Concurrent Binary Search Trees Design and Optimizations



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Overview

Introduction

Design Approaches

Linearizability

Binary Search Tree

Related Works

Lock Based Binary Search Tree

Lock Free Binary Search Tree

Local recovery

Wait free search

Experimental Evaluation

Future Work

Introduction

- ► CPUs aren't getting faster (memory wall, ILP wall and power wall)
- ► Shift towards multicore and manycore

Problem

How to keep all the cores busy?

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Solutions
Parallel computing

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Solutions
Parallel computing
Concurrent computing

Concurrency vs Parallelism

Concurrency is not parallelism (it's better!!)

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Parallel Computing

- decades of research done
- Example Matrix-Matrix Multiplication
- **do** lot of things simultaneously
- cannot be done on a single CPU
- deterministic control flow
- is about speedup
- hard to debug

Concurrent Computing

- Relatively new
- Example A web crawler, mouse/keyboard
- deal lot of things simultaneously
- can be done on a single CPU
- non-deterministic control flow
- ▶ is about hiding latency
- very hard to debug

Designing Concurrent Data Structures

- Shared-memory multiprocessors concurrently execute multiple threads
- ► Threads communicate and synchronize through data structures in shared memory
- ▶ Threads can interleave in exponential number of ways
- Concurrent data structure must preserve its properties for all possible interleavings

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fetchAndIncrement: sequential fetchAndIncrement: Using locks

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fetchAndIncrement: sequential

fetchAndIncrement: Using locks

```
repeat

| r0ld = x;

| rNew = r0ld+1;

until (x.compareAndSwap(rOld,rNew));
```

fetchAndIncrement: using atomic instructions

compareAndSwap updates(atomically) the value of x to rNew only if the read value of x is equal to rOld. Returns true if it succeeds in updating the value of

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▶ Blocking Algorithms

Non-Blocking Algorithms

How to handle contention among threads?

- Blocking Algorithms
 - use locks to resolve contention
 - coarse grained or fine grained locking
 - easier to design
 - weaker progress guarantees (thread owns a lock)
 - are prone to deadlock, priority inversion
- Non-Blocking Algorithms

How to handle contention among threads?

Blocking Algorithms

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Non-Blocking Algorithms

- use atomic (Read-Modify-Write) instructions to resolve contention. E.g. Compare-And-Swap(CAS) instruction
- lock-free or wait-free
- stronger progress guarantees (operation owns a lock helping)
- deadlock or priority inversion not possible
- harder to design

Linearizability

Linearizability requires two properties:

- the object (or data structure) be sequentially consistent¹
- the total ordering which makes it sequentially consistent respect the real-time ordering among the operations in the execution

respecting real-time ordering - if an operation op_1 completed before another operation op_2 , then op_1 must be ordered before op_2

Linearizability

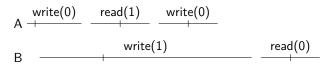
- linearization point a distinct point between a method invokation and response where the method appears to have taken effect instantaneously
- ▶ Order the method calls based on their linearization points
- Resulting order should be in the sequential specification of the object.

Linearizability - Examples

$$\begin{array}{ccc} A & \frac{\mathsf{read}(1)}{} & & \mathsf{write}(2) & & \mathsf{read}(2) \end{array}$$

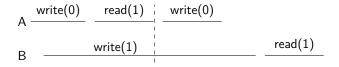
A history of a sequential object

Linearizability - Examples



A history of a concurrent object - linearizable

Linearizability - Examples



A history of a concurrent object - not linearizable

Binary Search Tree - Defintion

A binary search tree (BST) is a data structure which meets the following requirements:

- ▶ it is a binary tree (a node can contain atmost two children)
- each node contains a key k
- ▶ left subtree of a node contains keys lesser than *k*
- ▶ right subtree of a node contains keys greater than k

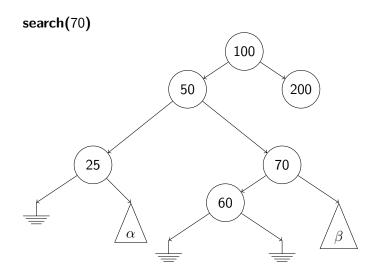
Binary Search Tree - Defintion

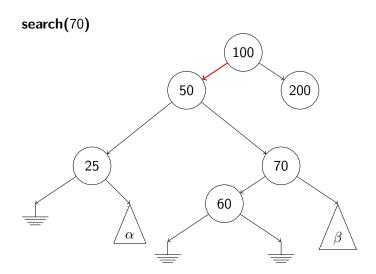
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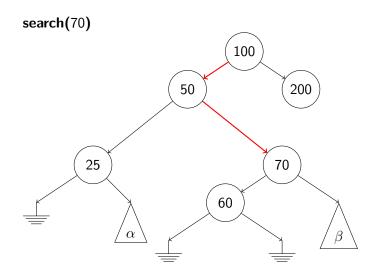
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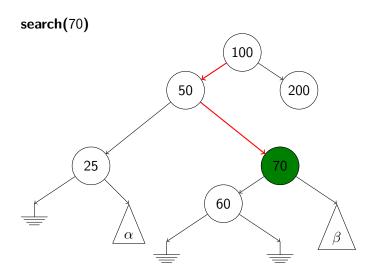
Operations on a BST

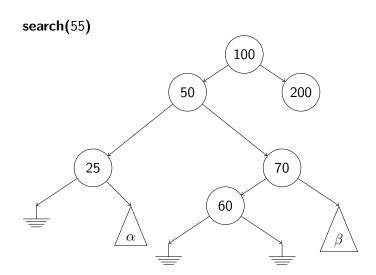
- **search**(k) returns *true* only if key k is present in the tree
- ▶ insert(k) inserts k into the tree if it does not already exist
- delete(k) deletes k from the tree if it already exist

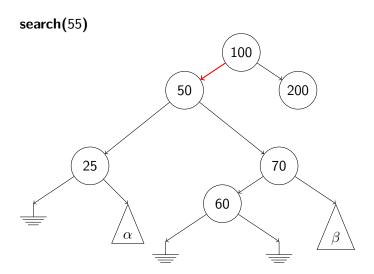


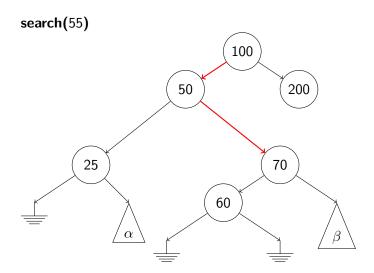


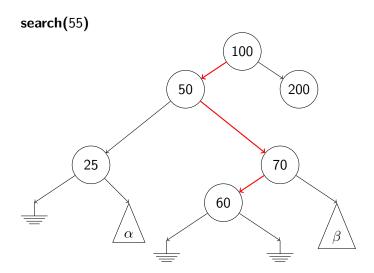


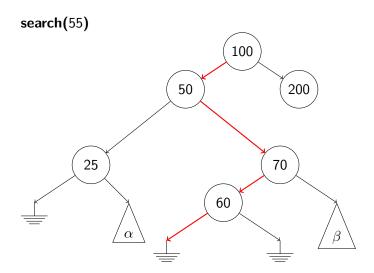


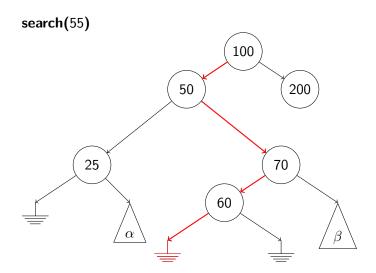






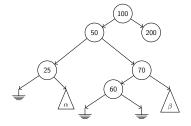






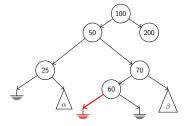
BST - Insert

insert(55)



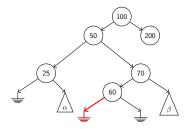
BST - Insert

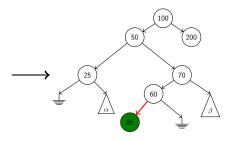
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BST - Insert

insert(55)



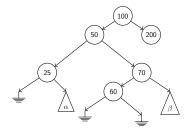


Types of delete

- simple removing a node which has atmost one child
- complex removing a node which has exactly two children

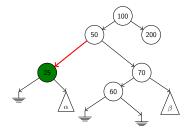
BST - Simple Delete

delete(25)



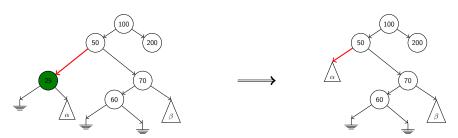
BST - Simple Delete

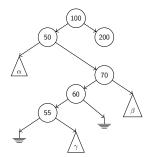
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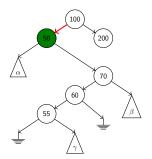


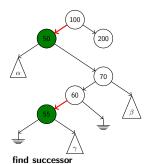
BST - Simple Delete

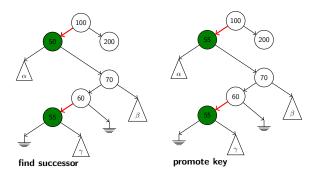
delete(25)

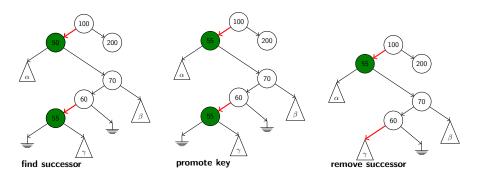












Related Works

| # | Algorithm Type | Works At | BST Type | Authors |
|---|-------------------|-------------|-------------|-----------------------------|
| 1 | lock free | node level | external | Ellen et.al[PODC'10] |
| 2 | lock free | node level | internal | Howley & Jones[SPAA'12] |
| 3 | lock free | edge level | external | Natarajan &Mittal[PPoPP'14] |
| 4 | lock based | node level | internal | Arbel & Attiya[PODC'14] |
| 5 | lock based | node level | internal | Drachsler et.al[PPoPP'14] |

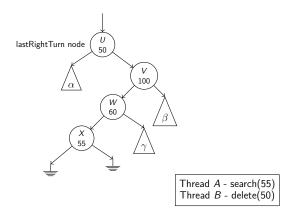
Contributions

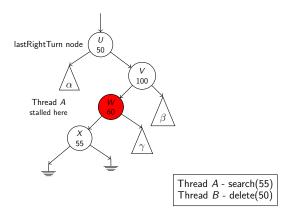
- combine edge-based locking with internal representation of BST
- optimistic tree traversal

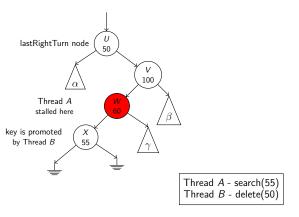
- common workloads have more searches than updates
 - design is optimized for searches
 - search operations are oblivious to locks

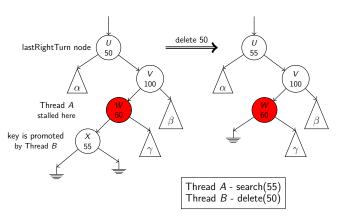
- common workloads have more searches than updates
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- ► Any real life workload will have more inserts than deletes
 - insert operations do not obtain any locks
 - performs only one atomic operation

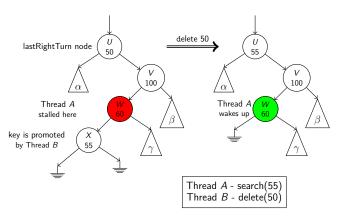
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- Any real life workload will have more inserts than deletes
 - insert operations do not obtain any locks
 - performs only one atomic operation
- removal of a node in a concurrent BST is challenging
 - delete operations uses locks
 - locks can be obtained on nodes or edges
 - locking edges instead of nodes increases concurrency

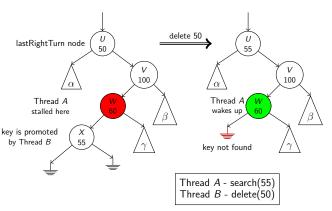




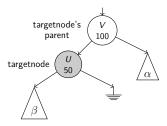


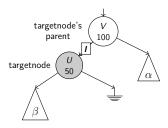


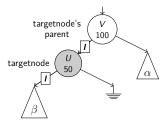


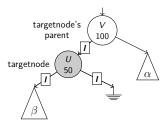


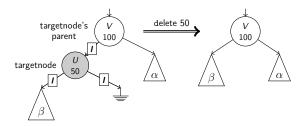
Keep track of last right turn node and its key. If search terminates at a NULL node, check if the current key in the last right turn node has changed. If yes restart the operation from root.

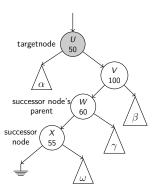


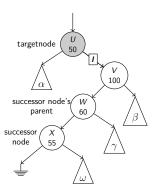


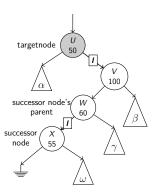


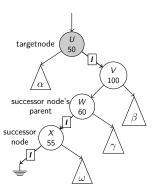


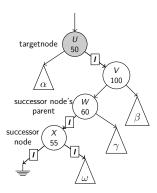


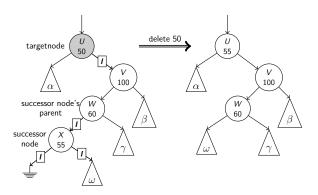




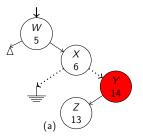






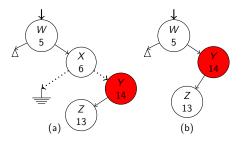


A scenario in which the last right turn node is removed



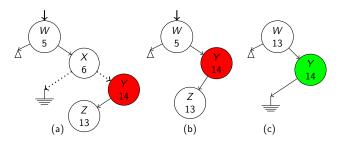
ightharpoonup Search(13) gets stalled at Y in (a). Its last right turn node is X

A scenario in which the last right turn node is removed



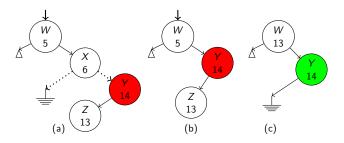
▶ Delete(6) removes *X* from the tree in (b). The key stored in *X* is still 6

A scenario in which the last right turn node is removed



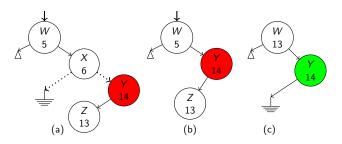
▶ Delete(5) results in 13 moving up the tree from Z to W in (c). When search(13) wakes up, it will miss 13 as the key in the last right turn node has not changed

A scenario in which the last right turn node is removed



▶ In the first traversal search(13) saw the node X

A scenario in which the last right turn node is removed



- ▶ In the second traversal there are two cases
 - case1, search(13) did not find X save the traversal and restart
 - case2, search(13) did find X use the results of previous traversal

Lock Free BST[ICDCN'15]

Contributions

- combine edge-based locking with internal representation of BST
- optimistic tree traversal

Lock Free BST[ICDCN'15]

Contributions

- combine edge-based locking with internal representation of BST
- optimistic tree traversal
- ▶ lock-free algorithm

Lock Free BST[ICDCN'15]

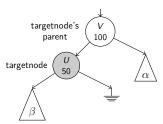
- search and inserts are same as in lock Based BST
- ► to maintain lock-free property, if an insert or delete operation fails, it helps a pending delete operation(if needed)

pseudocode for delete

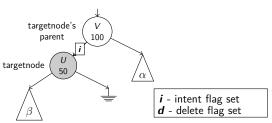
```
locate the node to delete;
flag the children edges for deletion;
if simple delete then
| make the parent point to the non-null child atomically;
else // complex delete
| find the successor;
  flag the children edges of successor for promotion;
  promote key;
  remove successor by a simple delete;
  replace node with a fresh copy;
end
```

- flag is owned by an operation
- ▶ if a thread which installed the flag is stalled, other threads can help complete the operation

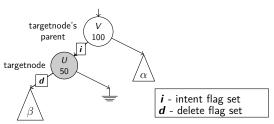
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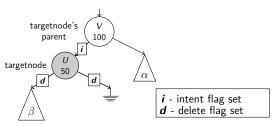
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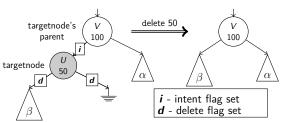
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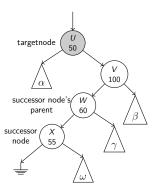


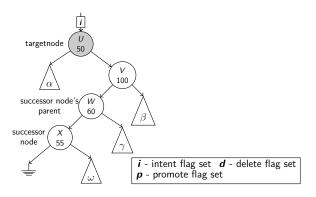
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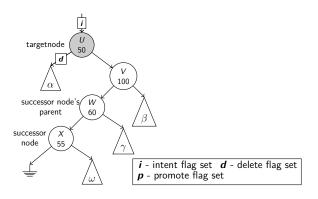


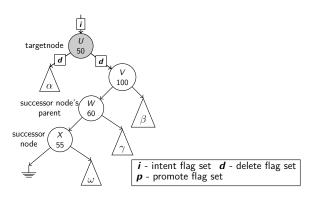
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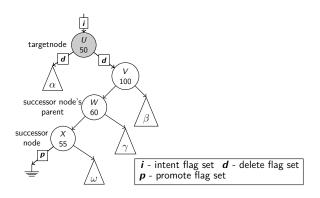


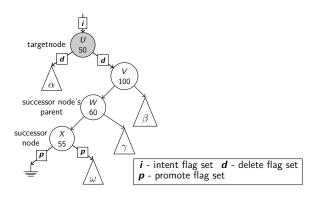


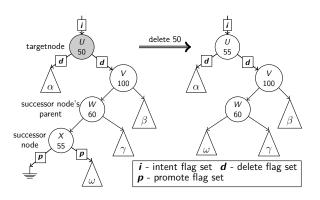


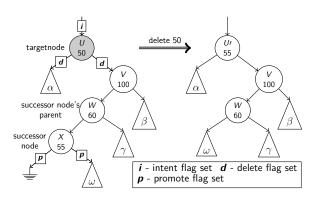












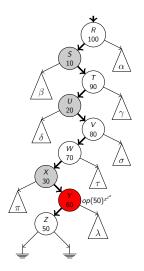
Local recovery[PPoPP'16 Poster]

Overview

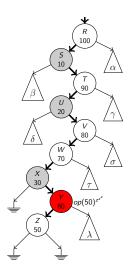
- ▶ a general technique for local recovery for concurrent BSTs
- reduces tree traversal cost during failures by restarting closer to an operation's window

Motivation

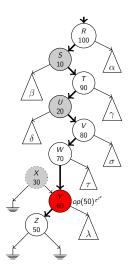
- in most concurrent BSTs, execution phase of an operation have constant time complexity
- seek phase is where an operation may end up spending most of its time (esp for large trees)
- this technique reduces the seek time



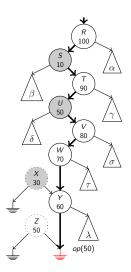
Operation op(50) is suspended at node Y during its traversal



All keys in subtree π are deleted one-by-one

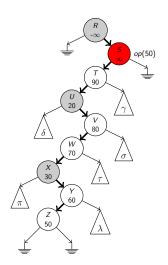


Key 30 is deleted (simple delete); node X is removed



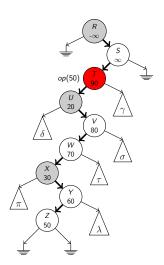
Key 20 is deleted (complex delete); key 20 is replaced with key 50 in node $\it U$ and node $\it Z$ is removed

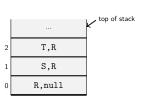
- a stack to keep track of anchor nodes of all nodes in the traversal path
- reduces tree traversal cost during failures by restarting closer to an operation's window

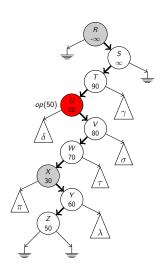


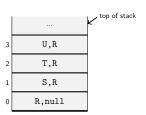


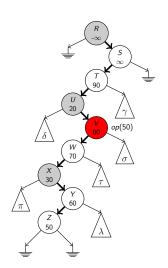
Operation op(50) starting at R and suspended at S along with the stack

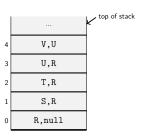


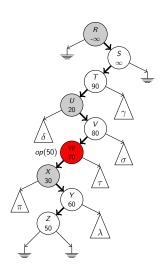


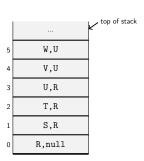


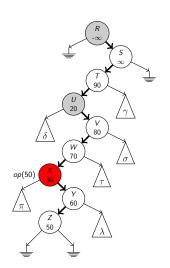


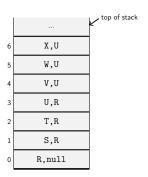


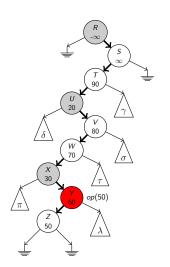


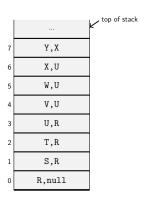


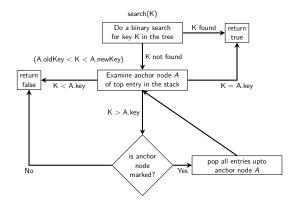




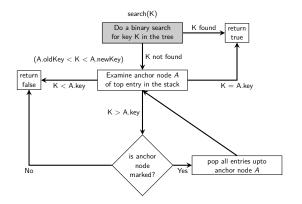




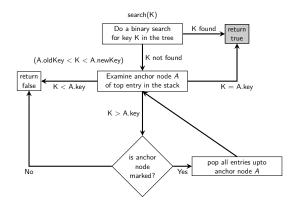




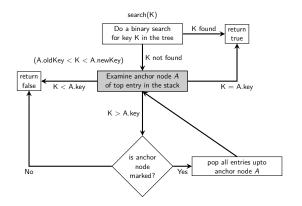
Sequence of steps in a search operation



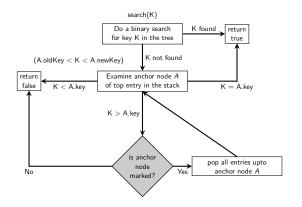
Sequence of steps in a search operation



Sequence of steps in a search operation

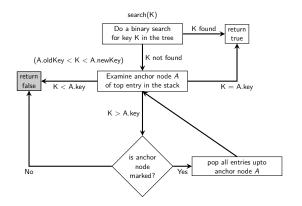


Sequence of steps in a search operation



Sequence of steps in a search operation

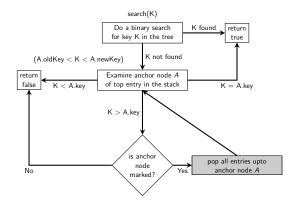
search operations do not restart



Sequence of steps in a search operation

Search

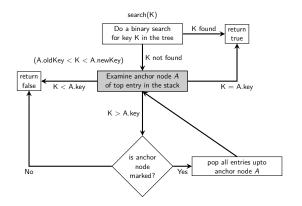
search operations do not restart



Sequence of steps in a search operation

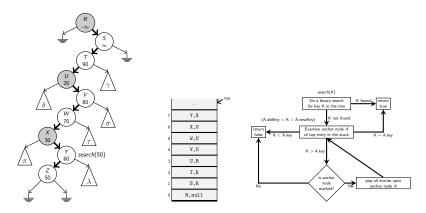
Search

search operations do not restart



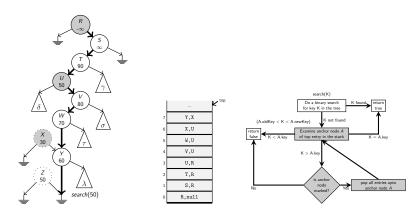
Sequence of steps in a search operation

Search - consistent anchors



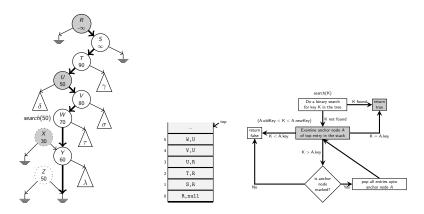
Operation search(50) starting at R and suspended at Y along with the stack

Search - consistent anchors



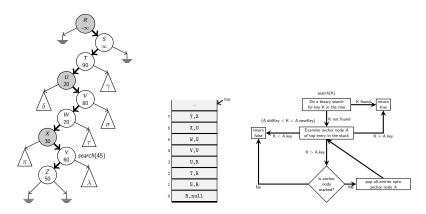
Key 30 is deleted; key 20 is deleted & replaced with key 50 in node ${\it U}$ and node ${\it Z}$ is removed

Search - consistent anchors



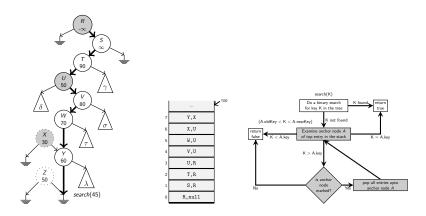
Pop upto marked anchor node X. Top of stack is now W. Examine anchor node ${\cal U}$

Search - inconsistent anchor



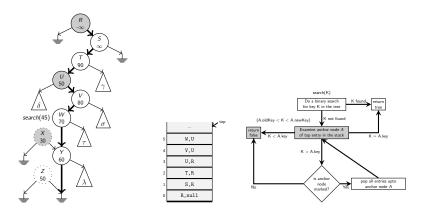
Operation search(45) starting at R and suspended at Y along with the stack

Search - inconsistent anchor



Key 30 is deleted; key 20 is deleted & replaced with key 50 in node $\it U$ and node $\it Z$ is removed

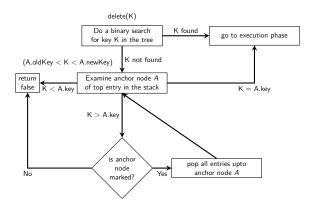
Search - inconsistent anchor



Pop upto marked anchor node X. Top of stack is now W. Examine anchor node U. A.oldKey(20) < K(45) < A.newKey(50). Inconsistent anchor

Delete

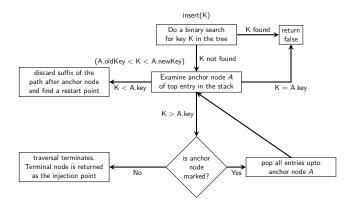
A delete operation do not restart except when there is a failure in the execution phase



Sequence of steps in a delete operation

Insert

An insert operation needs to restart only if one of the anchor nodes in the path has become inconsistent



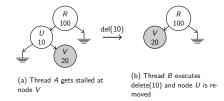
Sequence of steps in an insert operation

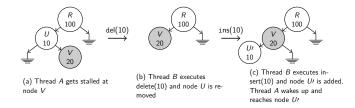
wait-free - every thread is able to complete its operations in a finite number of steps over an infinite period of time

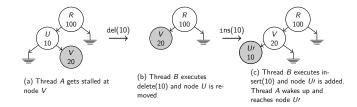
- two light-weight techniques to make search operations for concurrent internal BSTs, wait-free
- low additional overhead
- no write instructions on share memory
- minimizes cache traffic

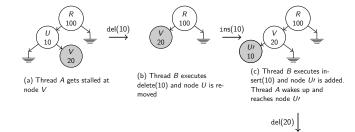


(a) Thread A gets stalled at node V



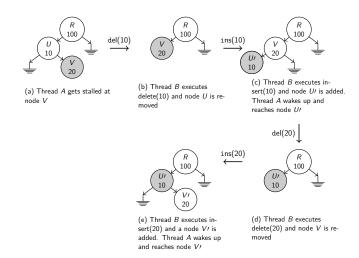


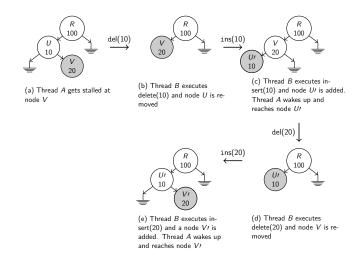




(d) Thread *B* executes delete(20) and node *V* is removed

100





- as long as a key is continuously present in the tree, its distance from root is monotonically non-increasing
- if a key is not found after visiting a "certain" number of nodes in the tree, then traversal stops
- sufficient to examine the path traversed to check whether or not the key has moved up
- ▶ In case the key is not continuously present in the tree, it is acceptable to return either:
 - present linearized after the insert operation that added the key to the tree
 - not present linearized after the delete operation that removed the key from the tree

when to stop?

when to stop?

Each process maintains two counters:

- insert counter number of true inserts
- delete counter number of true deletes

IC[i] and DC[i] denote the number of insert and delete operations, respectively, process P_i has performed so far

- ▶ insert counter incremented before adding a key
- delete counter incremented before removing a key
- insert (delete) counter at a process is an upper (lower) bound on the number of keys that the process has added to (removed from) the tree

```
read and aggregate delete counter values of all processes DC = \sum_{i=1}^{p} DC[i]; read and aggregate insert counter values of all processes IC = \sum_{i=1}^{p} IC[i]; IC - DC \ge actual tree size as IC \le actual inserts and DC \ge actual deletes;
```

pseudocode: waitFreeSearch

 ${\it IC-DC}$ gives an upper bound on number of keys to traverse before stopping the search operation

With Modification to Tree Node

- previous approach time complexity depends on tree size
- this approach time complexity depends on the tree height
- but needs modifications to tree node structure
- each node has a timestamp on when it was created
- timestamp \(\rangle \text{process id, process sequence number} \rangle \)
- process sequence number is incremented before a node is added to the tree

With Modification to Tree Node

```
read current sequence number of all processes; let label[i] denote the sequence number of proceess p_i; stop the downward traversal of the tree once a node with timestamp \langle i, v \rangle such that v > labels[i] is encountered;
```

pseudocode: waitFreeSearch

Experimental Setup

To compare the performance of various concurrent BSTs we considered the following parameters:

- Maximum Tree Size
 - ▶ key space size varied from 2¹³ (8Ki) to 2²⁴ (16Mi).
- Relative Distribution of Operations
 - ▶ Read-Dominated (90% search, 9% insert and 1% delete)
 - ► Mixed (70% search, 20% insert and 10% delete)
 - ▶ Write-Dominated (0% search, 50% insert and 50% delete)
- Maximum degree of Contention
 - number of threads that can concurrently operate on the tree
 - we collected data for 32 threads

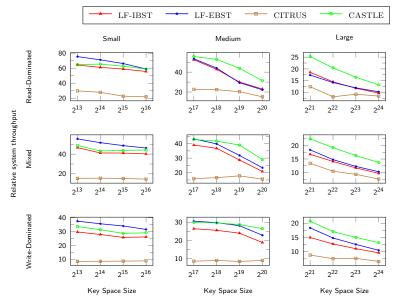
Experimental Setup

- ► Throughput computed as millions of operations per second (MOPS)
- each trial was run for 2 minutes
- Average over 5 trials
- pre-populated the tree to 50% of its maximum size to capture steady state behaviour
- beginning of each run consisted of a 1 second "warm-up" phase whose numbers were excluded in the computed statistics to avoid initial caching effects
- ▶ The machine we used is a Dell PowerEdge R820 server with 4 Intel E5-4650 @ 2.70GHz 8-core processors (32 cores in total) and 1TB of DDR3 memory with HT disabled. 256KB L2 and 20MB shared L3

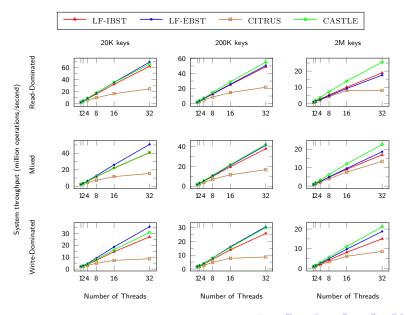
Other Concurrent BSTs

- a lock-free internal BST by Howley and Jones[SPAA'12], denoted by HJ-BST
- a lock-free external BST by Natarajan and Mittal[PPoPP'14], denoted by NM-BST
- RCU-based internal BST by Arbel and Attiya[PODC'14], denoted by CITRUS

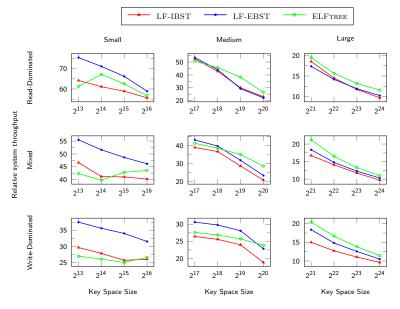
Lock Based BST - key sweep - absolute



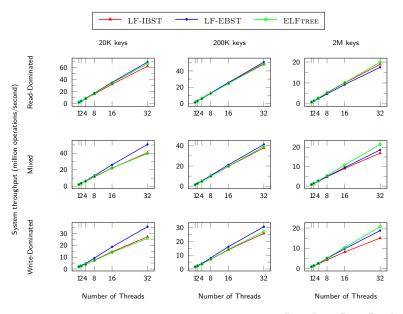
Lock Based BST - thread sweep - absolute



Lock Free BST - key sweep - absolute



Lock Free BST - thread sweep - absolute



Results Summary

Comparison of different concurrent BSTs in the absence of contention

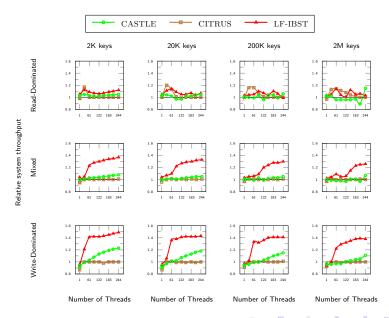
speedup is calculated over the second best algorithm

| | Speedup | |
|-----------------|----------------|---------------|
| Workload | Lock Based BST | Lock Free BST |
| Read-Dominated | 46% | 27% |
| Mixed | 33% | 22% |
| Write-Dominated | 26% | 13% |

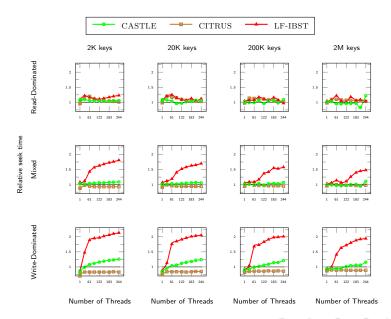
Local recovery

- helpful only for high contention cases
- uniform distribution usually causes less contention
- zipf distribution (a power-law distribution) causes high contention
- experiments run on a 61 core coprocessor
- 4 hardware threads per core 244 total threads

Local recovery - Throughput - relative



Local recovery - Seek Time - relative



Future Work

- analyze our local recovery algorithm (amortized time complexity)
- develop concurrent K-ary BST which can improve spatial locality
- work on other data structures like tries, bloom filters, etc.
- evaluate using real workloads.

Thank you