Solution Given any element a in A, both a and a must be either odd or even, so that $(a, a) \in$ R. Further, $(a, b) \in$ R $\Rightarrow$ both a and b must be either odd or even $\Rightarrow (b, a) \in$ R. Similarly, $(a, b) \in$ R and $(b, c) \in$ R $\Rightarrow$ all elements a, b, c, must be either even or odd simultaneously $\Rightarrow (a, c) \in$ R. Hence, R is an equivalence relation. Further, all the elements of {1, 3, 5, 7} are related to each other, as all the elements of this subset are odd. Similarly, all the elements of the subset {2, 4, 6} are related to each other, as all of them are even. Also, no element of the subset {1, 3, 5, 7} can be related to any element of {2, 4, 6}, as elements of {1, 3, 5, 7} are odd, while elements of {2, 4, 6} are even.

EXERCISE 1.1

1. Determine whether each of the following relations are reflexive, symmetric and transitive:
   (i) Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as
   $$R = \{(x, y) : 3x - y = 0\}$$
   (ii) Relation R in the set **N** of natural numbers defined as
   $$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$
   (iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as
   $$R = \{(x, y) : y \text{ is divisible by } x\}$$
   (iv) Relation R in the set **Z** of all integers defined as
   $$R = \{(x, y) : x - y \text{ is an integer}\}$$
   (v) Relation R in the set A of human beings in a town at a particular time given by
   (a) R = {(x, y) : x and y work at the same place}
   (b) R = {(x, y) : x and y live in the same locality}
   (c) R = {(x, y) : x is exactly 7 cm taller than y}
   (d) R = {(x, y) : x is wife of y}
   (e) R = {(x, y) : x is father of y}
2. Show that the relation R in the set **R** of real numbers, defined as R = $\{(a, b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.
3. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = $\{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.
4. Show that the relation R in **R** defined as R = $\{(a, b) : a \le b\}$, is reflexive and transitive but not symmetric.
5. Check whether the relation R in **R** defined by R = $\{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive.

© NCERT not to be republished

6. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

7. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by

R = {(a, b) : $|a - b|$ is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

9. Show that each of the relation R in the set A = $\{x \in \mathbf{Z} : 0 \le x \le 12\}$, given by

(i) R = {(a, b) : $|a - b|$ is a multiple of 4}

(ii) R = {(a, b) : $a = b$}

is an equivalence relation. Find the set of all elements related to 1 in each case.

10. Give an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

11. Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P $\neq$ (0, 0) is the circle passing through P with origin as centre.

12. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

13. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

14. Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L_1, L_2) : L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

© NCERT not to be republished

15. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

16. Let R be the relation in the set **N** given by R = $\{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.

(A) $(2, 4) \in$ R (B) $(3, 8) \in$ R (C) $(6, 8) \in$ R (D) $(8, 7) \in$ R

1.3 Types of Functions

The notion of a function along with some special functions like identity function, constant function, polynomial function, rational function, modulus function, signum function etc. along with their graphs have been given in Class XI.

Addition, subtraction, multiplication and division of two functions have also been studied. As the concept of function is of paramount importance in mathematics and among other disciplines as well, we would like to extend our study about function from where we finished earlier. In this section, we would like to study different types of functions.

Consider the functions f_1, f_2, f_3 and f_4 given by the following diagrams.

In Fig 1.2, we observe that the images of distinct elements of X_1 under the function f_1 are distinct, but the image of two distinct elements 1 and 2 of X_1 under f_2 is same, namely b. Further, there are some elements like e and f in X_2 which are not images of any element of X_1 under f_1, while all elements of X_3 are images of some elements of X_1 under f_3. The above observations lead to the following definitions:

Definition 5 A function $f : X \to Y$ is defined to be *one-one* (or *injective*), if the images of distinct elements of X under f are distinct, i.e., for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$. Otherwise, f is called *many-one*.

The function f_1 and f_4 in Fig 1.2 (i) and (iv) are one-one and the function f_2 and f_3 in Fig 1.2 (ii) and (iii) are many-one.

Definition 6 A function $f : X \to Y$ is said to be *onto* (or *surjective*), if every element of Y is the image of some element of X under f, i.e., for every $y \in Y$, there exists an element x in X such that $f(x) = y$.

The function f_3 and f_4 in Fig 1.2 (iii), (iv) are onto and the function f_1 in Fig 1.2 (i) is not onto as elements e, f in X_2 are not the image of any element in X_1 under f_1.

© NCERT not to be republished