Three Dimensional Geometry

12^{th} Maths - Chapter 11

This is Problem-3 from Exercise 11.1

1. If a line has the direction ratios –18, 12, –4, then what are its direction cosines ?

   Solution: The direction cosines are the cosines of the angles formed by the given vector with the respective axes, let $\mathbf{A}$ be the given vector

$$\mathbf{A} = \begin{pmatrix} -18 \\ 12 \\ -4 \end{pmatrix} \tag{1}$$

   The Directional vectors of x, y and z axes are given respectively

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

   The magnitudes for $\mathbf{A}$ and directional vectors $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ are

$$\|\mathbf{A}\| = 22, \|\mathbf{e_1}\| = \|\mathbf{e_2}\| = \|\mathbf{e_3}\| = 1 \tag{3}$$

   The Direction cosines are given by

$$\cos\theta_i = \frac{\mathbf{A}^\top \mathbf{e_i}}{\|\mathbf{A}\| \, \|\mathbf{e_i}\|} \tag{4}$$

$$\text{where } i = 1, 2, 3 \tag{5}$$

So for different values of $\cos\theta_i$ the direction cosines of vector $\mathbf{A}$ are

$$\cos\theta_1 = \frac{\begin{pmatrix}-18 & 12 & -4\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}}{22} = \frac{-9}{11} \tag{6}$$

$$\cos\theta_2 = \frac{\begin{pmatrix}-18 & 12 & -4\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}}{22} = \frac{6}{11} \tag{7}$$

$$\cos\theta_3 = \frac{\begin{pmatrix}-18 & 12 & -4\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}{22} = \frac{-2}{11} \tag{8}$$

Let vector $\mathbf{B}$ be unit vector in the direction of $\mathbf{A}$

$$\mathbf{B} = \begin{pmatrix}\cos\theta_1\\\cos\theta_2\\\cos\theta_3\end{pmatrix} \tag{9}$$

then magnitude of $\mathbf{B}$ is,

$$\|\mathbf{B}\| = \sqrt{\left(\frac{-9}{11}\right)^2 + \left(\frac{6}{11}\right)^2 + \left(\frac{-2}{11}\right)^2} = 1 \tag{10}$$