AOS 180 – Modeling Task 7
Rising thermal
Due date: May 24$^{\text{th}}$11:00AM

Problem description

We are interested in the solution of the two-dimensional Boussinesq equation system

$$\frac{\partial \omega}{\partial t} = -J(\psi, \omega) - \frac{g}{\theta_0}\frac{\partial \theta}{\partial x} \tag{1}$$

$$\frac{\partial \theta}{\partial t} = -J(\psi, \theta) \tag{2}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = -\omega, \tag{3}$$

on the domain $0 \leq x \leq L_x$ and $0 \leq z \leq L_z$. For initial condition we will prescribe a simple uniform circular warm bubble of radius r_0 rising in a neutrally stratified atmosphere. This can be accomplished by prescribing a constant potential temperature $\theta(x,z) = \theta_0$ everywhere except within the bubble. We will set the bubble center at (x_0, z_0) and give it a temperature $\theta = \theta_0 + \Delta\theta$. In other words, you will initialize your potential temperature field with

$$\theta(x,z) = \begin{cases} \theta_0 + \Delta\theta & \text{if } \left[(x-x_0)^2 + (z-z_0)^2\right]^{1/2} \leqslant r_0 \\ \theta_0 & \text{otherwise} \end{cases}. \tag{4}$$

For this problem, we will use periodic boundary conditions in the x direction and solid walls in the z direction. For the two walls, we will specify $\omega = 0$, $\psi = 0$, and $\theta = \theta_0$ at all times.

On this modeling task we will focus on validating our code and making sure our results are in agreement with those obtained by other authors. For this, you will use the simulation presented in section 5b of the paper by Smolarkiewicz and Pudykiewicz (1992) as a base for comparison (you do not need to read the entire paper, section 5b is enough). Note that they do not use the vorticity-streamfunction formulation and that their numerical scheme is much more accurate than ours. I uploaded the paper to CCLE for your convenience.

Numerical approach

We will use the centered second-order finite difference scheme to discretize all 3 equations with the Arakawa discretization of the Jacobian. Once again we will use the leapfrog scheme for time advancement of advection together with Euler forward with a time step equal to $2\Delta t$ for the artificial diffusion (of course, you can use AB3 or RK3 or any other scheme if you prefer). We will use SOR to solve the Poisson equation. Note that this code is very similar to Modeling Task 6. A note to think about: you can actually treat the baroclinic production of vorticity implicitly if you want. All you have to do is to advance the temperature equation first. (Does this help? Try if you have time. In principle it should be better, but I have not tried).

For the validation simulation, we want to match as closely as possible the configuration used by Smolarkiewicz and Pudykiewicz (1992). We will use: $L_x = L_z = 2\,\mathrm{km}$, $r_0 = 250\,\mathrm{m}$, $\theta_0 = 300\,\mathrm{K}$, $\Delta\theta = 0.5\,\mathrm{K}$ and the initial center of the bubble at $x_0 = L_x/2$ and $z_0 = 260\,\mathrm{m}$. I suggest you use 201 grid points in each direction to match the spatial resolution used by the authors. You cannot match the time step because that would make your integration scheme unstable. Choose your time step based on the stability criteria for advection and diffusion required for your discretization scheme. The velocity scale U employed by the authors is based on the initial condition and is given by $U = \sqrt{2r_0 g\Delta\theta/\theta_0}$. With this information, you should be able to determine the timescale $T = r_0/U$ and decide the most appropriate times to compare your solution to theirs. Integrate your solution for a total time $T_{tot} = 620\,\mathrm{s}$.

The solution presented in Figure 3 has no artificial (or real) diffusion, so you should start with no diffusion as well. Because our discretization scheme is not great, your solution at the end of the integration will look more like Figure 4a than Figure 3c (likely not as bad as 4a). The authors also show in Figure 4c a solution using the same scheme from 4a but including artificial diffusion to keep the numerical errors (computational mode, nonlinear instability, etc.) in check. They quantify the amount of diffusion (actually viscosity) in terms of a Reynolds number $\mathrm{Re} = 2r_0U/\nu \approx 1500$ (here ν is your artificial viscosity). Use this as a guide when including diffusion in your model (start from a value similar to what they used and then adjust it up or down to get a result that you are satisfied with when comparing to Figure 4c). Note that you can choose to add or not artificial diffusion to the temperature equation as well (I recommend you do). If you do so, I suggest you use the same diffusivity value you used for the vorticity equation (set the temperature diffusivity $k_\theta = \nu$; this is what they mean by Prandtl number $\mathrm{Pr} = \nu/k_\theta = 1$).

Once you have your code working and you are happy with the diffusivity, the next step is to check how sensitive your simulation is to the grid resolution. I suggest you run at least two additional simulations, one on a coarser grid and the other on a finer grid. In addition to comparing the position and the shape of the thermal at a fixed time, I suggest you also output some quantitative information as a function of time (e.g. the maximum vertical velocity and/or the maximum temperature) for comparison. For these 3 main simulations, I suggest you run your simulation a bit longer (maybe $T_{tot} = 1200\,\mathrm{s}$ or $T_{tot} = 1500\,\mathrm{s}$) so that you can see the second stage of thermal development as well.

I suggest you always keep track of the maximum Courant and Neumann numbers, as well as the number of iterations required for the Poisson equation in each time step (you may need to adjust your tolerance to get meaningful results).

Report

Please include in your report:

i. Figures: Figures that briefly document your solutions. In particular, I want to see figures comparing your 3 solutions to those in Figures 3c and 4c of the paper.

ii. Documentation: Please include in your report a brief description of your implementation and document your choice of diffusivity.

iii. Discussion: Please briefly discuss your results. I would like you to compare your validation case with the figures presented by Smolarkiewicz and Pudykiewicz (1992) together with a discussion of the capabilities and limitations of your code. Please also illlustrate and discuss the effects of grid resolution on your results.

Please upload a **.pdf** file with your report and **.pdf** file with your code to the dropbox that will be available on the CCLE website for the course. If you also want to upload your original code, that is fine. Please, do not upload word files or other formats, as they do not work well in other operational systems.