Logistic Regression

Logistic Function - $f(x) = {}^{L}/_{1 + e^{-k(x-x_0)}}$

L –> Curves Max Values

K -> Growth Rate/Steepness of Curve

x_0 -> Sigmoid Value at mid-point

<u>Ideal Scenario –</u>

L=1

K=1

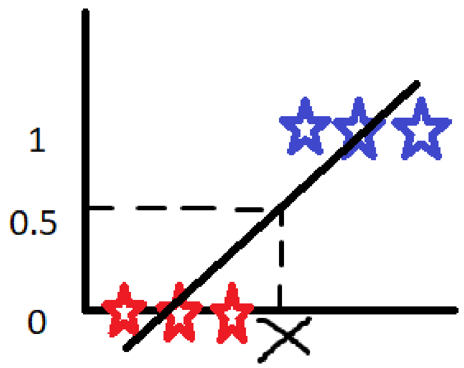
x_0=0

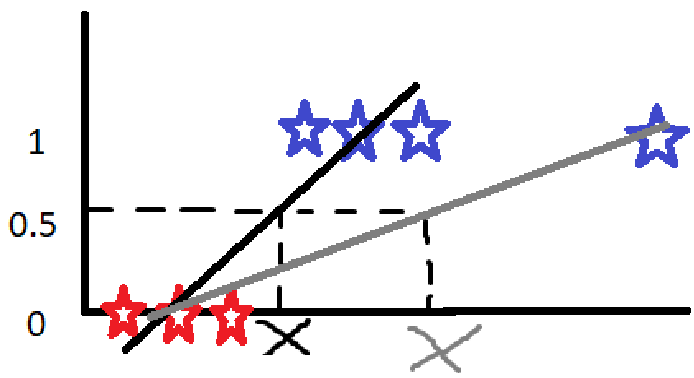
Equation Becomes – $f(x) = 1/1 + e^{-x}$ = $e^{x}/1 + e^{x}$

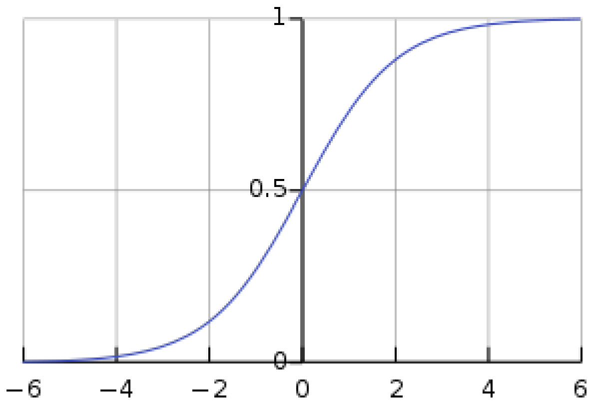
At x = 0, 1/1+1 = 0.5

At x = +∞, f(x) = 1

At x = -∞, f(x) = 0







Applying to ML-

Hypothesis - $h_{\ominus}(x) = {}^{1}/_{1 + e\, - \ominus^{Tx}}$

In the above equation X & ⊖ are two vectors. If X has 2 features then the term $\ominus^{Tx}$ resembles to –

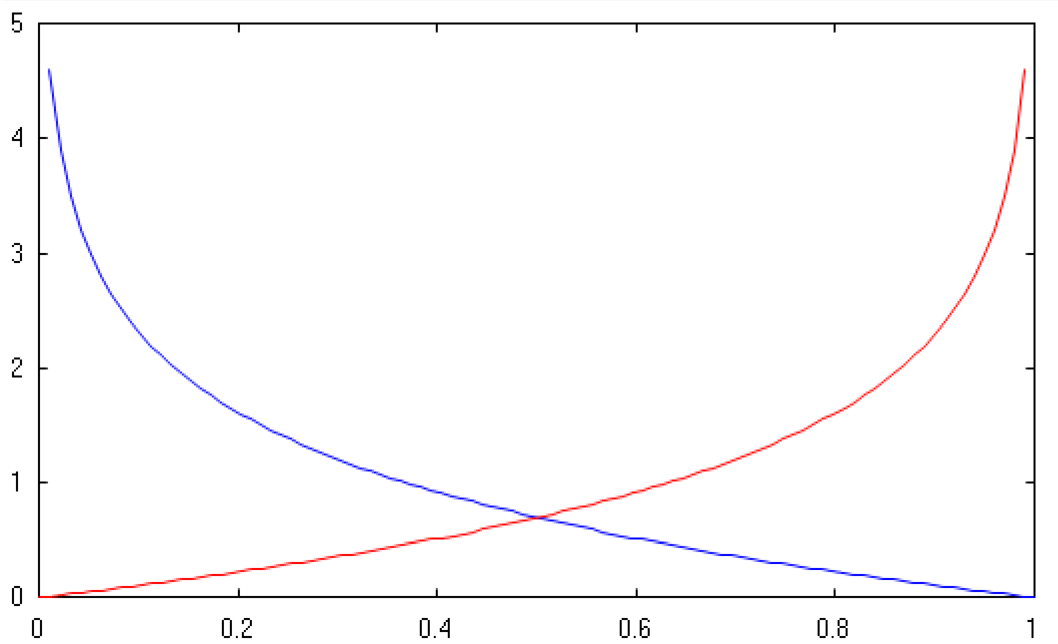
$$\theta_0 + \theta_1 x_1 + \theta_2 x_2$$

H(x) >=0.5 => y=1

H(x) < 0.5 => y=0

Cost Function –

$$\mathrm{cost}(h_\theta(x), y) = \begin{cases} -\log h_\theta(x) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



Y=1, h(x) =0 => -log(0) => -∞

Y=0, h(x)=1 => -log(0) => -∞

P(y=1|x=0) = -∞

P(y=0|x=1) = -log(1-1) = -∞

Final Cost Function for all samples –

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$
$$= -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

For y=0, the first part of the eqn. disappears and for y=1 2nd part disappears. This resembles to the earlier cost function only.

Confusion Matrix:

n=165	**Predicted: NO**	**Predicted: YES**	
Actual: NO	TN = 50	FP = 10	60
Actual: YES	FN = 5	TP = 100	105
	55	110	

Few Basic Terms:

Accuracy -
TP+TN/TP+FP+FN+TN

Precision - Correctly predicted positive observations/ Total Predicted Positive
Precision = TP/TP+FP

Recall (Sensitivity) - Correctly predicted positive observations/ All Actual Positive
Recall = TP/TP+FN

F1 score - F1 Score is the weighted average of Precision and Recall.

F1 Score = 2*(Recall * Precision) / (Recall + Precision)