Example 1 Write the solution set of the equation $x^2 + x - 2 = 0$ in roster form.

Solution The given equation can be written as

$$(x - 1)\ (x + 2) = 0, \text{ i. e., } \quad x = 1, -2$$

Therefore, the solution set of the given equation can be written in roster form as $\{1, -2\}$.

Example 2 Write the set $\{x : x \text{ is a positive integer and } x^2 < 40\}$ in the roster form.

Solution The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is $\{1, 2, 3, 4, 5, 6\}$.

Example 3 Write the set $A = \{1, 4, 9, 16, 25, \ldots\}$ in set-builder form.

Solution We may write the set A as

$$A = \{x : x \text{ is the square of a natural number}\}$$

Alternatively, we can write

$$A = \{x : x = n^2, \text{ where } n \in \mathbf{N}\}$$

Example 4 Write the set $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\}$ in the set-builder form.

Solution We see that each member in the given set has the numerator one less than the denominator. Also, the numerator begin from 1 and do not exceed 6. Hence, in the set-builder form the given set is

$$\left\{x : x = \frac{n}{n+1}, \text{ where } n \text{ is a natural number and } 1 \le n \le 6\right\}$$

Example 5 Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form :

(i)	$\{P, R, I, N, C, A, L\}$	(a) $\{x : x$ is a positive integer and is a divisor of 18$\}$
(ii)	$\{0\}$	(b) $\{x : x$ is an integer and $x^2 - 9 = 0\}$
(iii)	$\{1, 2, 3, 6, 9, 18\}$	(c) $\{x : x$ is an integer and $x + 1 = 1\}$
(iv)	$\{3, -3\}$	(d) $\{x : x$ is a letter of the word PRINCIPAL$\}$

Solution Since in (d), there are 9 letters in the word PRINCIPAL and two letters P and I are repeated, so (i) matches (d). Similarly, (ii) matches (c) as $x + 1 = 1$ implies $x = 0$. Also, 1, 2 ,3, 6, 9, 18 are all divisors of 18 and so (iii) matches (a). Finally, $x^2 - 9 = 0$ implies $x = 3, -3$ and so (iv) matches (b).

Example 6 State which of the following sets are finite or infinite :

(i) $\{x : x \in \text{N} \text{ and } (x - 1)(x - 2) = 0\}$

(ii) $\{x : x \in \text{N} \text{ and } x^2 = 4\}$

(iii) $\{x : x \in \text{N} \text{ and } 2x - 1 = 0\}$

(iv) $\{x : x \in \text{N} \text{ and } x \text{ is prime}\}$

(v) $\{x : x \in \text{N} \text{ and } x \text{ is odd}\}$

Solution (i) Given set = $\{1, 2\}$. Hence, it is finite.

(ii) Given set = $\{2\}$. Hence, it is finite.

(iii) Given set = ϕ. Hence, it is finite.

(iv) The given set is the set of all prime numbers and since set of prime numbers is infinite. Hence the given set is infinite

(v) Since there are infinite number of odd numbers, hence, the given set is infinite.

Example 7 Find the pairs of equal sets, if any, give reasons:

$A = \{0\}$, $B = \{x : x > 15 \text{ and } x < 5\}$,

$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$,

$E = \{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$.

Solution Since $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that, $A \neq B$, $A \neq C$, $A \neq D$, $A \neq E$.

Since $B = \phi$ but none of the other sets are empty. Therefore $B \neq C$, $B \neq D$ and $B \neq E$. Also $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.

Since $E = \{5\}$, $C = E$. Further, $D = \{-5, 5\}$ and $E = \{5\}$, we find that, $D \neq E$. Thus, the only pair of equal sets is C and E.

Example 8 Which of the following pairs of sets are equal? Justify your answer.

(i) X, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".

(ii) $A = \{n : n \in Z \text{ and } n^2 \leq 4\}$ and $B = \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$.

Solution (i) We have, X = {A, L, L, O, Y}, B = {L, O, Y, A, L}. Then X and B are equal sets as repetition of elements in a set do not change a set. Thus,

$$X = \{A, L, O, Y\} = B$$

(ii) $A = \{-2, -1, 0, 1, 2\}$, $B = \{1, 2\}$. Since $0 \in A$ and $0 \notin B$, A and B are not equal sets.

Example 9 Consider the sets

$$\phi,\ A = \{ 1, 3 \},\quad B = \{1, 5, 9\},\quad C = \{1, 3, 5, 7, 9\}.$$

Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets:

(i) $\phi \ldots B$ (ii) $A \ldots B$ (iii) $A \ldots C$ (iv) $B \ldots C$

Solution (i) $\phi \subset B$ as ϕ is a subset of every set.

(ii) $A \not\subset B$ as $3 \in A$ and $3 \notin B$

(iii) $A \subset C$ as $1, 3 \in A$ also belongs to C

(iv) $B \subset C$ as each element of B is also an element of C.

Example 10 Let $A = \{ a, e, i, o, u\}$ and $B = \{ a, b, c, d\}$. Is A a subset of B ? No. (Why?). Is B a subset of A? No. (Why?)

Example 11 Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$?. If not, give an example.

Solution No. Let $A = \{1\}$, $B = \{\{1\}, 2\}$ and $C = \{\{1\}, 2, 3\}$. Here $A \in B$ as $A = \{1\}$ and $B \subset C$. But $A \not\subset C$ as $1 \in A$ and $1 \notin C$.

Note that an element of a set can never be a subset of itself.

Example 12 Let A = { 2, 4, 6, 8} and B = { 6, 8, 10, 12}. Find $A \cup B$.

Solution We have $A \cup B = \{ 2, 4, 6, 8, 10, 12\}$

Note that the common elements 6 and 8 have been taken only once while writing $A \cup B$.

Example 13 Let A = { *a, e, i, o, u* } and B = { *a, i, u* }. Show that $A \cup B = A$

Solution We have, $A \cup B = \{ a, e, i, o, u \} = A$.

This example illustrates that union of sets A and its subset B is the set A itself, i.e., if $B \subset A$, then $A \cup B = A$.

Example 14 Let X = {Ram, Geeta, Akbar} be the set of students of Class XI, who are in school hockey team. Let Y = {Geeta, David, Ashok} be the set of students from Class XI who are in the school football team. Find $X \cup Y$ and interpret the set.

Solution We have, $X \cup Y$ = {Ram, Geeta, Akbar, David, Ashok}. This is the set of students from Class XI who are in the hockey team or the football team or both.

Example 15 Consider the sets A and B of Example 12. Find A ∩ B.

Solution We see that 6, 8 are the only elements which are common to both A and B. Hence A ∩ B = { 6, 8 }.

Example 16 Consider the sets X and Y of Example 14. Find X ∩ Y.

Solution We see that element 'Geeta' is the only element common to both. Hence, X ∩ Y = {Geeta}.

Example 17 Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = { 2, 3, 5, 7 }. Find A ∩ B and hence show that A ∩ B = B.

Solution We have A ∩ B = { 2, 3, 5, 7 } = B. We note that B ⊂ A and that A ∩ B = B.

Example 18 Let $A = \{ 1, 2, 3, 4, 5, 6\}$, $B = \{ 2, 4, 6, 8 \}$. Find $A - B$ and $B - A$.

Solution We have, $A - B = \{ 1, 3, 5 \}$, since the elements 1, 3, 5 belong to A but not to B and $B - A = \{ 8 \}$, since the element 8 belongs to B and not to A.
We note that $A - B \neq B - A$.

Example 19 Let V = { *a, e, i, o, u* } and B = { *a, i, k, u*}. Find V – B and B – V

Solution We have, V – B = { *e, o* }, since the elements *e, o* belong to V but not to B and B – V = { *k* }, since the element *k* belongs to B but not to V.

We note that V – B ≠ B – V. Using the set-builder notation, we can rewrite the definition of difference as

$$A - B = \{ x : x \in A \text{ and } x \notin B \}$$

The difference of two sets A and B can be represented by Venn diagram as shown in Fig 1.8.

The shaded portion represents the difference of the two sets A and B.

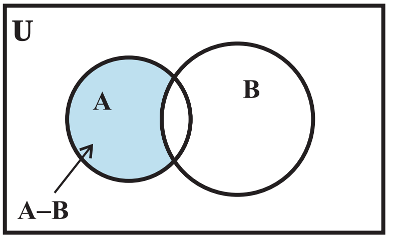

Fig 1.8

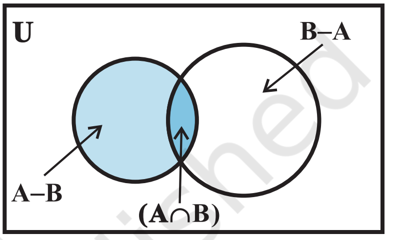

Fig 1.9

Remark The sets A – B, A ∩ B and B – A are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set as shown in Fig 1.9.

Example 20 Let U = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and A = $\{1, 3, 5, 7, 9\}$. Find A′.

Solution We note that 2, 4, 6, 8, 10 are the only elements of U which do not belong to A. Hence $A' = \{2, 4, 6, 8, 10\}$.

Example 21 Let U be universal set of all the students of Class XI of a coeducational school and A be the set of all girls in Class XI. Find A′.

Solution Since A is the set of all girls, A′ is clearly the set of all boys in the class.

Note If A is a subset of the universal set U, then its complement A′ is also a subset of U.

Again in Example 20 above, we have $A' = \{2, 4, 6, 8, 10\}$

Hence $(A')' = \{x : x \in U \text{ and } x \notin A'\}$

$= \{1, 3, 5, 7, 9\} = A$

It is clear from the definition of the complement that for any subset of the universal set U, we have $(A')' = A$

Now, we want to find the results for $(A \cup B)'$ and $A' \cap B'$ in the followng example.

Example 22 Let U = $\{1, 2, 3, 4, 5, 6\}$, A = $\{2, 3\}$ and B = $\{3, 4, 5\}$.
Find A′, B′, $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.

Solution Clearly $A' = \{1, 4, 5, 6\}$, $B' = \{1, 2, 6\}$. Hence $A' \cap B' = \{1, 6\}$

Also $A \cup B = \{2, 3, 4, 5\}$, so that $(A \cup B)' = \{1, 6\}$

$$(A \cup B)' = \{1, 6\} = A' \cap B'$$

It can be shown that the above result is true in general. If A and B are any two subsets of the universal set U, then

$(A \cup B)' = A' \cap B'$. Similarly, $(A \cap B)' = A' \cup B'$. These two results are stated in words as follows :

The complement of the union of two sets is the intersection of their complements and the complement of the intersection of two sets is the union of their complements. These are called *De Morgan's laws*. These are named after the mathematician De Morgan.

The complement A′ of a set A can be represented by a Venn diagram as shown in Fig 1.10.

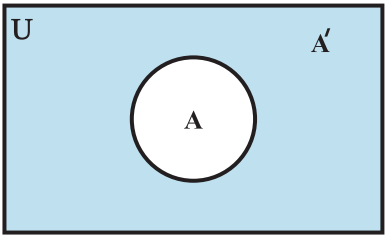

Fig 1.10

The shaded portion represents the complement of the set A.

Miscellaneous Examples

Example 23 Show that the set of letters needed to spell " CATARACT " and the set of letters needed to spell " TRACT" are equal.

Solution Let X be the set of letters in "CATARACT". Then

$$X = \{ C, A, T, R \}$$

Let Y be the set of letters in " TRACT". Then

$$Y = \{ T, R, A, C, T \} = \{ T, R, A, C \}$$

Since every element in X is in Y and every element in Y is in X. It follows that X = Y.

Example 24 List all the subsets of the set $\{ -1, 0, 1 \}$.

Solution Let $A = \{ -1, 0, 1 \}$. The subset of A having no element is the empty set ϕ. The subsets of A having one element are $\{ -1 \}$, $\{ 0 \}$, $\{ 1 \}$. The subsets of A having two elements are $\{-1, 0\}$, $\{-1, 1\}$, $\{0, 1\}$. The subset of A having three elements of A is A itself. So, all the subsets of A are ϕ, $\{-1\}$, $\{0\}$, $\{1\}$, $\{-1, 0\}$, $\{-1, 1\}$, $\{0, 1\}$ and $\{-1, 0, 1\}$.

Example 25 Show that $A \cup B = A \cap B$ implies $A = B$

Solution Let $a \in A$. Then $a \in A \cup B$. Since $A \cup B = A \cap B$, $a \in A \cap B$. So $a \in B$. Therefore, $A \subset B$. Similarly, if $b \in B$, then $b \in A \cup B$. Since

$A \cup B = A \cap B$, $b \in A \cap B$. So, $b \in A$. Therefore, $B \subset A$. Thus, $A = B$