \\ 2 \end{pmatrix}$	Third point
D	?	Desired point
E	?	Desired point

Table 1

Given,

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4} \quad (1)$$

From (1),

$$\frac{AD}{AB} = \frac{1}{4} \quad (2)$$

$$\frac{AD}{BD} = \frac{1}{3} \quad (3)$$

Point **D** divides **AB** in the ratio of $n = 1 : 3$. Using Section formula,

$$\mathbf{D} = \frac{\mathbf{A} + n\mathbf{B}}{1 + n} \quad (4)$$

Substituting the values **A**, **B** and n in (4),

$$\mathbf{D} = \frac{\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 \\ 5 \end{pmatrix}}{1 + \frac{1}{3}} \quad (5)$$

$$= \frac{1}{1 + \frac{1}{3}} \left(\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right) \quad (6)$$

$$= \frac{3}{4} \begin{pmatrix} 4 + \frac{1}{3} \\ 6 + \frac{5}{3} \end{pmatrix} \quad (7)$$

$$= \begin{pmatrix} \frac{13}{4} \\ \frac{23}{4} \end{pmatrix} \quad (8)$$

From (1),

$$\frac{AE}{AC} = \frac{1}{4} \quad (9)$$

$$\frac{AE}{CE} = \frac{1}{3} \quad (10)$$

Point **E** divides **AC** in the ratio of $n = \frac{1}{3}$. Using Section formula,

$$\mathbf{E} = \frac{\mathbf{A} + n\mathbf{C}}{1 + n} \quad (11)$$

Substituting the values \mathbf{A}, \mathbf{C} and n in (11),

$$\mathbf{E} = \frac{\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 7 \\ 2 \end{pmatrix}}{1 + \frac{1}{3}} \quad (12)$$

$$= \frac{1}{1 + \frac{1}{3}} \left(\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 7 \\ 2 \end{pmatrix} \right) \quad (13)$$

$$= \frac{3}{4} \begin{pmatrix} 4 + \frac{7}{3} \\ 6 + \frac{2}{3} \end{pmatrix} \quad (14)$$

$$= \begin{pmatrix} \frac{19}{4} \\ \frac{20}{4} \end{pmatrix} \quad (15)$$

The $\text{ar}(\text{ADE})$ can be expressed as

$$\text{ar}(ABD) = \frac{1}{2} \|(\mathbf{A} - \mathbf{D}) \times (\mathbf{A} - \mathbf{E})\| \quad (16)$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} \frac{13}{4} \\ \frac{23}{4} \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \end{pmatrix} \quad (17)$$

$$\mathbf{A} - \mathbf{E} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} \frac{19}{4} \\ \frac{20}{4} \end{pmatrix} = \begin{pmatrix} \frac{-3}{4} \\ 1 \end{pmatrix} \quad (18)$$

Substituting the values of (17) and (18) in (16),

$$\text{ar}(ADE) = \frac{1}{2} \begin{vmatrix} \frac{3}{4} & \frac{-3}{4} \\ \frac{1}{4} & 1 \end{vmatrix} = \frac{15}{32} \quad (19)$$

The $\text{ar}(\text{ABC})$ can be expressed as

$$\text{ar}(ABC) = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{B} - \mathbf{C})\| \quad (20)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (21)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} \quad (22)$$

Substituting the values of (21) and (22) in (20),

$$\text{ar}(ABC) = \frac{1}{2} \begin{vmatrix} 3 & -6 \\ 1 & 3 \end{vmatrix} = \frac{15}{2} \quad (23)$$

Comparing $\text{ar}(\text{ADE})$ with $\text{ar}(\text{ABC})$,

$$\frac{\text{ar}(\text{ADE})}{\text{ar}(\text{ABC})} = \frac{\frac{15}{32}}{\frac{15}{2}} \quad (24)$$

$$= \frac{15}{32} \times \frac{2}{15} = \frac{1}{16} \quad (25)$$

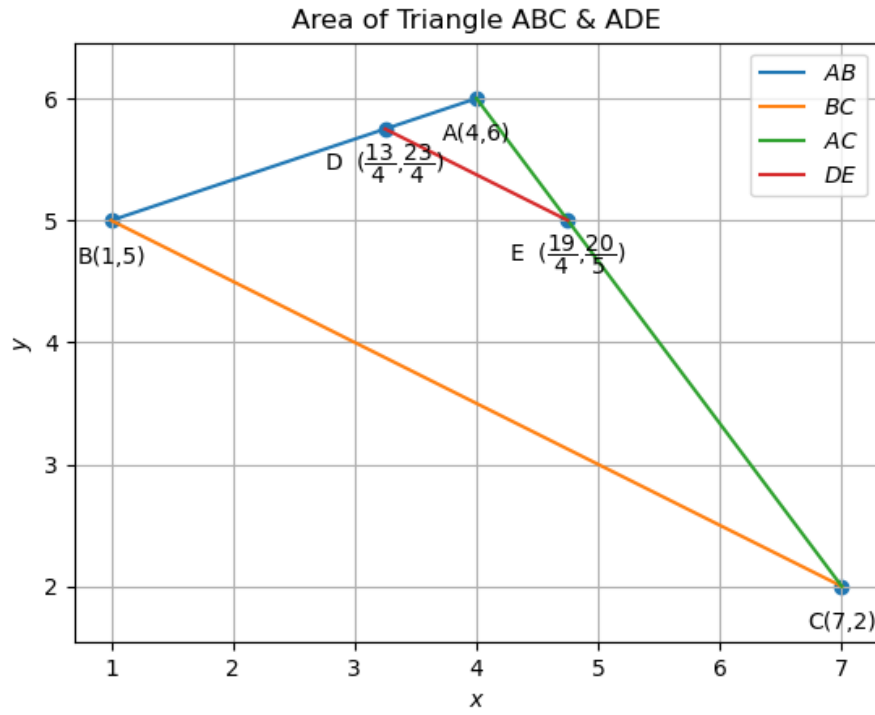


Figure 1