EXERCISE 1.1

1. Which of the following are sets ? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world.

2. Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

(i) 5. . .A (ii) 8 . . . A (iii) 0. . .A

(iv) 4. . . A (v) 2. . .A (vi) 10. . .A

3. Write the following sets in roster form:

(i) A = $\{x : x$ is an integer and $-3 \leq x < 7\}$

(ii) B = $\{x : x$ is a natural number less than 6$\}$

(iii) C = $\{x : x$ is a two-digit natural number such that the sum of its digits is 8$\}$

(iv) D = $\{x : x$ is a prime number which is divisor of 60$\}$

(v) E = The set of all letters in the word TRIGONOMETRY

(vi) F = The set of all letters in the word BETTER

4. Write the following sets in the set-builder form :

(i) {3, 6, 9, 12} (ii) {2,4,8,16,32} (iii) {5, 25, 125, 625}

(iv) {2, 4, 6, . . .} (v) {1,4,9, . . .,100}

5. List all the elements of the following sets :

(i) A = $\{x : x$ is an odd natural number$\}$

(ii) B = $\{x : x$ is an integer, $-\frac{1}{2} < x < \frac{9}{2}\}$

(iii) C = $\{x : x$ is an integer, $x^2 \leq 4\}$

(iv) D = $\{x : x$ is a letter in the word "LOYAL"$\}$

(v) E = $\{x : x$ is a month of a year not having 31 days$\}$

(vi) F = $\{x : x$ is a consonant in the English alphabet which precedes $k\}$.

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) {1, 2, 3, 6} (a) $\{x : x$ is a prime number and a divisor of 6$\}$

(ii) {2, 3} (b) $\{x : x$ is an odd natural number less than 10$\}$

(iii) {M,A,T,H,E,I,C,S} (c) $\{x : x$ is natural number and divisor of 6$\}$

(iv) {1, 3, 5, 7, 9} (d) $\{x : x$ is a letter of the word MATHEMATICS$\}$.

EXERCISE 1.2

1. Which of the following are examples of the null set

(i) Set of odd natural numbers divisible by 2

(ii) Set of even prime numbers

(iii) $\{ x : x$ is a natural numbers, $x < 5$ and $x > 7 \}$

(iv) $\{ y : y$ is a point common to any two parallel lines$\}$

2. Which of the following sets are finite or infinite

(i) The set of months of a year

(ii) $\{1, 2, 3, \ldots\}$

(iii) $\{1, 2, 3, \ldots 99, 100\}$

(iv) The set of positive integers greater than 100

(v) The set of prime numbers less than 99

3. State whether each of the following set is finite or infinite:

(i) The set of lines which are parallel to the x-axis

(ii) The set of letters in the English alphabet

(iii) The set of numbers which are multiple of 5

(iv) The set of animals living on the earth

(v) The set of circles passing through the origin (0,0)

4. In the following, state whether A = B or not:

(i) A = { a, b, c, d } B = { d, c, b, a }

(ii) A = { 4, 8, 12, 16 } B = { 8, 4, 16, 18}

(iii) A = {2, 4, 6, 8, 10} B = { x : x is positive even integer and $x \leq 10$}

(iv) A = { x : x is a multiple of 10}, B = { 10, 15, 20, 25, 30, . . . }

5. Are the following pair of sets equal ? Give reasons.

(i) A = {2, 3}, B = {x : x is solution of $x^2 + 5x + 6 = 0$}

(ii) A = { x : x is a letter in the word FOLLOW}

B = { y : y is a letter in the word WOLF}

6. From the sets given below, select equal sets :

A = { 2, 4, 8, 12}, B = { 1, 2, 3, 4}, C = { 4, 8, 12, 14}, D = { 3, 1, 4, 2}

E = {–1, 1}, F = { 0, a}, G = {1, –1}, H = { 0, 1}

EXERCISE 1.3

1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces :

(i) $\{2, 3, 4\}$ . . . $\{1, 2, 3, 4, 5\}$ (ii) $\{a, b, c\}$ . . . $\{b, c, d\}$

(iii) $\{x : x$ is a student of Class XI of your school$\}$ . . . $\{x : x$ student of your school$\}$

(iv) $\{x : x$ is a circle in the plane$\}$ . . . $\{x : x$ is a circle in the same plane with radius 1 unit$\}$

(v) $\{x : x$ is a triangle in a plane$\}$ . . . $\{x : x$ is a rectangle in the plane$\}$

(vi) $\{x : x$ is an equilateral triangle in a plane$\}$ . . . $\{x : x$ is a triangle in the same plane$\}$

(vii) $\{x : x$ is an even natural number$\}$ . . . $\{x : x$ is an integer$\}$

2. Examine whether the following statements are true or false:

(i) $\{a, b\} \not\subset \{b, c, a\}$

(ii) $\{a, e\} \subset \{x : x$ is a vowel in the English alphabet$\}$

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$

(iv) $\{a\} \subset \{a, b, c\}$

(v) $\{a\} \in \{a, b, c\}$

(vi) $\{x : x$ is an even natural number less than 6$\} \subset \{x : x$ is a natural number which divides 36$\}$

3. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ (iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$ (v) $1 \subset A$ (vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$ (viii) $\{1, 2, 3\} \subset A$ (ix) $\phi \in A$

(x) $\phi \subset A$ (xi) $\{\phi\} \subset A$

4. Write down all the subsets of the following sets

(i) $\{a\}$ (ii) $\{a, b\}$ (iii) $\{1, 2, 3\}$ (iv) ϕ

5. Write the following as intervals :

(i) $\{x : x \in R, -4 < x \leq 6\}$ (ii) $\{x : x \in R, -12 < x < -10\}$

(iii) $\{x : x \in R, 0 \leq x < 7\}$ (iv) $\{x : x \in R, 3 \leq x \leq 4\}$

6. Write the following intervals in set-builder form :

(i) $(-3, 0)$ (ii) $[6, 12]$ (iii) $(6, 12]$ (iv) $[-23, 5)$

7. What universal set(s) would you propose for each of the following :

(i) The set of right triangles. (ii) The set of isosceles triangles.

8. Given the sets A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}, which of the following may be considered as universal set (s) for all the three sets A, B and C

(i) {0, 1, 2, 3, 4, 5, 6}

(ii) ϕ

(iii) {0,1,2,3,4,5,6,7,8,9,10}

(iv) {1,2,3,4,5,6,7,8}

EXERCISE 1.4

1. Find the union of each of the following pairs of sets :

(i) X = {1, 3, 5} Y = {1, 2, 3}

(ii) A = [*a, e, i, o, u*} B = {*a, b, c*}

(iii) A = {*x* : *x* is a natural number and multiple of 3}
B = {*x* : *x* is a natural number less than 6}

(iv) A = {*x* : *x* is a natural number and $1 < x \leq 6$ }
B = {*x* : *x* is a natural number and $6 < x < 10$ }

(v) A = {1, 2, 3}, B = ϕ

2. Let A = { *a, b* }, B = {*a, b, c*}. Is A $\subset$ B ? What is A $\cup$ B ?

3. If A and B are two sets such that A $\subset$ B, then what is A $\cup$ B ?

4. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8 }and D = { 7, 8, 9, 10 }; find

(i) A $\cup$ B (ii) A $\cup$ C (iii) B $\cup$ C (iv) B $\cup$ D

(v) A $\cup$ B $\cup$ C (vi) A $\cup$ B $\cup$ D (vii) B $\cup$ C $\cup$ D

5. Find the intersection of each pair of sets of question 1 above.

6. If A = { 3, 5, 7, 9, 11 }, B = {7, 9, 11, 13}, C = {11, 13, 15}and D = {15, 17}; find

(i) A $\cap$ B (ii) B $\cap$ C (iii) A $\cap$ C $\cap$ D

(iv) A $\cap$ C (v) B $\cap$ D (vi) A $\cap$ (B $\cup$ C)

(vii) A $\cap$ D (viii) A $\cap$ (B $\cup$ D) (ix) (A $\cap$ B) $\cap$ (B $\cup$ C)

(x) (A $\cup$ D) $\cap$ (B $\cup$ C)

7. If A = $\{x : x$ is a natural number $\}$, B = $\{x : x$ is an even natural number$\}$
C = $\{x : x$ is an odd natural number$\}$ and D = $\{x : x$ is a prime number $\}$, find

(i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$
(iv) $B \cap C$ (v) $B \cap D$ (vi) $C \cap D$

8. Which of the following pairs of sets are disjoint

(i) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \le x \le 6\}$
(ii) $\{ a, e, i, o, u \}$ and $\{ c, d, e, f \}$
(iii) $\{x : x$ is an even integer $\}$ and $\{x : x$ is an odd integer$\}$

9. If A = $\{3, 6, 9, 12, 15, 18, 21\}$, B = $\{ 4, 8, 12, 16, 20 \}$,
C = $\{ 2, 4, 6, 8, 10, 12, 14, 16 \}$, D = $\{5, 10, 15, 20 \}$; find

(i) A – B (ii) A – C (iii) A – D (iv) B – A
(v) C – A (vi) D – A (vii) B – C (viii) B – D
(ix) C – B (x) D – B (xi) C – D (xii) D – C

10. If X= $\{ a, b, c, d \}$ and Y = $\{ f, b, d, g\}$, find

(i) X – Y (ii) Y – X (iii) $X \cap Y$

11. If **R** is the set of real numbers and **Q** is the set of rational numbers, then what is **R** – **Q**?

12. State whether each of the following statement is true or false. Justify your answer.

(i) $\{ 2, 3, 4, 5 \}$ and $\{ 3, 6\}$ are disjoint sets.
(ii) $\{ a, e, i, o, u \}$ and $\{ a, b, c, d \}$ are disjoint sets.
(iii) $\{ 2, 6, 10, 14 \}$ and $\{ 3, 7, 11, 15\}$ are disjoint sets.
(iv) $\{ 2, 6, 10 \}$ and $\{ 3, 7, 11\}$ are disjoint sets.

EXERCISE 1.5

1. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = { 1, 2, 3, 4}, B = { 2, 4, 6, 8 } and C = { 3, 4, 5, 6 }. Find (i) A′ (ii) B′ (iii) (A ∪ C)′ (iv) (A ∪ B)′ (v) (A′)′ (vi) (B – C)′

2. If U = { *a, b, c, d, e, f, g, h*}, find the complements of the following sets :

(i) A = {*a, b, c*} (ii) B = {*d, e, f, g*}

(iii) C = {*a, c, e, g*} (iv) D = { *f, g, h, a*}

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) {*x* : *x* is an even natural number} (ii) { *x* : *x* is an odd natural number }

(iii) {*x* : *x* is a positive multiple of 3} (iv) { *x* : *x* is a prime number }

(v) {*x* : *x* is a natural number divisible by 3 and 5}

(vi) { *x* : *x* is a perfect square } (vii) { *x* : *x* is a perfect cube}

(viii) { *x* : *x* + 5 = 8 } (ix) { *x* : 2*x* + 5 = 9}

(x) { *x* : *x* ≥ 7 } (xi) { *x* : *x* ∈ N and 2*x* + 1 > 10 }

4. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = {2, 4, 6, 8} and B = { 2, 3, 5, 7}. Verify that

(i) (A ∪ B)′ = A′ ∩ B′ (ii) (A ∩ B)′ = A′ ∪ B′

5. Draw appropriate Venn diagram for each of the following :

(i) (A ∪ B)′, (ii) A′ ∩ B′, (iii) (A ∩ B)′, (iv) A′ ∪ B′

6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A′?

7. Fill in the blanks to make each of the following a true statement :

(i) A ∪ A′ = . . . (ii) ϕ′ ∩ A = . . .

(iii) A ∩ A′ = . . . (iv) U′ ∩ A = . . .

Miscellaneous Exercise on Chapter 1

1. Decide, among the following sets, which sets are subsets of one and another:
$A = \{ x : x \in \mathbf{R}$ and x satisfy $x^2 - 8x + 12 = 0 \}$,
$B = \{ 2, 4, 6 \}$, $C = \{ 2, 4, 6, 8, \ldots \}$, $D = \{ 6 \}$.

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x \in A$ and $A \in B$, then $x \in B$
(ii) If $A \subset B$ and $B \in C$, then $A \in C$
(iii) If $A \subset B$ and $B \subset C$, then $A \subset C$
(iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$
(v) If $x \in A$ and $A \not\subset B$, then $x \in B$
(vi) If $A \subset B$ and $x \notin B$, then $x \notin A$

3. Let A, B, and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

4. Show that the following four conditions are equivalent :

(i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$

5. Show that if $A \subset B$, then $C - B \subset C - A$.

6. Show that for any sets A and B,
$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = (A \cup B)$

7. Using properties of sets, show that

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$.

8. Show that $A \cap B = A \cap C$ need not imply $B = C$.

9. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that A = B.
(**Hints** $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use Distributive law)

10. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.