CSE 473|573 Fall 2016 Homework set #1

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Ans: 1 a) Distance map for 4 neighbour distance

Step-1
top-down
left-right
right-left.
down-top

	5	4			1	2	
5	4	3	2	1	0	1	2
4	3	3	2	1	0	0	1
	2	2			1	1	
	1	1			2	2	
1	0	0	1	2	3	3	4
1	0	1	2	3	4	4	5
	1	2			5	5	

Take the minimum distance while traversing.

Step-2
Above-left
below-right.

6	5	4	3	2	1	2	3
5	4	3	2	1	0	1	2
4	3	3	2	1	0	0	1
3	2	2	3	2	1	1	2
2	1	1	2	3	2	2	3
1	0	0	1	2	3	3	4
1	0	1	2	3	4	4	5
2	1	2	3	4	5	5	6

(5) Distance map for 8 neighborhood.

Step-1

~~abo~~ top-down

down-top

left to right

right to left

	5	4		1	1	1	
5	4	3	2	1	0	1	1
	3	3	2	1	0	0	1
	2	2		1	1	1	1
1	1	1	1	2	2	2	2
1	0	0	1	2	3	3	3
1	0	1	1	2	3	4	4
1	1	1		3	4	5	5

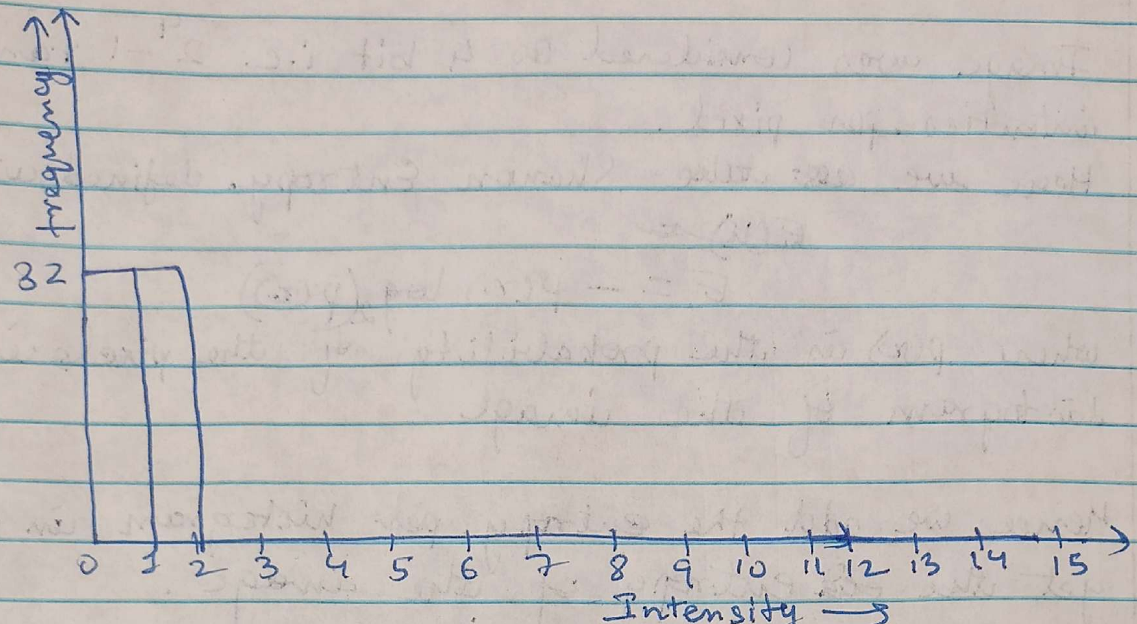
Take the minimum distance while traversing

Step 2

take the minimum distance through all the direction. including ~~horizontally~~, vertical, horizontal and daignol.

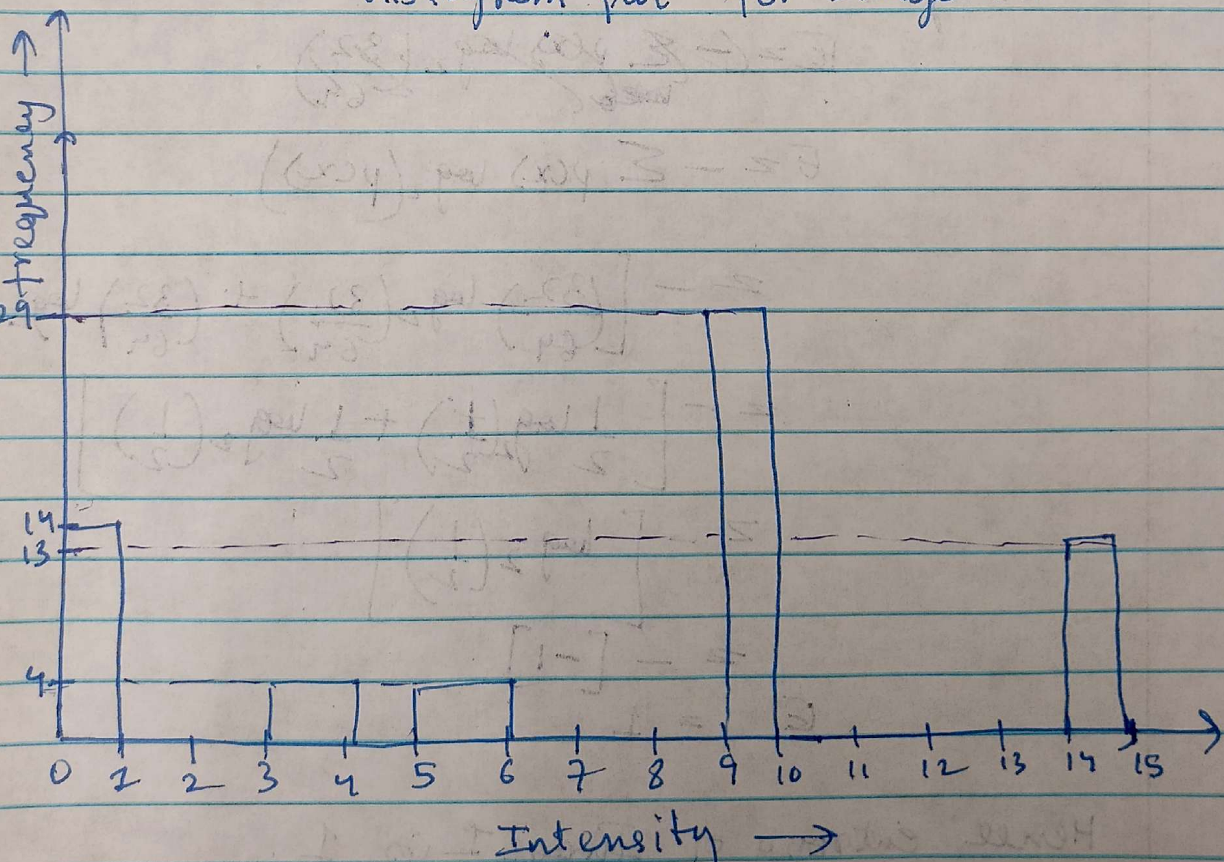
5	4	3	2	1	1	1	2
4	4	3	2	1	0	1	1
3	3	3	2	1	0	0	1
2	2	2	2	1	1	1	1
1	1	1	1	2	2	2	2
1	0	0	1	2	3	3	3
1	0	1	1	2	3	4	4
1	1	1	2	3	4	5	5

Ans: 2 a)



Histogram plot for Image 1.

b)



Histogram plot for Imag ~~2~~ 2.

In the above graphs the following histograms contains no. of pixels

Image 1
01 → 32
02 → 32

Image 2
01 → 14   06 → 04   15 → 13
04 → 04   10 → 29

Image was considered as 4 bit i.e. $2^4 - 1$ range of intenties per pixel.
Here we ~~as~~ take Shanon Entropy defined as

$$\cancel{E(H) =}$$

$$E = -p(x)\log_2(p(x))$$

where $p(x)$ is the probability of the pixels in the histogram of the image

Hence we add the entropy per histogram in order to get the ~~the~~ entropy of the image.

For Image 1 :-

$$\cancel{E = \left(-\sum p(x)\log_2\left(\frac{32}{64}\right)\right)}.$$

$$E = -\sum p(x)\log_2(p(x)).$$

$$= -\left[\left(\frac{32}{64}\right)\log_2\left(\frac{32}{64}\right) + \left(\frac{32}{64}\right)\log_2\left(\frac{32}{64}\right)\right]$$

$$= -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$

$$= -\left[\log_2\left(\frac{1}{2}\right)\right]$$

$$= -[-1]$$

$$E = 1$$

Hence entropy of Image 1 is 1.

for Image 2:

$$E = -\sum p(x_i) \log(p(x_i))$$

$$E = -\left[\left(\frac{14}{64}\right)\log_2\left(\frac{14}{64}\right) + \left(\frac{4}{64}\right)\log_2\left(\frac{4}{64}\right) + \left(\frac{4}{64}\right)\log_2\left(\frac{4}{64}\right) + \left(\frac{29}{64}\right)\log_2\left(\frac{29}{64}\right) + \left(\frac{13}{64}\right)\log_2\left(\frac{13}{64}\right)\right].$$

$$= -\left[-0.4796 - 0.2500 - 0.2500 - 0.5175 - 0.4671\right]$$

$$= +1.9642$$

Hence entropy of Image 2 is 1.9642.

(c) Since ~~we~~ both images ~~use~~ have different graylevels, both ~~have diff~~ ~~wi~~ will have different values of entropy.. ~~Ei~~ Entropy ~~de~~ gives change in average energy levels of the neighbourhood pixels i.e. higher the change, higher the entropy.

In image 1, there are only 2 ~~gra~~ graylevels 1 & 2. Hence the entropy is lower than Image 2 which ~~has~~ have ~~of~~ 5 graylevels. Also there are very erratic changes among the neighbourhood pixels.

Ans: 3 Given

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \quad \text{—— (1)}$$

$$h[n] = \left(\frac{1}{3}\right)^n u[n] \quad \text{—— (2)}$$

$$x[n] = u[n] \quad \text{—— (3)}$$

(a) $y_1[n] = x[n] * h[n]$

$$= \sum_{k=-1}^{1} x[k] * h[n-k]$$

$$= x[-1] * h[n+1] + x[0] * h[n-0] + x[1] * h[n-1]$$

$$= u[-1] * h[n+1] + u[0] * h[n] + u[1] * h[n-1]$$

$= \cancel{u[0]}$ (Using (3))

$$= h[n] + h[n-1] \quad \text{(Using (4))}$$

Here we will take $n = -1, 0, 1$

If $n = -1$

$$= h[-1] + h[-1-1]$$

$$= \left(\frac{1}{3}\right)^{-1} u[-1] + \left(\frac{1}{3}\right)^{-2} u[-2] \quad \text{(Using (2))}$$

$$= 0$$

If $n = 0$

$$= h[0] + h[-1]$$

$$= \left(\frac{1}{3}\right)^{0} u[0] + \left(\frac{1}{3}\right)^{-1} u[-1] \quad \text{(Using (2))}$$

$$= 1$$

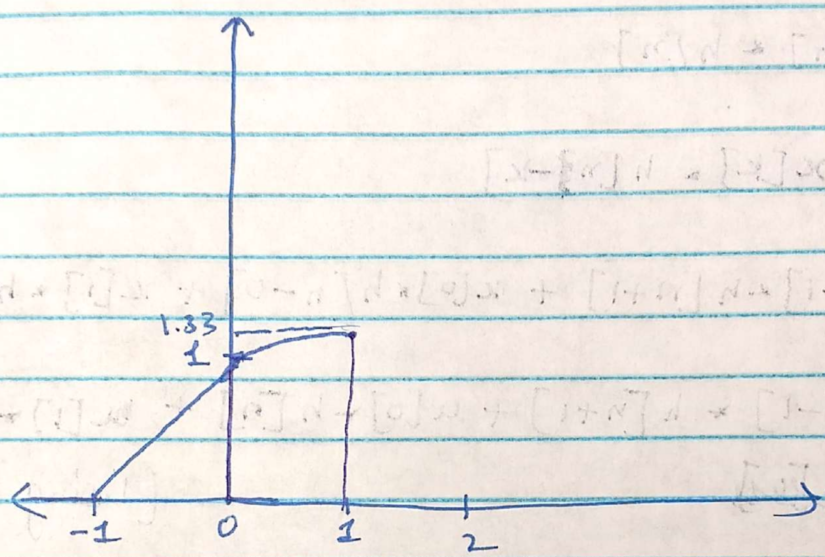
$\frac{1}{9} + \frac{1}{3\times3}$

$n = 1$

$y_1[n] = h[1] + h[0]$

$= (\frac{1}{3})^1 u[1] + (\frac{1}{3})^0 u[0]$

$= \frac{1}{3} + 1 = \frac{4}{3}$



(b) $y_2[n] = x[n-9] * h[n]$

$= \sum_{k=-1}^{1} x[k-9] * h[n-k]$

$= \sum_{k=-1}^{1} u[k-9] * h[n-k]$

Since all the terms will contribute towards 0 till $k=9$, we'll take the case from $n=9$ i.e. 9, 10, 11

$= \sum_{k=9}^{11} u[k-9] * h[n-k]$

$= u[0] * h[n-9] + u[1] \times h[n-10] + u[2] * h[n-11]$

$= h[n-9] + h[n-10] + h[n-11].$

taking $n = 9, 10, 11$.

$n = 9$

$= h[0] + h[-1] + h[-2]$

$= \left(\frac{1}{3}\right)^0 u[0] + \left(\frac{1}{3}\right)^{-1} u[-1] + \left(\frac{1}{3}\right)^{-2} u[-2]$

$= 1.$

$n = 10$

$= h[1] + h[0] + h[-1]$

$= \left(\frac{1}{3}\right)^1 u[1] + \left(\frac{1}{3}\right)^0 u[0] + \left(\frac{1}{3}\right)^{-1} u[-1]$

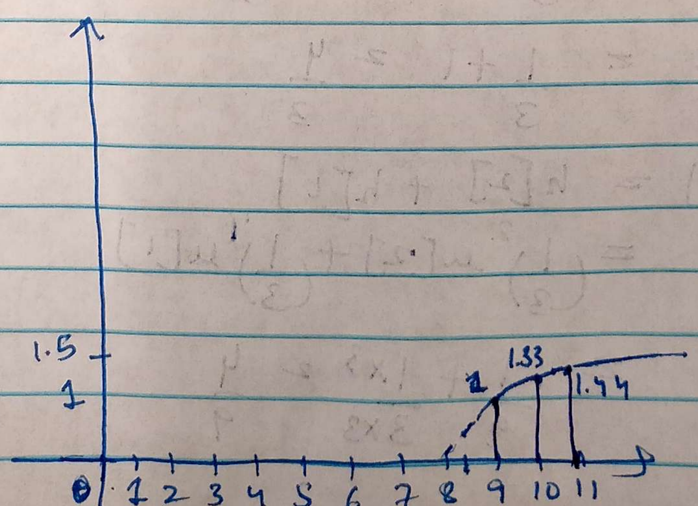
$= \frac{1}{3} + 1 = \frac{4}{3} =$

$n = 11$

$= h[2] + h[1] + h[0]$

$= \left(\frac{1}{3}\right)^2 u[2] + \left(\frac{1}{3}\right)^1 u[1] + \left(\frac{1}{3}\right)^0 u[0]$

$= \frac{1}{9} + \frac{1^{\times 3}}{3\times 3} + \frac{1^{\times 9}}{1\times 9} = \frac{13}{9} = 1.44$



(c) $y_3[n] = x[-n] * h[n]$

$$= \sum_{K=-1}^{1} x[-K] \times h[n-K]$$

$$= \sum_{K=-1}^{1} u[-K] \times h[n-K]$$

$$= u[1]\,h[n+1] + u[0]\,h[n] + u[-1]\,h[n-1]$$

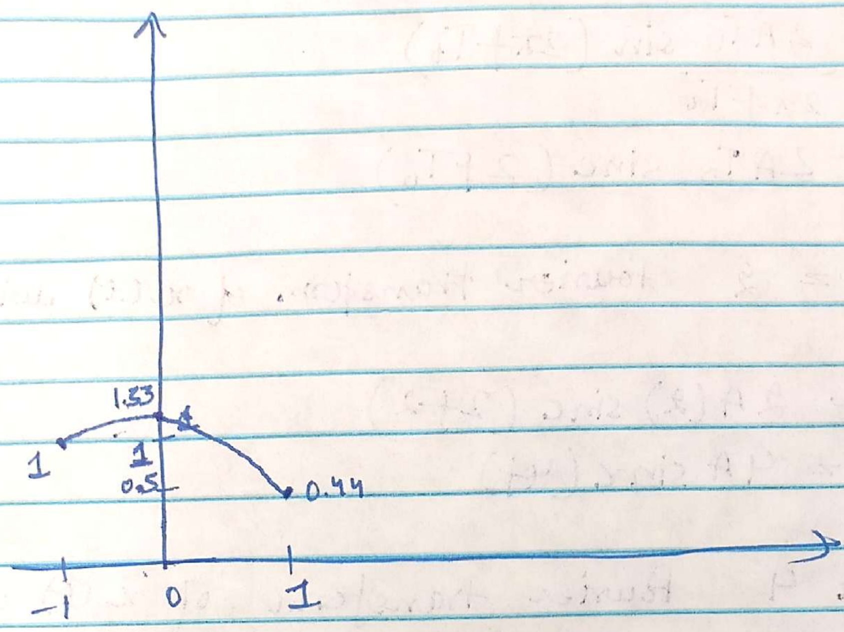
$$y_3[n] = h[n+1] + h[n]$$

Here we will take the case $n = -1, 0, 1$

$$y_3[-1] = h[0] + h[-1]$$
$$= \left(\frac{1}{3}\right)^0 u[0] + 0 = 1$$

$$y_3[0] = h[1] + h[0]$$
$$= \left(\frac{1}{3}\right)^1 u[1] + \left(\frac{1}{3}\right)^0 u[0]$$
$$= \frac{1}{3} + 1 = \frac{4}{3}$$

$$y_3[1] = h[2] + h[1]$$
$$= \left(\frac{1}{3}\right)^2 u[2] + \left(\frac{1}{3}\right)^1 u[1]$$
$$= \frac{1}{9} + \frac{1 \times 3}{3 \times 3} = \frac{4}{9}$$



ms : 4

$$x(t) = \begin{cases} 1 & |t| \le T_0 \\ 0 & |t| \ge T_0 \end{cases}$$

(e) ~~note~~ $f(x(t)) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt$

$$= \int_{-T_0}^{T_0} A e^{-2\pi i f t} dt$$

$$= \frac{A}{-2\pi i f} \left[e^{-2\pi i f t} \Big|_{-T_0}^{T_0} \right]$$

$$= \frac{A}{-2\pi i f} \left[e^{2\pi i f T_0} - e^{2\pi i f T_0} \right]$$

$$= \frac{A T_0}{\pi f T_0} \left[\frac{e^{2\pi i f \cancel{T_0}} - e^{-2\pi i f \cancel{T_0}}}{2i} \right]$$

$$= \cancel{\frac{A T_0}{\pi f T_0} \sin(2\pi f T_0)} \quad \left(\sin x = \frac{e^{ix} - e^{-ix}}{2i} \right)$$

$$= \frac{A T_0}{\pi f T_0} \sin(2\pi f T_0) \quad \left(\sin x = \frac{e^{ix} - e^{-ix}}{2i} \right)$$

$= \cancel{2 A T_0}$

$$= \frac{2AT_0 \sin(2\pi f T_0)}{2\pi f T_0}$$

$$= 2AT_0 \operatorname{sinc}(2fT_0)$$

(a) for $T_0 = 2$ fourier transform of $x(t)$ will be.

$$F(x(t)) = 2A(2)\operatorname{sinc}(2f2)$$
$$= 4A \operatorname{sinc}(4f)$$

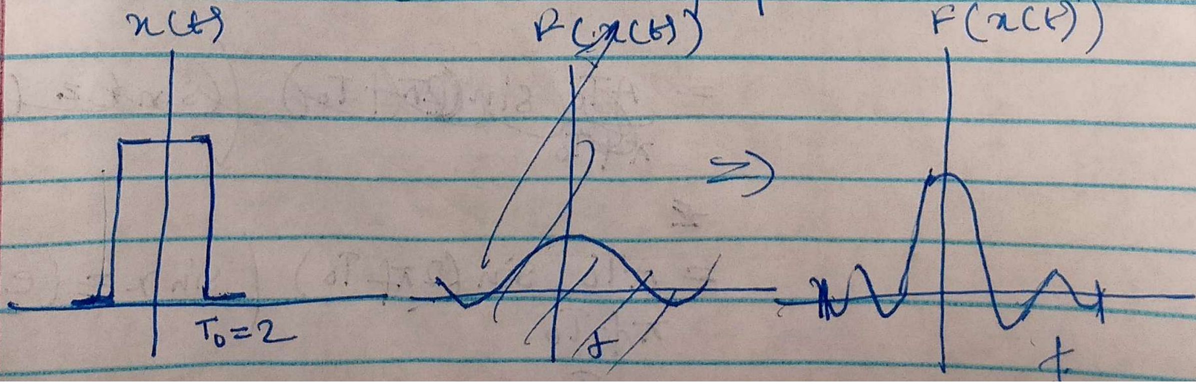
(b) for $T_0 = 4$ fourier transform of $x(t)$ will be

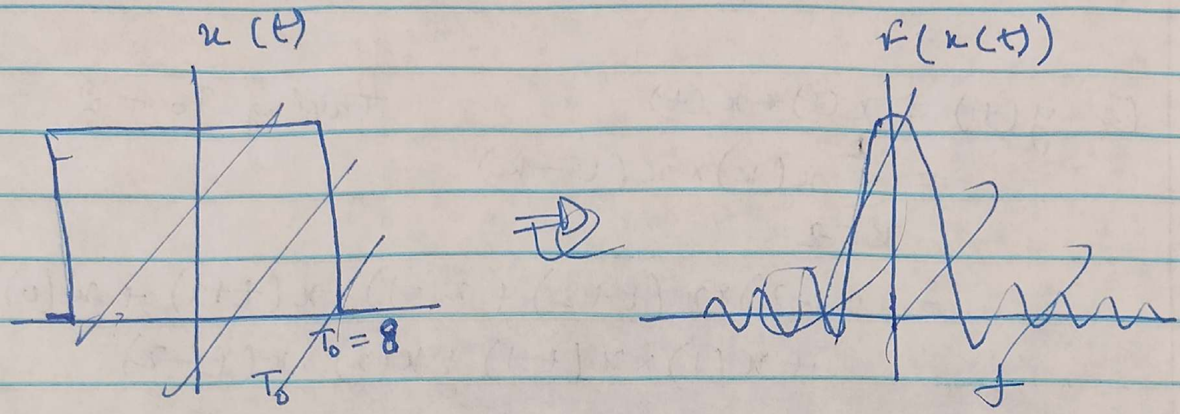
$$F(x(t)) = 2A(4)\operatorname{sinc}(2f4)$$
$$= 8A \operatorname{sinc}(8f)$$

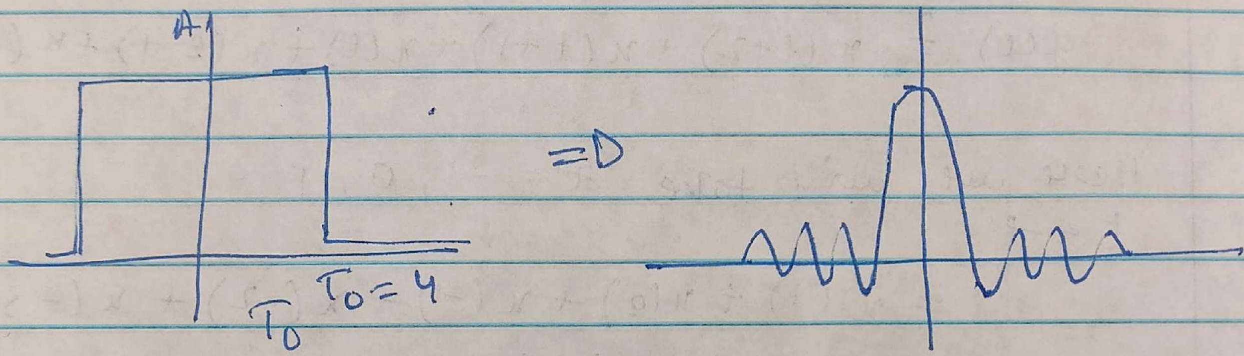
(c) for $T_0 = 8$ fourier transform of $x(t)$ will be

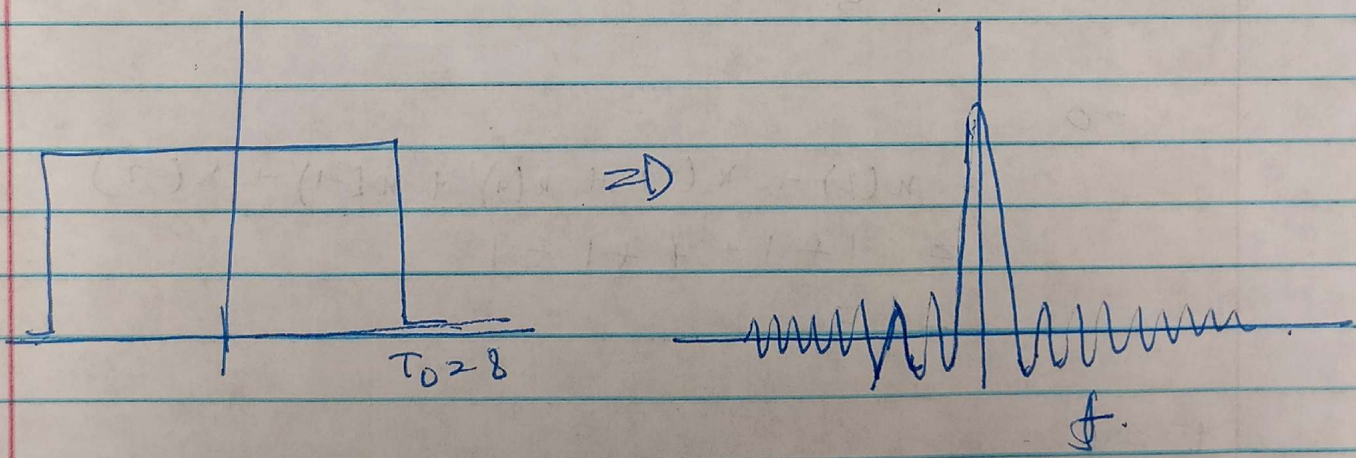
$$= 2A(8)\operatorname{sinc}(2f8)$$
$$= 16A \operatorname{sinc}(16f)$$

(d) from the above values of T_0 we can see that as the value of T_0 increases, the amplitude and the frequency also increases. ~~Also~~ ~~the~~ Since it it the rectangular pulse, more ~~the~~ the value of T_0, the narrower will be the graph.









As the To value get increased, the graph becomes more narrower.

(2) $y(t) = x(t) * x(t)$

$= \sum_{k=} x(t) \cdot x(t)$

$= u\sum$

(e) $y(t) = x(t) * x(t)$ Taking $T_0 = 2$.

$$= \sum_{k=-2}^{2} x(k) \times x(t-k)$$

$$= x(-2) \times x(t+2) + x(-1) \times x(t+1) + x(0) \times x(t) + x(1) \times x(t-1) + x(2) \times x(t-2)$$

$$y(t) = x(t+2) + x(t+1) + x(t) + x(t-1) + x(t-2)$$

Here we will take $t = -1, 0, 1$

$t = -1$

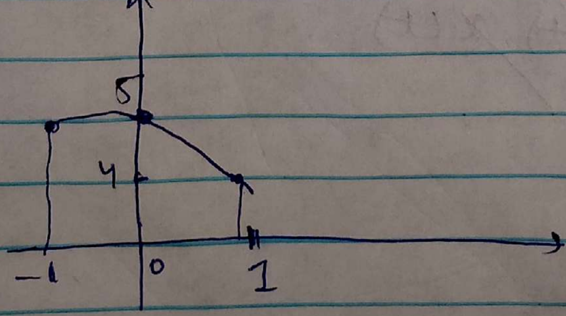
$$= x(1) + x(0) + x(-1) + x(-2) + x(-3)$$
$$= 1 + 1 + 1 + 1 + 1$$
$$= 5$$

$t = 0$

$$= x(2) + x(1) + x(0) + x(-1) + x(-2)$$
$$= 1 + 1 + \cancel{+} + 1 + 1$$
$$= 5$$

$t = 1$

$$= x(3) + x(2) + x(1) + x(0) + x(-1)$$
$$= 0 + 1 + 1 + 1 + 1$$
$$= 4$$



Forier Transform

$$F(y(t)) = \int y(t)\, e^{-2\pi i f t}\, dt$$

$$y(t) = x(t) * x(t)$$

$$= \cancel{\sum_{k=-1}^{1} x(k) * x(t-k)}$$

~~Exxla~~

$$= \sum_{k=-1}^{1} x(k) \cdot x(t-k)$$

$$= x(-1)\, x(t+1) + x(0) \cdot x(t) + x(1) \cdot x(t-1)$$

$$= x(t+1) + x(t) + x(t-1)$$

$$F(y(t)) = \int_{-T_0}^{T_0} y(t)\, e^{-2\pi i f t}$$

$$= \int_{-T_0}^{T_0} x(t+1) + \int_{-T_0}^{T_0} x(t) + \int_{-T_0}^{T_0} x(t-1)$$

$$= \int_{-2}^{2} x(3) + \int_{-2}^{2} x(2) + \int_{-2}^{2} x(1)$$

$$= \cancel{\int_{-2}^{2} x(2) + \int_{-2}^{2} x(1)} = 0 + \int_{-2}^{2} x(2) + \int_{-2}^{2} x(1)$$

~~$= 2A(3)\operatorname{sinc}(2f \cdot 3) + \int 2A \operatorname{sinc}(2f)$~~

$$= 2A(2)\operatorname{sinc}(2f \cdot 2) + 2A\operatorname{sinc}(f)$$

$$= \cancel{4A \operatorname{sinc}(8f)}$$

$$= 4A\operatorname{sinc}(4f) + 2A\operatorname{sinc}(2f).$$

Q 5 (a) Assuming the aperture as the rectangular shape taking T_1 & T_2 as horizontal & vertical size of the camera

$$H(u,v) = \iint h(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$= \int_{-T_1/2}^{T_1/2} e^{-j2\pi ux}\, dx \int_{-T_2/2}^{T_2/2} e^{-j2\pi vy}\, dy$$

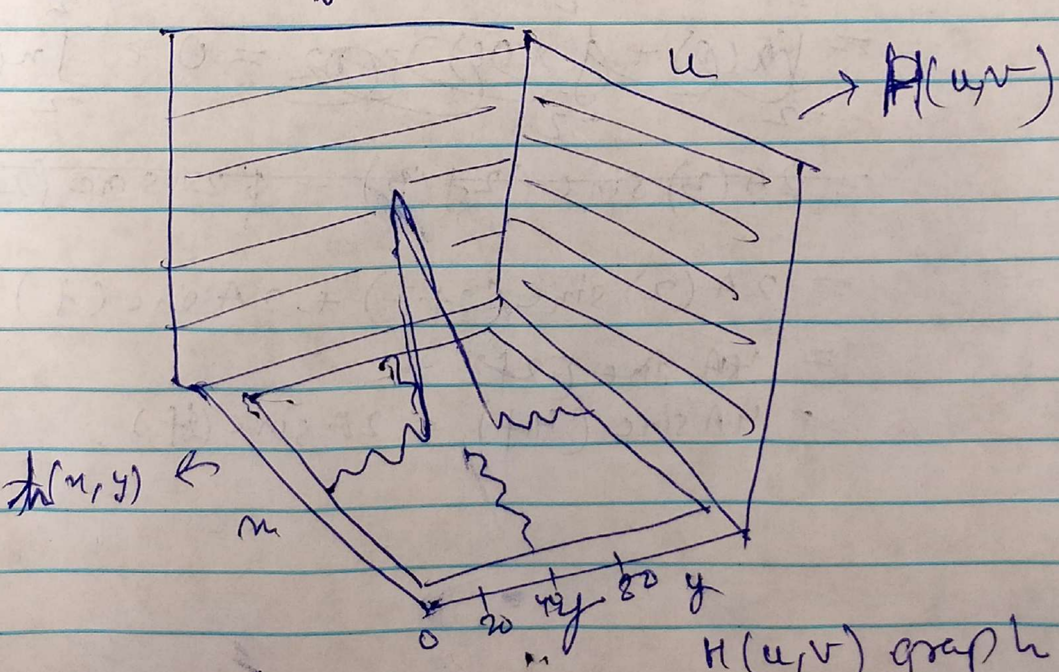
$$= \left.\frac{e^{-j2\pi ux}}{j2\pi u}\right|_{-T_1/2}^{T_1/2} \left.\frac{e^{-j2\pi vy}}{-j2\pi v}\right|_{-T_2/2}^{T_2/2}$$

$$= \frac{1}{-j2\pi u}\left[e^{-juT_1} - e^{juT_1}\right] \frac{1}{-j2\pi v}\left[e^{-jvT_1} - e^{jvT_2}\right]$$

$$= T_1 T_2 \left[\frac{\sin(\pi T_1 u)}{\pi T_1 u}\right]\left[\frac{\sin(\pi T_2 v)}{\pi T_2 v}\right]$$

$$= T_1 T_2 \sin(\pi T_1 u) \sin(\pi T_2 v)$$

(b)



H(u,v) graph

(c) As the value of T_1 & T_2 increases, the ~~the~~ frequency also increases. Here T_1 & T_2 are the horizontal & vertical dimensions of the ~~camper~~ camera's aperture. ~~the~~ The value $\frac{1}{T_1 T_2}$ ranges from $\pm T_1/2$ to $\pm T_2/2$, as the value T_1 & T_2 increases, the amplitude of the wave in the frequency domain decreases.

Scanned by CamScanner