

Straight Lines

12th Maths - Chapter 10

This is Problem-12 from Exercise 10.2

1. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

Solution: The direction cosines are the cosines of the angles formed by the given vector with the respective axes, let \mathbf{A} be the given vector

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad (1)$$

The Directional vectors of x, y and z axes are given respectively

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2)$$

The magnitudes for \mathbf{A} and directional vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are

$$\|\mathbf{A}\| = \sqrt{14}, \|\mathbf{e}_1\| = \sqrt{1}, \|\mathbf{e}_2\| = \sqrt{1}, \|\mathbf{e}_3\| = \sqrt{1} \quad (3)$$

The Direction cosines can be expressed as

$$\cos \theta_i = \frac{\mathbf{A}^\top \mathbf{e}_i}{\|\mathbf{A}\| \|\mathbf{e}_i\|} \quad (4)$$

$$\text{where } i = 1, 2, 3 \quad (5)$$

So for different values of i the direction cosines of vector \mathbf{A} are

$$\cos \theta_1 = \frac{(1 \ 2 \ 3) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\sqrt{14}} = \frac{1}{\sqrt{14}} \quad (6)$$

$$\cos \theta_2 = \frac{(1 \ 2 \ 3) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{14}} = \frac{2}{\sqrt{14}} \quad (7)$$

$$\cos \theta_3 = \frac{(1 \ 2 \ 3) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{14}} = \frac{3}{\sqrt{14}} \quad (8)$$