# Risk Management

Philippe Jorion

School of Business, the University of California, Irvine, Irvine, California 92697-3125; email: pjorion@uci.edu

Annu. Rev. Financ. Econ. 2010. 2:347–65

First published online as a Review in Advance on July 16, 2010

The *Annual Review of Financial Economics* is online at financial.annualreviews.org

This article's doi:
10.1146/annurev-financial-073009-104045



1941-1367/10/1205-0347$20.00

## Key Words

risk measurement, market risk, value at risk, stress tests, credit risk

## Abstract

Modern risk management systems were developed in the early 1990s to provide centralized risk measures at the top level of financial institutions. These are based on a century of theoretical developments in risk measures. In particular, value at risk (VAR) has become widely used as a statistical measure of market risk based on current positions. This methodology has been extended to credit risk and operational risk. This article reviews the benefits and limitations of these models. In spite of all these advances, risk methods are poorly adapted to measure liquidity risk and systemic risk.

## INTRODUCTION

Modern risk management emerged as a distinct discipline in the mid 1990s. It developed as a result of derivatives disasters, which prodded the industry toward better control of financial risks. In fact, risk had always been an integral consideration of finance. The difference was the development of new functions specifically designed for risk management and distinct from the usual business lines. This was made possible by advances in risk methodologies, coupled with computer processing capabilities, which made it possible to measure risk at the top levels of institutions. By now, most financial institutions have a chief risk officer reporting directly to top management. Risk management methods were initially applied to market risk then extended to liquidity, credit, and operational risk. This article reviews and evaluates the progress made in the field of financial risk management.

## FOUNDATIONS OF RISK MANAGEMENT

Modern risk management builds on a century of advances in analytical risk management tools, which are described in **Table 1**. Consider the simplest financial asset: a risk-free bond that promises to make a fixed stream of payments. Due to movements in the yield, or discount rate, the market value of this bond fluctuates over time. This creates financial risk or dispersion in the unexpected outcomes. Conceptually, this risk can be separated into two drivers. The first is exposure or sensitivity of the bond price to movements in the risk factor. The second is the range of possible movements in the risk factors, which can be described by a probability distribution function with parameters estimated by statistical analysis. Risk measurement brings together these two drivers by describing the range of possible losses on the position. Thus, it builds on the knowledge of financial instruments, financial markets, as well as statistics. Risk management requires a broad set of skills.

**Table 1  The evolution of analytical risk management tools**

| Year | Tool |
|---|---|
| 1938 | Macaulay's bond duration |
| 1952 | Markowitz's mean-variance framework |
| 1963 | Sharpe's one-factor beta model |
| 1966 | Multiple-factor models |
| 1973 | BS option pricing model and Greeks |
| 1982 | ARCH models |
| 1992 | Heath-Jarrow-Morton term structure models |
| 1993 | Value at risk |
| 1994 | RiskMetrics™ |
| 1997 | CreditMetrics™ |

ARCH, autoregressive conditional heteroskedastic; BS, Black & Scholes.

## EXPOSURES

### Bond Duration

The foundations of risk measurement were built one market at a time. In the fixed-income market, the most commonly used measure of exposure is duration. Duration is a measure of the linear sensitivity of bond prices to movements in the yield curve. This concept was first developed by Macaulay (1938) and has become the cornerstone of fixed-income portfolio management.

Duration is particularly useful because it can be computed directly from information about the characteristics of a bond. For example, a risk-free, 10-year, 5% coupon bond has a duration of $D = 8.1$ years when the yield is also 5%. The change in the bond price d$P$ can be related to the change in the yield d$y$ by

$$\mathrm{d}P \approx [-DP/(1+y)] \times \mathrm{d}y. \tag{1}$$

This leads to a very useful rule of thumb. If the yield increases by 1%, for example, the bond price will fall, reflecting that the fixed coupon of 5% is now lower than the coupon of 6% on a new bond. Equation 1 shows that the price will fall by approximately $[-8.1 \times \$100/(1.05)] \times 1\% = -7.7\%$. This illustrates the general principle that the change in the price (gain or loss) depends on the exposure and the movement in the risk factor:

$$\text{Change in Price} = [\text{Exposure}] \times \text{Change in Risk Factor}.$$

In addition, this duration measure aggregates easily to the entire bond portfolio. The portfolio duration is simply the weighted average of the duration for individual positions and it neatly encapsulates the total exposure. To understand the risk of this portfolio, the risk manager must evaluate the possible range of movements in yields, using statistical tools.

One must view the duration approach as a first approximation. It assumes that the yield curve is flat and is subject to parallel shocks. In practice, yield curve movements are more complex. Heath et al. (1992) provide a general framework to model interest rates that can be used to price bonds and their derivatives and also provides sensitivities to risk factors.

### Stock Beta

In the stock market, beta is the measure of exposure of individual stocks to the market. Beta, or systematic risk $\beta_i$, is estimated from a time-series regression of individual stock returns $R_{it}$ on the market $R_{Mt}$:

$$R_{it} = \alpha_i + \beta_i\, R_{Mt} + e_{it}. \tag{2}$$

Focusing on beta ignores the residual term $e_{it}$. As an example, consider shares in Procter & Gamble (P&G). This is a conservative, consumer stock with low beta. Assume that this is estimated at 0.5 relative to the S&P 500 stock market index. If the market falls by 10%, P&G would be expected to lose $0.5 \times 10\% = 5\%$. Unlike duration, beta is an estimated coefficient and thus subject to measurement error. Its value depends on the actual observations in the measurement window, the length of the sample, and the sampling frequency. As with duration, beta can be aggregated at the level of the portfolio, for which beta is simply the weighted average of individual stock betas.

## Option Greeks

Options are very common financial instruments. This type of derivative gives the right but not the obligation to buy or sell a particular asset at a particular price, at or before a particular point in time. Black & Scholes (1973), henceforth BS, developed a breakthrough option pricing model, for which they were awarded the 1997 Nobel Prize in Economics, along with Robert Merton. The model provides a particularly elegant formula for pricing options, based on a remarkably small set of inputs.

This model also generates closed-form solution for exposures to inputs, which are called the Greeks. Define $f$ as the value of the option, $S$ as the value of the underlying asset, and $\Delta$ as the linear exposure of the option price to $S$. We then have

$$df \approx \Delta\ dS. \tag{3}$$

As an example, consider a purchase of a call option to buy one share of P&G in three months. The stock price currently trades at \$100. The BS model price is \$4.20, and the delta is 0.57. So, if the stock price went down by \$1.00, the call would lose approximately $0.57 \times \$1.00 = \$0.57$. More generally, the BS model gives the quadratic exposure to the stock price as well as the exposures to other risk factors, especially the implied volatility. Again, these exposures can be aggregated across the portfolio for all options with the same underlying risk factor.

Exposure equations for bonds, stocks, and options have a similar structure. This assumes one risk factor only, however. To measure portfolio risk, we also need to aggregate risks across various categories and factors.

# RISK AGGREGATION AND PRICING

In a portfolio context, what matters is the total risk of the portfolio. This concept is not new. The mean-variance framework developed by Harry Markowitz (1952) emphasizes the importance of measuring risk in a total portfolio context. It also explicitly considers the trade-off between higher return and higher risk. This view was the foundation for modern risk management, for which Markowitz was awarded the 1990 Nobel Prize in Economics.

By its nature, centralized risk measurement is a large-scale aggregation problem, which tends to involve a large number of parameters. Consider, for example, the problem of allocating assets among the thousands of stocks available for investment. Generally, a covariance matrix that lists all pairwise correlations can describe joint movements in stock prices. The problem, however, is that the number of coefficients in this matrix rises with the square of the number of stocks. Too many coefficients create several problems, ranging from numerical instability in the covariance matrix to loss of intuitive understanding of the economic drivers of risk. As a result, simplifications are required.

## Pricing

William Sharpe (1964) simplified the covariance structure of stocks with a one-factor model. This reduced the dimensionality of the covariance matrix to a number of coefficients that increases linearly with the number of stocks. Such simplification is at the heart of mapping methods in modern risk measurement. Sharpe then showed that the only risk that should be priced in equilibrium (i.e., when demand equals supply) is systematic risk, or the beta to the market. This theory of asset pricing, known as the capital asset pricing model, is the reason why Sharpe was awarded the 1990 Nobel Prize in Economics.

Following Sharpe's one-factor model, multiple-factor models were developed, starting with King (1966). These provide a deeper and more realistic description of common movements due to several risk factors, which is still parsimonious. Ross (1976) later developed a model that relies on the existence of a small number of pervasive factors for pricing assets without arbitrage opportunities.

### Portfolio Diversification

Diversification is a powerful method to manage risk. Consider a portfolio with $N = 2$ assets. Define $w_i$ as the weight for asset $i$, $\sigma_i$ as the volatility, and $\rho_{ij}$ as the pairwise correlation. The portfolio variance $V$ can be computed as the weighted sum of individual variances and of the covariance between the two stocks:

$$V(R_p) = w_1{}^2\ V(R_1) + w_2{}^2\ V(R_2) + 2 \times w_1\ w_2\ COV(R_1,R_2)$$

$$V(R_p) = w_1{}^2\ \sigma_1{}^2 + w_2{}^2\ \sigma_2{}^2 + 2 \times w_1\ w_2\ \rho_{12}\ \sigma_1\, \sigma_2. \tag{4}$$

The number of pairwise correlations is $N(N–1)/2$, which can be very large as $N$ increases. In the simple homogeneous case, in which all position weights and parameters are the same, the portfolio volatility can be expressed as

$$\sigma_p = \sigma\sqrt{\frac{1}{N} + (1 - \frac{1}{N})\rho}. \tag{5}$$

This shows that the risk of a large and well-balanced portfolio converges to $\sigma_p = \sigma\sqrt{\rho}$ as $N$ increases. If the correlation is zero, the portfolio is perfectly diversified. This is an unrealistic example, however, because in practice, correlations across financial assets are mostly positive. Even so, choosing assets with low correlations should help lower portfolio risk.

Diversification has limitations, however. In practice, the correlation coefficients are estimated and can change over time, but errors in correlation estimates can have serious consequences. If correlations are viewed as low, the portfolio manager will think that portfolio risk is low as well. As a result, the portfolio could be leveraged to increase the rate of return to equity. If actual correlations go up, however, losses on individual positions will cluster, leading to a large loss that could bankrupt the institution. This explains the failure of Long-Term Capital Management, a hedge fund that lost \$4.7 billion in 1998 due to a combination of high leverage and increased correlations across positions (see Jorion 2000).

More generally, banks are susceptible to the same problem because they are highly leveraged. Time and again, banks have failed because they underestimated the correlations across their assets in times of stress. Indeed, this is one of the most important sources of financial disasters.

## DERIVATIVES FOR RISK MANAGEMENT

In parallel with these academic developments, the financial industry was creating new financial instruments to manage, to hedge, or to speculate on financial risks. Derivatives are private contracts whose value derives from some underlying price or index. Instruments such as forwards, futures, or swaps generally have linear exposures to risk factors. In contrast, options are nonlinear instruments. Derivatives can be traded on organized exchanges or in over-the-counter (OTC) markets.

## Hedging

Consider for instance a farmer who is planning to sell a wheat harvest at the end of the season. He worries about a drop in the price of wheat, in which case he would suffer a loss. This risk can be hedged by shorting wheat futures in the amount of the expected harvest. This position creates a stream of payments that generates a gain should the price of wheat fall. Normally, this gain should hedge the loss in the value of the harvest.

More generally, exposures can be used to determine the size of the hedge. Equation 2 can be used to determine the number of futures contracts to buy or sell to protect against stock market risk. Consider for instance an investor who wants to hedge a position in P&G stocks, valued at \$1 million, by shorting stock index futures. Each S&P 500 futures contract has a notional of \$250 times the current value of the index, which we assume is 1,000. The beta of P&G is 0.5, whereas the beta of the stock index futures is close to 1.0. The optimal number of contracts $N$ is then obtained by setting the dollar beta of the entire portfolio, stocks plus futures, to zero. In other words, we must have $0.5 \times \$1{,}000{,}000 - N(1.0 \times \$250{,}000) = 0$. The solution is to sell $N = 2$ contracts.

In practice, the movement in the price of the asset to be hedged may not parallel that of the hedging instrument. For example, our farmer sells a particular grade of wheat, which may not be the one that corresponds to that of the futures contract. This is an example of basis risk, which occurs when the cash and futures prices diverge. In this situation, an optimal hedge can be derived by minimizing the variance of the hedged position. Typically, the hedge ratio is based on historical relationships, assuming that these remain stable over time.

## Speculation

The same markets can be used for outright speculation. For example, an investor who thinks that the price of wheat will go up could establish a long position in wheat futures. If the price of wheat indeed goes up, the position should generate a profit.

Alternatively, this view could be implemented through the cash market, by buying wheat. However, this is more cumbersome and costly. The speculator would have to take physical delivery of wheat and store it. Derivative contracts are more cost effective. They also allow greater leverage than in cash markets. In other words, a speculator would only have to put up an initial margin that is a small fraction of the dollar value of the contract.

The ability to use leverage is a double-edged sword, however. Leverage can be used to magnify the risk. Using cash markets, the initial outlay represents the worst possible loss, which is very unlikely. For example, a speculator could buy \$1,000,000 worth of wheat. In the worst case, the price of wheat could go to zero, even though this is not likely. With futures, the speculator may have to put up only \$50,000 in initial margin for a \$1,000,000 futures position. If the difference, or \$950,000, is invested in a risk-free deposit account, the futures position is economically equivalent to a long position of \$1,000,000 in cash markets. The initial margin can be lost but can be supplemented by additional funds from the deposit account. On the one hand, a drop of 20% in the price of wheat, for example, will hit equally the position in wheat or the synthetic position in futures plus deposits. On the other hand, the speculator may be tempted to increase the size of the futures position, given the low margin requirements. This will increase the expected return but also the risk. Thus, derivatives require good risk management systems.

Even though they are driven by their own profit motives, speculators do serve useful social functions. They provide liquidity to markets, which lowers transaction costs for everybody. They are willing to take the opposite side to hedgers. Finally, trading on exchange creates transaction prices, which are widely disseminated across the world. These price signals convey useful information to financial markets.

## Market Growth

The need for efficient risk management tools led to rapid growth in the market for derivatives. These markets can be described in terms of their notional outstanding amounts, which represent the amount of exposure exchanged. We started having reliable market statistics in 1986. Before that, markets were smaller or even nonexistent. **Table 2** shows that derivatives markets have grown from $1 trillion to $652 trillion from 1986 to 2008.

A number of spectacular losses followed the expansion of these markets in the early 1990s, however. In 1993, Metallgesellschaft lost $1.53 billion from a failed hedge using oil futures. In 1994, Orange County lost $1.8 billion by leveraging up the portfolio and investing in OTC derivatives. In 1995, Barings lost $1.3 billion from unauthorized speculation in stock index futures. These cases threatened to create a backlash against derivatives because some users apparently did not fully appreciate their risks. In response, the industry devised more comprehensive and intuitive measures of risk.

**Table 2 Global derivatives markets: outstanding contracts ($ billion)**

| Instruments, by location and type | 1986 | 2008 |
|---|---|---|
| Exchange-traded instruments | 583 | 59,798 |
| Interest rate | 516 | 54,432 |
| Currency | 18 | 5.138 |
| Stock index | 49 | 228 |
| OTC instruments | 500 | 591,963 |
| Interest rate swaps | 400 | 418,678 |
| Currency swaps | 100 | 49,753 |
| Credit default swaps | 0 | 41,868 |
| Equity, commodity | 0 | 10,921 |
| **Total** | **1,083** | **651,761** |

Source: ISDA and Bank for International Settlements. OTC, over-the-counter.

## MARKET RISK MEASUREMENT METHODS

### Value at Risk

This led to the birth of value at risk (VAR) in 1993 as part of the best practices for measuring risks involving derivatives. VAR was developed at J.P. Morgan in response to the request of its chairman, "At the close of business each day, tell me what the market risks are across all businesses and locations." VAR provides a global view of risk on the basis of the most current positions. It summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence. For instance, at that time, J.P. Morgan's daily VAR was approximately \$15 million at the 95% level of confidence. In other words, the bank should not have expected to lose more than \$15 million in 95 days out of 100. This number describes the risk of the entire trading portfolio of the bank, as Markowitz had suggested.

To compute VAR, the institution has to set up systems to collect position and market data for the entire firm. From this information, its risk manager can build the probability density function of profits and losses over the risk horizon *T*. As shown in **Figure 1**, VAR is just a quantile of the distribution. Specifying a confidence level *c*, this is the cutoff point such that the area to its left is precisely $1 - c$.

If the distribution is normal, or more generally elliptical, the quantile can be derived from the standard deviation $\sigma$ and a multiplier that is distribution specific. For a normal distribution, for instance, the standard normal deviate that corresponds to the 99% confidence level is $\alpha = 2.33$. Therefore, the 99% VAR can be obtained by multiplying $\sigma$ by 2.33.

J.P. Morgan unveiled its RiskMetrics™ system in October 1994. Available free on the Internet, RiskMetrics provided a data feed for computing VAR, as well as a detailed technical manual (see J.P. Morgan 1996). The widespread availability of data immediately engaged the industry and spurred academic and practitioner research into risk management.

VAR took hold as a universally accepted measure of risk because of its simplicity. It summarizes the downside risk in one measure that is easy to understand. Because it relies on a statistical distribution, it is sometimes called a statistical measure of risk. This methodology has spread to most financial institutions. Risk systems are also increasingly used

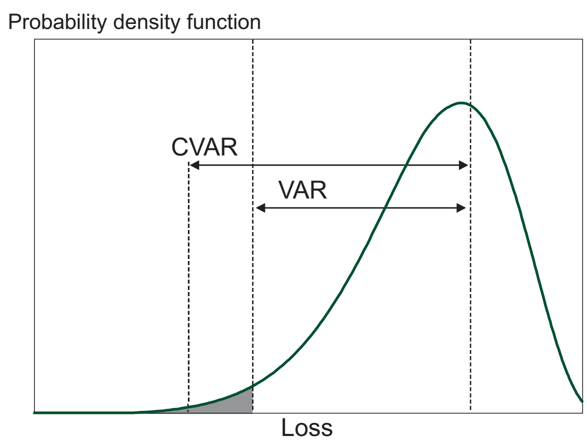


**Figure 1**

Summarizing the distribution of losses. CVAR, conditional value at risk; VAR, value at risk.

not only to measure risk passively but also to control risk. Position limits are now routinely based on VAR numbers and stress-test results.

The impetus for using VAR models came from the industry but also from financial regulators. Notably, starting in 1998, the Basel Committee on Banking Supervision (1996) allowed commercial banks to use their internal VAR models as the basis for their market risk charge. This led to the widespread acceptance of VAR.

## Limitations of Value at Risk and Risk Models

Initially, VAR was used as a relative measure of risk, for example, to compare risk profiles for different traders, business units, or across time. For this application, the choice of the horizon and confidence level is relatively arbitrary as long as these parameters are the same. The risk manager could choose, for example, a 95% confidence level and daily horizon. VAR ignores the magnitude of the loss beyond the quantile.

VAR has been increasingly used as an absolute measure of risk, as the worst loss if the portfolio were to be liquidated over the horizon at a given level of confidence. This requires more conservative parameters, such as a higher confidence level and longer horizon, both of which increase the VAR number. The horizon should be sufficiently long to allow an orderly sale. In other words, it should be longer for less liquid assets. Even so, the cost of making an error in this case can be very high. Underestimating VAR means that the portfolio is more likely to be wiped out than expected.

Another limitation of VAR is that it assumes that the positions are fixed over the horizon. This is common to all position-based risk measures. In practice, the portfolio is changing over time, which could increase or decrease the risk profile relative to VAR.

Finally, risk models view the institution as a price-taker. Risk distributions are built from recent price movements, which involve typical sales and purchases. If the institution liquidates a very large portfolio, the sale could move prices adversely and result in a loss that is greater than the VAR measure. Furthermore, if other institutions are forced to sell the same assets simultaneously, the loss could be even bigger. Such diseconomies-of-scale effects are not captured by risk models.

## Conditional Value at Risk

VAR summarizes the distribution of profits and losses by one number. The entire distribution, however, is of interest. We should examine whether the distribution is symmetric or skewed. Skewness to the left implies a greater probability of large losses than gains. Another question is, "What is the expected loss when VAR is exceeded?" This is known as the conditional VAR (CVAR). This must be greater than VAR by construction. In most cases, CVAR is only slightly greater than VAR. In other cases, however, it can be much larger. In such situations, VAR limits may not be effective, as rare losses could bankrupt the institution.

As a risk measure, CVAR has other useful theoretical properties. Ideally, risk measures should obey several properties to be coherent, as suggested by Artzner et al. (1999): (*a*) If a portfolio has lower returns for all states of the world, its risk must be greater (monotonicity); (*b*) adding cash to the portfolio should reduce its risk by this amount (translation invariance); (*c*) scaling a portfolio should simply scale its risk (homogeneity); and (*d*) merging portfolios cannot increase risk (subadditivity). VAR does not obey the last property (except when distributions are elliptical), unlike CVAR. Merging two portfolios could produce a higher VAR than the sum of the two separate VAR measures. So, in theory, CVAR is a better risk measure. In practice, CVAR measures are often used by the insurance industry but rarely by financial institutions. In addition, portfolio risk does go up when the size of the portfolio increases to a point at which its trades can move markets. Thus, traditional risk measures ignore diseconomies of scale.

## Tail Losses and Stress Tests

The VAR framework can also be used to uncover progressively larger but rarer losses by increasing the confidence level. However, the empirical distribution becomes more irregular in the extreme tails due to the scarcity of data. To some extent, this problem can be alleviated by extreme value theory, which provides a theoretically sound analytical density function for smoothing the tails (see McNeil et al. 2005). Even so, VAR measures at high confidence levels such as 99.9% are intrinsically less reliable than those at confidence levels such as 95% or even 99%.

In addition, historical data can have severe limitations. The history may not reflect plausible structural breaks. For instance, a trader may have a position on a foreign currency that is fixed against the dollar and for which there is no history of devaluations. Because the recent history shows no volatility, a VAR measure based on this would mistakenly indicate that the position is riskless, which may not be the case.

This is why VAR systems must be supplemented by stress-test scenarios. The risk manager needs to devise scenarios derived from history, perhaps from a very long time ago, and from prospective events. Stress tests can be used to estimate the effects of these scenarios on the portfolio. Such measures of risk are nonstatistical, yet are essential to evaluate the risk profile of the portfolio.

## Time Variation in Risk

Another issue with statistical risk models is that distributions are typically constructed from a short window of recent history, typically one to four years of daily data for commercial bank trading portfolios. This approach basically places the same weight on each day and does not capture well time variation in risk, which is a prevalent feature of financial time series.

Indeed, researchers have developed new models of time variation in risk. The best known example is the generalized autoregressive conditional heteroskedastic model. This is an extension of the autoregressive conditional heteroskedastic (ARCH) model, which was developed by Robert Engle (1982) and for which he received the 2003 Nobel Prize in Economics. Such models are widely employed for predicting time variation in risk because they are parsimonious and fit the data rather well.

A particularly simple example is the exponentially weighted moving average (EWMA) model, which works as follows: We start with an initial forecast for the variance $h_t$ and then observe the innovations in the risk factor $R_t$. The forecast for tomorrow is then constructed as

$$h_{t+1} = \lambda \; h_t + (1 - \lambda) \; R_t{}^2. \tag{6}$$

This not only ensures some persistence in the forecast due to the use of the previous variance but also accounts for recent shocks. The relative weight is determined by the decay factor λ, which is between 0 and 1. The model is called exponential because the weights on previous observations decrease exponentially.

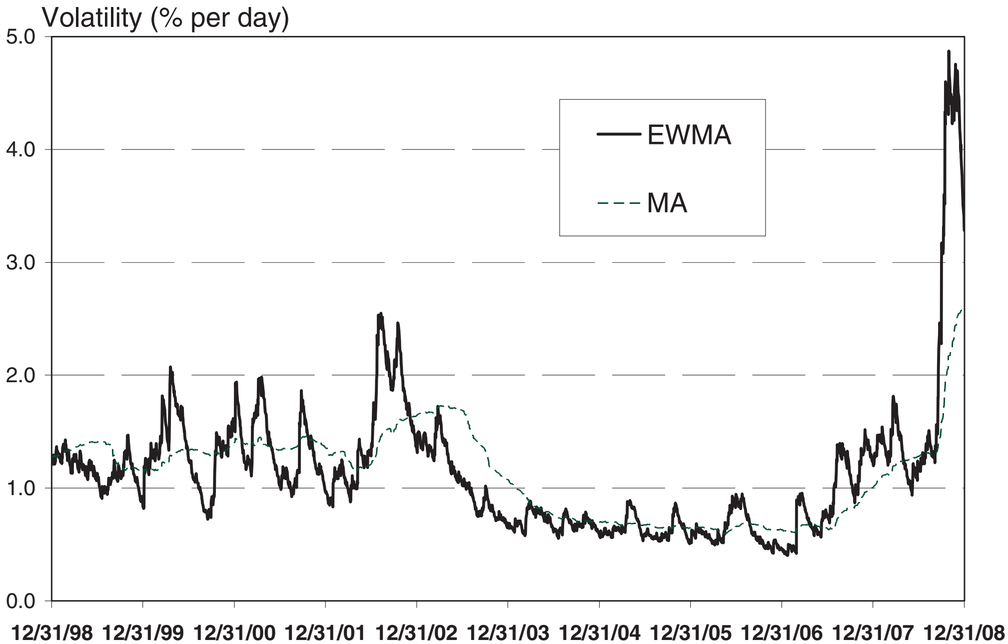


**Figure 2**

Forecast of daily volatility for S&P 500 Index using an exponentially weighted moving average (EWMA) and moving average (MA).

**Figure 2** illustrates the daily volatility forecast for a stock index. The solid line represents the EWMA model. This shows wide swings in the volatility over the short term but also over longer-term cycles. In particular, volatility was elevated in 2002 and 2008 but was much lower than normal from 2005 to 2006.

The graph also shows a moving average (MA) forecast with a fixed window of 250 days, which is typical of bank trading models. This second forecast responds very slowly to a more volatile environment. In 2008, in particular, this model systematically underestimated the more accurate EWMA forecast. As a result, most banks experienced too many large losses during that year.

## RISK MEASUREMENT SYSTEMS

### Position-Based Risk Measurement

That risk measures are now position based is a new aspect of modern risk management. Traditionally, risk was measured from historical return information. Today, however, trading portfolios turn over very quickly. As a result, returns-based risk measures give incomplete information. This is because today's risk profile may be very different from that over a recent history.

However, position-based risk measures are rather complex to build. They involve large-scale structural bottom-up models that aggregate risks from individual position data. The choice of the model reflects a trade-off between speed and accuracy. A portfolio may contain millions of positions that would be difficult to model individually. An alternative approach is to choose a set of risk factors that span the risk space effectively.

The structure of risk measurement systems is described in **Figure 3**. In the first component (positions), all the positions are collected from the front office. Instruments are then mapped on the risk factors, replacing the value of all positions by exposures. These exposures are aggregated across the entire portfolio. The second component (risk factors) involves modeling the distribution of risk factors, which is updated from current prices. Finally, the last component (risk engine) combines the exposures with the statistical distribution of risk factors, which gives a distribution of profits and losses for the portfolio.

This modern risk architecture has many more uses than reporting a single VAR number, however. Once this system is in place, the risk manager can examine easily the effect of extreme or unusual situations. In fact, stress testing is an important complement to VAR because VAR only provides an estimate of losses under normal market conditions (i.e., at a prespecified confidence level). Stress scenarios are just hypothetical realizations of the risk factors. More generally, the risk manager can evaluate the effect of changing models and parameters in the position-mapping or risk-factor-modeling component of their system.

## Design of the System

Mapping is the process by which the current values of the portfolio positions are replaced by global exposures on the risk factors. The risk manager must decide on a number of risk factors that effectively spans the risk space for the portfolio strategy. This choice reflects a delicate balance. Too many risk factors will unnecessarily complicate and perhaps slow down the system. Too few will cause the systems to miss important risks. The number of risk factors must take into account the trading strategies. Portfolios of options, for example, require implied volatilities as risk factors. In general, more complex strategies require

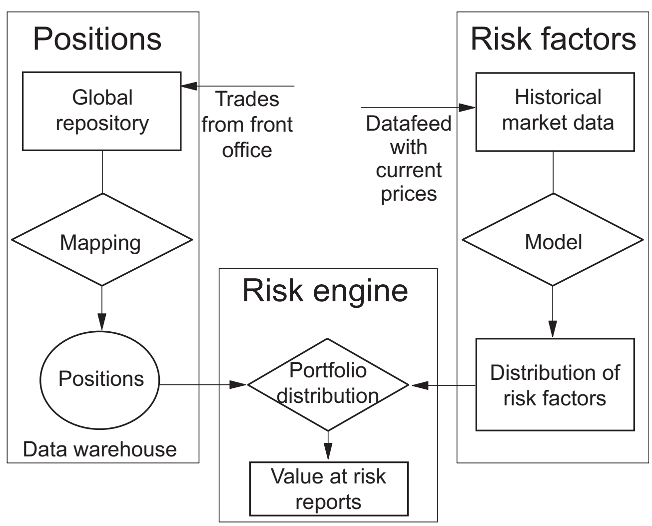


**Figure 3**

Structure of risk measurement systems.

more risk factors. Thus, the design of the risk management process is by necessity subjective. It must also adapt to a changing trading environment.

After mapping, the next step is to construct the distribution of portfolio returns, using a choice among three methods. Methods can be separated into local valuation methods and full valuation methods. Local valuation methods approximate movements in the value of the positions using exposures. Full valuation methods require repricing the positions under the new risk factor values. Consider, for example, a position in a bond. A local valuation method would price the bond at the current point, then approximate movements from the duration and movements in the yield. In contrast, a full valuation method would reprice the bond for each hypothetical value in the yield. Local valuation methods are faster and simpler but less accurate than full valuation methods. If the portfolio contains mainly linear instruments, local valuation may be sufficient. Otherwise, full valuation is needed. Jorion (2006) describes the methods in more detail.

## Value at Risk Models: Delta-Normal Method

The simplest approach is the delta-normal method, sometimes called the variance-covariance method. This assumes that the portfolio exposures are linear in the risk factors and that the risk factors are jointly normally distributed. This leads to an analytical expression for VAR, which is measured from the portfolio variance

$$\sigma^2 \, (R_{p,t+1}) = x_t' \sum\nolimits_{t+1} x_{t}, \tag{7}$$

where $\sum_{t+1}$ is the forecast of the covariance matrix over the horizon; $x_t$ represents the current dollar exposures on the risk factors, obtained via mapping; and $\sigma^2$ is the forecast variance of the portfolio dollar return $R_{p,t+1}$. This shows that changes in the portfolio risk are driven either by changes in the positions or by changes in the distribution of the risk factors.

VAR is directly obtained from the dollar volatility and the standard normal deviate $\alpha$ that corresponds to the confidence level $c$:

$$\mathrm{VAR}^{DN} = \alpha \, \sigma \, (R_{p,t+1}).$$

More generally, this method can be adapted to portfolio distributions that differ from the normal distribution, by changing $\alpha$.

On the one hand, this method is simple and quick to compute. It incorporates time variation in risk easily. On the other hand, it does not account for nonlinear effects, such as with options, and may underestimate the occurrence of large losses because of its reliance on a normal distribution. In practice, most financial series have heavier tails than the normal distribution, as pointed out by Fama (1965).

## Value at Risk Models: Historical Simulation

Historical simulation (HS) applies movements in risk factors observed over a recent window (e.g., 250 days) to the current portfolio. We observe data from 1 to $t$, which is the current time. The current portfolio value, $P_t$, is a function of the current value of the $N$ risk factors $f_{i,t}$:

$$P_t = P[f_{1,t}, f_{2,t}, \ldots, f_{N,t}].$$

We sample the factor movements from the historical distribution without replacement. The factor movements are drawn jointly for each day, which preserves the historical pattern of correlations across risk factors. Call $\Delta f_i^k$ the hypothetical movement in risk factor $i$ for the simulation numbered $k$. These are drawn from the historical distribution

$$\Delta f_i^k \sim \{\Delta f_{i,1}, \Delta f_{i,2}, \ldots, \Delta f_{i,t}\}. \tag{8}$$

From this, we can construct hypothetical factor values, starting from the current one, $f_i^k = f_{i,t} + \Delta f_i^k$, for all $i = 1, \ldots, N$. This new set of risk factor values can be used to construct a hypothetical value of the current portfolio under the new scenario $P^k = P[f_1{}^k, f_2{}^k, \ldots, f_N{}^k]$, using full valuation. Finally, the risk manager can compute changes in portfolio values to derive the distribution of profits and losses.

This method is easy to explain and is intuitive because each return is associated to an actual realization of risk factors. Going back to a particular date gives insight into what could go wrong. The method assumes independently and identically distributed historical factor changes. Otherwise, it makes no specific distributional assumption, other than assuming that the recent past is relevant. This is an improvement over the normal distribution because financial data typically contain fat tails. In addition, because the portfolio can be priced under full revaluation, this approach can handle options and other nonlinear instruments. This explains why this is perhaps the most popular VAR method.

However, HS typically relies on a short historical moving window, such as one year, to infer movements in market prices. This may be too short to provide adequate representation of the risk space. In addition, this method does not adequately track time variation in risk because it places equal weights on each observation. **Figure 2** showed that the risk forecast of the moving average model with 250 days is not sufficiently responsive to current events.

## Value at Risk Models: Monte Carlo Simulation

Monte Carlo simulation (MCS) was introduced in finance by Boyle (1977) in the context of pricing options. MCS is similar to HS, except that the movements in risk factors are generated by drawings from a prespecified distribution,

$$\Delta f^k \sim g(\theta), \tag{9}$$

where $g$ is the joint distribution function with parameters $\theta$. From this distribution, the risk manager samples pseudo-random values for the risk factors. As in the case of HS, these are used to price the portfolio for different scenarios, which generates a distribution of portfolio returns.

On the one hand, this method is the most flexible because it allows full revaluation as well as complex distributions for the risk factors. On the other hand, it is computationally more onerous. Monte Carlo methods also create inherent sampling variability because of the randomization process. Different sequences of random numbers will lead to different results. It may take a large number of iterations to converge to a stable VAR measure. Finally, the Monte Carlo method requires users to make detailed assumptions about the form and parameters of the stochastic process for the risk factors. This makes the output less amenable to economic interpretation than other methods. Therefore, it is subject to model risk. Risk managers need to understand how sensitive the results are to the model assumptions.

### Backtesting

The final step in the risk measurement process is to provide a feedback loop to evaluate the quality of the output. Backtesting is defined as a process for systematically comparing VAR measures with the subsequent daily returns. Each exceedance is called an exception. Assume, for example, that VAR is measured at the 99% level of confidence. If the model is well specified, we should observe on average 2.5 exceptions over the course of a year, or 1% of 250 trading days during a year. In practice, some deviation around this number is to be expected. If there are too many exceptions, however, the risk manager will have to conclude that the model is flawed. Thus, backtesting provides essential feedback about the accuracy of the VAR system.

## CREDIT RISK

VAR methods were initially developed to measure market risk, which is the risk of losses due to movements in the level or volatility of market prices. In the late 1990s, these techniques were extended to credit risk and operational risk. Credit risk is the risk of losses due to the fact that counterparties may be unwilling or unable to fulfill their contractual obligations. This is a major source of risk for the economy and has proved much more damaging than episodic losses due to market risk. Time and again, countries have experienced crises in their banking sector due to credit losses that required expensive recapitalizations.

### Portfolio Credit Risk Models

Portfolio credit risk models are much more complex than market risk models. They require models for the probability of default (PD) for each borrower in the portfolio, for the credit exposure (CE) to that borrower, and for the loss given default. The interactions among all these random variables must be taken into account, which is no easy affair. In addition, some of the required parameters, such as the PD, are not directly observable and must be inferred indirectly.

A major advance in credit risk was the development of so-called structural models, which was initiated by Merton (1974). The capital structure of the firm is split into equity and debt. Merton interpreted equity as a call option on the value of a firm's assets. If the value of the assets is sufficient to repay the debt, the residual value goes to equity. In contrast, a debt that cannot be repaid is in default, in which case equity holders receive nothing. Structural models can be used to infer the PD from observed firm variables, including the value of equity, the value of debt, and the stock volatility. For instance, as the value of the equity falls, the default probability increases.

Another approach is the class of reduced-form models, initially developed by Jarrow & Turnbull (1995). This approach assumes that default is driven by an intensity, or PD per unit time, that depends on state variables. Compared with structural models, reduced-form models assume that the risk manager has incomplete information about the firm's condition.

These advances led to the development of portfolio credit risk models toward the end of the 1990s. Notable examples are Moody's KMV and CreditMetrics$^{TM}$. A key feature of these models is the specification for the default correlations. These can be driven by estimated equity correlations, or using factor models, as shown by Vasicek (1984). Portfolio credit risk models generate a distribution of losses, such as in **Figure 1**, which can be summarized by a VAR measure, typically over a one-year horizon and at the 99.9% confidence level.

## Credit Default Swaps

For the first time, institutions could measure their total credit risk, based on the current positions. These models quickly spread to the banking sector. The ability to measure credit risk created a need to manage this risk. This led to the growth of the credit default swap market, which is a market to exchange CEs. The market, which is described by Hull (2008), expanded quickly because of its low transaction costs and also because it allowed short positions in credit, which was not feasible previously. Buying a credit default swap creates a profit if the credit event (e.g., default) happens. This is akin to shorting a corporate bond, but much more practical. **Table 2** shows that by 2008, the outstanding notional amount for credit default swaps was approximately $42 trillion, several times the size of the corporate bond markets.

## Implications

Credit risk models led to fundamental changes in the banking industry. Banks realized that holding loans on their books immobilized large amounts of capital and was not very profitable. In response, banks moved to a model in which they could reap fees from the origination of loans, which were then sold to independent entities called special purpose vehicles (SPVs) that were distributed to others. This led to the creation of the structured credit industry, in which pools of debt are put together and sold as securities with various levels of priority on the collateral.

Structured credit has its roots in the collateralized mortgage obligation (CMO) market developed in the early 1980s. Mortgage loans were placed in pools and sold to investors in the form of tranches with different priority claims. The same principle applies to collateralized debt obligations (CDOs). The methodology of credit risk portfolio measurement was applied to CDOs, allowing rapid pricing of the various tranches. This spurred an exponential growth of this market until 2007. CMOs, CDOs, and other structured products are reviewed in Fabozzi (2005).

This growth revealed flaws, however. On the one hand, these new instruments should have dispersed risk to investors and financial institutions across the globe. This is a positive development because it made the banking system more diversified. On the other hand, this dispersion of risk did not work when banks had to absorb on their balance sheets the SPVs that they had originated and that were in danger of failing. In addition, this securitization process led to laxer credit standards that aggravated the scale of losses. Lenders were more interested in generating fee income than the ultimate creditworthiness of borrowers because losses would be borne by somebody else, after all. The Senior Supervisors Group (2009) provides a good review of lessons to be drawn from the banking crisis.

### Limitations of Credit Risk Models

Credit risk models inherit the same limitations as traditional VAR systems. In addition, they face specific issues. Defaults seem to cluster more than the models predict. This could reflect direct linkages among firms that are not captured by these abstract models.

For instance, it is difficult to account fully for counterparty risk. It is not enough to know your counterparty; you need to know your counterparty's counterparties too. Haldane (2009) calls these effects network externalities. For example, understanding the full consequences of Lehman's failure would have required information on the entire topology of the financial network. It will be difficult for risk models to capture these interactions. These effects also explain why regulators want to impose more transparency in financial markets and are trying to move derivatives trading to clearinghouses. Clearinghouses offer many benefits: (*a*) They allow netting of counterparty exposures, (*b*) they lower exposures through margins and regular marking to market, and (*c*) they can help regulators identify institutions with exposures that could potentially create systemic risk. In practice, clearinghouses need to be sufficiently capitalized and manage their risk conservatively; otherwise, they could themselves be a source of systemic risk.

## OPERATIONAL RISK

Risk measurement techniques are now being extended to operational risk, which is the risk of losses resulting from inadequate or failed internal processes, people, and systems or from external events. Operational risk has caused large losses to financial institutions. In 1995, a single rogue trader caused a loss of $1.3 billion at Barings Bank, which led to its failure. This was a case of internal fraud, which is a category of operational risk. As a result, the Basel Committee on Banking Supervision (2005) mandated a new capital charge against operational risk. This forced the industry to pay attention and even try to measure operational risk.

The most advanced methods for measuring operational risk are based on the frequency of losses over a horizon as well as the severity of losses when they happen. These two statistical distributions are combined into a distribution of losses, which is summarized by the worst loss at a high confidence level. Operational risk methods are reviewed in Chernobai et al. (2007).

The measurement of operational risk is still controversial, however. Data on large operational risk losses are scarcer than for other types of risk. In addition, losses may not be applicable to banks with different control environments. Even so, institutions that are now measuring operational risk find that this often leads to improvements in internal processes.

## ENTERPRISE RISK MANAGEMENT

Finally, risk measurement techniques are now applied at the highest level of the institution, across several risk categories, which leads to the concept of enterprise-wide risk management. Economic capital (EC) is the amount of capital an institution would voluntarily set aside to support its business activities. This is typically estimated as a VAR measure for the distribution of profits and losses of current positions, but at a very high confidence level such as 99.97% over a year. The goal is to help determine whether the institution has sufficient equity capital to absorb a large, but plausible, loss.

Such measures, however, are very different from the original design of VAR models, the goal of which was to provide a loss measure over a short horizon at a reasonable confidence level (e.g., 95% or 99%). EC measures are less reliable for a number of reasons. First, quantiles at higher confidence levels are statistically less precise because there are fewer data observations in the tails. Second, the fixed position assumption is less tenable over a longer horizon. Third, there may not be sufficient data over a variety of business cycles to represent risk adequately. Finally, all of the risks, including their interactions, must be captured. Risk managers should be keenly aware of these limitations.

## CONCLUSIONS

Modern risk management techniques have transformed the financial industry. Risk can now be measured at the top level of an institution, using position-level information. VAR is now commonly used as a measure of the worst loss under normal market conditions. Modern risk management systems can implement stress tests easily.

In spite of its quantitative aspects, risk management is still largely an art form, however. The design of risk architecture involves many trade-offs. The goal of risk systems should be to produce reasonably accurate estimates of risk at a reasonable cost and within a reasonable time frame. Most importantly, the risk manager needs to be keenly aware of limitations in quantitative risk measures.

Finally, quantitative risk measurement tools are not effective with some important sources of risk, such as liquidity risk. This is the risk of loss due to the inability to meet payments obligations, which may force early liquidation of assets at fire-sale prices. Creditors may refuse to roll over their funding, creating bank runs such as those that befell Northern Rock and Bear Stearns. More generally, no model can fully account for systemic risk, which arises when default by one institution has a cascading effect on other firms, thus posing a threat to the viability of the entire financial system. Systemic risk should be handled by the central bank, which is effectively becoming the risk manager of last resort.

## DISCLOSURE STATEMENT

The author is not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

## LITERATURE CITED

Artzner P, Delbaen F, Eber JM, Heath D. 1999. Coherent measures of risk. *Math. Finance* 9:203–28

Basel Committee on Banking Supervision. 1996. *Amendment to the Capital Accord to Incorporate Market Risk*. Basel: BIS. 56 pp.

Basel Committee on Banking Supervision. 2005. *International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. Basel: BIS. 333 pp.

Black F, Scholes M. 1973. The pricing of options and corporate liabilities. *J. Polit. Econ.* 81:637–59

Boyle P. 1977. Options: A Monte Carlo approach. *J. Financ. Econ.* 4:323–38

Chernobai R, Rachev S, Fabozzi F. 2007. *Operational Risk*. New York: Wiley

Engle R. 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* 50:987–1007

Fabozzi FJ. 2005. *The Handbook of Fixed Income Securities*. New York: McGraw-Hill. 1339 pp.

Fama E. 1965. The behavior of stock market prices. *J. Bus.* 38:34–105

Haldane AG. 2009. *Why Banks Failed the Stress Test*. London: Bank Engl. 23 pp.

Heath D, Jarrow R, Morton A. 1992. Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation. *Econometrica* 60:77–105

Hull JC. 2008. *Options, Futures, and Other Derivatives*. Upper Saddle River, NJ: Prentice Hall. 744 pp. 7th ed.

Jarrow R, Turnbull S. 1995. Pricing derivatives on financial securities subject to default risk. *J. Finance* 50:53–86

Jorion P. 2000. Risk management lessons from long-term capital management. *Eur. Financ. Manag.* 6:277–300

Jorion P. 2006. *Value at Risk: The New Benchmark for Managing Financial Risk*. New York: McGraw-Hill. 600 pp. 3rd ed.

J.P. Morgan, Reuters Ltd. 1996. *RiskMetrics™—Technical Document*. New York: Morgan Guaranty Trust Co. New York. 296 pp. 4th ed.

King B. 1966. Market and industry factors in stock price behavior. *J. Bus.* 39:139–90

Macaulay FR. 1938. *The Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856*. New York: Natl. Bur. Econ. Res. 618 pp.

Markowitz H. 1952. Portfolio selection. *J. Finance* 7:77–91

Merton R. 1974. On the pricing of corporate debt: the risk structure of interest rates. *J. Finance* 29:449–70

McNeil AJ, Frey R, Embrechts P. 2005. *Quantitative Risk Management: Concepts, Techniques and Tools*. Princeton: Princeton Univ. Press. 538 pp.

Ross S. 1976. The arbitrage theory of capital asset pricing. *J. Econ. Theory.* 13:341–60

Senior Supervisors Group. 2009. Risk management lessons from the global banking crisis of 2008. *Senior Supervisors Group Rep.*, BIS, Basel, Switz.

Sharpe WF. 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. *J. Finance* 19:425–42

Vasicek OA. 1984. *Credit valuation*. White pap., Moody's, LLC San Francisco. 16 pp.