Scientific Computing (MA322)

Lab 05

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Question 1

1. Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

Use the Newton backward-difference formula to approximate $f(0.65)$.

Using Newton's Forward Difference for calculating f(0.05)

```
>> lab5Q1forward
The approximate value of f(0.05) is: 1.05125880
>>
```

Using Newton's Backward Difference for calculating f(0.65)

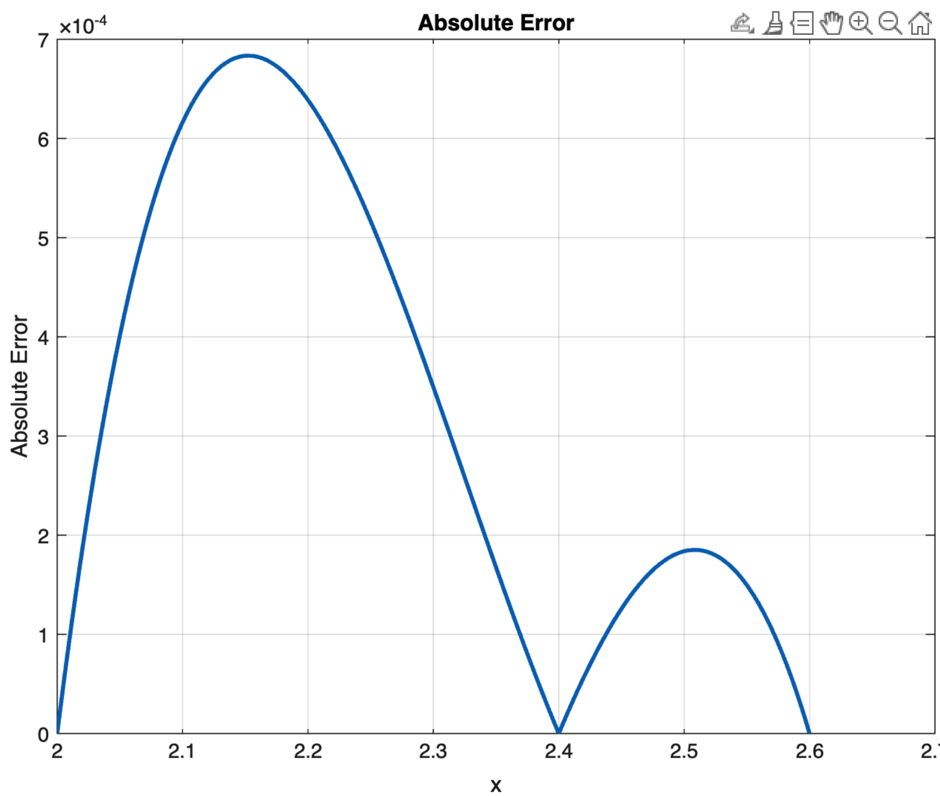
```
>> Lab5Q1backward
The approximate value of f(0.65) is: 1.91555052
>>
```

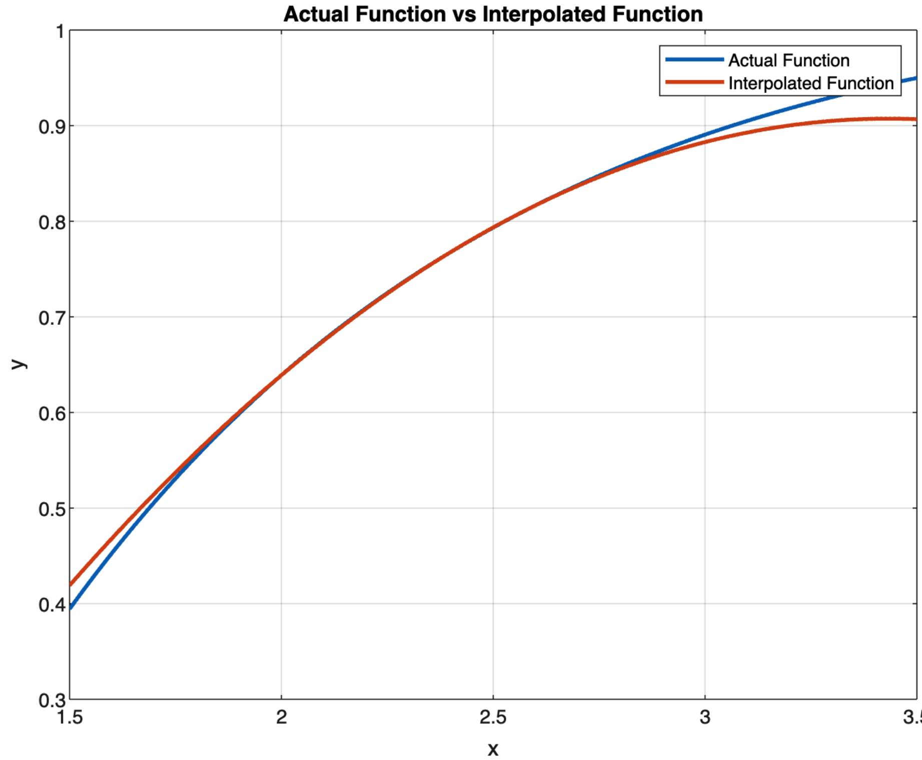
Question 2

2. Construct the Lagrange interpolating polynomial for the function $f(x) = \sin(\ln x)$, and find a bound for the absolute error on the interval $[x_0, x_2]$, where $x_0 = 2.0$, $x_1 = 2.4$, $x_2 = 2.6$.

```
>> Lab5q2
Maximum absolute error in range of [2,2.6] is : 0.00068350
>>
```

Absolute Error vs x graph (here absolute error is abs(actual function(x) - interpolating_polynomial(x))





Question 3

3. Use appropriate Lagrange interpolating polynomials to approximate each of the following:

   a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

   b. $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$

A part

```
>> Lab5Q3parta
Approximate value of f(0.43) using lagrange interpolation comes out to be : 2.36060473
>>
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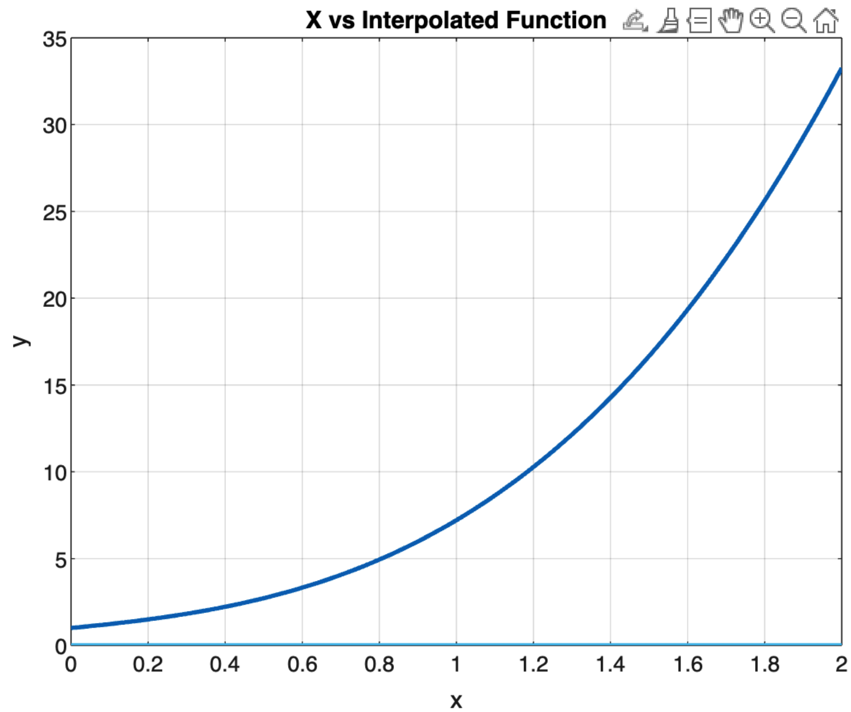

B Part

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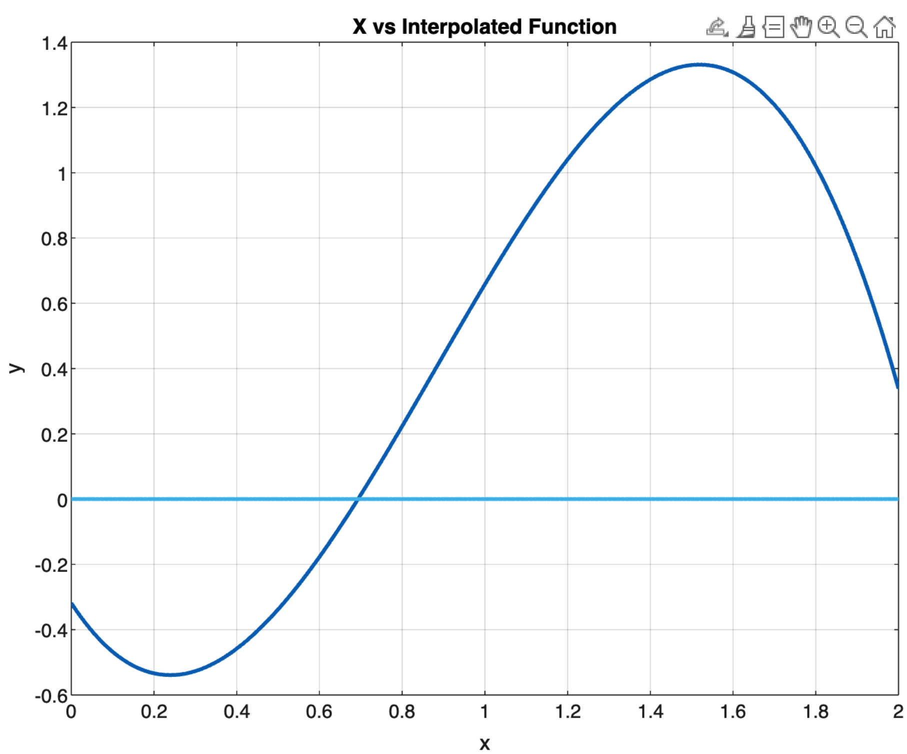
>> Lab5Q3partb
Approximate value of f(0.9) using lagrange interpolation comes out to be : 0.44198500
>>
```



Question 4

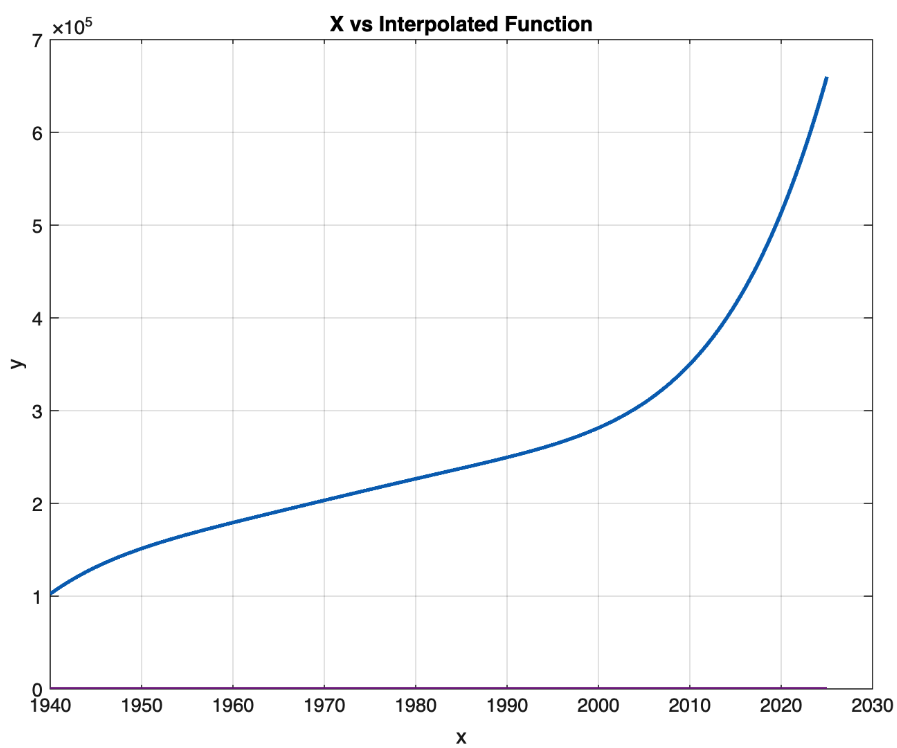
4. A census of the population of the United States is taken every 10 years. The following table lists the population, in thousands of people, from 1950 to 2000:

Year	1950	1960	1970	1980	1990	2000
Populations (in thousands)	151,326	179,323	203,302	226,542	249,633	281,422

Use appropriate divided differences to approximate the population in the years 1940, 1975, 2020.

```
Approximate population in 1940: 102397 thousands
Approximate population in 1975: 2.150428e+05 thousands
Approximate population in 2020: 513443 thousands
Divided Difference Table:
151326.000000000
179323.000000000        2397.900000000
203302.000000000        2324.000000000  -0.745000000
226542.000000000        2309.100000000  43.490000000    0.000000000
249633.000000000        3178.900000000  0.000000000     0.000000000     0.000000000
281422.000000000        0.000000000     0.000000000     0.000000000     0.000000000     0.000000000
>>
```

As can be seen there are some 0 values, although they won't be exactly 0, but those values will be very small nonetheless.



Question 5

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>> Lab5Q5
Approximate value of f(0.2) using lagrange interpolation comes out to be : -5.77858959
Approximate value of f(0.2) using Newton's divided difference comes out to be : -5.77858959

After Adding point x=1.1
Approximate value of f(0.2) using lagrange interpolation comes out to be : -5.77859865
Approximate value of f(0.2) using Newton's divided difference comes out to be : -5.77859865
>>
```

So I did get the same result with both the polynomials.

This is due to the fact that

The Newton interpolating polynomial has also degree n and passes also through the (n+1) given data points. In fact, being expanded in powers , this polynomial is the same as the Largange interpolating polynomial.

The difference between Newton and Lagrange interpolating polynomials lies only in the computational aspect. The advantage of Newton interpolation is the use of nested multiplication and the relative easiness to add more data points for higher-order interpolating polynomials.

We can also look at one theorem known as Weierstrass theorem(which also kinda shows the equivalence)

(Weierstrass's theorem) If *f* is continuous on [a, b] then, given any $\varepsilon > 0$, there exists a polynomial p such that $|f(x) - p(x)| < \varepsilon$ for all $x \in [a,b]$.