

# Vector Algebra

## 12<sup>th</sup> Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. If unit vector  $\vec{a}$  makes angles  $\frac{\pi}{3}$  with  $\hat{i}$ ,  $\frac{\pi}{4}$  with  $\hat{j}$  and an acute angle  $\theta$  with  $\hat{k}$ , then find  $\theta$  and hence, the components of  $\vec{a}$ .

**Solution:** Let  $\mathbf{A}$  be the given vector, in terms of their direction cosines

$$\mathbf{A} = \begin{pmatrix} \cos \theta_1 \\ \cos \theta_2 \\ \cos \theta_3 \end{pmatrix} \quad (1)$$

then,

$$\cos \theta_1 = \cos \frac{\pi}{3} \quad (2)$$

$$= \frac{1}{2} \quad (3)$$

$$\cos \theta_2 = \cos \frac{\pi}{4} \quad (4)$$

$$= \frac{1}{\sqrt{2}} \quad (5)$$

As  $\mathbf{A}$  is unit vector then

$$\|\mathbf{A}\| = 1 \quad (6)$$

$$\sqrt{\cos^2 \theta_1 + \cos^2 \theta_2 + \cos^2 \theta_3} = 1 \quad (7)$$

$$\sqrt{\frac{1}{2} + \frac{1}{2} + \cos^2 \theta_3} = 1 \quad (8)$$

$$\cos \theta_3 = \pm \frac{1}{2} \quad (9)$$

As  $\theta_3$  is an acute angle

$$\theta_3 = 60^\circ, \cos \theta_3 = \frac{1}{2} \quad (10)$$

$$\text{Hence } \mathbf{A} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \quad (11)$$