

Three Dimensional Geometry

12th Maths - Chapter 11

This is Problem-3 from Exercise 11.1

1. A tangent PQ at a point of a circle of radius 5cm meets a line through the centre O at a point Q so that $OQ=12$ cm then length of PQ is

Solution: The input parameters for this problem are available in Table (1)

Symbol	Value	Description
r	5	Radius
\mathbf{O}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Centre \mathbf{O}
\mathbf{P}	$r \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$	Point \mathbf{P}
d	12	Lenght of OQ

Table 1

The distance form origin to point \mathbf{Q} is given by

$$\|\mathbf{Q}\|^2 = d^2 \quad (1)$$

Then equation of line is given as

$$(\mathbf{Q} - \mathbf{P})^\top \mathbf{P} = 0 \quad (2)$$

$$\mathbf{P}^\top \mathbf{Q} = \|\mathbf{P}\|^2 = r^2 \quad (3)$$

$$\mathbf{P}^\top \mathbf{Q} = 25 \quad (4)$$

For $\theta = 0^\circ$ The point \mathbf{P} is given by

$$\mathbf{P} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (5)$$

Now substituting the value of \mathbf{P} in (4) gives

$$\begin{pmatrix} 5 & 0 \end{pmatrix} \mathbf{Q} = 25 \quad (6)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{Q} = 5 \quad (7)$$

$$\mathbf{Q} = \begin{pmatrix} 5 \\ \mu \end{pmatrix} \quad (8)$$

$$\mathbf{Q} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (9)$$

The (9) can be expressed in the form of parametric equation

$$\mathbf{Q} = \mathbf{A} + \mu \mathbf{m} \quad (10)$$

Then substituting (10) in (1) yeilds,

$$\implies (\mathbf{A} + \mu \mathbf{m})^\top (\mathbf{A} + \mu \mathbf{m}) = d^2 \quad (11)$$

$$\implies \mathbf{A}^\top \mathbf{A} + (\mu \mathbf{m})^\top \mu \mathbf{m} + \mathbf{A}^\top \mu \mathbf{m} + (\mu \mathbf{m})^\top \mathbf{A} = d^2 \quad (12)$$

$$\implies \|\mathbf{A}\|^2 + \mu^2 \|\mathbf{m}\|^2 + 2\mu \mathbf{A}^\top \mathbf{m} = d^2 \quad (13)$$

$$\implies \mu^2 \|\mathbf{m}\|^2 + 2\mu \mathbf{A}^\top \mathbf{m} + \|\mathbf{A}\|^2 = d^2 \quad (14)$$

where

$$\mathbf{A} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \text{ and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (15)$$

substituting the values of \mathbf{A} and \mathbf{m} in (14) gives

$$\implies \mu^2(1) + 2\mu \begin{pmatrix} 5 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 25 = 144 \quad (16)$$

$$\implies \mu^2 = 119 \quad (17)$$

$$\implies \mu = \sqrt{119} \quad (18)$$

$$\implies \mu = \pm 10.908 \quad (19)$$

substituting the value of μ in (9) yeilds

$$\mathbf{Q}_1 = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + 10.908 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (20)$$

$$\mathbf{Q}_2 = \begin{pmatrix} 5 \\ 0 \end{pmatrix} - 10.908 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (21)$$

$$\mathbf{Q}_1 = \begin{pmatrix} 5 \\ 10.908 \end{pmatrix}, \mathbf{Q}_2 = \begin{pmatrix} 5 \\ -10.908 \end{pmatrix} \quad (22)$$

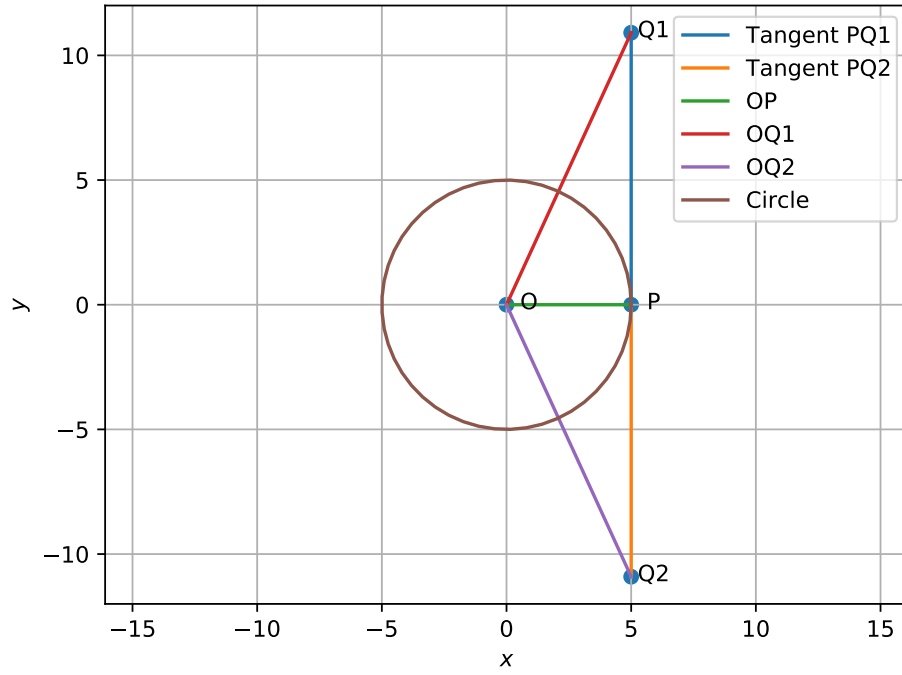


Figure 1