**Find the Maxima and Minima of an Equation**

Extrema of a function are the maximum and minimum points. Maxima and minima are the highest and lowest values of a function within a set of ranges. The largest value of the function in the full range is called the absolute maximum, and the smallest value is called the absolute minimum, while the least value is known as the absolute minima. Other peaks and minima of a function are known as local maxima and minima, and they are not the absolute maxima and minima of the function. Let’s take a closer look at local maxima and minima, absolute maxima and minima, and how to calculate the function’s maximum and minima.

**Maxima and Minima Points:**

The place in a given interval where the values of the function near that point are always less than the value of the function at that point is known as a local maxima. Local minima, on the other hand, are points where the function’s values around that point are bigger than the function’s value at that point.

**Maxima and Minima properties:**

If f(x) is a continuous function in its domain, then there should be at least one maximum and one minimum between equal values of f(x) .

The maxima and minima alternate. In other words, there is one minimum between two maxima and vice versa.

If f(x) approaches infinity as x approaches an or b, and f'(x) = 0 for exactly one value x, namely c between a and b, then f(c) is the lowest and least value. If f(x) tends to – as x tends to an or b, f(c) is the greatest and most extreme value.

**Maxima and Minima Examples:**

**Example 1:** Find the points of maxima and minima of a function:

y = 2x3 – 3x2 + 6

**Solution**

Given function: y = 2x3 – 3x2 + 6

Using the second order derivative test to find a function’s maximum and minimum:

Taking the first derivative of:

y = 2x3 – 3x2 + 6 —– (eq 1)

Differentiate both sides (eq 1), w.r.t x.

⇒ dy/dx = d(2x3)/dx – d(3x2)/dx + d(6)/dx

⇒ dy/dx = 6x2 – 6x + 0

⇒ dy / dx = 6x2 – 6x …. (eq 2)

To find crucial points, put dy/dx = 0.

⇒ 6x2 – 6x = 0

⇒ 6x (x – 1) = 0

⇒ x = 0,1

The critical points are 0 & 1.

Differentiating both sides in eqn 2 w.r.t x.

⇒ d2y/dx2 = d(6x2)/dx – d(6x)/dx

⇒ d2y/dx2 = 12x – 6

Put the x values together and obtain the maximum or minimum value.

At x = 0, d2y/dx2 = 12(0) – 6 = -6 < 0, hence x = 0 is a point of maxima

At x = 1, d2y/dx2 = 12(1) – 6 = 6 > 0, hence x = 1 is a point of minima

The function’s maximum value is x = 0 and its minimum value is x = 1.

**Example 2:** Using the maxima and minima formulas, find the extrema and extremum value of the preceding function: f(x) = -3x2 + 4x + 7.

**Solution:**

Using the second order derivative test to find a function’s maximum and minimum:

Given function: f(x) = -3x2 + 4x + 7 —————- (eq 1)

On both sides of (eq 1), differentiate w.r.t x.

⇒ dy/dx = d(-3x2)/dx + d(4x)/dx + d(7)/dx

⇒ dy/dx = – 6x + 4

dy/dx = 0 helps find critical points.

⇒ -6x + 4 = 0 ——-(eq 2)

⇒x = 2/3

The critical point is 2/3.

w.r.t. x, differentiate both sides of (eq 2)

⇒ d2y/dx2 = d(-6x)/dx + d(4)/dx

⇒ d2y/dx2 = -6

Since d2y/dx2 < 0, the given curve will have maxima at x = 2/3.

At x = 2/3, the maximum value of f(x) is

f(2/3) = -3(2/3)

2 + 4(2/3) + 7 = -4/3 + 8/3 + 7 = 25/3

The maxima of the function is at x = 2/3 and maximum value is 25/3.

Saddle points are critical points of a function where the function's gradient (first derivative) is zero, but the second derivative test fails to classify them as either maxima or minima. Geometrically, at a saddle point, the function resembles a saddle shape.

In more detail, at a saddle point, the function has a flat tangent plane in some directions and slopes upwards in others. This behavior makes saddle points neither maximum nor minimum points. Instead, they represent points of inflection, where the function changes concavity.

Mathematically, a saddle point occurs when the second derivative (or its equivalent in higher dimensions) changes sign along different directions. This change in concavity indicates that the function neither has a local maximum nor a local minimum at that point.

In the context of optimization, saddle points can pose challenges because they can slow down convergence or mislead optimization algorithms. However, they also provide valuable information about the function's behavior, especially in complex systems with multiple variables.