DEPARTMENT OF MECHANICAL ENGINEERING
College of Engineering Thalassery

ME202 Advanced Mechanics of Solids

Tutorial-3: Stress-strain relations and 2D problems in elasticity

1. Using the stress strain relations, strain compatibility relations, and equations of equilibrium, derive the relationship for plain strain $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\sigma_x+\sigma_y)=\frac{-1}{1-\nu}\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}\right)$ X and Y are body force components along x and y directions respectively and ν is the Poisson's ratio.

2. For steel, the following data is applicable: $E = 206\times10^6 kPa, \quad G = 80\times10^6 kPa$. For the given strain matrix at a point, determine the stress matrix.

$$[\epsilon_{ij}] = \begin{bmatrix} 0.001 & 0 & -0.002 \\ 0 & -0.003 & 0.003 \\ -0.002 & 0.003 & 0 \end{bmatrix}$$

3. A thin rubber sheet is enclosed between two fixed hard steel plates (see Fig.1). Friction between the rubber and steel faces is negligible. If the rubber plate is subjected to stresses σ_x and σ_y as shown, determine the strains ϵ_x and ϵ_y and also the stress σ_z.

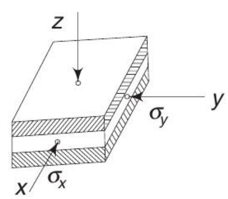

Figure 1: Problem-3

4. Investigate what problem solved by $\phi=\frac{F}{d^3}xy^2(3d-2y)$ applied to the region included in $y=0, y=d, x=0$ on the side x positive.

5. Determine whether the following $\phi=\frac{3F}{4h}\left(xy-\frac{xy^3}{3h^2})\right)+\frac{P}{2}y^2$ can be used as a stress function.If so, determine the components of stress represented by it.

6. Obtain the expressions for σ_x and σ_y in terms of Lame's constants, strain components and dilation.

7. Show that $\phi = A\left(xy^3-\frac{3}{4}xyh^2\right)$ is an Airy's stress function, where A and h are constants. Also show that it represents the stress distribution in a cantilever beam loaded at free end with a point load. Find the value of A if b and h are width and depth respectively.

8. The state of stress at a point is given by $[\sigma_{ij}] = \begin{bmatrix} 9 & 0 & 3 \\ 0 & -10 & 1 \\ 3 & 1 & 112 \end{bmatrix}\times10^3 kPa$ compute the strain tensor for the material with $E = 206\times10^6 kPa, \quad and \quad G = 80\times10^6 kPa$

9. Prove that $\phi = Ax^3$ represents Airy's stress function of a 2D problem. Find the stress components.

10. Obtain a polynomial stress function such that σ_y varies linearly in the x direction only. All the other components of stress are equal to zero.