No of Pages : 2 Course Code : 12XW22

Roll No:
(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

SEMESTER EXAMINATIONS, MAY - 2013

MSc – SOFTWARE ENGINEERING Semester : 2

12XW22 APPLIED LINEAR ALGEBRA

Time : 3 Hours **Maximum Marks : 100**

INSTRUCTIONS:
1. Group I, Group II and Group III questions should be answered in the Main Answer Book.
2. Ignore the box titled as **"Answers for Group III"** in the Main Answer Book.
3. Answer **ALL** questions from GROUP – I.
4. Answer any **4** questions from GROUP – II.
5. Answer any **ONE** question from GROUP – III.

GROUP - I **Marks : 10 × 3 = 30**

1. In order to know that a linear system is consistent with unique solution, what would you have to know about the pivot columns in the augmented matrix of the system?

2. Is it possible to have a vector space *with exactly two distinct vectors* in it? Justify.

3. Is the set of all points on the circle $x_1{}^2 + x_2{}^2 = 1$ a subspace of R^2? Why or why not?

4. Give an example to each of the following: i) a set of vectors that spans R^2 but not linearly independent; ii) a linearly independent set of vectors that does not span R^3.

5. What is the standard matrix of the transformation on R^2 that first rotates each vector through an angle 60° anti-clockwise, then projects it on the x-axis, and then reflects it about the line $y = x$?

6. Let $Ax = \overline{0}$ be a homogeneous linear system of three equations in three unknowns. If the column space of A is a line through the origin, what kind of geometric object is the null space of A? What kind of geometric object is the column space of A^T?

7. Find two vectors u and v in R^2 that are orthonormal with respect to the inner product $\langle u, v\rangle = 3u_1v_1 + 2u_2v_2$ but are not orthonormal with respect to the Euclidean inner product.

8. Indicate whether each of the following statements is always true or sometimes false. Justify your answer by giving a logical argument or a counterexample.
   i) A linearly dependent set of vectors in an inner product space cannot be orthonormal.
   ii) Every matrix with a nonzero determinant has a QR-decomposition.

9. The characteristic polynomial of some matrix A is found to be $(\lambda - 1)(\lambda - 3)^2(\lambda - 4)^3$. What is the order of A? Is A invertible? How many eigen-spaces does A have?

10. What is Singular Value Decomposition (SVD) of a matrix? Why do we need it?

GROUP - II **Marks : 4 × 12.5 = 50**

11. An economy consists of three sectors - Coal, Power, and Steel. The output from each sector is distributed among the various sectors as in Table 1. Formulate a system of linear equations that leads to prices at which each sector's income matches its expenses. Solve the system to determine the equilibrium prices.

TABLE 1

		Input to		
		Coal	Power	Steel
Output from	Coal	0	0.6	0.4
	Power	0.4	0.1	0.5
	Steel	0.6	0.2	0.2

12. Show that the set of all real matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition and scalar multiplication defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space.

13. Let $T: R^5 \rightarrow R^4$ be the transformation defined by $T[(x_1, x_2)] = (x_1 + 2x_2, \ -x_1, \ 0)$. Find the matrix of T with respect to the bases $B = \{ u_1, \ u_2 \}$ and $C = \{ v_1, \ v_2, \ v_3 \}$ where $u_1 = (1, 3)$, $u_2 = (-2, 4)$, $v_1 = (1, 1, 1)$, $v_2 = (2, 2, 0)$, and $v_3 = (3, 0, 0)$. Using the matrix obtained, find the image of $(8, 3)$ under T.

14. To measure the takeoff performance of an airplane, the horizontal position of the plane was measured every 2 seconds from $t = 0$ to $t = 10$. The positions (in feet) are 0, 29.9, 104.7, 222.0, 380.4, and 517.7. Fit a least-squares quadratic $y = \beta_0 + \beta_1 t + \beta_2 t^2$ for the data. Use the result to estimate the velocity of the plane when $t = 4.5$.

15. Applying Gram-Schmidt process, obtain an ortho-normal basis for the Euclidean inner product space R^4 from the basis $B = \{(1, 0, 1, 1), (1, 1, 0, 1), (1, 1, 1, 0), (0, 1, 1, 1)\}$.

GROUP - III **Marks : 1 × 20 = 20**

16. a) Let $C_1, C_2, \ldots, C_5$ denote the columns of the matrix $A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix}$ and $B = [\, C_1 \quad C_2 \quad C_4 \,]$.

   i) Explain why C_3 and C_5 are in the column space of B.

   ii) Find a set of vectors that spans the null space of A.

   iii) Let $T: R^5 \rightarrow R^4$ be the transformation defined by $T[x] = Ax$. Explain why T is neither one-one nor onto. [14]

   b) Determine the Kernel, Range, Nullity, and Rank of the transformation defined in (iii) above. Also verify Rank-Nullity theorem. [6]

17. a) Discuss the geometric interpretation of eigen-values and eigen-vectors. Mention any two real-life applications of eigen-values and eigen-vectors. [6]

   b) Diagonalize the matrix $A = \begin{bmatrix} 9 & -3 & 3 \\ -3 & 6 & -6 \\ 3 & -6 & 6 \end{bmatrix}$ if possible. [14]

FD/RL /END/