AOS 180 – Final Project
Due date: June 12^{th}11:59PM

Final project documentation

As part of your final project, you are required to upload 3 files to CCLE:

i. Presentation: please upload either a **.pptx** or a **.pdf** file of your slides that you used for your final presentation. It really does not matter if you include movies that I cannot play, but I need to be able to read all the information.

ii. Animation of you "favorite" simulation: Please upload a decent quality animation of your favorite simulation from your final project.

iii. Documentation of you "favorite" simulation: Please include a detailed description of your favorite simulation, with enough information for someone else to be able to reproduce your results. This is part of the "reproducibility" of numerical experiments in science, and it is a really important component of our job. I have prepared a summary of the information you need to provide below. You may need to add more information that is specific to your project, or there may be items that are not applicable to your project.

Summary of information

Here is the minimum information you need to provide about your favorite simulation. Please keep in mind that you are trying to give someone else **all** the information required to reproduce your results. Note that this is supposed to be a very brief description, probably about one-page in length.

i. Mathematical problem: write down the full set of equations you are solving and clearly indicate the domain size and all the boundary conditions used for all variables on all boundaries of the domain. Also clearly indicate all the details in the initial condition and provide the total time of integration. Finally, makes sure you also list all the values of the constants you used with units (e.g., gravitational acceleration, viscosity and diffusivity, etc.).

ii. Numerical discretization: indicate the methods you used for spatial discretization and time discretization (no need to write down equations here), and the method used for the Poisson solver. Also list the number of grid points in each direction, the grid sizes, the time step used, and the tolerance for the Poisson solver.

iii. References: List the papers or other materials you used. In particular, if you are comparing your simulations to published results, make sure this reference is clearly indicated.

Go over your code once you finish writing down your summary, to make sure that all the information has been reported. See an example of summary for the simulation in Homework 7 on the next page.

Simulation of a 2D hot bubble

i. Mathematical problem:

- Equations: 2D incompressible Navier-Stokes in vorticity-streamfunction formulation with the advection written in Jacobian form

$$\begin{aligned}\frac{\partial \omega}{\partial t} &= -J(\psi,\omega) - \frac{g}{\theta_0}\frac{\partial \theta}{\partial x} + \nu \nabla^2 \omega \\ \frac{\partial \theta}{\partial t} &= -J(\psi,\theta) + K\nabla^2\theta \\ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} &= -\omega,\end{aligned}$$

  with $g = 9.81\,\mathrm{m/s}$, $\nu = 0.25\,\mathrm{m^2/s}$ and $K = 0.25\,\mathrm{m^2/s}$.
- Domain: $0 \le x \le L_x$ and $0 \le z \le L_z$ with $L_x = L_z = 2\,\mathrm{km}$
- Boundary conditions: periodic in x for all variables (ψ, ω, and θ) and walls in z ($\psi = 0$, $\omega = 0$, and $\theta = \theta_0$ on both walls; $\theta_0 = 300\,\mathrm{K}$)
- Initial condition: hot bubble centered in x near the bottom of the domain:

$$\theta(x,z) = \begin{cases} \theta_0 + \Delta\theta & \text{if } \left[(x-x_0)^2 + (z-z_0)^2\right]^{1/2} \leqslant r_0 \\ \theta_0 & \text{otherwise} \end{cases}$$

  where $r_0 = 250\,\mathrm{m}$, $\Delta\theta = 0.5\,\mathrm{K}$ and $x_0 = L_x/2$ and $z_0 = 260\,\mathrm{m}$. Fluid starts from rest ($\omega = 0$ and $\psi = 0$). Total integration time $T_{tot} = 1200\,\mathrm{s}$.

ii. Numerical discretization:

- Spatial discretization: second-order centered differences for all spatial derivatives in all 3 equations using 201 grid points in each direction ($\Delta x = \Delta z = 10\,\mathrm{m}$). Arakawa discretization for the Jacobian to mitigate nonlinear instability.
- Time discretization: Split-operator combining Leapfrog for advection and buoyancy and Euler-Forward with $2\Delta t$ for diffusion ($\Delta t = 0.5\,\mathrm{s}$). Note: used Euler-Forward for first time step.
- Poisson solver: SOR iterative solver with estimated optimal α and $tol = 10^{-10}$. Used ψ from previous time step as initial guess for solver.

iii. References: Simulation setup following section 5b of Smolarkiewicz, P.K. and Pudykiewicz, J.A., 1992. A class of semi-Lagrangian approximations for fluids. Journal of the Atmospheric Sciences, 49(22), pp.2082-2096.