

EXERCISE 3.3

Prove that:

1. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$

2. $2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$

3. $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$

4. $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3} = 10$

5. Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Prove the following:

6. $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$

7. $\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$

8. $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$

9. $\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x)\right] = 1$

10. $\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$

11. $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$

12. $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

14. $\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$

15. $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

16. $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

17. $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

18. $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$

19. $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

20. $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$

21. $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

22. $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

23. $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

24. $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

25. $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$