Scientific Computing (MA322)

Lab 10

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Question 1

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1. Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following tables.

(a).

x	$f(x)$	$f'(x)$
0.5	0.4794	
0.6	0.5646	
0.7	0.6442	

(b).

x	$f(x)$	$f'(x)$
1.0	1.0000	
1.2	1.2625	
1.4	1.6595	

Approximating each entry of f'(x) using both forward and backward difference formula, we have

```
>> Lab10Q1
Q1(a):

Forward difference value at 0.5 is 0.852000
Forward difference value at 0.6 is 0.796000

Backward difference value at 0.6 is 0.852000
Backward difference value at 0.7 is 0.796000

Q1(b):

Forward difference value at 1.0 is 1.312500
Forward difference value at 1.2 is 1.985000

Backward difference value at 1.2 is 1.312500
Backward difference value at 1.4 is 1.985000
>>
```

Question 2:

2. The data in Exercise 1 were taken from the following functions. Compute the actual errors in Exercise 1, and find error bounds using the error formulas.
(a). $f(x) = \sin x$, (b). $f(x) = x^2 \ln x + 1$.

Approximating each entry of f'(x) using both forward and backward difference formulae and taking the maximum error bound between two nodal points as the error bound (i.e. in case of forward, error bound will be the maximum error between nodal point xi and xi+h and is given by h/2*f''(eta))

Q2a

```
>> Lab10Q2
Q2(a):

at x=0.5:
Approximate value at 0.5 is 0.852000
Actual value at 0.5 is 0.877583
Absolute value of error is 0.025583
Error bound is 0.028232

at x=0.6:
Approximate value at 0.6 is 0.796000
Actual value at 0.6 is 0.825336
Absolute value of error is 0.029336
Error bound is 0.032211
```

Q2b

```
Q2(b):

at x=1.0:
Forward difference value at 1.0 is 1.312500
Actual value at 1.0 is 1.000000
Absolute value of error is 0.312500
Error bound is 0.336464

at x=1.2:
Forward difference value at 1.2 is 1.985000
Actual value at 1.2 is 1.637572
Absolute value of error is 0.347428
Error bound is 0.367294
>>
```

Question 3:

3. In a circuit with impressed voltage $\mathcal{E}(t)$ and inductance L, Kirchhoff's first law gives the relationship

$$\mathcal{E}(t) = L\frac{di}{dt} + Ri,$$

where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain:

t	1.00	1.01	1.02	1.03	1.0
i	3.10	3.12	3.14	3.18	3.24

where t is measured in seconds, i is in amperes, the inductance L is a constant 0.98 henries, and the resistance is 0.142 ohms. Approximate the voltage $\mathcal{E}(t)$ when $t = 1.00, 1.01, 1.02, 1.03,$ and 1.04.

```
>> Lab10Q3
Approx Voltage at x = 1.00 is : 2.40020000
Approx Voltage at x = 1.01 is : 2.40304000
Approx Voltage at x = 1.02 is : 4.36588000
Approx Voltage at x = 1.03 is : 6.33156000
Approx Voltage at x = 1.04 is : 6.34008000
>>
```

Question 4:

4. Use Explicit-Euler's method to approximate the solutions for each of the following initial-value problems.

   (a) $y' = 1 + y/t$, $1 \le t \le 2$, $y(1) = 2$ with $h = 0.25$,

   (b) $y' = \cos 2t + \sin 3t$, $0 \le t \le 1$, $y(0) = 1$ with $h = 0.25$,

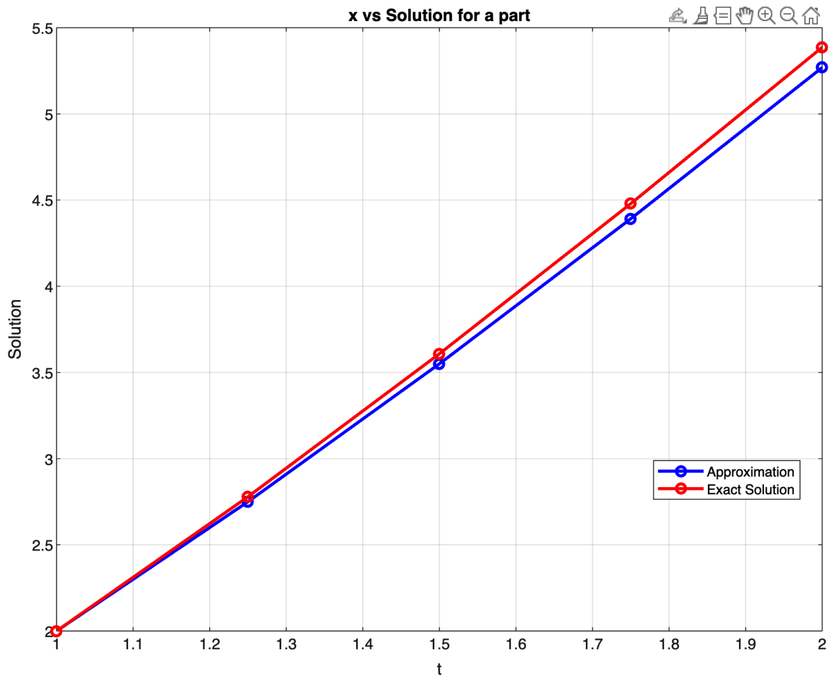
   (c) $y' = -y + ty^{1/2}$, $2 \le t \le 3$, $y(2) = 2$ with $h = 0.25$,

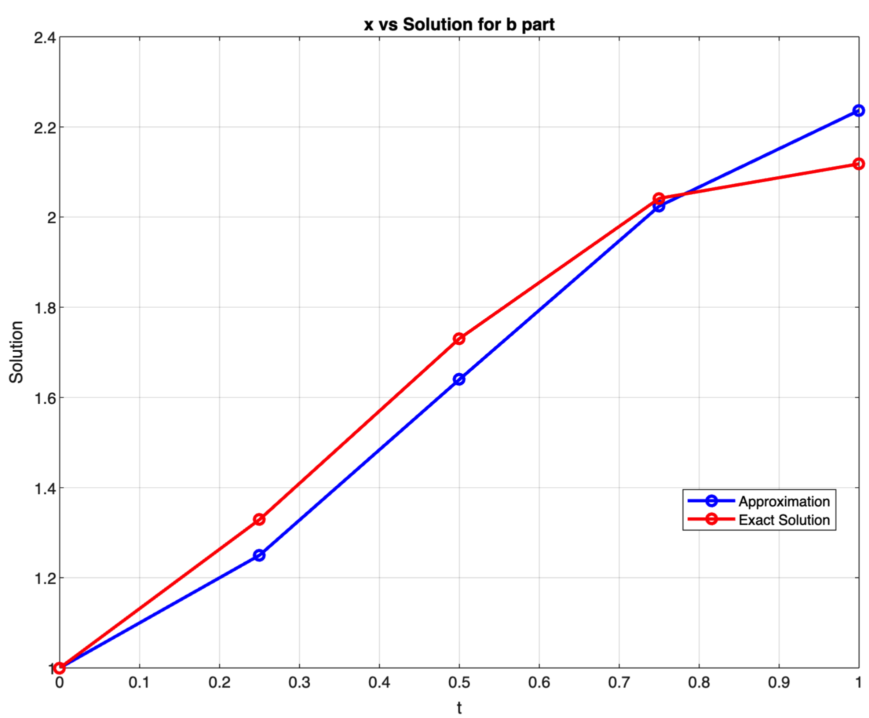
Approximating solutions for each sub part of the problem using Explicit Euler Method with relevant step sizes h we visualise

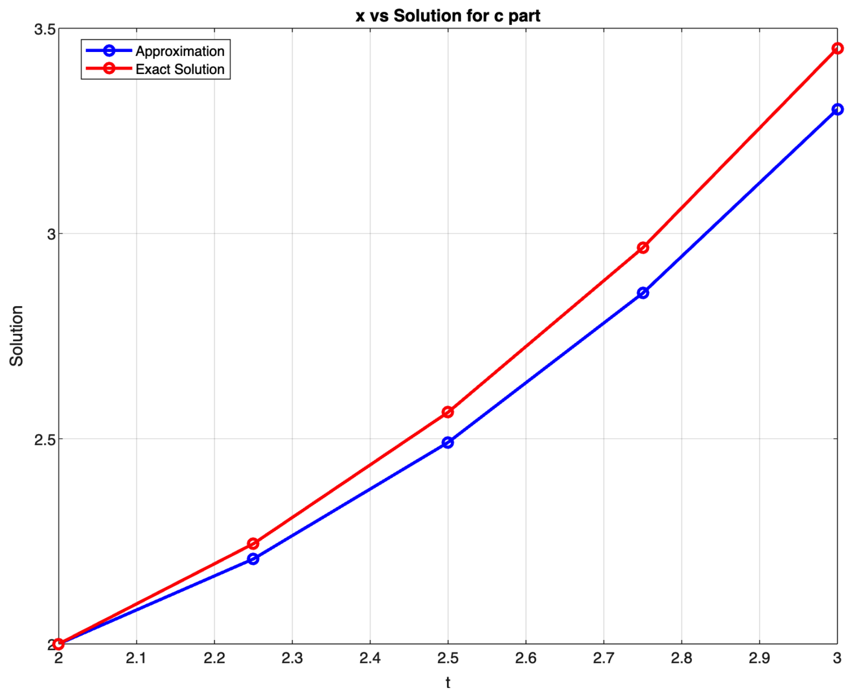
1) **The solution agains t, comparing the approximate and actual solution.**
2) **A plot showing the absolute errors obtained by comparing the approximation with the actual solution.**
3) **Variation of N versus log2(EN/E2N) where N varies and max error across all observation represent error bound for N value**
4) **Log log plot where we depict log(En) adjusting N and considering error bounds**

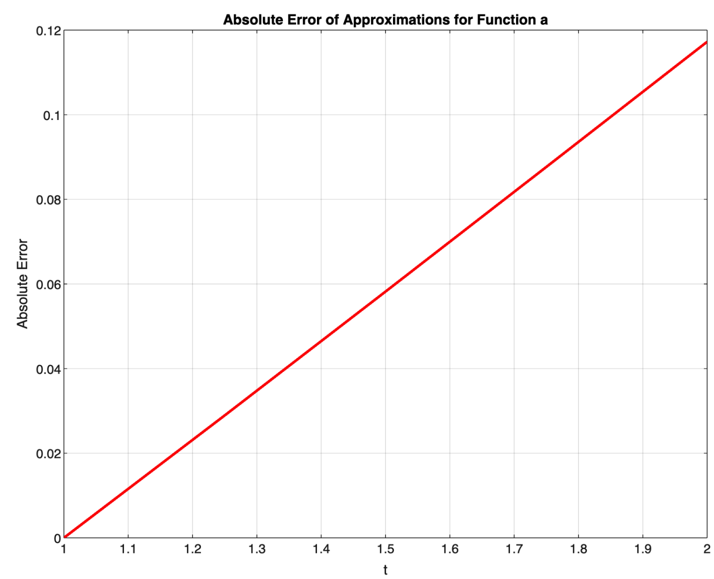
```
>> Lab10Q4
For Part A
t        Exact   Approx_sol      Error(abs)
1        2.0000      2.0000          0.0000
2        2.7789      2.7500          0.0289
3        3.6082      3.5500          0.0582
4        4.4793      4.3917          0.0877
5        5.3863      5.2690          0.1172

For Part B
t        Exact   Approx_sol      Error(abs)
1        1.0000      1.0000          0.0000
2        1.3291      1.2500          0.0791
3        1.7305      1.6398          0.0907
4        2.0415      2.0243          0.0172
5        2.1180      2.2365          0.1185

For Part C
t        Exact   Approx_sol      Error(abs)
1        2.0000      2.0000          0.0000
2        2.2441      2.2071          0.0370
3        2.5645      2.4910          0.0735
4        2.9652      2.8547          0.1105
5        3.4513      3.3026          0.1487
```

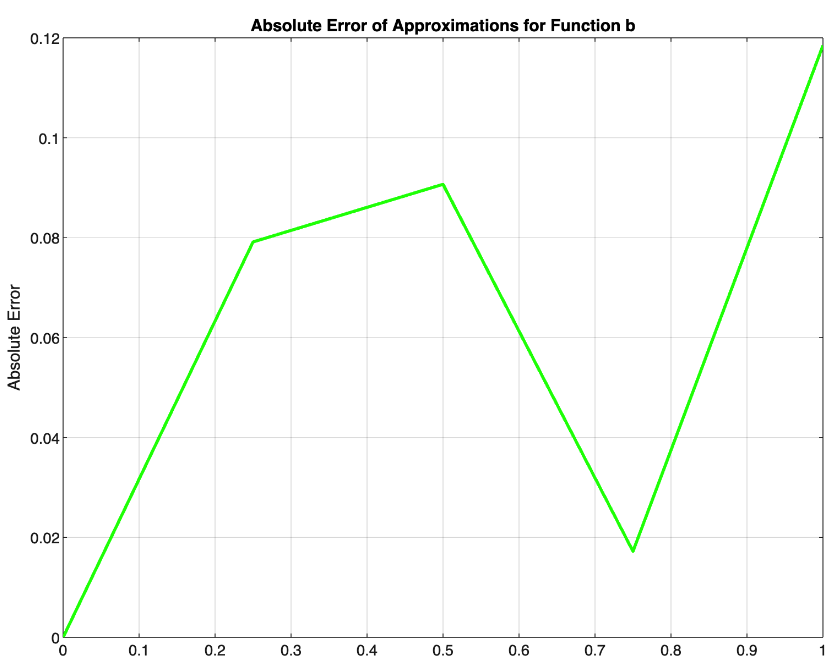
```
For Part A
N         E(N)     Order
2         0.2196           0.9055
4         0.1172           0.9533
8         0.0606           0.9770
16        0.0308           0.9886
32        0.0155           0.9943
64        0.0078           0.9972
128       0.0039           0.9986
256       0.0020           0.9993

For Part B
N         E(N)     Order
2         0.1509           0.3492
4         0.1185           0.7690
8         0.0695           0.8981
16        0.0373           0.9518
32        0.0193           0.9765
64        0.0098           0.9884
128       0.0049           0.9942
256       0.0025           0.9971

For Part C
N         E(N)     Order
2         0.3020           1.0221
4         0.1487           1.0111
8         0.0738           1.0056
16        0.0367           1.0028
32        0.0183           1.0014
64        0.0092           1.0007
128       0.0046           1.0003
256       0.0023           1.0002
>>
```

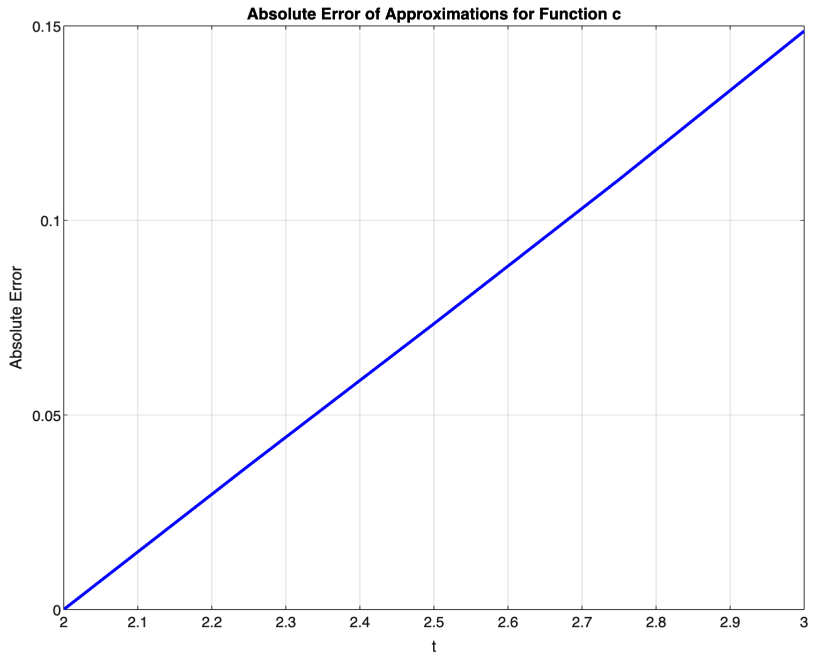

x vs Solution for a part
Solution
t
Approximation
Exact Solution
5.5
5
4.5
4
3.5
3
2.5
2
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


x vs Solution for b part
Solution
t
Approximation
Exact Solution
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

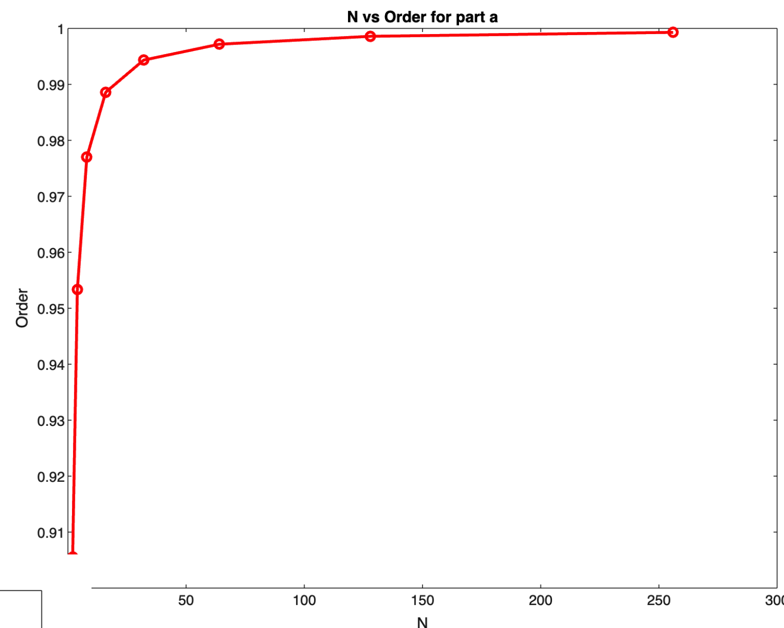

x vs Solution for c part
Approximation
Exact Solution
Solution
t

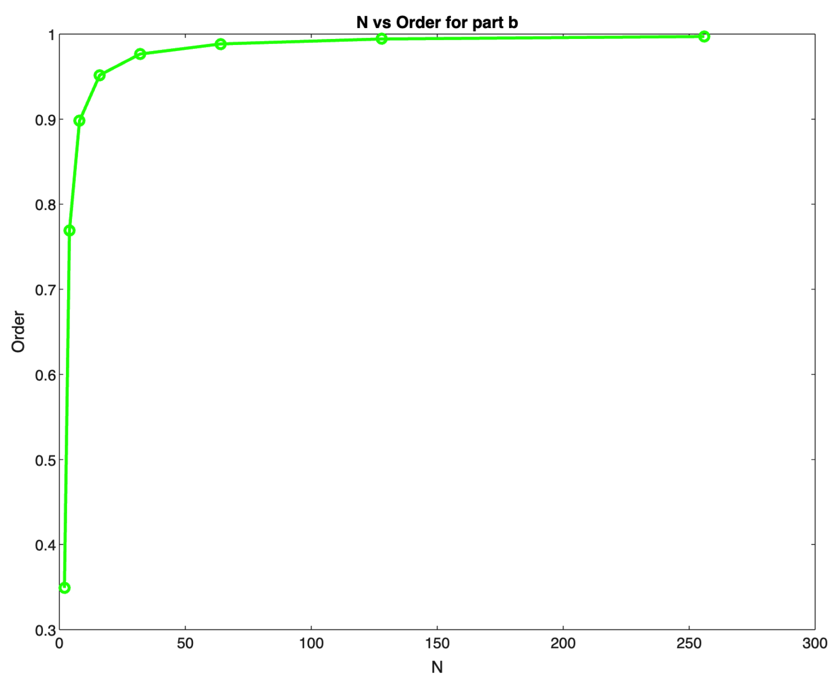

Absolute Error of Approximations for Function a
Absolute Error
t

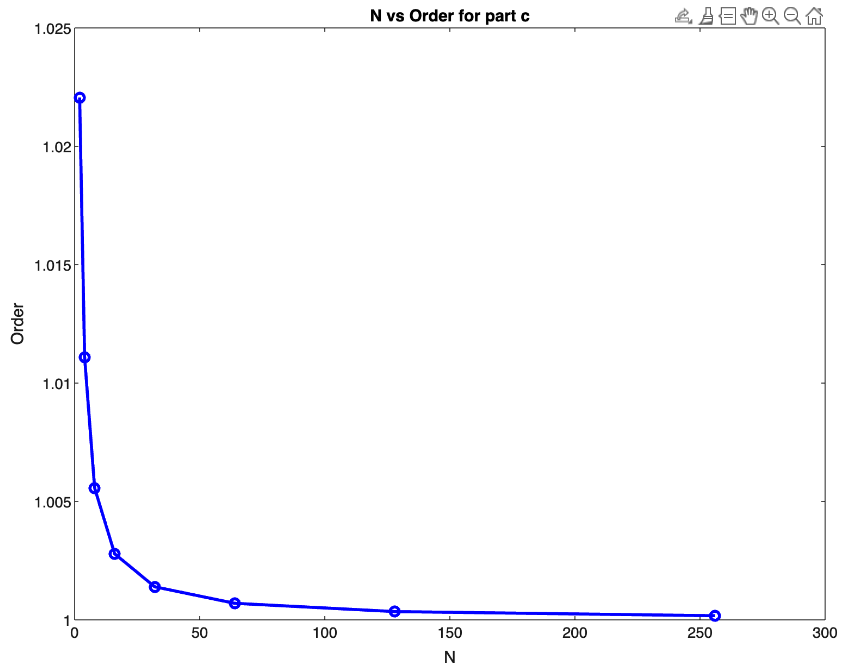

Absolute Error of Approximations for Function b
Absolute Error

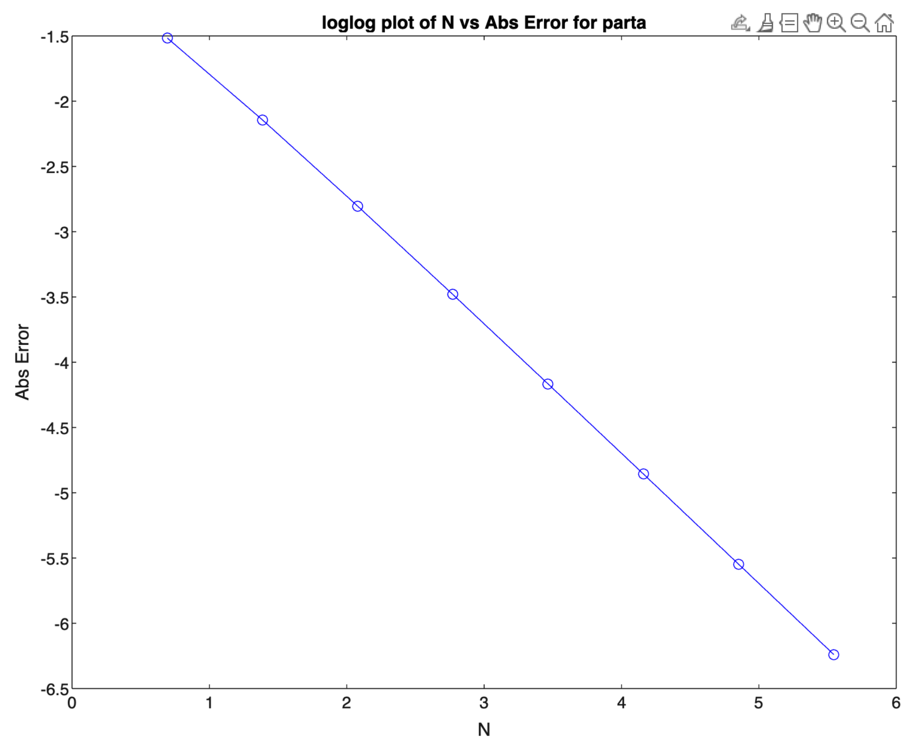

Absolute Error of Approximations for Function c
Absolute Error
t
0.15
0.1
0.05
0
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3

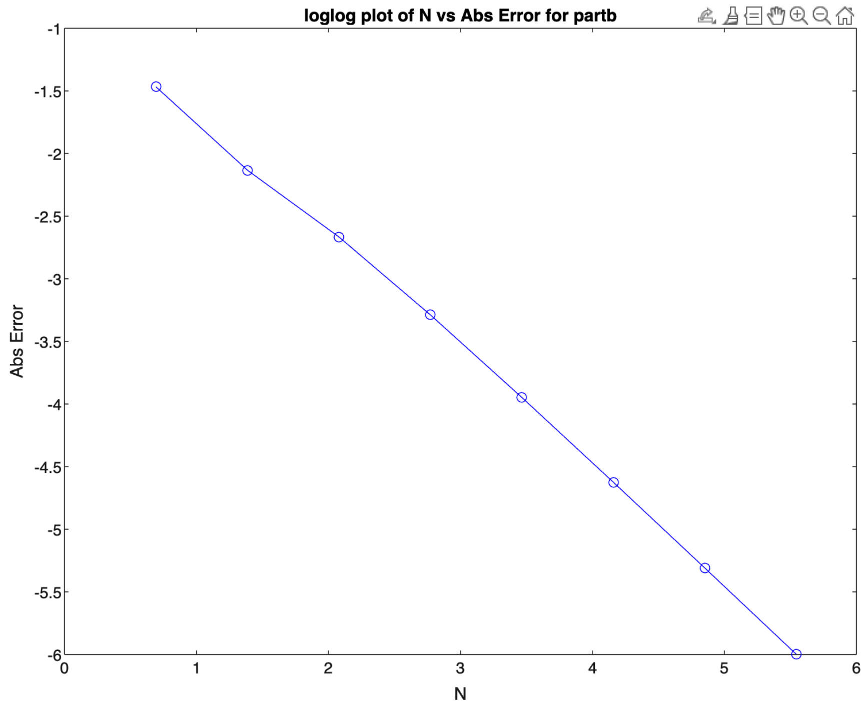
]


N vs Order for part a
Order
N
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
50
100
150
200
250
300

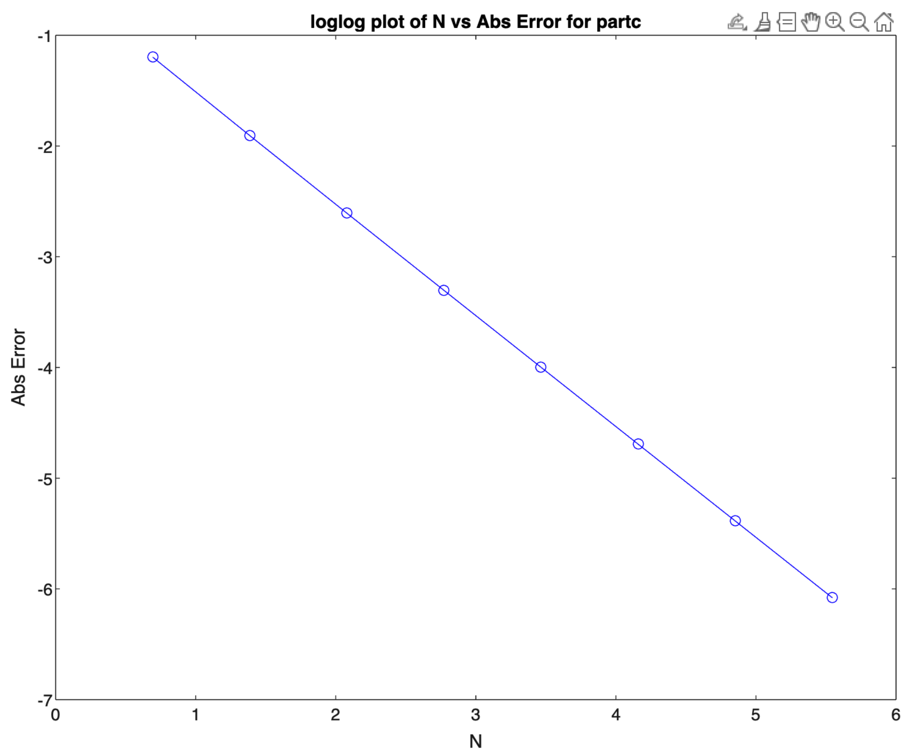

N vs Order for part b
Order
N
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
50
100
150
200
250
300


N vs Order for part c
Order
N
1.025
1.02
1.015
1.01
1.005
1
0
50
100
150
200
250
300


loglog plot of N vs Abs Error for parta
Abs Error
N
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
0
1
2
3
4
5
6



loglog plot of N vs Abs Error for partb
Abs Error
N
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
0
1
2
3
4
5
6



loglog plot of N vs Abs Error for partc
Abs Error
N
-1
-2
-3
-4
-5
-6
-7
0
1
2
3
4
5
6

Question 5:

5. Given the initial-value problem

$$y' = -y + t + 1,\ 0 \leq t \leq 5,\ y(0) = 1,$$

which has solution $y(t) = e^{-t} + t$. Approximate $y(5)$ using Explicit-Euler's method with $h = 0.2, h = 0.1$, and $h = 0.05$.

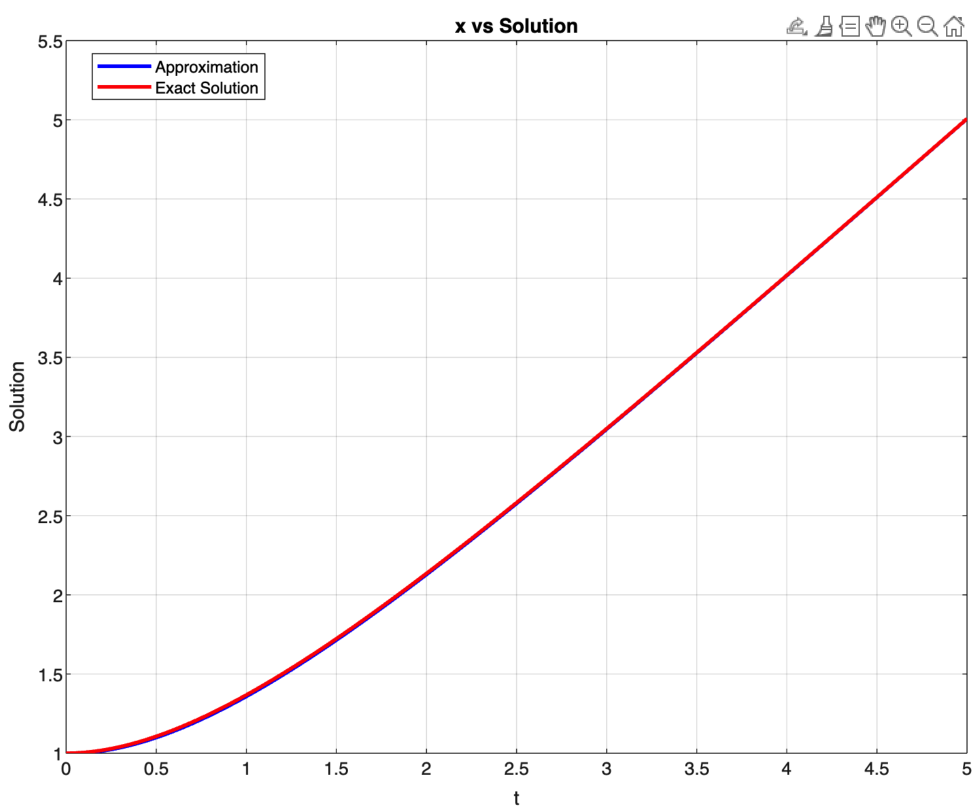
Approximating the value of f(5) using Explicit Euler's method for various step sizes h = 0.2,0.1,0.05, we have

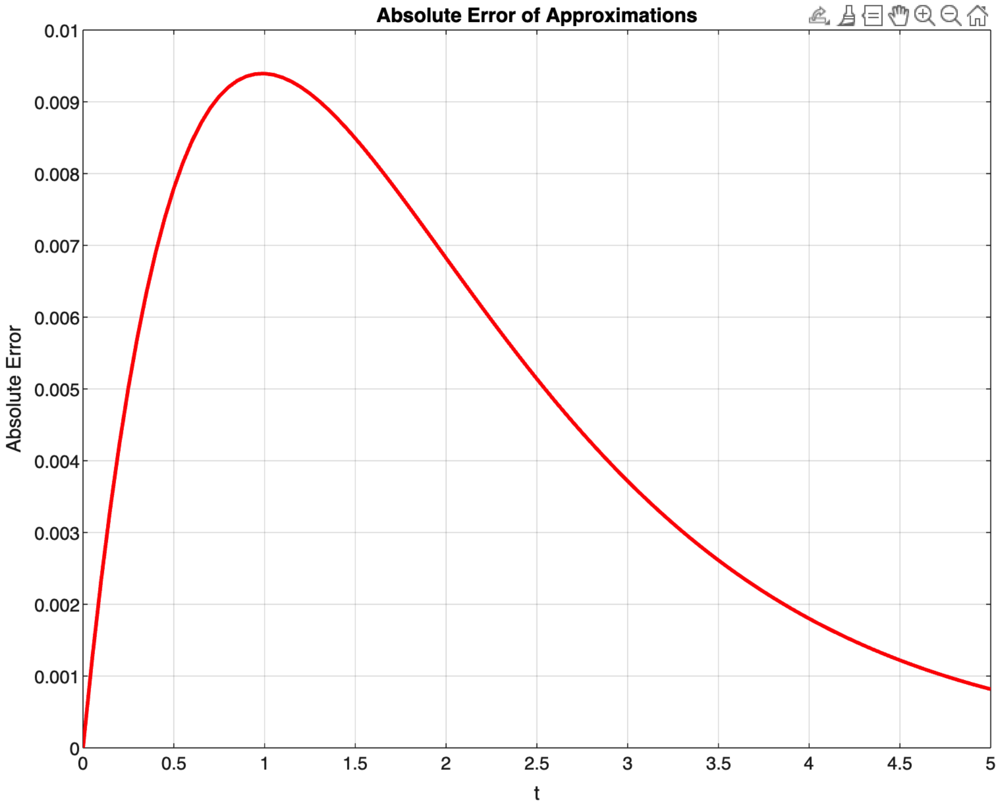
The approximate value of f(5) using h = 0.2 is 5.003778, whereas the exact value is 5.006738 and the absolute error then is 0.002960.

The approximate value of f(5) using h = 0.1 is 5.005154, whereas the exact value is 5.006738 and the absolute error then is 0.001584.

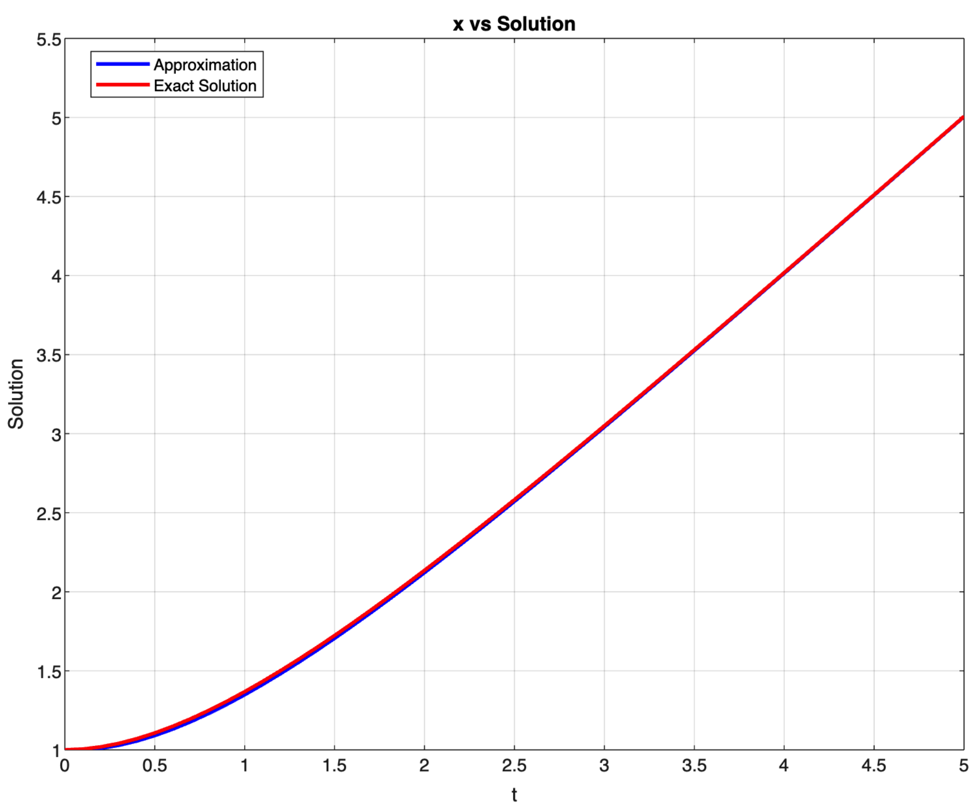
The approximate value of f(5) using h = 0.0.5 is 5.005921, whereas the exact value is 5.006738 and the absolute error then is 0.000817.

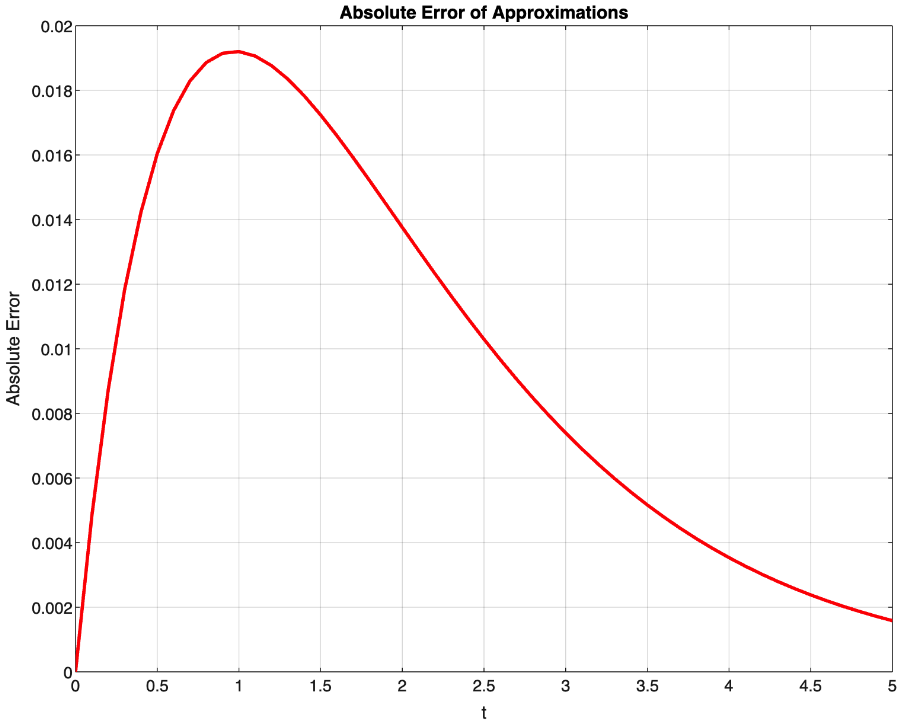
For h=0.5



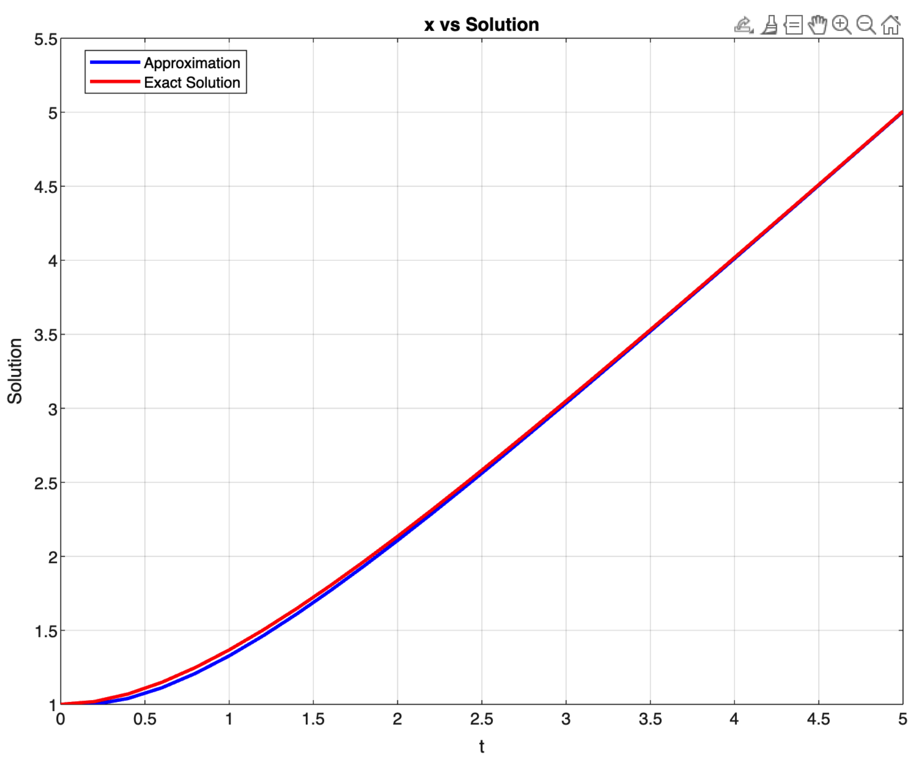

Absolute Error of Approximations
Absolute Error
t
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

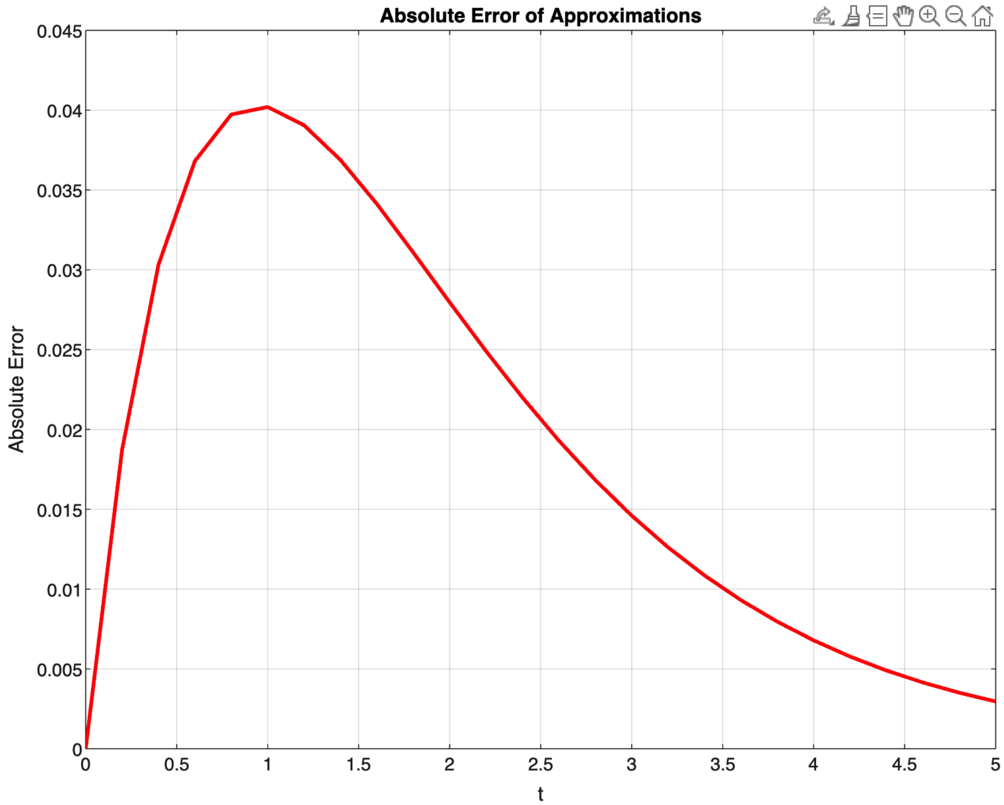
For h=0.1

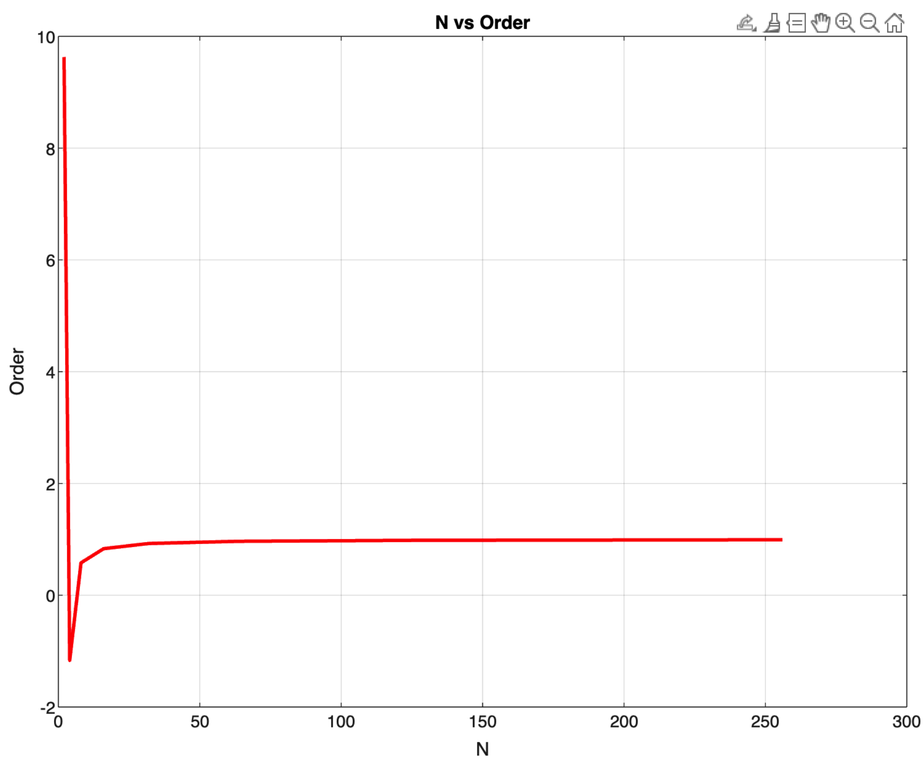

x vs Solution
Approximation
Exact Solution
Solution
t
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

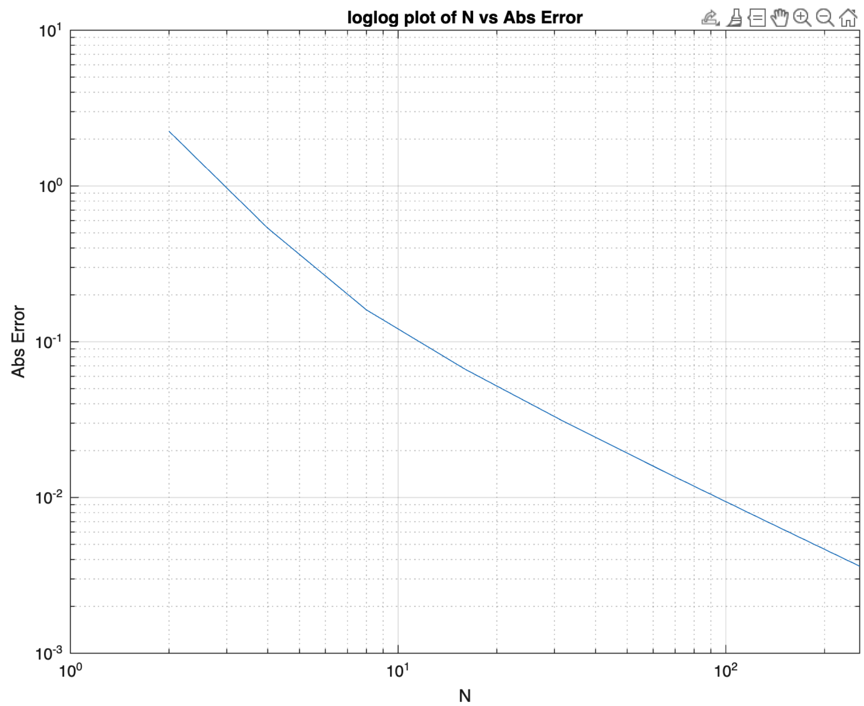

Absolute Error of Approximations
Absolute Error
t
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

For h=0.2


x vs Solution
Approximation
Exact Solution
Solution
t
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Absolute Error of Approximations
Absolute Error
t
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


N vs Order
Order
N
10
8
6
4
2
0
-2
50
100
150
200
250
300




```
>> Lab10Q5
N          E(N)    Order
2          2.2433          9.6297
4          0.0028          -1.1644
8          0.0063          0.5796
16         0.0042          0.8328
32         0.0024          0.9259
64         0.0013          0.9651
128        0.0006          0.9831
256        0.0003          0.9917
>> Lab10Q5
N          E(N)    Order
2          2.2433          9.6297
4          0.0028          -1.1644
8          0.0063          0.5796
16         0.0042          0.8328
32         0.0024          0.9259
64         0.0013          0.9651
128        0.0006          0.9831
256        0.0003          0.9917
>>
```