CONCURRENT BINARY SEARCH TREES: DESIGN AND OPTIMIZATIONS

by

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To my parents and sister

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PREFACE

This dissertation was produced in accordance with guidelines which permit the inclusion as part of the dissertation the text of an original paper or papers submitted for publication. The dissertation must still conform to all other requirements explained in the "Guide for the Preparation of Master's Theses and Doctoral Dissertations at The University of Texas at Dallas." It must include a comprehensive abstract, a full introduction and literature review, and a final overall conclusion. Additional material (procedural and design data as well as descriptions of equipment) must be provided in sufficient detail to allow a clear and precise judgment to be made of the importance and originality of the research reported.

It is acceptable for this dissertation to include as chapters authentic copies of papers already published, provided these meet type size, margin, and legibility requirements. In such cases, connecting texts which provide logical bridges between different manuscripts are mandatory. Where the student is not the sole author of a manuscript, the student is required to make an explicit statement in the introductory material to that manuscript describing the student's contribution to the work and acknowledging the contribution of the other author(s). The signatures of the Supervising Committee which precede all other material in the dissertation attest to the accuracy of this statement.

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Over the last decade processor clock speeds have hit a wall. But the demand for performance improvements continued to grow. So there has been a major shift towards multi-core and many-core processors. However, most of the software applications running on multi-core processors do not fully utilize all the cores. Rewriting the entire software stack for large applications seems impractical. On the other hand, improving the performance of common data structures which are the building blocks of any software application seemed feasible. This motivated the design of concurrent data structures.

Designing a concurrent data structure is far more challenging than its sequential counterpart because threads executing concurrently may interleave in exponential possible ways. A concurrent data structure should preserve its equivalent sequential specifications for all such interleavings.

In this work, we focus on concurrent binary search trees. We present a blocking and a non-blocking algorithm for concurrent manipulation of a binary search tree in an asynchronous shared memory system that supports search, insert and delete operations. We also provide a general technique to optimize them. Our technique is sufficiently general in the sense that it can be applied to a variety of concurrent binary search trees based on both blocking and non-blocking approaches.

Moreover, we also present several techniques to make search operations on such binary search trees *wait-free*. Our techniques have the advantage that a search operation does not need to perform any write instruction on shared memory thereby minimizing the cache traffic.

Experiments indicate that our algorithms perform best in most cases. And our optimization technique improves performance of our algorithms and other existing algorithms.

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CHAPTER 1

INTRODUCTION

With the growing prevalence of multi-core, multi-processor systems, concurrent data structures are becoming increasingly important. In such a data structure, multiple processes may need to operate on the data structure at the same time. Contention between different processes must be managed in such a way that all operations complete correctly and leave the data structure in a valid state.

Concurrency is often managed using locks. A lock can be used to achieve mutual exclusion, which can then be used to ensure that any updates to the data structure or a portion of it are performed by one process at a time. This makes it easier to design a lock-based concurrent data structure and reason about its correctness. Moreover, this also makes it easier to implement a lock-based data structure and debug it than its lock-free counterpart. Lock-based algorithms for concurrent versions of many important data structures for storing and managing shared data have been developed including linked lists, queues, priority queues, hash tables and skiplists (*e.g.*, [15, 16, 19, 20, 21, 23]).

However, locks are blocking; while a process is holding a lock, no other process can access the portion of the data structure protected by the lock. If a process stalls while it is holding a lock, then the lock may not be released for a long time. This may cause other processes to wait on the stalled process for extended periods of time. As a result, lock-based implementations of concurrent data structures are vulnerable to problems such as deadlock, priority inversion and convoying [16].

Non-blocking algorithms avoid the pitfalls of locks by using special (hardware-supported) *read-modify-write* instructions such as *load-link/store-conditional* *(LL/SC)* and *compare-and-swap (CAS)* [16]. Non-blocking implementations of many common data structures such as queues, stacks, linked lists, hash tables and search trees have been proposed (*e.g.*, [3, 4, 9, 10, 11, 13, 16, 18, 21, 24, 25, 26]).

Binary search tree is one of the fundamental data structures for organizing *ordered* data that supports search, insert and delete operations [8]. A binary search tree may be unbalanced (different leaf nodes may be at very different depths) or balanced (all leaf nodes are at roughly the same depth). A balanced binary search tree provides better worst-case guarantees about the cost of performing an operation on the tree. However, in many cases, the overhead of keeping the tree balanced, especially in a concurrent environment, may incur significant overhead. As a result, in many cases, an unbalanced binary search tree outperforms a balanced binary search tree in practice. In this work, our focus is on developing an efficient concurrent algorithms for an *unbalanced* binary search tree.

Concurrent algorithms for unbalanced binary search trees have been proposed in [2, 6, 9, 10, 11, 18, 25]. Algorithms in [2, 9] are blocking (or lock-based), whereas those in [6, 10, 11, 18, 25] are non-blocking (or lock-free). Also, algorithms in [2, 6, 9, 18] use internal representation of a search tree in which all nodes store data, where as those in [10, 11, 25] use an external representation of a search tree in which only leaf nodes store data (data stored in internal nodes is used for routing purposes only).

Algorithms that use internal representation of a search tree have to address the problem that arises due to a key moving from one location in the tree to another. This occurs when the key undergoing deletion resides in a binary node, which requires it to be either replaced with its predecessor (next smallest key) or its successor (next largest key). As a result, an operation traversing the tree may fail to find the target key both at its old location and at its new location, even though the target key was continuously present in the tree. Different algorithms use different approaches to handle the problem arising due to key movement. The algorithm by Drachsler *et al.* in [9] maintains a *sorted* linked list of all the keys in the tree. If the traversal of the tree fails to find a given key, then an operation traverses the linked list to look for the key. The algorithm by Arbel and Hattiya [2] uses the RCU (Read-Copy-Update) framework (first employed in Linux kernels) to allow reads to occur concurrently with updates.

Most of the concurrent algorithms (for BSTs) that have proposed so far use a naïve approach and simply restart the traversal from the root of the tree [2, 9, 10, 18, 25]. This is especially undesirable if the tree has large height, and the overhead of repeatedly traversing the tree may dominate all other overheads of performing an operation.

Recently, a few algorithms have been proposed in which an operation attempts to recover from a failure *locally* [6, 11]. The algorithm in [11], which is based on external representation and builds upon the algorithm in [10], maintains a stack of the nodes visited during the traversal of the tree and simply restarts from the last "unmarked" node (a node is marked before it is removed from the tree). Intuitively, this works because, in an external search tree, keys *do not* move from one location in the tree to another. However, in a search tree based on internal representation, keys may move from one location (in the tree) to another. This occurs when the key undergoing deletion resides in a binary node, which requires it to be either replaced with its predecessor (next smallest key) or its successor (next largest key). This causes two problems. First, an operation traversing the tree may fail to find the target key both at its old location and at its new location, even though the target key was continuously present in the tree. Second, it is not always safe to simply restart from the last unmarked node in the stack since the key may have moved to an ancestor of such a node. Their restart approach is, however, sufficiently general that it can be applied to other concurrent search trees based on external representation [25].

The algorithm in [6], which is based on internal representation, uses *backlink pointers* to find its way and recover from a failure while executing an operation. Their restart approach appears to be customized for their concurrent search tree and is not clear how it can be extended to other concurrent search trees based on internal representation.

1.1 Contributions

First we present a new *lock-based* algorithm for concurrent manipulation of a binary search tree in an asynchronous shared memory system that supports search, insert and delete operations. Our algorithm is based on an internal representation of a search tree as in [2, 9]. However, as in [25], it operates at edge-level (locks edges) rather than at node-level (locks nodes); this minimizes the contention window of a write operation and improves the system throughput. Further, in our algorithm, (i) a search operation uses only read and write instructions, (ii) an insert operation does not acquire any locks, and (iii) a delete operation only needs to lock up to four edges in the absence of contention. Our experiments indicate that our lock-based algorithm outperforms existing algorithms for a concurrent binary search tree—blocking as well as non-blocking—for medium-sized and larger trees, achieving up to 59% higher throughput than the next best algorithm.

Second we extend the previous algorithm to develop a new *lock-free* algorithm. It combines ideas from two existing lock-free algorithms, namely those by Howley and Jones [18] and Natarajan and Mittal [25], and is especially *optimized for the conflict-free scenario*. Like Howley and Jones' algorithm, it uses internal representation of a search tree in which all nodes store keys. Also, like Natarajan and Mittal's algorithm, it operates at edge-level rather than node-level and does not use a separate explicit object for enabling coordination among conflicting operations. As a result, it inherits benefits of both the lock-free algorithms. Specifically, when compared to modify operations of Howley and Jones' internal binary search tree, its modify operations (a) have a smaller contention window, (b) allocate fewer objects, (c) execute fewer atomic instructions, and (d) have a smaller memory footprint. Our experiments indicate that our new lock-free algorithm outperforms other lock-free algorithms in most cases, providing up to 35% improvement in some cases over the next best algorithm.

Third, we present a general approach for local recovery that enables a process to quickly recover from a failure while performing an operation by restarting the traversal from a point "close" to the operation's window rather than the root of the tree. Our approach can be applied to many existing concurrent algorithms for maintaining binary search trees using internal representation—blocking as well as non-blocking—such as those in [2, 9, 18, 28]. Our local recovery approach uses only local variables and does not require modifying a tree node (of the original algorithm) to store any additional information. Using experimental evaluation, we demonstrate that our local recovery approach can yield significant speed-ups for many concurrent algorithms.

Finally, we present two light-weight techniques to make search operations for concurrent binary search trees based on internal representation, such as those in those in [2, 9, 18, 27, 28], *wait-free* with low additional overhead. Both of our techniques have the desirable feature that a search operation does not need to perform any write instructions on the share memory thereby minimizing the cache coherence traffic.

1.2 Dissertation Roadmap

This dissertation organized as follows. We describe our system model in Chapter 2. Our lock-based algorithm for a binary search tree is described in Chapter 3. Our lock-free algorithm for a binary search tree is described in Chapter 4. Our general technique for local recovery is described in Chapter 5. The experimental evaluation of different concurrent algorithms for a binary search tree is described in Chapter 7. Finally Chapter 8 concludes the dissertation and outlines directions for future research.

CHAPTER 2

SYSTEM MODEL

2.1 Binary Search Tree

We assume that a binary search tree (BST) implements a dictionary abstract data type and supports *search*, *insert* and *delete* operations. For convenience, we refer to the insert and delete operations as *modify* operations. A search operation explores the tree for a given key and returns true if the key is present in the tree and false otherwise. An insert operation adds a given key to the tree if the key is not already present in the tree. Duplicate keys are not allowed in our model. A delete operation removes a key from the tree if the key is indeed present in the tree. In both cases, a modify operation returns true if it changed the set of keys present in the tree (added or removed a key) and false otherwise.

A binary search tree satisfies the following properties:

(a) the left subtree of a node contains only nodes with keys less than the node's key,

(b) the right subtree of a node contains only nodes with keys greater than or equal to the node's key, and

(c) the left and right subtrees of a node are also binary search trees.

2.2 Synchronization Primitives

We assume an asynchronous shared memory system that, in addition to read and write instructions, also supports compare-and-swap (CAS) atomic instruction. A compare-and-swap instruction takes three arguments: *address*, *old* and *new*; it compares the contents of a memory location (*address*) to a given value (*old*) and, only if they are the same, modifies the contents of that location to a given new value (*new*). The CAS instruction is commonly available in many modern processors such as Intel 64 and AMD64.

We also use locks and assume that the following properties hold true about the locks

(a) safe: it satisfies the mutual exclusion property, *i.e.*, at most one process can hold the lock at any time, and

(b) live: it satisfies the deadlock freedom property, *i.e.*, if the lock is free and one or more processes attempt to acquire the lock, then some process is eventually able to acquire the lock.

2.3 Proof of correctness

To demonstrate the correctness of our algorithm, we use *linearizability* [17] for the safety property and *deadlock-freedom* [16] for the liveness property. Broadly speaking, linearizability requires that an operation should appear to take effect instantaneously at some point during its execution. Deadlock-freedom requires that some process with a pending operation be able to complete its operation eventually.

PART I

DESIGN

CHAPTER 3

LOCK BASED CONCURRENT BINARY SEARCH TREE

3.1 The Lock-Based Algorithm

We first provide an overview of our algorithm. We then describe the algorithm in more detail and also give its pseudo-code. For ease of exposition, we describe our algorithm assuming no memory reclamation, which can be performed using the well-known technique of hazard pointers [22].

3.1.1 Overview of the Algorithm

Every operation in our algorithm uses *seek* function as a subroutine. The seek function traverses the tree from the root node until it either finds the target key or reaches a non-binary node whose next edge to be followed points to a null node. We refer to the path traversed by the operation during the seek as the *access-path*, and the last node in the access-path as the *terminal node*. The operation then compares the target key with the stored key (the key present in the terminal node). Depending on the result of the comparison and the type of the operation, the operation either terminates or moves to the execution phase. In certain cases in which a key may have moved upward along the access-path, the seek function may have to restart and traverse the tree again; details about restarting are provided later. We now describe the next steps for each of the type of operation one-by-one.

Search: A search operation starts by invoking seek operation. It returns `true` if the stored key matches the target key and `false` otherwise.

Insert: An insert operation starts by invoking seek operation. It returns false if the target key matches the stored key; otherwise, it moves to the execution phase. In the execution phase, it attempts to insert the key into the tree as a child node of the last node in the access-path using a CAS instruction. If the instruction succeeds, then the operation returns true; otherwise, it restarts by invoking the seek function again.

Delete: A delete operation starts by invoking seek function. It returns false if the stored key does not match the target key; otherwise, it moves to the execution phase. In the execution phase, it attempts to remove the key stored in the terminal node of the access-path. There are two cases depending on whether the terminal node is a binary node (has two children) or not (has at most one child). In the first case, the operation is referred to as *complex delete operation*. In the second case, it is referred to as *simple delete operation*. In the case of simple delete, the terminal node is removed by changing the pointer at the parent node of the terminal node. In the case of complex delete, the key to be deleted is replaced with the *next largest* key in the tree, which will be stored in the *leftmost node* of the *right subtree* of the terminal node.

3.1.2 Details of the Algorithm

As in most algorithms, to make it easier to handle special cases, we use sentinel keys and sentinel nodes. The structure of an empty tree with only sentinel keys (denoted by ∞_1 and ∞_2 with $\infty_1 < \infty_2$) and sentinel nodes (denoted by $\mathbb{R}$ and $\mathbb{S}$) is shown in Figure 3.1.

Our algorithm, like the one in [25], operates at edge level. A delete operation obtains ownership of the edges it needs to work on by locking them. To enable locking of an edge, we steal a bit from the child addresses of a node referred to as *lock-flag*. We also steal another bit from the child addresses of a node to indicate that the node is undergoing deletion and will be removed from the tree. We denote this bit by *mark-flag*. Finally, to avoid the ABA

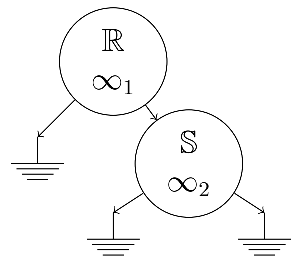


Figure 3.1: Sentinel keys and nodes ($\infty_1 < \infty_2$)

problem, as in Howley and Jones [18], we use *unique* null pointers. To that end, we steal yet another bit from the child address, referred to as *null-flag*, and use it to indicate whether the address points to a null or a non-null address. So, when an address changes from a non-null value to a null value, we only set the null-flag and the contents of the address are not otherwise modified. This ensures that all null pointers are unique.

We next describe the details of the seek operation, which is executed by all operations (search as well as modify) after which we describe the details of the execution phase of insert and delete operations.

The Seek Phase

A seek function keeps track of the node in the access-path at which it took the last "right turn" (*i.e.*, it last followed a right edge). Let this "right turn" node be referred to as *anchor node* when the traversal reaches the terminal node. Note that the terminal node is the node whose key matched the target key or whose next child edge is set to a null address. For an illustration, please see Figure 3.2. In the latter case (stored key does not match the target key), it is possible that the key may have moved up in the tree. To ascertain that the seek function did not miss the key because it may have moved up during the traversal, we use the following set of conditions that are *sufficient* (but not necessary) to guarantee that the seek function did not miss the key. First, the anchor node is still part of the tree. (For an illustration, see Figure 3.3) Second, the key stored in the anchor node has not changed since it first encountered the anchor node during the (current) traversal. To check for the above two conditions, we determine whether the anchor node is undergoing deletion by examining it right child edge. We discuss the two cases one-by-one.

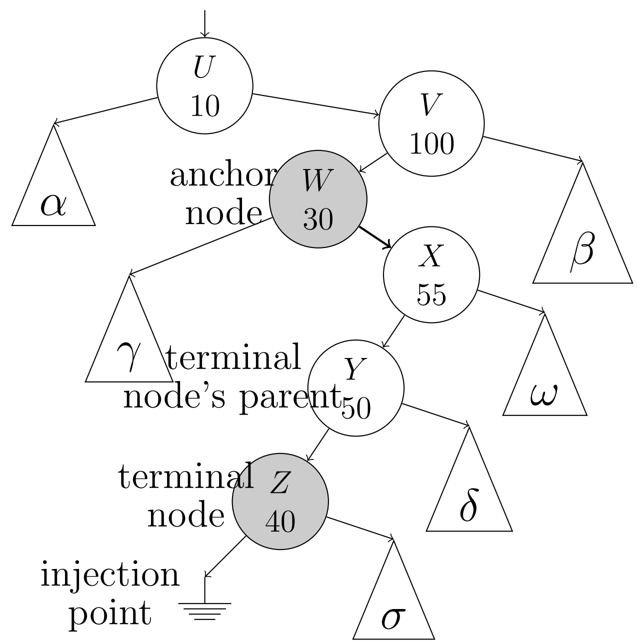


Figure 3.2: Nodes in the access path of seek

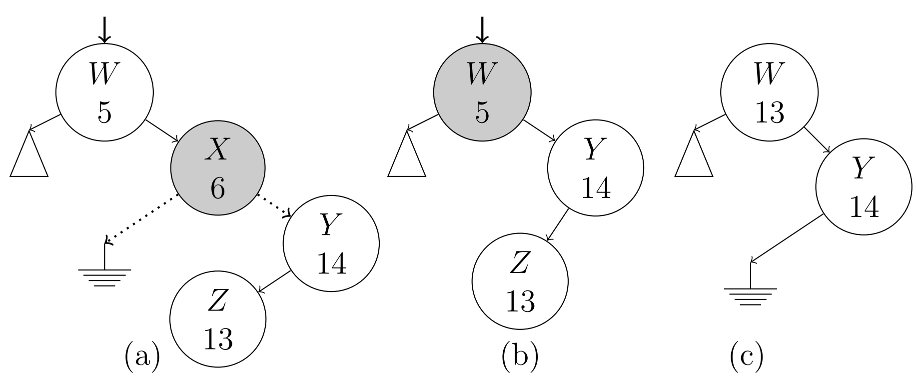


Figure 3.3: A case when last right turn node is no longer part of the tree

(a) *Right child edge not marked:* In this case, the anchor node is still part of the tree. We next check whether the key stored in the anchor node has changed. If the key has not changed, then the seek function returns the results of the (current) traversal, which consists of three addresses: (i) the address of the terminal node, (ii) the address of its parent, and (iii) the null address stored in the child field of the terminal node that caused the traversal to terminate. The last address is required to ensure that an insert operation works correctly (specifically to ascertain that the child field of the terminal node has not undergone any change since the completion of the traversal). We refer to it as the *injection point* of the insert operation. On the other hand, if the key has changed, then the seek function restarts from the root of the tree. A possible optimization is that the seek function restarts only if the target key is now less than the anchor node's key.

(b) *Right child edge marked:* In this case, we compare the information gathered in the current traversal about the anchor node with that in the previous traversal, if one exists. Specifically, if the anchor node of the previous traversal is same as that of the current traversal and the keys found in the anchor node in the two traversals also match, then the seek function terminates, but returns the results of the previous traversal (instead of that of the current traversal). This is because the anchor node was definitely part of the tree during the previous traversal since it was reachable from the root of the tree at the beginning of the current traversal. Otherwise, the seek function restarts from the root of the tree.

For insert and delete operations, we refer to the terminal node as the *target node*.

The Execution Phase of an Insert Operation

In the execution phase, an insert operation creates a new node containing the target key. It then adds the new node to the tree at the injection point using a CAS instruction. For an illustration, see Figure 3.4. If the CAS instruction succeeds, then (the new node becomes a part of the tree and) the operation terminates; otherwise, the operation restarts from the seek phase. Note that the insert operations are lock-free.

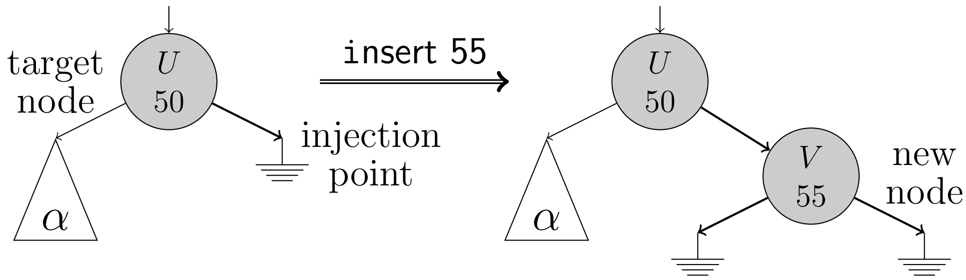


Figure 3.4: An illustration of an insert operation

The Execution Phase of a Delete Operation

The execution of a delete operation starts by checking if the target node is a binary node or not. If it is a binary node, then the delete operation is classified as complex; otherwise it is classified as simple.

For a tree node X, let $X.parent$ denote its parent node, and $X.left$ and $X.right$ denote its left and right child node, respectively. Also, hereafter in this section, let T denote the target node of the delete operation under consideration.

(a) *Simple Delete:* In this case, either $T.left$ or $T.right$ is pointing to a null node. Note that both $T.left$ and $T.right$ may be pointing to null nodes in which case T will be a leaf node. Without loss of generality, assume that $T.right$ is a null node. The removal of T involves locking the following three edges: $\langle T.parent, T\rangle$, $\langle T, T.left\rangle$ and $\langle T, T.right\rangle$. For an illustration, see Figure 3.5.

A lock on an edge is obtained by setting the lock-flag in the appropriate child field of the parent node using a CAS instruction. For example, to lock the edge $\langle X, Y\rangle$, where Y is the left child of X, the lock-flag in the left child of X is set to one. If all the edges are locked successfully, then the operation validates that the key stored in the target node still matches the target key. If the validation succeeds, then the operation marks both the children edges of T to indicate that T is going to be removed from the tree. Next, it changes the child pointer at $T.parent$ that is pointing to T to point to

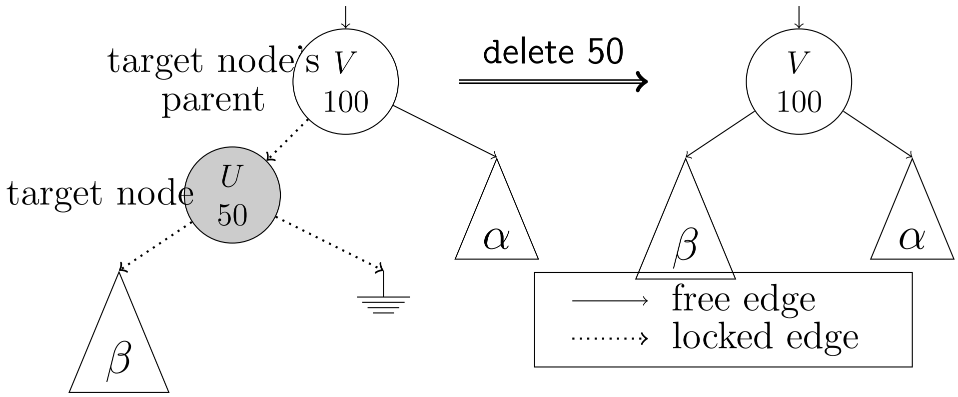


Figure 3.5: An illustration of a simple delete operation

$T.left$ using a simple write instruction. Finally, the operation releases all the locks and returns true.

(b) *Complex Delete:* In this case, both $T.left$ and $T.right$ are pointing to non-null nodes. The operation locates the next largest key in the tree, which is the smallest key in the subtree rooted at the right child of T. We refer to this key as the *successor key* and the node storing this key as the *successor node*. Hereafter in this section, let S denote the successor node. Deletion of the key stored in T involves copying the key stored in S to T and then removing S from the tree. To that end, the following edges are locked by setting the lock-flag on the edge using a CAS instruction: $\langle T, T.right\rangle$, $\langle S.parent, S\rangle$, $\langle S, S.left\rangle$ and $\langle S, S.right\rangle$. For an illustration, see Figure 3.6. Note that the first two edges may be same which happens if the successor node is the right child of the target node. Also, since we do not lock the left edge of the target node, the left edge may change and may possibly start pointing to a null address. But, that does not impact the correctness of the complex delete operation.

If all the edges are locked successfully, then the operation validates that the key stored in the target node still matches the target key. If the validation succeeds, then the operation copies the key stored in S to T, and marks both the children edges of S to indicate that S is going to be removed from the tree. Next, it changes the child pointer

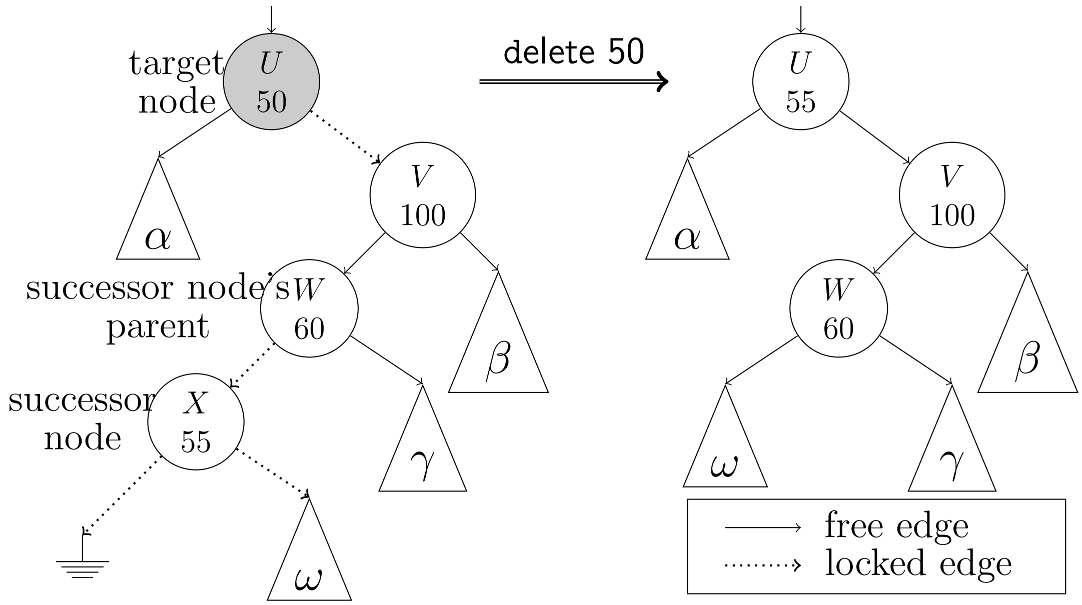


Figure 3.6: An illustration of a complex delete operation

at *S.parent* that is pointing to S to point to *S.right* using a simple write instruction. Finally, the operation releases all the locks and returns true.

In both cases (simple as well as complex delete), if the operation fails to obtain any of the locks, then it releases all the locks it was able to acquire up to that point, and restarts from the seek phase. Also, after obtaining all the locks, if the key validation fails, then it implies that some other delete operation has removed the key from the tree while the current execution phase was in progress. In that case, the given delete operation releases all the locks, and simply returns false. Note that using a CAS instruction for setting the lock-flag also enables us to *validate that the child pointer has not changed* since it was last observed in a single step.

3.1.3 Formal Description

We refer to our algorithm as CASTLE (Concurrent Algorithm for Binary Search Tree by Locking Edges).

Algorithm 1: Data Structures Used

```
// a tree node
struct Node {
    Key key;
    { boolean, boolean, boolean, NodePtr } child[2];
    // each child field contains four subfields: lFlag, mFlag, nFlag and address
};
// used to store the results of a tree traversal
struct SeekRecord {
    NodePtr node;
    NodePtr parent;
    NodePtr nullAddress;
};
// used to store information about an anchor node
struct AnchorRecord {
    NodePtr node;
    Key key;
};
// used to store information about an edge to lock
struct LockRecord {
    NodePtr node;
    enum { LEFT, RIGHT } which;
    { boolean, NodePtr } addressSeen;
    // addressSeen contains two subfields: nFlag and address
};
// local seek record used when looking for a node
SeekRecordPtr seekTargetKey, seekSuccessorKey;
// local array used to store the set of edges to lock
LockRecord lockArray[4];
```

A pseudo-code of our algorithm is given in Algorithms 1-7. Different data structures used in our algorithm are shown in Algorithm 1. Besides tree node, we use three additional records: (a) *seek record:* to store the outcome of a tree traversal both when looking for the target key and the successor key, (b) *anchor record:* to store information about the anchor node during the seek phase, and (c) *lock record:* to store information about a tree edge that needs to be locked.

The pseudo-code for the seek function is shown in Algorithm 2. The pseudo-codes for search, insert and delete operations are shown in Algorithm 3, Algorithm 4 and Algorithm 5, respectively. Algorithm 6 contains the pseudo-code for locking and unlocking a set of tree

Algorithm 2: Seek Function

```
Seek( key, seekRecord )
begin
    while true do
        // initialize the variables used in the traversal
        pNode := ℝ;     cNode := 𝕊;
        address := 𝕊→child[RIGHT].address;
        anchorRecord := {ℝ, ∞_1};
        while true do
            // reached terminal node; read the key stored in the current node
            cKey := cNode→key;
            if key = cKey then
                seekRecord := {cNode, pNode, address};
                return;
            which := key < cKey ? LEFT : RIGHT;
            // read the next address to dereference along with mark and null flags
            ⟨∗, ∗, nFlag, address⟩ := cNode→child[which];
            if nFlag then                    // the null flag is set; reached terminal node
                aNode := anchorRecord→node;
                if aNode→child[RIGHT].mFlag then
                    // the anchor node is marked; it may no longer be part of the tree
                    if anchorRecord = pAnchorRecord then
                        // the anchor record of the current traversal matches that of
                           the previous traversal
                        seekRecord := pSeekRecord;
                        return;
                    else break;
                else                         // the anchor node is definitely part of the tree
                    if aNode→key < key then                        // seek can terminate now
                        seekRecord := {cNode, pNode, address};
                        return;
                    else break;

            // update the anchor record if needed
            if which = RIGHT then
                // the next edge to be traversed is a right edge; keep track of
                   current node and its key
                anchorRecord := {cNode, cKey};

            // traverse the next edge
            pNode := cNode;     cNode : =address;
```

Algorithm 3: Search Operation

```
boolean Search( key )
begin
    Seek( key, seekTargetKey );
    node := seekTargetKey→node;
    if node→key = key then
        return true;                                   // key found
    else
        return false;                                  // key not found
```

Algorithm 4: Insert Operation

```
boolean Insert( key )
begin
    while true do
        Seek( key, seekTargetKey );
        node := seekTargetKey→node;
        if node→key = key then
            return false;                              // key found
        else
            // key not found; add the key to the tree
            newNode := create a new node;
            // initialize its fields
            newNode→key := key;
            newNode→child[LEFT] := ⟨0_l, 0_m, 1_n, null⟩;
            newNode→child[RIGHT] := ⟨0_l, 0_m, 1_n, null⟩;
            // determine which child field (left or right) needs to be modified
            which := key < node→key ? LEFT : RIGHT;
            // fetch the address observed by the seek function in that field
            address := seekTargetKey→nullAddress;
            result := CAS( node→child[which],
                           ⟨0_l, 0_m, 1_n, address⟩,
                           ⟨0_l, 0_m, 0_n, newNode⟩ );
            if result then
                // new key successfully added to the tree
                return true;
```

Algorithm 5: Delete Operation

```
boolean DELETE( key )
begin
    while true do
        SEEK( key, seekTargetKey );
        node := seekTargetKey→node;
        if node→key ≠ key then                                          // key not found
            return false;
        else               // key found; read contents of target node's children fields
            lField := CLEARFLAGS( node→child[LEFT] );
            rField := CLEARFLAGS( node→child[RIGHT] );
            if lField.nFlag  or  rField.nFlag then                 // simple delete operation
                parent := seekTargetKey→parent;
                if key < parent→key then  which := LEFT;
                else  which := RIGHT;
                lockArray[0] := {parent, which, ⟨0, node⟩};
                lockArray[1] := {node, LEFT, lField};
                lockArray[2] := {node, RIGHT, rField};
                result := LOCKALL( lockArray, 3 );
                if result then                  // all locks acquired; perform the operation
                    if node→key = key then         // key still matches; remove the node
                        REMOVECHILD( parent, which ); match := true;
                    else  match := false;
                    UNLOCKALL( lockArray, 3 );
                    return match;
            else                 // complex delete operation; locate the successor node
                FINDSMALLEST(node, rField.address, seekSuccessorKey);
                sNode := seekSuccessorKey→node; sParent := seekSuccessorKey→parent;
                // determine the edges to be locked
                lockArray[0] := {node, RIGHT, rField};
                if node ≠ sParent then
                    // successor node is not the right child of target node
                    lockArray[1] := {sParent, LEFT, ⟨0, sNode⟩}; size := 4;
                else  size := 3 ;
                lField := CLEARFLAGS( sNode→child[LEFT] );
                rField := CLEARFLAGS( sNode→child[RIGHT] );
                lockArray[size − 2] := {sNode, LEFT, lField};
                lockArray[size − 1] := {sNode, RIGHT, rField};
                result := LOCKALL( lockArray, size );
                if result then                  // all locks acquired; perform the operation
                    if node→key = key then
                        // key still matches; copy key in successor node to target node
                        node→key := sNode→key;
                        REMOVECHILD( sParent, LEFT ); match := true;
                    else  match := false;
                    UNLOCKALL( lockArray, size );
                    return match;
```

Algorithm 6: Lock and Unlock Functions

```
boolean LockAll( lockArray, size )
begin
    for i ← 0 to size − 1 do
        // acquire lock for the i-th entry
        node := lockArray[i].node;
        which := lockArray[i].which;
        lockedAddress := lockArray[i].addressSeen;
        lockedAddress.lFlag := true;
        // set the lock flag in the child edge
        result := CAS(node→child[which], lockArray[i].addressSeen, lockedAddress);
        if not (result) then
            // release all the locks acquired so far
            UnlockAll( lockArray, i − 1 );
            return false;
    return true;

UnlockAll( lockArray, size )
begin
    for i ← size − 1 to 0 do
        node := lockArray[i].node;
        which := lockArray[i].which;
        // clear the lock flag in the child edge
        node→child[which].lFlag := false;
```

edges, as specified in an array. Finally, Algorithm 7 contains the pseudo-codes for three helper functions used by a delete operation, namely: (a) ClearFlags: to clear lock and mark flags from a child field, (b) FindSmallest: to locate the smallest key in a subtree, and (c) RemoveChild: to remove a given child of a node.

In the pseudo-code, to improve clarity, we sometimes use subscripts l, m and n to denote lock, mark and null flags, respectively.

3.1.4 Correctness Proof

It is convenient to treat insert and delete operations that do not change the tree as search operations. We call a tree node *active* if it is reachable from the root of the tree. We call a tree node *passive* if it was active earlier but is not active any more. Note that, before an active node is made passive by a delete operation, both its children edges are *marked*. Also,

Algorithm 7: Helper Functions used by Delete Operation

```
word ClearFlags( word field )
begin
    newField := field with lock and mark flags cleared;
    return newField;

FindSmallest( parent, node, seekRecord )
begin
    // initialize the variables used in the traversal
    pNode := parent;      cNode := node;
    while true do
        ⟨*, *, nFlag, address⟩ := cNode→child[LEFT];
        if not (nFlag) then
            // visit the next node
            pNode := cNode;      cNode := address;
        else
            // reached the successor node
            seekRecord := {cNode, pNode, address};
            break;

RemoveChild( parent, which )
hu begin
    // determine the address of the child to be removed
    node := parent→child[which];
    // mark both the children edges of the node to be removed
    node→child[LEFT].mFlag := true;
    node→child[RIGHT].mFlag := true;
    // determine whether both the child pointers of the node to be removed are null
    if node→child[LEFT].nFlag  and  node→child[RIGHT].nFlag then
        // set the null flag only
        parent→child[which].nFlag := true;
    else
        // switch the pointer at the parent to point to its appropriate grandchild
        if node→child[LEFT].nFlag then
            address := node→child[RIGHT].address;
        else  address := node→child[LEFT].address ;
        parent→child[which].address := address;
```

a CAS instruction performed on an edge (by either an insert operation or a delete operation as part of locking) is successful only if the edge is unmarked. As a result, clearly, if an insert operation completes successfully, then its target node was active when its edge was modified to make the new node (containing the target key) a part of the tree. Likewise, if a delete operation completes successfully, then all the nodes involved in the operation (up to three nodes) were active when their edges were locked.

All Executions are Linearizable

We show that an arbitrary execution of our algorithm is linearizable by specifying the *linearization point* of each operation. Note that the linearization point of an operation is the point during its execution at which the operation appeared to have taken effect. Our algorithm supports three types of operations: search, insert and delete. We now specify the linearization point of each operation.

1. *Insert operation:* The operation is linearized at the point at which it performed the successful CAS instruction that resulted in its target key becoming part of the tree.

2. *Delete operation:* There are two cases depending on whether the delete operation is simple or complex. If the operation is simple delete, then the operation is linearized at the point at which a successful write step was performed at the parent of the target node that resulted in the target node becoming passive. Otherwise, it is linearized at the point at which the original key of the target node was replaced with its successor key.

3. *Search operation:* There are two cases depending on whether the target node was active when the operation read the key stored in the node. If the target node was not active, then the operation is linearized at the point at which the target node became passive. Otherwise, it is linearized at the point at which the read step was performed.

It can be easily verified that, for any execution of the algorithm, the sequence of operations obtained by ordering operations based on their linearization points is legal, *i.e.*, all operations in the sequence satisfy their specification.

Thus we have:

Theorem 1. *Every execution of our algorithm is linearizable.*

All Executions are Deadlock-Free

We say that the system is in a *quiescent state* if no modify operation completes hereafter. We say that the system is in a *potent state* if it has one or more pending modify operations. Note that quiescence is a *stable property*; once the system is in a quiescent state, it stays in a quiescent state. We show that our algorithm is deadlock-free by proving that a potent state is necessarily non-quiescent.

Note that, in a quiescent state, no edges in the tree can be marked. This is because a delete operation marks edges only after it has successfully obtained all the locks, after which it is guaranteed to complete. This also implies that the tree cannot undergo any changes now because that would imply eventual completion of a modify operation. Thus, once a system has reached a quiescent state, all modify operation currently pending repeatedly alternate between seek and execution phases. We say that the system is in a *strongly-quiescent state* if all pending modify operations started their most recent seek phase *after* the system became quiescent. Note that, like quiescence, strong quiescence is also a stable property. Now, once the system has reached a strongly quiescent state, the following can be easily verified. First, for a given modify operation, every traversal of the tree in the seek phase returns the same target node. Second, for a given delete operation, the set of edges it needs to lock remains the same.

Now, assume that the system eventually reaches a state that is both potent and quiescent. Clearly, from this state, the system will eventually reach a state that is potent and strongly-quiescent. Note that a delete operation in our algorithm locks edges in a *top-down*, *left-right* manner. As a result, there cannot be a "cycle" involving delete operations. If a delete operation continues to fail in the execution phase, then it is necessarily because it tried to acquire lock on an already locked edge. (Recall that the set of edges does not change any more and there are no marked edges in the tree.) We can construct a chain of operations such that each operation in the chain tried to lock an edge already locked by the next operation in the chain. Clearly, the length of the chain is bounded. This implies that the last operation in the chain is guaranteed to obtain all the locks and will eventually complete. This contradicts the fact that the system is in a quiescent state.

Thus, we have:

Theorem 2. *Every execution of our algorithm is deadlock-free.*

CHAPTER 4

LOCK FREE CONCURRENT BINARY SEARCH TREE

4.1 The Lock-Free Algorithm

For ease of exposition, we describe our algorithm assuming no memory reclamation.

4.1.1 Overview of the Algorithm

Every operation in our algorithm uses *seek* function as a subroutine. The seek function traverses the tree from the root node until it either finds the target key or reaches a non-binary node whose next edge to be followed points to a null node. We refer to the path traversed by the operation during the seek as the *access-path*, and the last node in the access-path as the *terminal node*. The operation then compares the target key with the stored key (the key present in the terminal node). Depending on the result of the comparison and the type of the operation, the operation either terminates or moves to the execution phase. In certain cases in which a key may have moved upward along the access-path, the seek function may have to restart and traverse the tree again; details about restarting are provided later. We now describe the next steps for each of the operations one-by-one.

Search A search operation starts by invoking seek operation. It returns `true` if the stored key matches the target key and `false` otherwise.

Insert An insert operation ((shown in Figure 4.1)) starts by invoking seek operation. It returns `false` if the target key matches the stored key; otherwise, it moves to the execution phase. In the execution phase, it attempts to insert the key into the tree as a child node of

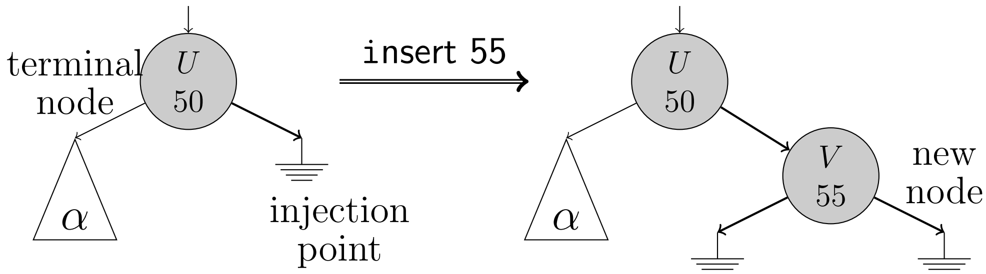


Figure 4.1: An illustration of an insert operation.

the last node in the access-path using a CAS instruction. If the instruction succeeds, then the operation returns true; otherwise, it restarts by invoking the seek function again.

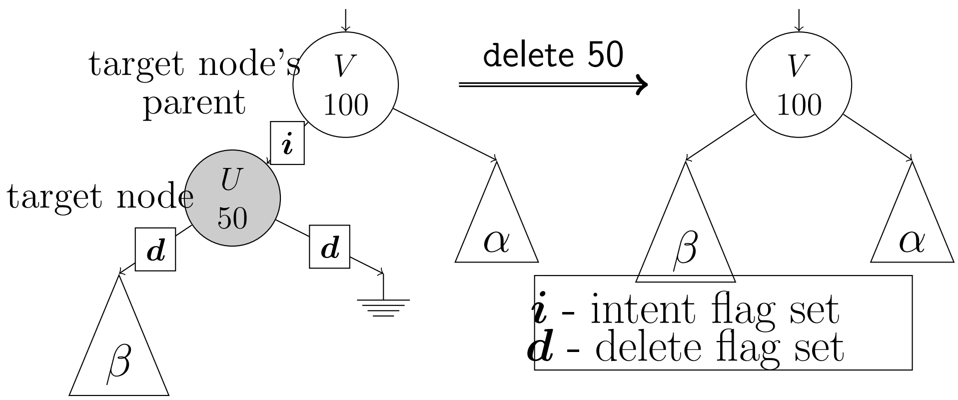


Figure 4.2: An illustration of a simple delete operation.

Delete A delete operation starts by invoking seek function. It returns false if the stored key does not match the target key; otherwise, it moves to the execution phase. In the execution phase, it attempts to remove the key stored in the terminal node of the access-path. There are two cases depending on whether the terminal node is a binary node (has two children) or not (has at most one child). In the first case, the operation is referred to as *complex delete operation*. In the second case, it is referred to as *simple delete operation*. In the case of simple delete (shown in Figure 4.2), the terminal node is removed by changing the pointer at the parent node of the terminal node. In the case of complex delete (shown in Figure 4.3),

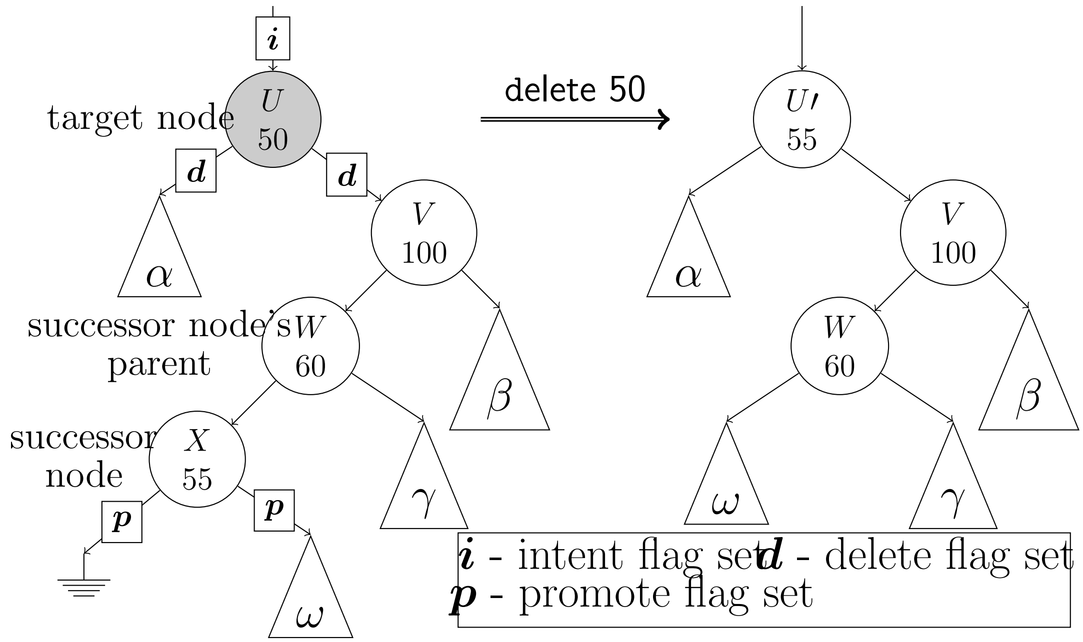


Figure 4.3: An illustration of a complex delete operation.

the key to be deleted is replaced with the *next largest* key in the tree, which will be stored in the *leftmost node* of the *right subtree* of the terminal node.

4.1.2 Details of the Algorithm

As in most algorithms, we use sentinel keys and three sentinel nodes to handle the boundary cases easily. The structure of an empty tree with only sentinel keys (denoted by ∞_0, ∞_1 and ∞_2 with $\infty_0 < \infty_1 < \infty_2$) and sentinel nodes (denoted by $\mathbb{R}$, $\mathbb{S}$ and $\mathbb{T}$) is shown in Figure 4.4.

Our algorithm, like the one in [25], operates at edge level. A delete operation obtains ownership of the edges it needs to work on by marking them. To enable marking, we steal bits from the child addresses of a node. Specifically, we steal *three* bits from each child address to distinguish between three types of marking: (i) marking for *intent*, (ii) marking for *deletion* and (iii) marking for *promotion*. The three bits are referred to as *intent-flag*, *delete-flag* and *promote-flag*. To avoid the ABA problem, as in Howley and Jones [18], we

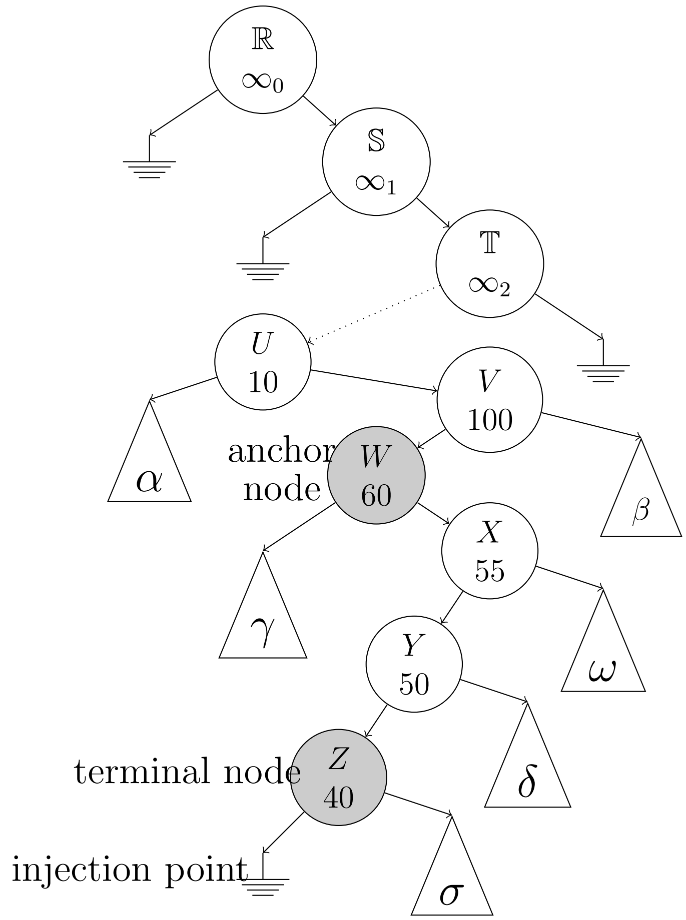


Figure 4.4: Nodes in the access path of seek along with sentinel keys and nodes ($\infty_0 < \infty_1 < \infty_2$)

use *unique* null pointers. To that end, we steal another bit from the child address, referred to as *null-flag*, and use it to indicate whether the address field contains a null or a non-null value. So, when an address changes from a non-null value to a null value, we only set the null-flag and the contents of the address field are not otherwise modified. This ensures that all null pointers are unique.

Finally, we also steal a bit from the key field to indicate whether the key stored in a node is the original key or the replacement key. This information is used in a complex delete operation to coordinate helping among processes.

We next describe the details of the seek function, which is used by all operations (search as well as modify) to traverse the tree after which we describe the details of the execution phase of insert and delete operations.

The Seek Phase

A seek function keeps track of the node in the access-path at which it took the last "right turn" (*i.e.*, it last followed a right edge). Let this "right turn" node be referred to as *anchor node* when the traversal reaches the terminal node. Note that the terminal node is the node whose key matched the target key or whose next child edge is set to a null address. For an illustration, please see Figure 4.4. In the latter case (stored key does not match the target key), it is possible that the key may have moved up in the tree. To ascertain that the seek function did not miss the key because it may have moved up during the traversal, we use the following set of conditions that are *sufficient* (but not necessary) to guarantee that the seek function did not miss the key. First, the anchor node is still part of the tree. Second, the key stored in the anchor node has not changed since it first encountered the anchor node during the (current) traversal. To check for the above two conditions, we determine whether the anchor node is undergoing removal (either delete or promote flag set) by examining its right child edge. We discuss the two cases one-by-one.

(a) *Right child edge not marked:* In this case, the anchor node is still part of the tree. We next check whether the key stored in the anchor node has changed. If the key has not changed, then the seek function returns the results of the (current) traversal, which consists of three addresses: (i) the address of the terminal node, (ii) the address of its parent, and (iii) the null address stored in the child field of the terminal node that caused the traversal to terminate. The last address is required to ensure that an insert operation works correctly (specifically to ascertain that the child field of the terminal node has not undergone any change since the completion of the traversal). We refer to it as the *injection point* of the insert operation. On the other hand, if the key has changed, then the seek function restarts from the root of the tree.

(b) *Right child edge marked:* In this case, we compare the information gathered in the current traversal about the anchor node with that in the previous traversal, if one exists. Specifically, if the anchor node of the previous traversal is same as that of the current traversal and the keys found in the anchor node in the two traversals also match, then the seek function terminates, but returns the results of the previous traversal (instead of that of the current traversal). This is because the anchor node was definitely part of the tree during the previous traversal since it was reachable from the root of the tree at the beginning of the current traversal. Otherwise, the seek function restarts from the root of the tree.

The seek function also keeps track of the *second-to-last* edge in the access-path (whose endpoints are the parent and grandparent nodes of the terminal node), which is used for helping, if there is a conflict. For insert and delete operations, we refer to the terminal node as the *target node*.

The Execution Phase of an Insert Operation

In the execution phase, an insert operation creates a new node containing the target key. It then adds the new node to the tree at the injection point using a CAS instruction. If the CAS instruction succeeds, then (the new node becomes a part of the tree and) the operation terminates; otherwise, the operation determines if it failed because of a *conflicting* delete operation in progress. If there is no conflicting delete operation in progress, then the operation restarts from the seek phase; otherwise it performs helping and then restarts from the seek phase. Figure 4.5 shows a flow chart describing the sequence of steps of an insert operation.

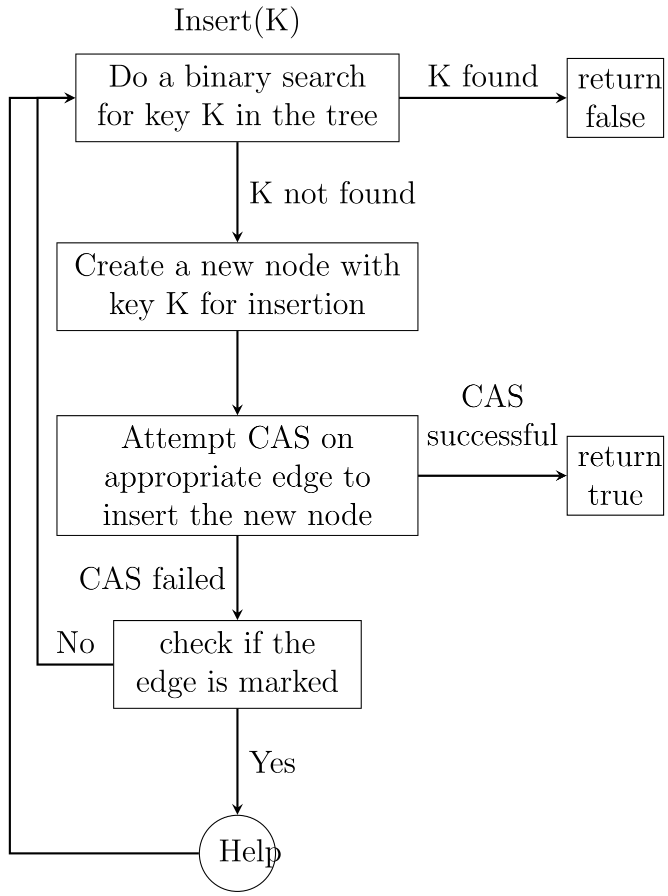


Figure 4.5: ELFTREE - Sequence of steps in an insert operation

The Execution Phase of a Delete Operation

The execution of a delete operation starts in *injection mode*. Once the operation has been injected into the tree, it advances to either *discovery mode* or *cleanup mode* depending on the type of the delete operation.

Injection Mode In the injection mode, the delete operation marks the three edges involving the target node as follows: (i) It first sets the intent-flag on the edge from the parent of the target node to the target node using a CAS instruction. (ii) It then sets the delete-flag on the left edge of the target node using a CAS instruction. (iii) Finally, it sets the delete-flag on the right edge of the target node using a CAS instruction. If the CAS instruction fails at any step, the delete operation performs helping, and either repeats the same step or restarts from the seek phase. Specifically, the delete operation repeats the same step when setting the delete-flag as long as the target node has not been claimed as the successor node by another delete operation. In all other cases, it restarts from the seek phase.

We maintain the invariant that an edge, once marked, cannot be unmarked. After marking both the edges of the target node, the operation checks whether the target node is a binary node or not. If it is a binary node, then the delete operation is classified as complex; otherwise it is classified as simple. Note that the type of the delete operation cannot change once all the three edges have been marked as described above. If the delete operation is complex, then it advances to the discovery mode after which it will advance to the cleanup mode. On the other hand, if it is simple, then it directly advances to the cleanup mode (and skips the discovery mode). Eventually, the target node is either removed from the tree (if simple delete) or replaced with a "new" node containing the next largest key (if complex delete).

For a tree node X, let $X.parent$ denote its parent node, and $X.left$ and $X.right$ denote its left and right child node, respectively. Also, hereafter in this section, let T denote the target node of the delete operation under consideration.

Discovery Mode In the discovery mode, a complex delete operation performs the following steps:

1. **Find Successor Key:** The operation locates the next largest key in the tree, which is the smallest key in the subtree rooted at the right child of T. We refer to this key as the *successor key* and the node storing this key as the *successor node*. Hereafter in this section, let S denote the successor node.
2. **Mark Child Edges of Successor Node:** The operation sets the promote-flag on both the child edges of S using a CAS instruction. Note that the left child edge of S will be

null. As part of marking the left child edge, we also store the address of T (the target node) in the edge. This is done to enable helping in case the successor node is obstructing the progress of another operation. In case the CAS instruction fails while marking the left child edge, the operation repeats from step 1 after performing helping if needed. On the other hand, if the CAS instruction fails while marking the right child edge, then the marking step is repeated after performing helping if needed.

3. **Promote Successor Key:** The operation replaces the target node's original key with the successor key. At the same time, it also sets the mark bit in the key to indicate that the current key stored in the target node is the replacement key and not the original key.
4. **Remove Successor Node:** The operation removes S (the successor node) by changing the child pointer at$S.parent$ that is pointing to S to point to the right child of S using a CAS instruction. If the CAS instruction succeeds, then the operation advances to the cleanup mode. Otherwise, it performs helping if needed. It then finds S again by performing another traversal of the tree starting from the right child of T. If the traversal fails to find S (recall that the left edge of S is marked for promotion and contains the address of T), then S has already been removed from the tree by another operation as part of helping, and the delete operation advances to the cleanup mode. On advancing to the cleanup mode, the operation sets a flag in T indicating that S has been removed from the tree (and T can now be replaced with a new node) so that other operations trying to help it know not to look for S.

Figure 4.6 shows a flow chart describing the sequence of steps of a delete operation.

Cleanup Mode There are two cases depending on whether the delete operation is simple or complex.

(a) **Simple Delete:** In this case, either $T.left$ or $T.right$ is pointing to a null node. Note that both $T.left$ and $T.right$ may be pointing to null nodes (which in turn will imply

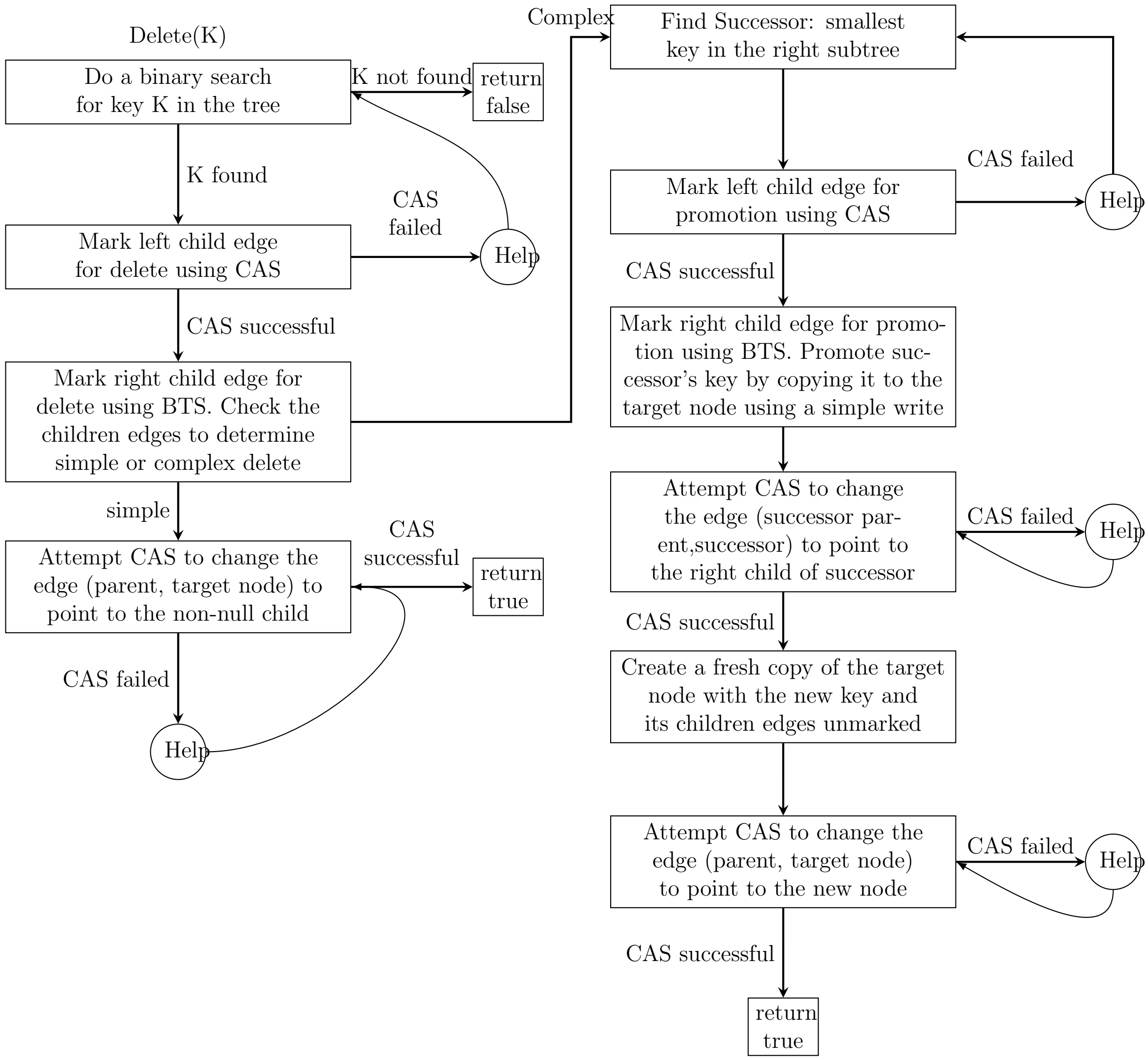


Figure 4.6: ELFTREE - Sequence of steps in a delete operation

Algorithm 8: Data Structures Used

```
struct Node {
    {Boolean, Key} mKey;
    {Boolean, Boolean, Boolean, Boolean, NodePtr} child[2];
    Boolean readyToReplace;
};
struct Edge {
    NodePtr parent, child;
    enum which { LEFT, RIGHT };
};
struct SeekRecord {
    Edge lastEdge, pLastEdge, injectionEdge;
};
struct AnchorRecord {
    NodePtr node;
    Key key;
};
struct StateRecord {
    Edge targetEdge, pTargetEdge;
    Key targetKey, currentKey;
    enum mode { INJECTION, DISCOVERY, CLEANUP };
    enum type { SIMPLE, COMPLEX } ;
    // the next field stores pointer to a seek record; it is used for finding the
       successor if the delete operation is complex
    SeekRecordPtr successorRecord;
};
// object to store information about the tree traversal when looking for a given key
   (used by the seek function)
SeekRecordPtr targetRecord := new seek record;
// object to store information about process' own delete operation
StateRecordPtr myState := new state;
```

that T is a leaf node). Without loss of generality, assume that $T.right$ is a null node. The removal of T involves changing the child pointer at $T.parent$ that is pointing to T to point to $T.left$ using a CAS instruction. If the CAS instruction succeeds, then the delete operation terminates; otherwise, it performs another seek on the tree. If the seek function either fails to find the target key or returns a terminal node different from T, then T has been already removed from the tree (by another operation as part of helping) and the delete operation terminates; otherwise, it attempts to remove T from

Algorithm 9: Seek Function

```
Seek( key, seekRecord )
begin
    pAnchorRecord := {𝕊, ∞₁};
    while true do
        // initialize all variables used in traversal
        pLastEdge := {ℝ, 𝕊, RIGHT};      lastEdge := {𝕊, 𝕋, RIGHT};
        curr := 𝕋;      anchorRecord := {𝕊, ∞₁};
        while true do
            // read the key stored in the current node
            ⟨∗, cKey⟩ := curr → mKey;
            // find the next edge to follow
            which := key < cKey ? LEFT: RIGHT;
            ⟨n, ∗, d, p, next⟩ := curr → child[which];
            // check for the completion of the traversal
            if key = cKey  or n then
                // either key found or no next edge to follow; stop the traversal
                seekRecord → pLastEdge := pLastEdge;
                seekRecord → lastEdge := lastEdge;
                seekRecord → injectionEdge := {curr, next, which};
                if key = cKey then // keys match
                    return;
                else break;
            if which = RIGHT then
                // next edge to be traversed is a right edge; keep track of the
                   current node and its key
                anchorRecord := ⟨curr, cKey⟩;
            // traverse the next edge
            pLastEdge := lastEdge;      lastEdge := {curr, next, which};      curr := next;
        // key was not found; check if can stop
        ⟨∗, ∗, d, p, ∗⟩ := anchorRecord.node → child[RIGHT];
        if not (d)  and not (p) then
            // anchor node is still part of the tree; check if anchor node's key has
               changed
            ⟨∗, aKey⟩ := anchorRecord.node → mKey;
            if anchorRecord.key = aKey then  return;
        else
            // check if the anchor record (the node and its key) matches that of the
               previous traversal
            if pAnchorRecord = anchorRecord then
                // return the results of the previous traversal
                seekRecord := pSeekRecord;
                return;
        // store the results of the traversal and restart
        pSeekRecord := seekRecord;      pAnchorRecord := anchorRecord;
```

Algorithm 10: Search Operation

```
Boolean SEARCH( key )
begin
    SEEK( key, mySeekRecord );
    node := mySeekRecord→lastEdge.child;
    ⟨∗, nKey⟩ := node→mKey;
    if nKey = key then return true;
    else return false;
```

Algorithm 11: Insert Operation

```
Boolean INSERT( key )
begin
    while true do
        SEEK( key, targetRecord );
        targetEdge := targetRecord→lastEdge;
        node := targetEdge.child;
        ⟨∗, nKey⟩ := node→mKey;
        if key = nKey then return false;
        // create a new node and initialize its fields
        newNode := create a new node;
        newNode→mKey := ⟨0_m, key⟩;
        newNode→child[LEFT] := ⟨1_n, 0_i, 0_d, 0_p, null⟩;
        newNode→child[RIGHT] := ⟨1_n, 0_i, 0_d, 0_p, null⟩;
        newNode→readyToReplace := false;
        which := targetRecord→injectionEdge.which;
        address := targetRecord→injectionEdge.child;
        result := CAS(node→child[which], ⟨1_n, 0_i, 0_d, 0_p, address⟩, ⟨0_n, 0_i, 0_d, 0_p, newNode⟩);
        if result then return true;
        // help if needed
        ⟨∗, ∗, d, p, ∗⟩ := node→child[which];
        if d then HELPTARGETNODE( targetEdge ) ;
        else if p then HELPSUCCESSORNODE( targetEdge ) ;
```

Algorithm 12: Delete Operation

```
Boolean DELETE( key )
begin
    // initialize the state record
    myState→targetKey := key;        myState→currentKey := key;
    myState→mode := INJECTION;

    while true do
        SEEK( myState→currentKey, targetRecord );
        targetEdge := targetRecord→lastEdge;        pTargetEdge := targetRecord→pLastEdge;
        ⟨∗, nKey⟩ := targetEdge.child→mKey;

        if myState→currentKey ≠ nKey then
            // the key does not exist in the tree
            if myState→mode = INJECTION then return false;
            else return true;

        // perform appropriate action depending on the mode
        if myState→mode = INJECTION then
            // store a reference to the target edge
            myState→targetEdge := targetEdge;
            myState→pTargetEdge := pTargetEdge;
            // attempt to inject the operation at the node
            INJECT( myState );

        // mode would have changed if injection was successful
        if myState→mode ≠ INJECTION then
            // check if the target node found by the seek function matches the one
               stored in the state record
            if myState→targetEdge.child ≠ targetEdge.child then  return true;
            // update the target edge information using the most recent seek
            myState→targetEdge := targetEdge;

        if myState→mode = DISCOVERY then
            // complex delete operation; locate the successor node and mark its child
               edges with promote flag
            FINDANDMARKSUCCESSOR( myState );
        if myState→mode = DISCOVERY then
            // complex delete operation; promote the successor node's key and remove
               the successor node
            REMOVESUCCESSOR( myState );

        if myState→mode = CLEANUP then
            // either remove the target node (simple delete) or replace it with a new
               node with all fields unmarked (complex delete)
            result := CLEANUP( myState );
            if result then return true;
            else
                ⟨∗, nKey⟩ := targetEdge.child→mKey;
                myState→currentKey := nKey;
```

Algorithm 13: Injecting a Deletion Operation

```
Inject( state )
begin
    targetEdge := state→targetEdge;
    // try to set the intent flag on the target edge
    // retrieve attributes of the target edge
    parent := targetEdge.parent;
    node := targetEdge.child;
    which := targetEdge.which;
    result := CAS( parent→child[which],
                   ⟨0_n, 0_i, 0_d, 0_p, node⟩,
                   ⟨0_n, 1_i, 0_d, 0_p, node⟩ );
    if not (result) then
        // unable to set the intent flag; help if needed
        ⟨*, i, d, p, address⟩ := parent→child[which];
        if i then HelpTargetNode( targetEdge ) ;
        else if d then
            HelpTargetNode( state→pTargetEdge );
        else if p then
            HelpSuccessorNode( state→pTargetEdge );
        return;
    // mark the left edge for deletion
    result := MarkChildEdge( state, LEFT );
    if not (result) then return;
    // mark the right edge for deletion; cannot fail
    MarkChildEdge( state, RIGHT );
    // initialize the type and mode of the operation
    InitializeTypeAndUpdateMode( state );
```

the tree again using possibly the new parent information returned by seek. This process may be repeated multiple times.

(b) **Complex Delete:** Note that, at this point, the key stored in the target node is the replacement key (the successor key of the target key). Further, the key as well as both the child edges of the target node are marked. The delete operation attempts to replace target node with a *new* node, which is basically a copy of target node except that all its fields are unmarked. This replacement of T involves changing the child pointer at $T.parent$ that is pointing to T to point to the new node. If the CAS instruction succeeds, then the delete operation terminates; otherwise, as in the case of simple delete,

Algorithm 14: Locating the Successor Node

```
FindAndMarkSuccessor( state )
begin
    // retrieve the addresses from the state record
    node := state→targetEdge.child;
    seekRecord := state→successorRecord;

    while true do
        // read the mark flag of the key in the target node
        ⟨m, ∗⟩ := node→mKey;
        // find the node with the smallest key in the right subtree
        result := FindSmallest( state );

        if m or not (result) then
            // successor node had already been selected before the traversal or the
               right subtree is empty
            break;

        // retrieve the information from the seek record
        successorEdge := seekRecord→lastEdge;
        left := seekRecord→injectionEdge.child;

        // read the mark flag of the key under deletion
        ⟨m, ∗⟩ := node→mKey;
        if m then // successor node has already been selected
            continue;

        // try to set the promote flag on the left edge
        result := CAS( successorEdge.child →
                       child[LEFT],
                       ⟨1_n, 0_i, 0_d, 0_p, left⟩,
                       ⟨1_n, 0_i, 0_d, 1_p, node⟩ );
        if result then break;

        // attempt to mark the edge failed; recover from the failure and retry if
           needed
        ⟨n, ∗, d, ∗, ∗⟩ := successorEdge.child→child[LEFT];
        if n and d then
            // the node found is undergoing deletion; need to help
            HelpTargetNode( successorEdge );

    // update the operation mode
    UpdateMode( state );
```

Algorithm 15: Removing the Successor Node

```
RemoveSuccessor( state )
begin
    // retrieve addresses from the state record
    node := state→targetEdge.child;
    seekRecord := state→successorRecord;
    // extract information about the successor node
    successorEdge := seekRecord→lastEdge;

    // ascertain that seek record for successor node contains valid information
    ⟨∗, ∗, ∗, p, address⟩ := successorEdge.child→child[LEFT];
    if not (p) or (address ≠ node) then
        node→readyToReplace := true;
        UpdateMode( state );
        return;

    // mark the right edge for promotion if unmarked
    MarkChildEdge( state, RIGHT );

    // promote the key
    node→mKey := ⟨1_m, successorEdge.child→mKey⟩;
    while true do
        // check if the successor is the right child of the target node itself
        if successorEdge.parent = node then
            // need to modify the right edge of target node whose delete flag is set
            dFlag := 1;        which := RIGHT;
        else
            dFlag := 0;        which := LEFT;

        ⟨∗, i, ∗, ∗, ∗⟩ := successorEdge.parent→child[which];

        ⟨n, ∗, ∗, ∗, right⟩ := successorEdge.child→child[RIGHT];
        oldValue := ⟨0_n, i, dFlag, 0_p, successorEdge.child⟩;
        if n then                      // only set the null flag; do not change the address
            newValue := ⟨1_n, 0_i, dFlag, 0_p, successorEdge.child⟩;
        else                           // switch the pointer to point to the grand child
            newValue := ⟨0_n, 0_i, dFlag, 0_p, right⟩ ;

        result := CAS(successorEdge.parent→child[which],oldValue, newValue);
        if result or dFlag then break;
        ⟨∗, ∗, d, ∗, ∗⟩ := successorEdge.parent→child[which];
        pLastEdge := seekRecord→pLastEdge;
        if d and (pLastEdge.parent ≠ null) then
            HelpTargetNode( pLastEdge );

        result := FindSmallest( state );
        lastEdge := seekRecord→lastEdge;
        if not (result) or lastEdge.child ≠ successorEdge.child then
            break;                     // the successor node has already been removed

        else successorEdge := seekRecord→lastEdge ;

    node→readyToReplace := true;
    UpdateMode( state );
```

Algorithm 16: Cleaning Up the Tree

```
Boolean Cleanup( state )
begin
    ⟨parent, node, pWhich⟩ := state→targetEdge;
    if state→type = COMPLEX then
        // replace the node with a new copy in which all fields are unmarked
        ⟨∗, nKey⟩ := node→mKey;
        newNode→mKey := ⟨0_m, nKey⟩;
        // initialize left and right child pointers
        ⟨∗, ∗, ∗, ∗, left⟩ := node→child[LEFT];
        newNode→child[LEFT] := ⟨0_n, 0_i, 0_d, 0_p, left⟩;
        ⟨n, ∗, ∗, ∗, right⟩ := node→child[RIGHT];
        if n then
            newNode→child[RIGHT] := ⟨1_n, 0_i, 0_d, 0_p, null⟩;
        else newNode→child[RIGHT] := ⟨0_n, 0_i, 0_d, 0_p, right⟩ ;
        // initialize the arguments of CAS instruction
        oldValue := ⟨0_n, 1_i, 0_d, 0_p, node⟩;
        newValue := ⟨0_n, 0_i, 0_d, 0_p, newNode⟩;
    else // remove the node
        // determine to which grand child will the edge at the parent be switched
        if node→child[LEFT] = ⟨1_n, ∗, ∗, ∗, ∗⟩ then
            nWhich := RIGHT;
        else nWhich := LEFT;
        // initialize the arguments of the CAS instruction
        oldValue := ⟨0_n, 1_i, 0_d, 0_p, node⟩;
        ⟨n, ∗, ∗, ∗, address⟩ := node→child[nWhich];
        if n then // set the null flag only
            newValue := ⟨1_n, 0_i, 0_d, 0_p, node⟩;
        else // change the pointer to the grand child
            newValue := ⟨0_n, 0_i, 0_d, 0_p, address⟩ ;

    result := CAS( parent→child[pWhich],
                   oldValue, newValue );
    return result;
```

Algorithm 17: Mark Child Edge

```
Boolean MarkChildEdge( state, which )
begin
    if state→mode = INJECTION then
        edge := state→targetEdge;
        flag := DELETE_FLAG;
    else
        edge := (state→successorRecord)→lastEdge;
        flag := PROMOTE_FLAG;
    node := edge.child;
    while true do
        ⟨n, i, d, p, address⟩ := node→child[which];
        if i then
            helpeeEdge := {node, address, which};
            HelpTargetNode( helpeeEdge );
            continue;
        else if d then
            if flag = PROMOTE_FLAG then
                HelpTargetNode( edge );
                return false;
            else return true;
        else if p then
            if flag = DELETE_FLAG then
                HelpSuccessorNode( edge );
                return false;
            else return true;
        oldValue := ⟨n, 0_i, 0_d, 0_p, address⟩;
        newValue := oldValue | flag;
        result := CAS( node → child[which],
                       oldValue,
                       newValue );
        if result then break;
    return true;
```

Algorithm 18: Find Smallest

```
Boolean FindSmallest( state )
begin
    // find the node with the smallest key in the subtree rooted at the right child
       of the target node
    node := state→targetEdge.child;
    seekRecord := state→seekRecord;
    ⟨n, ∗, ∗, ∗, right⟩ := node→child[RIGHT];
    if n then // the right subtree is empty
        return false;

    // initialize the variables used in the traversal
    lastEdge := ⟨node, right, RIGHT⟩;
    pLastEdge := ⟨node, right, RIGHT⟩;
    while true do
        curr := lastEdge.child;
        ⟨n, ∗, ∗, ∗, left⟩ := curr→child[LEFT];
        if n then // reached the node with the smallest key
            injectionEdge := ⟨curr, left, LEFT⟩;
            break;

        // traverse the next edge
        pLastEdge := lastEdge;
        lastEdge := ⟨curr, left, LEFT⟩;

    // initialize seek record and return
    seekRecord→lastEdge := lastEdge;
    seekRecord→pLastEdge := pLastEdge;
    seekRecord→injectionEdge := injectionEdge;
    return true;
```

it performs another seek on the tree, this time looking for the successor key. If the seek function either fails to find the successor key or returns a terminal node different from T, then T has been already replaced (by another operation as part of helping) and the delete operation terminates. Otherwise, it attempts to replace T again using possibly the new parent information returned by seek. This process may be repeated multiple times.

Discussion It can be verified that, in the absence of conflict, a delete operation performs three atomic instructions in the injection mode, three in the discovery mode (if delete is complex), and one in the cleanup mode.

Algorithm 19: Helper Routines

```
InitializeTypeAndUpdateMode( state )
begin
    // retrieve the target node's address from the state record
    node := state→targetEdge.child;
    ⟨lN, ∗, ∗, ∗, ∗⟩ := node→child[LEFT];
    ⟨rN, ∗, ∗, ∗, ∗⟩ := node→child[RIGHT];
    if lN or rN then
        // one of the child pointers is null
        ⟨m, ∗⟩ := node→mKey;
        if m then state→type := COMPLEX;
        else state→type := SIMPLE;
    else // both child pointers are non-null
        state→type := COMPLEX;
    UpdateMode( state );

UpdateMode( state )
begin
    // update the operation mode
    if state→type = SIMPLE then // simple delete
        state→mode := CLEANUP;
    else // complex delete
        node := state→targetEdge.child;
        if node→readyToReplace then
            state→mode := CLEANUP;
        else state→mode := DISCOVERY;
```

Helping

To enable helping, as mentioned earlier, whenever traversing the tree to locate either a target key or a successor key, we keep track of the *last two* edges encountered in the traversal. When a CAS instruction fails, depending on the reason for failure, helping is either performed along the last edge or the second-to-last edge.

4.1.3 Formal Description

A pseudo-code of our algorithm is given in Algorithms 8-20.

Algorithm 8 describes the data structures used in our algorithm. Besides Node, three important data types in our algorithm are: Edge, SeekRecord and StateRecord. The data

Algorithm 20: Helping Conflicting Delete Operations

```
HelpTargetNode( helpeeEdge )
begin
    // intent flag must be set on the edge
    // obtain new state record and initialize it
    state→targetEdge := helpeeEdge;
    state→mode := INJECTION;
    // mark the left and right edges if unmarked
    result := MarkChildEdge( state, LEFT );
    if not (result) then return;
    MarkChildEdge( state, RIGHT );
    InitializeTypeAndUpdateMode( state );
    // perform the remaining steps of a delete operation
    if state→mode = DISCOVERY then
        FindAndMarkSuccessor( state );
    if state→mode = DISCOVERY then
        RemoveSuccessor( state );
    if state→mode = CLEANUP then Cleanup( state ) ;
HelpSuccessorNode( helpeeEdge )
begin
    // retrieve the address of the successor node
    parent := helpeeEdge.parent;
    node := helpeeEdge.child;
    // promote flat must be set on the successor node's left edge
    // retrieve the address of the target node
    ⟨*, *, *, *, left⟩ := node→child[LEFT];
    // obtain new state record and initialize it
    state→targetEdge := {null, left, _};
    state→mode := DISCOVERY;
    seekRecord := state→successorRecord;
    // initialize the seek record in the state record
    seekRecord→lastEdge := helpeeEdge;
    seekRecord→pLastEdge := {null, parent, _};
    // promote the successor node's key and remove the successor node
    RemoveSuccessor( state );
    // no need to perform the cleanup
```

type Edge is a structure consisting of three fields: the two endpoints and the direction (left or right). The data type SeekRecord is a structure used to store the results of a tree traversal. The data type StateRecord is a structure used to store information about a delete operation (*e.g.*, target edge, type, current mode, etc.). Note that only objects of type Node are shared between processes; objects of all other types (*e.g.*, SeekRecord, StateRecord) are *local* to a process and not shared with other processes.

The pseudo-code of the seek function is described in Algorithm 9, which is used by all the operations. The pseudo-codes of the search, insert and delete operations are given in Algorithm 10, Algorithm 11 and Algorithm 12, respectively. A delete operation executes function INJECT in injection mode, functions FINDANDMARKSUCCESSOR and REMOVESUCCESSOR in discovery mode and function CLEANUP in cleanup mode. Their pseudo-codes are given in Algorithm 13, Algorithm 14, Algorithm 15 and Algorithm 16, respectively. The pseudo-codes for helper routines (used by multiple functions) are given in Algorithm 18, Algorithm 17 and Algorithm 19. Finally, the pseudo-codes of functions used to help other (conflicting) delete operations are given in Algorithm 20.

4.1.4 Correctness Proof

It can be shown that our algorithm satisfies linearizability and lock-freedom properties [16]. Broadly speaking, linearizability requires that an operation should appear to take effect instantaneously at some point during its execution. Lock-freedom requires that some process should be able to complete its operation in a finite number of its own steps. It is convenient to treat insert and delete operations that do not change the tree as search operations. We call a tree node *active* if it is reachable from the root of the tree. We call a tree node *passive* if it was active earlier but is not active any more. It can be verified that, if an insert operation completes successfully, then its target node was active when it performed the successful CAS instruction on the node's child edge. Likewise, if a delete operation completes successfully, then its target node was active when it marked the node's left edge for deletion. Further, for a complex delete, the successor node was active when it marked the node's left edge for promotion.

All Executions are Linearizable

We show that an arbitrary execution of our algorithm is linearizable by specifying the *linearization point* of each operation. Note that the linearization point of an operation is the point during its execution at which the operation appeared to have taken effect. Our algorithm supports three types of operations: search, insert and delete. We now specify the linearization point of each operation.

1. *Insert operation:* The operation is linearized at the point at which it performed the successful CAS instruction that resulted in its target key becoming part of the tree.
2. *Delete operation:* There are two cases depending on whether the delete operation is simple or complex. If the operation is simple delete, then the operation is linearized at the point at which a successful CAS instruction was performed at the parent of the target node that resulted in the target node becoming passive. Otherwise, it is linearized at the point at which the original key of the target node was replaced with its successor key.
3. *Search operation:* There are two cases depending on whether the target node was active when the operation read the key stored in the node. If the target node was not active, then the operation is linearized at the point at which the target node became passive. Otherwise, it is linearized at the point at which the read action was performed.

It can be easily verified that, for any execution of the algorithm, the sequence of operations obtained by ordering operations based on their linearization points is legal, *i.e.*, all operations in the sequence satisfy their specification. This establishes that *our algorithm generates only linearizable executions.*

All Executions are Lock-Free

We say that the system is in a *quiescent state* if no modify operation completes hereafter. We say that the system is in a *potent state* if it has one or more pending modify operations. Note that a quiescence is a *stable* property; once the system is in a quiescent state, it stays in a quiescent state. We show that our algorithm is lock-free by proving that a potent state is necessarily non-quiescent provided assuming that some process with a pending modify operation continues to take steps.

Assume, by the way of contradiction, that there is an execution of the system in which the system eventually reaches a state that is potent as well as quiescent. Note that, once the system has reached a quiescent state, it will eventually reach a state after which the tree will not undergo any structural changes. This is because a modify operation makes at most two structural changes to the tree. So, if the tree is undergoing continuous structural changes, then it clearly implies that modify operations are continuously completing their responses, which contradicts the assumption that the system is in a quiescent state. Further, on reaching such a state, the system will reach a state after which no new edges in the tree are marked. Again, this is because a modify operation marks at most four edges and the set of edges in the tree does not change any more. We call such a system state after which neither the set of edges nor the set of *marked* edges in the tree change any more as a *strongly quiescent state*. Note that, like quiescence, strong quiescence is also a stable property.

From the above discussion, it follows that the system in a quiescent state will eventually reach a state that is strongly quiescent. Consider the search tree in such a strongly quiescent state. It can be verified that no more modify operations can now be injected into the tree, and, moreover, all modify operations already injected into the tree are delete operations currently "stuck" in either discovery or cleanup mode. Now, consider a process, say p, that continues to take steps to execute either its own operation or another operation blocking its progress (directly or indirectly) as part of helping. Consider the recursive chain of the *helpee* operations that p proceeds to help in order to complete its own operation. Let α_i denote the i^{th} helpee operation in the chain. It can be shown that:

Lemma 1. *Let $\mathcal{C}_D$ denote the set of all complex delete operations already injected into the tree that are "stuck" in the discovery mode. Then,*

1. $\alpha_1 \in \mathcal{C}_D$, *and*
2. *Suppose p is currently helping α_i for some $i \geq 1$ and assume that $\alpha_i \in \mathcal{C}_D$. Let α_{i+1} denote the next operation that p selects to help. Then, (a) α_{i+1} exists, (b) $\alpha_{i+1} \in \mathcal{C}_D$, and (c) the target node of α_{i+1} is at strictly larger depth than the target node of α_i.*

Using the above lemma, we can easily construct a chain of distinct helpee operations whose length exceeds the number of processes—a contradiction. This establishes that *our algorithm only generates lock-free executions.*

PART II

OPTIMIZATIONS

CHAPTER 5

LOCAL RECOVERY FOR CONCURRENT BINARY SEARCH TREES

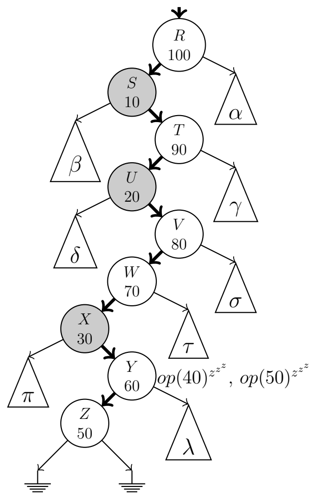
5.1 The Local Recovery Algorithm

We first present the main idea behind the algorithm and then provide its pseudo-code with more details.

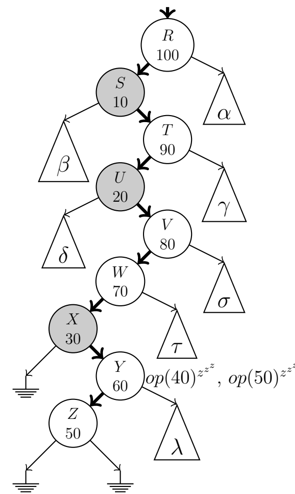
5.1.1 Overview of the Algorithm

As mentioned earlier, every operation on a BST involves first traversing the tree from top to down starting from the root node and following either the left or the right child pointer until either the target key is found or a null pointer is encountered (termination condition). Depending on the outcome of the traversal and the type of the operation, the tree may then need to be modified to actually realize the operation. We refer to the period during which the tree is being traversed as *seek phase*. Further, we refer to the period during which the tree is being modified as *execution phase*.

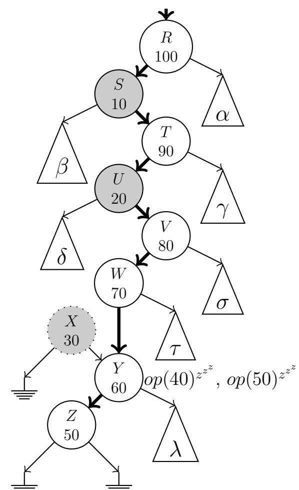
During the seek phase, the target key may move from its current location to a new location up the tree. As a result, the traversal may miss the key both at its old location as well as its new location. For an illustration, see Figure 5.1. In the illustration, key 50 has moved up by five nodes. In most concurrent BST algorithms, if it is suspected that the key may have moved up the tree, then the traversal is simply restarted from the root node. Different algorithms use different approaches to detect possible key movement. For example, in [18], the traversal is restarted if the *last right-turn* node is detected to have undergone some change. For example, in Figure 5.1a, the last right-turn node for the operation $op(50)$



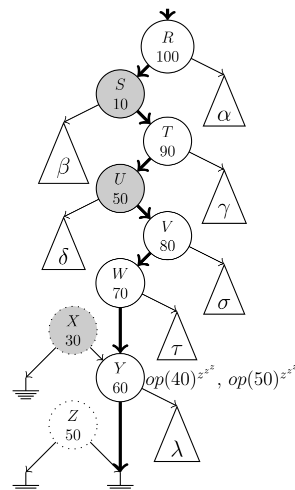
(a) Operation $op(50)$ is suspended at node Y during its traversal.



(b) All keys in subtree π are deleted one-by-one.



(c) Key 30 is deleted (simple delete); node X is removed.



(d) Key 20 is deleted (complex delete); key 20 is replaced with key 50 in node U and node Z is removed.

Figure 5.1: An illustration of a key moving up the tree

is node X. On reaching the terminal node in Figure 5.1d, after resuming running, $op(50)$ needs to restart since X has since been removed from the tree.

A *re*-traversal of the tree may also be required if the operation encounters any failure during the execution phase. For example, in [28], which is a lock-based algorithm, execution phase is aborted if, after locking the relevant edges, the validation step fails. This happens if the portion of the tree that lies within the "operation's window", which typically consists of a small constant number of nodes, has undergone some change since it was last observed. In that case, the operation moves to the seek phase again.

In most concurrent BST algorithms, (a single instance of) the execution phase of an operation typically tends to have constant time complexity. The seek phase is where an operation may end up spending most of its time especially if the tree is large. Hence, it is desirable to make the seek phase of an operation more efficient by: (i) reducing the number of restarts due to "suspected" key movement, and (ii) restarting the traversal from a point "close" to the operation's window. This leads to two separate but related questions that any local recovery algorithm needs to address. First, "If a key is not found, then does the traversal need to restart?". Second, "If the traversal needs to be restarted, then from which node should the traversal restart?"

Consider an operation α currently traversing the tree; let $\Pi(\alpha)$ denote the path taken by α so far. For example, in Figure 5.1a, considering only the subtree shown in the figure, $\Pi(op(50)) = \langle R, S, T, U, V, W, X, Y \rangle$. At each node in the path (except the last node), α either followed the left or the right child pointer. We say that a node in the path is an *anchor* node if the operation followed its *right* child pointer; otherwise we say that is a *non-anchor* node. For example, in the path $\Pi(op(50))$, nodes S, U and X are anchor nodes, whereas nodes R, T, V, W and Y are non-anchor nodes. We assume that the first anchor node in a traversal path is always a *sentinel* node that is never marked (not shown in the figure). Of course, as an operation is traversing the tree, the tree may undergo changes as a result of which the path taken by the operation may no longer be correct. For example, in Figure 5.1d, the new access-path of $op(50)$ in the subtree, which is obtained from the subtree in Figure 5.1a after applying several delete operations, is now given by $\langle R, S, T, U \rangle$.

Note that, since in a complex delete operation we assume that the key being deleted is replaced with its successor key, the value of a key stored in a node can only increase. Therefore, the child pointer followed by an operation at a node, if it is still part of the access-path, may change (from right to left) for an anchor node but cannot change for a non-anchor node. For example, as shown in Figure 5.1a, the node U is an anchor node for $op(40)$. But due to the changes made to the tree, the key at U has now become 50, as shown in Figure 5.1d. Hence, the pointer that $op(40)$ now needs to follow at U is left and not right. We say that an anchor node is *consistent* with respect to an operation if its key is still less than the operation's key; otherwise, we say that it is *inconsistent*. For example, in Figure 5.1d, anchor nodes S and X are still consistent with respect to $op(40)$ but node U has become inconsistent with respect to $op(40)$.

Clearly, in case an operation needs to restart, *no node* in the path *after an inconsistent anchor node* in general can serve as a restart point since the path taken and the path that needs to be taken may now diverge. This implies that, to find a restart point, a local recovery algorithm should locate the *shallowest* inconsistent anchor node in the path (with the least depth) and discard the suffix of the path after such a node. Moreover, a restart point has to be a node that is still a part of the tree.

This leads to the following approach to find a restart point for an operation α when needed. Find a node C in the path taken so far by α such that the following two conditions hold. First, C is not marked. Second, every anchor node in the path preceding C is consistent with respect to α. To check for the second condition, it is not necessary to examine every anchor node in the path preceding C as stated in the following lemma.

Lemma 2. *Consider an operation α and let $\Pi(\alpha)$ denote the path taken by α when traversing the tree. Let A be an anchor node in $\Pi(\alpha)$ and let $\sigma = A_0, A_1, \ldots, A_k$, where $A_k = A$, denote the sequence of anchor nodes in $\Pi(\alpha)$ up to and including A. Then, if A is unmarked and consistent with respect to α, then, for every i with $0 \leq i \leq k$, A_i is also consistent with respect to α. Moreover, the access-path of α in the current tree includes A.*

We say that an anchor node is *critical* with respect to a node C in the path if it is the *closest* preceding anchor node to C that is also unmarked. For example, in Figure 5.1c, the critical anchor node with respect to node Y is node U since node X is marked. Using the above lemma, the second condition can now be replaced with the following: the critical anchor node with respect to C in the path, say A, as well as every anchor node in the path that lies between A and C, which will be marked, is consistent with respect to α.

The next lemma states a useful property about an inconsistent anchor node.

Lemma 3. *Consider an operation α with target key k and let $\Pi(\alpha)$ denote the path taken by α when traversing the tree. Let A be an anchor node in $\Pi(\alpha)$. Assume that A is now inconsistent with respect to α. Then, at the time A became inconsistent, the tree did not contain k.*

To see why the above lemma holds, let k_{old} (k_{new}) be the key stored in A just before (after) A became inconsistent. Clearly, $k_{old} < k < k_{new}$. Let t denote the time just after which k_{old} was replaced with k_{new} in A. Note that k_{new} must be the next smallest key in the right subtree of A at time t. This implies that the right subtree of A did not contain k at time t. Further, from Lemma 2, A is on the access-path of α at time t. Hence, we can conclude that the tree does not contain k at time t. Note that, if a key is not present in the tree at some point while a search/delete operation is in progress, it is acceptable for the operation to say that key was not found. In this case, the operation will be linearized after the delete operation that removed the key from the tree.

When an operation fails to find the target key after traversing the tree from top to bottom, it examines the path it took to check whether or not the key has moved up the tree and/or a re-traversal is required. To that end, it examines the anchor nodes in the reverse order in which they were visited, starting from the one closest to the terminal node. We now discuss the behavior of each operation one-by-one.

Search Operation: A search operation *does not* need to restart. When it examines an anchor node as mentioned above, there are three possibilities. First, if the anchor node's key matches the target key, then the key has been found and the operation terminates. Second, if the anchor node's key is greater than the target key (the anchor node has become inconsistent), then the operation concludes that the key is not present in the tree and terminates. Finally, if the anchor node's key is smaller than the target key (the anchor node is still consistent), then the operation terminates if the node is not marked; otherwise it moves to the preceding anchor node and repeats the comparison.

Insert Operation: An insert operation needs to restart only if one of the anchor nodes in the path has become inconsistent. When it examines an anchor node as mentioned above, there are three possibilities. First, if the anchor node's key matches the target key, then the key has been found and the operation terminates. Second, if the anchor node's key is greater than the target key (the anchor node has become inconsistent), then it discards the suffix of the path after the anchor node and restarts the traversal from a restart point. Finally, if the anchor node's key is smaller than the target key (the anchor node is still consistent), then the traversal terminates if the node is not marked (the terminal node is returned as the injection point); otherwise it moves to the preceding anchor node and repeats the comparison.

Delete Operation: A delete operation also *does not* need to restart except when there is a failure in the execution phase. When it examines an anchor node as mentioned above,

Algorithm 21: Data Structures Used

```
// Used to store information about a node visited during tree traversal
struct StackEntry {
    NodePtr node;
    enum Direction which;
    integer anchor;
};
// Used to store the path from the root node to the current node in the tree
struct State {
    StackEntry[ ] stack;
    integer top;
};
// Used to store information about the operation currently in progress
struct OpRecord {
    enum Type type;
    Key key;
    State targetStack, successorStack;
    NodePtr injectionPoint;
    // algorithm-specific fields
};
// Used to store the outcome of a tree traversal
struct SeekRecord{
    // algorithm-specific fields (e.g., target node and its parent)
};
```

there are three possibilities. First, if the anchor node's key matches the target key, then the key has been found and the operation moves to the execution phase. Second, if the anchor node's key is greater than the target key (the anchor node has become inconsistent), then the operation concludes that the key is not present in the tree and terminates. Finally, if the anchor node's key is smaller than the target key (the anchor node is still consistent), then the traversal terminates if the node is not marked; otherwise it moves to the preceding anchor node and repeats the comparison.

5.1.2 Details of the Algorithm

A pseudo-code of the local recovery algorithm is given in Pseudo-codes 21-28. The pseudo-code only shows the seek phase of an algorithm and not its execution phase since the exe-

Algorithm 22: Functions for Manipulating Traversal Stack

```
// Returns the number of elements in the stack
integer Size( state )
begin
    return state→top + 1;

// Returns the topmost node in the stack
NodePtr GetTop( state )
begin
    {stack, top} := state;
    return stack[top]→node;

// Returns the second topmost node in the stack
NodePtr GetSecondToTop( state )
begin
    {stack, top} := state;
    return stack[top − 1]→node;

// Adds the given node to the stack along with its anchor node
AddToTop( state, node, which )
begin
    {stack, top} := state;
    // find the anchor node
    if which = RIGHT then anchor := top ;
    else anchor := stack[top]→anchor ;
    // push the node into the stack
    stack[top + 1] := {node, which, anchor};
    state→top := top + 1;

// Removes the topmost node from the stack
RemoveFromTop( state )
begin
    {stack, top} := state;
    // update the anchor node of the penultimate entry if needed
    anchor := stack[top − 1]→anchor;
    if stack[top]→anchor < stack[anchor]→anchor then
        stack[anchor]→anchor := stack[top]→anchor;
    // pop the node from the stack
    state→top := top − 1;

// Pops the stack until a given entry
RemoveUntil( state, index )
begin
    state→top := index;
```

cution phase is algorithm-specific. We have also moved the pseudo-code for local recovery when looking for a successor key to the appendix due to lack of space.

Algorithm 23: Functions for Manipulating Traversal Stack (Continued)

```
// Remember the critical node (to avoid locating it again)
RememberCritical( state, critical )
begin
    {stack, top} := state;
    anchor := stack[top]→anchor;
    if critical < stack[anchor]→anchor then
        stack[anchor]→anchor := critical;

// Returns a given entry in the stack
{ NodePtr, enum Direction, integer }
                GetFullEntry( state, index )
begin
    {stack, top} := state;
    if index = ⊤ then return stack[top] ;
    else return stack[index] ;

// initializes the traversal stack
InitializeTraversalState( state, type )
begin
    if type = TARGET_STACK then
        // initialize the stack using sentinel nodes
        // sentinel nodes are never removed from the stack
        // a sentinel node is always a safe starting point for the traversal
    else state→top := -1 ;
```

The local recovery algorithm assumes that the original algorithm supports the following functions: (a) GetKey( ), IsMarked( ) and GetChild( ) returns the various attributes of a tree node, (b) IsNull( ) returns true if a reference is null and false otherwise, (c) GetAddress( ) returns the node address stored in a reference, if non-null, (d) Move( ) enables the original algorithm to move along an edge, which may invoke helping and restarting of the traversal as in [18], (e) NeedCleanParentNode( ) returns true if the operation needs the parent node to be clean and have no operation in progress (needed for a delete operation since it needs to modify a child pointer at the parent node), and (f) PopulateSeekRecord( ) copies the relevant information from the traversal state required by the algorithm into a seek record.

Pseudo-code 21 shows the data structures used by the local recovery algorithm. Note that all the data structures shown in Pseudo-code 21 are *local* to a process not shared among processes. A process uses three main data structures, namely State, OpRecord and SeekRecord. A State (lines 473-476) is essentially a stack used to store the nodes visited during tree traversal when looking for a key (target or successor). Note that the traversal stack satisfies the last-in-first-out (LIFO) semantics but our algorithm sometimes uses it in a non-traditional way by accessing entries in the middle of the stack. One way to implement such an "augmented" stack is to use an auto-resizing vector provided as part of C++ STL library or Java package. Each entry in a traversal stack (lines 468-472) stores the address of the node, the location of its closest anchor node (within the stack's vector) and whether the node is a left or right child of its parent. An OpRecord (lines 477-482) stores information about the operation such as type and key as well two stacks: one used when looking for the target key (all operations) and one used when looking for the successor key (only complex delete operations). Finally, a SeekRecord (lines 483-484) is used to return the outcome of a tree traversal to the original algorithm. Its fields are algorithm-specific. For example, for CASTLE, SeekRecord contains three fields: (a) two addresses, namely those of the target node and its parent, and (b) the contents of the injection point where an insert operation needs to attach the new node.

Pseudo-code 22 shows the functions used to manipulate a traversal stack. The function Size (lines 485-487) returns the number of entries in the stack. The functions GetTop (lines 488-491) and GetSecondToTop (lines 492-495) return the address of the node stored in the topmost entry and the entry below it, respectively. The function AddToTop (lines 496-502) adds an entry to the top of the stack while RemoveFromTop (lines 503-509) removes an entry from the top of the stack. The function RemoveUntil (lines 510-512) removes the entries from the top of the stack until a given point. The function Remember-Critical (lines 513-517) updates the anchor field of the anchor node of the topmost entry in the stack. The function GetFullEntry (lines 520-524 returns all the three fields of a given entry in the stack (may not be the topmost entry). The function InitializeTraversalState (lines 525-528) initializes a traversal stack. The stack for target key

Pseudo-codes 24 & 25 shows the functions used to find the target key by a search operation. The function SeekForSearch (lines 560-566) first traverses the tree starting from the root node (line 562). If the traversal fails to locate the key, then the key may have moved up the tree. To address this possibility, the function examines the traversal stack to determine whether or not that is the case (line 564). The function Traverse (lines 529-534) first initializes the traversal stack (line 532) and then, starting from the topmost node in the stack (line 533), follows either the left or the right child pointer (line 536) until it either finds the key (line 538) or encounters a null pointer (line 539). It also populates the traversal stack as it moves (line 542). The function ExamineStack(lines 543-559) examines the anchor nodes stored in the stack in the reverse order in which they were visited, starting from the anchor node closest to the topmost node in the traversal stack (line 547). If the anchor node's key matches the target key, then the function returns true (lines 552-554). If the anchor node is no longer consistent or is unmarked, then the function returns false (lines 555-556). Otherwise, the function backtracks and examines the preceding anchor node in the stack (lines 557-558).

Pseudo-codes 26-27 & 28 show the functions used to find the target key by a modify (insert or delete) operation. The function SeekForModify (lines 603-631) first backtracks to a safe node in the stack (line 608). Initially, the starting point is typically a sentinel node which is a safe node. The function then traverses the tree from top to down by following either the left or the right child pointer (line 613) until it either finds the key or encounters a null pointer (lines 615-624). In case the terminal node's key is greater than the target key, the function checks whether the path stored in the traversal stack is still valid (line 617). If not, the traversal is restarted. As the traversal moves down the tree, the function also populates the traversal stack (lines 625-629). The function ValidatePath (lines 567-570) checks whether or not the path stored in the stack is still valid. To that end, it examines the anchor nodes in the stack in the reverse order in which they were visited, starting from the anchor node closest to the topmost node in the traversal stack. There are three possible cases. First, the anchor node is still consistent (lines 574-578). In this case, the path is deemed to be valid if the anchor node is unmarked; otherwise, the function moves to the preceding anchor node. Second, the anchor node is no longer consistent (lines 580-583). In this case, the path is deemed to be invalid. However, if the operation is a delete operation, then it can be deduced that the key did not exist in the tree continuously and the function returns indicating that the key was not found (thereby causing the operation to terminate). Finally, the anchor node's key matches the target key (lines 586-588). In this case, if the anchor node is marked and the operation is a delete operation, then the path is deemed to be invalid (and further backtracking is required). This is because the key may be in the process of moving up the tree. Otherwise, the function returns indicating that the key was found. The function FindASafeNode (lines 589-591) finds a safe node on the path stored in the stack from which the operation can restart its traversal. To that end, it backtracks to an unmarked node with a clean parent if required (lines 592-600). It then checks whether or not the remaining path in the stack is still valid (line 601). If not, it repeats the above-mentioned steps.

Pseudo-code 29 shows the function SeekForSuccessor used to locate the successor key by a complex delete operation (lines 632-667). The function first backtracks to an unmarked node with a clean parent if required (lines 636-645). It then checks whether or not the successor key is still needed by invoking NeedSuccessorKey function (line 646). The function NeedSuccessorKey returns a reference, which is null if the successor key is no longer needed and contains the address of the target node's right child otherwise. This address is used as a traversal point if the stack only contains a single entry (the node whose key needs to be replaced). If the successor key is still needed, then the function repeatedly follows the left child pointer until it encounters a null pointer (lines 655-664). While moving down the tree, the function also populates the traversal stack (line 661).

Algorithm 24: Seek Function for Target Key (Search Operation)

```
// Traverses the tree starting from the root until either the key is found or a null
   pointer is encountered
boolean Traverse( opRecord, seekRecord )
begin
    state := opRecord→targetStack;
    // initialize the stack and the variables used in the traversal
    InitializeTraversalState( state, TARGET_STACK );
    current := GetTop( state );
    // traverse the tree (starting from current)
    while true do
        key := GetKey( current );
        which := opRecord→key < key ? LEFT : RIGHT;
        // read the next address to de-reference
        reference := GetChild( current, which );
        if opRecord→key = key then return true ;
        if IsNull( reference ) then return false ;
        // traverse the next edge
        address := GetAddress( reference );
        current := address;
        // push the next node to be visited into the stack
        AddToTop( state, address, which );

// Checks if the key being searched for has moved up in the path stored in the stack
boolean ExamineStack( opRecord, seekRecord )
begin
    result := false;
    state := opRecord→targetStack;
    // start with the anchor closest to the topmost node in the stack
    {∗, ∗, critical} := GetFullEntry( state, ⊤ );
    while true do
        // retrieve the node and its closest anchor node from the stack
        {node, ∗, anchor} := GetFullEntry( state, critical );
        // read the attributes of the node
        marked := IsMarked( node );
        key := GetKey( node );
        if opRecord→key = key then
            // the key stored in the node matches the one being searched for
            result := true;
            break;
        else if (opRecord→key < key) or not (marked) then
            // the target key did not exist continuously in the tree
            break;
        else // examine the preceding anchor node
            critical := anchor;
    return result;
```

Algorithm 25: Seek Function for Target Key (Search Operation) (continued)

```
// Looks for a given key in the tree (invoked by a search operation)
boolean SeekForSearch( opRecord, seekRecord )
begin
    // traverse the tree from top to down
    result := Traverse( opRecord, seekRecord );
    if not (result) then
        // check if the key has moved up in the path stored in the stack
        result := ExamineStack( opRecord, seekRecord );
    // return the outcome
    PopulateSeekRecord( seekRecord, state );
    return result;
```

Algorithm 26: validate path

```
// Determines if the path stored in the stack is still valid
// Returns one of the following four values:
// { SAFE, NOT_SAFE, FOUND, NOT_FOUND}
// May backtrack along the path under certain situations
enum Outcome ValidatePath( opRecord, state )
begin
    // check if any of the anchor nodes in the stack has become inconsistent
       starting with the one immediately preceding the topmost node in the stack
    {*, *, critical} := GetFullEntry( state, ⊤ );
    while true do
        // retrieve the node and its anchor from the stack
        {node, *, anchor} := GetFullEntry( state, critical );
        // read the attributes of the node
        marked := IsMarked( node );
        key := GetKey( node );
        if opRecord→key > key then
            // the anchor node is still consistent
            if not (marked) then
                // the access-path is still valid
                RememberCritical( state, critical );
                return SAFE;
            else // need to check the previous anchor node
                critical := anchor;
        else if opRecord→key < key then
            // the anchor node is no longer consistent
            if opRecord→type = DELETE then
                // the target key did not exist continuously in the tree
                return NOT_FOUND;
            else // the path is not valid
                RemoveUntil( state, critical );
                return NOT_SAFE;
        else // the two keys match
            RemoveUntil( state, critical );
            // stop the traversal
            return FOUND;
```

Algorithm 27: find a safe node

```
// Backtracks along the path stored in the stack until a suitable restart point is
   found
enum Outcome FindASafeNode( opRecord, state )
begin
    while true do
        // backtrack until an unmarked node
        current := GetTop( state );
        while IsMarked( current ) do
            RemoveFromTop( state );
            current := GetTop( state );

        // check if the algorithm needs a clean parent node
        if NeedCleanParentNode( opRecord, current ) then
            parent := GetSecondToTop( state );
            if not (IsClean( parent )) then
                // need to backtrack even further
                RemoveFromTop( state );
                continue;

        // check if the topmost node in the stack is a suitable restart point
        result := ValidatePath( opRecord, state );
        if result ≠ NOT_SAFE then return result ;
```

Algorithm 28: Seek Function for Target Key (Modify Operation)

```
// Looks for a given key in the tree (invoked by insert/delete operation)
boolean SeekForModify( opRecord, seekRecord )
begin
    state := opRecord→targetStack;
    result := NOT_SAFE;
    while result = NOT_SAFE do
        // backtrack to a suitable restart point in the path stored in the stack
        result := FindASafeNode( opRecord, state );
        if result ∈ {FOUND, NOT_FOUND} then break;
        // traverse the tree starting from the topmost node in the stack
        current := GetTop( state );
        while true do
            key := GetKey( current );
            which := opRecord→key < key ? LEFT : RIGHT;
            // read the next address to de-reference
            reference := GetChild( current, which );
            if (opRecord→key = key) or IsNull( reference ) then
                // either stop or backtrack & restart
                if opRecord→key < key then
                    // check if the path traversed is still valid
                    result := ValidatePath( opRecord, state );
                else // determine what value to return
                    result := FOUND;
                    if opRecord→key ≠ key then
                        // remember the address read
                        opRecord→injectionPoint :=
                                GetAddress( reference );
                        result := NOT_FOUND;
                // terminate the current traversal
                break;
            // traverse the next edge
            address := GetAddress( reference );
            restart := Move( current, address, which );
            if restart then
                // the algorithm wants to restart the traversal
                break;
            // push the node visited into the stack
            AddToTop( state, address, which );
    // return the outcome
    PopulateSeekRecord( seekRecord, opRecord );
    return (result = FOUND ? true: false);
```

Algorithm 29: Seek Function for Successor Key

```
// Looks for the next largest key with respect to a given key
boolean SeekForSuccessor( opRecord, seekRecord )
begin
    // the stack used in locating the successor key is initialized before this
       function is invoked
    state := opRecord→successorStack;
    while true do
        // backtrack until either an unmarked node or the stack becomes empty
        while (Size( state ) > 1) do
            current := GetTop( state );
            if not (IsMarked( current )) then break;
            else RemoveFromTop( state ) ;

        // backtrack further if a clean parent is needed but the parent is not clean
        if (Size( state ) ≥ 2) then
            if NeedCleanParentNode( opRecord, current ) then
                // the parent node should be a clean node
                parent := GetSecondToTop( state );
                if not (IsClean( parent )) then
                    RemoveFromTop( state );
                    continue;

        // check if the successor key is still needed
        reference := NeedSuccessorKey( opRecord );
        if IsNull( reference ) then                 // successor key no longer required
            return false;
        current := GetTop( state );
        if (Size( state ) = 1) then
            // visit the node pointed to by the reference returned by
               NeedSuccessorKey function
            which := RIGHT;
        else             // follow the left child node of the top node, if it exists
            reference := GetChild( current, LEFT );
            which := LEFT;

        repeat                                            // stop if reference is null
            if IsNull( reference ) then break;
            // obtain the address of the node
            address := GetAddress( reference );
            // traverse the edge
            restart := Move( current, address, which );
            if restart then           // the algorithm wants to restart the traversal
                break;
            // push the node visited into the stack
            AddToTop( state, address, which );
            current := address;
            // determine the next node to be visited
            reference := GetChild( current, LEFT );
            which := LEFT;
        until true;
    // return the outcome
    PopulateSeekRecord( seekRecord, opRecord );
    return true;
```

CHAPTER 6

WAIT FREE SEARCH

6.1 No Modification to Tree Node

Due to the limited manner in which the tree can evolve in the concurrent algorithms described in [2, 9, 18, 27, 28], it is possible to design a light-weight wait-free algorithm for a search operation for all of the algorithms. The main property we use is that as long as a key is *continuously* present in the tree, its distance from the root of the tree is *monotonically non-increasing*. As a result, if a key is not found after visiting a "certain" number of nodes in the tree, then the traversal can stop and it is sufficient to examine the path traversed to check whether or not the key has moved up. In case the key is not continuously present in the tree, while a search operation is in progress, it is acceptable to return either of the outcomes—present or not present—to the application. In the first case, the search operation can be linearized after the insert operation that added the key to the tree. In the second case, it can be linearized after the delete operation that removed the key from the tree.

The main question is: "How do we *efficiently* determine the number of nodes to visit in the tree before stopping the downward traversal *without missing* the key that is continuously present in the tree?" To that end, we maintain two arrays with one entry for each process in the array, denoted by IC and DC. Roughly speaking, entries $IC[i]$ and $DC[i]$ denote the number of insert and delete operations, respectively, process P_i has performed so far. A process increments its insert counter before adding a key to the tree and its delete counter after removing a key from the tree. As a result, the insert (delete) counter at a process is an upper (lower) bound on the number of keys that the process has added to (removed from) the tree. Before starting downward traversal for a search operation, a process first reads the delete counter values of all processes and then reads the insert counter values of all processes. It then computes an estimate for the number of nodes to visit as the sum of all the insert counter values minus the sum of all the delete counter values. We show that the estimate computed by a process is *safe* in the sense that a search operation will not miss a key continuously present in the tree while the operation is in progress.

To show that our algorithm works, we introduce some notation. Consider a time t *after* an operation has read all the delete counter values but *before* its starts reading any of the insert counter values. Let $I_{t,i}$ ($D_{t,i}$) denote the *actual* number of keys added to (removed from) the tree by process P_i at or before time t. Also, let $IC[i]$ ($DC[i]$) denote the value read for $IC[i]$ ($DC[i]$) by the operation. Note that, the way counters are maintained, $IC[i] \geq I_{t,i}$ and $DC[i] \leq D_{t,i}$. Also, let S_t and Δ_t denote the *actual* size of the tree and the *actual* distance of the target key from the root of the tree, respectively, at time t. Clearly, we have:

$$S_t = \sum_{0 \leq i < p} I_{t,i} - \sum_{0 \leq i < p} D_{t,i} \quad \text{and} \quad \Delta_t \leq S_t$$

Thus we have:

$$\sum_{0 \leq i < p} IC[i] - \sum_{0 \leq i < p} DC[i] \;\geq\; \sum_{0 \leq i < p} I_{t,i} - \sum_{0 \leq i < p} D_{t,i} \;=\; S_t$$

In other words, the estimate computed by our algorithm is an upper bound on the actual distance of the key from the root of tree when the operation starts traversing the tree (which is monotonically non-increasing).

A pseudo-code of the algorithm is given in Pseudo-code 30. In the pseudo-code, p denotes the number of processes. To amortize the overhead of reading $O(p)$ counters, an operation first visits p nodes in the tree. If it does not find the key, then it reads the counter values and proceeds as described above. Thus $O(p)$ overhead is incurred only for "large" trees.

Some advantages of our approach are as follows. First, it works even if a key space is unbounded. Second, it does not require a search operation to perform any write instruction on shared memory. Third, it does not require a modify operation to perform any additional atomic instruction or helping (besides that performed by the original algorithm).

Algorithm 30: Seek Function for Target Key based on Estimating Tree Size

```
integer IC[p];
integer DC[p];
// Traverses the tree starting from the root node but visits a limited number of
   nodes
boolean LimitedTraverse( opRecord, seekRecord, limit )
begin
    // similar to Traverse except that the while loop from lines 534-542 is executed
       at most limit times

// A wait-free seek function for a search operation based on computing an
   upper-bound on tree size
boolean WFSeekForSearchBOSize( opRecord, seekRecord )
begin
    result := LimitedTraverse( opRecord, seekRecord, p );
    if not (result) then
        D := DC[0] + DC[1] + ··· + DC[p − 1];
        I := IC[0] + IC[1] + ··· + IC[p − 1];
        S := I − D;
        result := LimitedTraverse( opRecord, seekRecord, S );
    if not (result) then
        // examine the stack
        result := ExamineStack( opRecord, seekRecord );
    // return the outcome
    PopulateSeekRecord( seekRecord, state );
    return result;
```

6.2 With Modification to Tree Node

A disadvantage of the previous approach is that the time complexity of a search operation depends on the tree size. We now describe another approach to achieve wait-freedom for which the time complexity of a search operation depends on the tree height. This approach, however, requires modifying tree node to store a time-stamp of when the node was created. It consists of the identifier of the process that created the node and the process-specific sequence number (which is incremented before the node is added to the tree). This time-stamp is copied if a node is *replaced* with a new node (in a complex delete operation) as in [27]. Before a search operation starts traversing the tree, it reads the current sequence number values of all processes. Let $labels[i]$ denote the value read for process P_i. The

Algorithm 31: Seek Function for Target Key based on Time-Stamps

```
integer labels[p];

// Traverses the tree starting from the root node but stops if recently added key is
   found
boolean TimeStampTraverse( opRecord, seekRecord, labels )
begin
    // similar to Traverse except that the while loop from lines 534-542 is
       terminated as soon as a node with a ``recent'' time-stamp is encountered
    // specifically, the following lines are inserted between lines 534 & 535
    ⟨pid, label⟩ := node→timeStamp;
    if label > labels[pid] then break;

// A wait-free seek function for a search operation based on estimating tree height
boolean WFSeekForSearchBOHeight( opRecord, seekRecord )
begin
    result := LimitedTraverse( opRecord, seekRecord, p );
    if not (result) then
        copyOfLabels := labels;
        result := TimeStampTraverse( opRecord,
                                     seekRecord,
                                     copyOfLabels );
    if not (result) then
        // examine the stack
        result := ExamineStack( opRecord, seekRecord );
    // return the outcome
    PopulateSeekRecord( seekRecord, state );
    return result;
```

operation then stops the downward traversal of the tree once it encounters a node with time-stamp $\langle i, v\rangle$ such that $v > labels[i]$. Clearly, this node and its descendents were added to the tree after the operation read the sequence number value of process P_i. A pseudo-code of the algorithm is given in Pseudo-code 31.

CHAPTER 7

EXPERIMENTAL EVALUATION

We now describe the results of the comparative evaluation of different implementations of a concurrent BST using simulated workloads. This chapter is organized as follows. Performance evaluation of our lock-based algorithm is described in Section 7.2 followed by our lock-free algorithm described in Section 7.3. Performance evaluation of our local recovery technique is described in Section 7.4.

7.1 Experimental Setup

We conducted our experiments on a single large-memory node in stampede[1] cluster at TACC (Texas Advanced Computing Center). This node is a Dell PowerEdge R820 server with 4 Intel E5-4650 8-core processors (32 cores in total) and 1TB of DDR3 memory. Hyper-threading has been disabled on the node. It runs CentOS 6.3 operating system.

To better understand the scalability of our algorithms we also conducted experiments on a single Intel Xeon Phi SE10P Coprocessor[2] having 61 1.1 GHz cores with 4 hardware threads per core and 8GB of GDDR5 memory.

We used Intel C/C++ compiler (version 2013.2.146) with optimization flag set to O3. We used GNU Scientific Library to generate random numbers. We used Intel's *TBB Malloc* [29] as the dynamic memory allocator since it provided superior performance to C/C+ default allocator in a multi-threaded environment.

[1] https://www.tacc.utexas.edu/systems/stampede

[2] http://www.intel.com/content/www/us/en/processors/xeon/xeon-phi-detail.html

To compare the performance of different implementations, we considered the following parameters:

1. **Relative Distribution of Operations:** We considered three different workload distributions: (a) *read-dominated:* 90% search, 9% insert and 1% delete, (b) *mixed:* 70% search, 20% insert and 10% delete, and (c) *write-dominated:* 0% search, 50% insert and 50% delete.
2. **Maximum Degree of Contention:** This depends on number of threads that can concurrently operate on the tree. On 32 core machine, we varied the number of threads from 1 to 32 in powers of two. On 61 core machine we varied the number of threads from 1 to 244 in multiples of 61.
3. **Maximum Tree Size:** This depends on the size of the key space. To get the peak throughput, we set the number of threads to be the value where the peak performance is achieved and we varied key space size from 2^{13} (8Ki) to 2^{24} (16Mi). To understand the scalability of the algorithms, we varied the number of threads and considered four different key ranges: 2,000 (2k), 20,000 (20K), 200,000 (200K) and 2 million (2M) keys.

We compared the performance of different algorithms with respect to *system throughput*, given by the number of operations executed per unit time. In each run of the experiment, we ran each algorithm for 10 seconds, and calculated the total number of operations completed by the end of the run to determine the system throughput. The results were averaged over 10 runs. To capture only the steady state behaviour, we *pre-populated* the tree to 50% of its maximum size, prior to starting a simulation run. The beginning of each run consisted of a 1 second "warm-up" phase whose numbers were excluded in the computed statistics to avoid initial caching effects.

7.2 Lock based tree

In this section we evaluate CASTLE against three other implementations of a concurrent BST, namely those based on:

(i) the lock-free internal BST by Howley and Jones [18], denoted by LF-IBST,

(ii) the lock-free external BST by Natarajan and Mittal [25], denoted by LF-EBST, and

(iii) the RCU-based internal BST by Arbel and Attiya [2], denoted by CITRUS.

Note that CITRUS is a blocking implementation. The above three implementations were obtained from their respective authors. All implementations were written in C/C++. In our experiments, none of the implementations used garbage collection to reclaim memory. The experimental evaluation in [18, 25] showed that, in all cases, either LF-IBST or LF-EBST outperformed the concurrent BST implementation based on Ellen *et al.*'s lock-free algorithm in [10]. So we did not consider it in our experiments.

The results of our experiments are shown in Figure 7.1 and Figure 7.2. In Figure 7.1, the absolute value of the system throughput is plotted against the number of threads (varying from 1 to 32 in powers of 2). Here each column represents a specific workload (read-dominated, mixed or write-dominated) and each row represents a specific key space size (50K, 500K or 5M). In Figure 7.2, the relative value of the system throughput with respect to that of LF-IBST is plotted against the key space size. Here each column represents a range of key space sizes (small, medium and large) and each row represents a specific workload. As the peak performance for all the four algorithms (for all cases) occurred at 32 threads, we set the number of threads to 32 while varying the key space size from 2^{13} to 2^{24}.

As both Figure 7.1 and Figure 7.2 show, for smaller key space sizes, LF-EBST achieves the best system throughput. This is not surprising since LF-EBST is optimized for high contention scenarios. For medium and large key space sizes, CASTLE achieves the best

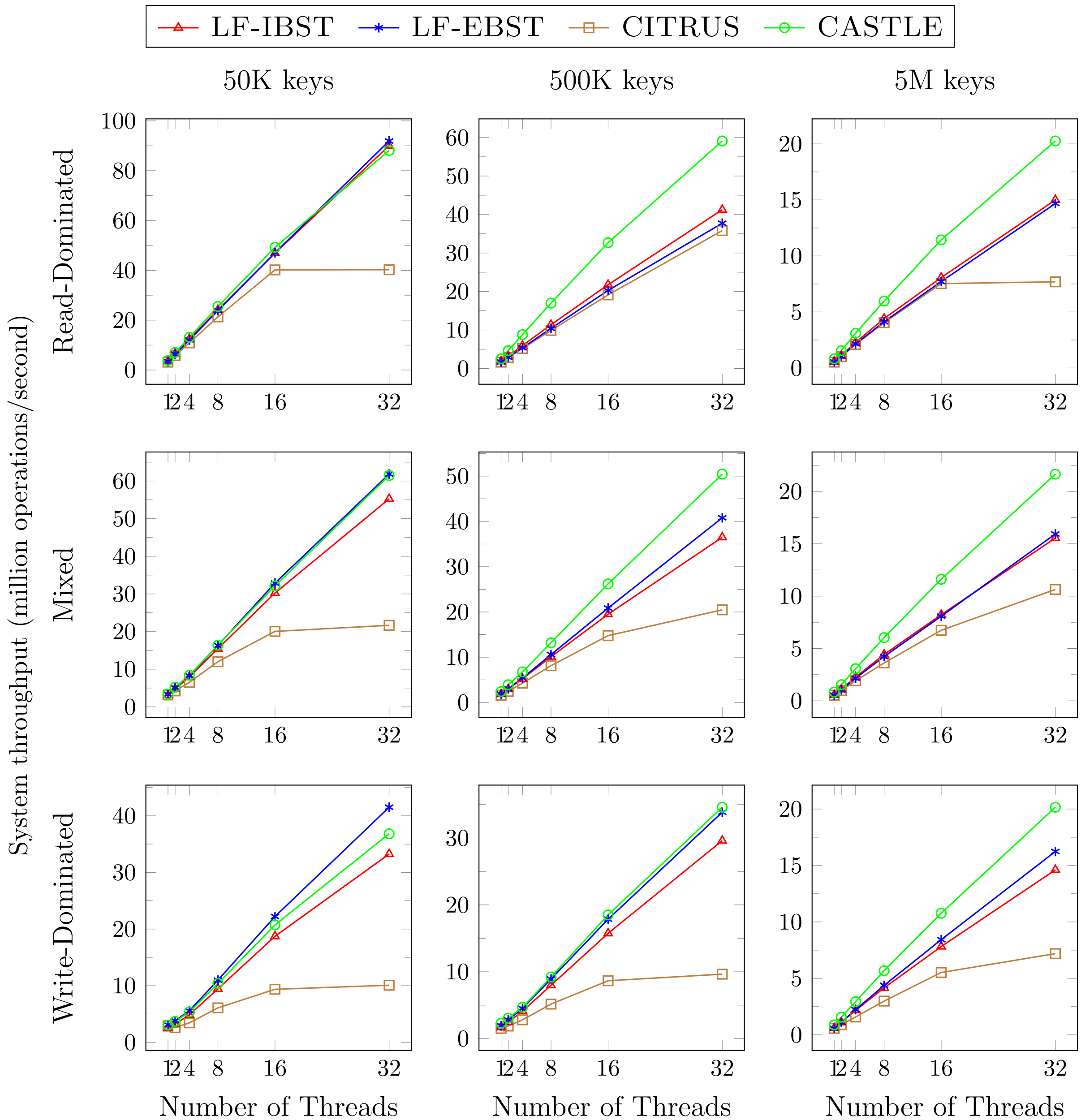


Figure 7.1: Comparison of system throughput of different algorithms. Each row represents a key space size and each column represents a workload type. Higher the throughput, better the performance of the algorithm.

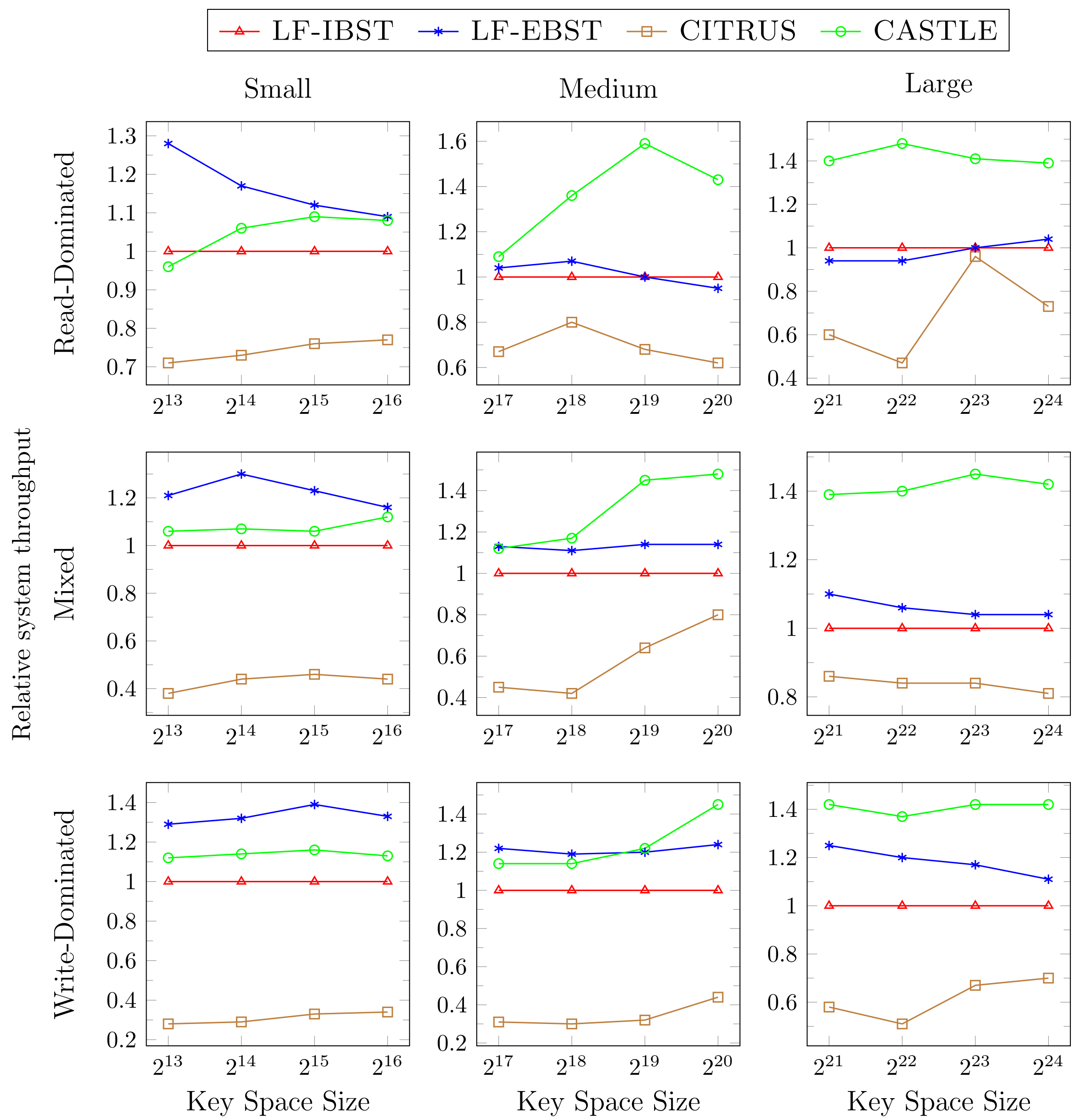


Figure 7.2: Comparison of system throughput of different algorithms *relative to that of* LF-IBST at 32 threads. Each row represents a workload type. Each column represents a range of key space size. Higher the ratio, better the performance of the algorithm.

system throughput for all three workload types in almost all the cases (except when the workload is write-dominated and the key space size is in the lower half of the medium range). The maximum gap between CASTLE and the next best performer is around 59% which occurs at 500K key space size, read-dominated workload and 32 threads.

We believe that some of the reasons for the better performance of CASTLE over the other three concurrent algorithms, especially when the contention is relatively low, are as follows. First, as explained in [25], operating at edge-level rather than at node-level reduces the contention among operations. Second, using a CAS instruction for locking an edge also validates that the edge has not undergone any change since it was last observed. Third, locking edges as late as possible minimizes the blocking effect of the locks.

Table 7.1: Comparison of different concurrent algorithms in the absence of contention.

Algorithm	**Number of Objects Allocated**		**Number of Synchronization Primitives Executed**	
	Insert	**Delete**	**Insert**	**Delete**
LF-IBST	2	1	3	simple: 4 complex: 9
LF-EBST	2	0	1	3
CASTLE	1	0	1	simple: 3 complex: 4

Table 7.1 shows a comparison of LF-IBST, LF-EBST and CASTLE with respect to the number of objects allocated dynamically and the number of synchronization primitives executed per modify operation in the absence of contention. We omitted CITRUS in this comparison since it based on a different framework. As Table 7.1 shows, our algorithm allocates fewer objects dynamically than the two lock-free algorithms (one for insert operations and none for a delete operations). Further, again as Table 7.1 shows, our algorithm executes much fewer synchronization primitives than LF-IBST. It executes the same number of synchronization primitives as LF-EBST for insert and simple delete operations and only one more for complex delete operations. This is important since a synchronization primitive is usually much more expensive to execute than a simple read or write instruction. Finally, we

observed in our experiments that CASTLE had a smaller memory footprint than all the three implementations (by a factor of two or more) since it uses internal representation of a search tree and allocates fewer objects dynamically. As a result, it was likely able to benefit from caching to a larger degree than the other algorithms.

In our experiments, we observed that, for key space sizes larger than 10K, the likelihood of an operation restarting was extremely low (less than 0.1%) even for a write-dominated workload. This implies that, in at least 99.9% of the cases, an operation was able to complete without encountering any conflicts. Thus, for key space sizes larger than 10K, we expect CASTLE to outperform the implementation based on the lock-free algorithm described in [11], which is basically derived from the one in [10].

7.3 Lock free tree

In this section we evaluate ELFTREE against three other implementations of a concurrent BST, namely those based on:

(i) the lock-free internal BST by Howley and Jones [18], denoted by LF-IBST,

(ii) the lock-free external BST by Natarajan and Mittal [25], denoted by LF-EBST, and

(iii) the RCU-based internal BST by Arbel and Attiya [2], denoted by CITRUS.

The results of our experiments are shown in Figure 7.3 and Figure 7.4. In Figure 7.3, each row represents a specific workload (read-dominated, mixed or write-dominated) and each column represents a specific key space size; *small* (8Ki to 64Ki), *medium* (128Ki to 1Mi) and *large* (2Mi to 16Mi). Figure 7.4 shows the scaling with respect to the number of threads for key space size of 2^{19} (512Ki). We do not show the numbers for CITRUS in the graphs as it had the worst performance among all implementations (slower by a factor of four in some cases). This is not surprising as CITRUS is optimized for read operations (*e.g.*, 98% reads & 2% updates) [2].

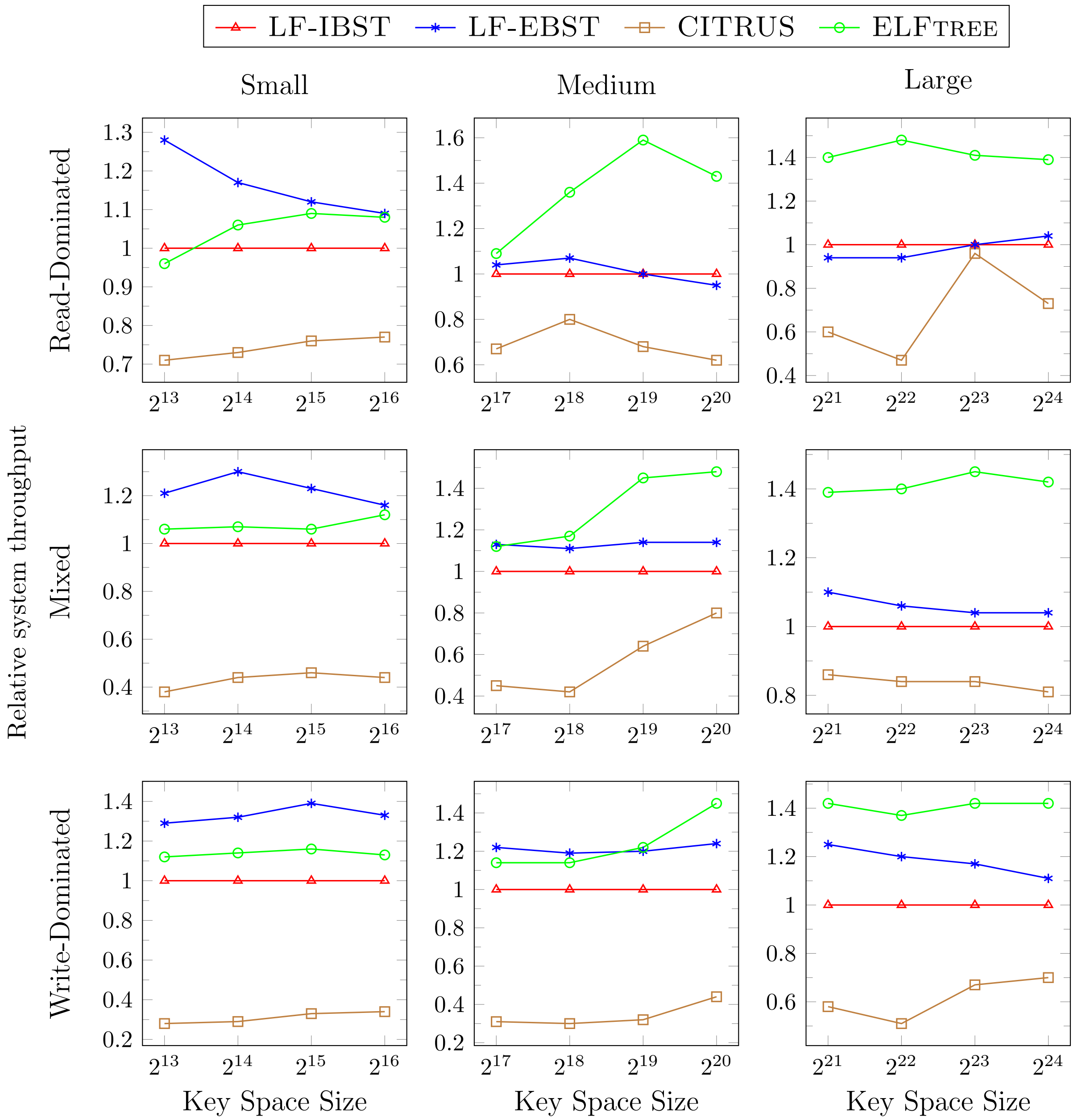


Figure 7.3: Comparison of system throughput of different algorithms *relative to that of* LF-IBST at 32 threads. Each row represents a workload type. Each column represents a range of key space size. Higher the ratio, better the performance of the algorithm.

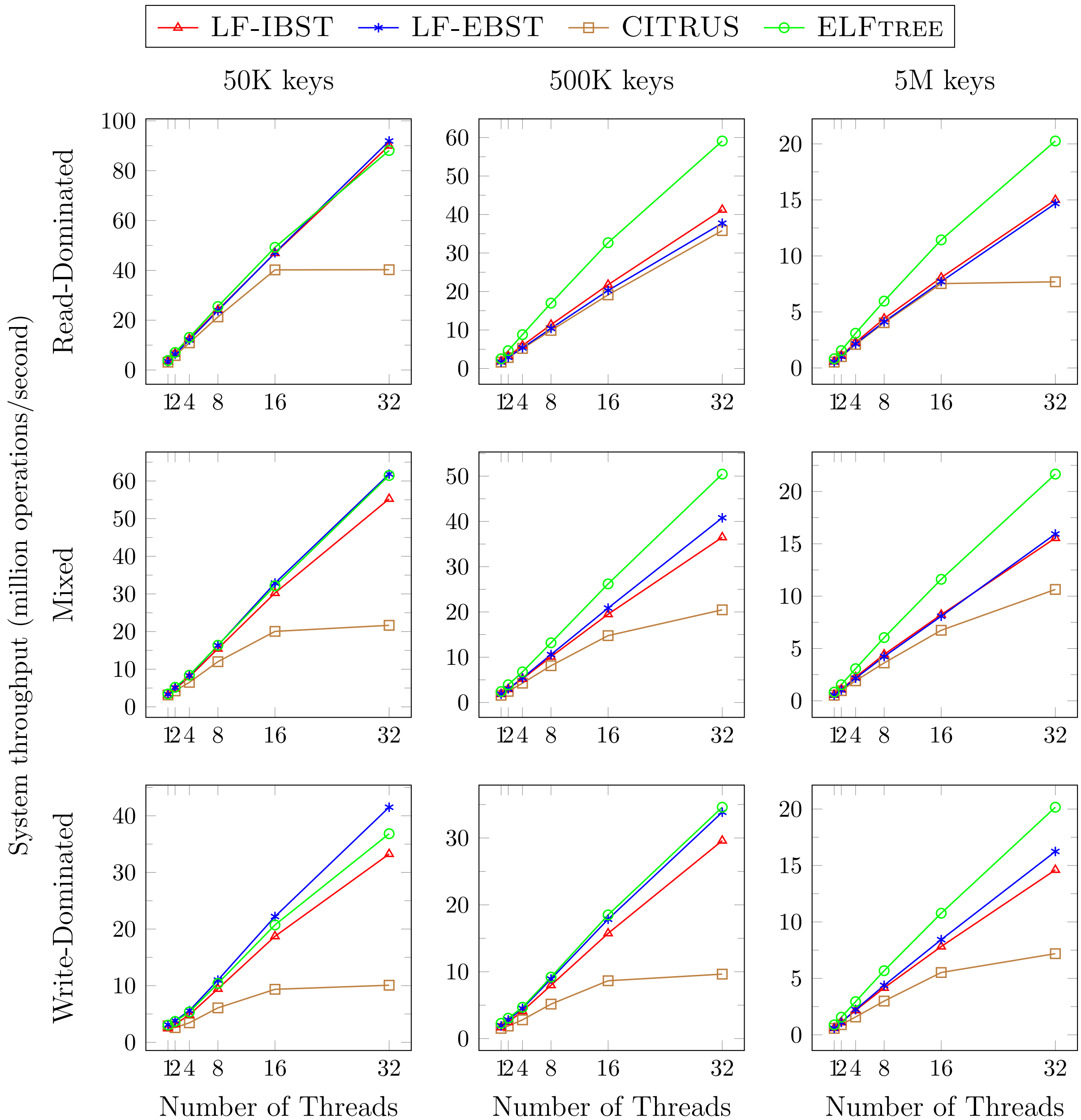


Figure 7.4: Comparison of system throughput of different algorithms. Each row represents a key space size and each column represents a workload type. Higher the throughput, better the performance of the algorithm.

As the graphs show, ELFtree achieved nearly same or higher throughput than the other two implementations for medium and large key space sizes (except for medium key space size with write-dominated workload). Specifically, at 32 threads and for a read-dominated workload, ELFtree had 35% and 24% higher throughput than the next best performer for key space sizes of 512Ki and 1Mi, respectively. Also, at 32 threads and for a mixed workload, ELFtree had 27% and 19% higher throughput than the next best performer for key space sizes of 1Mi and 2Mi, respectively. Overall, ELFtree outperformed the next best implementation by as much as 35%; it outperformed LF-IBST by as much as 44% and LF-EBST by as much as 35% (both achieved for medium key space sizes). For large key space sizes, the overhead of traversing the tree appears to dominate the overhead of actually modifying the operation's window, and the gap between various implementations becomes smaller.

Table 7.2: Comparison of different lock-free algorithms in the absence of contention.

Algorithm	**Number of Objects Allocated**		**Number of Atomic Instructions Executed**	
	Insert	**Delete**	**Insert**	**Delete**
LF-IBST	2	simple: 1 complex: 1	3	simple: 4 complex: 9
LF-EBST	2	0	1	3
ELFtree	1	simple: 0 complex: 1	1	simple: 4 complex: 7

There are several reasons why ELFtree outperformed the other two implementations in many cases. First, as Table 7.2 shows, our algorithm allocates fewer objects than the two other algorithms on average considering the fact that the fraction of insert operations will generally be larger than the fraction of delete operations in any realistic workload. Further, we observed in our experiments that the number of simple delete operations outnumbered the number of complex delete operations by two to one, and our algorithm does not allocate any object for a simple delete operation. Second, again as Table 7.2 shows, our algorithm executes the same number of atomic instructions as in [25] for insert operations; and, in all the cases, executes same or fewer atomic instructions than in [18]. This is important since an atomic instruction is more expensive to execute than a simple read or write instruction. Third, we observed in our experiments that ELFTREE had a smaller memory footprint than the other two implementations (by almost a factor of two) since it uses internal representation and allocates fewer objects. As a result, it was likely able to benefit from caching to a larger degree than LF-IBST and LF-EBST.

7.4 Impact of local recovery

In this section we evaluate our local recovery technique.

To show that our local recovery algorithm is sufficiently general, we implemented it for three different concurrent internal BSTs, namely those based on: (i) the lock-free BST by Howley and Jones [18], denoted by LF-IBST, (ii) the lock-based BST by Ramachandran and Mittal [28], denoted by CASTLE and (iii) the RCU (Read-Copy-Update) lock-based BST by Arbel and Attiya [2], denoted by CITRUS. These implementations were chosen so that we covered both lock-free and lock-based approaches. We choose two lock-based implementations as one is based on locking edges [28] and the other is based on locking nodes using RCU framework [2].

Usually uniform key distribution (where all keys have same frequency of occurrence) have been used to evaluate concurrent BSTs. But in many of the real world workloads, keys have skewed distribution [7] where some keys are more popular than others. Zipfian distribution, a type of power-law distribution simulates this behavior [5, 12, 14]. It is characterized by a parameter α which usually lies between 0.5 and 1 [1, 5]. In our experiments we used both uniform and Zipfian distributions to evaluate the local recovery algorithm.

To better understand the effect of local recovery, we also measure *seek time* which is defined as the total time an operation spends on tree traversal including all restarts as well as stack processing time. For both uniform and Zipfian distribution CASTLE and LF-IBST reached peak performance at 244 threads while CITRUS reached its peak performance at 61 threads for smaller trees and at 122 and 183 threads for larger trees. Performance comparison of these algorithms is not shown here as we are more interested in studying the impact of our local recovery algorithm on each of these algorithms.

For uniform distribution, the performance gain is marginal and, in many cases, is actually slightly worse due to the overhead of stack maintenance (graphs are provided in the appendix). This is not surprising because, for small trees, even though contention is higher, seek time is small to begin with and any benefit of local recovery is nullified by additional overhead of stack maintenance. For larger trees, even though seek time is larger, contention is low as key accesses are spread evenly.

Figure 7.5 shows the behavior for Zipfian distribution with α=1. In general, Zipfian distribution causes more contention than uniform distribution. So even for small trees for which seek times are smaller, we still see performance gains for mixed and write-dominated workloads. Figure 7.6 shows how seek time improves (reduces) due to local recovery.

From Figures 7.5 & 7.6, we see a clear correlation between seek time and system throughput. As seek time reduces due to local recovery, the system throughput also improves. We see maximum improvement for LF-IBST. Since CASTLE and CITRUS are lock-based algorithms, no helping is performed during tree traversal. But in LF-IBST, during the tree traversal, if a pending operation is seen it is helped and then the current operation is restarted. This results in frequent restarts and hence local recovery improves performance by a larger margin.

Table 7.3: Effect of local recovery on system throughput. Positive number indicates a gain while a negative number indicates a drop.

Algorithm	**Uniform (max%, min%)**	**Zipfian (max%, min%)**
CASTLE	(6, -7)	(23, -11)
CITRUS	(23, -11)	(20, -13)
LF-IBST	(9, -6)	(49, -6)

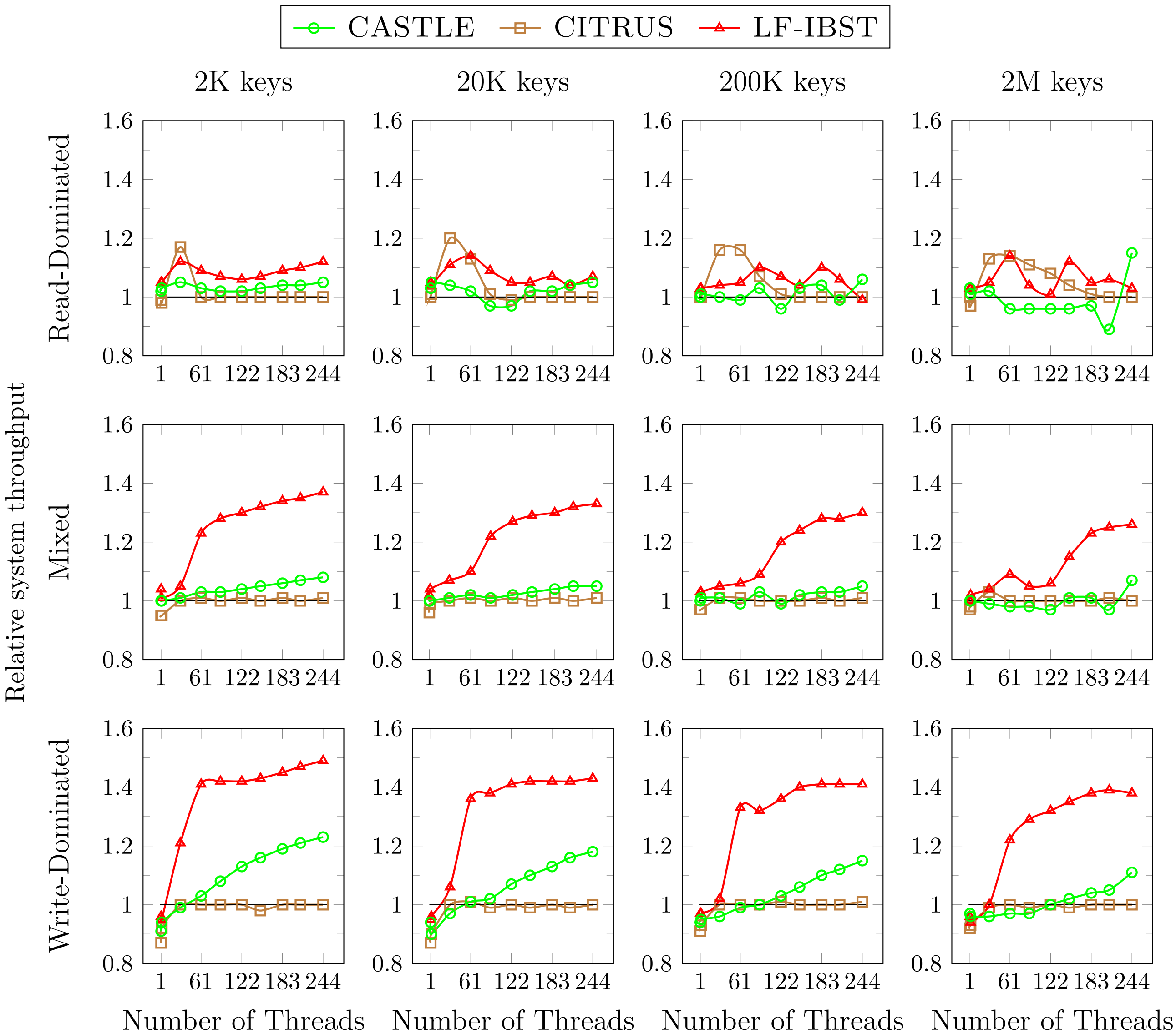


Figure 7.5: Effect of local recovery on system throughput when varying the number of threads from 1 to 244 for zipf distribution. Each row represents a workload type and each column represents a key space size. Higher the relative throughput, better the performance of the algorithm with local recovery.

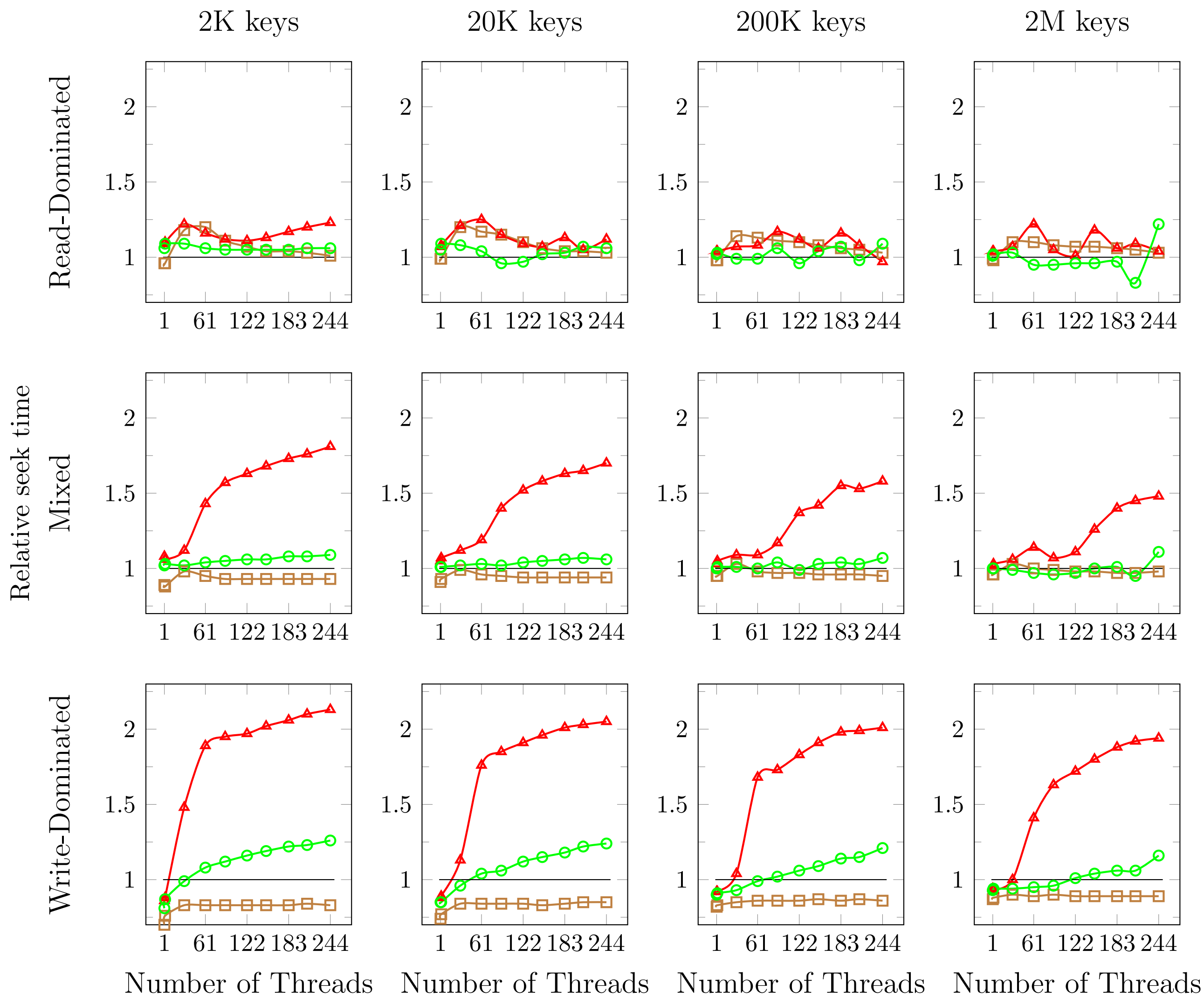


Figure 7.6: Effect of local recovery on seek time when varying the number of threads from 1 to 244 for zipf distribution. Each row represents a workload type and each column represents a key space size. Higher the relative seek time, better the performance of the algorithm with local recovery.

Table 7.3 summarizes the performance gap (with respect to system throughput) between the base algorithm and its extension using local recovery for uniform and Zipfian distributions.

CHAPTER 8
CONCLUSION

In this dissertation we presented a blocking and a non-blocking algorithm for concurrent manipulation of a binary search tree in an asynchronous shared memory system that supports search, insert and delete operations.

Our *lock-based* algorithm is very simple and looks almost identical to a sequential algorithm. In contrast to other lock-based algorithms, it locks edges rather than nodes. This minimizes the contention window of an operation and improves the system throughput. Since the locks are based on edges and as we steal bits from the children edges, the tree node structure is identical to a sequential tree node. This keeps the memory foot print low and reduces the impact of *memory-allocation*. A desirable feature of this algorithm is that its search and insert operations are lock-free; they do not obtain any locks. As indicated by our experiments, our algorithm has the best performance—compared to other concurrent algorithms for a binary search—when the contention is relatively low. Specifically, it achieved the best performance for medium-sized and larger trees with mixed workloads and read-dominated workloads.

Our *lock-free* algorithm combined ideas from two existing lock-free algorithms and is especially *optimized for the conflict-free scenario*. Specifically, when compared to modify operations exiting internal binary search trees, its modify operations (a) have a smaller contention window, (b) allocate fewer objects, (c) execute fewer atomic instructions, and (d) have a smaller memory footprint. Our experiments indicated that our new lock-free algorithm outperforms other lock-free algorithms in most cases.

We also presented a new approach to recover from such failures more efficiently in a concurrent binary search tree based on internal representation using *local recovery* by restarting the traversal from the "middle" of the tree in order to locate an operation's window. Our approach is sufficiently general in the sense that we were able to apply it to a variety of concurrent binary search trees based on both blocking and non-blocking approaches.

We also presented a framework to allow a concurrent algorithm for maintaining an internal BST to recover locally when traversing the tree to locate a key. Our framework is sufficiently general that we were able to apply it to a variety of concurrent binary search trees based on both blocking and non-blocking approaches. we showed by experiments that our local recovery framework improved the performance of concurrent BST algorithms under non-uniform key distribution (*e.g.*, Zipfian) for many different workloads.

As a future work, we would like to analyze our local recovery algorithm (and possibly refine it if needed) so that, when applied to a *non-blocking* BST, it yields a concurrent BST whose operations have provably low amortized time complexity. Also, we would like to analyze the effect of local recovery for other standard non-uniform distributions like normal and Poisson on real workloads.

We also would like to extend the ideas used in our algorithms and our local recovery technique to other data structures. A simple extention would be to apply them to k-ary search trees and then extend it further to B-trees. We also plan to explore other data structures like Bloom-filters which are commonly used in big-data applications.

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