



Miscellaneous Exercise on Chapter 4

1. Evaluate: $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$.
2. For any two complex numbers z_1 and z_2 , prove that $\operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$.

3. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form .
4. If $x-iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.
5. If $z_1 = 2-i, z_2 = 1+i$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$.
6. If $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.
7. Let $z_1 = 2-i, z_2 = -2+i$. Find
- (i) $\operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) $\operatorname{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$.
8. Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.
9. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.
10. If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$.
11. If α and β are different complex numbers with $|\beta|=1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.
12. Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.
13. If $(a+ib)(c+id)(e+if)(g+ih) = A+iB$, then show that $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$
14. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m .