

## ▼ Keras – MLPs on MNIST

```
1 # if you keras is not using tensorflow as backend set "KERAS_BACKEND=tensorflow" use this command
2 from keras.utils import np_utils
3 from keras.datasets import mnist
4 import seaborn as sns
5 from keras.initializers import RandomNormal
```

```
1 %matplotlib inline
2 %matplotlib notebook
3 import matplotlib.pyplot as plt
4 import numpy as np
5 import time
6 # https://gist.github.com/greydanus/f6eee59eaf1d90fcb3b534a25362cea4
7 # https://stackoverflow.com/a/14434334
8 # this function is used to update the plots for each epoch and error
9 def plt_dynamic(x, vy, ty, fig, ax, colors=['b']):
10     ax.plot(x, vy, 'b', label="Validation Loss")
11     ax.plot(x, ty, 'r', label="Train Loss")
12     plt.legend()
13     plt.grid()
14     fig.canvas.draw()
```

```
1 # the data, shuffled and split between train and test sets
2 (X_train, y_train), (X_test, y_test) = mnist.load_data()
```

```
1 print("Number of training examples :", X_train.shape[0], "and each image is of shape (%d, %d)"%(X_train.shape[1], X_train.shape[2]))
2 print("Number of training examples :", X_test.shape[0], "and each image is of shape (%d, %d)"%(X_test.shape[1], X_test.shape[2]))
```

➡ Number of training examples : 60000 and each image is of shape (28, 28)  
Number of training examples : 10000 and each image is of shape (28, 28)

```
1 # if you observe the input shape its 2 dimensional vector
2 # for each image we have a (28*28) vector
3 # we will convert the (28*28) vector into single dimensional vector of 1 * 784
4
5 X_train = X_train.reshape(X_train.shape[0], X_train.shape[1]*X_train.shape[2])
6 X_test = X_test.reshape(X_test.shape[0], X_test.shape[1]*X_test.shape[2])
```

```
1 # after converting the input images from 3d to 2d vectors
2
3 print("Number of training examples :", X_train.shape[0], "and each image is of shape (%d)"%(X_train.shape[1]))
4 print("Number of training examples :", X_test.shape[0], "and each image is of shape (%d)"%(X_test.shape[1]))
```

➡ Number of training examples : 60000 and each image is of shape (784)  
Number of training examples : 10000 and each image is of shape (784)

```
1 # An example data point
2 print(X_train[0])
```

➡

[illegible]

```
1 # if we observe the above matrix each cell is having a value between 0-255
2 # before we move to apply machine learning algorithms lets try to normalize the data
3 #  $X \Rightarrow (X - X_{min}) / (X_{max} - X_{min}) = X / 255$ 
4
5 X_train = X_train/255
6 X_test = X_test/255
```

```
1 # example data point after normalizing
2 print(X_train[0])
```



[illegible]

[illegible]

```
1 # here we are having a class number for each image
2 print("Class label of first image :", y_train[0])
3
4 # lets convert this into a 10 dimensional vector
5 # ex: consider an image is 5 convert it into 5 => [0, 0, 0, 0, 0, 1, 0, 0, 0, 0]
6 # this conversion needed for MLPs
7
8 Y_train = np_utils.to_categorical(y_train, 10)
9 Y_test = np_utils.to_categorical(y_test, 10)
10
11 print("After converting the output into a vector :", Y_train[0])
```

```

➡ Class label of first image : 5
   After converting the output into a vector : [0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]

```

## Softmax classifier

```

1 # https://keras.io/getting-started/sequential-model-guide/
2
3 # The Sequential model is a linear stack of layers.
4 # you can create a Sequential model by passing a list of layer instances to the constructor:
5
6 # model = Sequential([
7 #     Dense(32, input_shape=(784,)),
8 #     Activation('relu'),
9 #     Dense(10),
10 #     Activation('softmax'),
11 # ])
12
13 # You can also simply add layers via the .add() method:
14
15 # model = Sequential()
16 # model.add(Dense(32, input_dim=784))
17 # model.add(Activation('relu'))
18
19 ###
20
21 # https://keras.io/layers/core/
22
23 # keras.layers.Dense(units, activation=None, use_bias=True, kernel_initializer='glorot_uniform',
24 # bias_initializer='zeros', kernel_regularizer=None, bias_regularizer=None, activity_regularizer=None,
25 # kernel_constraint=None, bias_constraint=None)
26
27 # Dense implements the operation: output = activation(dot(input, kernel) + bias) where
28 # activation is the element-wise activation function passed as the activation argument,

```

```

29 # kernel is a weights matrix created by the layer, and
30 # bias is a bias vector created by the layer (only applicable if use_bias is True).
31
32 # output = activation(dot(input, kernel) + bias) => y = activation(WT. X + b)
33
34 #####
35
36 # https://keras.io/activations/
37
38 # Activations can either be used through an Activation layer, or through the activation argument supported by all forward layers:
39
40 # from keras.layers import Activation, Dense
41
42 # model.add(Dense(64))
43 # model.add(Activation('tanh'))
44
45 # This is equivalent to:
46 # model.add(Dense(64, activation='tanh'))
47
48 # there are many activation functions available ex: tanh, relu, softmax
49
50
51 from keras.models import Sequential
52 from keras.layers import Dense, Activation
53

```

```

1 # some model parameters
2
3 output_dim = 10
4 input_dim = X_train.shape[1]
5
6 batch_size = 128
7 nb_epoch = 20

```

```

1 %matplotlib inline
2 def loss_plot(model_name):
3     score = model_name.evaluate(X_test, Y_test, verbose=0)
4     print('Test score:', score[0])
5     print('Test accuracy:', score[1])
6
7     fig, ax = plt.subplots(1,1, figsize=(10,6))
8     ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')
9     ax.set_title('Variation of Loss with epochs')
10
11     # list of epoch numbers
12     x = list(range(1,nb_epoch+1))
13
14     # print(history.history.keys())
15     # dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
16     # history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
17
18     # we will get val_loss and val_acc only when you pass the paramter validation_data
19     # val_loss : validation loss
20     # val_acc : validation accuracy
21
22     # loss : training loss
23     # acc : train accuracy
24     # for each key in history.history we will have a list of length equal to number of epochs
25
26
27     vy = history.history['val_loss']
28     ty = history.history['loss']
29     plt_dynamic(x, vy, ty, fig, ax)

```

```

1 def weight_plot(model_name):
2     w_after = model_name.get_weights()
3
4     h1_w = w_after[0].flatten().reshape(-1,1)
5     h2_w = w_after[2].flatten().reshape(-1,1)
6     out_w = w_after[4].flatten().reshape(-1,1)
7
8     fig = plt.figure(figsize=(15,7))
9     fig.suptitle("Weight matrices after model trained")
10    plt.subplot(1, 3, 1)
11    ax = sns.violinplot(y=h1_w,color='b')
12    plt.xlabel('Hidden Layer 1')
13
14    plt.subplot(1, 3, 2)
15    ax = sns.violinplot(y=h2_w, color='r')
16    plt.xlabel('Hidden Layer 2 ')
17
18    plt.subplot(1, 3, 3)
19    ax = sns.violinplot(y=out_w,color='y')
20    plt.xlabel('Output Layer ')
21    plt.show()

```

## ▼ Architecture 1 : Layer1(600), Layer2(100)

### i) MLP + ReLU + ADAM

```

1 import warnings
2 warnings.filterwarnings("ignore")

```

```

1 model_relu = Sequential()

```

```

2 model_relu1.add(Dense(600, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.062, seed=None))
3 model_relu1.add(Dense(100, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.125, seed=None)) )
4 model_relu1.add(Dense(output_dim, activation='softmax'))
5
6 print(model_relu1.summary())
7
8 model_relu1.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
9
10 history = model_relu1.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

```

Model: "sequential\_12"

Layer (type)	Output Shape	Param #
dense_42 (Dense)	(None, 600)	471000
dense_43 (Dense)	(None, 100)	60100
dense_44 (Dense)	(None, 10)	1010
Total params: 532,110		
Trainable params: 532,110		
Non-trainable params: 0		

None

Train on 60000 samples, validate on 10000 samples

```

Epoch 1/20
60000/60000 [=====] - 5s 89us/step - loss: 0.2137 - acc: 0.9351 - val_loss: 0.1095 - val_acc: 0.9665
Epoch 2/20
60000/60000 [=====] - 4s 62us/step - loss: 0.0815 - acc: 0.9755 - val_loss: 0.0956 - val_acc: 0.9713
Epoch 3/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0523 - acc: 0.9841 - val_loss: 0.0708 - val_acc: 0.9760
Epoch 4/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0341 - acc: 0.9890 - val_loss: 0.0684 - val_acc: 0.9798
Epoch 5/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0279 - acc: 0.9913 - val_loss: 0.0741 - val_acc: 0.9779
Epoch 6/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0187 - acc: 0.9941 - val_loss: 0.0639 - val_acc: 0.9809
Epoch 7/20
60000/60000 [=====] - 4s 59us/step - loss: 0.0150 - acc: 0.9954 - val_loss: 0.0711 - val_acc: 0.9788
Epoch 8/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0142 - acc: 0.9957 - val_loss: 0.0800 - val_acc: 0.9794
Epoch 9/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0149 - acc: 0.9948 - val_loss: 0.0872 - val_acc: 0.9779
Epoch 10/20
60000/60000 [=====] - 4s 58us/step - loss: 0.0119 - acc: 0.9960 - val_loss: 0.0853 - val_acc: 0.9800
Epoch 11/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0102 - acc: 0.9964 - val_loss: 0.0839 - val_acc: 0.9796
Epoch 12/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0110 - acc: 0.9962 - val_loss: 0.0901 - val_acc: 0.9780
Epoch 13/20
60000/60000 [=====] - 3s 58us/step - loss: 0.0113 - acc: 0.9962 - val_loss: 0.0879 - val_acc: 0.9795
Epoch 14/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0074 - acc: 0.9975 - val_loss: 0.0951 - val_acc: 0.9791
Epoch 15/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0115 - acc: 0.9965 - val_loss: 0.0745 - val_acc: 0.9822
Epoch 16/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0063 - acc: 0.9979 - val_loss: 0.1070 - val_acc: 0.9766
Epoch 17/20
60000/60000 [=====] - 3s 58us/step - loss: 0.0069 - acc: 0.9978 - val_loss: 0.0906 - val_acc: 0.9803
Epoch 18/20
60000/60000 [=====] - 3s 58us/step - loss: 0.0065 - acc: 0.9979 - val_loss: 0.0869 - val_acc: 0.9806
Epoch 19/20
60000/60000 [=====] - 4s 59us/step - loss: 0.0090 - acc: 0.9972 - val_loss: 0.0911 - val_acc: 0.9809
Epoch 20/20
60000/60000 [=====] - 4s 59us/step - loss: 0.0060 - acc: 0.9980 - val_loss: 0.0855 - val_acc: 0.9838

```

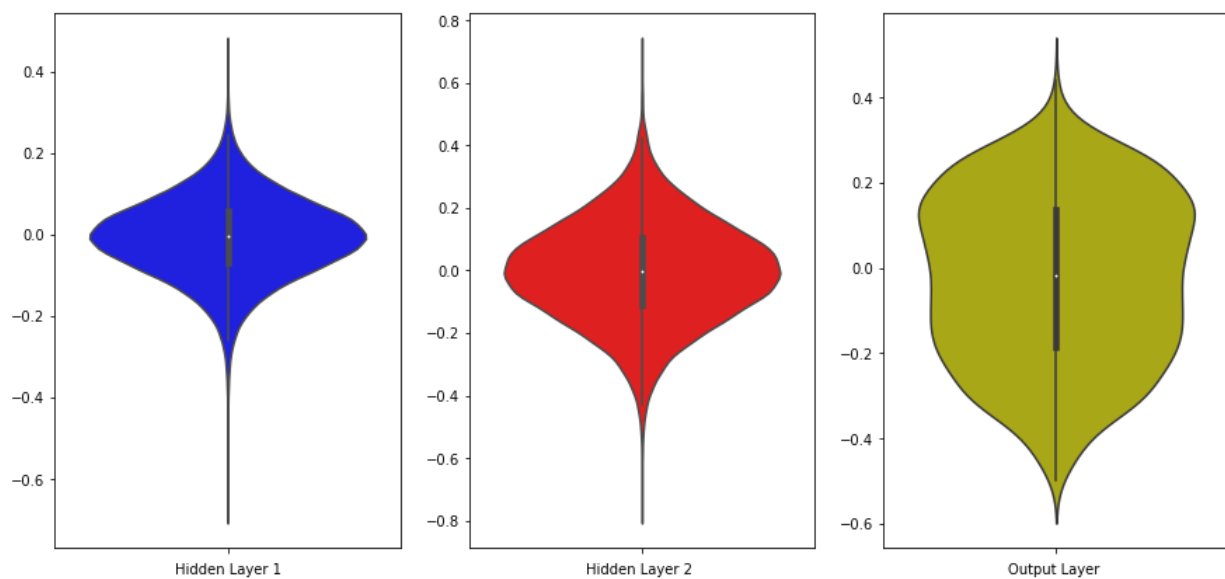
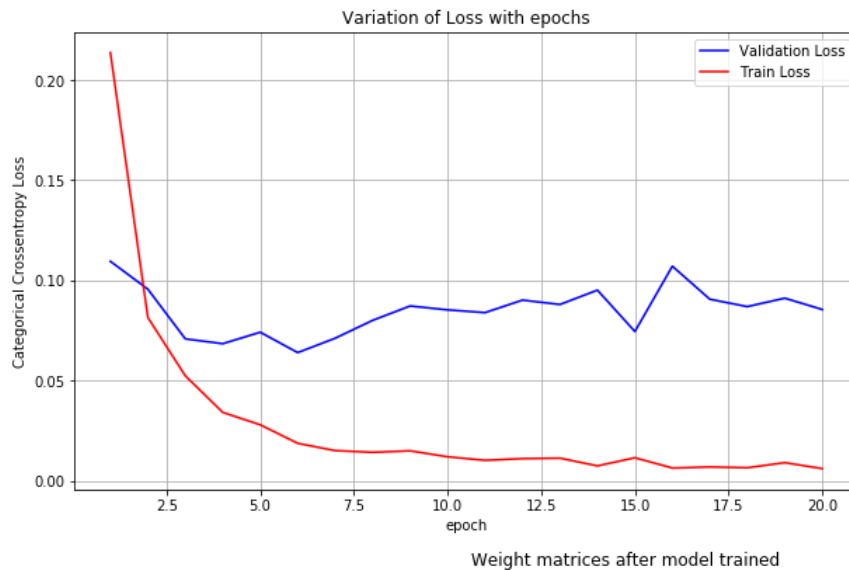
```

1 loss_plot(model_relu1)
2 weight_plot(model_relu1)

```

→

Test score: 0.08545361435633295  
Test accuracy: 0.9838



## ii) MLP + Batch-Norm on hidden Layers + AdamOptimizer

```
1 # Multilayer perceptron
2
3 # https://intoli.com/blog/neural-network-initialization/
4 # If we sample weights from a normal distribution  $N(0, \sigma)$  we satisfy this condition with  $\sigma = \sqrt{2/(n_i + n_{i+1})}$ .
5 # h1 =>  $\sigma = \sqrt{2/(n_i + n_{i+1})} = 0.039 \Rightarrow N(0, \sigma) = N(0, 0.039)$ 
6 # h2 =>  $\sigma = \sqrt{2/(n_i + n_{i+1})} = 0.055 \Rightarrow N(0, \sigma) = N(0, 0.055)$ 
7 # h3 =>  $\sigma = \sqrt{2/(n_i + n_{i+1})} = 0.120 \Rightarrow N(0, \sigma) = N(0, 0.120)$ 
8
9 from keras.layers.normalization import BatchNormalization
10
11 model_batch1 = Sequential()
12
13 model_batch1.add(Dense(600, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.039, seed=None)))
14 model_batch1.add(BatchNormalization())
15
16 model_batch1.add(Dense(100, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.55, seed=None)))
17 model_batch1.add(BatchNormalization())
18
19 model_batch1.add(Dense(output_dim, activation='softmax'))
20
21
22 model_batch1.summary()
```



Model: "sequential\_13"

Layer (type)	Output Shape	Param #
=====		
dense_45 (Dense)	(None, 600)	471000
=====		
batch_normalization_17 (Batch Normalization)	(None, 600)	2400
=====		
dense_46 (Dense)	(None, 100)	60100
=====		
batch_normalization_18 (Batch Normalization)	(None, 100)	400
=====		
dense_47 (Dense)	(None, 10)	1010
=====		
Total params: 534,910		
Trainable params: 533,510		
Non-trainable params: 1,400		
=====		

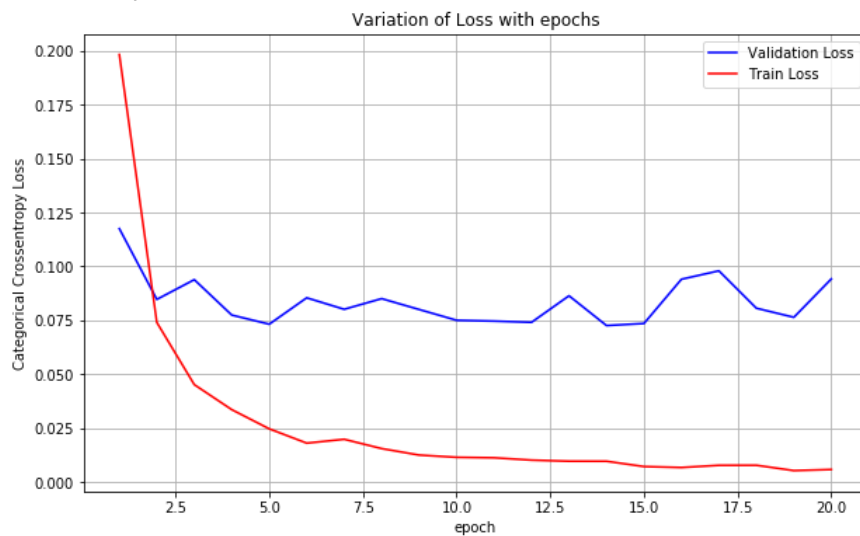
```
1 model_batch1.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
2
3 history = model_batch1.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
4 loss_plot(model_batch1)
5 weight_plot(model_batch1)
```



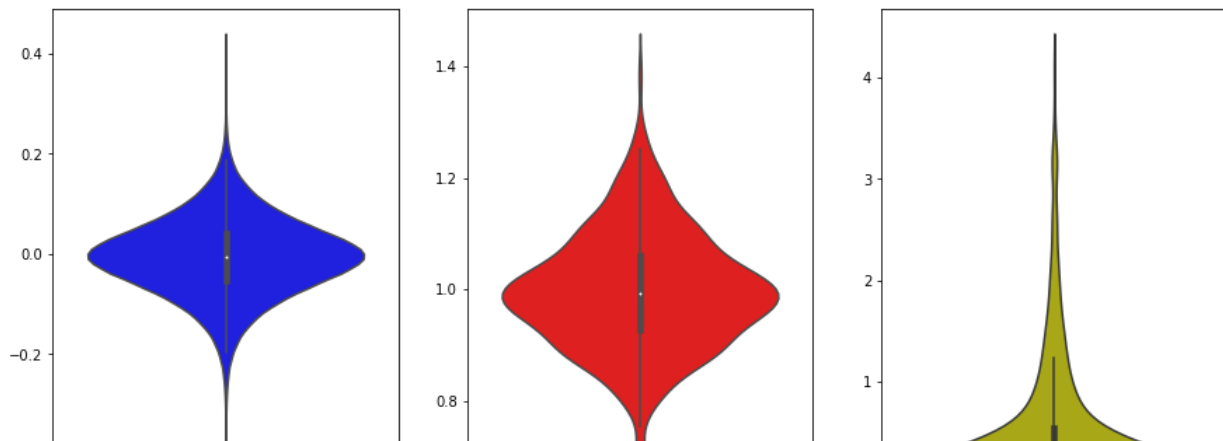


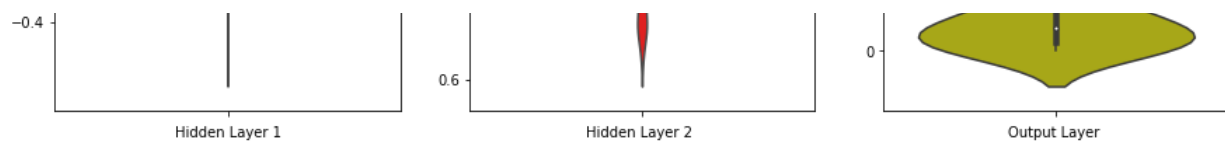
Train on 60000 samples, validate on 10000 samples

Epoch 1/20  
60000/60000 [=====] - 9s 151us/step - loss: 0.1981 - acc: 0.9432 - val\_loss: 0.1175 - val\_acc: 0.961  
Epoch 2/20  
60000/60000 [=====] - 6s 98us/step - loss: 0.0740 - acc: 0.9783 - val\_loss: 0.0847 - val\_acc: 0.9719  
Epoch 3/20  
60000/60000 [=====] - 6s 103us/step - loss: 0.0452 - acc: 0.9870 - val\_loss: 0.0938 - val\_acc: 0.969  
Epoch 4/20  
60000/60000 [=====] - 6s 100us/step - loss: 0.0336 - acc: 0.9897 - val\_loss: 0.0774 - val\_acc: 0.975  
Epoch 5/20  
60000/60000 [=====] - 6s 95us/step - loss: 0.0247 - acc: 0.9926 - val\_loss: 0.0732 - val\_acc: 0.9788  
Epoch 6/20  
60000/60000 [=====] - 6s 105us/step - loss: 0.0181 - acc: 0.9946 - val\_loss: 0.0854 - val\_acc: 0.975  
Epoch 7/20  
60000/60000 [=====] - 6s 108us/step - loss: 0.0199 - acc: 0.9939 - val\_loss: 0.0801 - val\_acc: 0.975  
Epoch 8/20  
60000/60000 [=====] - 7s 109us/step - loss: 0.0156 - acc: 0.9955 - val\_loss: 0.0850 - val\_acc: 0.977  
Epoch 9/20  
60000/60000 [=====] - 7s 109us/step - loss: 0.0126 - acc: 0.9962 - val\_loss: 0.0800 - val\_acc: 0.975  
Epoch 10/20  
60000/60000 [=====] - 7s 113us/step - loss: 0.0115 - acc: 0.9965 - val\_loss: 0.0750 - val\_acc: 0.978  
Epoch 11/20  
60000/60000 [=====] - 7s 110us/step - loss: 0.0113 - acc: 0.9964 - val\_loss: 0.0746 - val\_acc: 0.980  
Epoch 12/20  
60000/60000 [=====] - 6s 106us/step - loss: 0.0102 - acc: 0.9969 - val\_loss: 0.0741 - val\_acc: 0.980  
Epoch 13/20  
60000/60000 [=====] - 6s 106us/step - loss: 0.0097 - acc: 0.9970 - val\_loss: 0.0864 - val\_acc: 0.977  
Epoch 14/20  
60000/60000 [=====] - 7s 114us/step - loss: 0.0097 - acc: 0.9969 - val\_loss: 0.0726 - val\_acc: 0.981  
Epoch 15/20  
60000/60000 [=====] - 7s 113us/step - loss: 0.0072 - acc: 0.9975 - val\_loss: 0.0735 - val\_acc: 0.979  
Epoch 16/20  
60000/60000 [=====] - 7s 111us/step - loss: 0.0068 - acc: 0.9979 - val\_loss: 0.0940 - val\_acc: 0.975  
Epoch 17/20  
60000/60000 [=====] - 7s 110us/step - loss: 0.0078 - acc: 0.9975 - val\_loss: 0.0979 - val\_acc: 0.976  
Epoch 18/20  
60000/60000 [=====] - 7s 112us/step - loss: 0.0078 - acc: 0.9975 - val\_loss: 0.0806 - val\_acc: 0.979  
Epoch 19/20  
60000/60000 [=====] - 7s 110us/step - loss: 0.0054 - acc: 0.9984 - val\_loss: 0.0764 - val\_acc: 0.982  
Epoch 20/20  
60000/60000 [=====] - 6s 106us/step - loss: 0.0059 - acc: 0.9981 - val\_loss: 0.0942 - val\_acc: 0.979  
Test score: 0.09417202328130224  
Test accuracy: 0.9796



Weight matrices after model trained





### iii) MLP + Dropout + AdamOptimizer

```

1 # https://stackoverflow.com/questions/34716454/where-do-i-call-the-batchnormalization-function-in-keras
2
3 from keras.layers import Dropout
4
5 model_drop1 = Sequential()
6
7 model_drop1.add(Dense(600, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.039, seed=None)))
8 model_drop1.add(BatchNormalization())
9 model_drop1.add(Dropout(0.5))
10
11 model_drop1.add(Dense(100, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.55, seed=None)))
12 model_drop1.add(BatchNormalization())
13 model_drop1.add(Dropout(0.5))
14
15 model_drop1.add(Dense(output_dim, activation='softmax'))
16
17
18 model_drop1.summary()

```

Model: "sequential\_14"

Layer (type)	Output Shape	Param #
=====		
dense_48 (Dense)	(None, 600)	471000
batch_normalization_19 (Batch Normalization)	(None, 600)	2400
dropout_15 (Dropout)	(None, 600)	0
dense_49 (Dense)	(None, 100)	60100
batch_normalization_20 (Batch Normalization)	(None, 100)	400
dropout_16 (Dropout)	(None, 100)	0
dense_50 (Dense)	(None, 10)	1010
=====		
Total params: 534,910		
Trainable params: 533,510		
Non-trainable params: 1,400		

```

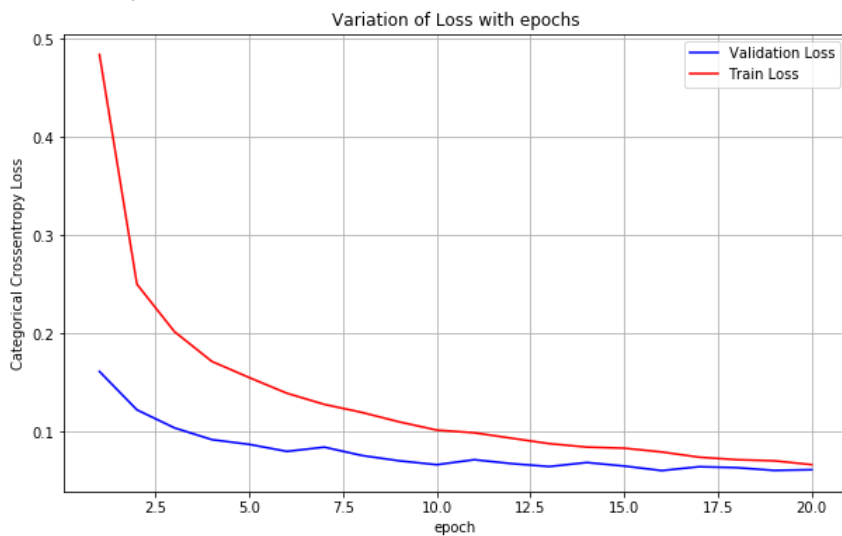
1 model_drop1.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
2
3 history = model_drop1.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
4 loss_plot(model_drop1)
5 weight_plot(model_drop1)

```

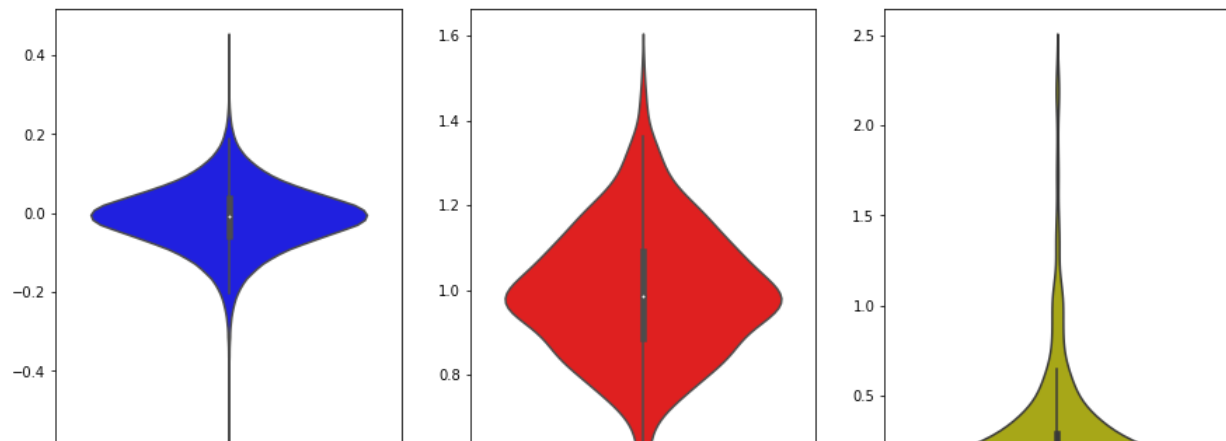


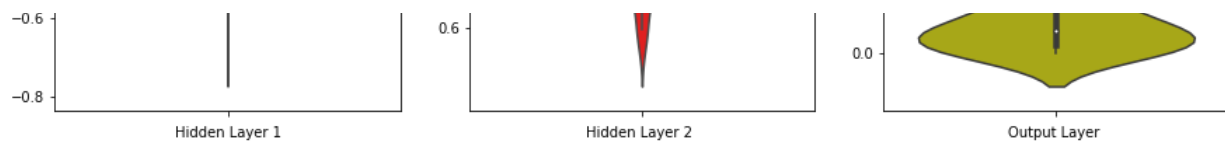
Train on 60000 samples, validate on 10000 samples

```
Epoch 1/20
60000/60000 [=====] - 10s 164us/step - loss: 0.4840 - acc: 0.8543 - val_loss: 0.1615 - val_acc: 0.95
Epoch 2/20
60000/60000 [=====] - 6s 99us/step - loss: 0.2502 - acc: 0.9259 - val_loss: 0.1224 - val_acc: 0.9597
Epoch 3/20
60000/60000 [=====] - 6s 97us/step - loss: 0.2020 - acc: 0.9401 - val_loss: 0.1041 - val_acc: 0.9685
Epoch 4/20
60000/60000 [=====] - 6s 102us/step - loss: 0.1717 - acc: 0.9490 - val_loss: 0.0921 - val_acc: 0.971
Epoch 5/20
60000/60000 [=====] - 6s 101us/step - loss: 0.1552 - acc: 0.9533 - val_loss: 0.0873 - val_acc: 0.973
Epoch 6/20
60000/60000 [=====] - 6s 99us/step - loss: 0.1393 - acc: 0.9583 - val_loss: 0.0802 - val_acc: 0.9752
Epoch 7/20
60000/60000 [=====] - 6s 100us/step - loss: 0.1280 - acc: 0.9612 - val_loss: 0.0845 - val_acc: 0.974
Epoch 8/20
60000/60000 [=====] - 7s 110us/step - loss: 0.1198 - acc: 0.9635 - val_loss: 0.0760 - val_acc: 0.977
Epoch 9/20
60000/60000 [=====] - 7s 122us/step - loss: 0.1102 - acc: 0.9659 - val_loss: 0.0706 - val_acc: 0.978
Epoch 10/20
60000/60000 [=====] - 7s 118us/step - loss: 0.1018 - acc: 0.9693 - val_loss: 0.0666 - val_acc: 0.979
Epoch 11/20
60000/60000 [=====] - 7s 120us/step - loss: 0.0991 - acc: 0.9694 - val_loss: 0.0717 - val_acc: 0.978
Epoch 12/20
60000/60000 [=====] - 7s 116us/step - loss: 0.0935 - acc: 0.9708 - val_loss: 0.0677 - val_acc: 0.979
Epoch 13/20
60000/60000 [=====] - 7s 118us/step - loss: 0.0880 - acc: 0.9725 - val_loss: 0.0648 - val_acc: 0.980
Epoch 14/20
60000/60000 [=====] - 7s 116us/step - loss: 0.0845 - acc: 0.9737 - val_loss: 0.0688 - val_acc: 0.979
Epoch 15/20
60000/60000 [=====] - 8s 129us/step - loss: 0.0834 - acc: 0.9739 - val_loss: 0.0652 - val_acc: 0.981
Epoch 16/20
60000/60000 [=====] - 7s 121us/step - loss: 0.0796 - acc: 0.9744 - val_loss: 0.0606 - val_acc: 0.981
Epoch 17/20
60000/60000 [=====] - 7s 117us/step - loss: 0.0743 - acc: 0.9780 - val_loss: 0.0646 - val_acc: 0.981
Epoch 18/20
60000/60000 [=====] - 6s 108us/step - loss: 0.0717 - acc: 0.9777 - val_loss: 0.0636 - val_acc: 0.982
Epoch 19/20
60000/60000 [=====] - 6s 105us/step - loss: 0.0706 - acc: 0.9780 - val_loss: 0.0608 - val_acc: 0.982
Epoch 20/20
60000/60000 [=====] - 6s 96us/step - loss: 0.0667 - acc: 0.9794 - val_loss: 0.0615 - val_acc: 0.9816
Test score: 0.061509001989616084
Test accuracy: 0.9816
```



Weight matrices after model trained





## ▼ Architecture 2 : Layer1(500), Layer2(250), Layer3(125)

### i) MLP + ReLU + ADAM

```

1 import warnings
2 warnings.filterwarnings("ignore")

3 model_relu2 = Sequential()
4 model_relu2.add(Dense(500, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.062, seed=None)))
5 model_relu2.add(Dense(250, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.125, seed=None)))
6 model_relu2.add(Dense(125, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.125, seed=None)))
7 model_relu2.add(Dense(output_dim, activation='softmax'))
8
9 print(model_relu2.summary())
10
11 model_relu2.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
12 history = model_relu2.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

```



Model: "sequential\_15"

Layer (type)	Output Shape	Param #
dense_51 (Dense)	(None, 500)	392500
dense_52 (Dense)	(None, 250)	125250
dense_53 (Dense)	(None, 125)	31375
dense_54 (Dense)	(None, 10)	1260
Total params: 550,385		
Trainable params: 550,385		
Non-trainable params: 0		

None

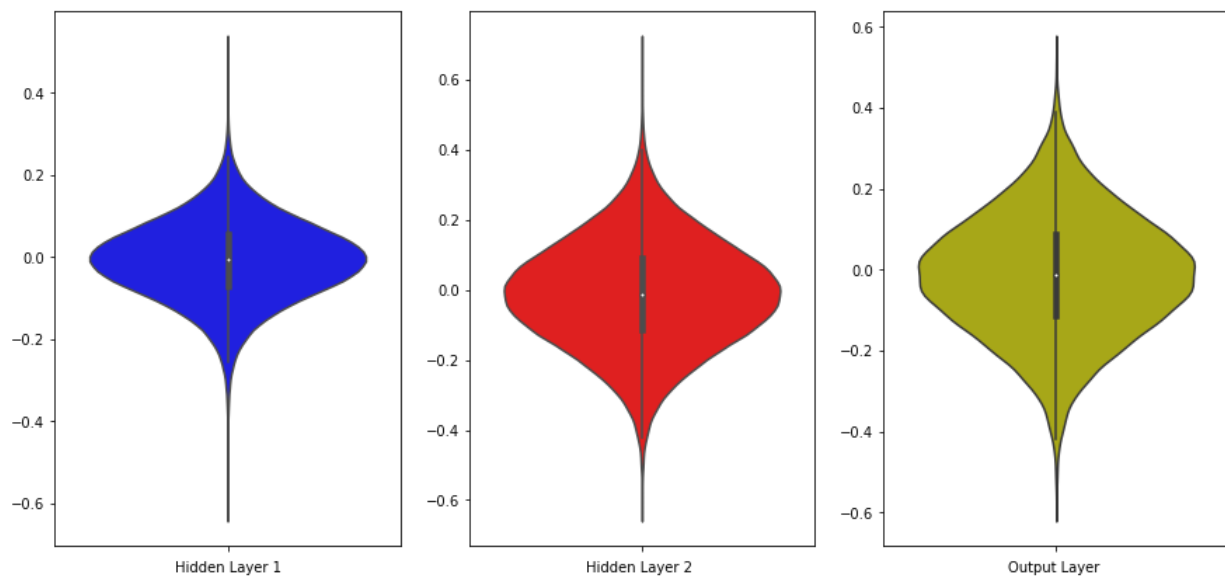
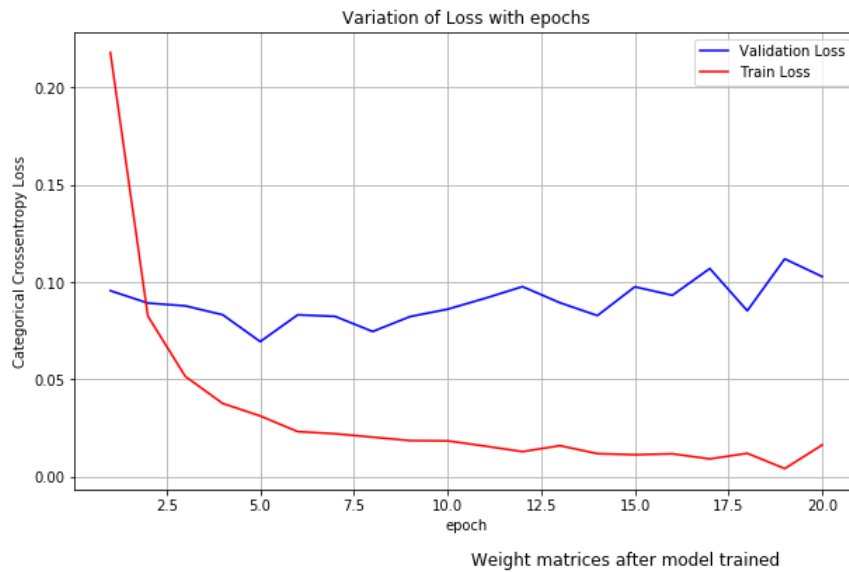
Train on 60000 samples, validate on 10000 samples

Epoch 1/20  
60000/60000 [=====] - 7s 113us/step - loss: 0.2180 - acc: 0.9334 - val\_loss: 0.0955 - val\_acc: 0.971  
Epoch 2/20  
60000/60000 [=====] - 4s 74us/step - loss: 0.0825 - acc: 0.9746 - val\_loss: 0.0892 - val\_acc: 0.9725  
Epoch 3/20  
60000/60000 [=====] - 4s 71us/step - loss: 0.0514 - acc: 0.9834 - val\_loss: 0.0877 - val\_acc: 0.9720  
Epoch 4/20  
60000/60000 [=====] - 4s 71us/step - loss: 0.0375 - acc: 0.9879 - val\_loss: 0.0831 - val\_acc: 0.9748  
Epoch 5/20  
60000/60000 [=====] - 4s 67us/step - loss: 0.0311 - acc: 0.9895 - val\_loss: 0.0693 - val\_acc: 0.9787  
Epoch 6/20  
60000/60000 [=====] - 4s 69us/step - loss: 0.0230 - acc: 0.9924 - val\_loss: 0.0831 - val\_acc: 0.9781  
Epoch 7/20  
60000/60000 [=====] - 4s 70us/step - loss: 0.0220 - acc: 0.9925 - val\_loss: 0.0823 - val\_acc: 0.9802  
Epoch 8/20  
60000/60000 [=====] - 4s 70us/step - loss: 0.0202 - acc: 0.9932 - val\_loss: 0.0745 - val\_acc: 0.9801  
Epoch 9/20  
60000/60000 [=====] - 4s 71us/step - loss: 0.0184 - acc: 0.9936 - val\_loss: 0.0822 - val\_acc: 0.9799  
Epoch 10/20  
60000/60000 [=====] - 4s 70us/step - loss: 0.0183 - acc: 0.9936 - val\_loss: 0.0860 - val\_acc: 0.9796  
Epoch 11/20  
60000/60000 [=====] - 4s 70us/step - loss: 0.0156 - acc: 0.9946 - val\_loss: 0.0916 - val\_acc: 0.9797  
Epoch 12/20  
60000/60000 [=====] - 4s 74us/step - loss: 0.0128 - acc: 0.9959 - val\_loss: 0.0976 - val\_acc: 0.9790  
Epoch 13/20  
60000/60000 [=====] - 4s 74us/step - loss: 0.0158 - acc: 0.9950 - val\_loss: 0.0893 - val\_acc: 0.9805  
Epoch 14/20  
60000/60000 [=====] - 4s 73us/step - loss: 0.0117 - acc: 0.9961 - val\_loss: 0.0827 - val\_acc: 0.9823  
Epoch 15/20  
60000/60000 [=====] - 4s 72us/step - loss: 0.0112 - acc: 0.9965 - val\_loss: 0.0976 - val\_acc: 0.9809  
Epoch 16/20  
60000/60000 [=====] - 4s 71us/step - loss: 0.0116 - acc: 0.9962 - val\_loss: 0.0932 - val\_acc: 0.9803  
Epoch 17/20  
60000/60000 [=====] - 4s 73us/step - loss: 0.0090 - acc: 0.9972 - val\_loss: 0.1069 - val\_acc: 0.9782  
Epoch 18/20  
60000/60000 [=====] - 4s 74us/step - loss: 0.0119 - acc: 0.9963 - val\_loss: 0.0852 - val\_acc: 0.9813  
Epoch 19/20  
60000/60000 [=====] - 4s 72us/step - loss: 0.0040 - acc: 0.9987 - val\_loss: 0.1119 - val\_acc: 0.9799  
Epoch 20/20  
60000/60000 [=====] - 4s 73us/step - loss: 0.0162 - acc: 0.9952 - val\_loss: 0.1028 - val\_acc: 0.9808

```
1 loss_plot(model_relu2)
2 weight_plot(model_relu2)
```



Test score: 0.10277052689156026  
Test accuracy: 0.9808



## ii) MLP + Batch Normalization + Dropout + AdamOptimizer

```
1 # https://stackoverflow.com/questions/34716454/where-do-i-call-the-batchnormalization-function-in-keras
2
3 from keras.layers import Dropout
4
5 model_drop2 = Sequential()
6
7 model_drop2.add(Dense(500, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.039, seed=None)))
8 model_drop2.add(BatchNormalization())
9 model_drop2.add(Dropout(0.5))
10
11 model_drop2.add(Dense(250, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.55, seed=None)))
12 model_drop2.add(BatchNormalization())
13 model_drop2.add(Dropout(0.5))
14
15 model_drop2.add(Dense(125, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.55, seed=None)))
16 model_drop2.add(BatchNormalization())
17 model_drop2.add(Dropout(0.5))
18
19 model_drop2.add(Dense(output_dim, activation='softmax'))
20
21
22 model_drop2.summary()
```



Model: "sequential\_16"

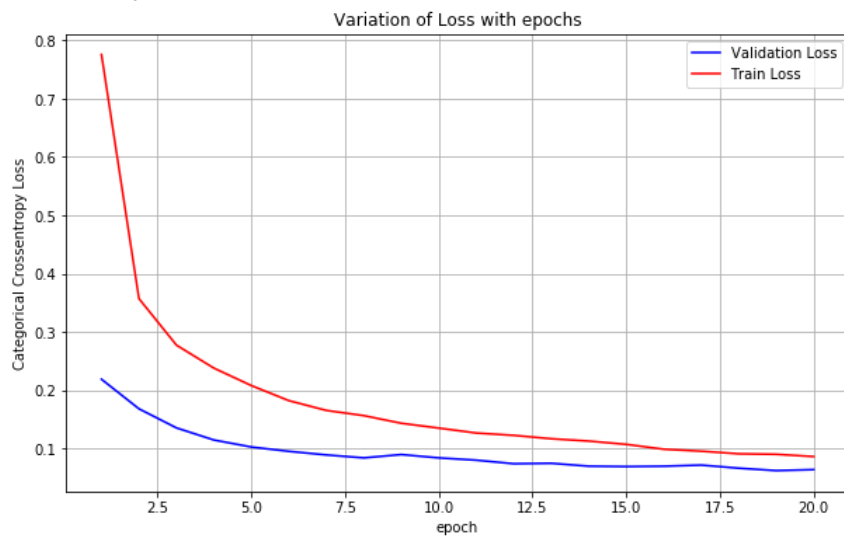
Layer (type)	Output Shape	Param #
=====		
dense_55 (Dense)	(None, 500)	392500
batch_normalization_21 (Batch Normalization)	(None, 500)	2000
dropout_17 (Dropout)	(None, 500)	0
dense_56 (Dense)	(None, 250)	125250
batch_normalization_22 (Batch Normalization)	(None, 250)	1000
dropout_18 (Dropout)	(None, 250)	0
dense_57 (Dense)	(None, 125)	31375
batch_normalization_23 (Batch Normalization)	(None, 125)	500
dropout_19 (Dropout)	(None, 125)	0
dense_58 (Dense)	(None, 10)	1260
=====		
Total params: 553,885		
Trainable params: 552,135		
Non-trainable params: 1,750		

```
1 model_drop2.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
2
3 history = model_drop2.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
4 loss_plot(model_drop2)
5 weight_plot(model_drop2)
```

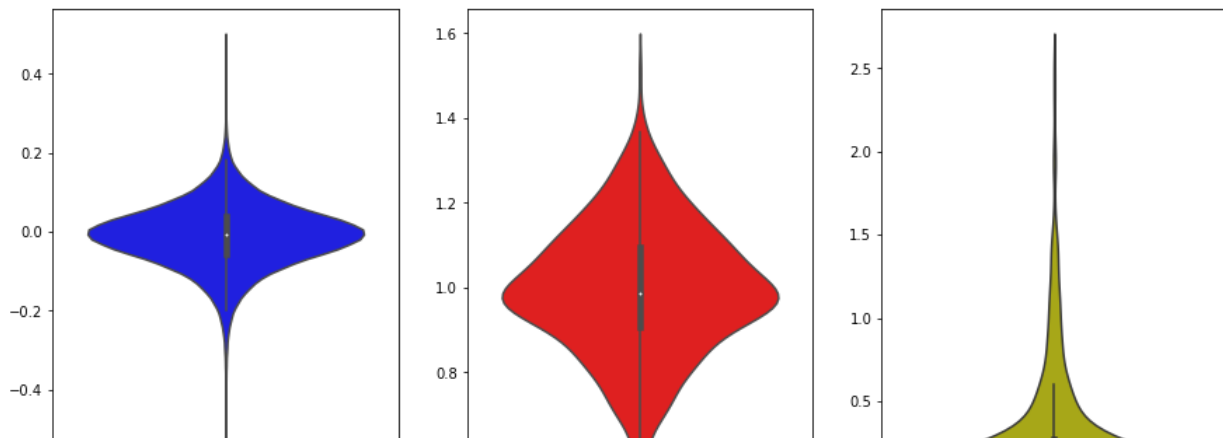


Train on 60000 samples, validate on 10000 samples

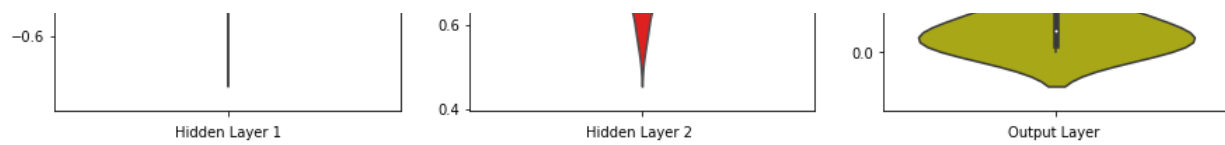
```
Epoch 1/20
60000/60000 [=====] - 12s 206us/step - loss: 0.7758 - acc: 0.7553 - val_loss: 0.2188 - val_acc: 0.93
Epoch 2/20
60000/60000 [=====] - 9s 143us/step - loss: 0.3571 - acc: 0.8941 - val_loss: 0.1681 - val_acc: 0.947
Epoch 3/20
60000/60000 [=====] - 9s 144us/step - loss: 0.2773 - acc: 0.9179 - val_loss: 0.1353 - val_acc: 0.959
Epoch 4/20
60000/60000 [=====] - 9s 142us/step - loss: 0.2378 - acc: 0.9301 - val_loss: 0.1146 - val_acc: 0.963
Epoch 5/20
60000/60000 [=====] - 9s 144us/step - loss: 0.2079 - acc: 0.9394 - val_loss: 0.1025 - val_acc: 0.968
Epoch 6/20
60000/60000 [=====] - 9s 143us/step - loss: 0.1820 - acc: 0.9472 - val_loss: 0.0951 - val_acc: 0.970
Epoch 7/20
60000/60000 [=====] - 9s 143us/step - loss: 0.1653 - acc: 0.9516 - val_loss: 0.0889 - val_acc: 0.972
Epoch 8/20
60000/60000 [=====] - 8s 137us/step - loss: 0.1564 - acc: 0.9542 - val_loss: 0.0839 - val_acc: 0.973
Epoch 9/20
60000/60000 [=====] - 8s 140us/step - loss: 0.1432 - acc: 0.9573 - val_loss: 0.0896 - val_acc: 0.972
Epoch 10/20
60000/60000 [=====] - 9s 142us/step - loss: 0.1350 - acc: 0.9607 - val_loss: 0.0839 - val_acc: 0.975
Epoch 11/20
60000/60000 [=====] - 8s 133us/step - loss: 0.1265 - acc: 0.9628 - val_loss: 0.0800 - val_acc: 0.977
Epoch 12/20
60000/60000 [=====] - 8s 127us/step - loss: 0.1225 - acc: 0.9637 - val_loss: 0.0739 - val_acc: 0.977
Epoch 13/20
60000/60000 [=====] - 8s 127us/step - loss: 0.1167 - acc: 0.9655 - val_loss: 0.0746 - val_acc: 0.978
Epoch 14/20
60000/60000 [=====] - 8s 126us/step - loss: 0.1127 - acc: 0.9670 - val_loss: 0.0697 - val_acc: 0.978
Epoch 15/20
60000/60000 [=====] - 8s 128us/step - loss: 0.1071 - acc: 0.9678 - val_loss: 0.0690 - val_acc: 0.980
Epoch 16/20
60000/60000 [=====] - 8s 126us/step - loss: 0.0987 - acc: 0.9707 - val_loss: 0.0696 - val_acc: 0.979
Epoch 17/20
60000/60000 [=====] - 8s 126us/step - loss: 0.0954 - acc: 0.9714 - val_loss: 0.0715 - val_acc: 0.980
Epoch 18/20
60000/60000 [=====] - 8s 128us/step - loss: 0.0909 - acc: 0.9730 - val_loss: 0.0662 - val_acc: 0.981
Epoch 19/20
60000/60000 [=====] - 8s 126us/step - loss: 0.0900 - acc: 0.9726 - val_loss: 0.0621 - val_acc: 0.982
Epoch 20/20
60000/60000 [=====] - 8s 126us/step - loss: 0.0862 - acc: 0.9739 - val_loss: 0.0637 - val_acc: 0.981
Test score: 0.06373635350148543
Test accuracy: 0.9818
```



Weight matrices after model trained







▼ Architecture 3 : Layer1(700), Layer2(350), Layer3(200), Layer4(100), Layer5(50)

### i) MLP + ReLU + ADAM

```
1 import warnings
2 warnings.filterwarnings("ignore")

1 model_relu3 = Sequential()
2 model_relu3.add(Dense(700, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.062, seed=None)))
3 model_relu3.add(Dense(350, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.125, seed=None)))
4 model_relu3.add(Dense(200, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.25, seed=None)))
5 model_relu3.add(Dense(100, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.1, seed=None)))
6 model_relu3.add(Dense(50, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.5, seed=None)))
7 model_relu3.add(Dense(output_dim, activation='softmax'))
8
9 print(model_relu3.summary())
10
11 model_relu3.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
12
13 history = model_relu3.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```



Model: "sequential\_17"

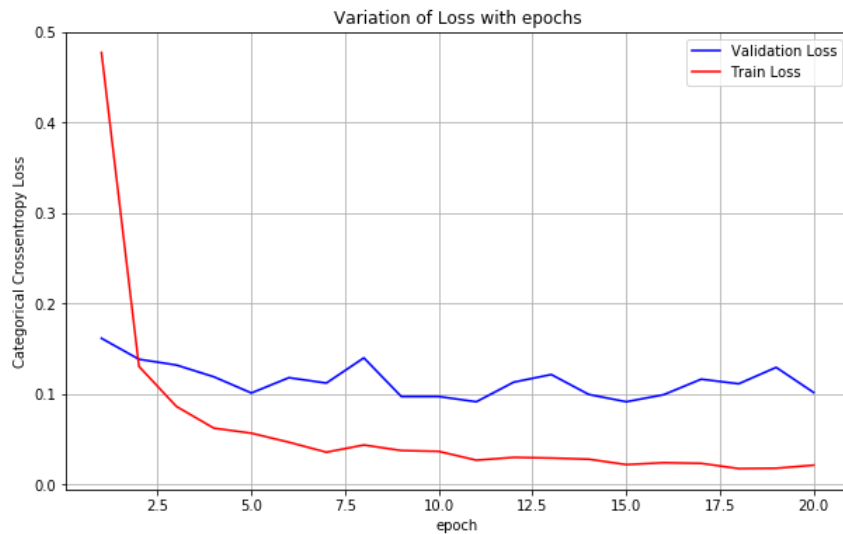
Layer (type)	Output Shape	Param #
dense_59 (Dense)	(None, 700)	549500
dense_60 (Dense)	(None, 350)	245350
dense_61 (Dense)	(None, 200)	70200
dense_62 (Dense)	(None, 100)	20100
dense_63 (Dense)	(None, 50)	5050
dense_64 (Dense)	(None, 10)	510
Total params: 890,710		
Trainable params: 890,710		
Non-trainable params: 0		

None  
Train on 60000 samples, validate on 10000 samples  
Epoch 1/20  
60000/60000 [=====] - 8s 133us/step - loss: 0.4770 - acc: 0.9003 - val\_loss: 0.1612 - val\_acc: 0.951  
Epoch 2/20  
60000/60000 [=====] - 5s 77us/step - loss: 0.1302 - acc: 0.9619 - val\_loss: 0.1381 - val\_acc: 0.9604  
Epoch 3/20  
60000/60000 [=====] - 5s 77us/step - loss: 0.0859 - acc: 0.9739 - val\_loss: 0.1317 - val\_acc: 0.9617  
Epoch 4/20  
60000/60000 [=====] - 5s 76us/step - loss: 0.0619 - acc: 0.9807 - val\_loss: 0.1187 - val\_acc: 0.9691  
Epoch 5/20  
60000/60000 [=====] - 5s 80us/step - loss: 0.0564 - acc: 0.9819 - val\_loss: 0.1008 - val\_acc: 0.9728  
Epoch 6/20  
60000/60000 [=====] - 5s 79us/step - loss: 0.0464 - acc: 0.9849 - val\_loss: 0.1177 - val\_acc: 0.9682  
Epoch 7/20  
60000/60000 [=====] - 5s 77us/step - loss: 0.0354 - acc: 0.9881 - val\_loss: 0.1118 - val\_acc: 0.9709  
Epoch 8/20  
60000/60000 [=====] - 5s 78us/step - loss: 0.0433 - acc: 0.9867 - val\_loss: 0.1396 - val\_acc: 0.9664  
Epoch 9/20  
60000/60000 [=====] - 5s 76us/step - loss: 0.0372 - acc: 0.9885 - val\_loss: 0.0969 - val\_acc: 0.9741  
Epoch 10/20  
60000/60000 [=====] - 5s 78us/step - loss: 0.0363 - acc: 0.9888 - val\_loss: 0.0969 - val\_acc: 0.9745  
Epoch 11/20  
60000/60000 [=====] - 5s 79us/step - loss: 0.0265 - acc: 0.9920 - val\_loss: 0.0911 - val\_acc: 0.9767  
Epoch 12/20  
60000/60000 [=====] - 5s 78us/step - loss: 0.0296 - acc: 0.9908 - val\_loss: 0.1128 - val\_acc: 0.9737  
Epoch 13/20  
60000/60000 [=====] - 5s 76us/step - loss: 0.0289 - acc: 0.9911 - val\_loss: 0.1212 - val\_acc: 0.9730  
Epoch 14/20  
60000/60000 [=====] - 5s 77us/step - loss: 0.0276 - acc: 0.9913 - val\_loss: 0.0992 - val\_acc: 0.9749  
Epoch 15/20  
60000/60000 [=====] - 5s 78us/step - loss: 0.0217 - acc: 0.9933 - val\_loss: 0.0911 - val\_acc: 0.9794  
Epoch 16/20  
60000/60000 [=====] - 5s 77us/step - loss: 0.0237 - acc: 0.9930 - val\_loss: 0.0989 - val\_acc: 0.9794  
Epoch 17/20  
60000/60000 [=====] - 5s 77us/step - loss: 0.0230 - acc: 0.9936 - val\_loss: 0.1160 - val\_acc: 0.9764  
Epoch 18/20  
60000/60000 [=====] - 5s 80us/step - loss: 0.0172 - acc: 0.9947 - val\_loss: 0.1110 - val\_acc: 0.9766  
Epoch 19/20  
60000/60000 [=====] - 5s 77us/step - loss: 0.0176 - acc: 0.9949 - val\_loss: 0.1291 - val\_acc: 0.9745  
Epoch 20/20  
60000/60000 [=====] - 5s 78us/step - loss: 0.0210 - acc: 0.9939 - val\_loss: 0.1013 - val\_acc: 0.9776

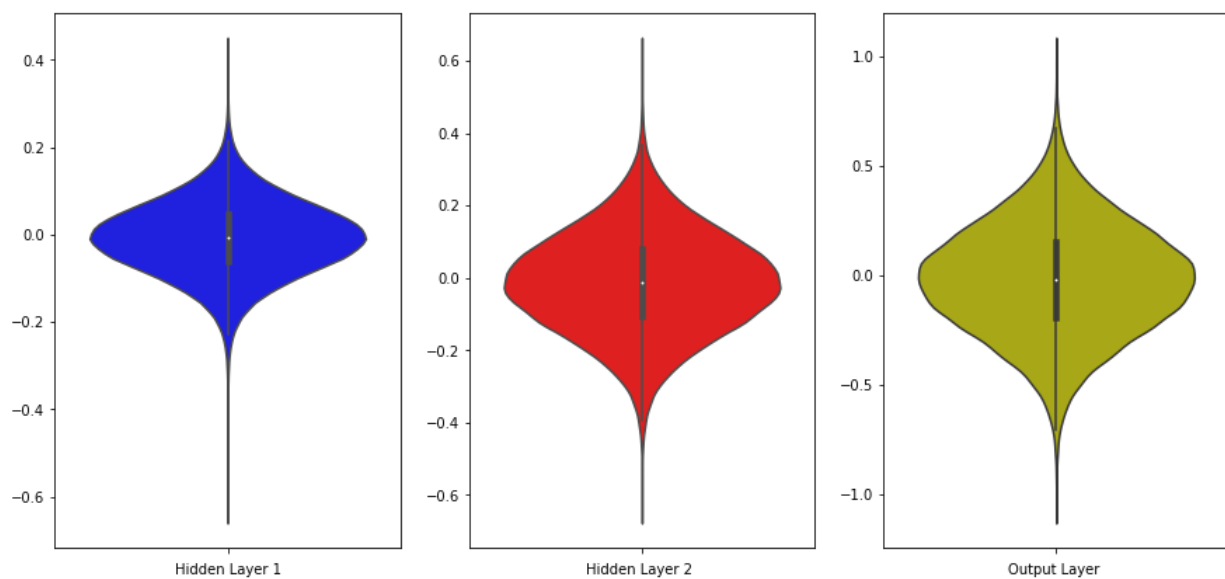
```
1 loss_plot(model_relu3)
2 weight_plot(model_relu3)
```



Test score: 0.10133317776405511  
Test accuracy: 0.9776



Weight matrices after model trained



## ii) MLP + Batch Normalization + Dropout + AdamOptimizer

```
1 # https://stackoverflow.com/questions/34716454/where-do-i-call-the-batchnormalization-function-in-keras
2
3 from keras.layers import Dropout
4
5 model_drop3 = Sequential()
6
7 model_drop3.add(Dense(700, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.039, seed=None)))
8 model_drop3.add(BatchNormalization())
9 model_drop3.add(Dropout(0.5))
10
11 model_drop3.add(Dense(350, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.25, seed=None)))
12 model_drop3.add(BatchNormalization())
13 model_drop3.add(Dropout(0.5))
14
15 model_drop3.add(Dense(200, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.55, seed=None)))
16 model_drop3.add(BatchNormalization())
17 model_drop3.add(Dropout(0.5))
18
19 model_drop3.add(Dense(100, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.15, seed=None)))
20 model_drop3.add(BatchNormalization())
21 model_drop3.add(Dropout(0.5))
22
23 model_drop3.add(Dense(50, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.3, seed=None)))
24 model_drop3.add(BatchNormalization())
25 model_drop3.add(Dropout(0.5))
26
27 model_drop3.add(Dense(output_dim, activation='softmax'))
28
29
30 model_drop3.summary()
```



Model: "sequential\_18"

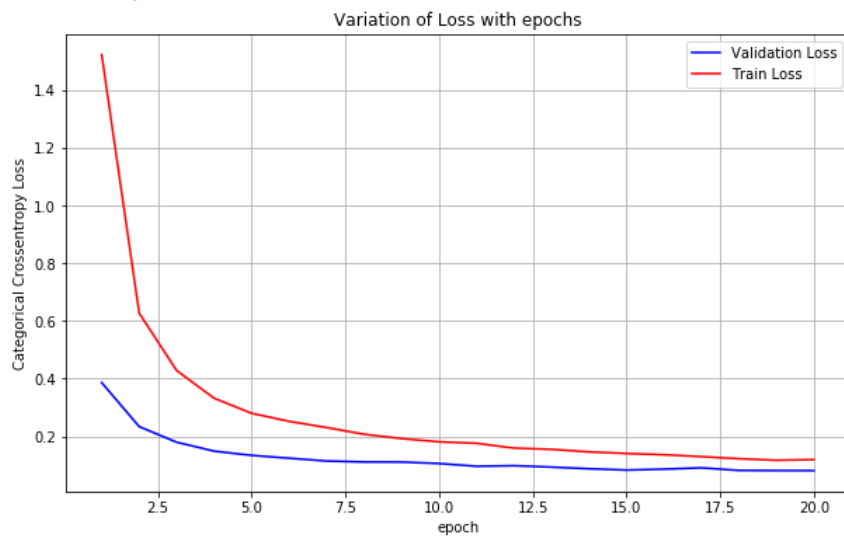
Layer (type)	Output Shape	Param #
=====		
dense_65 (Dense)	(None, 700)	549500
batch_normalization_24 (Batch Normalization)	(None, 700)	2800
dropout_20 (Dropout)	(None, 700)	0
dense_66 (Dense)	(None, 350)	245350
batch_normalization_25 (Batch Normalization)	(None, 350)	1400
dropout_21 (Dropout)	(None, 350)	0
dense_67 (Dense)	(None, 200)	70200
batch_normalization_26 (Batch Normalization)	(None, 200)	800
dropout_22 (Dropout)	(None, 200)	0
dense_68 (Dense)	(None, 100)	20100
batch_normalization_27 (Batch Normalization)	(None, 100)	400
dropout_23 (Dropout)	(None, 100)	0
dense_69 (Dense)	(None, 50)	5050
batch_normalization_28 (Batch Normalization)	(None, 50)	200
dropout_24 (Dropout)	(None, 50)	0
dense_70 (Dense)	(None, 10)	510
=====		
Total params: 896,310		
Trainable params: 893,510		
Non-trainable params: 2,800		

```
1 model_drop3.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
2
3 history = model_drop3.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
4 loss_plot(model_drop3)
5 weight_plot(model_drop3)
```

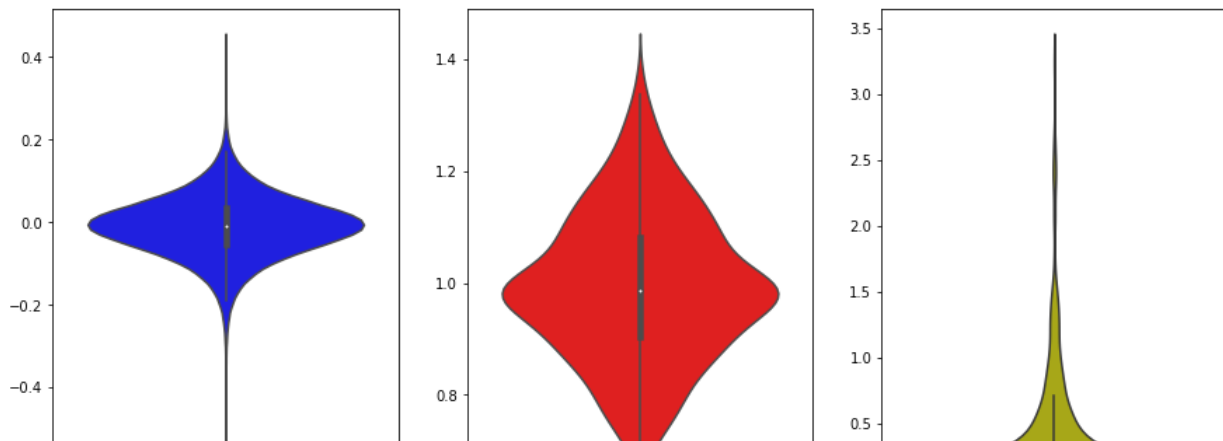


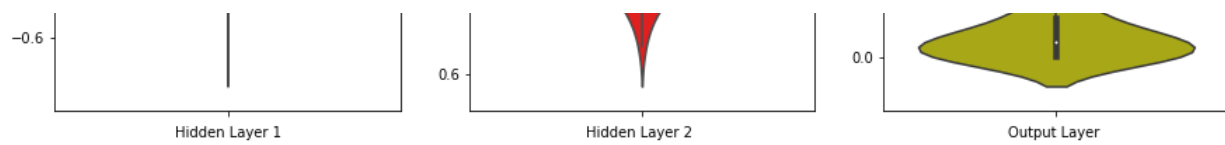
Train on 60000 samples, validate on 10000 samples

```
Epoch 1/20
60000/60000 [=====] - 15s 250us/step - loss: 1.5225 - acc: 0.5005 - val_loss: 0.3861 - val_acc: 0.89
Epoch 2/20
60000/60000 [=====] - 11s 176us/step - loss: 0.6269 - acc: 0.8081 - val_loss: 0.2339 - val_acc: 0.93
Epoch 3/20
60000/60000 [=====] - 11s 178us/step - loss: 0.4280 - acc: 0.8796 - val_loss: 0.1796 - val_acc: 0.94
Epoch 4/20
60000/60000 [=====] - 10s 175us/step - loss: 0.3320 - acc: 0.9116 - val_loss: 0.1487 - val_acc: 0.95
Epoch 5/20
60000/60000 [=====] - 11s 178us/step - loss: 0.2799 - acc: 0.9259 - val_loss: 0.1342 - val_acc: 0.96
Epoch 6/20
60000/60000 [=====] - 11s 176us/step - loss: 0.2519 - acc: 0.9355 - val_loss: 0.1243 - val_acc: 0.96
Epoch 7/20
60000/60000 [=====] - 11s 178us/step - loss: 0.2305 - acc: 0.9401 - val_loss: 0.1146 - val_acc: 0.96
Epoch 8/20
60000/60000 [=====] - 11s 180us/step - loss: 0.2072 - acc: 0.9464 - val_loss: 0.1113 - val_acc: 0.97
Epoch 9/20
60000/60000 [=====] - 11s 183us/step - loss: 0.1922 - acc: 0.9512 - val_loss: 0.1109 - val_acc: 0.97
Epoch 10/20
60000/60000 [=====] - 12s 200us/step - loss: 0.1811 - acc: 0.9542 - val_loss: 0.1060 - val_acc: 0.97
Epoch 11/20
60000/60000 [=====] - 13s 208us/step - loss: 0.1759 - acc: 0.9556 - val_loss: 0.0963 - val_acc: 0.97
Epoch 12/20
60000/60000 [=====] - 12s 206us/step - loss: 0.1592 - acc: 0.9609 - val_loss: 0.0984 - val_acc: 0.97
Epoch 13/20
60000/60000 [=====] - 13s 215us/step - loss: 0.1550 - acc: 0.9612 - val_loss: 0.0935 - val_acc: 0.97
Epoch 14/20
60000/60000 [=====] - 13s 215us/step - loss: 0.1461 - acc: 0.9639 - val_loss: 0.0877 - val_acc: 0.97
Epoch 15/20
60000/60000 [=====] - 13s 221us/step - loss: 0.1406 - acc: 0.9645 - val_loss: 0.0832 - val_acc: 0.97
Epoch 16/20
60000/60000 [=====] - 13s 219us/step - loss: 0.1363 - acc: 0.9666 - val_loss: 0.0864 - val_acc: 0.97
Epoch 17/20
60000/60000 [=====] - 14s 225us/step - loss: 0.1298 - acc: 0.9680 - val_loss: 0.0910 - val_acc: 0.97
Epoch 18/20
60000/60000 [=====] - 13s 222us/step - loss: 0.1224 - acc: 0.9698 - val_loss: 0.0820 - val_acc: 0.98
Epoch 19/20
60000/60000 [=====] - 13s 217us/step - loss: 0.1172 - acc: 0.9707 - val_loss: 0.0816 - val_acc: 0.98
Epoch 20/20
60000/60000 [=====] - 13s 212us/step - loss: 0.1195 - acc: 0.9709 - val_loss: 0.0813 - val_acc: 0.98
Test score: 0.08133343692359049
Test accuracy: 0.9811
```



Weight matrices after model trained





## Observations:

1. Batch Normalization and Dropout layers reduce the variance of the model(prevent it from overfitting to the train data). The performance of the MLP on train data is much better with BN and Dropout.
2. All the models are able to minimize the loss and give close to 98% accuracy after just 15 epochs.
3. For MNIST data, even a shallow NN works well, there's not much difference in the performance of models on increasing the depth of the neural network.