Scientific Computing (MA322)

Lab 08

Name- Arush Gupta   Roll No-210123008

Question 1

1. Approximate the following integrals using the Rectangle rule:

a. $\int_{0.5}^{1} x^4 dx$ b. $\int_{0}^{0.5} \frac{2}{x-4} dx$ c. $\int_{1}^{1.6} \frac{2x}{x^2-4} dx$

d. $\int_{0}^{\pi/4} e^{3x} \sin 2x dx$ e. $\int_{0.75}^{1.3} \left((\sin x)^2 - 2x \sin x + 1\right) dx.$

Approximating the value of each integral using the rectangle rule and considering the initial point x0 for approximate on i.e. f(x0), we have

```
>> Lab8Q1
A part
Approx Value of integral  using Rectangle rule comes out to be 0.03125

B part
Approx Value of integral  using Rectangle rule comes out to be -0.25000

C part
Approx Value of integral  using Rectangle rule comes out to be -0.40000

D part
Approx Value of integral  using Rectangle rule comes out to be 0.00000

E part
Approx Value of integral  using Rectangle rule comes out to be 0.05574

>>
```

Question 2

2. Use the Midpoint rule, Trapezoidal rule and Simpson's rule to approximate the integrals given in Exercise 1.

Using the midpoint rule to approximate the values of the integrals, i.e. considering f((x0+x1)/2) to be the constant approximation

Using the trapezoidal rule to approximate the values of the integrals, i.e. considering 0.5*(f(x0) + f(x1))*(x1-x0) to be the approximation

Using the Simpson's rule to approximate the values of the integrals, i.e. considering (b-a)/6 *(f(x0) + 4*f((x0+x1)/2) + f(x1)) to be the approximation

```
>> Lab8Q2
A part
Approx Value of integral using Midpoint rule comes out to be 0.15820

Approx Value of integral using Trapezoidal rule comes out to be 0.26562

Approx Value of integral using Simpsons 1/3rd rule comes out to be 0.19401

B part
Approx Value of integral using Midpoint rule comes out to be -0.26667

Approx Value of integral using Trapezoidal rule comes out to be -0.26786

Approx Value of integral using Simpsons 1/3rd rule comes out to be -0.26706

C part
Approx Value of integral using Midpoint rule comes out to be -0.67532

Approx Value of integral using Trapezoidal rule comes out to be -0.86667

Approx Value of integral using Simpsons 1/3rd rule comes out to be -0.73911

D part
Approx Value of integral using Midpoint rule comes out to be 1.80391

Approx Value of integral using Trapezoidal rule comes out to be 4.14326

Approx Value of integral using Simpsons 1/3rd rule comes out to be 2.58370

E part
Approx Value of integral using Midpoint rule comes out to be 0.01161

Approx Value of integral using Trapezoidal rule comes out to be 0.02824

Approx Value of integral using Simpsons 1/3rd rule comes out to be 0.01715

>>
```

Question 3

3. Compute π from an integral of the form $\int_0^1 \frac{4}{1+x^2}dx$ by using Rectangle, Trapezoidal, Simpson's one-third and three-eighth rules. Compare and explain these numerical results to the true solution. Simpson's three-eighth rule is given by

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] - \frac{3h^5}{80}f^4(\xi), \quad \text{where } x_0 < \xi < x_3.$$

```
>> Lab8Q3
Approx Value of integral  using Rectangle rule comes out to be 4.00000

Absolute errors for Rectangle Rule is 0.85841

Approx Value of integral using Trapezoidal rule comes out to be 3.00000

Absolute errors for Trapezoidal Rule is 0.14159

Approx Value of integral using Simpsons 1/3rd rule comes out to be 3.13333

Absolute errors for Simpson's 1/3rd Rule is 0.00826

Approx area using Simpsons 3/8th rule comes out to be 3.13846

Absolute errors for  Simpson's 3/8th Rule is 0.00313

>>
```

Question 4

4. We want to approximate $\int_1^2 f(x)dx$ given the table of the values

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
$f(x)$	10	8	7	6	5

Compute an estimate by the composite trapezoid rule.

A composite rule fundamentally involves dividing the interval into multiple segments, and within each of these segments, we apply a specific approximation technique. By doing this, we calculate the integrals for each segment and then sum them up to approximate the integral over the entire interval. Generally, we anticipate that this approach would yield a lower error for continuous functions

Applying the composite trapezoidal rule, we have

```
>> Lab8Q4
Approximate estimate using composite Trapezoidal rule comes out to be 7.12500
>>
```

Question 5

5. Determine the value of n and h required to approximate

$$\int_1^2 x \ln x \, dx$$

to within 10^{-5} and compute the approximation. Use (a) Composite Trapezoidal rule (b) Composite Simpson's rule (c) Composite Midpoint rule.

For obtaining the value of n and h in this question, we can iterate over possible values of n starting from 1 while having a stopping criterion of maximum number of iterations possible for the value of n

```
>> Lab8Q5
A part Trapezoidal rule
Value of n for which error comes out less than 1e-5 is 77  and h comes out to be 0.01299
Value of corresponding integral is 0.63630

B part  Composite Simpsons rule
Value of n for which error comes out less than 1e-5 is 6  and h comes out to be 0.16667
Value of corresponding integral is 0.63630

C part  Composite Midpoint rule
Value of n for which error comes out less than 1e-5 is 54  and h comes out to be 0.01852
Value of corresponding integral is 0.63628
```

Composite Midpoint rule is given by

ntervals can be written with its error term as

$$\int_a^b f(x)\,dx = 2h\sum_{j=0}^{n/2} f(x_{2j}) + \frac{b-a}{6}h^2 f''(\mu).$$

Composite Trapezoidal rule is given by

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right] - \frac{b-a}{12}h^2 f''(\mu).$$

Composite Simpsons rule is given by following algorithm

To approximate the integral $I = \int_a^b f(x)\,dx$:

INPUT endpoints a, b; even positive integer n.

OUTPUT approximation XI to I.

Step 1 Set $h = (b-a)/n$.

Step 2 Set $XI0 = f(a) + f(b)$;
$XI1 = 0$; (*Summation of* $f(x_{2i-1})$.)
$XI2 = 0$. (*Summation of* $f(x_{2i})$.)

Step 3 For $i = 1, \ldots, n-1$ do Steps 4 and 5.

Step 4 Set $X = a + ih$.

Step 5 If i is even then set $XI2 = XI2 + f(X)$
else set $XI1 = XI1 + f(X)$.

Step 6 Set $XI = h(XI0 + 2 \cdot XI2 + 4 \cdot XI1)/3$.

Step 7 OUTPUT (XI);
STOP.

We see that for the composite simpson's rule, the N value converges quickly i.e. N = 6. Whereas for the Composite Midpoint rule and Composite Trapezoidal rules, we obtain N = 54 and N = 77 respectively. The stopping criteria used was the error in approximation should be less than 1e-5.

Question 6

6. A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?

We can Write Distance as area under speed time curve, given all these speed time points, we approximate the area using Composite Trapezoidal Rule.

```
>> Lab8Q6
Approximate estimate using composite Trapezoidal rule comes out to be 9855.00000
>>
```