Roll No:
(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004
SEMESTER EXAMINATIONS,   MAY 2017
MSc - SOFTWARE SYSTEM   Semester : 2
15XW21 PROBABILITY AND STATISTICS

Time : 3 Hours   **Maximum Marks : 100**

INSTRUCTIONS:
1. Answer **ALL** questions from GROUP – I.
2. Answer any **FOUR** questions from GROUP – II.
3. Answer any **ONE** question from GROUP – III.
4. Ignore the box titled as **"Answers for Group III"** in the Main Answer Book.
5. **Statistical table may be permitted.**

GROUP - I   **Marks : 10 x 3 = 30**

1. Which of the following statements are true? If a statement is true, prove it. If it is false, give a counterexample.

   (i) If $P(A) + P(B) + P(C) = 1$, then the events A, B, C are mutually exclusive.

   (ii) If $P(A \cup B \cup C) = 1$, then A, B and C are mutually exclusive events.

2. Identify the following experiments decide whether it would be appropriate to use a sample space which is finite, countably infinite, or continuous:

   (i) The amount of cosmic radiation to which passengers are exposed during a transcontinental jet flight is measured by means of a suitable counter.

   (ii) In a torture test, a watch is dropped from a tall building until it stops running.

   (iii) A study is made to determine in how many of 450 airplane accidents the main cause is pilot error.

3. A certain basketball player makes a foul shot with probability 0.45. What is the probability that

   (i) his first basket occurs on the sixth shot ?

   (ii) his first and second baskets occur on his fourth and eighth shots, respectively?

4. The radius of a sphere is a random number between 2 and 4. What is the probability that its volume is at most 36π?

5. Let the joint probability mass function of random variable X and Y be given by

$$p(x, y) = \begin{cases} \dfrac{1}{7} x^2 y, & if (x, y) = (1,1),(1,2),(2,1) \\ 0, & elsewhere . \end{cases}$$

   Are X and Y independent? Why or why not?

6. Let $X_1, X_2,…, X_7$ denote a random sample from a population having mean μ and variance σ^2. Consider the following estimators of μ: $\Theta_1 = (X_1, X_2,…, X_7)/7$, $\Theta_2 = (2 X_1 - X_6 + X_4)/2$ Which estimator is best? In what sense is it best?

7. Suppose that n=100 random samples of water from a fresh water lake were taken and the calcium concentration (milligrams per liter) measured. A 95% confidence interval on the mean calcium concentration is 0.49< μ<0.82. Consider the statement:" There is a

95% chance that μ is between 0.49 and 0.82." Is this statement correct? Justify your answer.

8. Suppose that we want to test the null hypothesis that an antipollution device for cars is effective. Explain under what conditions we would commit a Type I error and under what conditions we would commit a Type II error.

9. Sketch the scatter diagram for the following situations: High linear correlation, moderate or low linear correlation and no linear correlation

10. Write the limitations of ANOVA one way test.

GROUP - II   Marks : 4 x 12.5 = 50

11 a) There are five boys and six girls in a class. For an oral exam, their teacher calls them one by one and randomly. (i) What is the probability that the boys and the girls alternate? (ii) What is the probability that the boys are called first? Compare the answers to parts (i) and (ii).   (6.5)

b) Jim has three cars of different models: A, B, and C. The probabilities that models A, B, and C use over 3 gallons of gasoline from Jim's house to his work are 0.25, 0.32, and 0.53, respectively. On a certain day, all three of Jim's cars have 3 gallons of gasoline each. Jim chooses one of his cars at random, and without paying attention to the amount of gasoline in the car drives it toward his office. What is the probability that he makes it to the office?   (6)

12. a) The sales of a convenience store on a randomly selected day are X thousand dollars, where X is a random variable with a distribution function of the following form:

$$F(t) = \begin{cases} 0, & t < 0 \\ \left(\dfrac{1}{2}\right)t^2 & 0 \leq t < 1 \\ k(4t - t^2), & 1 \leq t < 2 \\ 1, & t \geq 2 \end{cases}$$

Suppose that this convenience store's total sales on any given day are less than $2000.

i) Find the value of k

ii) Let A and B be the events that tomorrow the store's total sales are between 500 and 1500 dollars, and over 1000 dollars, respectively. Find P(A) and P(B).

iii) Are A and B independent?   (6.5)

b) In a lottery every week, 2,00,000 tickets are sold for Rs. 1 apiece. If 4000 of these tickets pay off Rs. 30 each, 500 pay off Rs. 800 each, one ticket pays off Rs. 1,20,000 and no ticket pays off more than one prize, what is the expected value of the winning amount for a player with a single ticket?   (6)

13. a) The life time of a TV tube (in years) is an exponential random variable with mean 10. What is the probability that the average life time of a random sample of 36 TV tubes is at least 10.5?   (6.5)

b) Let the joint probability mass function of random variable X and Y be given by (6)

$$P(x, y) = \begin{cases} \dfrac{1}{25}(x^2 + y^2), & if\ x = 1, 2,\ y = 0, 1, 2 \\ 0, & elsewhere. \end{cases}$$

Find P(X>Y), P(X+Y<2), P(X+Y=2), P(X=2/Y=1) and E(X/Y=1)

14. a) The advertised claim for batteries for cell phones is set at 48 operating hours, with proper charging procedures. A study of 5000 batteries is carried out and 15 stop operating prior to 48 hours. So these experimental results support the claim that less than 0.2 percent of the company's batteries will fail during the advertised time period, with proper charging procedures? Use a hypothesis-testing procedure with α=0.01.   (6.5)

b) The overall distance traveled by a golf ball is tested by hitting the ball with Iron Byron, a mechanical golfer with a swing that is said to emulate the legendary champion, Byron Nelson. Ten randomly selected balls of two different brands are tested and the overall distance measured. The data follow:

Brand 1: 275, 286, 287, 281, 283, 271, 279, 275, 263, 267

Brand 2: 258, 244, 260, 265, 273, 281, 271, 270, 263, 268

Test the hypothesis that both brands of ball have equal mean overall distance. Use α=0.05.   (6)

15. a) A company has to choose among three pension plans. Management wishes to know whether the preference for plans is independent of job classification and wants to use α=0.05. The opinions of a random sample of 500 employees are show in table 1.   (6.5)

		Pension Plan			
		1	2	3	Total
Job classification	Salaried workers	160	140	40	340
	Hourly workers	40	60	60	160
	Total	200	200	100	500

Table 1

b) Construct the linear regression line y = ax + b for the following data.   (6)

x	2.5	3.0	3.5	4.0	4.5	5.0	5.5
y	4.32	4.83	5.27	5.74	6.26	6.79	7.23

GROUP - III   **Marks : 1 x 20 = 20**

16 . The joint probability density function of X and Y is given by

$$f(x, y) = \begin{cases} 8xy & if\ 0 < y \le x \le 1 \\ 0, & elsewhere. \end{cases}$$

(i) Calculate the marginal probability density functions of X and Y, respectively.

(ii) Calculate E(X) and E(Y)

(iii) Find $f_{X/Y}((x/y)$ and $f_{Y/X}(y/x)$

(iv) Calculate $E(Y/X = x)$ and $\text{var}(Y/X = x)$

17. a) The following is the distribution of the hourly number of trucks arriving at a company's warehouse:

Trucks arriving per hour	0	1	2	3	4	5	6	7	8
Frequency	52	151	130	102	45	12	5	1	2

Find the mean of this distribution and using it as the parameter λ, fit a Poisson distribution. Test the goodness of fit at the 0.05 level of significance.

b) Six samples of each of four types of cereal grain grown in a certain region were analyzed to determine thiamin content, resulting in the following data :

Wheat   5.2   4.5   6.0   6.1   6.7   5.8

Barley   6.5   8.0   6.1   7.5   5.9   5.6

Maize   5.8   4.7   6.4   4.9   6.0   5.2

Oats   8.3   6.1   7.8   7.0   5.5   7.2

Does the data suggest that the grains differ with respect to true average thiamin content? Use $\alpha = 0.05$.

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FD/RL