Miscellaneous Exercise on Chapter 3

Prove that:

1. $2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13}+\cos\dfrac{3\pi}{13}+\cos\dfrac{5\pi}{13}=0$

2. $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x = 0$

3. $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\dfrac{x+y}{2}$

4. $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \dfrac{x-y}{2}$

5. $\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cos 2x \sin 4x$

6. $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

7. $\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \dfrac{x}{2} \cos \dfrac{3x}{2}$

Find $\sin \dfrac{x}{2}$, $\cos \dfrac{x}{2}$ and $\tan \dfrac{x}{2}$ in each of the following :

8. $\tan x = -\dfrac{4}{3}$, x in quadrant II

9. $\cos x = -\dfrac{1}{3}$, x in quadrant III

10. $\sin x = \dfrac{1}{4}$, x in quadrant II