1.

  The only events that are independent of themselves are those with probability either 0 or 1. That follows from the fact that a number is its own square if and only if it's either 0 or 1. The only way a random variable X can be independent of itself is if for every measurable set A, either Pr(X∈A)=1 or Pr(X∈A)=0.

2.

Let A and B be independent events, and let A and C be independent events. How do I show that A and B∪C are independent events as well?

According to the definition of independent events, A and B∪C are independent if and only if

P(A∩(B∪C))=P(A)P(B∪C).

Since A and B and A and C are independent, I know that

P(A∩B)=P(A)P(B) and P(A∩C)=P(A)P(C).

However, I have no idea how to solve this. I attempted to apply the probability rules I know but got nowhere.