

Straight Lines

12th Maths - Chapter 10

This is Problem-12 from Exercise 10.2

1. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

Solution: The direction cosines are the cosines of the angles formed by the given vector with the respective axes, let \mathbf{A} be the given vector

$$\mathbf{A} = \hat{i} + 2\hat{j} + 3\hat{k} \quad (1)$$

The magnitude of the given vector is given by,

$$\|\mathbf{A}\| = \sqrt{1^2 + 2^2 + 3^2} \quad (2)$$

$$\|\mathbf{A}\| = \sqrt{14} \quad (3)$$

The direction cosines of \mathbf{A} can be expressed as

$$x = \frac{(i.A)}{\|\mathbf{A}\|} \quad (4)$$

$$y = \frac{(j.A)}{\|\mathbf{A}\|} \quad (5)$$

$$z = \frac{(k.A)}{\|\mathbf{A}\|} \quad (6)$$

The dot product of the unit vectors in the direction of the x, y and z axes with vector are expressed as

$$i.(\hat{i} + 2\hat{j} + 3\hat{k}) = 1 \quad (7)$$

$$j.(\hat{i} + 2\hat{j} + 3\hat{k}) = 2 \quad (8)$$

$$k.(\hat{i} + 2\hat{j} + 3\hat{k}) = 3 \quad (9)$$

So the direction cosines of vector $\hat{i} + 2\hat{j} + 3\hat{k}$ are

$$x = \frac{(i.A)}{\|\mathbf{A}\|} = \frac{1}{\sqrt{14}} \quad (10)$$

$$y = \frac{(j.A)}{\|\mathbf{A}\|} = \frac{2}{\sqrt{14}} \quad (11)$$

$$z = \frac{(k.A)}{\|\mathbf{A}\|} = \frac{3}{\sqrt{14}} \quad (12)$$