Scientific Computing (MA322)

Lab 03

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Assumption - 1)Since Real(Actual) roots cannot be calculated perfectly , I used fzero function of matlab to estimate zero of a function and used that number as base ie Actual root(Real root though cant be calculated)

2) While calculating Residues initially I assumed X0=0 and X1 to be my initial approximation.

Question-1

-Used epsilon as 1e-4
- Used Nmax as 10000
- Used Initial approximation as x = 2
- Used Actual solution as `1.85558453`

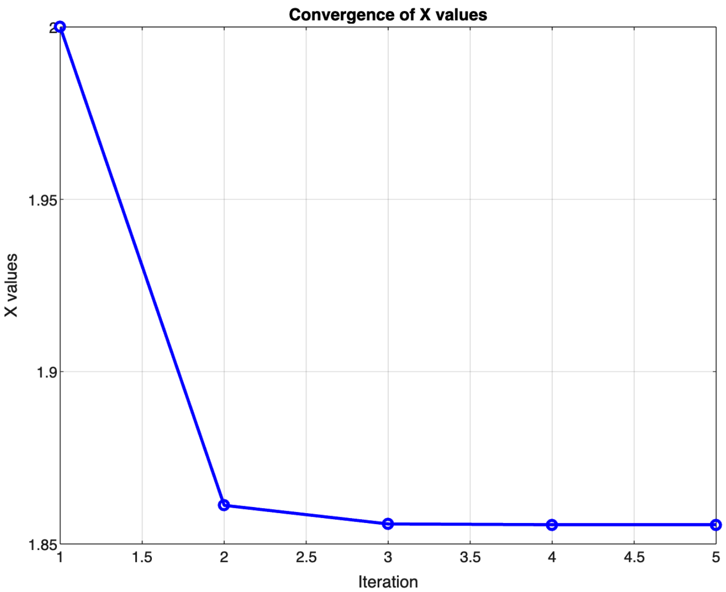
Here we had to implement Fixed point Iteration method. With some rearrangement I got the below mentioned equation.

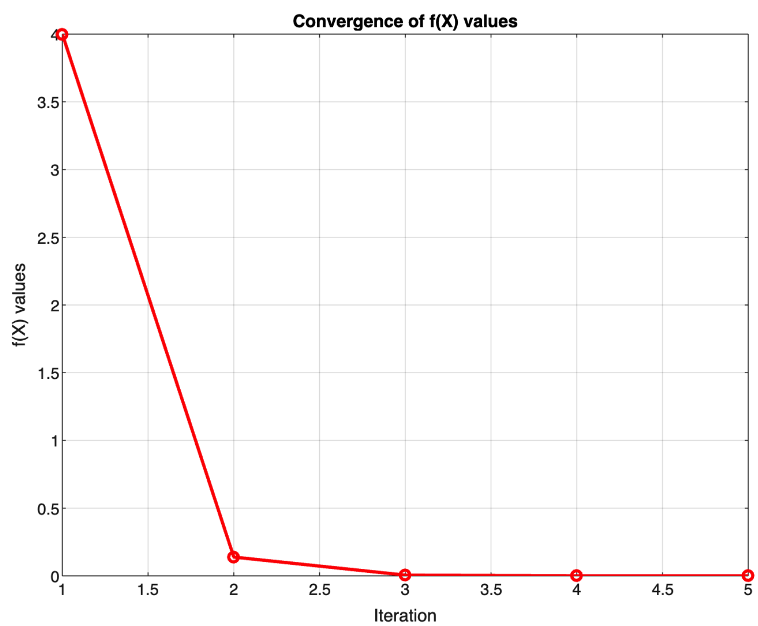
X = (X+10)^(0.25)

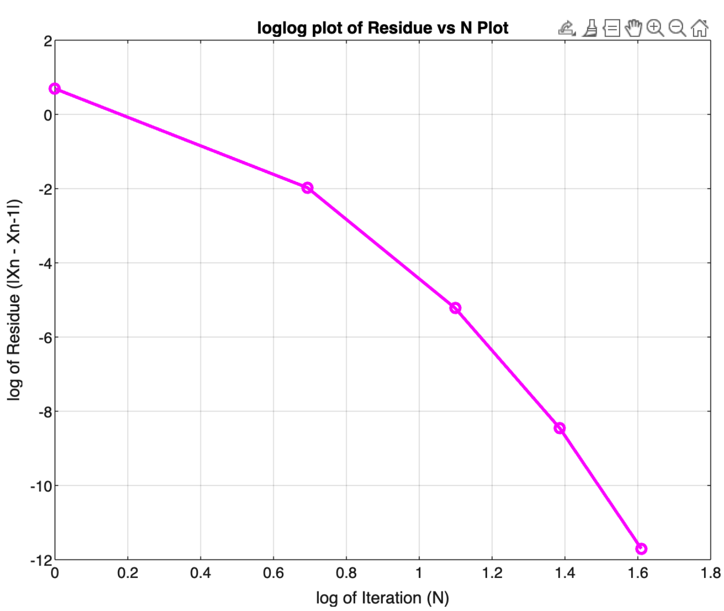
So here g(X) = (X+10)^(0.25)

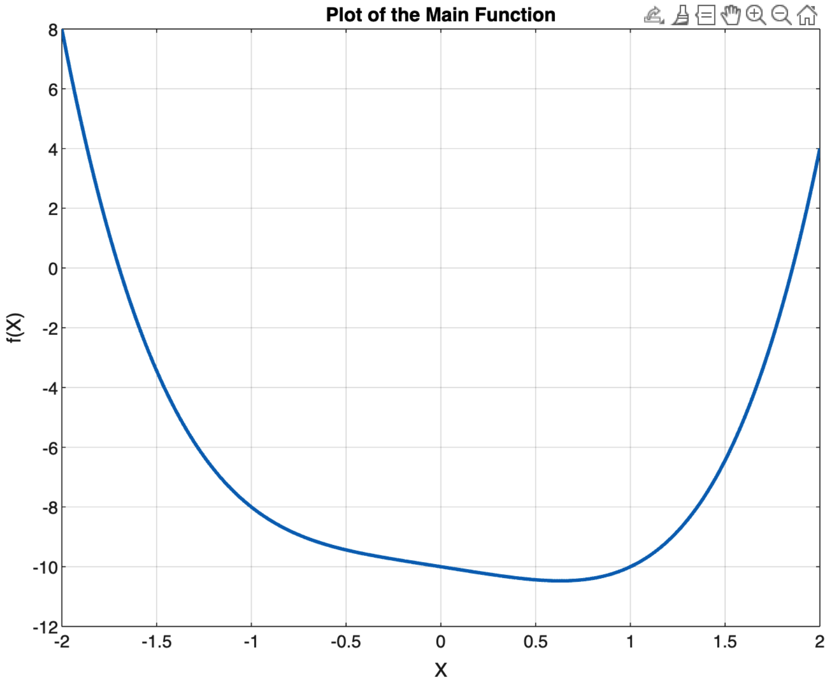
Motivation for choosing this g(x) was that given a range of x i.e. [1,2] , the range of g(x) was subset of [1,2]. Moreover g(x) was continuous in this interval.

```
Below Values are up to 8 decimal places
Iteration       X               f(x)            Actual Solution         Absolute Error          Residue
1               2.00000000      4.00000000      1.85558453      0.14441547              2.00000000
2               1.86120972      0.13879028      1.85558453      0.00562519              0.13879028
3               1.85580460      0.00540512      1.85558453      0.00022007              0.00540512
4               1.85559314      0.00021146      1.85558453      0.00000861              0.00021146
5               1.85558487      0.00000827      1.85558453      0.00000034              0.00000827
5               1.85558454      0.00000827      1.85558453      0.00000001              0.00000860
Approximate Solution comes out to be 1.85558454
>> |
```


Convergence of X values
X values
Iteration
2
1.95
1.9
1.85
1
1.5
2
2.5
3
3.5
4
4.5
5


Convergence of f(X) values
f(X) values
Iteration
4
3.5
3
2.5
2
1.5
1
0.5
0
1
1.5
2
2.5
3
3.5
4
4.5
5


loglog plot of Residue vs N Plot
log of Residue (lXn - Xn-1l)
log of Iteration (N)
2
0
-2
-4
-6
-8
-10
-12
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8



Question-2

For this Question I Tried 2 approaches

1) **Manually choosing p and then checking how many iterations are taking place until X converges and then choosing that p for which number of iterations were minimum (ie rate of convergence was maximum)**

2) **Taking a new function u(x) = f(x)/f'(x). We can easily prove that alpha(root of f(x) of multiplicity p) is actually the root of u(x) . Moreover its multiplicity is 1 which means we can apply standard newtons formula on u(x)**

Here xn = xn-1 - (f(xn-1)*(f'(xn-1))/(f'(xn-1)*f'(xn-1) - f(xn-1)* f''(xn-1))

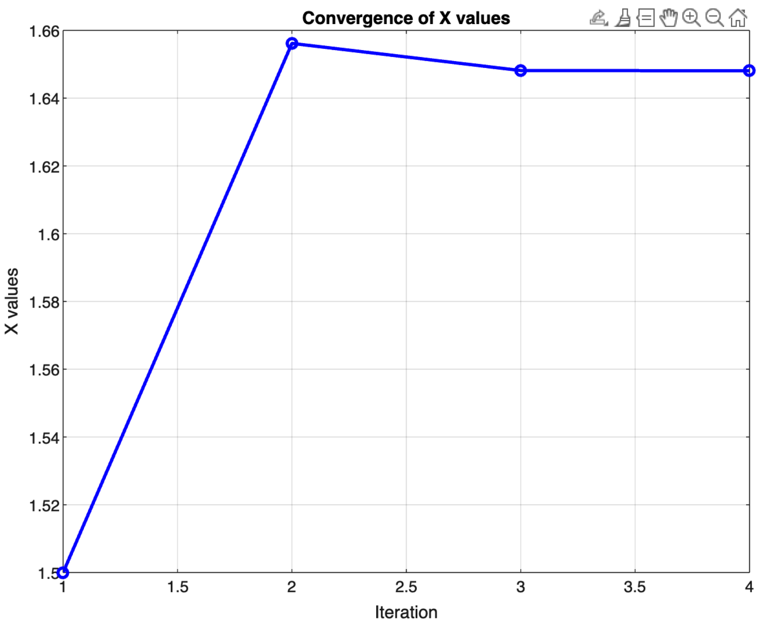
I Implemented 2nd method in code , though I tried first method as well

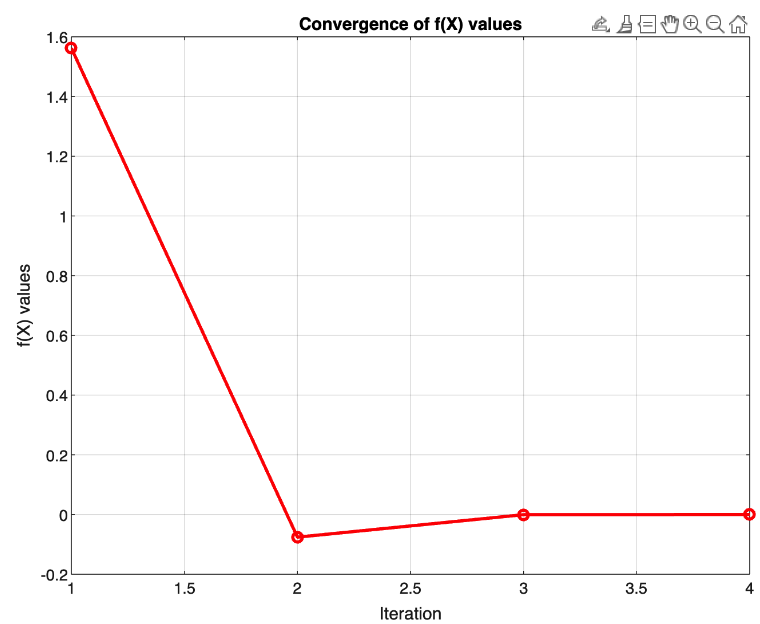
Question-2 Part A

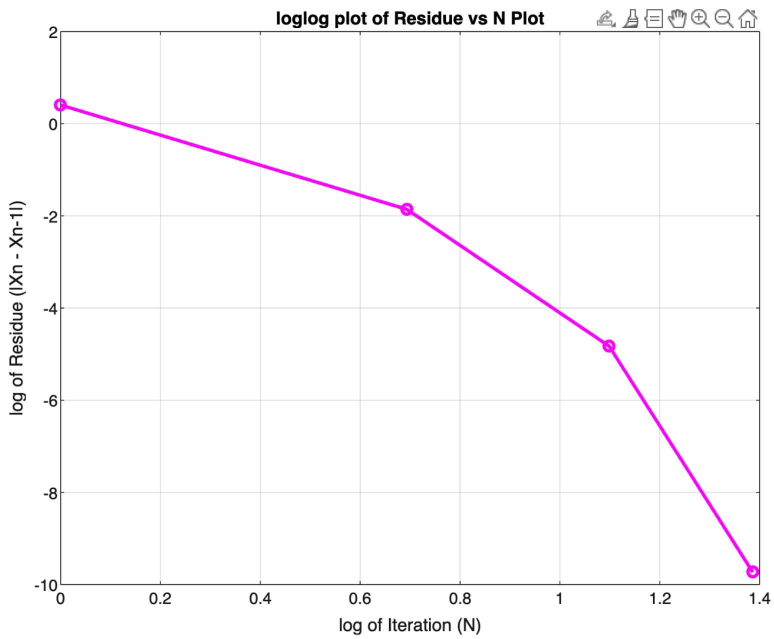
-Used epsilon as 1e-5
- Used Nmax as 10000
- Used Initial approximation as x = 1.5
- Used Actual solution as 1.64809537

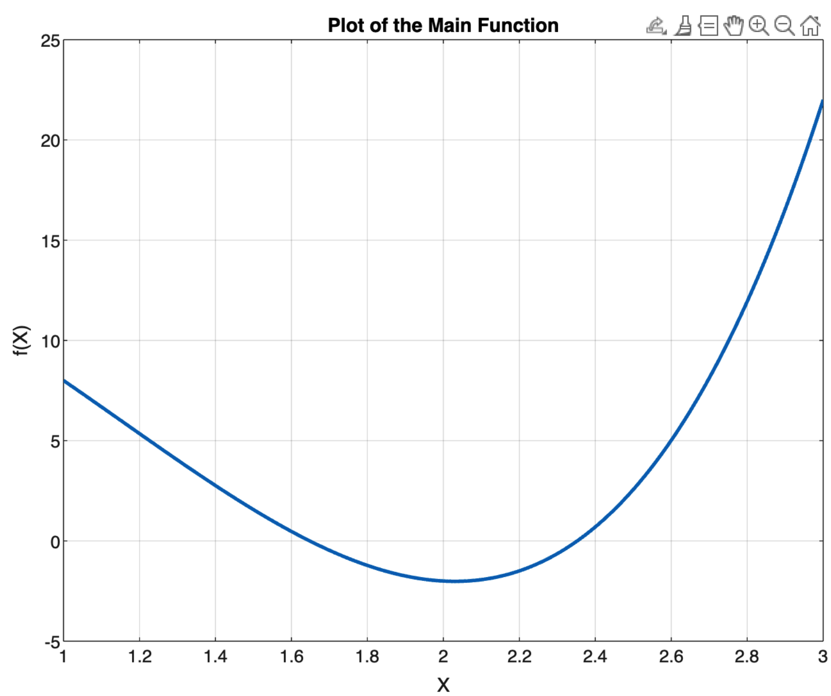
So for this part Convergence of modified Newton method is fastest when multiplicity is taken to be 1,implying quadratic convergence of the equation.

```
Below Values are up to 8 decimal places
Iteration        X                f(x)              Actual Solution         Absolute Error          Residue
1                1.50000000       1.56250000        1.64809537       0.14809537                1.50000000
2                1.65616513       -0.07582180       1.64809537       0.00806976                0.15616513
3                1.64815476       -0.00056207       1.64809537       0.00005940                0.00801036
4                1.64809537       -0.00000003       1.64809537       0.00000000                0.00005940
Approximate Solution comes out to be 1.64809537
```







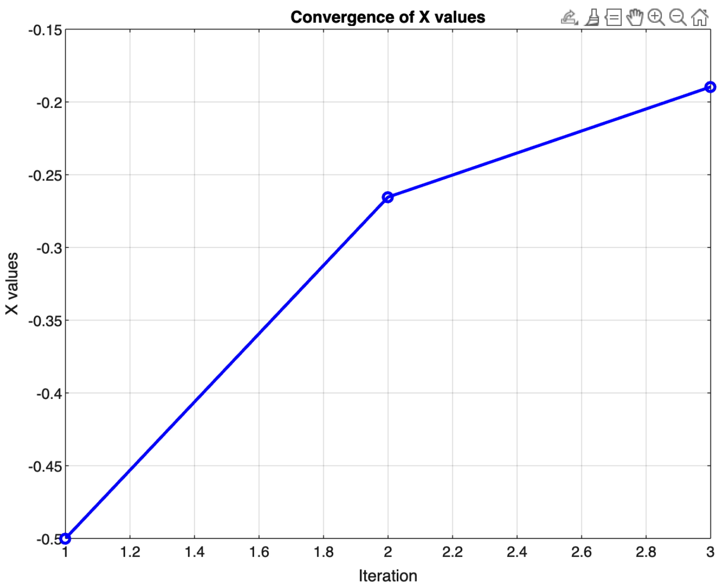


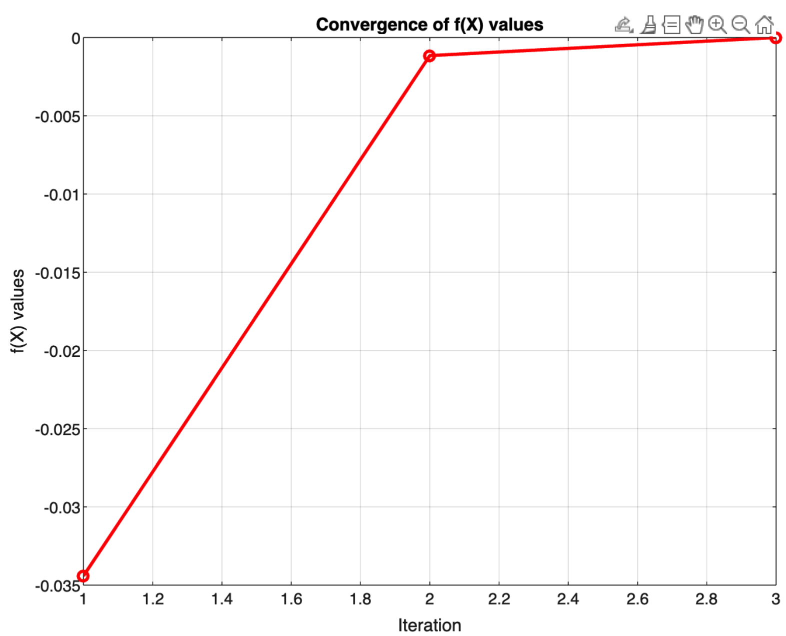
Question-2 Part B

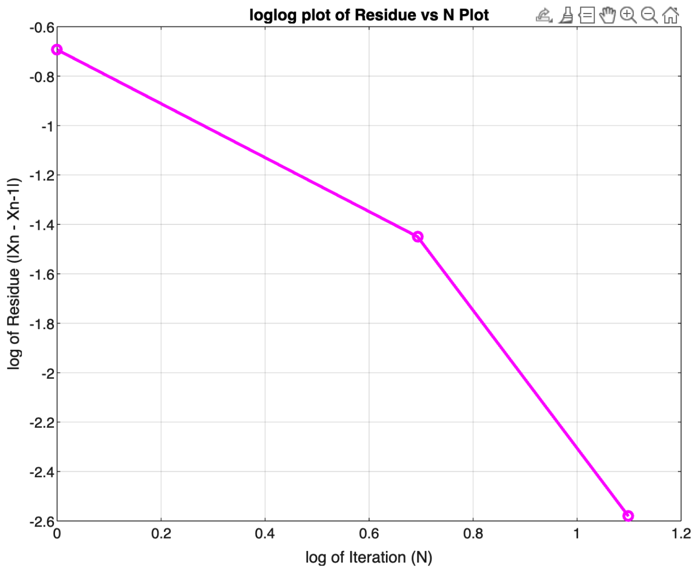
-Used epsilon as 1e-5
- Used Nmax as 10000
- Used Initial approximation as x = -0.5
- Used Actual solution as -0.18325332

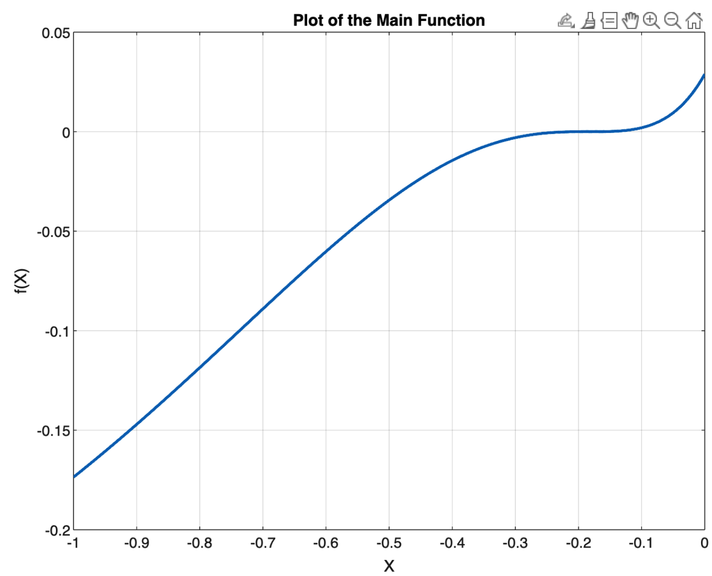
So for this part Convergence of modified Newton method is fastest when multiplicity is taken to be 3 ,implying quadratic convergence of the equation.
If Standard Newton method was to be applied then It took more iterations than above

```
Below Values are up to 8 decimal places
Iteration        X               f(x)             Actual Solution          Absolute Error             Residue
1                -0.50000000     -0.03441303      -0.18325332      0.31674668                 0.50000000
2                -0.26536892     -0.00115684      -0.18325332      0.08211560                 0.23463108
3                -0.18964449     -0.00000068      -0.18325332      0.00639117                 0.07572443
Approximate Solution comes out to be -0.18329709
```







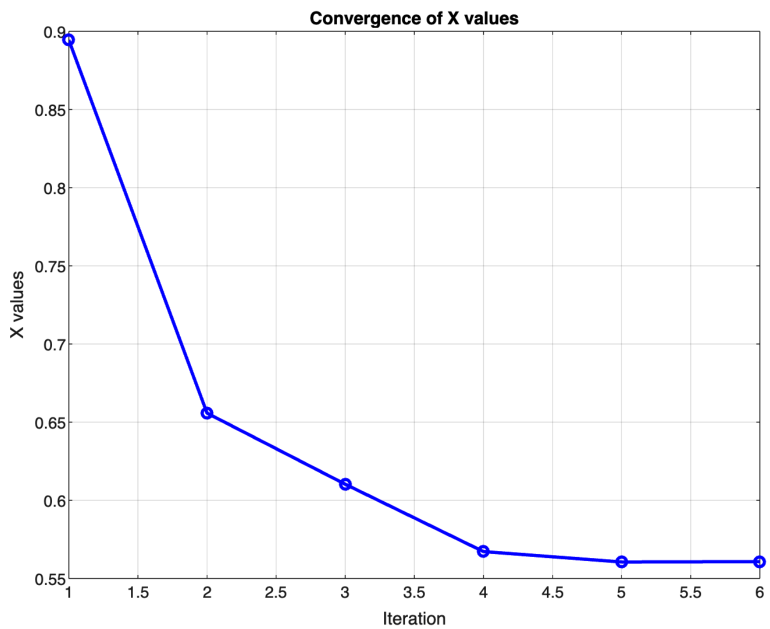


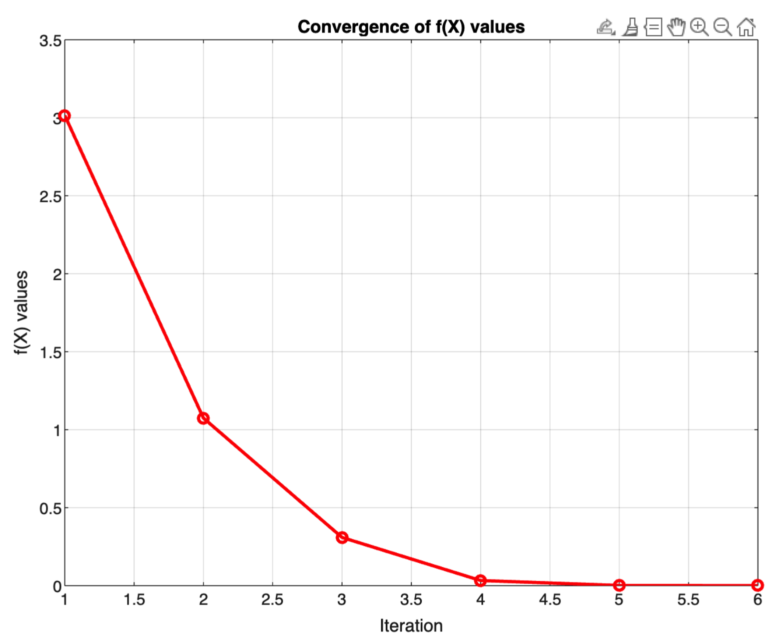
Question-3

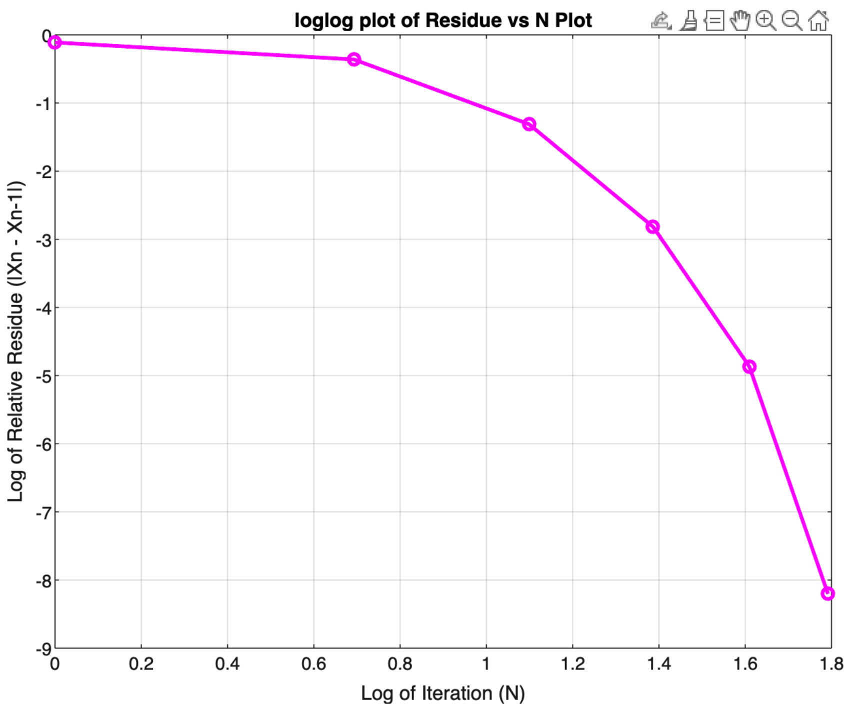
Question-3 Part A

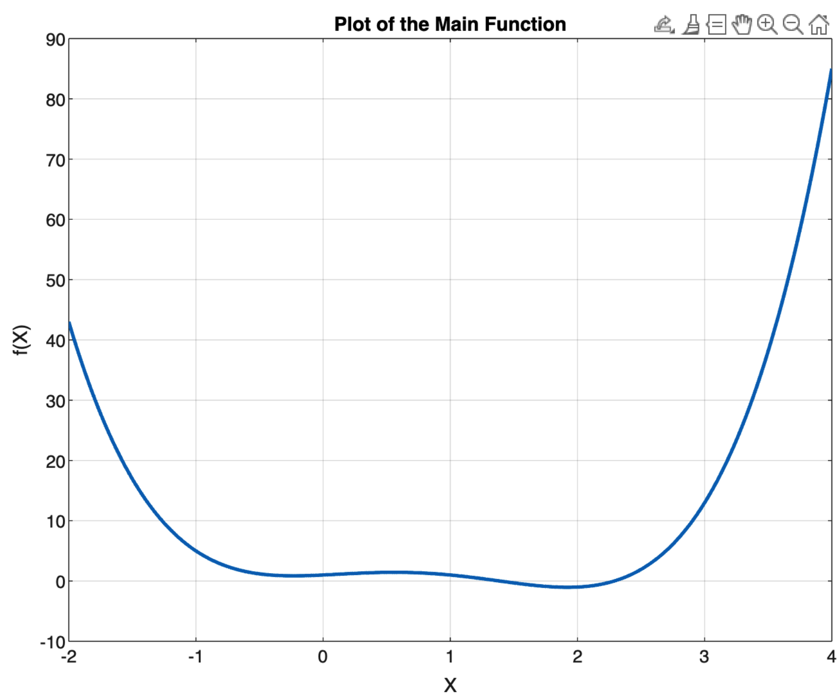
-Used epsilon as 1e-5
- Used Nmax as 10000
- Used Initial approximation as x0=-0.5, x1=0, x2=0.5
- Used Actual solution as `-0.33909284 + -0.44663010i`

```
Below Values are up to 8 decimal places
Iteration            X                     f(x)                          Actual Solution               Absolute Error       Residue
1              -0.10000000 + -0.88881944i     -0.01120000 + -3.01487555i     -0.33909284 + -0.44663010i     0.50268957      0.89442719
2              -0.55004347 + -0.35708179i     0.36935955 + 1.00897545i       -0.33909284 + -0.44663010i     0.22917039      0.69662332
3              -0.33017777 + -0.51327608i     -0.25053571 + -0.18108851i     -0.33909284 + -0.44663010i     0.06723961      0.26969905
4              -0.33975100 + -0.45421869i     -0.03084244 + -0.01093611i     -0.33909284 + -0.44663010i     0.00761708      0.05982827
5              -0.33882677 + -0.44656569i     0.00071234 + -0.00091970i      -0.33909284 + -0.44663010i     0.00027376      0.00770861
6              -0.33909323 + -0.44662907i     0.00000331 + 0.00000329i       -0.33909284 + -0.44663010i     0.00000110      0.00027390
Approximate Solution comes out to be -0.33909323 + -0.44662907i
```





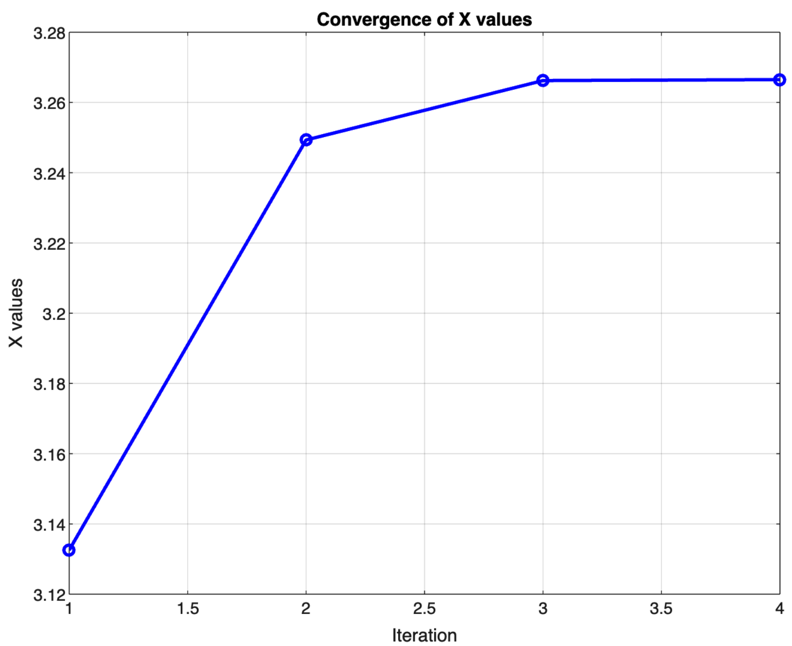

loglog plot of Residue vs N Plot
Log of Relative Residue (|Xn - Xn-1|)
Log of Iteration (N)
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

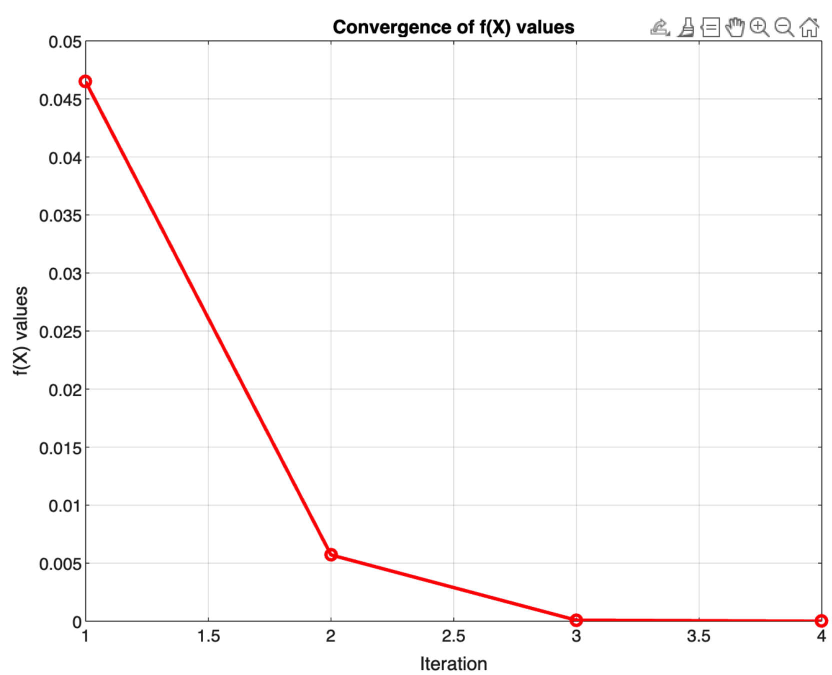

Plot of the Main Function
f(X)
X
90
80
70
60
50
40
30
20
10
0
-10
-2
-1
0
1
2
3
4

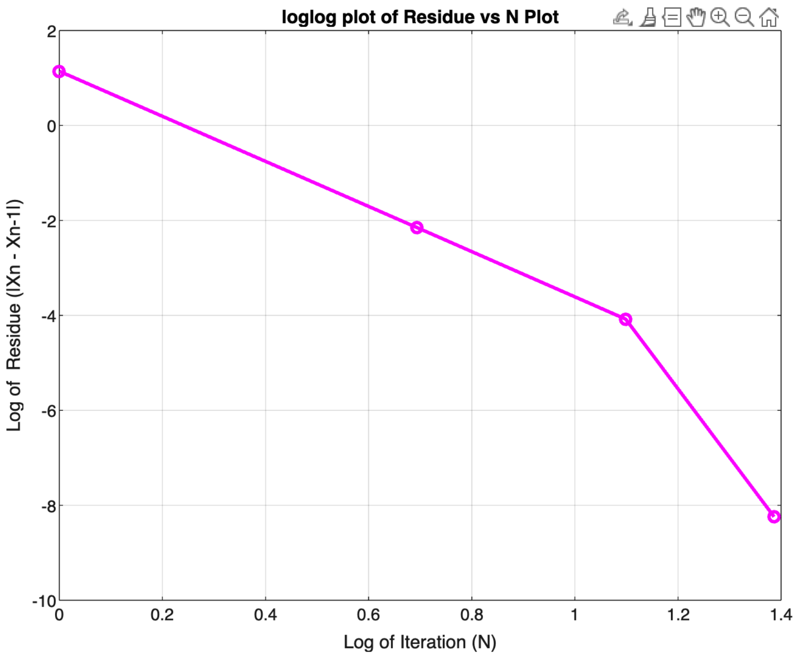
Question-3 Part B

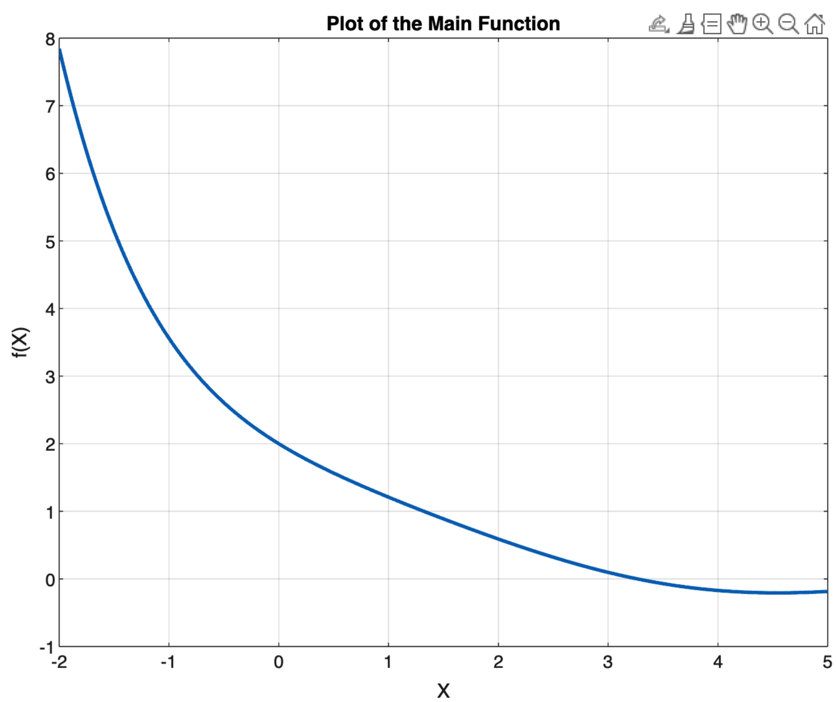
-Used epsilon as 1e-5
- Used Nmax as 10000
- Used Initial approximation as x0=1, x1=1.5, x2=2
- Used Actual solution as 3.266500436785624 + 0.000000000i

```
Below Values are up to 8 decimal places
Iteration            X                       f(x)                        Actual Solution                Absolute Error          Residue
1             3.13257322 + 0.00000000i     0.04648465 + 0.00000000i     3.26650044 + 0.00000000i       0.13392722              3.13257322
2             3.24932038 + 0.00000000i     0.00571073 + 0.00000000i     3.26650044 + 0.00000000i       0.01718006              0.11674716
3             3.26623603 + 0.00000000i     0.00008732 + 0.00000000i     3.26650044 + 0.00000000i       0.00026440              0.01691566
4             3.26649992 + 0.00000000i     0.00000017 + 0.00000000i     3.26650044 + 0.00000000i       0.00000052              0.00026388
Approximate Solution comes out to be 3.26649992 + 0.00000000i
>>
```









Question-4

-Used epsilon as 1e-4
- Used Nmax as 10000
- Used Initial approximation as x = 8
- Used Actual solution as `5.56776436`

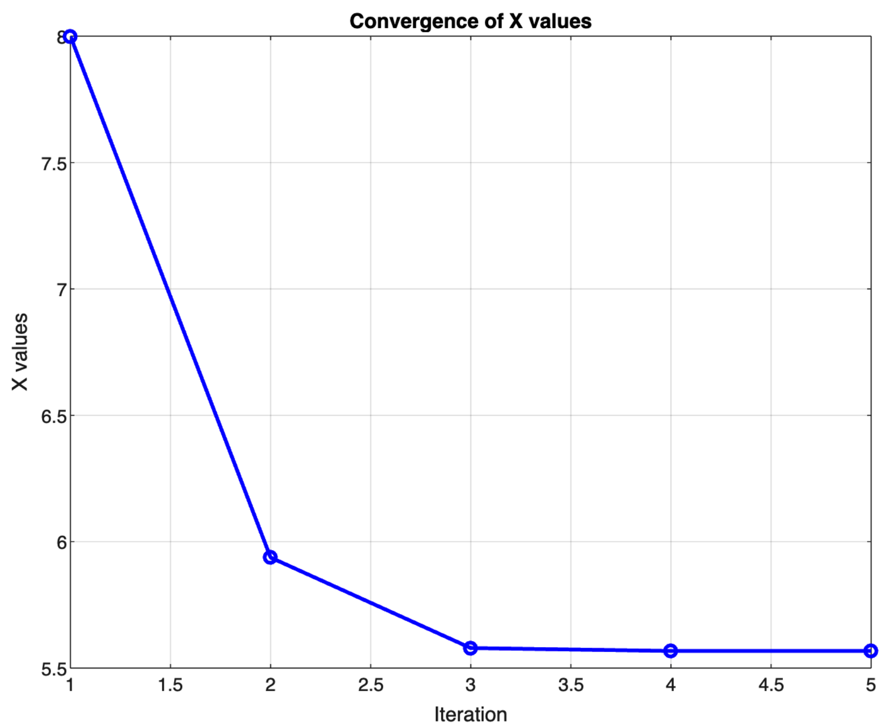
Here we had to implement Fixed point Iteration method. With some rearrangement I got the below mentioned equation.

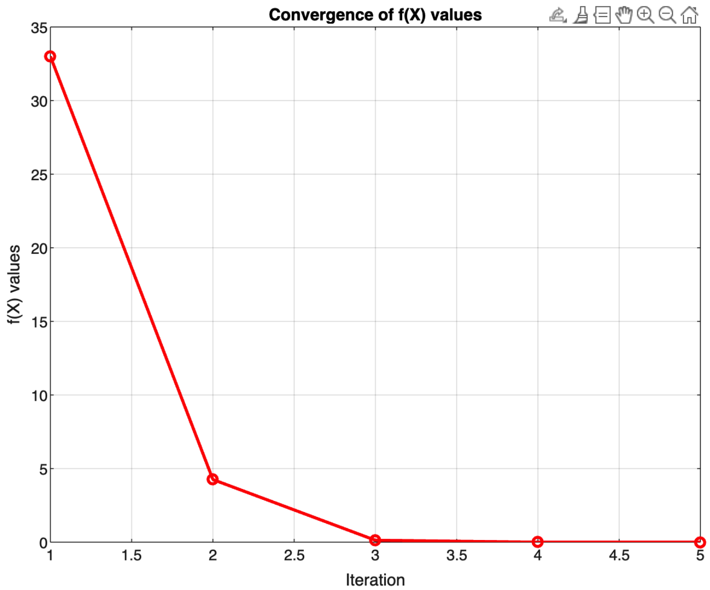
X = ½ * (X+31/X)

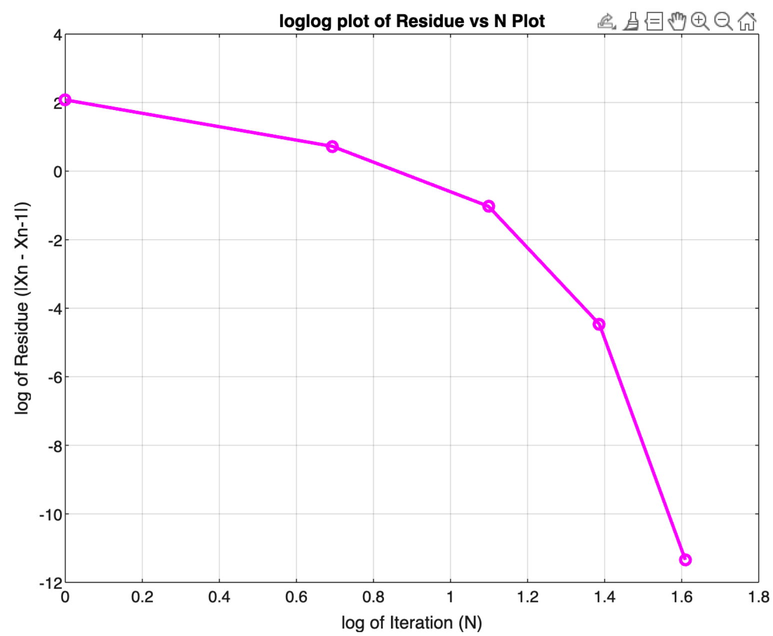
So here g(X) = ½ * (X+31/X)

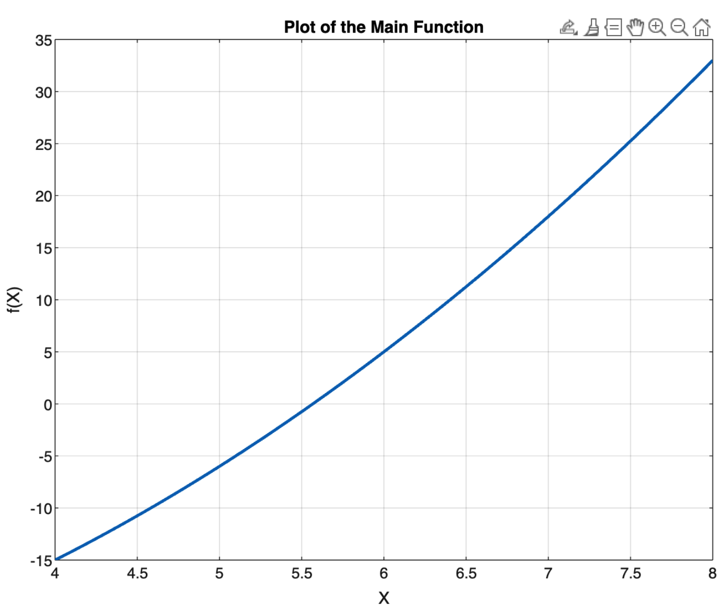
Motivation for choosing this g(x) was that given a range of x i.e. [5,8] , the range of g(x) was subset of [5,8]. Moreover g(x) was continuous in this interval.

```
Below Values are up to 8 decimal places
Iteration        X               f(x)             Actual Solution          Absolute Error           Residue
1                8.00000000      33.00000000      5.56776436       2.43223564               8.00000000
2                5.93750000      4.25390625       5.56776436       0.36973564               2.06250000
3                5.57927632      0.12832421       5.56776436       0.01151195               0.35822368
4                5.56777624      0.00013225       5.56776436       0.00001188               0.01150008
5                5.56776436      0.00000000       5.56776436       0.00000000               0.00001188
Approximate Solution comes out to be 5.56776436
>>
```




Convergence of f(X) values
f(X) values
Iteration
35
30
25
20
15
10
5
0
1
1.5
2
2.5
3
3.5
4
4.5
5


loglog plot of Residue vs N Plot
log of Residue (|Xn - Xn-1|)
log of Iteration (N)
4
2
0
-2
-4
-6
-8
-10
-12
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


Plot of the Main Function
f(X)
X
35
30
25
20
15
10
5
0
-5
-10
-15
4
4.5
5
5.5
6
6.5
7
7.5
8