CS 440 - Introduction to Artificial Intelligence
Homework 6

Due Date: Thursday, Oct 26, 12:30pm

1. A heuristic function $h(n)$ is said to be *consistent* if, for every node n and every successor n' of n generated by any action a, the estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n', i.e.
$$h(n) \leq c(n, a, n') + h(n')$$
where $c(n, a, n')$ is the step cost of getting to n' by taking action a at n.

   (a) Prove that if a heuristic is consistent it must be admissible.

   Ans. Let $k(n)$ be the cost of the cheapest path from n to the goal node. We will proove by induction on the number of steps to the goal that $h(n) \leq k(n)$.

   Base case: If there are 0 steps to the goal from node n, then n is a goal and therefore $h(n) = 0 \leq k(n)$.

   Induction step: If n is i steps away from the goal, there must exist some successor n' of n generated by some action a s.t. n' is on the optimal path from n to the goal (via action a) and n' is $i - 1$ steps away from the goal. Therefore,
$$h(n) \leq c(n, a, n') + h(n')$$
   But by the induction hypothesis, $h(n') \leq k(n')$. Therefore,
$$h(n) \leq c(n, a, n') + k(n') = k(n)$$
   since n' is on the optimal path from n to the goal via action a.

   QED

   (b) Construct an admissible heuristic for some search problem, that is not consistent.

   Ans Consider a search problem where the states are nodes along a path $P = n_0, n_1, \ldots, n_m$ where n_0 is the start state, n_m is the goal state and there is one action from each state n_i which gives n_{i+1} as a successor with cost 1. The cheapest cost to the goal from a state i is then $k(n_i) = m - i$. Define a heuristic function as follows:
$$h(n_i) = m - 2\lceil i/2 \rceil$$
   For all states n_i, $h(n_i) \leq k(i)$, and so h is admissible. However, if i is odd, then $h(n_i) = h(n_{i+1}) > 1 + h((n_{i+1})$. Thus h is not consistent.

2. Victor has been murdered, and Arthur, Bertram, and Carleton are suspects. Arthur says he did not do it. He says that Bertram was the victim's friend but that Carleton hated the victim. Bertram says he was out of town the day of the murder, and besides he didn't even know the guy. Carleton says he is innocent and he saw Arthur and Bertram with the victim just before the murder. Assuming that everyone–except possibly for the murderer–is telling the truth, use resolution to solve the crime.

Ans. Let Am, Bm, Cm be the propositions that A, B, C respectively is the murderer. Let Ao, Bo, Co be the propositions that A, B, C respectively was out of town at the time of the murder. Let Ah, Bh, Ch be the propositions that A, B, C respectively were friends with the victim . The statements we have from a suspects are true iff he is not the murderer. Therefore:

$$Am \vee (Bf \wedge \neg Cf)$$

$$Bm \vee (\neg Bf \wedge Bo)$$

$$Cm \vee (\neg Bo \wedge \neg Ao)$$

Also, there is only one murderer:

$$Am \oplus Bm \oplus Cm$$

Converting the above into CNF,

$$Am \vee Bf \tag{1}$$

$$Am \vee \neg Cf \tag{2}$$

$$Bm \vee Bo \tag{3}$$

$$Bm \vee \neg Bf \tag{4}$$

$$Cm \vee \neg Bo \tag{5}$$

$$Cm \vee \neg Ao \tag{6}$$

$$\neg Am \vee \neg Bm \tag{7}$$

$$\neg Am \vee \neg Cm \tag{8}$$

$$\neg Bm \vee \neg Cm \tag{9}$$

Resolving 1,4 we get

$$Am \vee Bm \tag{10}$$

Resolving 3,5 we get

$$Bm \vee Cm \tag{11}$$

Resolving 10, 8 we get

$$Bm \vee \neg Cm \tag{12}$$

Resolving 11, 12 we get

$$Bm \tag{13}$$

Therefore, Bertram is the murderer.