Scientific Computing (MA322)

Lab 04

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Assumption - 1) While calculating Residues initially I assumed X0=0 and X1 to be my initial approximation.In other words initial point of residues are actual approximations

2) Here H1,H2,H3 represents Absolute value of Residues and Graph Plotted is log log plot of Residues and Iterations.

3) |F(x)| is L1 norm of all function values

Question-1

-Used epsilon as 1e-5
- Used Nmax as 6
- Used Initial approximation as x = [1 , 1 , 1]

1. Starting with $(1, 1, 1)^T$, carry out six iterations of Newton's method for finding a root of the non-linear system

$$\begin{cases} x_1 x_2 = x_3^2 + 1, \\ x_1 x_2 x_3 + x_2^2 = x_1^2 + 2, \\ e^{x_1} + x_3 = e^{x_2} + 3. \end{cases}$$

```
>> Lab4Q1
Below Values are up to 8 decimal places
Iteration       X1              X2              X3              H1              H2              H3              |F(x)|
0               1.00000000      1.00000000      1.00000000      1.00000000      1.00000000      1.00000000      4.00000000
1               2.18932610      1.59847516      1.39390063      1.18932610      0.59847516      0.39390063      3.57427867
2               1.85058965      1.44425142      1.27822400      0.33873645      0.15422374      0.11567663      0.51948761
3               1.78016120      1.42443598      1.23929244      0.07042845      0.01981544      0.03893156      0.01726113
4               1.77767471      1.42396093      1.23747382      0.00248649      0.00047505      0.00181862      0.00002346
5               1.77767192      1.42396060      1.23747112      0.00000279      0.00000033      0.00000270      0.00000000
6               1.77767192      1.42396060      1.23747112      0.00000000      0.00000000      0.00000000      0.00000000
Approximate Solution after 6 Iteration comes out to be 1.77767192 1.42396060 1.23747112
>>
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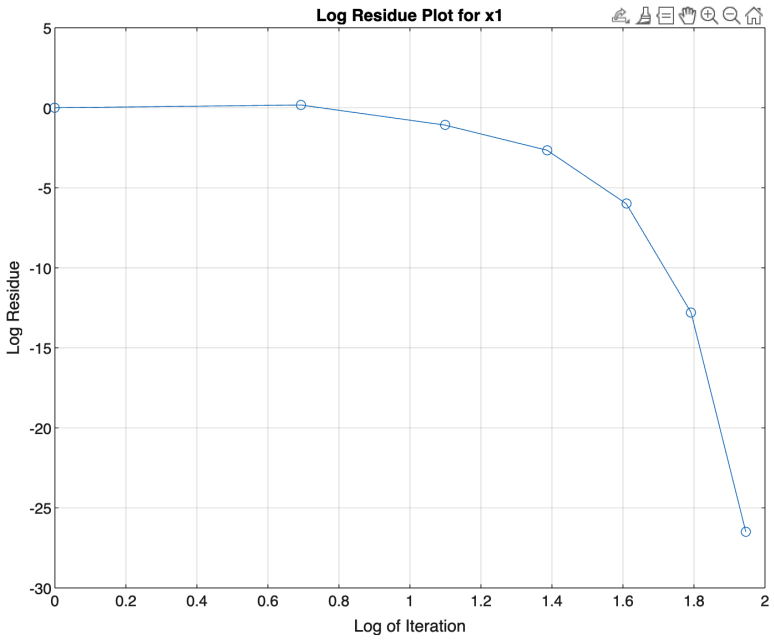
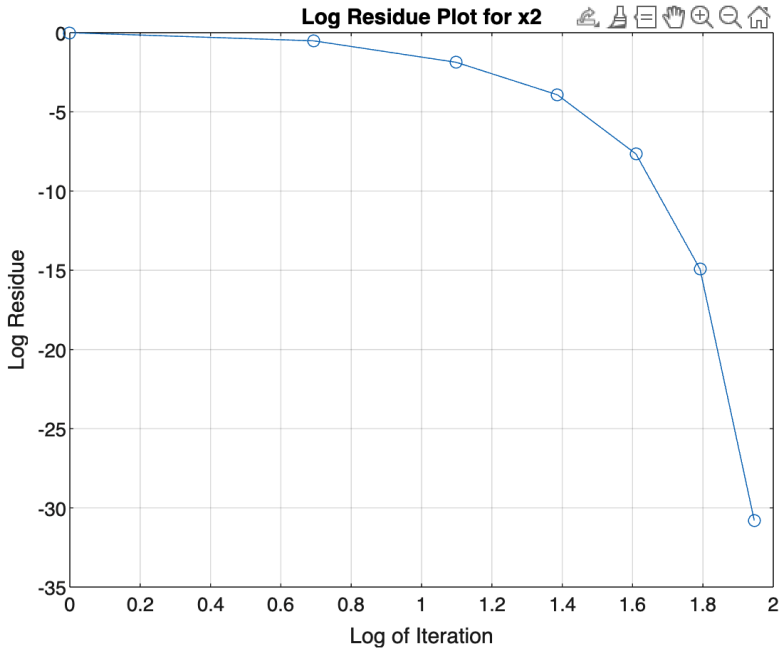
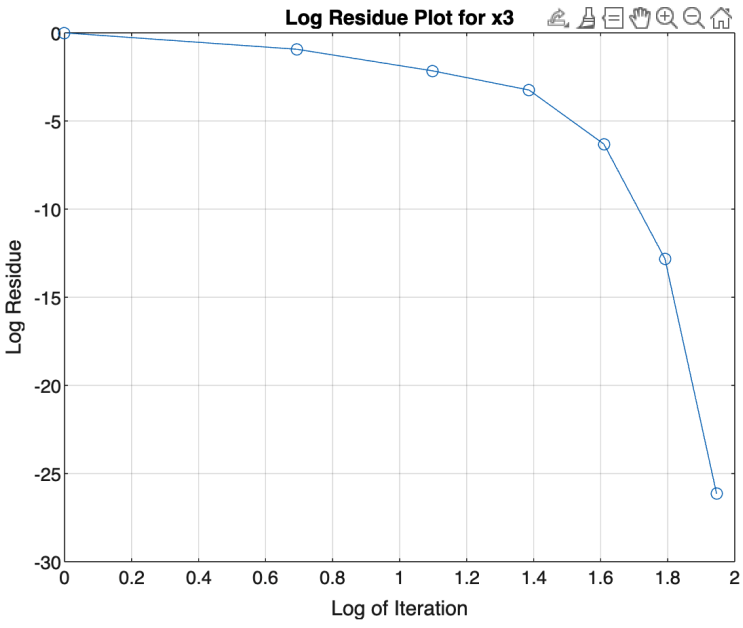
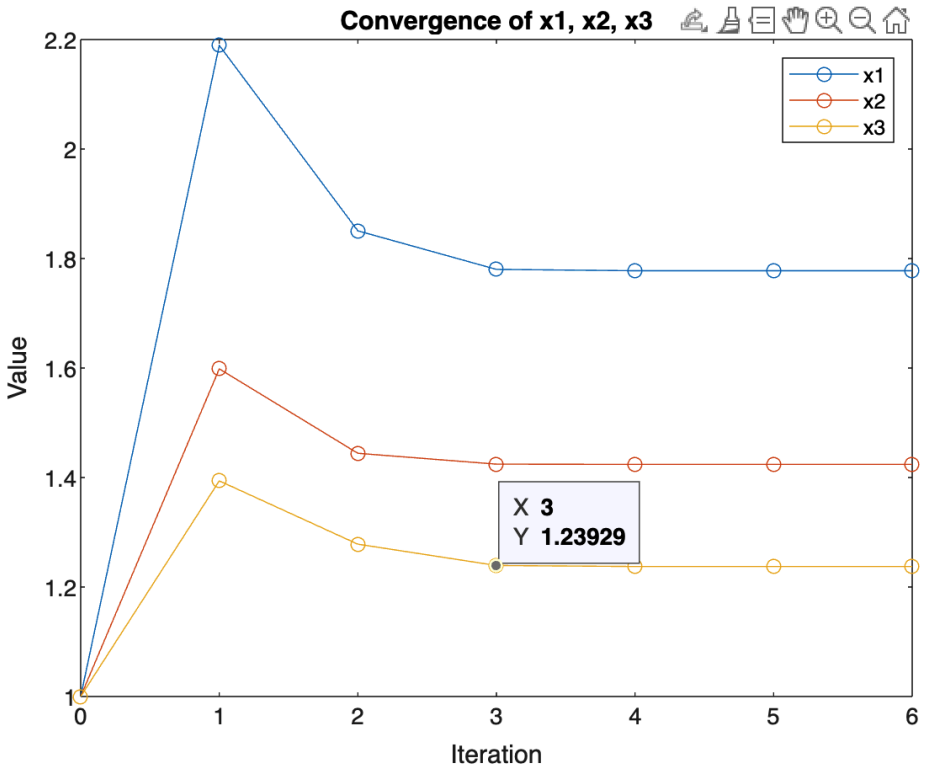

Log Residue Plot for x1
Log Residue
Log of Iteration
5
0
-5
-10
-15
-20
-25
-30
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Log Residue Plot for x2
Log Residue
Log of Iteration
0
-5
-10
-15
-20
-25
-30
-35
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Log Residue Plot for x3
Log Residue
Log of Iteration
0
-5
-10
-15
-20
-25
-30
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



Question-2

2. Perform two iterations of Newton's method in part (a) and five iterations in part (b): (a). Starting with (0,1) (b). Starting with (-1,4)

$$(a).\quad \begin{cases} 4x_1^2 - x_2^2 = 0, \\ 4x_1x_2^2 - x_1 = 1. \end{cases} \qquad (b).\quad \begin{cases} 1 + x^2 - y^2 + e^x\cos(y) = 0, \\ 2xy + e^x\sin(y) = 0. \end{cases}$$

Question-2 A part

-Used epsilon as 1e-6
- Used Nmax as 2
- Used Initial approximation as x = [0 , 1]

```
>> Lab4Q2parta
Below Values are up to 8 decimal places
Iteration       X1              X2              H1              H2              |F(x)|
0               0.00000000      1.00000000      0.00000000      1.00000000      2.00000000
1               0.33333333      0.50000000      0.33333333      0.50000000      1.19444444
2               0.54166667      1.25000000      0.20833333      0.75000000      2.23263889
Approximate Solution after 2 Iteration comes out to be 0.54166667 1.25000000
>>
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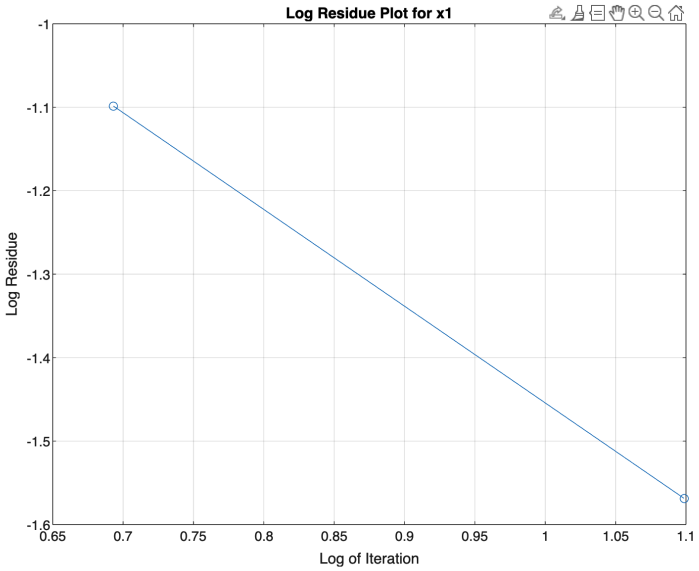
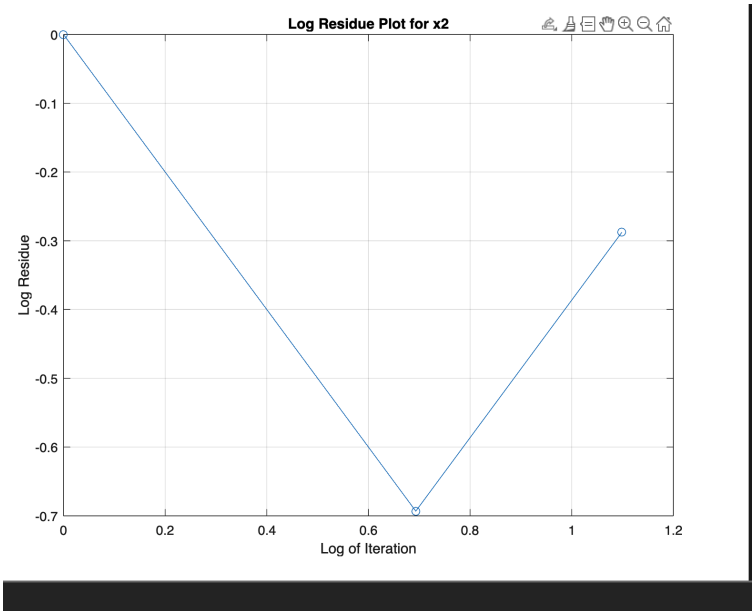
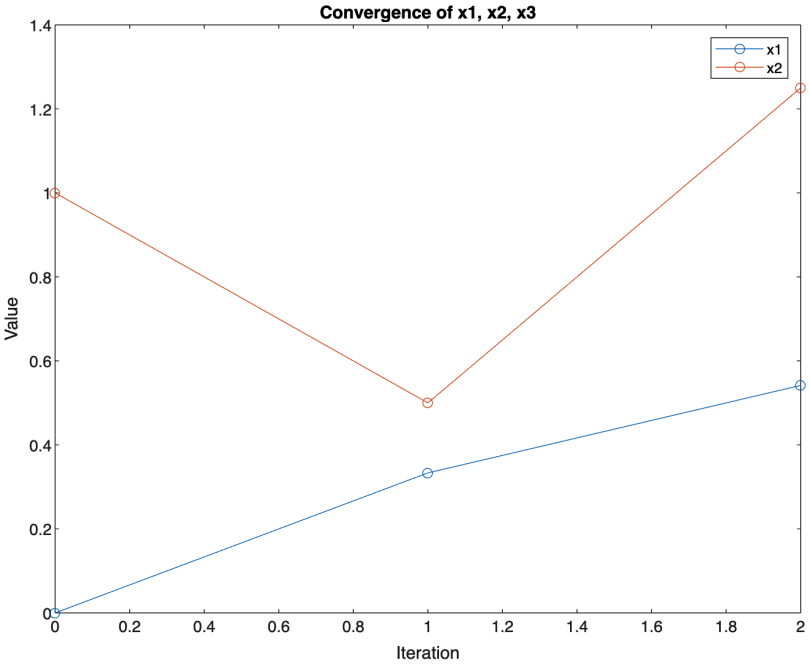
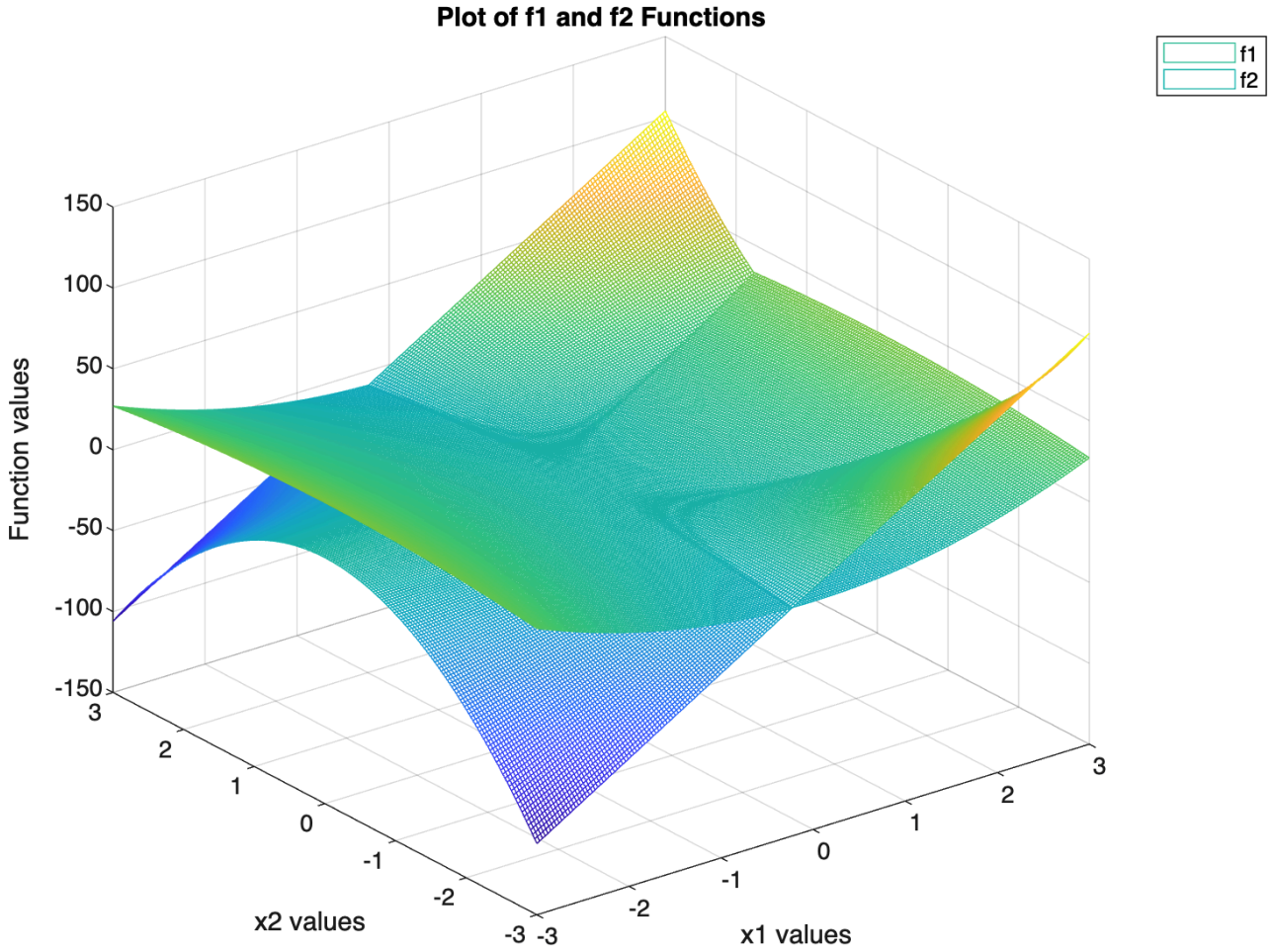

Log Residue Plot for x1
Log Residue
Log of Iteration
-1
-1.1
-1.2
-1.3
-1.4
-1.5
-1.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
1.05
1.1


Log Residue Plot for x2
Log Residue
Log of Iteration
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
0.2
0.4
0.6
0.8
1
1.2


Convergence of x1, x2, x3
x1
x2
Value
Iteration
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



Question-2 B part

-Used epsilon as 1e-6
- Used Nmax as 5
- Used Initial approximation as x = [-1 , 4]

$-1,4)$

$$(b). \quad \begin{cases} 1 + x^2 - y^2 + e^x \cos(y) & = 0, \\ 2xy + e^x \sin(y) & = 0. \end{cases}$$

to the following nonlinear systems in the

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>> Lab4Q2partb
Below Values are up to 8 decimal places
Iteration       X1              X2              H1              H2              |F(x)|
0               -1.00000000     4.00000000      1.00000000      4.00000000      22.51887413
1               -0.50470314     2.01204674      0.49529686      1.98795326      4.53653253
2               -0.37486720     1.30833023      0.12983594      0.70371652      0.70991305
3               -0.30130709     1.17576390      0.07356010      0.13256632      0.03257385
4               -0.29317801     1.17263436      0.00812909      0.00312954      0.00012065
5               -0.29316269     1.17265982      0.00001532      0.00002546      0.00000000
Approximate Solution after 5 Iteration comes out to be -0.29316269 1.17265982
>>
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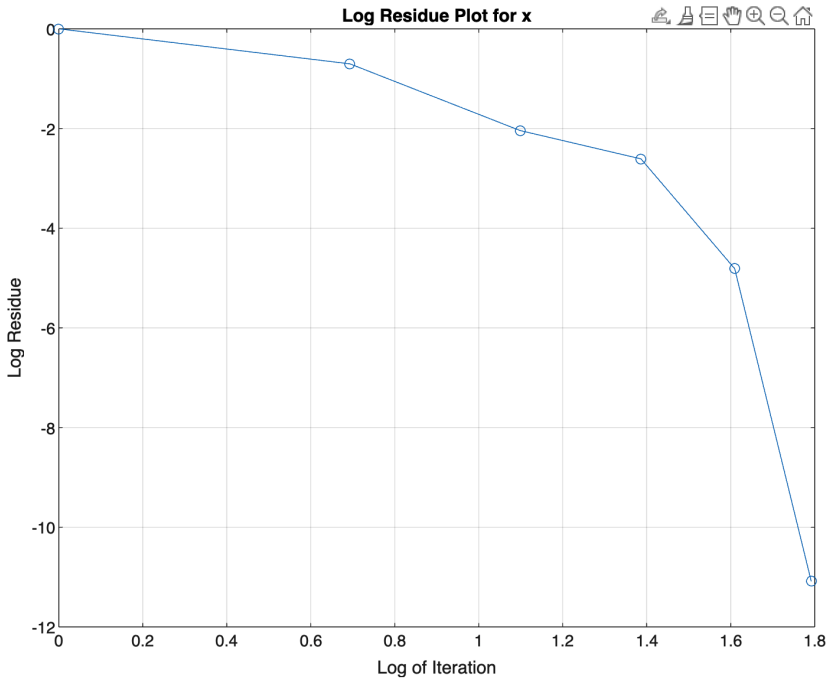
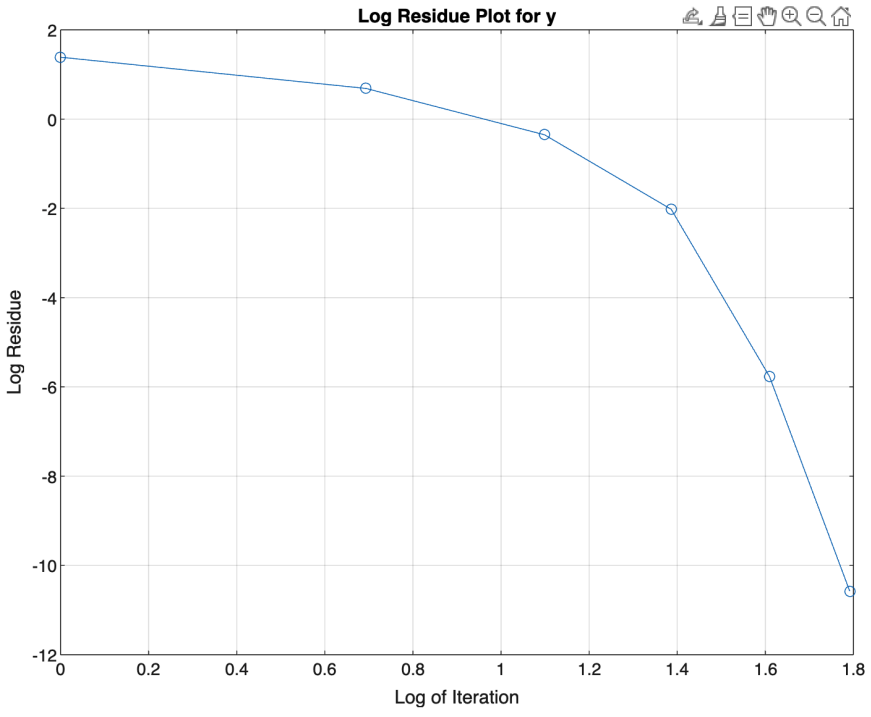
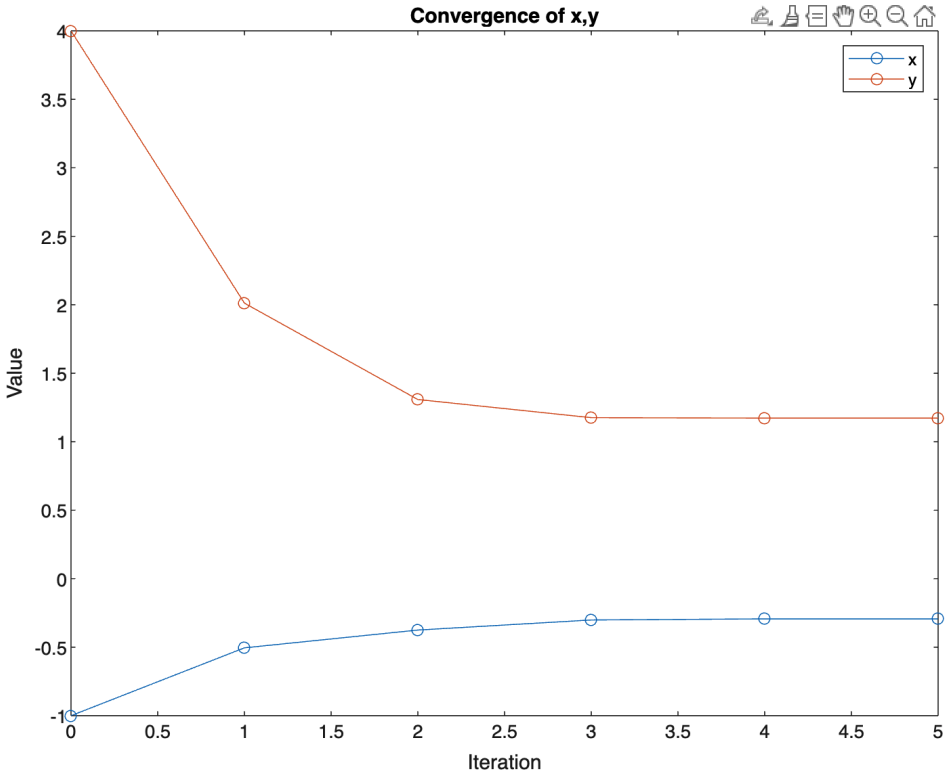
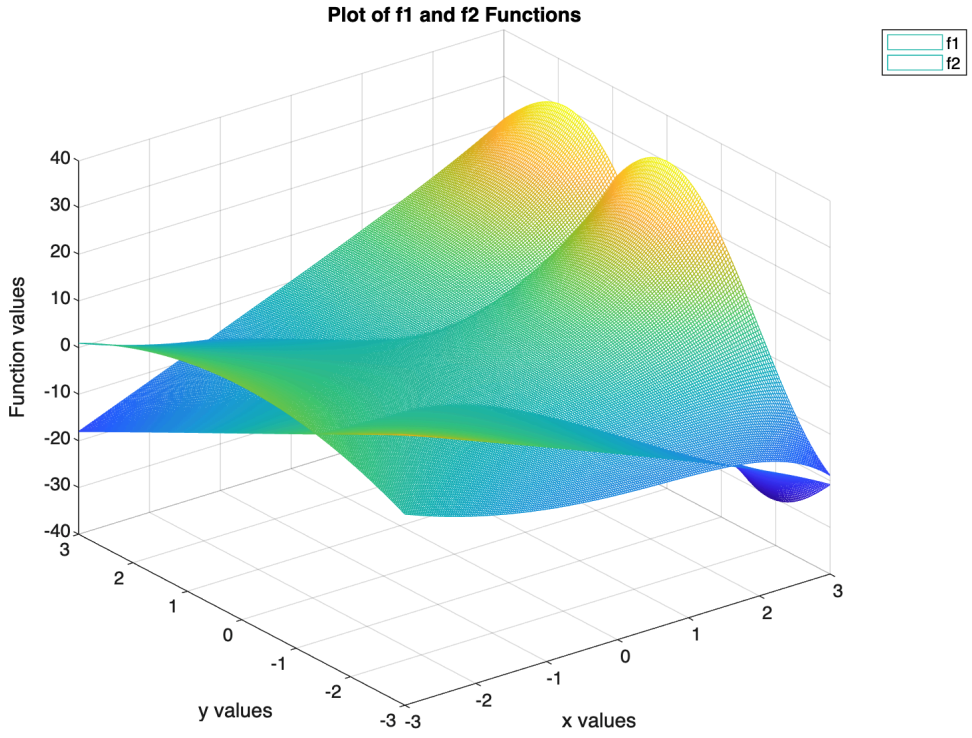





Convergence of x,y
x
y
Value
Iteration
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Plot of f1 and f2 Functions
f1
f2
Function values
y values
x values
40
30
20
10
0
-10
-20
-30
-40
3
2
1
0
-1
-2
-3

Question-3

-Used epsilon or delta as 1e-6
- Used Nmax as 10000
- Used Initial approximation as x = [0 , 0 , 0]

3. Use Newton's method to find a solution to the following nonlinear systems in the given domain. Iterate until $\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_\infty < 10^{-6}$.

$$\begin{cases} 6x_1 - 2\cos(x_2 x_3) - 1 = 0, \\ 9x_2 + \sqrt{x_1^2 + \sin(x_3) + 1.06} + 0.9 = 0, \\ 60x_3 + 3e^{-x_1 x_2} + 10\pi - 3 = 0. \end{cases}$$

Use $\mathbf{x}^{(0)} = (0, 0, 0)^T$.

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Lab4Q3
Below Values are up to 8 decimal places
Iteration      X1             X2             X3             H1             H2             H3             |F(x)|
0              0.00000000     0.00000000     0.00000000     0.00000000     0.00000000     0.00000000     36.34548955
1              0.50000000     -0.18614233    -0.52359878    0.50000000     0.18614233     0.52359878     0.42683024
2              0.49815781     -0.19960682    -0.52882640    0.00184219     0.01346449     0.00522762     0.00009498
3              0.49814468     -0.19960590    -0.52882598    0.00001313     0.00000093     0.00000042     0.00000000
4              0.49814468     -0.19960590    -0.52882598    0.00000000     0.00000000     0.00000000     0.00000000
Approximate Solution comes out to be 0.49814468 -0.19960590 -0.52882598
>>
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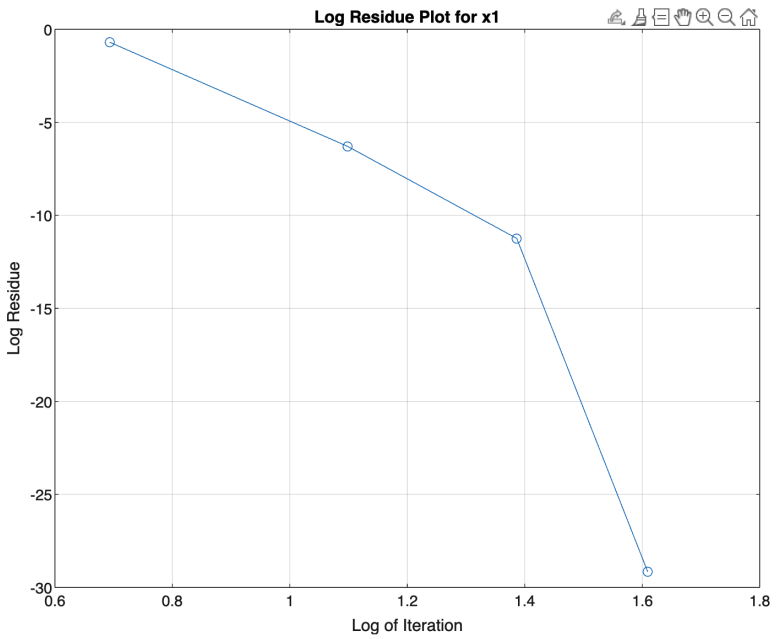
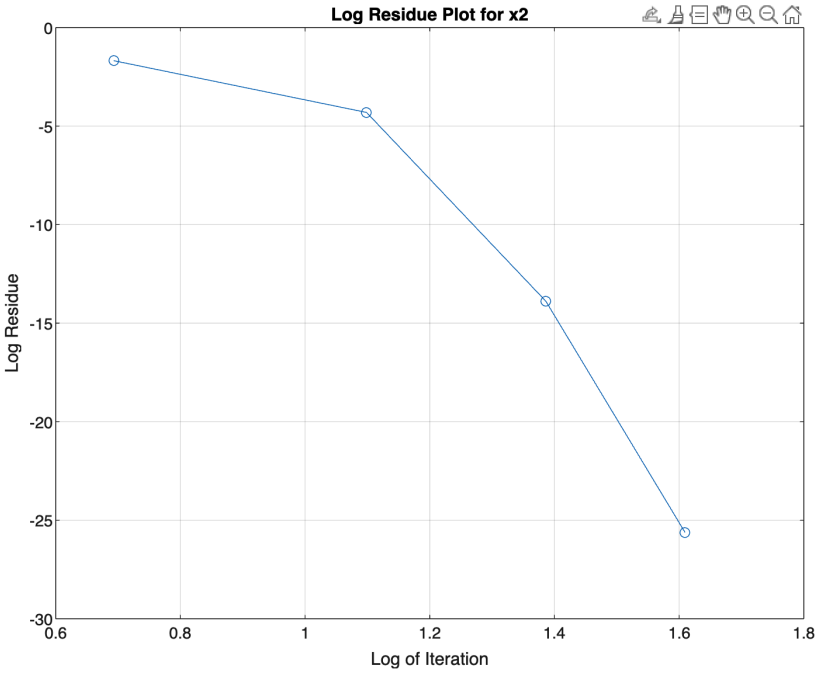
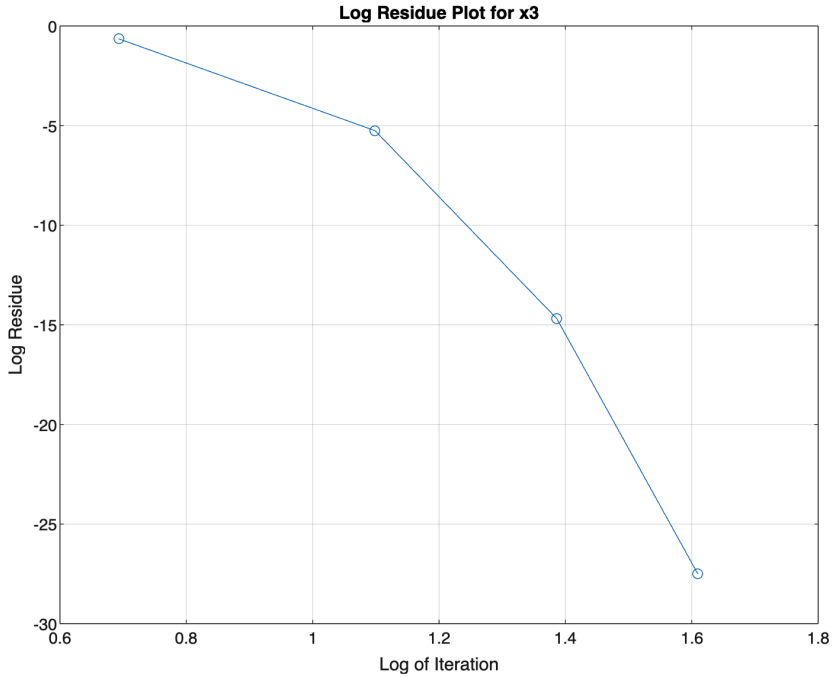
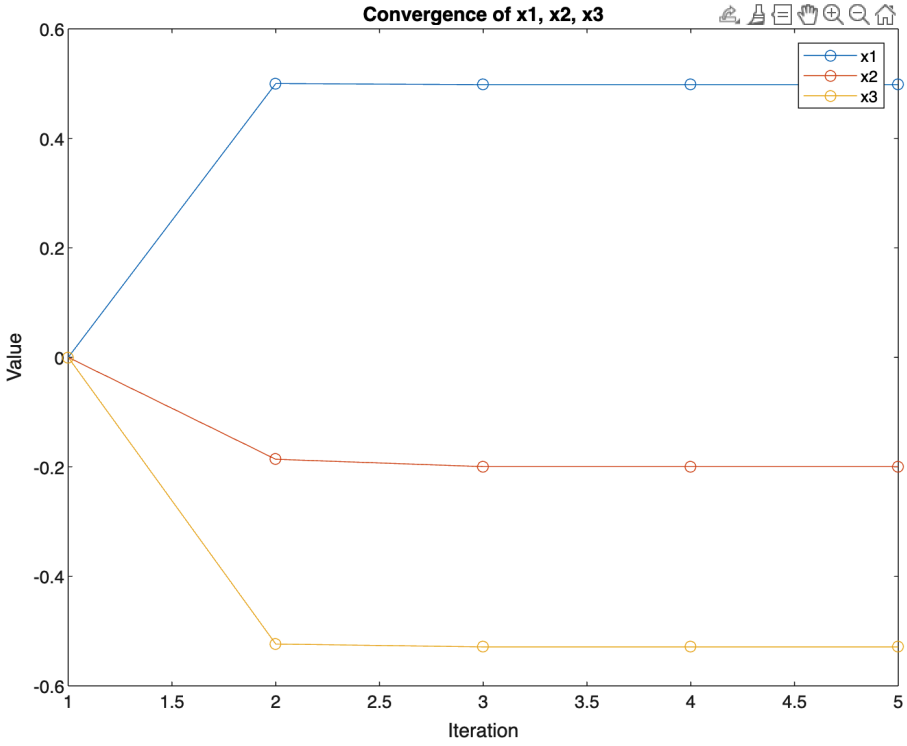



Log Residue Plot for x2
Log Residue
Log of Iteration
0
-5
-10
-15
-20
-25
-30
0.6
0.8
1
1.2
1.4
1.6
1.8


Log Residue Plot for x3
Log Residue
Log of Iteration
0
-5
-10
-15
-20
-25
-30
0.6
0.8
1
1.2
1.4
1.6
1.8


Convergence of x1, x2, x3
x1
x2
x3
Value
Iteration
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1
1.5
2
2.5
3
3.5
4
4.5
5