No of Pages : 2 **Course Code : 12XW22**

Roll No:
(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004
SEMESTER EXAMINATIONS, AUGUST / SEPTEMBER - 2015
MSc - SOFTWARE SYSTEMS Semester : 2
12XW22 APPLIED LINEAR ALGEBRA

Time : 3 Hours **Maximum Marks : 100**

INSTRUCTIONS:
1. Answer **ALL** questions from GROUP – I.
2. Answer any **FOUR** questions from GROUP – II.
3. Answer any **ONE** question from GROUP – III.
4. Ignore the box titled as **“Answers for Group III”** in the Main Answer Book.

GROUP - I **Marks : 10 x 3 = 30**

1. In a given linear system of equations, we could also define elementary column operations in analogy with the elementary row operations. What can you say about the effect of elementary column operations of the linear system?
2. Prove or disprove : If A is any square matrix, then the system $AX = 0$ has only the trivial solution.
3. Find a vector that is orthogonal to both $u = (-6, 4, 2)$ and $v = (3, 1, 5)$.
4. Let V be the set of all 2×2 matrices of the form $\begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix}$ with the standard addition and scalar multiplication. Is V a vector space under the given operations? If not, list all axioms that fail to hold.
5. When is a function $f: X \to Y$ called an operator?
6. Why is not $(2,-3, 1)$, $(4, 1, 1)$, $(0, -7, 1)$ a basis for $\mathbb{R}^3$?
7. Sketch the unit circle in $\mathbb{R}^2$ using the inner product $<u, v> = \frac{1}{4}u_1v_1 + \frac{1}{16}u_2v_2$ on $\mathbb{R}^2$. What is the significance of the underlined word ‘unit’?
8. Let $\mathbb{R}^3$ have the Euclidean inner product. Let $u = (1, 1, -1)$ and $v = (6, 7, -15)$. If $\|ku + v\| = 13$, what is k? Show details of your answer.
9. Find the sum and product of the eigen values of the matrix $\begin{pmatrix} -7 & a & b \\ 0 & 4 & -3 \\ 0 & 0 & 2 \end{pmatrix}$.
10. What is diagonalization of a matrix? Give an example.

GROUP - II **Marks : 4 x 12.5 = 50**

11. a) Let $A = \begin{pmatrix} 4.50 & 3.55 \\ 3.55 & 2.80 \end{pmatrix}$, $b_1 = \begin{pmatrix} 5.2 \\ 4.1 \end{pmatrix}$, $b_2 = \begin{pmatrix} 5.2 \\ 4.0 \end{pmatrix}$. Solve $Ax = b_1$, $Ax = b_2$ and compare the solutions obtained. Verify whether the system $AX = b_1$ is ill-conditioned or not using the condition number of A. (6.5)

    b) Find an LU-decomposition of $\begin{pmatrix} 6 & -2 & 0 \\ 9 & -1 & 1 \\ 3 & 7 & 5 \end{pmatrix}$. (6)

12. a) State and prove Dimension theorem for matrices. (6.5)

b) Verify Dimension theorem for $A = \begin{pmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{pmatrix}$. (6)

13. a) Find a basis for the null space of $A = \begin{pmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$. (6.5)

b) Find a subset of $v_1 = (1, -2, 0, 3)$, $v_2 = (2, -5, -3, 6)$, $v_3 = (0, 1, 3, 0)$, $v_4 = (2, -1, 4, -7)$, $v_5 = (5, -8, 1, 2)$ that form a basis for the space spanned by these vectors. Also express each vector not in the basis as a linear combination of the basis vectors. (6)

14. a) Find a QR-decomposition of $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. (6.5)

b) Find the least-squares solution of the linear system $AX = b$ given by

$$\begin{aligned} x_1 - x_2 &= 4 \\ 3x_1 + 2x_2 &= 1 \\ -2x_1 + 4x_2 &= 3. \end{aligned}$$ (6)

15. If u and v are vectors in an inner product space V, and if k is any scalar, then prove the following :

(i) $\|ku\| = |k|\,\|u\|$ (ii) $\|u+v\| \le \|u\| + \|v\|$ (iii) $d(u, v) \ge 0$ (iv) $d(u, v) = d(v, u)$

(v) $d(u, v) \le d(u, w) + d(w, v)$.

GROUP - III **Marks : 1 x 20 = 20**

16. a) For each of the following operators in $\mathbb{R}^3$, write the standard matrix, find their eigen values and the corresponding eigenvectors.

(i) contraction with factor k on $\mathbb{R}^3$, $0 \le k \le 1$.

(ii) reflection about the xy–plane.

(iii) orthogonal projection on the xy–plane. (10)

b) Find bases for the eigenspaces of $A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$. (10)

17. a) Find an orthogonal matrix that diagonalizes $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$. (8)

b) Find the singulal value decomposition for $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$. (12)

CSK. /END/