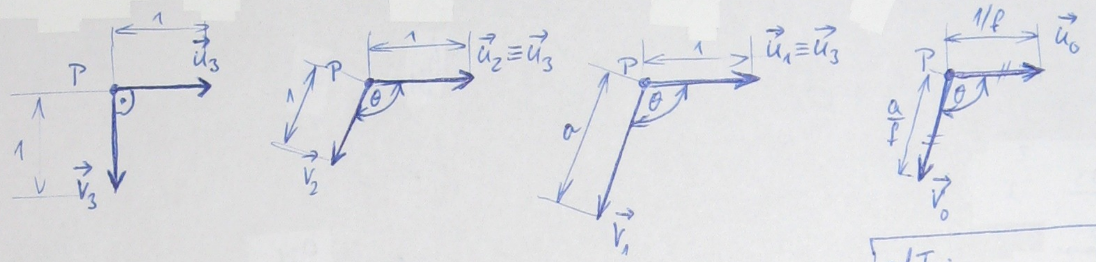
FILIP JAREŠ, TDV, 3.10.2011

TASK: DERIVE THE CAMERA CALIBRATION MATRIX K (HOMEWORK 1)

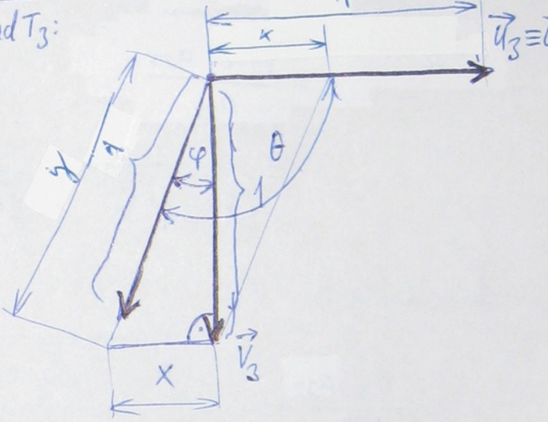
notation: $\vec{x}_{\mathcal{B}}$ – column matrix of coordinates of the vector $\vec{x}$ with respect to the basis $\mathcal{B}$

K maps an orthogonal system to a skewed one

coordinate systems: $\mathcal{B}_3$ $\mathcal{B}_2$ $\mathcal{B}_1$ $\mathcal{B}_0$



ad T_3:



$$\vec{x}_{\mathcal{B}_2} = T_3\, \vec{x}_{\mathcal{B}_3} \qquad T_3 = \begin{bmatrix} \vec{u}_{3_{\mathcal{B}_2}} & \vec{v}_{3_{\mathcal{B}_2}} \end{bmatrix} = \begin{bmatrix} 1 & \operatorname{tg}\varphi \\ 0 & \frac{1}{\cos\varphi} \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 1 & -\operatorname{cotg}\theta \\ 0 & \frac{1}{\sin\theta} \end{bmatrix}$$

$$\vec{v}_3 = x\vec{u}_2 + y\vec{v}_2 \qquad \theta = \frac{\pi}{2} + \varphi$$

$$x = \operatorname{tg}\varphi = -\operatorname{cotg}\theta$$

$$y = \frac{1}{\cos\varphi} = \frac{1}{\sin\theta}$$

$$\vec{v}_3 = -\operatorname{cotg}(\theta)\cdot\vec{u}_2 + \sin(\theta)\cdot\vec{v}_2$$

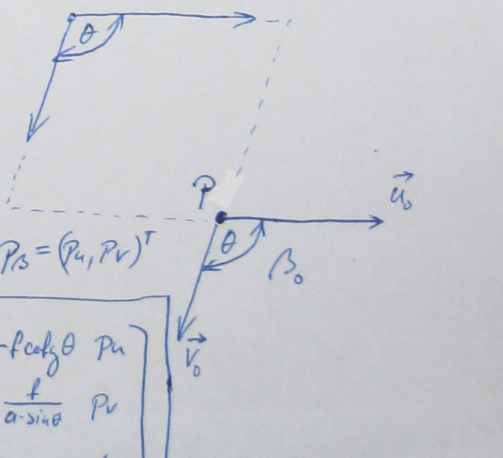
$$\vec{x}_{\mathcal{B}_1} = T_2\, \vec{x}_{\mathcal{B}_2} \qquad T_2 = \begin{bmatrix} \vec{u}_{2_{\mathcal{B}_1}} & \vec{v}_{2_{\mathcal{B}_1}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{a} \end{bmatrix}$$

$$\vec{x}_{\mathcal{B}_0} = T_1\, \vec{x}_{\mathcal{B}_1} \qquad T_1 = \begin{bmatrix} \vec{u}_{1_{\mathcal{B}_0}} & \vec{v}_{1_{\mathcal{B}_0}} \end{bmatrix} = \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix}$$

$$\vec{x}_{\mathcal{B}_0} = \underbrace{T_1 \cdot T_2 \cdot T_3}_{T}\, \vec{x}_{\mathcal{B}_3} \qquad T = T_1 \cdot T_2 \cdot T_3 = \begin{bmatrix} f & -f\operatorname{cotg}\theta \\ 0 & \frac{f}{a\cdot\sin\theta} \end{bmatrix}$$

$$\underline{\vec{x}_{\mathcal{B}_0}} = \begin{pmatrix} u_0 \\ v_0 \\ 1 \end{pmatrix}, \quad \underline{\vec{x}_{\mathcal{B}}} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

– (coordinates of homogenous representation of vector $\vec{x}$ w.t.r. to bases $\mathcal{B}_0, \mathcal{B}$)



$$\underline{\vec{x}_{\mathcal{B}}} = \overbrace{\begin{bmatrix} 1 & 0 & p_u \\ 0 & 1 & p_v \\ 0 & 0 & 1 \end{bmatrix}}^{T_0} \underline{\vec{x}_{\mathcal{B}_0}} \; ; \qquad \underline{x_{\mathcal{B}}} = K \cdot \underline{\vec{x}_{\mathcal{B}_3}}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & & p_u \\ & 1 & p_v \\ & & 1 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \\ 1 \end{pmatrix}$$

$$\boxed{K = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{T_1 \cdot T_2 \cdot T_3} & \begin{matrix} p_u \\ p_v \end{matrix} \\ \hline 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} f & -f\operatorname{cotg}\theta & p_u \\ 0 & \frac{f}{a\cdot\sin\theta} & p_v \\ 0 & 0 & 1 \end{bmatrix}}$$

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$Q = K \cdot R$ MATRIX DECOMPOSITION

$$Q \cdot R_{32} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & +s \\ 0 & -s & c \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Ad R_{32}:

$$c a_{32} - s a_{33} = 0 \quad , \quad c^2 + s^2 = 1$$

$$c a_{32} = s a_{33} \qquad s^2 = 1 - c^2$$

$$c a_{32} = \sqrt{1-c^2}\, a_{33} \qquad (\#)$$

nebezpečná úprava(!)

$$(a_{32}^2 + a_{33}^2) c^2 = a_{33}^2$$

$$c^2 = \frac{a_{33}^2}{(a_{32}^2 + a_{33}^2)}$$

$$c \overset{?}{\in} \left\{ \frac{a_{33}}{\sqrt{a_{32}^2 + a_{33}^2}} \, , \, \cancel{\frac{-a_{33}}{\sqrt{a_{32}^2 + a_{33}^2}}} \right\}$$

Zkusit dosadit do (#):

$$\frac{-a_{33}}{\sqrt{a_{32}^2 + a_{33}^2}} \cdot a_{32} \overset{?}{=} \sqrt{1 - \frac{a_{33}^2}{a_{32}^2 + a_{33}^2}} \cdot a_{33}$$

$$\frac{-a_{33} \cdot a_{32}}{\sqrt{a_{32}^2 + a_{33}^2}} \neq \frac{a_{32} \cdot a_{33}}{\sqrt{a_{32}^2 + a_{33}^2}}$$

$$c = \frac{a_{33}}{\sqrt{a_{32}^2 + a_{33}^2}} \qquad s = \frac{a_{32}}{\sqrt{a_{32}^2 + a_{33}^2}}$$

Ad $R_{31} = \begin{bmatrix} c & 0 & c \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}$

$$c b_{31} - s b_{33} = 0 \qquad c^2 + s^2 = 1$$

$$\Longrightarrow$$

$$c = \frac{b_{33}}{\sqrt{b_{31}^2 + b_{33}^2}} \qquad s = \frac{b_{31}}{\sqrt{b_{31}^2 + b_{31}^2}}$$

Ad $R_{21} = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$Q \cdot R_{32} \cdot R_{31} = c_{ij}$$

$$c c_{21} - s c_{22} = 0$$

$$c = \frac{c_{22}}{\sqrt{c_{21}^2 + c_{22}^2}} \qquad s = \frac{c_{21}}{\sqrt{c_{21}^2 + c_{22}^2}}$$

Contents

Known matrices K*R produce Q

```
K = [...
    1 0 400;
    0 1 600;
    0 0 1];

R = [0.8047   -0.5059   -0.3106; ...
     0.3106    0.8047   -0.5059; ...
     0.5059    0.3106    0.8047];

Q = K*R
```

```
Q =

  203.1647  123.7341  321.5694
  303.8506  187.1647  482.3141
    0.5059    0.3106    0.8047
```

Now, we will decompose Q back to K*R where R = R32*R31*R21

```
% multiplication Q*R32 has to produce result with zero on position 3,2
den = sqrt(Q(3,2)^2 + Q(3,3)^2);
c = Q(3,3)/den;
s = Q(3,2)/den;

R32 = [ ...
    1  0 0; ...
    0  c s; ...
    0 -s c]

B = Q*R32;

% multiplication Q*R32*Q31 has to produce result
% with zeros on positions 3,1 and 3,2
den = sqrt(B(3,1)^2 + B(3,3)^2);
c = B(3,3)/den;
s = B(3,1)/den;

R31 = [ ...
     c 0 s; ...
     0 1 0; ...
    -s 0 c]

C = B*R31;

% now K = Q*R32*R31*R21 should be upper triangular
den = sqrt(C(2,1)^2 + C(2,2)^2);
```

```
c = C(2,2)/den;
s = C(2,1)/den;

R21 = [ ...
     c s 0; ...
    -s c 0; ...
     0 0 1]

R = R32*R31*R21;
K = C*R21;          % K = Q*R


R32 =

    1.0000         0         0
         0    0.9329    0.3601
         0   -0.3601    0.9329


R31 =

    0.8626         0    0.5059
         0    1.0000         0
   -0.5059         0    0.8626


R21 =

    0.9329    0.3601         0
   -0.3601    0.9329         0
         0         0    1.0000
```

Results:

K =

```
   1.0000   -0.0000  399.9899
        0    1.0000  599.9848
   0.0000    0.0000    1.0000
```

R = R32*R31*R21

```
   0.8047    0.3106    0.5059
  -0.5059    0.8047    0.3106
  -0.3106   -0.5059    0.8047
```

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VANISHING POINT, p. 39

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$$\underline{m}_\infty = \lim_{\lambda \to \pm\infty} P \begin{bmatrix} x_0 + \lambda \vec{d} \\ 1 \end{bmatrix} = \lim_{\lambda \to \pm\infty} \lambda \left[Q\vec{d} + \frac{1}{\lambda}\left[Q x_0 + \vec{q} \right] \right]$$

$$\boxed{P \begin{bmatrix} x_0 + \lambda \vec{d} \\ 1 \end{bmatrix} = \begin{bmatrix} Q & \vec{q} \end{bmatrix} \begin{bmatrix} x_0 + \lambda d \\ 1 \end{bmatrix} = Q(x_0 + \lambda \vec{d}) + q = Q x_0 + \lambda Q \vec{d} + q}$$

$$\underline{\underline{m}}_\infty \simeq \lim_{\lambda \to \pm\infty} \frac{1}{\lambda} \lambda \left[Q\vec{d} + \frac{1}{\lambda}\left[Q x_0 + \vec{q} \right] \right] = \lim_{\lambda \to \pm\infty} Q\vec{d} + \frac{1}{\lambda}\left[Q x_0 + \vec{q} \right] = \underline{\underline{Q\vec{d}}}$$

$$\boxed{\underline{m}_\infty \simeq Q\vec{d}}$$