Miscellaneous Exercise on Chapter 1

1. Show that the function $f : \mathbf{R} \to \{x \in \mathbf{R} : -1 < x < 1\}$ defined by $f(x) = \dfrac{x}{1 + |x|}$, $x \in \mathbf{R}$ is one one and onto function.

2. Show that the function $f : \mathbf{R} \to \mathbf{R}$ given by $f(x) = x^3$ is injective.

3. Given a non empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows:

   For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on P(X)? Justify your answer.

4. Find the number of all onto functions from the set $\{1, 2, 3, ..., n\}$ to itself.

5. Let A = $\{-1, 0, 1, 2\}$, B = $\{-4, -2, 0, 2\}$ and $f, g : A \to B$ be functions defined by $f(x) = x^2 - x$, $x \in A$ and $g(x) = 2\left|x - \dfrac{1}{2}\right| - 1$, $x \in A$. Are f and g equal?

   Justify your answer. (Hint: One may note that two functions $f : A \to B$ and $g : A \to B$ such that $f(a) = g(a) \ \forall \ a \in A$, are called equal functions).

6. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(A) 1   (B) 2   (C) 3   (D) 4

7. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is

(A) 1   (B) 2   (C) 3   (D) 4