Bisection Method

Computational Methods Lab ME15L1

Department of Mechanical Engineering
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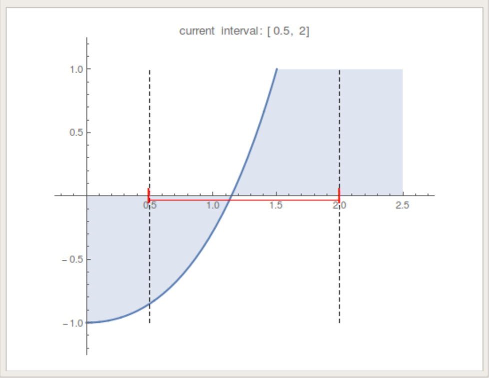
Outline

Introduction

- Bracketing Methods
  - **Bisection**
  - False Position
- Open Methods
  - Successive Iteration
  - Newton-Raphson
  - Secant

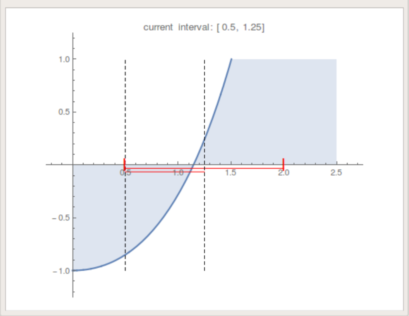
Method

Solve $f(x) = e^x - 2 - x$
$f(0.5) = -0.851279$ & $f(2) = 3.389056$



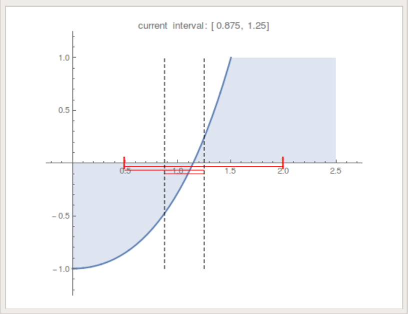
Method - Contd.

$$x1 = 0.5, x2 = 2$$
$$x = \frac{0.5 + 2}{2} = 1.25$$
$$f(0.5) = -0.851279 \ \&$$
$$f(1.25) = 0.240343$$
$$f(0.5) \times f(1.25) < 0$$
$$x2 = 1.25$$



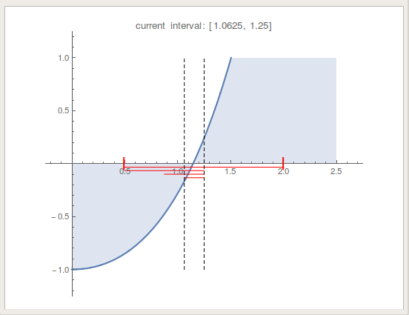
Method - Contd.

$$x1 = 0.5, x2 = 1.25$$
$$x = \frac{0.5 + 1.25}{2} = 0.875$$
$$f(0.5) = -0.851279 \ \&$$
$$f(0.875) = -0.476125$$
$$f(0.5) \times f(0.875) > 0$$
$$x1 = 0.875$$



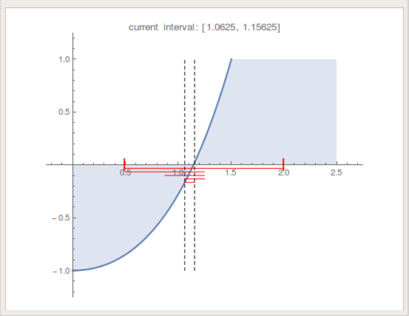
Method - Contd.

$$x1 = 0.875, x2 = 1.25$$
$$x = \frac{0.875 + 1.25}{2} = 1.0625$$
$$f(0.875) = -0.476125 \ \&$$
$$f(1.0625) = -0.168904$$
$$f(0.875) \times f(1.0625) > 0$$
$$x1 = 1.0625$$



Method - Contd.

$$x1 = 1.0625, x2 = 1.25$$
$$x = \frac{1.0625 + 1.25}{2} = 1.15625$$
$$f(1.0625) = -0.168904 \ \&$$
$$f(1.15625) = 0.021743$$
$$f(1.0625) \times f(1.15625) < 0$$
$$x2 = 1.15625$$



Method - Stopping Criteria

Error calculation

$$e = \left| \frac{x - xold}{x} \right|$$

Method - Contd.

Iteration	$x1$	$x2$	x	$f(x)$	e
1	0.500000	2.000000	1.250000	0.240343	0.600000
2	0.500000	1.250000	0.875000	-0.476125	0.428571
3	0.875000	1.250000	1.062500	-0.168904	0.176471
4	1.062500	1.250000	1.156250	0.021743	0.081081
5	1.062500	1.156250	1.109375	-0.076912	0.042254
6	1.109375	1.156250	1.132812	-0.028437	0.020690
7	1.132812	1.156250	1.144531	-0.003563	0.010239
8	1.144531	1.156250	1.150391	0.009036	0.005093
9	1.144531	1.150391	1.147461	0.002723	0.002553

Algorithm

1. Start
2. Read $x1$, $x2$, e *Here x1 and x2 are initial guesses e is the absolute error i.e. the desired degree of accuracy*
3. Compute: $f1 = f(x1)$ and $f2 = f(x2)$, assign: $xold = x1$
4. If $(f1 \times f2) > 0$, then display initial guesses are wrong and go to (11). Otherwise continue.
5. $x = (x1 + x2)/2$
6. If $|(x - xold)/x| < e$, then display x and go to (11).
7. Else, $f = f(x)$
8. If $((f \times f1) > 0)$, then $x1 = x$ and $f1 = f$.
9. Else, $x2 = x$ and $f2 = f$.
10. $xold = x$ Go to (5). *Now the loop continues with new values.*
11. Stop

Example-1

Find a positive root of $xe^x = 1$, which lies between 0 and 1.

Example-1 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	0.000000	1.000000	0.500000	-0.175639	1.000000

Example-1 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	0.000000	1.000000	0.500000	-0.175639	1.000000
2	0.500000	1.000000	0.750000	0.587750	0.333333

Example-1 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	0.000000	1.000000	0.500000	-0.175639	1.000000
2	0.500000	1.000000	0.750000	0.587750	0.333333
3	0.500000	0.750000	0.625000	0.167654	0.200000

Example-1 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	0.000000	1.000000	0.500000	-0.175639	1.000000
2	0.500000	1.000000	0.750000	0.587750	0.333333
3	0.500000	0.750000	0.625000	0.167654	0.200000
4	0.500000	0.625000	0.562500	-0.012782	0.111111

Example-1 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	0.000000	1.000000	0.500000	-0.175639	1.000000
2	0.500000	1.000000	0.750000	0.587750	0.333333
3	0.500000	0.750000	0.625000	0.167654	0.200000
4	0.500000	0.625000	0.562500	-0.012782	0.111111
5	0.562500	0.625000	0.593750	0.075142	0.052632
6	0.562500	0.593750	0.578125	0.030619	0.027027
7	0.562500	0.578125	0.570312	0.008780	0.013699
8	0.562500	0.570312	0.566406	-0.002035	0.006897
9	0.566406	0.570312	0.568359	0.003364	0.003436
10	0.566406	0.568359	0.567383	0.000662	0.001721
11	0.566406	0.567383	0.566895	-0.000687	0.000861
12	0.566895	0.567383	0.567139	-0.000013	0.000430
13	0.567139	0.567383	0.567261	0.000325	0.000215

Example-2

Find a positive root of $x^3 - x - 1$.

Example-2 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	1.000000	2.000000	1.500000	0.875000	0.333333

Example-2 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	1.000000	2.000000	1.500000	0.875000	0.333333
2	1.000000	1.500000	1.250000	-0.296875	0.200000

Example-2 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	1.000000	2.000000	1.500000	0.875000	0.333333
2	1.000000	1.500000	1.250000	-0.296875	0.200000
3	1.250000	1.500000	1.375000	0.224609	0.090909

Example-2 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	1.000000	2.000000	1.500000	0.875000	0.333333
2	1.000000	1.500000	1.250000	-0.296875	0.200000
3	1.250000	1.500000	1.375000	0.224609	0.090909
4	1.250000	1.375000	1.312500	-0.051514	0.047619

Example-2 - Solution

Iteration	$x1$	$x2$	x	$f(x)$	e
1	1.000000	2.000000	1.500000	0.875000	0.333333
2	1.000000	1.500000	1.250000	-0.296875	0.200000
3	1.250000	1.500000	1.375000	0.224609	0.090909
4	1.250000	1.375000	1.312500	-0.051514	0.047619
5	1.312500	1.375000	1.343750	0.082611	0.023256
6	1.312500	1.343750	1.328125	0.014576	0.011765
7	1.312500	1.328125	1.320312	-0.018711	0.005917
8	1.320312	1.328125	1.324219	-0.002128	0.002950
9	1.324219	1.328125	1.326172	0.006209	0.001473
10	1.324219	1.326172	1.325195	0.002037	0.000737
11	1.324219	1.325195	1.324707	-0.000047	0.000369
12	1.324707	1.325195	1.324951	0.000995	0.000184
13	1.324707	1.324951	1.324829	0.000474	0.000092

Algorithm

1. Start
2. Read $x1$, $x2$, e *Here x1 and x2 are initial guesses e is the absolute error i.e. the desired degree of accuracy*
3. Compute: $f1 = f(x1)$ and $f2 = f(x2)$, assign: $xold = x1$
4. If $(f1 \times f2) > 0$, then display initial guesses are wrong and go to (11). Otherwise continue.
5. $x = \dfrac{x1f(x2) - x2f(x1)}{f(x2) - f(x1)}$
6. If $|(x - xold)/x| < e$, then display x and go to (11).
7. Else, $f = f(x)$
8. If $((f \times f1) > 0)$, then $x1 = x$ and $f1 = f$.
9. Else, $x2 = x$ and $f2 = f$.
10. $xold = x$ Go to (5). *Now the loop continues with new values.*
11. Stop