Circles

10^{th} Maths - Chapter 10

This is Problem-3 from Exercise 1

1. A tangent **PQ** at a point **P** of a circle of radius 5 cm meets a line through the centre **O** at a point **Q** so that **OQ** = 12 cm. Length **PQ** is

   Solution:
   The input parameters for this problem are available in Table (1)

Symbol	**Value**	**Description**
r	5	Radius of the Circle
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Centre of the circle
OP	5	Length of OP
OQ	12	Lenght of OQ
PQ	?	Length of PQ
θ	$\cos\theta = \frac{5}{12}$	Angle

Table 1

Let,

$$\mathbf{O} - \mathbf{P} = p \tag{1}$$
$$\mathbf{O} - \mathbf{Q} = q \tag{2}$$
$$\mathbf{P} - \mathbf{Q} = o \tag{3}$$

by triangle law of vector addition

$$p = q + o \tag{4}$$
$$o = p - a \tag{5}$$

Now magnitude of o is given by

$$\|o\|^2 = \|p - q\|^2 \tag{6}$$
$$= \|p\|^2 + \|q\|^2 - 2p.a^\top \tag{7}$$
$$= 25 + 144 - 2(|p|\,|a|)(\cos\theta) \tag{8}$$

Since PQ is a tanget then

$$OP \perp PQ \tag{9}$$

Then $\triangle OPQ$ is a right angle triangle, From $\triangle OPQ$,

$$\cos\theta = \frac{5}{12} \tag{10}$$

Then substituting (10) in (8) yeilds

$$\|o\|^2 = 169 - 2(12)(5)\left(\frac{5}{12}\right) \tag{11}$$
$$\|o\| = \sqrt{119} \tag{12}$$

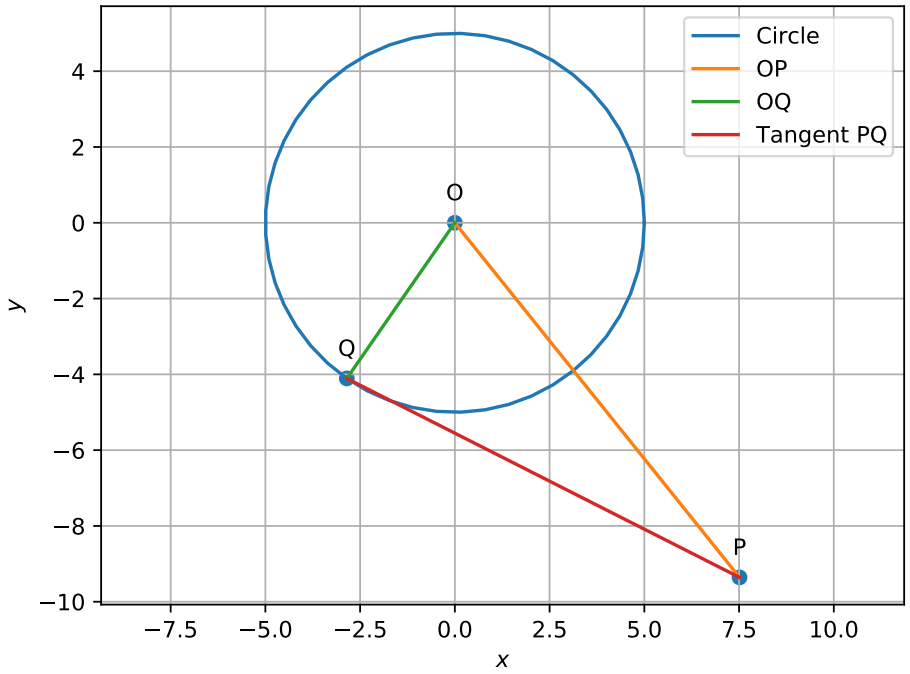
Circle
OP
OQ
Tangent PQ
O
Q
P
4
2
0
−2
−4
−6
−8
−10
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x
y

Figure 1