Scientific Computing (MA322)

Lab 12

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Question 1

1. Consider the following IVP's:

   (a) $y' = 1 + y/t + (y/t)^2$, $1 \leq t \leq 3$, $y(1) = 0$ with $h = 0.2$; actual solution $y(t) = t\tan(\ln t)$.

   (b) $y' = -ty + 4ty^{-1}$, $0 \leq t \leq 1$, $y(0) = 1$ with $h = 0.1$; actual solution $y(t) = \sqrt{4 - 3e^{-t^2}}$.

Use Adams-Bashforth and Adams-Moulton methods to approximate the solutions to the IVPs given in Question 1.

(a) Use exact starting values.

Compare the results to the actual values.

Adams Bashforth fourth order scheme

$$w_0 = \alpha, \quad w_1 = \alpha_1, \quad w_2 = \alpha_2, \quad w_3 = \alpha_3,$$

$$w_{i+1} = w_i + \frac{h}{24}[55f(t_i, w_i) - 59f(t_{i-1}, w_{i-1}) + 37f(t_{i-2}, w_{i-2}) - 9f(t_{i-3}, w_{i-3})],$$

Adams Moulton third order scheme

$$w_0 = \alpha, \quad w_1 = \alpha_1, \quad w_2 = \alpha_2,$$

$$w_{i+1} = w_i + \frac{h}{24}[9f(t_{i+1}, w_{i+1}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})],$$

A part

Adams - Bashforth 4 step

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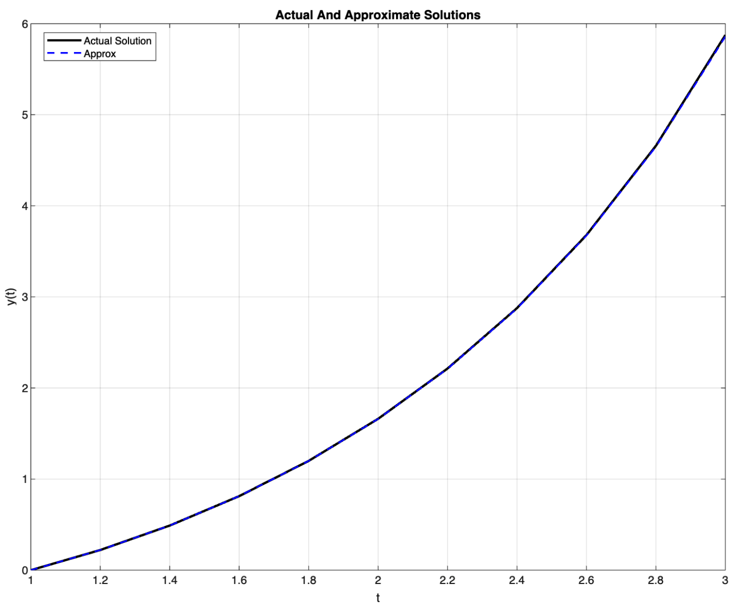
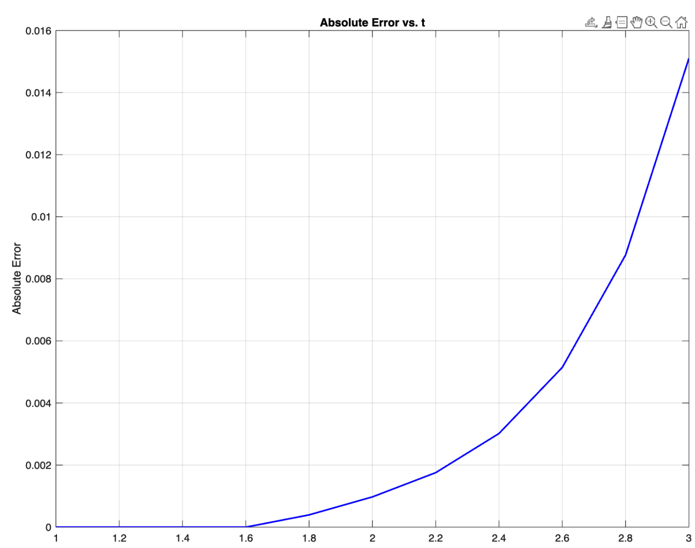
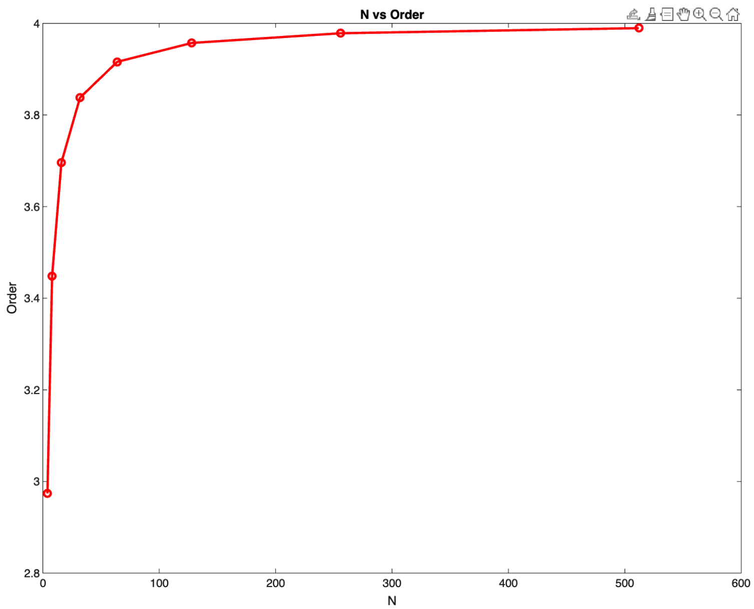
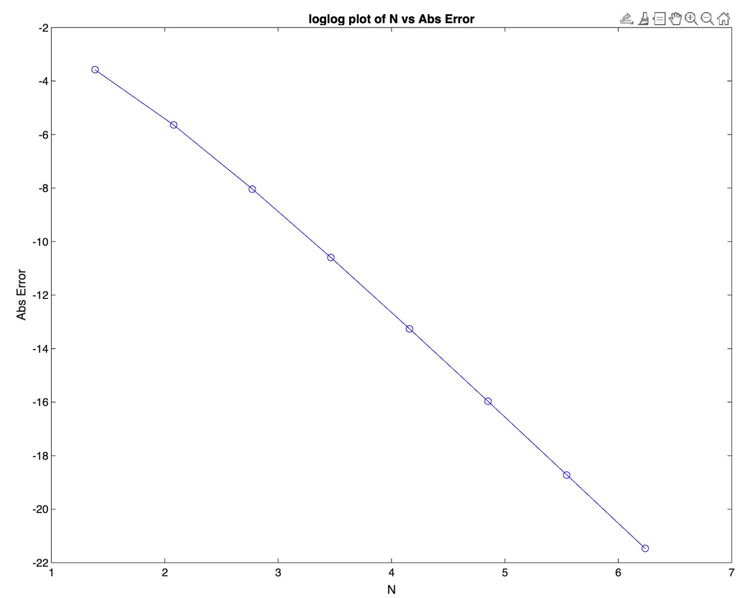
>> Lab12Q1
```

t	Actual Value	Approximate Value	Absolute Error
1.0000	0.000000	0.000000	0.000000
1.2000	0.221243	0.221243	0.000000
1.4000	0.489682	0.489682	0.000000
1.6000	0.812753	0.812753	0.000000
1.8000	1.199439	1.199044	0.000395
2.0000	1.661282	1.660307	0.000975
2.2000	2.213502	2.211746	0.001756
2.4000	2.876551	2.873534	0.003017
2.6000	3.678475	3.673329	0.005146
2.8000	4.658665	4.649897	0.008768
3.0000	5.874100	5.858999	0.015101



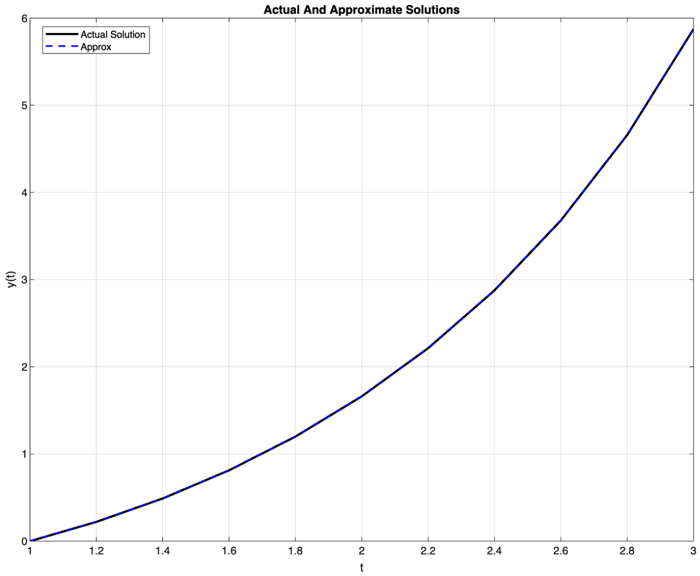




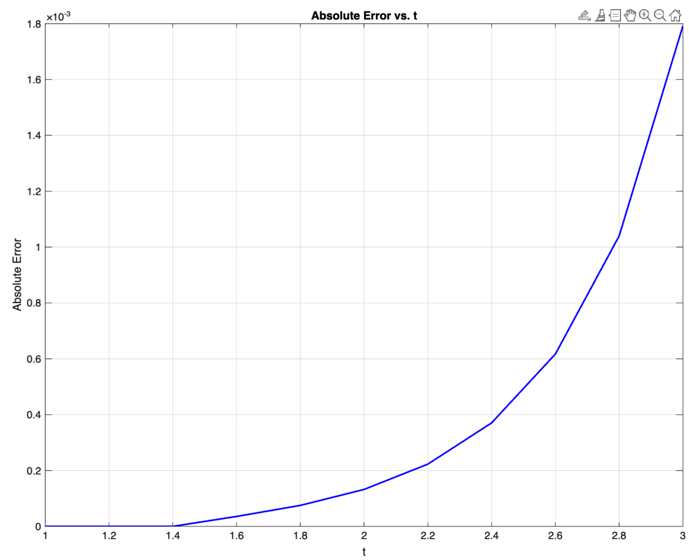


Adams - Moulton 3-step

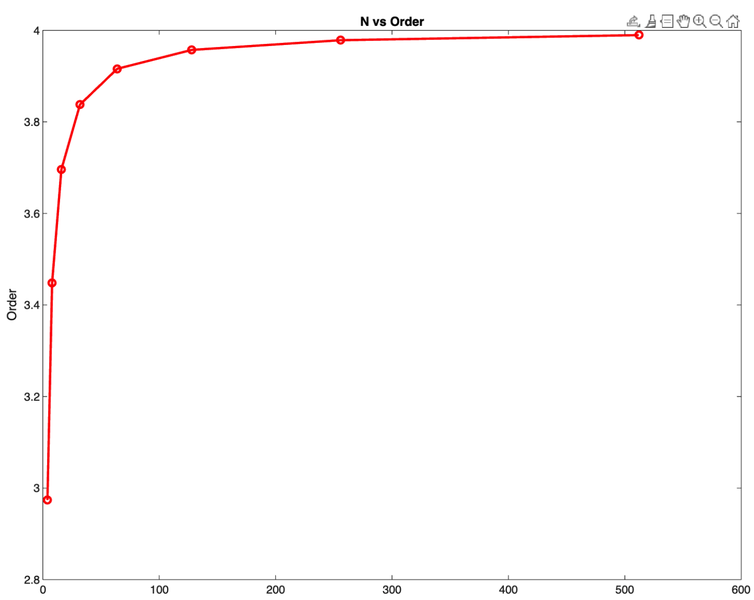
t	Actual Value	Approximate Value	Absolute Error
1.0000	0.000000	0.000000	0.000000
1.2000	0.221243	0.221243	0.000000
1.4000	0.489682	0.489682	0.000000
1.6000	0.812753	0.812788	0.000036
1.8000	1.199439	1.199514	0.000075
2.0000	1.661282	1.661414	0.000132
2.2000	2.213502	2.213725	0.000223
2.4000	2.876551	2.876922	0.000371
2.6000	3.678475	3.679092	0.000617
2.8000	4.658665	4.659705	0.001040
3.0000	5.874100	5.875892	0.001792



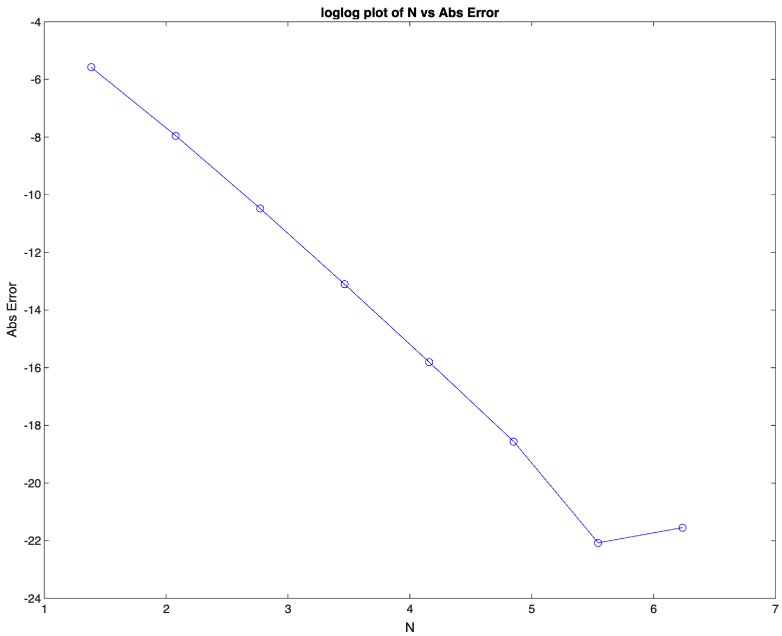
Actual And Approximate Solutions
Actual Solution
Approx
y(t)
t



Absolute Error vs. t
×10^-3
Absolute Error
t



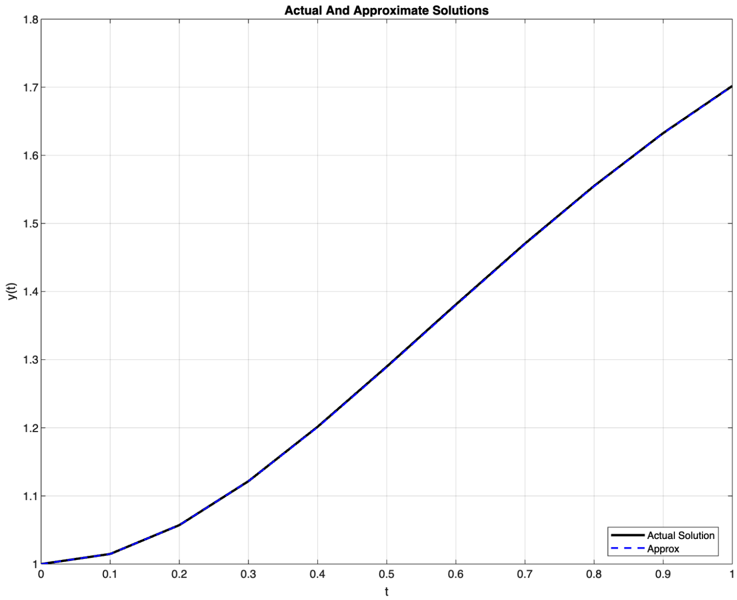
N vs Order
Order
N

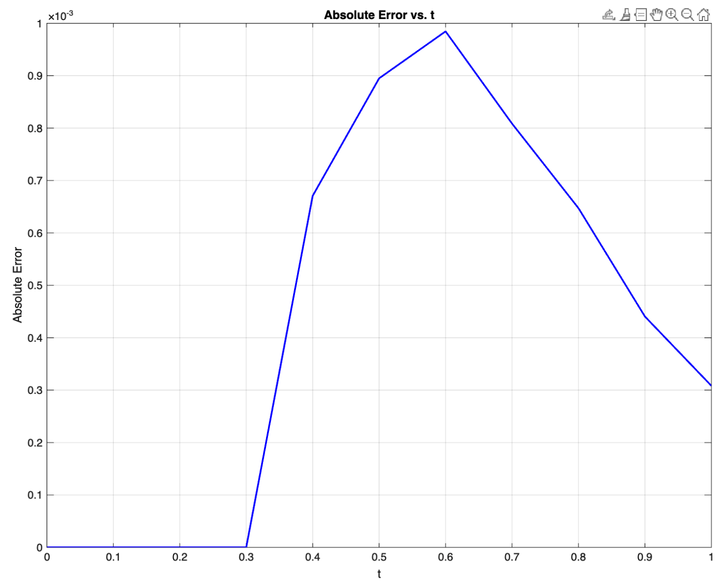


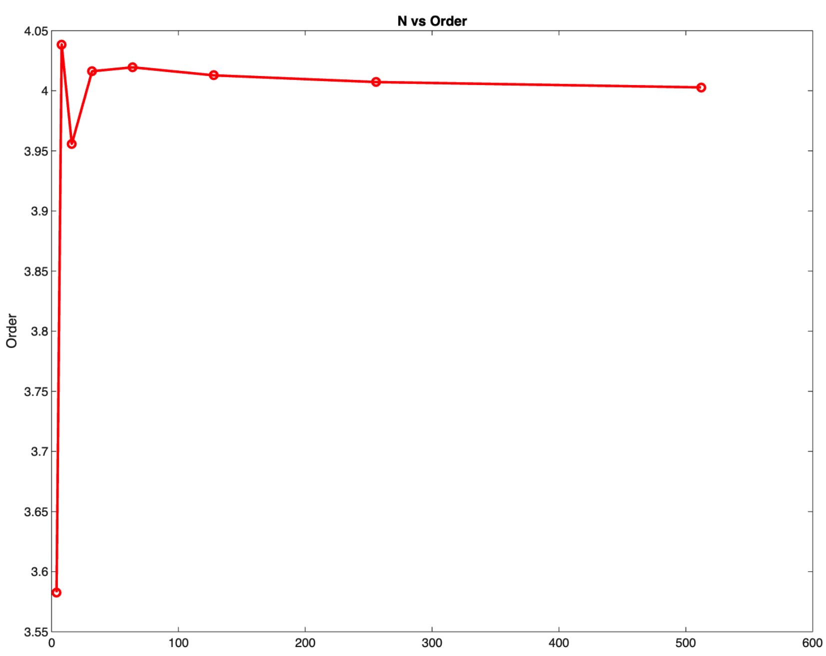
B part

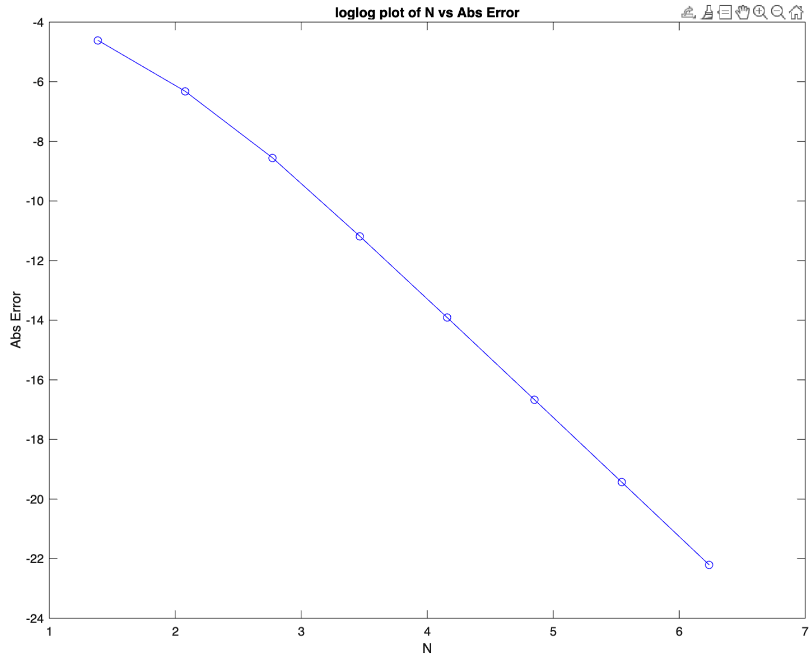
Adams - Bashforth 4 step

t	Actual Value	Approximate Value	Absolute Error
0.0000	1.000000	1.000000	0.000000
0.1000	1.014815	1.014815	0.000000
0.2000	1.057181	1.057181	0.000000
0.3000	1.121698	1.121698	0.000000
0.4000	1.201486	1.200815	0.000671
0.5000	1.289805	1.288910	0.000895
0.6000	1.380931	1.379947	0.000985
0.7000	1.470415	1.469607	0.000809
0.8000	1.555031	1.554384	0.000647
0.9000	1.632613	1.632173	0.000440
1.0000	1.701870	1.701562	0.000308


Actual And Approximate Solutions
y(t)
t
Actual Solution
Approx

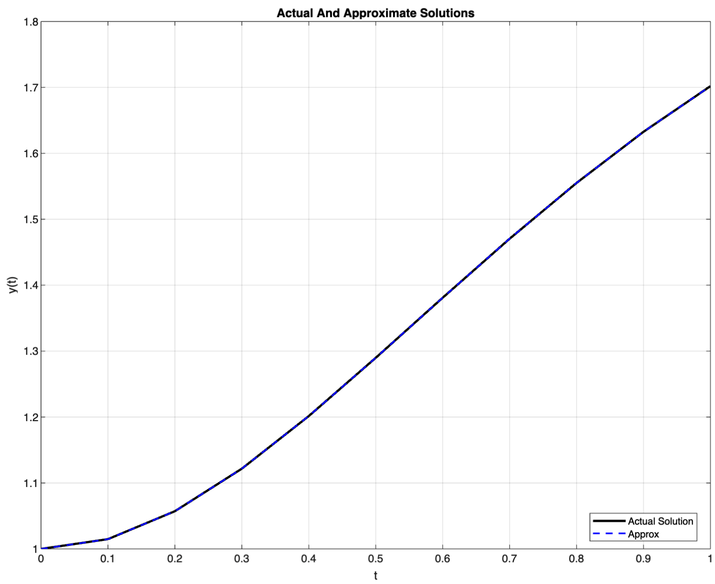

Absolute Error vs. t
×10-3
Absolute Error
t

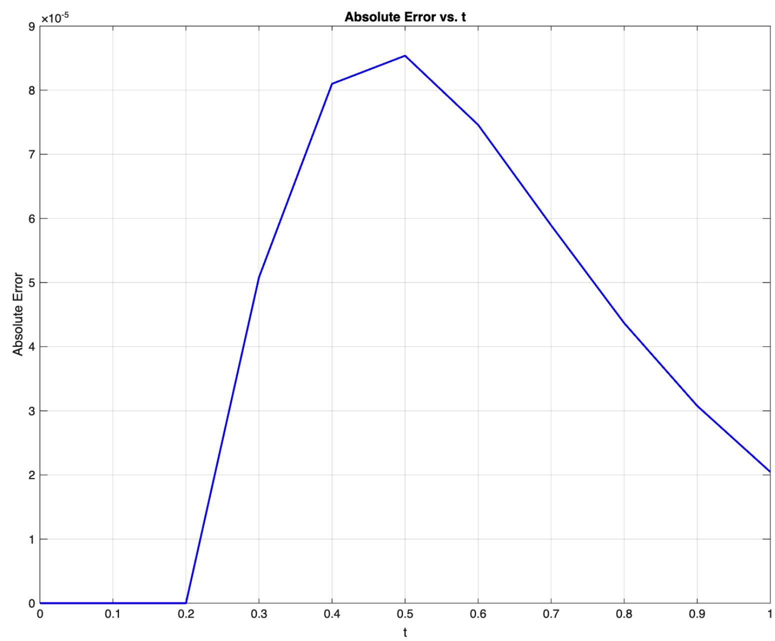

N vs Order
Order
N

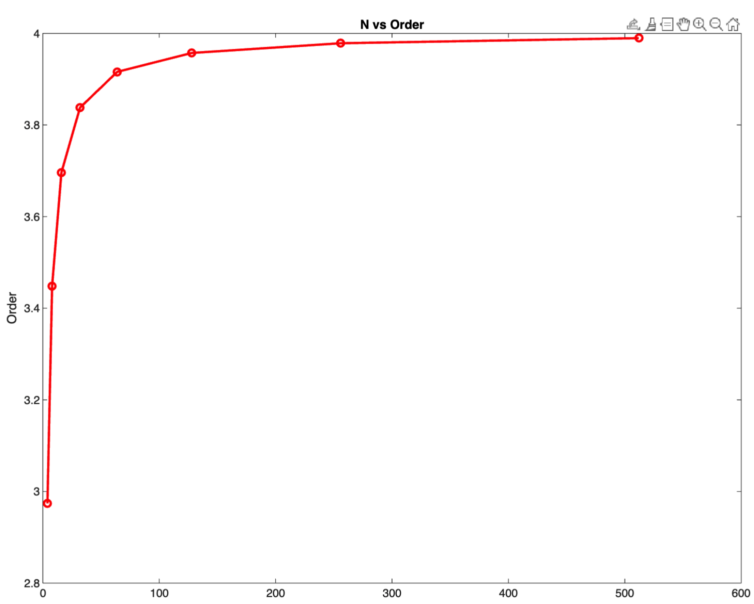


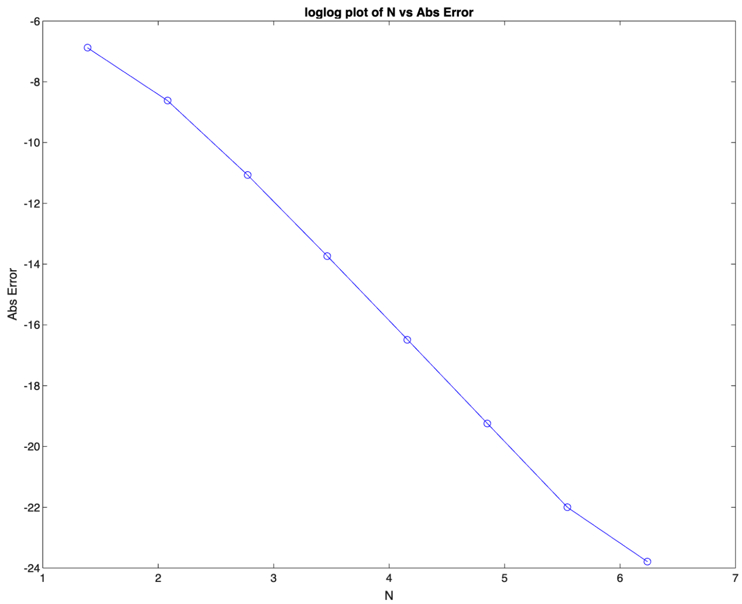
Adams - Moulton 3-step

```
t         | Actual Value | Approximate Value | Absolute Error
   0.0000 |     1.000000 |          1.000000 |       0.000000
   0.1000 |     1.014815 |          1.014815 |       0.000000
   0.2000 |     1.057181 |          1.057181 |       0.000000
   0.3000 |     1.121698 |          1.121749 |       0.000051
   0.4000 |     1.201486 |          1.201567 |       0.000081
   0.5000 |     1.289805 |          1.289891 |       0.000085
   0.6000 |     1.380931 |          1.381006 |       0.000075
   0.7000 |     1.470415 |          1.470474 |       0.000059
   0.8000 |     1.555031 |          1.555075 |       0.000044
   0.9000 |     1.632613 |          1.632644 |       0.000031
   1.0000 |     1.701870 |          1.701891 |       0.000020
>>
```


Actual And Approximate Solutions
y(t)
t
Actual Solution
Approx


Absolute Error vs. t
Absolute Error
t
×10^-5


N vs Order
Order
N



Question 2

2. Consider the following problem

$$y' = -2y + 1,\ 0 \leq t \leq 1,\ y(0) = 1.$$

Solve it by using the following methods:

(a) Explicit Euler.

(b) Implicit Euler.

(c) $y_{n+1} = y_{n-1} + 2hf(t_n, y_n)$.

For h=0.1

```
>> Lab12Q2
For explicit Euler
t        | Actual Value | Approximate Value | Absolute Error
  0.0000 |     1.000000 |          1.000000 |       0.000000
  0.1000 |     0.909365 |          0.900000 |       0.009365
  0.2000 |     0.835160 |          0.820000 |       0.015160
  0.3000 |     0.774406 |          0.756000 |       0.018406
  0.4000 |     0.724664 |          0.704800 |       0.019864
  0.5000 |     0.683940 |          0.663840 |       0.020100
  0.6000 |     0.650597 |          0.631072 |       0.019525
  0.7000 |     0.623298 |          0.604858 |       0.018441
  0.8000 |     0.600948 |          0.583886 |       0.017062
  0.9000 |     0.582649 |          0.567109 |       0.015541
  1.0000 |     0.567668 |          0.553687 |       0.013981

For implicit Euler
t        | Actual Value | Approximate Value | Absolute Error
  0.0000 |     1.000000 |          1.000000 |       0.000000
  0.1000 |     0.909365 |          0.916667 |       0.007301
  0.2000 |     0.835160 |          0.847222 |       0.012062
  0.3000 |     0.774406 |          0.789352 |       0.014946
  0.4000 |     0.724664 |          0.741127 |       0.016462
  0.5000 |     0.683940 |          0.700939 |       0.016999
  0.6000 |     0.650597 |          0.667449 |       0.016852
  0.7000 |     0.623298 |          0.639541 |       0.016243
  0.8000 |     0.600948 |          0.616284 |       0.015336
  0.9000 |     0.582649 |          0.596904 |       0.014254
  1.0000 |     0.567668 |          0.580753 |       0.013085

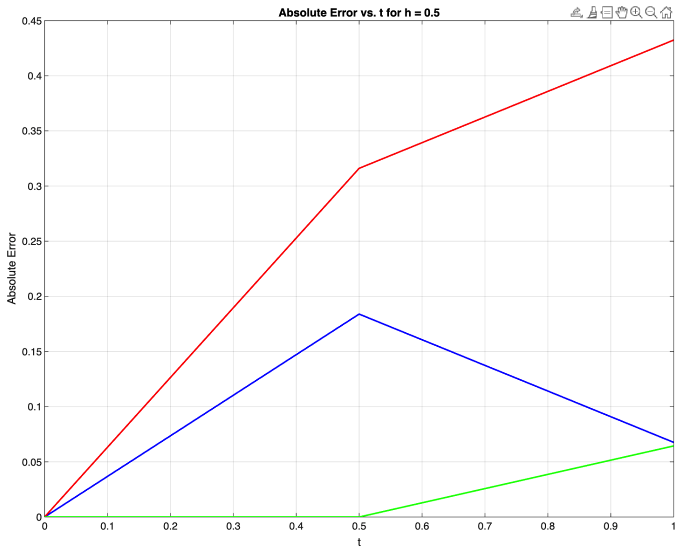
For Central Difference
t        | Actual Value | Approximate Value | Absolute Error
  0.0000 |     1.000000 |          1.000000 |       0.000000
  0.1000 |     0.909365 |          0.909365 |       0.000000
  0.2000 |     0.835160 |          0.836254 |       0.001094
  0.3000 |     0.774406 |          0.774864 |       0.000458
  0.4000 |     0.724664 |          0.726308 |       0.001644
  0.5000 |     0.683940 |          0.684341 |       0.000401
  0.6000 |     0.650597 |          0.652572 |       0.001975
  0.7000 |     0.623298 |          0.623312 |       0.000013
  0.8000 |     0.600948 |          0.603247 |       0.002299
  0.9000 |     0.582649 |          0.582013 |       0.000637
  1.0000 |     0.567668 |          0.570442 |       0.002775
>>
```

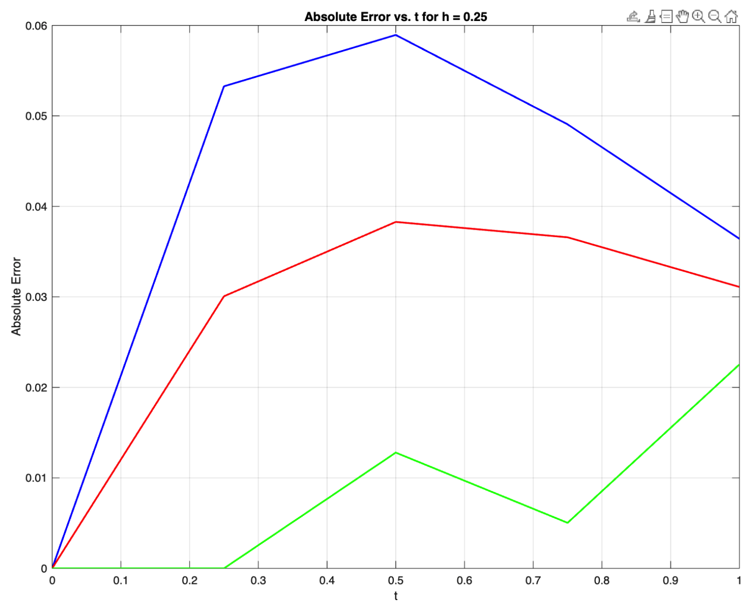
We can analyse the stability by observing the plots of approximate value error vs nodal points. We check the asymptotic behaviour of plot for different values of h, to check whether it increases exponentially or not.

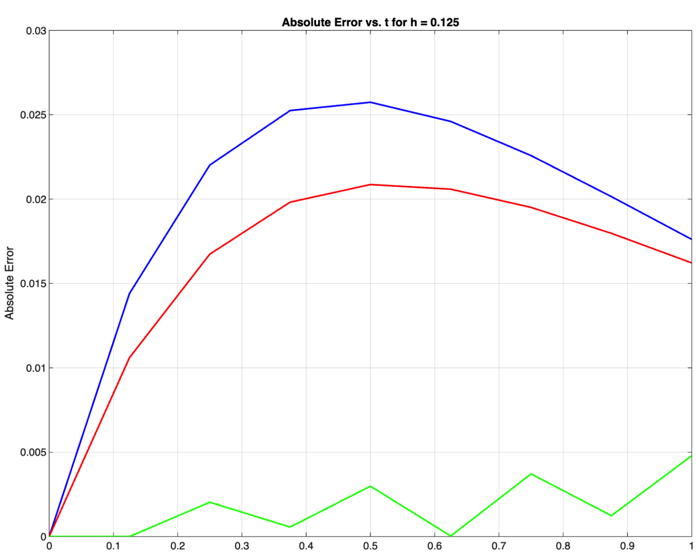
Blue line denotes Explicit Euler

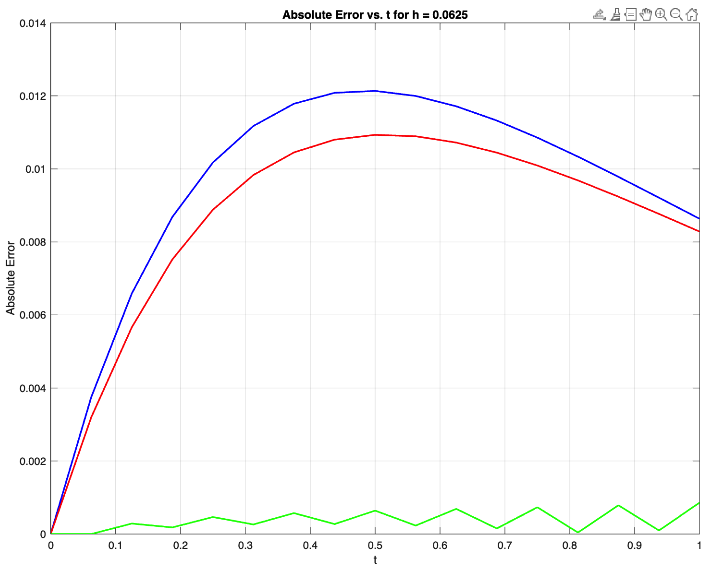
Red line denotes Implicit Euler

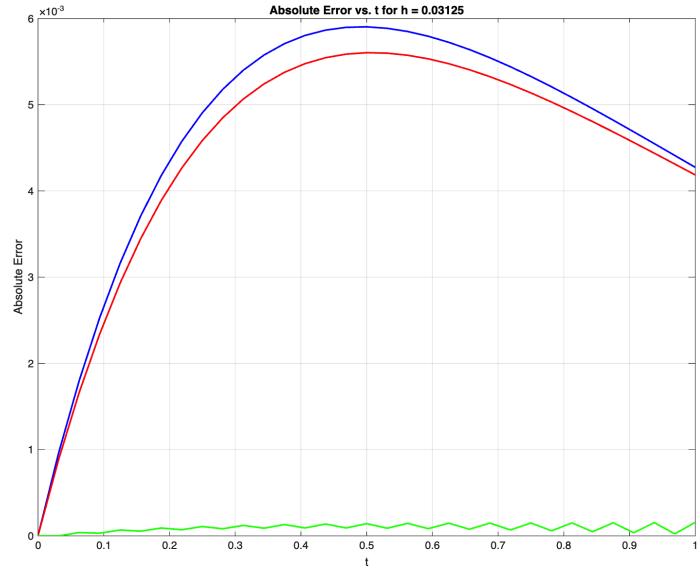
Green line denot central difference

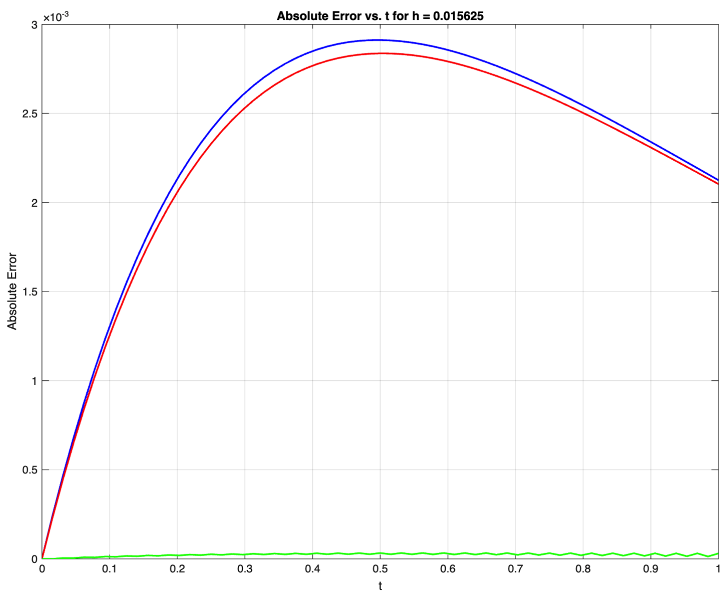

Absolute Error vs. t for h = 0.5
Absolute Error
t
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

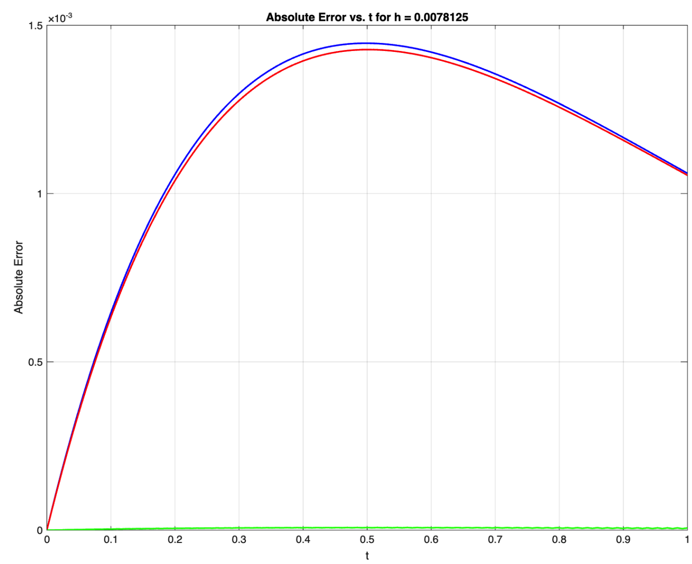

Absolute Error vs. t for h = 0.25
Absolute Error
t
0 0.01 0.02 0.03 0.04 0.05 0.06
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

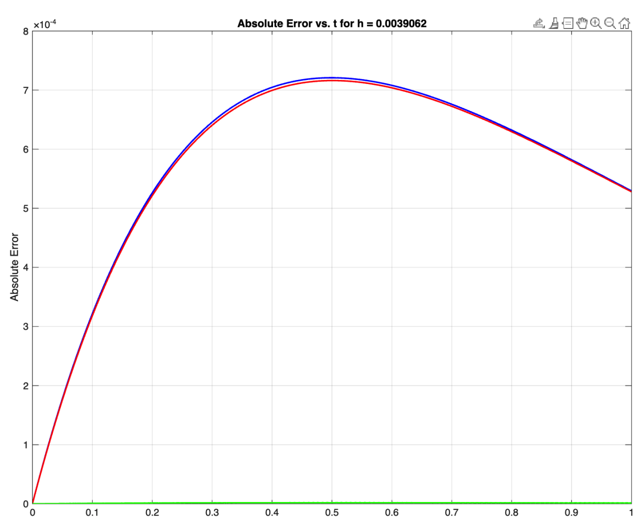

Absolute Error vs. t for h = 0.125
Absolute Error
t
0 0.005 0.01 0.015 0.02 0.025 0.03
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Absolute Error vs. t for h = 0.0625
Absolute Error
t


Absolute Error vs. t for h = 0.03125
Absolute Error
t


Absolute Error vs. t for h = 0.015625
Absolute Error
t


Absolute Error vs. t for h = 0.0078125
Absolute Error
t


Absolute Error vs. t for h = 0.0039062
Absolute Error
t

The Explicit Euler's method is conditionally stable for the IVP of the form y' = c*y, y(0) = y0 with the stability region of step size being the interval (0,2/|c|).

The given solution can be split into a homogeneous solution as well as a par□cular solu□on and for the homogeneous problem we have the constant c = 2.

So the region of stability for the Explicit Euler Scheme is the interval (0,2/2) which is (0,1)

The Implicit Euler's method is unconditionally stable for the IVP of the form y' = c*y, y(0) = y0 which means the region of stability for the step size h = (0,inf)

The Central Difference method is of order 2 and there is an extraneous root that is associated when we solve the IVP, which is oscillatory in nature and its amplitude is also increasing. The extraneous term is of the form $(-1)_n$ exp(2*tn) .

We can easily tell that the central difference method is unstable, while the Euler method, both, are stable as error does not increase exponentially in later case.

Question 3

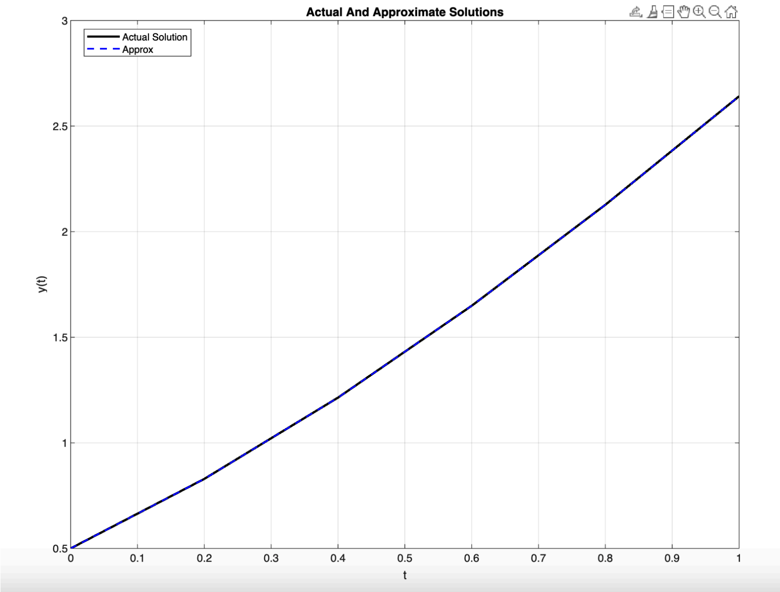
3. Consider the IVP

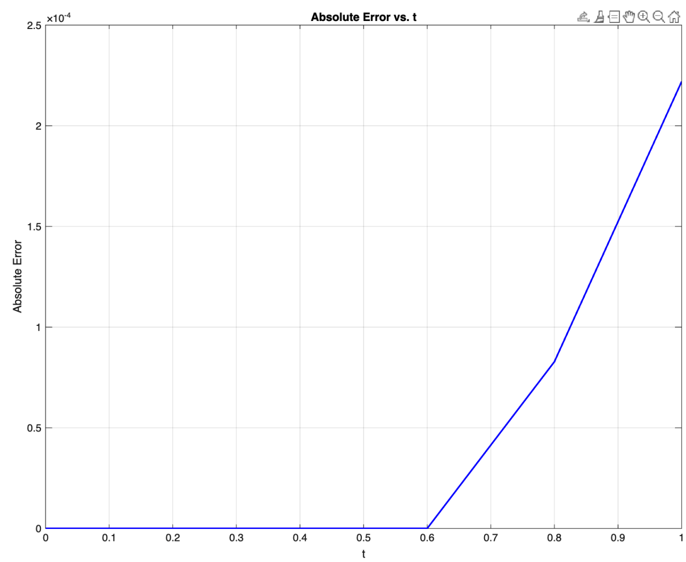
$$y' = y - t^2 + 1,\ 0 \le t \le 1,\ y(0) = 0.5.$$

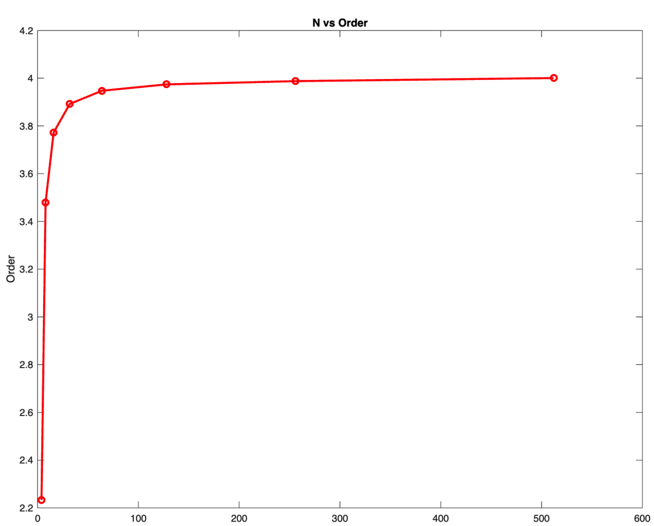
Use the exact solution $y(t) = (t+1)^2 - 0.5e^t$ to get the starting values and $h = 0.2$ to compare the approximations got by implementing the explicit Adams-Bashforth four-step method and the implicit Adams-Moulton three-step method.

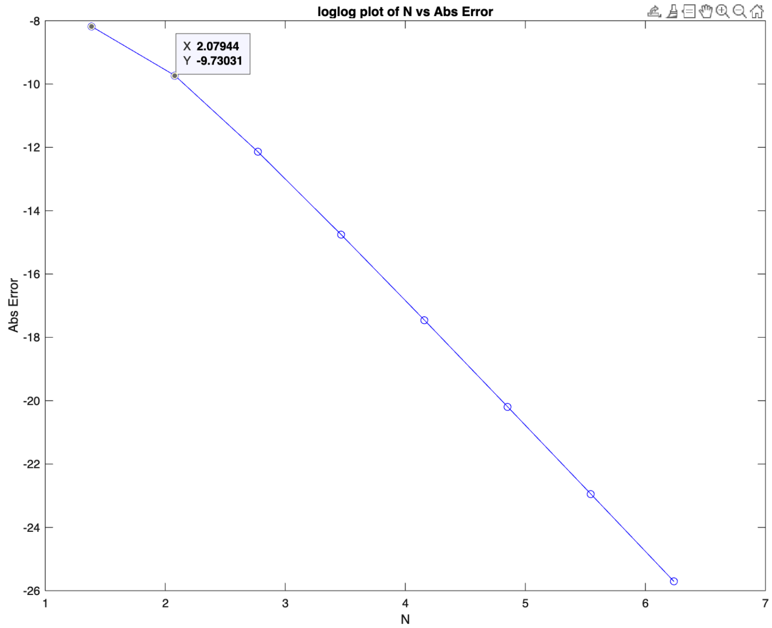
Adams - Bashforth 4 step

```
>> Lab12Q3
t         | Actual Value | Approximate Value | Absolute Error
   0.0000 |     0.500000 |          0.500000 |       0.000000
   0.2000 |     0.829299 |          0.829299 |       0.000000
   0.4000 |     1.214088 |          1.214088 |       0.000000
   0.6000 |     1.648941 |          1.648941 |       0.000000
   0.8000 |     2.127230 |          2.127312 |       0.000083
   1.0000 |     2.640859 |          2.641081 |       0.000222
```


Actual And Approximate Solutions
Actual Solution
Approx
y(t)
t
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

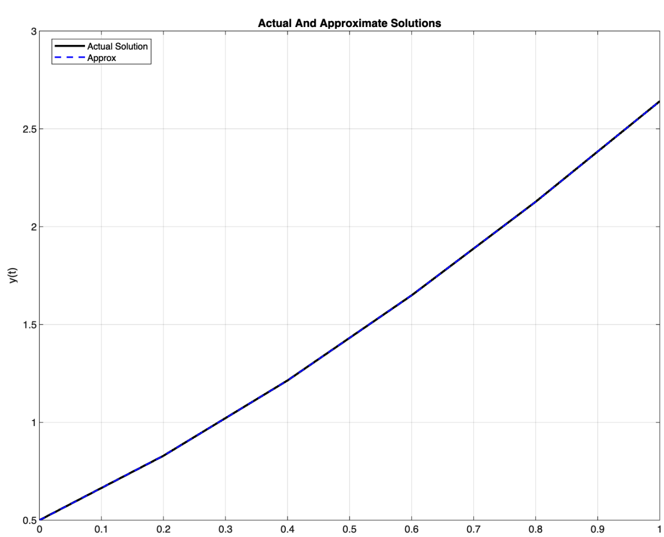

Absolute Error vs. t
×10⁻⁴
Absolute Error
t
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

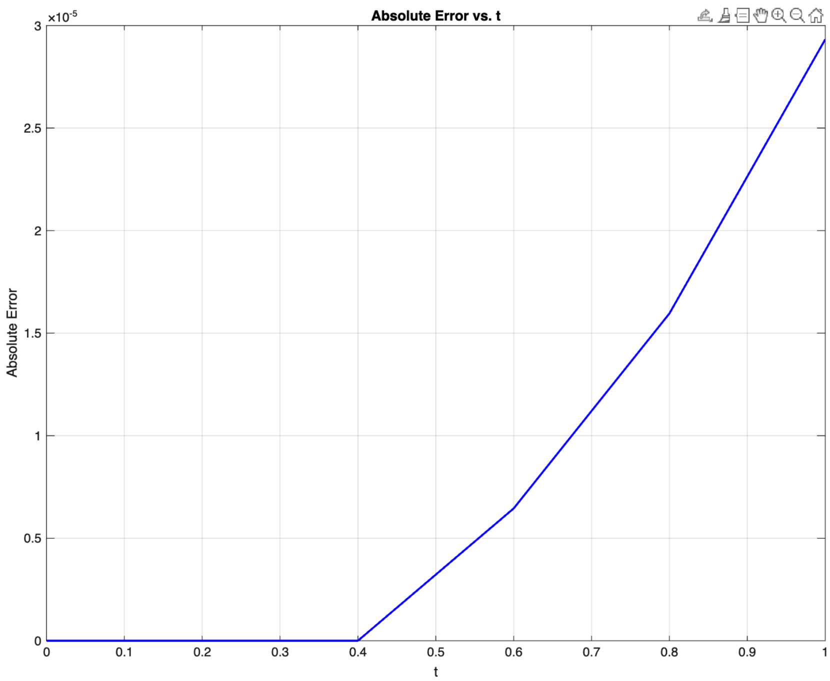

N vs Order
Order
N
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
0
100
200
300
400
500
600

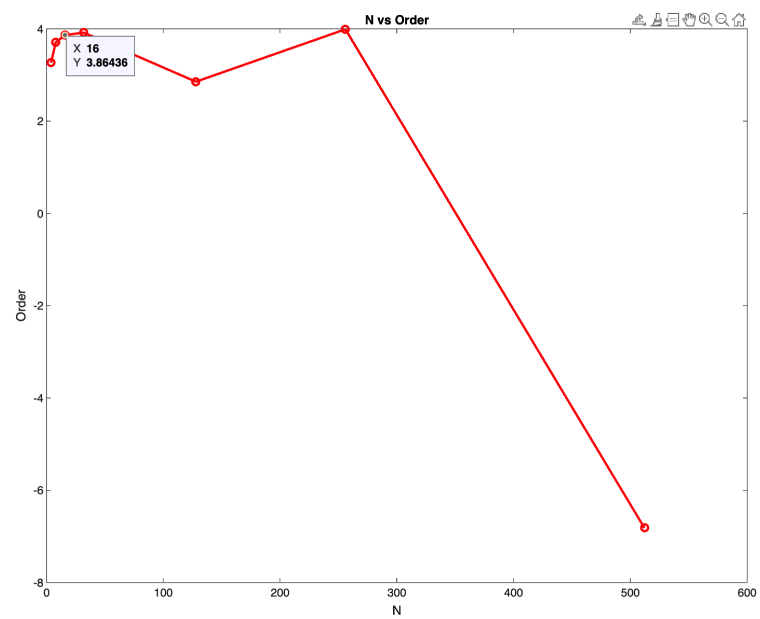


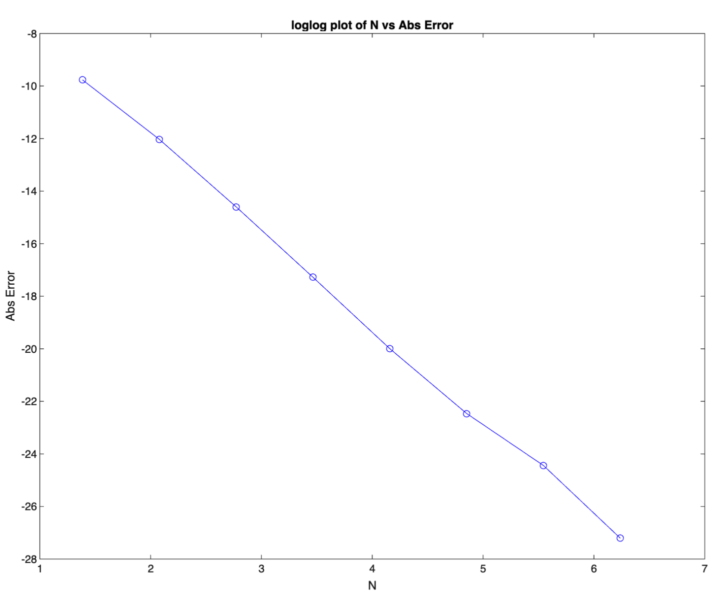
Adams - Moulton 3-step

t	Actual Value	Approximate Value	Absolute Error
0.0000	0.500000	0.500000	0.000000
0.2000	0.829299	0.829299	0.000000
0.4000	1.214088	1.214088	0.000000
0.6000	1.648941	1.648934	0.000006
0.8000	2.127230	2.127214	0.000016
1.0000	2.640859	2.640830	0.000029




Absolute Error vs. t
×10^-5
Absolute Error
t

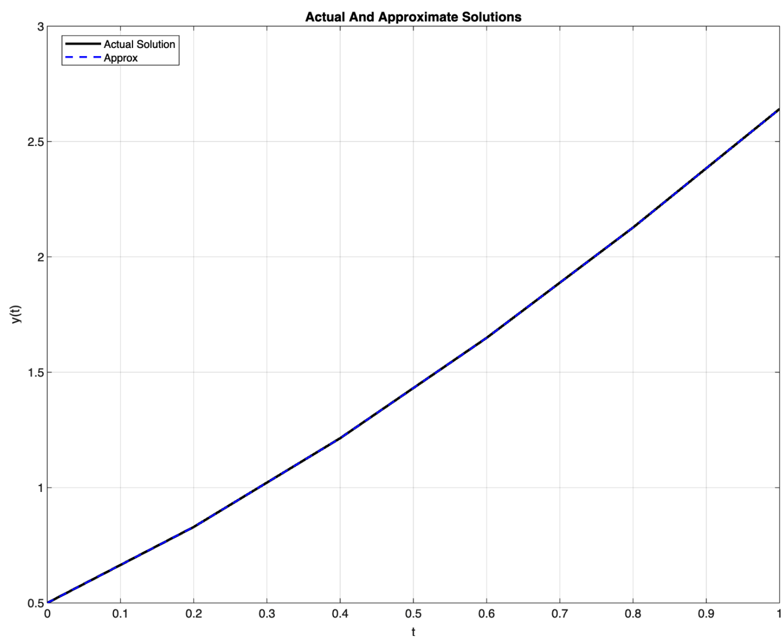

N vs Order
X 16
Y 3.86436
Order
N

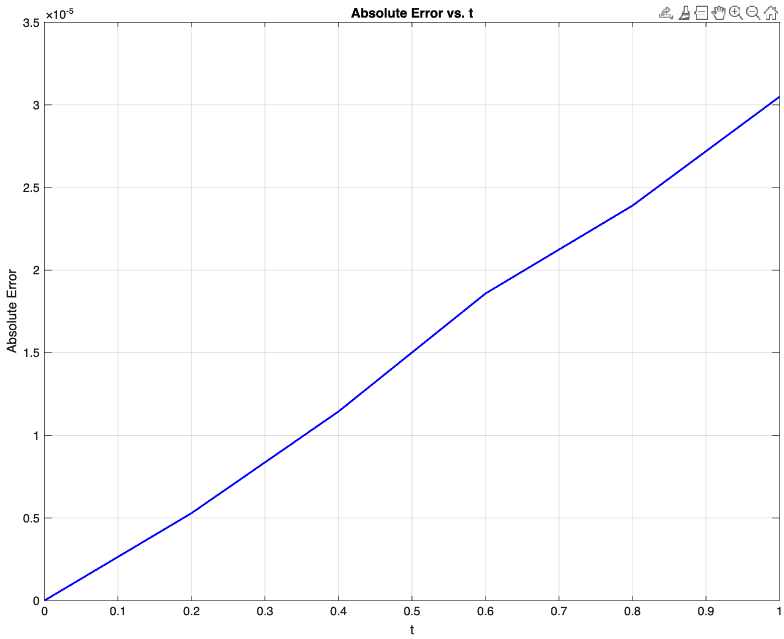

loglog plot of N vs Abs Error
Abs Error
N

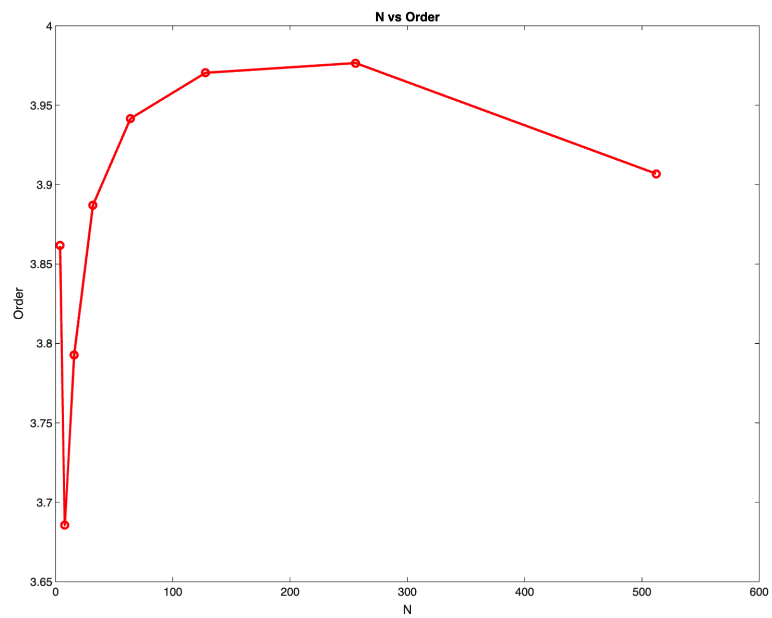
Question 4

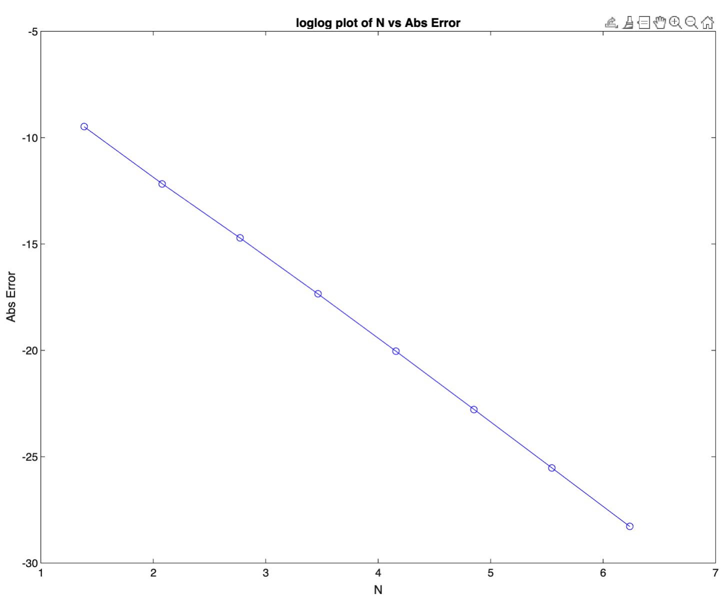
4. Apply the Adams fourth-order predictor-corrector method with $h = 0.2$ and starting values from the Runge-Kutta fourth order method to the IVP given in Question 3.

```
>> Lab12Q4
t          | Actual Value | Approximate Value | Absolute Error
  0.0000 |      0.500000 |           0.500000 |        0.000000
  0.2000 |      0.829299 |           0.829293 |        0.000005
  0.4000 |      1.214088 |           1.214076 |        0.000011
  0.6000 |      1.648941 |           1.648922 |        0.000019
  0.8000 |      2.127230 |           2.127206 |        0.000024
  1.0000 |      2.640859 |           2.640829 |        0.000030
>>
```




Absolute Error vs. t
Absolute Error
t
×10^-5


N vs Order
Order
N


loglog plot of N vs Abs Error
Abs Error
N