Vector Algebra

12th Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. If unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.

   Solution: Let $\mathbf{A}$ be the given vector,

   $$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{1}$$

   As $\mathbf{A}$ makes angles with respect to direction cosines then,

   $$a_1 = \cos\theta_1 \tag{2}$$

   $$= \cos\frac{\pi}{3} = \frac{1}{2} \tag{3}$$

   $$a_2 = \cos\theta_2 \tag{4}$$

   $$= \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \tag{5}$$

   As $\mathbf{A}$ is unit vector then

   $$\|\mathbf{A}\| = 1 \tag{6}$$

   $$\sqrt{a_1^2 + a_2^2 + a_3^2} = 1 \tag{7}$$

   $$\sqrt{\frac{1}{2}^2 + \frac{1}{\sqrt{2}}^2 + \cos^2\theta_3} = 1 \tag{8}$$

   $$\cos\theta_3 = \pm\frac{1}{2} \tag{9}$$

As θ_3 is an acute angle

$$\theta_3 = 60°, a_3 = \cos 60° = \frac{1}{2} \tag{10}$$

$$\text{Hence } \mathbf{A} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \tag{11}$$