Example 1 Write the first three terms in each of the following sequences defined by the following:

(i) $a_n = 2n + 5$, (ii) $a_n = \frac{n-3}{4}$.

Solution (i) Here $a_n = 2n + 5$

Substituting $n = 1, 2, 3$, we get

$$a_1 = 2(1) + 5 = 7,\ a_2 = 9,\ a_3 = 11$$

Therefore, the required terms are 7, 9 and 11.

(ii) Here $a_n = \frac{n-3}{4}$. Thus, $a_1 = \frac{1-3}{4} = -\frac{1}{2},\ a_2 = -\frac{1}{4},\ a_3 = 0$

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Hence, the first three terms are $-\frac{1}{2}, -\frac{1}{4}$ and 0.

Example 2 What is the 20th term of the sequence defined by

$$a_n = (n - 1)(2 - n)(3 + n)\ ?$$

Solution Putting $n = 20$, we obtain

$$a_{20} = (20 - 1)(2 - 20)(3 + 20)$$
$$= 19 \times (-18) \times (23) = -7866.$$

Example 3 Let the sequence a_n be defined as follows:

$$a_1 = 1, \quad a_n = a_{n-1} + 2 \text{ for } n \geq 2.$$

Find first five terms and write corresponding series.

Solution We have

$$a_1 = 1,\ a_2 = a_1 + 2 = 1 + 2 = 3,\ a_3 = a_2 + 2 = 3 + 2 = 5,$$
$$a_4 = a_3 + 2 = 5 + 2 = 7,\ a_5 = a_4 + 2 = 7 + 2 = 9.$$

Hence, the first five terms of the sequence are 1,3,5,7 and 9. The corresponding series is $1 + 3 + 5 + 7 + 9 + \ldots$

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Example 4 Find the 10^{th} and n^{th} terms of the G.P. 5, 25,125,… .

Solution Here $a = 5$ and $r = 5$. Thus, $a_{10} = 5(5)^{10-1} = 5(5)^9 = 5^{10}$

and $\quad a_n = ar^{n-1} = 5(5)^{n-1} = 5^n$.

Example 5 Which term of the G.P., 2,8,32, ... up to n terms is 131072?

Solution Let 131072 be the n^{th} term of the given G.P. Here $a = 2$ and $r = 4$.

Therefore $\quad 131072 = a_n = 2(4)^{n-1} \quad$ or $\quad 65536 = 4^{n-1}$

This gives $\quad 4^8 = 4^{n-1}$.

So that $n - 1 = 8$, i.e., $n = 9$. Hence, 131072 is the 9^{th} term of the G.P.

Example 6 In a G.P., the 3^{rd} term is 24 and the 6^{th} term is 192.Find the 10^{th} term.

Solution Here, $a_3 = ar^2 = 24$ ... (1)

and $\quad a_6 = ar^5 = 192$ ... (2)

Dividing (2) by (1), we get $r = 2$. Substituting $r = 2$ in (1), we get $a = 6$.

Hence $a_{10} = 6\,(2)^9 = 3072$.

Example 7 Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+...$

Solution Here $a = 1$ and $r = \frac{2}{3}$. Therefore

$$S_n = \frac{a\,(1-r^n)}{1-r} = \frac{\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}} = 3\left[1-\left(\frac{2}{3}\right)^n\right]$$

In particular, $\quad S_5 = 3\left[1-\left(\frac{2}{3}\right)^5\right] = 3\times\frac{211}{243} = \frac{211}{81}$.

Example 8 How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}, ...$ are needed to give the sum $\frac{3069}{512}$?

Solution Let n be the number of terms needed. Given that $a = 3$, $r = \frac{1}{2}$ and $S_n = \frac{3069}{512}$

Since $\quad S_n = \frac{a\,(1-r^n)}{1-r}$

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Therefore $$\frac{3069}{512}=\frac{3(1-\frac{1}{2^n})}{1-\frac{1}{2}}=6\left(1-\frac{1}{2^n}\right)$$

or $$\frac{3069}{3072} = 1-\frac{1}{2^n}$$

or $$\frac{1}{2^n} = 1-\frac{3069}{3072}=\frac{3}{3072}=\frac{1}{1024}$$

or $2^n = 1024 = 2^{10}$, which gives $n = 10$.

Example 9 The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is – 1. Find the common ratio and the terms.

Solution Let $\frac{a}{r}$, a, ar be the first three terms of the G.P. Then

$$\frac{a}{r} + ar + a = \frac{13}{12} \qquad \text{... (1)}$$

and $$\left(\frac{a}{r}\right)(a)\,(ar) = -1 \qquad \text{... (2)}$$

From (2), we get $a^3 = -1$, i.e., $a = -1$ (considering only real roots)

Substituting $a=-1$ in (1), we have

$$-\frac{1}{r}-1-r=\frac{13}{12} \text{ or } 12r^2 + 25r + 12 = 0.$$

This is a quadratic in r, solving, we get $r = -\frac{3}{4}$ or $-\frac{4}{3}$.

Thus, the three terms of G.P. are : $\frac{4}{3}, -1, \frac{3}{4}$ for $r = \frac{-3}{4}$ and $\frac{3}{4}, -1, \frac{4}{3}$ for $r = \frac{-4}{3}$,

Example10 Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.

Solution This is not a G.P., however, we can relate it to a G.P. by writing the terms as

$$S_n = 7 + 77 + 777 + 7777 + \text{... to } n \text{ terms}$$

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$$= \frac{7}{9}[9+99+999+9999+ \text{...to } n \text{ term}]$$

$$= \frac{7}{9}[(10-1)+(10^2-1)+(10^3-1)+(10^4-1)+ \text{...} n \text{ terms}]$$

$$= \frac{7}{9}[(10+10^2+10^3+\text{...} n \text{ terms})-(1+1+1+\text{...} n \text{ terms})]$$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1}-n\right] = \frac{7}{9}\left[\frac{10\,(10^n-1)}{9}-n\right].$$

Example 11 A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

Solution Here $a = 2$, $r = 2$ and $n = 10$

Using the sum formula $\quad S_n = \dfrac{a\,(r^n-1)}{r-1}$

We have $\quad S_{10} = 2(2^{10} - 1) = 2046$

Hence, the number of ancestors preceding the person is 2046.

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Example12 Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

Solution Let G_1, G_2,G_3 be three numbers between 1 and 256 such that

1, G_1,G_2,G_3 ,256 is a G.P.

Therefore $256 = r^4$ giving $r = \pm 4$ (Taking real roots only)

For $r = 4$, we have $G_1 = ar = 4$, $G_2 = ar^2 = 16$, $G_3 = ar^3 = 64$

Similarly, for $r = -4$, numbers are -4,16 and -64.

Hence, we can insert 4, 16, 64 between 1 and 256 so that the resulting sequences are in G.P.

Example 13 If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

Solution Given that $$\text{A.M.} = \frac{a+b}{2} = 10 \qquad \text{... (1)}$$

and $$\text{G.M.} = \sqrt{ab} = 8 \qquad \text{... (2)}$$

From (1) and (2), we get

$$a + b = 20 \qquad \text{... (3)}$$
$$ab = 64 \qquad \text{... (4)}$$

Putting the value of a and b from (3), (4) in the identity $(a-b)^2 = (a+b)^2 - 4ab$, we get

$$(a-b)^2 = 400 - 256 = 144$$

or $$a - b = \pm 12 \qquad \text{... (5)}$$

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Solving (3) and (5), we obtain

$$a = 4,\ b = 16 \text{ or } a = 16,\ b = 4$$

Thus, the numbers a and b are 4, 16 or 16, 4 respectively.

Miscellaneous Examples

Example 14 If a, b, c, d and p are different real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)\, p + (b^2 + c^2 + d^2) \leq 0$, then show that a, b, c and d are in G.P.

Solution Given that

$$(a^2 + b^2 + c^2)\, p^2 - 2\,(ab + bc + cd)\, p + (b^2 + c^2 + d^2) \leq 0 \quad \text{... (1)}$$

But L.H.S.

$$= (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2),$$

which gives $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \geq 0$ ... (2)

Since the sum of squares of real numbers is non negative, therefore, from (1) and (2), we have, $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

or $ap - b = 0,\ bp - c = 0,\ cp - d = 0$

This implies that $\dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = p$

Hence a, b, c and d are in G.P.

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