2481

No of Pages : 2 Course Code : 18XW21

Roll No:
(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

SEMESTER EXAMINATIONS, APRIL 2019

MSc - SOFTWARE SYSTEMS Semester : 2

18XW21 APPLIED LINEAR ALGEBRA

Time : 3 Hours **Maximum Marks : 100**

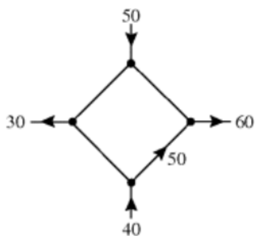
INSTRUCTIONS:
1. Answer **ALL** questions. Each question carries 20 Marks.
2. Subdivision (a) carries 3 marks each, subdivision (b) carries 7 marks each and subdivision (c) carries 10 marks each.
3. Course Outcome : Table

Qn.1	CO1	Qn.2	CO2	Qn.3	CO3	Qn.4	CO4	Qn.5	CO5

1. a) Support or refute: Every homogeneous system of linear equations is consistent.

b) (i) Derive an equation involving $g, h,$ and k that makes the following augmented matrix consistent $\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$. (3)

(ii) A network in which the flow rate and direction of flow in certain branches are given below. Deduce the flow rates and directions of flow in the remaining branches.



(4)

c) Determine an LU-Decomposition of the matrix, $A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$.

2. a) Does $sin\ 3x$ lie in the space spanned by $\{sin\ x, sin^3\ x\}$? Justify.

b) (i) Comment: The vectors $v_1 = (1, 1, 2, -1)$, $v_2 = (4, 9, 9, -4)$ and $v_3 = (5, 8, 9, -5)$ in R^4 form a linearly independent set. (3)

(ii) Let V be a vector space over a field F. Prove that a non-empty subset W of V is a subspace of V if and only it $\alpha u + \beta v \in W$ for all $u, v \in W$ and $\alpha, \beta \in F$. (4)

c) (i) Find a subset of vectors $v_1 = (1, -2, 0, 3)$, $v_2 = (2, -5, -3, 6)$, $v_3 = (0, 1, 3, 0)$, $v_4 = (2, -1, 4, 7)$ and $v_5 = (5, -8, 1, 2)$ that forms a basis for the space spanned by these vectors. Express each vector not in the basis as a linear combination of the basis vectors.

(OR)

(ii) Verify that the set of all triples $(x,\ y,\ z)$ of real numbers with the operations of addition and scalar multiplication defined by $(x,\ y,\ z) + (x',\ y',\ z') = (x + x' + 1,\ y + y' + 1,\ z + z' + 1)$ and $k(x, y, z) = (0,0,0)$ form a vector space or not. If not, list all axioms that fail to hold.

3. a) Let V be a vector space and T be a linear transformation defined on V. Prove that kernel of T is a subspace of V..

b) (i) Prove or disprove: Composition of two linear transformations is also a linear transformation. (3)

(ii) Consider the basis $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2\}$ for R^2, where $u_1 = (1, 0)$, $u_2 = (0, 1)$, $v_1 = (2,\ 1)$ and $v_2 = (-3, 4)$. Find the transition matrix from B' to B. (4)

c) Consider the basis $S = \{v_1, v_2,\ v_3\}$ for R^3, where $v_1 = (1, 1, 1)$, $v_2 = (1, 1, 0)$ and $v_3 = (1, 0, 0)$ and let $T: R^3 \to R^3$ be a linear operator such that $T(v_1) = (-1, 2, 4)$ $T(v_2) = (0, 3, 2)$ and $T(v_3) = (1, 5, -1)$. Find a formula for $T(x_1,\ x_2, x_3)$ and use that formula to find $T(2,\ 4, -1)$.

4. a) Sketch the unit circle in the inner product space $V = R^2$ under the inner product $\langle u, v\rangle = 5u_1v_1 + 9u_2v_2$ where $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in R^2.

b) (i) Define an inner product $\langle p(x), q(x)\rangle = \int_0^1 p(x)q(x)dx$. Verify that this satisfies each of the axioms for an inner product. (3)

(ii) If $\|u\| = 3, \|v\| = 2$ and $\langle u, v\rangle = 0$ then compute the value of $\langle u - 2v, 3u + 4v\rangle$. (4)

c) (i) If $S = \{v_1, v_2, \dots, v_n\}$ is an orthonormal set of nonzero vectors in an inner product space V, then prove that S is an orthonormal basis for V and for any vector u in V $u = \langle u, v_1\rangle v_1 + \langle u, v_2\rangle v_2 + \cdots + \langle u, v_n\rangle v_n$

(OR)

(ii) Find the QR – decomposition of the matrix , $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$.

5. a) If A is a $n \times n$ matrix such that one of its eigenvalues is zero, then A is an invertible matrix. Comment on this statement.

b) (i) Give the geometrical interpretation of eigenvectors in R^2. (3)

(ii) Determine the bases for the eigen spaces of the matrix $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$. (4)

c) Find a singular value decomposition for $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$.

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FD/RL