Straight Lines

11^{th} Maths - Chapter 10

This is Problem-1 from Exercise 10.4

1. Find the values of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is
   (a) Parallel to the x-axis
   (b) Parallel to the y-axis
   (c) Passing through the origin

   Solution: Given line is

$$(k-3)x-(4-k^2)y+k^2-7k+6=0 \tag{1}$$

   this equation can be expressed in the form of

$$\mathbf{n}^\top \mathbf{x} = c \tag{2}$$

$$\text{where } \mathbf{n} = \begin{pmatrix} k-3 \\ -4+k^2 \end{pmatrix}, c = -k^2+7k-6 \tag{3}$$

   then (1) can be expressed as

$$\begin{pmatrix} k-3 & -4+k^2 \end{pmatrix} \mathbf{x} = -k^2+7k-6 \tag{4}$$

   (a) Parallel to x-axis

   The normal vector of x-axis is given by

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5}$$

Equating the $\mathbf{n}$ to the normal vector of x-axis

$$\begin{pmatrix} k-3 \\ -4+k^2 \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

$$k-3=0 \tag{7}$$

$$k=3 \tag{8}$$

Substituting the value of k in (4) then equation of line parallel to x-axis is given by

$$\begin{pmatrix} 0 & 5 \end{pmatrix} \mathbf{x} = 6 \tag{9}$$

The line parallel to x-axis is shown is Figure (1)

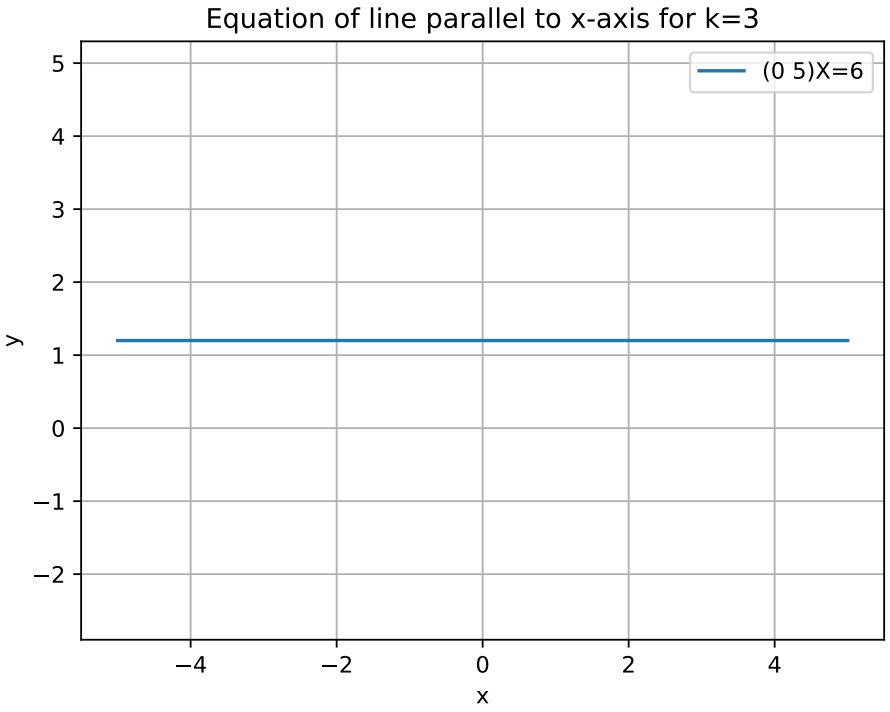

Figure 1

(b) Parallel to y-axis

The normal vector of y-axis is given by

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{10}$$

Equating the $\mathbf{n}$ to the normal vector of y-axis

$$\begin{pmatrix} k-3 \\ -4+k^2 \end{pmatrix} = \beta \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{11}$$

$$-4 + k^2 = 0 \tag{12}$$

$$k = \pm 2 \tag{13}$$

Substituting the value of k in (4) then equation of line parallel to y-axis is given by

$$\text{for } k = 2 \tag{14}$$

$$\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} = 4 \tag{15}$$

$$\text{for } k = -2 \tag{16}$$

$$\begin{pmatrix} -5 & 0 \end{pmatrix} \mathbf{x} = -24 \tag{17}$$

The line parallel to y-axis is shown is Figure (2)

(c) Passing through the origin

When line is passing through origin $(0, 0)$ then x and y coordinates are equal to 0, then

$$\begin{pmatrix} k-3 & -4+k^2 \end{pmatrix} \mathbf{x} = -k^2 + 7k - 6 \tag{18}$$

$$0 = -k^2 + 7k - 6 \tag{19}$$

$$\implies k = 1 \text{ or } k = 6 \tag{20}$$

Substituting the value of k in (4) then equation of line parallel to y-axis is given by

$$\text{for } k = 1 \tag{21}$$

$$\begin{pmatrix} -2 & -3 \end{pmatrix} \mathbf{x} = 0 \tag{22}$$

$$\text{for } k = 6 \tag{23}$$

$$\begin{pmatrix} 3 & 32 \end{pmatrix} \mathbf{x} = 0 \tag{24}$$

The line passing through origin $(0, 0)$ is shown is Figure (3)

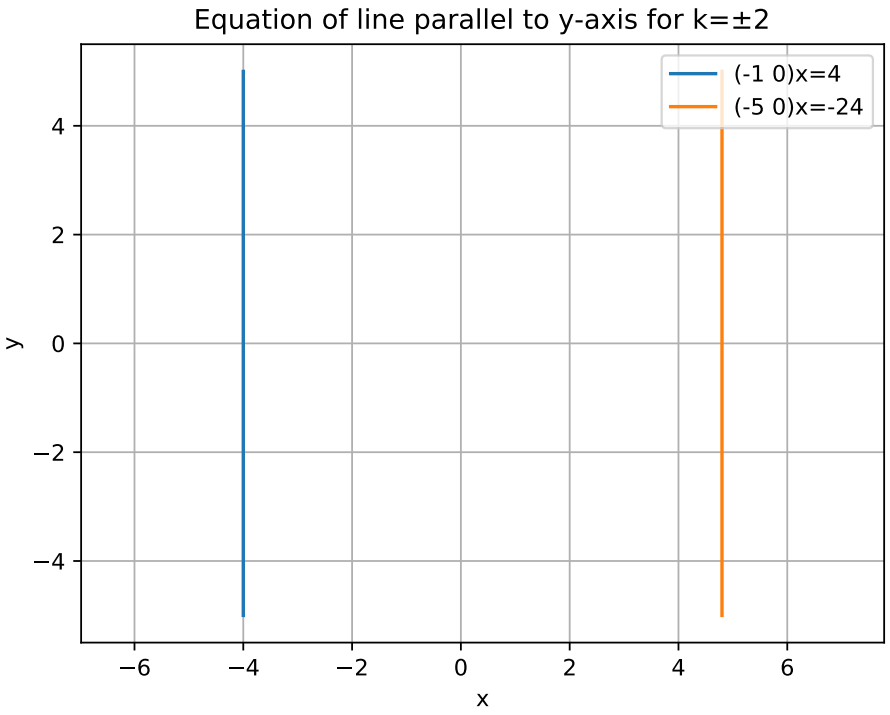
Equation of line parallel to y-axis for k=±2
(-1 0)x=4
(-5 0)x=-24
y
x
4
2
0
−2
−4
−6
−4
−2
0
2
4
6

Figure 2

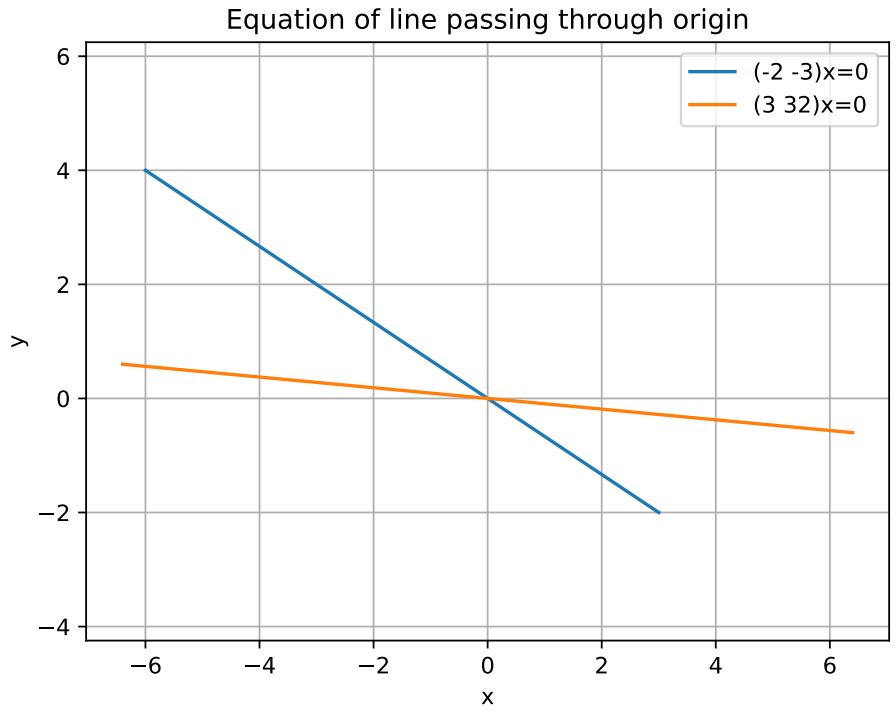
Equation of line passing through origin
(-2 -3)x=0
(3 32)x=0
6
4
2
0
−2
−4
y
−6
−4
−2
0
2
4
6
x

Figure 3