Probability - Part 1

Definitions, Rules And Types

A way to measure uncertainty


Life is uncertain, my friend!
Yes, but you can calculate how uncertain it is!

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Events, Sample Spaces and Probability

An **experiment** is an act or process of observation that leads to a single outcome that cannot be predicted with certainty.

Events, Sample Spaces and Probability

A **sample point, also referred to as an event**, is the most basic outcome of an experiment.

Events, Sample Spaces and Probability

A **sample space** of an event is the collection of all sample points. Roll a single die. The possible outcomes and hence the event space are:

S: {1, 2, 3, 4, 5, 6}

So What Is Probability?

A **probability** associated with an event is given by the number of favourable outcomes divided by the total number of possible outcomes.

So the probability of getting an even number when you roll a die

= N{2,4,6}/N{1,2,3,4,5,6} or 0.5

Img src: Pinterest

Probability Rules

1. A probability associated with any given event must be between 0 and 1.

$$0 \leq p_i \leq 1$$

2. The probabilities of all the sample points must sum up to 1.

$$\sum_{i=1}^{n} p_i = 1$$

Img src: Deccan Chronicle

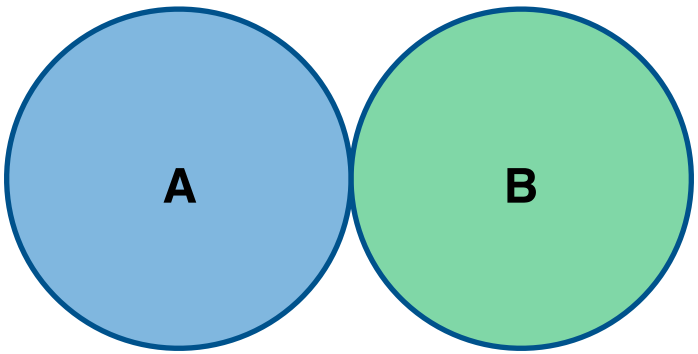
The Additive Rule

The probability of the union of events *A* and *B* is the sum of the probabilities of *A* and *B* minus the probability of the intersection of *A* and *B*:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually Exclusive Events

Two events or outcomes are called mutually exclusive if they cannot occur at the same time. So effectively the intersection space between A and B contains so sample points. Therefore P(AnB) = 0. For example the outcomes 1 and 2 are mutually exclusive events in the roll of a die.



If A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

Complementary Events

Two events or outcomes are called complementary if they are mutually exclusive and together constitute all possible outcomes of an experiment. For example the event of getting head or a tail on the valid toss of a coin: (Forget Sholay!)

$$P(A) + P(A^C) = 1$$

$$P(A) = 1 - P(A^C)$$

$$P(A^C) = 1 - P(A)$$

Img src: Wikipedia

Conditional Probability

Additional information or other events occurring may have an impact on the probability of an event.

The conditional probability of **A** given **B** is the probability that an event, A, will occur given that another event, B, has occurred is given by:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \; if \; P(B) > 0$$

Conditional Probability

For an example: after rolling a dice, you blindly guessed 2 as the face value of the dice. Your chance of being correct is 1/6. Now suppose somebody said that the face value is an even number. Immediately the chance of being correct increased to 1/3.

Here event **A** is occurrence of 2 and event **B** is occurrence of any even number.
$P(B)$ = chance of occurring any of {2, 4, 6} from {1, 2, 3, 4, 5, 6} = 3/6 = 1/2.

$AB = \{2\}\{2, 4, 6\} = \{2\} = A$

$P(A)$ = chance of occurring {2} from {1, 2, 3, 4, 5, 6} = 1/6.
Hence $P(A|B) = (1/6)/(1/2) = 1/3$

The Multiplicative Rule

The conditional probability formula can be rearranged into the Multiplicative Rule of Probability to find joint probability.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B)P(A \mid B) \quad \text{or}$$

$$P(A \cap B) = P(A)P(B \mid A)$$

Independent Events

If the chance of event A occurring s the same regardless of whether or not an outcome B occurs, then the outcomes A and B are said to be independent of one another. The events of getting 6 on two successive roll of a die are independent of each other.

$$P(A) = P(A \mid B)$$

If A and B are independent events, then:

$$P(A \cap B) = P(A)P(B)$$

If A, B, and C are independent given that an event X has occurred, then

$$P(A \cap B \cap C) = P(A \mid X)\ P(B \mid X)\ P(C \mid X)$$