Quantum-Inspired Swarm Modeling with Kronecker Products

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Key words:

QUANTUM-INSPIRED SWARM MODELING WITH KRONECKER PRODUCTS

1. Separable (Independent) Robot States

Assume each robot i has a state vector $|\psi_i\rangle \in \mathbb{R}^d$. The full swarm state is the tensor product:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle$$

This represents a fully separable swarm: each robot evolves independently.

For example, if each $|\psi_i\rangle \in \mathbb{R}^2$, and we have 3 robots:

$$|\psi_1\rangle = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad |\psi_2\rangle = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad |\psi_3\rangle = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Then the full swarm state is:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \in \mathbb{R}^8$$

2. Entangled (Coupled) Robot States

In contrast, if the system state cannot be written as a tensor product of individual states:

$$|\Psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle$$

then the robots are *entangled*, i.e., coupled. For example:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle \right)$$

This state cannot be factorized, and reflects shared or interdependent processing.

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3. Dynamics via Hamiltonian Evolution

Quantum-inspired swarm dynamics can be modeled as:

$$\frac{d}{dt} |\Psi\rangle = -iH |\Psi\rangle$$

where H is the global Hamiltonian. For independent dynamics, H is local:

$$H = H_1 \otimes I \otimes I + I \otimes H_2 \otimes I + \ldots$$

Coupling can be introduced via interaction terms, e.g.,

$$H_{12} = J \cdot (\sigma_x \otimes \sigma_x)$$

where σ_x is the Pauli-X matrix and J is a coupling constant.

4. Summary Table

Concept	**Interpretation in Swarm**
Kronecker product	Stack of independent robot states
Tensor product separability	Independent (decoupled) processing
Entanglement	Interdependent (coupled) processing
Local Hamiltonian	Self-dynamics only
Interaction Hamiltonian	Coupling/influence between robots
$\lvert\dot{\Psi}\rangle = -iH \lvert\Psi\rangle$	Swarm state evolution