

Straight Lines

11th Maths - Chapter 10

This is Problem-1 from Exercise 10.4

1. Find the values of k for which the line $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$ is
 - (a) Parallel to the x -axis
 - (b) Parallel to the y -axis
 - (c) Passing through the origin

Solution: Given line is

$$(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0 \quad (1)$$

this equation can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c \quad (2)$$

$$\text{where } \mathbf{n} = \begin{pmatrix} k-3 \\ 4-k^2 \end{pmatrix}, c = -k^2 + 7k - 6 \quad (3)$$

- (a) Parallel to x -axis

the equation of line parallel to x -axis is given by

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \quad (4)$$

where \mathbf{A} and \mathbf{n}^\top is

$$\mathbf{n}^\top = (k-2 \quad 4-k^2) \quad (5)$$

$$\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (6)$$

Substituting the value of \mathbf{n}^\top and \mathbf{A} in (4)

$$(k-3 \quad 4-k^2) \left(\mathbf{x} - \begin{pmatrix} a \\ b \end{pmatrix} \right) = 0 \quad (7)$$

When line is parallel to x -axis then $y = 0$ and $b = 0$

$$k-3(x-a) = 0 \quad (8)$$

$$k-3 = 0 \quad (9)$$

$$k = 3 \quad (10)$$

Substituting the value of k in (1) then equation of line parallel to x -axis is given by

$$5y + 4 = 0 \quad (11)$$

$$5y = -4 \quad (12)$$

$$y = \frac{-4}{5} \quad (13)$$

The line parallel to x -axis is shown in Figure (1)

(b) Parallel to Y -axis

When line is parallel to y -axis then $(x-a) = 0$

$$4-k^2(x-b) = 0 \quad (14)$$

$$k = \sqrt{4} \quad (15)$$

$$k = \pm 2 \quad (16)$$

Substituting the value of k in (1) then equation of line parallel to y -axis is given by

$$\text{for } k = 2 \quad (17)$$

$$-x - 4 = 0 \quad (18)$$

$$x = -4 \quad (19)$$

$$\text{for } k = -2 \quad (20)$$

$$-5x + 24 = 0 \quad (21)$$

$$5x = 24 \quad (22)$$

$$x = \frac{24}{5} \quad (23)$$

The line parallel to y -axis is shown in Figure (2)

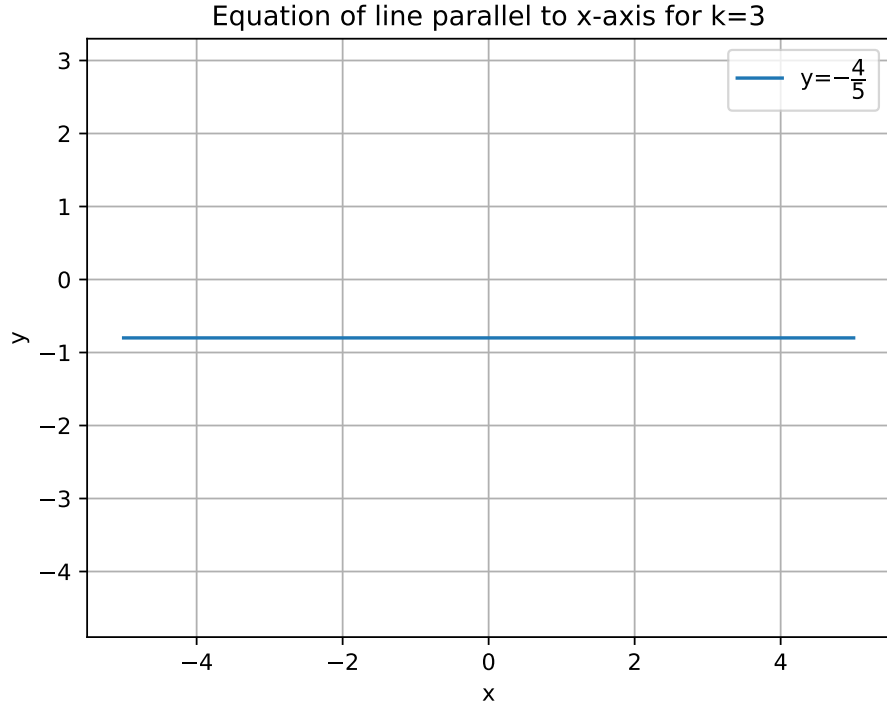


Figure 1

(c) Passing through the origin

When line is passing through origin $(0, 0)$ then x and y coordinates are equal to 0, from (2)

$$\mathbf{n}^\top \mathbf{x} = c \quad (24)$$

Substituting the value of n^\top and c in

$$(k - 3 \quad 4 - k^2) \mathbf{x} = -k^2 + 7k - 6 \quad (25)$$

$$0 = -k^2 + 7k - 6 \quad (26)$$

$$\implies k = 1 \text{ or } k = 6 \quad (27)$$

Substituting the value of k in (1) then equation of line parallel to

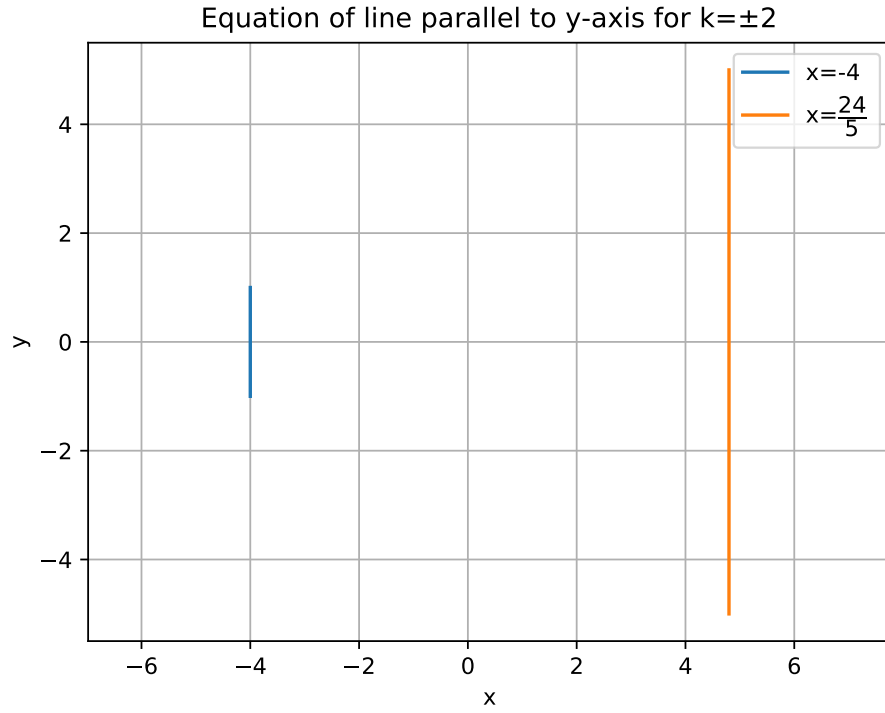


Figure 2

y -axis is given by

$$\text{for } k = 1 \quad (28)$$

$$-2x - 3y = 0 \quad (29)$$

$$\text{for } k = 6 \quad (30)$$

$$-3x - 32y = 0 \quad (31)$$

The line passing through origin $(0,0)$ is shown in Figure (3)

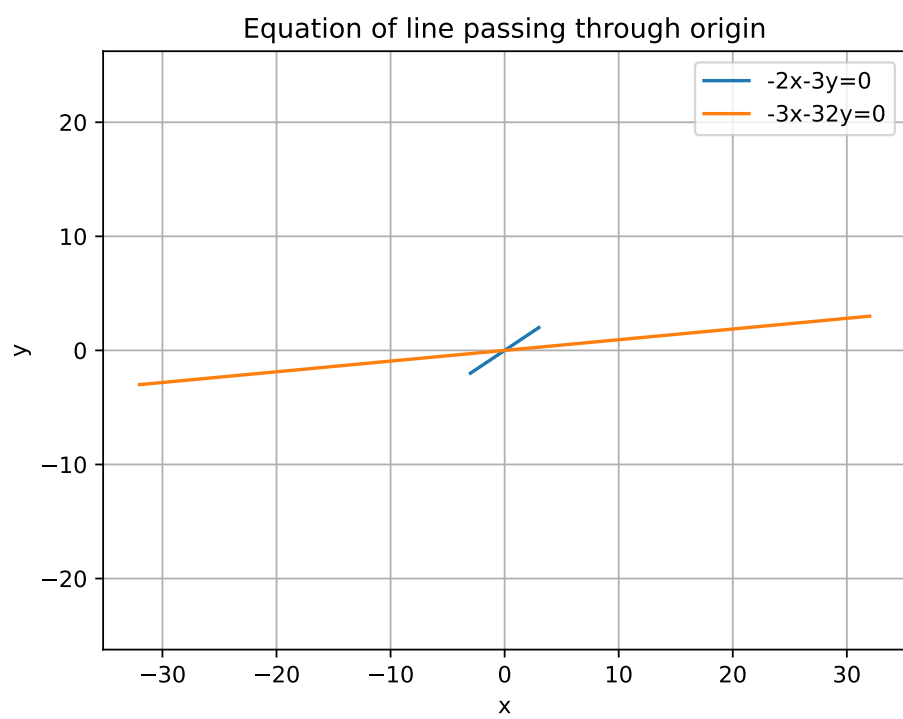


Figure 3