Concurrent Binary Search Trees
Design and Optimizations

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Overview

Introduction

- CPUs aren't getting faster (memory wall, ILP wall and power wall)
- Shift towards multicore and manycore

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How to keep all the cores **busy**?

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Parallel computing

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Concurrent computing

Concurrency vs Parallelism

Concurrency is not parallelism (it's better!!)

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Parallel Computing

- decades of research done
- Example - Matrix-Matrix Multiplication
- **do** lot of things simultaneously
- cannot be done on a single CPU
- **deterministic** control flow
- is about **speedup**
- **hard** to debug

Concurrent Computing

- Relatively new
- Example - A web crawler, mouse/keyboard
- **deal** lot of things simultaneously
- can be done on a single CPU
- **non-deterministic** control flow
- is about **hiding latency**
- **very hard** to debug

Designing Concurrent Data Structures

- Shared-memory multiprocessors concurrently execute multiple threads
- Threads communicate and synchronize through data structures in shared memory
- Threads can interleave in exponential number of ways
- Concurrent data structure must preserve its properties for all possible interleavings

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fetchAndIncrement: Using locks

```
repeat
    rOld = x;
    rNew = rOld+1;
until (x.compareAndSwap(rOld,rNew));
```

fetchAndIncrement: using atomic instructions

compareAndSwap updates(atomically) the value of x to *rNew* only if the read value of x is equal to *rOld*. Returns *true* if it succeeds in updating the value of x

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  - use locks to resolve contention
  - coarse grained or fine grained locking
  - easier to design
  - weaker progress guarantees (thread owns a lock)
  - are prone to deadlock, priority inversion

- Non-Blocking Algorithms

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- Non-Blocking Algorithms
  - use atomic (Read-Modify-Write) instructions to resolve contention. E.g. Compare-And-Swap(CAS) instruction
  - lock-free or wait-free
  - stronger progress guarantees (operation owns a lock - helping)
  - deadlock or priority inversion not possible
  - harder to design

Linearizability

Linearizability requires two properties:

- the object (or data structure) be sequentially consistent[1]
- the total ordering which makes it sequentially consistent respect the *real-time ordering* among the operations in the execution

respecting real-time ordering - if an operation op_1 completed before another operation op_2, then op_1 must be ordered before op_2

[1] *the result of any execution is the same as if the operations of all the processors were executed in some sequential order, and the operations of each individual processor appear in this sequence in the order specified by its program*

Linearizability

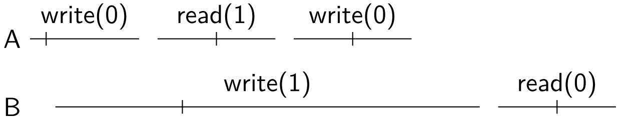
- *linearization point* - a distinct point between a method invokation and response where the method appears to have taken effect instantaneously
- Order the method calls based on their linearization points
- Resulting order should be in the sequential specification of the object.

Linearizability - Examples

A read(1) write(2) read(2)

A history of a sequential object

Linearizability - Examples



A history of a concurrent object - linearizable

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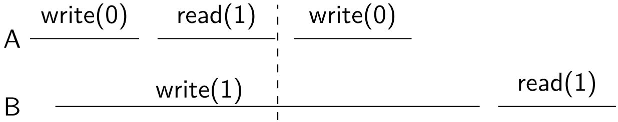

A history of a concurrent object - not linearizable

Binary Search Tree - Defintion

A *binary search tree* (BST) is a data structure which meets the following requirements:

- it is a binary tree (a node can contain atmost two children)
- each node contains a key k
- left subtree of a node contains keys lesser than k
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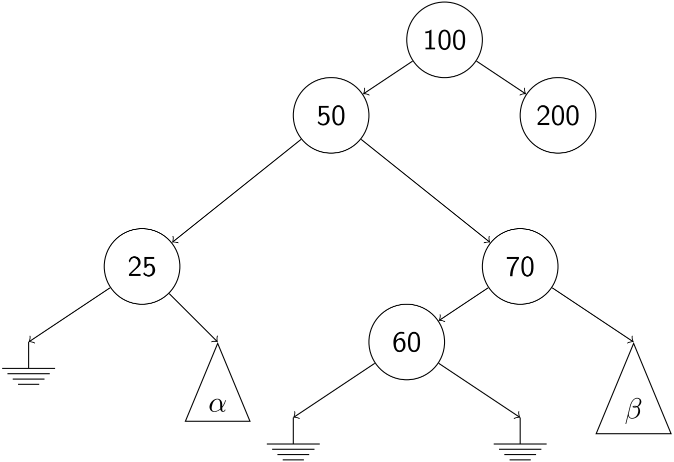
Operations on a BST

- **search(k)** - returns *true* only if key k is present in the tree
- **insert(k)** - inserts k into the tree if it does not already exist
- **delete(k)** - deletes k from the tree if it already exist

BST - Search

search(70**)**



BST - Search

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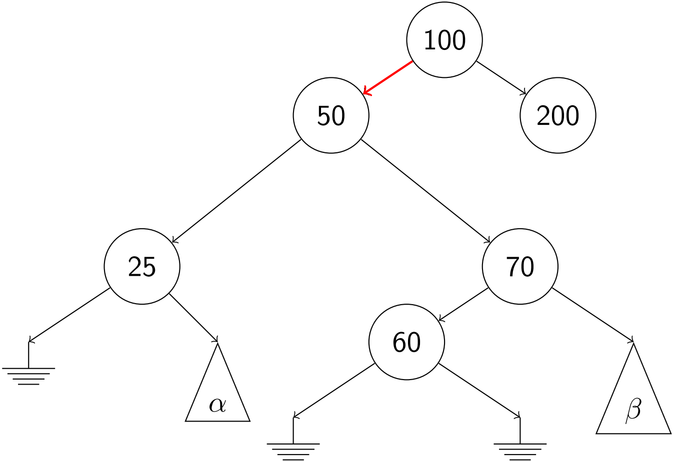

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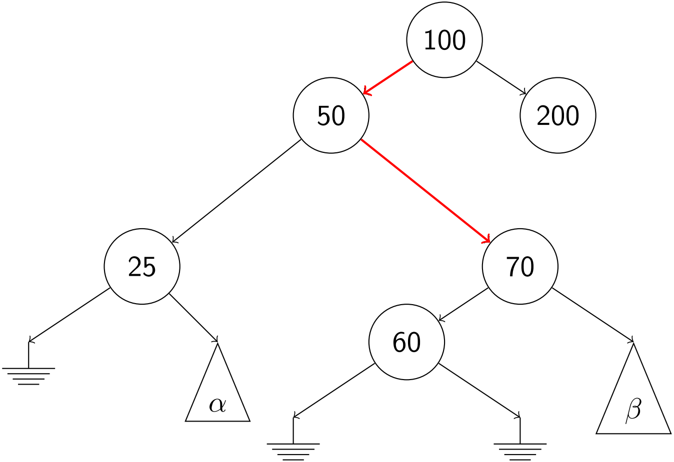

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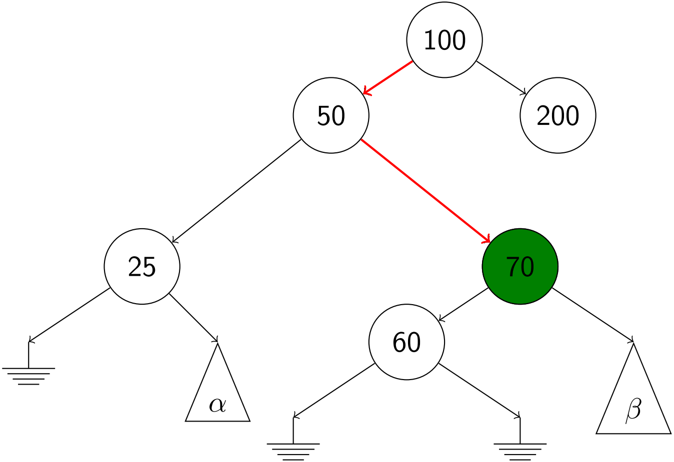

BST - Search

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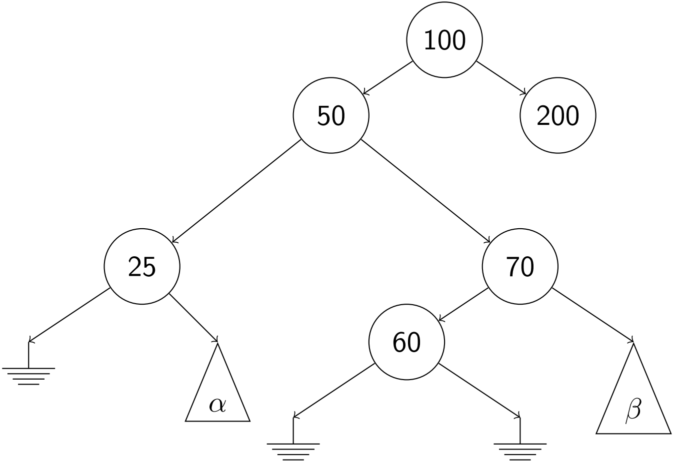

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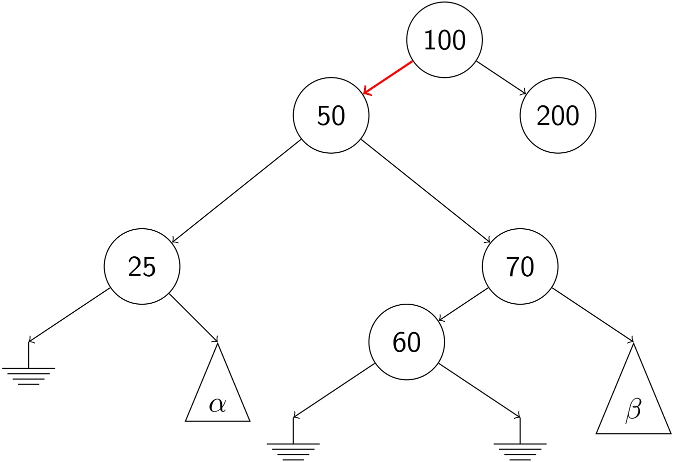

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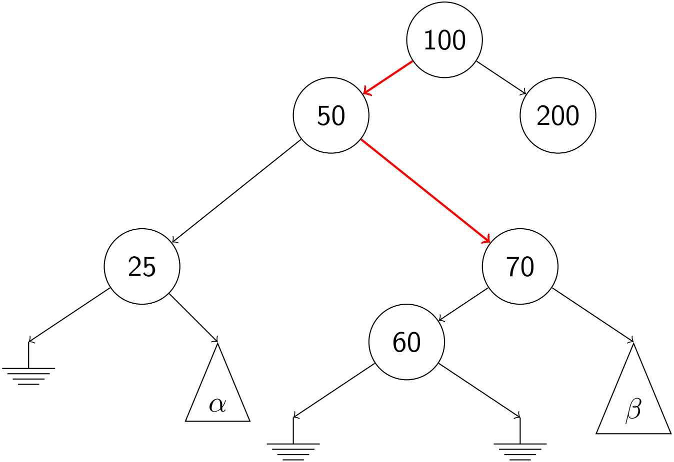

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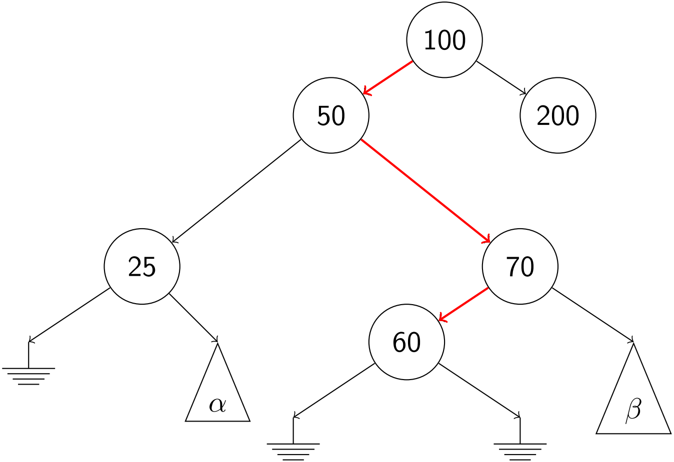

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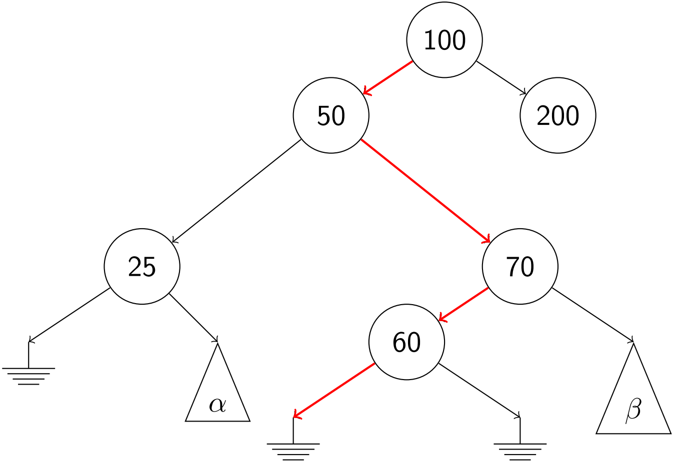

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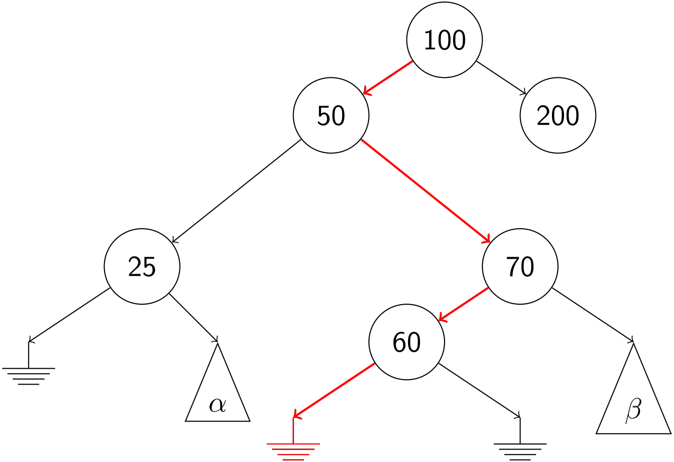

BST - Insert

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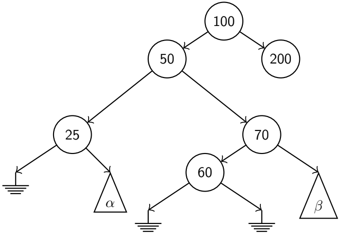

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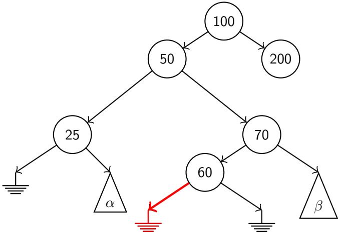

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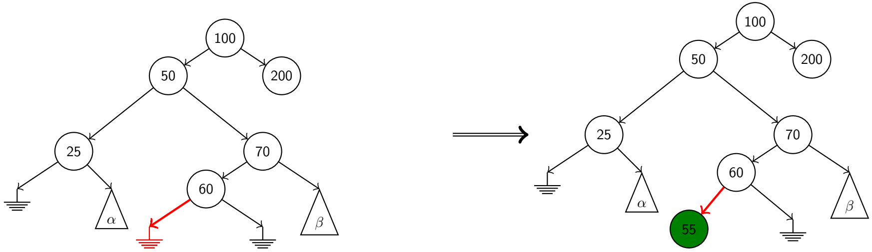
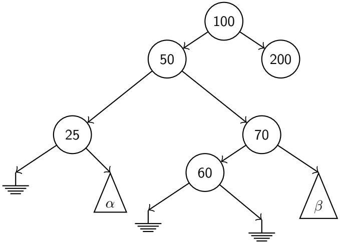


Types of delete

- simple - removing a node which has atmost one child
- complex - removing a node which has exactly two children

BST - Simple Delete

delete(25**)**



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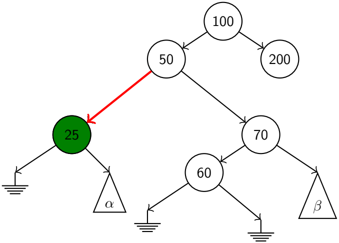

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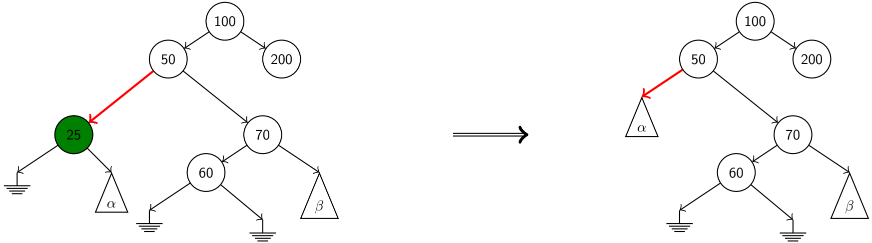
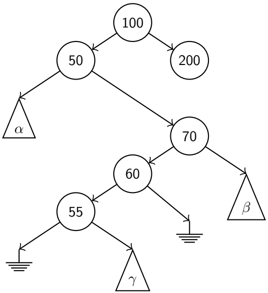


BST - Complex Delete

delete(50**)**



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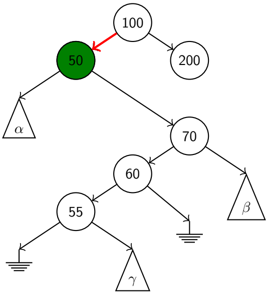

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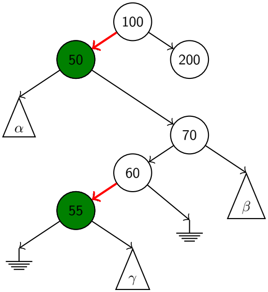

find successor

BST - Complex Delete

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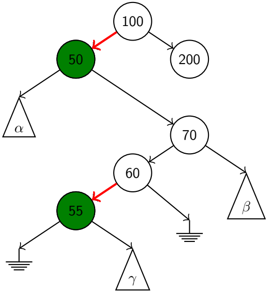

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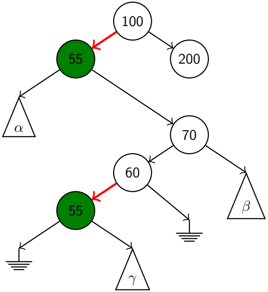

promote key

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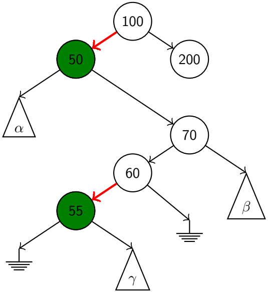

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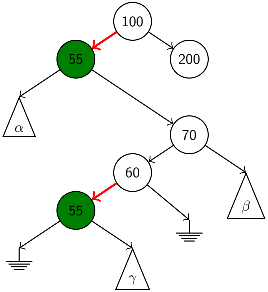

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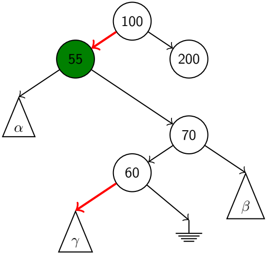

remove successor

Related Works

#	Algorithm Type	Works At	BST Type	Authors
1	lock free	node level	external	Ellen et.al[PODC'10]
2	lock free	node level	internal	Howley & Jones[SPAA'12]
3	lock free	edge level	external	Natarajan &Mittal[PPoPP'14]
4	lock based	node level	internal	Arbel & Attiya[PODC'14]
5	lock based	node level	internal	Drachsler et.al[PPoPP'14]

Lock Based BST[PPoPP'15 Poster]

Contributions

- combine edge-based locking with internal representation of BST
- optimistic tree traversal

Lock Based BST[PPoPP'15 Poster]

- common workloads have more searches than updates
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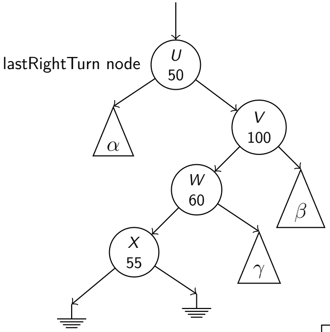
  - design is optimized for searches
  - search operations are oblivious to locks
- Any real life workload will have more inserts than deletes
  - insert operations do not obtain any locks
  - performs only one atomic operation
- removal of a node in a concurrent BST is challenging
  - delete operations uses locks
  - locks can be obtained on nodes or edges
  - locking edges instead of nodes increases concurrency

Lock Based BST - Challenges in search



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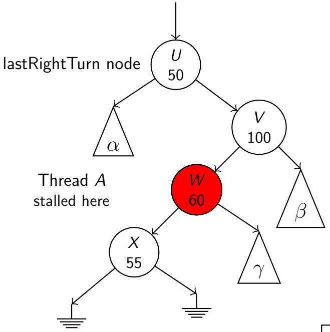

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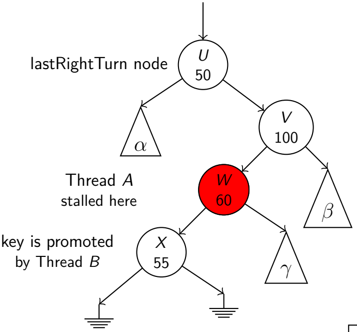

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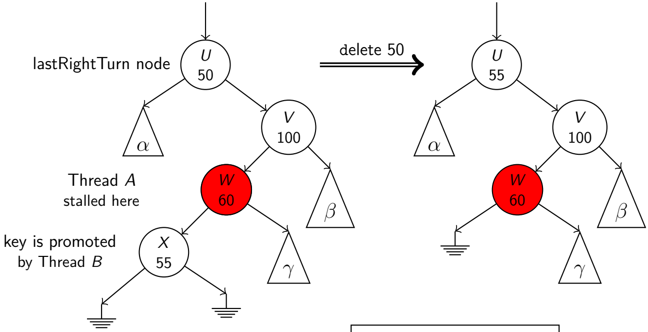

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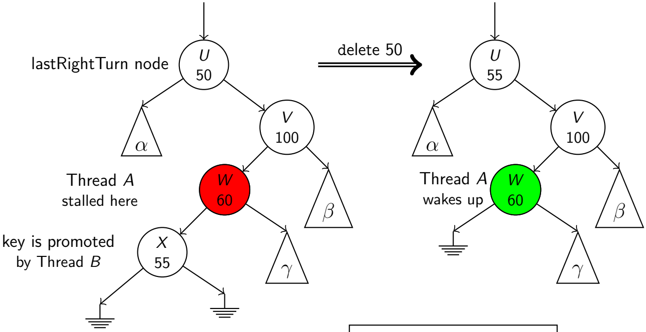

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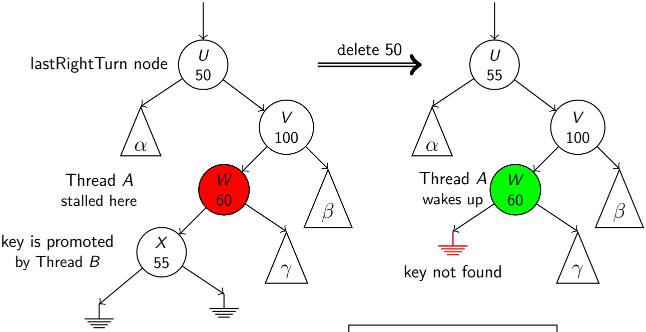

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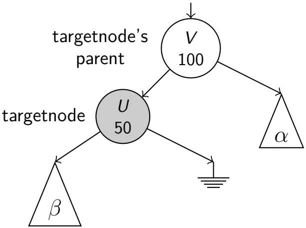
Keep track of last right turn node and its key. If search terminates at a NULL node, check if the current key in the last right turn node has changed. If yes restart the operation from root.

Lock Based BST - Simple Delete



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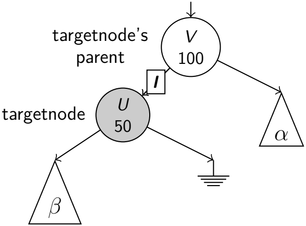

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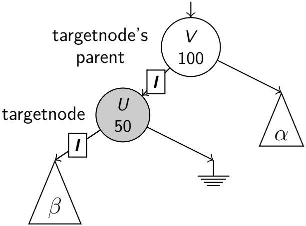

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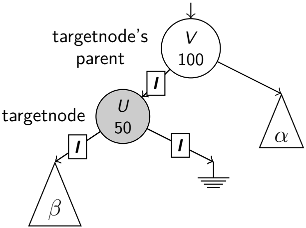

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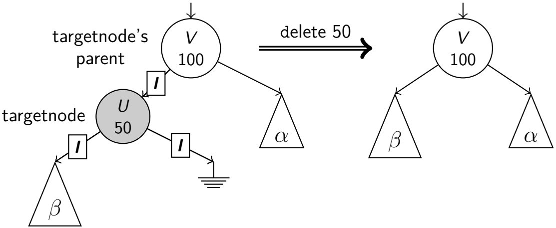

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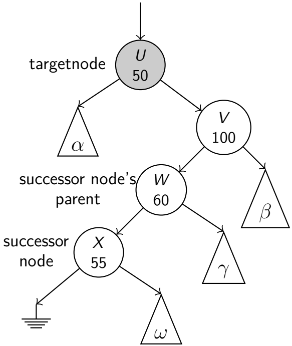

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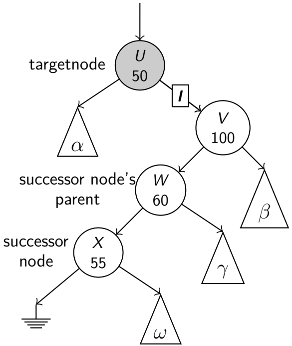

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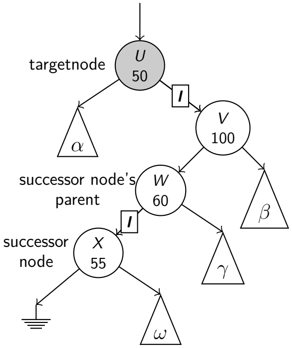

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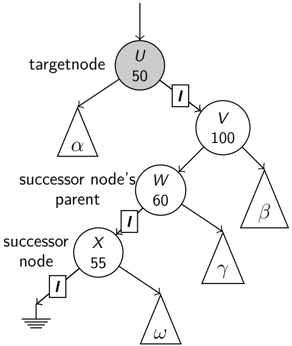

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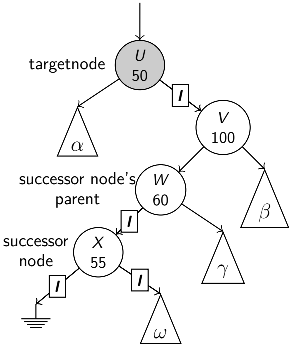

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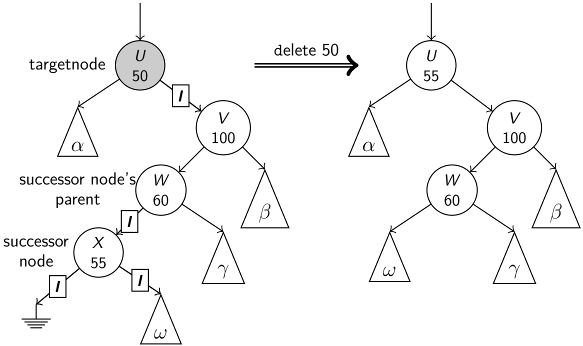
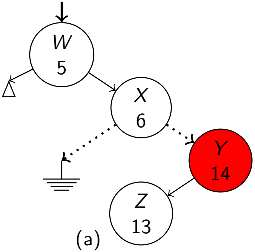


Lock Based BST - More challenges in search

A scenario in which the last right turn node is removed



(a)

- Search(13) gets stalled at Y in (a). Its last right turn node is X

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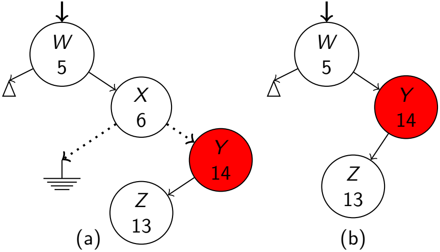

- Delete(6) removes X from the tree in (b). The key stored in X is still 6

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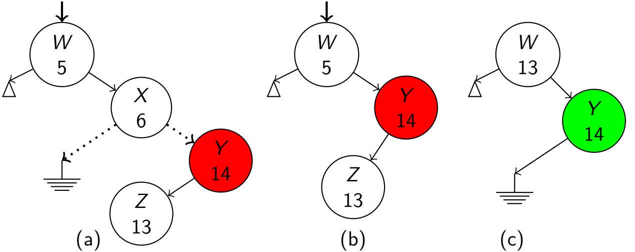

- Delete(5) results in 13 moving up the tree from Z to W in (c). When search(13) wakes up, it will miss 13 as the key in the last right turn node has not changed

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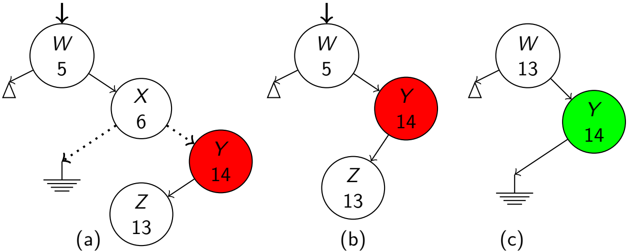
A scenario in which the last right turn node is removed



- In the first traversal search(13) saw the node X

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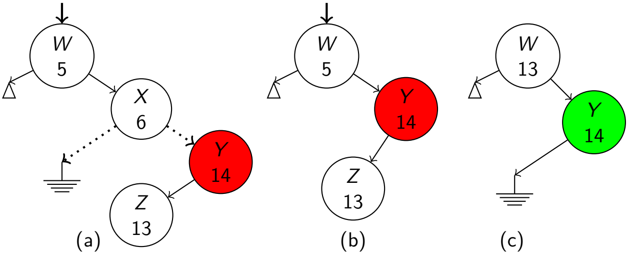

- In the second traversal there are two cases
  - case1, search(13) did not find X - save the traversal and restart
  - case2, search(13) did find X - use the results of previous traversal

Lock Free BST[ICDCN'15]

Contributions

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Lock Free BST[ICDCN'15]

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- optimistic tree traversal
- lock-free algorithm

Lock Free BST[ICDCN'15]

- search and inserts are same as in lock Based BST
- to maintain lock-free property, if an insert or delete operation fails, it helps a pending delete operation(if needed)

pseudocode for delete

```
locate the node to delete;
flag the children edges for deletion;
if simple delete then
    make the parent point to the non-null child atomically;
else // complex delete
    find the successor;
    flag the children edges of successor for promotion;
    promote key;
    remove successor by a simple delete;
    replace node with a fresh copy;
end
```

Lock Free BST - Simple Delete

- flag is owned by an operation
- if a thread which installed the flag is stalled, other threads can help complete the operation

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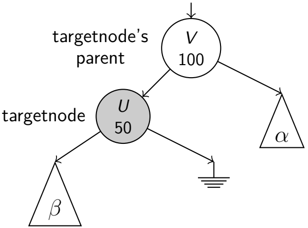

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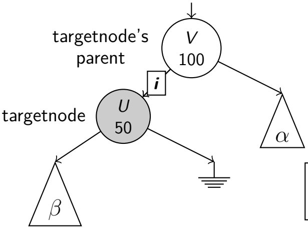

i - intent flag set
d - delete flag set

Lock Free BST - Simple Delete

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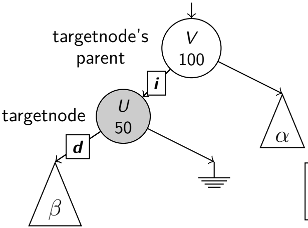

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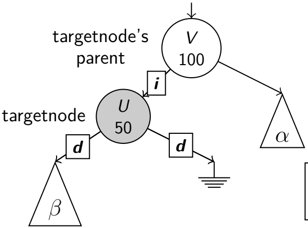

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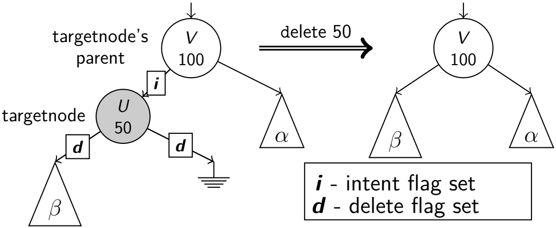
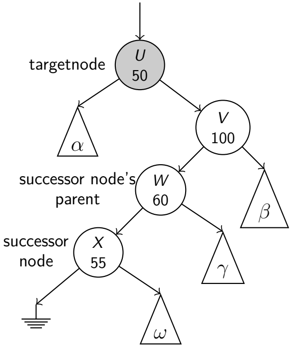
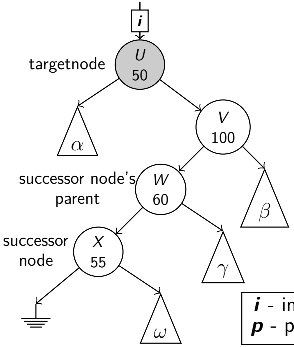


Lock Free BST - Complex Delete

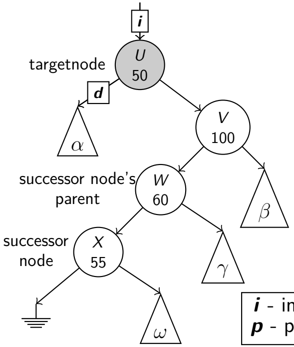


Lock Free BST - Complex Delete



i - intent flag set ***d*** - delete flag set
p - promote flag set

Lock Free BST - Complex Delete



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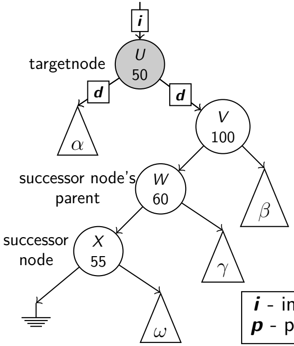
Lock Free BST - Complex Delete



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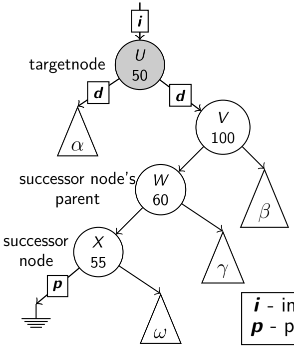

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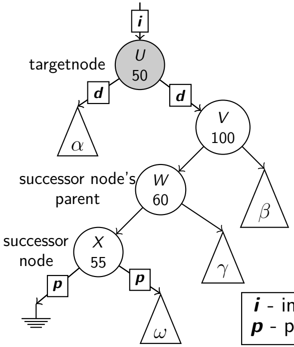

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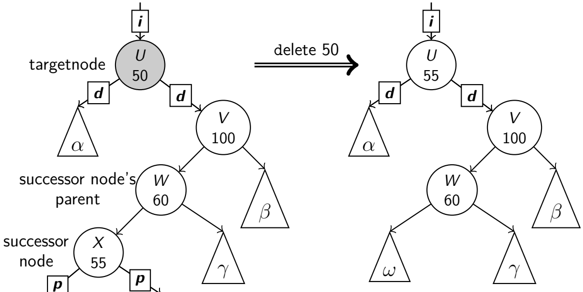
Lock Free BST - Complex Delete



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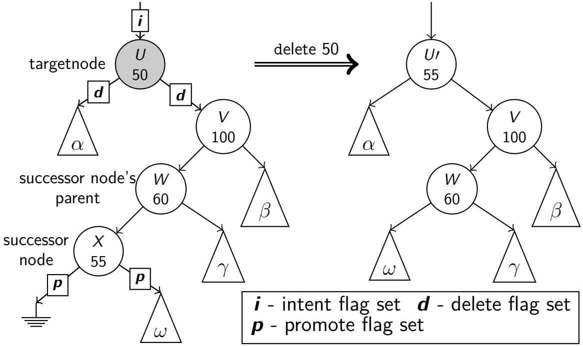
Lock Free BST - Complex Delete



i - intent flag set ***d*** - delete flag set ***p*** - promote flag set

Local recovery[PPoPP'16 Poster]

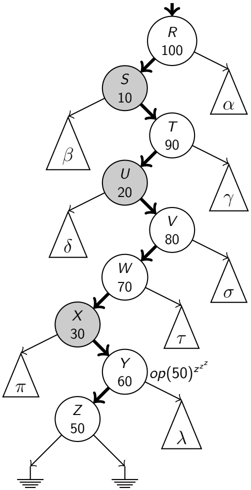
Overview

- a general technique for local recovery for concurrent BSTs
- reduces tree traversal cost during failures by restarting closer to an operation's window

Motivation

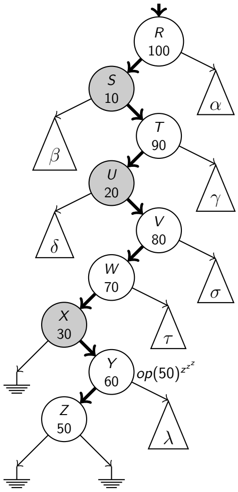
- in most concurrent BSTs, execution phase of an operation have constant time complexity
- seek phase is where an operation may end up spending most of its time (esp for large trees)
- this technique reduces the seek time

Example



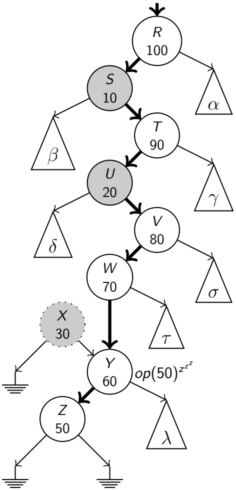
Operation $op(50)$ is suspended at node Y during its traversal

Example



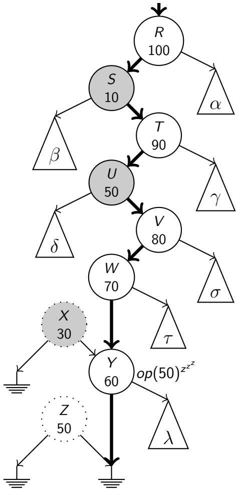
All keys in subtree π are deleted one-by-one

Example



Key 30 is deleted (simple delete); node X is removed

Example

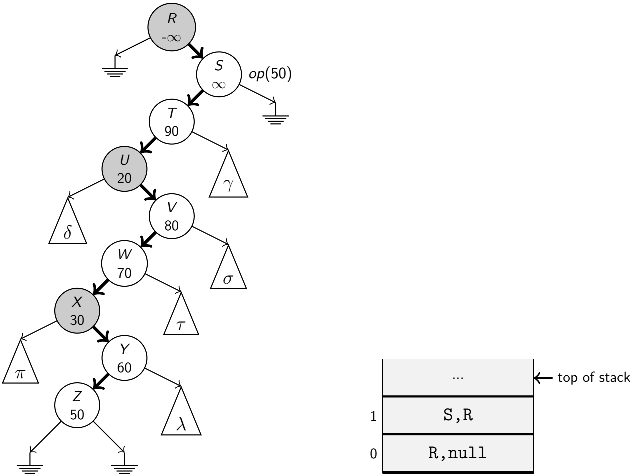


Key 20 is deleted (complex delete); key 20 is replaced with key 50 in node U and node Z is removed

Traversal Stack

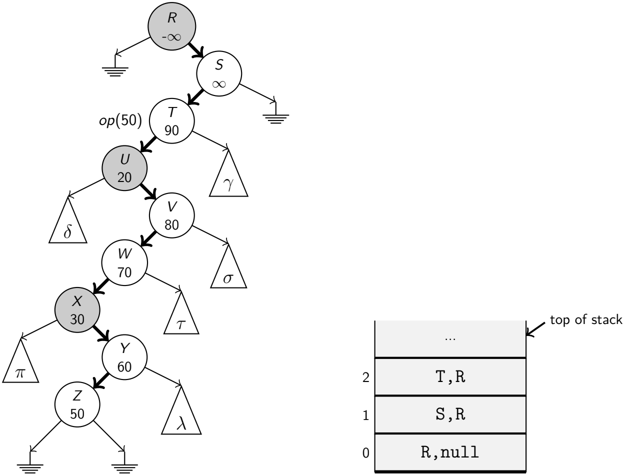
- a stack to keep track of anchor nodes of all nodes in the traversal path
- reduces tree traversal cost during failures by restarting closer to an operation's window

Traversal Stack

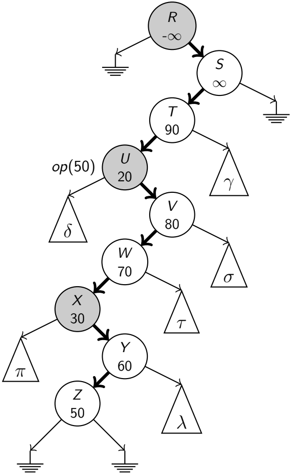


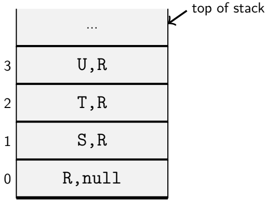
Operation $op(50)$ starting at R and suspended at S along with the stack

Traversal Stack

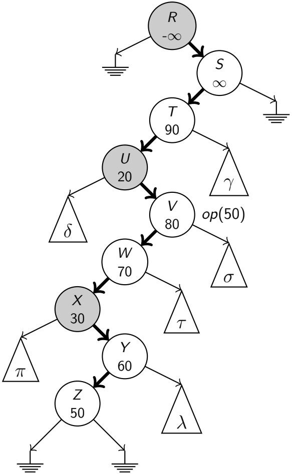


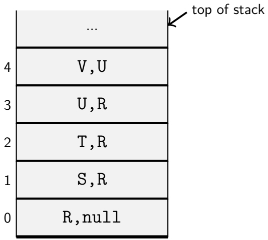
Traversal Stack



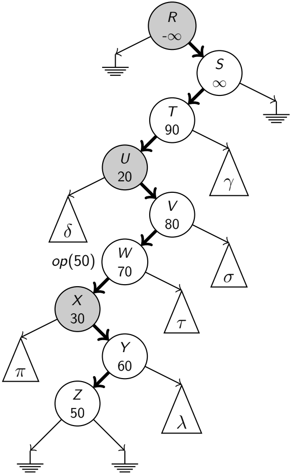


Traversal Stack





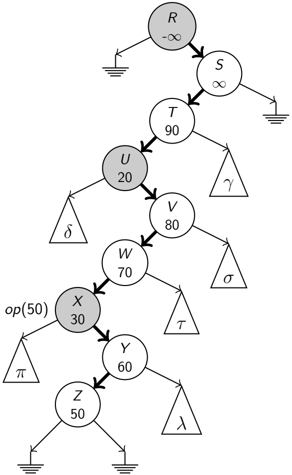
Traversal Stack



top of stack

	...
5	W,U
4	V,U
3	U,R
2	T,R
1	S,R
0	R,null

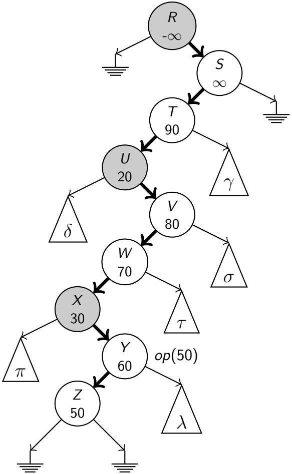
Traversal Stack



top of stack

	...
6	X,U
5	W,U
4	V,U
3	U,R
2	T,R
1	S,R
0	R,null

Traversal Stack

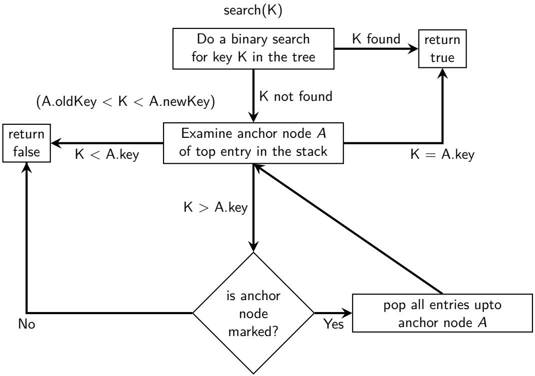


top of stack

	...
7	Y,X
6	X,U
5	W,U
4	V,U
3	U,R
2	T,R
1	S,R
0	R,null

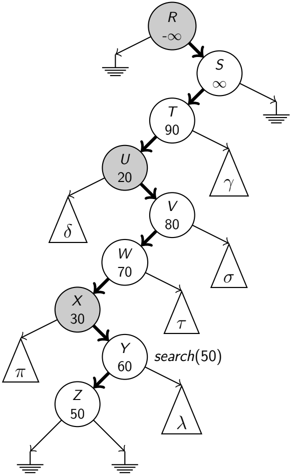
Search

search operations do not restart



Sequence of steps in a search operation

Search - consistent anchors

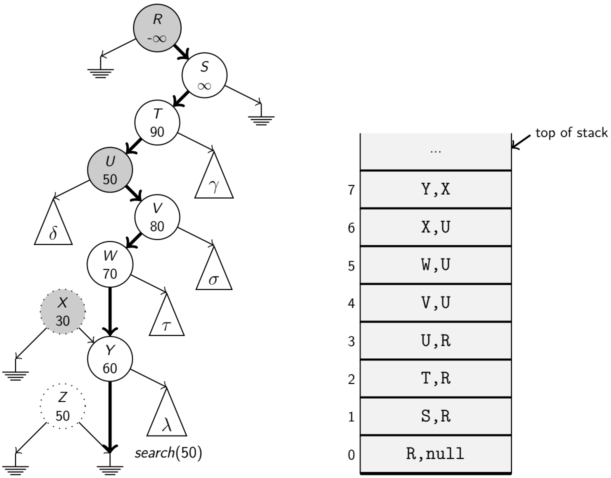


top of stack

	...
7	Y,X
6	X,U
5	W,U
4	V,U
3	U,R
2	T,R
1	S,R
0	R,null

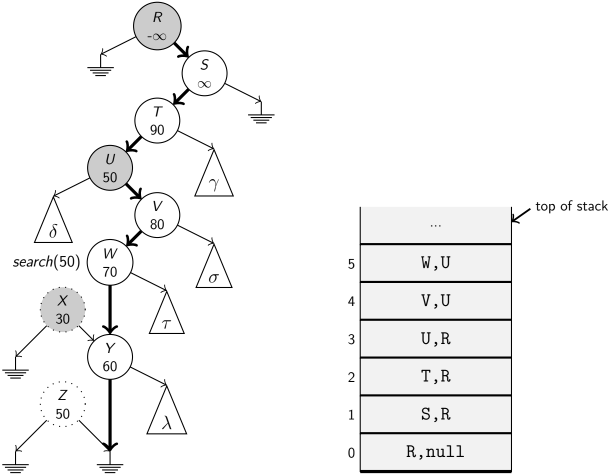
Operation *search*(50) starting at R and suspended at Y along with the stack

Search - consistent anchors



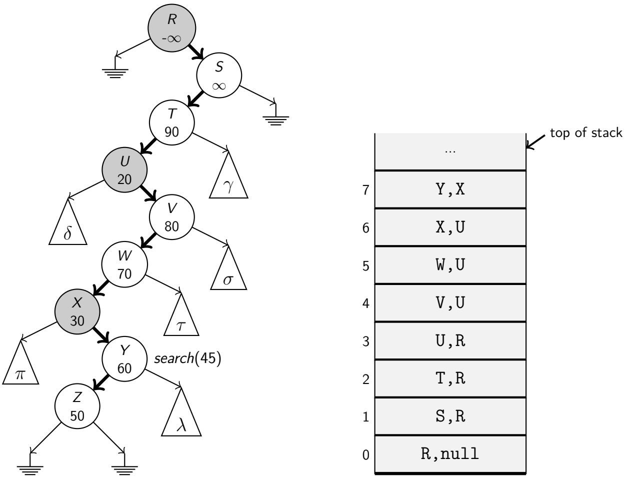
Key 30 is deleted;key 20 is deleted & replaced with key 50 in node U and node Z is removed

Search - consistent anchors



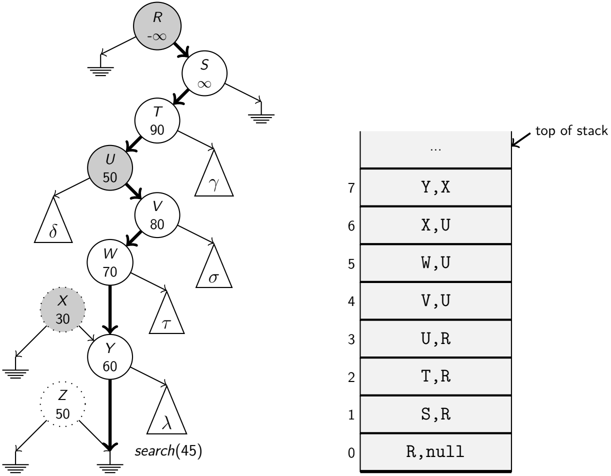
Pop upto marked anchor node X. Top of stack is now W. Examine anchor node U

Search - inconsistent anchor



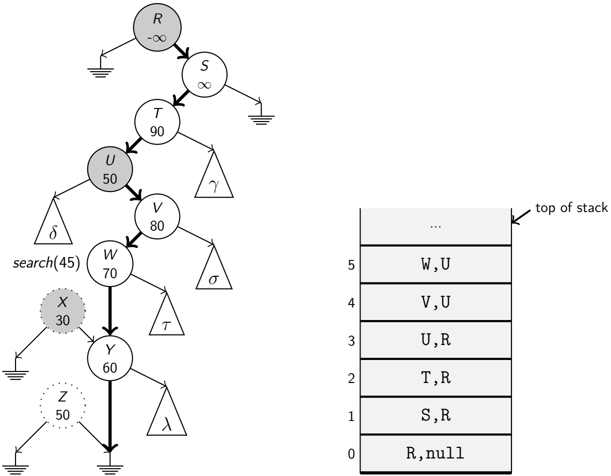
Operation *search*(45) starting at R and suspended at Y along with the stack

Search - inconsistent anchor



Key 30 is deleted;key 20 is deleted & replaced with key 50 in node U and node Z is removed

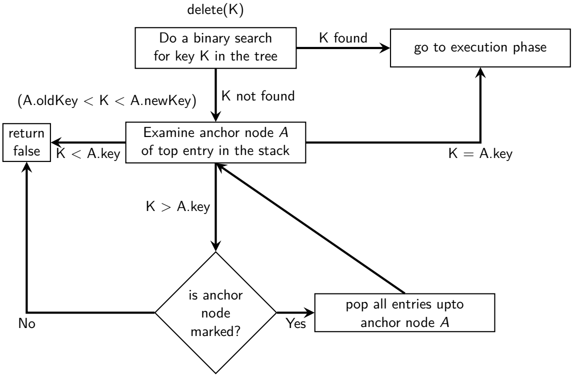
Search - inconsistent anchor



Pop upto marked anchor node X. Top of stack is now W. Examine anchor node U. A.oldKey(20) $<$ K(45) $<$ A.newKey(50). Inconsistent anchor

Delete

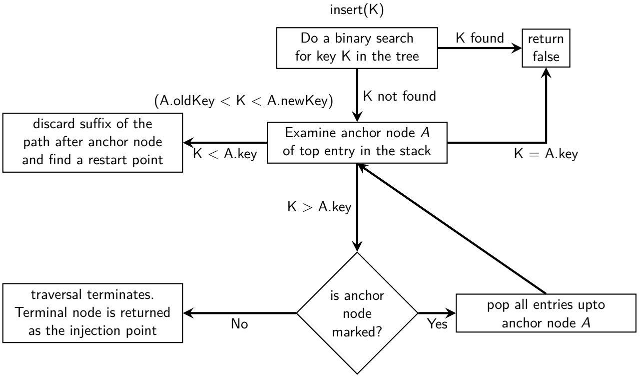
A delete operation do not restart except when there is a failure in the execution phase



Sequence of steps in a delete operation

Insert

An insert operation needs to restart only if one of the anchor nodes in the path has become inconsistent



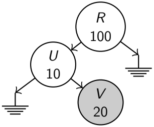
Sequence of steps in an insert operation

Wait Free Search

wait-free - every thread is able to complete its operations in a finite number of steps over an infinite period of time

- two light-weight techniques to make search operations for concurrent internal BSTs, *wait-free*
- low additional overhead
- no write instructions on share memory
- minimizes cache traffic

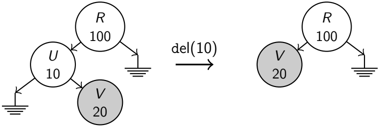
Wait Free Search



(a) Thread A gets stalled at node V

A scenario in which contains operation is not wait-free

Wait Free Search

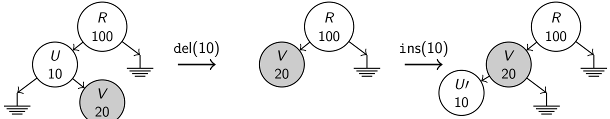


(a) Thread A gets stalled at node V

(b) Thread B executes delete(10) and node U is removed

A scenario in which contains operation is not wait-free

Wait Free Search



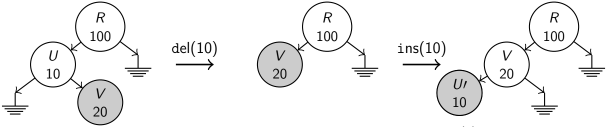
(a) Thread *A* gets stalled at node *V*

(b) Thread *B* executes delete(10) and node *U* is removed

(c) Thread *B* executes insert(10) and node *U′* is added. Thread *A* wakes up and reaches node *U′*

A scenario in which contains operation is not wait-free

Wait Free Search



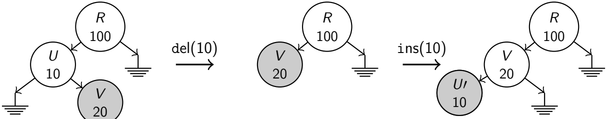
(a) Thread A gets stalled at node V

(b) Thread B executes delete(10) and node U is removed

(c) Thread B executes insert(10) and node $U\prime$ is added. Thread A wakes up and reaches node $U\prime$

A scenario in which contains operation is not wait-free

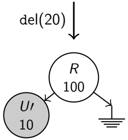
Wait Free Search



(a) Thread A gets stalled at node V

(b) Thread B executes delete(10) and node U is removed

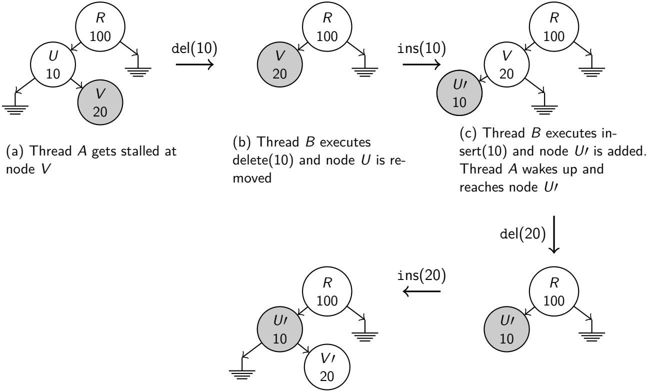
(c) Thread B executes insert(10) and node U' is added. Thread A wakes up and reaches node U'



(d) Thread B executes delete(20) and node V is removed

A scenario in which contains operation is not wait-free

Wait Free Search



(a) Thread *A* gets stalled at node *V*

(b) Thread *B* executes delete(10) and node *U* is removed

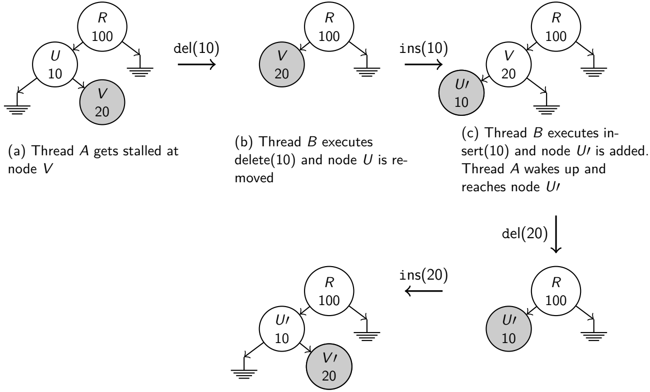
(c) Thread *B* executes insert(10) and node *U′* is added. Thread *A* wakes up and reaches node *U′*

(d) Thread *B* executes delete(20) and node *V* is removed

(e) Thread *B* executes insert(20) and a node *V′* is added. Thread *A* wakes up and reaches node *V′*

A scenario in which contains operation is not wait-free

Wait Free Search



(a) Thread A gets stalled at node V

(b) Thread B executes delete(10) and node U is removed

(c) Thread B executes insert(10) and node $U\prime$ is added. Thread A wakes up and reaches node $U\prime$

(d) Thread B executes delete(20) and node V is removed

(e) Thread B executes insert(20) and a node $V\prime$ is added. Thread A wakes up and reaches node $V\prime$

A scenario in which contains operation is not wait-free

No Modification to Tree Node

- as long as a key is *continuously* present in the tree, its distance from root is *monotonically non-increasing*
- if a key is not found after visiting a "certain" number of nodes in the tree, then traversal stops
- sufficient to examine the path traversed to check whether or not the key has moved up
- In case the key is not continuously present in the tree, it is acceptable to return either:
  - present - linearized after the insert operation that added the key to the tree
  - not present - linearized after the delete operation that removed the key from the tree

No Modification to Tree Node

when to stop?

No Modification to Tree Node

when to stop?

Each process maintains two counters:

- insert counter - number of true inserts
- delete counter - number of true deletes

$IC[i]$ and $DC[i]$ denote the number of insert and delete operations, respectively, process P_i has performed so far

No Modification to Tree Node

- insert counter incremented before adding a key
- delete counter incremented before removing a key
- insert (delete) counter at a process is an upper (lower) bound on the number of keys that the process has added to (removed from) the tree

No Modification to Tree Node

```
read and aggregate delete counter values of all processes DC = Σ_{i=1}^{p} DC[i];
read and aggregate insert counter values of all processes IC = Σ_{i=1}^{p} IC[i];
IC − DC ≥ actualtreesize as IC ≤ actual inserts and DC ≥ actual deletes;
```

pseudocode: waitFreeSearch

$IC - DC$ gives an upper bound on number of keys to traverse before stopping the search operation

With Modification to Tree Node

- previous approach - time complexity depends on tree size
- this approach - time complexity depends on the tree height
- but needs modifications to tree node structure
- each node has a timestamp on when it was created
- timestamp - $\langle$process id,process sequence number$\rangle$
- process sequence number is incremented before a node is added to the tree

With Modification to Tree Node

```
read current sequence number of all processes;
let label[i] denote the sequence number of procecess p_i;
stop the downward traversal of the tree once a node with timestamp ⟨i,v⟩
such that v > labels[i] is encountered;
```

pseudocode: waitFreeSearch

Experimental Setup

To compare the performance of various concurrent BSTs we considered the following parameters:

- Maximum Tree Size
  - key space size varied from 2^{13} (8Ki) to 2^{24} (16Mi).
- Relative Distribution of Operations
  - Read-Dominated (90% search, 9% insert and 1% delete)
  - Mixed (70% search, 20% insert and 10% delete)
  - Write-Dominated (0% search, 50% insert and 50% delete)
- Maximum degree of Contention
  - number of threads that can concurrently operate on the tree
  - we collected data for 32 threads

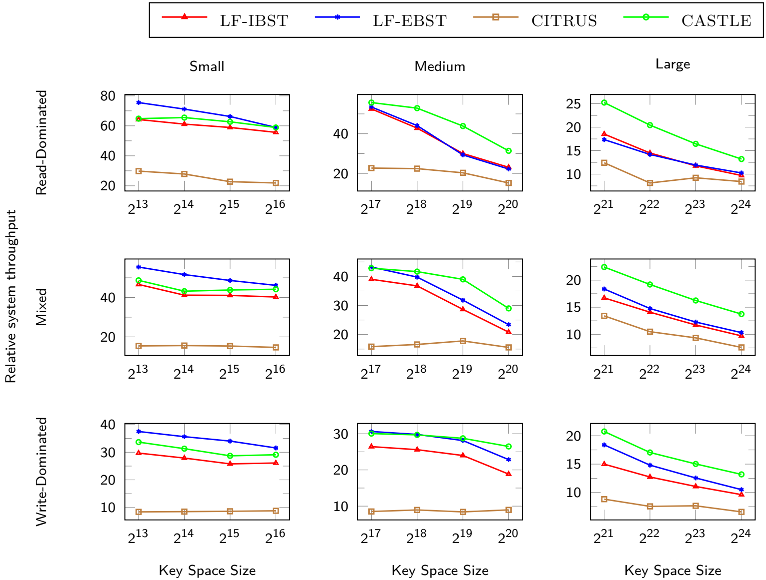
Experimental Setup

- Throughput computed as millions of operations per second (MOPS)
- each trial was run for 2 minutes
- Average over 5 trials
- *pre-populated* the tree to 50% of its maximum size to capture steady state behaviour
- beginning of each run consisted of a 1 second "warm-up" phase whose numbers were excluded in the computed statistics to avoid initial caching effects
- The machine we used is a Dell PowerEdge R820 server with 4 Intel E5-4650 @ 2.70GHz 8-core processors (32 cores in total) and 1TB of DDR3 memory with HT disabled. 256KB L2 and 20MB shared L3

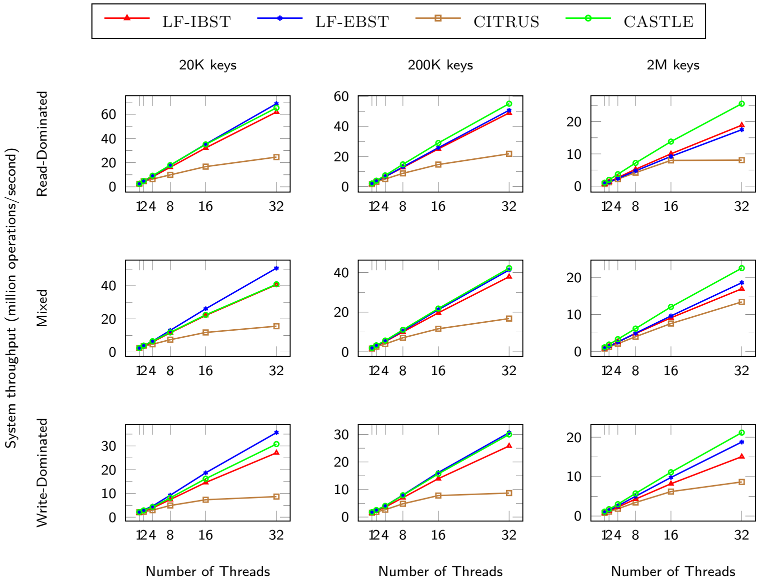
Other Concurrent BSTs

- a lock-free internal BST by Howley and Jones[SPAA'12], denoted by HJ-BST
- a lock-free external BST by Natarajan and Mittal[PPoPP'14], denoted by NM-BST
- RCU-based internal BST by Arbel and Attiya[PODC'14], denoted by CITRUS

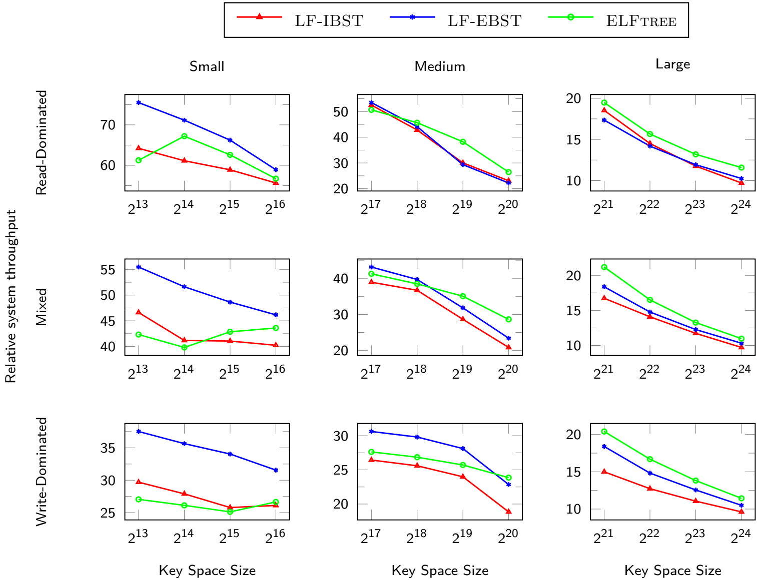
Lock Based BST - key sweep - absolute



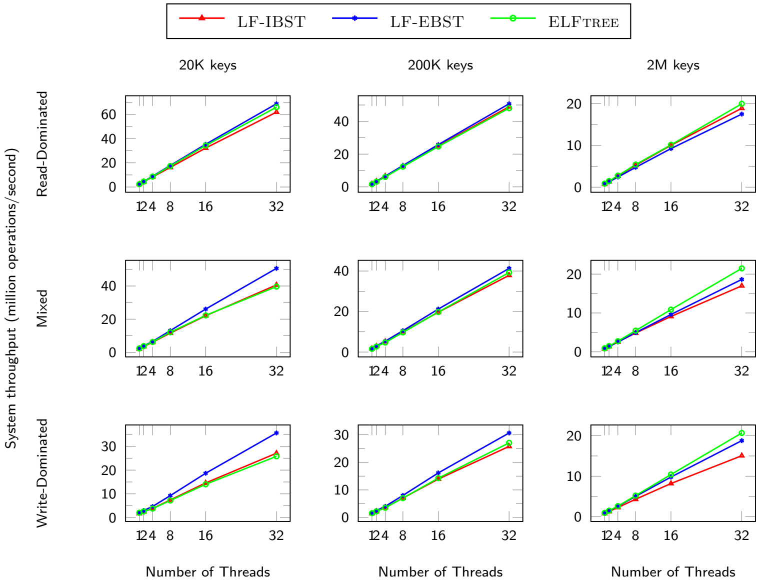
Lock Based BST - thread sweep - absolute



Lock Free BST - key sweep - absolute



Lock Free BST - thread sweep - absolute



Results Summary

Comparison of different concurrent BSTs in the absence of contention

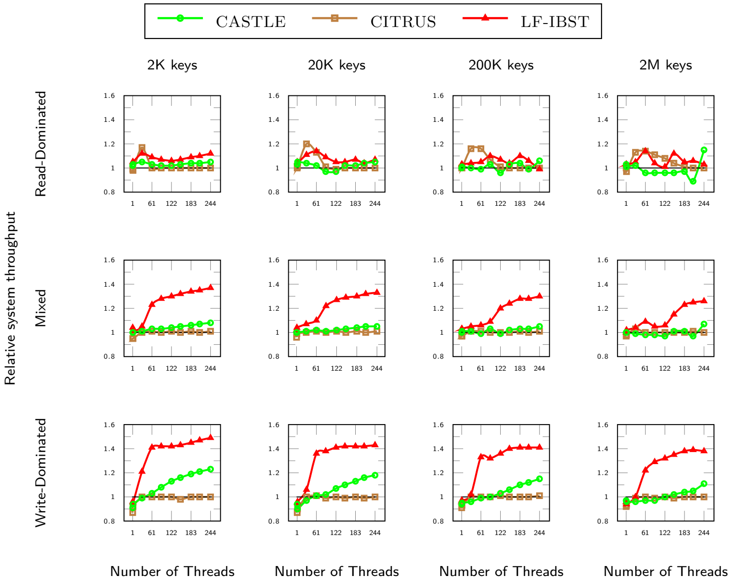
- speedup is calculated over the second best algorithm

	Speedup	
Workload	**Lock Based BST**	**Lock Free BST**
Read-Dominated	46%	27%
Mixed	33%	22%
Write-Dominated	26%	13%

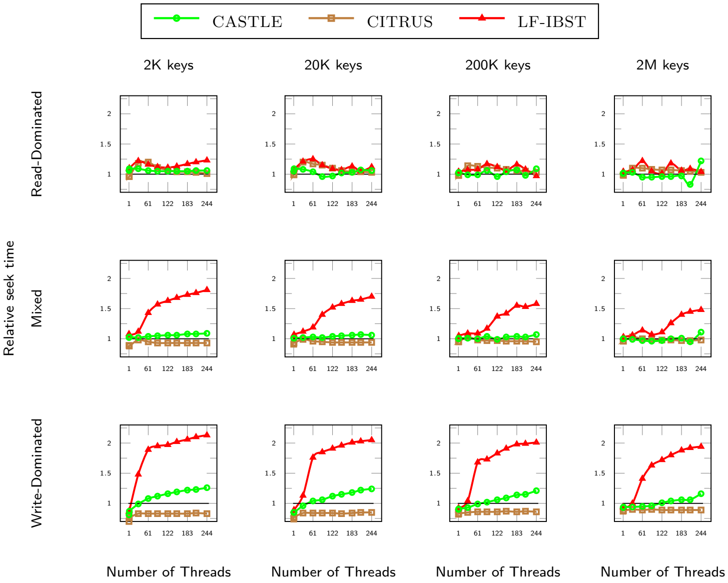
Local recovery

- helpful only for high contention cases
- uniform distribution usually causes less contention
- zipf distribution (a power-law distribution) causes high contention
- experiments run on a 61 core coprocessor
- 4 hardware threads per core - 244 total threads

Local recovery - Throughput - relative



Local recovery - Seek Time - relative



Future Work

- analyze our local recovery algorithm (amortized time complexity)
- develop concurrent K-ary BST which can improve spatial locality
- work on other data structures like tries, bloom filters, etc.
- evaluate using real workloads.

Thank you