

Conditional Probability



Instructor



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Conditional Probability



Simply put, conditional probability is the probability of an event to occur given that another event has occurred.

Probability that an
event will occur

Given that another
event has occurred

Conditional Probability



Good friends, Niki and Tasha love hanging out in cafés. The probability of Niki or Tasha ordering juice is 0.3 and 0.5 respectively. Probability that Niki orders juice given that Tasha orders juice is 0.6.

What is the probability that Niki will order juice:

1. When Niki is in the café on her own
2. When Niki is with Tasha and Tasha orders juice

Conditional Probability



From the definition of the problem, the solutions are as follows:

1. Probability of Niki ordering juice on her own or $P(\text{Niki}) = 0.3$
2. Probability of Niki ordering juice given that Tasha orders juice or $P(\text{Niki}|\text{Tasha}) = 0.6$
3. Or the conditional probability that Niki orders juice given that Tasha orders juice = 0.6

Conditional Probability – Definitions and Formulae



The **conditional probability** of an event B is the probability that the event will occur given the knowledge that an event A has already occurred. It is denoted by $P(B|A)$.

1. For 2 interdependent events A and B,

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

2. If A and B are independent events,
then $P(B|A) = P(B)$ and $P(A|B) = P(A)$

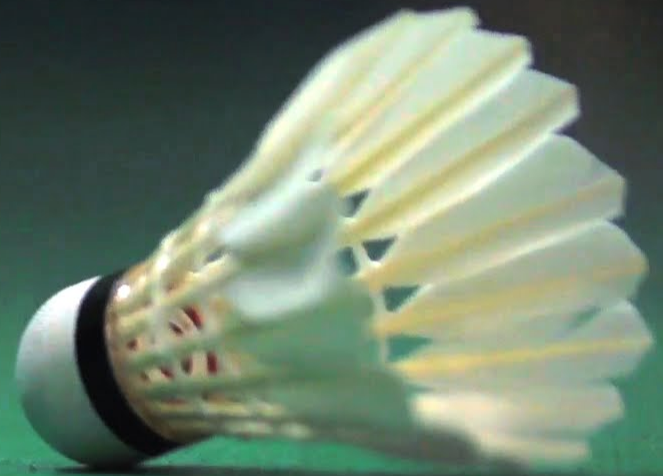
Therefore for 2 independent events A and B,

$$P(A \cap B) = P(A|B) \cdot P(B) = P(A) \cdot P(B)$$

Conditional Probability - Solved Example



Naina is an ace badminton player who has made it to the Olympics final. Her opponent in the final will be the winner of the semi-final match between Lee and Indu. Naina's probability of beating Indu is 0.4 and that of beating Lee is 0.8. Indu's probability of winning against Lee is 0.3.



What is the probability that Naina wins the Olympics final?

Conditional Probability - Solved Example

To simplify the notations let the event that Indu makes it to final) be denoted by I, Lee making it to final is denoted by L and Naina winning final is denoted by N.

The probability that Naina wins in the final is given by:
 $P(\text{Naina Wins Final}) = P(\text{Naina defeats Indu in final} \cup \text{Naina defeats Lee in final})$

$$\begin{aligned}\text{Therefore, } P(N) &= P(NnI) + P(NnL) \\ &= P(N|I) \cdot P(I) + P(N|L) \cdot P(L) \\ &= 0.4 \times 0.3 + 0.8 \times 0.7 \\ &= 0.68\end{aligned}$$



Conditional Probability - Solved Example



Good friends, Niki and Tasha love hanging out in cafés. The probability of Niki or Tasha ordering juice is 0.3 and 0.5 respectively. Probability that Niki orders juice given that Tasha orders juice is 0.6.

What is the probability of Tasha ordering juice given that Niki orders juice or $P(\text{Tasha} | \text{Niki})$?

Conditional Probability - Solved Example



From the definition of the problem, we know

$$P(\text{Niki}) = 0.3, P(\text{Tasha}) = 0.5 \text{ and } P(\text{Niki}|\text{Tasha}) = 0.6$$

$$P(\text{Niki} \cap \text{Tasha}) = P(\text{Niki}|\text{Tasha}) \cdot P(\text{Tasha}) = 0.5 \times 0.6 = 0.3$$

$$P(\text{Tasha}|\text{Niki}) = P(\text{Niki} \cap \text{Tasha}) / P(\text{Niki}) = 1$$

Or the conditional probability that Tasha orders juice given that Niki orders juice is 1.

Conditional Probability - Exercise

In a neighborhood, the vaccination rate is 70%. The probability that a child survives their 5th birthday is 0.9 if he is vaccinated. What is the probability that little Rehan, who lives in the neighborhood survives his 5th birthday?

