No of Pages : 3 Course Code : 12XW22

Roll No:
(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

SEMESTER EXAMINATIONS, MAY - 2015

MSc – SOFTWARE SYSTEMS Semester : 2

12XW22 APPLIED LINEAR ALGEBRA

Time : 3 Hours **Maximum Marks : 100**

INSTRUCTIONS:

1. Answer **ALL** questions from GROUP – I.
2. Answer any **FOUR** questions from GROUP – II.
3. Answer any **ONE** question from GROUP – III.
4. Ignore the box titled as **"Answers for Group III"** in the Main Answer Book.

GROUP - I **Marks : 10 x 3 = 30**

1. Consider the following definition of a matrix: "An m x n matrix over the field F is a function A from the set of pairs of integers (i, j), $1 \le i \le m$, $1 \le j \le n$, into the field F". Does the definition of a matrix hold good if F is a group? Justify your answer.
2. If A is an invertible symmetric matrix, then verify whether A^{-1} is symmetric or not.
3. Let V be the set of all pairs of real numbers (x, y) with the operations $(x, y) + (x', y') = (x + x', y + y')$ and $k(x, y) = (kx, 2ky)$. Is V a vector space under the given operations? If not, list all axioms that fail to hold.
4. Why is the transformation $T : P_2(x) \to P_3(x)$, defined by $T(p(x)) = xp(x)$ not an isomorphism? (Notations are usual).
5. When is a function $f : X \to Y$ called an operator? If $f : M_{nn} \to R$ is a function that maps an n x n matrix A into its determinant, $f(A) = |A|$, is f a linear operator? Give reason.
6. If $T_A : R^n \to R^m$ is the transformation defined by $T(x) = Ax$, find the kernel of T_A, and the range of T_A. What is the relation between the dimensions of the kernel and range of T_A?
7. Sketch the unit circle in R^2 using the inner product $\langle u, v \rangle = \frac{1}{4} u_1 v_1 + \frac{1}{16} u_2 v_2$ on R^2. What is the significance of the underlined word 'unit'?
8. Use vector norm to prove that a triangle that is inscribed in a circle so that it has a diameter for a side must be a right triangle.

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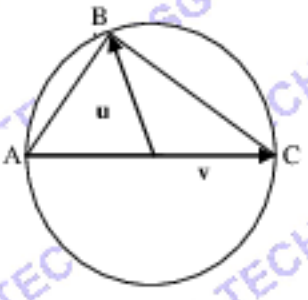

9. Write any two reasons to justify that $\lambda = 0, \lambda = 1, \lambda = 3$ cannot be the eigenvalues of an invertible matrix $A = \begin{pmatrix} 0 & 1 & 3 \\ a & 1 & b \\ c & 1 & 1 \end{pmatrix}$, where *a*, *b* and *c* are scalars.

10. Prove or disprove: Matrices with repeated eigenvalues are nondiagonalizable.

GROUP - II Marks : 4 x 12.5 = 50

11. a) Let $A = \begin{pmatrix} 4.50 & 3.55 \\ 3.55 & 2.80 \end{pmatrix}$, $b_1 = \begin{pmatrix} 5.2 \\ 4.1 \end{pmatrix}$, $b_2 = \begin{pmatrix} 5.2 \\ 4.0 \end{pmatrix}$. Solve $Ax = b_1$, $Ax = b_2$ and compare the solutions. Verify whether the system $Ax = b_1$ is ill–conditioned or not using the condition number of *A*. (6.5)

b) Find an *LU*–decomposition of the coefficient matrix of the following system and hence solve the system. $\begin{pmatrix} 1 & 3 & 4 \\ 3 & 10 & 10 \\ 2 & 4 & 11 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}$. (6)

12. a) If $A\boldsymbol{x} = \boldsymbol{0}$ is a homogenous linear system of *m* equations in *n* unknowns, then prove that the set of solution vectors is a subspace of R^n. (6.5)

b) Find the rank and nullity of the matrix given below and verify the Dimension theorem for A. (6)

$$A = \begin{pmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{pmatrix}$$

13. a) Determine whether $\mathbf{b} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ is in the column space of $A = \begin{pmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. If so, express ***b*** as a linear combination of the column vectors of *A*. (6.5)

b) Consider the bases $B = \{u_1, u_2\}$ and $B' = \{u'_1, u'_2\}$ for R^2, where

$u_1 = (-1, 0)$ $u_2 = (0, 1)$ $u'_1 = (1, 1)$ $u'_2 = (2, 1)$

i) Find the transition matrix from B' to B.

ii) Find the coordinate vector $[v]_B$ of a vector v if the coordinate vector $[v]_{B'} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$. (6)

14. a) Check whether the matrix $A \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$ has a QR–decomposition. If possible, find the decomposition. (6.5)

b) Find the least squares solution of the linear system $A\mathbf{x} = \mathbf{b}$, and find the orthogonal projection of $\mathbf{b}$ onto the column space of A.

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \\ 3 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}. \quad (6)$$

15. a) If u and v are vectors in a real inner product space, then prove that $|\langle \mathbf{u}, \mathbf{v} \rangle| \leq \|\mathbf{u}\| \|\mathbf{v}\|$. (6.5)

b) If u, v and w are vectors in an inner product space V and if k is any scalar, then prove that (i) $\|k\mathbf{u}\| = |k| \|\mathbf{u}\|$ (ii) $d(\mathbf{u}, \mathbf{v}) \leq d(\mathbf{u}, \mathbf{w}) + d(\mathbf{w}, \mathbf{v})$ where 'd' stands for distance. (6)

GROUP - III Marks : 1 x 20 = 20

16. a) For each of the following operators in R^3, write the standard matrix, find their eigenvalues and the corresponding eigenvectors. (10)

i) Reflection about the xy– plane.

ii) Reflection about the xz– plane.

iii) Orthogonal projection on the yz– plane.

b) Find the eigenvalues and bases for the eigenspaces of the linear operator (10)

$T: P_2 \to P_2$ defined by $T(a + bx + cx^2) = -2c + (a + 2b + c)x + (a + 3c)x^2$.

17. a) Find an orthogonal matrix that diagonalizes $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$. (8)

b) Find the Singular Value Decomposition for $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$. (12)

SR /END/