Scientific Computing (MA322)

Lab 09

Name- Arush Gupta Roll No-210123008

Note:To assess the error, I computed the definite integral using MATLAB's built-in int(f, a, b) function and compared it with the estimated integral.

For question 3, we were tasked with implementing Gaussian Quadrature, so I employed both Gauss-Legendre and Gauss-Lobatto Quadrature method

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Question 1

1. Approximate the following integrals using Gaussian quadrature with $n = 2$, and compare your results to the exact values of the integrals:

$$a. \quad \int_{1}^{1.5} x^2 \ln x dx \quad b. \quad \int_{0}^{0.35} \frac{2}{x^2 - 4} dx$$

```
>> Lab9Q1

Part A
Estimate of Integral using Gaussian Lagrange Method is 0.228074
Absolute Error comes out to be  0.035815
Estimate of Integral using Gaussian Lobatto Method is 0.228074
Absolute Error comes out to be  0.035815

Part B
Estimate of Integral using Gaussian Lagrange Method is -0.177764
Absolute Error comes out to be  0.000944
Estimate of Integral using Gaussian Lobatto Method is -0.177764
Absolute Error comes out to be  0.000944
>>
```

Question 2

2. Approximate the following integrals using Gaussian quadrature with $n = 2, 3, 4, 5$, uniformly spaced data points of the respective intervals:

$$a.\quad \int_0^{\pi/4} e^{3x} \sin 2x dx \quad b.\quad \int_1^{1.6} \frac{2x}{x^2-4} dx$$

```
>> Lab9Q2

Part A


For N= 2
Estimate of Integral using Gaussian Lagrange Method is 4.143260
Absolute Error comes out to be  1.554631
Estimate of Integral using Gaussian Lobatto Method is 4.143260
Absolute Error comes out to be  1.554631

For N= 3
Estimate of Integral using Gaussian Lagrange Method is 2.583696
Absolute Error comes out to be  0.004932
Estimate of Integral using Gaussian Lobatto Method is 2.583696
Absolute Error comes out to be  0.004932

For N= 4
Estimate of Integral using Gaussian Lagrange Method is 2.585789
Absolute Error comes out to be  0.002840
Estimate of Integral using Gaussian Lobatto Method is 2.587786
Absolute Error comes out to be  0.000843

For N= 5
Estimate of Integral using Gaussian Lagrange Method is 2.587968
Absolute Error comes out to be  0.000660
Estimate of Integral using Gaussian Lobatto Method is 2.588623
Absolute Error comes out to be  0.000005
```

We observe that the approximations by Newton-Cotes and Gauss-Lobatto Quadrature are same for n = 2,3 and after that, Gauss-Lobatto Quadrature is giving a better approximation.

```
Part B


For N= 2
Estimate of Integral using Gaussian Lagrange Method is -0.866667
Absolute Error comes out to be  0.132697
Estimate of Integral using Gaussian Lobatto Method is -0.866667
Absolute Error comes out to be  0.132697

For N= 3
Estimate of Integral using Gaussian Lagrange Method is -0.739105
Absolute Error comes out to be  0.005136
Estimate of Integral using Gaussian Lobatto Method is -0.739105
Absolute Error comes out to be  0.005136

For N= 4
Estimate of Integral using Gaussian Lagrange Method is -0.736428
Absolute Error comes out to be  0.002459
Estimate of Integral using Gaussian Lobatto Method is -0.734204
Absolute Error comes out to be  0.000235

For N= 5
Estimate of Integral using Gaussian Lagrange Method is -0.734157
Absolute Error comes out to be  0.000187
Estimate of Integral using Gaussian Lobatto Method is -0.733980
Absolute Error comes out to be  0.000011
>>
```

We observe that the approximations by Newton-Cotes and Gauss-Lobatto Quadrature are same for n = 2,3 and after that, Gauss-Lobatto Quadrature is giving a better approximation.

Question 3

3. Approximate $\int_{-1}^{1} e^x \sin x dx$ and $\int_{-1}^{1} e^x \cos x dx$ using Gaussian quadrature with $n = 2$ and $n = 4$.

```
>> Lab9Q3

Part A


For N= 2
Estimate of Integral using Gaussian Legendre Method is 0.665844
Absolute Error comes out to be  0.002350
Estimate of Integral using Gaussian Lobatto Method is 1.977795
Absolute Error comes out to be  1.314302

For N= 4
Estimate of Integral using Gaussian Legendre Method is 0.663493
Absolute Error comes out to be  0.000000
Estimate of Integral using Gaussian Lobatto Method is 0.662818
Absolute Error comes out to be  0.000676

Part B


For N= 2
Estimate of Integral using Gaussian Legendre Method is 1.962973
Absolute Error comes out to be  0.029551
Estimate of Integral using Gaussian Lobatto Method is 1.667460
Absolute Error comes out to be  0.265961

For N= 4
Estimate of Integral using Gaussian Legendre Method is 1.933417
Absolute Error comes out to be  0.000005
Estimate of Integral using Gaussian Lobatto Method is 1.933467
Absolute Error comes out to be  0.000045
>>
```

Here, we observe that Gauss-Legendre Quadrature is giving better approximations than Gauss- Lobatto Quadrature.

Question 4

4. Consider using Gauss-Legendre quadrature to integrate

$$a. \quad \int_0^1 e^{-x^2} dx \quad b. \quad \int_{-4}^4 \frac{1}{1+x^2} dx$$

with $n = 2, 4, 6$ node-point formulas.

```
>> Lab9Q4

Part A


For N= 2
Estimate of Integral using Gaussian Legendre Method is 0.74659469
Absolute Error comes out to be  0.00022944

For N= 4
Estimate of Integral using Gaussian Legendre Method is 0.74682447
Absolute Error comes out to be  0.00000034

For N= 6
Estimate of Integral using Gaussian Legendre Method is 0.74682413
Absolute Error comes out to be  0.00000000
```

```
Part B


For N= 2
Estimate of Integral using Gaussian Legendre Method is 1.26315789
Absolute Error comes out to be  1.38847743

For N= 4
Estimate of Integral using Gaussian Legendre Method is 2.04728501
Absolute Error comes out to be  0.60435032

For N= 6
Estimate of Integral using Gaussian Legendre Method is 2.41168893
Absolute Error comes out to be  0.23994640
>>
```

For Part A there is less variation for increasing N, but for part B there is large variation for increasing N.

This is due to behaviour of the function itself

For part A when N=6 error is very small in 1e-11 range.