Example 1

If $(x+1, y-2) = (3,1)$, find the values of x and y.

Solution: Since the ordered pairs are equal, the corresponding elements are equal.

Therefore, $x+1=3$ and $y-2=1$.

Solving these equations, we get $x=2$ and $y=3$.

Example 2

If $P=\{a,b,c\}$ and $Q=\{r\}$, form the sets $P\times Q$ and $Q\times P$. Are these two products equal?

Solution: By the definition of the Cartesian product:

$P\times Q=\{(a,r),(b,r),(c,r)\}$

$Q\times P=\{(r,a),(r,b),(r,c)\}$

Since, by the definition of equality of ordered pairs, the pair (a,r) is not equal to the pair (r,a), we conclude that $P\times Q=Q\times P$.

However, the number of elements in each set will be the same.

Example 3

Let $A=\{1,2,3\}$, $B=\{3,4\}$ and $C=\{4,5,6\}$. Find:

(i) $A\times(B\cap C)$

(ii) $(A\times B)\cap(A\times C)$

(iii) $A\times(B\cup C)$

(iv) $(A\times B)\cup(A\times C)$

Solution:

(i) By the definition of the intersection of two sets, $(B\cap C)=\{4\}$.

Therefore, $A\times(B\cap C)=\{(1,4),(2,4),(3,4)\}$.

(ii) Now $(A\times B)=\{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$

And $(A\times C)=\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

Therefore, $(A \times B) \cap (A \times C) = \{(1,4),(2,4),(3,4)\}$.

(iii) Since, $(B \cup C) = \{3,4,5,6\}$, we have

$A \times (B \cup C) = \{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$.

(iv) Using the sets $A \times B$ and $A \times C$ from part (ii) above, we obtain

$(A \times B) \cup (A \times C) = \{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$.

Example 4

If $P = \{1,2\}$, form the set $P \times P \times P$.

Solution: We have, $P \times P \times P = \{(1,1,1),(1,1,2),(1,2,1),(1,2,2),(2,1,1),(2,1,2),(2,2,1),(2,2,2)\}$.

Example 5

If R is the set of all real numbers, what do the Cartesian products $R \times R$ and $R \times R \times R$ represent?

Solution: The Cartesian product $R \times R$ represents the set $R \times R = \{(x,y) : x,y \in R\}$, which represents the coordinates of all the points in two-dimensional space.

The Cartesian product $R \times R \times R$ represents the set $R \times R \times R = \{(x,y,z) : x,y,z \in R\}$, which represents the coordinates of all the points in three-dimensional space.

Example 6

If $A \times B = \{(p,q),(p,r),(m,q),(m,r)\}$, find A and B.

Solution:

A = set of first elements $= \{p,m\}$

B = set of second elements $= \{q,r\}$.

Example 7

Let $A = \{1,2,3,4,5,6\}$. Define a relation R from A to A by $R = \{(x,y) : y = x+1\}$.

(i) Depict this relation using an arrow diagram.

(ii) Write down the domain, codomain and range of R.

Solution:

(i) By the definition of the relation, R={(1,2),(2,3),(3,4),(4,5),(5,6)}.

(ii) The domain is {1,2,3,4,5}.

The range is {2,3,4,5,6}.

The codomain is {1,2,3,4,5,6}.

Example 8

The Fig 2.6 shows a relation between the sets P and Q. Write this relation (i) in set-builder form, (ii) in roster form. What is its domain and range?

Solution: The relation R is "x is the square of y".

(i) In set-builder form, $R=\{(x,y): x \text{ is the square of } y, x\in P, y\in Q\}$.

(ii) In roster form, $R=\{(9,3),(9,-3),(4,2),(4,-2),(25,5),(25,-5)\}$.

The domain of this relation is {4,9,25}.

The range of this relation is $\{-5,-3,-2,2,3,5\}$.

The set Q is the codomain of this relation.

Example 9

Let A={1,2} and B={3,4}. Find the number of relations from A to B.

Solution: We have, $A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$.

Since $n(A\times B)=4$, the number of subsets of $A\times B$ is 24.

Therefore, the number of relations from A into B will be 24.

Example 10

Let N be the set of natural numbers and the relation R be defined on N such that $R=\{(x,y):y=2x, x,y\in N\}$. What is the domain, codomain and range of R? Is this relation a function?

Solution: The domain of R is the set of natural numbers N. The codomain is also N.

The range is the set of even natural numbers.

Since every natural number n has one and only one image, this relation is a function.

Example 11

Examine each of the following relations given below and state in each case, giving reasons whether it is a function or not?

(i) R={(2,1),(3,1),(4,2)}

(ii) R={(2,2),(2,4),(3,3),(4,4)}

(iii) R={(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)}

Solution:

(i) Since 2, 3, 4 are the elements of the domain of R having their unique images, this relation R is a function.

(ii) Since the same first element 2 corresponds to two different images 2 and 4, this relation is not a function.

(iii) Since every element has one and only one image, this relation is a function.

Example 12

Let N be the set of natural numbers. Define a real valued function f:N→N by f(x)=2x+1. Using this definition, complete the table given below.

X	1	2	3	4	5	6	7
y	f(1)=...	f(2)=...	f(3)=...	f(4)=...	f(5)=...	f(6)=...	f(7)=...

Solution: The completed table is given by:

X	1	2	3	4	5	6	7
y	f(1)=3	f(2)=5	f(3)=7	f(4)=9	f(5)=11	f(6)=13	f(7)=15

Example 13

Define the function f:R→R by y=f(x)=x2,x∈R. Complete the Table given below by using this definition. What is the domain and range of this function? Draw the graph of f.

X	-4	-3	-2	-1	0	1	2	3	4
y=f(x)=x2									

Solution: The completed Table is given below:

X	-4	-3	-2	-1	0	1	2	3	4
y=f(x)=x2	16	9	4	1	0	1	4	9	16

Domain of f={x:x∈R}.

Range of f={x2:x∈R}.

Example 14

Draw the graph of the function f:R→R defined by f(x)=x3,x∈R.

Solution: We have f(0)=0,f(1)=1,f(−1)=−1,f(2)=8,f(−2)=−8,f(3)=27,f(−3)=−27, etc.

Therefore, f={(x,x3):x∈R}. The graph of f is given in Fig 2.11.

Example 15

Define the real valued function f:R−{0}→R defined by f(x)=x1,x∈R−{0}. Complete the Table given below using this definition. What is the domain and range of this function?

x	-2	-1.5	-1	-0.5	0.25	0.5	1	1.5	2
y=x1	...	...	...	...	...	...	...	...	...

Solution: The completed Table is given by:

x	-2	-1.5	-1	-0.5	0.25	0.5	1	1.5	2

y=x1	-0.5	-0.67	-1	-2	4	2	1	0.67	0.5

The domain is all real numbers except 0 and its range is also all real numbers except 0.

Example 16

Let f(x)=x2 and g(x)=2x+1 be two real functions. Find (f+g)(x),(f−g)(x),(fg)(x),(gf)(x).

Solution:

(f+g)(x)=x2+2x+1

(f−g)(x)=x2−2x−1

(fg)(x)=x2(2x+1)=2x3+x2

(gf)(x)=2x+1x2, x=−21.

Example 17

Let f(x)=x and g(x)=x be two functions defined over the set of non-negative real numbers. Find (f+g)(x),(f−g)(x),(fg)(x) and (gf)(x).

Solution:

(f+g)(x)=x+x

(f−g)(x)=x−x

(fg)(x)=x(x)=x23

(gf)(x)=xx=x−21, x=0.

Example 18

Let R be the set of real numbers. Define the real function f:R→R by f(x)=x+10 and sketch the graph of this function[cite:1 333, 334].

Solution: Here f(0)=10,f(1)=11,f(2)=12,…,f(10)=20, etc., and f(−1)=9,f(−2)=8,…,f(−10)=0 and so on. The shape of the graph of the given function assumes the form as shown in Fig 2.16.

Example 19

Let R be a relation from Q to Q defined by R={(a,b):a,b∈Q and a−b∈Z}. Show that:

(i) $(a,a) \in R$ for all $a \in Q$

(ii) $(a,b) \in R$ implies that $(b,a) \in R$

(iii) $(a,b) \in R$ and $(b,c) \in R$ implies that $(a,c) \in R$

Solution:

(i) Since $a-a=0 \in Z$, it follows that $(a,a) \in R$.

(ii) $(a,b) \in R$ implies that $a-b \in Z$. So, $b-a=-(a-b) \in Z$. Therefore, $(b,a) \in R$.

(iii) $(a,b) \in R$ and $(b,c) \in R$ implies that $a-b \in Z$ and $b-c \in Z$.

So, $a-c=(a-b)+(b-c) \in Z$. Therefore, $(a,c) \in R$.

Example 20

Let $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a linear function from Z into Z. Find $f(x)$.

Solution: Since f is a linear function, $f(x)=mx+c$.

Also, since $(1,1) \in f$ and $(0,-1) \in f$, we have:

$f(1)=m(1)+c=1 \Rightarrow m+c=1$ (Equation 1)

$f(0)=m(0)+c=-1 \Rightarrow c=-1$ (Equation 2)

Substitute $c=-1$ into Equation 1:

$m+(-1)=1 \Rightarrow m=2$.

So, $f(x)=2x-1$.

Let's verify this with the other points:

For $(2,3)$:$f(2)=2(2)-1=4-1=3$. This matches.

For $(-1,-3)$:$f(-1)=2(-1)-1=-2-1=-3$. This matches.

Thus, $f(x)=2x-1$.

Example 21

Find the domain of the function

$$f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$$

Solution:
We are given:

$$f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$$

To find the domain, we must exclude values of x that make the denominator zero.

Factor the denominator:

$$x^2 - 5x + 4 = (x - 4)(x - 1)$$

So, the function is **undefined at** $x = 1$ and $x = 4$.
Hence, the **domain** is:

$$\boxed{\mathbb{R} \setminus \{1, 4\}}$$

Example 22

The function f is defined by

$$f(x) = \begin{cases} 1 - x, & x < 0 \\ 0, & x = 0 \\ x + 1, & x > 0 \end{cases}$$

Draw the graph of $f(x)$.

Solution:
Break down the function:

- For $x < 0$: $f(x) = 1 - x$
  Examples:
  $f(-4) = 5, f(-3) = 4, f(-2) = 3, f(-1) = 2$

- For $x = 0$: $f(0) = 0$

- For $x > 0$: $f(x) = x + 1$
  Examples:
  $f(1) = 2, f(2) = 3, f(3) = 4, f(4) = 5$

This is a piecewise function forming a **V-shaped graph**, symmetric about the origin, with a "corner" at $(0, 0)$.