

Coordinate Geometry

10th Maths - Chapter 7

This is Problem-4 from Exercise 7.3

1. Find the area of quadrilateral whose vertices, taken in order, are $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Solution: The input parameters for this problem are available in Table (1)

Symbol	Value	Description
A	$\begin{pmatrix} -4 \\ -2 \end{pmatrix}$	First point
B	$\begin{pmatrix} -3 \\ -5 \end{pmatrix}$	Second point
C	$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$	Third point
D	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	Fourth point

Table 1

By joining **B** to **D**, you will get two triangles **ABD** and **BCD**.
In general, the ar(ABD) can be expressed as

$$\frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{D})\| \quad (1)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (2)$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \end{pmatrix} \quad (3)$$

Substituting the values of (2) and (3) in (1),

$$\frac{1}{2} \begin{vmatrix} -1 & 3 \\ -6 & -5 \end{vmatrix} = \frac{23}{2} \quad (4)$$

Also, the ar(BCD) can be expressed as

$$\frac{1}{2} \|(\mathbf{B} - \mathbf{C}) \times (\mathbf{B} - \mathbf{D})\| \quad (5)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \end{pmatrix} \quad (6)$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} \quad (7)$$

Substituting the values of (6) and (7) in (5),

$$\frac{1}{2} \begin{vmatrix} -6 & -3 \\ -5 & -8 \end{vmatrix} = \frac{33}{2} \quad (8)$$

Area of Quadrilateral **ABCD** = ar(ABD) + ar(BCD),

$$\frac{23}{2} + \frac{33}{2} = 28 \quad (9)$$

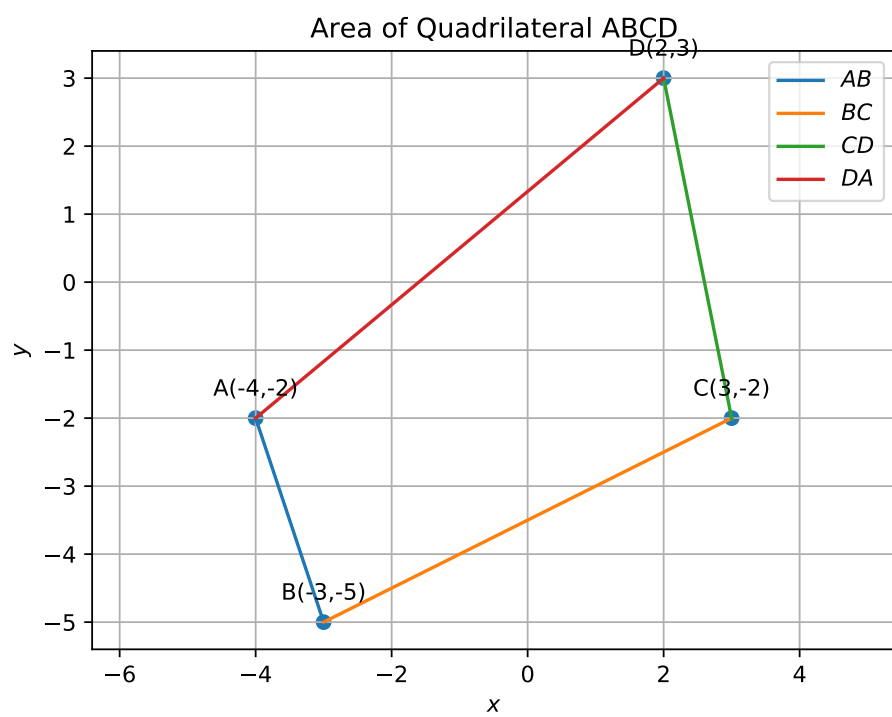


Figure 1