Coordinate Geometry

10^{th} Maths - Chapter 7

This is Problem-7 from Exercise 7.1

1. The point on the x-axis which is equidistant from $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

Solution: The input parameters for this problem are available in If \mathbf{x} lies on the x-axis and is equidistant from the points \mathbf{A} and \mathbf{B} ,

$$\|\mathbf{x} - \mathbf{A}\| = \|\mathbf{A} - \mathbf{B}\| \tag{1}$$

$$\implies \|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \tag{2}$$

which can be expressed as

$$(\mathbf{x} - \mathbf{A})^{\top} (\mathbf{x} - \mathbf{A}) = (\mathbf{x} - \mathbf{B})^{\top} (\mathbf{x} - \mathbf{B})$$

$$\implies \|\mathbf{x}\|^{2} - 2\mathbf{x}^{\top}\mathbf{A} + \|\mathbf{A}\|^{2}$$

$$= \|\mathbf{x}\|^{2} - 2\mathbf{x}^{\top}\mathbf{B} + \|\mathbf{B}\|^{2} \quad (3)$$

which can be simplified to obtain

$$\mathbf{x} = x\mathbf{e}_1 \tag{4}$$

where

$$x = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^{\mathsf{T}} \mathbf{e}_1}$$
 (5)

now substituting the A and B values in eq.5

$$(\mathbf{A} - \mathbf{B})^{\top} = \left(\begin{pmatrix} 2 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 9 \end{pmatrix} \right)^{\top} = \begin{pmatrix} 4 & -14 \end{pmatrix}$$
 (6)

$$\|\mathbf{A}\|^2 = 21\tag{7}$$

$$\left\|\mathbf{B}\right\|^2 = 85\tag{8}$$

upon substituting the values in eq 5. the value of x = -7. Hence, the desired point is $\begin{pmatrix} -7 \\ 0 \end{pmatrix}$.

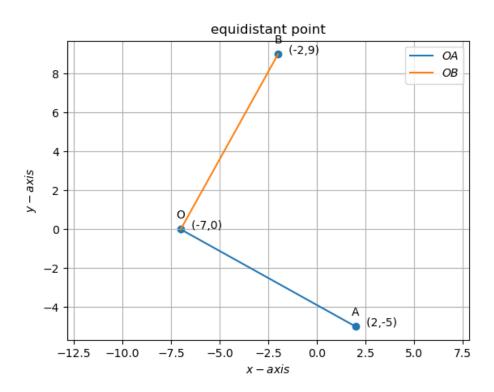


Figure 1