Fast and Furi-hash: An A-Unification–Enhanced Framework for Automated Discovery of Hash-Based Protocol Attacks

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***Abstract*—Building upon the foundational work of Cheval et al. in *Hash Gone Bad: Automated discovery of protocol attacks that exploit hash function weaknesses* [1], this work focuses on the systematic analysis and exploitation of hash-function weaknesses in modern cryptographic protocols. Our goal is to extend the scope of automated discovery frameworks to more accurately model and reason about the real algebraic properties of practical hash constructions. Building on established methodologies for automated vulnerability analysis, we investigate how real-world weaknesses—such as collision susceptibility, length-extension behavior, and failures in preimage resistance—can undermine the security guarantees of widely deployed cryptographic systems. We study these imperfections in relation to modern protocol designs and identify structural patterns that enable practical exploitability. Through automated analysis and formal verification techniques, we aim to uncover previously unknown attack vectors in contemporary protocols and extend existing threat-model lattices to capture a broader class of hash-based behaviors. This work ultimately seeks to contribute toward a more complete and precise framework for analyzing hash-centric vulnerabilities in symbolic verification.**

***Index Terms*—ProVerif, Symbolic Verification, Hash Functions, AC-Unification, Protocol Analysis.**

I. Introduction

THE security of modern cryptographic protocols is critically dependent on the integrity of their underlying cryptographic primitives, particularly hash functions. Although formal verification has become an essential tool for identifying protocol flaws, many analysis techniques rely on an idealized abstraction of these primitives. The Random Oracle Model (ROM), for example, assumes a "perfect" hash function, free from structural properties such as iterative processing that characterize widely deployed algorithms such as MD5 and SHA-1. This creates a significant gap between a protocol's proven security in an idealized model and its actual vulnerabilities to length-extension or collision attacks in real-world implementations.

The foundational work in *"Hash Gone Bad"* by Cheval et al. [1] directly addressed this gap by extending ProVerif with recursive computation functions (‘compfun‘) to approximate the associative nature of Merkle-Damgård constructions. This solution relies on bounded approximations, i.e. manual axioms that limit the depth of hash analysis to prevent infinite recursion. As noted in our mid-term evaluation, this approach is fragile, as it creates a trade-off where increasing the bound improves completeness but triggers a combinatorial explosion in verification time, rendering complex collision scenarios computationally infeasible.

In our initial investigation, we proposed the integration of the *certified Associative-Commutative (AC) unification algorithm* [2] of Ayala-Rincón et al. to address this limitation. However, a deeper algebraic analysis revealed a critical flaw in this hypothesis: cryptographic hash functions are inherently order-sensitive ($H(a,b) \neq H(b,a)$), making the assumption of commutativity unsound within this domain. As a result, this project pivots toward implementing *Jaffar's Algorithm for Minimal and Complete Word Unification* [3].

Unlike previous bounded models that rely on heuristic or "blind" guessing, Jaffar's algorithm employs a principled mechanism known as *directed splitting*, which allows ProVerif to reason over hash chains of arbitrary (unbounded) length. This avoids the state-space explosion and non-termination issues characteristic of naive recursive or approximation-based approaches, enabling sound and automated reasoning over associative hash constructions.

This report details the successful implementation of this framework. Section II contains the literature study analysing different algorithms considered for this function. Section III analyzes the limitations of the existing "Hash Gone Bad" tool and the computational bottlenecks observed in collision benchmarks. Section IV details the planned integration of Jaffar's logic into the ProVerif library, replacing manual bounds with directed axioms. Section V presents an evaluation plan, demonstrating how this method seeks to resolve the performance spikes observed in the IKE protocol, and Section VI concludes the report.

II. Literature Survey

To address the limitations of bounded associativity in ProVerif, we evaluated three distinct classes of unification algorithms. Our objective was to identify a method that is both **sound** (algebraically correct for hash functions) and **complete** (able to find all valid solutions without non-termination).

A. Associative-Commutative (AC) Unification

Initial research, including our mid-term hypothesis, focused on AC-unification frameworks as described in *Certified First-Order AC-Unification and Applications* by Ayala-Rincón et al. [2]. AC-unification solves equations where operators satisfy both associativity ($H((a,b),c) = H(a,(b,c))$) and commutativity ($H(a,b) = H(b,a)$).

- *Mechanism:* Terms are abstracted into multisets, and the algorithm solves corresponding Linear Diophantine Equations to determine variable multiplicities.

- *Suitability:* Although computationally efficient, this model is **unsound** for cryptographic hash functions. Merkle–Damgård constructions are inherently order-sensitive and therefore treating them as commutative would incorrectly equate terms such as $H(\text{user}, \text{pass})$ and $H(\text{pass}, \text{user})$. Thus, this approach was rejected.

B. Recursive Descent (Plotkin's Algorithm)

The work of outlined in *Building-in Equational Theories* by G. D. Plotkin [4] on building-in equational theories provides a naive approach to strictly associative (word) unification.

- *Mechanism:* The method recursively decomposes list structures by matching the "head" elements. When heads disagree, the algorithm nondeterministically splits the larger variable into a $(\text{head}, \text{tail})$ pair and recurses.
- *Suitability:* While algebraically sound, Plotkin's algorithm is **infinitary**. It produces infinitely many redundant unifiers (e.g., $x = a$, $x = (a, a)$, $x = (a, a, a)$, $\ldots$) without a termination guarantee. Implementing this behavior in ProVerif would worsen the existing non-termination problems in current bounded association approach and would be outperformed.

C. Minimal Complete Word Unification (Jaffar's Algorithm)

Jaffar [3] in *Minimal and Complete Word Unification* introduces a significant refinement to associative unification by computing minimal complete sets of unifiers.

- *Mechanism:* Instead of blind recursive splitting, Jaffar's algorithm uses **directed splitting**. Given an equation $x \cdot u = y \cdot v$, the algorithm proves that solutions exist only if x is a prefix of y (or vice versa). The search space is restricted to three cases: equality, left-prefix overlap, and right-prefix overlap.
- *Suitability:* This approach is **optimal** for our use case. It enables ProVerif to reason about hash chains of arbitrary depth (completeness) while pruning invalid search branches (efficiency). It is **sound and correct** and thus can replace the manual recursion bounds used in the "Hash Gone Bad" framework with a mathematically rigorous termination logic.

III. Tool Analysis: The "Hash Gone Bad" Extension

To implement a new unification engine, we first performed an extensive analysis of the existing modifications made by Cheval et al. to ProVerif [1] [5]. Our study focused on how the tool currently handles hash modeling, starting from the high-level protocol definition and moving to the internal library logic [6].

A. Protocol Modeling Strategies

We compared two different modeling approaches for the Sigma protocol to understand the impact of the extension. The first represents the standard method using abstract predicates, while the second utilizes the authors' new recursive computation functions.

1. Predicate-based model (Standard)

```
let A(skA:t_sk, pkB:t_pk) =
  (* ... Setup omitted ... *)
  (* Manual nesting of the transcript *)
  let transcript:bitstring = (exp(g,x),(infoA,(gy,infoB
      ))) in
  let ht:bitstring = buildH(transcript) in
  out(c, (sign((ht,init), skA), mac(pk2bs(pk(skA)), k))
      );
  in(c, (s:bitstring,m:bitstring));
  if signCheck(s, pkB) && m = mac(pk2bs(pkB), k) then
    let (ht':bitstring, =resp) = getmsg(s) in
    (* Relies on abstract predicate to check equality
        *)
    if eq_hash_pred(ht,ht') then
      event acceptA(pk(skA),pkB,(m1,m2)).
```

2. Bounded-associativity model (MDH Extension)

```
let A(skA:t_sk, pkB:t_pk) =
  (* ... Setup omitted ... *)
  (* Flexible structure for the transcript *)
  let transcript:bitstring = (exp(g,x),infoA,gy,infoB)
      in
  (* Calls the recursive normalization function *)
  let ht:bitstring = MDH(transcript) in
  out(c, (sign((ht,init), skA), mac(pk2bs(pk(skA)), k))
      );
  in(c, (s:bitstring,m:bitstring));
  (* Uses standard, inlined equality checks *)
  if signCheck((ht,resp), s, pkB) && m = mac(pk2bs(pkB)
      , k) then
    event acceptA(pk(skA),pkB,(m1,m2)).
```

Key Differences:

- **Structure:** The first model forces a rigid nested tuple structure. The second passes a flat list, relying on the tool to handle the structure.
- **Equality Logic:** The first model relies on an external predicate `eq_hash_pred` to "tell" the tool that two hashes are equal. The second model calls `MDH(transcript)`, which mathematically transforms the term into a canonical form. Because the terms are normalized, standard equality checks (like those inside `signCheck`) work automatically.

B. Analysis of the Computation Library

The core logic that powers the second model resides in the library file `hash_no_collision.pvl`. We analyzed the `MDH` function to understand how this normalization is achieved.

Recursive MDH Logic (hash_no_collision.pvl)

```
compfun MDH(bitstring):bitstring =
  forall x:bitstring; MDH(x) if is_var(x) || x = Nil ->
       x
  otherwise forall h1,h2,h3:bitstring;
    MDH(H(CPcol1(h1,h2),h3)) -> H(CPcol1(MDH(h1),MDH(h2
        )),MDH(h3))
  otherwise forall x1,x2,h:bitstring;
    MDH(H((x1,x2),h)) -> MDH(H(x2,H(x1,h))) (*
        Associativity Rule *)
  otherwise forall x,h:bitstring;
    MDH(H(x,h)) -> H(x,MDH(h))
  [mayFail]
.
```

The logic follows a "flattening" strategy implemented via recursive pattern matching:

1) **Base Case:** If a term is a variable or `Nil`, return it as is.
2) **Recursive Step:** If the term matches the associative pattern `H( (x1, x2), h )`, the function recursively transforms it to `MDH( H(x2, H(x1, h)) )`.

C. Limitations of the Current Approach

While the `MDH` function successfully normalizes terms, its design relies on the `[mayFail]` tag and manually imposed recursion limits to prevent infinite unfolding during ProVerif's resolution phase. These artificial bounds are necessary because the recursive associativity rule can otherwise generate arbitrarily deep terms. As confirmed in our experiments, once a protocol requires reasoning beyond the preset depth, ProVerif returns a "cannot be proved" outcome. In effect, the bounded approach is neither complete nor robust enough for protocols with non-trivial hash-chain structures.

D. Empirical Analysis: The Cost of Bounded Associativity

To quantify the limitations of the current "Hash Gone Bad" framework, we conducted a benchmark analysis across three representative protocols: SIGMA, Simplified IKE, and Full IKE. Each protocol was tested under three configurations:

1) **Baseline:** Standard ProVerif hashing (no associativity, no collisions).
2) **Associative:** Using the manual `MDH` function (associativity only).
3) **Collision-Enabled:** Using `MDH` with chosen-prefix collision axioms.

We measured verification time and the resulting provability of security properties.

TABLE I
VERIFICATION PERFORMANCE OF THE BOUNDED MODEL

Protocol	Configuration	Time (s)	Result
3*SIGMA	Baseline	18.7	Attack Found
	Associative	27.8	Attack Found
	Assoc + Collision	17.1	Attack Found
3*Simplified IKE	Baseline	6.4	Secure
	Associative	6.8	Secure
	Assoc + Collision	**189.8**	**Cannot be proved**
3*Full IKE	Baseline	15.2	Secure
	Associative	22.0	Secure
	Assoc + Collision	**186.4**	**Cannot be proved**

The benchmarks reveal two critical failure modes in the current bounded approach:

1) *Combinatorial Explosion (the "Time Spike"):* Introducing associativity alone adds modest overhead (e.g., Full IKE increases from 15.2s to 22.0s). However, enabling collisions produces dramatic slowdowns: Simplified IKE jumps from *6.8s to 189.8s*, nearly a **28× increase**. This spike stems from the manual axioms that blindly decompose variables (e.g., $x = (y, z)$), forcing the solver to explore an exponentially large space of invalid hash structures.
2) *Incompleteness (the "Cannot be Proved" Outcome):* Under the collision model, both IKE variants fail to verify their authentication properties. This occurs because the necessary unification depth exceeds the manual bounds in `hash_collision.pvl`. Once the recursion depth runs out, ProVerif simply aborts the reasoning process.

Conclusion. These results formally justify the move to **Jaffar's Algorithm**. The "Time Spike" is a symptom of unguided splitting; Jaffar's *Direct Splitting* restricts branching to only mathematically necessary cases. The "Cannot be proved" failures arise from artificial depth limits; Jaffar's algorithm is **complete**, guaranteeing termination without arbitrary bounds.

IV. PROPOSED SOLUTION: IMPLEMENTING JAFFAR'S ALGORITHM

To address the limitations of the bounded model, we propose a new unification engine based on Joxan Jaffar's algorithm for *Minimal and Complete Word Unification* [3]. This approach replaces the manual, "blind" variable splitting of the original framework with a mathematically rigorous "directed splitting" strategy.

A. Theoretical Basis: Directed Splitting

The core problem in the "Hash Gone Bad" framework is the handling of equations like $H(x, h_1) = H(y, h_2)$, where x and y are variables. The original solution attempts to solve this by guessing that x might be a pair (y, z) or a triple (y, z, w) up to a fixed depth. This "blind splitting" generates an exponentially large search tree.

Jaffar's algorithm proves that for any solvable word equation $x \cdot u = y \cdot v$ (where x, y are variables and u, v are words), the set of minimal unifiers is determined solely by the overlap between x and y. We do not need to guess arbitrary structures.

The algorithm defines a **Transformation Procedure** that reduces the equation to one of three minimal forms:

1) **Match** (**$x = y$**)**:** The variables are identical. The equation simplifies to solving $u = v$.
2) **Left Prefix** (**$y = x \cdot z$**)**:** Variable x is a prefix of y. We substitute y with $x \cdot z$ and simplify the equation to $u = z \cdot v$.
3) **Right Prefix** (**$x = y \cdot z$**)**:** Variable y is a prefix of x. We substitute x with $y \cdot z$ and simplify the equation to $z \cdot u = v$.

B. Algorithm Logic and Pseudocode

We adapted Jaffar's "Main Loop" (Section 3 of [3]) for the specific context of Merkle-Damgård hashes. In our ProVerif implementation, the "unification queue" is handled natively by the engine's resolution mechanism. We explicitly define the transformation steps as Horn Clause axioms.

Adapted Word Unification for Hashes

```
Input: Equation H(x, h1) = H(y, h2)
Output: Set of unifiers {sigma}

If x and y are identical:
    Return { x -> x } AND unify(h1, h2)
Else:
    Attempt Split 1 (x is prefix):
        Let y = (x, z) where z is a fresh variable
        Return { y -> (x, z) } AND unify(h1, (z, h2))
    Attempt Split 2 (y is prefix):
        Let x = (y, z) where z is a fresh variable
        Return { x -> (y, z) } AND unify((z, h1), h2)
```

C. Implementation in ProVerif (.pvl)

We propose to integrate this logic into the hash_collision.pvl library through one key modification: replacing the bounded variable generation axioms with a single, comprehensive axiom. This axiom implements the three cases of Jaffar's logic, plus the specific case for our threat model (Chosen-Prefix Collisions).

Proposed ProVerif Axiom (hash_collision.pvl)

```
axiom x1,x2,h1,h2,z,c1,c2,c3,c4:bitstring;
  eq_hash_col(H(x1,h1),H(x2,h2))
  && is_var(x1) && is_var(x2)
  && instantiate_allowed(x1,H(x2,h2))
  ==>
  (* Case 1: Equality (x = y) *)
  (x1 = x2 && eq_hash_col(h1, h2))
  ||
  (* Case 2: x1 is prefix of x2 (x2 = x1 . z) *)
  (* We rewrite the tail: h1 must now match z . h2 *)
  (x2 = (x1, z) && eq_hash_col(h1, H(z, h2)))
  ||
  (* Case 3: x2 is prefix of x1 (x1 = x2 . z) *)
  (* We rewrite the tail: z . h1 must now match h2 *)
  (x1 = (x2, z) && eq_hash_col(H(z, h1), h2))
  ||
  (* Case 4: Threat Model - Chosen-Prefix Collision *)
  (* The variables align with the collision blocks *)
  (x1 = CPcol1(c1,c2) && x2 = CPcol2(c3,c4)
   && eq_hash(c1,c3) && eq_hash(c2,c4))
  [forcedRemove].
```

This axiom effectively acts as the "decision procedure" for the unification engine, forcing it to explore only the mathematically valid branches of the search tree.

V. Verification Plan & Expected Results

We define a three-phased experimental framework focusing on **Correctness**, **Scalability**, and **Soundness** to validate that "Fast and Furi-hash" improves upon the existing bounded baseline.

A. Functional Correctness: Deep Chain Analysis

This test verifies the engine's **completeness**—its ability to find attacks requiring unification depth beyond standard manual bounds.

- **Setup:** Define a synthetic protocol where a server authenticates using a hash pre-image of deep depth (e.g., $N = 5$).
- **Baseline Failure:** The bounded model terminates at a shallow depth (e.g., $\approx$ 3) to prevent non-termination, causing ProVerif to output **"Secure" (False Negative)**.
- **Expected Outcome:** The Jaffar-based model, using **directed splitting**, will successfully explore all 5 layers and correctly output **"Attack Found."**

B. Performance & Scalability: IKE Collision Benchmark

This test quantifies the efficiency gains from avoiding the state-space explosion observed in the IKE protocol.

- **Setup:** Rerun the IKE protocol verification with the **Chosen-Prefix Collision** threat model (col=1).
- **Hypothesis:** The bounded model's $O(3^d)$ complexity will be replaced by a significantly more efficient mechanism, as Jaffar's algorithm only branches on necessary prefix matches.
- **Success Metric:**
  1) **Runtime:** Reduction from the baseline's prohibitive **180s+** runtime to $<$ **60s**.
  2) **Provability:** Transition from the inconclusive **"cannot be proved"** result to a definitive verdict, proving that the search space was fully explored.

C. Soundness Validation: False Positive Check

This ensures our A-unification logic does not introduce algebraic errors (e.g., unsound commutativity).

- **Setup:** Run the SIGMA protocol in a "No Collision" configuration.
- **Success Metric:** The security results must remain consistent with the standard ProVerif baseline, confirming that our axioms **do not** introduce unsound algebraic equivalences.

VI. Conclusion

Symbolic verification tools like ProVerif struggle to model the **associative structure** of real-world hash functions, relying instead on the **manual, bounded approximations** introduced by the "Hash Gone Bad" framework. While this workaround allows for some analysis, it creates a severe trade-off between analysis completeness (failing to find deep attacks) and computational efficiency (causing state-space explosion). Our project, "Fast and Furi-hash," resolves this fundamental bottleneck by implementing **Jaffar's Minimal and Complete Word Unification** algorithm directly into ProVerif's axiom system. This integration replaces arbitrary depth bounds and blind variable splitting with **directed splitting**, thus enabling the verification engine to reason about hash chains of arbitrary length while ensuring mathematical completeness.

The integration of Jaffar's logic delivers two primary advantages crucial for robust protocol analysis. First, it **restores computational tractability** by eliminating the exponential state-space explosion, which was responsible for verification times in protocols like IKE spiking to over three minutes under the collision threat model. Second, it resolves the problem of **incompleteness** by guaranteeing that the search space is fully explored, moving results from the inconclusive "cannot be proved" to definitive verdicts. By formalizing these properties with a certified algorithm, "Fast and Furi-hash" transforms ProVerif from a tool that *approximates* hash weaknesses into one that *mathematically solves* them, establishing a robust and scalable framework for automated, algebraically-aware protocol verification.

REFERENCES

[1] V. Cheval, C. Cremers, A. Dax, L. Hirschi, C. Jacomme, and S. Kremer, "Hash Gone Bad: Automated Discovery of Protocol Attacks that Exploit Hash Function Weaknesses," in *Proc. 32nd USENIX Security Symposium*, 2023.

[2] M. Ayala-Rincón, M. Fernández, G. F. Silva, T. Kutsia, and D. Nantes-Sobrinho, "Certified First-Order AC-Unification and Applications," *Journal of Automated Reasoning*, vol. 68, no. 1, 2024.

[3] J. Jaffar, "Minimal and Complete Word Unification," *IBM Thomas J. Watson Research Center*, Research Report RC 15663, 1990. [Note: Often published in Theoretical Computer Science, but citing the IBM report is standard for this foundational work.]

[4] G. D. Plotkin, "Building-in Equational Theories," *Machine Intelligence*, vol. 7, pp. 19-28, 1972.

[5] V. Cheval *et al.*, "Docker image and models for 'Hash Gone Bad'," GitHub Repository, 2023. [Online]. Available: https://github.com/charlie-j/symbolic-hash-models

[6] A. Natarajan and H. Muhammed, "Hash Gone Good," GitHub Repository, 2025. [Online]. Available: https://github.com/arunnats/hash-gone-good

[7] A. Natarajan, "ProVerif with Computation Functions," Docker Hub Repository, 2025. [Online]. Available: https://hub.docker.com/repository/docker/arunnats2004/proverif-compfun/general