RAYLEIGH-TAYLOR INSTABILITY

ARUSHI SINHA

The Problem

- Overview
- Goals
- Set-Up

Methods

- AB3
- RK4

Stability & Convergence

Literature Comparison

Conclusions & Limitations

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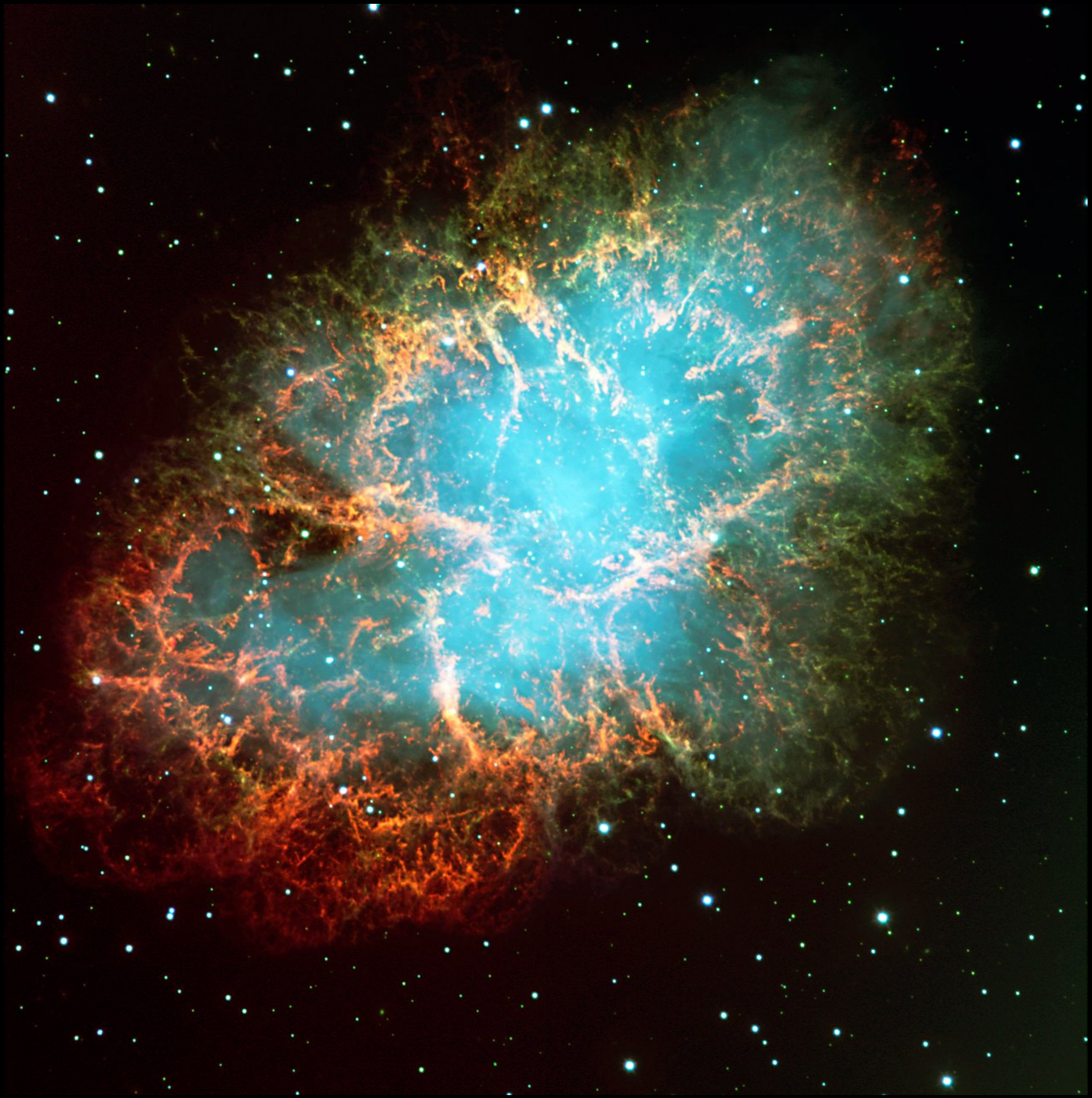
- **Literature Comparison**
- **Conclusions & Limitations**

The R-T Instability

- instability of an interface between two fluids
- stratified due to differing densities
- lighter fluid below displaces heavier fluid on top

- e.g: water above oil; mushroom clouds from volcanic eruptions; supernovae



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The Goal

- simulate R-T instability
  - two methods
  - two resolutions
- make it look like this!

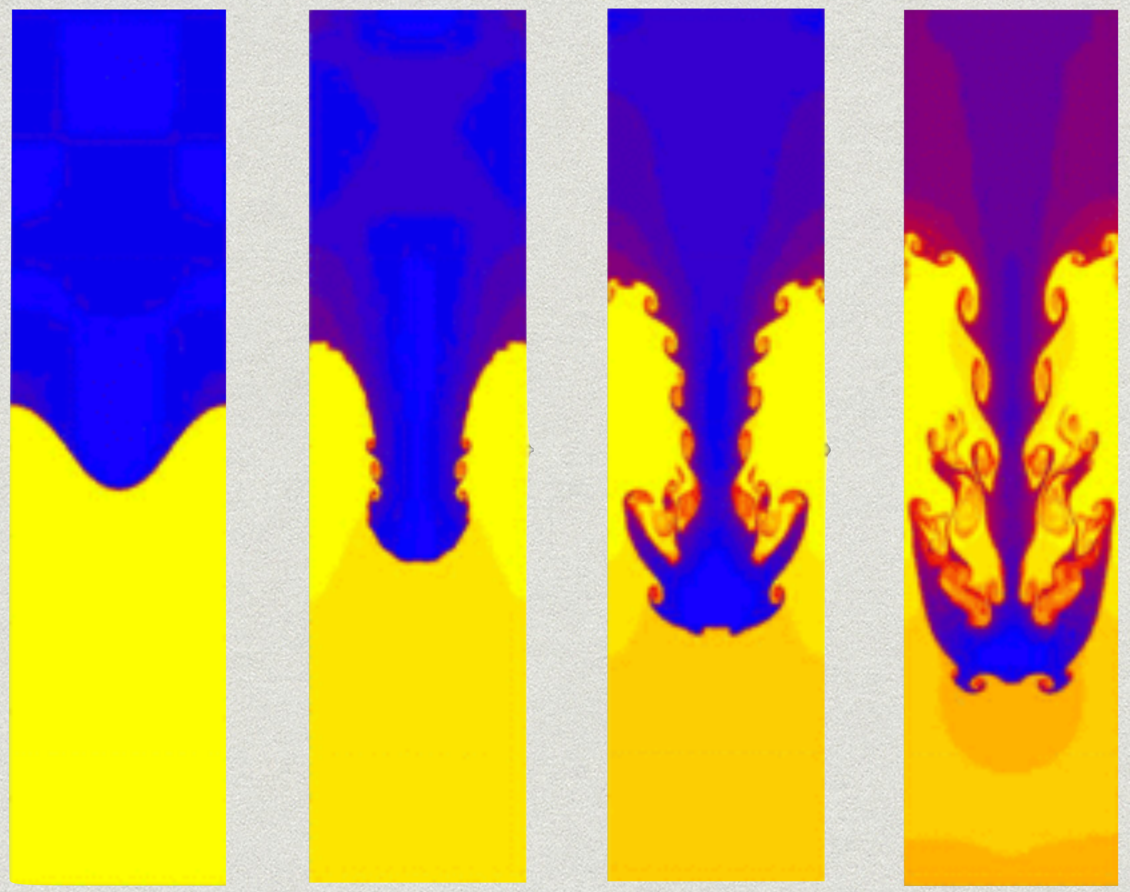
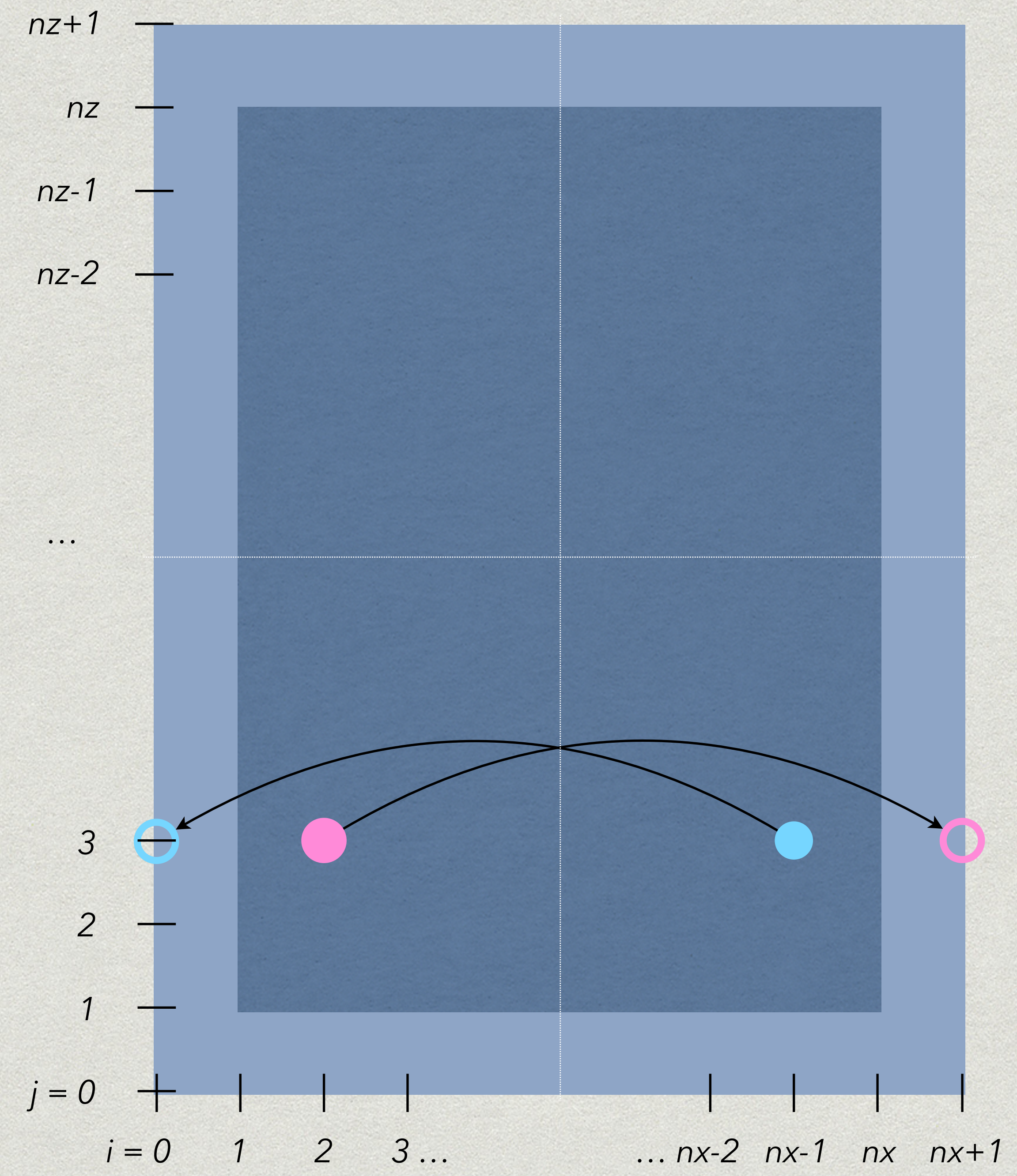


FIGURE 1:
Li, Shengtai & Hui Li. "Parallel AMR Code for Compressible MHD or HD Equations". Los Alamos National Laboratory. Retrieved 2006-09-05

The Set-Up

- z: reflective / wall (Dirichlet BC)
- x: periodic
- cosine perturbation
- graduated Δθ



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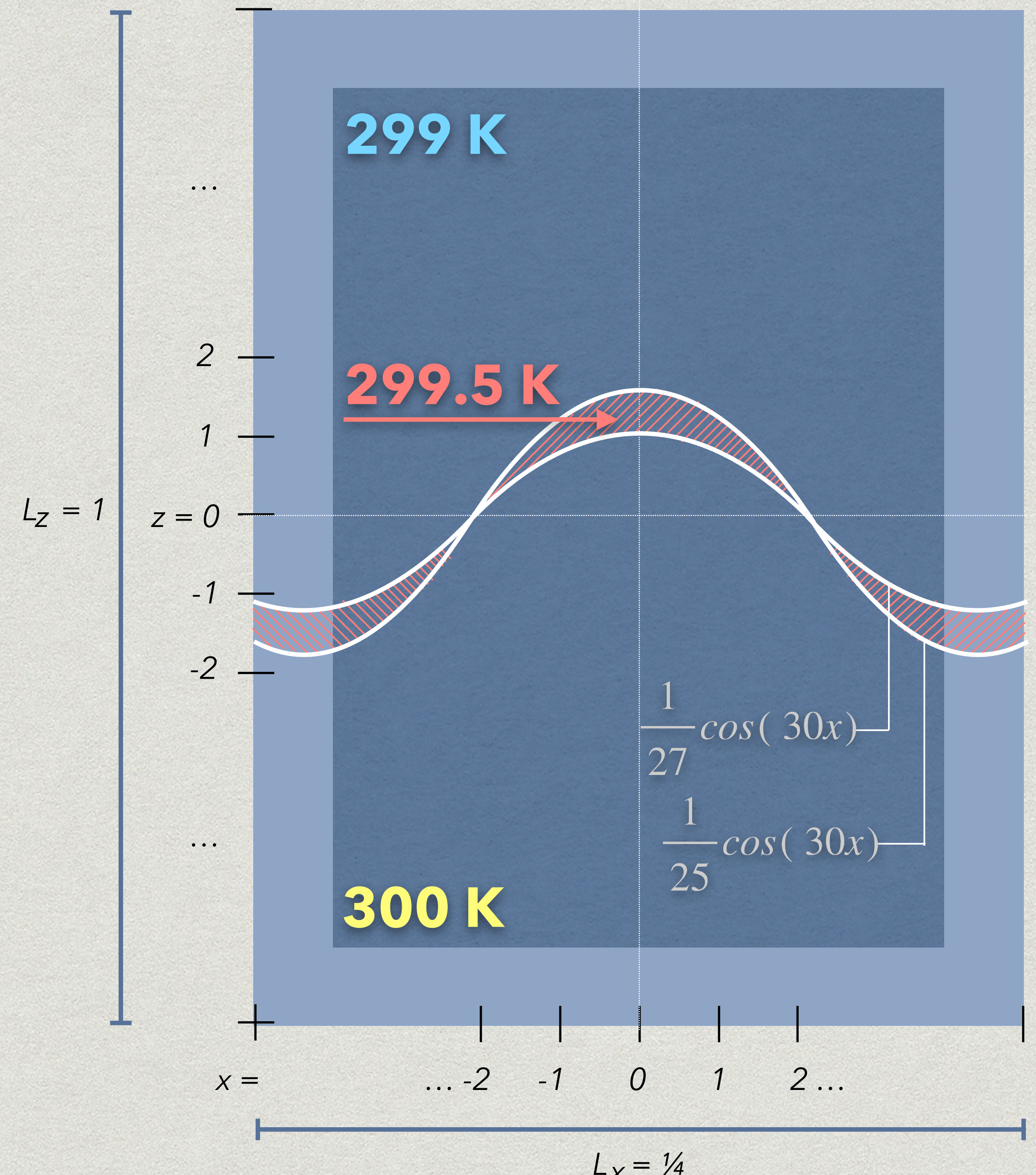

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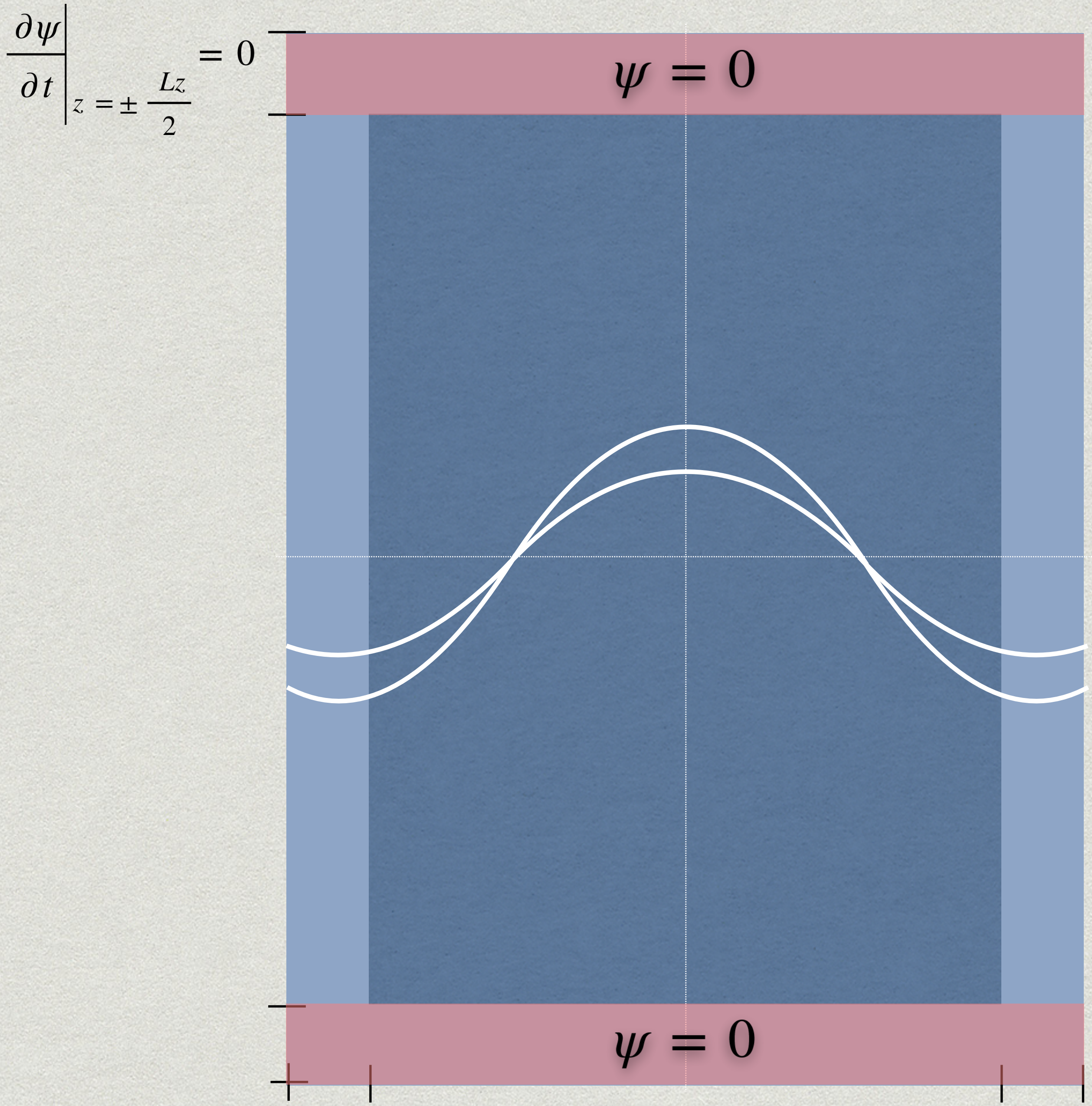

The Set-Up

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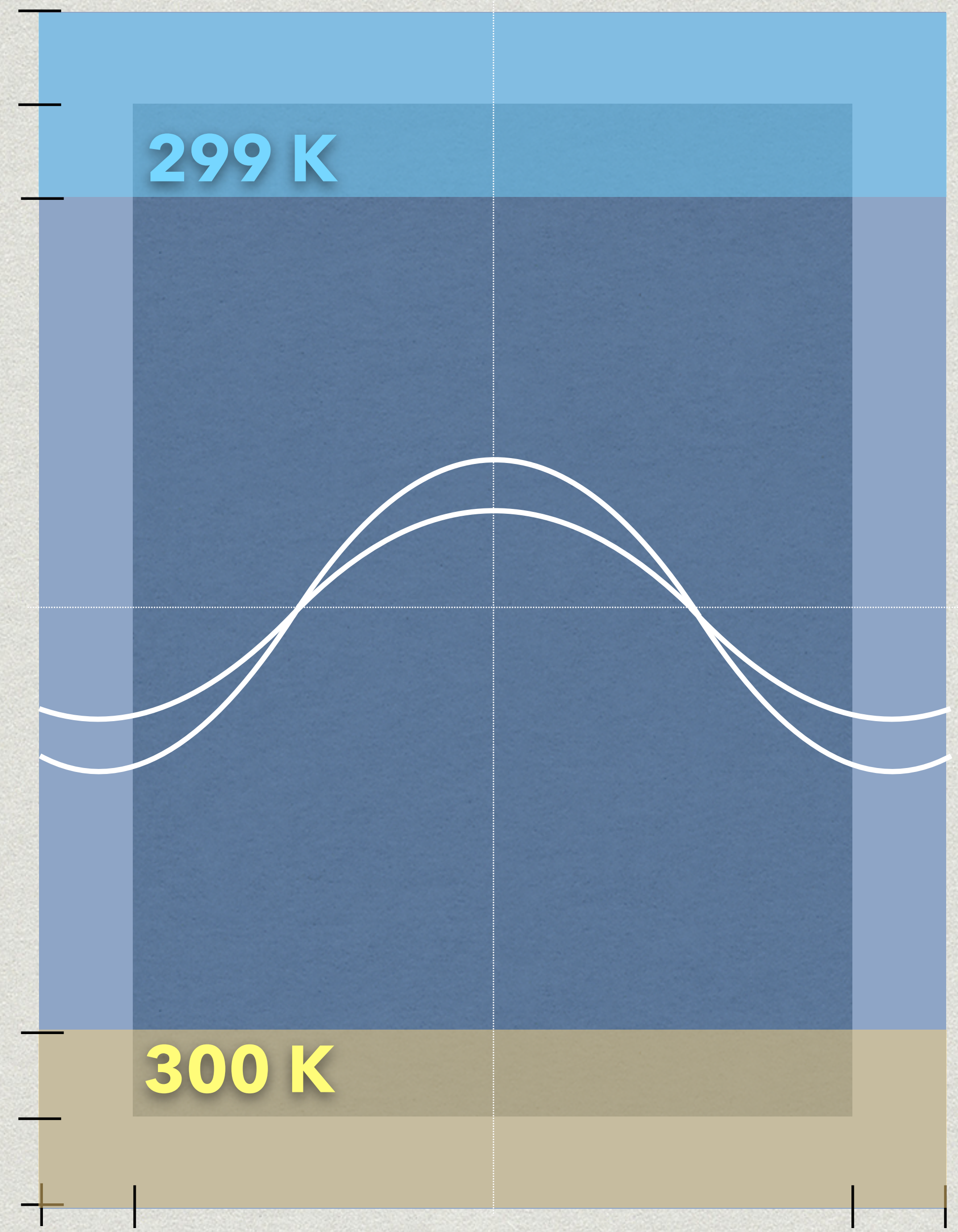

$$\left.\frac{\partial \theta}{\partial z}\right|_{z=\pm\frac{Lz}{2}} = 0$$

Some Constants

- Atwood number << 1
- Prandtl number = 1
  - $\nu = \kappa = 1.58 \times 10^{-6}$
  - $\beta = 207 \times 10^{-6}$ ~ water

BOUSSINESQ APPROXIMATION

$$\frac{\partial \omega}{\partial t} = -J(\psi, \omega) + \beta g \frac{\partial \theta}{\partial x} + \nu \nabla^2 \omega$$

$$\frac{\partial \theta}{\partial t} = -J(\psi, \theta) + \kappa \nabla^2 \theta$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = \omega$$

BOUSSINESQ APPROXIMATION

$$\frac{\partial \omega}{\partial t} = \boxed{-J(\psi, \omega)} + \beta g \frac{\partial \theta}{\partial x} + \boxed{\nu \nabla^2 \omega}$$

$$\frac{\partial \theta}{\partial t} = \boxed{-J(\psi, \theta)} + \boxed{\kappa \nabla^2 \theta}$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2}} = \omega$$

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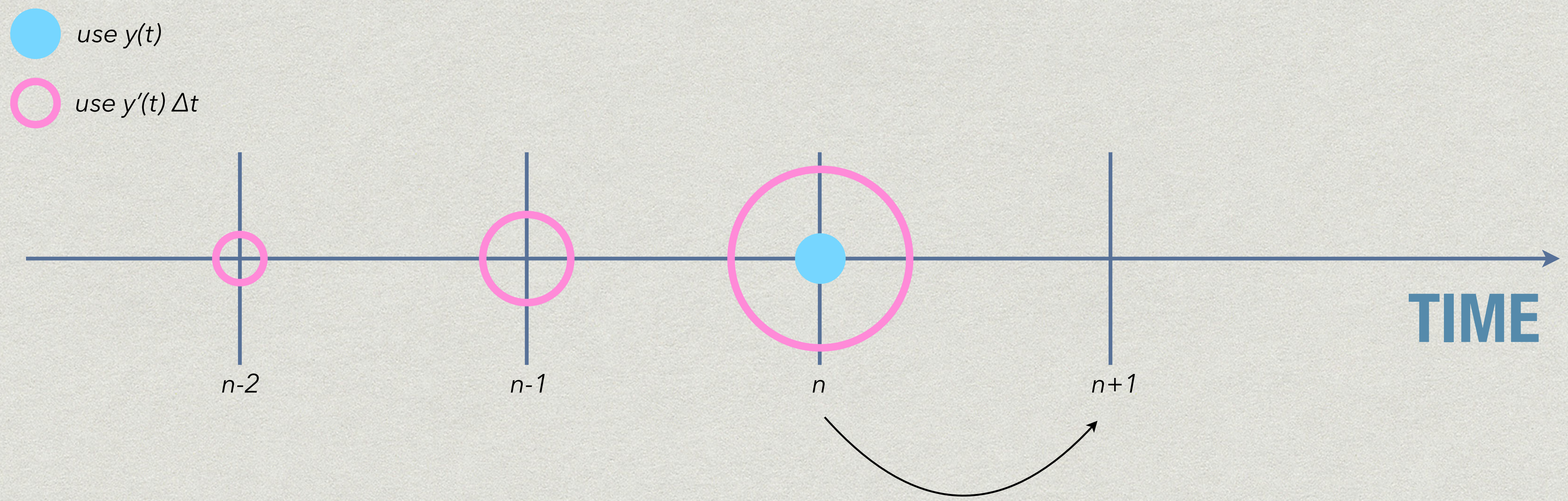
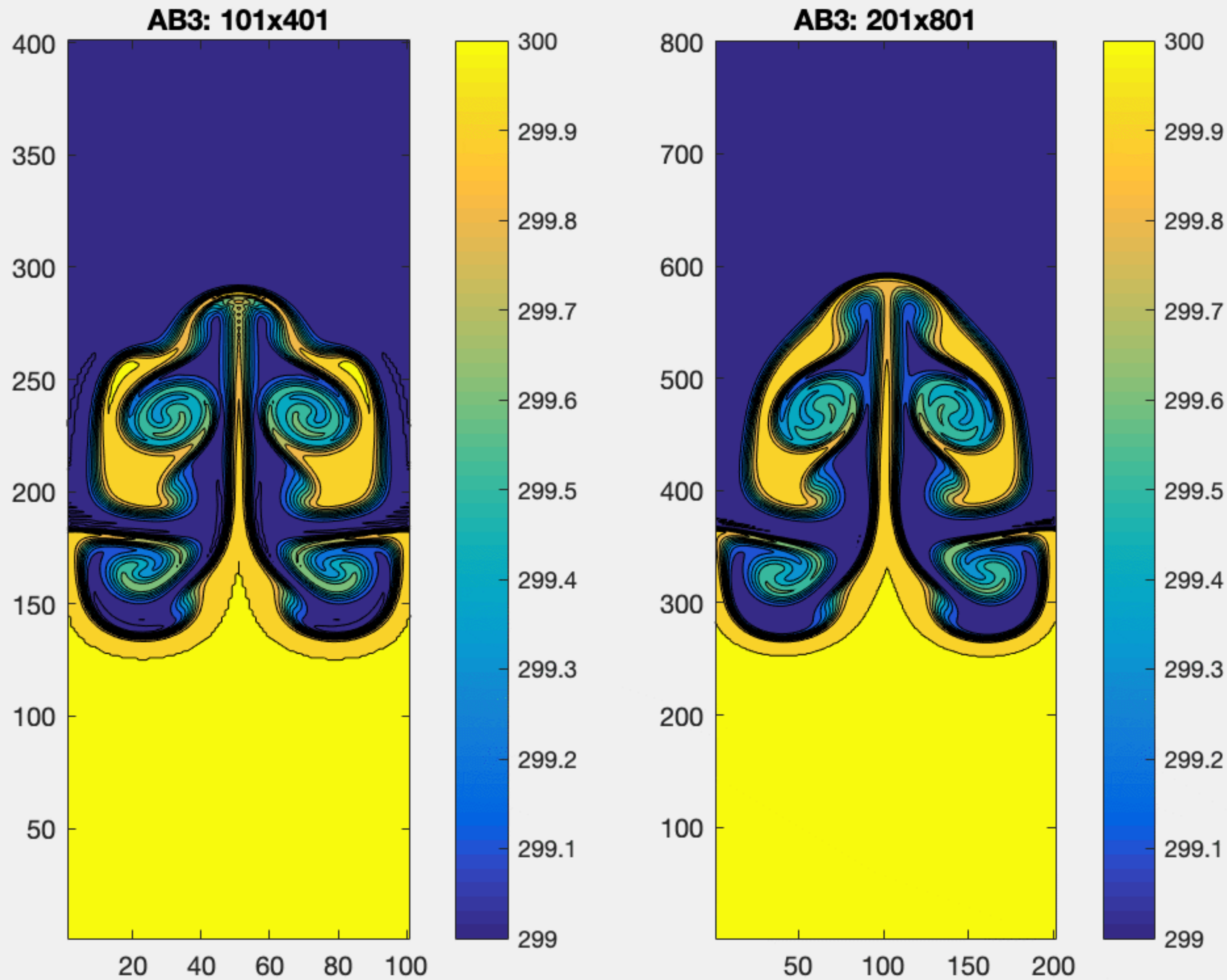
AB3



- $\mathcal{O}(\Delta t^3)$
- computational mode
- $\mathbb{C} < 1$



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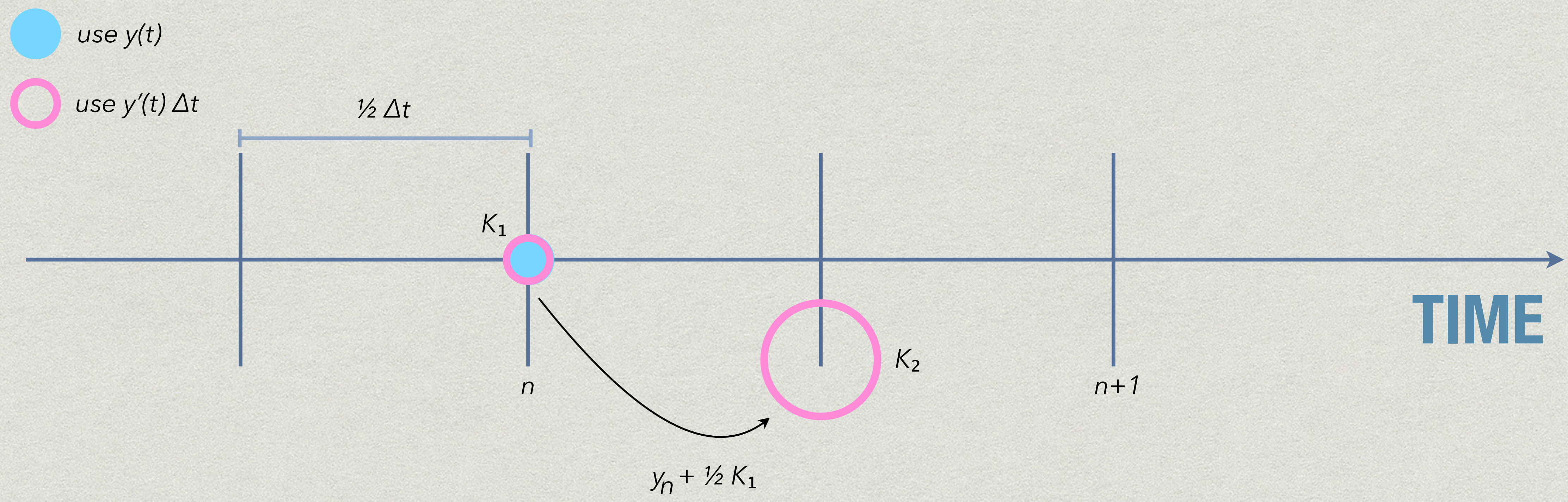
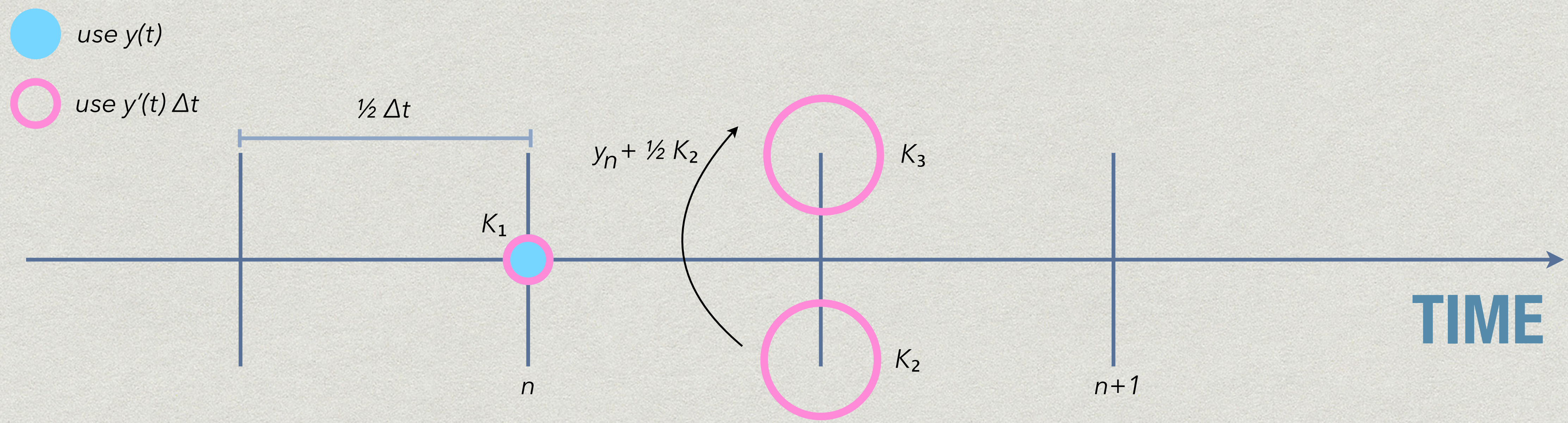
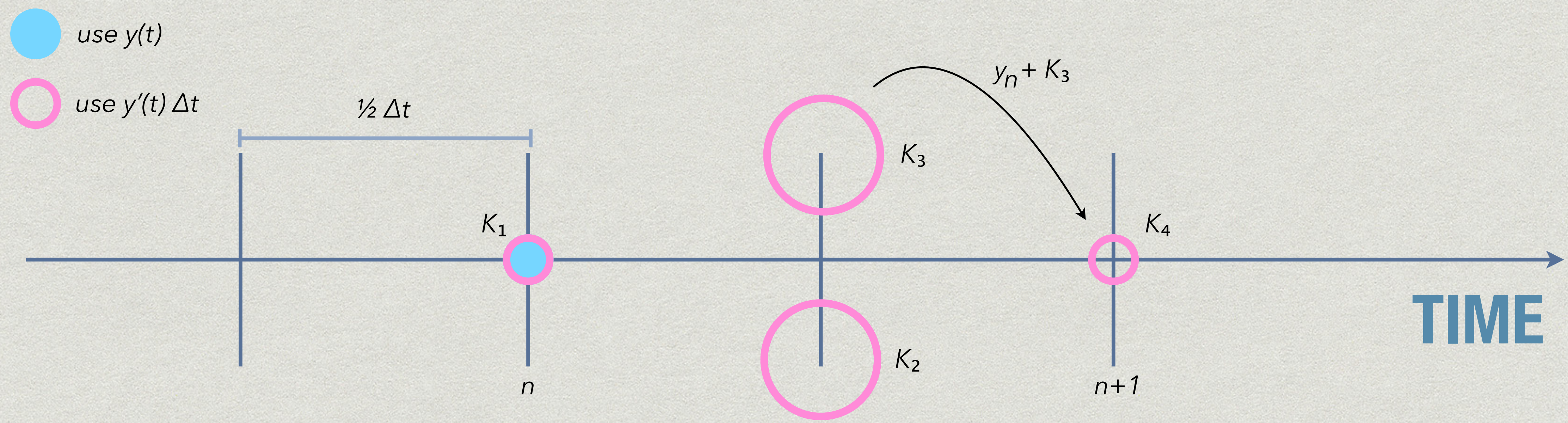
RK4



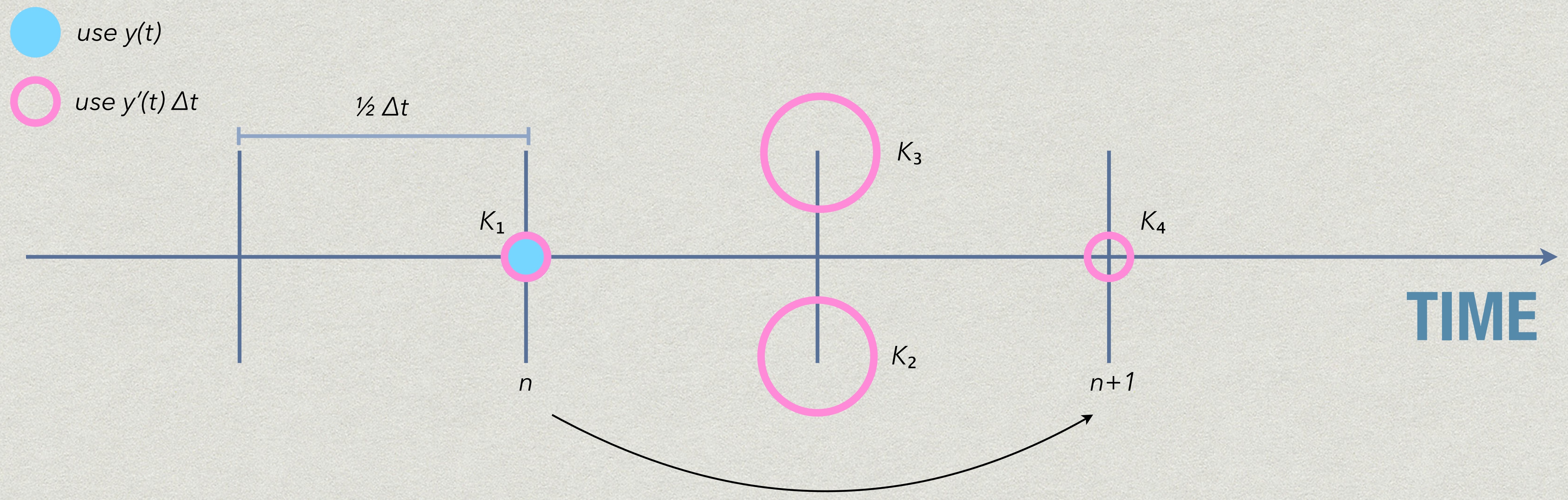
RK4



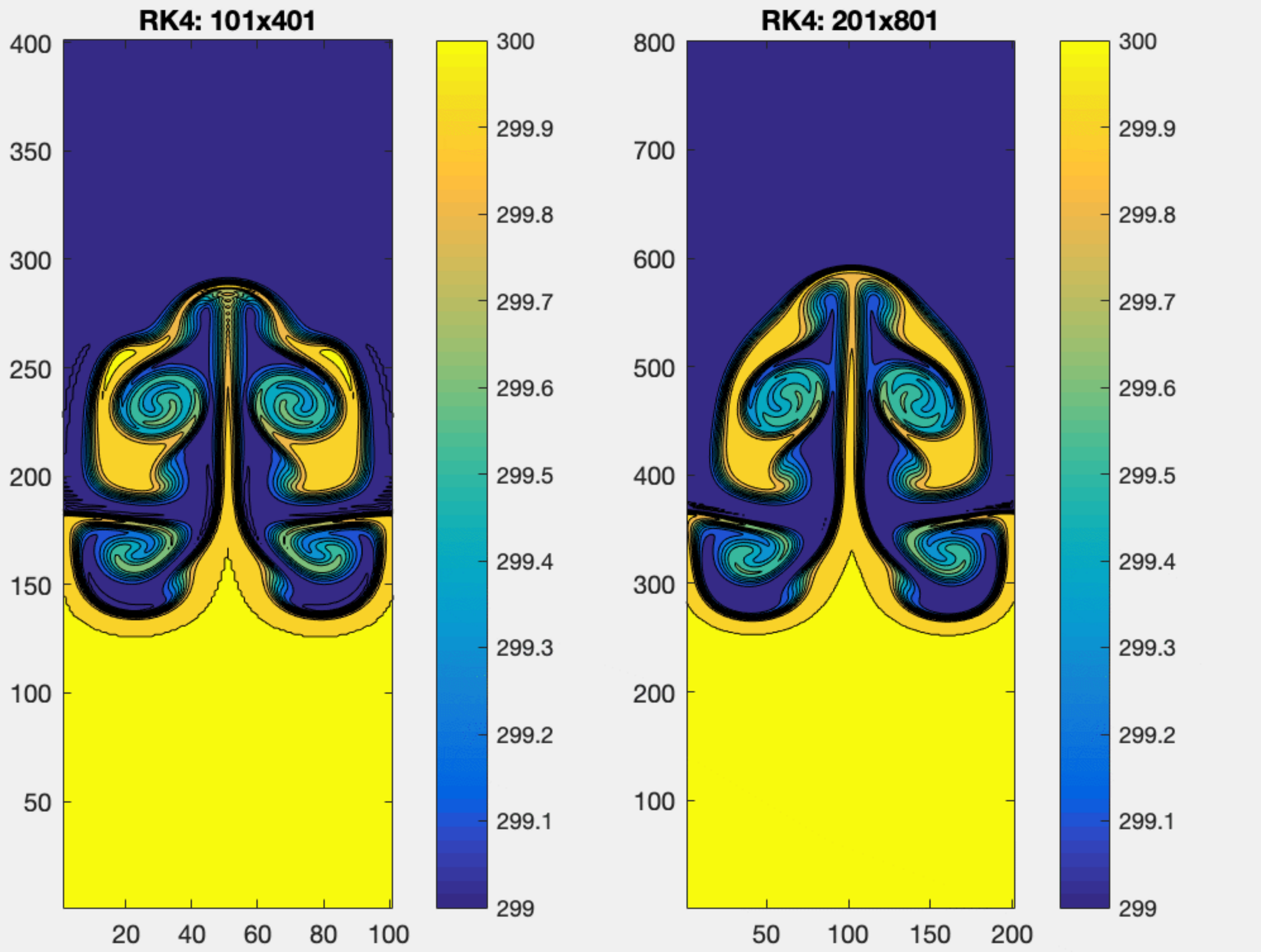
RK4



RK4



- $\mathcal{O}(\Delta t^4)$
- no computational mode
- $\mathbb{C} < 2.8$



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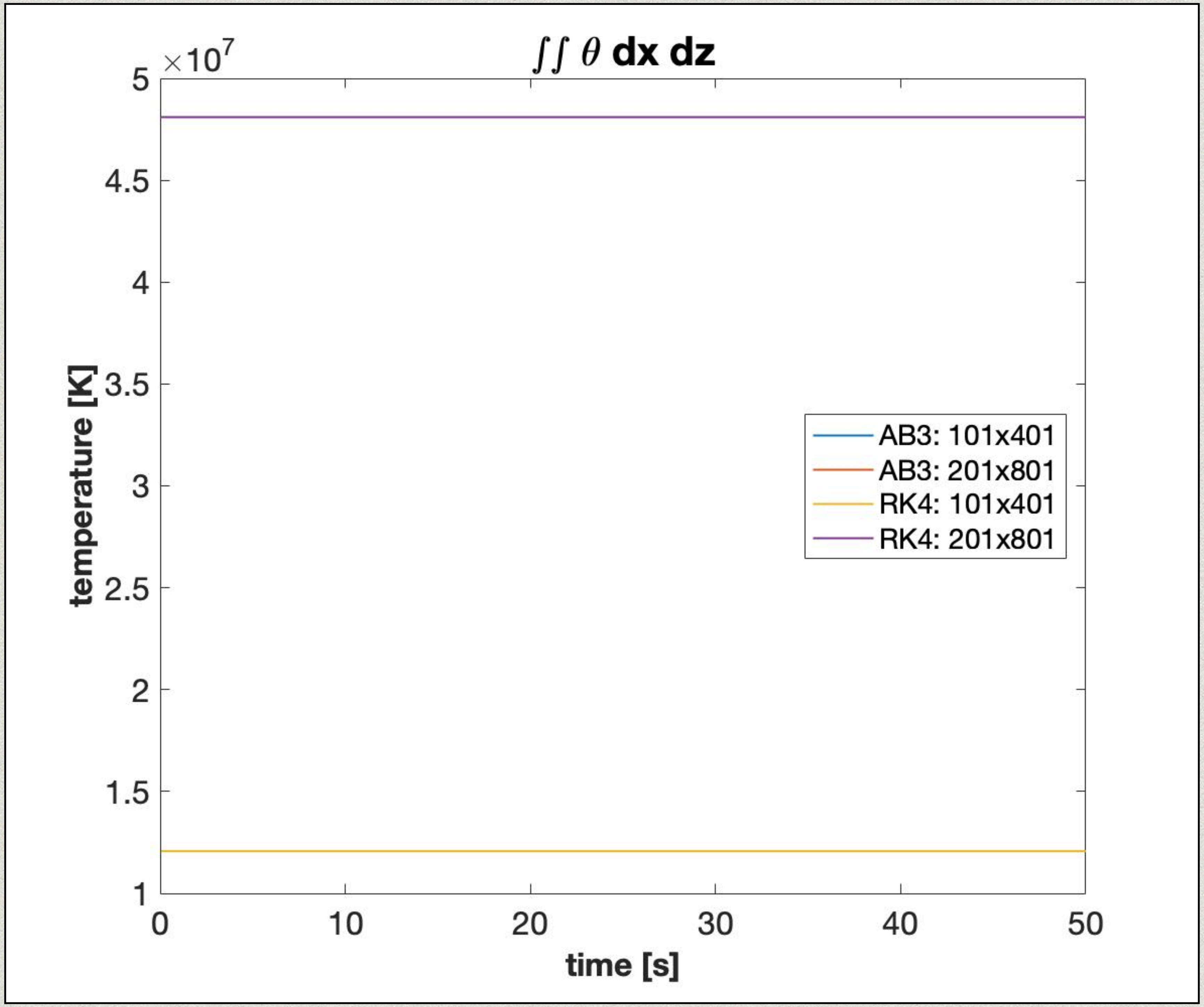
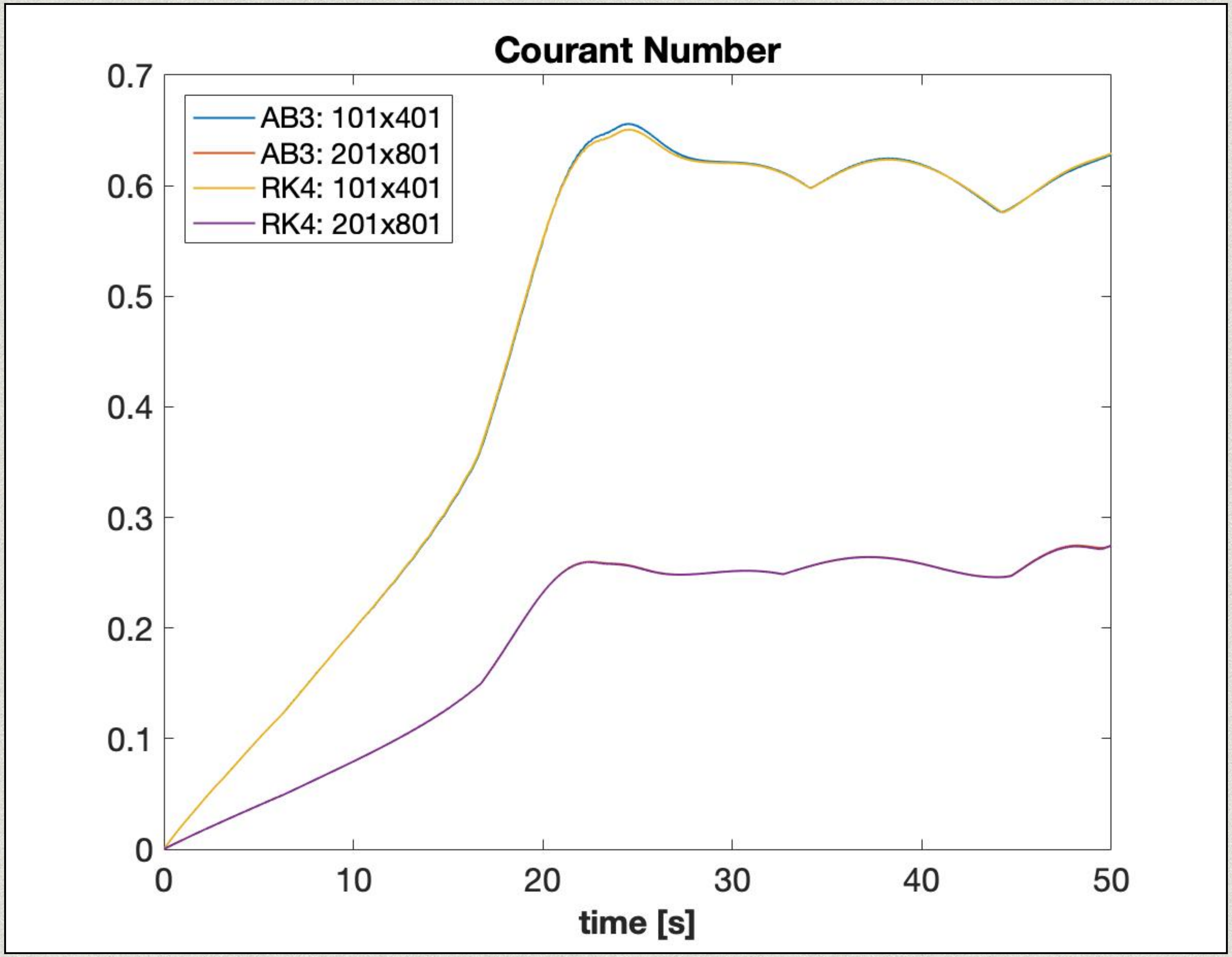
Stability

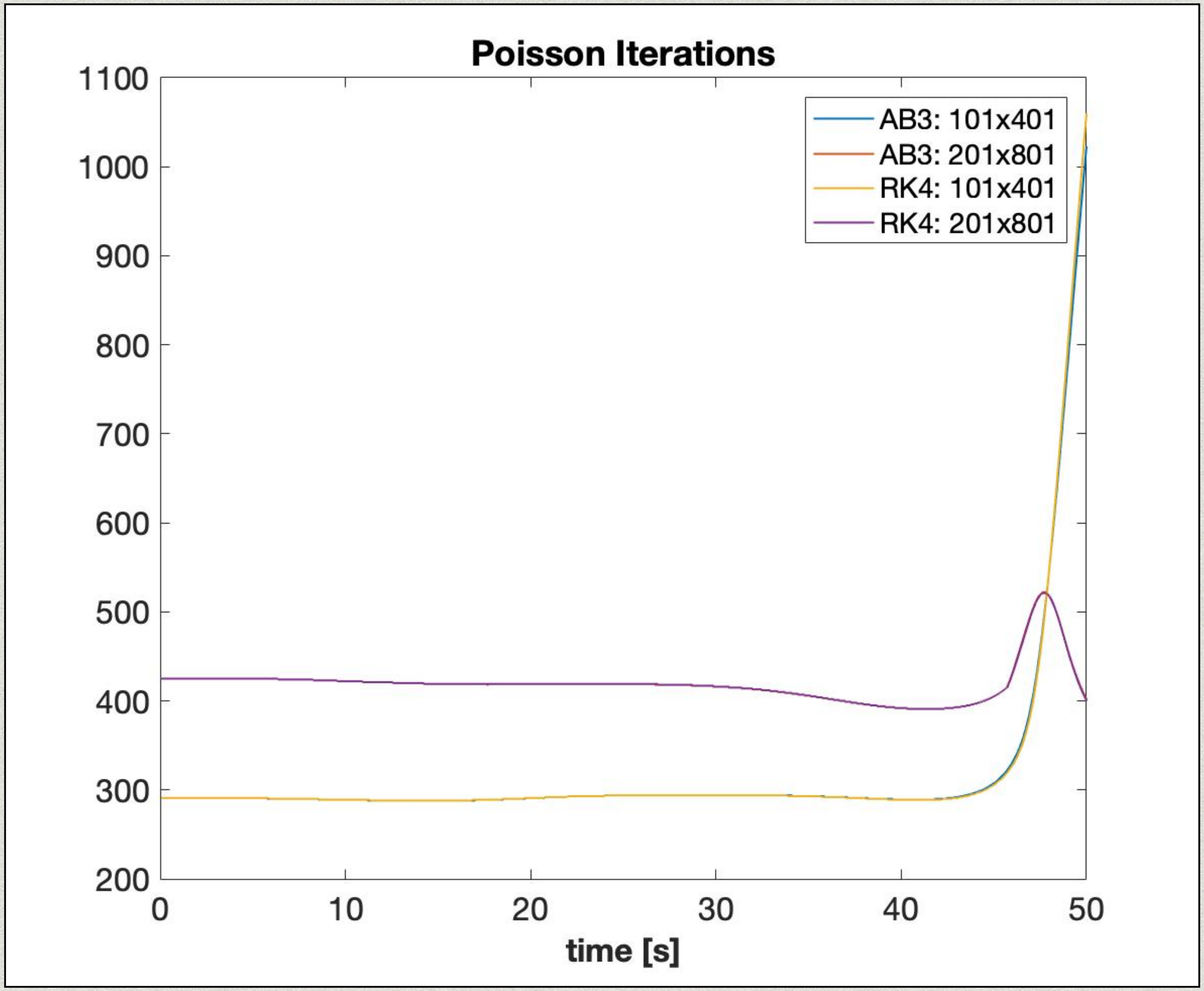

$\int\int \theta$ dx dz
$\times 10^7$
5
4.5
4
3.5
3
2.5
2
1.5
1
temperature [K]
0
10
20
30
40
50
time [s]
AB3: 101x401
AB3: 201x801
RK4: 101x401
RK4: 201x801

Stability



Courant Number
AB3: 101x401
AB3: 201x801
RK4: 101x401
RK4: 201x801
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
time [s]

Convergence



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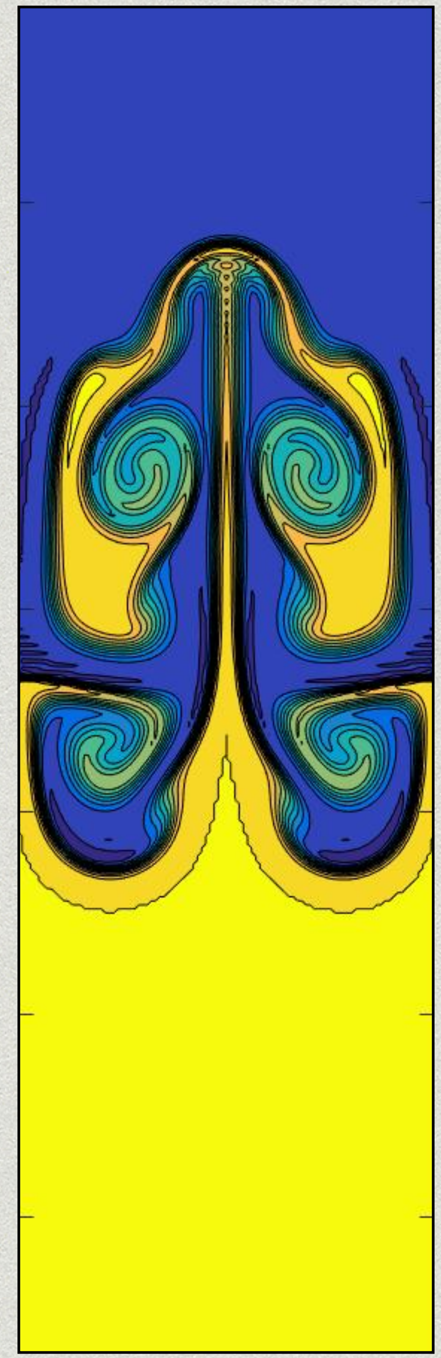
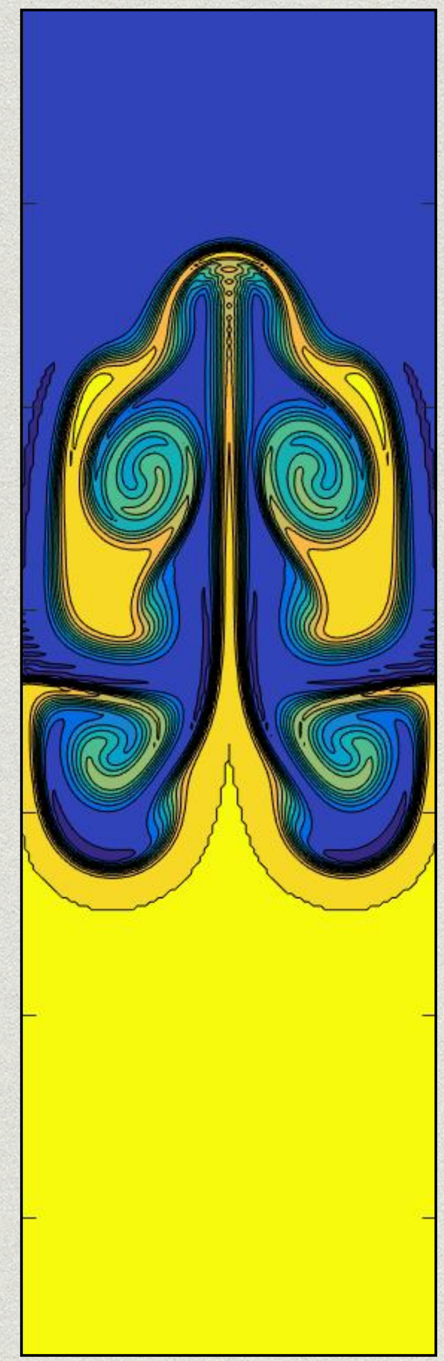
A Pseudo-spectral Miscible R-T Instability

- Spatial discretization:
    - Fourier on x (periodic)
    - Chebyshev on z (wall)
- Time discretization: AB3
- Laplacian: Crank-Nicholson
- Atwood number << 1, Schmidt, Prandtl numbers = 1
    - $\nu = \kappa = 10^{-3}$
- Perturbation unknown

$\nu = \kappa = 10^{-3}$





$\nu = \kappa$

$= 1.58 \times 10^{-6}$

10^{-3} blows up

$s = 16 > 1/4$

$\uparrow \Delta x$ & Δz, $\downarrow \Delta t$

FIGURE 2:
Young, Y.-N., H. Tufo, A. Dubey, R. Rosner. "On the miscible Rayleigh–Taylor instability: two and three dimensions". University of Chicago. Received 1999-12-15

AB3: 101x401
t = 36.5 s

RK4: 101x401
t = 36.5 s

FIGURE 2:
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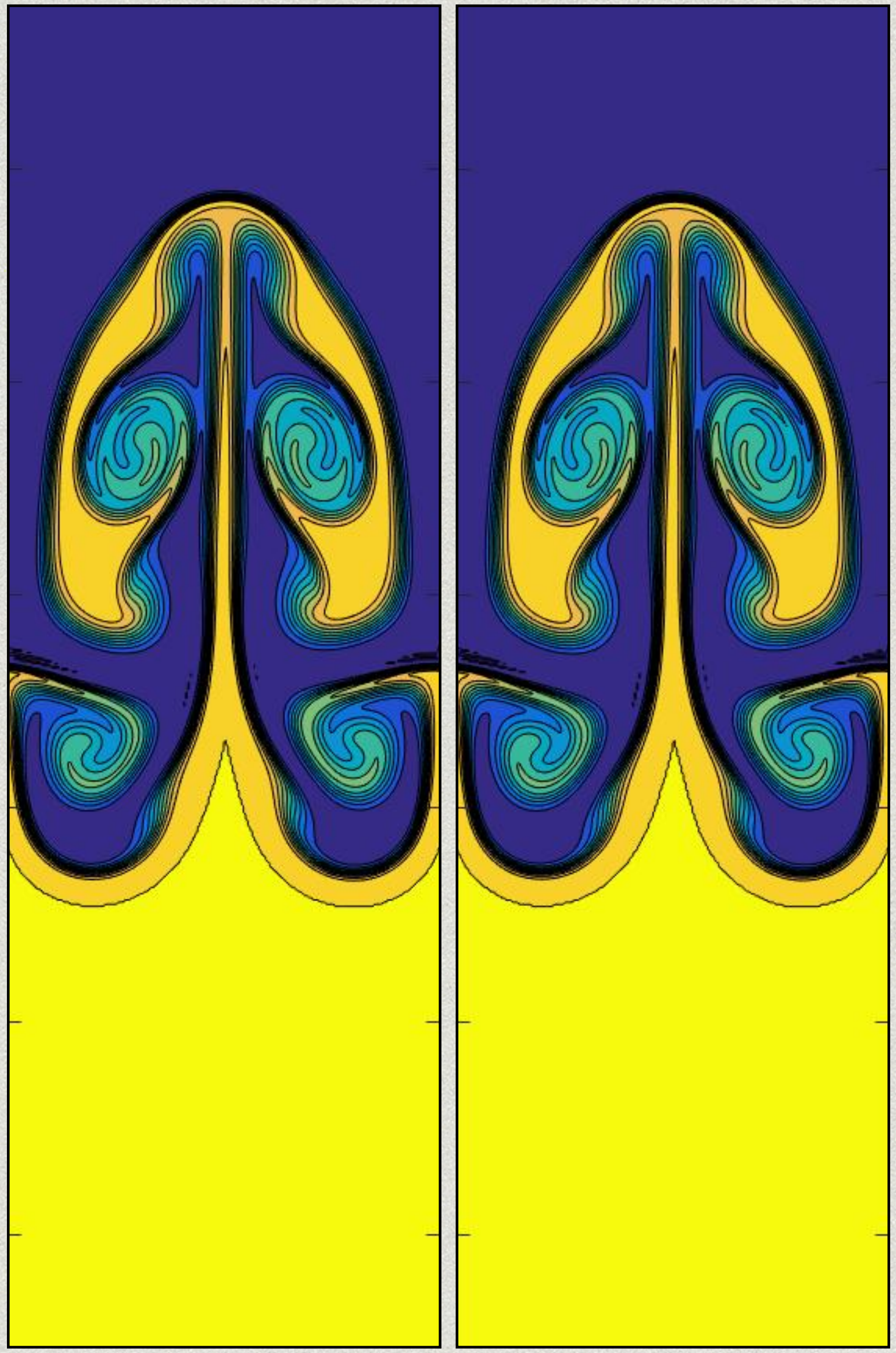
$\nu = \kappa = 10^{-3}$



AB3: 201x801
t = 36.5 s

RK4: 201x801
t = 36.5 s

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$= 1.58 \times 10^{-6}$

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Conclusions

- AB3 is faster than RK4 → similar results
    - limited duration of stability at low resolutions
    - asymmetry at higher resolutions
- tradeoff: increase diffusion but coarsen grid

Improvements

- Adjusting diffusion
- Kinetic energy V time
- Longer period of observation
- Adjusting Reynold's number
- Multiple modes
- Amplitude of perturbation
- Simulate randomness

Thank you!

Any Questions?