

EXERCISE 1.1

1. Determine whether each of the following relations are reflexive, symmetric and transitive:
 - (i) Relation R in the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as
$$R = \{(x, y) : 3x - y = 0\}$$
 - (ii) Relation R in the set \mathbf{N} of natural numbers defined as
$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$
 - (iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as
$$R = \{(x, y) : y \text{ is divisible by } x\}$$
 - (iv) Relation R in the set \mathbf{Z} of all integers defined as
$$R = \{(x, y) : x - y \text{ is an integer}\}$$
 - (v) Relation R in the set A of human beings in a town at a particular time given by
 - (a) $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$
 - (b) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$
 - (c) $R = \{(x, y) : x \text{ is exactly } 7 \text{ cm taller than } y\}$
 - (d) $R = \{(x, y) : x \text{ is wife of } y\}$
 - (e) $R = \{(x, y) : x \text{ is father of } y\}$
2. Show that the relation R in the set \mathbf{R} of real numbers, defined as
$$R = \{(a, b) : a \leq b^2\}$$
is neither reflexive nor symmetric nor transitive.
3. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as
$$R = \{(a, b) : b = a + 1\}$$
is reflexive, symmetric or transitive.
4. Show that the relation R in \mathbf{R} defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.
5. Check whether the relation R in \mathbf{R} defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

6. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.
7. Show that the relation R in the set A of all the books in a library of a college, given by $R = \{(x, y) : x \text{ and } y \text{ have same number of pages}\}$ is an equivalence relation.
8. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.
9. Show that each of the relation R in the set $A = \{x \in \mathbf{Z} : 0 \leq x \leq 12\}$, given by
 - (i) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$
 - (ii) $R = \{(a, b) : a = b\}$
 is an equivalence relation. Find the set of all elements related to 1 in each case.
10. Give an example of a relation. Which is
 - (i) Symmetric but neither reflexive nor transitive.
 - (ii) Transitive but neither reflexive nor symmetric.
 - (iii) Reflexive and symmetric but not transitive.
 - (iv) Reflexive and transitive but not symmetric.
 - (v) Symmetric and transitive but not reflexive.
11. Show that the relation R in the set A of points in a plane given by $R = \{(P, Q) : \text{distance of the point } P \text{ from the origin is same as the distance of the point } Q \text{ from the origin}\}$, is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.
12. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1 , T_2 and T_3 are related?
13. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?
14. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

15. Let R be the relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer.
- (A) R is reflexive and symmetric but not transitive.
 - (B) R is reflexive and transitive but not symmetric.
 - (C) R is symmetric and transitive but not reflexive.
 - (D) R is an equivalence relation.
16. Let R be the relation in the set \mathbf{N} given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.
- (A) $(2, 4) \in R$ (B) $(3, 8) \in R$ (C) $(6, 8) \in R$ (D) $(8, 7) \in R$