Miscellaneous Exercise on Chapter 2

1. The relation f is defined by $f(x) = \begin{cases} x^2, 0 \le x \le 3 \\ 3x, 3 \le x \le 10 \end{cases}$

The relation g is defined by $g(x) = \begin{cases} x^2, 0 \le x \le 2 \\ 3x, 2 \le x \le 10 \end{cases}$

Show that f is a function and g is not a function.

2. If $f(x) = x^2$, find $\dfrac{f(1.1) - f(1)}{(1.1 - 1)}$.

3. Find the domain of the function $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

4. Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x-1)}$.

5. Find the domain and the range of the real function f defined by $f(x) = |x - 1|$.

6. Let $f = \left\{ \left(x, \dfrac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$ be a function from **R** into **R**. Determine the range of f.

7. Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\dfrac{f}{g}$.

8. Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a function from **Z** to **Z** defined by $f(x) = ax + b$, for some integers a, b. Determine a, b.

9. Let R be a relation from **N** to **N** defined by $R = \{(a, b) : a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?

(i) $(a,a) \in R$, for all $a \in \mathbf{N}$ (ii) $(a,b) \in R$, implies $(b,a) \in R$

(iii) $(a,b) \in R$, $(b,c) \in R$ implies $(a,c) \in R$.

Justify your answer in each case.

10. Let $A = \{1,2,3,4\}$, $B = \{1,5,9,11,15,16\}$ and $f = \{(1,5), (2,9), (3,1), (4,5), (2,11)\}$ Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case.

11. Let f be the subset of $\mathbf{Z} \times \mathbf{Z}$ *defined by* $f = \{(ab, a + b) : a, b \in \mathbf{Z}\}$. Is f a function from $\mathbf{Z}$ to $\mathbf{Z}$? Justify your answer.

12. Let $A = \{9,10,11,12,13\}$ and let $f: A \rightarrow \mathbf{N}$ be defined by $f(n)$ = the highest prime factor of n. Find the range of f.