

Statistical Learning Assignment

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Question 1:

Let X_1, X_2, X_3, X_4, X_5 be independent $U(0, 1)$ random variables. Let $X = X_1 + X_2 + X_3$ and $Y = X_3 + X_4 + X_5$. Use the `runif()` function to simulate 1000 trials of each of these variables. Use these to estimate $\text{Cov}(X, Y)$.

Solution:

Step 1: Invoking seed function to use the same random sample for every trial

```
set.seed(900)
```

Step 2: Formation of 5 independent matrix variables with range (0,1)

```
x1<-matrix(runif(1000,0,1),ncol=1)
x2<-matrix(runif(1000,0,1),ncol=1)
x3<-matrix(runif(1000,0,1),ncol=1)
x4<-matrix(runif(1000,0,1),ncol=1)
x5<-matrix(runif(1000,0,1),ncol=1)
```

Step 3: Computation of X and Y

```
X<-x1+x2+x3
Y<-x3+x4+x5
```

Step 4: Computation of covariance of X and Y parameters

```
cov(X,Y)
```

```
##           [,1]
## [1,] 0.08979664
```

The covariance value shows positive correlation in this test and it will be different for each and every trial if different random values for every trial are used i.e if the seed function is not used

Question 2:

The random variable X takes values -1, 0, 1 with probabilities $1/8, 2/8, 5/8$ respectively.

(a) Compute $E(X)$.

Solution:

x	-1	0	1
p(x)	1/8	2/8	5/8

$E(X)$ = Expected value of X

$$E(X) = \sum_{\text{all } x} xp(x)$$

$$E(X) = (-1*(1/8)) + (0*(2/8)) + (1*(5/8))$$

$$E(X) = 0.5$$

(b) Given the pmf of $Y = X^2$ and use it to compute $E(Y)$.

Solution:

$$Y = X^2$$

x	-1	0	1
p(x)	1/8	2/8	5/8
x^2	1	0	1

$$E(Y) = E(X^2) = \sum_{\text{all } x} f(x)p(x)$$

$$E(Y) = (1*(1/8)) + (0*(2/8)) + (1*(5/8))$$

$$E(Y) = 0.75$$

(c) Instead, compute $E(X^2)$ directly from an extended table.

Solution:

$$E(X^2) = (1*(1/8)) + (0*(2/8)) + (1*(5/8))$$

$$E(X^2) = 0.75$$

(d) Compute $\text{Var}(X)$.

Solution:

$$V(X) = \text{Variance of } X$$

$$E(X) = 0.5$$

$$E(X^2) = 0.75$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\text{Var}(X) = 0.75 - (0.5^2)$$

$$\text{Var}(X) = 0.5$$