EXERCISE 1.2

1. Show that the function $f : \mathbf{R}_* \to \mathbf{R}_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where $\mathbf{R}_*$ is the set of all non-zero real numbers. Is the result true, if the domain $\mathbf{R}_*$ is replaced by $\mathbf{N}$ with co-domain being same as $\mathbf{R}_*$?
2. Check the injectivity and surjectivity of the following functions:
   (i) $f : \mathbf{N} \to \mathbf{N}$ given by $f(x) = x^2$
   (ii) $f : \mathbf{Z} \to \mathbf{Z}$ given by $f(x) = x^2$
   (iii) $f : \mathbf{R} \to \mathbf{R}$ given by $f(x) = x^2$
   (iv) $f : \mathbf{N} \to \mathbf{N}$ given by $f(x) = x^3$
   (v) $f : \mathbf{Z} \to \mathbf{Z}$ given by $f(x) = x^3$
3. Prove that the Greatest Integer Function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

4. Show that the Modulus Function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

5. Show that the Signum Function $f : \mathbf{R} \to \mathbf{R}$, given by

$$f(x) = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ \quad 1, \text{ if } x < 0 \end{cases}$$

is neither one-one nor onto.

6. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.
   (i) $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 3 - 4x$
   (ii) $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 1 + x^2$

8. Let A and B be sets. Show that $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is bijective function.

9. Let $f : \mathbf{N} \to \mathbf{N}$ be defined by $f(n) = \begin{cases} \dfrac{n+1}{2}, \text{ if } n \text{ is odd} \\ \dfrac{n}{2}, \text{ if } n \text{ is even} \end{cases}$ for all $n \in \mathbf{N}$.

   State whether the function f is bijective. Justify your answer.

10. Let A = $\mathbf{R}$ – {3} and B = $\mathbf{R}$ – {1}. Consider the function $f : A \to B$ defined by $f(x) = \left(\dfrac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

11. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = x^4$. Choose the correct answer.
    (A) f is one-one onto
    (B) f is many-one onto
    (C) f is one-one but not onto
    (D) f is neither one-one nor onto.

12. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = 3x$. Choose the correct answer.
    (A) f is one-one onto
    (B) f is many-one onto
    (C) f is one-one but not onto
    (D) f is neither one-one nor onto.

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