Linear Algebra Refresher

Basics of Tensor

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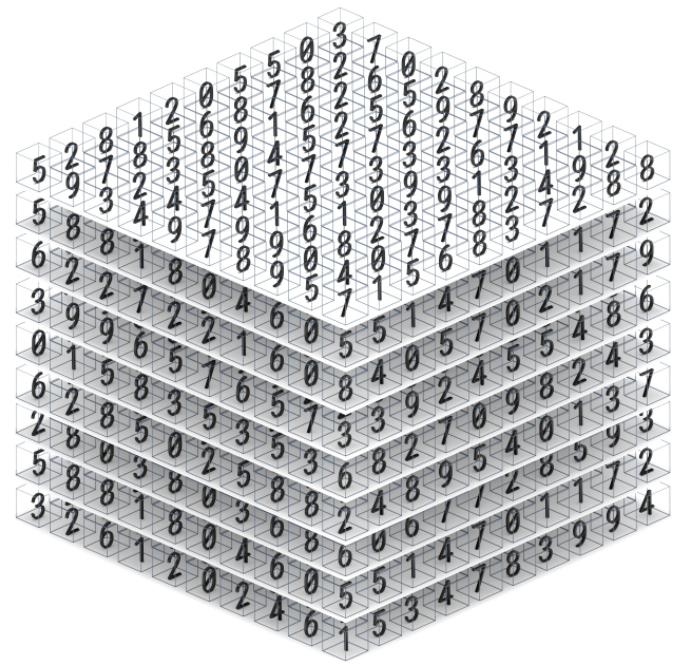
What is a Tensor?

In many cases, we need an array with more than two axes. In the general case, an array of numbers arranged on a regular grid with a variable number of axes is known as a tensor.



Structurally a tensor can be thought of as a generalised matrix. Dynamically, a tensor elements transform uniformly according to certain rules under a change of coordinates.

A tensor in 3 dimensions is a 3D matrix or cube of numbers

Scalar, Vector, Matrix and Tensor

So how are scalars, vectors, matrices and tensors related?

A vector is a first order tensor, a matrix is a second order **tensor** or two dimensional tensor. A tensor may just have a single number, in which case it is as a **tensor** of order zero, or simply a scalar.

Tensor notation is like matrix notation with capital letter represent a tensor and elements are represented with small letters with subscript.

A three-dimensional tensor can be represented as $A = ((a_{i,j,k}))$

$$1$$

Scalar

$$\begin{bmatrix} 1 & 2 \end{bmatrix}$$

Vector

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Matrix

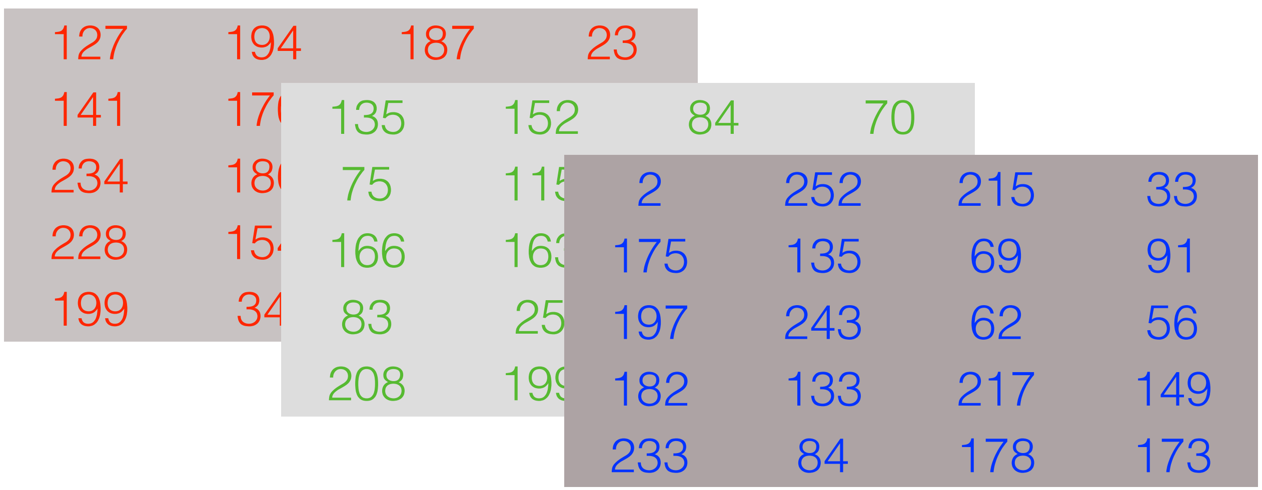
$$\begin{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} & \begin{bmatrix} 3 & 2 \\ 4 & 4 \end{bmatrix} \\ \begin{bmatrix} 5 & 2 \\ 8 & 6 \end{bmatrix} & \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \end{bmatrix}$$

Tensor

Tensor – Applications in Deep Learning

Tensors are used as the fundamental data structure in Deep Learning. In computer vision, tensor data structures are used to represent multi-channel colour images and extract colour features. Tensor data structures are also used to represent node inputs, outputs and weights in Artificial Neural Networks.

For example, following is a tensor representation of a 3 channels (RGB) image. Order of the tensor is 3×5×4.



Basic Tensor Operations – Addition and Subtraction

Tensor Addition of two tensors **A** and **B** is simply the element wise addition of two tensors resulting in a matrix **C** of same dimension.

Let $A = (a_1,a_2)$ and $B = (b_1,b_2)$
Then $\mathbf{C = A + B}$
$= (a_1+b_1,\ a_2+b_2)$

Similarly Tensor Subtraction of two tensors **A** and **B** is simply the element wise subtraction of two tensors resulting in a matrix C of same dimension. Therefore:

$\mathbf{C = A - B}$
$= (a_1-b_1,\ a_2-b_2)$

Basic Tensor Operations - Product

Consider a tensor **A** with m dimensions and tensor **B** with n dimensions, the product of these tensors will be a new tensor **C** with q + r dimensions.

Let $A = (a_1, a_2)$ and $B = (b_1, b_2)$
Then C = A (x) B

$$C = \begin{pmatrix} a_1 * [b_1, b_2] \\ a_2 * [b_1, b_2] \end{pmatrix}$$

$$\text{Or } C = \begin{pmatrix} a_1 * b_1 & a_1 * b_2 \\ a_2 * b_1 & a_2 * b_2 \end{pmatrix}$$