EXERCISE 8.2

1. Find the 20th and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$
2. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.
3. The 5th, 8th and 11th terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.
4. The 4th term of a G.P. is square of its second term, and the first term is – 3. Determine its 7th term.
5. Which term of the following sequences:

   (a) $2, 2\sqrt{2}, 4, \ldots$ is 128 ? (b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729 ?

   (c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is $\frac{1}{19683}$?

6. For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?

Find the sum to indicated number of terms in each of the geometric progressions in Exercises 7 to 10:

7. 0.15, 0.015, 0.0015, ... 20 terms.
8. $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$ n terms.
9. $1, -a, a^2, -a^3, \ldots$ n terms (if $a \neq -1$).
10. $x^3, x^5, x^7, \ldots$ n terms (if $x \neq \pm 1$).
11. Evaluate $\sum_{k=1}^{11} (2 + 3^k)$
12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.
13. How many terms of G.P. 3, 3^2, 3^3, … are needed to give the sum 120?
14. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.
15. Given a G.P. with $a = 729$ and 7th term 64, determine S_7.

16. Find a G.P. for which sum of the first two terms is – 4 and the fifth term is 4 times the third term.
17. If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.
18. Find the sum to n terms of the sequence, 8, 88, 888, 8888… .
19. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.
20. Show that the products of the corresponding terms of the sequences a, ar, ar^2, $...ar^{n-1}$ and A, AR, AR^2, … AR^{n-1} form a G.P, and find the common ratio.
21. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4^{th} by 18.
22. If the p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that
$$a^{q-r}\, b^{r-p} c^{p-q} = 1.$$
23. If the first and the n^{th} term of a G.P. are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.
24. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.
25. If a, b, c and d are in G.P. show that
$(a^2 + b^2 + c^2)\,(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.
26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.
27. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.
28. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.
29. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.
30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour ?

31. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

32. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.