



Introduction to Logarithms

In its simplest form, a logarithm answers the question:

How many of *one number* do we multiply to get *another number*?

Example: How many **2s** do we multiply to get **8**?

Answer: $2 \times 2 \times 2 = 8$, so we had to multiply **3** of the **2s** to get **8**

So the logarithm is **3**

How to Write it

We write "the number of 2s we need to multiply to get 8 is 3" as:

$$\log_2(8) = 3$$

So these two things are the same:

$$\underbrace{2 \times 2 \times 2}_{3} = 8 \quad \leftrightarrow \quad \log_2(8) = 3$$

The number we multiply is called the "base", so we can say:

- "the logarithm of 8 with base 2 is 3"
- or "log base 2 of 8 is 3"
- or "the base-2 log of 8 is 3"

Notice we are dealing with three numbers:

- the **base**: the number we are multiplying (a "2" in the example above)
- how often to use it in a multiplication (3 times, which is the **logarithm**)
- The number we want to get (an "8")

More Examples

Example: What is $\log_5(625)$... ?

We are asking "how many 5s need to be multiplied together to get 625?"

$5 \times 5 \times 5 \times 5 = 625$, so we need 4 of the 5s

Answer: $\log_5(625) = 4$

Example: What is $\log_2(64)$... ?

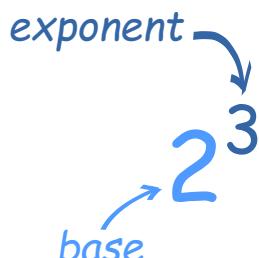
We are asking "how many 2s need to be multiplied together to get 64?"

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, so we need 6 of the 2s

Answer: $\log_2(64) = 6$

Exponents

Exponents and Logarithms are related, let's find out how ...



The **exponent** says **how many times** to use the number in a multiplication.

In this example: $2^3 = 2 \times 2 \times 2 = 8$

(2 is used 3 times in a multiplication to get 8)

So a logarithm answers a question like this:

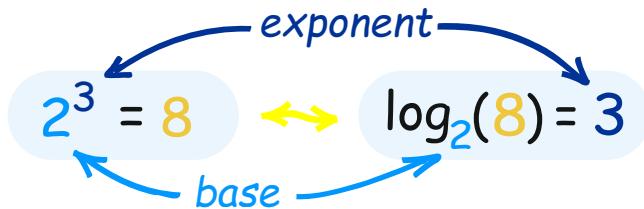
$$2^? = 8$$

In this way:

$$\begin{array}{c} 2^3 = 8 \\ \downarrow \quad \swarrow \quad \searrow \\ \log_2(8) = 3 \end{array}$$

The logarithm tells us what the exponent is!

In that example the "base" is 2 and the "exponent" is 3:



So the logarithm answers the question:

What exponent do we need
(for one number to become another number) ?

The **general** case is:

$$\begin{array}{c} a^x = y \\ \downarrow \quad \swarrow \quad \searrow \\ \log_a(y) = x \end{array}$$

Example: What is $\log_{10}(100)$... ?

$$10^2 = 100$$

So an exponent of 2 is needed to make 10 into 100, and:

$$\log_{10}(100) = 2$$

Example: What is $\log_3(81)$... ?

$$3^4 = 81$$

So an exponent of 4 is needed to make 3 into 81, and:

$$\log_3(81) = 4$$

Common Logarithms: Base 10

Sometimes a logarithm is written **without** a base, like this:

$$\log(100)$$

This **usually** means that the base is really 10.



It is called a "common logarithm". Engineers love to use it.

On a calculator it is the "log" button.

It is how many times we need to use 10 in a multiplication, to get our desired number.

Example: $\log(1000) = \log_{10}(1000) = 3$

Natural Logarithms: Base "e"

Another base that is often used is [e \(Euler's Number\)](#) which is about 2.71828.



This is called a "natural logarithm". Mathematicians use this one a lot.

On a calculator it is the "ln" button.

It is how many times we need to use "e" in a multiplication, to get our desired number.

Example: $\ln(7.389) = \log_e(7.389) \approx 2$

Because $2.71828^2 \approx 7.389$

But Sometimes There Is Confusion ... !

Mathematicians use "log" (instead of "ln") to mean the natural logarithm. This can lead to confusion:

Example	Engineer Thinks	Mathematician Thinks	
$\log(50)$	$\log_{10}(50)$	$\log_e(50)$	confusion
$\ln(50)$	$\log_e(50)$	$\log_e(50)$	no confusion
$\log_{10}(50)$	$\log_{10}(50)$	$\log_{10}(50)$	no confusion

So, be careful when you read "log" that you know what base they mean!

Logarithms Can Have Decimals

All of our examples have used whole number logarithms (like 2 or 3), but logarithms can have decimal values like 2.5, or 6.081, etc.

Example: what is **log₁₀(26)** ... ?



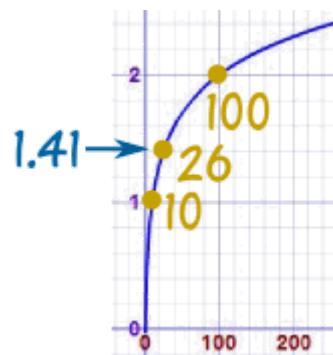
Get your calculator, type in **26** and press **log**

Answer is: **1.41497...**

The logarithm is saying that $10^{1.41497\dots} = 26$
(10 with an exponent of **1.41497...** equals 26)

This is what it looks like on a graph:

See how nice and smooth the line is.



Read [Logarithms Can Have Decimals](#) to find out more.

Negative Logarithms

Negative? But logarithms deal with multiplying.
What is the opposite of multiplying? **Dividing!**

A negative logarithm means how many times **to divide** by the number.

We can have just one divide:

Example: What is $\log_8(0.125)$... ?

Well, $1 \div 8 = 0.125$,

So $\log_8(0.125) = -1$

Or many divides:

Example: What is $\log_5(0.008)$... ?

$1 \div 5 \div 5 \div 5 = 5^{-3}$,

So $\log_5(0.008) = -3$

It All Makes Sense

Multiplying and Dividing are all part of the same simple pattern.

Let us look at some Base-10 logarithms as an example:

Number	How Many 10s	Base-10 Logarithm
.. etc..		
1000	$1 \times 10 \times 10 \times 10$	$\log_{10}(1000) = 3$
100	$1 \times 10 \times 10$	$\log_{10}(100) = 2$
10	1×10	$\log_{10}(10) = 1$
1	1	$\log_{10}(1) = 0$

0.1	$1 \div 10$	$\log_{10}(0.1) = -1$
0.01	$1 \div 10 \div 10$	$\log_{10}(0.01) = -2$
0.001	$1 \div 10 \div 10 \div 10$	$\log_{10}(0.001) = -3$
.. etc..		

Looking at that table, see how positive, zero or negative logarithms are really part of the same (fairly simple) pattern.

The Word

"Logarithm" is a word made up by Scottish mathematician John Napier (1550-1617), from the Greek word *logos* meaning "proportion, ratio or word" and *arithmos* meaning "number", ... which together makes "ratio-number" !

[Question 1](#) [Question 2](#) [Question 3](#) [Question 4](#) [Question 5](#)
[Question 6](#) [Question 7](#) [Question 8](#) [Question 9](#) [Question 10](#)