

MDL ASSIGNMENT 3 PART 1

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Variables Used

Roll Numbers: 2019101120 and 2019111030

$$x = 1 - ((1030\%30) + 1)/100 = 0.89 \quad y = (30\%4) + 1 = 3$$

Observation Probabilities:

Observation	Probability
P (Observation = RED State = RED)	0.85
P (Observation = GREEN State = GREEN)	0.9

Initial States: S_1, S_3, S_6 with equal probability

Calculation

Initial Belief State: $[\frac{1}{3}, 0, \frac{1}{3}, 0, 0, \frac{1}{3}]$

Step 1: Right with Observation Green

$$P(\text{Green}|\text{Right}) = (0.89 * 0.9 + 0.11 * 0.15) * \frac{1}{3} + (0.89 * 0.9 + 0.11 * 0.9) * \frac{1}{3} + (0.89 * 0.15 + 0.11 * 0.9) * \frac{1}{3} = 65$$

From S_1 , going right can lead to S_1 or S_2 .

$$P(S_1) = 0.11, P(S_2) = 0.89$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$P(\text{Green}|\text{Right}) = 0.89 * 0.9 + 0.11 * 0.15 = 0.8175$$

$$b(s_1) = \frac{0.11 * 0.15}{0.65} * \frac{1}{3} \approx 0.008461538461538461$$

$$b(s_2) = \frac{0.89 * 0.90}{0.65} * \frac{1}{3} \approx 0.4107692307692308$$

From S_3 , going right can lead to S_4 or S_2 .

$$P(S_2) = 0.11, P(S_4) = 0.89$$

$$P(\text{Green}|\text{Green}) = 0.9$$

$$b(s_2) = \frac{0.11*0.90}{0.65} * \frac{1}{3} \approx 0.05076923076923077$$

$$b(s_4) = \frac{0.89*0.90}{0.65} * \frac{1}{3} \approx 0.4107692307692308$$

From S_6 going right can lead to S_6 or S_5

$$P(S_5) = 0.11, P(S_6) = 0.89$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$b(S_5) = \frac{0.11*0.9}{0.65} * \frac{1}{3} \approx 0.05076923076923077$$

$$b(S_6) = \frac{0.89*0.15}{0.65} * \frac{1}{3} \approx 0.06846153846153846$$

Belief State after Step 1: [0.008461538461538461, 0.46153846153846156, 0, 0.4107692307692308, 0.05076923076923077, 0.06846153846153846]

Step 2: Left with Observation Red

$$P(\text{Red}|\text{Left}) = (0.89 * 0.85 + 0.11 * 0.1) * 0.008461538461538461 + (0.89 * 0.85 + 0.11 * 0.85) * 0.46153846153846156 + (0.89 * 0.85 + 0.11 * 0.1) * 0.4107692307692308 + (0.89 * 0.1 + 0.11 * 0.85) * 0.05076923076923077 + (0.89 * 0.1 + 0.11 * 0.85) * 0.06846153846153846 = 0.735826923076923$$

From S_1 going left can lead to S_1 or S_2

$$P(S_1) = 0.89, P(S_2) = 0.11$$

$$P(\text{Red}|\text{Red}) = 0.85, P(\text{Red}|\text{Green}) = 0.1$$

$$b(S_1) = \frac{0.89*0.85}{0.735826923076923} * 0.008461538461538461 \approx 0.00869926560907404$$

$$b(S_2) = \frac{0.11*0.1}{0.735826923076923} * 0.008461538461538461 \approx 0.000126492956642187$$

From S_2 going left can lead to S_1 or S_3

$$P(S_1) = 0.89, P(S_3) = 0.11$$

$$P(\text{Red}|\text{Red}) = 0.85, P(\text{Red}|\text{Green}) = 0.1$$

$$b(S_1) = \frac{0.89*0.85}{0.735826923076923} * 0.46153846153846156 \approx 0.47450539685858406$$

$$b(S_3) = \frac{0.11*0.85}{0.735826923076923} * 0.46153846153846156 \approx 0.058646734443195785$$

From S_4 going left can lead to S_3 or S_5

$$P(S_3) = 0.89, P(S_5) = 0.11$$

$$P(\text{Red}|\text{Red}) = 0.85, P(\text{Red}|\text{Green}) = 0.1$$

$$b(S_3) = \frac{0.89 \cdot 0.85}{0.735826923076923} * 0.4107692307692308 \approx 0.42230980320413986$$

$$b(S_5) = \frac{0.11 \cdot 0.1}{0.735826923076923} * 0.4107692307692308 \approx 0.006140658076993441$$

From S_5 , going left can lead to S_4 and S_6

$$P(S_4) = 0.89, P(S_6) = 0.11$$

$$P(\text{Red} | \text{Red}) = 0.85, P(\text{Red} | \text{Green}) = 0.1$$

$$b(S_4) = \frac{0.89 \cdot 0.1}{0.735826923076923} * 0.05076923076923077 \approx 0.006140658076993441$$

$$b(S_6) = \frac{0.11 \cdot 0.85}{0.735826923076923} * 0.05076923076923077 \approx 0.006451140788751536$$

From S_6 , going left can lead to S_5 or S_6

$$P(S_5) = 0.89, P(S_6) = 0.11$$

$$P(\text{Red}|\text{Red}) = 0.85, P(\text{Red}|\text{Green}) = 0.1$$

$$b(S_5) = \frac{0.89 \cdot 0.1}{0.735826923076923} * 0.06846153846153846 \approx 0.008280584376551763$$

$$b(S_6) = \frac{0.11 \cdot 0.85}{0.735826923076923} * 0.06846153846153846 \approx 0.008699265609074042$$

Belief State after Step 2: [0.4832046624676581, 0.000126492956642187, 0.48095653764733565, 0.006140658076993441, 0.014421242453545204, 0.015150406397825578]

Step 3: Left with Observation Green

$$\begin{aligned} P(\text{Green} | \text{Left}) &= (0.89 \cdot 0.15 + 0.11 \cdot 0.90) * 0.4832046624676581 + (0.89 \cdot 0.15 + 0.11 \cdot 0.15) * \\ &0.000126492956642187 + (0.89 \cdot 0.90 + 0.11 \cdot 0.90) * 0.48095653764733565 + (0.89 \cdot 0.15 + \\ &0.11 \cdot 0.90) * 0.006140658076993441 + (0.89 \cdot 0.90 + 0.11 \cdot 0.15) * 0.014421242453545204 + \\ &(0.89 \cdot 0.90 + 0.11 \cdot 0.15) * 0.015150406397825578 = 0.5708274677887255 \end{aligned}$$

From S_1 going left can lead to S_1 or S_2

$$P(S_1) = 0.89, P(S_2) = 0.11$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$b(S_1) = \frac{0.89 \cdot 0.15}{0.5708274677887255} * 0.4832046624676581 \approx 0.11300756547214362$$

$$b(S_2) = \frac{0.11 \cdot 0.90}{0.5708274677887255} * 0.4832046624676581 \approx 0.08380336315911775$$

From S_2 going left can lead to S_1 or S_3

$$P(S_1) = 0.89, P(S_3) = 0.11$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$b(S_1) = \frac{0.89*0.15}{0.5708274677887255} * 0.000126492956642187 \approx 0.00002958303631944724$$

$$b(S_3) = \frac{0.11*0.15}{0.5708274677887255} * 0.000126492956642187 \approx 0.0000036563303316170746$$

From S_3 going left can lead to S_2 or S_4

$$P(S_2) = 0.89, P(S_4) = 0.11$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$b(S_2) = \frac{0.89*0.90}{0.5708274677887255} * 0.48095653764733565 \approx 0.6748907654144336$$

$$b(S_4) = \frac{0.11*0.90}{0.5708274677887255} * 0.48095653764733565 \approx 0.08341346538830077$$

From S_4 going left can lead to S_3 or S_5

$$P(S_3) = 0.89, P(S_5) = 0.11$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$b(S_3) = \frac{0.89*0.15}{0.5708274677887255} * 0.006140658076993441 \approx 0.0014361219449622567$$

$$b(S_5) = \frac{0.11*0.90}{0.5708274677887255} * 0.006140658076993441 \approx 0.0010649893075001006$$

From S_5 going left can lead to S_4 or S_6

$$P(S_4) = 0.89, P(S_6) = 0.11$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$b(S_4) = \frac{0.89*0.90}{0.5708274677887255} * 0.014421242453545204 \approx 0.020236263769922706$$

$$b(S_6) = \frac{0.11*0.15}{0.5708274677887255} * 0.014421242453545204 \approx 0.0004168518754103928$$

From S_6 going left can lead to S_5 or S_6

$$P(S_5) = 0.89, P(S_6) = 0.11$$

$$P(\text{Green}|\text{Green}) = 0.9, P(\text{Green}|\text{Red}) = 0.15$$

$$b(S_5) = \frac{0.89*0.90}{0.5708274677887255} * 0.015150406397825578 \approx 0.021259445645930037$$

$$b(S_6) = \frac{0.11*0.15}{0.5708274677887255} * 0.015150406397825578 \approx 0.00043792865562777226$$

Belief State after Step 3: [0.11303714850846307, 0.7586941285735513, 0.0014397782752938737, 0.10364972915822349, 0.022324434953430138,

0.0008547805310381651]