

EXERCISE 3.3

1. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$,

(ii) $(A - B)' = A' - B'$

3. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$

(ii) $(A - B)' = A' - B'$

4. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$

5. For the matrices A and B, verify that $(AB)' = B'A'$, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

6. If (i) $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then verify that $A' A = I$

(ii) If $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$, then verify that $A' A = I$

7. (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

9. Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

10. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

$$(i) \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Choose the correct answer in the Exercises 11 and 12.

11. If A, B are symmetric matrices of same order, then $AB - BA$ is a

(A) Skew symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix

12. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, and $A + A' = I$, then the value of α is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) π

(D) $\frac{3\pi}{2}$