

# Regression

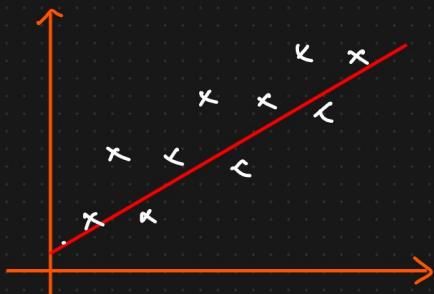
① Mean Square Error (MSE), MAE, RMSE ✓

② Ridge Regression ✓

③ Lasso Regression ✓

④ ElasticNet ✓

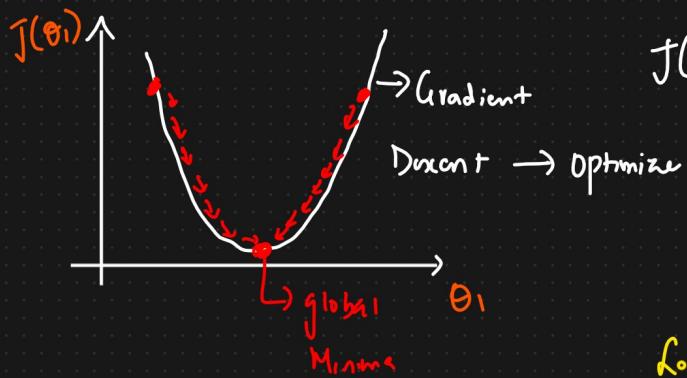
⑤ Practicals. [Simple Implementation]



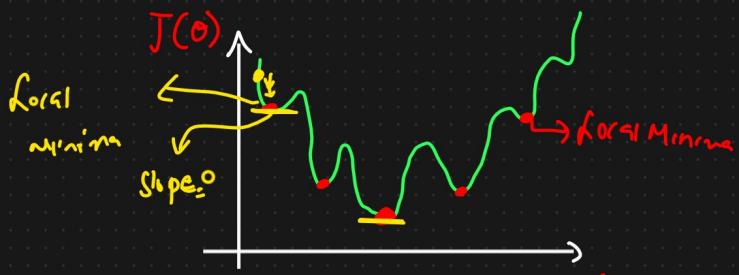
① Mean Squared Error (MSE)

② MAE

③ RMSE



$$J(\theta_0, \theta_1) = \frac{1}{n} \sum_{i=1}^n (y_i - h_\theta(x))^2$$



① Mean Square Error [cost function].

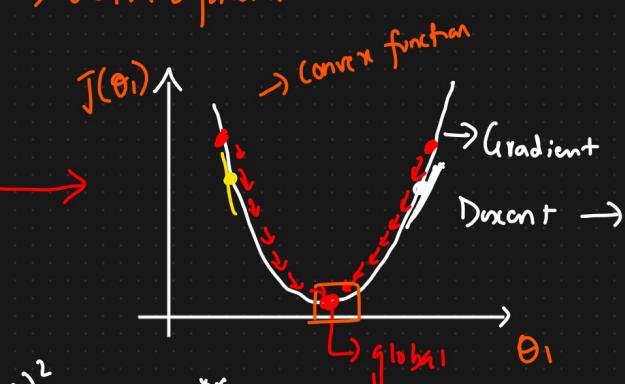
$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \Rightarrow \text{Quadratic function.}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

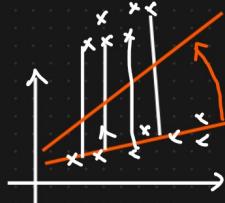
$$ax + by + c = 0$$

$$y = mx + c$$

$$\left[ (y_i - \hat{y}_i)^2 \right] \stackrel{\text{INR}}{\rightarrow} \left[ (y_i - \hat{y}_i)^2 \right] \stackrel{\text{FREOR}}{\uparrow \uparrow}$$



$$\begin{aligned} ax + by + c &= 0 \\ by &= -ax - c \\ y &= \left[ \frac{-a}{b} \right] x \left[ \frac{-c}{b} \right] \end{aligned}$$



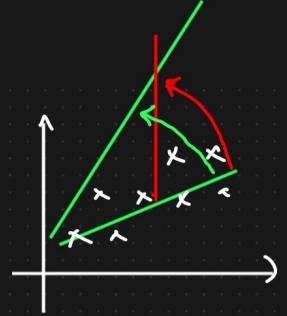
$$\text{MSE} \quad (y_i - \hat{y}_i)^2 \uparrow \uparrow$$

### Advantages

- ① It is differentiable.
- ② It has one local and one global Minima.

### Disadvantages

- ① Not Robust to outliers  
*{Affected by outliers}*
- ② MSE changes the unit



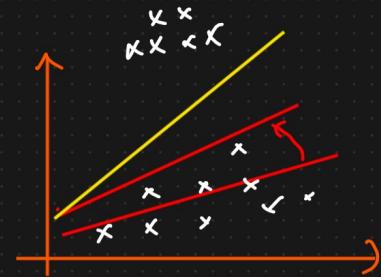
### ② Mean Absolute Error (MAE)

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \Rightarrow$$

#### Advantages

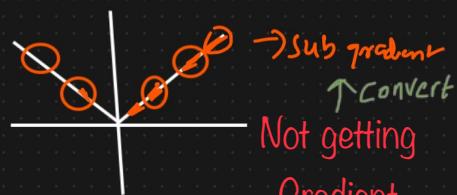
- ① Robust to outliers *hence*
- ② If will be in same unit

ERROR ↑  
But not as much as MSE



#### Disadvantage

Time Complexity is more for → optimizer -



### ③ Root Mean Square Error

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \Rightarrow \text{Advantages And Disadvantages.}$$

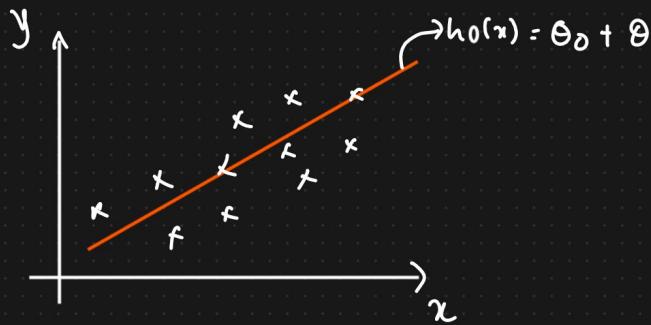
Performance metric =  $R^2$  and Adjusted  $R^2$

Cost function = MSE, MAE, RMSE, Huber loss ← DL.

Split gradient by gradient and find then converge

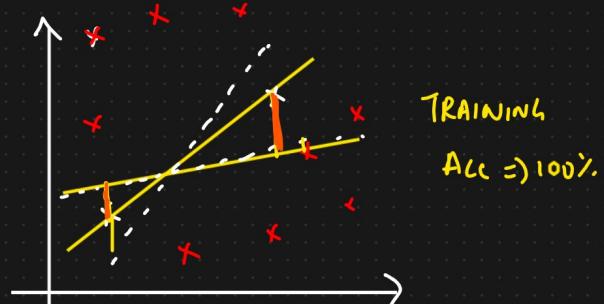
# Ridge, Lasso and ElasticNet

## Linear Regression

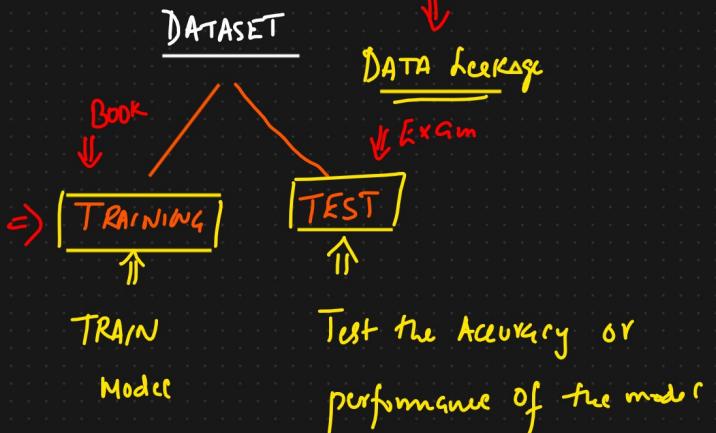


$$\text{Cost function} = \frac{1}{n} \sum_{i=1}^n (y_i - h_\theta(x))^2$$

① Ridge Regression ( $L_2$  Regularization) → Reducing Overfitting }



Overfitting  
Training Data Acc 100% ↑↑



$$\text{Cost fn} = \frac{1}{n} \sum_{i=1}^n (y_i - h_\theta(x_i))^2 + \lambda \sum_{i=1}^n (\text{slope}_i)^2$$

$\lambda > 0$

Hyperegument  
Coefficients

$$+ 1 * [(\theta_1)^2 + (\theta_2)^2 + (\theta_3)^2] \Rightarrow \underline{\text{tvc value}}$$

$$\lambda = 1$$

Cost fn = +ve values ↓↓↓

Coefficients ↓↓

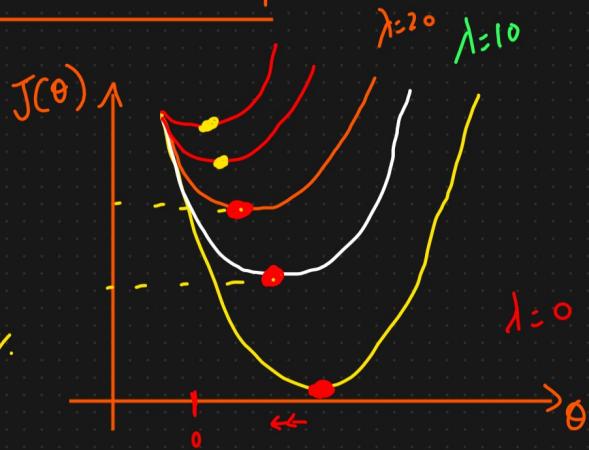
$$\lambda = 1, 2, 3, 4, 10, 20, 30, 40, 50$$

Relationship between  $\lambda$  and Slope

TRAINING

$$\text{Accuracy} = 100\%$$

TRAINING ERROR



$$\text{Cost} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \frac{\lambda}{n} \sum_{i=1}^n (\text{Slope})^2$$

$\lambda \uparrow \uparrow$  Slope ↓↓

$\lambda \uparrow \uparrow \uparrow$  Slope ↓↓↓

$\lambda \uparrow \uparrow \uparrow$  Error ↑↑↑  
Cost ↑↑↑

② Lasso Regression ( $\lambda_1$ , Regularization) → Feature Selection

$$\text{Cost fn: } \frac{1}{n} \sum_{i=1}^n (y_i - h_\theta(x)_i)^2 + \left[ \lambda \sum_{i=1}^n |\text{Slope}| \right] \leftarrow \text{for Regularization}$$

Unit change in  $x_1$ ,

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$$

0.65 change in output

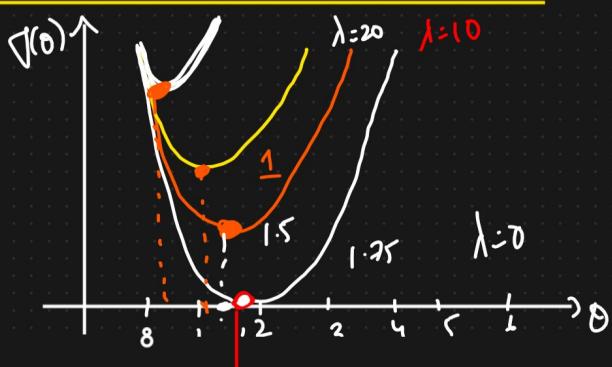
$$h_\theta(x) = 0.52 + \underbrace{0.65 x_1}_{\uparrow} + \underbrace{0.72 x_2}_{\uparrow} + \underbrace{0.12 x_3}_{\downarrow}$$

$\Downarrow$   
Lasso Regression

unit change in  $x_3$

0.12 change in output

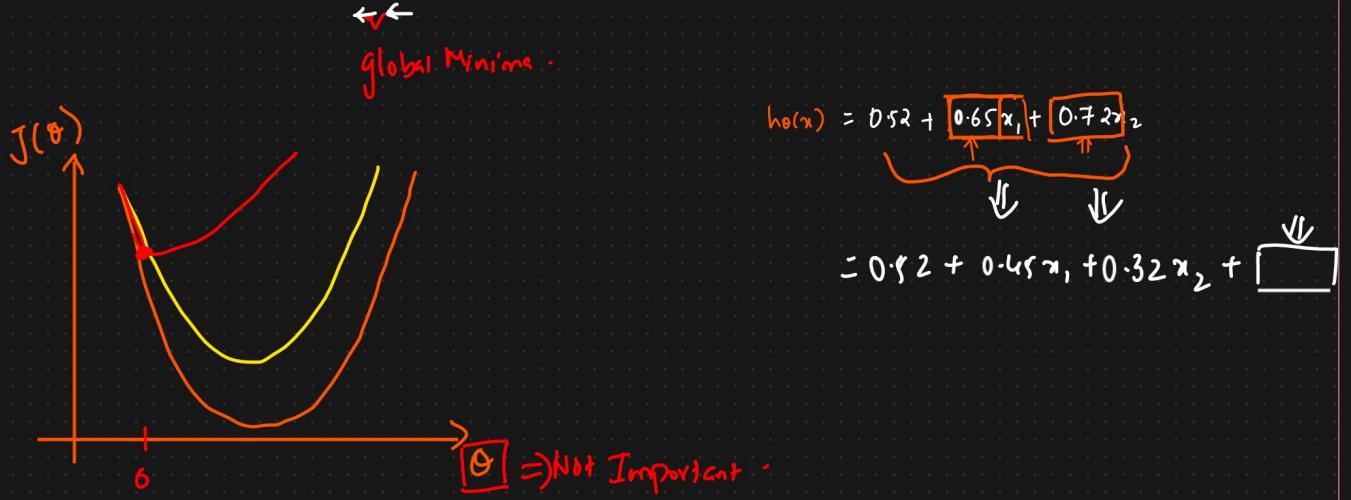
Relationship between  $\lambda$  and |slope|



$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$$

$$h_\theta(x) = 0.52 + \boxed{0.65 x_1} + \boxed{0.72 x_2} + \boxed{0.12 x_3}$$

$$\begin{matrix} 1 \\ 0 \\ 0.12, 0.20, 0.50 \end{matrix}$$



③ ElasticNet       $\begin{cases} \text{① Reducing overfitting} \\ \text{② Feature Selection} \end{cases} \quad \left. \begin{array}{l} \text{Ridge + Lasso} \\ \text{Ridge} \\ \text{Lasso} \end{array} \right\}$

$$\text{Cost fn} = \frac{1}{n} \sum_{i=1}^n (y_i - h_\theta(x_i))^2 + \boxed{\lambda_1 \sum_{i=1}^n (\text{slope}_i)^2} + \boxed{\lambda_2 \sum_{i=1}^n |\text{slope}_i|}$$

$\downarrow$                                    $\downarrow$

$\lambda_1, \lambda_2$  [Hyperparameters],      Reducing Overfitting      Feature Selection.

