Discrete Math Chapter 5 (Attempt) Solutions

Question 1. (Exercise 5.1) Explain why the following is logically correct:
1. Everyone loves my baby;
2. My baby loves only me;
3. Therefore, I am my own baby.

Solution:
From statement 1, "everyone" includes the baby itself, so the baby loves itself. But statement 2 says the baby only loves one person: me. Therefore, if the baby loves itself and only loves me, I must be the baby. Hence, I am my own baby.

Question 2. (Nearly Exercise 5.3) Determine which of the following are statements. If so, identify whether they are true or false.
Solution:
(a) $\sin(\pi) = 0$ — This is a true statement.
(b) If x is an integer, then x is positive — This is a false statement (e.g., $x = 0$ or $x = -1$).
(c) "The sets $\mathbb{Z}$ and $\mathbb{Q}$" — Not a statement; this is a noun phrase.
(d) "The sets $\mathbb{Z}$ and $\mathbb{Q}$ both contain $\sqrt{2}$" — False. $\mathbb{Z}$ and $\mathbb{Q}$ do not contain $\sqrt{2}$ since it is irrational.

Question 3. (Exercise 5.5) Consider the open sentence:

$$\frac{2n^2 + 5 + (-1)^n}{2} \text{ is prime.}$$

Solution:
For $n = 1$:

$$\frac{2(1)^2 + 5 + (-1)}{2} = \frac{6}{2} = 3 \Rightarrow \text{prime.}$$

For $n = 5$:

$$\frac{2(5)^2 + 5 + (-1)^5}{2} = \frac{50 + 5 - 1}{2} = \frac{54}{2} = 27 \Rightarrow \text{not prime.}$$

So it is true for $n = 1$ and false for $n = 5$.

Question 4. (Exercise 5.8 parts (f)-(h)) Rewrite each sentence in the form "If P, then Q."
Solution:
(f) If a tree has m edges, then it has $m + 1$ vertices.
(g) If an integer is not odd, then it is even.
(h) If a geometric series has ratio r with $|r| \geq 1$, then the series diverges.

Question 5. (Exercise 5.9(d)) Rewrite as a biconditional.
Solution:
N is a normal subgroup of G if and only if $Ng = gN$ for all $g \in G$.

Question 6. (Exercise 5.10 parts (g)-(h)) Negate the following statements.
Solution:
(g) Original: For all $\varepsilon > 0$ there exists some $\delta > 0$ such that $|x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon$.
Negation: There exists $\varepsilon > 0$ such that for all $\delta > 0$, there exists x with $|x - a| < \delta$ and $|f(x) - f(a)| \geq \varepsilon$.
(h) Original: If I pass Algebra I and Analysis I this semester, then I will take Algebra II or Analysis II next semester.
Negation: I pass Algebra I and Analysis I this semester, and I will not take Algebra II and I will not take Analysis II next semester.

Question 7. (Nearly Exercise 5.18(d)) Construct a truth table for $\sim (\sim P \vee Q)$.
Solution:

P	Q	$\sim P$	$\sim P \vee Q$	$\sim (\sim P \vee Q)$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	F

Question 8. (Exercise 5.18(f)) Construct a truth table for $(P \wedge Q) \vee \sim R$.
Solution:

P	Q	R	$P \wedge Q$	$\sim R$	$(P \wedge Q) \vee \sim R$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

Question 9. (Exercise 5.22(g)) Show that $P \Rightarrow Q \equiv \sim P \vee Q$.
Solution:

P	Q	$P \Rightarrow Q$	$\sim P$	$\sim P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Since the final columns match, the two expressions are logically equivalent.

Question 10. (Exercise 5.26(d)) Determine whether $(P \vee Q) \vee (\sim P \wedge \sim Q)$ is a tautology.
Solution:

P	Q	$P \vee Q$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	Final Expr
T	T	T	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	F	T	T	T	T

All rows in the final column are true. Thus, this is a tautology.