Example 1 Solve $30x < 200$ when

(i) x is a natural number, (ii) x is an integer.

Solution We are given $30x < 200$

or $\frac{30x}{30} < \frac{200}{30}$ (Rule 2), i.e., $x < 20/3$.

(i) When x is a natural number, in this case the following values of x make the statement true.

1, 2, 3, 4, 5, 6.

The solution set of the inequality is {1,2,3,4,5,6}.

(ii) When x is an integer, the solutions of the given inequality are

..., – 3, –2, –1, 0, 1, 2, 3, 4, 5, 6

The solution set of the inequality is {...,–3, –2,–1, 0, 1, 2, 3, 4, 5, 6}

Example 2 Solve $5x - 3 < 3x + 1$ when

(i) x is an integer, (ii) x is a real number.

Solution We have, $5x - 3 < 3x + 1$

or $5x - 3 + 3 < 3x + 1 + 3$ (Rule 1)

or $5x < 3x + 4$

or $5x - 3x < 3x + 4 - 3x$ (Rule 1)

or $2x < 4$

or $x < 2$ (Rule 2)

(i) When x is an integer, the solutions of the given inequality are

..., – 4, – 3, – 2, – 1, 0, 1

(ii) When x is a real number, the solutions of the inequality are given by $x < 2$, i.e., all real numbers x which are less than 2. Therefore, the solution set of the inequality is $x \in (-\infty, 2)$.

We have considered solutions of inequalities in the set of natural numbers, set of integers and in the set of real numbers. Henceforth, unless stated otherwise, we shall solve the inequalities in this Chapter in the set of real numbers.

Example 3 Solve $4x + 3 < 6x + 7$.

Solution We have, $\quad 4x + 3 < 6x + 7$

or $\quad 4x - 6x < 6x + 4 - 6x$

or $\quad -2x < 4 \quad$ or $\quad x > -2$

i.e., all the real numbers which are greater than -2, are the solutions of the given inequality. Hence, the solution set is $(-2, \infty)$.

Example 4 Solve $\frac{5-2x}{3} \leq \frac{x}{6} - 5$.

Solution We have

$$\frac{5-2x}{3} \leq \frac{x}{6} - 5$$

or $\quad 2\,(5 - 2x) \leq x - 30.$

or $\quad 10 - 4x \leq x - 30$

or $\quad -5x \leq -40, \quad$ i.e., $x \geq 8$

Thus, all real numbers x which are greater than or equal to 8 are the solutions of the given inequality, i.e., $x \in [8, \infty)$.

Example 5 Solve $7x + 3 < 5x + 9$. Show the graph of the solutions on number line.

Solution We have $7x + 3 < 5x + 9$ or

$$2x < 6 \text{ or } x < 3$$

The graphical representation of the solutions are given in Fig 5.1.

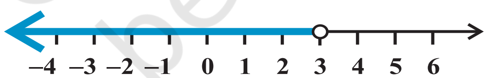

Fig 5.1

Example 6 Solve $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$. Show the graph of the solutions on number line.

Solution We have

$$\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$$

or $\quad \frac{3x-4}{2} \geq \frac{x-3}{4}$

or $\quad 2\,(3x - 4) \geq (x - 3)$

or $\qquad 6x - 8 \geq x - 3$

or $\qquad 5x \geq 5$ or $x \geq 1$

The graphical representation of solutions is given in Fig 5.2.

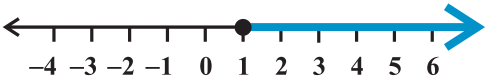

Fig 5.2

Example 7 The marks obtained by a student of Class XI in first and second terminal examination are 62 and 48, respectively. Find the minimum marks he should get in the annual examination to have an average of at least 60 marks.

Solution Let x be the marks obtained by student in the annual examination. Then

$$\frac{62+48+x}{3} \geq 60$$

or $\qquad 110 + x \geq 180$

or $\qquad x \geq 70$

Thus, the student must obtain a minimum of 70 marks to get an average of at least 60 marks.

Example 8 Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

Solution Let x be the smaller of the two consecutive odd natural number, so that the other one is x +2. Then, we should have

$x > 10 \qquad$ ... (1)

and $\quad x + (x + 2) < 40 \qquad$ ... (2)

Solving (2), we get

$2x + 2 < 40$

i.e., $\quad x < 19 \qquad$ ... (3)

From (1) and (3), we get

$10 < x < 19$

Since x is an odd number, x can take the values 11, 13, 15, and 17. So, the required possible pairs will be

(11, 13), (13, 15), (15, 17), (17, 19)

Miscellaneous Examples

Example 9 Solve $-8 \leq 5x - 3 < 7$.

Solution In this case, we have two inequalities, $-8 \leq 5x - 3$ and $5x - 3 < 7$, which we will solve simultaneously. We have $-8 \leq 5x - 3 < 7$

or $\quad -5 \leq 5x < 10 \quad$ or $\quad -1 \leq x < 2$

Example 10 Solve $-5 \leq \dfrac{5-3x}{2} \leq 8$.

Solution We have $\quad -5 \leq \dfrac{5-3x}{2} \leq 8$

or $\quad -10 \leq 5 - 3x \leq 16 \quad$ or $\quad -15 \leq -3x \leq 11$

or $\quad 5 \geq x \geq -\dfrac{11}{3}$

which can be written as $\dfrac{-11}{3} \leq x \leq 5$

Example 11 Solve the system of inequalities:

$$3x - 7 < 5 + x \qquad \text{... (1)}$$

$$11 - 5x \leq 1 \qquad \text{... (2)}$$

and represent the solutions on the number line.

Solution From inequality (1), we have

$$3x - 7 < 5 + x$$

or $\quad x < 6 \qquad$ ... (3)

Also, from inequality (2), we have

$$11 - 5x \leq 1$$

or $\quad -5x \leq -10 \quad$ i.e., $x \geq 2 \qquad$ ... (4)

If we draw the graph of inequalities (3) and (4) on the number line, we see that the values of x, which are common to both, are shown by bold line in Fig 5.3.

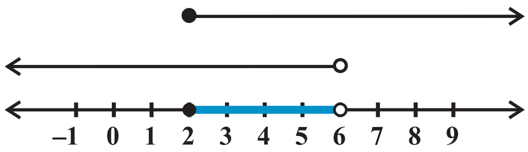

Fig 5.3

Thus, solution of the system are real numbers x lying between 2 and 6 including 2, i.e.,

$$2 \leq x < 6$$

Example 12 In an experiment, a solution of hydrochloric acid is to be kept between 30° and 35° Celsius. What is the range of temperature in degree Fahrenheit if conversion formula is given by $C = \frac{5}{9}(F - 32)$, where C and F represent temperature in degree Celsius and degree Fahrenheit, respectively.

Solution It is given that $30 < C < 35$.

Putting $\quad C = \frac{5}{9}(F - 32)$, we get

$$30 < \frac{5}{9}(F - 32) < 35,$$

or $\quad \frac{9}{5} \times (30) < (F - 32) < \frac{9}{5} \times (35)$

or $\quad 54 < (F - 32) < 63$

or $\quad 86 < F < 95.$

Thus, the required range of temperature is between 86° F and 95° F.

Example 13 A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Solution Let x litres of 30% acid solution is required to be added. Then

Total mixture = $(x + 600)$ litres

Therefore $\quad 30\%\, x + 12\%$ of $600 > 15\%$ of $(x + 600)$

and $\quad 30\%\, x + 12\%$ of $600 < 18\%$ of $(x + 600)$

or $\quad \frac{30x}{100} + \frac{12}{100}(600) > \frac{15}{100}(x + 600)$

and $\quad \frac{30x}{100} + \frac{12}{100}(600) < \frac{18}{100}(x + 600)$

or $\quad 30x + 7200 > 15x + 9000$

and $\quad 30x + 7200 < 18x + 10800$

or $\quad 15x > 1800$ and $12x < 3600$

or $\quad x > 120$ and $x < 300$,

i.e. $\quad 120 < x < 300$

Thus, the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.