

Vector Algebra

12th Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. If unit vector \vec{a} makes angles $\frac{\pi}{3}$ with \hat{i} , $\frac{\pi}{4}$ with \hat{j} and an acute angle θ with \hat{k} , then find θ and hence, the components of \vec{a} .

Solution: Let \mathbf{A} be the given vector, in terms of their direction cosines

$$\mathbf{A} = \begin{pmatrix} \cos \theta_1 \\ \cos \theta_2 \\ \cos \theta_3 \end{pmatrix} \quad (1)$$

then,

$$\cos \theta_1 = \cos \frac{\pi}{3} \quad (2)$$

$$\Rightarrow \frac{1}{2} \quad (3)$$

$$\cos \theta_2 = \cos \frac{\pi}{4} \quad (4)$$

$$\Rightarrow \frac{1}{\sqrt{2}} \quad (5)$$

As \mathbf{A} is unit vector then

$$\|\mathbf{A}\| = 1 \quad (6)$$

$$\sqrt{\cos^2 \theta_1 + \cos^2 \theta_2 + \cos^2 \theta_3} = 1 \quad (7)$$

$$\sqrt{\frac{1}{2} + \frac{1}{2} + \cos^2 \theta_3} = 1 \quad (8)$$

$$\cos \theta_3 = \pm \frac{1}{2} \quad (9)$$

As θ_3 is an acute angle

$$\theta_3 = 60^\circ, \cos \theta_3 = \frac{1}{2} \quad (10)$$

$$\text{Hence } \mathbf{A} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \quad (11)$$