Simple Proof of the Pythagorean Theorem

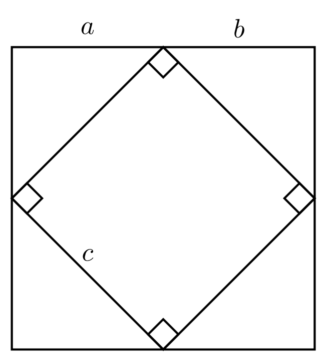
Theorem Statement

The Pythagorean Theorem states that, given a right triangle with a hypotenuse of length c and legs of lengths a and b, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides:

$$c^2 = a^2 + b^2$$

Algebraic Proof

Consider a right triangle with side lengths a, b, and hypotenuse c. Construct a large square with side length $a + b$. The remaining area in the middle forms a smaller inner square with side length c.



The area A_S of the outer square is:

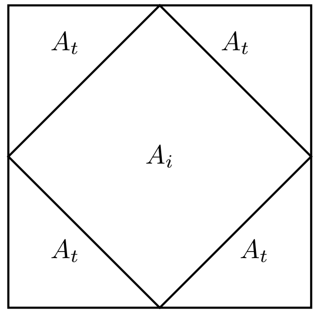
$$A_S = (a + b)^2 \tag{1}$$
$$A_S = a^2 + 2ab + b^2 \tag{2}$$

The area A_t of each of the triangles is:

$$A_t = \frac{1}{2}ab \tag{3}$$

The area A_i of the inner square is:

$$A_i = c^2 \tag{4}$$



Thus, the area A_i of the inner square is the difference between the area of the outer square and the sum of the areas of the four inner triangles:

$$A_i = A_S - 4A_t \tag{5}$$
$$\tag{6}$$

Subtituting the values of A_S and A_t calculated above:

$$A_i = a^2 + 2ab + b^2 - 4 \cdot \frac{1}{2}ab \tag{7}$$
$$A_i = a^2 + 2ab + b^2 - 2ab \tag{8}$$
$$A_i = a^2 + b^2 \tag{9}$$
$$c^2 = a^2 + b^2 \tag{10}$$