# Concurrent Binary Search Trees Design and Optimizations



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#### Design

Lock Based Binary Search Tree Lock Free Binary Search Tree

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**Evaluation** 

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**Evaluation** 

**Future Work** 

#### Introduction

- CPUs aren't getting faster (memory wall, ILP wall and power wall)
- Shift towards multicore and manycore

#### **Problem**

How to keep all the cores **busy**?

#### Introduction

- CPUs aren't getting faster (memory wall, ILP wall and power wall)
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#### **Problem**

How to keep all the cores **busy**?

Solution

Concurrent computing

# Concurrent computing

- Example A web crawler, mouse/keyboard
- deal lot of things simultaneously
- can be done on a single CPU
- non-deterministic control flow
- is about hiding latency
- very hard to debug

# Designing Concurrent Data Structures

- Shared-memory multiprocessors concurrently execute multiple threads
- ► Threads communicate and synchronize through data structures in shared memory
- ▶ Threads can interleave in exponential number of ways
- Concurrent data structure must preserve its properties for all possible interleavings

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Here are some possible implementations of this function

```
r1 = x;
inc(r1);
x = r1;
```

fetchAndIncrement: sequential

Let x be a shared counter which can be incremented using a function fetchAndIncrement()

Here are some possible implementations of this function

```
r1 = x;
inc(r1);
x = r1;

acquire(lock);
r1 = x;
inc(r1);
x = r1;
release(lock);
```

fetchAndIncrement: sequential fetchAndIncrement: Using locks

Let x be a shared counter which can be incremented using a function fetchAndIncrement()

Here are some possible implementations of this function

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r1 = x;
inc(r1);
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acquire(lock);
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release(lock);
```

fetchAndIncrement: sequential

fetchAndIncrement: Using locks

fetchAndIncrement: using atomic instructions

compareAndSwap updates(atomically) the value of x to rNew only if the read value of x is equal to rOld. Returns true if it succeeds in updating the value of

How to handle contention among threads?

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▶ Blocking Algorithms

Non-Blocking Algorithms

# How to handle contention among threads?

- Blocking Algorithms
  - use locks to resolve contention
  - coarse grained or fine grained locking
  - easier to design
  - weaker progress guarantees (locking)
  - are prone to deadlock, priority inversion
- Non-Blocking Algorithms

# How to handle contention among threads?

# Blocking Algorithms

- use locks to resolve contention
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- easier to design
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## Non-Blocking Algorithms

- use atomic (Read-Modify-Write) instructions to resolve contention. E.g. Compare-And-Swap(CAS) instruction
- lock-free or wait-free
- stronger progress guarantees (helping)
- deadlock or priority inversion not possible
- harder to design

a correctness condition for concurrent objects

#### a correctness condition for concurrent objects

- methods take time
- methods intervals with a start point (invocation) and an end point (response)
- history sequence of method invocations and responses

Linearizability gives a total ordering of a history

#### Linearizability gives a total ordering of a history

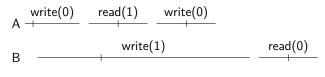
- ordering of method calls w.r.t calls on the same thread should remain unchanged
- overlapping method calls can be ordered based on the history
- non-overlapping method calls across threads should preserve real-time ordering

# Linearizability - Examples

$$\begin{array}{ccc} A & \frac{\mathsf{read}(1)}{} & & \mathsf{write}(2) & & \mathsf{read}(2) \end{array}$$

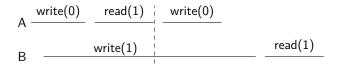
A history of a sequential object

# Linearizability - Examples



A history of a concurrent object - linearizable

# Linearizability - Examples



A history of a concurrent object - not linearizable

#### Binary Search Tree - Defintion

A binary search tree (BST) is a data structure which meets the following requirements:

- ▶ it is a binary tree (a node can contain atmost two children)
- each node contains a key k
- ▶ left subtree of a node contains keys lesser than *k*
- ▶ right subtree of a node contains keys greater than k

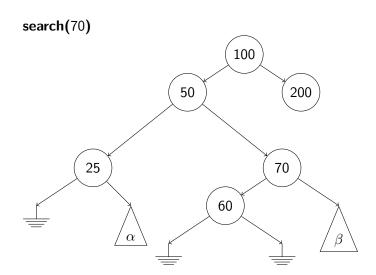
## Binary Search Tree - Defintion

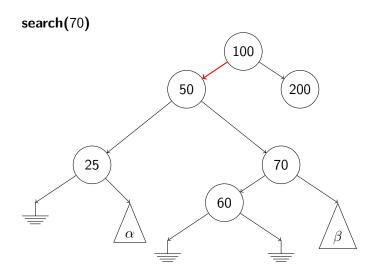
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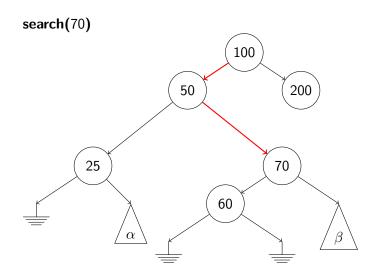
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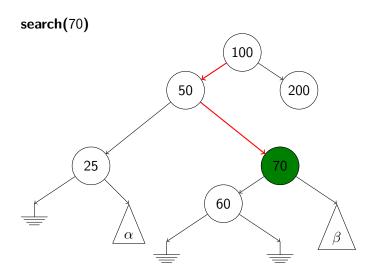
#### Operations on a BST

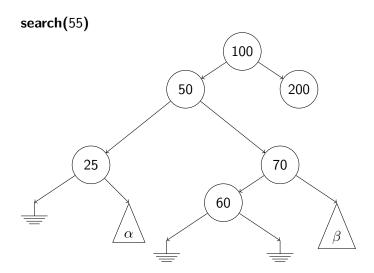
- **search**(k) returns *true* only if key k is present in the tree
- ▶ insert(k) inserts k into the tree if it does not already exist
- delete(k) deletes k from the tree if it already exist

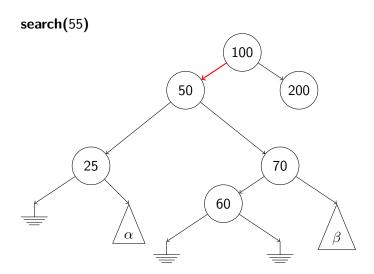


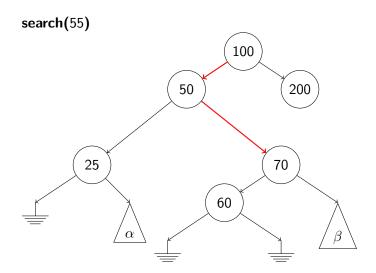


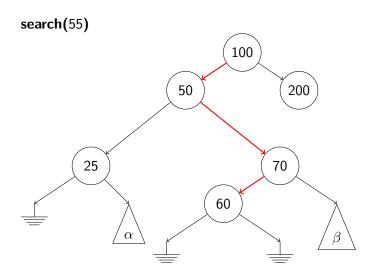


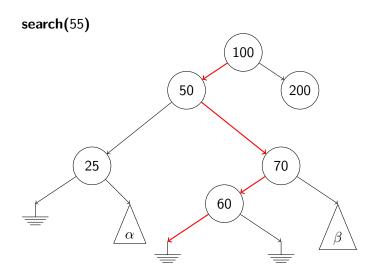


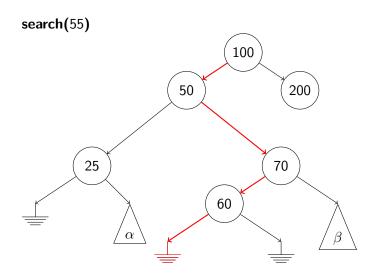






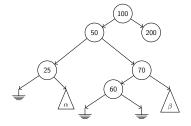






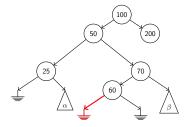
#### BST - Insert

### insert(55)



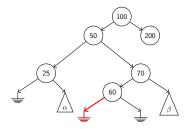
#### BST - Insert

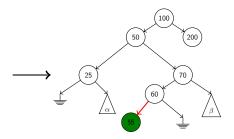
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#### BST - Insert

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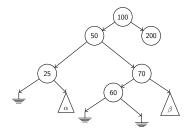


#### Types of delete

- simple removing a node which has atmost one child
- complex removing a node which has exactly two children

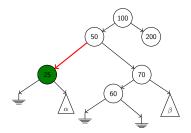
### BST - Simple Delete

### delete(25)



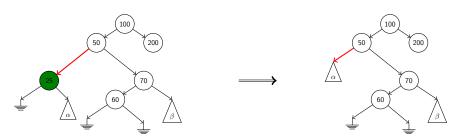
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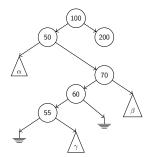
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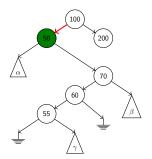


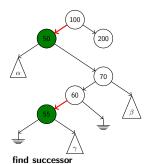
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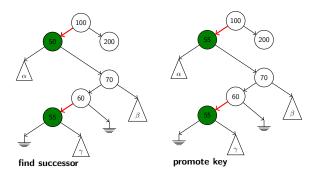
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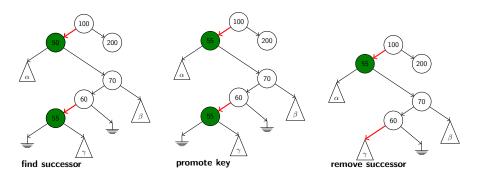












#### Related Works

#	Algorithm Type	Works At	BST Type	Authors
1	lock free	node level	external	Ellen et.al[PODC'10]
2	lock free	node level	internal	Howley & Jones[SPAA'12]
3	lock free	edge level	external	Natarajan &Mittal[PPoPP'14]
4	lock based	node level	internal	Arbel & Attiya[PODC'14]
5	lock based	node level	internal	Drachsler et.al[PPoPP'14]

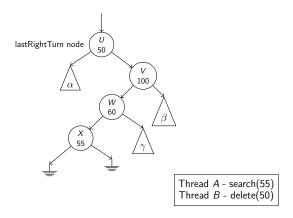
#### Contributions

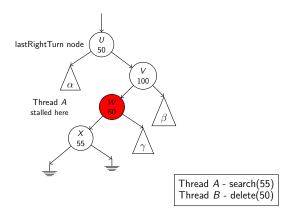
- combine edge-based locking with internal representation of BST
- optimistic tree traversal

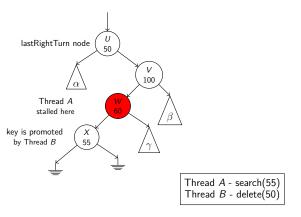
- common workloads have more searches than updates
  - design is optimized for searches
  - search operations are oblivious to locks

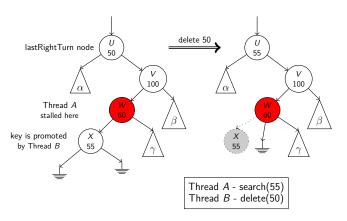
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- ▶ Any real life workload will have more inserts than deletes
  - insert operations do not obtain any locks
  - performs only one atomic operation

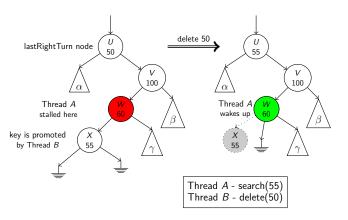
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- ▶ Any real life workload will have more inserts than deletes
  - insert operations do not obtain any locks
  - performs only one atomic operation
- removal of a node in a concurrent BST is challenging
  - delete operations uses locks
  - locks can be obtained on nodes or edges
  - locking edges instead of nodes increases concurrency

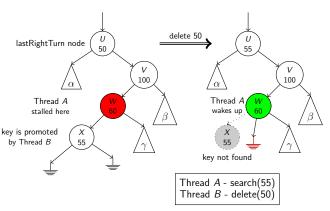




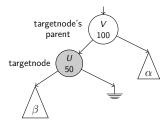


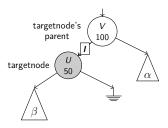


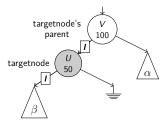


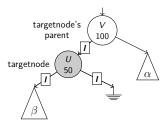


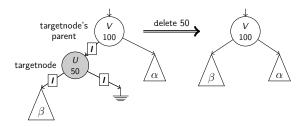
Keep track of last right turn node and its key. If search terminates at a NULL node, check if the current key in the last right turn node has changed. If yes restart the operation from root.

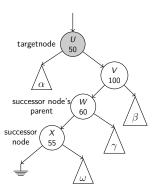


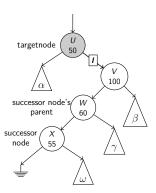


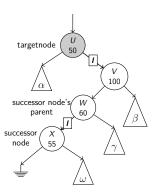


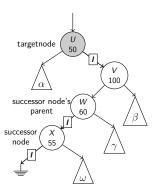




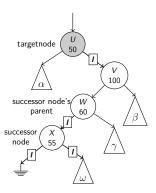




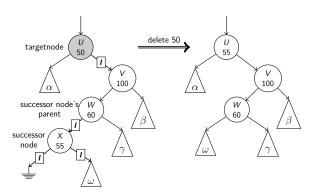




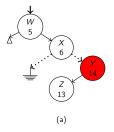
## Lock Based BST - Complex Delete



# Lock Based BST - Complex Delete

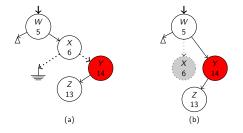


A scenario in which the last right turn node is removed



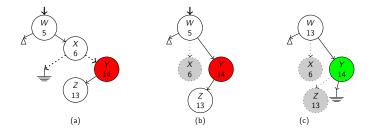
▶ Search(13) gets stalled at Y in (a). Its last right turn node is X

A scenario in which the last right turn node is removed



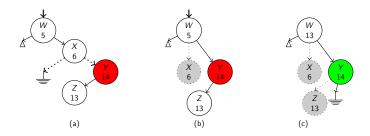
▶ Delete(6) removes X from the tree in (b). The key stored in X is still 6

A scenario in which the last right turn node is removed



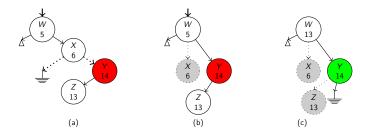
▶ Delete(5) results in 13 moving up the tree from Z to W in (c). When search(13) wakes up, it will miss 13 as the key in the last right turn node has not changed

A scenario in which the last right turn node is removed



▶ In the first traversal search(13) saw the node X

A scenario in which the last right turn node is removed



- In the second traversal there are two cases
  - case1, search(13) did not find X save the traversal and restart
  - case2, search(13) did find X use the results of previous traversal

## Lock Free BST[ICDCN'15]

#### Contributions

- combine edge-based locking with internal representation of BST
- optimistic tree traversal

## Lock Free BST[ICDCN'15]

#### Contributions

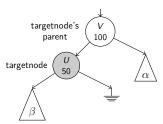
- combine edge-based locking with internal representation of BST
- optimistic tree traversal
- ▶ lock-free algorithm

#### Lock Free BST[ICDCN'15]

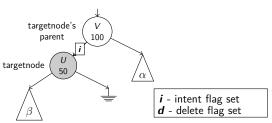
- search and inserts are same as in lock Based BST
- to maintain lock-free property, if an insert or delete operation fails, it helps a pending delete operation(if needed)

- flag is owned by an operation
- ▶ if a thread which installed the flag is stalled, other threads can help complete the operation

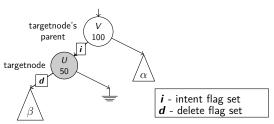
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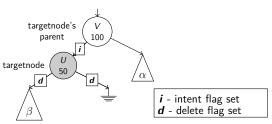
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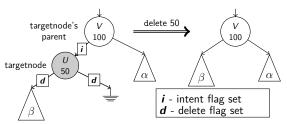
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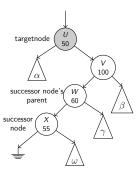


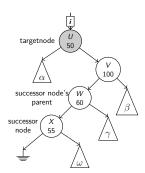
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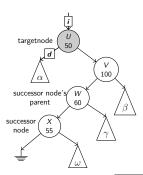
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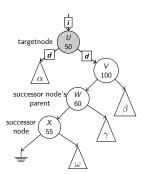




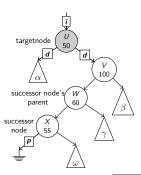
i - intent flag set d - delete flag set
 p - promote flag set



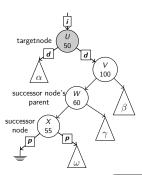
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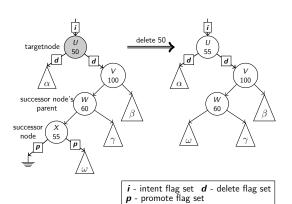
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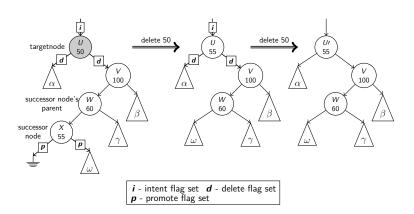


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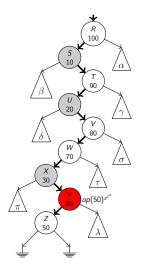
## Local recovery[PPoPP'16 Poster]

#### Overview

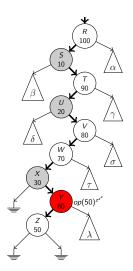
- ▶ a general technique for local recovery for concurrent BSTs
- reduces tree traversal cost during failures by restarting closer to an operation's window

#### Motivation

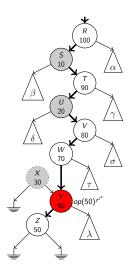
- in most concurrent BSTs, execution phase of an operation have constant time complexity
- seek phase is where an operation may end up spending most of its time (esp for large trees)
- this technique reduces the seek time



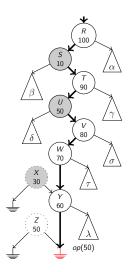
Operation op(50) is suspended at node Y during its traversal



All keys in subtree  $\pi$  are deleted one-by-one

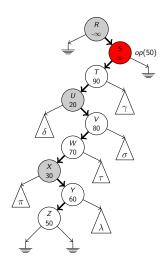


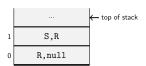
Key 30 is deleted (simple delete); node X is removed



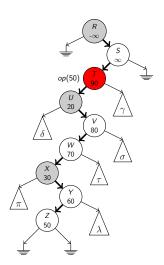
Key 20 is deleted (complex delete); key 20 is replaced with key 50 in node  $\it U$  and node  $\it Z$  is removed

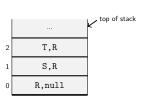
- a stack to keep track of anchor nodes of all nodes in the traversal path
- reduces tree traversal cost during failures by restarting closer to an operation's window

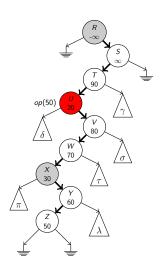


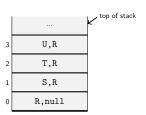


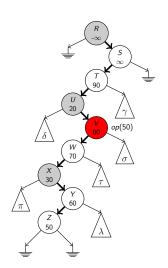
Operation op(50) starting at R and suspended at S along with the stack

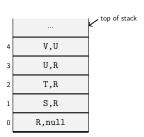


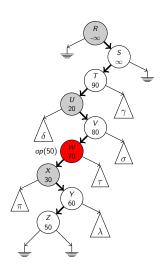


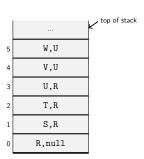


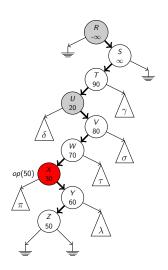


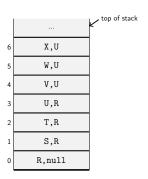




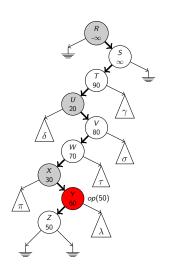


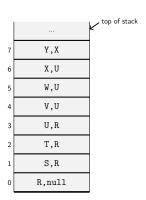


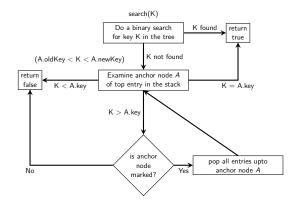




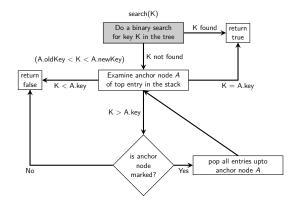
## Traversal Stack



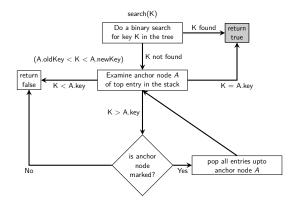




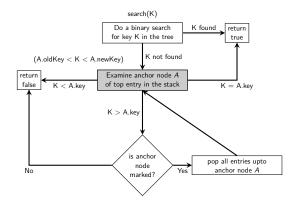
Sequence of steps in a search operation



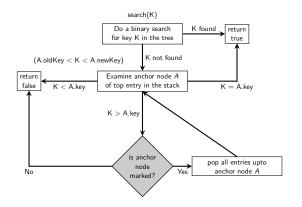
Sequence of steps in a search operation



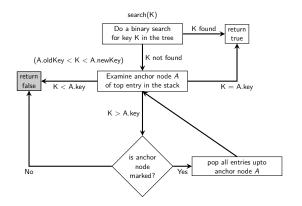
Sequence of steps in a search operation



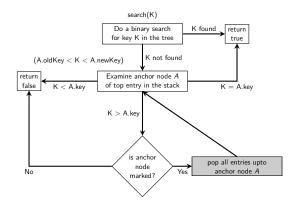
Sequence of steps in a search operation



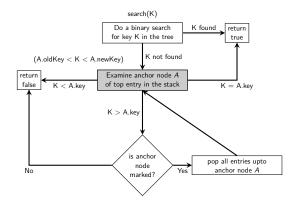
Sequence of steps in a search operation



Sequence of steps in a search operation

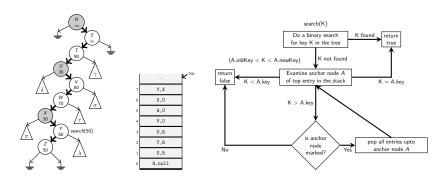


Sequence of steps in a search operation



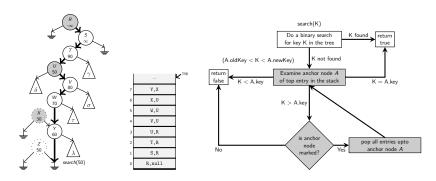
Sequence of steps in a search operation

#### Search - consistent anchors



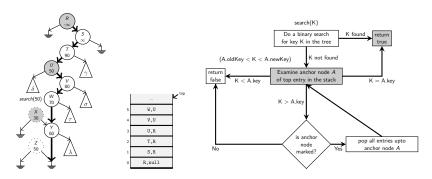
Operation search(50) starting at R and suspended at Y along with the stack

#### Search - consistent anchors



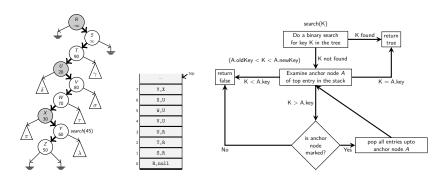
Key 30 is deleted; key 20 is deleted & replaced with key 50 in node  $\it U$  and node  $\it Z$  is removed

#### Search - consistent anchors



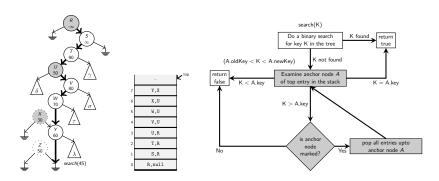
Pop upto marked anchor node X. Top of stack is now W. Examine anchor node  ${\it U}$ 

#### Search - inconsistent anchor



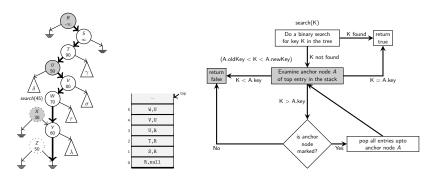
Operation search(45) starting at R and suspended at Y along with the stack

#### Search - inconsistent anchor



Key 30 is deleted; key 20 is deleted & replaced with key 50 in node  $\it U$  and node  $\it Z$  is removed

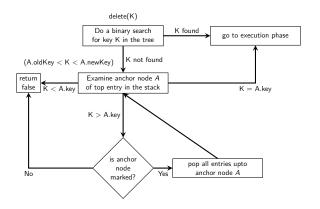
#### Search - inconsistent anchor



Pop upto marked anchor node X. Top of stack is now W. Examine anchor node U. A.oldKey(20) < K(45) < A.newKey(50). Inconsistent anchor

#### Delete

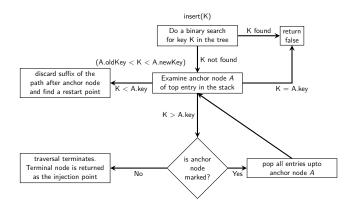
A delete operation do not restart except when there is a failure in the execution phase



Sequence of steps in a delete operation

#### Insert

An insert operation needs to restart only if one of the anchor nodes in the path has become inconsistent



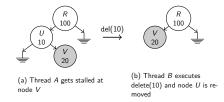
Sequence of steps in an insert operation

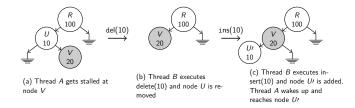
wait-free - every thread is able to complete its operations in a finite number of steps over an infinite period of time

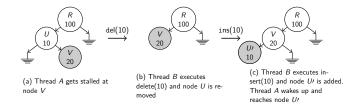
- two light-weight techniques to make search operations for concurrent internal BSTs, wait-free
- low additional overhead
- no write instructions on share memory
- minimizes cache traffic

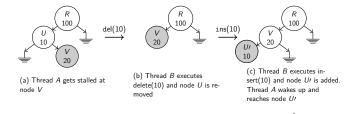


(a) Thread  $\boldsymbol{A}$  gets stalled at node  $\boldsymbol{V}$ 



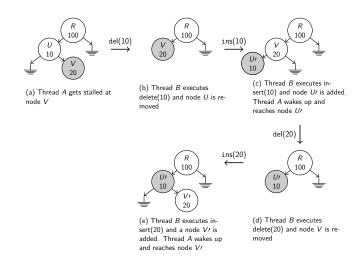


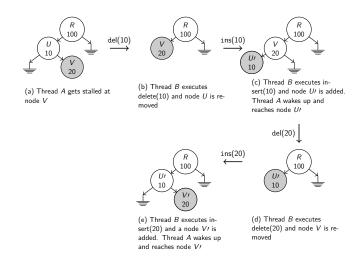






(d) Thread B executes delete(20) and node V is removed





- as long as a key is continuously present in the tree, its distance from root is monotonically non-increasing
- if a key is not found after visiting a "certain" number of nodes in the tree, then traversal stops
- sufficient to examine the path traversed to check whether or not the key has moved up
- ▶ In case the key is not continuously present in the tree, it is acceptable to return either:
  - present linearized after the insert operation that added the key to the tree
  - not present linearized after the delete operation that removed the key from the tree

when to stop?

#### when to stop?

Each process maintains two counters:

- insert counter number of true inserts
- delete counter number of true deletes

IC[i] and DC[i] denote the number of insert and delete operations, respectively, process  $P_i$  has performed so far

- ▶ insert counter incremented before adding a key
- delete counter incremented before removing a key
- insert (delete) counter at a process is an upper (lower) bound on the number of keys that the process has added to (removed from) the tree

```
read and aggregate delete counter values of all processes DC = \sum_{i=1}^{p} DC[i]; read and aggregate insert counter values of all processes IC = \sum_{i=1}^{p} IC[i]; IC - DC \ge actual tree size as IC \ge actual inserts and DC \ge actual deletes;
```

pseudocode: waitFreeSearch

IC-DC gives an upper bound on number of keys to traverse before stopping the search operation

#### With Modification to Tree Node

- previous approach time complexity depends on tree size
- this approach time complexity depends on the tree height
- but needs modifications to tree node structure
- each node has a timestamp on when it was created
- timestamp \( \rangle \text{process id, process sequence number} \rangle \)
- process sequence number is incremented before a node is added to the tree

#### With Modification to Tree Node

```
read current sequence number of all processes; let label[i] denote the sequence number of proceess p_i; stop the downward traversal of the tree once a node with timestamp \langle i, v \rangle such that v > labels[i] is encountered;
```

pseudocode: waitFreeSearch

## Experimental Setup

To compare the performance of various concurrent BSTs we considered the following parameters:

- Maximum Tree Size
  - ▶ key space size varied from 2<sup>13</sup> (8Ki) to 2<sup>24</sup> (16Mi).
- Relative Distribution of Operations
  - ▶ Read-Dominated (90% search, 9% insert and 1% delete)
  - ► Mixed (70% search, 20% insert and 10% delete)
  - ▶ Write-Dominated ( 0% search, 50% insert and 50% delete)
- Maximum degree of Contention
  - number of threads that can concurrently operate on the tree
  - we collected data for 32 threads

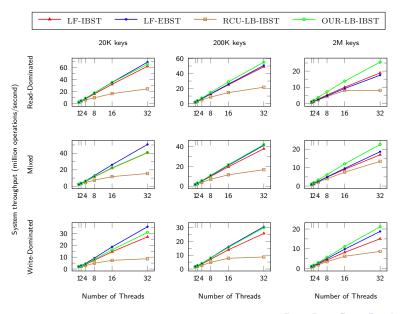
## Experimental Setup

- ► Throughput computed as millions of operations per second (MOPS)
- each trial was run for 2 minutes
- Average over 5 trials
- pre-populated the tree to 50% of its maximum size to capture steady state behaviour
- beginning of each run consisted of a 1 second "warm-up" phase whose numbers were excluded in the computed statistics to avoid initial caching effects
- ▶ The machine we used is a Dell PowerEdge R820 server with 4 Intel E5-4650 @ 2.70GHz 8-core processors (32 cores in total) and 1TB of DDR3 memory with HT disabled. 256KB L2 and 20MB shared L3

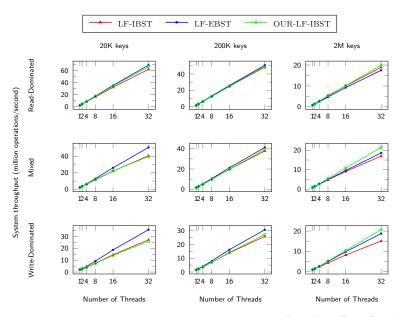
#### Other Concurrent BSTs

- ▶ a lock-free internal BST by Howley and Jones[SPAA'12], denoted by LF-IBST
- a lock-free external BST by Natarajan and Mittal[PPoPP'14], denoted by LF-EBST
- RCU-based internal BST by Arbel and Attiya[PODC'14], denoted by RCU-LB-IBST

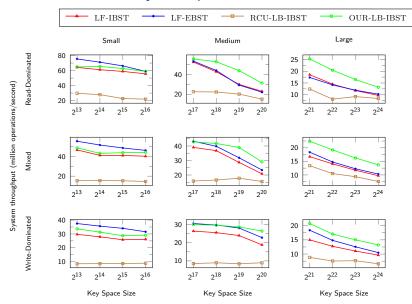
## Lock Based BST - thread sweep



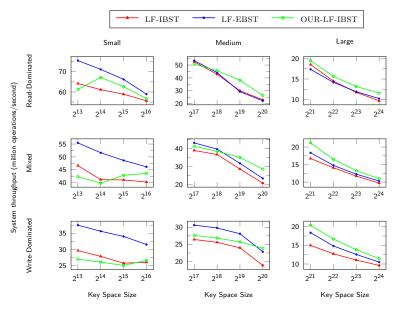
# Lock Free BST - thread sweep



## Lock Based BST - key sweep



## Lock Free BST - key sweep



## Results Summary

Comparison of different concurrent BSTs in the absence of contention

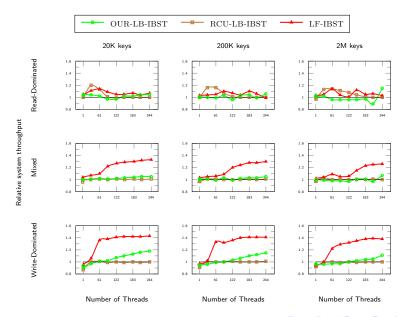
speedup is calculated over the second best algorithm

	Speedup	
Workload	Lock Based BST	Lock Free BST
Read-Dominated	46%	27%
Mixed	33%	22%
Write-Dominated	26%	13%

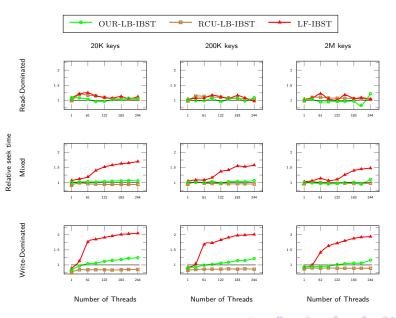
## Local recovery

- helpful only for high contention cases
- uniform distribution usually causes less contention
- zipf distribution (a power-law distribution) causes high contention
- experiments run on a 61 core coprocessor
- 4 hardware threads per core 244 total threads

## Local recovery - Throughput - relative



## Local recovery - Seek Time - relative



#### Future Work

- analyze our local recovery algorithm (amortized time complexity)
- develop concurrent K-ary BST which can improve spatial locality
- work on other data structures like tries, bloom filters, etc.
- evaluate using real workloads.

# Thank you