A MAXIMUM LIKELIHOOD CLASSIFICATION METHOD FOR IMAGE SEGMENTATION CONSIDERING SUBJECT VARIABILITY

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ABSTRACT

In this paper, we present a new statistical model for Maximum Likelihood Classification (MLC) algorithm to improve the image segmentation/classification performance. MLC has been widely used in many classification applications. For supervised MLC, the parameters of the statistical model are obtained from the training dataset at the learning step. However, in the previous studies, the feature values of different classes are assumed to have similar distributions for different subjects. This is not true in many real world situations. The considerable differences across subjects have not obtained much attention before. To conquer this difficulty, we model the mean of feature values of each subject and the feature values as two groups of dependent random variables. This is made possible by using a bivariate Gaussian mixture model to fit the image data of different subjects. In this way, class membership depends on both the feature values and another random variable that captures subject-specific information. We apply our method to simulated image data and our experimental results show that the proposed model could improve the classical supervised MLC segmentation results when there are considerable differences across subjects.

***Index Terms*—** Image segmentation, maximum likelihood classification, Gaussian mixture model, bivariate Gaussian distribution.

1. INTRODUCTION

Image segmentation is of great interest in a variety of applications. Maximum likelihood classification (MLC), as one of the most important segmentation algorithms has been widely used in many applications, including some medical image processing problems. For MLC algorithm, there are two groups of unknowns: the statistical model parameters, such as the mean and variance, and the class labels. In the literature, there are two possible solutions to estimate these unknowns. One solution is to estimate the two groups of unknowns in an alternating fashion by Expectation-Maximization (EM) algorithm as illustrated in [1]. In [2], Zhang proposed to apply the mean field theory in EM procedures for classification. In [3], a method to estimate the two groups of unknown simultaneously for medical image segmentation was presented. The other solution, which is supervised, is to learn the model parameters from training data [4].

The supervised statistical model, which is efficient and simple, is considered in this paper. In traditional supervised MLC, the model parameters are calculated from the training dataset. Then, this classifier is applied to the test subject. However, there is an implicit assumption that we assume the image distributions of different classes from different subjects are similar. Unfortunately, this assumption may not be satisfied in some applications, such as most medical imaging applications. The traditional supervised MLC does not consider the variability between different subjects/patients. To overcome this difficulty, we propose a new statistical model modeling the intensity values and the mean intensity values of each subject as bivariate random variables. In this way, the class label does not only depend on the intensity value but also depends on another random variable that captures the subject-specific information.

This paper is organized as follows. In Section 2, we briefly review the MLC algorithm with Gaussian mixture model. The proposed method is presented in Section 3. Section 4 presents computer simulations and visual comparisons between the classical supervised MLC and the proposed MLC methods. Finally, conclusions and future work are given in Section 5.

2. MAXIMUM LIKELIHOOD CLASSIFICATION

The MLC considers two random fields, $K = K_s$, $s \in S$ and $Y = Y_s$, $s \in S$, where S is the image dataset, the measured data to be segmented is a realization of Y denoted by y and the desired segmentation result k is a unobservable field, or called hidden field. It contains the class of each pixel, and is a realization of K denoted by k. For binary classification problems, we have $K = 1$ or 2. The estimator of $\hat{K}$ of K is obtained by maximizing the probability of each class $p(y|K)$. That is:

$$\hat{K} = \arg\max_{k} p(y|K). \tag{1}$$

Depending on the choice of $p(y|K)$, different MLC and segmentation methods can be constructed. A common probabilistic model used is the Gaussian mixture model, and extension to other distributions is possible by using the corresponding PDFs.

For a simplified development, we consider binary segmentation problem, background and a single object with means and variances denoted by μ_1, μ_2, σ_1^2 and σ_2^2. For Gaussian mixture model, we have:

$$\begin{aligned}\hat{K} &= \arg\max_k p(y|K=k) \\ &= \arg\max_k \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\frac{-(y_s-\mu_k)^2}{2\sigma_k^2}, \end{aligned} \quad (2)$$

which is equivalent to

$$\begin{aligned}\hat{K} &= \arg\min_k -\log p(y|K=k) \\ &= \arg\min_k \left[\log\sigma_k + \frac{(y-\mu_k)^2}{2\sigma_k^2}\right]. \end{aligned} \quad (3)$$

In the literature, the parameters of the probabilistic model μ_1, μ_2, σ_1^2 and σ_2^2 can be estimated by Expectation-Maximization (EM) algorithm, or learned from the training dataset.

For the supervised learning procedure, there is an implicit assumption that the image distributions of different classes of different subjects/patients are similar. The segmentation performance would suffer if this assumption is not satisfied. However, in many real world segmentation problems, this assumption is not satisfied. In order to overcome this problem, we propose a new statistical model to incorporate the subject-specific information by using a bivariate Gaussian distribution to fit the image data as explained next. In our method, the class membership is modeled as a function of another random variable which captures a particular subject's image distribution.

3. PROPOSED PROBABILISTIC MODEL

In the new statistical model, we consider two groups of random variables: the measured data, or intensity value y, and the mean intensity values of each subject/patient z that represents subject-specific information. We assume y and z belong to a bivariate Gaussian distribution. That is:

$$\begin{aligned}p(y,z|K=k) &= \frac{1}{2\pi|\Sigma_k|^{1/2}} \times \\ & \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_k)\Sigma_k^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_k)^T\right] \end{aligned} \quad (4)$$

where $\boldsymbol{\mu}_k$ and Σ_k are the mean vector and covariance matrix for the kth class,

$$\begin{aligned}\boldsymbol{x} &= \left[\begin{array}{c} y \\ z \end{array}\right], \\ \boldsymbol{\mu}_k &= \left[\begin{array}{c} \mu_{yk} \\ \mu_{zk} \end{array}\right], \\ \Sigma_k &= \left[\begin{array}{cc} \sigma_{xk}^2 & C_{yz}^k \\ C_{zy}^k & \sigma_{zk}^2 \end{array}\right], \end{aligned} \quad (5)$$

and $C_{yz}^k = C_{zy}^k = E[(y-\mu_{yk})(z-\mu_{zk})]$. For bivariate Gaussian model, the MLC can be expressed as:

$$\begin{aligned}\hat{K} &= \arg\max_k p(y,z|K=k) \\ &= \arg\max_k \frac{1}{2\pi|\Sigma_k|^{\frac{1}{2}}} \\ & \quad \times \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_k)\Sigma_k^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_k)^T\right] \end{aligned} \quad (6)$$

which is equivalent to

$$\begin{aligned}\hat{K} &= \arg\min_k -\log p(y,z|K=k) \\ &= \arg\min_k \left[(\boldsymbol{x}-\boldsymbol{\mu}_k)\Sigma_k^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_k)^T \right. \\ & \quad \left. + \log|\Sigma_k|\right]. \end{aligned} \quad (7)$$

The model parameters $\boldsymbol{\mu}_k$ and Σ_k are calculated from training dataset using ML estimation based on sample means and variances.

In the proposed statistical model, we consider the differences across subjects, and incorporate this information by assuming the mean intensity value of each patient and the intensity value of each pixel belong to a bivariate Gaussian distribution. This new model could better fit the image data when there are considerable variations between subjects. Because, in such circumstances, the probability of each pixel belongs to a certain class is not only dependent on its intensity value, but also depends on the subject it belongs to. In another words, the random field K does not only depend on the measured data Y, but also depends on the particular subject Z.

4. EXPERIMENTAL RESULTS

4.1. Simulated Image Data

To illustrate the efficacy of the proposed method, we apply our method to a simulated image dataset which consists of 30 subjects. Each subject is a gray-scale image with two regions: a background and an object. The intensity values of the background and the object both belong to Gaussian distribution. The mean intensity values of the background and object regions of each subject are provided in Fig. 1. The variances of the background and object regions are 100 and 225 respectively for all 30 subjects. From Fig. 1-Fig. 3, we can see that the intensity values of the background and object regions of different subjects may be distributed at different intensity ranges. This is a realistic scenario in many real world applications. For the 14th subject, whose histogram shown in Fig. 2, it has higher intensity values overall; while the 15th subject, whose histogram illustrated in Fig. 3, has lower intensity values overall. When there is a considerable difference between subjects, we could expect poor segmentation results if the subject-specific information is not taken into account. Fig. 2 and 3 also demonstrate the importance of modeling the

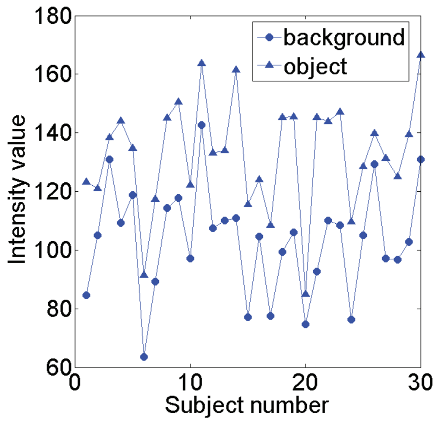


Fig. 1. The mean intensity values of the background and object for 30 subjects. The triangles are the mean intensity values for the background, and the circles the mean intensity values for the object.

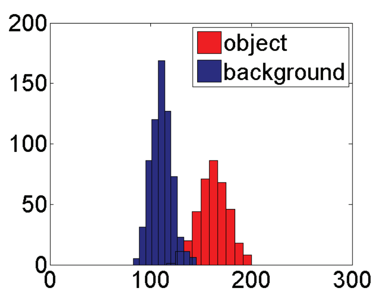


Fig. 2. Histogram of the 14th subject.

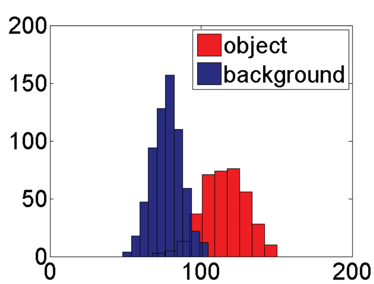


Fig. 3. Histogram of the 15th subject.

Method	Specificity	Sensitivity	DSC
Classical MLC	0.788	0.726	0.675
Proposed method	0.907	0.826	0.831

Table 1. Comparison of specificity, sensitivity and DSC values for 30 subjects' segmentation results obtained by classical supervised MLC and the proposed MLC approach.

class membership as a random field which depends on both intensity values and the particular subject.

4.2. Experimental Results

The differences between subjects/patients induces error to the image segmentation when classical approach is used. In traditional supervised MLC, we do not consider the variability between the subjects, and directly combine the information from the training data. Therefore, the false alarm will be high for the subjects with higher intensity values, such as the 14th subject, shown in Fig. 4(a)-(b). The miss rate will be high for the subjects with lower intensity values, such as the 15th subject, provided in Fig. 5(a)-(b). Fig. 4 and 5 give a visual comparisons for the two example subjects between the classical supervised MLC and the proposed approach. The segmentation results demonstrate that the proposed method has superior segmentation performance.

In addition to visual comparison, we also provide a quantitative performance comparison between the proposed method and the classical supervised MLC algorithm. The specificity, sensitivity, and dice measure (DSC) values are calculated for evaluation:

$$\text{Specificity} = \frac{DTN}{TN}, \quad (8)$$

$$\text{Sensitivity} = \frac{DTP}{TP}, \quad (9)$$

$$\text{DSC} = \frac{2 \cdot DTP}{DP + TP}, \quad (10)$$

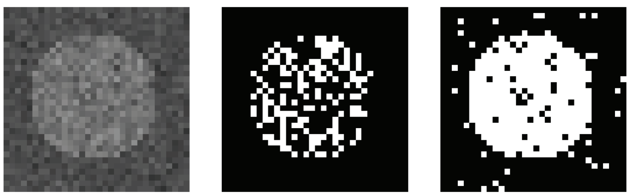
where DTN is the number of the detected true negative pixels, TN for the true negative, DTP the detected true positive, TP the true positive, and DP the detected positive. Table 1 provides the mean of specificity, sensitivity and DSC values of the 30 subjects' segmentation results obtained by the classical supervised MLC and the proposed method. It shows that the specificity, sensitivity are both improved by the proposed method. The DSC values for the 30 subjects are given in Fig. 6, and it proves that for most subjects, the DSC values are considerably improved.

5. CONCLUSIONS

In this paper, a novel method based on supervised maximum likelihood classification is presented for image segmentation.

(a) (b) (c)

Fig. 4. Segmentation results comparison between the classical supervised MLC and the proposed method. Part (a) is the image of the 14th subject, and part (b) the segmentation result of the classical supervised MLC, (c) the segmentation result of the proposed method.



(a) (b) (c)

Fig. 5. The same as Fig.4 for the 15th subject.

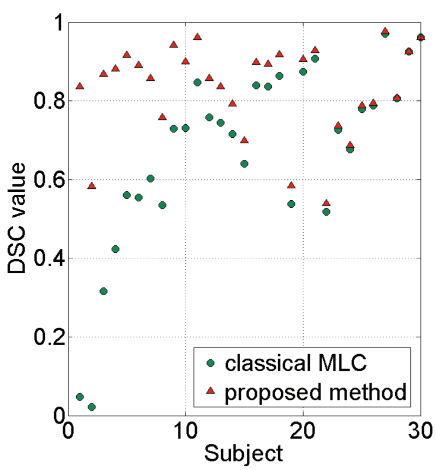


Fig. 6. The DSC values for all 30 subjects (sorted by the improvement obtained the proposed method). The triangles represent DSC values obtained by the proposed method, and circles the DSC values obtained by the classical supervised MLC.

Although many supervised image segmentation methods have been developed and applied for various areas, the differences across the subjects and the consequent error induced to the segmentation results have not been paid much attention before. In many real world applications, especially in medical image segmentation problems, the feature values of different classes of different subjects/patients may not have similar distributions. In the previous studies, the subjects in the training dataset are directly combined for classifier learning, the probability of each pixel belongs to a certain class is only dependent with its feature values.

Instead of consider all the subjects' image data as a Gaussian mixture model, we assume the mean intensity value of each subject and the intensity value as a bivariate random variable and use a bivariate Gaussian mixture model to fit the image data. In the new statistical model, the mean intensity value of each subject is assumed to belong to another Gaussian distribution. In the proposed model, the probability of each pixel belongs to a certain class is dependent on its intensity value and the mean intensity value of the subject. We apply our method to simulated image data as provided in Section 4. The qualitative and quantitative comparisons between the proposed method and the classical supervised MLC demonstrate that the new model could improve the segmentation performance when there is a considerable differences between subjects.

Future work includes extending our method to multispectral image segmentation problems and applying it to real medical image datasets.

6. REFERENCES

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