

Straight Lines

11th Maths - Chapter 10

This is Problem-1 from Exercise 10.4

1. Find the values of k for which the line $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$ is
 - (a) Parallel to the x -axis
 - (b) Parallel to the y -axis
 - (c) Passing through the origin

Solution: Given line is

$$(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0 \quad (1)$$

this equation can be expressed in the form of

$$\mathbf{n}^\top \mathbf{x} = c \quad (2)$$

$$\text{where } \mathbf{n} = \begin{pmatrix} k-3 \\ -4+k^2 \end{pmatrix}, c = -k^2 + 7k - 6 \quad (3)$$

then (1) can be expressed as

$$(k-3 \quad -4+k^2) \mathbf{x} = -k^2 + 7k - 6 \quad (4)$$

- (a) Parallel to x -axis

The normal vector of x -axis is given by

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (5)$$

Equating the \mathbf{n} to the normal vector of x -axis

$$\begin{pmatrix} k-3 \\ -4+k^2 \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (6)$$

$$0 = \alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} -4+k^2 \\ k-3 \end{pmatrix} \quad (7)$$

$$k = 3 \quad (8)$$

Substituting the value of k in (4) then equation of line parallel to x -axis is given by

$$(0 \ 5) \mathbf{x} = 6 \quad (9)$$

The line parallel to x -axis is shown in Figure (1)

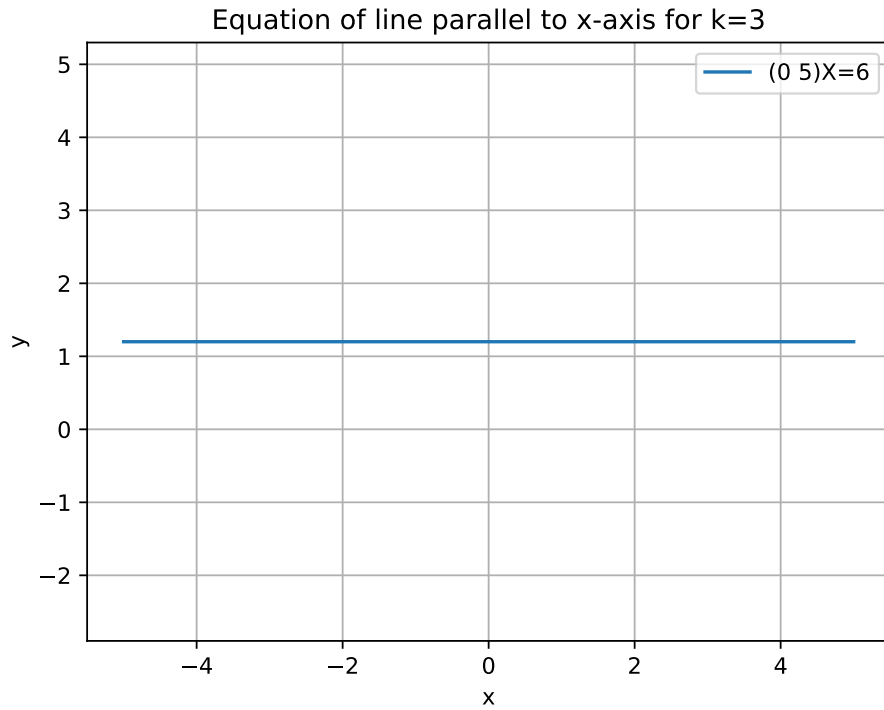


Figure 1

(b) Parallel to y -axis

The normal vector of y -axis is given by

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (10)$$

Equating the \mathbf{n} to the normal vector of y -axis

$$\begin{pmatrix} k-3 \\ -4+k^2 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (11)$$

$$0 = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} -4+k^2 \\ k-3 \end{pmatrix} \quad (12)$$

$$k = \pm 2 \quad (13)$$

Substituting the value of k in (4) then equation of line parallel to y -axis is given by

$$\text{for } k = 2 \quad (14)$$

$$(-1 \ 0) \mathbf{x} = 4 \quad (15)$$

$$\text{for } k = -2 \quad (16)$$

$$(-5 \ 0) \mathbf{x} = -24 \quad (17)$$

The line parallel to y -axis is shown in Figure (2)

(c) Passing through the origin

When line is passing through origin $(0, 0)$ then x and y coordinates are equal to 0, then

$$(k-3 \ -4+k^2) \mathbf{x} = -k^2 + 7k - 6 \quad (18)$$

$$0 = -k^2 + 7k - 6 \quad (19)$$

$$\implies k = 1 \text{ or } k = 6 \quad (20)$$

Substituting the value of k in (4) then equation of line parallel to y -axis is given by

$$\text{for } k = 1 \quad (21)$$

$$(-2 \ -3) \mathbf{x} = 0 \quad (22)$$

$$\text{for } k = 6 \quad (23)$$

$$(3 \ 32) \mathbf{x} = 0 \quad (24)$$

The line passing through origin $(0, 0)$ is shown in Figure (3)

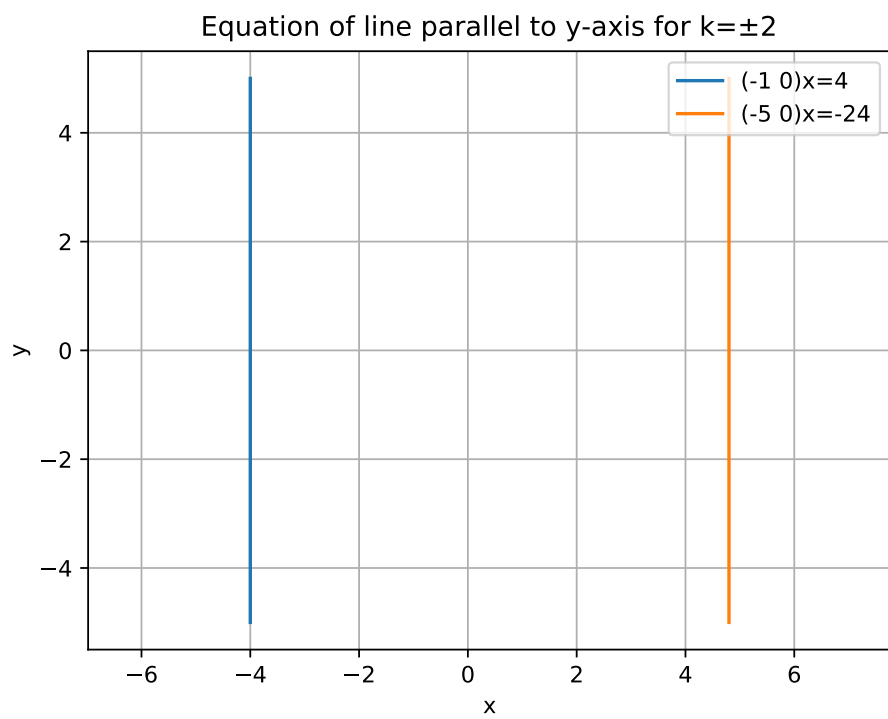


Figure 2

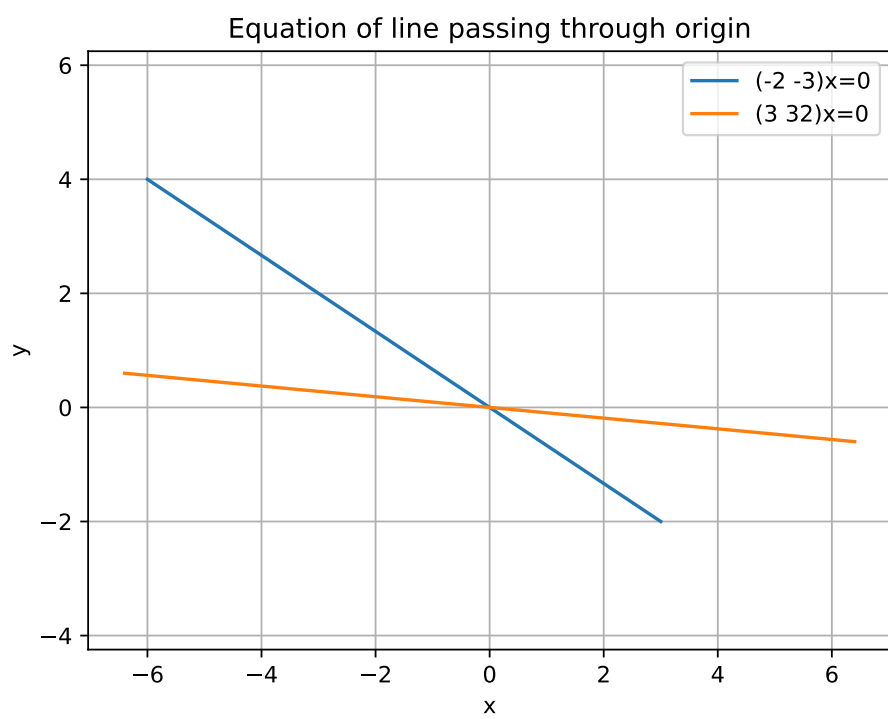


Figure 3