

Straight Lines

12th Maths - Chapter 11

This is Problem-3 from Exercise 11.1

1. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines ?

Solution: The direction cosines are the cosines of the angles formed by the given vector with the respective axes, let \mathbf{A} be the given vector

$$\mathbf{A} = \begin{pmatrix} -18 \\ 12 \\ -4 \end{pmatrix} \quad (1)$$

The Directional vectors of x, y and z axes are given respectively

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2)$$

The magnitudes for \mathbf{A} and directional vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are

$$\|\mathbf{A}\| = 22, \|\mathbf{e}_1\| = \|\mathbf{e}_2\| = \|\mathbf{e}_3\| = 1 \quad (3)$$

The Direction cosines are given by

$$\cos \theta_i = \frac{\mathbf{A}^\top \mathbf{e}_i}{\|\mathbf{A}\| \|\mathbf{e}_i\|} \quad (4)$$

$$\text{where } i = 1, 2, 3 \quad (5)$$

So for different values of $\cos \theta_i$ the direction cosines of vector \mathbf{A} are

$$\cos \theta_1 = \frac{(-18 \ 12 \ -4) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{22} = \frac{-9}{11} \quad (6)$$

$$\cos \theta_2 = \frac{(-18 \ 12 \ -4) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}{22} = \frac{6}{11} \quad (7)$$

$$\cos \theta_3 = \frac{(-18 \ 12 \ -4) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{22} = \frac{-2}{11} \quad (8)$$