Miscellaneous Exercise on Chapter 1

1. Decide, among the following sets, which sets are subsets of one and another:
$A = \{ x : x \in \mathbf{R}$ and x satisfy $x^2 - 8x + 12 = 0 \}$,
$B = \{ 2, 4, 6 \}$, $C = \{ 2, 4, 6, 8, \ldots \}$, $D = \{ 6 \}$.

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x \in A$ and $A \in B$, then $x \in B$
(ii) If $A \subset B$ and $B \in C$, then $A \in C$
(iii) If $A \subset B$ and $B \subset C$, then $A \subset C$
(iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$
(v) If $x \in A$ and $A \not\subset B$, then $x \in B$
(vi) If $A \subset B$ and $x \notin B$, then $x \notin A$

3. Let A, B, and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

4. Show that the following four conditions are equivalent :

(i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$

5. Show that if $A \subset B$, then $C - B \subset C - A$.

6. Show that for any sets A and B,
$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = (A \cup B)$

7. Using properties of sets, show that

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$.

8. Show that $A \cap B = A \cap C$ need not imply $B = C$.

9. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that A = B.
(**Hints** $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use Distributive law)

10. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.