Straight Lines

12th Maths - Chapter 10

This is Problem-12 from Exercise 10.2

1. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

   Solution: The direction cosines are the cosines of the angles formed by the given vector with the respective axes, let **A** be the given vector

$$A = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1}$$

   The magnitude of the given vector is given by,

$$\|\mathbf{A}\| = \sqrt{1^2 + 2^2 + 3^2} \tag{2}$$
$$\|\mathbf{A}\| = \sqrt{14} \tag{3}$$

   The direction cosines of **A** can be expressed as

$$a = \frac{(i.A)}{\|\mathbf{A}\|} \tag{4}$$
$$b = \frac{(j.A)}{\|\mathbf{A}\|} \tag{5}$$
$$c = \frac{(k.A)}{\|\mathbf{A}\|} \tag{6}$$

   The dot product of the unit vectors in the direction of the x, y and z axes with vector are expressed as

$$i.(\hat{i} + 2\hat{j} + 3\hat{k}) = 1 \tag{7}$$
$$j.(\hat{i} + 2\hat{j} + 3\hat{k}) = 2 \tag{8}$$
$$k.(\hat{i} + 2\hat{j} + 3\hat{k}) = 3 \tag{9}$$

So the direction cosines of vector $\hat{i} + 2\hat{j} + 3\hat{k}$ are

$$a = \frac{(i.A)}{\|\mathbf{A}\|} = \frac{1}{\sqrt{14}} \tag{10}$$

$$b = \frac{(j.A)}{\|\mathbf{A}\|} = \frac{2}{\sqrt{14}} \tag{11}$$

$$c = \frac{(k.A)}{\|\mathbf{A}\|} = \frac{3}{\sqrt{14}} \tag{12}$$