Scientific Computing (MA322)

Lab 07

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Question 1

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1. By using *Hermite* interpolation, construct an approximating polynomial for the following data which is generated using the function $f(x) = \ln(\exp(x) + 2)$:

x	$f(x)$	$f'(x)$
-1	0.86199480	0.15536240
-0.5	0.95802009	0.23269654
0	1.0986123	0.33333333
0.5	1.2943767	0.45186776

Approximate $f(0.25)$ and calculate the absolute error.

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>> Lab7Q1
Hermite interpolation Table:
x1:     0.86199480      0.15536240      0.07337636      0.01583112      -0.00014728     -0.00089244     -0.00007672     0.00006864
x2:     0.86199480      0.19205058      0.08129192      0.01568384      -0.00103972     -0.00100752     0.00002624      0.00000000
x3:     0.95802009      0.23269654      0.09697576      0.01464412      -0.00255100     -0.00096816     0.00000000      0.00000000
x4:     0.95802009      0.28118442      0.10429782      0.01209312      -0.00351916     0.00000000      0.00000000      0.00000000
x5:     1.09861230      0.33333333      0.11639094      0.00857396      0.00000000      0.00000000      0.00000000      0.00000000
x6:     1.09861230      0.39152880      0.12067792      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000
x7:     1.29437670      0.45186776      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000
x8:     1.29437670      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000


Approximate value of f(0.25) using hermite interpolation comes out to be : 1.18906976
Absolute error at x=0.25 comes out to be : 0.00000017
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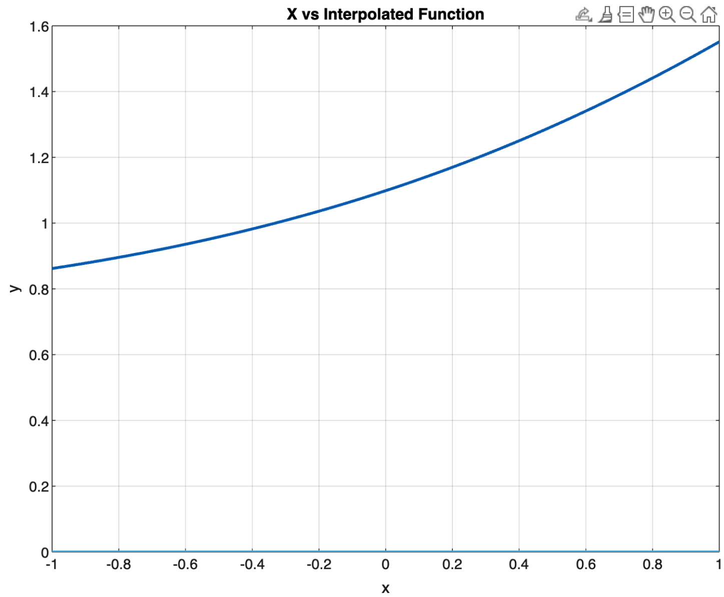
We can observe the interpolating polynomial by the following plot: -



Question 2

2. A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

(a) Use a *Hermite* polynomial to predict the position of the car and its speed when $t = 10$ seconds.

(b) Use the derivative of the *Hermite* polynomial to determine whether the car ever exceeds a 55 mi/h speed limit on the road. If so, what is the first time the car exceeds this speed?

(c) What is the predicted maximum speed for the car?

Taking the function f (t) as distance and f'(t) as speed, we can approximate the given data with the help of Hermite interpolation

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>> Lab7Q2
Part-(a)


Approximate position  of car at time = 10 seconds  using hermite interpolation comes out to be : 742.50283910 fe


Approximate speed  of car at time = 10 seconds  using hermite interpolation comes out to be : 48.38173636 feets/
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Part-(b)

Car exceeds given speed at time = 5.64933 seconds with speed = 80.667325 feets/seconds
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Part-(c)

Max Speed of the car comes out to be 12.371955 feets/seconds and it was achieved at time 119.41734 seconds
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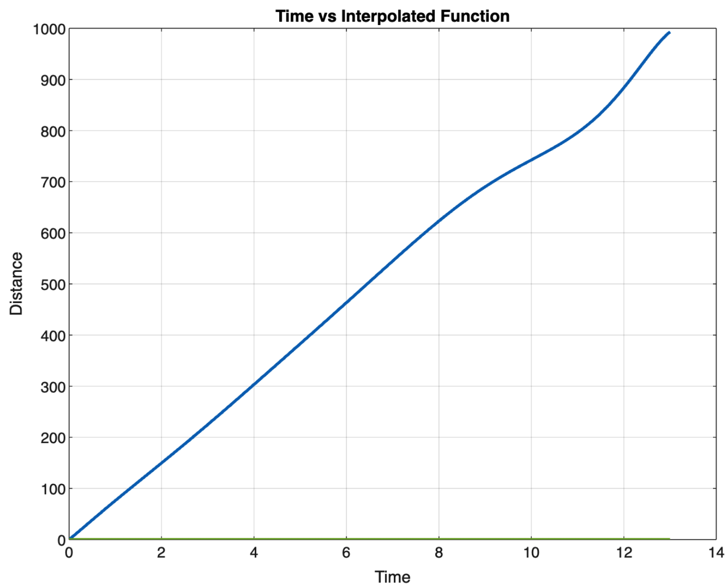

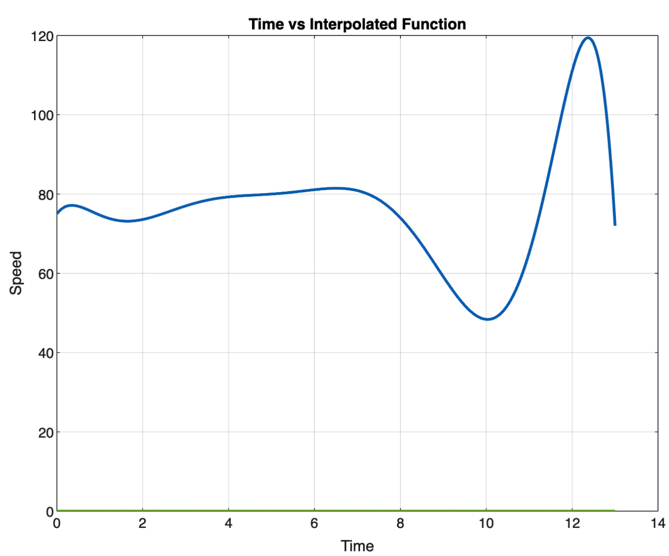

Taking 55 mi/h i.e., 80.6667 feet/sec as speed threshold, we can observe that speed clearly exceeds the threshold multiple times. Also, it first exceeds the threshold somewhere between t=5 sec and t=6 sec.

Question 3

3. Use the cubic splines for the value of $x = 0$ and $x = 0.5$ to approximate $f(x)$ and $f'(x)$, and calculate the actual error.

   (a) $f(x) = e^{2x}$; approximate $f(0.43)$ and $f(0.43)$.

   (b) $f(x) = \ln(e^x + 2)$; approximate $f(0.25)$ and $f(0.25)$.

Since, it was not mentioned which cubic spline is to be used, I used both natural and clamped cubic spline to approximate the following polynomials at given values. For clamped spline, we already know the function explicitly, so we can differentiate and find the values of f'(x0) and f'(xn) required in clamped cubic spline interpolation.

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>> Lab7Q3
Part-(a)

Exact value of f(0.43) is 2.363161.
Exact value of f'(0.43) is 4.726321.

Approximate value of f(0.43) by natural cubic spline is 2.477722.
Approximate value of f'(0.43) by natural cubic spline is 3.436564.

Actual Error in f(0.43) for natural spline is 1.145617e-01.
Actual Error in f'(0.43) for natural spline is 1.289758e+00.

Approximate value of f(0.43) by clamped cubic spline is 2.362071.
Approximate value of f'(0.43) by clamped cubic spline is 4.751932.

Actual Error in f(0.43) for clamped spline is 1.089680e-03.
Actual Error in f'(0.43) for clamped spline is 2.561015e-02.

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Part-(b)

Exact value of f(0.25) is 1.189070.
Exact value of f'(0.25) is 0.390991.

Approximate value of f(0.25) by natural cubic spline is 1.196495.
Approximate value of f'(0.25) by natural cubic spline is 0.391529.

Actual Error in f(0.25) for natural spline is 7.424598e-03.
Actual Error in f'(0.25) for natural spline is 5.376463e-04.

Approximate value of f(0.25) by clamped cubic spline is 1.189086.
Approximate value of f'(0.25) by clamped cubic spline is 0.390994.

Actual Error in f(0.25) for clamped spline is 1.650865e-05.
Actual Error in f'(0.25) for clamped spline is 3.103286e-06.
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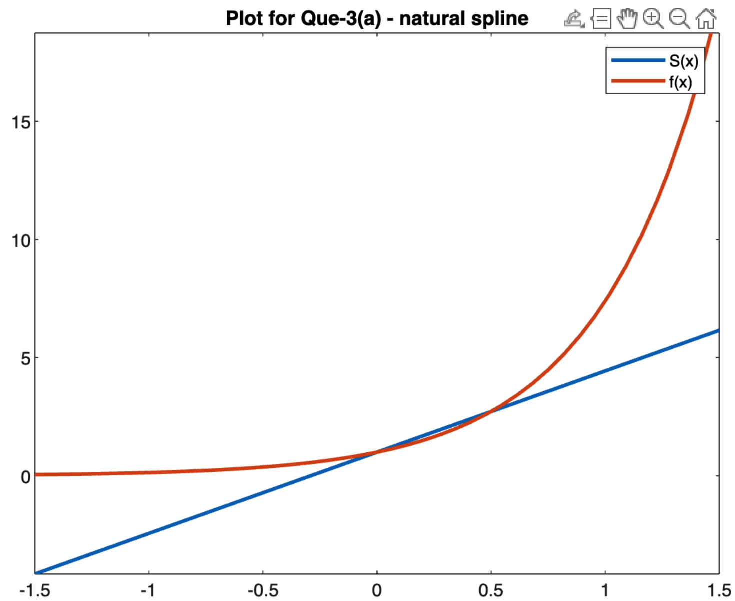
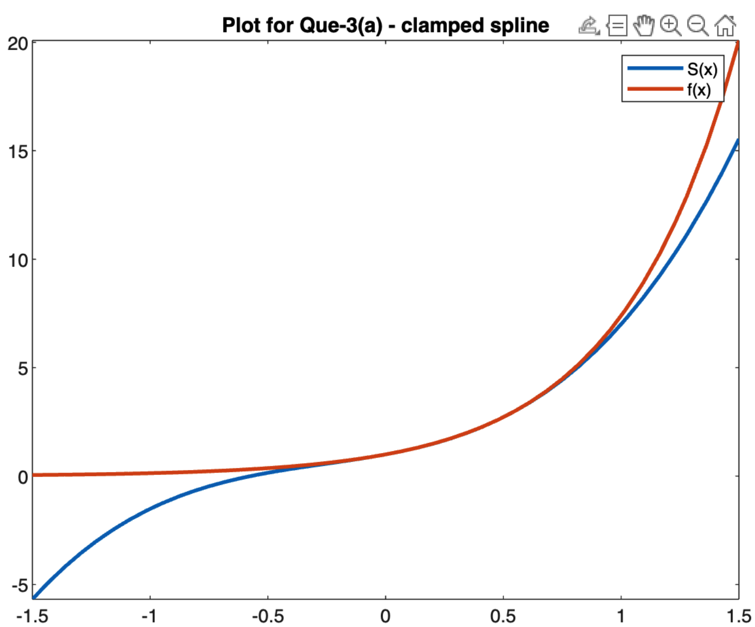
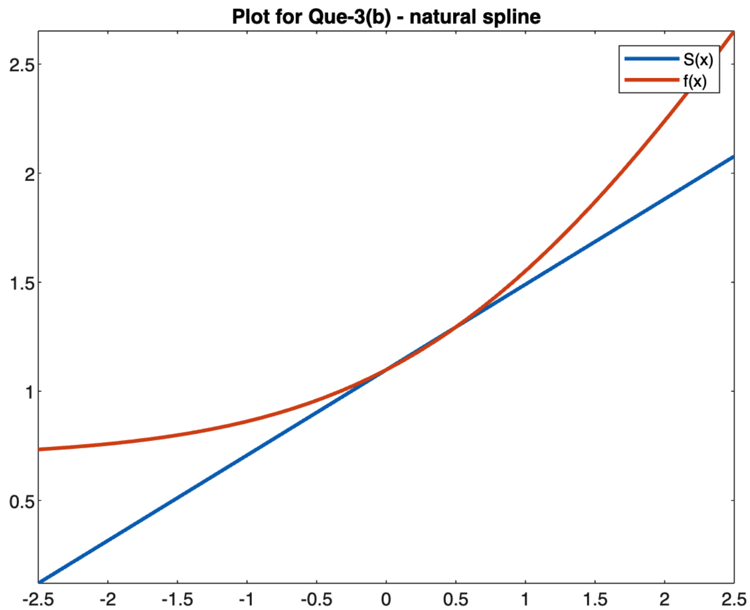
The plot of natural cubic spline for Part A can be observed as follows:



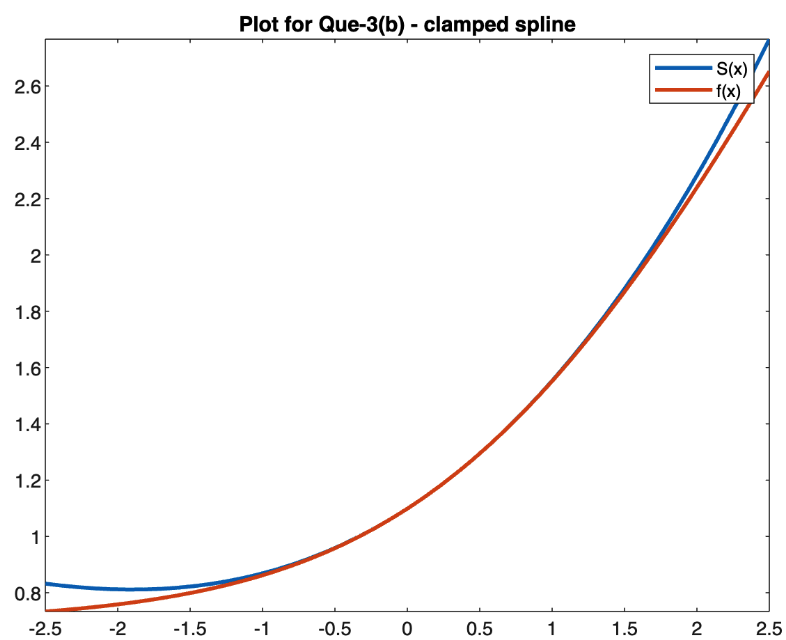
The plot for clamped cubic spline for part A can be observed as follows: -



The plot of natural cubic spline for Part B can be observed as follows: -



The plot for clamped cubic spline for part B can be observed as follows: -



In both the cases, we observe that the clamped cubic spline is fitting and approximating the function in a better way than natural spline.

Question 4

4. Construct the clamped cubic spline using the data of Exercise 1 and the fact that

(a) $f'(-1) = 0.15536240$ and $f'(0.5) = 0.45186276$.

Si(x) = ai + bi(x – xi) + ci(x – xi)2 + di(x – xi)3 for i = 1,2,3

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a =

    0.8620    0.9580    1.0986

b =

    0.1554    0.2327    0.3334

c =

    0.0654    0.0894    0.1119

d =

    0.0160    0.0150    0.0088
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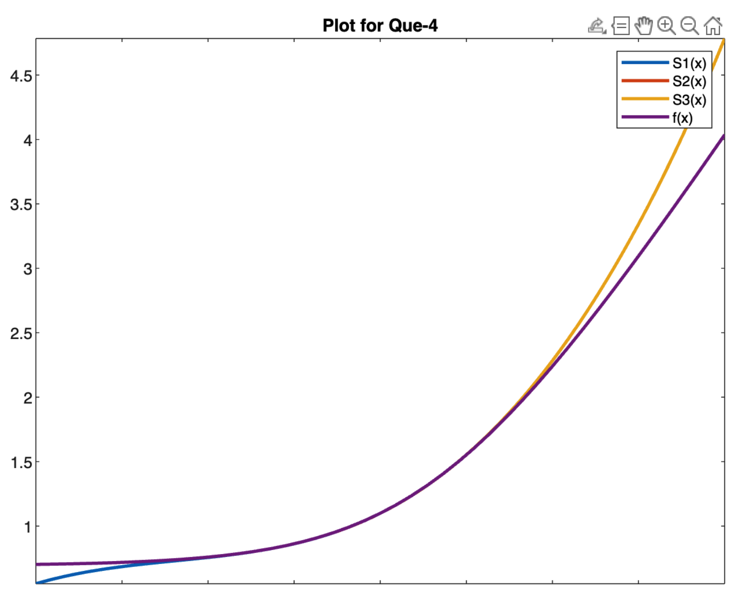

Question 5

5. A car traveling along a straight road is clocked at a number of points. The data from th observations is given in the following table where the time is in seconds, and the distanc in feet:

Time	0	3	5	8	13
Distance	0	225	383	623	993

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(a) Use *the natural cubic spline* interpolation to predict the position of the car and it speed when $t = 10$ seconds.

(b) Use *the clamped cubic spline* interpolation to predict the position of the car and it speed when $t = 10\,sec$. In this case use the fact that the speed at the beginning an the end of the race is 75 and 72 feet per second, respectively.

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>> Lab7Q5
Part-(a)

Approximate position of car by natural spline at t=10sec is :- 774.863900
Approximate speed of car by natural spline at t=10sec is :- 74.160996

Part-(b)

Approximate position of car by clamped spline at t=10sec is :- 774.838407
Approximate speed of car by clamped spline at t=10sec is :- 74.160265

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