Coordinate Geometry

10^{th} Maths - Chapter 7

This is Problem-4 from Exercise 7.3

1. Find the area of quadrilateral whose vertices, taken in order, are $\begin{pmatrix}-4\\-2\end{pmatrix}$, $\begin{pmatrix}-3\\-5\end{pmatrix}$, $\begin{pmatrix}3\\-2\end{pmatrix}$ and $\begin{pmatrix}2\\3\end{pmatrix}$.
Solution: The input parameters for this problem are available in Table (1)

Symbol	**Value**	**Description**
A	$\begin{pmatrix}-4\\-2\end{pmatrix}$	First point
B	$\begin{pmatrix}-3\\-5\end{pmatrix}$	Second point
C	$\begin{pmatrix}3\\-2\end{pmatrix}$	Third point
D	$\begin{pmatrix}2\\3\end{pmatrix}$	Fourth point

Table 1

By joining **B** to **D**, you will get two triangles **ABD** and **BCD**.
In general, the ar(ABD) can be expressed as

$$ar(ABD) = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{D})\right\| \tag{1}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{2}$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \end{pmatrix} \tag{3}$$

Substituting the values of (2) and (3) in (1),

$$ar(ABD) = \frac{1}{2} \begin{vmatrix} -1 & 3 \\ -6 & -5 \end{vmatrix} = \frac{23}{2} \tag{4}$$

Also, the ar(BCD) can be expressed as

$$ar(BCD) = \frac{1}{2} \left\| (\mathbf{B} - \mathbf{C}) \times (\mathbf{B} - \mathbf{D}) \right\| \tag{5}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \end{pmatrix} \tag{6}$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} \tag{7}$$

Substituting the values of (6) and (7) in (5),

$$ar(BCD) = \frac{1}{2} \begin{vmatrix} -6 & -3 \\ -5 & -8 \end{vmatrix} = \frac{33}{2} \tag{8}$$

Area of Quadrilateral **ABCD** = ar(ABD) + ar(BCD),

$$\frac{23}{2} + \frac{33}{2} = 28 \tag{9}$$

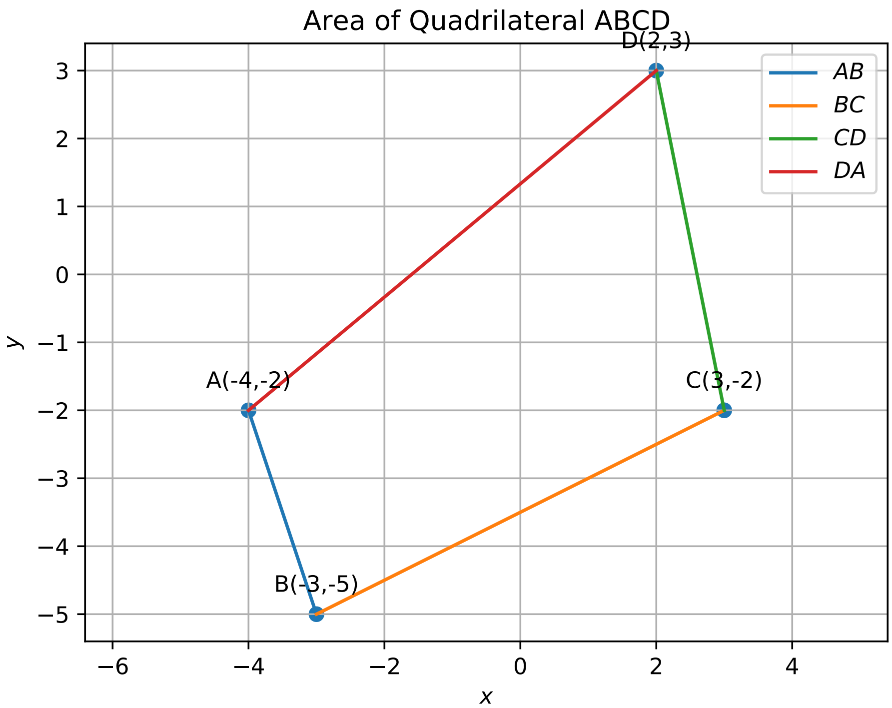
Area of Quadrilateral ABCD
D(2,3)
A(-4,-2)
C(3,-2)
B(-3,-5)
AB
BC
CD
DA
3
2
1
0
−1
−2
−3
−4
−5
−6
−4
−2
0
2
4
x
y

Figure 1