International Baccalaureate Math HL

TITLE: CIRCUMFERENCE OF THE ORBIT OF THE MOON

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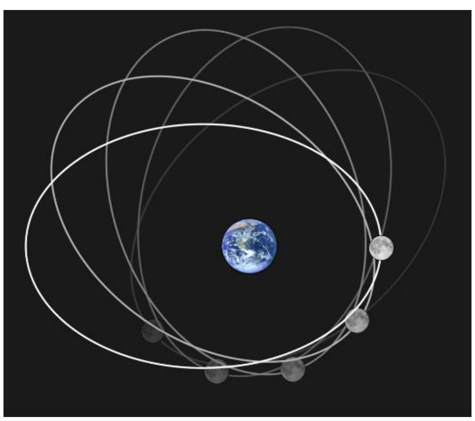
Introduction

I have always been immensely interested in astronomy. I thought of various topics for example- the mapping of constellations, calculating the distances of the nearest galaxies but one topic stuck out to me the most. Some time ago I was reading about the orbit of the moon and I got to know that the orbit's position and circumference changes on a daily basis. This made me wonder whether I could use this mathematics exploration to find the circumference of the orbit of the moon even with these changes. Many of my peers thought it would not be possible to do so, however, I didn't give up.

Terms we need to know

1. Apogee: Farthest distance of the moon from the earth.[1]
2. Perigee: Shortest distance of the moon from the earth.[1]
3. Eccentricity of the moon: Deviation of a curve or orbit from circularity.[2]

The moon changing



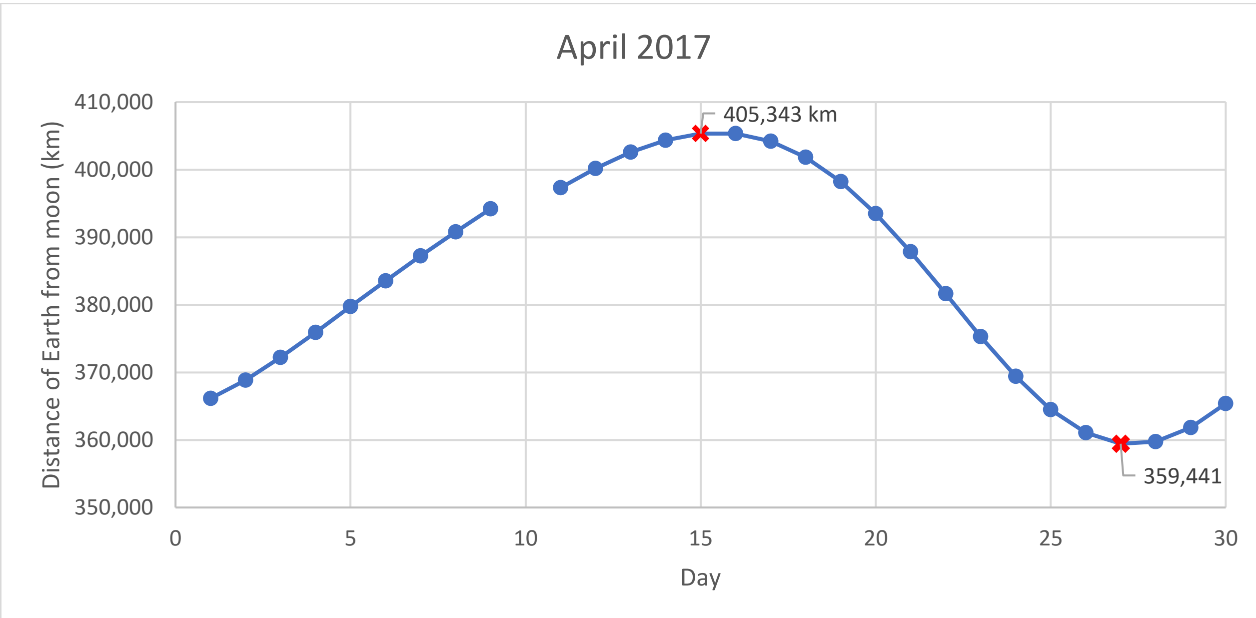
This is known as apsidal precession. This is when the orbit never stays in the same position and while the moon itself rotates so does the orbit. This causes various changes to the distance of the moon from the earth.

[3]

[1] https://en.wikipedia.org/wiki/Orbit_of_the_Moon
[2] Google Dictionary
[3] https://en.wikipedia.org/wiki/Orbit_of_the_Moon#/media/File:Moon_apsidal_precession.png

Readings

I will be using the month of April in the year 2017 for the calculations.[4]



Using these distances which were calculated from satellite readings I will be finding the circumference of the moon.

Firstly, I will take out the apogee and perigee.

The red crosses on the graph above signify the apogee and perigee.

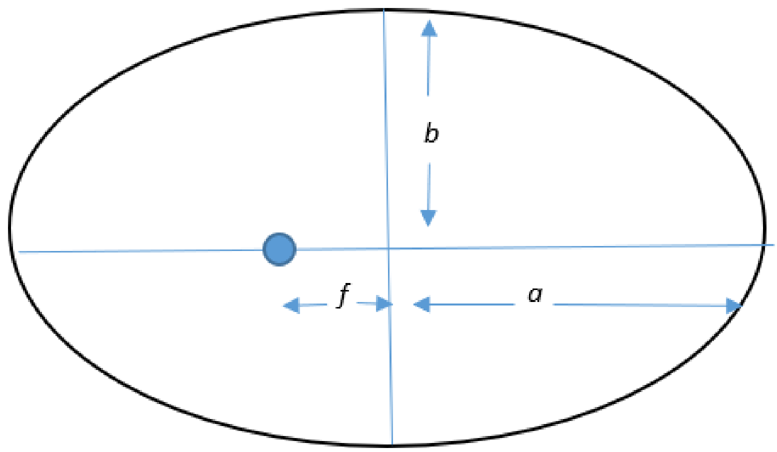
Apogee in April 2017: 405343 km

Perigee in April 2017: 359441 km

[4] https://www.timeanddate.com/moon/india/new-delhi?month=4&year=2017

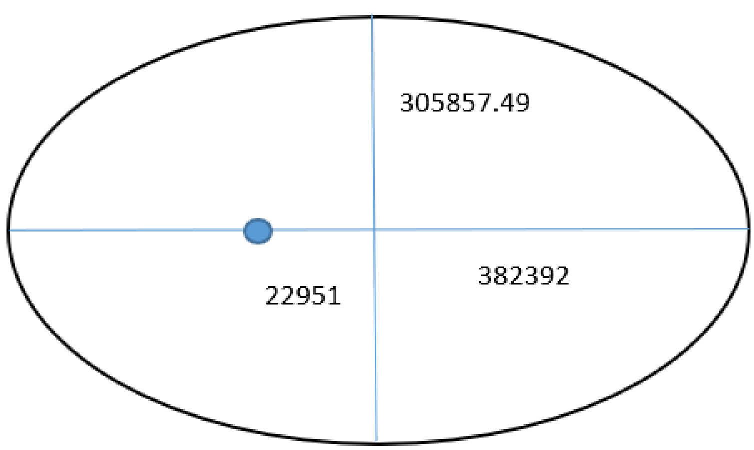
Eccentricity[5]

To find the circumference I first need to find the values of *a* and *b* using eccentricity.



e=$\frac{f}{a}$ (e=eccentricity)

e=$\sqrt{1-\frac{b^2}{a^2}}$



[5] http://slideplayer.com/slide/10089366/

So first I will find the value of a.

a= $\frac{apogee+perigee}{2}$

a=$\frac{359441+405343}{2}$

a=382392 km

f= Apogee- a

=405343-382392

=22951

e=$\frac{f}{a}$

e=$\frac{382392}{22951}$

e≈0.6002

Using the formula e=$\sqrt{1-\frac{b^2}{a^2}}$, we will find the value of b.

0.6002=$\sqrt{1-\frac{b^2}{382392^2}}$

b≈305857.4857

Thus, by doing so I have found the values of the *a* and *b*. This same process can be done for any month of the year.

Circumference of the moon

Ramanujan's formula[6]

Srinivasa Ramanujan created a formula which enables us to find the perimeter of an ellipse. I will be using this formula to find out the circumference of the orbit of the moon.

The formula[6]-

$$h=\frac{(a-b)^2}{(a+b)^2}$$

$$p\approx \pi(a+b)\left(1+\frac{3h}{10+\sqrt{4-3h}}\right)$$

Plugging in a and b-

$$h=\frac{(382392-305857.49)^2}{(382392+305857.49)^2}$$

$$h\approx \frac{5857531877.7776}{473687354578.53}$$

$$h\approx 0.0124$$

$$p\approx \pi(382392+305857.49)\left(1+\frac{3\times 0.0123658}{10+\sqrt{4-3\times 0.0124}}\right)$$

$$p\approx 2168889.0520 \text{ km}$$

[6] https://www.mathsisfun.com/geometry/ellipse-perimeter.html

Infinite series[7]

This is another way of finding the perimeter of the orbit but this one is more accurate.

$$p = \pi(a+b)\sum_{n=0}^{\infty}\binom{0.5}{n}^2 h^n$$

Using the same value of h from before

h=$\frac{(382392-305857.49)^2}{(382392+305857.49)^2}$

h≈ $\frac{5857531877.7776}{473687354578.53}$

h≈0.0124

$$p = \pi(382392+305857.49)\sum_{n=0}^{\infty}\binom{0.5}{n}^2 0.0124^n$$

We have to check if $p = \pi(382392+305857.49)\sum_{n=0}^{\infty}\binom{0.5}{n}^2 0.0124^n$

converges or not.

First check convergence of $\sum_{n=0}^{\infty}(\frac{0.5!}{n!(0.5-n)!})^2 0.0124^n$

[7] https://www.mathsisfun.com/geometry/ellipse-perimeter.html

$0.5!^2 = .7854$

Opening the brackets: $\sum_{n=0}^{\infty} \frac{0.7854 \times 0.0124^n}{n!^2 (0.5-n)!^2}$

Taking the constant out: $0.7853981635 \sum_{n=0}^{\infty} \frac{0.0124^n}{n!^2 (0.5-n)!^2}$

Now, using Ratio Test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{0.0124^{(n+1)}}{(n+1)!^2 \left(0.5-(n+1)\right)!^2}}{\frac{0.0124^n}{n!^2 (0.5-n)!^2}}\right|$$

Dividing the fractions: $\frac{0.0124^{(n+1)} \times n!^2 (0.5-n)!^2}{(n+1)!^2 \left(0.5-(n+1)\right)!^2 \times 0.0124^n}$

$$= \frac{0.0124 \times n!^2 (0.5-n)!^2}{(n+1)!^2 \left(0.5-(n+1)\right)!^2}$$

$$= \frac{0.0124 \times n!^2 (0.5-n)!^2}{(n+1)!^2 (-n-0.5)!^2}$$

$$\lim_{n\to\infty} \left|\frac{0.0124 \times n!^2 (0.5-n)!^2}{(n+1)!^2 (-n-0.5)!^2}\right| = \frac{79}{6389}$$

$$\frac{79}{6389} < 1$$

According to the ratio test it converges.

Now, checking the convergence of $\sum_{n=0}^{\infty}\left|(\frac{0.5!}{n!(0.5-n)!})^2 0.0124^n\right|$

$=\sum_{n=0}^{\infty}\frac{0.7854e^{-4.3929n}}{n!^2(0.5-1\times n)!^2}$

$$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{e^{-4.3929(n+1)}}{(n+1)!^2(0.5-1\times(n+1))!^2}}{\frac{0.7854e^{-4.3929n}}{n!^2(0.5-1\times n)!^2}}\right|$$

$=\frac{n!^2(-n+0.5)!^2}{e^{4.3929}(n+1)!^2(-(n+1)+0.5)!^2}$

$=0.124\ \frac{n!^2(-n+0.5)!^2}{(n+1)!^2(-(n+1)+0.5)!^2}$

(Note: $\frac{1}{e^{4.39289}}=0.124$)

$=\frac{0.0124n!^2(0.5-n)!^2}{(n+1)!^2(-n-0.5)!^2}$

$$\lim_{n\to\infty}\left|\frac{0.0124n!^2(0.5-n)!^2}{(n+1)!^2(-n-0.5)!^2}\right|=\frac{103}{8330}$$

$\frac{103}{8330}<1$

According to the ratio test it converges.

Now, to solve the equation we expand the infinite series.

$$p = \pi(a+b)(1 + \frac{1}{4}h + \frac{1}{64}h^2 + \frac{1}{256}h^3 + \frac{25}{16384}h^4 + \frac{49}{65536}h^5 + \frac{441}{1048576}h^6)$$

$h \approx 0.0124$

p= 2168889.0555 km

Limitations

An issue with this exploration could be the rounded off values. The values being used will never be 100% accurate since the decimal value is never ending.

Conclusion

Through this exploration I was able to find a concrete formula from which you can find the distance the moon covers, when the moon takes one revolution around the earth, for any month. All you need to know is the apogee and perigee (the farthest and shortest distance respectively) of the moon during that month which you can acquire from satellite readings. The reading will have different values every month due to the way the moon moves. We can do say by both Ramanujan's formula and the infinite series, but, the infinite series will give a more accurate reading.

Bibliography

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