# Vector Algebra

## $12^{th}$ Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. If unit vector $\overrightarrow{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle $\theta$ with $\hat{k}$, then find $\theta$ and hence, the components of $\overrightarrow{a}$.

   **Solution:** Let $\mathbf{A}$ be the given vector, in terms of their direction cosines

$$\mathbf{A} = \begin{pmatrix} \cos\theta_1 \\ \cos\theta_2 \\ \cos\theta_3 \end{pmatrix} \tag{1}$$

   then,

$$\implies \cos\theta_1 = \cos\frac{\pi}{3} \tag{2}$$

$$= \frac{1}{2} \tag{3}$$

$$\implies \cos\theta_2 = \cos\frac{\pi}{4} \tag{4}$$

$$= \frac{1}{\sqrt{2}} \tag{5}$$

   As $\mathbf{A}$ is unit vector then

$$\|\mathbf{A}\| = 1 \tag{6}$$

$$\sqrt{\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3} = 1 \tag{7}$$

$$\sqrt{\frac{1}{2}^2 + \frac{1}{\sqrt{2}}^2 + \cos^2\theta_3} = 1 \tag{8}$$

$$\cos\theta_3 = \pm\frac{1}{2} \tag{9}$$

As $\theta_3$ is an acute angle

$$\theta_3 = 60^\circ, \cos\theta_3 = \frac{1}{2} \tag{10}$$

$$\text{Hence } \mathbf{A} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \tag{11}$$