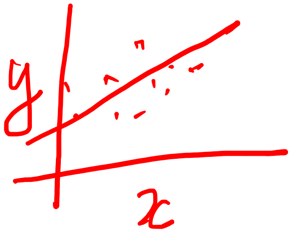
Logistic Regression
logistic classification

Arunkumar Nair

Logistic Regression

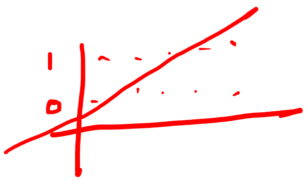


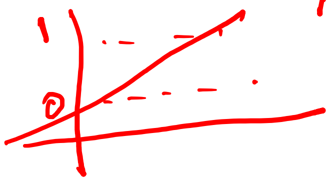
$y = mx + c \Rightarrow -\infty$ to $\infty \Rightarrow -.5$ ---- $+5$ ---

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ ----

Logis.R = Yes or No

Yes	No
1	0





Log	exp

$10^0 = 1 \rightarrow \log_{10} 1 = 0$

$10^1 = 10 \rightarrow \log_{10} 10 = 1$

$10^2 = 100$ $\quad \log_{10} 100 = 2$

$10^3 = 1000$

$10^4 = 10000$

10^5

$\log_{10} 0 = 1$

$\log_{10} 1 = 10$

$\log_{10} 100 = 2$ =

$10^0 = 1$

$10^1 = 10$

$10^2 = 100$

$10^3 = 1000$

$10^4 = 10000$

$10^5 = 100,000$

$10^6 = 1,000,000$

$\log_{10} 1 = 0$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

$\log_{10} 10000 = 4$

$\log_{10} 100,000 = 5$

0

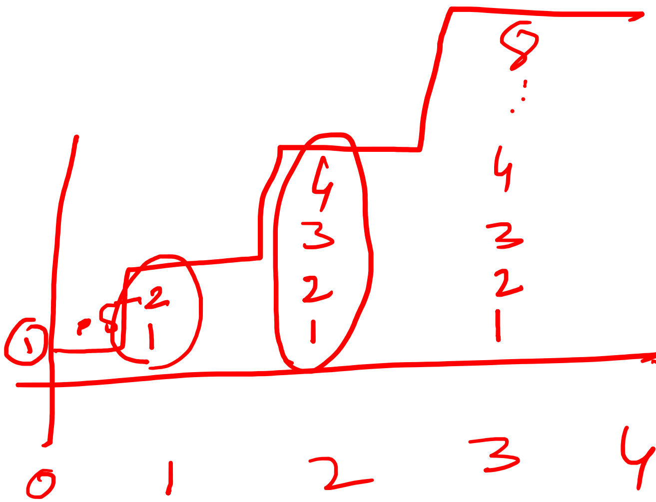
1

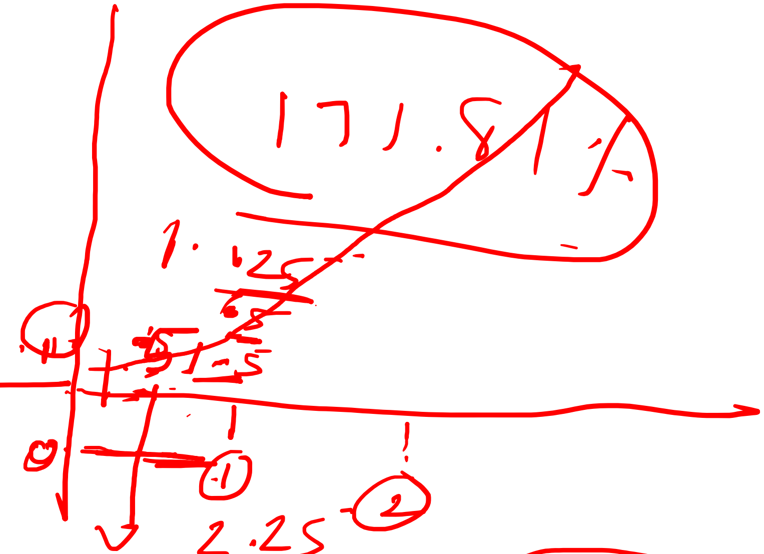
2

3

4

5





100 %

$$\left(1+\frac{r}{n}\right)^{n} = \left(1+\frac{Rate}{x}\right)^{2} \Rightarrow$$

6m

$$e = 2.718$$

1 —	2.2
2	2.25
3	2.441
12 =	2.613
365 =	2.7146
1,000 =	2.7169
1m =	2.718...

Log

1

10^1

$100 \to 10^2$

$1000 \to 10^3$

10000

$\vdots$

$\to \log_{10} 1 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

$= 4$

Expo

10^1

10^2

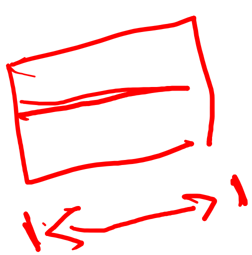
10^3

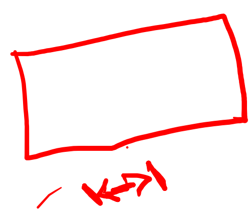
$8 = 2^3$

$2 \times 2 \times 2$

$\log_2 8 = 3$

$2^3 = 8$



log 

Log formula's

$$\log(ab) = \log(a) + \log(b)$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

$$\log(a^b) = b * \log(a)$$

$$\log\left(\frac{1}{x^a}\right) = -a$$

$$\log(\text{odd ratio}) = \text{logit}$$

COMMON LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12

Probability

$P(x) = 0$ to 1

$$P = \frac{\text{Number of events}}{\text{Total num of events}}$$

$$\text{odd} = \frac{\text{Number of events that will occur}}{\text{No. events that will NOT occur}}$$

$$= \frac{\text{Success}}{\text{Failure}}$$

$$= \frac{P}{1-P}$$

$$=$$

Probability

odd

2

E

H T

1 ?

$$\Rightarrow \frac{P(H)}{P(C)} = \frac{P(E)}{Total} = \frac{1}{2}$$

$$\Rightarrow \frac{.5}{.5} = 1$$

$$\log(odd) = \ln\left(\frac{P}{1-P}\right)$$

Logistic Regression

Logistic Regression, also known as **Logit Regression** or **Logit** Model, is a mathematical model used in statistics to estimate (guess) the probability of an event occurring having been given some previous data. **Logistic Regression** works with binary data, where either the event happens (1) or the event does not happen (0).

Logistic Regression

Putting z value to sigmoid function

Linear Regression Equation

$$z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k$$

Sigmoid Function

$$p = \frac{1}{1 + e^{-z}}$$

$$p = \frac{e^z}{e^z + 1}$$

$$p = \frac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k}}{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k} + 1}$$

Replace p in odd ratio and solve

$$\text{Odds Ratio S} = \frac{\textit{Probability of Success}}{\textit{Probability of Failure}} = \frac{p}{1-p}$$

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$$S = \frac{\dfrac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k}}{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k} + 1}}{1 - \dfrac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k}}{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k} + 1}}$$

$$S = e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k}$$

Take log each side and solve

$$\mathbf{Ln(S)} = \boldsymbol{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k}$$

Transformed into Linear Regression known as log of Odds

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Transformed into Linear Regression known as log of Odds

$$\frac{\dfrac{e^{\beta_0}}{e^{\beta_0}+1}}{\dfrac{1}{1} - \dfrac{e^{\beta_0}}{e^{\beta_0}+1}}$$

$$\frac{\dfrac{e^{\beta_0+1}}{e^{\beta_0}+1}}{1 + e^{\beta_0}+1}$$

$\log(-86)$ $\cdot 2 \cdot$

2.45

$$S = \frac{\dfrac{e^{\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\dots+\beta_kx_k}}{e^{\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\dots+\beta_kx_k}+1}}{1-\dfrac{e^{\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\dots+\beta_kx_k}}{e^{\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\dots+\beta_kx_k}+1}}$$

$$\log(S) = \log\left(e^{\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\dots+\beta_kx_k}\right)$$

Take log each side and solve

$$\text{Ln(S)} = \boldsymbol{\beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \dots + \beta_kx_k}$$

Transformed into Linear Regression known as log of Odds

$$S = \frac{\dfrac{e^{\beta_0}}{e^{\beta_0}+1}}{\dfrac{1}{1} - \dfrac{e^{\beta_0}}{e^{\beta_0}+1}} = \frac{e^{\beta_0}}{e^{\beta_0}+1} \div \left(\frac{1}{1} - \frac{e^{\beta_0}}{e^{\beta_0}+1}\right)$$

$$\frac{1 * e^{\beta_0}+1 - e^{\beta_0} * 1}{(1 * e^{\beta_0}+1)}$$

Case-Study Data

We are provided a sample of 1000 customers.

We need to predict the probability whether a **customer of a Particular Age** will buy (y) a sim card or

not.

As we've a categorical outcome variable, we'll use logistic regression.

Linear to Logistic – (a)

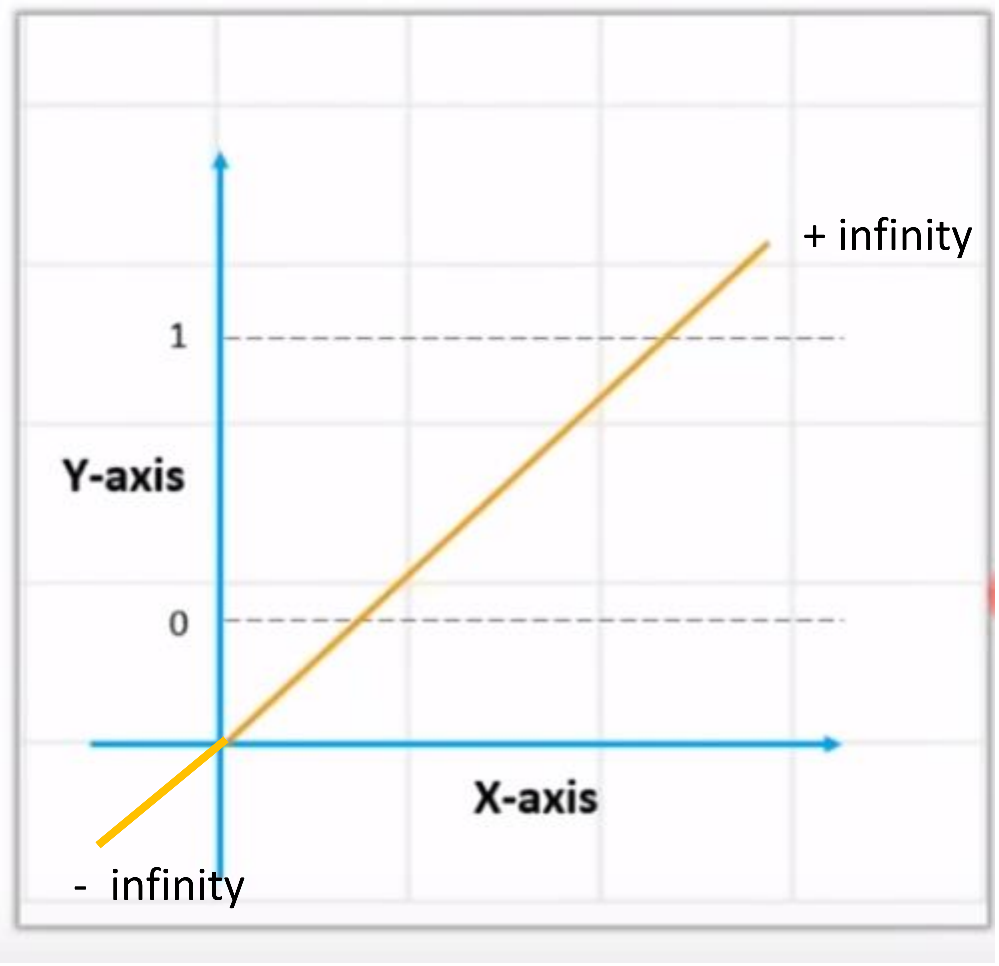
- To start with logistic regression, first write the simple linear regression equation with dependent variable enclosed in a link function:

$$Y = c + m X$$

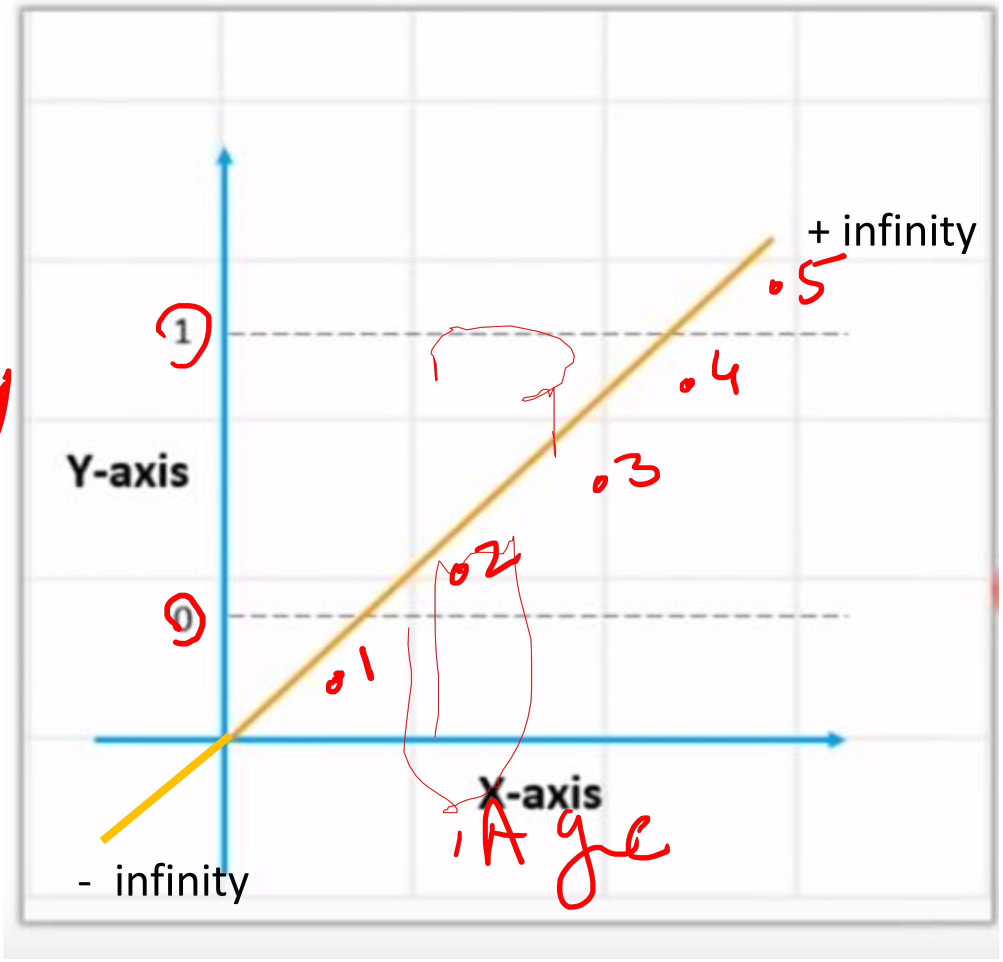
$$g(y) = \beta o + \beta(\text{Age}) \text{———} \quad (a)$$

For understanding, consider '*Age*' as independent variable.

Linear Regression

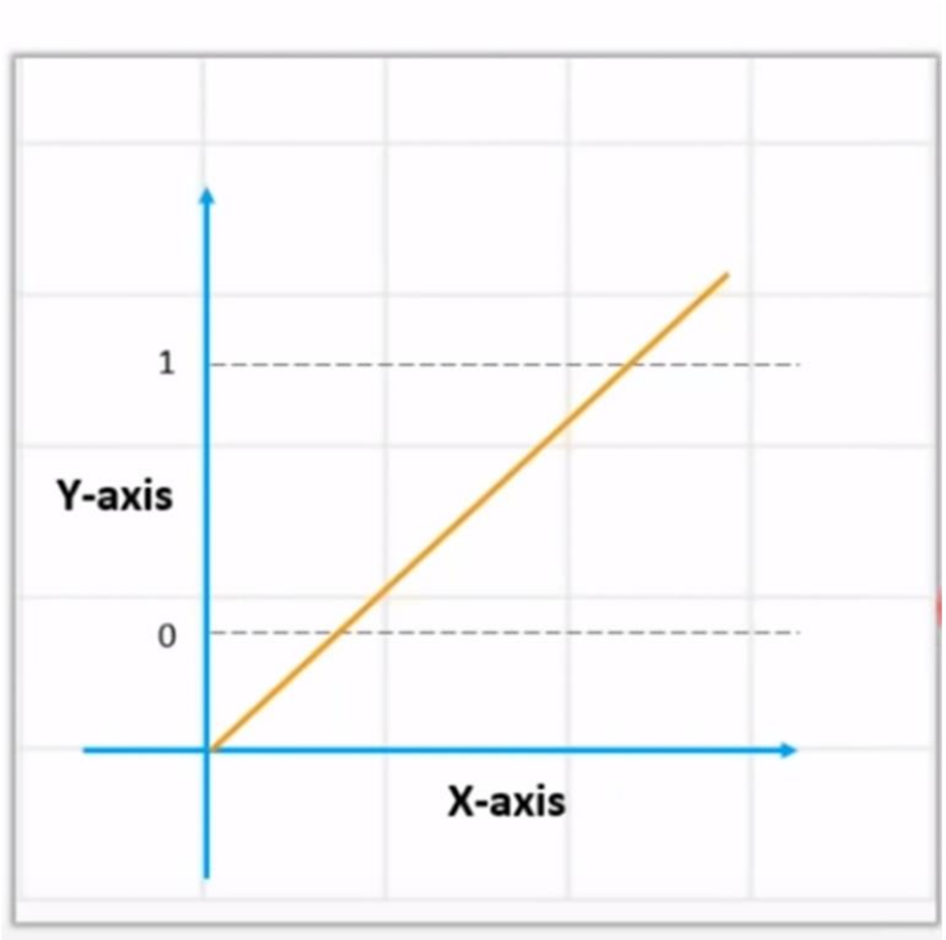


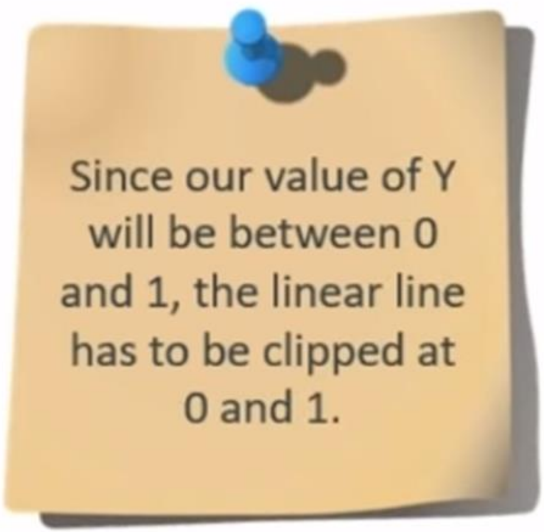
Linear Regression



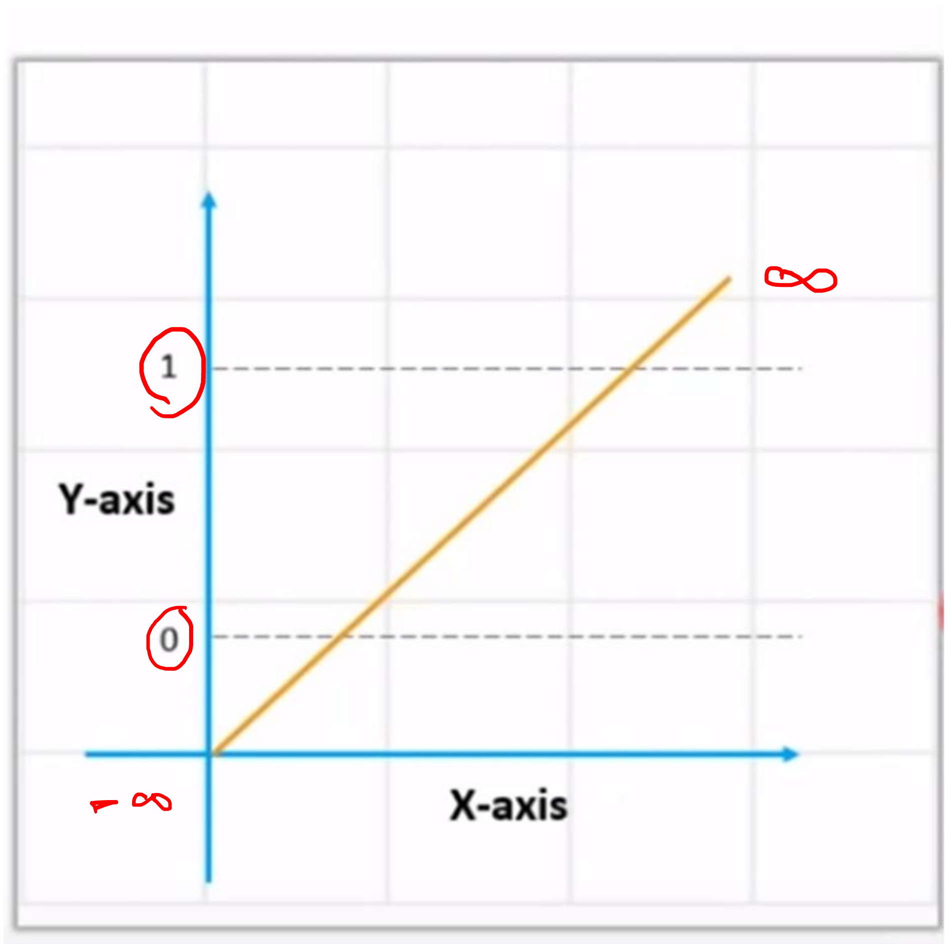
1 or 0

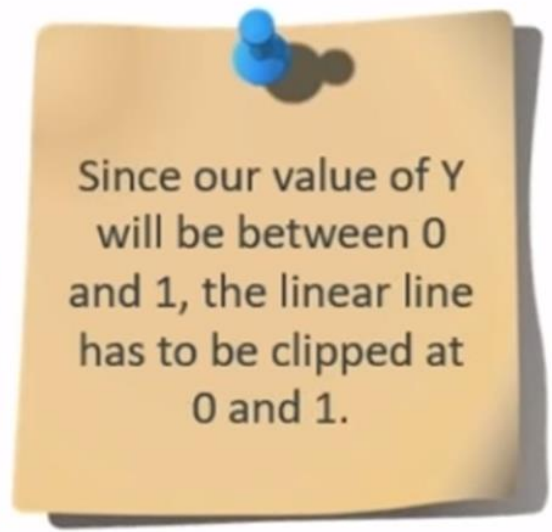
Linear regression equation: $\mathbf{y = \beta 0 + \beta 1X1 + \beta 2X2 \ldots . + \beta nXn}$



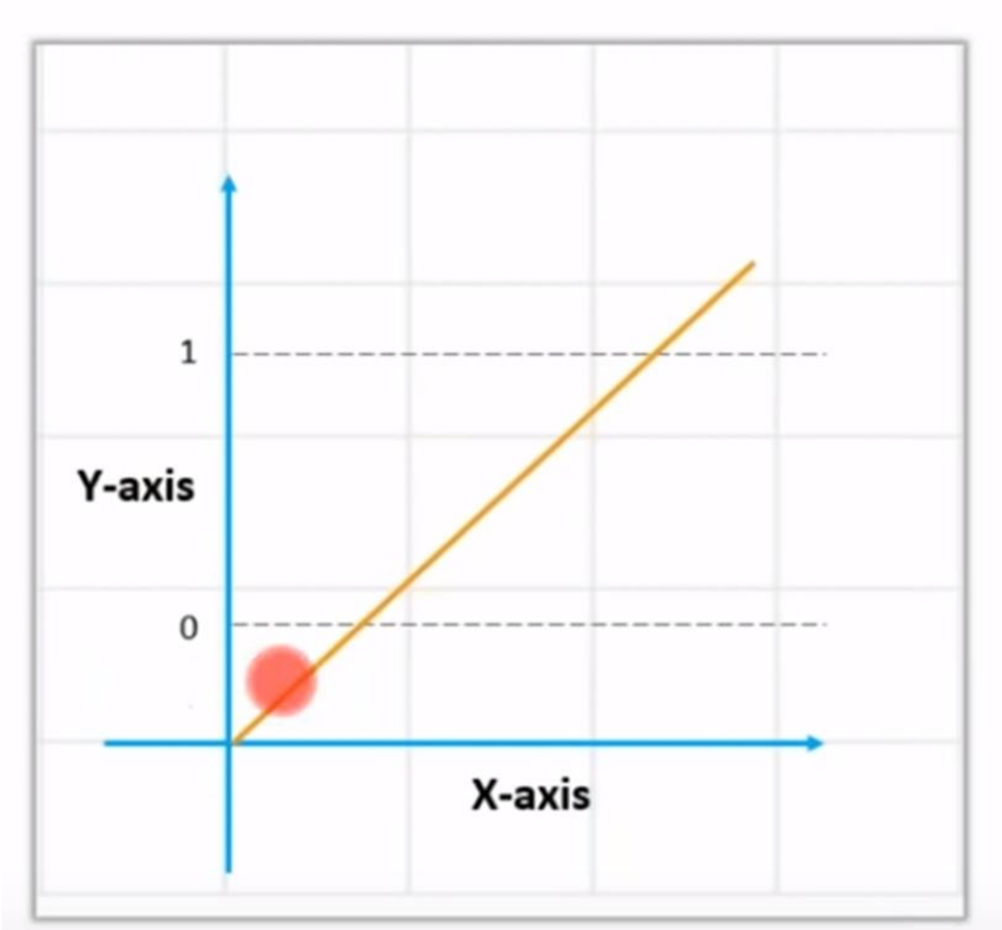


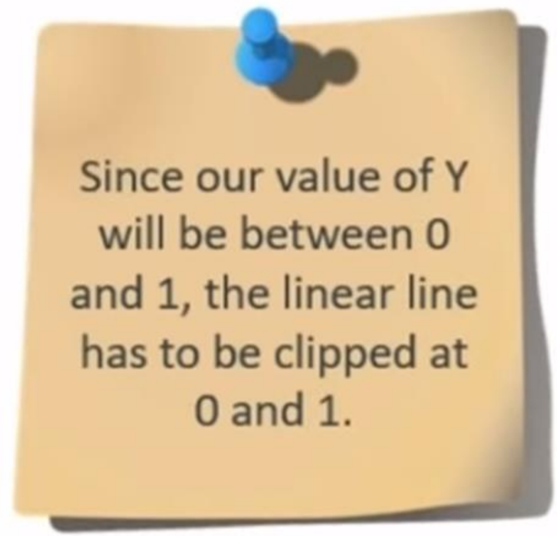
Linear regression equation: **$y = \beta 0 + \beta 1X1 + \beta 2X2 \ldots. + \beta nXn$**



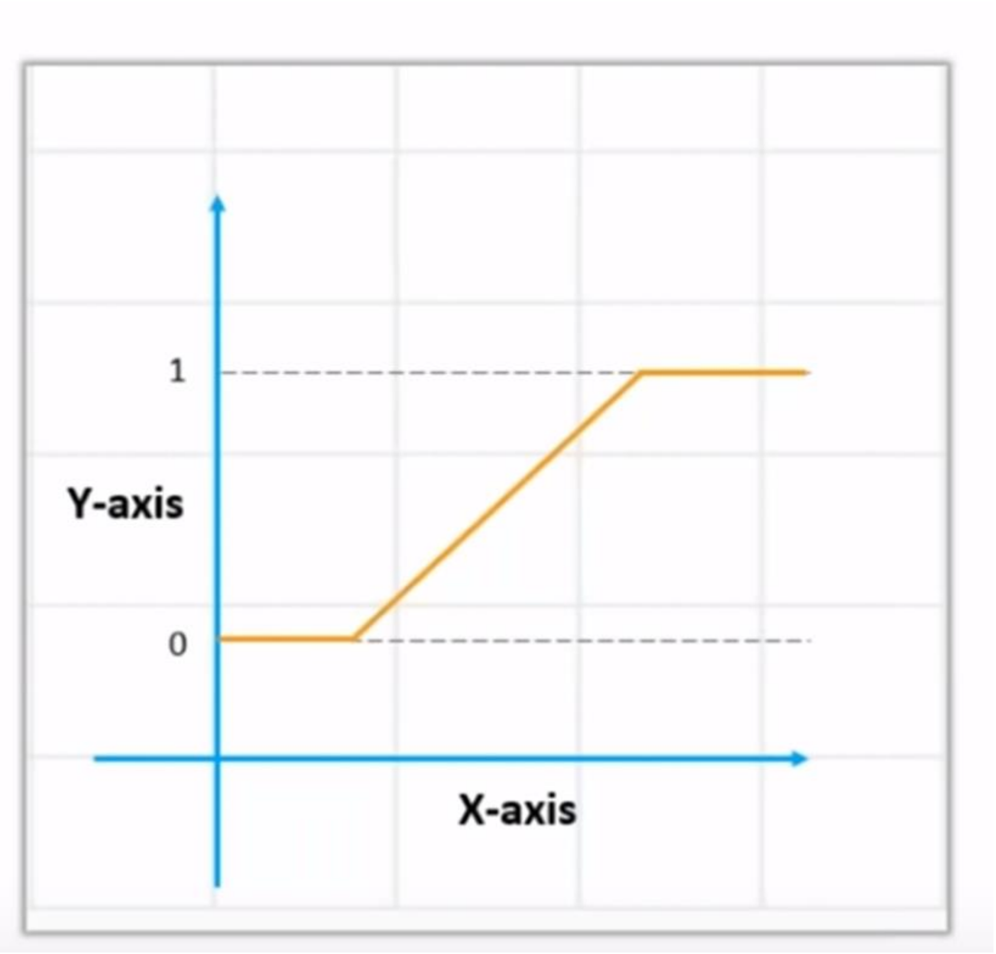


Value of Y – between 0 and 1





How to get the value of 0 and 1



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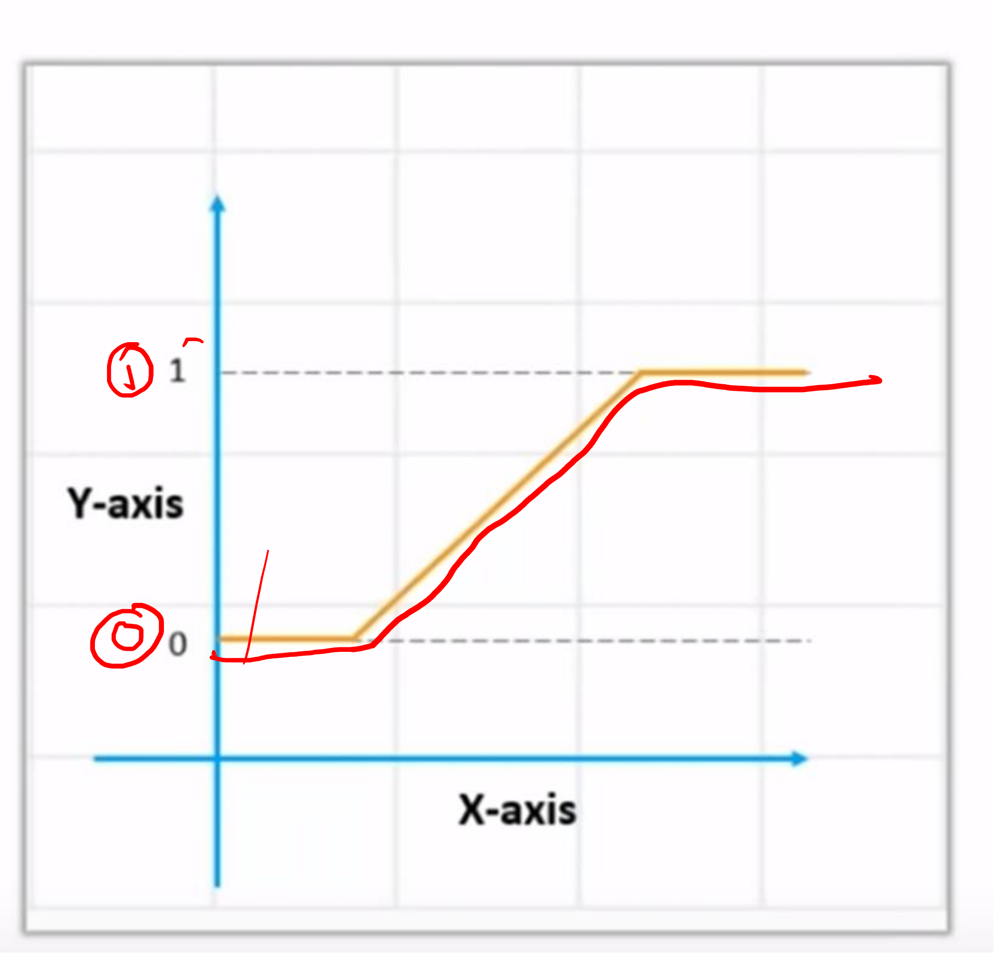

How to get the value of 0 and 1?
Use Sigmoid

- We Apply sigmoid function on the linear regression equation to get the S-curve so that it lies between 0 and 1

Sigmoid function: $\boldsymbol{p = 1 / 1 + e^{-y}}$

- A sigmoid function is a mathematical function/equation having a characteristic "S"-shaped curve or sigmoid curve.

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Convert Linear to Logistics

- Linear regression equation: **$y = \beta 0 + \beta 1X1 + \beta 2X2 .... + \beta nXn$**
- Sigmoid function: ***$p = 1 / 1 + e^{-y}$***

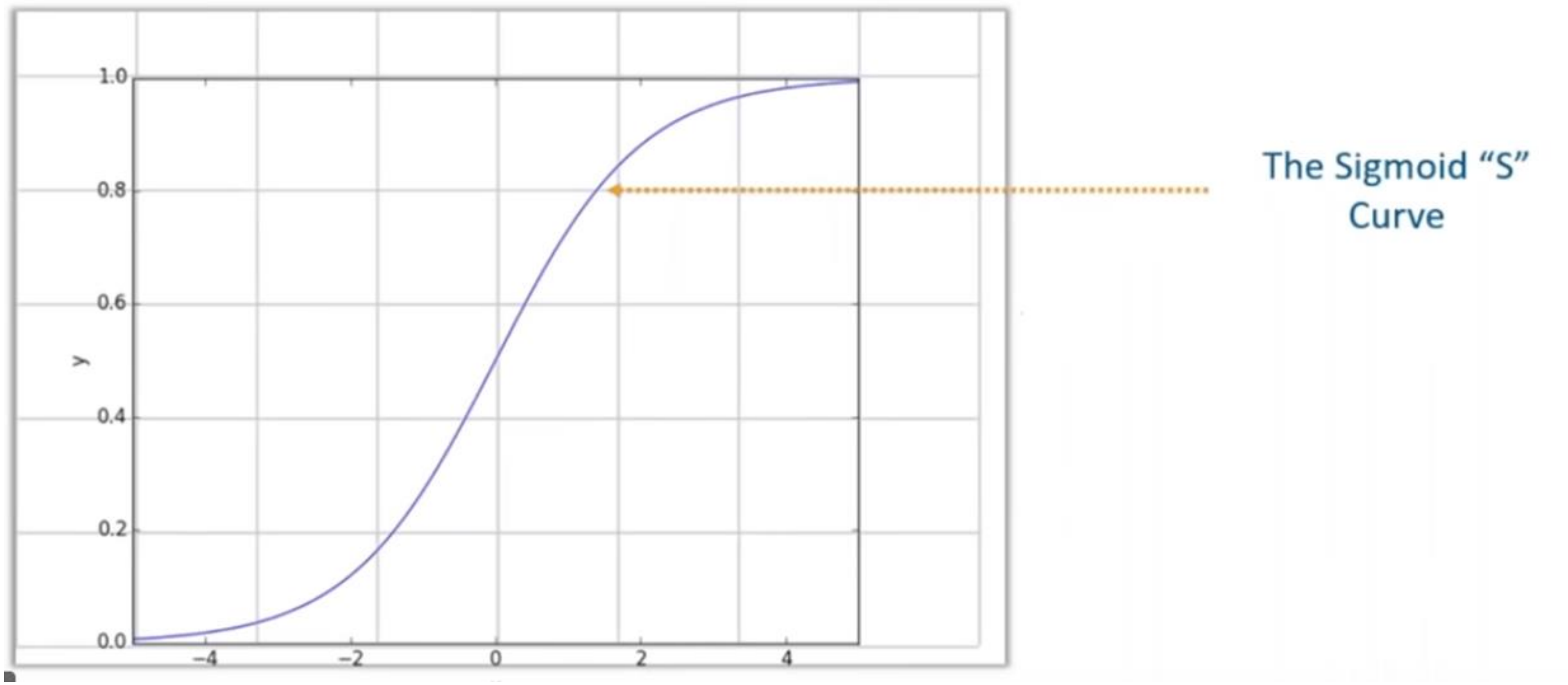
  e^{-y} y is replaced
- Logistic Regression equation: ***$p = 1 / 1 + e^{-(\beta 0 + \beta 1X1 + \beta 2X2 .... + \beta nXn)}$***

Convert Linear to Logistics

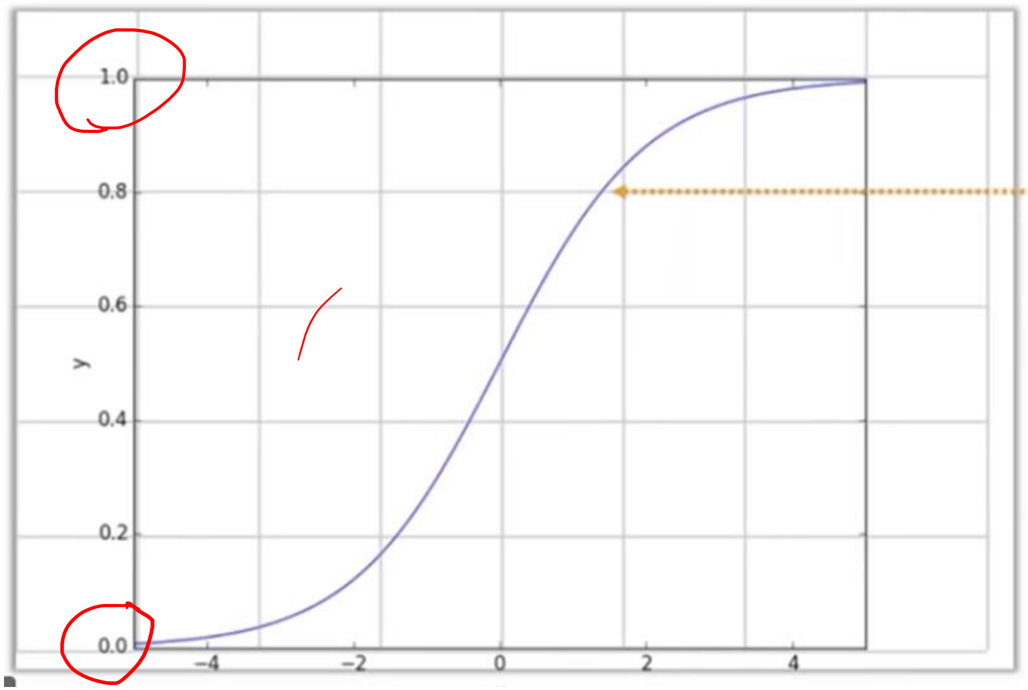
- Linear regression equation: **y = β0 + β1X1 + β2X2 …. + βnXn**
- Sigmoid function: ***p*** **= 1 / 1 + e^{-y}**

  e^{-y} y is replaced
- Logistic Regression equation: ***p*** **= 1 / 1 + e$^{-(β0 + β1X1 + β2X2 …. + βnXn)}$**

Sigmoid – S-curve



Sigmoid – S-curve



The Sigmoid "S" Curve

Sigmoid

- Sigmoid curve converts any value from

-infinity to +infinity to (0 to 1)

- Sigmoid will output:

‘0’ as x approaches $-\infty$

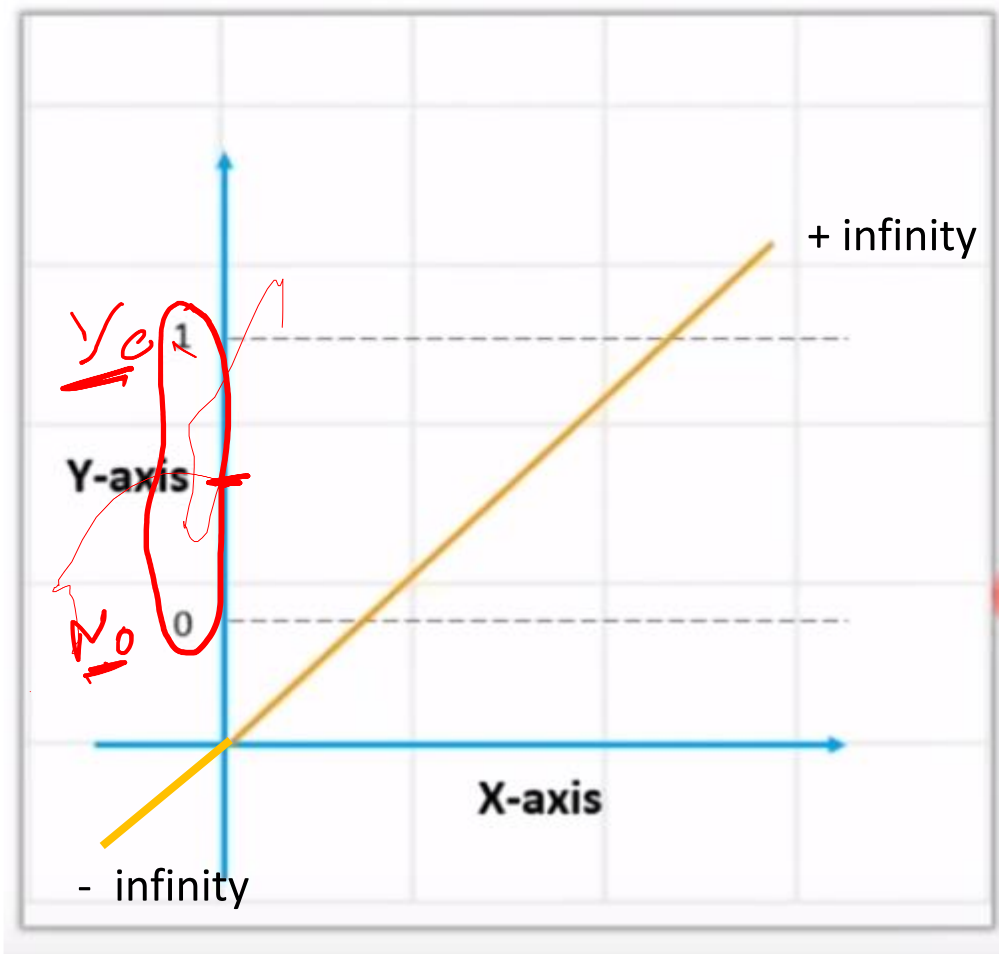
‘1’ as x approaches $+\infty$

Linear Regression

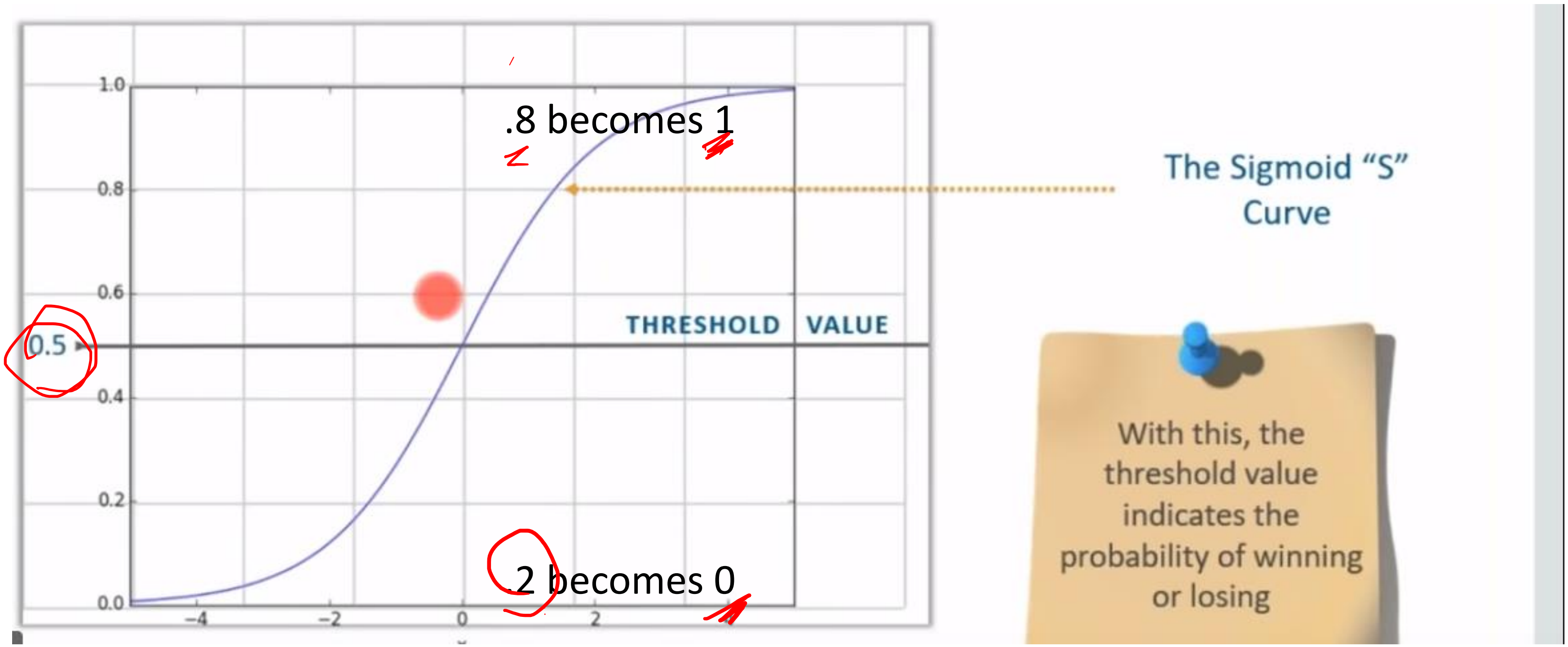
3 1 2 Yes

2

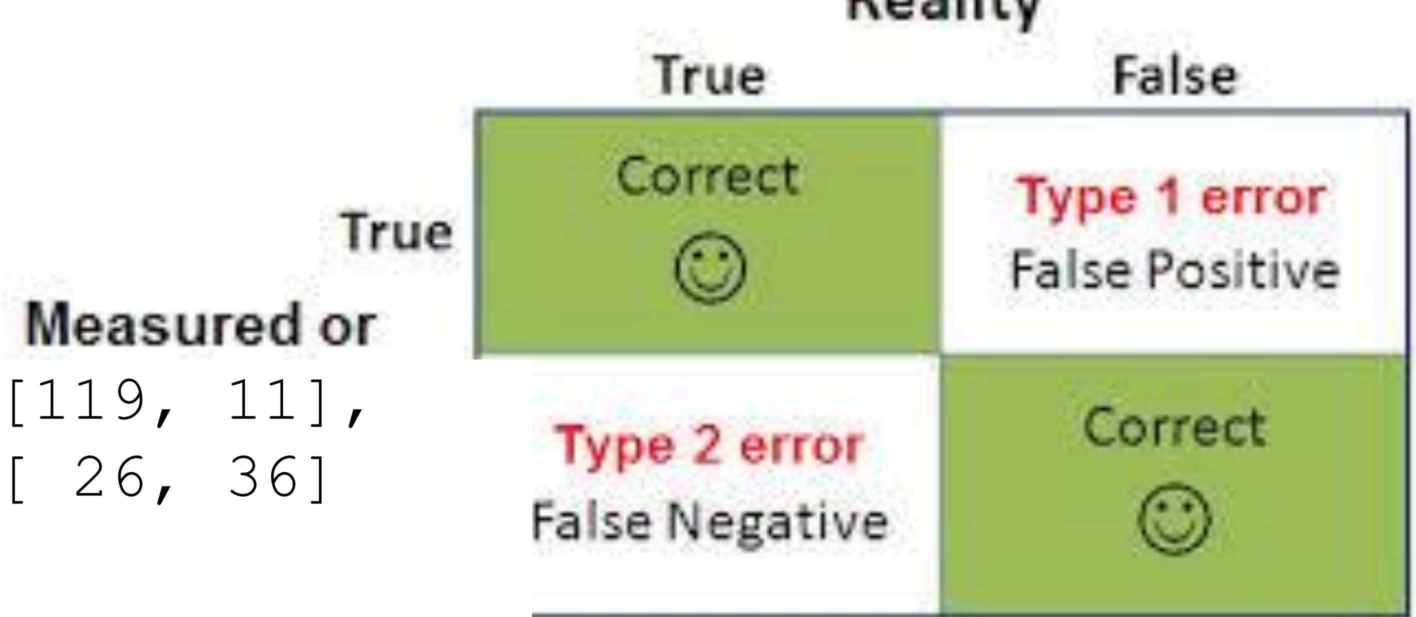
1 0 1 No



Probability values for the answers

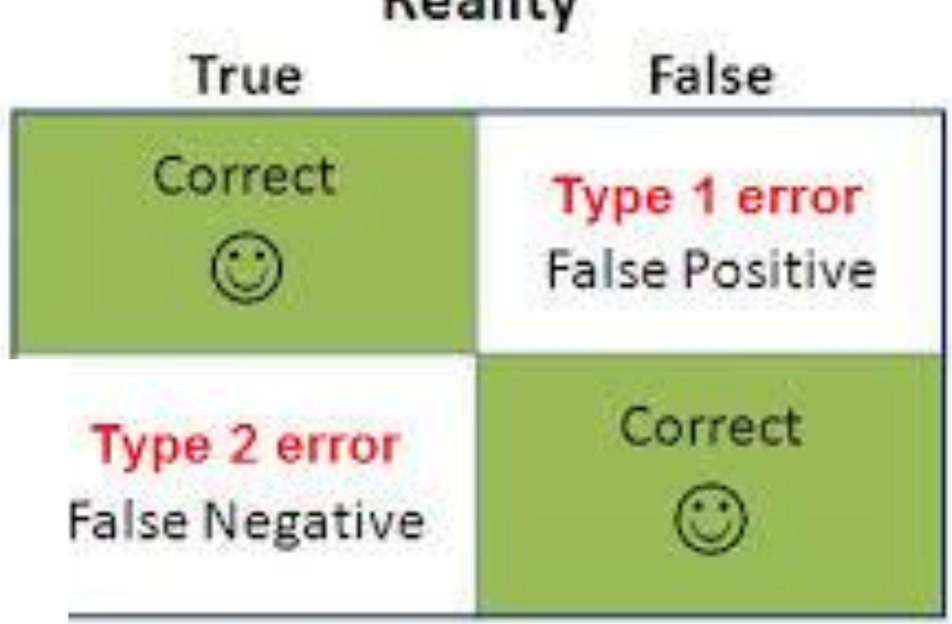


		Reality	
		True	False
Measured or Perceived	True	Correct ☺	Type 1 error False Positive
	False	Type 2 error False Negative	Correct ☺




```
[[119, 11],
 [ 26, 36]
```

Accuracy=
(TP + TN)/
(TP+TN+FP+FN)

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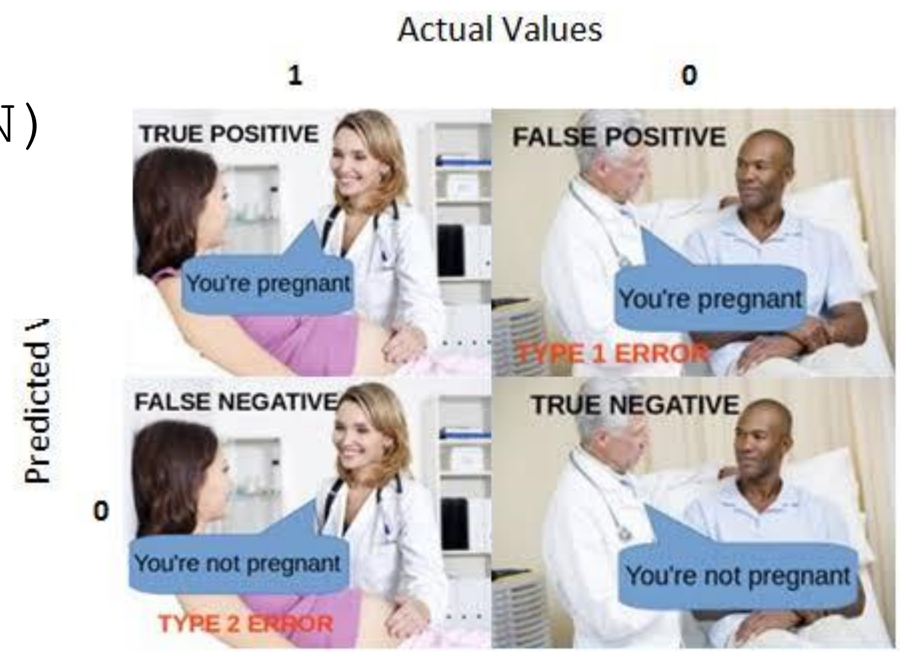

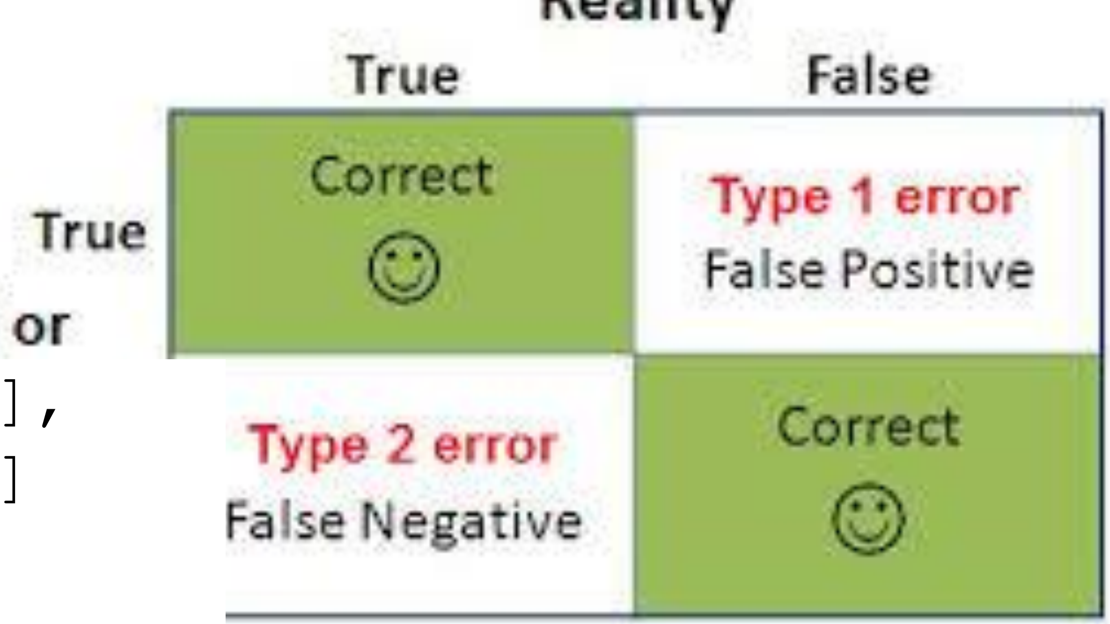

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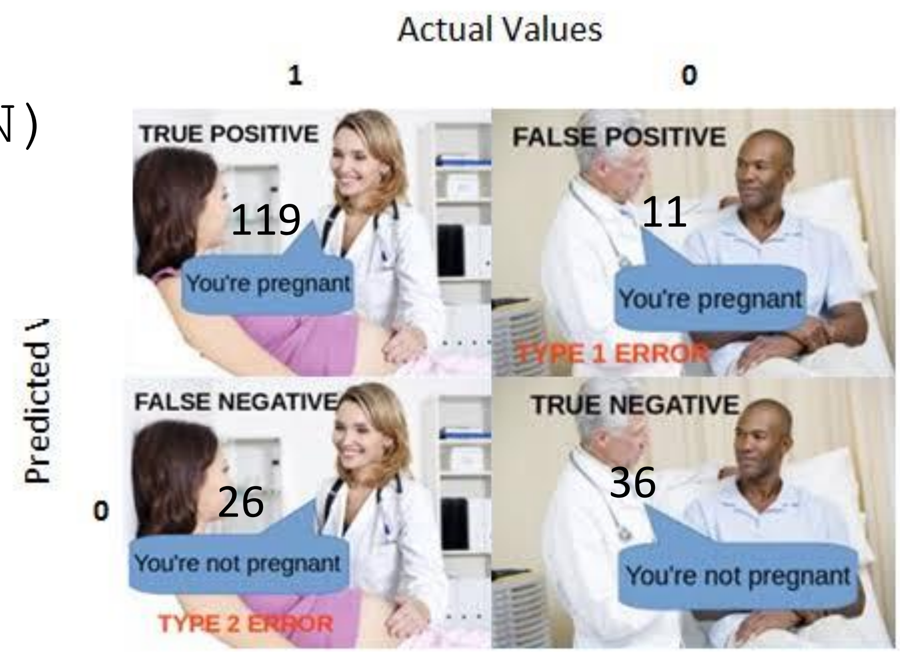
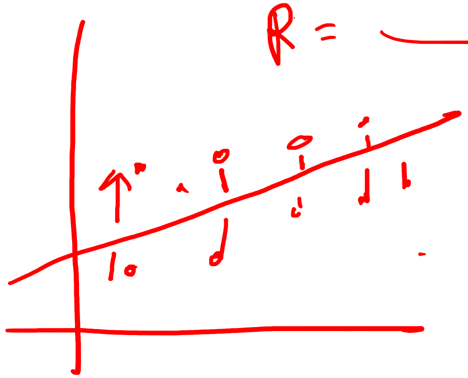
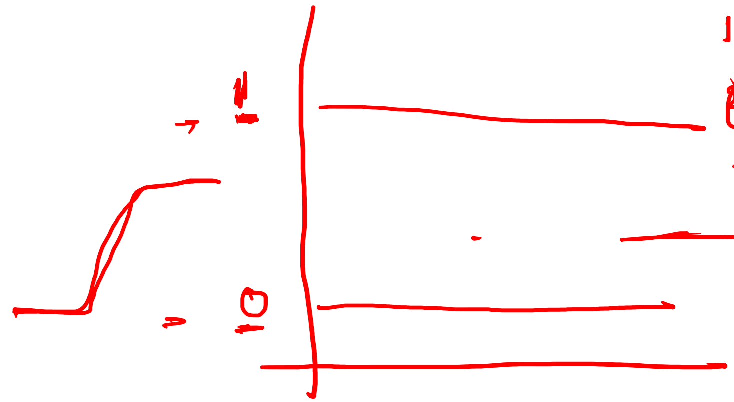


$R^2 =$



$y - y$



A	P	
0	0	✓
1	0	×
0	1	×
1	1	✓

$y \log y + 1 - y \log(1 - y)$

False Positive Rate (caused by Type I Error)

Type1 Error- not to worry

Sometimes, this error might translate to a simple case where a person is predicted to have some bacterial infection while actually that might not be the case. The medication to treat simple bacterial infections might not be very dangerous and is believed to have very mild or no side effects on the patient.

So, in such cases, we might not worry much about the Type I error..

False Positive Rate (caused by Type I Error)

Type1 Error- if it is cancer then we Need to worry

Things can get complicated and serious if Type 1 error happens in a scenario where a person *not* suffering from cancer is diagnosed to have cancer. This can be really dangerous and sometimes fatal due to the high doses of radiation and chemotherapy that a patient can be exposed to.

True Negative Rate (or Specificity)

True Negative Rate (or Specificity)is a metric that tells us how often the model predicts ‘no’ for an actual ‘no’. It is equivalent to 1 minus False Positive Rate.

False Negative Rate (caused by Type II Error):

Number of items the model wrongly predicted ‘no’ out of the total actual ‘yes’. This metric is especially important in most binary classification problems, as it tells us the frequency with which a positive instance is wrongly identified as negative. For example, if a cancer patient is wrongly diagnosed as not having cancer, that individual would either go undiagnosed or misdiagnosed. Similarly, identifying a fraudulent transaction as non-fraudulent can cause several serious repercussions for a bank. Hence, whenever we intend our model to be a diagnostic aid, we would always want this metric to be as low as possible.

Precision — Out of all the examples that predicted as positive, how many are really positive?
positive predictive value

		Real	
		Positive	Negative
Predicted	Positive	True Positive (tp)	False Positive (fp)
	Negative	False Negative (fn)	True Negative (tn)

$$\boldsymbol{precision} = \frac{tp}{tp + fp}$$

$$\boldsymbol{recall} = \frac{tp}{tp + fn}$$

$$\boldsymbol{specificity} = \frac{tn}{tn + fp}$$

Recall — Out of all the positive examples, how many are predicted as positive?

Specificity — Out of all the people that do not have the disease, how many got negative results?

Recall and Sensitivity are the same.

$$\textbf{Precision} = \frac{\text{True Positive}}{\text{Actual Results}} \quad \text{or} \quad \frac{\text{True Positive}}{\text{True Positive + False Positive}}$$

Precision — Out of all the examples that predicted as positive, how many are really positive?
positive predictive value

		Real	
		Positive	Negative
Predicted	Positive	True Positive (tp)	False Positive (fp)
	Negative	False Negative (fn)	True Negative (tn)

$$precision = \frac{tp}{tp + fp}$$

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Recall — Out of all the positive examples, how many are predicted as positive?

Specificity — Out of all the people that do not have the disease, how many got negative results?

Recall and Sensitivity are the same.

$$\textbf{Recall} = \frac{\text{True Positive}}{\text{Predicted Results}} \quad \text{or} \quad \frac{\text{True Positive}}{\text{True Positive + False Negative}}$$

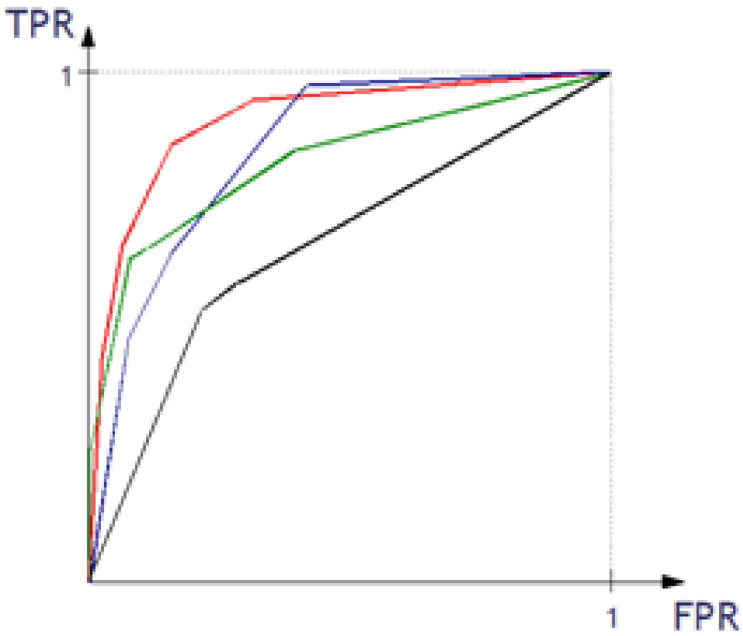
What is the ROC curve?

The Receiver Operating Characteristic (ROC) curve is one of the methods for visualizing classification quality, which shows the dependency between TPR (True Positive Rate) and FPR (False Positive Rate).

predicted → real ↓	*Class_pos*	*Class_neg*
Class_pos	TP	FN
Class_neg	FP	TN

$$\text{TPR (sensitivity)} = \frac{\text{TP}}{\text{TP}+\text{FN}}$$

$$\text{FPR (1-specificity)} = \frac{\text{FP}}{\text{TN}+\text{FP}}$$



Thanks

End