MA-374 Lab Assignment 1

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Question 1

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(base) arushgupta@depressed-guy Fe2lab % python '/Users/arushgupta/Desktop/Fe2lab/Lab1/q1.py'
+-----+--------------+--------------+
|   M |  call option |   put option |
+=====+==============+==============+
|   1 |      38.1676 |      19.9417 |
+-----+--------------+--------------+
|   5 |      34.9065 |      16.6806 |
+-----+--------------+--------------+
|  10 |      33.625  |      15.3991 |
+-----+--------------+--------------+
|  20 |      33.8594 |      15.6335 |
+-----+--------------+--------------+
|  50 |      33.9812 |      15.7553 |
+-----+--------------+--------------+
| 100 |      34.0112 |      15.7852 |
+-----+--------------+--------------+
| 200 |      34.0196 |      15.7937 |
+-----+--------------+--------------+
| 400 |      34.0191 |      15.7932 |
+-----+--------------+--------------+
```

Using No Arbitrage Principle

First I checked whether given values adhere to No Arbitrage Principle or not For that
Up_factor > R > Down_factor where R is $e^{r\Delta t}$, Up_factor is $e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$ and Down_factor is $e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$

Solving this inequality we get

$\sigma \cdot \text{root}(\Delta t) - \frac{1}{2}\cdot\sigma\cdot\sigma\cdot\Delta t > 0 > -\sigma \cdot \text{root}(\Delta t) - \frac{1}{2}\cdot\sigma\cdot\sigma\cdot\Delta t$

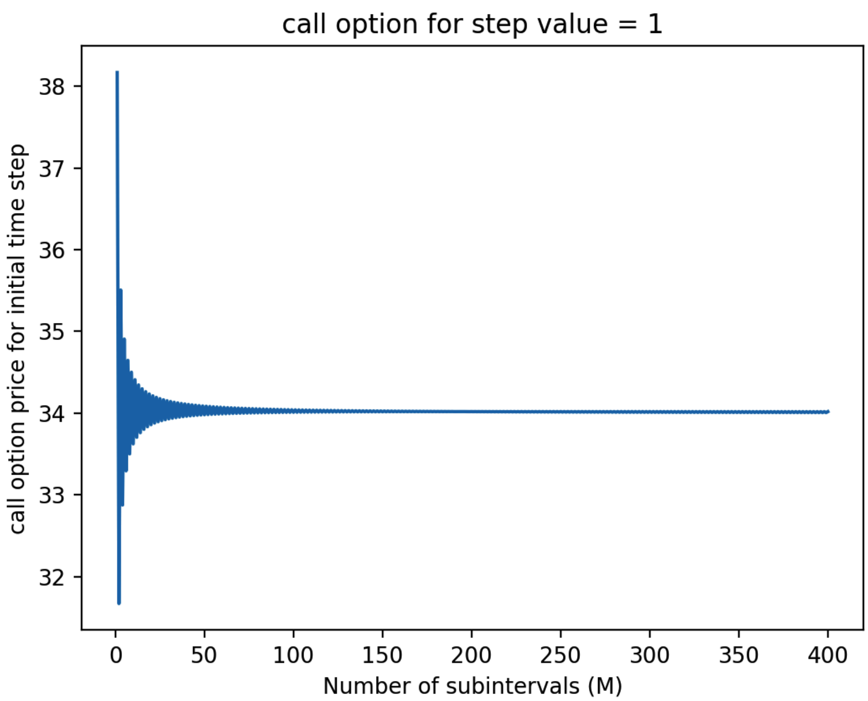
The second inequality always holds true. Thus we get

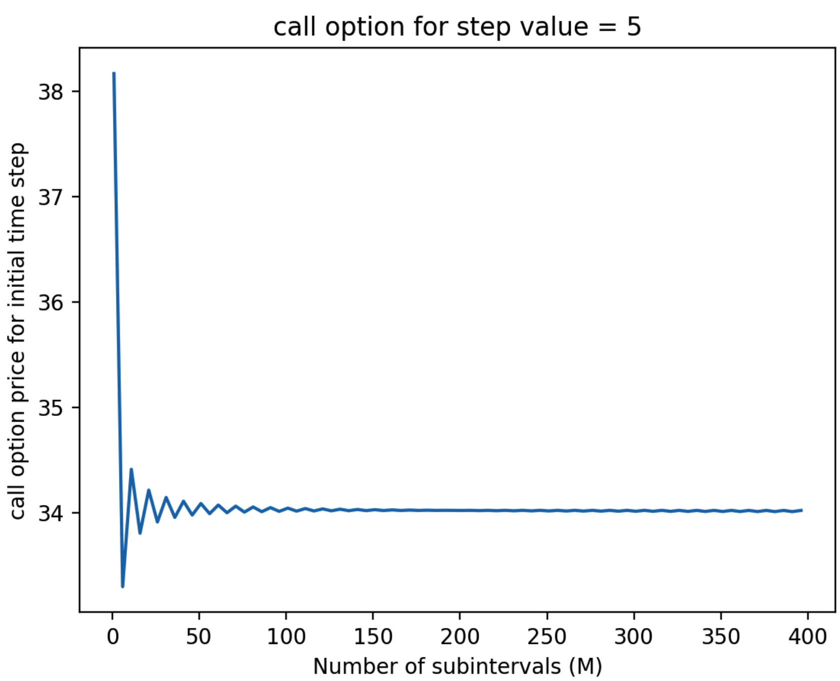
$\sigma \cdot \text{root}(\Delta t) > \frac{1}{2}\cdot\sigma\cdot\sigma\cdot\Delta t \Rightarrow \Delta t < 4/(\sigma\cdot\sigma)$
Since $\Delta t = T/M$
We get $M > T\cdot\sigma\cdot\sigma/4$ ie $M > 0.1125$
Since M is natural number as steps will be natural number only , we it doesn't have lower bound constraint as well as M is natural number ie M>=1 which is true

So Finally We get M has no upper bound it can be infinitely large and it doesn't have lower bound as well

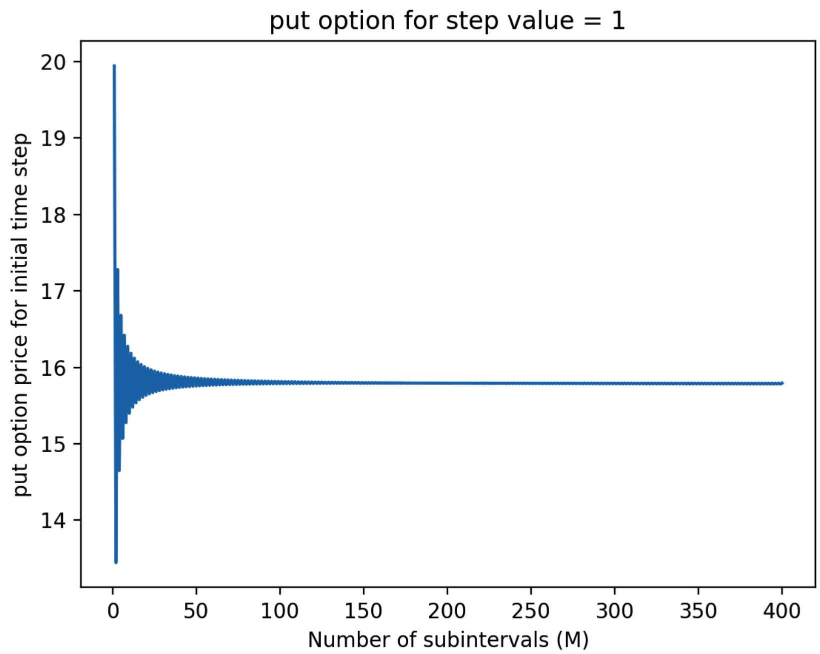
Question 2

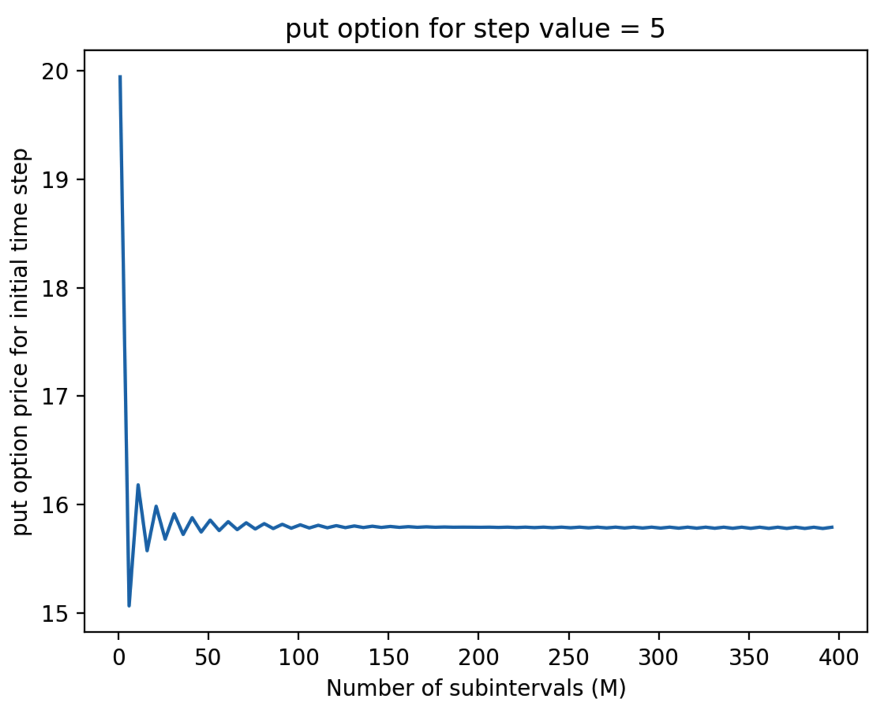
The plots for the Call Option are:





The plots for the Put Option are:





Observations: 1) In each case, the graphs converge to a particular value. For call option, the convergent price is 34.019 For put option, the convergent price is 15.793.
2. The convergence of the plot is faster when step value is 5. The deviations from the convergence value is higher when the value of M is less.

Question 3

Time	Call option
0	[33.859449488493816]
2	[15.095872513879756, 31.89325322224636, 59.95876890092256]
4	[5.154831129992465, 13.469716242796961, 29.803955121326922, 57.69999468717514, 100.6626657133613]
6	[1.125003214520916, 4.121404621027411, 11.767496962598829, 27.57320423638378, 55.295355678567226, 98.43886924880026, 160.61138775301703]
12	[0.0, 0.0, 0.0, 0.11833014485168807, 1.2359711338578985, 6.148520463426524, 19.725206220102628, 46.976187784851014, 91.19343329629585, 154.84169905359852, 242.03018282001355, 359.9341837907892, 519.0996888507186]
18	[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 8.149173872616732, 36.251494491245246, 83.95057683153273, 149.14960563525577, 237.1590889113628, 355.9594650618288, 516.3231991518775, 732.7915980291059, 1024.9933728154078, 1419.4245121000376]

Time	Put option
0	[15.633531710991265]
2	[24.67281716153605, 15.487143431401373, 8.479204228539842]
4	[35.96530361639752, 24.98328656939405, 15.269432108574831, 8.004223459740736, 3.5041738979719703]
6	[48.304950835193225, 36.970072066516444, 25.270959639777356, 14.963371872697072, 7.4362620091378195, 2.998249745266071, 0.9424265244113352]
12	[78.22822279375713, 72.35769482612884, 64.43331094390453, 53.85484171072241, 40.53331384641619, 25.95502392526393, 13.221828652306378, 4.958185582926971, 1.2357022342387147, 0.17210275688518697, 0.008705281628291725, 0.0, 0.0]
18	[95.53406311515673, 93.12931642139074, 89.88324791682145, 85.50151375593354, 79.58679130640232, 71.60275111353512, 60.82542413915251, 46.2775544006557, 26.639984302677387, 8.28121121914694, 0.6015461682626716, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]

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(base) arushgupta@depressed-guy Fe2lab %
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Observation

1) At time t option price can take 1+ $t/\Delta t$ unique values,since $t/\Delta t + 1$ different asset prices are available according to the Binomial Model.
2) As time period increases maximum option price tends to increase and minimum option price tends to decrease.