EXERCISE 2.2

Prove the following:

1. $3\sin^{-1} x = \sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

2. $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x),\ x \in \left[\frac{1}{2}, 1\right]$

Write the following functions in the simplest form:

3. $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x},\ x \neq 0$

4. $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ 0 < x < \pi$

5. $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right),\ \frac{-\pi}{4} < x < \frac{3\pi}{4}$

6. $\tan^{-1}\frac{x}{\sqrt{a^2 - x^2}},\ |x| < a$

7. $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right),\ a > 0;\ \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

Find the values of each of the following:

8. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

9. $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right],\ |x| < 1,\ y > 0 \text{ and } xy < 1$

Find the values of each of the expressions in Exercises 16 to 18.

10. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **11.** $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

12. $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

13. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

14. $\sin\left(\frac{\pi}{3}-\sin^{-1}(-\frac{1}{2})\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

15. $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$ is equal to

(A) π (B) $-\frac{\pi}{2}$ (C) 0 (D) $2\sqrt{3}$