Scientific Computing (MA322)

Lab 06

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Question 1

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1. The following data represents the function $f(x) = \exp(x)$.

x	1.0	1.5	2.0	2.5
$f(x)$	2.7183	4.4817	7.3819	12.1825

Estimate the value of $f(2.25)$ using the (i) Newton's forward difference interpolation and (ii) Newton's backward difference interpolation. Compare with the exact value.

Using Newton's Forward Difference for calculating f(2.25)

```
>> Lab6Q1forward
forward interpolating table is given below
1.00    2.7183  1.7634  1.1368  0.7636
1.50    4.4817  2.9002  1.9004
2.00    7.3819  4.8006
2.50    12.1825


The actual value of f(2.25) is: 9.48773584
The approximate value of f(2.25) is: 9.49692500
>>
```

Using Newton's Backward Difference for calculating f(2.25)

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>> Lab6Q1backward

The actual value of f(2.25) is: 9.48773584
The approximate value of f(2.25) is: 9.49692500
>>
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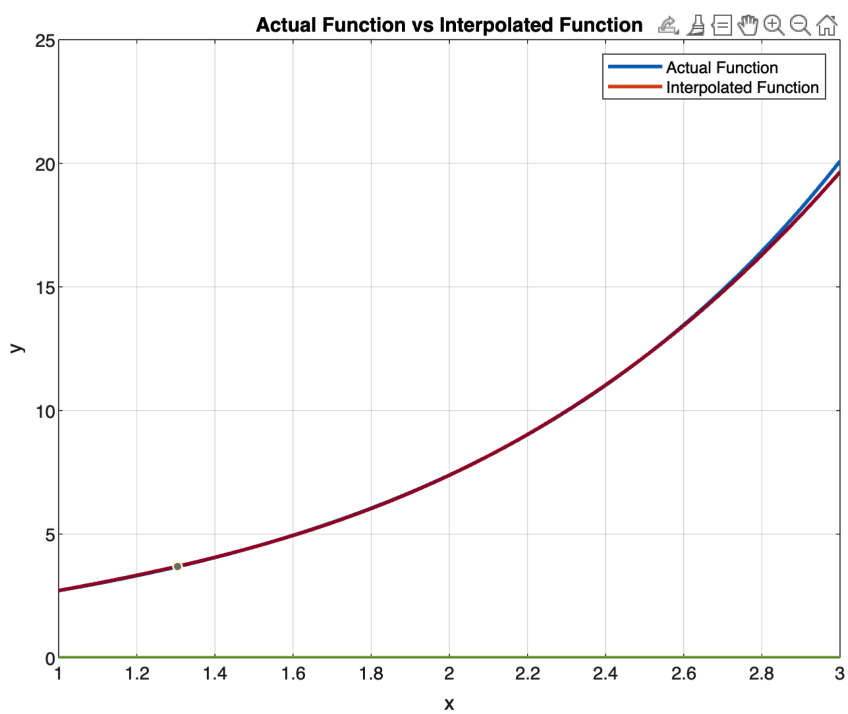

We observe that we get the same value from both the techniques and also it is very close to the actual value of f(2.25).

This is because final expression of the polynomial in the both the cases is same, only difference is way of representation of those polynomials.

Question 2

2. Use Newton forward-diiference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials

   a. $f(0.43)$ if $f(0)=1,\ f(0.25)=1.64872,\ f(0.5)=2.71828,\ f(0.75)=4.48169$

   b. $f\left(\frac{-1}{3}\right)$ if $f(-0.75)=-0.07181250, f(-0.5)=-0.02475000, f(-0.25)=0.33493750, f(0)=1.10100000$

   Also plot the obtained interpolating polynomials.

1

A part

```
>> Lab6Q2parta
forward interpolating table for degree 1 is given below
0.00    1.0000

The approximate value of f(0.43) using interpolating polynomial of degree 1.000000 : 2.11579840

forward interpolating table for degree 2 is given below
0.00    1.0000  0.6487
0.25    1.6487

The approximate value of f(0.43) using interpolating polynomial of degree 2.000000 : 2.37638253

forward interpolating table for degree 3 is given below
0.00    1.0000  0.6487  0.4208
0.25    1.6487  1.0696
0.50    2.7183

The approximate value of f(0.43) using interpolating polynomial of degree 3.000000 : 2.36060473
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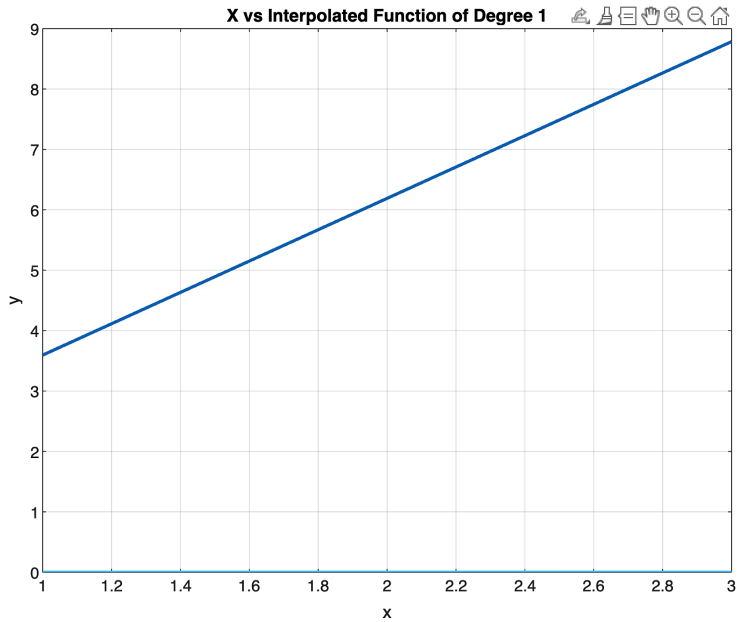
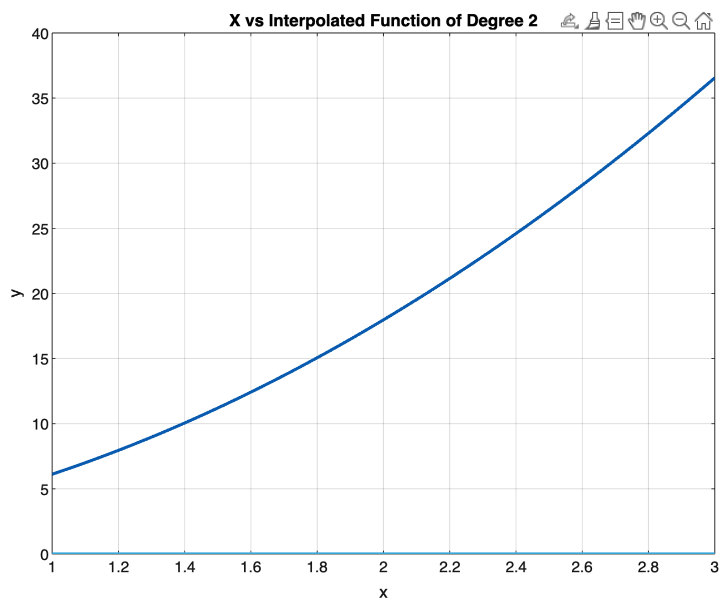
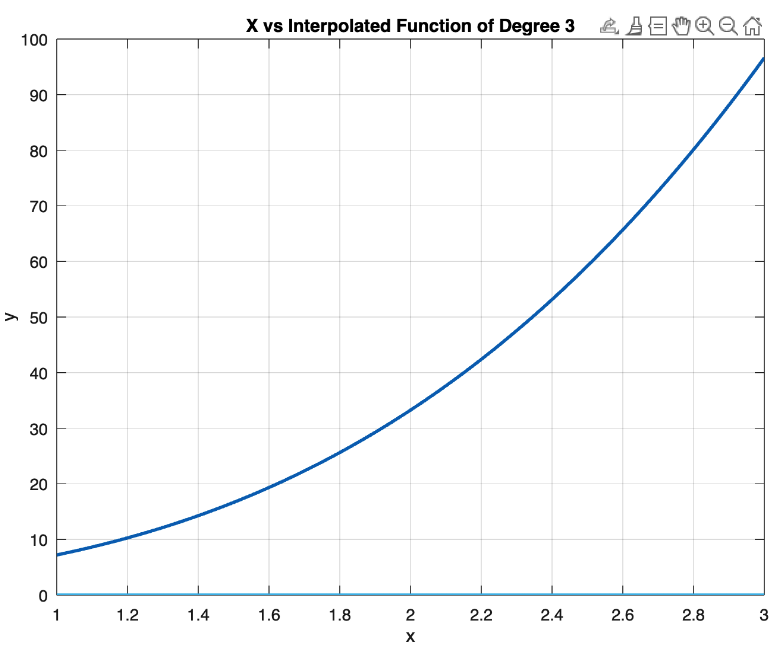
Graphs of polynomials are as follows :


X vs Interpolated Function of Degree 1
y
x


X vs Interpolated Function of Degree 2
y
x


X vs Interpolated Function of Degree 3
y
x

B part

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>> Lab6Q2partb
Forward interpolating table for degree 1 is given below
-0.75   -0.0718

The approximate value of f(0.43) using interpolating polynomial of degree 1 : 0.00662563

Forward interpolating table for degree 2 is given below
-0.75   -0.0718 0.0471
-0.50   -0.0248

The approximate value of f(0.43) using interpolating polynomial of degree 2 : 0.18031105

Forward interpolating table for degree 3 is given below
-0.75   -0.0718 0.0471  0.3126
-0.50   -0.0248 0.3597
-0.25   0.3349

The approximate value of f(0.43) using interpolating polynomial of degree 3 : 0.17452408

>>
```
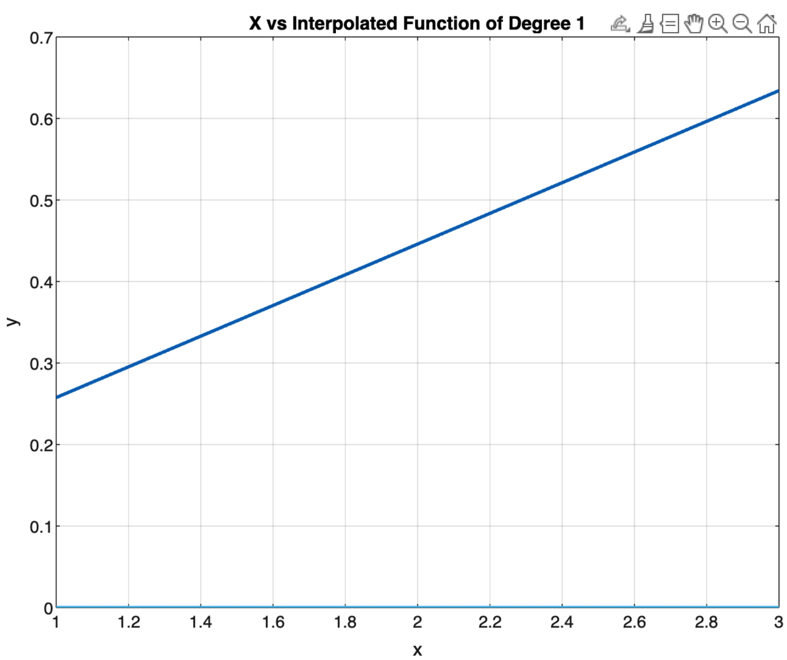
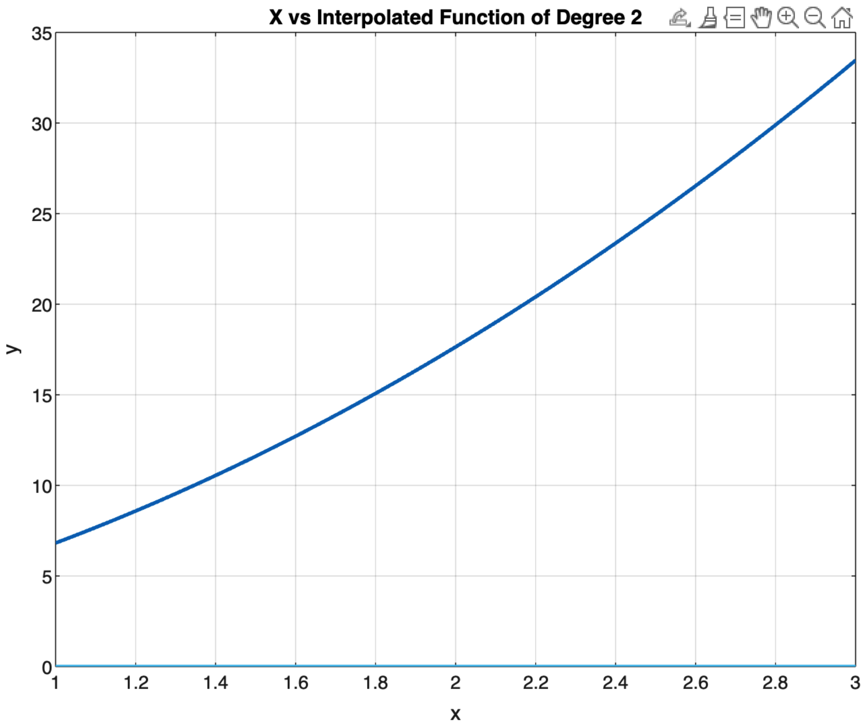
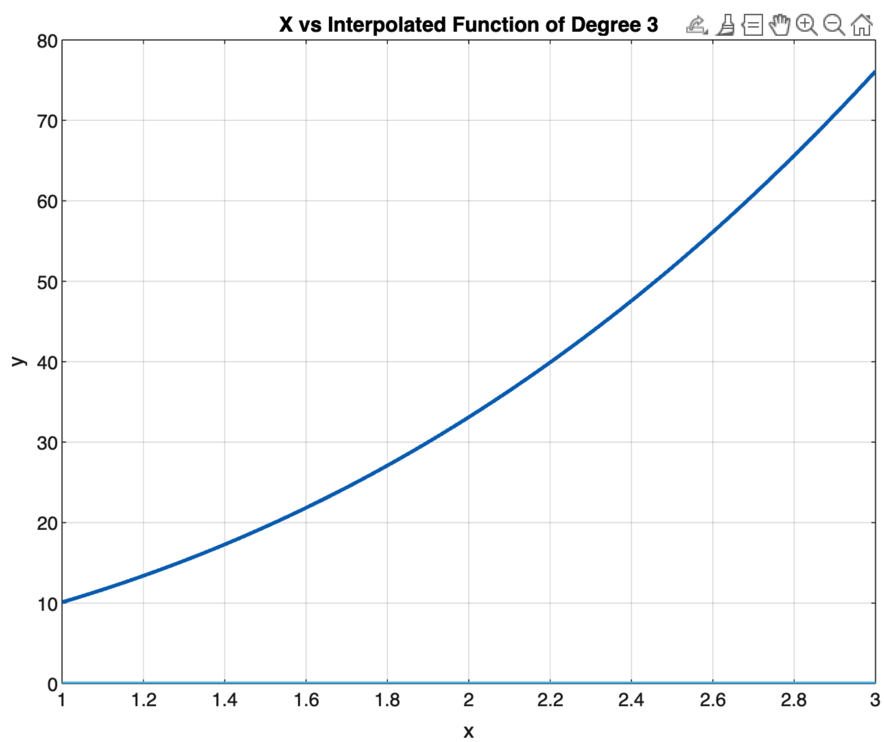

Graphs of polynomials are as follows :




X vs Interpolated Function of Degree 2
35
30
25
20
15
10
5
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
x
y


X vs Interpolated Function of Degree 3
80
70
60
50
40
30
20
10
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
x
y

Question 3

3. Let $f(x) = \dfrac{1}{1+x^2}$ for $-5 \le x \le 5$. For each $n = 1, 2, \ldots, 10$, let $h = 10/n$ and $y_n = P_n(1+\sqrt{10})$, where $P_n(x)$ is the interpolating polynomial for $f(x)$ at the nodes $x_0^{(n)}, x_1^{(n)}, \ldots, x_n^{(n)}$ and $x_j^{(n)} = -5 + jh$, for each $j = 0, 1, \ldots, n$. Does the sequence $\{y_n\}$ appear to converge to $f(1+\sqrt{10})$? Explain your observations with reasons.

Take P_n as Lagrange interpolant, Newton-forward and Newton-backward.

We get the values from the different technique as follows:

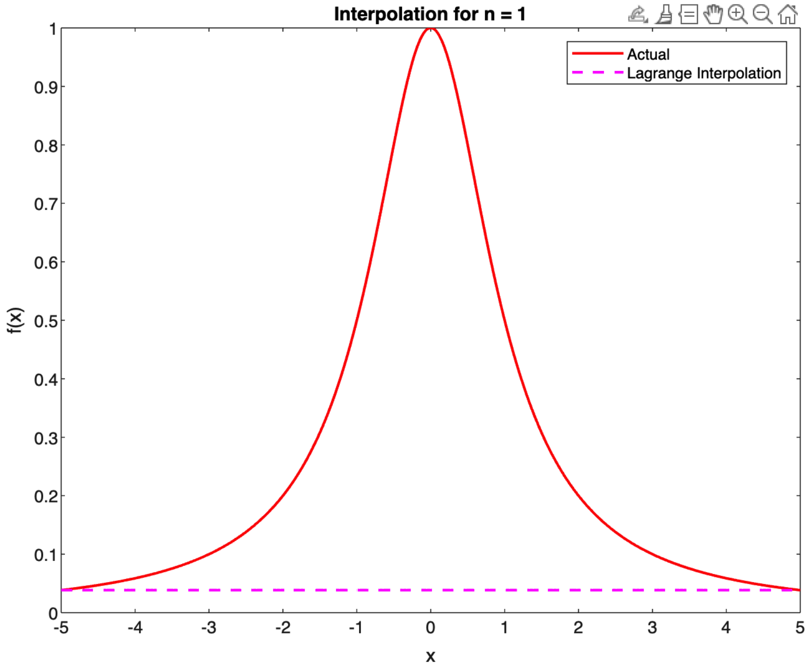
```
>> Lab6Q3
Main Table:
         n        Forward          Backward          Lagrange
         1    0.0384615385     0.0384615385     0.0384615385
         2    0.3336709492     0.3336709492     0.3336709492
         3    0.1166052060     0.1166052060     0.1166052060
         4   -0.3717596394    -0.3717596394    -0.3717596394
         5   -0.0548918740    -0.0548918740    -0.0548918740
         6    0.6059346282     0.6059346282     0.6059346282
         7    0.1902492330     0.1902492330     0.1902492330
         8   -0.5133526169    -0.5133526169    -0.5133526169
         9   -0.0668173424    -0.0668173424    -0.0668173424
        10    0.4483348123     0.4483348123     0.4483348123
>>
```

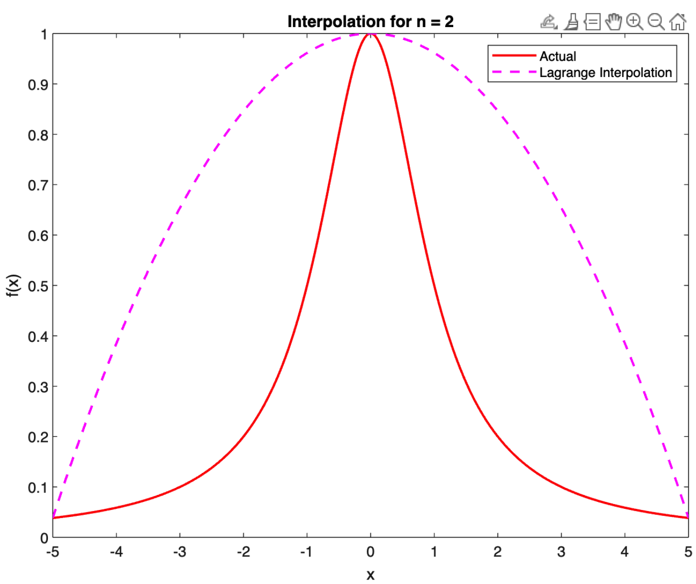
We clearly observe that the values don’t converge, since the actual value is 0.054572.

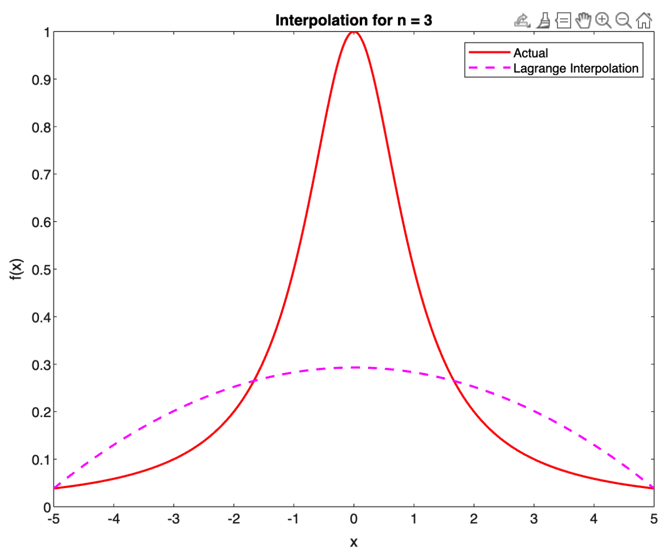
This is because the shape of the function f(x) = 1/(1+x2)

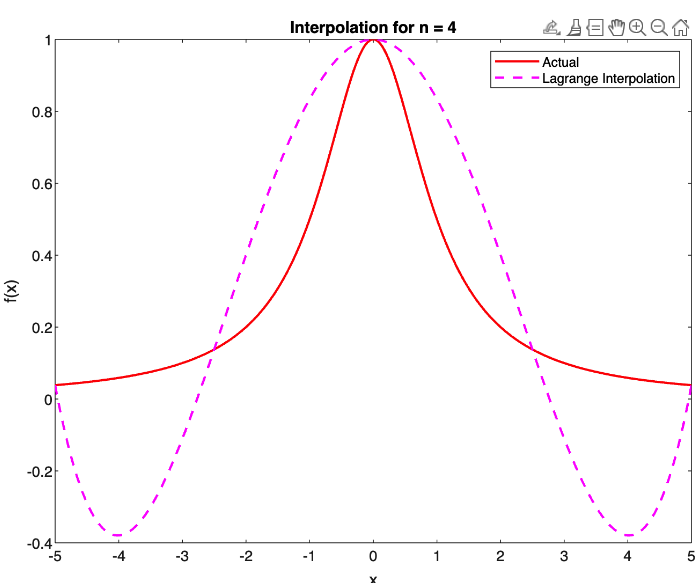
And as we increase the degree of polynomial, we get the following interpolating polynomial:

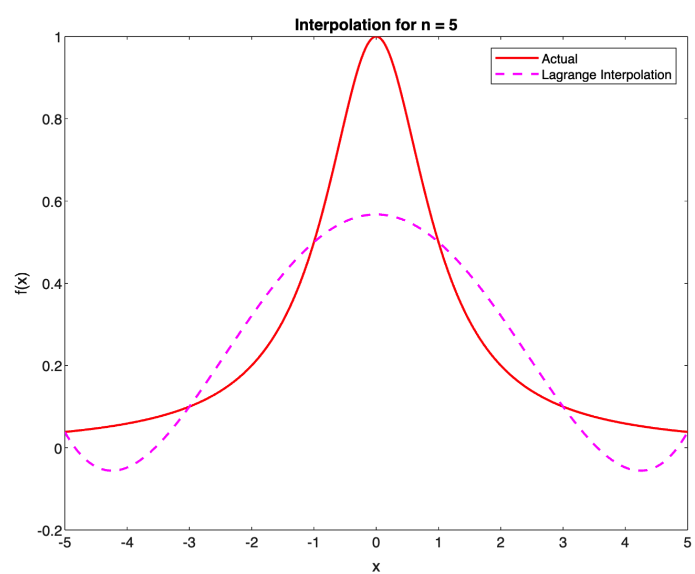
Sidenote (I am showing only lagrange polynomial graph, simply because We would get the same graph from Newton’s Forward and Newton’s Backward interpolation as well)

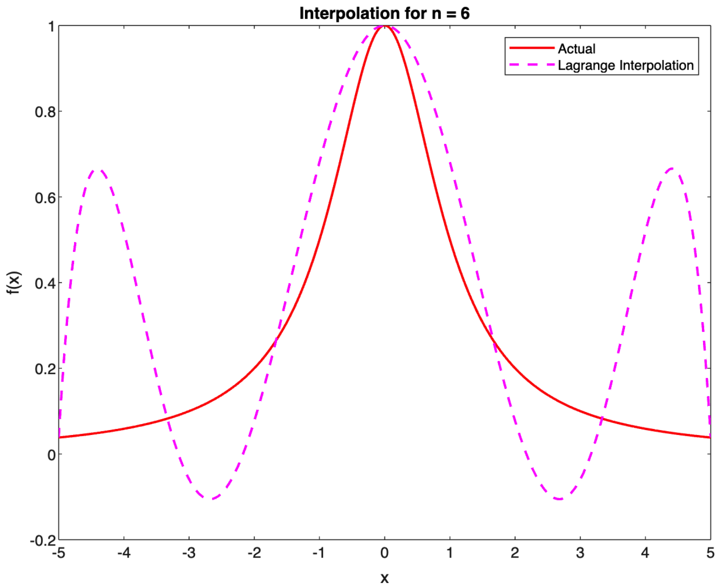

Interpolation for n = 1
Actual
Lagrange Interpolation
f(x)
x
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
-5 -4 -3 -2 -1 0 1 2 3 4 5

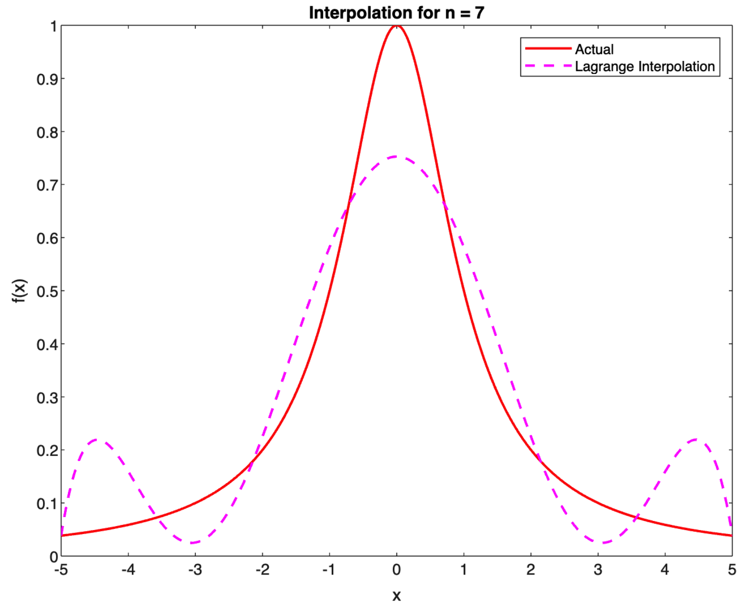

Interpolation for n = 2
Actual
Lagrange Interpolation
f(x)
x
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
-5 -4 -3 -2 -1 0 1 2 3 4 5

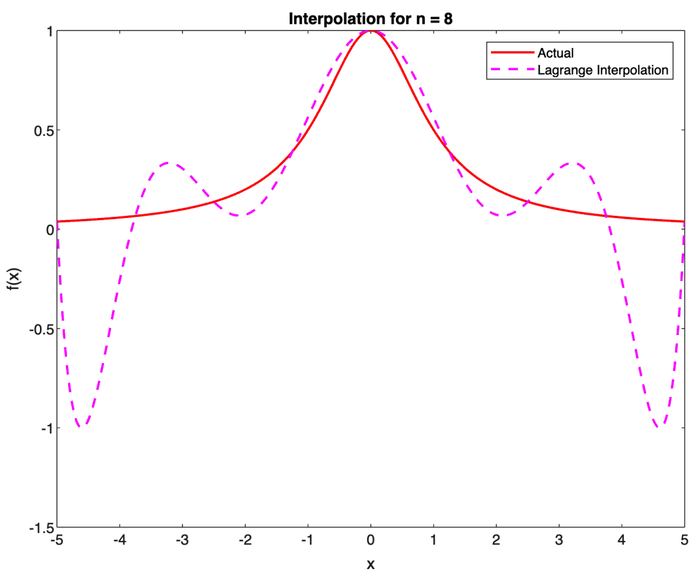

Interpolation for n = 3
Actual
Lagrange Interpolation
f(x)
x
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
-5 -4 -3 -2 -1 0 1 2 3 4 5

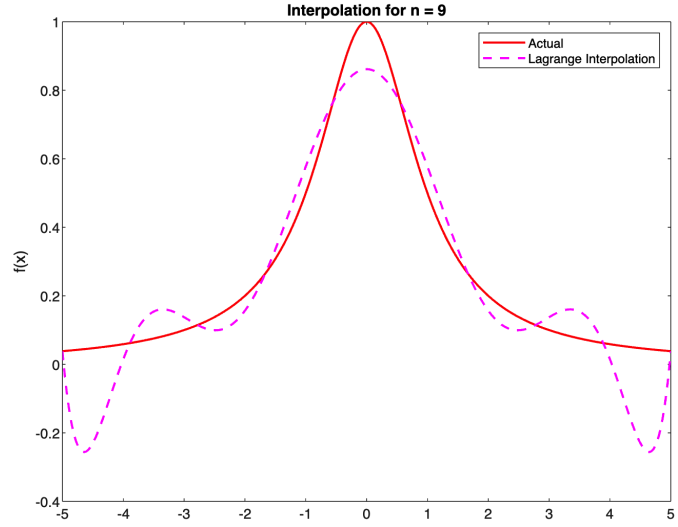

Interpolation for n = 4
Actual
Lagrange Interpolation
f(x)
x
-0.4 -0.2 0 0.2 0.4 0.6 0.8 1
-5 -4 -3 -2 -1 0 1 2 3 4 5

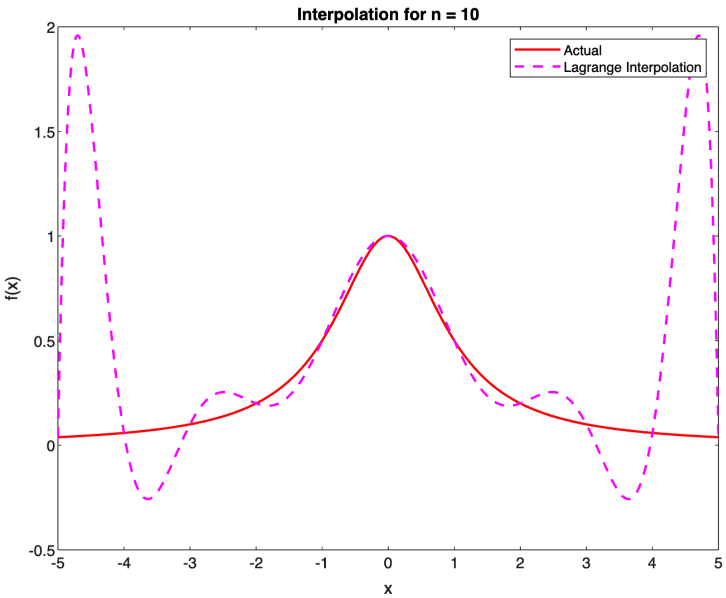

Interpolation for n = 5
Actual
Lagrange Interpolation
f(x)
x
-0.2 0 0.2 0.4 0.6 0.8 1
-5 -4 -3 -2 -1 0 1 2 3 4 5


Interpolation for n = 6
Actual
Lagrange Interpolation
f(x)
x


Interpolation for n = 7
Actual
Lagrange Interpolation
f(x)
x


Interpolation for n = 8
Actual
Lagrange Interpolation
f(x)
x


Interpolation for n = 9
Actual
Lagrange Interpolation
f(x)



It's evident that increasing the degree of the interpolating polynomial doesn't lead to a convergence between the graphs. This lack of convergence is clearly observed as there's no apparent resemblance or similarity between the graphs despite the polynomial degree being increased.