Algebraic Proof of the Pythagorean Theorem

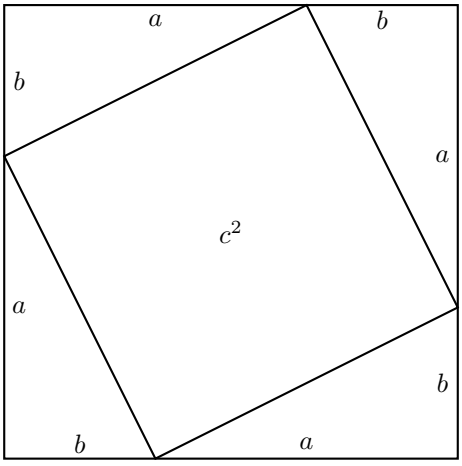
Theorem Statement

The Pythagorean theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides:

$$c^2 = a^2 + b^2$$

Algebraic Proof

Consider a right triangle with sides a, b, and hypotenuse c. We construct a large square with side length $a + b$ and fit four identical right triangles inside it. The remaining area in the middle forms a smaller square with side length c.



The area of the large square is:

$$\text{Area of large square} = (a + b)^2 = a^2 + 2ab + b^2$$

The area of the four triangles is:

$$\text{Area of four triangles} = 4 \times \frac{1}{2}ab = 2ab$$

The area of the inner square is:

$$\text{Area of inner square} = c^2$$

Now, the total area of the large square can also be written as the sum of the areas of the four triangles and the inner square:

$$a^2 + 2ab + b^2 = 2ab + c^2$$

Canceling $2ab$ from both sides:

$$a^2 + b^2 = c^2$$

Thus, we have proven the Pythagorean theorem:

$$c^2 = a^2 + b^2$$