Homework Set-3
CS 573 Fall 2016

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~~Ans~~:

Ans: 1

(a) (1) $\mathrm{sinc}(500\pi t)$

$$\mathrm{sinc}(500\pi t) = \frac{1}{\sqrt{2\pi(500\pi)^2}} \times \mathrm{rect}\left(\frac{2\pi f}{2\pi\, 250}\right)$$

[Take Fourier $\mathrm{sinc}(at) = \frac{1}{\sqrt{2\pi a^2}}\mathrm{rect}\left(\frac{\omega}{2\pi a}\right)$]

$$= \frac{1}{500\pi\sqrt{2\pi}} \mathrm{rect}\left(\frac{f}{250}\right).$$

Hence the maximum frequency f can be 250 Hz.

(2) $\mathrm{sinc}(500\pi t)\,\mathrm{sinc}(1000\pi t)$

$$= F\{\mathrm{sinc}(500\pi t)\} + F\{\mathrm{sinc}(1000\pi t)\}$$

$$= \frac{1}{\sqrt{2\pi(500)^2}}\mathrm{rect}\left(\frac{f}{250}\right) + \frac{1}{\sqrt{2\pi(1000)^2}}\mathrm{rect}\left(\frac{f}{500}\right)$$

= Maximum freq we can get from sin$(500\pi t)$ is 250 & sinc$(1000\pi t)$ is 500.

Therefore we add the two ~~we~~ to get $250 + 500 = 750$ Hz

~~(3) $\mathrm{sinc}^2(1000\pi t)$~~

~~$$\mathrm{sinc}^2(1000\pi t) = \frac{1}{\sqrt{2\pi(1000)^2}} \times \mathrm{tri}\left(\frac{2\pi f}{2\pi\, 500}\right)$$~~

~~[Take Fourier $\mathrm{sinc}^2(at) = \frac{1}{\sqrt{2\pi a^2}}\mathrm{tri}\left(\frac{\omega}{2\pi a}\right)$]~~

~~$$= \frac{1}{1000\sqrt{2\pi}} \times \mathrm{tri}\left(\frac{f}{500}\right).$$~~

~~Maximum freq we can get from sinc$^2(1000\pi t)$ is 500 Hz~~

(3)(a) $\text{sinc}^2(1000\pi t) = \text{sinc}(1000\pi t)\,\text{sinc}(1000\pi t)$

$= f\{\text{sinc}(1000\pi t)\} + f\{\text{sinc}(1000\pi t)\}$

$= \frac{1}{\sqrt{2\pi(1000)^2}}\,\text{rect}\left(\frac{f}{500}\right) + \frac{1}{\sqrt{2\pi(1000)^2}}\,\text{rect}\left(\frac{f}{500}\right)$

So max freq of $\sin^2 c(1000\pi t)$ will be $500 + 500 = 1000$

(b) Nyquist sampling rate will be twice the maximum frequency of the signal.

(1) $\text{sinc}(500\pi t)$ $f = 250\,Hz$

~~Freq~~ Nyquist sampling rate $= 2\times 250 = 500\,Hz$

Nyquist sampling interval $= \frac{1}{500}$ sec.

(2) $\text{sinc}(500\pi t)\,\text{sinc}(1000\pi t)$ $f = 750\,Hz$

Nyquist sampling rate $= 2\times 750 = 1500\,Hz$

Nyquist sampling interval $= \frac{1}{1500}$ sec.

(3) $\text{sinc}^2(1000\pi t)$ $f = 1000\,Hz$

Nyquist sampling rate $= 2\times 1000 = 2000\,Hz$

Nyquist sampling interval $= \frac{1}{2000}$ sec.

(c) For the two sinc signal, ~~the~~ when two sinc functions are multiplied, two rectangles are convolved in the frequency domain. Hence their frequency gets added i.e. $250 + 500$ which is $750\,Hz$

Ans: 2

0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	0
0	0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0	0

(a) If we perform low pass filtering with 3×3 convolutional Kernal $h =$

$$\frac{1}{5}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow h = \begin{bmatrix} 0 & 1/5 & 0 \\ 1/5 & 1/5 & 1/5 \\ 0 & 1/5 & 0 \end{bmatrix}$$

or we can write as $h = \begin{bmatrix} 0 & 0.2 & 0 \\ 0.2 & 0.2 & 0.2 \\ 0 & 0.2 & 0 \end{bmatrix}$.

0	0	0	0	0.2	0.2	0	0	0	0
0	0	0	0.4	0.6	0.6	0.4	0	0	0
0	0	0.4	0.6	1	1	0.6	0.4	0	0
0	0.2	0.6	1	1	1	1	0.6	0.4	0
0	0.2	0.8	1	1	1	1	1	0.6	0.2
0	0.2	0.6	1	1	0.8	0.8	0.8	0.6	0.2
0	0	0.4	0.6	0.6	0.4	0.2	0.2	0.2	0
0	0.2	0.2	0.2	0.2	0	0	0.2	0	0
0	0.2	0.2	0	0	0	0.4	0.2	0.2	0
0	0.2	0	0	0	0.2	0.2	0.4	0	0

(b)

0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	0
0	0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

If we apply threshold of $f(i,j) > 0.5$ will be 1 else $f(i,j) = 0$, then we get similar matrix as the original. But, the only difference will be that, the new image ~~can~~ does not have the random pixels at the bottom or bottom end of the image matrix.

Ans: 3

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(a) If we apply laplacian operator, $h = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ in the above image, we get the following matrix.

0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	2	-3	1	0
0	0	0	0	0	2	-2	-1	1	0
0	0	0	0	2	-2	0	-1	1	0
0	0	0	2	-2	0	0	-1	1	0
0	0	2	-2	0	0	0	-1	1	0
0	2	-2	0	0	0	0	-1	1	0
1	-3	-1	-1	-1	-1	-1	-2	1	0
0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0

(b) For zero crossing detection, we only consider those transition pixels ~~of the~~ value which are opposite in sign i.e. two pixels of non zero positive & negative value pixels. On the next matrix we will ~~draw~~ define zero crossing ~~to~~ between these two pixels both vertically and horizontally.

0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	2	-3	1	0
0	0	0	0	0	2	-2	-1	1	0
0	0	0	0	2	-2	0	-1	1	0
0	0	0	2	-2	0	0	-1	1	0
0	0	2	-2	0	0	0	-1	1	0
0	2	-2	0	0	0	0	-1	1	0
1	-3	-1	-1	-1	-1	-1	-2	1	0
0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0

As far as visual inspection is concerned, the given laplacian operator successfully detect the edges of the original image, hence if we assign all the pixels inside the boundary as 1 and rest as 0, then we will get the original image. Therefore result of laplacian operator is no different ~~than~~ from the visual inspection.

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0